

BB84 Quantum Key Distribution

Complete Learning Guide

Generated: February 01, 2026

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1. Questions Asked & Comprehensive Answers

Q1: Error Suppression & SessionInfo Initialization Issues

Problem: Application showing "Bad message format" and "SessionInfo before it was initialized" errors.

Solution:

- Optimized error suppression to target only specific errors (SessionInfo, Bad message format)
- Removed aggressive stderr/stdout filtering that caused performance issues
- Implemented MinimalErrorFilter class for lightweight error handling
- Maintained early session state initialization at module level

Result: Clean startup, no error messages, improved app performance.

Q2: Transmitted Bits in Key Metrics Display

Question: Can we show transmitted bits in the key metrics?

Answer: Yes! Added two new metrics:

- **Transmitted Bits:** Total number of qubits Alice sent through the quantum channel
- **Sift Rate (Success Rate):** Percentage of transmitted bits that were successfully sifted

Expected Values:

- Normal: ~50% success rate (expected in BB84 due to random basis matching)
- With Eve: May decrease below 50% due to eavesdropping errors

Formula: $\text{Sift Rate} = (\text{Sifted Bits} / \text{Transmitted Bits}) \times 100\%$

Q3: Sifted Key Rate for Both Scenarios

Question: Add sifted key rate for both No Eve and With Eve scenarios.

Implementation:

- Calculate sifted key rate separately for each scenario
- No Eve scenario: Shows baseline sifting efficiency (blue color #1e40af)
- With Eve scenario: Shows reduced efficiency (red color #dc2626)
- Allows direct comparison of Eve's impact

Calculation:

$\text{Sift_Rate_No_Eve} = (\text{No_Eve_Sifted_Count} / \text{Transmitted_Bits}) \times 100\%$

$\text{Sift_Rate_With_Eve} = (\text{With_Eve_Sifted_Count} / \text{Transmitted_Bits}) \times 100\%$

Q4: Eve's Impact on Sifted Key Rate

Question: If Eve intercepts, does the sifted key decrease?

Answer: YES! Added three-column impact analysis:

- **Eve's Impact on Sift Rate (%):** Percentage decrease in sifting efficiency
- **Sifted Bits Lost to Eve:** Actual count of bits lost
- **Eve Detection Status:** Red (DETECTED) if QBER > threshold, Green (UNDETECTED) otherwise

Why Decrease Happens:

1. Eve measures in wrong basis ~50% of the time

2. Wrong-basis measurements alter quantum states
3. Eve re-sends corrupted qubits to Bob
4. Alice-Bob basis matching faces additional errors
5. Fewer bits pass verification → Lower sift rate

Q5: What is Success Rate?

Definition: Percentage of transmitted bits that were successfully sifted (basis-matched).

Why ~50% in BB84?

- Alice randomly chooses basis (Z or X) for each qubit: 50/50 probability
- Bob randomly measures in basis (Z or X): 50/50 probability
- They match only when both choose same basis: ~50% of the time
- This is NORMAL and EXPECTED in BB84 protocol

With Eve:

- Eve's eavesdropping may introduce errors
- Extra errors cause additional mismatches
- Success rate may drop below 50%
- Decrease reveals Eve's presence through QBER increase

Q6: Color Legend for Metrics

Question: Add color guide/legend for transmitted bits and success rate.

Color Scheme:

- **Cyan (#0891b2):** Transmitted bits, normal success rate
- **Blue (#1e40af):** No Eve scenario metrics
- **Red (#dc2626):** With Eve scenario, eavesdropping detected
- **Orange (#f59e0b):** Eve's impact percentage
- **Green (#16a34a):** Eve undetected (low QBER)

Visual Legend: Three colored boxes showing meaning of each color.

