EconS 305: Intermediate Microeconomics w/o Calculus
Homework 3:
Market Analysis, Monopoly and Perfect Competition

Due: Friday, June 5th, 2020 at 5:00pm via Blackboard

- Please submit all homework solutions in the order the questions are presented and as **one .PDF**.

- Please **show all calculations** as these exercises are meant to refine your quantitative tool set. If I can not follow your calculations or it seems as you just "copy and pasted" answers from the internet, I will be deducting half the points from that solution.

1. **A Market Welfare Analysis of a Tax using Linear Demand and Supply Curves**

   Consider a demand curve for rice $Q^D = 22 - 2P$ and a supply curve for rice $Q^S = 3P - 23$, where both quantities are measured in pounds.

(a) Find the before tax market equilibrium (i.e. Q^* and P^*).

The market equilibrium is

$$Q^*, P^* = (4\,lbs.\,,\ \$9)$$

(b) Find the pre-tax consumer surplus.

$$\begin{aligned} CS &= \frac{1}{2}(4)(\$11 - \$9) \\ &= \$4 \end{aligned}$$

(c) Find the pre-tax producer surplus.

$$\begin{aligned} PS &= \frac{1}{2}(4)(\$9 - \$7.67) \\ &= \$2.67 \end{aligned}$$

(d) What is the total social welfare in the market of rice?

$$SW = CS + PS$$

$$SW = \$4 + \$2.67$$

$$SW = \$6.67$$

(e) Now, assume that we are going to apply a \$.50 tax to each pound of rice sold, which means that each rice producer will have pay \$.50 to the government for every pound sold. This means that the price the buyer will pay is $P_b = P_s + \$.50$. What is the equilibrium price received by the buyer (P_b) and producer (P_s), respectively? (Hint: the new demand curve is $Q^D = 22 - 2(P_b)$ and the new supply curve for rice $Q^S = 3P_s - 23$. Solve for P_s first, then plug in for P_b.)

$$P_s = \frac{44}{5} \approx \$8.80$$

$$\implies P_b \approx \$8.80 + \$.50 \approx \$9.30$$

(f) What is the new quantity demanded and supplied in the new equilibrium with the tax? Show that they are equivalent using the prices you just found.

$$\begin{aligned} Q_2^D &= 22 - 2(P_b) & \& \quad & Q_2^S &= 3(P_s) - 23 \\ Q_2^D &= 22 - 2(\$9.30) & \& \quad & Q_2^S &= 3(\$8.80) - 23 \\ Q_2^D &= 3.4 & \& \quad & Q_2^S &= 3.4 \end{aligned}$$

Which are both the same to guarantee the market equilibrium.

(g) What is the Consumer Surplus after the tax?

$$CS = \frac{1}{2}(3.4)(\$11 - \$9.30)$$

$$\implies CS \approx \$2.89$$

(h) What is the Producer Surplus after the tax?

$$PS = \frac{1}{2}(3.4)(\$8.80 - \$7.67)$$

$$\implies PS \approx \$1.92$$

(i) What is the Total Social Welfare after the tax? What is the loss in Social Welfare because of the tax?

$$SW = CS + PS = \$2.89 + \$1.92$$

$$= \$4.81$$

$$\implies Loss = SW_{before} - SW_{after} = \$6.66 - \$4.81 = \$1.85$$

(j) What is the Government Revenues?

$$GovtRevenues = .50(3.4) = \$1.70$$

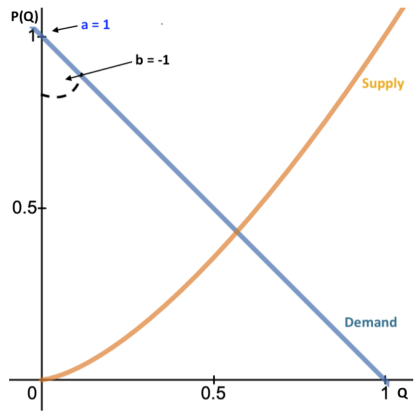
(k) Calculate the Dead Weight Loss (DWL) from the tax.

$$\$1.85 - \$1.7 = \$.15$$

2. **The Basic Case of a Monopoly with Fixed Costs**

Consider a monopolist facing a linear inverse demand function of $p(Q) = a - b(Q)$, where $a > c$, and total cost function of $TC(Q) = cQ + F$. We interpret a as the intercept, or the choke price consumers are willing to pay for $Q = 0$, and b as the slope of the inverse demand curve. Graphically, it can be represented as

