

# *EconS 305: Intermediate Microeconomics w/o Calculus*

## *Homework 4:*

### *Pricing, Imperfect Competition and Game Theory*

*Due: Friday, June 12th, 2020 at 5:00pm via Blackboard*

- Please submit all homework solutions in the order the questions are presented and as **one .PDF**.

- Please **show all calculations** as these exercises are meant to refine your quantitative tool set. If I can not follow your calculations or it seems as you just “copy and pasted” answers from the internet, I will be deducting half the points from that solution.

#### 1. Price Discrimination: Block Pricing (2nd Degree Price Discrimination)

Consider a monopolist looking to price discriminate by setting a price for the first units sold, and a different price for the remaining units sold. The first  $q_1$  units will be sold to the first consumers, until the stock runs out, and the remaining  $(q_2 - q_1)$  units will be the quantity provided to the remaining consumers. The firm faces the same inverse demand curve of  $p(q_i) = a - b(q_i)$  for all of the consumers, where  $a > c$  and  $i \in \{1, 2\}$ , and has a constant marginal cost of  $c$ . Notice that the first consumers will be paying a different price per unit for the quantity they consume, and that the remaining consumers will be paying a different price per unit for the quantity they consume.

We represent the Profit Maximization Problem ( $PMP_i$ ) for the firm as a two part problem where they maximize profits for each consumer  $i$  as:

$$\begin{aligned}\max_{q_1 \geq 0} \pi_1 &= p(q_1)q_1 - cq_1 \\ \max_{q_2 \geq 0} \pi_2 &= p(q_2)(q_2 - q_1) - c(q_2 - q_1)\end{aligned}$$

Where, we can combine these two PMPs into one function such that the monopolist maximizes the two choice variables of  $q_1$  &  $q_2$  in one problem

$$\begin{aligned}\max_{q_1, q_2 \geq 0} \pi &= \pi_1 + \pi_2 \\ &= p(q_1)q_1 - cq_1 + p(q_2)(q_2 - q_1) - c(q_2 - q_1)\end{aligned}$$

$$= (a - b(q_1))q_1 - cq_1 + (a - b(q_2))(q_2 - q_1) - c(q_2 - q_1)$$

**CALCULUS PART:**

From here, we can take our derivatives and set them equal to zero

$$\frac{\partial \pi(q_1, q_2)}{\partial q_1} = a - 2bq_1 - c - a + bq_2 + c = 0 \quad (1)$$

$$\frac{\partial \pi(q_1, q_2)}{\partial q_2} = a - 2bq_2 - c + bq_1 = 0 \quad (2)$$

where we now have two equations ((1) and (2)), and two choice variable  $(q_1, q_2)$  to solve for.

**CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.**

- (a) Find the firm's optimal allocation of quantities  $(q_1, q_2)$  to maximize its profit in equilibrium (i.e. find  $q_1^*, q_2^*$ ). Do any of these quantities look familiar? Identify it and state why it is familiar to you as an economic analyst.
- (b) Find the the equilibrium prices the firm will be charging for each one of the quantities (i.e. find  $p(q_1^*), p(q_2^*)$ ). Which one is larger? Does it have the same relationship as the two equilibrium quantities derived in part a?
- (c) Find both of the firm's profits in equilibrium (i.e. find  $\pi_1^*$  and  $\pi_2^*$ ). Do you notice anything familiar about the profits coming from  $\pi_2^*$ ? Finally, add these profits up to get total profits of the price discriminating firm (i.e. find  $\pi^* = \pi_1^* + \pi_2^*$ ).
- (d) Derive the profits for the standard monopolist model (as we did in HW 3-Question 2) without fixed costs, and compare these profits to the price discriminating monopolist profits you got in part c. Which profit is larger, and what does this tell you about a monopolist's incentive to price discriminate?

## 2. Modeling a Cartel Between Two Firms

Consider two firms acting as a cartel. In economics, we typically model a cartel as  $N$  firms collectively acting as a monopoly, and then evenly dividing the profits among each firm participating in the cartel. This implies that we can calculate each firm's profits from participating in a cartel as  $\pi^{Cartel} = \frac{\pi^{Monopolist}}{N}$ . So, we consider first the monopolist's problem where they face a linear inverse demand function of  $p(Q) = a - b(Q)$ , where  $a > c$ , and a total cost function of  $TC(Q) = cQ + F$ . We can interpret the fixed cost ( $F$ ) as perhaps some "entry" fee, and we interpret  $c$  as the cost the firm has to pay per each unit of output produced (i.e. the marginal cost).

We can represent the Profit Maximization Problem (*PMP*) for firm as:

$$\begin{aligned} \max_{Q \geq 0} \quad & \pi = p(Q)Q - (cQ + F) \\ \implies \max_{Q \geq 0} \quad & \pi = [a - bQ]Q - (cQ + F) \end{aligned}$$

### CALCULUS PART:

From here, we can take our derivatives and set them equal to zero

$$\frac{\partial \pi(Q)}{\partial Q} = a - 2bQ - c = 0 \quad (3)$$

where we now have one equation ((3)), and one choice variable ( $Q$ ) to solve for.

### CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.

- (a) Find the firm's optimal allocation of production ( $Q$ ) to maximize its profit in equilibrium (i.e. find  $Q^*$ ). After getting your  $Q^*$ , please divide this by  $N = 2$  as this is the amount produced by each firm (i.e.  $q_i^{Cartel} = \frac{Q^*}{N}$ ).
- (b) What is the equilibrium price the firm will receive (i.e. find  $p(Q^*)$ ?)
- (c) What is the optimal profit function of the firm (i.e. find  $\pi^*(Q^*)$ ).
- (d) Calculate the profits that each firm gets from participating in the cartel (i.e. find  $\pi^{Cartel}$  where  $N = 2$ ).

### 3. A Cournot Game of Competing in Quantities w/ Fixed Costs - A Game Theory Extension

Consider two firms competing a la Cournot in a market with an inverse demand function of  $p(Q) = a - b(Q)$ , where  $Q = q_i + q_j$  and  $a > c$ , and a total cost function of  $TC_i(q_i) = F + c_i q_i$ . Notice that each firm has the same fixed cost ( $F$ ) but their marginal costs ( $c_i$ ) are not equal to each other (i.e.  $c_i \neq c_j$ ). This means these homogeneous product producing firms have asymmetric costs, and we can represent the Profit Maximization Problem ( $PMP_i$ ) for firm  $i$  as:

#### CALCULUS PART:

$$\begin{aligned} \max_{q_i \geq 0} \pi_i &= [a - b(q_i + q_j)] q_i - (F + c_i q_i) \\ \frac{\partial \pi_i(q_i, q_j)}{\partial q_i} &= a - 2bq_i - bq_j - c_i = 0 \end{aligned} \quad (4)$$

And through symmetry we know that firm  $j$ 's PMP is

$$\begin{aligned} \max_{q_j \geq 0} \pi_j &= [a - b(q_i + q_j)] q_j - (F + c_j q_j) \\ \frac{\partial \pi_j(q_i, q_j)}{\partial q_j} &= a - 2bq_j - bq_i - c_j = 0 \end{aligned} \quad (5)$$

where we now have two equations ((2) and (3)), and two choice variables ( $q_i$  and  $q_j$ ) to solve for.

#### CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.

- (a) Find the optimal equilibrium allocation for each firm when they are competing a la Cournot. That is, find  $q_i^*$  and  $q_j^*$ . Note that this solution is analogous to the answer you derived in Question 3 of Homework 3.
- (b) Now, consider that the firm's have symmetric costs (i.e.  $c_i = c_j = c$ ) in the competitive equilibrium and for all analyses from here on out. Find the competitive equilibrium quantities (i.e. find  $q_i^*$  and  $q_j^*$ ). Again, note that this solution is analogous to the answer you derived in Question 3 of Homework 3.
- (c) Find the equilibrium price (i.e.  $p(Q^*) = a - b(Q^*)$ ). Again, note that this solution is analogous to the answer you derived in Question 3 of Homework 3.
- (d) Find the equilibrium profits (i.e.  $\pi^*$ ). Again, note that this solution is analogous to the answer you derived in Question 3 of Homework 3.
- (e) Now, assume that both firms are pooling resources and acting as cartel. Please re-write the equilibrium profits ( $\pi^{Cartel}$ ) you found in Question #2. The answer should be the same answer you got in Question #2.

- (f) Now, consider that one of the firms in the cartel unilaterally deviates from the cartel to compete in quantities (i.e. sets their quantity at the competitive level instead of the cartel level). Derive the profits from deviating ( $\pi_i^{Dev}$ ) and simplify the expression. The key here is to remember that when deviation occurs, the deviating firm sets their quantities at a level as if they were competing a la Cournot and leaves the other firm operating as a cartel. This implies that the deviating firm's profits ( $\pi^D$ ) become

$$\pi_i^{Dev} = (a - b(q_i^{Cournot} + q_i^{Cartel}))(q_i^{Cournot}) - c(q_i^{Cournot}) - F$$

- (g) Similar to part f, we can find the profits of the firm that remains in the cartel while the other firm deviates. We will call these profits  $\pi_i^{NDev}$  (i.e. does not deviate) such that

$$\pi_i^{NDev} = (a - b(q_i^{Cournot} + q_i^{Cartel}))(q_i^{Cartel}) - c(q_i^{Cartel}) - F$$

Use this formula to find the profits of the firm that is being deviated upon (i.e.  $\pi_i^{NDev}$ ).

- (h) Take the four different equilibrium profit functions you found ( $\pi^{Cartel}$ ,  $\pi^{Cournot}$ ,  $\pi^{Dev}$  and  $\pi^{NDev}$ ), set all fixed costs equal to zero (i.e.  $F = 0$ ), and compare them mathematically (i.e. from most profit gained to least profit gained). Once the comparison is done, plug these equations into a matrix following the matrix template given below. This is called a normal form game, and this allows us to determine the “best response” for each firm when trying to decide to participate in a cartel or compete in quantities.

