Question #1 (Math)

1A)

Let,

$$Q^D = Q^S$$

$$\Rightarrow \quad 22 - 2P = 3P - 23$$

$$5P = 45$$

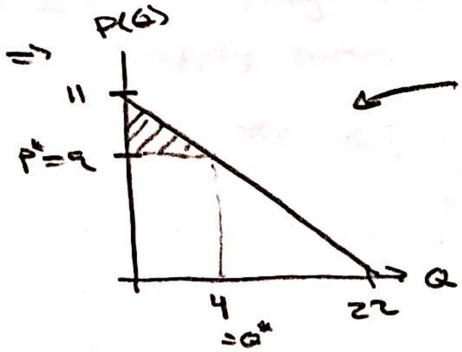
$$\Rightarrow \quad P^* = 45/5 = \$9$$

plugging back into $Q^D \Rightarrow Q^D = 22 - 2(9) = 22 - 18 = 4$

$$\Rightarrow Q^{D*} = 4 \text{ lbs.}$$

1b) Since we have linear demand & supply curves, we can calculate CS using the area of a triangle.

$\Rightarrow$



① $Q^D = 0$

$\Rightarrow 0 = 22 - 2P$

$\Rightarrow P = 11$

② $P = 0$

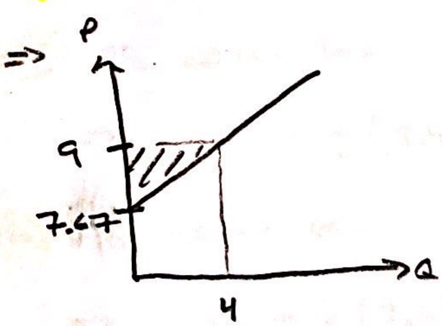
$\Rightarrow Q^D = 22$

$$\Rightarrow CS = \frac{1}{2} bh$$

$$\Rightarrow \quad = \frac{1}{2}(4)(\$11 - \$9) = \$4$$

1c) PS, similarly using the supply curve

$\Rightarrow$



①

$Q^S = 0$

$\Rightarrow 0 = 3P - 23$

$\Rightarrow P = 23/3$

≈ 7.67

② Don't need

$$\Rightarrow PS = \frac{1}{2} bh$$

$$= \frac{1}{2} 4(\$9 - \$7.67) \approx \$2.66$$

1d) Total Social Welfare

$$\Rightarrow \quad SW = CS + PS \quad \Rightarrow \quad \$4 + \$2.66 = \$6.66$$

1e) Let $P_b = (P_s + .50)$

$\Rightarrow$ $22 - 2(P_s + .50) = 3P_s - 23$

$22 - 2P_s - 1 = 3P_s - 23$

$\Rightarrow$ $21 - 2P_s = 3P_s - 23$

$44 = 5P_s$

$\Rightarrow P_s^* = 44/5 = \$8.80$

$\therefore$ The buyers face

$P_b^* = 8.80 + .5 = \$9.30$

1f) If we plug the buyers price into demand & sellers price into supply curve we get the same quantity

$\Rightarrow Q_2^D = 22 - 2P_b^* \quad \& \quad Q_2^S = 3(P_s^*) - 23$

$= 22 - 2(9.30) \quad\quad = 3(8.80) - 23$

$= 3.4 \quad\quad = 3.4$

same

$\Rightarrow$ 3.4 lbs of rice will be sold after tax.

1g) CS after tax

choke

$\Rightarrow CS = \frac{1}{2}(3.4)(\$11 - \$9.30) = \frac{1}{2}(3.4)(\$1.7)$

$\approx \$2.89$

1h) PS after tax

choke

$\Rightarrow PS = \frac{1}{2}(3.4)(8.80 - 7.67) = \frac{1}{2}(3.4)(\$1.13)$

$\approx \$1.921$

1i) $SW^{Tax} = \$2.89 + \$1.921$

$\approx \$4.81$

$\Rightarrow \$6.66 - 4.81 = \$1.85$ lost surplus from tax

$\Rightarrow$

1j) Government Revenue

$\Rightarrow \quad .50(3.4) = \$1.7$

1k) Dead weigh loss (DWL)

$\Rightarrow \quad 1.85 - 1.7 = \$.15$ as a dead weight loss from tax.

NOTE:
A Different way to calculate DWL is:

$$DWL = \frac{1}{2}(Q_1 - Q_2) * (P_D - P_S)$$
$$= \frac{1}{2}(Q_1 - Q_2) * (tax)$$
$$= \frac{1}{2}(4 - 3.4)(.50) = .15 \quad //$$

$\Rightarrow$ For every $1 increase in DWL, we get a tax return (i.e. revenue) of $11.33

$$\Rightarrow \quad \frac{1.7}{.15} \approx \$11.33$$

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Question #2 (Math)

