# Nash Equilibrium Examples

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(1/2)

## ① Prisoners Dilemma

| | | P2 Don't Confess | P2 Confess |
|---|---|---|---|
| P1 | Don't Confess | 3,3 | 0,4 |
| | Confess | 4,0 | 1,1 |

Consider a one shot game where the players move simultaneously. They can either chose to confess or don't confess, and their payoffs are listed according to the outcome of the game. All players completly observe the game, & know all the rules to the game. Players cannot coordinate.

=> Best Responses

Player 1: - If player 2 plays Don't confess, I play Confess (4>3).
- If player 2 plays Confess, I play Confess (1>0).

Player 2: - If player 1, plays Don't Confess, I play Confess (4>3).
- If player 1 plays Confess, I play Confess (1>0).

Therefore, The Nash Equilibrium of this game is

$$psNE = \{(\underbrace{Confess}_{P1}, \underbrace{Confess}_{P2})\}$$

## ② Battle of the Sexes

| | | P2 Football | P2 Opera |
|---|---|---|---|
| P1 | Football | 3,1 | 0,0 |
| | Opera | 0,0 | 1,3 |

Same Rules as Prisoners Dilemma.
Still not allowed to coordinate.

=> Best Responses

P1: - If P2 plays Football, I play Football (3>0).
- If P2 plays Opera, I play Opera (1>0).

P2: - If P1 plays Football, I play Football (1>0).
- If P1 plays Opera, I play Opera (3>0).

Therefor, psNE = {(Football, Football), (Opera, Opera)}

Slide 11.2

## Collusion & Cartels

Consider, - Firms make identical products
- Industry firms agree to coordinate their quantity & pricing decisions
- No firm deviates from decision

w/ Inverse Demand Function

$$P(Q) = 20 - Q$$

Marginal cost (MC)

$$MC = \$4 \quad \text{per input/output}$$

$\Rightarrow$ $MR = 20 - 2(Q)$

$\Rightarrow$ $\pi$ max conditions

$$MR = MC$$

$\Rightarrow$ $20 - 2Q = 4$

$\Rightarrow$ $16 = 2Q$

$\Rightarrow$ $Q^* = 8$

$\Rightarrow$ $P(Q^*) = 20 - 8 = \$12$

$\Rightarrow \pi^* = (12)(8) - (4)(8)$

$= (12-4)(8)$

$= (8)(8) = 64$

$\Rightarrow$ If firms work together to get monopoly profit, they will split output & $\pi$s to get

formula: $q_i^{*\,cartel} = Q^*/N \Rightarrow q_1^* = q_2^* = 8/2 = 4$ where $N = 2$

$$\pi^{cartel} = \frac{\pi^{monopol}}{N} \Rightarrow \pi_1^{cartel} = \pi_2^{cartel} = 64/2 = \$32$$

∴ Individual quantity produced is 4 units, & Individual profits gained are \$32.

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## Do firms have incentive to deviate?

As a firm, should I produce 5 units instead of 4?

$\Rightarrow \overset{*}{Q} = 4+5$

$\Rightarrow P(Q^*) = (20-(4+5)) = 20-9 = \$11$

$\Rightarrow \pi^{*monopoly} = ((11)-(4))(9) = (7)(9) = \$63$ profits

**BUT**, Profit from firm 1 $\Rightarrow (11-4)5 = 7*5 = \$35$

& Profit from firm 2 $\Rightarrow (11-4)4 = 7*4 = \$28$

$\Rightarrow$ If I deviate as a firm, I will get \$35 instead of \$32

$\Rightarrow \pi^{*Deviation} > \pi^{*Cartel}$

$\Rightarrow$ Firms have incentive to deviate (i.e. break cartel).

## Cournot Competition Between Saudi Arabia & Iran

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Note; $e$ = Iran

Consider,

- Two countries competing in oil quantities.
- Both w/ Marginal costs (MC) of $c = 20$
- Inverse Demand $P(Q) = (200 - 3(Q))$ where ① $Q = q_{SA} + q_e$ ② $a = 200$ ③ $b = 3$

$$\Rightarrow P(Q) = 200 - 3(q_{SA} + q_e)$$
$$= 200 - 3q_{SA} - 3q_e$$

$$\Rightarrow \pi_{SA} = \underbrace{(200 - 3(q_{SA} + q_e))}_{P(Q)} q_{SA} - 20 q_{SA}$$

$$\Rightarrow \pi_e = (200 - 3(q_{SA} + q_e)) q_e - 20 q_e$$

$\Rightarrow$ $MR_{SA}$: use same MR formula, but only change slope of SA

$$\Rightarrow MR_{SA} = (200 - \underbrace{3(2) q_{SA}} - 3q_e)$$
$$= (200 - 6q_{SA} - 3q_e)$$

