Chapter 12

Game Theory

Introduction (1/3)

12

Chapter 11 introduced the concept of strategic interaction among firms that have some market power but compete with each other.

Game theory is focused on understanding strategic interaction.

Chapter Outline

Introduction (2/3)

Game theory is the study of strategic interactions among two or more economic actors.

- Examine behavior when players are making **strategic decisions**—actions based on anticipation of others' actions.
- Concepts can be applied to market situations, as well as any number of human (and nonhuman) interactions.

Some things to remember:

- Game theory is all about seeing the world through the eyes of your opponent.
- As with consumer theory, we assume players are rationally self-interested.
- Finally, rules often determine the outcome of a game; therefore, it is very important to understand the timing of moves, allowable actions, and so on.

Introduction (3/3)

We will study three basic categories of games.

Simultaneous games

- The participants choose their actions simultaneously without knowing their opponents' strategies.
- The Cournot and Bertrand models from Chapter 11 are simultaneous games.

Repeated games

- A series of simultaneous games among the same set of economic actors
- Successful collusion (e.g., functioning cartels) is made possible by the cyclic repetition of output decisions.

Sequential games

- Games in which players take turns making decisions
- The Stackelberg model from Chapter 11 is a sequential game.

What is a Game? (1/4)

12.1

Every game shares three elements:

1. **Players**
    - A player is a participant in an economic game who must decide his or her actions based on the actions of others.
2. **Strategy**
    - The action taken by a player
    - Strategies may be simple or complex, and they depend on the actions (anticipated or actual) of other players.
3. **Payoff**
    - The outcome a player receives from playing the game
    - The payoff to one player depends on the actions of other players; otherwise, the former would have no incentive to act strategically.

What is a Game? (2/4)

12.1

Dominant and Dominated Strategies

Predicting behavior in games relies on finding the **optimal strategy** for the different players.

- The action that results in the highest expected payoff

Dominant strategy

- A winning strategy for a player, regardless of his or her opponents' strategies
  - If a player has a dominant strategy, that strategy is always chosen.

Dominated strategy

- A losing strategy for a player, regardless of his or her opponents' strategies
  - Dominated strategies are never chosen, and once identified, can be ignored.
- If there is a dominant strategy, all other strategies are dominated.

If there is a dominated strategy, must there be a dominant strategy?

– Not necessarily

What is a Game? (3/4)

12.1

A **payoff matrix** is a table that lists the players, strategies, and payoffs of an economic game.

- For example, consider the decisions facing Warner Bros. and Disney regarding advertising for animated movies.
  - It is reasonable to assume that animated movies are competing with one another for young viewers.

What is a Game? (4/4)

12.1

Table 12.1: An Advertising Game

	DISNEY: Advertise	DISNEY: Don't Advertise
WARNER BROTHERS: Advertise	250, 250	550, −80
WARNER BROTHERS: Don't Advertise	−80, 550	320, 320

In this game, Warner Bros. (payoffs are in left values) has a dominant strategy of advertising, regardless of what Disney does.

Nash Equilibrium in Simultaneous Games (1/12)

When individuals or firms engaged in strategic competition have dominant strategies, determining equilibrium is easy.

However, in most games, the optimal strategy for a player depends on the actions of the other player or players.

- In these cases, we rely on the stronger concept of **Nash equilibrium** introduced in the last chapter.
- The Nash equilibrium requires that an action be the best thing a player can do *given the action the opponent is taking.*

The first step to finding a Nash equilibrium is to understand the three elements of the game: players, strategies, and payoffs.

- A common way to do this is to use the **normal form:** organize an economic game into its players, strategies, and payoffs in a payoff matrix.

Nash Equilibrium in Simultaneous Games (2/12)

12.2

<u>Tips for Finding a Nash Equilibrium:</u> The Check Method

- Always analyze players' strategy and payoff combinations one at a time.

Call our players Row and Column, and assume each has two actions available. What should Row do?

- If Column chooses action A, what is the best action for Row? Check off the payoff for that action.
- If Column chooses action B, what is the best action for Row? Check off that option.
    - Repeat these steps for Column.

If one of the boxes has two check marks, that is the Nash equilibrium.

Nash Equilibrium in Simultaneous Games (3/12): Question 1

12.2

Samuel's Coffee and Janice's Coffee are deciding whether to build a new store, expand by adding space, or leave operations as they are. Their payoff matrix (with amounts listed in dollars of profit per day):

Janice's

Samuel's

	Open	Expand	Leave as is
Open	250 , 250	180 , 260	200 , 300
Expand	150 , 210	150 , 230	175 , 260
Leave as is	200 , 200	160 , 400	150 , 250

Which of the following is **true**?

