Question #1 (Math)

1a) using equations (1) & (2) we get

$$BRF_i \equiv q_i(q_j) = \frac{(a-c_i)}{2b} - \frac{1}{2}q_j$$

&

$$BRF_j \equiv q_j(q_i) = \frac{(a-c_j)}{2b} - \frac{1}{2}q_i$$

combining

$$\Rightarrow \quad q_i = \frac{(a-c_i)}{2b} - \frac{1}{2}\left[\frac{(a-c_j)}{2b} - \frac{1}{2}q_i\right]$$

$$\Rightarrow \quad q_i = \frac{(a-c_i)}{2b} - \frac{(a-c_j)}{4b} + \frac{1}{4}q_i$$ multiply by 4b

$$\Rightarrow \quad 4bq_i = 2(a-c_i) - (a-c_j) + bq_i$$

$$\Rightarrow \quad 3bq_i = a - 2c_i + c_j$$

$$\Rightarrow \quad q_i^* = \frac{(a-2c_i+c_j)}{3b}$$ also known as $q_i^{Cournot}$

$$\Rightarrow \quad q_j^* = \frac{(a-c_j)}{2b} - \frac{1}{2}\left[\frac{(a-2c_i+c_j)}{3b}\right]$$

$$= \frac{(a-c_j)}{2b} - \frac{(a-2c_i+c_j)}{6b}$$

$$= \frac{3(a-c_j) - (a-2c_i+c_j)}{6b}$$

$$= \frac{2a-4c_j+2c_i}{6b} = \frac{2(a-2c_j+c_i)}{6b}$$

$$= \frac{(a-2c_j+c_i)}{3b} \quad //$$

$$\Rightarrow \quad (q_i^*, q_j^*) = \left(\frac{(a-2c_i+c_j)}{3b}, \frac{(a-2c_j+c_i)}{3b}\right) \quad //$$

aka $(q_i^{Cournot}, q_j^{Cournot})$

$\Rightarrow$

1b) Let $c_i = c_j = c$

$$\Rightarrow \quad (q_i^*, q_j^*) = \left(\frac{a-c}{3b}, \frac{a-c}{3b}\right)$$

1c)
$$P(Q^*) = \left(a - \cancel{b}\left(\frac{(a-c)}{3\cancel{b}} + \frac{(a-c)}{3\cancel{b}}\right)\right)$$

$$= \left(\frac{3a - 2(a-c)}{3}\right) = \frac{a+2c}{3} \quad //$$

1d)
$$\pi_i^* = \underbrace{\left(\left(\frac{a+2c}{3}\right) - c\right)}_{P(Q^*)-C}\underbrace{\frac{(a-c)}{3b}}_{q_i^*} - F$$

$$= \left(\frac{(a+2c-3c}{3}\right)\left(\frac{(a-c)}{3b}\right) - F$$

$$= \frac{(a-c)^2}{9b} - F$$

$\boxed{\text{Note, matches general form s.t. } \frac{(a-c)^2}{(N+1)^2 b} - F}$

where, $\pi_i^* = \pi_j^*$ due to symmetry

1e)
$$Q^* = \frac{(a-c)}{2b} \Rightarrow q_i^* = \frac{(a-c)}{4b} \equiv q^{cartel}$$

$$\Rightarrow \quad P(Q^*) = \frac{(a+c)}{3}$$

$$\Rightarrow \quad \pi^{cartel} = \frac{\pi^{Monopoly}}{N}$$

$$\Rightarrow \quad \pi^{Monopoly} = \frac{(a-c)^2}{4b} - F$$

$$\Rightarrow \quad \pi^{cartel} = \frac{(a-c)^2}{8b} - \frac{F}{2}$$

1f)
$$\pi_i^{Dev} = \left(a - b\left(\frac{(a-c)}{3b} + \frac{(a-c)}{4b}\right)\right)\frac{(a-c)}{3b} - c\frac{(a-c)}{3b} - F$$

$$\Rightarrow \quad \left(\frac{12(a-c)}{12} - \frac{7(a-c)}{12}\right)\frac{(a-c)}{3b} - F$$

$$\Rightarrow \quad \frac{5(a-c)^2}{36b} - F \equiv \pi_i^{Dev} \equiv \pi_j^{Dev}$$

