Investment, Time, and Insurance

Introduction (1/1)

14

This chapter explores the combined role of risk, uncertainty, and time in economic decision making.

Chapter Outline

Present Discounted Value Analysis (1/10)

14.1

When the benefits and/or costs of actions occur at different points in time or accrue over time, it is necessary to compare outcomes.

The common way to do this is through the use of **present discounted value (PDV)**.

- a mathematical concept that allows us to compare costs and benefits over time in a way that puts all present and future financial values on equal footing.

Why should we discount future payments?

– We value the payments less than if we received them today.
– Money received today could be invested and earn a return.

How should we discount future payments?

– Use interest rates that could be earned on current payments.

Present Discounted Value Analysis (2/10)

14.1

Interest Rates

Think of a savings account that pays a 4% annual rate of interest.

- **Interest:** a periodic payment tied to an amount of assets borrowed or lent
- **Interest rate:** interest expressed as a fraction or percentage of the principal (deposit)

Assume you have $100 in your saving account. This is the **principal**.

- The amount of assets on which interest payments are paid

If you are offered a risk-free investment opportunity that requires you to withdraw and lend the $100 for one year, what is the minimum return (payment in excess of the loan) you should ask for upon repayment?

  – You should ask for at least $4, as you would have earned that in your savings account.

Present Discounted Value Analysis (3/10)

14.1

What happens when payments occur more than one period in the future?

- Interest is *compounded.*
- **Compound interest:** a calculation of interest based on the sum of the original principal and the interest paid over past periods

Imagine you are given an initial principal amount A_0, and the interest rate is r.

- After a single period, we add the interest to find the new principal:

$$A = A \text{ x } (1 + r)$$

- After two periods, the principal grows to:

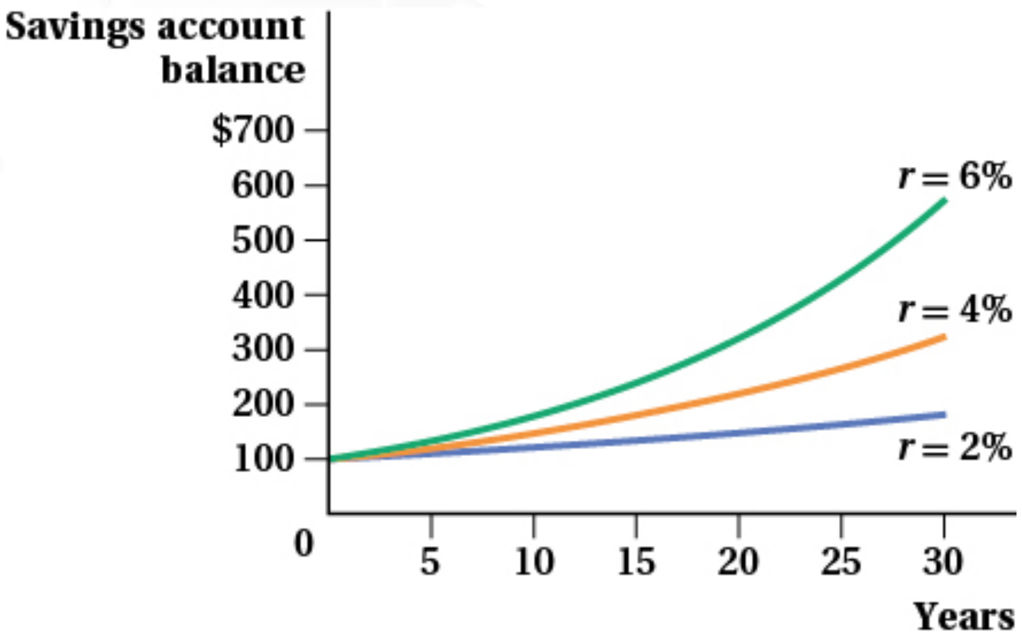
$$A = A \text{ x } (1 + r) \text{ x } (1 + r) = A \text{ x } (1 + r)^2$$

- This process is generalized, so that the value in any period t is:

$$V_t = A \text{ x } (1 + r)^t$$

Present Discounted Value Analysis (4/10)

Figure 14.1 Compound Interest



As time goes on, compounding interest leads to an accelerating growth of savings.