A. Samuel does not have a dominant strategy.
B. Samuel's dominated strategy is "Open New Store."
C. Samuel's dominated strategy is "Expand."
D. Samuel's dominated strategy is "Leave as is."

Nash Equilibrium in Simultaneous Games (3/12): Question 1 – Correct Answer

12.2

Samuel's Coffee and Janice's Coffee are deciding whether to build a new store, expand by adding space, or leave operations as they are. Their payoff matrix (with amounts listed in dollars of profit per day):

Janice's

Samuel's		Open	Expand	Leave as is
	Open	250 , 250	180 , 260	200 , 300
	Expand	150 , 210	150 , 230	175 , 260
	Leave as is	200 , 200	160 , 400	150 , 250

Which of the following is **true**?

A. Samuel does not have a dominant strategy.

B. Samuel's dominated strategy is "Open New Store." (correct answer)

C. Samuel's dominated strategy is "Expand."

D. Samuel's dominated strategy is "Leave as is."

Nash Equilibrium in Simultaneous Games (4/12): Question 2

12.2

Samuel's Coffee and Janice's Coffee are deciding whether to build a new store, expand by adding space, or leave operations as they are. Their payoff matrix (with amounts listed in dollars of profit per day):

Janice's

Samuel's	Open	Expand	Leave as is
Open	250 , 250	180 , 260	200 , 300
Expand	150 , 210	150 , 230	175 , 260
Leave as is	200 , 200	160 , 400	150 , 250

Which of the following is true **for Janice**?

A. Janice does not have a dominated strategy.

B. "Open New Store" is a dominated strategy.

C. "Expand" is a dominated strategy.

D. "Leave as is" is a dominated strategy.

Nash Equilibrium in Simultaneous Games (4/12): Question 2 – Correct Answer

12.2

Samuel's Coffee and Janice's Coffee are deciding whether to build a new store, expand by adding space, or leave operations as they are. Their payoff matrix (with amounts listed in dollars of profit per day):

Samuel's \ Janice's	Open	Expand	Leave as is
Open	250 , 250	180 , 260	200 , 300
Expand	150 , 210	150 , 230	175 , 260
Leave as is	200 , 200	160 , 400	150 , 250

Which of the following is true **for Janice**?

A. Janice does not have a dominated strategy.

B. "Open New Store" is a dominated strategy. (correct answer)

C. "Expand" is a dominated strategy.

D. "Leave as is" is a dominated strategy.

Nash Equilibrium in Simultaneous Games (5/12): Question 3

12.2

Samuel's Coffee and Janice's Coffee are deciding whether to build a new store, expand by adding space, or leave operations as they are. Their payoff matrix (with amounts listed in dollars of profit per day):

Janice's

Samuel's	Open	Expand	Leave as is
Open	250 , 250	180 , 260	200 , 300
Expand	150 , 210	150 , 230	175 , 260
Leave as is	200 , 200	160 , 400	150 , 250

Which of the following is **true**?

A. There are no Nash equilibria in this game.
B. Samuel "Leave as is" and Janice "Expand" is a Nash equilibrium.
C. Samuel "Open" and Janice "Open" is a Nash equilibrium.
D. Samuel "Open" and Janice "Leave as is" is a Nash equilibrium.

Nash Equilibrium in Simultaneous Games (5/12): Question 3 – Correct Answer

Samuel's Coffee and Janice's Coffee are deciding whether to build a new store, expand by adding space, or leave operations as they are. Their payoff matrix (with amounts listed in dollars of profit per day):

Janice's

Samuel's		Open	Expand	Leave as is
	Open	250 , 250	180 , 260	200 , 300
	Expand	150 , 210	150 , 230	175 , 260
	Leave as is	200 , 200	160 , 400	150 , 250

Which of the following is **true**?

A. There are no Nash equilibria in this game.

B. Samuel "Leave as is" and Janice "Expand" is a Nash equilibrium.

C. Samuel "Open" and Janice "Open" is a Nash equilibrium.

D. Samuel "Open" and Janice "Leave as is" is a Nash equilibrium. (correct answer)

Nash Equilibrium in Simultaneous Games (6/12)

12.2

Predicting outcomes of a game is relatively straightforward when there is only one Nash equilibrium. What if there is more than one?

Multiple Equilibria

Consider the Warner Bros./Disney rivalry discussed previously.

- Suppose that firms are deciding when to release a film.
- The available periods are May, December, and March.

