Homework #5

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For all questions, choose the **best** answer.

Assume starting index is 1.

1. When using quicksort to sort an array of length *n*, what is the asymptotic running time if the pivot is always chosen to be the *k*th selected element of the list, where *k* = O(1)? When using the term "*k*th selected" in this context, we mean that the element would be the *k*th element if the list were sorted (like in lecture).

   If you choose k that's not in the middle (i.e. median), runtime will not be good (worst case). Especially if the pivot is the smallest or largest number in the array. The runtime of this algorithm would be the same as the worst case runtime as the typical quicksort algorithm: O(n^2).

   a. $O(n^2)$
   b. $O(n)$
   c. $O(n \log n)$
   d. $O(n \log \log n)$
   e. $O(n^2 \log n)$

2. When using quicksort to sort an array of length *n*, what is the asymptotic running time if the pivot is always chosen to be the *k*th selected element of the list, where $k = \frac{n}{2}$? When using the term "*k*th selected" in this context, we mean that the element would be the *k*th element if the list were sorted (like in lecture).

   k being n/2 is the best case for the choice of pivot because it splits the array in two equal parts. Best case runtime for quicksort is O(n log n).

   a. $O(n^2)$
   b. $O(n)$
   c. $O(n \log \log n)$
   d. $O(n \log n)$
   e. $O(n^2 \log n)$

3. Let L = [5, 8, 16, 2, 7, 11, 0, 9, 3, 4, 6, 8, 3, 15, 6, 5, 12, 4, 8, 36, 25, 17, 39, 21, 42]

   What is the set $\{m_i\}$ (the medians of five)?

   Break up the array in groups of 5. Then sort them. Then find the median. Then those medians will make up the array in your answer.

   Highlighting every other group of 5 to clearly show the separation.

   L = [5; 8; 16; 2; 7; 11; 0; 9; 3; 4; 6; 8; 3; 15; 6; 5; 12; 4; 8; 36; 25; 17; 39; 21; 42]

   Sort the groups of 5 in ascending order.

   [2, 5, **7**, 8, 16]

   [0, 3, **4**, 9, 11]

   [3, 6, **6**, 8, 15]

   [4, 5, **8**, 12, 36]

   [17, 21, **25**, 39, 42]

   Now take the medians and create an array from it (medians bolded above):

   [7, 4, 6, 8, 25]

   a. [1, 5, 10, 15, 19]
   b. [3, 8, 13, 18, 22]
   c. [16, 9, 3, 4, 17]
   d. [7, 4, 6, 8, 25]
   e. [1, 3, 16, 7, 9]

4. Let L = [5, 8, 16, 2, 7, 11, 0, 9, 3, 4, 6, 8, 3, 15, 6, 5, 12, 4, 8, 36, 25, 17, 39, 21, 42]

   What is L_1?

   Sort the medians of medians array from #3 in ascending order:
   [4, 6, 7, 8, 25]

   Then take the median of the above array, which is 7.

   L1 is all of the values in the array that is less than 7:

   [5, 2, 0, 3, 4, 6, 3, 6, 5, 4]

a. [5, 2, 0, 3, 4, 6, 3, 6, 5, 4]
b. [4, 6]
c. [2, 0, 3, 3]
d. [8, 16, 11, 9, 15, 12, 8, 36, 25, 17, 39, 25]
e. [2, 0, 3, 3, 8, 16]

5. Let L = [5, 8, 16, 2, 7, 11, 0, 9, 3, 4, 6, 8, 3, 15, 6, 5, 12, 4, 8, 36, 25, 17, 39, 21, 42]

   What is SELECT(L, 12)?

   Sort the array and output the value in the 12^{th} position.

   Sorted array is:

   [0, 2, 3, 3, 4, 4, 5, 5, 6, 6, 7, 8, 8, 8, 9, 11, 12, 15, 16, 17, 21, 25, 36, 39, 42]

   12^{th} position is 8.

   a. 7
   b. 8
   c. 9
   d. 12
   e. 11