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1) Relate $f(n) = n(n+1) = n^2 + n$ to $f(n-1) = (n-1)(n-1+1) = (n-1).n = n^2 - n$

We have $f(n-1) = n^2 - n = \underbrace{n^2 + n} - 2n = f(n) - 2n$.

$f(0) = 0.1 = 0$

$f(1) = 1.2 = 2$

$\Rightarrow f(n) = f(n-1) + 2n$: recursive case $n \geqslant 2$

2) Basic step: $n = 1$:

$f_1 = 1$; $f_{2n} = f_2 = 1$ ✓

Inductive step: Assume $f_1 + f_3 + f_5 + \ldots + f_{2n-1} = f_{2n}$ for all $n \leq k$, $k \in \mathbb{Z}$ arbitrary. Show $f_1 + f_3 + f_5 + \ldots + f_{2k-1} + f_{2k+1} = f_{2(k+1)} = f_{2k+2}$

By inductive assumption: $f_1 + f_3 + \ldots + f_{2k-1} = f_{2k}$

$f_1 + f_3 + f_5 + \ldots + f_{2k-1} + f_{2k+1} = f_{2k} + f_{2k+1} = f_{2k+2}$ by recursive definition of the Fibonaci sequence.