

# Producer Behavior

# Introduction (1/2)

We now turn to the supply side of the supply and demand model.

## Chapter Outline

- 6.1** The Basics of Production
- 6.2** Production in the Short Run
- 6.3** Production in the Long Run
- 6.4** The Firm's Cost-Minimization Problem
- 6.5** Returns to Scale
- 6.6** Technological Change
- 6.7** The Firm's Expansion Path and the Total Cost Curve
- 6.8** Conclusion

# Introduction (2/2)

## 3 Key Questions:

1. How do firms decide whether and how much to produce?
2. How do firms choose between inputs such as capital and labor?
3. How does the timeframe of analysis affect firm decisions?

**These questions are fundamental to understanding how supply responds to changing market conditions.**

# The Basics of Production (1/5)

**Production** describes the process by which a person, company, government, or non-profit agency uses inputs to create a good or service for which others are willing to pay.

- **Final goods** - a good that is bought by a consumer (e.g., bread).
- **Intermediate goods** - a good that is used to produce another good (e.g., wheat used to produce bread).

Start with a **production function**.

- A mathematical relationship that describes how much output can be made from different combinations of inputs
- Similar to a utility function for consumers, except more tangible

# The Basics of Production (2/5)

## **Simplifying Assumptions about Firms' Production Behavior**

1. The firm produces a single good.
  
2. The firm has already chosen which product to produce.
  
3. Firms minimize costs associated with every level of production.
  - Necessary condition for profit maximization
  
4. Only two inputs are used in production: capital and labor.
  - Capital: buildings, equipment, etc.
  - Labor: All human resources

# The Basics of Production (3/5)

5. In the **short run**, firms can choose the amount of labor employed, but capital is assumed to be fixed in total supply.
  - Short run: period of time in which one or more inputs used in production cannot be changed
  - Fixed inputs: inputs that cannot be changed in the short run
  - Variable inputs: inputs that can be changed in the short run
  - Long run: the amount of time necessary for all inputs into production to be fully adjustable
6. The more inputs the firm uses, the more output it makes.

# The Basics of Production (4/5)

7. Inputs are characterized by **diminishing returns**.
  - If the amount of capital is held constant, each additional worker eventually produces less incremental output than the last, and vice versa.
8. The firm can employ unlimited capital and labor at fixed prices,  
*and*
9. Capital markets are well functioning (the firm is not budget-constrained).

# The Basics of Production (5/5)

## Production Functions

- Describe how output is made from different combinations of inputs,

$$Q = f(K, L)$$

where  $Q$  is the quantity of output,  $K$  is the quantity of capital used, and  $L$  is the quantity of labor used.

A common *functional form* used in economics is referred to as the *Cobb–Douglas* production function,

$$Q = K^\alpha L^\beta$$

where the quantity of each input, each raised to a power (usually less than one), are multiplied together.

# Production in the Short Run (1/9)

The **short run** refers to the case in which the level of capital is fixed, typically expressed as:

$$Q = f(\bar{K}, L)$$

Consider how production changes as we vary the amount of labor.

An important production metric is **average product**:

- Total output divided by the total amount of an input used
- The average product of labor is given by the equation:

$$AP_L = \frac{Q}{L}$$

# Production in the Short Run (2/9)

**Marginal product** refers to the additional output that a firm can produce using an additional unit of an input (holding use of the other input constant).

- Generally assumed to fall as more of an input is used – referred to diminishing marginal product

The **marginal product of labor**,  $MP_L$ , is given as:

$$MP_L = \frac{\Delta Q}{\Delta L}$$

# Production in the Short Run (3/9)

Consider the production function  $Q = K^{0.5} L^{0.5}$   
where capital is fixed at four units. Plug 4 into the function

$$Q = 4^{0.5} L^{0.5} = 2L^{0.5}$$

Table 6.1 calculates the average and marginal product of labor for this production function.

Both Table 6.1 and Figure 6.1 reveal the common assumption of **diminishing marginal product** associated with production inputs.

- As a firm employs more of one input, while holding all others fixed, the marginal product of that input will fall.

# Production in the Short Run (4/9)

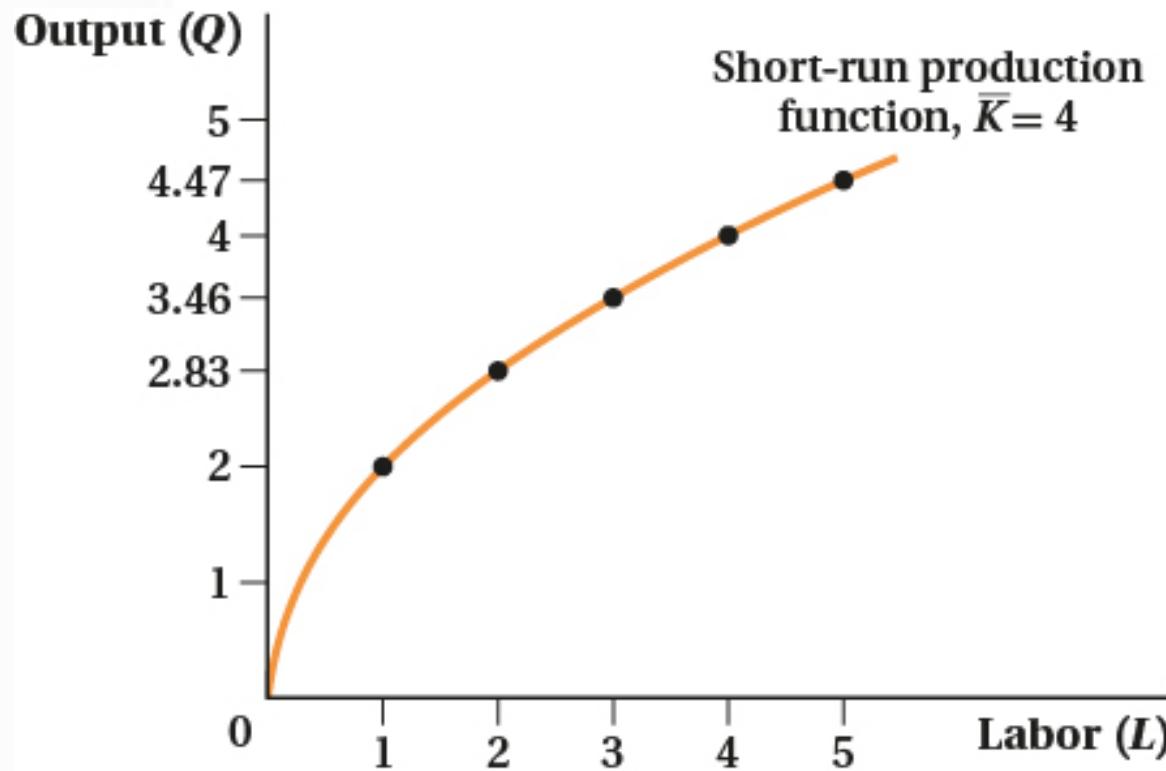
**Table 6.1: An Example of a Short-Run Production Function**

