Homework #4

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For all questions, choose the **best** answer.

All questions pertain to the Fast Fourier Transform algorithm as taught in class.

1. Adjust the following polynomial so that the FFT algorithm can obtain its point value representation:

Solution:
5x^8 + 3x^2 + 2x + 81

n = 8 + 1 = 9

But 9 is not a power of 2. The smallest number that's a power of 2 and greater than 9 is 16.

So we would need to have n = 16. In order to do that, we would need to add higher ordered termed zero coefficient elements to the polynomial. Also include missing terms with zero coefficients.

0x^15 + 0x^14 +0x^13 + 0x^12 + 0x^11 + 0x^10 + 0x^9 + 5x^8 + 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 3x^2 + 2x + 81

$$5x^8 + 3x^2 + 2x + 81$$

a. $5x^8 + 3x^2 + 2x + 81$
b. $5x^8 + 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 3x^2 + 2x + 81$
c. $0x^{15} + 0x^{14} + 0x^{13} + 0x^{12} + 0x^{11} + 0x^{10} + 0x^9 + 5x^8 + 3x^2 + 2x + 81$
d. $5x^8 + 3x^2 + 2x + 81 + 0$
e. $0x^{15} + 0x^{14} + 0x^{13} + 0x^{12} + 0x^{11} + 0x^{10} + 0x^9 + 5x^8 + 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 3x^2 + 2x + 81$

2. What is n in the polynomial from question 1?
   a. 8
   b. 9
   c. 16
   d. 15
   e. 4

3. What is ω when n = 32?
   a. -1
   b. 0.707106781 + 0.707106781 i
   c. 0.98078528 + 0.195090322 i
   d. i
   e. 1

4. When n = 16, what is Ω?

Solution:
$$\omega = e^{2\pi i/16} = 0.923879533 + 0.382683432 \text{ i}$$

$$\Omega = \left(\omega^{\frac{16}{2}-1}, \omega^{\frac{16}{2}-2}, \omega^{\frac{16}{2}-3}, \omega^{\frac{16}{2}-4}, \omega^{\frac{16}{2}-5}, \omega^{\frac{16}{2}-6}, \omega^{\frac{16}{2}-7}, \omega^{\frac{16}{2}-8}\right)$$

$$= (\omega^7, \omega^6, \omega^5, \omega^4, \omega^3, \omega^2, \omega, 1)$$

(-0.923879533 + 0.382683432 i, -0.707106781 + 0.707106781 i, -0.382683432 + 0.923879533 i, i, 0.382683432 + 0.923879533 i, 0.707106781 + 0.707106781 i, 0.923879533 + 0.382683432 i, 1)

   a. (-0.923879533 + 0.382683432 i, -0.707106781 + 0.707106781 i, -0.382683432 + 0.923879533 i, i, 0.382683432 + 0.923879533 i, -i, 0.923879533 + 0.382683432 i, 1)
   b. (-0.923879533 + 0.382683432 i, -0.707106781 + 0.707106781 i, -0.382683432 + 0.923879533 i, i, 0.382683432 + 0.923879533 i, 0.707106781 + 0.707106781 i, 0.923879533 + 0.382683432 i, 1)
   c. (-0.923879533 + 0.382683432 i, 0.707106781 + 0.707106781 i, -0.382683432 + 0.923879533 i, i, 0.382683432 + 0.923879533 i, 0.707106781 + 0.707106781 i, 0.923879533 + 0.382683432 i, 1)
   d. (-0.923879533 + 0.382683432 i, -0.707106781 - 0.707106781 i, -0.382683432 + 0.923879533 i, i, 0.382683432 + 0.923879533 i, 0.707106781 + 0.707106781 i, 0.923879533 + 0.382683432 i, 1)
   e. (-0.923879533 + 0.382683432 i, -0.707106781 + 0.707106781 i, 0.382683432 + 0.923879533 i, i, 0.382683432 + 0.923879533 i, 0.707106781 + 0.707106781 i, 0.923879533 + 0.382683432 i, 1)

5. What is the point-value representation of $22x^3 + x^2 + 9x - 3$?
   a. (-4+13i, -33, -4-13i, 29)
   b. (-4+13i, 33, -4-13i, 29)
   c. (-4+13i, -33, 4-13i, 29)
   d. (-4+13i, -33, -4-13i, -29)
   e. (-4+13i, -33, -4-13i, 33)

$22x^3 + x^2 + 9x - 3$

$22x^3 + 9x$

$x^2 - 3$

22

9

1

-3

22

9

1

-3

9-1*22.9+1*22
9-22.9+22
(-13, 31)

-3-1*1.-3+1*1
-3-1, -3+1 = (-4, -2)

(-4, -2)-(i, 1)*(-13, 31).(-4, -2)+(i, 1)*(-13, 31)
=-4-i*(-13), -2-1*31. -4+i*(-13), -2+1*31
=-4+13i, -33, -4-13i, 29