

Anatomy of C-D Production/Utility Functions in Three Dimensions

[Peter Fuleky](#)

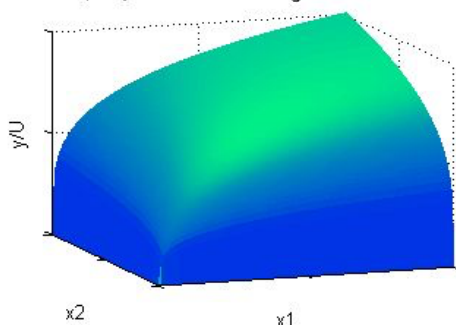
Department of Economics, University of Washington

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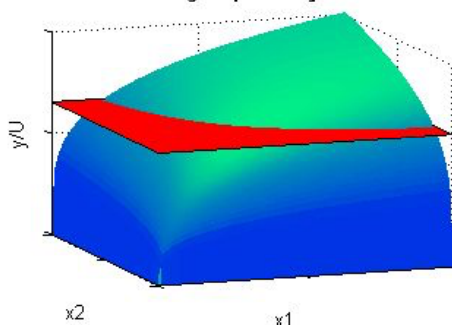
Decreasing returns to scale

(Strongly concave production/utility function)

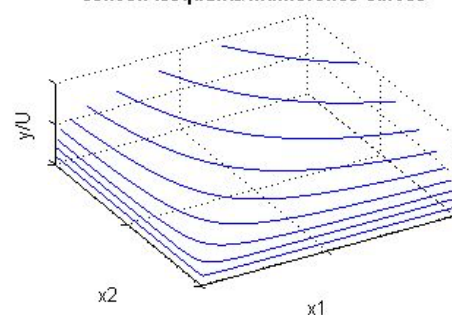
production/utility function = $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.25$ - decreasing returns to scale



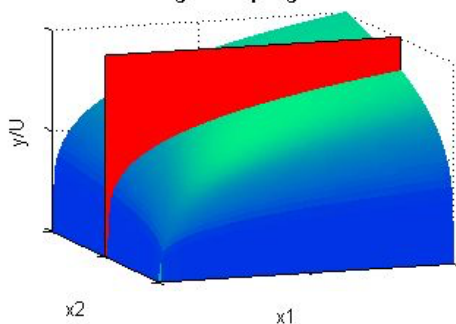
holding output/utility fixed



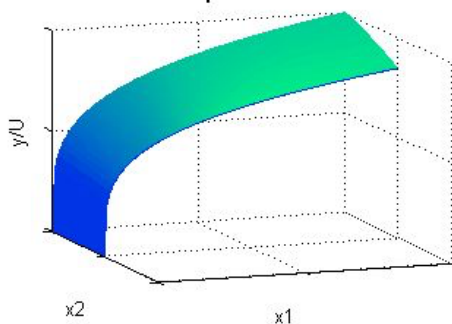
convex isoquants/indifference curves



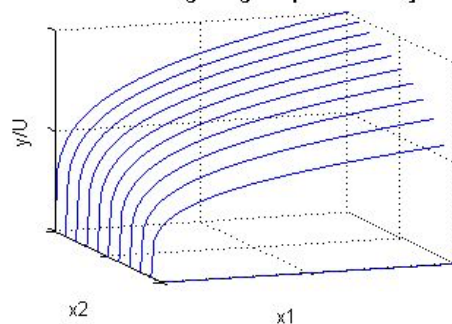
holding one input/good fixed



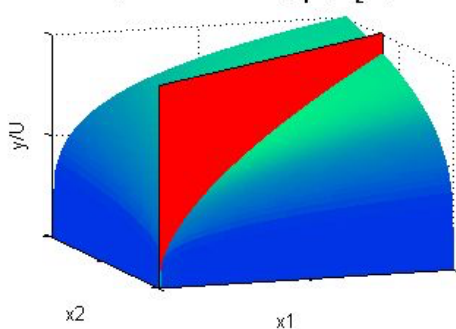
slice parallel to axis



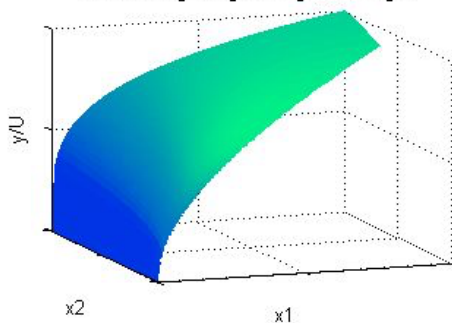
diminishing marginal product/utility



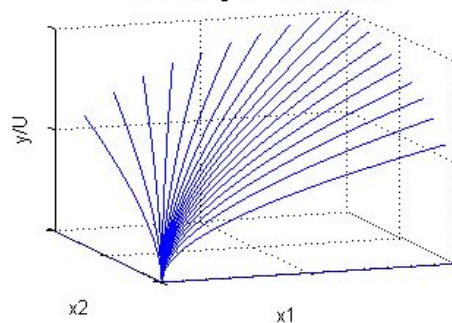
increasing both inputs/goods
by the same factor ($x_1^0=3, x_2^0=2$)



slice along a ray through the origin



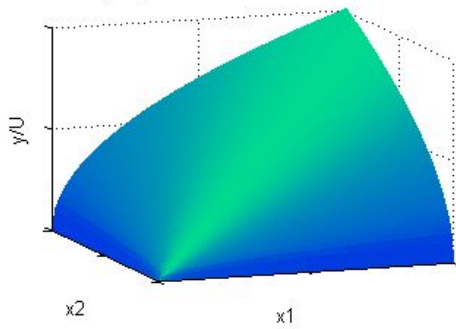
decreasing returns to scale



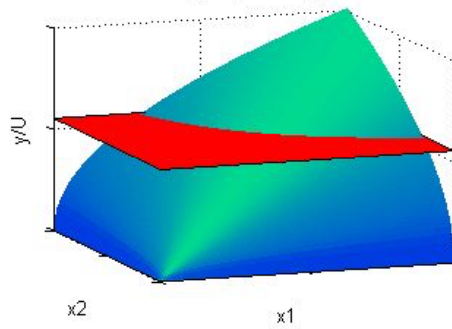
Constant returns to scale

(Weakly concave production/utility function)

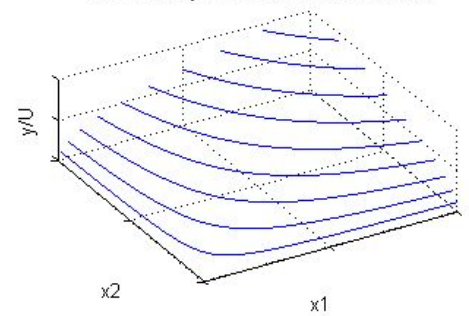
production/utility function = $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.5$ - constant returns to scale