Figure 1: The Linear Demand Curve



We can interpret the fixed cost (F) as perhaps some "entry" fee, and we interpret c as the marginal cost the firm has to pay according to how much output they produce. We can represent the Profit Maximization Problem (PMP) for firm as:

$$\max_{Q \geq 0} \quad \pi = p(Q)Q - (cQ + F)$$

$$\implies \max_{Q \geq 0} \quad \pi = [a - bQ]\, Q - (cQ + F)$$

CALCULUS PART:

From here, we can take our derivatives and set them equal to zero

$$\frac{\partial \pi(Q)}{\partial Q} \quad = \quad a - 2bQ - c = 0 \tag{1}$$

where we now have one equation ((1)), and one choice variable (Q) to solve for.

CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.

(a) Find the firm's optimal allocation of production (Q) to maximize its profit in equilibrium (i.e. find Q^*).
The optimal allocation of capital and labor respectively, is

$$Q^* = \frac{(a - c)}{2b}$$

(b) What is the equilibrium price the firm will receive (i.e. find $p(Q^*)$?

$$p(Q^*) = \frac{(a+c)}{2}$$

(c) What is the optimal profit function of the firm (i.e. find $\pi^*(Q^*)$

$$\pi^*(Q^*) = p(Q^*)Q^* - cQ^* - F$$

$$\implies \pi^*(Q^*) = \frac{(a-c)^2}{4b} - F$$

(d) What is the level of fixed cost in which the firm will choose to continue to operate?

$$F < \frac{(a-c)^2}{4b}$$

(e) Will the firm produce if $a < c$? Careful when answering this question.

The answer is no, because the optimal quantity produced (Q^*)will be negative, which is not feasible. A firm can not produce negative output, even though their profit function is technically yielding something "positive."

3. **A Cournot Game of Competing in Quantities w/ Fixed Costs**

Consider two firms competing a la Cournot in a market with an inverse demand function of $p(Q) = a - b(Q)$, where $Q = q_i + q_j$ and $a > c$, and a total cost function of $TC_i(q_i) = F + c_i q_i$. Notice that each firm has the same fixed cost (F) but their marginal costs (c_i) are not equal to each other (i.e. $c_i \neq c_j$). This means these homogeneous product producing firms have asymmetric costs, and we can represent the Profit Maximization Problem (PMP_i) for firm i as:

CALCULUS PART:

$$\max_{q_i \geq 0} \quad \pi_i = \left[a - b(q_i + q_j)\right] q_i - (F + c_i q_i)$$

$$\frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = a - 2bq_i - bq_j - c_i = 0 \tag{2}$$

And through symmetry we know that firm j's PMP is

$$\max_{q_j \geq 0} \quad \pi_j = \left[a - b(q_i + q_j)\right] q_j - (F + c_j q_j)$$

$$\frac{\partial \pi_j(q_i, q_j)}{\partial q_j} = a - 2bq_j - bq_i - c_j = 0 \tag{3}$$

where we now have two equations ((2) and (3)), and two choice variables (q_i and q_j) to solve for.

CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.

(a) Before you solve for the optimal equilibrium allocations, find the Best Response Functions ($BRFs$) for each firm (i.e. find $q_i(q_j)$ and $q_j(q_i)$). How does the firm respond in their own quantities with respect to an increase in a, b, c_i, and q_j?

$$BRF_i \equiv q_i(q_j) = \frac{(a - c_i)}{2b} - \frac{1}{2} q_j$$

$$\implies \qquad BRF_j \equiv q_j(q_i) = \frac{(a - c_j)}{2b} - \frac{1}{2} q_i$$

Where, intuitively we can see that as the demand curve shifts right, which increases our demand curve intercept (a), the firm responds with a higher quantity. As the slope of the demand curve (b) increases, which means it would become more negative, the firms responds is to decrease it's own output. As the firm's own marginal cost (c_i) increases, it makes sense that the firm would have to respond by also decreasing it's own quantity produced. Lastly, when firm j increases their quantity produced, it is the best response of firm i to decrease production.