$\Rightarrow$

1g)

$$\pi_i^{NDev} = \left(a - b\left(\frac{(a-c)}{3b} + \frac{(a-c)}{4b}\right)\right)\frac{(a-c)}{4b} - c\,\frac{(a-c)}{4b} - F$$

$$\Rightarrow \quad = \left(\frac{12(a-c)}{12} - \frac{7(a-c)}{12}\right)\frac{(a-c)}{4b} - F$$

$$= \frac{5(a-c)}{48b} - F \equiv \pi_i^{NDev} \equiv \pi_j^{NDev}$$

1h) <u>Summarizing</u>

$$\pi_i^{N} = \pi_i^{Cournot} = \frac{(a-c)^2}{9b} - F, \qquad \pi_i^{Cartel} = \frac{(a-c)^2}{8b} - \frac{F}{2}$$

$$\pi_i^{Dev} = \frac{5(a-c)^2}{36b} - F, \qquad \pi_i^{NDev} = \frac{5(a-c)^2}{48b} - F$$

Let $F = 0$ (ie. no fixed costs)

$$\Rightarrow \quad \frac{5(a-c)^2}{36b} > \frac{(a-c)^2}{8b} > \frac{(a-c)^2}{9b} > \frac{5(a-c)^2}{48b}$$

where we can multiply by $1 = \frac{5}{5}$ & cancel

$$\Rightarrow \quad \underbrace{\frac{(a-c)^2}{36b}}_{\pi_i^{Dev}} > \underbrace{\frac{(a-c)^2}{40b}}_{\pi_i^{Cartel}} > \underbrace{\frac{(a-c)^2}{45b}}_{\pi_i^{Cournot}} > \underbrace{\frac{(a-c)^2}{48b}}_{\pi_i^{NDev}}$$

1 h) (continued)

$\Rightarrow$

Firm j

Firm i	Cartel	Compete
Cartel	$\pi_i^{Cartel}, \pi_j^{Cartel}$	$\pi_i^{NDev}, \pi_j^{Dev}$
Compete	$\pi_i^{Dev}, \pi_j^{NDev}$	$\pi_i^{Cournot}, \pi_j^{Cournot}$

plugging in values

$\Rightarrow$

Firm j

Firm i	Cartel	Compete
Cartel	$\frac{(a-c)^2}{40b}, \frac{(a-c)^2}{40b}$	$\frac{(a-c)^2}{48b}, \frac{(a-c)^2}{36b}$
Compete	$\frac{(a-c)^2}{36b}, \frac{(a-c)^2}{48b}$	$\frac{(a-c)^2}{45b}, \frac{(a-c)^2}{45b}$

1 i)

where, doing our Best Response Analysis s.t.

Firm i: ① If Firm j plays Cartel, I play Compete ($\pi_i^{Dev} > \pi_i^{Cartel}$)

② If Firm j plays Compete, I play Compete ($\pi_i^{Cournot} > \pi_i^{NDev}$)

Firm j: ① If Firm i plays Cartel, I play Compete ($\pi_j^{Dev} > \pi_j^{Cartel}$)

② If Firm i plays Compete, I play Compete ($\pi_j^{Cournot} > \pi_j^{NDev}$)

$\Rightarrow$

Firm j

Firm i	Cartel	Compete
Cartel	$\pi_i^{Cartel}, \pi_j^{Cartel}$	$\pi_i^{NDev}, \underline{\pi_j^{Dev}}$
Compete	$\underline{\pi_i^{Dev}}, \pi_j^{NDev}$	$\underline{\pi_i^{Cournot}}, \underline{\pi_j^{Cournot}}$