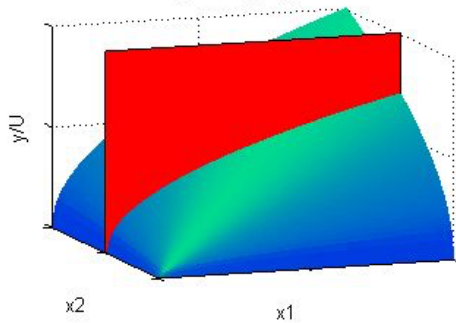
holding output/utility fixed



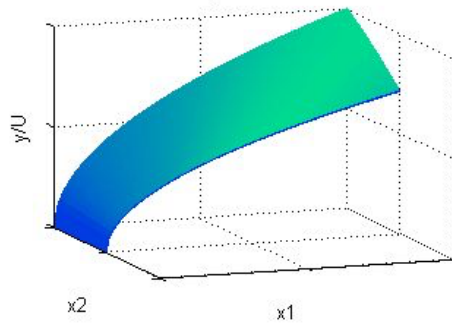
convex isoquants/indifference curves



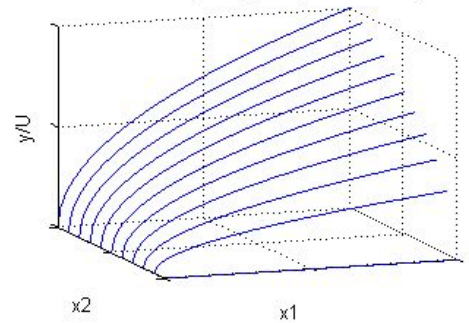
holding one input/good fixed



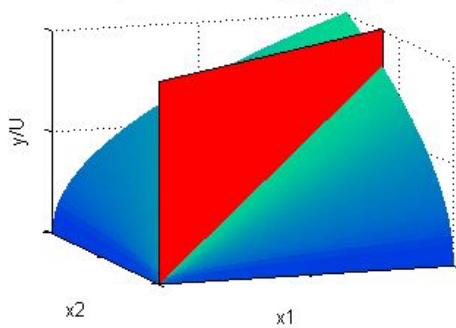
slice parallel to axis



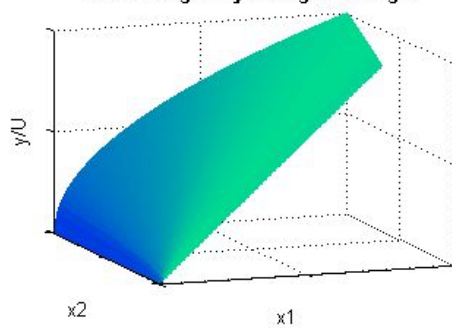
diminishing marginal product/utility



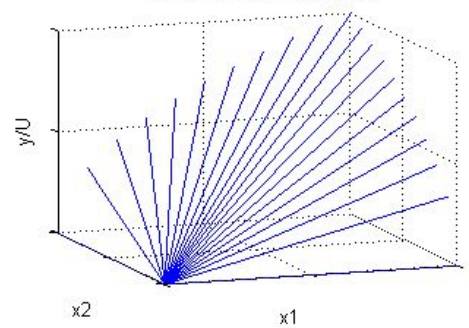
increasing both inputs/goods
by the same factor ($x_1^0=3, x_2^0=2$)



slice along a ray through the origin

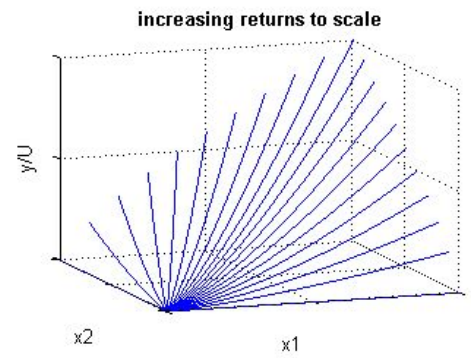
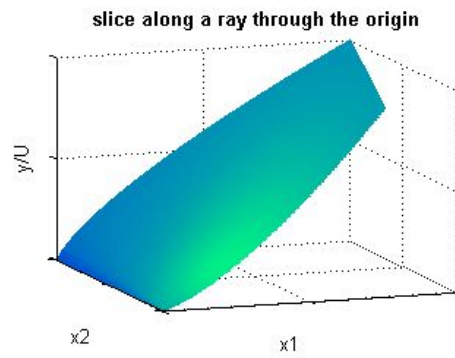
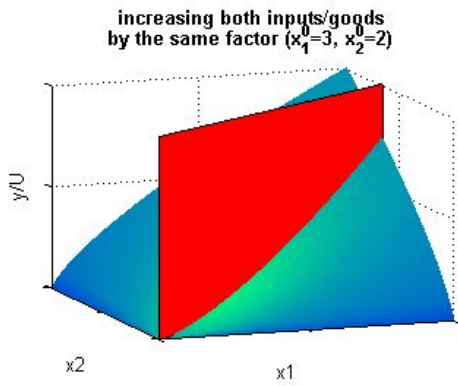
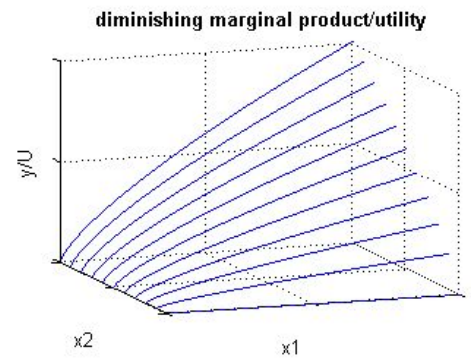
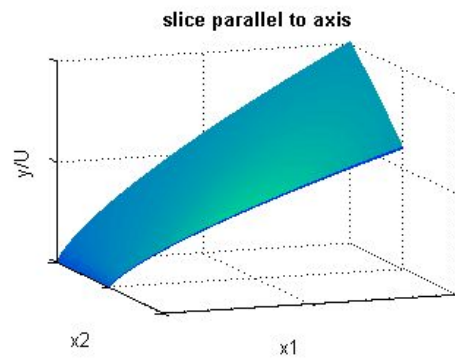
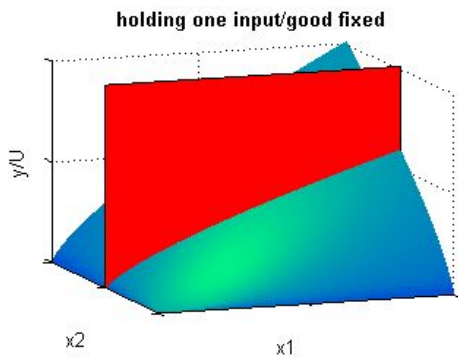
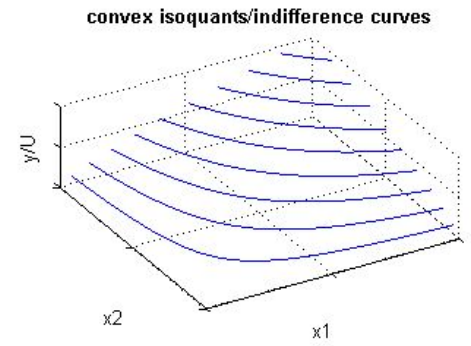
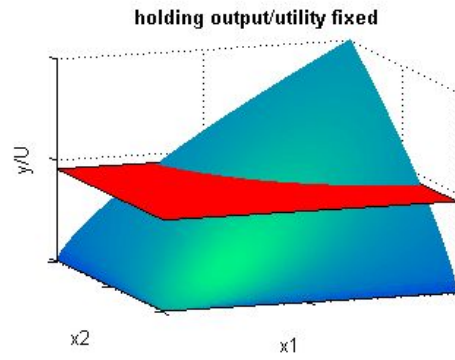
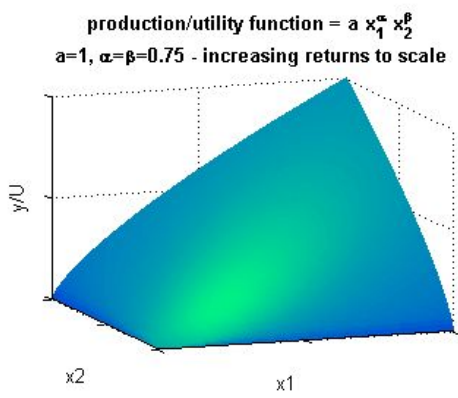


