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Problem c

The image shows handwritten mathematical work on grid paper. At the top, there are two mappings, f and g , each represented as a mapping from a set $\{1, 2, 3, 4, 5\}$ to itself.

Mapping f :

- $1 \rightarrow 1$
- $2 \rightarrow 2$
- $3 \rightarrow 3$
- $4 \rightarrow 4$
- $5 \rightarrow 5$

Mapping g :

- $1 \rightarrow 3$
- $2 \rightarrow 2$
- $3 \rightarrow 1$
- $4 \rightarrow 4$
- $5 \rightarrow 5$

Below the mappings, the work shows a proof for $g(3) = 3$:

c) $g(3) = 3$
 From f) we can determine $g(\{3, 4\}) = \{1, 3, 5\}$
 and g) we can determine $g(\{3, 4\}) = \{2, 3, 5\}$ means $g(3)$ can be 3 or 5
 $g(3) \neq 5$ because if it is then clue i) gives $f^{-1}(\{3\}) = \{4, 5\}$, which contradicts clue d) stating that $f(\{1, 4, 5\}) = \{1, 4\}$
 Therefore, $g(3) = 3$

Next, the work shows a proof for $g(1) = 3$:

$g(1) = 3$
 From f) we can determine $g(\{1\}) = \{1, 3, 5\}$
 Knowing that $f(\{4, 5\}) = \{4\}$, from clue h) we get
 $g(f(\{2, 3, 4\})) = g(\{1, 3, 4\}) = \{3, 5\} \Rightarrow g(1)$ can be 3 or 5
 • Suppose $g(1) = 5$
 From clue i), we get $f^{-1}(g^{-1}(\{1, 4, 5\})) = f^{-1}(\{1, 4, 5\}) = \{4\}$
 but $f^{-1}(\{1, 4, 5\}) = \{1, 2, 4, 5\} \neq \{4\}$ given in clue i)
 Therefore $g(1) \neq 5$
 So $g(1) = 3$