(b) Find the optimal equilibrium allocation for each firm when they are competing a la Cournot. That is, find q_i^* and q_j^*. How does firm i's equilibrium allocation change with respect to an increase in their own marginal costs (c_i) and their opponents marginal cost (c_j)? Which increase is larger in absolute magnitude?

We find the equilibrium allocation by simultaneously solving for q_i^* and q_j^* using each firm's BRFs s.t.

$$q_i(q_j) = \frac{(a-c_i)}{2b} - \frac{1}{2}\left[\frac{(a-c_j)}{2b} - \frac{1}{2}q_i\right]$$

$$4bq_i = 2(a-c_i) - (a-c_i) + bq_i$$

$$\implies \quad q_i^* = \frac{(a-2c_i+c_j)}{3b} \quad and \quad q_j^* = \frac{(a-2c_j+c_i)}{3b}$$

by symmetry.

We can see that an increase in firm i's own marginal costs (c_i) decreases their optimal equilibrium output, and an increase in firm j's marginal cost increases firm i's equilibrium output. This can be seen as a positive externality, and we can see that the absolute magnitude of the effect of firm i's own marginal cost is greater than the absolute effect of firm j's marginal cost.

(c) Now, consider that the firm's have symmetric costs (i.e. $c_i = c_j = c$) in the competitive equilibrium and for all analyses from here on out. Find the competitive equilibrium quantities (i.e. find q_i^* and q_j^*).

When costs are equivalent, we know quantities, price, and profits are all the same for every firm i. This implies we get the standard Cournot quantities of

$$q_i^* \equiv q_j^* = \frac{(a-c)}{3b}$$

Note that we can also label these quantities as $q_i^{Cournot}$ as they are the output of the Cournot Model.

(d) Find the equilibrium price (i.e. $p(Q^*) = a - b(Q^*)$).

$$p(Q^*) = a - b\left(\frac{(a-c)}{3b} + \frac{(a-c)}{3b}\right)$$

$$= a - \frac{2(a-c)}{3}$$

$$\implies p(Q^*) = \frac{(a+2c)}{3}$$

(e) Find the equilibrium profits (i.e. π^*).

Using q^* and p^* we get π^* s.t.

$$\pi^* = p^*q^* - (F + cq^*)$$

$$= \left(\frac{(a+2c)}{3}\right)\left(\frac{(a-c)}{3b}\right) - \left(F + c\left(\frac{(a-c)}{3b}\right)\right)$$

$$
\begin{aligned}
&= \left(\frac{(a+2c)}{3} - c\right)\left(\frac{(a-c)}{3b}\right) - F \\
&= \frac{(a-c)}{3}\frac{(a-c)}{3b} - F \\
\implies \quad & \pi^{*\,Cournot} = \frac{(a-c)^2}{9b} - F
\end{aligned}
$$

4. **A Cournot Game with N Firms Competing in Quantities w/ Fixed Costs**

Consider N firms competing a la Cournot in a market with an inverse demand function of $p(Q) = a - b(Q)$, where $Q = \sum_{i=1}^{N} q_i$ and $a > c$, and a total cost function of $TC_i(q_i) = F + cq_i$. Notice that each firm has the same fixed cost (F) and, for simplicity, their marginal costs (c) are equal to each other (i.e. $c_i = c_j = \cdots = c_N = c$). This means these homogeneous product producing firms have symmetric costs, and we can represent the Profit Maximization Problem (PMP_i) for firm i as:

CALCULUS PART:

$$\begin{aligned}
&\max_{q_i \geq 0} \ \pi_i = \left[a - b\left(\sum_{i=1}^{N} q_i\right)\right] q_i - (F + cq_i) \\
\Longrightarrow &\max_{q_i \geq 0} \ \pi_i = \left[a - b\left(q_i + \sum_{i \neq j}^{N} q_j\right)\right] q_i - (F + cq_i) \\
&\frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = a - 2bq_i - b\sum_{i \neq j}^{N} q_j - c = 0 \qquad (4)
\end{aligned}$$

where we now have a symmetric equation ((4)) and one choice variable for each firm i (q_i) to solve for.

CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.

