Homework #3

Instructor: Ali Sharifian

Fall 2021

For all questions, choose the **best** answer.

For all questions, unless otherwise stated, assume the below matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$B = \begin{bmatrix} 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 \\ 19 & 20 & 21 & 22 \\ 23 & 24 & 25 & 26 \end{bmatrix}$$

1. Which of the below most closely represents the first (top most level) C_{21} when executing SQUARE-MATRIX-MULTIPLY-RECURSIVE(A, B)?

   a. C_{21} = SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}$) + SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix}, \begin{bmatrix} 21 & 22 \\ 25 & 26 \end{bmatrix}$)

   b. C_{21} = SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}$) + SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix}, \begin{bmatrix} 19 & 20 \\ 23 & 24 \end{bmatrix}$)

   c. C_{21} = SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix}, \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}$) + SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix}, \begin{bmatrix} 19 & 20 \\ 23 & 24 \end{bmatrix}$)

   d. C_{21} = SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix}, \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}$) + SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}, \begin{bmatrix} 19 & 20 \\ 15 & 16 \end{bmatrix}$)

   e. C_{21} = SQUARE-MATRIX-MULTIPLY-RECURSIVE$\left(\begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix}, \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}\right)$+ SQUARE-MATRIX-MULTIPLY-RECURSIVE$\left(\begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}, \begin{bmatrix} 19 & 20 \\ 23 & 24 \end{bmatrix}\right)$

2. Which of the below most closely represents the first (top most level) C_{12} when executing SQUARE-MATRIX-MULTIPLY-RECURSIVE(A, B)?

   a. C_{12} = SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix}1 & 2\\5 & 6\end{bmatrix}, \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix}$) + SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix}3 & 4\\7 & 8\end{bmatrix}, \begin{bmatrix}21 & 22\\25 & 26\end{bmatrix}$)

   b. C_{12} = SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix}1 & 2\\5 & 6\end{bmatrix}, \begin{bmatrix}13 & 14\\17 & 18\end{bmatrix}$) + SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix}3 & 4\\7 & 8\end{bmatrix}, \begin{bmatrix}21 & 22\\25 & 26\end{bmatrix}$)

   c. C_{12} = SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix}9 & 10\\13 & 14\end{bmatrix}, \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix}$) + SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix}9 & 10\\13 & 14\end{bmatrix}, \begin{bmatrix}19 & 20\\23 & 24\end{bmatrix}$)

   d. C_{12} = SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix}9 & 10\\13 & 14\end{bmatrix}, \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix}$) + SQUARE-MATRIX-MULTIPLY-RECURSIVE($\begin{bmatrix}11 & 12\\15 & 16\end{bmatrix}, \begin{bmatrix}19 & 20\\15 & 16\end{bmatrix}$)

   e. C_{12} = SQUARE-MATRIX-MULTIPLY-RECURSIVE$\left(\begin{bmatrix}9 & 10\\13 & 14\end{bmatrix}, \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix}\right)$+ SQUARE-MATRIX-MULTIPLY-RECURSIVE$\left(\begin{bmatrix}11 & 12\\15 & 16\end{bmatrix}, \begin{bmatrix}19 & 20\\23 & 24\end{bmatrix}\right)$

3. Using Strassen's algorithm from the textbook to multiply matrices A and B, which of the below most closely represents the top most P_7 value?

   a. $\begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} + \begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}13 & 14\\17 & 18\end{bmatrix} - \begin{bmatrix}9 & 10\\13 & 14\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} - \begin{bmatrix}9 & 10\\13 & 14\end{bmatrix} * \begin{bmatrix}13 & 14\\17 & 18\end{bmatrix}$

   b. $\begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} + \begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}13 & 14\\17 & 18\end{bmatrix} - \begin{bmatrix}9 & 10\\13 & 14\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} + \begin{bmatrix}9 & 10\\13 & 14\end{bmatrix} * \begin{bmatrix}13 & 14\\17 & 18\end{bmatrix}$

   c. $\begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} + \begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}13 & 14\\17 & 18\end{bmatrix} + \begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} + \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} * \begin{bmatrix}13 & 14\\17 & 18\end{bmatrix}$

   d. $\begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} + \begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}13 & 14\\17 & 18\end{bmatrix} - \begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} - \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} * \begin{bmatrix}13 & 14\\17 & 18\end{bmatrix}$

   e. $\begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} + \begin{bmatrix}1 & 2\\5 & 6\end{bmatrix} * \begin{bmatrix}19 & 20\\23 & 24\end{bmatrix} - \begin{bmatrix}9 & 10\\13 & 14\end{bmatrix} * \begin{bmatrix}11 & 12\\15 & 16\end{bmatrix} - \begin{bmatrix}9 & 10\\13 & 14\end{bmatrix} * \begin{bmatrix}19 & 20\\23 & 24\end{bmatrix}$

4. Which of the below statements are true?
    I. If matrix M = [2] and N = [11, 12, 13], then you can compute the multiplication of M*N using Strassen's algorithm from the textbook.
    II. Strassen's algorithm has a better runtime than the Square Matrix Multiply Recursive algorithm because it executes lg 7 recursions in every Strassen algorithm execution.
    III. If matrix M = [2] and N = [11, 12, 13], then you can compute the multiplication of M*N using the Square Matrix Multiply algorithm from the textbook.
    IV. If matrix $X = \begin{bmatrix} 1 & 2 & 3 & 4 & 4 & 5 \\ 9 & 8 & 7 & 6 & 5 & 4 \\ 2 & 2 & 2 & 2 & 5 & 2 \\ -51 & 4 & 7 & 8 & 6 & 2 \\ 6 & 1 & 1 & 1 & 7 & 1 \\ 5 & 9 & 8 & 8 & 8 & 6 \end{bmatrix}$ and $Y = \begin{bmatrix} 11 & 4 & 22 & 33 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 2 & 2 & 2 & 2 \\ -51 & 2 & 4 & 7 & 8 & 2 \\ 61 & 1 & 1 & 1 & 1 & 1 \\ 45 & 2 & 49 & 58 & 58 & 16 \end{bmatrix}$ then you can compute the multiplication of X*Y using Strassen's algorithm from the textbook.

    a. I and IV
    b. II only
    c. II and III
    d. II and IV
    e. None of the statements are true.

5. If you wanted to multiply two 16x16 matrices. How many recursive calls would be made using Strassen's algorithm and how many using the Square Matrix Multiply Recursive algorithm?
    a. Strassen: 2401, Square Matrix Multiply Recursive: 4096
    b. Strassen: 236, Square Matrix Multiply Recursive: 343
    c. Strassen: 2418, Square Matrix Multiply Recursive: 4096
    d. Strassen: 268435456, Square Matrix Multiply Recursive: 4294967296
    e. Strassen: 2419, Square Matrix Multiply Recursive: 4096