2a)

Question # 2 (Math)

using equations (8) & (9)

=> Solve for λ & set equal

=> $$\frac{\alpha q_1^{(\alpha-1)} \cancel{q_2^{\beta}}}{P_1} = \frac{\beta \cancel{q_1^{\alpha}} q_2^{(\beta-1)}}{P_2} \equiv \lambda$$

=> $$q_2 = \frac{\beta}{\alpha} \frac{P_1}{P_2} q_1 \quad \text{where } q_2(q_1) \qquad (*)$$

=> Plug into (10)

=> $$P_1 q_1 + \cancel{P_2}\left(\frac{\beta}{\alpha} \cdot \frac{P_1}{\cancel{P_2}} q_1\right) = M$$

=> $$P_1 q_1 + \frac{\beta}{\alpha} P_1 q_1 = M$$

=> $$P_1 q_1 \left(1 + \frac{\beta}{\alpha}\right) = M$$

=> $$q_1^* = \frac{\alpha}{(\alpha+\beta)} \frac{M}{P_1}$$

plug into (*)

$$q_2^* = \frac{\beta}{\cancel{\alpha}} \frac{\cancel{P_1}}{P_2} \left(\frac{\cancel{\alpha}}{(\alpha+\beta)} \frac{M}{\cancel{P_1}}\right)$$

$$= \frac{\beta}{(\alpha+\beta)} \frac{M}{P_2}$$

=> $$q_1^*, q_2^* = \left(\frac{\alpha}{(\alpha+\beta)} \frac{M}{P_1}, \frac{\beta}{(\alpha+\beta)} \frac{M}{P_2}\right)$$

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3a) using equation (11) & (12) we can solve for λ s.t.

Question #3 (Math)

(11) (12)

$$\Rightarrow \frac{\alpha}{q_1} = \lambda P_1(1+\tau) \quad \& \quad \frac{\beta}{q_2} = \lambda P_2$$

$$\Rightarrow \frac{\alpha}{q_1 P_1(1+\tau)} = \lambda \quad \& \quad \frac{\beta}{q_2 P_2} = \lambda$$

Combining

$$\Rightarrow \frac{\alpha}{q_1 P_1(1+\tau)} = \frac{\beta}{q_2 P_2} \equiv \lambda$$

$$\Rightarrow q_2 = \frac{\beta}{\alpha} \frac{P_1(1+\tau)}{P_2} q_1 \quad (*)$$

where q_2 is is a function of q_1 s.t. $q_2(q_1)$

plugging into (13)

$$\Rightarrow P_1(1+\tau)q_1 + \cancel{P_2}\left(\frac{\beta}{\alpha}\frac{P_1(1+\tau)}{\cancel{P_2}} q_1\right) = M$$

$$\Rightarrow P_1(1+\tau)q_1 + \frac{\beta}{\alpha} P_1(1+\tau)q_1 = M$$

$$\Rightarrow P_1(1+\tau)q_1\left(1 + \frac{\beta}{\alpha}\right) = M$$

$$\Rightarrow P_1(1+\tau)q_1 = \frac{\alpha}{(\alpha+\beta)} M$$

$$\Rightarrow q_1^* = \frac{\alpha}{(\alpha+\beta)} \frac{M}{P_1(1+\tau)}$$

(q_1^*, q_2^*)

plugging into (*)

$$\Rightarrow q_2^* = \frac{\beta}{\cancel{\alpha}} \frac{\cancel{P_1(1+\tau)}}{P_2}\left(\frac{\cancel{\alpha}}{(\alpha+\beta)} \frac{M}{\cancel{P_1(1+\tau)}}\right) = \frac{\beta}{(\alpha+\beta)} \frac{M}{P_2} \quad //$$

4a) using equations (14) & (15)

Question #4 (Math)

$\Rightarrow$ (14) (15)

$$\frac{\alpha(q_1-\gamma_1)^{\cancel{\alpha-1}}(q_2-\gamma_2)^{\cancel{1-\alpha}}}{p_1\tau} = \frac{(1-\alpha)\cancel{(q_1-\gamma_1)^{\alpha}}\cancel{(q_2-\gamma_2)^{-\alpha}}}{p_2} \equiv \lambda$$

(both equal to λ)

$\Rightarrow$ solving for $q_2(q_1)$

$$(q_2-\gamma_2) = \frac{(1-\alpha)}{\alpha}\frac{p_1\tau}{p_2}(q_1-\gamma_1)$$

$$\Rightarrow \quad q_2 = \gamma_2 + \frac{(1-\alpha)}{\alpha}\frac{p_1\tau}{p_2}(q_1-\gamma_1) \qquad (*)$$

plugging into BC (16), & solve for q_1^*

$$\Rightarrow \quad p_1 q_1\tau + p_2\left(\gamma_2 + \frac{(1-\gamma)}{\alpha}\frac{p_1}{p_2}\tau(q_1-\gamma_1)\right) = M$$