(a) Before you solve for the optimal equilibrium allocation for firm i, find the Best Response Function ($BRFs$) for firm i (i.e. find $q_i\left(\sum_{i \neq j}^{N} q_j\right)$). How does the firm respond in their own quantity with respect to an increase in a, b, c and all other quantities (i.e. q_j)?

$$BRF_i \equiv q_i\left(\sum_{i \neq j}^{N} q_j\right) = \frac{(a-c)}{2b} - \frac{1}{2}\sum_{i \neq j}^{N} q_j$$

Where, intuitively we can see that as the demand curve shifts right, which increases our demand curve intercept (a), the firm responds with a higher quantity. As the slope of the demand curve (b) increases, which means it would become more negative, the firms responds is to decrease it's own output. As the firm's own marginal cost (c) increases, it makes sense that the firm would have to respond by also decreasing it's own quantity produced. Lastly, when firm j increases their quantity produced, it is the best response of firm i to decrease production. These results are analogous to the results of Question 2 & Question 3.

(b) Find the optimal equilibrium allocation for each firm i when they are competing a la Cournot. That is, find q_i^* for all $i \in \{1, 2, \ldots, N\}$. To do this, please invoke the assumption that the firms are symmetric in output (i.e. $q_i = q_j$), and that the sum of a constant is equal to the multiplying by the number of constants in the sum (i.e. $\sum_{i=1}^{N} q_i = Nq_i$ when $q_i = q_j$).

We find the equilibrium allocation by simultaneously solving for q_i^* using equation (4) s.t.

$$\frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = a - 2bq_i - b\sum_{i \neq j}^{N} q_j - c = 0$$

$$bq_i + bq_i + b\sum_{i \neq j}^{N} q_j = a - c$$

$$bq_i + b\sum_{i=1}^{N} q_i = a - c$$

Invoking symmetry in production output and the aforementioned assumption

$$bq_i + b(N)q_i = a - c$$

$$bq_i(1 + N) = a - c$$

$$\implies q_i^* = \frac{(a-c)}{b(N+1)} \qquad \forall \ i \in \{1, 2, \ldots, N\}$$

(c) Find the Aggregate Quantity Demanded (i.e. $Q^* = \sum_{i=1}^{N} q_i^*$)

$$Q^* = \frac{(a-c)N}{b(N+1)}$$

(d) Find the equilibrium price (i.e. $P(Q^*)$)

$$P(Q^*) = \frac{a + Nc}{(N+1)}$$

(e) Find the equilibrium profits for each firm (i.e. π^*).

$$\pi^* = \frac{(a-c)^2}{b(N+1)^2} - F$$

(f) Assuming that we are operating in a perfectly competitive equilibrium (i.e. set $\pi^* = 0$, find the optimal number of firms in the industry (i.e. solve for N^*). Does the equilibrium number of firms increase or decrease as the demand curve becomes more inealastic?

$$N^* = \frac{(a-c) - \sqrt{bF}}{\sqrt{bF}}$$

If the demand curve becomes more inealastic, then the slope (b) will increase and thus make it more negative. To conduct this analysis we manipulate the equation to look like

$$N^* = \frac{(a-c)}{b^2\sqrt{F}} - 1$$

which shows that as the slope of the demand curve gets larger (i.e $b \to \infty$) then the equilibrium number of firms will fall.

5. **Comparing Outputs and Profits across Market Structures**

- Please assume $a > c$ throughout the analysis.

(a) Take each optimal quantity produced from Questions 2-4, and compare them mathematically (i.e. $q_i^{Monopoly}$ ($<$ or $>$) $q_i^{Duopoly}$ ($<$ or $>$) $q_i^{Perfect\ Competition}$). Please rank them in terms of highest quantities to lowest, assuming that $N \geq 3$. What happens as the number of firms increases?

$$q_i^{Monopoly} > q_i^{Duopoly} > q_i^{Perfect\ Competition}$$
$$\frac{(a-c)}{2b} > \frac{(a-c)}{3b} > \frac{(a-c)}{(N+1)b}$$

where $N \geq 3$

Which means that as the number of firms increases, their individual quantity produced also decreases. This is due to them getting a smaller and smaller market share as firms enter the market. With that said, notice that aggregate output increases, which is consistent with our notes and the law of demand.