Capital, <i>K</i>	Labor, <i>L</i>	Output, <i>Q</i>	Marginal Product of Labor, $MP_L = \frac{\Delta Q}{\Delta L}$	Average Product of Labor, $AP_L = \frac{Q}{L}$
4	0	0.00	—	—
4	1	2.00	2.00	2.00
4	2	2.83	0.83	1.42
4	3	3.46	0.63	1.15
4	4	4.00	0.54	1.00
4	5	4.47	0.47	0.89

Goolsbee et al., *Microeconomics*, 3e, © 2020 Worth Publishers

# Production in the Short Run (5/9)

Figure 6.1 A Short-Run Production Function



# Production in the Short Run (6/9)

Returning to the mathematical representation of  $MP_L$ ,

$$MP_L = \frac{\Delta Q}{\Delta L} = \frac{f(\bar{K}, L + \Delta L) - f(\bar{K}, L)}{\Delta L}$$

and using the example production function  $Q = 4^{0.5} L^{0.5} = 2L^{0.5}$

$$MP_L = \frac{2(L + \Delta L)^{0.5} - 2L^{0.5}}{\Delta L}$$

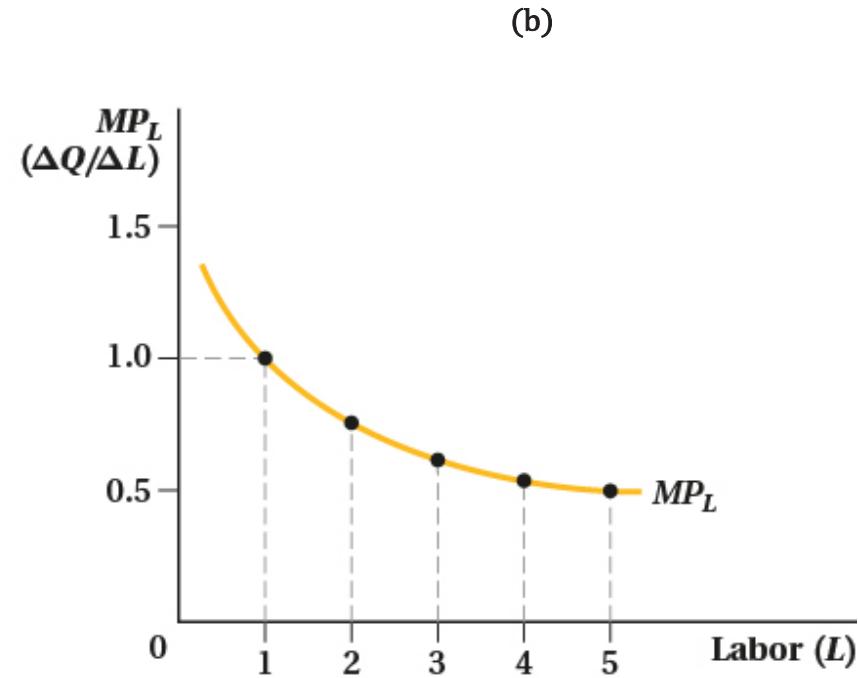
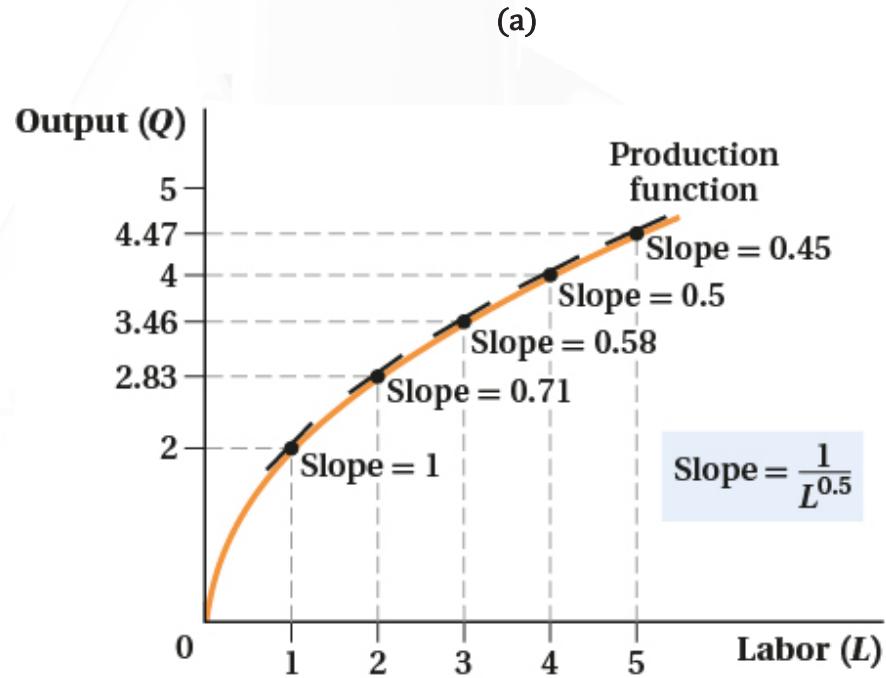
As  $\Delta L$  becomes very small, we use calculus to arrive at the equation for  $MP_L$ .

$$MP_L = \frac{df(\bar{K}, L)}{dL} = \frac{1}{L^{0.5}} = L^{-0.5}$$

This is seen most easily using a graph.