constant returns to scale



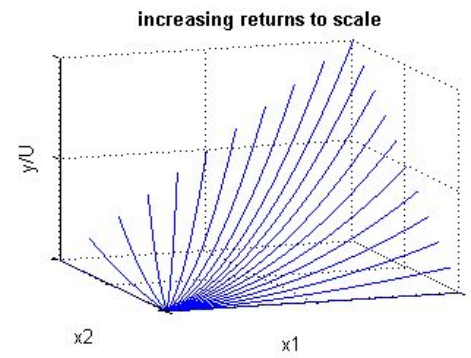
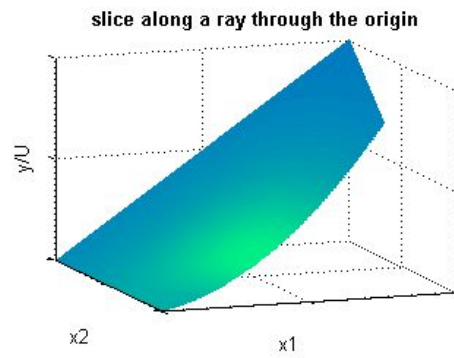
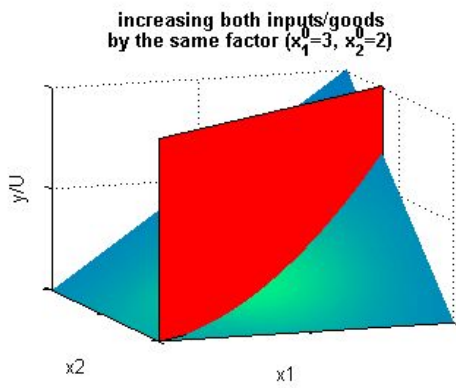
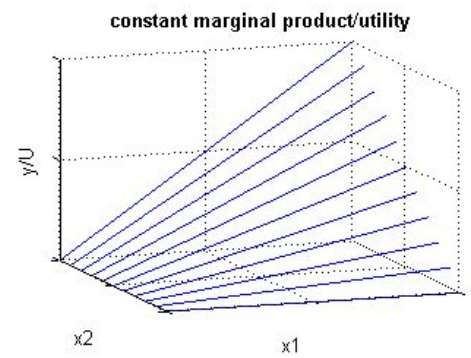
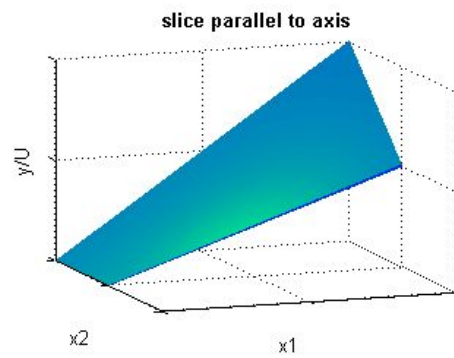
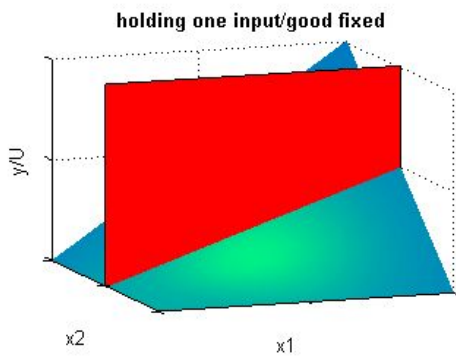
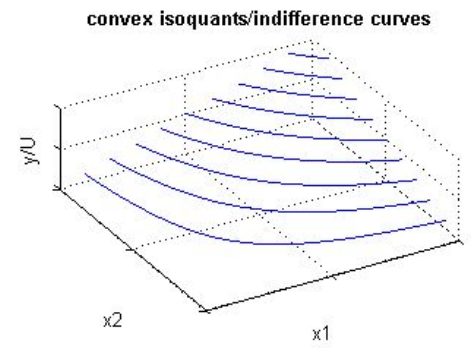
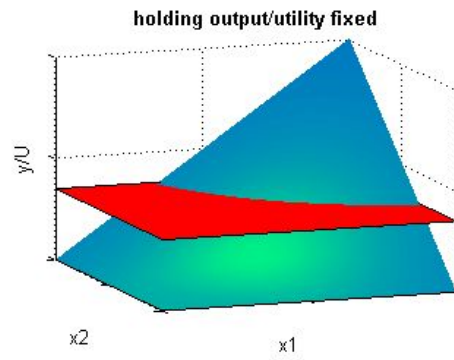
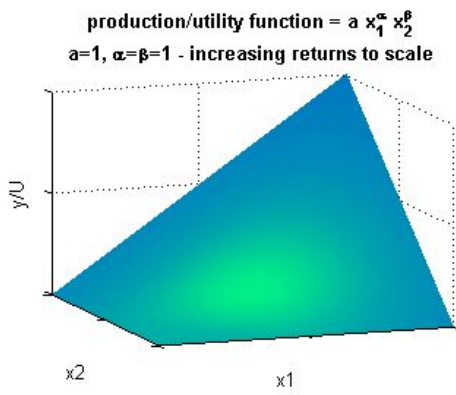
Increasing returns to scale with diminishing marginal product/utility

(Quasiconcave production/utility function)



Increasing returns to scale with constant marginal product/utility

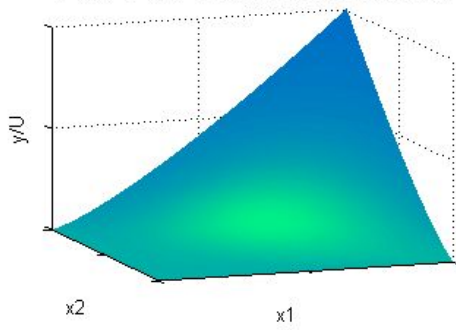
(Quasiconcave production/utility function)



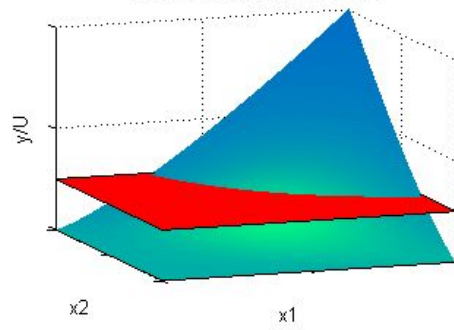
Increasing returns to scale with increasing marginal product/utility

(Quasiconcave production/utility function)

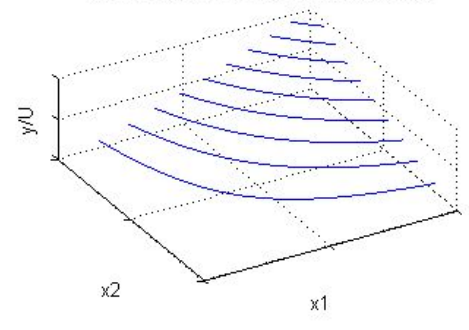
production/utility function = $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=1.25$ - increasing returns to scale