⇒

		<u>Firm j</u>	
		Cartel	Compete
<u>Firm i</u>	Cartel	$\pi_i^{Cartel}, \pi_j^{Cartel}$	$\pi_i^{NDev}, \pi_j^{Dev}$
	Compete	$\pi_i^{Dev}, \pi_j^{NDev}$	$\pi_i^{Cournot}, \pi_j^{Cournot}$

- (i) Find the Pure Strategy Nash Equilibrium ( $psNE$ ).

#### 4. Bertrand Price Competition - A Game Theory Extension

Consider two firms competing a la Bertrand in a market with demand  $Q = 1 - p$ . For simplicity, both firms face no marginal costs, i.e.,  $c = 0$ . Recall the rules of Bertrand competition: if firm  $i$  sets a lower price than firm  $j$ ,  $p_i < p_j$ , firm  $i$  captures the entire market ( $Q$ ) and makes profits of

$$\pi_i^* = p_i Q = p_i \underbrace{(1 - p_i)}_Q$$

while firm  $j$  gets zero sales, and thus, zero profits (i.e.  $\pi_j^* = 0$ ). If both firms set the same price,  $p_i = p_j$ , then both firms obtain half of the above profits,  $\frac{1}{2}p_i(1 - p_i)$ .

- (a) Find the competitive equilibrium price for every firm (i.e.  $p_i^*$ ), and the equilibrium profits for every firm (i.e.  $\pi_i^*$ ). Recall that in a one shot game, firms will undercut each other until they set a price equal to their marginal cost (i.e.  $p_i^* = c$ ).
- (b) Now assume that the firms form a cartel. This implies that the firms would jointly act as a monopolist and split the monopoly profits evenly. We set the monopolist's problem up as

$$\max_{p \geq 0} \pi = p \underbrace{(1 - p)}_Q = p - p^2$$

#### CALCULUS PART:

where from here, we can take our derivatives and set them equal to zero

$$\frac{\partial \pi(p)}{\partial p} = 1 - 2p = 0 \tag{6}$$

where we now have one equation ((6)), and one choice variable ( $p$ ) to solve for.

#### CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.

What is the price that every firm should set in order to maximize the profits of the cartel? Notice that the firm will not divide this price by the number of firms in the market as this would yield lower profits. Once the optimal price is found, find monopoly profits and the individual profits that every firm makes in the cartel. Please label them as  $\pi^{Cartel}$ .

- (c) Now assume that a firm tries to under-cut the other firm's price (*say, by a value of  $\epsilon$* ) in an attempt to capture the whole market. We can denote this price deviation as

$$p^D = p^C - \epsilon = \frac{1}{2} - \epsilon$$

What are the firm's profits if they deviate from the cartel agreement, and what are the firm's profits if they are the one that sticks to the cartel agreement? Please label them as  $\pi^{Dev}$  and  $\pi^{NDev}$ , respectively.

- (d) Using the profits found in parts a through c (i.e.  $\pi^{Bertrand}$ ,  $\pi^{Cartel}$ ,  $\pi^{Dev}$  and  $\pi^{NDev}$ ), please construct the same 2x2 normal form matrix you did in Question 3. For reference, the matrix is

$\Rightarrow$

		<u>Firm j</u>	
		Cartel	Compete
<u>Firm i</u>	Cartel	$\pi_i^{Cartel}, \pi_j^{Cartel}$	$\pi_i^{NDev}, \pi_j^{Dev}$
	Compete	$\pi_i^{Dev}, \pi_j^{NDev}$	$\pi_i^{Cournot}, \pi_j^{Cournot}$

where you can substitute  $\pi^{Bertrand}$  for  $\pi^{Cournot}$ .

- (e) Find the condition on  $\epsilon$  that will insure that the cartel stays intact.

**5. An Increase in the Elasticity of Demand Decreases the Monopolist's Markup Price for that Consumer**

- (a) Using the general definition of Marginal Revenue (i.e.  $MR = p + \frac{\Delta p}{\Delta q}q$ ), please derive the Lerner Index. Please show all calculations. What does the Lerner Index tell us about the monopolist's price?
- (b) Using the Lerner Index you derived in part a, please derive the Inverse Elasticity Pricing Rule (IEPR). Please show all calculations.
- (c) Assuming that the elasticity of demand for consumer 1 is greater than the elasticity of demand for consumer 2 in absolute value (i.e.  $|\epsilon_1^D| > |\epsilon_2^D|$ , and thus is more elastic), please prove that the markup price for consumer 1 is less than the markup price for consumer 2. Please do not prove this trivially, and make sure you show all steps.

*Hint:* Start by assuming that the  $p(MC, \epsilon_2^D) > p(MC, \epsilon_1^D)$ .