# Production in the Short Run (7/9)

Figure 6.2 Deriving the Marginal Product of Labor



# Production in the Short Run (8/9): Question 1

A firm has the Cobb-Douglas production function  $Q = K^{0.25}L^{0.5}$ , and K is fixed at 81 units. What is the average product of labor when L is equal to 100 units?

- A. 0.3
- B. 3.0
- C. 1.2
- D. 2.5

# Production in the Short Run (8/9): Question 1 – Correct Answer

A firm has the Cobb-Douglas production function:  $Q = K^{0.25}L^{0.5}$ , and K is fixed at 81 units. What is the average product of labor when L is equal to 100 units?

- A. 0.3 (correct answer)
- B. 3.0
- C. 1.2
- D. 2.5

# Production in the Short Run (9/9): Question 2

Capital ( $K$ )	Labor ( $L$ )	Output ( $Q$ )
6	9	50
6	10	54
6	11	56

**What is the marginal product of labor of the 10<sup>th</sup> laborer?**

- A. 54
- B. 4
- C. 104
- D. 2

# Production in the Short Run (9/9): Question 2 – Correct Answer

Capital ( $K$ )	Labor ( $L$ )	Output ( $Q$ )
6	9	50
6	10	54
6	11	56

**What is the marginal product of labor of the 10<sup>th</sup> laborer?**

- A. 54
- B. 4 (correct answer)
- C. 104
- D. 2

# Production in the Long Run (1/2)

For our purposes, the **long run** is defined as a period of time during which all inputs into production are fully adjustable.

Table 6.2 describes a long-run production function in which two inputs, capital and labor, are used to produce various quantities of a product.

- Columns represent different quantities of labor.
- Rows represent different quantities of capital.
  - **Each cell in the table shows the quantity of output produced with the labor and capital represented by the column and row values.**

# Production in the Long Run (2/2)

	Units of Labor, $L: 1$	Units of Labor, $L: 2$	Units of Labor, $L: 3$	Units of Labor, $L: 4$	Units of Labor, $L: 5$
Units of Capital, $K: 1$	1.00	1.41	1.73	2.00	2.24
Units of Capital, $K: 2$	1.41	2.00	2.45	2.83	3.16
Units of Capital, $K: 3$	1.73	2.45	3.00	3.46	3.87
Units of Capital, $K: 4$	2.00	2.83	3.46	4.00	4.47
Units of Capital, $K: 5$	2.24	3.16	3.87	4.47	5.00

## The Firm's Cost-Minimization Problem (1/15)

Recall, the third assumption about production behavior: *Firms minimize the cost of production.*

Cost minimization refers to the firm's goal of producing a specific quantity of output at minimum cost.

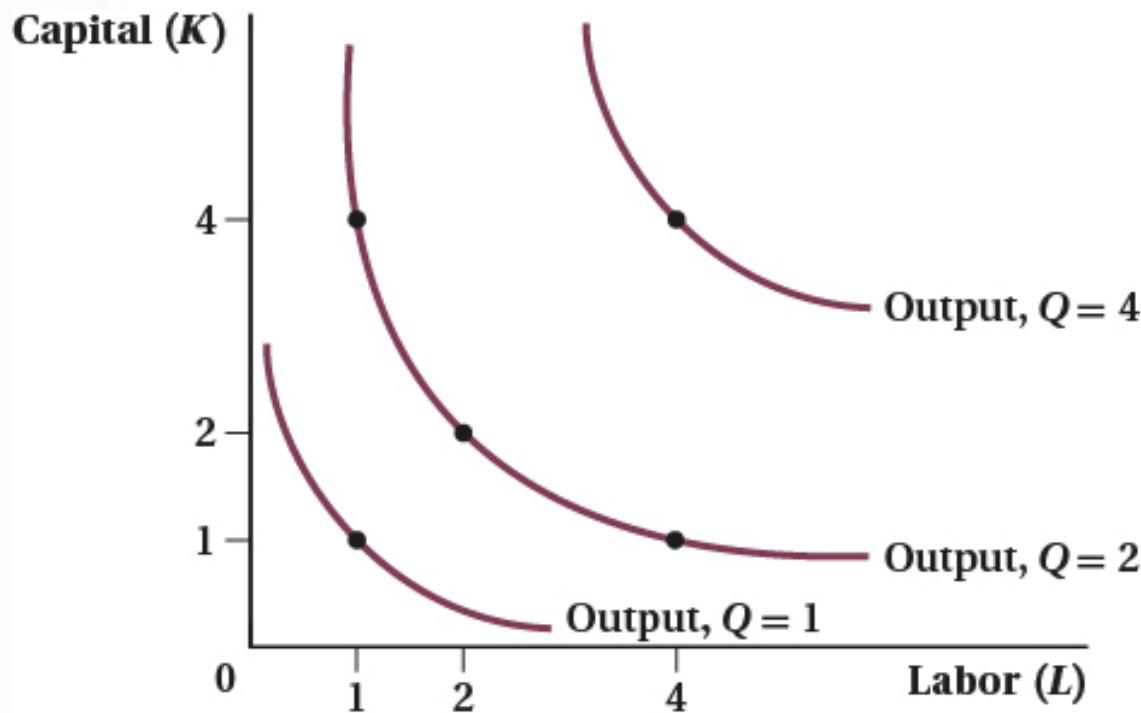
- This is an example of a *constrained optimization* problem.
- The firm will minimize costs subject to a specific amount of output that must be produced.

The cost minimization model requires two concepts: **isoquants** and **isocost** lines.

- An isoquant is a curve representing combinations of inputs that allow a firm to make a particular quantity of output.