2a) using equation 1 & solving for Q

$\Rightarrow \quad a - 2bQ - c = 0$

$\Rightarrow \quad 2bQ = a - c$

$\Rightarrow \quad Q^* = \frac{(a-c)}{2b}$

2b) plugging in Q^* into $P(Q)$

$\Rightarrow \quad P(Q^*) = (a - bQ^*)$

$$= a - \cancel{b}\left(\frac{(a-c)}{2\cancel{b}}\right)$$

$$= a - \frac{(a-c)}{2}$$

$$= \frac{2a - a + c}{2} = \frac{a+c}{2}$$

2c) plugging Q^* into π

$\Rightarrow \quad \pi^* = P(Q^*)Q^* - cQ^* - F$

$$\Rightarrow \quad = \left(\frac{(a+c)}{2}\right)\left(\frac{(a-c)}{2b}\right) - c\left(\frac{(a-c)}{2b}\right) - F$$

$$= \left(\frac{a+c}{2} - c\right)\left(\frac{(a-c)}{2b}\right) - F$$

$$= \frac{(a-c)}{2}\frac{(a-c)}{2b} - F \quad = \frac{(a-c)^2}{4b} - F$$

2d) At what Fixed cost will they operate?

$\Rightarrow$ If $0 < \pi^* \quad \Rightarrow \quad 0 < \frac{(a-c)^2}{4b} - F$

$$\Rightarrow \quad F < \frac{(a-c)^2}{4b}$$

3a) using equations (2) & (3) we get

Question #3 (Math)

(2)

$\Rightarrow a - 2bq_i - bq_j - c_i = 0$

$\Rightarrow 2bq_i = a - c_i - bq_j$

$\Rightarrow BRF_i \equiv q_i(q_j) = \frac{(a-c_i)}{2b} - \frac{1}{2}q_j$

(3)

$\Rightarrow a - 2bq_j - q_i - c_j = 0$

$\Rightarrow 2bq_j = a - c_j - q_i$

$\Rightarrow BRF_j \equiv$

$q_j(q_i) = \frac{a-c_j}{2b} - \frac{1}{2}q_i$

3b)

using these BRFs

$$\Rightarrow q_i = \frac{(a-c_i)}{2b} - \frac{1}{2}\left[\frac{(a-c_j)}{2b} - \frac{1}{2}q_i\right]$$

$$\Rightarrow q_i = \frac{(a-c_i)}{2b} - \frac{(a-c_j)}{4b} + \frac{1}{4}q_i$$

$$\Rightarrow 4bq_i = 2(a-c_i) - (a-c_j) + bq_i$$

multiplying by 4b on both sides.

$$\Rightarrow 3bq_i = 2a - a - 2c_i + c_j$$

$$\Rightarrow 3bq_i = a - 2c_i + c_j$$

$$\Rightarrow q_i^* = \frac{a - 2c_i + c_j}{3b}$$

plugging back into BRF_j

$$\Rightarrow q_j^* = \frac{(a-c_j)}{2b} - \frac{1}{2}\left[\frac{a-2c_i+c_j}{3b}\right]$$

$$q_j^* = \frac{(a-c_j)}{2b} - \frac{(a-2c_i+c_j)}{6b}$$

Note the symmetry.

$$\Rightarrow q_j^* = \frac{(3a - 3c_j - a + 2c_i - c_j)}{6b}$$

$$q_j^* = \frac{(2a - 4c_j + 2c_i)}{6b}$$

$$q_j^* = \frac{2(a - 2c_j + c_i)}{6b} = \frac{(a - 2c_j + c_i)}{3b}$$

3c) Note the symmetric costs

$$\Rightarrow \quad q_i^* = \frac{(a-2c+c)}{3b} = \frac{(a-c)}{3b}$$ SAME

&

$$q_j^* = \frac{(a-2c+c)}{3b} = \frac{(a-c)}{3b}$$

3d)

$\Rightarrow$ IS $P(Q) = (a-bQ)$ $\quad q_i^* + q_j^*$

$$\Rightarrow \quad = \left(a - b\left(\frac{(a-c)}{3b} + \frac{(a-c)}{3b}\right)\right)$$

$$= \left(a - \frac{2(a-c)}{3}\right)$$

$$P(Q^*) = \left(\frac{3a-2a+2c}{3}\right) = \frac{a+2c}{3} \quad //$$

3e) π^*

$$\Rightarrow \quad \pi^* = p^* q^* - (F + cq^*)$$

$$= (p^* - c) q^* - F$$

$$= \left(\frac{a+2c}{3} - c\right)\left(\frac{(a-c)}{3b}\right) - F$$

$$= \frac{(a-c)}{3} \frac{(a-c)}{3b} - F$$

$$\Rightarrow \pi^* = \frac{(a-c)^2}{9b} - F$$

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(Question #4)
Math

4a) using equation #4

$$\Rightarrow \quad a - 2bq_i - b\sum_{i\neq j}^{N} q_j - c = 0$$

$$\Rightarrow \quad 2bq_i = a - c - b\sum_{i\neq j}^{N} q_j$$

$$\Rightarrow BRF_i \equiv q_i\left(\sum_{i\neq j}^{N} q_j\right) = \frac{a-c}{2b} - \frac{1}{2}\sum_{i\neq j}^{N} q_j$$

where,

$$\sum_{i\neq j}^{N} q_i = (q_1 + q_2 + \ldots + q_{i-1} + q_{i+1} + \ldots + q_N)$$

Note that there are $N-1$ quantities included in ~~here~~ because q_i is not included.