Nash Equilibrium in Simultaneous Games (7/12)

12.2

Table 12.3: Selecting a Release Date

	Disney's Opening Date Choice: May	Disney's Opening Date Choice: December	Disney's Opening Date Choice: March
Warner Brothers' Opening Date Choice: May	100, 100	✓ 600, 400 ✓	✓ 600, 200
Warner Brothers' Opening Date Choice: December	✓ 400, 600 ✓	0, 0	400, 200
Warner Brothers' Opening Date Choice: March	200, 600 ✓	200, 400	−100, −100

Disney and Warner Bros. have symmetrical payoffs, and it appears there are two Nash equilibria so no pure strategy.

- December/May and May/December

What about March?

- March is a dominated strategy for each firm.
- This means we can simplify the game.

Nash Equilibrium in Simultaneous Games (8/12)

12.2

Table 12.4: Selecting a Release Date (Simplified Game)

	Disney's Opening Date Choice: May	**Disney's Opening Date Choice: December**
Warner Brothers' Opening Date Choice: May	100, 100	✓ 600, 400 ✓
Warner Brothers' Opening Date Choice: December	✓ 400, 600 ✓	0, 0

There are now two equilibria.

- If Disney believes Warner Bros. will be releasing in May, Disney should release in December, and vice versa.
- Neither firm has an incentive to deviate from these two release date combinations.

With the tools presented thus far, there is no way to determine which equilibrium occurs.

Nash Equilibrium in Simultaneous Games (9/12)

12.2

Mixed Strategies

As highlighted by the previous slides, sometimes there is no pure strategy equilibrium. In these cases, a player's best option may be to choose actions randomly from a set of pure strategies. This is called a **mixed strategy**.

- A strategy in which the player randomizes his or her actions

Consider the case of a player taking a penalty kick in soccer.

- For simplicity, assume the kicker can kick the ball either left or right.
- The goalie must choose which way to dive to save the goal.
- If the goalie and kicker choose the same side, the goal is saved; otherwise, it is scored.

Nash Equilibrium in Simultaneous Games (10/12)

12.2

Mixed Strategies

The payoff matrix for this game:

	Goalie: Left	Goalie: Right
Kicker: Left	0, 1 ✓	✓ 1, 0
Kicker: Right	✓ 1, 0	0, 1 ✓

No single strategy combination has two checks.

- There is no pure strategy equilibrium.
- However, there is a Nash equilibrium.
- The Nash equilibrium calls for each player to randomize across the two strategies.

Nash Equilibrium in Simultaneous Games (11/12)

12.2

Mixed Strategies

The payoff matrix for this game:

	Goalie: Left	Goalie: Right
Kicker: Left	0, 1 ✓	✓ 1, 0
Kicker: Right	✓ 1, 0	0, 1 ✓

Because payoffs are symmetrical, the only possible equilibrium is for each player to play each strategy with a 50% probability.

- If Kicker plays left 80% of the time, Goalie will always go left.
  - But then Kicker has an incentive to go right.
- With asymmetrical payoffs, the probabilities associated with the mixed equilibrium will be different.

Nash Equilibrium in Simultaneous Games (12/12)

12.2

The Maximin Strategy (Or: What If My Opponent Is an Idiot?)

The idea of a Nash equilibrium is based on rational decision making by players.

What if players make systematic errors? (In other words, they don't appear to be optimizing their own payoffs.)

- A rational player facing less than rational opponents may find a more conservative strategy to be appealing.

A **maximin strategy** is one in which a player minimizes his or her exposure to loss.

- The idea is to choose the strategy with the largest minimum payoff across all possible strategies for your opponent or opponents.

Repeated Games (1/8)

12.3

Finitely Repeated Games

When a simultaneous game is played repeatedly, strategies are temporal, consisting of a strategy for each period.

To analyze finitely repeated games, we use **backward induction**.

- Solving a multistep game by first solving the last step and then working backward

Again, consider the prisoner's dilemma that arises when Warner Bros. and Disney decide whether or not to advertise.

- Suppose there are two planned movies for each firm—*Wonder Woman 2* versus *Black Panther 2*—in the first period.
- Total payoffs are maximized when both firms decide not to advertise.

Can the prisoner's dilemma be avoided in two periods?

  – If firms know how many periods there will be, without a commitment device they will both have an incentive to cheat prior to the last period.

Repeated Games (2/8)

12.3

Infinitely Repeated Games

Infinitely repeated games sometimes sustain cooperation; however, they are more difficult to analyze.

Rather than attempting to determine the optimal strategy for Disney and Warner Bros. from induction, instead consider a specific strategy and decide whether it constitutes a Nash equilibrium.

Assume the two companies plan to continue to release *Wonder Woman* and *Black Panther* movies indefinitely.

