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Problem 2

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$p$  : we repeat the process infinitely

$q$  : we will never end up with nine zeros

a) By proof-by-contradiction, we assume  $p$  and  $\neg q$ . That is, we assume if we repeat the process, at some point we will have all nine zeros.

b) Assume that after repeating the process, we have the circle that contains nine zeros eventually.

We obtain nine zeros if all the bits in the previous step were the same, which means all bits were 0s or all were 1s.

\* In the case all bits in previous step were 0s:

- We have the same case as current case. It means all the bits of the circle in the step that is before this step were the same.

- Going backward, we can conclude that all the bits have to be the same through each step, including in the initial circle. However, it creates contradiction because the initial circle has five ones and four zeros, which can not produce the circle having all the same bits in the next step.

\* In the case all bits in previous step were 1s:

- That means every pair of neighboring bits was different.

As we have 9 bits and either 1 or 0 will be alternatively assigned (to make different pair of bits), the first and the last bits would be the same. As the result, there would be a 0 put between the first and the last bits, while the other pairs would give 1s. Thus, there is no way to produce a circle with all nine ones.

Therefore, we can never get nine zeros.