As the interest rate rises, savings rise even faster.

Present Discounted Value Analysis (5/10)

14.1

The Rule of 72

A simple rule of thumb allows us to compute the time it will take for a principal amount to double when it is earning a positive interest rate.

- Simply divide the number 72 by the annual interest rate.
- For instance, with a 4% rate of interest, the principal should double every 18 years: 72 / 4 = 18.

Present Discounted Value

- To evaluate how future payments compare with payments today, reverse the calculation.
- A delayed payment is less valuable than a payment today because a payment today can be put to work immediately, earning income.
- The *opportunity cost* of waiting for a payment is this loss of income.

Present Discounted Value Analysis (6/10)

14.1

Present Discounted Value: An Example

- Consider a payment of $104 one year from now. The interest rate is 6%.

How much is this worth to you in today's dollars?

- Using the equation for compounded interest,

$$V_t = A \text{ x } (1 + r)^t$$

$$\$104 = A \text{ x } (1 + 0.06)^1$$

$$A = \frac{\$104}{(1.06)^1} = \$98.11$$

- The original principal amount that makes us indifferent between payment today and payment in one year is the present discounted value.
- Generalizing:

$$PDV = \frac{V_t}{(1 + r)^t}$$

Present Discounted Value Analysis (7/10): Question 1

14.1

What is the present discounted value of a payment of $500 in 10 years (10 periods) if the interest rate is 3%?

A. $372.05
B. $375.66
C. $4,952.12
D. $4,972.55

Present Discounted Value Analysis (7/10): Question 1 – Correct Answer

14.1

What is the present discounted value of a payment of $500 in 10 years (10 periods) if the interest rate is 3%?

- **A. $372.05 (correct answer)**
- B. $375.66
- C. $4,952.12
- D. $4,972.55

Present Discounted Value Analysis (8/10)

14.1

Present Discounted Value of Payment Streams

- If payments occur as a stream (for instance, a stream of dividends over *T* periods), the present discounted value can be computed as:

$$PDV = \frac{M}{(1+r)} + \frac{M}{(1+r)^2} + \cdots \frac{M}{(1+r)^T}$$

- For example, a scholarship pays $1,000 in four installments; the first installment arrives today and the next three come one, two, and three years from today:

$$PDV = \$1{,}000 + \frac{\$1{,}000}{(1+r)} + \frac{\$1{,}000}{(1+r)^2} + \frac{\$1{,}000}{(1+r)^3}$$

Present Discounted Value Analysis (9/10)

14.1

Present Discounted Value: A Special Case

- A special case involves identical payments that occur every period over an indefinite period.
- Let the fixed payment be *M*. The present discounted value of a *never-ending* stream of payments is given by:

$$PDV = \frac{M}{(1+r)} + \frac{M}{(1+r)^2} + \frac{M}{(1+r)^3} \dots = \frac{M}{r}$$

Present Discounted Value Analysis (10/10): Question 2

14.1

What is the present discounted value of an infinite stream of payments of $20 if the interest rate is 2%?

A. $100
B. $1,000
C. $10,000
D. $100,000

Present Discounted Value Analysis (10/10): Question 2 – Correct Answer

14.1

What is the present discounted value of an infinite stream of payments of $20 if the interest rate is 2%?

A. $100
B. $1,000 (correct answer)
C. $10,000
D. $100,000

Evaluating Investment Choices (1/5)

14.2

Net Present Value

In the previous section, there were no costs associated with obtaining the right to a payment stream.

Many actions are associated with costs and benefits that are temporally decoupled. How does one decide whether to proceed?

When an investment stream is associated with payments as well as receipts, how can we place a value on it?

- **Net present value (NPV) analysis** is the use of the present discounted value to evaluate the expected long-term return on an investment.
- NPV analysis allows us to determine whether the benefits of an investment exceed the costs.

Evaluating Investment Choices (2/5)

14.2

Net Present Value

You are deciding whether to purchase skis.

- Up-front cost is $500.
- Periodic (annual) saving is $200 for three years.
- Interest rate is 4%.

Should you purchase the skis?