4b) using the assumption ① such that

$q_i = q_j$

$\Rightarrow$ equation (4) becomes

$$a - 2bq_i - b\sum_{i=1}^{(N-1)} q_i - c = 0$$

where

Assumption ②

$$\sum_{i=1}^{N} c = Nc$$

when c = constant

$$\Rightarrow \quad a - bq_i - \left(bq_i + b\sum_{i=1}^{(N-1)} q_i\right) - c = 0$$

$$\Rightarrow \quad a - bq_i - b\sum_{i=1}^{N} q_i - c = 0$$

$$\Rightarrow \quad bq_i + b\sum_{i=1}^{N} q_i = a - c$$

$$\Rightarrow \quad bq_i + Nbq_i = a - c$$

$$\Rightarrow \quad bq_i(1+N) = a - c$$

$$\Rightarrow \quad q_i^* = \frac{(a-c)}{(N+1)b}$$

4c) using the same assumption above

$$\Rightarrow \quad Q^* = \sum_{i=1}^{N} q_i^* = Nq_i^* = \frac{N(a-c)}{(N+1)b}$$

4d) $P(Q^*) = (a - bQ^*)$

$$= \left(a - \cancel{b}\left(\frac{N(a-c)}{(N+1)\cancel{b}}\right)\right)$$

$$= \frac{a(N+1) - N(a-c)}{N+1}$$

$$= \frac{\cancel{aN} + a - \cancel{Na} + Nc}{(N+1)} = \frac{a+Nc}{(N+1)}$$

4e)

$$\pi_i^* = \underbrace{(a - bQ^*)}_{P(Q^*)} q_i^* - c q_i^* - F$$

$$= (P(Q^*) - c) q_i^* - F$$

$$= \left(\frac{a+Nc}{(N+1)} - c\right)\left(\frac{(a-c)}{(N+1)b}\right) - F$$

$$= \left(\frac{a + \cancel{Nc} - \cancel{Nc} - c}{(N+1)}\right)\left(\frac{a-c}{(N+1)b}\right) - F$$

$$= \frac{(a-c)}{(N+1)} \frac{(a-c)}{(N+1)b} - F$$

$$= \underbrace{\frac{(a-c)^2}{(N+1)^2 b}}_{\text{Producers Surplus (PS)}} - F$$

4f) In a Perfectly Competitive Equilibrium $\pi^* = 0$

$$\Rightarrow \quad 0 = \frac{(a-c)^2}{(N+1)^2 b} - F$$

$$\Rightarrow \quad bF = \left(\frac{(a-c)}{(N+1)}\right)^2$$

$$\Rightarrow \quad \sqrt{bF} = \frac{(a-c)}{(N+1)}$$

$$\Rightarrow \quad \sqrt{bF}\,N + \sqrt{bF} = (a-c)$$

$$\Rightarrow \quad \sqrt{bF}\,N = (a-c) - \sqrt{bF}$$

$$\Rightarrow \quad N^* = \frac{(a-c) - \sqrt{bF}}{\sqrt{bF}} \quad \text{//}$$

Question #5
Math

5a) Comparing q_i^*'s

$\Rightarrow$ $q_i^m > q_i^D > q_i^{PC}$

Because $\frac{(a-c)}{2b} > \frac{(a-c)}{3b} > \frac{(a-c)}{(N+1)b}$ if $N \geq 3$

$\Rightarrow$ as # of firms increases $(N \to \infty)$, individual output increases.

5b) $P(Q^m) > P(Q^D) > P(Q^{PC})$

$\Rightarrow$ $\underbrace{\frac{(a+c)}{2} > \frac{(a+2c)}{3}} > \frac{(a+Nc)}{(N+1)}$ if $N \geq 3$

Proof ①
$\frac{(a+c)}{2} > \frac{(a+2c)}{3}$

$\Rightarrow 3(a+c) > 2(a+2c)$

$\Rightarrow 3a+3c > \cancel{2a}+4c$

$\Rightarrow a > c$ which holds by assumption

Proof ②
$\frac{(a+2c)}{3} > \frac{(a+Nc)}{(N+1)}$

$(N+1)(a+2c) > 3a+3Nc$

$Na+\cancel{2Nc}+\cancel{a}+2c > 3a+3Nc$

$\Rightarrow Na+2c > a+Nc$

$a(N-1) > c(N-2)$ where $a>c$ & $(N-1)>(N-2)$

$\Rightarrow P(Q^D) > P(Q^{PC})$ //

π^*'s

$\Rightarrow$

5c) $\pi^{M} > \pi^{Duopoly} > \pi^{PC}$

$$\Rightarrow \quad \frac{(a-c)^2}{4b} - \cancel{F} > \frac{(a-c)^2}{9b} - \cancel{F} > \frac{(a-c)^2}{(N+1)^2 b} - \cancel{F}$$

$$\Rightarrow \quad \frac{(a-c)^2}{4b} > \frac{(a-c)^2}{9b} > \frac{(a-c)^2}{(N+1)^2 b}$$ which holds if $N \geq 2$