$$p_1 q_1\tau + p_1\tau(q_1-\gamma_1)\frac{(1-\alpha)}{\alpha} + p_2\gamma_2 = M$$

$$p_1 q_1\tau + p_1\tau\frac{(1-\alpha)}{\alpha} - p_1\tau\gamma_1\frac{(1-\alpha)}{\alpha} + p_2\gamma_2 = M$$

$$p_1 q_1\tau\left(1+\frac{(1-\alpha)}{\alpha}\right) - p_1\tau\gamma_1\frac{(1-\alpha)}{\alpha} + p_2\gamma_2 = M$$

$$p_1 q_1\tau\left(\frac{1}{\alpha}\right) - p_1\tau\gamma_1\frac{(1-\alpha)}{\alpha} + p_2\gamma_2 = M$$

$$\Rightarrow \quad \frac{p_1 q_1\tau}{\alpha} = p_1\tau\gamma_1\frac{(1-\alpha)}{\alpha} - p_2\gamma_2 + M$$

$$\Rightarrow \quad q_1^* = \gamma_1(1-\alpha) + \frac{\alpha(M-p_2\gamma_2)}{p_1\tau}$$

plug back into $(*)$ to get q_2^*

$\Rightarrow$

$\Rightarrow$ from $\oplus$

$$q_2 = \gamma_2 + \frac{(1-\alpha)}{\alpha}\frac{P_1\tau}{P_2}\left(\gamma_1(1-\alpha) + \frac{\alpha(M-P_2\gamma_2)}{P_1\tau} - \gamma_1\right)$$

$$q_2 = \gamma_2 + \frac{(1-\alpha)}{\alpha}\frac{P_1\tau}{P_2}\left(\frac{\alpha(M-P_2\gamma_2)}{P_1\tau} - \alpha\gamma_1\right)$$

$$q_2 = \gamma_2 + \frac{(1-\alpha)(M-P_2\gamma_2)}{P_2} - \frac{\cancel{\alpha}\gamma_1(1-\alpha)P_1\tau}{\cancel{\alpha}P_2}$$

$$\Rightarrow q_2 = \gamma_2 + \frac{(1-\alpha)(M-P_2\gamma_2-\gamma_1P_1\tau)}{P_2}$$

$$q_2 = \gamma_2 - (1-\alpha)\gamma_2 + \frac{(1-\alpha)(M-\gamma_1P_1\tau)}{P_2}$$

$$\Rightarrow q_2^* = \alpha\gamma_2 + \frac{(1-\alpha)(M-\gamma_1P_1\tau)}{P_2} \quad .//$$

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5a) using (17) & (18)

Question #5 (Math)

$$\Rightarrow \frac{\alpha q_1^{(\alpha-1)} \cancel{q_2^{\beta}} \cancel{q_3^{\gamma}}}{p_1} = \frac{\beta \cancel{q_1^{\alpha}} q_2^{\beta-1} \cancel{q_3^{\gamma}}}{p_2} \equiv \lambda$$

$$\Rightarrow q_2 = \frac{\beta}{\alpha}\frac{p_1}{p_2} q_1 \quad *1$$

& using (17) & (19)

$$\frac{\alpha q_1^{\alpha-1} \cancel{q_2^{\beta}} \cancel{q_3^{\gamma}}}{p_1} = \frac{\gamma \cancel{q_1^{\alpha}} \cancel{q_2^{\beta}} q_3^{\gamma-1}}{p_2}$$

$$\Rightarrow q_3 = \frac{\gamma}{\alpha}\frac{p_1}{p_3} q_1 \quad *2$$

plugging into BC (20)

$$\Rightarrow p_1 q_1 + \cancel{p_2}\left(\frac{\beta}{\alpha}\frac{p_1}{\cancel{p_2}} q_1\right) + \cancel{p_3}\left(\frac{\gamma}{\alpha}\frac{p_1}{\cancel{p_3}} q_1\right) = M$$

$$\Rightarrow p_1 q_1\left(1 + \frac{\beta}{\alpha} + \frac{\gamma}{\alpha}\right) = M$$

$$\Rightarrow q_1^* = \frac{\alpha}{(\alpha+\beta+\gamma)}\frac{M}{p_1}$$

plugging into *1

$$q_2 = \frac{\beta}{\cancel{\alpha}}\frac{\cancel{p_1}}{p_2}\left(\frac{\cancel{\alpha}}{(\alpha+\beta+\gamma)}\frac{M}{\cancel{p_1}}\right) \Rightarrow q_2^* = \frac{\beta}{(\alpha+\beta+\gamma)}\frac{M}{p_2}$$

& plugging q_1^* into *2

$$q_3 = \frac{\gamma}{\cancel{\alpha}}\frac{\cancel{p_1}}{p_2}\left(\frac{\cancel{\alpha}}{(\alpha+\beta+\gamma)}\frac{M}{\cancel{p_1}}\right) \Rightarrow q_3^* = \frac{\gamma}{(\alpha+\beta+\gamma)}\frac{M}{p_3}$$

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