## The Firm's Cost-Minimization Problem (2/15)

Figure 6.3 Isoquants



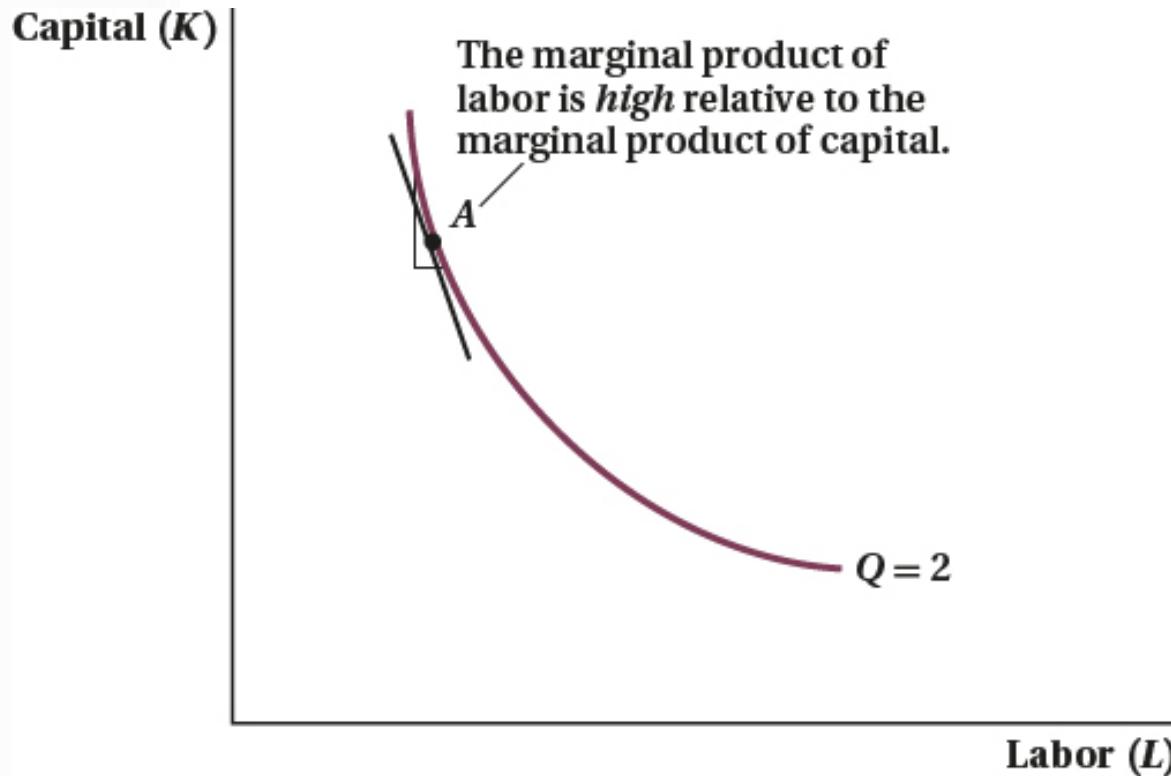
## The Firm's Cost-Minimization Problem (3/15)

The slope of an isoquant describes how inputs may be substituted to produce a fixed level of output.

This relationship is referred to as the **marginal rate of technical substitution**: the rate at which the firm can trade input  $X$  for input  $Y$ , holding output constant ( $MRTS_{XY}$ ).

## The Firm's Cost-Minimization Problem (4/15)

Figure 6.4 The Marginal Rate of Technical Substitution



## The Firm's Cost-Minimization Problem (5/15)

Mathematically,  $MRTS_{LK}$  can be derived from the condition that, along an isoquant, quantity of output produced is held constant.

$$\Delta Q = MP_L \times \Delta L + MP_K \times \Delta K = 0$$

Rearranging to find the slope of the isoquant yields the  $MRTS_{LK}$ .

$$MP_K \times \Delta K = -MP_L \times \Delta L \rightarrow MRTS_{LK} = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$$

Moving down an isoquant, the amount of capital used declines.

- $MRTS_{LK}$  describes the rate at which labor must be substituted for capital to hold the quantity produced constant.
- As you move down an isoquant, the slope gets smaller, meaning the firm has less capital and each unit is relatively more productive.

# The Firm's Cost-Minimization Problem (6/15)

## The Curvature of Isoquants: Substitutes and Complements

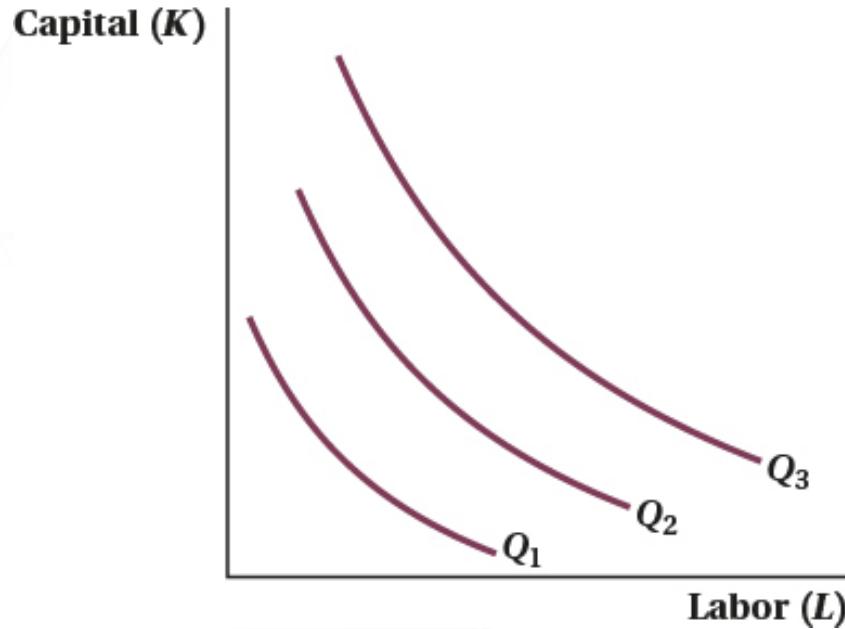
The shape of an isoquant reveals information about the relationship between inputs to production.

1. Relatively **straight** isoquants imply that the inputs are *relatively substitutable*.
  - $MRTS_{LK}$  does NOT vary much along the curve.
2. Relatively **curved** isoquants imply the inputs are *relatively complementary*.
  - $MRTS_{LK}$  varies greatly along the curve.