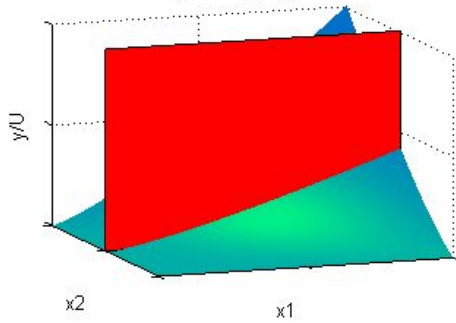
holding output/utility fixed



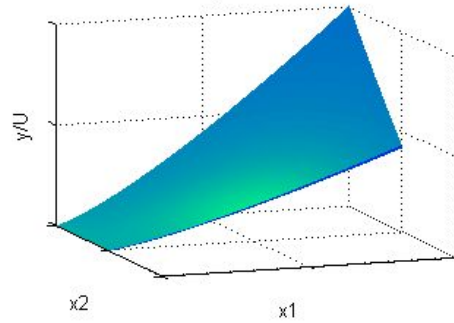
convex isoquants/indifference curves



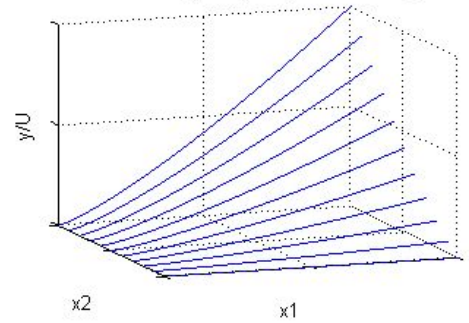
holding one input/good fixed



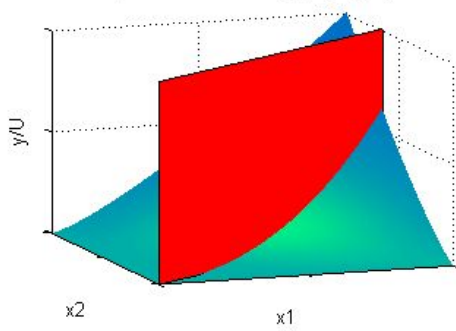
slice parallel to axis



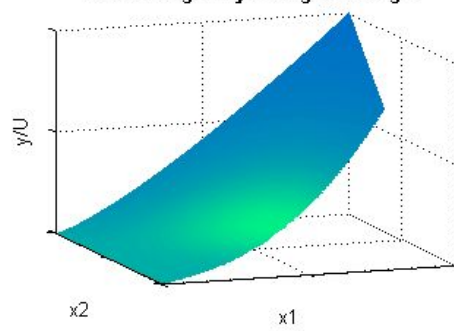
increasing marginal product/utility



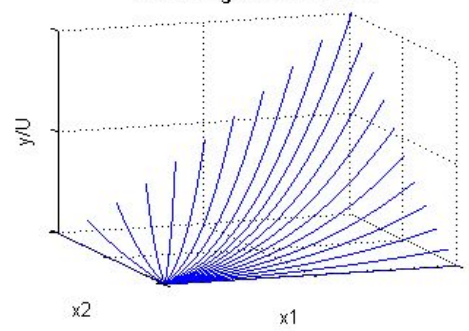
increasing both inputs/goods
by the same factor ($x_1^0=3, x_2^0=2$)



slice along a ray through the origin



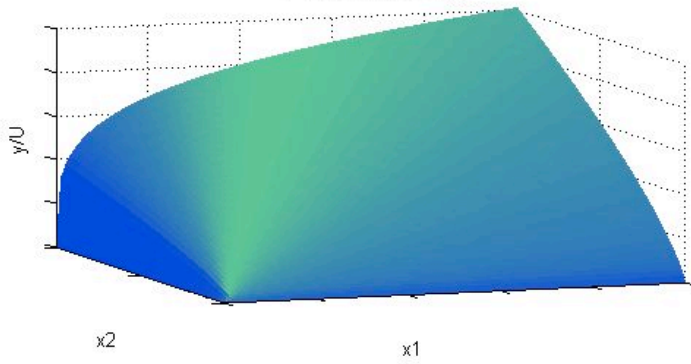
increasing returns to scale



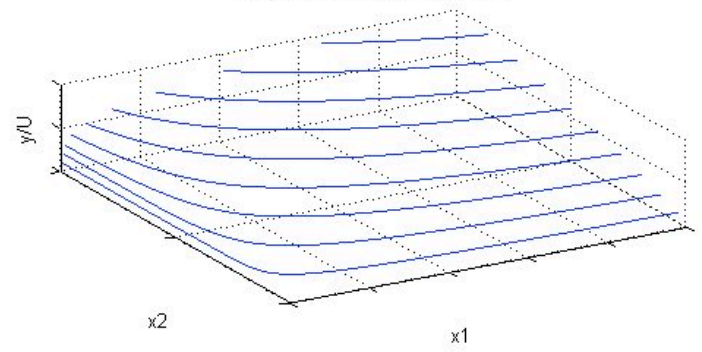
Non-symmetric production/utility function with constant returns to scale

production/utility function: $a x_1^\alpha x_2^\beta$

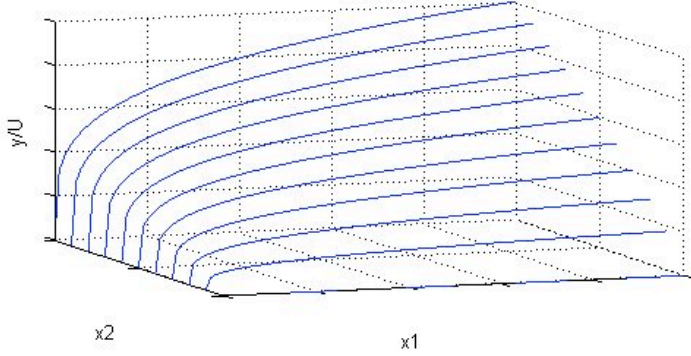
$a=1, \alpha=0.25, \beta=0.75$



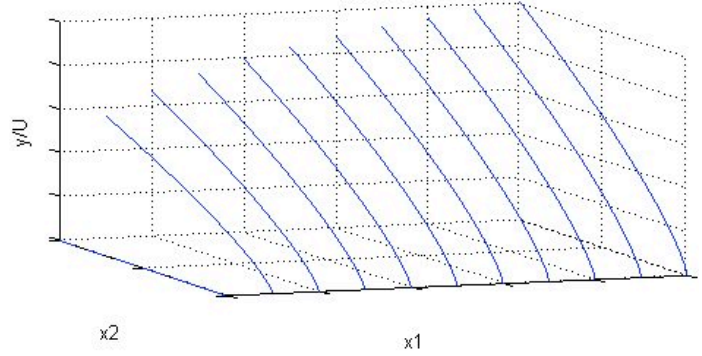
isoquants/indifference curves



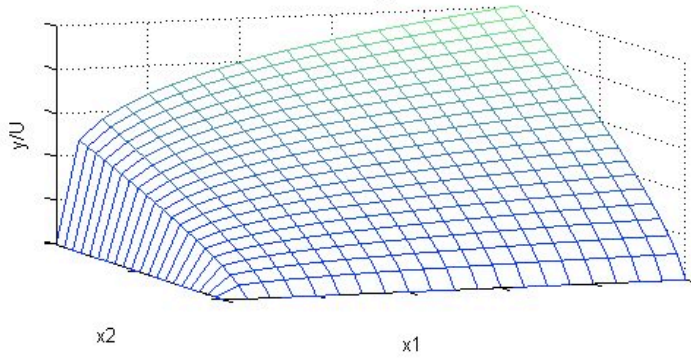
marginal properties (intensive margin)



