Nhi Ngoc Hong Pham
ID: 026078856

Problem 3:

a) With $x \in [0,8]$:

$\sqrt{0} = 0$; $\sqrt{8} = 2\sqrt{2} \approx 2.828 \rightarrow \quad 0 \leq \sqrt{x} \leq 2.83$

$\rightarrow \lfloor 0 \rfloor \leq \lfloor \sqrt{x} \rfloor \leq \lfloor 2.83 \rfloor$

$\rightarrow \quad 0 \leq \lfloor \sqrt{x} \rfloor \leq 2$

$\rightarrow \quad 3.5 \times 0 \quad \leq \quad 3.5 \lfloor \sqrt{x} \rfloor \quad \leq 3.5 \times 2$

$\rightarrow \quad 0 \quad \leq \quad f(x) \quad \leq 7$

$\rightarrow \quad f([0,8]) = [0,7]$

b) $5 \leq f(x) \leq 10$

$\rightarrow \frac{5}{3.5} \leq \frac{f(x)}{3.5} \leq \frac{10}{3.5}$

$\rightarrow \frac{10}{7} \leq \lfloor \sqrt{x} \rfloor \leq \frac{20}{7}$ (Note: $\frac{10}{7} \approx 1.43$ and $\frac{20}{7} \approx 2.86$)

$\rightarrow \lfloor \sqrt{x} \rfloor = 2$ (as $\lfloor \sqrt{x} \rfloor$ is a non-negative integer)

$\rightarrow \sqrt{x} \in [2,3)$

$\rightarrow \quad x \in [4,9)$

$\rightarrow \quad f^{-1}([5,10]) = [4,9)$

c) g is not one-to-one because $g(\{3,2,9\}) = 3+2+9 = 14 = g(\{2,5,7\})$

g is onto because every natural number in codomain can be the sum of one or more natural numbers.