Slide 12.2

Mixed Strategy Nash Equilibrium (MSNE)

		Goalie L	Goalie R
Kicker	L	0,1	1,0
	R	1,0	0,1

$\Rightarrow$

	L (q)	R ($1-q$)
L (p)	0,1	1,0
R ($1-p$)	1,0	0,1

Kicker

$\Rightarrow \quad EU_1(L) = EU_1(R)$

$\Rightarrow \quad q(0) + (1-q)(1) = q(1) + (1-q)0$

$\Rightarrow \quad 1-q = q$

$\Rightarrow \quad q^* = \frac{1}{2}$

Goalie $\quad EU_2(L) = EU_2(R)$

$\Rightarrow \quad p(1) + (1-p)0 = p(0) + (1-p)(1)$

$p = 1-p$

$\Rightarrow \quad p^* = \frac{1}{2}$

$$\Rightarrow MSNE = \{ \underbrace{(pL, (1-p)R)}_{\text{Kicker}}, \underbrace{(qL, (1-q)R)}_{\text{Goalie}} \}$$

$$\Rightarrow \quad = \{ (\tfrac{1}{2}L, \tfrac{1}{2}R), (\tfrac{1}{2}L, \tfrac{1}{2}R) \}$$ //

Repeated Games

First note we need to review what a geometric series is:

Def. A geometric series is

$$\sum_{K=0}^{\infty} \delta^K = \delta^0 + \delta^1 + \delta^2 + \delta^3 + \dots + \delta^\infty$$

such that,

It can be represented as: $\underbrace{\sum_{K=0}^{\infty} \delta^K = \frac{1}{(1-\delta)}}_{\text{Important!}}$ is $0 \le \delta < 1$

$\Rightarrow$

$$\sum_{K=1}^{\infty} \delta^K = \delta^1 + \delta^2 + \delta^3 + \dots$$

$$= \delta(1 + \delta^1 + \delta^2 + \delta^3 + \dots)$$

$$= \delta \underbrace{\sum_{K=0}^{\infty} \delta^K}_{\text{Geometric Series}}$$

$$\Rightarrow \quad = \delta \frac{1}{(1-\delta)} = \frac{\delta}{(1-\delta)}$$

$\Rightarrow$ From slide 12.3 (5/8):

Payoffs from Defecting		Payoffs From Cooperating
$\underbrace{550\delta^0}_{\text{Defect}} + \underbrace{\delta^1(250) + \delta^2(250) + \dots}_{\text{Compete in Adversity}}$	$<$	$320\delta^0 + 320\delta^1 + 320\delta^2 + \dots$

$$\Rightarrow 550 + \delta 250 \underbrace{(1 + \delta^1 + \delta^2 + \delta^3 + \dots)}_{\text{SERIES}} < 320 \underbrace{(\delta^0 + \delta^1 + \delta^2 + \dots)}_{\text{SERIES}}$$

$$\Rightarrow 550 + 250 \frac{\delta}{(1-\delta)} < 320 \frac{1}{(1-\delta)} \Rightarrow$$

$$\Rightarrow \quad 550 + 250\frac{\delta}{(1-\delta)} < 320\frac{1}{(1-\delta)}$$

$$\Rightarrow \quad 550(1-\delta) + 200\delta < 320$$

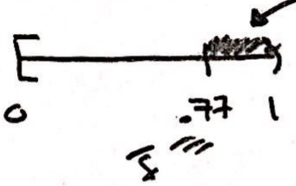
$$\Rightarrow \quad 550 - 300\delta < 320$$

$$\Rightarrow \quad 230 < 300\delta$$

$$\Rightarrow \quad \delta > \frac{23\cancel{0}}{30\cancel{0}} = \frac{23}{30} \approx .77 \equiv \bar{\delta}$$

$\Rightarrow$ If $\delta > \bar{\delta}$, then collusion can be sustained.

$\Rightarrow$ Graphically,



If δ is in here, collusion will be sustained.

Intuitively, If both WB & Disney are patient enough, the pareto optimal outcome will be sustained.