1a)

Using equation (1) & (2)

(1) $\Rightarrow \quad q_1 = \frac{1}{2} q_2$

plugging into (2)

$$\Rightarrow \quad q_2 = \frac{(a-c)}{2b} + \frac{1}{2} q_1$$

$$\Rightarrow \quad q_2 = \frac{(a-c)}{2b} + \frac{1}{2}\left[\frac{1}{2} q_2\right]$$

$$\Rightarrow \quad q_2 = \frac{(a-c)}{2b} + \frac{1}{4} q_2$$

$$\Rightarrow \quad 4 q_2 = \frac{4(a-c)}{2b} + q_2$$

$$\Rightarrow \quad q_2^* = \frac{4(a-c)}{6 \cdot 2b} = \frac{2(a-c)}{3b}$$

$$\Rightarrow \quad q_1^* = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}(a-c)}{3b}\right] = \frac{(a-c)}{3b}$$ ← Familiar?

Question #1 (Math)

1b) Prices

$$\Rightarrow \quad P(q_1^*) = a - \cancel{b}\left[\frac{(a-c)}{3\cancel{b}}\right] = \frac{2a+c}{3}$$

$$P(q_2^*) = a - \cancel{b}\left[\frac{2(a-c)}{3\cancel{b}}\right] = \frac{3a - 2a + 2c}{3} = \frac{a+2c}{3}$$

$\Rightarrow$ If $a > c$

$$P(q_1^*) > P(q_2^*)$$

$$\frac{\cancel{2}a + \cancel{c}}{\cancel{3}} > \frac{\cancel{a} + 2c}{\cancel{3}}$$

$\Rightarrow \quad a > c$ ✓ by assumption.

$\Rightarrow$

1c) πs

①
$$\pi_1 = \left(\frac{2a+c}{3}\right)\left(\frac{(a-c)}{3b}\right) - (c)\left(\frac{(a-c)}{3b}\right)$$ Factoring out

$$= \left(\frac{2a+c-3c}{3}\right)\left(\frac{(a-c)}{3b}\right)$$

Duopoly

$$= \frac{2(a-c)}{3}\frac{(a-c)}{3b} = \frac{2(a-c)^2}{9b} = 2\pi^C$$

②
$$\pi_2 = \underbrace{\left(\frac{a+2c}{3} - c\right)}_{p(q_2^*)-c}\left(\underbrace{\frac{2(a-c)}{3b}}_{q_2^*} - \underbrace{\frac{(a-c)}{3b}}_{q_1^*}\right)$$

$$= \left(\frac{a-c}{3}\right)\left(\frac{(a-c)}{3b}\right) = \frac{(a-c)^2}{9b} = \pi^{*C}$$

Profit in typical Cournot duopoly

$$\Rightarrow \pi^* = \pi_1^* + \pi_2^*$$

$$= \frac{2(a-c)^2}{9b} + \frac{(a-c)^2}{9b} = \frac{3(a-c)^2}{9b} = \frac{(a-c)^2}{3b} = \pi^{PD}$$

1d) using monopoly π's from HW2

$\underline{\text{wsget}}$, $$\pi^M < \pi^{PD}$$

$$\Rightarrow \frac{(a-c)^2}{4b} < \frac{(a-c)^2}{3b}$$ $\Rightarrow$ Firms have incentive to price $\underline{\text{discriminate}}$.

Question #2 (Math)

2a)

using (3)

$$\Rightarrow \quad 2bQ = a-c$$

$$\Rightarrow \quad Q^* = \frac{(a-c)}{2b}$$

$$\Rightarrow \quad q_i^{\text{Cartel}} = \frac{(a-c)}{2b}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad q_i^{\text{Cartel}} = \frac{(a-c)}{4b}$$

2b)

$$P(Q^*) = (a - b(Q^*))$$

$$= a - \cancel{b}\left(\frac{(a-c)}{2\cancel{b}}\right)$$

$$= \frac{2a-a+c}{2} = \frac{a+c}{2}$$

Same as the Monopoly

2c)

$$\pi^* = P(Q^*)Q^* - cQ^* - F$$

$$\Rightarrow \quad \left(\left(\frac{a+c}{2}\right) - c\right)\left(\frac{(a-c)}{2b}\right) - F$$

$$\Rightarrow \quad \left(\frac{(a-c)}{2}\right)\left(\frac{(a-c)}{2b}\right) - F = \frac{(a-c)^2}{4b} - F = \pi^{\text{Monopoly}}$$