- Using NPV analysis, we simply compute the net present discounted value of the purchase:

$$PDV = -500 + \frac{200}{(1+0.04)} + \frac{200}{(1+0.04)^2} + \frac{200}{(1+0.04)^3}$$

$$= -500 + 192.31 + 184.91 + 177.80 = \$55.02$$

Table 14.1: Payoff Structure of the Skis Purchase

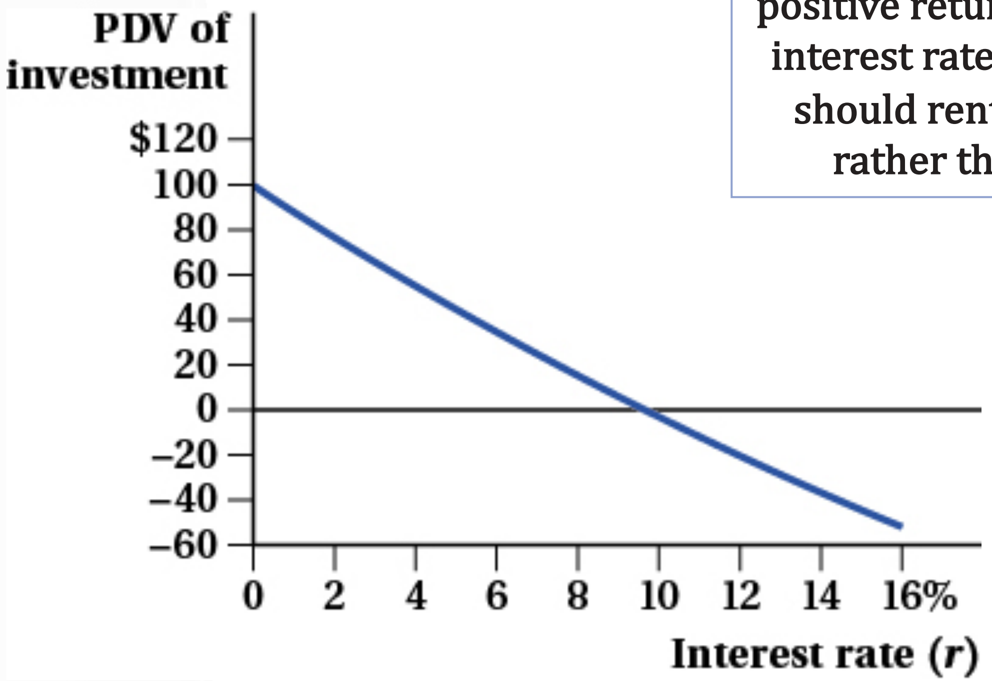
Period	Costs	Benefits
0	$500	$0
1	0	200
2	0	200
3	0	200

The positive NPV implies that, at an interest rate of 4%, the benefit exceeds the cost. You should purchase the skis.

Evaluating Investment Choices (3/5)

14.2

Figure 14.2 The PDV of a Pair of Skis



Investing in a pair of skis yields positive returns up until $r = 9.7\%$. At interest rates higher than 9.7%, you should rent skis each time you go rather than buy the new pair.

Evaluating Investment Choices (4/5): Question 1

14.2

A new assembly line costs $200,000 today, but it saves the firm $125,000 in one year and $80,000 in two years. If the interest rate is 2%, what is the net present value of the assembly line?

A. -$557.48
B. -$113.43
C. $980.39
D. $1,252.36

Evaluating Investment Choices (4/5): Question 1 – Correct Answer

A new assembly line costs $200,000 today, but it saves the firm $125,000 in one year and $80,000 in two years. If the interest rate is 2%, what is the net present value of the assembly line?

A. **-$557.48 (correct answer)**
B. -$113.43
C. $980.39
D. $1,252.36

Evaluating Investment Choices (5/5)

14.2

NPVs versus Payback Periods

Payback period is another way to evaluate investment projects.

- The length of time required for an investment's initial costs to be recouped in future benefits *without* discounting future flows

What is the payback period from the example of a ski purchase?

– 3 years

An important note: The payback calculation does not include forgone interest on the money spent on skis.

- That is, future payments are treated the same as current payments.

Table 14.1: Payoff Structure of the Skis Purchase

Period	Costs	Benefits
0	$500	$0
1	0	200
2	0	200
3	0	200

The Correct Interest Rate to Use, and Capital Markets (1/2)

14.3

Nominal Versus Real Interest Rates

The interest rates observed in the market capture two effects:

1. Price changes in the broader economy
2. The real rate of return to capital

The **nominal interest rate** is the rate quoted in the market.