$$Q^{Monopoly} < Q^{Duopoly} < Q^{Perfect\ Competition}$$
$$\frac{(a-c)}{2b} < \frac{2(a-c)}{3b} < \frac{N(a-c)}{(N+1)b}$$

where $N \geq 3$

(b) Take each optimal price you found from Questions 2-4, and compare them mathematically (i.e. $p(Q^*)^{Monopoly}$ ($<$ or $>$) $p(Q^*)^{Duopoly}$ ($<$ or $>$) $p(Q^*)^{Perfect\ Competition}$). Please rank them in terms of highest prices to lowest, assuming that $N \geq 3$. Which price is the greatest and which is the least? Is this different than the quantities ranking? If so, why is this?

$$p(Q^{Monopoly}) > p(Q^{Duopoly}) > p(Q^{Perfect\ Competition})$$
$$\frac{(a+c)}{2} > \frac{(a+c)}{3} > \frac{(a+Nc)}{(N+1)}$$

where $N \geq 3$

No, this is not the opposite of the quantity rankings, and the reason for the decrease in quantity output is because since more firms are entering the industry, the collective price they are getting per unit is falling, and thus the marginal revenues are less for each individual producer. With that said, notice the inverse relationship between market price and aggregate market output. This is expected due to the law of demand.

(c) Take each profit you found from Questions 2-4, and compare them mathematically (i.e. $\pi^{Monopoly}$ ($<$ or $>$) $\pi^{Duopoly}$ ($<$ or $>$) $\pi^{Perfect\ Competition}$). Please rank them in terms of highest profits to lowest, assuming that $N \geq 3$.

$$\pi_i^{Monopoly} > \pi_i^{Duopoly} > \pi_i^{Perfect\ Competition}$$
$$\frac{(a-c)^2}{4b} > \frac{(a-c)^2}{9b} > \frac{(a-c)^2}{(N+1)^2 b}$$

where $N \geq 3$

Question #1 (Math)

1A)

Let,

$$Q^D = Q^S$$

$$\Rightarrow \quad 22-2P = 3P-23$$

$$5P = 45$$

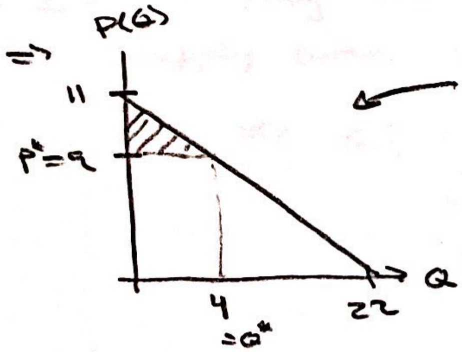
$$\Rightarrow \quad P^* = 45/5 = \$9$$

plugging back into $Q^D \Rightarrow Q^D = 22-2(9) = 22-18 = 4$

$$\Rightarrow \quad Q^{D*} = 4 \text{ lbs.}$$

1b) Since we have linear demand & supply curves, we can calculate CS using the area of a triangle.

$\Rightarrow$



① $Q^D=0$

$\Rightarrow 0 = 22-2P$

$\Rightarrow P = 11$

② $P=0$

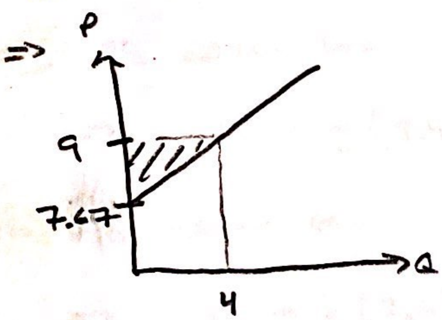
$\Rightarrow Q^D = 22$

$$\Rightarrow \quad CS = \tfrac{1}{2}bh$$

$$\Rightarrow \quad = \tfrac{1}{2}(4)(\$11-\$9) = \$4$$

1c) PS, similarly using the supply curve

$\Rightarrow$



①

$Q^S=0$

$\Rightarrow 0 = 3P-23$

$\Rightarrow P = 23/3$

≈ 7.67

② Don't need

$$\Rightarrow \quad PS = \tfrac{1}{2}bh$$

$$= \tfrac{1}{2}4(\$9-\$7.67) \approx \$2.66$$

1d) Total Social Welfare

$$\Rightarrow \quad SW = CS + PS \quad \Rightarrow \quad \$4 + \$2.66 = \$6.66$$

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1e) Let $P_b = (P_s + .50)$

$\Rightarrow$ $22 - 2(P_s + .50) = 3P_s - 23$

$22 - 2P_s - 1 = 3P_s - 23$

$\Rightarrow$ $21 - 2P_s = 3P_s - 23$

$44 = 5P_s$

$\Rightarrow P_s^* = 44/5 = \$8.80$

$\therefore$ The buyers face

$P_b^* = 8.80 + .5 = \$9.30$

1f) $\therefore$ If we plug the buyers price into demand & sellers price into supply curve we get the same quantity