- Possible strategy: Warner Bros. will not advertise in the first period and will continue not to advertise as long as Disney does not advertise either; if Disney advertises, Warner Bros. will start advertising and continue to do so forever.
- Disney does the same.
  - To determine whether this set of strategies is a Nash equilibrium, we must evaluate whether either firm can do better by defecting.

Repeated Games (3/8)

12.3

Consider the following single-period payoff matrix for the two firms. Payoffs are assumed to remain constant over time.

Table 12.4: The Single-Period Payoffs of an Infinitely Repeated Advertising Game

	Disney: Advertise	Disney: Don't Advertise
Warner Brothers: Advertise	250, 250	550, −80
Warner Brothers: Don't Advertise	−80, 550	320, 320

If both firms cooperate, the lower right box can be sustained, and periodic payoffs will be \$70 million greater for each firm than if both advertise.

- The benefits of cooperation can be viewed as an infinite stream of \$70 million per period for each firm, discounted to present-value terms. Payoffs are assumed to be constant over time.

Repeated Games (4/8)

12.3

How should Warner Bros. evaluate the costs and benefits of the different strategies when some payments occur in the future?

Because of impatience and positive interest rates, future payments are generally seen as less desirable than immediate payments.

- The future is *discounted* by a factor *d*, where *d* is a value between 0 and 1.
- A firm views $1 in the next period as being worth $*d* today.
- Higher values for *d* imply a greater consideration for future payments.
- When $d = 0$, the firm does not care at all about future payments.

Repeated Games (5/8)

12.3

Consider Warner Bros.' payoffs under the two strategies of cooperating (no advertising) or defecting (advertising).

Payoff from Defecting

$$550 + d \text{ x } (250) + d^2 \text{ x } (250) + \; d^3 \text{ x } (250) \ldots$$

Payoff from Cooperating

$$320 + d \text{ x } (320) + d^2 \text{ x } (320) + \; d^3 \text{ x } 320 \ldots$$

Payments further into the future are discounted at a higher rate.

Repeated Games (6/8)

12.3

If the payoff from cooperation is higher than the payoff from defection, cooperation (no advertising) can be sustained as a Nash equilibrium, or

$$320 + d \text{ x } (320) + d^2 \text{ x } (320) + d^3 \text{ x } (320) + \cdots >$$
$$550 + d \text{ x } (250) + d^2 \text{ x } (250) + d^3 \text{ x } (250) \ldots$$

Simplifying,

$$70 \text{ x } (d + d^2 + d^3 + \cdots) > 230$$
$$(d + d^2 + d^3 + \cdots) > 23/7$$

Repeated Games (7/8)

12.3

Using the mathematical identity for an infinite series when $0 \le d < 1$ and substituting into the condition for a Nash equilibrium:

$$d + d^2 + d^3 + \ldots = \frac{d}{1-d}$$

$$\frac{d}{(1-d)} > \frac{23}{7}$$

$$d > 0.77$$

So long as Disney and Warner Bros. view a dollar earned in the next period to be worth at least as much as $0.77 today, the cooperative Nash equilibrium can be sustained.

Repeated Games (8/8)

12.3

Underlying this analysis has been the assertion that if either Warner Bros. or Disney defects by advertising, both firms will advertise from that point forward. This strategy is often referred to as a **grim trigger strategy**.

- Cooperative play ends when one player cheats.

An alternative strategy in repeated games is known as **tit-for-tat**.

- The second player mimics the opponent's prior-period action in each round.
- The second player cheats if her opponent cheated in the preceding round and cooperates when her opponent cooperated in the previous round.

Sequential Games (1/6)

12.4

In many games, players make decisions one at a time. These types of games are called **sequential games**.

- One player moves first and other players observe this action before making their decisions.

The matrix approach to visually describing simultaneous-move games will not work for sequential games; instead, we use an **extensive form** or **decision tree**.

- Representation of a sequential game shows both the choice and timing of players' actions.

Let's return to the decisions facing Warner Bros. and Disney regarding release dates for the films *Black Panther 2* and *Wonder Woman 2.*

- There are three possible release months: March, May, and December.

Sequential Games (2/6)

12.4

The normal form shows us that we have two possible Nash equilibria.

Table 12.8: Selecting a Release Date

	Disney's Opening Date Choice: May	Disney's Opening Date Choice: December	Disney's Opening Date Choice: March
Warner Brothers' Opening Date Choice: May	100, 100	✓ 600, 400 ✓	✓ 600, 200
Warner Brothers' Opening Date Choice: December	✓ 400, 600 ✓	0, 0	400, 200
Warner Brothers' Opening Date Choice: March	200, 600 ✓	200, 400	−100, −100

- The two possible equilibria are May/December and December/May.

Sequential Games (3/6)

12.4

Now, assume that Warner Bros. moves first, committing to a release month that is observable to Disney.