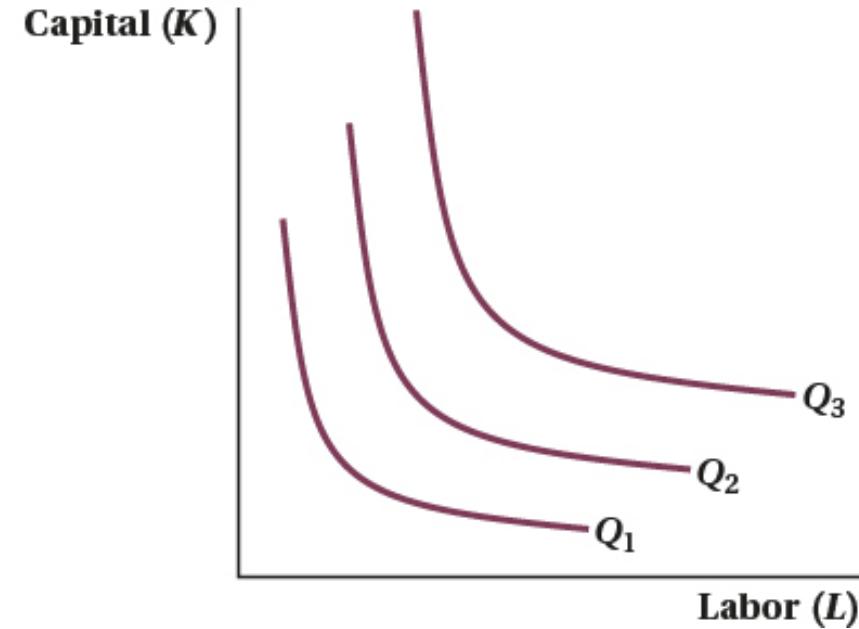
## The Firm's Cost-Minimization Problem (7/15)

Figure 6.5 The Shape of Isoquants Indicates the Substitutability of Inputs

(a) Inputs Are Close Substitutes



(b) Inputs Are Not Close Substitutes



# The Firm's Cost-Minimization Problem (8/15)

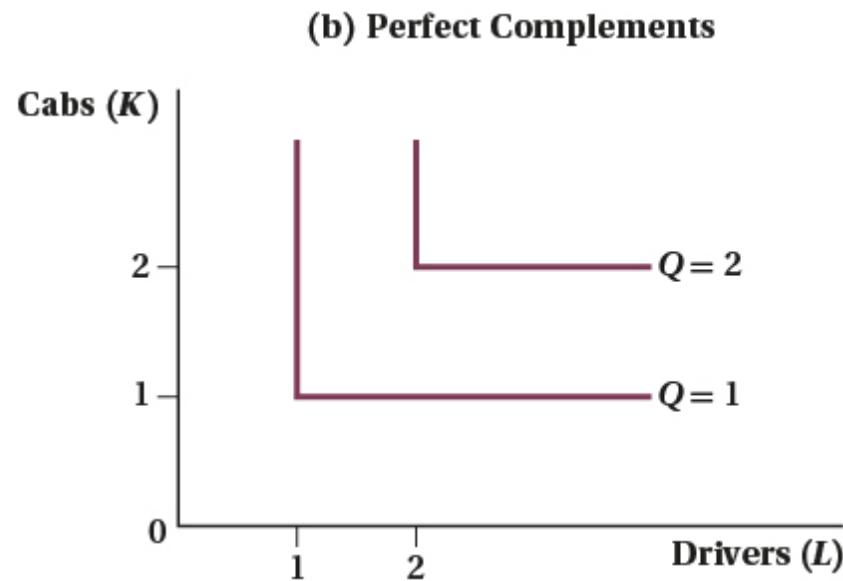
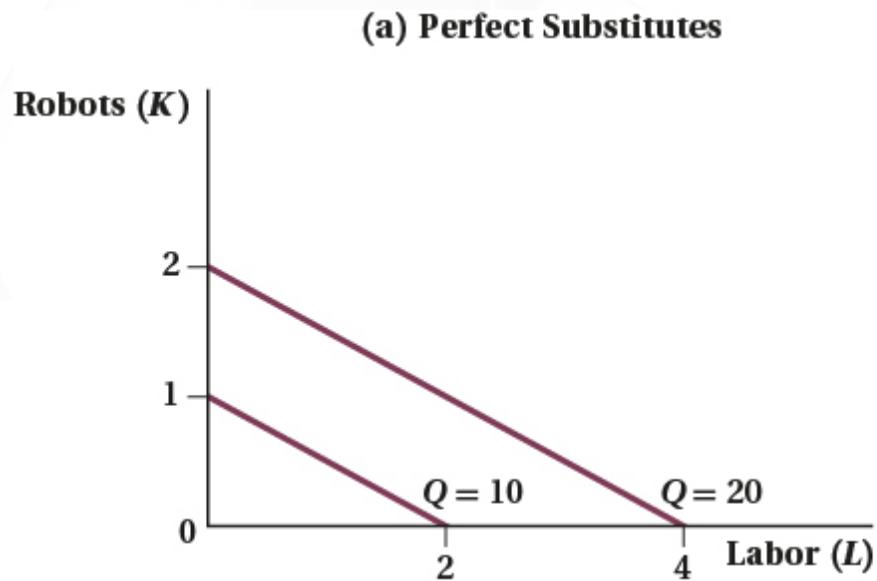
## The Curvature of Isoquants: Substitutes and Complements

To illustrate, consider extreme cases:

1. When inputs are **perfect substitutes**, they can be traded off in a constant ratio in a production process.
  - $MRTS$  is constant.
  - For example, labor and robots could be perfect substitutes in production (shown in Figure 6.6 – next slide).
2. When inputs are **perfect complements**, they must be used in a fixed ratio as part of a production process.
  - For example, cabs and drivers (shown in Figure 6.6 – next slide)

## The Firm's Cost-Minimization Problem (9/15)

Figure 6.6 Perfect Substitutes and Perfect Complements in Production



## The Firm's Cost-Minimization Problem (10/15)

An **isocost line** shows all of the input combinations that yield the same cost.

