Notes for 11/18

4.3 Budget Constraint

$\text{Income} = P_x Q_x + P_y Q_y$

$\boxed{\text{NOTE:}}$ In my homework I use $m = P_1 q_1 + P_2 q_2$

$\Rightarrow$ If, $I = P_x Q_x + P_y Q_y$

Then, $\Rightarrow P_y Q_y = I - P_x Q_x$

$$Q_y = \underbrace{\frac{I}{P_y}}_{b} - \underbrace{\frac{P_x}{P_y}}_{m} Q_x$$

analagous to $y = mx + b$

$\Rightarrow$ Plotting where, $I = 50$, $P_x = 5$, $P_y = 10$ $\Rightarrow Q_y = \frac{50}{10} - \frac{5}{10} Q_x$

$\Rightarrow$ Budget constraint:

$$Q_y = 5 - \frac{1}{2} Q_x$$

$\Rightarrow$ ① Set $Q_y = 0$

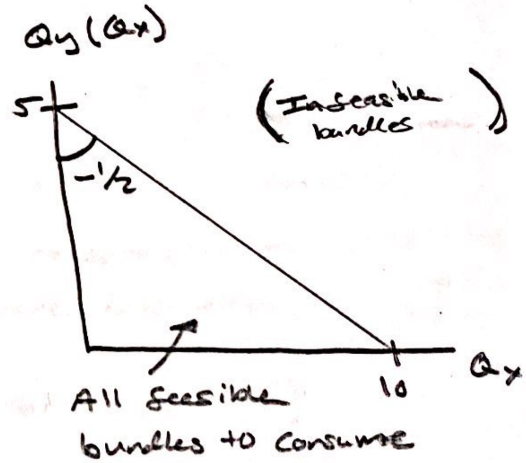
$\Rightarrow 0 = 5 - \frac{1}{2} Q_x$

$\Rightarrow 5 = \frac{1}{2} Q_x$

$\Rightarrow Q_x = 10$

② Set $Q_x = 0$

$\Rightarrow Q_y = 5$



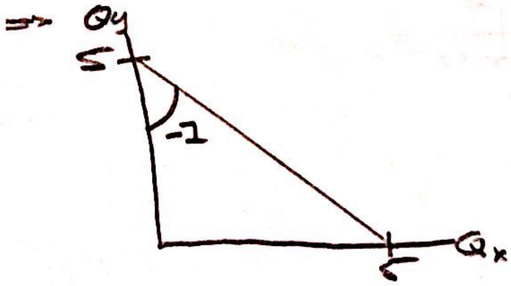
Slide 4/8: Shifts

$\Rightarrow$ (a) Double price of Lattes or $(P_x' = 2P_x)$

$\Rightarrow Q_y = \frac{50}{10} - \frac{5(2)}{10} Q_x$

$Q_y = 5 - (1) Q_x$

$\Rightarrow$

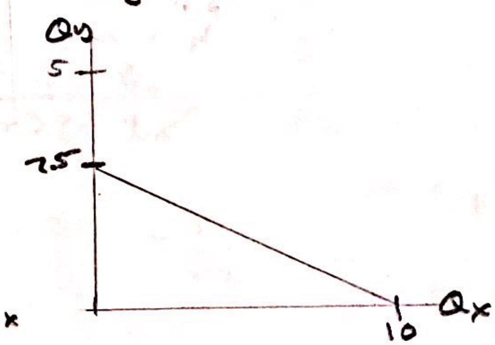


(b) Double prices Burritos

$\Rightarrow P_y' = 2P_y$ or $P_y' = 2(10) = 20$

$\Rightarrow Q_y = \frac{50}{20} - \frac{5}{20} Q_x$

$\Rightarrow Q_y = 5/2 - 1/4 Q_x$

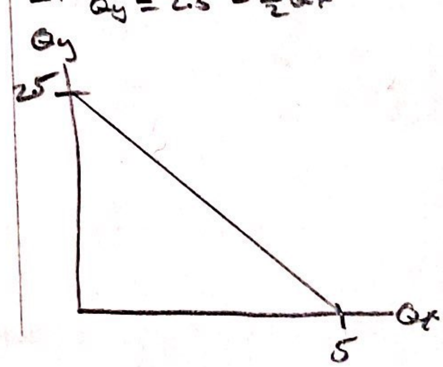


(c) Reduce Income by 1/2

$\Rightarrow I' = \frac{1}{2} I = \frac{1}{2}(50) = 25$

$\Rightarrow Q_y = \frac{25}{10} - \frac{5}{10} Q_x$

$\Rightarrow Q_y = 2.5 - \frac{1}{2} Q_x$



HW 1 Question 1

Note, in part (a) I set $\frac{MU_1}{P_1} = \lambda$ & $\frac{MU_2}{P_2} = \lambda$

Then, $\Rightarrow \frac{MU_1}{P_1} = \frac{MU_2}{P_2}$ which is the same in the slides.

Now, a little trick is if we have

$$\mathcal{L}(q_1, q_2; \lambda) = u(q_1, q_2) - \lambda \underbrace{[P_1 q_1 + P_2 q_2 - M]}_{BC}$$

$\Rightarrow$ ① $\frac{\partial \mathcal{L}(q_1, q_2)}{\partial q_1} \equiv MU_1 - \lambda P_1 \overset{set}{=} 0$

② $\frac{\partial \mathcal{L}(q_1, q_2)}{\partial q_2} \equiv MU_2 - \lambda P_2 = 0$

$$\left.\begin{array}{l} \Rightarrow ① \; MU_1 = \lambda P_1 \\ \& \; ② \; MU_2 = \lambda P_2 \end{array}\right\} \overset{combining}{\Rightarrow} -MRS_{1,2} \equiv \frac{MU_1}{MU_2} = \frac{P_1}{P_2}$$

since λ's cancel

$\Rightarrow$ Once we get $MRS_{1,2}$, we have the relationship we need to solve the problem in a different, yet more informative, way!

$\boxed{\text{Ex}}$ Let, $\mathcal{L}(q_1, q_2; \lambda) = q_1^{\alpha} q_2^{(1-\alpha)} - \lambda[P_1 q_1 + P_2 q_2 - M]$

$$\Rightarrow \left.\begin{array}{l} \frac{\partial \mathcal{L}}{\partial q_1} \equiv \alpha q_1^{\alpha-1} q_2^{1-\alpha} - \lambda P_1 \overset{set}{=} 0 \\ \frac{\partial \mathcal{L}}{\partial q_2} \equiv (1-\alpha) q_1^{\alpha} q_2^{-\alpha} - \lambda P_2 = 0 \end{array}\right\} \overset{combining}{\Rightarrow} -MRS_{1,2} \equiv \frac{\alpha q_1^{\alpha-1} q_2^{1-\alpha}}{(1-\alpha) q_1^{\alpha} q_2^{-\alpha}} = \frac{\cancel{\lambda} P_1}{\cancel{\lambda} P_2}$$

Simplifying,

$$-MRS_{1,2} \equiv \frac{\alpha q_1^{\alpha-1} q_2^{1-\alpha}}{(1-\alpha) \cancel{q_1^{\alpha}} \cancel{q_2^{-\alpha}}} = \frac{P_1}{P_2}$$

$$\Rightarrow \frac{\alpha}{(1-\alpha)} \frac{q_2}{q_1} = \frac{P_1}{P_2}$$

Note: What's this? This is what you plug into your BC to solve for optimal demand!

$$\Rightarrow q_2 = \frac{(1-\alpha)}{\alpha} \frac{P_1}{P_2} q_1$$

which is the amount you would get for good 2 for some amount of good 1