Question 1
(Math)

1a)
$$\frac{\alpha \cancel{P} \cancel{A} K^{(\alpha-1)} L^{(1-\alpha)}}{(1-\alpha) \cancel{P}\cancel{A} \cancel{K^{\alpha}} \cancel{L^{-\alpha}}} = \frac{P_K}{P_L}$$

$$\Rightarrow \quad L = \frac{(1-\alpha)}{\alpha} \frac{P_K}{P_L} K \quad \Leftarrow (L(K))$$

$\Rightarrow$ Plugging into (B)

$$P_K K + \cancel{P_L}\left(\frac{(1-\alpha)}{\alpha} \frac{P_K}{\cancel{P_L}} K\right) + F = TC$$

$$\Rightarrow \quad P_K K\left(1 + \frac{(1-\alpha)}{\alpha}\right) + F = TC$$

$$\Rightarrow \quad K^* = \frac{\alpha}{P_K}(TC - F)$$

$\Rightarrow$ Plugging K^* into $L(K)$

$$L^* = \frac{(1-\alpha)}{\cancel{\alpha}} \frac{\cancel{P_K}}{P_L}\left(\frac{\cancel{\alpha}}{\cancel{P_K}}(TC - F)\right)$$

$$\Rightarrow \quad L^* = \frac{(1-\alpha)}{P_L}(TC - F)$$

1B)
What happens if
$F > TC$?
$\Rightarrow$ Negative output,
Non feasible.

1C) Find, $\pi^* = PA(K^*)^{\alpha}(L^*)^{1-\alpha} - P_K K^* - P_L L^* - F$

$$\Rightarrow \pi^* = PA\left(\frac{\alpha}{P_K}(TC-F)\right)^{\alpha}\left(\frac{(1-\alpha)}{P_L}(TC-F)\right)^{1-\alpha} - P_K\left(\frac{\alpha}{P_K}(TC-F)\right) - P_L\left(\frac{(1-\alpha)}{P_L}(TC-F)\right) - F$$

$$\Rightarrow \quad = PA\left(\frac{\alpha}{P_K}\right)^{\alpha}\left(\frac{(1-\alpha)}{P_L}\right)^{1-\alpha}(TC-F) - \underbrace{\alpha(TC-F) - (1-\alpha)(TC-F)}_{= -(TC-F)} - F$$

$$\Rightarrow \pi^* = \left(PA\left(\frac{\alpha}{P_K}\right)^{\alpha}\left(\frac{(1-\alpha)}{P_L}\right)^{1-\alpha}\right)(TC-F) - TC$$

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1D) Set $\pi^* = 0$, assume we have a TC budget ($\bar{TC}$), & solve $\underline{F}$.

=> $\underline{\text{Need}}$ $0 < \left(PA\left(\frac{\alpha}{P_K}\right)^{\alpha}\left(\frac{(1-\alpha)}{P_L}\right)^{(1-\alpha)}\right)(\bar{TC}-F) - \bar{TC}$

=> Let $\gamma = PA\left(\frac{\alpha}{P_K}\right)^{\alpha}\left(\frac{(1-\alpha)}{P_L}\right)^{(1-\alpha)}$

=> $0 < \gamma(\bar{TC}-F) - \bar{TC}$

=> $\bar{TC} < \gamma\bar{TC} - \gamma F$

=> $\gamma F < \bar{TC}(\gamma - 1)$

=> $F < \frac{(\gamma-1)}{\gamma}\bar{TC}$ => If Fixed Costs are ~~less then~~ less than $\frac{(\gamma-1)}{\gamma}TC$, then the firm will find it profitable to enter since the entry fee (F) is not too high.

c)

Question #2 (Math)

2a) using equation (4) & (5) we have

(4) (5)

$$\Rightarrow \alpha PAK^{(\alpha-1)}L^{\beta} = P_K \quad \& \quad \beta PAK^{\alpha}L^{\beta-1} = P_L$$

where, we can use a trick (which is not necessary) & multiplying (4) on both sides by "K" & (5) on both sides "L" we have

$$\Rightarrow \alpha PAK^{\alpha}L^{\beta} = P_K K \quad (4) \quad \& \quad \beta PAK^{\alpha}L^{\beta} = P_L L$$

$$\Rightarrow PAK^{\alpha}L^{\beta} = \frac{P_K K}{\alpha} \quad \& \quad PAK^{\alpha}L^{\beta} = \frac{P_L L}{\beta}$$

Combining since both $= PAK^{\alpha}L^{\beta}$

$$\Rightarrow \frac{P_K K}{\alpha} = \frac{P_L L}{\beta}$$

solving for L

L as a function of K

$$\Rightarrow L = \frac{\beta}{\alpha}\frac{P_K}{P_L}K$$

plugging back into (4)

$$\Rightarrow \alpha PAK^{(\alpha-1)}\left(\frac{\beta}{\alpha}\frac{P_K}{P_L}K\right)^{\beta} = P_K$$