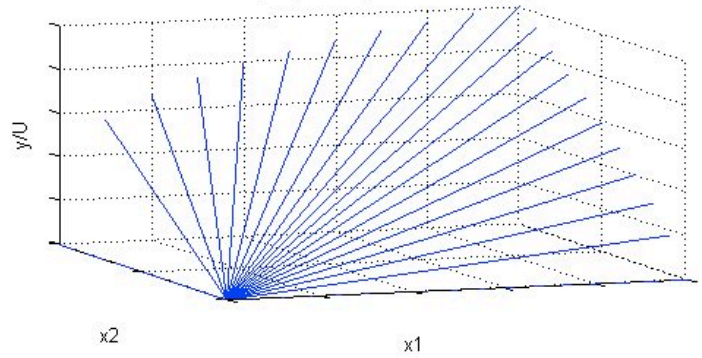
marginal properties (intensive margin)



meshgrid

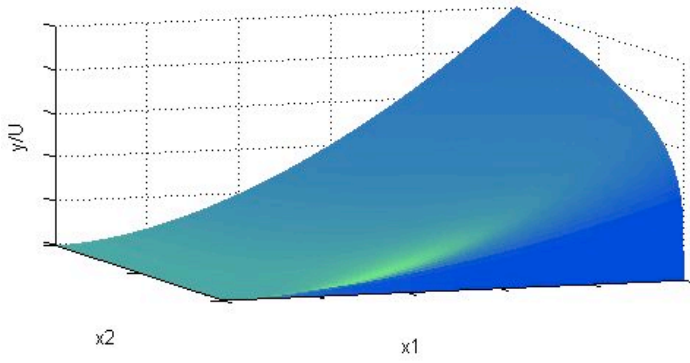


scale properties (extensive margin)

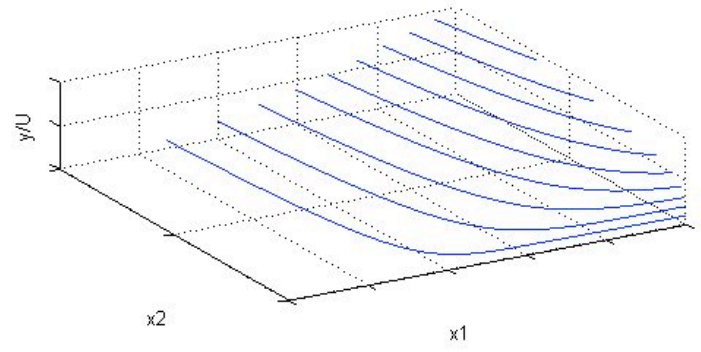


Non-symmetric production/utility function with increasing returns to scale

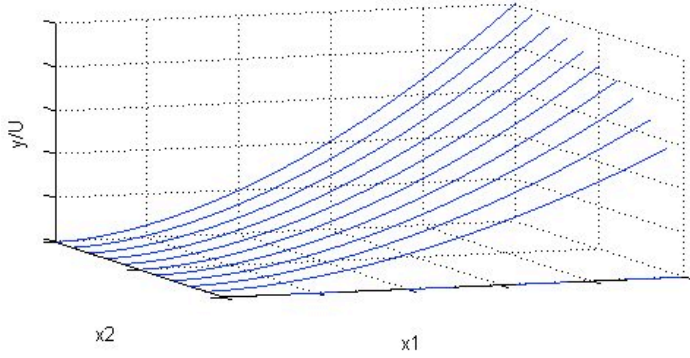
production/utility function: $a x_1^\alpha x_2^\beta$
 $a=1, \alpha=1.75, \beta=0.25$



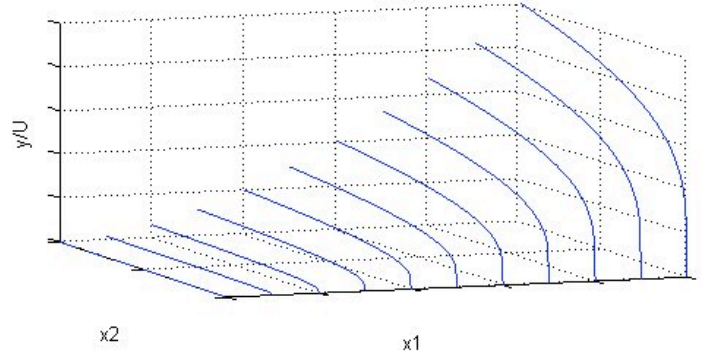
isoquants/indifference curves



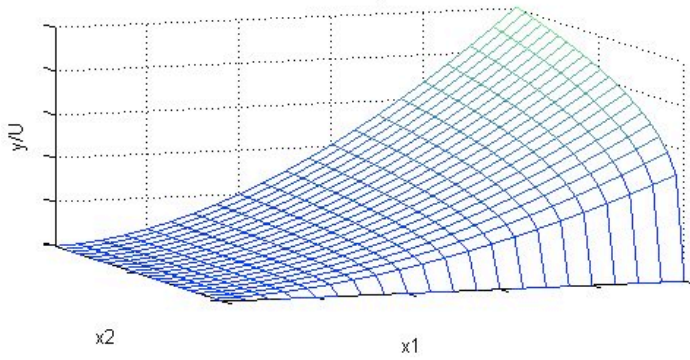
marginal properties (intensive margin)



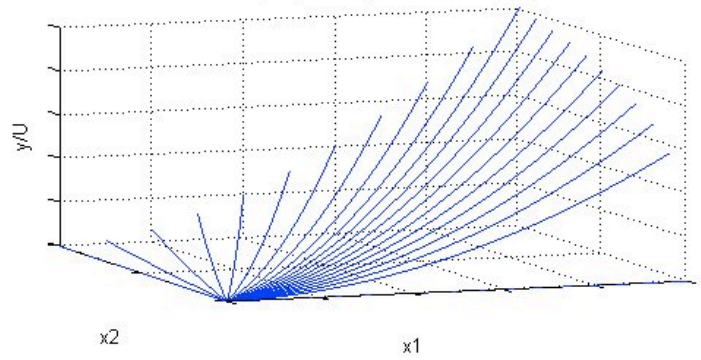
marginal properties (intensive margin)



meshgrid

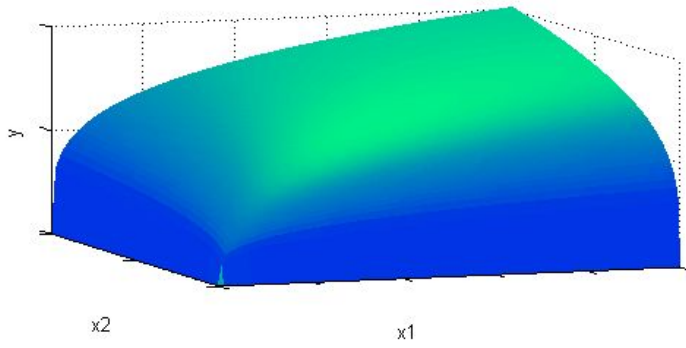


scale properties (extensive margin)