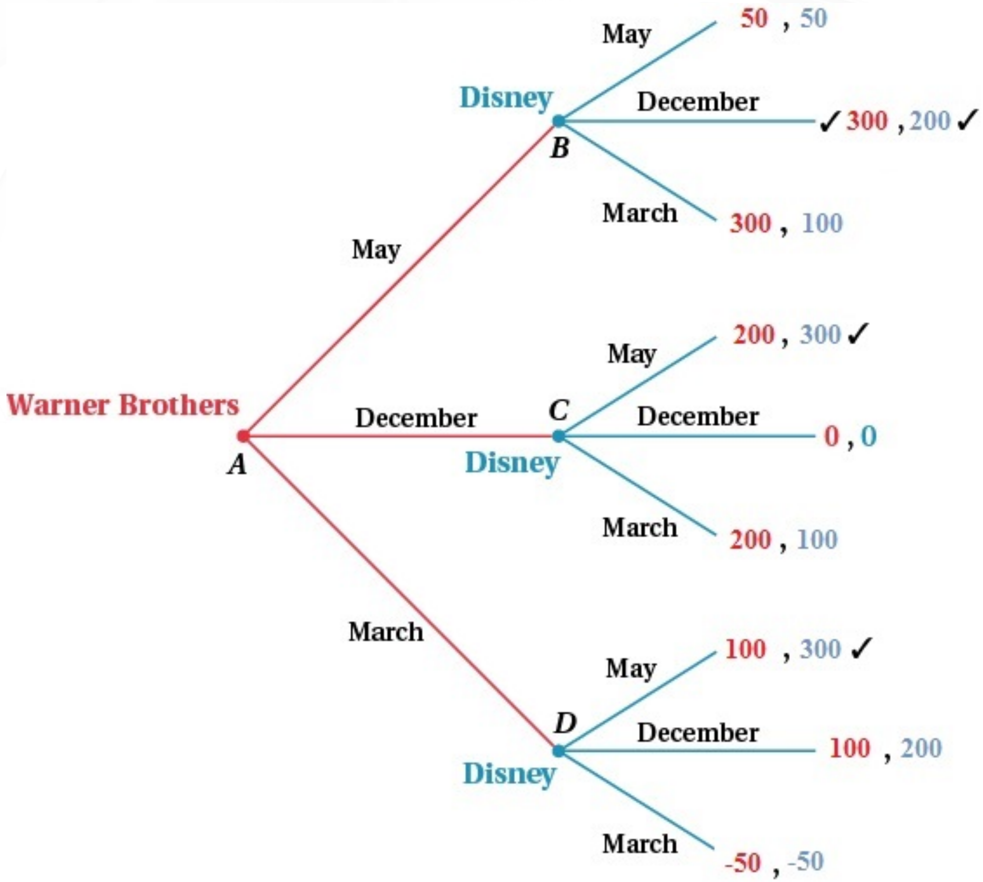
Use backward induction to evaluate the Nash equilibrium.

- For each strategy that Warner Bros. might choose, evaluate the best response by Disney.
- These outcomes are *subgame equilibria*.
- Warner Bros. chooses a strategy that results in the most favorable outcome, given knowledge of Disney's best response.

Sequential Games (4/6)

12.4

Figure 12.2 Decision Tree for Choosing a Release Date



If Warner Bros. chooses May, Disney chooses December.

Knowing Disney's best response to each release date, Warner Bros. will choose May, and that will be the Nash equilibrium.

If Warner Bros. chooses December or March, Disney will choose May at nodes C and D, respectively. In both cases, they will select May.

Sequential Games (5/6)

Play or Pass

Consider another type of sequential game with a different structure.