- Similar to the budget constraint facing consumers, using the equation

$$C = RK + WL$$

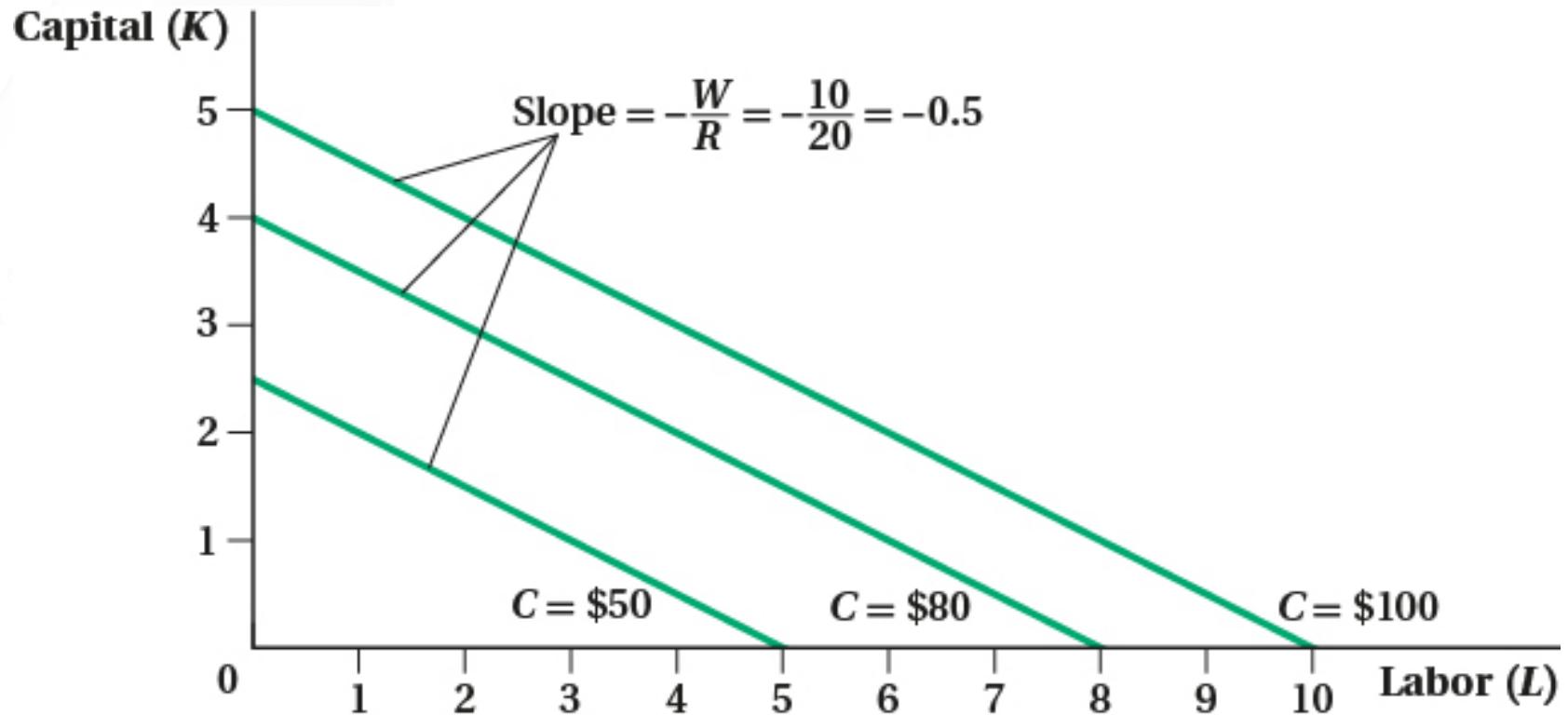
where  $C$  is total cost,  $R$  is the “rental rate” of capital, and  $W$  is the wage rate.

- Rearranging yields capital as a function of the rental rate, wage rate, and labor.

$$K = \frac{C}{R} - \frac{W}{R}L$$

# The Firm's Cost-Minimization Problem (11/15)

Figure 6.7 Isocost Lines



# The Firm's Cost-Minimization Problem (12/15)

## Identifying Minimum Cost: Combining Isoquants and Isocost Lines

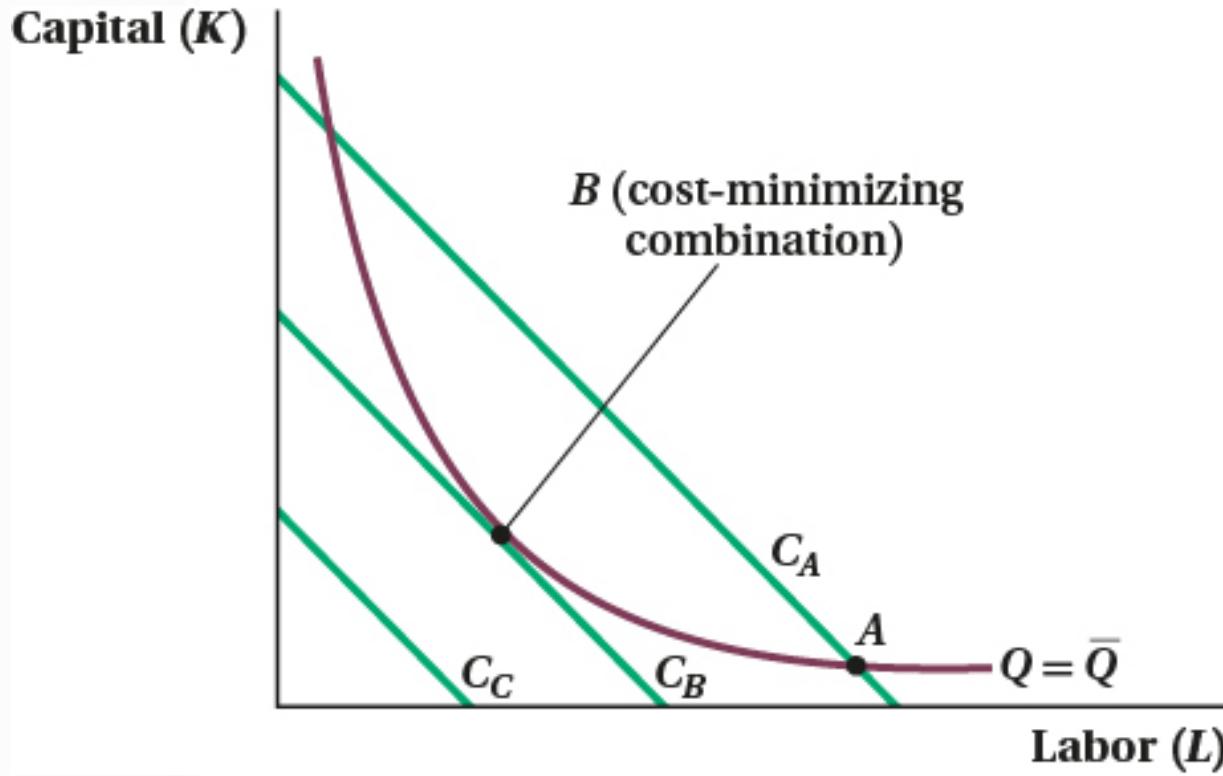
Remember, the firm's problem is one of *constrained minimization*.