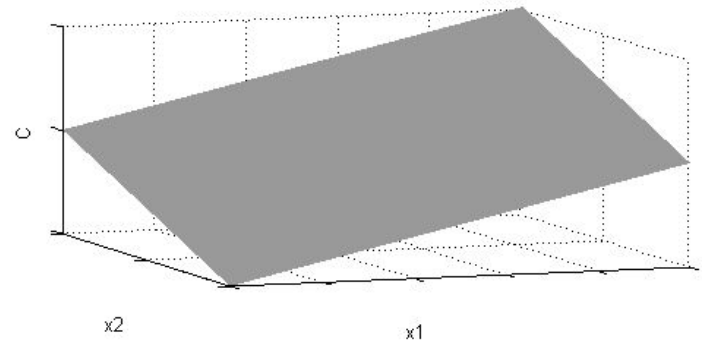


Profit maximization: production function with decreasing returns to scale

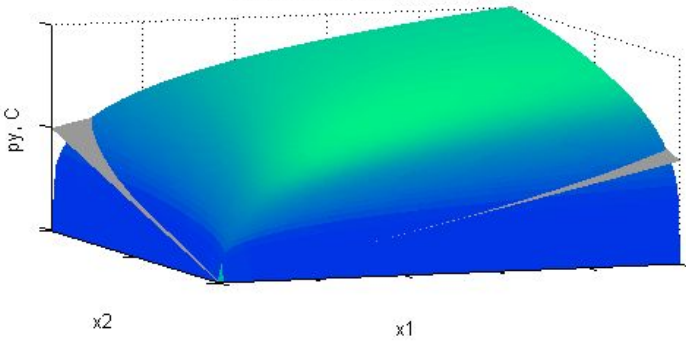
production function: $y = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.25$ - decreasing returns to scale



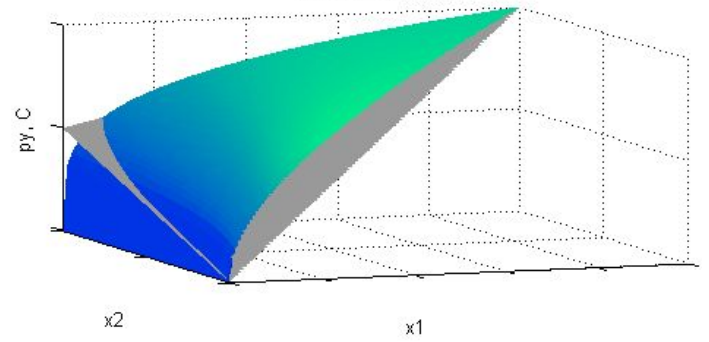
cost function: $C = w_1 x_1 + w_2 x_2$
 $w_1=w_2=1$



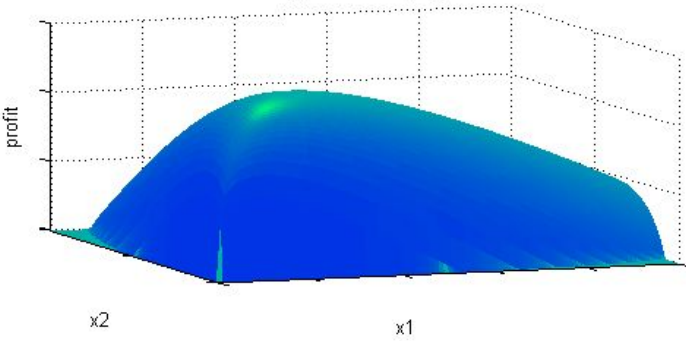
the two functions combined



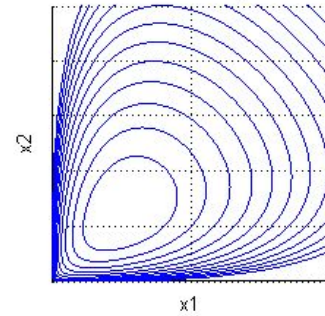
slice along a ray



the vertical difference: value of output - cost = profit
 only profit > 0 displayed

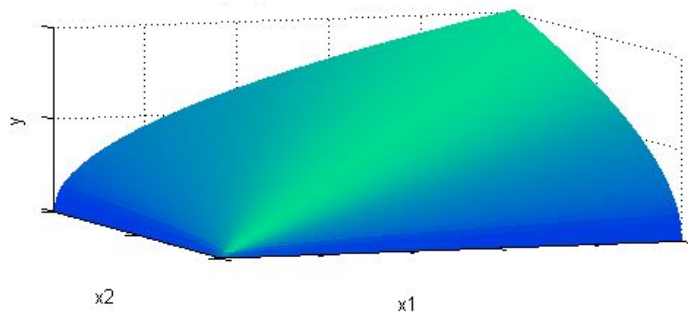


with decreasing returns to scale,
 an interior solution for maximum profit can be achieved

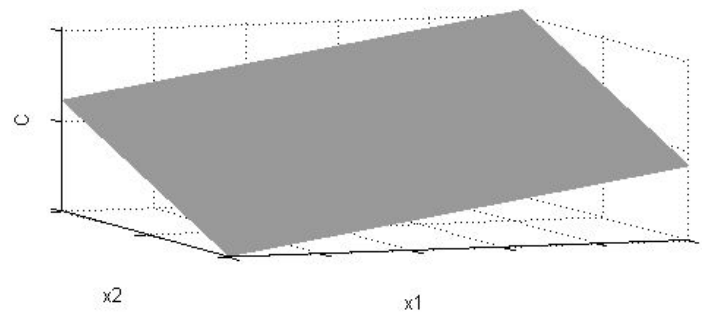


Profit maximization: production function with constant returns to scale

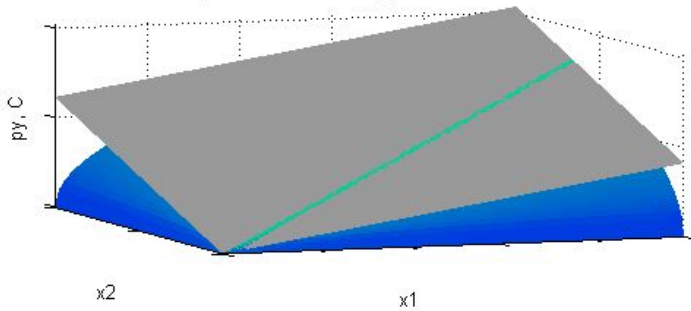
production function: $y = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.5$ - constant returns to scale



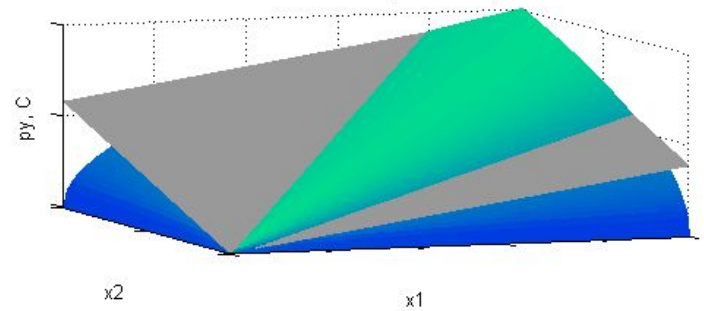
cost function: $C = w_1 x_1 + w_2 x_2$
 $w_1=2, w_2=3$



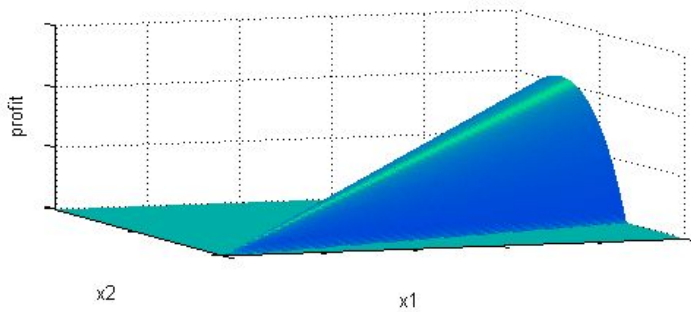
in a competitive market, price and wage adjust:
 $p = 2 \sqrt{w_1 w_2} = 2 \sqrt{6}$
 profit = value of output - cost = zero



