

# Homework #7

Instructor: Ali Sharifian

Fall 2021

For all questions, choose the **best** answer.

For questions 1 and 2, assume the following matrices and their corresponding dimensions:

$$A_1 = 3 \times 2$$

$$A_2 = 2 \times 4$$

$$A_3 = 4 \times 3$$

$$A_4 = 3 \times 5$$

$$A_5 = 5 \times 1$$

1. What is the minimum number of multiplications that the matrix multiplication  $A_1A_2A_3A_4A_5$  can be done in?

- a. 24
- b. 54
- c. 41
- d. 42
- e. 38

A1: 3x2

A2: 2x4

A3: 4x3

A4: 3x5

A5: 5x1

$3 \times 5 \times 1 = 15 \leftarrow A_4A_5$ , which is a  $3 \times 1$  matrix.

$4 \times 3 \times 1 = 12 \leftarrow A_3(A_4A_5)$ , which is a  $4 \times 1$  matrix.

$2 \times 4 \times 1 = 8 \leftarrow A_2(A_3(A_4A_5))$ , which is a  $2 \times 1$  matrix

$3 \times 2 \times 1 = 6 \leftarrow A_1(A_2(A_3(A_4A_5)))$ , which is a  $3 \times 1$  matrix.

Now we add up all of the multiplications that we did above.

$15 + 12 + 8 + 6 = 41 \leftarrow$  total number of multiplications for the minimum amount.

2. What is the parenthesization that would produce the minimum number of multiplications of  $A_1A_2A_3A_4A_5$ ?

- a.  $((((A_1A_2)A_3)A_4)A_5)$
- b.  $((A_1A_2)((A_3A_4)A_5))$
- c.  $(A_1(A_2(A_3(A_4A_5))))$
- d.  $((A_1A_2)(A_3A_4))A_5$

e.  $(A_1((A_2A_3)(A_4A_5)))$

For questions 3 to 5, assume the following constraints for the 0-1 Knapsack problem:

Capacity = 9

Item	Weight	Profit
1	5	5
2	2	2
3	2	1
4	6	4
5	1	2
6	4	6

In the 0-1 Knapsack problem, which is a problem that can be solved by dynamic programming, we want to maximize the total profit of the items taken, while not exceeding the weight capacity of the knapsack.

3. What is the maximum profit?

- a. 9
- b. 10
- c. 11
- d. 12
- e. 13

Two possible optimal solutions:

**First solution:**

$w_1 + w_6 = 5 + 4 = 9$  ← doesn't exceed capacity and meets it. That's what we want. We want the profit to be as maximum as possible and the capacity to be as close to 9 it as possible, but not exceed it.

$p_1 + p_6 = 5 + 6 = 11$  ← one possible combination of items to steal is to steal items 1 and 6.

**Second solution:**

$w_2 + w_3 + w_5 + w_6 = 2 + 2 + 1 + 4 = 9$  ← doesn't exceed capacity and meets it

$p_2 + p_3 + p_5 + p_6 = 2 + 1 + 2 + 6 = 11$

4. Which of the following statements are true:

- I. Taking items 1 and 6 will produce the maximum profit.
- II. Taking items 2, 3, 5, and 6 will produce the maximum profit.
- III. Taking items 4, 5, and 6 will produce the maximum profit.

- a. I
- b. II
- c. III
- d. I and II
- e. II and III

5. If the Capacity was 10 instead of 9, what would the maximum profit be?

- a. 9
- b. 10
- c. 11
- d. 12
- e. 13

If the knapsack capacity is 10, then a combination that would produce the max profit is stealing items 1, 5, and 6. That produces a total weight of  $5 + 1 + 4 = 10$ . The total profit is  $5 + 2 + 6 = 13$ .