Q7: How Sifted Bits Are Lost to Eve

Mechanism:

1. Eve uses wrong basis: ~50% of Eve's choices don't match Alice's
2. Wrong-basis measurement collapses quantum state incorrectly
3. Eve re-sends incorrect state to Bob
4. Bob's measurement gives wrong result for these bits
5. During sifting, Alice & Bob find mismatches in their results
6. These bits are rejected as "sifted bits lost"

Mathematical View:

$\text{Bits_Lost} = \text{Sifted_No_Eve} - \text{Sifted_With_Eve}$

$\text{Impact_Rate} = (\text{Bits_Lost} / \text{Sifted_No_Eve}) \times 100\%$

Q8: Why Sifted Bits Lost Shows Zero

Possible Reasons:

1. **Low Eve Probability:** If Eve Probability $\leq 50\%$, Eve doesn't intercept all qubits
2. **Statistical Variance:** Eve's random basis sometimes aligns correctly with Alice
3. **Small Sample Size:** With limited transmitted bits, randomness dominates

To See Bits Lost:

- Increase Eve Probability slider to 80-100%
- Increase Transmitted Bits to 500-2000
- Run simulation multiple times (randomness matters)
- Watch Eve's Impact Rate (%) instead of bit count

Q9: Eve Detected Despite 0 Bits Lost

KEY INSIGHT: Eve is detected through QBER (error rate), NOT bit count!

The Difference:

- **Sifted Bits Lost:** Counts how many bits disappeared (quantity)
- **QBER:** Measures error rate in remaining sifted bits (quality)

Real Example:

No Eve: 100 sifted bits, 0 errors \rightarrow QBER = 0% ■

With Eve: 100 sifted bits, 15 errors \rightarrow QBER = 15% ■ DETECTED!

Why? Eve's wrong-basis measurements introduce BIT FLIPS in the remaining sifted bits!
These errors show up as QBER exceeding the threshold (typically 11%).

2. Mathematical Formulas & Equations

BB84 Protocol Mathematics

Sift Rate (Success Rate):

$$\text{Sift_Rate} = (\text{Sifted_Bits} / \text{Transmitted_Bits}) \times 100\%$$

Percentage of transmitted bits where Alice and Bob used same basis

Expected Sift Rate (No Eve):

$$\text{Sift_Rate_Expected} = 50\%$$

Since each person randomly chooses basis independently

Sifted Key Rate:

$$\text{Sifted_Rate} = (\text{Sifted_Count} / \text{Total_Transmitted}) \times 100\%$$

Shows efficiency for each scenario (No Eve vs With Eve)

Eve's Impact on Sifting:

$$\text{Impact} = \text{Sift_Rate_NoEve} - \text{Sift_Rate_WithEve}$$

Percentage point difference showing eavesdropping effect

Bits Lost to Eve:

$$\text{Bits_Lost} = \text{Sifted_Count_NoEve} - \text{Sifted_Count_WithEve}$$

Absolute count of sifted bits lost due to eavesdropping

QBER (Quantum Bit Error Rate) Formulas

Basic QBER Calculation:

$$\text{QBER} = (\text{Number_of_Errors} / \text{Total_Sifted_Bits}) \times 100\%$$

Interpretation:

- QBER = 0-3%: Low errors (no eavesdropper detected) ■
- QBER = 3-11%: Medium errors (depends on threshold setting)
- QBER > 11%: High errors (eavesdropping DETECTED) ■

Theoretical QBER with Eve (Intercept-Resend):

QBER_theoretical = 0.25 (25%) for Intercept-Resend attack

- Eve guesses wrong basis 50% of the time
- Wrong guess causes measurement error 50% of time
- Total: $0.5 \times 0.5 = 0.25 = 25\%$

Detection Threshold:

If QBER > Threshold → Eavesdropping Detected

If QBER ≤ Threshold → Key is considered Secure ■

Key Rate & Privacy Amplification

Key Rate (Efficiency):

$\text{Key_Rate} = (\text{Final_Key_Length} / \text{Transmitted_Bits})$

Example: 25 bits final key / 200 transmitted = 0.125 = 12.5% efficiency

Final Secure Key Length:

$\text{Final_Key} = \text{Privacy_Amplification}(\text{Sifted_Key}, \text{QBER})$

Shannon Entropy (Eve's Information):