- A rate of return expressed in raw currency values without regard for how much purchasing power those values hold

The **real interest rate** is the rate of return in terms of purchasing power.

- Also called the *inflation-adjusted interest rate*

As long as inflation is not exorbitantly high, the real rate is approximately equal to the nominal rate net of inflation: $r \approx i - \pi$

The Correct Interest Rate to Use, and Capital Markets (2/2)

14.3

Using the Correct Rate

Remember, the rate used in NPV calculations should represent the *opportunity cost* of the investment.

If a firm is deciding whether to use cash to finance a new business venture, the relevant rate for the firm is the interest rate on savings available to the firm.

Evaluating Risky Investments (1/3)

14.4

NPV with Uncertainty: Expected Value

A second feature of many investment projects is uncertainty with respect to outcomes.

The most basic way to incorporate uncertainty into NPV analysis is through the use of **expected value** calculations.

- Expected value is the probability-weighted average payout.

The expected value of an investment is given by:

$$\text{Expected value} = (p_1 \times M_1) + (p_2 \times M_2) + \ldots + (p_N \times M_N)$$

where *p is* probability and *M* is the associated payout.

What must the p's add up to?

– Since they are probabilities, they must sum to 1 (or 100%).

Evaluating Risky Investments (2/3)

14.4

Consider a firm that is evaluating whether to invest in a risky project.

Table 14.4: Analyzing a Risky Investment

Benefit Payout	Probability of Payout
$0	0.2
$1 million	0.6
$2 million	0.2

The expected value of the investment is given by:

$$\text{Expected value} = (0.2 \times 0) + (0.6 \times \$1{,}000{,}000) + (0.2 \times \$2{,}000{,}000)$$

$$\text{Expected value} = 0 + \$600{,}000 + \$400{,}000 = \$1{,}000{,}000$$

Evaluating Risky Investments (3/3)

Risk and the Option Value of Waiting

Investments are risky because we cannot foresee what will happen to them.

As time progresses, some uncertainty is necessarily resolved.

- This implies that an informational value is associated with delaying investment decisions.
- The **option value of waiting** is the value created if an investor can postpone the investment decision until the uncertainty about an investment's return is wholly or partially resolved.

Uncertainty, Risk, and Insurance (1/11) 14.5

Expected Income, Expected Utility, and the Risk Premium

Our discussion of risk and uncertainty thus far has assumed that economic agents are risk neutral; that is, agents care only about the expected value of an investment.

However, in general we assume economic agents have a preference for less risk.

- This follows directly from the basic assumption of declining marginal utility.

Uncertainty, Risk, and Insurance (2/11) 14.5

Example

Adam's utility is a function of his income, which he can use to purchase goods. It takes the form $U = \sqrt{I}$, where income is measured in thousands.

- Suppose Adam's income is $100,000 a year but he faces a 50/50 chance of a tornado.
- If a tornado hits, his income is reduced to $36,000, as he has to pay for the damages it causes; if it does not hit, his income remains at $100,000.

Adam's expected income = $(0.5 \times \$36{,}000) + (0.5 \times \$100{,}000)$

- Expected income = $68,000

Adam's expected utility = $(0.5 \text{ x } 6) + (0.5 \text{ x } 10) = 3 + 5$

- Expected utility = 8

Uncertainty, Risk, and Insurance (3/11)

Example

If he had the amount of income from the expected payoff without the risk, his utility would have been higher.