Set $MR_{SA} = MC$ for $\pi$ max

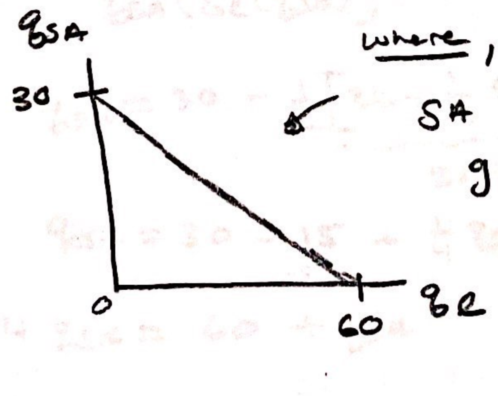
$$\Rightarrow MR_{SA} = MC$$

$$\Rightarrow 200 - 6q_{SA} - 3q_e = 20$$

Note that this is a function (ie $q_{SA}(q_e)$)

$$\Rightarrow \text{BRF}_{SA} = q_{SA} = \frac{180}{6} - \frac{3}{6} q_e = 30 - \frac{1}{2} q_e$$

plotting:




where, this function ($\text{BRF}_{SA}$) gives SA the optimal quantity to set given that Iran has set their quantity.

Same for Iran $\Rightarrow$

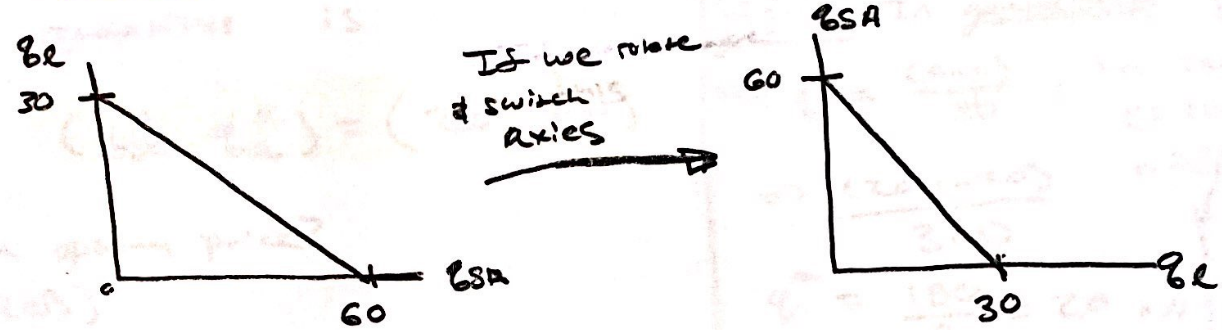
$\Rightarrow \quad MR_e = (200 - 3q_{SA} - 6q_e)$

Set $MR_e = MC$

$\Rightarrow \quad 200 - 3q_{SA} - 6q_e = 20$

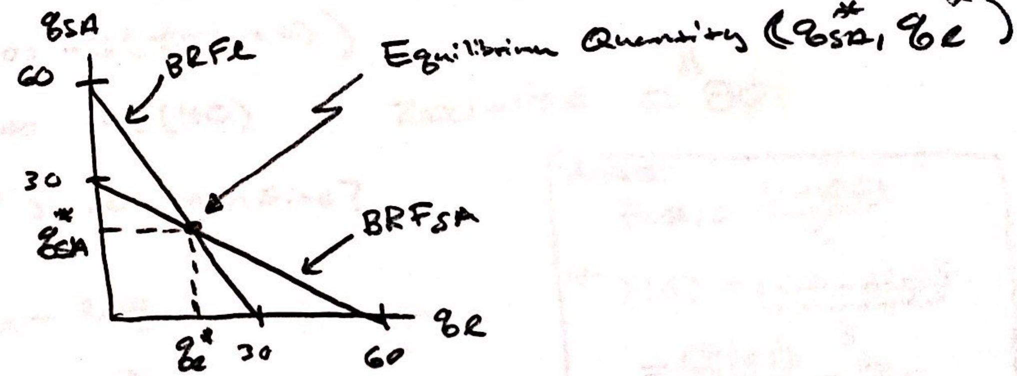
$\Rightarrow \quad BRF_e \equiv \quad q_e(q_{SA}) = 30 - \frac{1}{2}q_{SA}$

plotting




& If we lay this new plot on top of Saudi Arabia's plot

$\Rightarrow$




This means, we can use our BRFs to solve such that

$BRF_{SA}(BRF_e)$

$\Rightarrow \quad q_{SA}(q_e(q_{SA}))$ ⇐ Now just a function of $q_{SA}$

$\Rightarrow \quad q_{SA} = 30 - \frac{1}{2}\underbrace{\left[30 - \frac{1}{2}q_{SA}\right]}_{BRF_{SA}}$

$\Rightarrow \quad q_{SA} = 30 - 15 + \frac{1}{4}q_{SA}$

$\Rightarrow \quad 4q_{SA} = 60 + q_{SA} \quad \Rightarrow \quad q^*_{SA} = \frac{60}{3} = 20$

$\Rightarrow$

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Plugging $q_{SA}^*$ into BRFe

$\Rightarrow \quad q_e(q_{SA}^*) = 30 - \frac{1}{2}[20]$

$\Rightarrow \quad q_e^* = 30 - 10 = 20$ by symmetry

$\Rightarrow$ Optimal quantity to set when simultaneously setting quantities is

$(q_{SA}^*, q_e^*) = (20, 20)$

Note: In general form

$\Rightarrow q_i^* = \frac{(a-c)}{3b}$, if $a=200$, $c=20$, $b=3$

$\Rightarrow \frac{(200-20)}{3(3)}$

$q_i^* = \frac{180}{9} = 20$ .//

SAME!