<u>And solving for K</u>

$$\Rightarrow \alpha PAK^{(\alpha+\beta-1)}\left(\frac{\beta}{\alpha}\frac{P_K}{P_L}\right)^{\beta} = P_K$$

$$\Rightarrow K^{\alpha+\beta-1} = \frac{1}{\alpha PA}\left(\frac{\alpha}{\beta}\frac{P_L}{P_K}\right)^{\beta}P_K$$

$\Rightarrow$

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$$\Rightarrow \quad K^{\alpha+\beta-1} = \frac{1}{\alpha P A}\left(\frac{\alpha}{\beta}\frac{p_L}{p_K}\right)^{\beta} p_K$$

$$\Rightarrow \quad K^{\alpha+\beta-1} = \frac{1}{PA}\left(\frac{p_L}{\beta}\right)^{\beta}\left(\frac{p_K}{\alpha}\right)^{1-\beta}$$

$$\Rightarrow \quad K^{*} = \left[\frac{1}{PA}\left(\frac{p_L}{\beta}\right)^{\beta}\left(\frac{p_K}{\alpha}\right)^{1-\beta}\right]^{\frac{1}{\alpha+\beta-1}} \qquad (*)$$

Answer

pluggin $(*)$ into $L = \frac{\beta}{\alpha}\frac{p_K}{p_L}K$ from above

$$\Rightarrow \quad L^{*} = \frac{\beta}{\alpha}\frac{p_K}{p_L}\left[\frac{1}{PA}\left(\frac{p_L}{\beta}\right)^{\beta}\left(\frac{p_K}{\alpha}\right)^{1-\beta}\right]^{\frac{1}{\alpha+\beta-1}}$$

Part B: See Above

Part C: See Above

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Question # 3 (Math)

a) Using (6) + (7)

$$\Rightarrow \quad \frac{1}{3} P K^{-2/3} L^{\frac{1}{3}} = P_K \quad \& \quad \frac{1}{3} P K^{1/3} L^{-2/3} = P_L$$

$\Rightarrow$ Lets solve this the typical way...

$\Rightarrow$ Solve for K in (6)

$$\Rightarrow \quad K^{2/3} P_K = \frac{1}{3} P L^{1/3}$$

$$\Rightarrow \quad K^{2/3} = \frac{P}{3P_K} L^{1/3}$$

$$\Rightarrow \quad K = \left(\frac{P}{3P_K}\right)^{3/2} L^{3/6} \equiv \left(\frac{P}{3P_K}\right)^{3/2} L^{1/2}$$

Plugging into (7)

$$\Rightarrow \quad L^{2/3} = \frac{1}{3P_L} P \left(\left(\frac{P}{3P_K}\right)^{3/2} L^{1/2} \right)^{1/3}$$ (taking everything to the third power)

$$L^2 = \left(\frac{P}{3P_L}\right)^3 \left(\frac{P}{3P_K}\right)^{3/2} L^{1/2}$$

$$\Rightarrow \quad L^{3/2} = \left(\frac{P}{3P_L}\right)^3 \left(\frac{P}{3P_K}\right)^{3/2}$$

$$\Rightarrow \quad L^* = \left[\left(\frac{P}{3P_L}\right)^3 \left(\frac{P}{3P_K}\right)^{3/2} \right]^{2/3}$$

$$= \left(\frac{P}{3P_L}\right)^2 \frac{P}{3P_K} \equiv \frac{P^3}{27 P_L^2 P_K}$$

$\Rightarrow$ optimal labor choice is

$$L^* = \frac{P^3}{27 P_L^2 P_K}$$

Plugging L^* back into equation 6 (the simplified one) $\Rightarrow$

$$\Rightarrow \quad K^* = \left(\frac{P}{3P_K}\right)^{3/2}\left(L^*\right)^{1/2}$$

$$\Rightarrow \quad K^* = \left(\frac{P}{3P_K}\right)^{3/2}\left(\frac{P^3}{27P_L^2P_K}\right)^{1/2}$$

$$K^* = \left(P^{3/2}P^{3/2}\right)\left(\frac{1}{3P_K}\right)^{3/2}\left(\frac{1}{27P_L^2P_K}\right)^{1/2}$$

$$= P^{(3/2+3/2)}\left(\frac{1}{3^{3/2+3^{3/2}}}\right)\left(\frac{1}{P_K^{3/2+1/2}}\right)\left(\frac{1}{P_L}\right)$$

$$= P^3\left(\frac{1}{3^3P_K^2P_L}\right)$$

$$= \frac{P^3}{27P_K^2P_L}$$

3b) see key

$$\Rightarrow \quad \left(K^*, L^*\right) = \left(\frac{P^3}{27P_K^2P_L}, \frac{P^3}{27P_L^2P_K}\right)$$