2d)

$$\Rightarrow \quad \pi^{\text{Cartel}} = \frac{\pi^{\text{Monopoly}}}{N}$$

$$\Rightarrow \quad \pi^{\text{Cartel}} = \left(\frac{(a-c)^2}{4b} - F\right)/2 \qquad \text{where } N=2$$

$$\Rightarrow \quad = \frac{(a-c)^2}{8b} - \frac{F}{2}$$

3a)
<u>using</u> <u>(4) & (5)</u>

Question #3 (MATH)

$\Rightarrow$ (4) $2bq_i = a - c_i - bq_j$

$\Rightarrow BRF_i = q_i(q_j) = \frac{(a-c_i)}{2b} - \frac{1}{2}q_j$

Combining

(5) $2bq_j = a - c_j - bq_i$

$\Rightarrow BRF_j = q_j(q_i) = \frac{(a-c_j)}{2b} - \frac{1}{2}q_i$

$\Rightarrow q_i(q_j) = \frac{(a-c_i)}{2b} - \frac{1}{2}\left[\frac{(a-c_j)}{2b} - \frac{1}{2}q_i\right]$ everything is in terms of q_i

$\Rightarrow q_i = \frac{(a-c_i)}{2b} - \frac{(a-c_j)}{4b} + \frac{1}{4}q_i$

(x4) both sides

$\Rightarrow 4bq_i = 2(a-c_i) - (a-c_j) + b\cancel{q}_i$

$\Rightarrow 3bq_i = (2a-a) - 2c_i + c_j$

$\Rightarrow q_i^* = \frac{(a-2c_i+c_j)}{3b}$

Plug into other BRF_j

$\Rightarrow q_j(q_i^*) = \frac{(a-c_j)}{2b} - \frac{1}{2}\left[\frac{(a-2c_i+c_j)}{3b}\right]$

$q_j^* = \frac{(a-c_j)}{2b} - \frac{(a-2c_i)+c_j}{6b}$

$= \frac{3(a-c_j)-(a-2c_i+c_j)}{6b}$

$= \frac{(2a - 3c_j + 2c_i - c_j)}{6b}$

*Also, Cournot Cournot q_i, q_j by notation.

$\Rightarrow q_j^* = \frac{(2a - 4c_j + 2c_i)}{6b} = \frac{2(a-2c_j+c_i)}{6b} = \frac{(a-2c_j+c_i)}{3b}$

$\Rightarrow (q_i^*, q_j^*) = \left(\frac{(a-2c_i+c_j)}{3b}, \frac{(a-2c_j+c_i)}{3b}\right)$

3b) $\Rightarrow (q_i^*, q_j^*)$ become $\frac{(a-2c+c)}{3b}$ since $c_i = c_j = c$

$$\Rightarrow \left(q_i^{\text{cournot}}, q_j^{\text{cournot}}\right) = \left(\frac{a-c}{3b}, \frac{a-c}{3b}\right) \equiv q_i^*$$

3c)

$$P(Q^*) = a - b\left(\frac{(a-c)}{3b} + \frac{(a-c)}{3b}\right)$$

$$= a - \frac{2(a-c)}{3}$$

$$= \frac{(3a-2a+c)}{3} = \frac{a+c}{3}$$

⊛ Note, not a function of b.

3d)

aka q_i^{cournot}

$$\Rightarrow \pi_i^* = p^*(Q^*)q_i^* - (F + cq_i^*)$$

$$\Rightarrow \quad = \left(\frac{a+c}{3}\right)\left(\frac{(a-c)}{3b}\right) - \left(F + c\left(\frac{(a-c)}{3b}\right)\right)$$

$$= \left(\frac{a+c}{3} - c\right)\left(\frac{(a-c)}{3b}\right) - F$$

$$= \frac{(a-c)}{3}\frac{(a-c)}{3b} - F$$

$$\pi^{*\text{cournot}} = \frac{(a-c)^2}{9b} - F$$

Note the similarity w/

$$\boxed{{*}\text{Imperfect Competition} \quad \pi = \frac{(a-c)^2}{(N+1)^2 b} - F}$$

