Recall, For $Q = f(K,L) = AK^{\alpha}L^{(1-\alpha)}$

$$\Rightarrow \quad MRTS_{K,L} = \frac{\frac{\partial f}{\partial K}}{\frac{\partial f}{\partial L}} = \frac{A\alpha K^{\alpha-1}L^{(1-\alpha)}}{A(1-\alpha)K^{\alpha}L^{-\alpha}} = \left.\frac{P_K}{P_L}\right\} \text{Price Ratio of Inputs}$$

Re-writing

$$\Rightarrow \quad \frac{\cancel{A}\alpha K^{\alpha-1}L^{(1-\cancel{\alpha})}}{\cancel{A}(1-\alpha)\cancel{K^{\alpha}}\cancel{L^{-\alpha}}} = \frac{P_K}{P_L}$$

$$\Rightarrow \quad \frac{\alpha L}{(1-\alpha)\underbrace{K}_{\text{Input Mix!}}} = \frac{P_K}{P_L} \leftarrow \text{Also, slope of constraint}$$

If we solve for K^* & L^*

$\Rightarrow$ Optimal Input mix

$\Rightarrow$ Labor (L)

Slide 13.1 (2/12)

Recall,

$$MP_L = \frac{\partial f}{\partial L} = \frac{\Delta f}{\Delta L} = \frac{\Delta Q}{\Delta L} = A(1-\alpha)K^{\alpha}L^{-\alpha} \leftarrow \text{In Cobb-Douglas Example.}$$

$$MR = \frac{\partial TR}{\partial Q} = \frac{\Delta TR}{\Delta Q} = P \text{ (Price)}$$

since, $TR = PQ$

$$\Rightarrow \quad MRP_L = \underbrace{P}_{MR}\underbrace{A(1-\alpha)K^{\alpha}L^{-\alpha}}_{} \leftarrow \text{Total Benefit (i.e. Revenue) of adding one worker.}$$

$= MR * MP_L$

Note: In Homework 2 we got conditions of

$$MP_L \equiv A(1-\alpha)K^{\alpha}L^{-\alpha} = \underbrace{P_L}_{W \text{ in notes}}$$

$$\Rightarrow \quad \underline{MRP_L \equiv PA(1-\alpha)K^{\alpha}L^{-\alpha} = P_L}$$

$\Rightarrow$ Optimal amount of labor is employed!

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Slide 13.2 (7/10)

A Simple Labor – Leisure Model

✓ Notice: C & L are choice variables

– Cobb-Douglas utility

$$u(C,L) = C^{\alpha} L^{1-\alpha}$$

– w/ Budget Constraint

$$C = wh$$

&

$$T = h + L$$

$$\Rightarrow \quad C = w(T-L)$$

$$\Rightarrow \quad C = \underbrace{wT}_{\text{constant}} - wL$$

where,

L = # of Leisure hours

C = consumption goods in dollars

w = wage rate

h = hours worked

T = Total hours available to work (for example, in 1 day $\Rightarrow T = 24$)

$\Rightarrow$ @ 24 hrs total to work (i.e. $T=24$)

$$\Rightarrow \quad C = \underbrace{w(24)}_{\text{constant}} - \underbrace{w}_{\text{slope}}L$$

In form of $y = mx + b$

$\Rightarrow$ ① @ $L=0$

$$C = w24$$

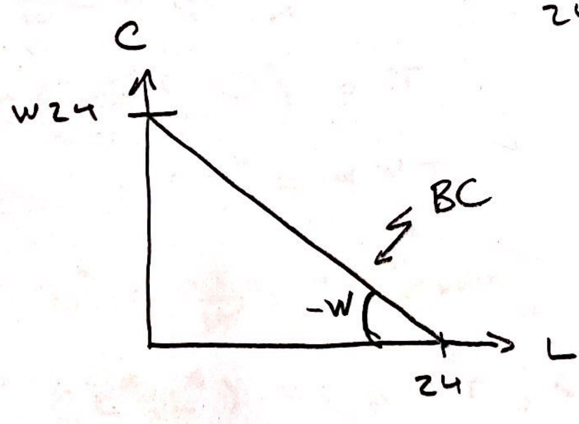
② @ $C=0$

$$0 = 24w - wL$$

$$\Rightarrow 24\cancel{w} = \cancel{w}L$$

$$24 = L$$

$\Rightarrow$



$\Rightarrow$ UMP is

$$\max_{C,L\geq 0} u(C,L) = C^{\alpha}L^{1-\alpha} \qquad \text{where}, \quad \alpha+\beta=1$$

$$\text{s.t.} \quad C = WT - WL \quad \text{or} \quad \frac{1}{W}C + L = T$$

$\Rightarrow$

$$\mathcal{L}: C^{\alpha}L^{1-\alpha} - \lambda\left[\tfrac{1}{W}C + L - T\right]$$

$$\left.\begin{aligned}\frac{\partial\mathcal{L}}{\partial C} &= \alpha C^{\alpha-1}L^{1-\alpha} - \lambda\frac{1}{W} = 0 \\ \frac{\partial\mathcal{L}}{\partial L} &= (1-\alpha)C^{\alpha}L^{-\alpha} - \lambda = 0\end{aligned}\right\} \quad \text{Combining (1) \& (2)}$$

$$\frac{\partial\mathcal{L}}{\partial \lambda} = \frac{1}{W}C + L - T = 0 \qquad (3)$$

$$\text{MRS} \equiv \frac{\alpha C^{\alpha-1}L^{1-\cancel{\alpha}}}{(1-\alpha)\cancel{C^{\alpha}}\cancel{L^{-\alpha}}} = \frac{\lambda\frac{1}{W}}{\lambda(1)}$$

$\leftarrow$ price ratio of consumption & Leisure ($1/w$)

$\Rightarrow$

$$\frac{\alpha}{(1-\alpha)}\frac{L}{C} = \frac{1}{W}$$

$$\Rightarrow L = \frac{C}{W}\frac{(1-\alpha)}{\alpha} \qquad (*)$$

Plug into (3)

$$\Rightarrow \frac{1}{W}C + \frac{C}{W}\frac{(1-\alpha)}{\alpha} = T$$

$$\Rightarrow C\left(\frac{\cancel{\alpha}+(1-\cancel{\alpha})}{\alpha W}\right) = T$$

$$C^* = \alpha W T$$

plug into (*)

$$L^* = \frac{1}{\cancel{W}}\frac{(1-\alpha)}{\cancel{\alpha}}\left[\cancel{\alpha}\cancel{W}T\right] = (1-\alpha)T$$

$$\Rightarrow (C^*, L^*) = (\alpha W T, (1-\alpha)T)$$