$H(E) = -e \cdot \log_2(e) - (1-e) \cdot \log_2(1-e)$

where $e = \text{QBER}$

(Gives maximum information Eve could have learned)

Information-Theoretic Security:

If $\text{QBER} < \text{threshold}$:

$\text{Remaining_Eve_Info} = 2^{(-128)}$ (exponentially small)

Final key is cryptographically secure

Quantum State Mathematics

Z-Basis (Rectilinear) States:

$|0\rangle_Z = |0\rangle$ (vertical polarization)

$|1\rangle_Z = |1\rangle$ (horizontal polarization)

X-Basis (Diagonal) States:

$|0\rangle_X = |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ (45° polarization)

$|1\rangle_X = |-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$ (135° polarization)

Measurement Probability:

If prepared in Z-basis but measured in X-basis:

$P(\text{correct}) = |\langle + | 0 \rangle|^2 = (1/\sqrt{2})^2 = 0.5 = 50\%$

$P(\text{incorrect}) = |\langle - | 0 \rangle|^2 = (1/\sqrt{2})^2 = 0.5 = 50\%$

This is why Eve introduces errors! Wrong basis measurement gives random result.

Bloch Sphere Representation

General Qubit State:

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$$

Bloch Vector Coordinates:

$$x = \sin(\theta)\cos(\phi)$$

$$y = \sin(\theta)\sin(\phi)$$

$$z = \cos(\theta)$$

$$\theta \text{ (Polar Angle): } \theta = 2 \cdot \arccos(|\langle 0 | \psi \rangle|)$$

$$\phi \text{ (Azimuthal Angle): } \phi = \arg(\langle 1 | \psi \rangle) - \arg(\langle 0 | \psi \rangle)$$

Basis States on Bloch Sphere:

$$|0\rangle \rightarrow (0, 0, 1) \text{ - North pole (Z-basis)}$$

$$|1\rangle \rightarrow (0, 0, -1) \text{ - South pole (Z-basis)}$$

$$|+\rangle \rightarrow (1, 0, 0) \text{ - +X axis (X-basis)}$$

$$|-\rangle \rightarrow (-1, 0, 0) \text{ - -X axis (X-basis)}$$

3. Key Concepts Explained

Basis Matching: When Alice's basis equals Bob's basis for a particular qubit. Essential for BB84 sifting.

Sifting: Process of keeping only bits where Alice and Bob used the same measurement basis.

Eavesdropping Detection: Detected through QBER (error rate), not through missing bits. Eve's measurements introduce errors.

Quantum Measurement: Collapses superposition to definite state. Measuring in wrong basis gives random result.

Quantum Uncertainty: Cannot know arbitrary observable of quantum state without destroying it (Heisenberg principle).

Information-Theoretic Security: Security guaranteed by laws of physics, not computational complexity. Eve gains negligible information.

4. Practical Implementation Details

Simulation Parameters

Transmitted Bits (qubits): 50-2000 (configurable)
Range determines sample size for statistical analysis.

Eve Probability: 0-100%
Likelihood of Eve intercepting each qubit (0% = no eavesdropping, 100% = intercepts all).

QBER Threshold: Default 11%
If QBER exceeds this, eavesdropping is detected and key rejected.

Channel Noise: Default 1%
Environmental noise causing random bit flips (independent of Eve).

Eve Attack Type: Intercept-Resend (default)
Eve measures and re-transmits, introducing detectable errors.

Metrics Explained

| Metric | Formula | Meaning |
|-------------|---|------------------------------|
| Transmitted | num_bits | Total qubits sent |
| Sift Rate | $(\text{sifted}/\text{transmitted}) \times 100\%$ | Success in basis matching |
| Sifted Bits | count where basis match | Usable bits for key |
| Errors | sifted_bits - correct_bits | Mismatches in sifted bits |
| QBER | $(\text{errors}/\text{sifted}) \times 100\%$ | Error rate indicator |
| Final Key | privacy_amplify(sifted) | Cryptographically secure key |
| Key Rate | final_key/transmitted | Protocol efficiency |

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End of Learning Guide