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Problem 1

By checking the numbers 1 to 70, it appears that numbers $60 \leq n \leq 70$ are all nonnegative linear combinations of 7 and 11. Use induction to prove its true for all $n \geq 60$, or $d = 60$

Basic step $n = 60 = 7(7) + 11(1)$

Inductive step : Assume $n$ is a nonnegative linear combination of 7's and 11's, for some $n \geq 60$. Show: $(n+1)$ is also a nonnegative linear combination of 7's and 11's

Since $n$ is a nonnegative linear combination of 7's and 11's, we can write it as $n = 7k_1 + 11k_2$, where $k_1, k_2 \geq 0$. Then

$$n + 1 = 7k_1 + 11k_2 + 1 = 7k_1 + 11k_2 - 7(3) + 11(2)$$

$$\Leftrightarrow n + 1 = 7(k_1 - 3) + 11(k_2 + 2)$$

Case 1: $k_1 \geq 3$. Then the above linear combination suffices since $(k_1 - 3) \geq 0$

Case 2: $k_1 = 2$. Then $n = 14 + 11k_2$, where $k_2 > 5$, since $n \geq 60$.

$$n + 1 = 11k_2 + 15 = 11k_2 - 7(1) + 11(2) = 11(k_2 + 2) - 7(1)$$

is a nonnegative linear combination of 7's and 11's

Case 3: $k_1 = 1$. Then $n = 7 + 11k_2$, where $k_2 > 5$, since $n \geq 60$.

$$n + 1 = 11k_2 + 8 = 11k_2 - 7(2) + 11(2) = 11(k_2 + 2) - 7(2)$$

is a nonnegative linear combination of 7's and 11's

Case 4: $k_1 = 0$. Then $n = 11k_2$, where $k_2 > 6$, since $n \geq 60$

$$n + 1 = 11k_2 + 1 = 11k_2 - 7(3) + 11(2) = 11(k_2 + 2) - 7(3)$$

is a nonnegative linear combination of 7's and 11's