$\Rightarrow Q_2^D = 22 - 2P_b^*$ & $Q_2^S = 3(P_s^*) - 23$

$= 22 - 2(9.30)$ $\qquad = 3(8.80) - 23$

$= 3.4$ $\qquad = 3.4$

same

$\Rightarrow$ 3.4 lbs of rice will be sold after tax.

1g) CS after tax

choke

$\Rightarrow CS = \frac{1}{2}(3.4)(\$11 - \$9.30) = \frac{1}{2}(3.4)(\$1.7)$

$\approx \$2.89$

1h) PS after tax

choke

$\Rightarrow PS = \frac{1}{2}(3.4)(8.80 - 7.67) = \frac{1}{2}(3.4)(\$1.13)$

$\approx \$1.921$

1i) $SW^{Tax} = \$2.89 + \$1.921$

$\approx \$4.81$

$\Rightarrow \$6.66 - 4.81 = \$1.85$ lost surplus from tax

$\Rightarrow$

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1j) Government Revenue

=> $.50(3.4) = \$1.7$

1k) Dead weigh loss (DWL)

=> $1.85 - 1.7 = \$.15$ as a dead weight loss from ~~tax~~.

NOTE:
A Different way to calculate DWL is:

$$DWL = \frac{1}{2}(Q_1 - Q_2) * (P_D - P_S)$$
$$= \frac{1}{2}(Q_1 - Q_2) * (tax)$$
$$= \frac{1}{2}(4 - 3.4)(.50) = .15 \quad //$$

=> For every $1^{\$}$ increase in DWL, we get a tax return (i.e. revenue) of $\$11.33$

=> $\frac{1.7}{.15} \approx \$11.33$

Question #2 (Math)

2a) using equation I & solving for Q

$$\Rightarrow \quad a - 2bQ - c = 0$$

$$\Rightarrow \quad 2bQ = a - c$$

$$\Rightarrow \quad Q^* = \frac{(a-c)}{2b}$$

2b) plugging in Q^* into $P(Q)$

$$\Rightarrow \quad P(Q^*) = (a - bQ^*)$$

$$= a - \cancel{b}\left(\frac{(a-c)}{2\cancel{b}}\right)$$

$$= a - \frac{(a-c)}{2}$$

$$= \frac{2a - a + c}{2} = \frac{a+c}{2}$$

2c) plugging Q^* into π

$$\Rightarrow \quad \pi^* = P(Q^*)Q^* - cQ^* - F$$

$$\Rightarrow \quad = \left(\frac{(a+c)}{2}\right)\left(\frac{(a-c)}{2b}\right) - c\left(\frac{(a-c)}{2b}\right) - F$$

$$= \left(\frac{a+c}{2} - c\right)\left(\frac{(a-c)}{2b}\right) - F$$

$$= \frac{(a-c)}{2}\frac{(a-c)}{2b} - F \quad = \frac{(a-c)^2}{4b} - F$$

2d) At what Fixed cost will they operate?

$$\Rightarrow \text{If } 0 < \pi^* \quad \Rightarrow \quad 0 < \frac{(a-c)^2}{4b} - F$$

$$\Rightarrow \quad F < \frac{(a-c)^2}{4b}$$

3a) using equations (2) & (3) we get

Question #3 (Math)

(2)

$\Rightarrow a - 2bq_i - bq_j - c_i = 0$

$\Rightarrow 2bq_i = a - c_i - bq_j$

$\Rightarrow BRF_i \equiv q_i(q_j) = \frac{(a-c_i)}{2b} - \frac{1}{2}q_j$

(3)

$\Rightarrow a - 2bq_j - q_i - c_j = 0$

$\Rightarrow 2bq_j = a - c_j - q_i$

$\Rightarrow BRF_j \equiv$

$q_j(q_i) = \frac{a-c_j}{2b} - \frac{1}{2}q_i$

3b)

using these BRFs

$\Rightarrow q_i = \frac{(a-c_i)}{2b} - \frac{1}{2}\left[\frac{(a-c_j)}{2b} - \frac{1}{2}q_i\right]$

$\Rightarrow q_i = \frac{(a-c_i)}{2b} - \frac{(a-c_j)}{4b} + \frac{1}{4}q_i$

$\Rightarrow 4bq_i = 2(a-c_i) - (a-c_j) + bq_i$ multiplying by 4b on both sides.

