Def Infinite Discounting

$$\Rightarrow \sum_{t=0}^{\infty} \beta^t = \beta^0 + \beta^1 + \beta^2 + \beta^3 + \cdots$$

$$\Rightarrow \sum_{t=0}^{\infty} \beta^t = \frac{1}{(1-\beta)}$$ by geometric sequence

$\beta \in [0,1]$

Def Compounded Interest

$$\Rightarrow A_0 (1+r)^t = A_0 \underbrace{\underbrace{(1+r)}_{t=1} (1+r)}_{t=2} (1+r)(1+r)\ldots$$

today principal

$$\Rightarrow V_t = A_0 (1+r)^t$$

where, Given that you take out a loan of A_0, V_t is the amount you will have to pay back at time (t) with an interest rate (r).

Or Given you lend out an initial amount A_0, V_t is your total return, including A_0, that you will get back

Def Present Discounted Value

$$\Rightarrow V_t = A_0 (1+r)^t$$

$$\Rightarrow PDV \equiv A_0 = \frac{V_t}{(1+r)^t}$$ which is how much you would have to pay today to get rid of the loan you took out for t periods at interest rate (r).

Or How much you would need to be paid right now to make you indifferent between the money you will gain in the future.

Present Discounted Value of Payment Streams

Given that we are to make/receive payments over time we get

$$PDV = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} m$$

$$\Rightarrow \quad PDV = \frac{1}{(1+r)^0} m + \frac{1}{(1+r)^1} m + \frac{1}{(1+r)^2} m + \cdots$$

$$PDV = m + \frac{1}{(1+r)} m + \frac{1}{(1+r)^2} m + \frac{1}{(1+r)^3} + \cdots$$

$$= m\left(\sum_{t=0}^{\infty} \frac{1}{(1+r)^t}\right)$$

$$= m \sum_{t=0}^{\infty} \left(\frac{1}{(1+r)}\right)^t$$

$$= m\left(\frac{1}{(1-(1+r))}\right) = \frac{m}{r} \quad \text{//}$$

By Geometric Sequence

=> Given that you have to make a stream of payoffs on a loan, the PDV is how much you would have to pay right now to buy out of a loan in which you are contracted into for t periods at interest rate r.

or How much you would have to get bought out at.

Net Present Value (Infinite)

$$PDV - \text{Investment} = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} m - C$$

Initial cost of Investment

$$= \frac{m}{r} - C$$

=>

Net Present Value w/ Different Cost & Benefits @ every period

$$\Rightarrow \quad NPV = (B_0 - C_0) + \frac{(B_1 - C_1)}{(1+r)} + \frac{(B_2 - C_2)}{(1+r)^2} + \dots$$

$$\Rightarrow \quad = \sum_{t=0}^{\infty} \frac{(B_t - C_t)}{(1+r)^t} \quad \text{(Infinite period)}$$

$$= \sum_{t=0}^{T} \frac{(B_t - C_t)}{(1+r)^t} \quad \text{(Finite period)}$$

Nominal vs. Real Interest Rates

Nominal: Rate quoted on the market. It is not relative to anything. We do not know the purchasing power of these values.

Real Interest Rate: Rate of return in terms of purchasing power. Also called inflation-adjusted interest rate.

Typically,

$$\underbrace{r}_{\text{Real}} \simeq \underbrace{i}_{\text{Nominal}} - \underbrace{\pi}_{\text{Inflation Rate}}$$

$\boxed{\text{Def}}$ Expected Value (Discrete Probability structure) — Slide 14.4 (1/2)

$\Rightarrow$ Let X be a RV s.t. $X \in \{x_1, x_2, \dots, x_n\}$

$\Rightarrow \quad E(x) = \sum_{i=1}^{n} p_i x_i$ where, ① p_i is the probability of event x_i happening.

② $\sum_{i=1}^{n} p_i = 1$

In the notes

$$E(M) = p_1 M_1 + p_2 M_2 + \dots + p_N M_N$$

$$= \sum_{i=1}^{N} p_i M_i$$

which will give you a pay out (M).

Expected Income, Expected Utility, & Risk Premium

Slide 14.5 (1/11)

- Risk neutral is when payoff function is constant.
- We usually assume economic agents are risk adverse.

↳ Comes from declining marginal utility.

$\Rightarrow$ utility function is increasing & a decreasing rate.

Examples: ① Cobb-Douglas where $0 < \alpha + \beta \leq 1$

② $u(x,y) = \sqrt{x}\sqrt{y} = \sqrt{xy} = x^{1/2}y^{1/2}$

③ CES utility

④ $\log(xy) = \log(x) + \log(y)$

Adam Example

Let $u(I) = \sqrt{I}$

$\Rightarrow E(u) = \frac{1}{2}\sqrt{36} + \frac{1}{2}\sqrt{100}$

$= \frac{1}{2}6 + \frac{1}{2}10 = 3+5 = 8$

Calculating the risk into total utility

$\Rightarrow E(I) = \frac{1}{2}36 + \frac{1}{2}100 = 68$

Calculating risk into only income.

plugging in 68 into the utility

$\Rightarrow Eu(68) = \sqrt{68} = 8.25$

$\Rightarrow$ What is the Income in which Adam needs to gain the same utility when including the risk into his decision?

$\Rightarrow 8 = \sqrt{x}$

$\Rightarrow 8^2 = x = 64$

CE

Now, to make him indifferent $\Rightarrow 68 - 64 = 4$