Adam's expected utility = $\sqrt{68}$

- Expected utility = 8.25

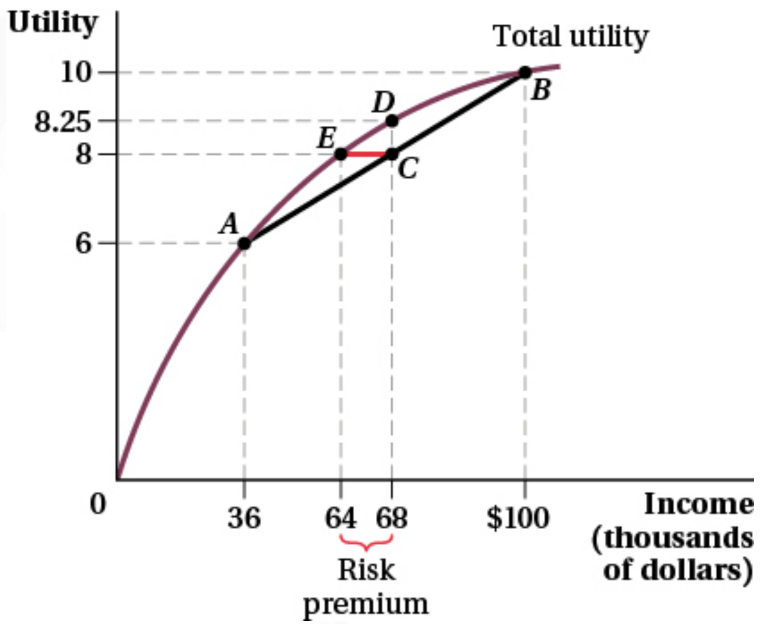
This means that the same expected payoff can give different amounts of utility depending on the riskiness of the of the underlying income levels.

Uncertainty, Risk, and Insurance (4/11) 14.5

Figure 14.3 A Risk-Averse Individual Will Pay to Avoid Risk

Because Adam is risk-averse, he is willing to give up as much as $4,000 in expected income (the distance from point *C* to point *E*) to have a certain income of $64,000.

A certain income of $64,000 also gives Adam a utility of 8 (point *E*).



Adam faces a 50% probability of an income of $100,000 (point *B*) and a 50% probability of an income of $36,000 (point *A*).

His expected income is $68,000 and his expected utility is 8 (point *C*).

Uncertainty, Risk, and Insurance (5/11)

14.5

Adam is indifferent between a certain income of $64,000 and the riskier expected income of $68,000; his **expected utility** is the same for each. Therefore, he would be willing to pay up to $4,000 to reduce the risk in his income stream to zero.

- This is because Adam gets less incremental utility from income at high levels and more at low levels. The lost utility associated with the low income is larger than the gained utility associated with the high income.
- As the variability of potential income increases, the risk premium increases.

Uncertainty, Risk, and Insurance (6/11) 14.5

An economic agent who is willing to pay to reduce risk is **risk-averse**.

- Expecting a utility loss from uncertainty or equivalently, being willing to pay for a risk reduction

- The **certainty equivalent** is the guaranteed income level at which a potential investor would receive the same expected utility as from an uncertain income.
  – For Adam, this would have been $64,000.

- A **risk premium** is the amount an investor must be compensated for bearing risk without taking a loss in expected utility.
  – For Adam, this would have been $4,000 ($68,000 − $64,000).

Uncertainty, Risk, and Insurance (7/11)

Insurance Markets

The fact that economic agents are risk-averse gives rise to **insurance**.

- Insurance is a payment to reduce a risk facing the payer.

Insurance offers compensation if an undesired outcome occurs.

- For example, the variability in Adam's income may be due to the risk that his business may be flattened. By purchasing tornado insurance, he can reduce the loss he will take if a tornado does, indeed, occur.

Complete or **full insurance** is a policy that leaves the insured individual equally well off regardless of the actual outcome.

- This is different from partial insurance, which is still valuable but does not eliminate risk.
- For instance, insurance policies often carry deductibles that must be paid by the insured.

Uncertainty, Risk, and Insurance (8/11)

Insurance Markets

While Adam may benefit from insurance, we must also consider the insurer.

- Insurance shifts risk from the insured to the insurer.

How do insurers avoid massive losses when there are, for example, a number of tornadoes?

Diversification reduces risk by combining investments with uncertain outcomes.

- Insurers tend to insure different types of risk (e.g., flood, fire, auto, life).
- The trick is to make sure risks are not too closely correlated.
- For instance, it would not be wise to provide fire insurance for every house in a tightly packed neighborhood, as it is possible that one fire could destroy all of them.

Uncertainty, Risk, and Insurance (9/11)

14.5

Insurance Markets

When the expected payouts or loss from a policy are equal to the expected premiums, a policy is deemed to be **actuarially fair**.