← underlining all Best Responses

$\Rightarrow$ Nash Equilibrium $= (q_i^{*Cournot}, q_j^{*Cournot}) = \left\{\left(\frac{a-c}{3b}, \frac{a-c}{3b}\right)\right\}$

1j)

$$GTS = \delta^0(\pi^{Dev}) + \delta^1(\pi^{Cournot}) + \delta^2(\pi^{Cournot}) + \delta^3(\pi^{Cournot}) + \dots$$

$$\Rightarrow \quad = (\pi^{Dev}) + \delta\pi^{Cournot}(1 + \delta + \delta^2 + \delta^3 + \dots)$$

$$= \pi^{Dev} + \delta\pi^{Cournot}\underbrace{\left(\sum_{k=0}^{\infty}\delta^k\right)}$$

Geometric series where $\sum_{k=0}^{\infty}\delta^k = \frac{1}{(1-\delta)}$

$$= \pi^{Dev} + \frac{\delta}{(1-\delta)}\pi^{Cournot}$$

//

1k)

$$\text{Cartel Payoffs} = \delta^0\pi^{Cartel} + \delta^1\pi^{Cartel} + \delta^2\pi^{Cartel} + \delta^3\pi^{Cartel} + \dots$$

$$\Rightarrow \quad = \pi^{Cartel}\underbrace{(1 + \delta + \delta^2 + \delta^3 + \dots)}_{\text{Geometric Series}}$$

$$\Rightarrow \quad = \pi^{Cartel}\frac{1}{(1-\delta)}$$

1l) The Cartel can be sustained iff

$$\text{Cartel Payoffs} > GTS$$

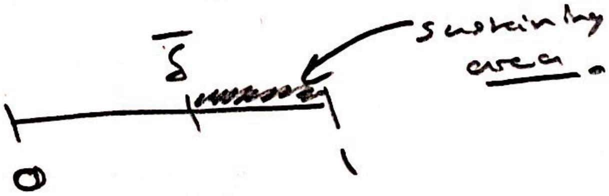
$$\Rightarrow \quad \pi^{Cartel}\frac{1}{(1-\delta)} > \pi^{Dev} + \frac{\delta}{(1-\delta)}\pi^{Cournot}$$

$$\Rightarrow \quad \pi^{Cartel} > \pi^{Dev}(1-\delta) + \delta\pi^{Cournot}$$

$$\Rightarrow \quad \delta(\pi^{Dev} - \pi^{Cournot}) > \pi^{Dev} - \pi^{Cartel}$$

$$\Rightarrow \quad \delta > \frac{\pi^{Dev} - \pi^{Cartel}}{\pi^{Dev} - \pi^{Cournot}} \equiv \bar{\delta}$$

Where is $\delta > \bar{\delta}$ Cartel Agreement will be sustained



$\Rightarrow$

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1m)

$$\delta > \frac{\frac{5(a-c)^2}{36b} - \frac{(a-c)^2}{8b}}{\frac{5(a-c)^2}{36b} - \frac{(a-c)^2}{9b}}$$

$$\Rightarrow \quad \delta > \frac{\frac{5}{36} - \frac{1}{8}}{\frac{5}{36} - \frac{1}{9}} = \frac{1/72}{1/36} = \frac{1/\cancel{72}}{2/\cancel{72}}$$

$$\Rightarrow \quad \delta > \bar{\delta} = \frac{1}{2}$$

$\Rightarrow$ If $\delta > 1/2$, then collusion will be sustained. //

Question #2 (Math)

2a) using equations (3) $ (4)

$$\Rightarrow \quad BRF_i = P_i(P_j) = \frac{(a+bc)}{2b} + \frac{1}{2b}P_j$$

$

$$BRF_j = P_j(P_i) = \frac{(a+bc)}{2b} + \frac{1}{2b}P_i$$

$\Rightarrow$ ① $a\uparrow \Rightarrow P_i \uparrow$

② $b\uparrow \Rightarrow P_i \downarrow$ since $\frac{a}{2b} + \frac{1}{2}c + \frac{1}{2b}P_j$ $\Rightarrow$ If $b\uparrow$ $P_i\downarrow$