$\Rightarrow 3bq_i = 2a - a - 2c_i + c_j$

$\Rightarrow 3bq_i = a - 2c_i + c_j$

$\Rightarrow q_i^* = \frac{a - 2c_i + c_j}{3b}$

plugging back into BRF_j

$\Rightarrow q_j^* = \frac{(a-c_j)}{2b} - \frac{1}{2}\left[\frac{a-2c_i+c_j}{3b}\right]$

$q_j^* = \frac{(a-c_j)}{2b} - \frac{(a-2c_i+c_j)}{6b}$

$\Rightarrow q_j^* = \frac{(3a-3c_j-a+2c_i-c_j)}{6b}$

$q_j^* = \frac{(2a-4c_j+2c_i)}{6b}$

$q_j^* = \frac{2(a-2c_j+c_i)}{6b} = \frac{(a-2c_j+c_i)}{3b}$

Note the symmetry.

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3c) Note the symmetric costs

$$\Rightarrow \quad q_i^* = \frac{(a-2c+c)}{3b} = \frac{(a-c)}{3b}$$ SAME

&

$$q_j^* = \frac{(a-2c+c)}{3b} = \frac{(a-c)}{3b}$$

3d)

$\Rightarrow$ If $P(Q) = (a - bQ)$ $\quad q_i^* + q_j^*$

$$\Rightarrow \quad = \left(a - b\left(\frac{(a-c)}{3b} + \frac{(a-c)}{3b}\right)\right)$$

$$= \left(a - \frac{2(a-c)}{3}\right)$$

$$P(Q^*) = \left(\frac{3a-2a+2c}{3}\right) = \frac{a+2c}{3} \quad //$$

3e) π^*

$$\Rightarrow \quad \pi^* = p^* q^* - (F + cq^*)$$

$$= (p^* - c)q^* - F$$

$$= \left(\frac{a+2c}{3} - c\right)\left(\frac{(a-c)}{3b}\right) - F$$

$$= \frac{(a-c)}{3}\frac{(a-c)}{3b} - F$$

$$\Rightarrow \pi^* = \frac{(a-c)^2}{9b} - F$$

(Question #4)
Math

4a) using equation #4

$$\Rightarrow \quad a - 2bq_i - b\sum_{i\neq j}^{N} q_j - c = 0$$

$$\Rightarrow \quad 2bq_i = a - c - b\sum_{i\neq j}^{N} q_j$$

$$\Rightarrow BRF_i \equiv q_i\left(\sum_{i\neq j}^{N} q_j\right) = \frac{a-c}{2b} - \frac{1}{2}\sum_{i\neq j}^{N} q_j$$

where,

$$\sum_{i\neq j}^{N} q_i = (q_1 + q_2 + \dots + q_{i-1} + q_{i+1} + \dots + q_N)$$

Note that there are $N-1$ quantities included in ~~here~~ because q_i is not included.

4b) using the assumption ① such that

$q_i = q_j$

$\Rightarrow$ equation (4) becomes

$$a - 2bq_i - b\sum_{i=1}^{(N-1)} q_i - c = 0$$

where (assumption ②)

$$\sum_{i=1}^{N} c = Nc$$

when c = constant

$$\Rightarrow \quad a - bq_i - \left(bq_i + b\sum_{i=1}^{(N-1)} q_i\right) - c = 0$$

$$\Rightarrow \quad a - bq_i - b\sum_{i=1}^{N} q_i - c = 0$$

$$\Rightarrow \quad bq_i + b\sum_{i=1}^{N} q_i = a - c$$

$$\Rightarrow \quad bq_i + Nbq_i = a - c$$

$$\Rightarrow \quad bq_i(1+N) = a - c$$

$$\Rightarrow \quad q_i^* = \frac{(a-c)}{(N+1)b}$$

4c) using the same assumption above

$$\Rightarrow \quad Q^* = \sum_{i=1}^{N} q_i^* = Nq_i^* = \frac{N(a-c)}{(N+1)b}$$