3c) Optimal Supply

$$\Rightarrow \quad Q^* = (K^*)^{1/3}(L^*)^{1/3} = \left(\frac{P^3}{27P_K^2P_L}\right)^{1/3}\left(\frac{P^3}{27P_KP_L^2}\right)^{1/3}$$

$$= \frac{P^2}{3^2P_KP_L} = \frac{P^2}{9P_KP_L}$$

3d) Optimal Profit

$$\Rightarrow \quad \pi^*(P, P_K, P_L) = PQ^* - P_KK^* - P_LL^*$$

$$= \frac{P^3}{9P_KP_L} - \frac{2P^3}{27P_KP_L}$$

$$= \left(\frac{3}{27} - \frac{2}{27}\right)\frac{P^3}{P_KP_L} = \frac{P^3}{27P_KP_L}$$

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Question #4 (Math)

4a) using (8) & (9), & solving for λ

$$\Rightarrow \frac{P_K}{\alpha A K^{\alpha-1} L^{1-\alpha}} = \lambda \quad \& \quad \frac{P_L}{(1-\alpha) A K^{\alpha} L^{-\alpha}} = \lambda$$

Combining & Flipping $\Rightarrow$ $$\frac{\alpha \cancel{A} K^{\alpha-1} L^{1-\alpha}}{P_K} = \frac{(1-\alpha) \cancel{A} \cancel{K^{\alpha}} \cancel{L^{-\alpha}}}{P_L}$$ ← same condition as Question 1

$$\Rightarrow \quad L = \frac{(1-\alpha)}{\alpha} \frac{P_K}{P_L} K \quad \text{(*)}$$

plugging into (10)

$$\Rightarrow \quad A K^{\alpha} \left(\frac{(1-\alpha)}{\alpha} \frac{P_K}{P_L} K \right)^{1-\alpha} = \bar{Q}$$

$$\Rightarrow \quad K^* = \frac{1}{A} \left(\frac{\alpha}{(1-\alpha)} \frac{P_L}{P_K} \right)^{1-\alpha} \bar{Q}$$

plugging into (*)

$$L^* = \frac{(1-\alpha)}{\alpha} \frac{P_K}{P_L} \left(\frac{1}{A} \left(\frac{\alpha}{(1-\alpha)} \frac{P_L}{P_K} \right)^{1-\alpha} \bar{Q} \right)$$

$$= \frac{1}{A} \left(\left(\frac{\alpha}{(1-\alpha)} \right)^{-1} \left(\frac{\alpha}{(1-\alpha)} \right)^{1-\alpha} \right) \left(\left(\frac{P_K}{P_L} \right)^{-1} \left(\frac{P_K}{P_L} \right)^{1-\alpha} \right) \bar{Q}$$

$$= \frac{1}{A} \left(\frac{\alpha}{(1-\alpha)} \frac{P_K}{P_L} \right)^{-\alpha} \bar{Q}$$

or $$L^* = \frac{1}{A} \left(\frac{(1-\alpha)}{\alpha} \frac{P_L}{P_K} \right)^{\alpha} \bar{Q}$$

$\Rightarrow$ In summary,

$$K^*(P_K, P_L, A, \alpha, \bar{Q}), L^*(P_K, P_L, A, \alpha, \bar{Q}) = \left(\frac{1}{A} \left(\frac{\alpha}{(1-\alpha)} \frac{P_L}{P_K} \right)^{1-\alpha} \bar{Q} \; , \; \frac{1}{A} \left(\frac{(1-\alpha)}{\alpha} \frac{P_K}{P_L} \right)^{\alpha} \bar{Q} \right)$$

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Question 5 (Math)

5a) i) A Fixed Cost is a cost that does not vary with output.

$\Rightarrow$ Set $Q=0$ $\Rightarrow$ $TC = 20(0)^2 + 8(0) + 90$

$\Rightarrow$ $\boxed{FC = 90}$

ii) $VC = TC - FC$

$\Rightarrow \underbrace{20Q^2 + 8Q + 90}_{TC} - \underbrace{90}_{FC}$

$\Rightarrow \boxed{VC = 20Q^2 + 8Q}$

iii) Average Total Cost (ATC)

$\Rightarrow \boxed{\frac{TC}{Q} = 20Q + 8 + 90/Q}$

iv) Average Variable Cost (AVC)

$\Rightarrow \frac{VC}{Q} = \frac{20Q^2 + 8Q}{Q} = \boxed{20Q + 8}$

5b) Minimum Average Total Cost (ATCmin) happens when

$ATC = MC$

$\Rightarrow 20Q + \cancel{8} + 90/Q = 40Q + \cancel{8}$

$90/Q = 20Q$

$\Rightarrow 90/20 = Q^2 \Rightarrow Q = \sqrt{9/2}$

$\boxed{= 3/\sqrt{2}}$

5c) Minimum AVC occures when

$AVC = MC$

$\Rightarrow 20Q + \cancel{8} = 40Q + \cancel{8}$

$\Rightarrow 20Q = 0$ $\Rightarrow$ @ $\boxed{Q=0}$

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