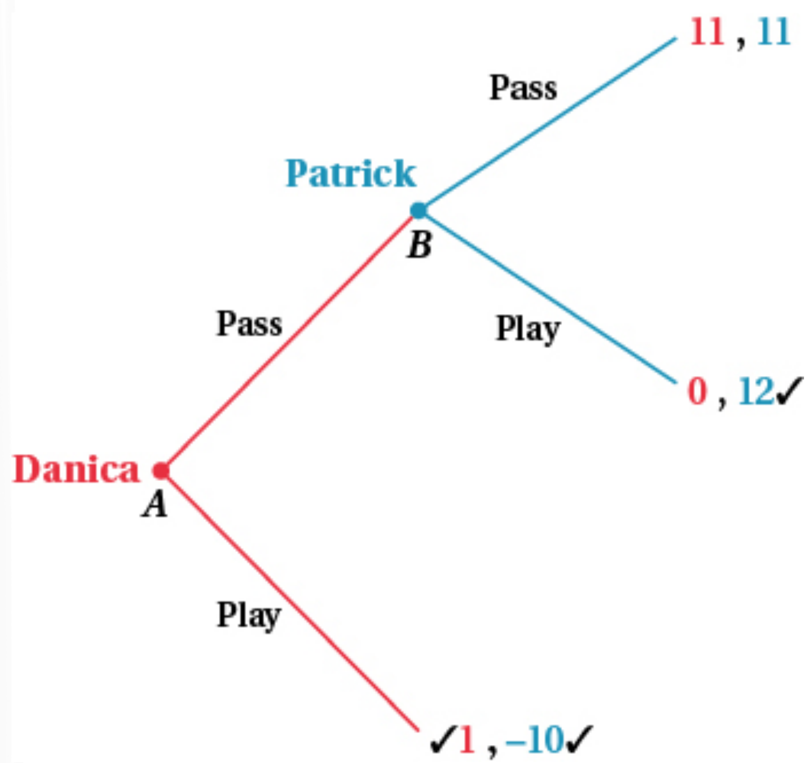
Contestants Danica and Patrick are playing a game called Play or Pass.

- Flip a coin to decide who moves first. Danica wins.
- Danica can choose to play, which ends the game with a payoff of 1 for her and −10 for Patrick, or to pass, which moves the game to Patrick.
- If Patrick chooses to play, he receives 12 and Danica receives 0; if he chooses to pass, each player receives 11.

Sequential Games (6/6)

12.4

Figure 12.3 Sequential Play or Pass Game



If Danica chooses to play, the game ends. If she chooses to pass, Patrick moves.

However, Patrick's best response is to play.

Therefore, Danica will always choose to play.

Strategic Moves, Credibility, and Commitment (1/8)

12.5

In Play or Pass, the equilibrium was clearly suboptimal for both players.

What types of changes to the game structure might allow players to reach a more efficient outcome?

- A side payment could be used.

The key is for players to use **strategic moves.**

- An action taken early in a game that favorably influences the ultimate outcome of the game

Strategic Moves, Credibility, and Commitment (2/8)

12.5

Side Payments

A simple strategic move is to offer a **side payment**, a bribe that influences the outcome of a strategic game.

Return to Play or Pass

- Danica has no incentive to pass because Patrick will always play, leaving Danica with nothing.

How might Danica and Patrick reach a better outcome through a side payment?

– Danica might offer Patrick a deal to pass rather than play.

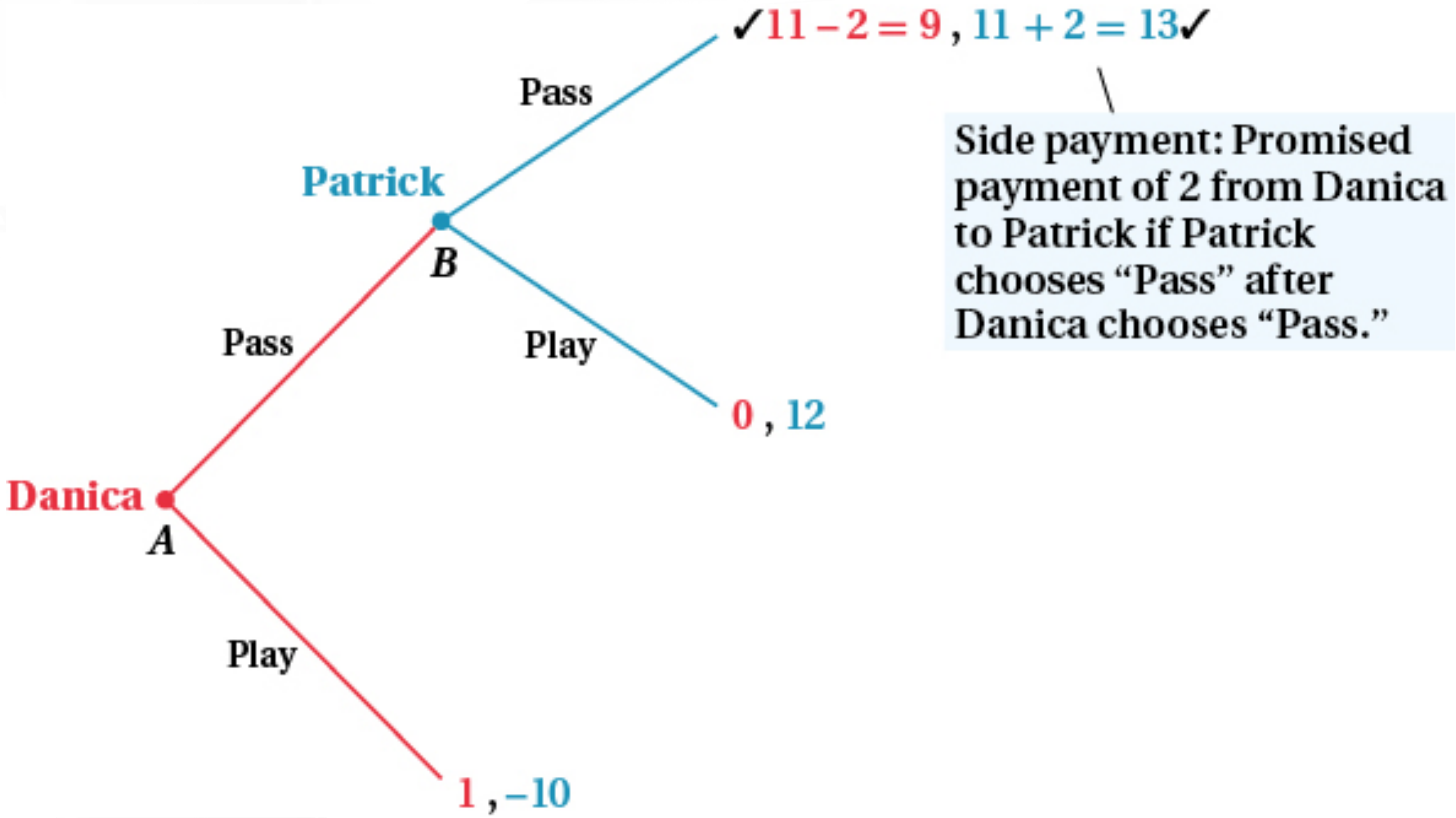
Suppose Danica promises to pass and give Patrick a side payment if Patrick will pass, too.