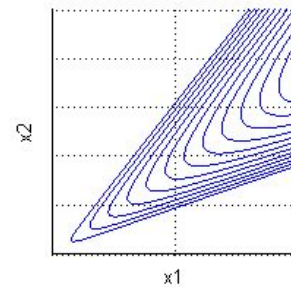
in a non-competitive market,
 we don't have to impose zero profit



profit in a non-competitive market
 profit = value of output - cost

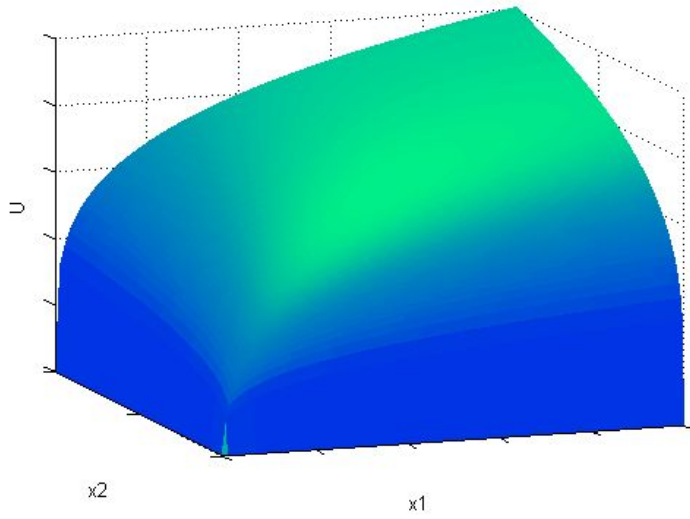


in a non-comp. mkt., profit could be infinite
 (but with increasing production,
 decreasing returns to scale eventually kick in)

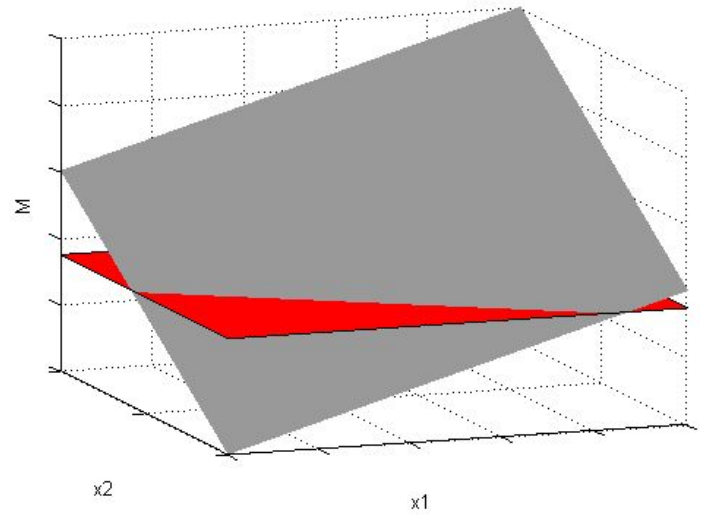


Utility maximization: utility function with decreasing returns to scale

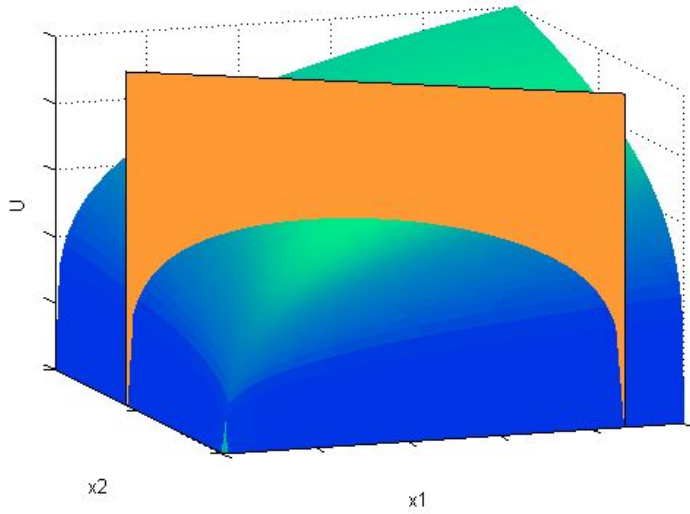
utility function: $U = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.25$ - decreasing returns to scale



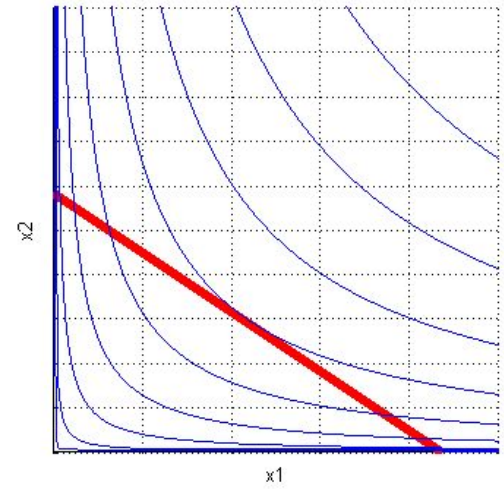
expenditure held constant at: $M^0 = p_1 x_1 + p_2 x_2$
 $p_1=2, p_2=3$



maximize utility subject to expenditure constraint

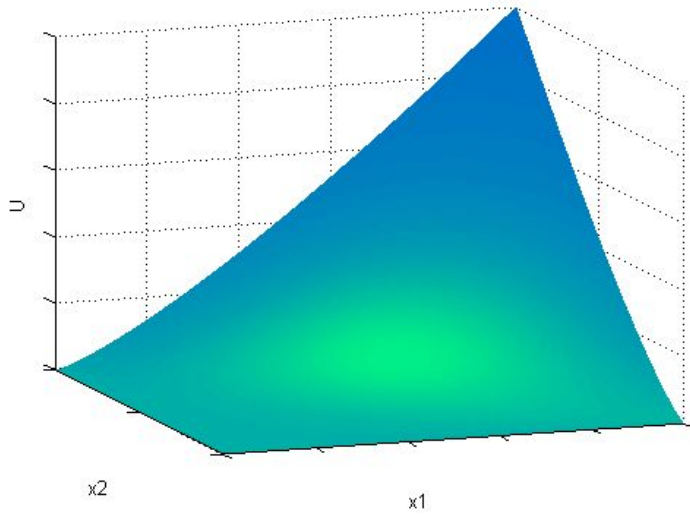


constrained maximum utility:
the tangency point of the constraint
and the indifference curve with the maximum value