- Description of an insurance policy with expected net payments equal to zero
- In competitive insurance markets, premiums will adjust downward toward the actuarially fair level, thereby reducing insurers' profits.

Uncertainty, Risk, and Insurance (10/11)

The Degree of Risk Aversion

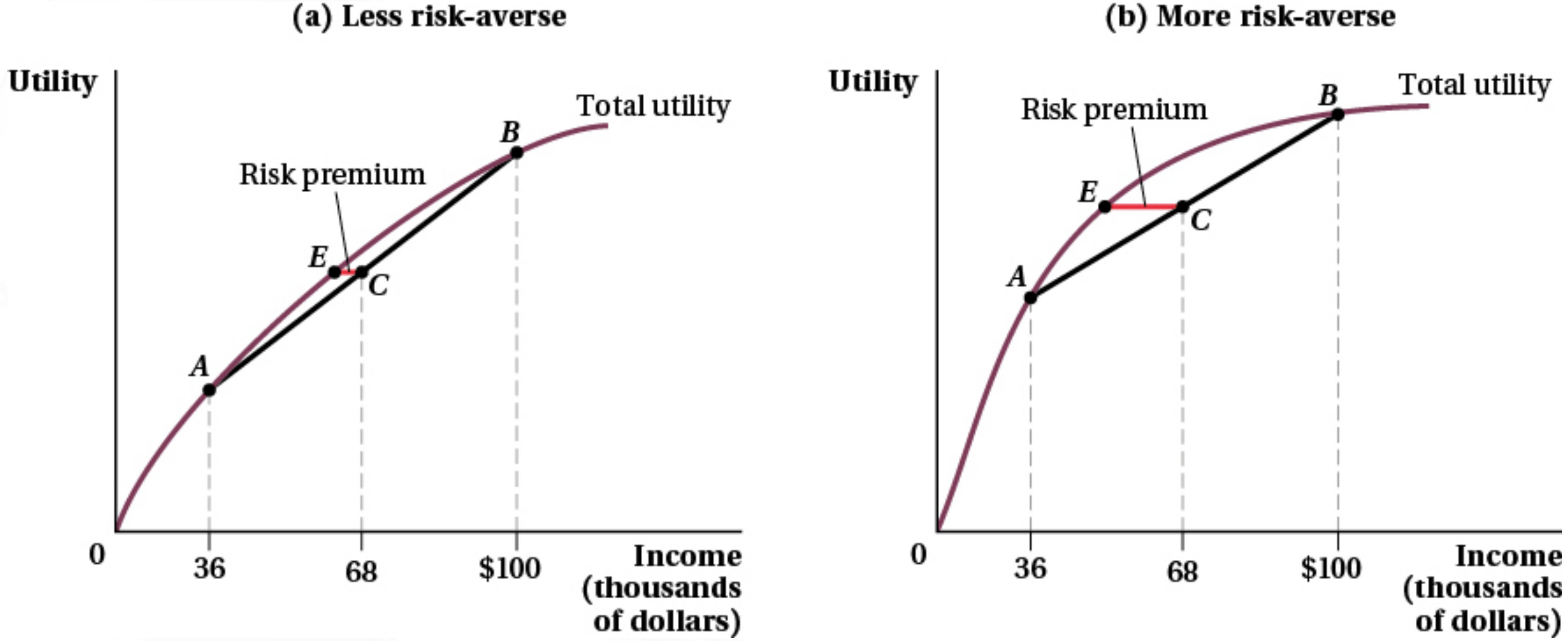
A concave utility function reflects the preferences of a risk-averse agent.

- This implies a link between the degree of risk aversion and the shape of a utility function.

Uncertainty, Risk, and Insurance (11/11)

14.5

Figure 14.4 Utility Functions and Risk Aversion



Consider different utility functions for Adam. Panel (a) is flatter and, therefore, less risk-averse.

Panel (b) has a stronger curvature, indicating risk aversion.

Conclusion (1/1)

14.6

In this chapter, we introduced the economic approach to two important aspects of many investment choices:

- Payments that occur over time
- Uncertain outcomes

For the former, we use discounting and net present value analysis. In considering uncertainty, we take into account the risk preferences of economic agents. We use expected values and expected utility to analyze risky outcomes.

In the next chapter, we introduce the concept of **general equilibrium**.