- Firms minimize costs subject to a given amount of production.
- Cost minimization is achieved by adjusting the ratio of capital to labor.
  - Similar to expenditure minimization in Chapter 4

Graphically, cost minimization requires tangency between the isoquant associated with the chosen level of production, and the lowest cost isocost line.

# The Firm's Cost-Minimization Problem (13/15)

Figure 6.10 Cost Minimization



# The Firm's Cost-Minimization Problem (14/15)

## Identifying Minimum Cost: Combining Isoquants and Isocost Lines

Mathematically, tangency occurs where the slope of the isocost line is equal to the slope of the isoquant, or:

$$-\frac{W}{R} = -\frac{MP_L}{MP_K} \rightarrow \frac{MP_K}{R} = \frac{MP_L}{W}$$

### What does this condition imply?

- Costs are minimized when the marginal product per dollar spent is equalized across inputs.

# The Firm's Cost-Minimization Problem (15/15)

## Cost Minimizing Condition:

- Costs are minimized when the marginal product per dollar spent is equalized across inputs.

$$-\frac{W}{R} = -\frac{MP_L}{MP_K} \rightarrow \frac{MP_K}{R} = \frac{MP_L}{W}$$

IF     1.  $\frac{MP_K}{R} > \frac{MP_L}{W}$                   or              2.  $\frac{MP_K}{R} < \frac{MP_L}{W}$

1. The marginal product per dollar spent on capital is higher than the marginal product per dollar spent on labor.
  - **More capital and less labor should be used in production.**
2. The marginal product per dollar spent on capital is less than the marginal product per dollar spent on labor.
  - **More labor and less capital should be used in production.**

# Returns to Scale (1/5)

**Returns to scale** refers to the change in output when all inputs are increased in the *same proportion*.

Returning to the Cobb–Douglas production function

$$Q = K^\alpha L^\beta$$

If  $\alpha = \beta = 0.5$ , then

$$Q = K^{0.5} L^{0.5}$$

- If  $K=2$  and  $L=2$ , then  $Q=2$ .

**What happens if the amount of capital and labor used both double?**

$$Q = 4^{0.5} 4^{0.5} = 2 \times 2 = 4$$

**Output Doubles!**

# Returns to Scale (2/5)

This relationship, whereby production increases proportionally with inputs, is called **Constant Returns to Scale**.

- If double inputs → output doubles

**Increasing Returns to Scale:** describes production for which changing all inputs by the same proportion changes output *more* than proportionally.

- If double inputs → output more than doubles

**Decreasing Returns to Scale:** describes production for which changing all inputs by the same proportion changes output *less* than proportionally.

- Double inputs → output increases by *less* than double.

# Returns to Scale (3/5)

Why might a firm experience ***increasing returns to scale***?

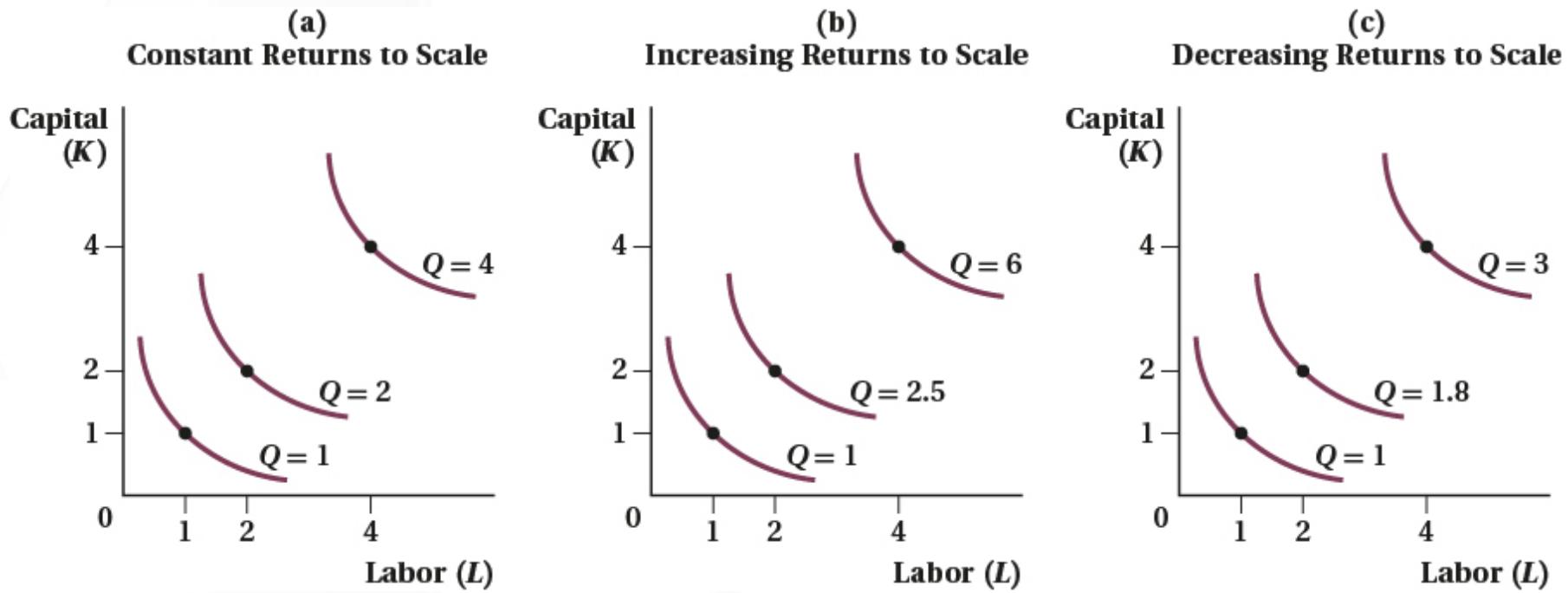
- **Fixed costs** (e.g., webpage management, advertising contracts) do not vary with output.
- **Learning by doing** may occur, whereby a firm develops more efficient processes as it expands or produces more output.