What is the optimal price? (i.e. $P(Q^*)$)

$\Rightarrow \quad P(Q^*) = (200 - 3(q_{SA}^* + q_e^*))$

$= (200 - 3(20+20))$

$= 200 - 3(40) = 200 - 120 = \$80$

Note:

$P(Q) = \frac{(a+2c)}{3}$

$\Rightarrow P(Q) = \frac{(200+2(20))}{3}$

$= \frac{(240)}{3} = \$80$ .//

What is optimal $\pi$'s for both firms?

$\Rightarrow \pi_{SA}^* = P(Q^*)q_{SA}^* - 20q_{SA}^*$

$= (80-20)q_{SA}^*$

$= (60)(20) = \$1200$

$\Rightarrow \pi_e^* = P(Q^*)q_e^* - 20q_e^*$

$= (80-20)$

$= (60)(20) = {}^*1200$

Summarizing

The Cournot Equilibrium allocations are

$(q_{SA}^*, q_e^*) = (20, 20)$

$\Rightarrow$ Price & $\pi^*$'s are

$P(Q^*) = (\$80)$

$(\pi_{SA}^*, \pi_e^*) = (\$1200, \$1200)$ .//

$\Rightarrow Q^* = 40$, & $\pi = \$2{,}400$

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## Stackelberg Competition (First Mover Advantage)

- Assume Saudi Arabia chooses its optimal quantity first
- Iran's incentives stay the same, but now SA's is a bit different.

⇒ Knowing that they act first, & knowing how Iran will respond, they anticipate this an optimize off of this assumption.

$$\Rightarrow P(Q) = 200 - 3q_{SA} - 3(\underbrace{q_{I}(q_{SA})}_{BRF_I})$$

$$\Rightarrow P(Q) = 200 - 3q_{SA} - 3\left[30 - \tfrac{1}{2}q_{SA}\right]$$

$$= 200 - 3q_{SA} - 90 + \tfrac{3}{2}q_{SA}$$

$$P(Q) = 110 - \tfrac{3}{2}q_{SA} \quad \text{where } a = 110,\ b = 3/2$$

$$\Rightarrow MR_{SA} = 110 - (2)\tfrac{3}{2}q_{SA}$$

$$= 110 - 3q_{SA}$$

⇒ set $MR_{SA}$ = MC

$$\Rightarrow 110 - 3q_{SA} = 20$$

$$\Rightarrow 3q_{SA} = 90 \Rightarrow q_{SA}^{*} = 30 > q_{SA}^{Cournot}$$

Plug into $BRF_I$

$$\Rightarrow q_I^{*} = 30 - \tfrac{1}{2}(30) = 15$$

$$\Rightarrow (q_{SA}^{*}, q_I^{*}) = (30, 15)$$

$$\Rightarrow P(Q^{*}) = (200 - 3(45)) = 200 - 135 = \$65$$

$$\Rightarrow \pi_{SA}^{*} = (65)(30) - (20)(30)$$
$$= (65-20)(30) = 45(30) = \$1350$$

$$\Rightarrow \pi_I^{*} = (65-20)(15) = \$675$$

⇒

# Product Differentiation (Bertrand Competition)

⇒ Burton

Kz

$q_B = 900 - 2P_B + P_K$

$q_K = 900 - 2P_K + P_B$

$\boxed{\text{Ex}}$

⇒ $P_B(q_B, P_K) = 450 - \frac{1}{2}q_K + \frac{1}{2}P_K$

Notice the possitive relationship in price.

Assume, $MC = c = 0$

⇒ $MR_B = 900 - 4P_B + P_K$

⇒ $\pi$ max condition: $MR_B = MC$

BRF$_B$

⇒ $900 - 4P_B + P_K = 0$

⇒ $P_B = \frac{900}{4} + \frac{1}{4}P_K \quad = 225 + \frac{1}{4}P_K$

By Symmetry,

⇒ $\underbrace{P_K(P_B)}_{BRF_K} = 225 + \frac{1}{4}P_B$

Solving,

⇒ $P_B = 225 + \frac{1}{4}\left[225 + \frac{1}{4}P_B\right]$

multiply by 4 again

⇒ $4P_B = 900 + 225 + \frac{1}{4}P_B$

⇒ $16P_B = 3600 + 900 + P_B$

⇒ $P_B^* = \frac{4500}{15} = 300$

⇒ $P_K^* = 300$