③ $c\uparrow \Rightarrow P_i \uparrow$

④ $P_j\uparrow \Rightarrow P_i \uparrow$

2b) Solving for P_i^* first

$$\Rightarrow \quad P_i = \frac{(a+cb)}{2b} + \frac{1}{2b}\left[\frac{(a+bc)}{2b} + \frac{1}{2b}P_i\right]$$

$$\Rightarrow \quad P_i = \frac{(a+cb)}{2b} + \frac{(a+bc)}{4b^2} + \frac{P_i}{4b^2}$$ multiply by $4b^2$

$$\Rightarrow \quad 4b^2 P_i = 2b(a+cb) + (a+cb) + P_i$$

$$\Rightarrow \quad P_i(4b^2-1) = (2b+1)(a+cb)$$

$$\Rightarrow \quad P_i^* = \frac{(2b+1)(a+cb)}{\underbrace{(4b^2-1)}_{\text{Diff of squares} = ((2b)^2 - (1)^2)}} = \frac{\cancel{(2b+1)}(a+cb)}{\cancel{(2b+1)}(2b-1)} = \frac{(a+cb)}{(2b-1)}$$

$$\Rightarrow \quad P_j^* = \frac{(a+bc)}{2b} + \frac{1}{2b}\left(\frac{a+cb}{2b-1}\right)$$

$$= \frac{(a+bc)(2b-1) + (a+cb)}{2b(2b-1)}$$

$$= \frac{(2b-\cancel{1}+\cancel{1})(a+bc)}{2b(2b-1)} = \frac{\cancel{2b}}{\cancel{2b}}\frac{(a+bc)}{(2b-1)}$$

$$\Rightarrow \quad (P_i^*, P_j^*) = \left(\frac{(a+cb)}{(2b-1)}, \frac{(a+cb)}{(2b-1)}\right)$$ ← prices are still the same :)

2c) $$q_i^* = a - b\left(\frac{(a+cb)}{(2b-1)}\right) + \left(\frac{(a+cb)}{(2b-1)}\right)$$

$$\Rightarrow \quad q_i^* = \left(\frac{a(2b-1)}{(2b-1)} - \frac{b(a+cb)}{(2b-1)} + \frac{(a+cb)}{(2b-1)}\right)$$

$$= \left(\frac{a(2b-1) + (1-b)(a+cb)}{(2b-1)}\right)$$

$$= \left(\frac{\cancel{2}ab - \cancel{a} + \cancel{a} + cb - \cancel{ab} - cb^2}{(2b-1)}\right)$$

or

$$q_i^* = \frac{(ab + cb - cb^2)}{(2b-1)} = \frac{b(a - cb + c)}{(2b-1)} = \frac{b(a - c(b-1))}{(2b-1)}$$

Through symmetry, we know $q_i^* = q_j^*$

$$\Rightarrow \quad q_j^* = \frac{b(a + c(1-b))}{(2b-1)}$$

when if $b = 1$

$\Rightarrow q_i^* = a$

where, this is actually a corner solution where $p_i^* = c$

2d)

$$\Rightarrow \pi_i^* = \left(p_i^* - c\right) q_i^*$$

$$= \left(\frac{(a+bc)}{(2b-1)} - c\right)\frac{b(a - c(b-1))}{(2b-1)}$$

$$= \left(\frac{a + bc - c(2b-1)}{(2b-1)}\right)\frac{b(a - c(b-1))}{(2b-1)}$$

$$= \left(\frac{a - cb + c}{(2b-1)}\right)\frac{b(a - c(b-1))}{(2b-1)} = \frac{b(a - c(b-1))^2}{(2b-1)^2}$$

$$\Rightarrow \pi_j^* = \frac{b(a - c(b-1))^2}{(2b-1)^2}$$ by symmetry. //

3a) using equations (5) & (6) Question #3 (math)