3e) From Q2

$$\pi^{\text{Cartel}} = \frac{(a-c)^2}{8b} - \frac{F}{2}$$

(f) $\Rightarrow$

3f)

$$\pi_i^{Dev} = \left(a - b\left(q_i^{Cournot} + q_i^{cartel}\right)\right)\left(q_i^{Cournot}\right) - c\left(q_i^{Cournot}\right) - F$$

$$= \left(a - b\left(\frac{(a-c)}{3b} + \frac{(a-c)}{4b}\right)\right)\left(\frac{(a-c)}{3b}\right) - c\left(\frac{(a-c)}{3b}\right) - F$$

$$\Rightarrow \; = \left(\frac{12a - 4(a-c) - 3(a-c) - 12c}{12}\right)\frac{(a-c)}{3b} - F$$

$$= \frac{12(a-c) - 7(a-c)}{12}\;\frac{(a-c)}{3b} - F$$

$$= \frac{5(a-c)}{36\,b} - F$$

3g) Similarly, assume you act as cartel while they compete

$$\Rightarrow \pi_i^{NDev} = \underbrace{\left(a - b\left(\frac{(a-c)}{3b} + \frac{(a-c)}{4b}\right)\right)}\left(\frac{(a-c)}{4b}\right) - c\left(\frac{(a-c)}{4b}\right) - F$$

$$= \frac{5(a-c)}{12}\;\frac{(a-c)}{4b} - F$$

$$= \frac{5(a-c)^2}{48\,b} - F$$

3h) <u>Summarizing</u>

$$\Rightarrow \pi_i^* = \pi_i^{Cournot} = \frac{(a-c)^2}{9b} - F \;, \quad \pi_i^{Cartel} = \frac{(a-c)^2}{8b} - \frac{F}{2}$$

$$\pi_i^{Dev} = \frac{5(a-c)^2}{36b} - F \;, \quad \pi_i^{ND} = \frac{5(a-c)}{48b} - F$$

Shutting down F (ie set F=0),

⟹ <u>Simplifying</u>, & <u>comparing</u>

Interesting

$$\underbrace{\frac{(a-c)^2}{36b}}_{\pi_i^{Dev}} > \underbrace{\frac{(a-c)^2}{40b}}_{\pi_i^{Cartel}} > \underbrace{\frac{(a-c)^2}{45b}}_{\pi_i^{Cournot} = \pi_i^* = \pi_i^{Compete}} > \underbrace{\frac{(a-c)^2}{48b}}_{\pi_i^{NDev}}$$

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3h (continued))

⇒

		Firm j	
		Cartel	Compete
Firm i	Cartel	$\pi_i^{Cartel}, \pi_j^{Cartel}$	$\pi_i^{NDev}, \pi_j^{Dev}$
	Compete	$\pi_i^{Dev}, \pi_j^{NDev}$	$\pi_i^{Cournot}, \pi_j^{Cournot}$

plugging in values

⇒

		Firm j	
		Cartel	Compete
Firm i	Cartel	$\frac{(a-c)^2}{40b}, \frac{(a-c)^2}{40b}$	$\frac{(a-c)^2}{48b}, \frac{(a-c)^2}{36b}$
	Compete	$\frac{(a-c)^2}{36b}, \frac{(a-c)^2}{48b}$	$\frac{(a-c)^2}{45b}, \frac{(a-c)^2}{45b}$

3i)

where, doing our Best Response Analysis s.t.

Firm i: ① If Firm j plays Cartel, I play Compete ($\pi_i^{Dev} > \pi_i^{Cartel}$)

② If Firm j plays Compete, I play Compete ($\pi_i^{Cournot} > \pi_j^{NDev}$)

Firm j: ① If Firm i plays Cartel, I play Compete ($\pi_j^{Dev} > \pi_j^{Cartel}$)

② If Firm i plays Compete, I play Compete ($\pi_j^{Cournot} > \pi_j^{NDev}$)

⇒

		Firm j	
		Cartel	Compete
Firm i	Cartel	$\pi_i^{Cartel}, \pi_j^{Cartel}$	$\pi_i^{NDev}, \underline{\pi_j^{Dev}}$
	Compete	$\underline{\pi_i^{Dev}}, \pi_j^{NDev}$	$\underline{\pi_i^{Cournot}}, \underline{\pi_j^{Cournot}}$

underlining all Best Responses

⇒ Nash Equilibrium $= (q_i^{*Cournot}, q_j^{*Cournot}) = \{(\frac{(a-c)}{3b}, \frac{(a-c)}{3b})\}$

Question #4 (Math)

4a) Bertrand Competitive Equilibrium

$\Rightarrow \quad p^* = c \quad$ where $\quad c = 0$

$\Rightarrow \quad p^* = 0$

$\Rightarrow \quad \pi^* = 0$

4b) using equation (6)

$\Rightarrow \quad 1 - 2p = 0$

$\Rightarrow \quad p^* = \frac{1}{2} = p^m = p^c$

$\Rightarrow \quad \pi^{\text{Monopoly}} = p^c(1 - p^c)$

$= \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$

$\Rightarrow \quad \pi_i^{\text{Cartel}} = \frac{\pi^{\text{Monoply}}}{N} = \frac{1}{4} * \frac{1}{2} \quad$ where $N = 2$