How much does Danica have to promise Patrick?

– A payoff of 2, as that will make Patrick better off with pass than play

Strategic Moves, Credibility, and Commitment (3/8)

Figure 12.4 A Side Payment Can Alter the Nash Equilibrium



Strategic Moves, Credibility, and Commitment (4/8)

12.5

Commitment

Side payments sound like a good idea, but in real life there may be incentives to renege.

Commitments that are not enforceable are often referred to as **noncredible threats**.

- A threat made in a game that is not rational for the player to actually follow through on and, as such, is an empty threat

To avoid this dilemma, a player must make a **credible commitment**.

- A signal to a player's opponent that the threat or promise is credible

Strategic Moves, Credibility, and Commitment (5/8)

12.5

<u>Entry Deterrence:</u> Credibility Applied

A common application of strategic moves in microeconomics is firms deterring other firms from entering an industry.

Consider the market for tablet computers.

- Assume Apple is the only company making tablets but Samsung is thinking about entering the market.

What option does Apple have to try to prevent Samsung from entering?

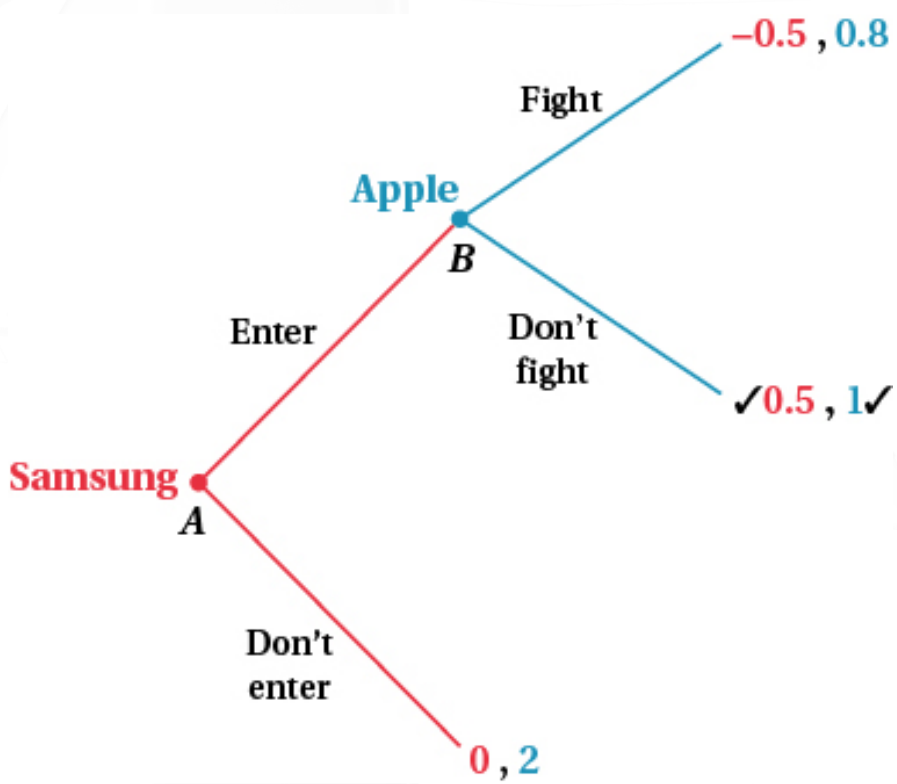
Apple could threaten a price war.

