Homework #2

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For all questions, choose the **best** answer.

1. n =
   I. $O(\sqrt{n})$
   II. $\theta(n \lg n)$
   III. $\Omega(n)$
   IV. $\Omega(\lg n)$
   V. $\theta(\mathrm{n}^2)$

Solution:
Assume c is a constant greater than 0.

I. n = $O(\sqrt{n})$? No. n is not upper bounded by $c\sqrt{n}$
II. n = $\theta(n \lg n)$? No. n doesn't have a lower bound of cn lg n.
III. n = $\Omega(n)$? Yes. n is lower bounded by cn.
IV. n = $\Omega(\lg n)$? Yes, n is lower bounded by c lg n.
V. n = $\theta(\mathrm{n}^2)$? No, n is not lower bounded by cn^2.

a. I only
b. II and V
c. III only
d. III and IV
e. IV and V

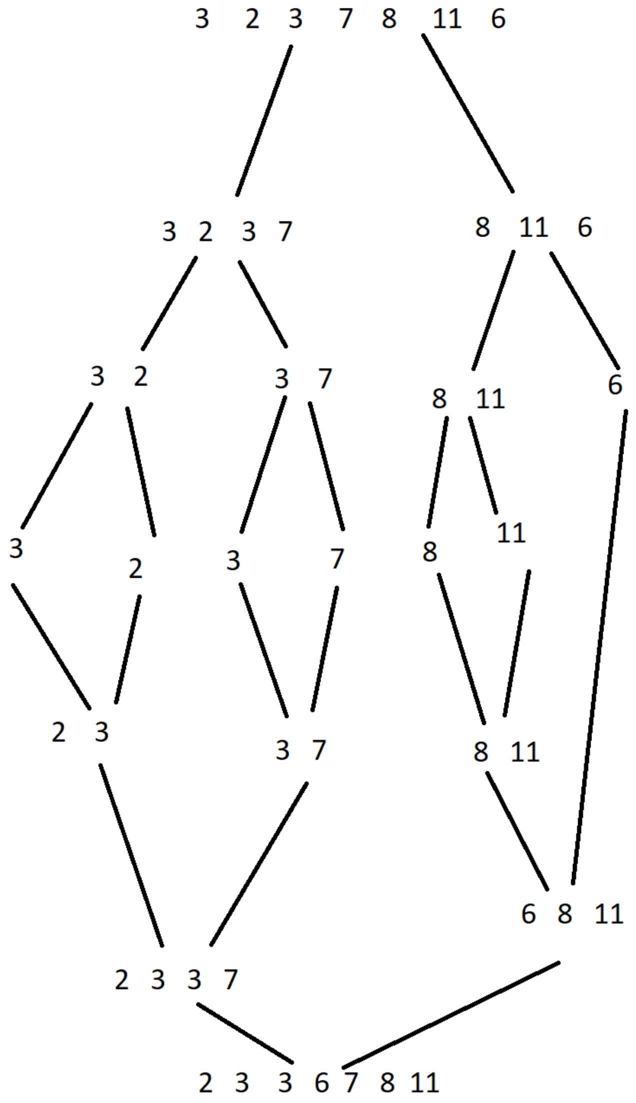
2. If you were to use merge sort on the below array of numbers using the Merge Sort algorithm from the book, which of the below statements are true? Assume index starts at 1.

   A = [3, 2, 3, 7, 8, 11, 6]

   I. The first recursive step has 3, 2, 3 as the left sub-array and 7, 8, 11, 6 as the right sub-array.
   II. After the base cases are reached, the sub-arrays produced are:
   [2, 3], [3, 7], [8, 11], and [6]
   III. The sub-arrays produced right before the outputted array are:
   [2, 3, 3] and [6, 7, 8, 11]

   a. I only

b. I and III
c. II only
d. III only
e. I and II



For questions 3 and 4, find the runtime. Use any approach you like.

3. T(n) = 100T(n/2) + n^2 log^5 n

   A=100
   B=2
   K=2
   P=5

   A > b^k

T(n) = Theta(n^6.64)

a) Theta(n^6.64 log^6 n)
b) Theta(n^6.64 log log n)
c) Theta(n^6.64)
d) Theta(n^2 log^5 n)
e) Theta(n^2)

4. T(n) = 4T(n/5) + n log^3 n

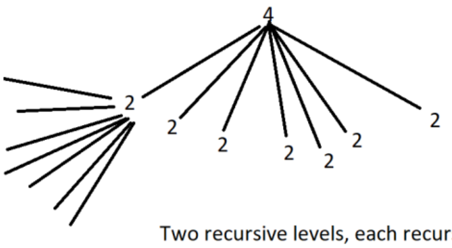
A=4
B=5
K=1
P=3

B^k= 5^1=5
4<5
A<b^k
p>=0
T(n) = Theta(n^1 log^3 n) = Theta(n log^3 n)

a) Theta(n^.86 log^4 n)
b) Theta(n^.86 log log n)
c) O(n)
d) Theta(n^.86)
e) Theta(n log^3 n)

5. If you wanted to multiply two 4x4 matrices using Strassen's algorithm for matrix multiplication, how many recursive calls would be made?

Solution:



Two recursive levels, each recursion has 7 recursive calls. 7^2 = 49

For brevity, the above tree screenshot shows just one of the n=2 having 7 recursions (far left), but in actuality, all 7 of them have 7 recursions.

a. 49

b. 50

c. 56

d. 64

e. 60