$= \frac{1}{8}$

4c)

$p^{Dev} = p^{\text{Cartel}} - \varepsilon = \frac{1}{2} - \varepsilon$

$\Rightarrow \quad \pi^{Dev} = p^D(1 - p^D)$

$= \left(\frac{1}{2} - \varepsilon\right)\left(1 - \left(\frac{1}{2} - \varepsilon\right)\right)$

$= \left(\frac{1}{2} - \varepsilon\right)\left(\frac{1}{2} + \varepsilon\right)$ ← Difference of squares

$= \frac{1}{4} - \varepsilon^2$

$\pi^{NDev} = 0$ since $Q^{NDev} = 0$

$\Rightarrow$

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3d)

$\Rightarrow$

	Cartel	Compete
Cartel	$\frac{1}{8}, \frac{1}{8}$	$0, \frac{1}{4}-\varepsilon^2$
Compete	$\frac{1}{4}-\varepsilon^2, 0$	$0, 0$

$\Rightarrow$ We want $\pi^{\text{Cartel}} > \pi^{\text{Dev}}$ to sustain collusion

$$\Rightarrow \quad \frac{1}{8} > \frac{1}{4}-\varepsilon^2$$

$$\Rightarrow \quad \varepsilon^2 > \frac{1}{8}$$

$$\varepsilon > \sqrt{1/8}$$

$$\Rightarrow \quad \varepsilon > \frac{1}{\sqrt{4}\sqrt{2}}$$

$$\varepsilon > \frac{1}{2\sqrt{2}} \equiv \bar{\varepsilon}$$

$\Rightarrow$ If $\varepsilon > \bar{\varepsilon}$ cartel will be sustained.

price decrease is too insane.

Question #5 (math)

5a) Knowing, $MR = p + \frac{\Delta P}{\Delta q} q$ & that $MR = MC$ is the monopolist's profit maximization condition

$$\Rightarrow \quad MR = MC$$

$$\Rightarrow \quad p + \frac{\Delta P}{\Delta Q} q = MC$$

multiply by 1

$$\Rightarrow \quad p + \frac{\Delta P}{\Delta Q} q \left(\frac{p}{p}\right) = MC$$

$$\Rightarrow \quad P + \frac{\Delta P}{\Delta q} \frac{q}{p} P = MC$$

since $\frac{\Delta P}{\Delta q} \frac{q}{p} = \frac{\Delta P/p}{\Delta q/q} = \frac{1}{\varepsilon_D}$

$$\Rightarrow \quad P + \frac{\Delta P/p}{\Delta q/q} P = MC$$

$$\Rightarrow \quad P + \frac{1}{\varepsilon_D} P = MC$$

$$\Rightarrow \quad P - MC = -\frac{1}{\varepsilon_D} P$$

$$\Rightarrow \quad \frac{(P - MC)}{P} = -\frac{1}{\varepsilon_D}$$

It tells us the % mark-up in price relative to the perfectly competitive equilibrium

5b) solve for p

$$\Rightarrow \quad \frac{(p - MC)}{p} = -\frac{1}{\varepsilon_D}$$

$$\Rightarrow \quad p + \frac{1}{\varepsilon_D} p = MC$$

$$\Rightarrow \quad p = \frac{\varepsilon_D}{(1 + \varepsilon_D)} MC \equiv \underline{IERP}$$

5c) Assuming $\varepsilon_1^D > \varepsilon_2^D$, we start our proof w/ what we are trying to prove such that

$$P_2 > P_1$$

$$\Rightarrow P(MC, \varepsilon_2^D) > P(MC, \varepsilon_1^D)$$

invoking IEPR

$$\Rightarrow \frac{\varepsilon_D^2}{(1+\varepsilon_D^2)} \cancel{MC} > \frac{\varepsilon_D^1}{(1+\varepsilon_D^1)} \cancel{MC}$$

$$\Rightarrow \frac{(1+\varepsilon_D^1)}{\varepsilon_D^1} > \frac{(1+\varepsilon_D^2)}{\varepsilon_D^2}$$

$$\Rightarrow \frac{1}{\varepsilon_D^1} + \cancel{1} > \frac{1}{\varepsilon_D^2} + \cancel{1}$$

$$\Rightarrow \frac{1}{\varepsilon_D^1} > \frac{1}{\varepsilon_D^2}$$

$$\Rightarrow \varepsilon_D^2 > \varepsilon_D^1$$

where If $\varepsilon_D^i < 0$ Then the sign flips and the result holds true when we take the absolute value

$$\Rightarrow |\varepsilon_D^2| < |\varepsilon_D^1|$$