Social and Economic Networks
WSU Python Working Group

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November 19, 2021

Measures of Connectivity

Betweenness Centrality:

The betweenness centrality of a node v is the sum of the fraction of all-pairs shortest paths that pass through v:

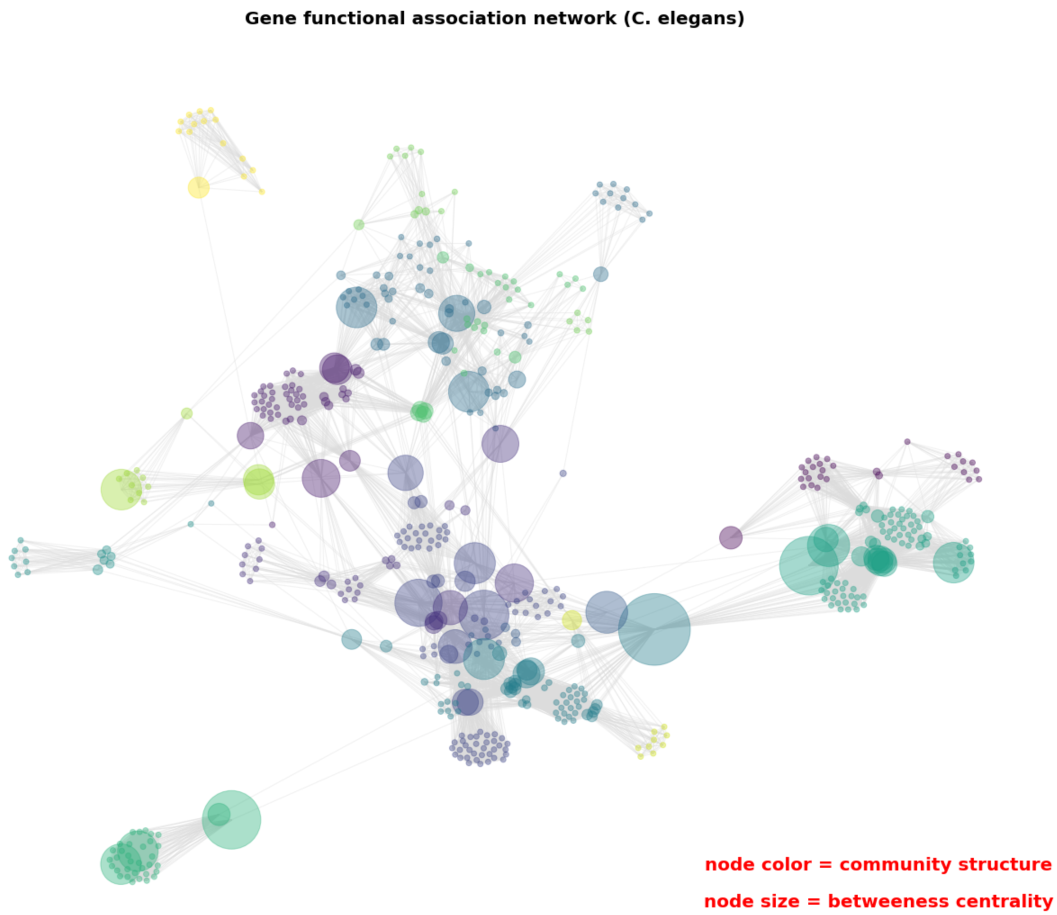
$$c_B(v) = \sum_{s,t \in V} \frac{\sigma(s,t|v)}{\sigma(s,t)}$$

where V is the set of nodes, $\sigma(s,t)$ is the number of shortest (s,t)-paths, and $\sigma(s,t|v)$ is the number of those paths passing through some node v other than s,t. If $s = t$, $\sigma(s,t) = 1$, and if $v \in \{s,t\}$, $\sigma(s,t|v) = 0$

https://networkx.org/documentation/stable/

Measures of Connectivity

Betweenness Centrality of Positive Gene Functional Associations



Measures of Connectivity

In-degree Centrality:

The in-degree centrality x_i of node i is given by: $x_i = \sum_k a_{k,i}$ or in matrix form (1 is a vector with all components equal to unity): $x = 1A$

The out-degree centrality y_i of node i is given by: $y_i = \sum_k a_{i,k}$ or in matrix form: $y = A1$.

https://networkx.org/documentation/stable/

Measures of Connectivity

Closeness Centrality:

Closeness centrality of a node u is the reciprocal of the average shortest path distance to u over all $n-1$ reachable nodes

$$C(u) = \frac{n-1}{\sum_{v=1}^{n-1} d(v,u)}$$

where $d(v,u)$ is the shortest-path distance between v and u, and n is the number of nodes that can reach u. Notice that the closeness distance function computes the incoming distance to u for directed graphs.