- Still must determine whether this is a credible threat

Strategic Moves, Credibility, and Commitment (6/8)

12.5

Figure 12.6 An Entry Game*



Apple has promised to fight a price war if Samsung enters the tablet market. Is this a credible threat?

If we look at the payouts for Apple, we can determine this is NOT a credible threat, as the payoff for not fighting is higher. Therefore, Samsung will choose to enter, and Apple will choose not to fight.

*Payoffs are in billions of dollars of profit.

Strategic Moves, Credibility, and Commitment (7/8)

12.5

<u>Entry Deterrence:</u> Credibility Applied

A common application of strategic moves in microeconomics is firms deterring other firms from entering an industry.

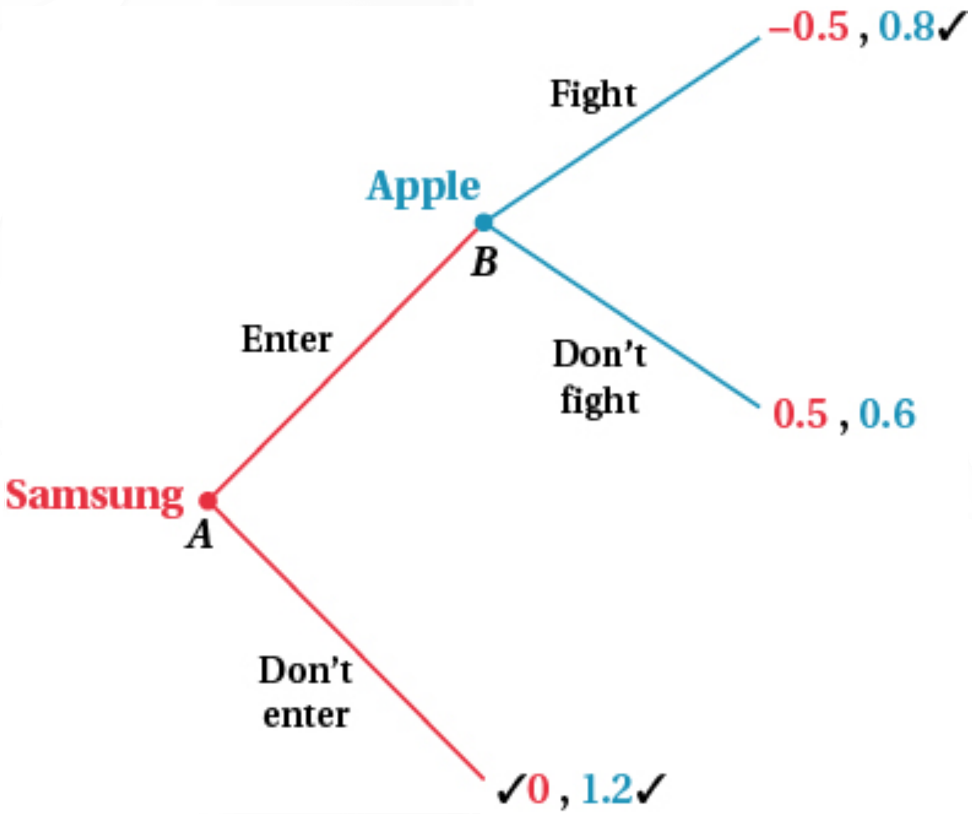
Again, consider the market for tablet computers.

- Assume Apple is the only company making tablets, but Samsung is thinking about entering the market.
- Rather than threatening just a price war, Apple could invest in excess capacity in anticipation of entry, putting itself in a better position for a price war.

Strategic Moves, Credibility, and Commitment (8/8)

12.5

Figure 12.7 Excess Capacity to Make a Credible Threat



Apple has invested in excess capacity, lowering its cost of engaging in a price war. Is Apple's threat of a price war credible?

Knowing this, Samsung will choose not to enter, and that will represent the Nash equilibrium.

Conclusion (1/1)

12.6

Game theory gives us a logical framework in which to understand an opponent's likely responses to your decisions, a framework that is useful in many strategic situations.

In the next chapter, we take a closer look at how firms and individuals make decisions over time and when there is *uncertainty* regarding outcomes.