$\Rightarrow$

$$BRF_i \equiv q_i(q_j) = \frac{(a-c)}{2b} - \frac{d}{2b} q_j$$

$$\Rightarrow BRF_j \equiv q_j(q_i) = \frac{(a-c)}{2b} - \frac{d}{2b} q_i$$

$\Rightarrow$

$a\uparrow \Rightarrow q_i \uparrow$

$b\uparrow \Rightarrow q_i \uparrow$ & $q_j \uparrow \Rightarrow q_i \downarrow$

$c\uparrow \Rightarrow q_i \downarrow$

$d\uparrow \Rightarrow q_i \downarrow$

3b) starting w/ q_i

$$\Rightarrow \quad q_i = \frac{(a-c)}{2b} - \frac{d}{2b}\left[\frac{(a-c)}{2b} - \frac{d}{2b} q_i\right]$$

$$\Rightarrow \quad q_i = \frac{(a-c)}{2b} - \frac{d(a-c)}{4b^2} + \frac{d^2}{4b^2} q_i$$

$$\Rightarrow \quad 4b^2 q_i = 2b(a-c) - d(a-c) + d^2 q_i$$

$$\Rightarrow \quad q_i(4b^2 - d^2) = (2b-d)(a-c)$$

$$\Rightarrow \quad q_i^* = \frac{(2b-d)(a-c)}{\underbrace{(4b^2-d^2)}_{\text{diff of squares}}} = \frac{(2b-d)(a-c)}{(2b-d)(2b+d)} = \frac{(a-c)}{(2b+d)}$$

$$\Rightarrow \quad q_j^* = \frac{(a-c)}{2b} - \frac{d}{2b}\left[\frac{(a-c)}{(2b+d)}\right]$$

$$= \frac{(a-c)(2b+d)}{2b(2b+d)} - \frac{d(a-c)}{2b(2b+d)}$$

$$= \frac{(a-c)(\cancel{2b})}{\cancel{2b}(2b+d)} = \frac{(a-c)}{(2b+d)}$$

Intuition	
$d \to b \Rightarrow q_i^* \to q_i$	Cournot
$d \to 0 \Rightarrow q_i^* \to q_i$	Monopoly

$$\Rightarrow \quad (q_i^*, q_j^*) = \left(\frac{(a-c)}{(2b+d)}, \frac{(a-c)}{(2b+d)}\right)$$

$\Rightarrow$ as products become more differentiated, they produce ∇ less $\Rightarrow$

3c) $P_i^* = a - b\left(\frac{(a-c)}{(2b+d)}\right) - d\left(\frac{(a-c)}{(2b+d)}\right)$

$$= \left(\frac{a(2b+d) - (b+d)(a-c)}{(2b+d)}\right)$$

$$\Rightarrow P_j^* = \left(\frac{a(2b+d) - (b+d)(a-c)}{(2b+d)}\right) \quad \text{by symmetry}$$

3d) $$\pi_i^* = \left(\frac{a(2b+d) - (b+d)(a-c)}{(2b+d)} - c\right)\left(\frac{(a-c)}{(2b+d)}\right)$$

$$\Rightarrow \quad = \left(\frac{(a-c)(2b+d) - (b+d)(a-c)}{(2b+d)}\right)\frac{(a-c)}{(2b+d)}$$

$$= \left(\frac{(a-c)(2b+\cancel{d}-b-\cancel{d})}{(2b+d)}\right)\frac{(a-c)}{(2b+d)}$$

$$\pi_i^* = \frac{b(a-c)}{(2b+d)}\frac{(a-c)}{(2b+d)} = \frac{b(a-c)^2}{(2b+d)^2}$$

$$\Rightarrow (\pi_i^*, \pi_j^*) = \left(\frac{b(a-c)^2}{(2b+d)^2}, \frac{b(a-c)^2}{(2b+d)^2}\right) \quad \text{by } \underline{\text{symmetry}}$$