4d) $P(Q^*) = (a - bQ^*)$

$$= \left(a - \cancel{b}\left(\frac{N(a-c)}{(N+1)\cancel{b}}\right)\right)$$

$$= \frac{a(N+1) - N(a-c)}{N+1}$$

$$= \frac{\cancel{aN} + a - \cancel{Na} + Nc}{(N+1)} = \frac{a + Nc}{(N+1)}$$

4e)

$$\pi_i^* = \underbrace{(a - bQ^*)}_{P(Q^*)} q_i^* - cq_i^* - F$$

$$= (P(Q^*) - c)\, q_i^* - F$$

$$= \left(\frac{a + Nc}{(N+1)} - c\right)\left(\frac{N(a-c)}{(N+1)b}\right) - F$$

$$= \left(\frac{a + \cancel{Nc} - \cancel{Nc} - c}{(N+1)}\right)\left(\frac{a-c}{(N+1)b}\right) - F$$

$$= \frac{(a-c)}{(N+1)} \frac{(a-c)}{(N+1)b} - F$$

$$= \underbrace{\frac{(a-c)^2}{(N+1)^2 b}}_{\text{Producers Surplus (PS)}} - F$$

4F) In a Perfectly Competitive Equilibrium $\pi^* = 0$

$$\Rightarrow \quad 0 = \frac{(a-c)^2}{(N+1)^2 b} - F$$

$$\Rightarrow \quad bF = \left(\frac{(a-c)}{(N+1)}\right)^2$$

$$\Rightarrow \quad \sqrt{bF} = \frac{(a-c)}{(N+1)}$$

$$\Rightarrow \quad \sqrt{bF}\, N + \sqrt{bF} = (a-c)$$

$$\Rightarrow \quad \sqrt{bF}\, N = (a-c) - \sqrt{bF}$$

$$\Rightarrow \quad N^* = \frac{(a-c) - \sqrt{bF}}{\sqrt{bF}} \quad \text{//}$$

Question #5
Math

5a) comparing q_i^*s

$\Rightarrow$ $q_i^M > q_i^D > q_i^{PC}$

Because $\frac{(a-c)}{2b} > \frac{(a-c)}{3b} > \frac{(a-c)}{(N+1)b}$ if $N \geq 3$

$\Rightarrow$ as # of firms increases $(N \to \infty)$, individual output increases.

5b) $P(Q^M) > P(Q^D) > P(Q^{PC})$

$$\Rightarrow \underbrace{\frac{(a+c)}{2} > \frac{(a+2c)}{3}} > \frac{(a+Nc)}{(N+1)} \quad \text{if } N \geq 3$$

Proof ①

$$\frac{(a+c)}{2} > \frac{(a+2c)}{3}$$

$\Rightarrow 3(a+c) > 2(a+2c)$

$\Rightarrow 3a+3c > \cancel{2a}+4c$

$\Rightarrow a > c$ which holds by assumption

Proof ②

$$\frac{(a+2c)}{3} > \frac{(a+Nc)}{(N+1)}$$

$(N+1)(a+2c) > 3a+3Nc$

$Na+\cancel{2Nc}+\cancel{a}+2c > 3a+3Nc$

$\Rightarrow$ $Na+2c > a+Nc$

$a(N-1) > c(N-2)$ where $a>c$ & $(N-1)>(N-2)$

$\Rightarrow P(Q^D) > P(Q^{PC})$ //

π^*s

$\Rightarrow$

5c)

$$\pi^{M} \geq \pi^{Duopoly} \geq \pi^{PC}$$

$$\Rightarrow \quad \frac{(a-c)^2}{4b} - \cancel{F} \geq \frac{(a-c)^2}{9b} - \cancel{F} \geq \frac{(a-c)^2}{(N+1)^2 b} - \cancel{F}$$

$$\Rightarrow \quad \frac{(a-c)^2}{4b} \geq \frac{(a-c)^2}{9b} \geq \frac{(a-c)^2}{(N+1)^2 b}$$ which holds if $N \geq 2$