

Nhi Ngoc Hong Pham

ID: 026078856

Part 3.

2a)

1. Basic step: $f(1) = 1 \geq 0.5(1.1)^1 = 0.55$: true

2. Inductive step: Assume $f(k) \geq 0.5(1.1)^k$ for all $1 \leq k \leq n$. Show $f(n+1) \geq 0.5(1.1)^{n+1} = (0.5)(1.1)^n(1.1) = \frac{11}{20}(1.1)^n = 0.55(1.1)^n$

3. $f(n+1) = f(n) + f(n-1)$ (By recursive-function definition)

$f(n+1) \geq 0.5(1.1)^n + 0.5(1.1)^{n-1}$ (By inductive assumption)

$f(n+1) \geq 0.5(1.1)^n \left(1 + \frac{1}{1.1}\right)$

$f(n+1) \geq \frac{21}{22}(1.1)^n > 0.55(1.1)^n$

$\rightarrow f(n+1) \geq 0.5(1.1)^{n+1}$ for all $n \geq 1$

2b) 1. You can choose the suit in 5 ways, and you can choose 6 cards from 10 in $C(10, 6) = 210$ ways

Thus, the total number of ways of getting a flush is:

$$5 \times 210 = 1050 \text{ ways}$$

2. Starting with A, 2, 3, 4, 5, 6, 7, 8, 9, 10. The number of consecutive groups of 6 cards is

For each sequence of 6, we can choose each card in 5 ways with 5 different suits $\rightarrow 5^6 = 15625$ ways

So the total is

3. To choose the suits for the group of 5 cards, we have 5 ways and they ~~the~~ have to be all different $\rightarrow 5! = 120$

We have 10 groups of 5 cards with different suits. So the total ways to place all 50 cards is $10 \times 120 = 1200$ ways