https://networkx.org/documentation/stable/

Preliminaries for Social and Economic Networks

Matrix of Beliefs $T_{n\times n}$

- Let there be a finite set of $N = \{1, \ldots, n\}$ *agents* or *nodes* that interact in a social network.
- The interactions between agents are defined by an $n \times n$ nonnegative row stochastic matrix $\mathbf{T}$, where each element of the $\mathbf{T}$ matrix T_{ik} represents the amount of attention agent i gives to agent k.
- The $\mathbf{T}$ matrix is called an interaction matrix, and T_{ik} can be seen as the weight or trust agent i puts on the current belief of agent k in constructing their belief for next period.

https://web.stanford.edu/ jacksonm/books.html

Preliminaries for Social and Economic Networks

The DeGroot (1974) Model

Each agent has belief $p_i^{(t)} \in \mathbb{R}$, where for simplicity it is assumed to be between $[0, 1]$, and the vector of all agent's beliefs is represented by an $n \times 1$ array $\mathbf{p}^{(t)}$. The belief updating rule is

$$\mathbf{p}^{(t)} = \mathbf{T}\mathbf{p}^{(t-1)}$$

which implies

$$\mathbf{p}^{(t)} = \mathbf{T}^t \mathbf{p}^{(0)} \tag{1}$$

where,

$$\mathbf{T}_{n\times n} = \begin{bmatrix} T_{11} & T_{12} & \dots & T_{1n} \\ T_{21} & T_{22} & \dots & T_{2n} \\ \vdots & \vdots & \dots & \vdots \\ T_{n1} & T_{n2} & \dots & T_{nn} \end{bmatrix} \quad \text{and} \quad \mathbf{p}^{(t)}_{n\times 1} = \begin{bmatrix} p_1^{(t)} \\ p_2^{(t)} \\ \vdots \\ p_n^{(t)} \end{bmatrix}.$$

https://web.stanford.edu/ jacksonm/books.html

Preliminaries for Social and Economic Networks

Consensus

A consensus is reached in the DeGroot model if and only if there is exactly one strongly connected and closed group of agents, $\mathbf{T}$ is aperiodic on that group, and this convergence happens for any initial belief vector $\mathbf{p}^{(0)}$.

Social Influence

Assuming that a consensus can be reached, the social influence of agent i in a society is the i^{th} entry in the converged vector of beliefs if and only if that belief vector sums to 1. More formally, and skipping a lot of theory, we seek a limiting vector $s \in [0, 1]^n$ such that

$$s\mathbf{T} = s$$

where, $\sum_{i=1}^{n} s_i = 1$, and s is the left-hand eigenvector of $\mathbf{T}$.

https://web.stanford.edu/ jacksonm/books.html

Finding the Influence Vector

An Example using The Left-hand Eigenvector Theory

Let,

$$\mathbf{T}_{3\times 3} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

this implies the s that solves $s\mathbf{T} = s$ is

$$\begin{bmatrix} 2/5 & 2/5 & 1/5 \end{bmatrix} \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2/5 & 2/5 & 1/5 \end{bmatrix}.$$

https://web.stanford.edu/ jacksonm/books.html

Another Way of Finding the Influence Vector

An Example Using the Power Iteration

Consider the same example from above where,

$$\mathbf{T}_{3\times 3} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{p}^{(0)}_{3\times 1} = \begin{bmatrix} p_1^{(0)} \\ p_2^{(0)} \\ p_3^{(0)} \end{bmatrix}.$$

We know that any matrix $\mathbf{T}$ should converge to a consensus $\mathbf{p}^{(\infty)}_{n\times 1}$ for any initial belief vector $\mathbf{p}^{(0)}_{n\times 1}$ such that

$$\mathbf{p}^{(\infty)}_{n\times 1} = \mathbf{T}^{\infty}\mathbf{p}^{(0)}_{n\times 1}$$

implying that if we multiply $\mathbf{T}$ to itself enough times, we should find a convergent matrix. This is apparent in this example since

$$\mathbf{T}^{\infty}_{3\times 3} = \begin{bmatrix} 2/5 & 2/5 & 1/5 \\ 2/5 & 2/5 & 1/5 \\ 2/5 & 2/5 & 1/5 \end{bmatrix} \text{where} \quad s = \begin{bmatrix} 2/5 & 2/5 & 1/5 \end{bmatrix}.$$

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References

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Note: If any citations need to be added, please feel free to e-mail.