3e) Let $d \to b$ s.t. $d = b$

$$\Rightarrow \pi_i^* = \frac{b(a-c)^2}{(2b+(b))^2}$$ plugging in

$$\Rightarrow \quad = \frac{b(a-c)^2}{(3b)^2} = \frac{\cancel{b}(a-c)^2}{9b^{\cancel{2}}} = \frac{(a-c)^2}{9b} \equiv \pi_i^{\underline{\text{Cournot}}}$$ //

3f) $\Rightarrow$ Let $d=0$

$$\Rightarrow \pi_i^* = \frac{b(a-c)^2}{(2b+(0))^2} = \frac{b(a-c)^2}{(2b)^2} = \frac{(a-c)^2}{4b} \equiv \pi_i^{\underline{\text{Monopoly}}}$$ //

Question #4 (Math)

4a) See key

4b) using (7) & (8)

$$\Rightarrow \frac{2P_A}{2t} = \frac{t+c}{2t} + \frac{1}{2t}P_B \qquad (7)$$

$$\Rightarrow BRF_A \equiv P_A(P_B) = \frac{t+c}{2} + \frac{1}{2}P_B$$

Combining

$$\Rightarrow \frac{2P_B}{2t} = \frac{t+c}{2t} + \frac{1}{2t}P_B \qquad (8)$$

$$\Rightarrow BRF_B \equiv P_B(P_A) = \frac{t+c}{2} + \frac{1}{2}P_A$$

$$\Rightarrow P_A = \frac{t+c}{2} + \frac{1}{2}\left[\frac{t+c}{2} + \frac{1}{2}P_A\right]$$

$$\Rightarrow P_A = \frac{t+c}{2} + \frac{(t+c)}{4} + \frac{1}{4}P_A$$

$$\Rightarrow 4P_A = 2(t+c) + (t+c) + P_A$$

$$\Rightarrow P_A^* = \frac{3(t+c)}{3} = t+c$$

Same (P_A^*, P_B^*)

$$\Rightarrow P_B^* = \frac{(t+c)}{2} + \frac{1}{2}[t+c] = t+c$$

Note, If $t=0$, $\Rightarrow$ we get $P_A^* = P_B^* = c$

Which is analagous to the bertrand model w/ transportation.

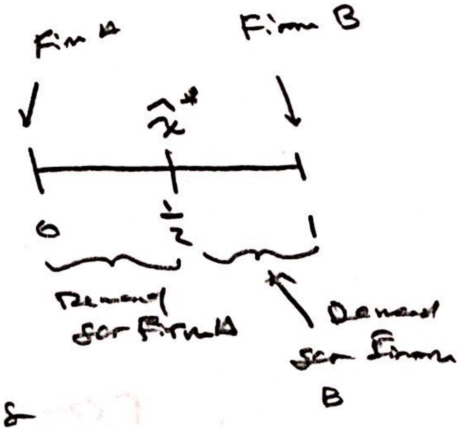
4c) To find $\hat{x}^* = Q_A^*(p_A^*, p_B^*)$ we plug in p_A^* & p_B^*

$$\Rightarrow \hat{x}^* = \left(\frac{p_B^* - p_A^*}{2t} + \frac{1}{2}\right)$$

$$= \left(\frac{(t+c) - (t+c)}{2t} + \frac{1}{2}\right)$$

$$= \left(\frac{0}{2t} + \frac{1}{2}\right) = \frac{1}{2}$$

$\Rightarrow$



$$\Rightarrow (1 - \hat{x}^*) = \left(1 - \frac{1}{2}\right) = \frac{1}{2}$$

$\Rightarrow$ They will split the $\underbrace{\text{consumers}}_{\text{(i.e. total demand)}}$ in half.