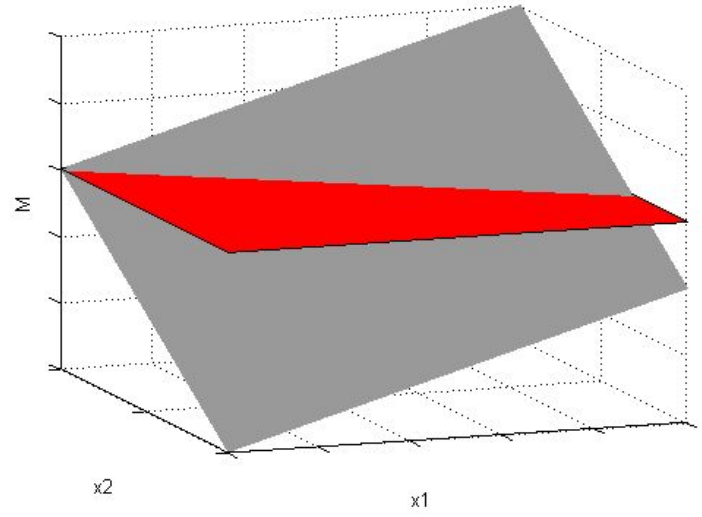


Utility maximization: utility function with increasing returns to scale

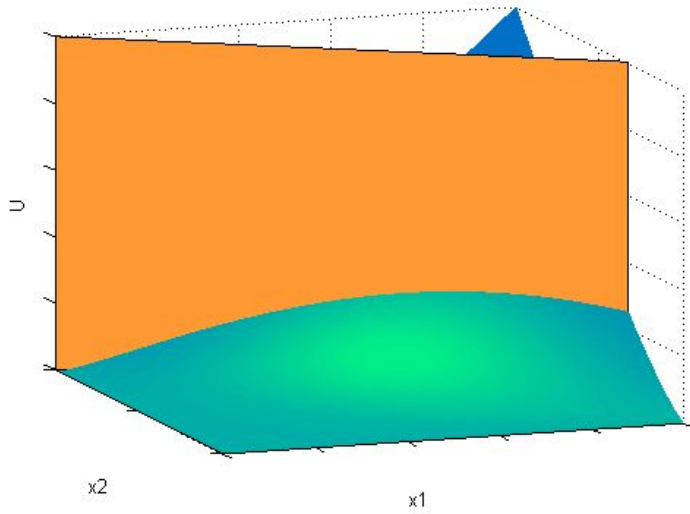
utility function: $U = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=1.25$ - increasing returns to scale



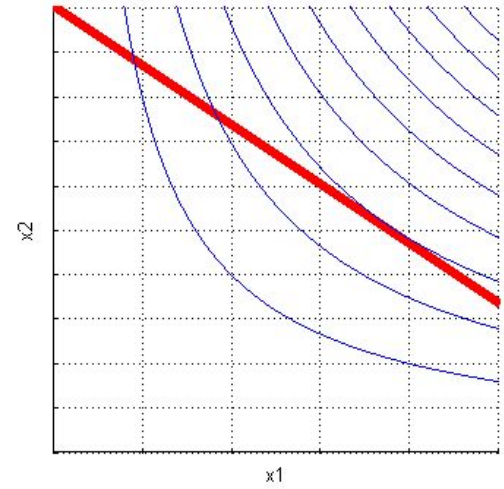
expenditure held constant: $M^0 = p_1 x_1 + p_2 x_2$
 $p_1=2, p_2=3$



maximize utility subject to expenditure constraint

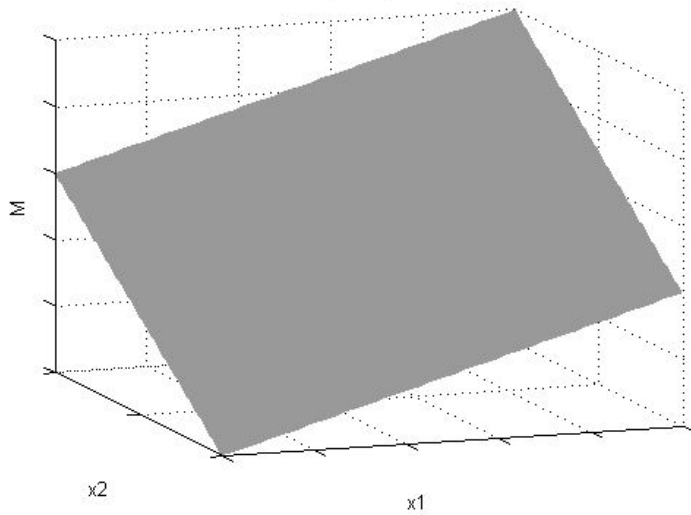


constrained maximum utility:
the tangency point of the constraint
and the indifference curve with the maximum value

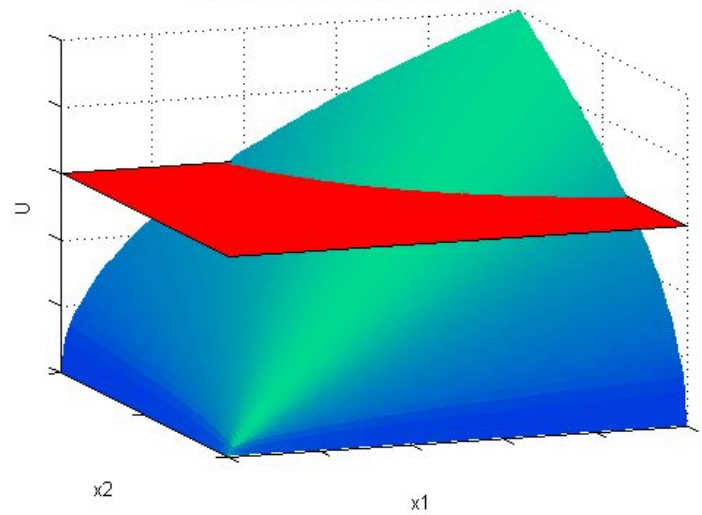


Expenditure minimization: utility function with constant returns to scale

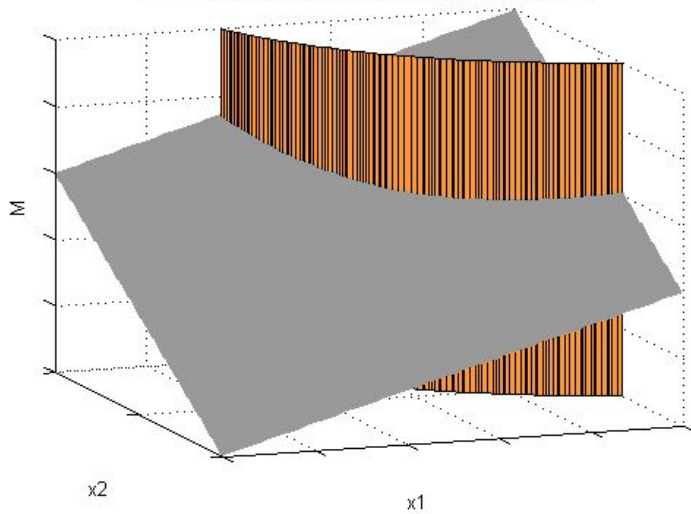
expenditure function: $M = p_1 x_1 + p_2 x_2$
 $p_1=2, p_2=3$



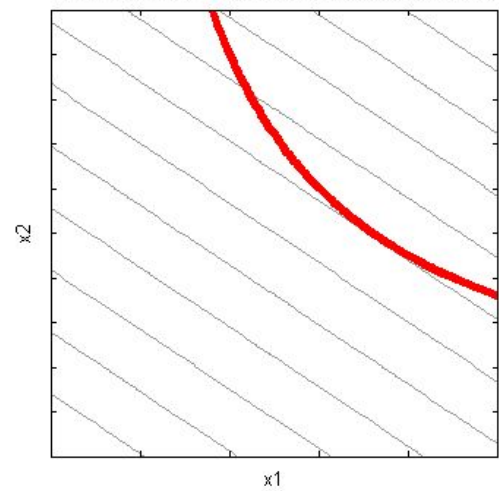
utility held constant at: $U^0 = a x_1^\alpha x_2^\beta$
 $a=1, \alpha=\beta=0.5$ - constant returns to scale



minimize expenditure subject to utility constraint



constrained minimum expenditure:
the tangency point of the constraint
and the isocost curve with the minimum value



Literature and further reading:

The Structure of Economics, 3rd ed., Eugene Silberberg
MATLAB Documentation, MathWorks

[Anatomy of CES Functions in 3D](#)
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