4d) $\pi_A^* = (p_A^* - c)\hat{x}^* = (t + c - c)\left(\frac{1}{2}\right)$

$$= \frac{t}{2}$$

Similarly

$$\pi_B^* = (p_B^* - c)(1 - \hat{x}^*) = (t + c - c)\left(\frac{1}{2}\right)$$

$$= \frac{t}{2}$$

$$\Rightarrow (\pi_A^*, \pi_B^*) = \left(\frac{t}{2}, \frac{t}{2}\right)$$

@ $t = 0$, we get $\pi_i^* = 0$, which is the same as the Bertrand competition w/o product differentiation outcome. //

5A) using (9),(10), & (11) Question #5 (Math)

$$\Rightarrow q_1^* = \frac{(a-c)}{2b} \qquad (11)$$

$$\Rightarrow BRF_2 \equiv q_2^*(q_1^*) = \frac{(a-c)}{2b} - \frac{1}{2}\left[\frac{(a-c)}{2b}\right]$$

$$= \frac{(a-c)}{2b} - \frac{(a-c)}{4b}$$

$$= \frac{2(a-c) - (a-c)}{4b}$$

$$= \frac{(a-c)}{4b}$$

$$\Rightarrow BRF_3 \equiv q_3^*(q_1^*, q_2^*) = \frac{(a-c)}{2b} - \frac{1}{2}\left[\frac{(a-c)}{2b} + \frac{(a-c)}{4b}\right]$$

$$= \frac{(a-c)}{2b} - \frac{1}{2}\left[\frac{3(a-c)}{4b}\right]$$

$$= \frac{(a-c)}{2b} - \frac{3(a-c)}{8b}$$

$$= \frac{4(a-c) - 3(a-c)}{8b} = \frac{(a-c)}{8b}$$

$$\Rightarrow (q_1^*, q_2^*, q_3^*) = \left(\frac{(a-c)}{2b}, \frac{(a-c)}{4b}, \frac{(a-c)}{8b}\right)$$

~~where~~, Quantities are decreasing due to the effect of the first movers advantage.

5b) $$p(Q^*) = \left(a - b\left(\frac{(a-c)}{2b} + \frac{(a-c)}{4b} + \frac{(a-c)}{8b}\right)\right)$$

$$= \left(a - \frac{7(a-c)}{8}\right) = \frac{a+7c}{8}$$

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5c)

$$\pi_1^* = \left(\frac{(a-c)}{4} - \frac{\cancel{b}}{4}\frac{(a-c)}{2\cancel{b}}\right)\frac{(a-c)}{2b}$$

$$= \frac{(a-c)}{8}\,\frac{(a-c)}{2b} = \frac{(a-c)^2}{16b}$$

$$\pi_2^* = \left[\frac{(a-c)}{2} - \frac{\cancel{b}}{2}\left(\frac{(a-c)}{2\cancel{b}} + \frac{(a-c)}{4\cancel{b}}\right)\right]\frac{(a-c)}{4b}$$

$$= \left[\frac{(a-c)}{2} - \frac{3(a-c)}{8}\right]\frac{(a-c)}{4b}$$

$$= \frac{(a-c)}{8}\,\frac{(a-c)}{4b} = \frac{(a-c)^2}{32b}$$

$$\Rightarrow \pi_3^* = \left[(a-c) - \cancel{b}\left(\frac{(a-c)}{2\cancel{b}} + \frac{(a-c)}{4\cancel{b}} + \frac{(a-c)}{8\cancel{b}}\right)\right]\frac{(a-c)}{8b}$$

$$= \left[(a-c) - \frac{7(a-c)}{8}\right]\frac{(a-c)}{8b}$$

$$= \frac{(a-c)}{8}\,\frac{(a-c)}{8b} = \frac{(a-c)^2}{64b}$$

$$\Rightarrow \quad \pi_1^* > \pi_2^* > \pi_3^*$$

$$\Rightarrow \quad \frac{(a-c)^2}{16b} > \frac{(a-c)^2}{32b} > \frac{(a-c)^2}{64b}$$ //