Generally, firms should not experience ***decreasing returns to scale***.

- When this phenomenon is observed in data, it often results from not accounting for all inputs (or attributes).

# Returns to Scale (4/5)

Figure 6.12 Returns to Scale



# Returns to Scale (5/5): Question 1

If a firm has the Cobb-Douglas production function  $Q = K^{0.75}L^{0.5}$ , it faces:

- A. efficient returns to scale.
- B. constant returns to scale.
- C. increasing returns to scale.
- D. decreasing returns to scale.

# Returns to Scale (5/5): Question 1 – Correct Answer

If a firm has the Cobb-Douglas production function  $Q = K^{0.75}L^{0.5}$ , it faces:

- A. Efficient returns to scale
- B. Constant returns to scale
- C. Increasing returns to scale (correct answer)
- D. Decreasing returns to scale

# Technological Change (1/2)

Examining firm-level production data over time reveals increasing output, even when input levels are held constant.

- The only way to explain this is by assuming some change to the production function.

This change is referred to as **Total Factor Productivity Growth**.

- An improvement in technology that changes the firm's production function such that more output is obtained from the same amount of inputs

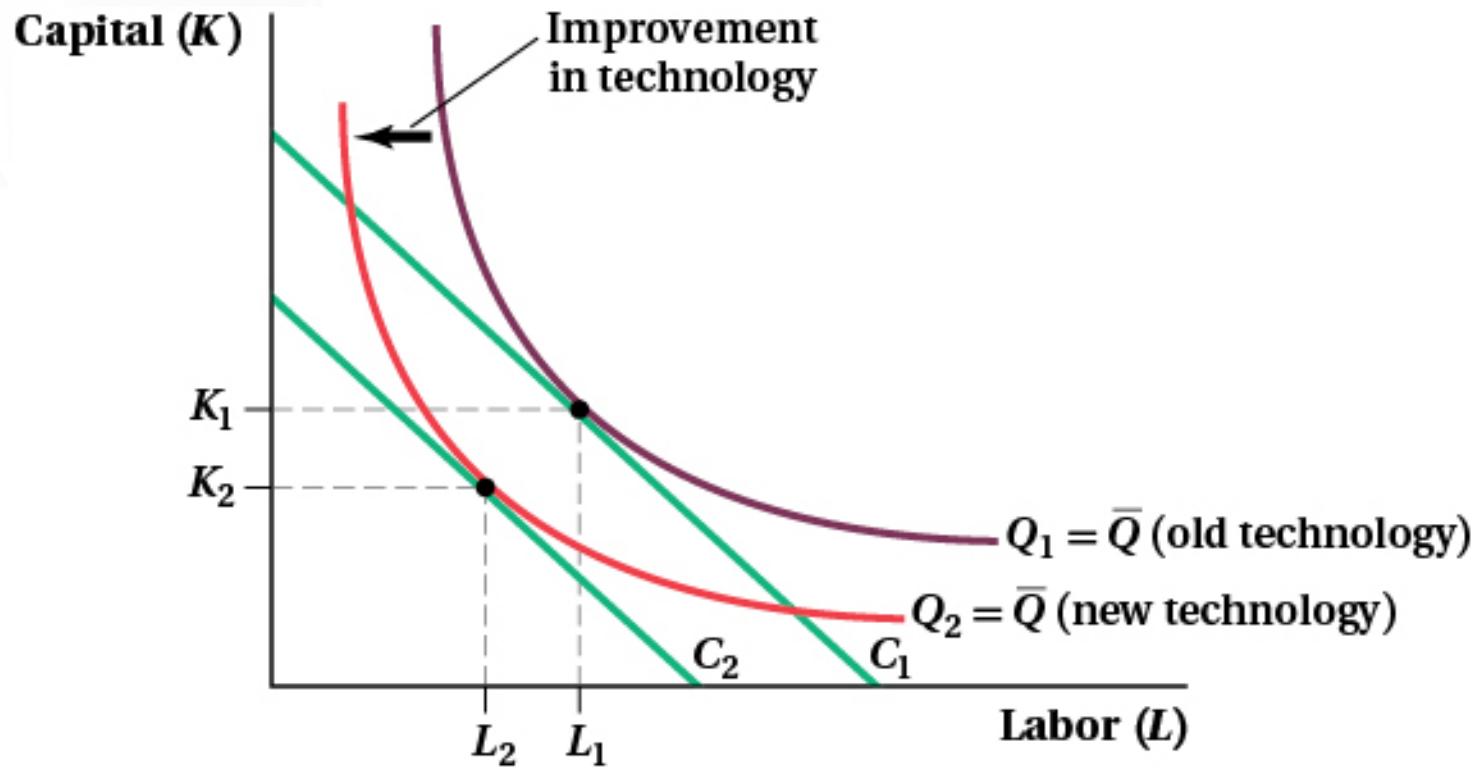
Often assumed to enter multiplicatively with production,

$$Q = Af(K, L)$$

where  $A$  is the level of total factor productivity.

# Technological Change (2/2)

Figure 6.13 The Impact of Technological Change



# The Firm's Expansion Path and Total Cost Curve (1/2)

So far, we have only focused on how firms minimize costs, subject to a fixed quantity of output.

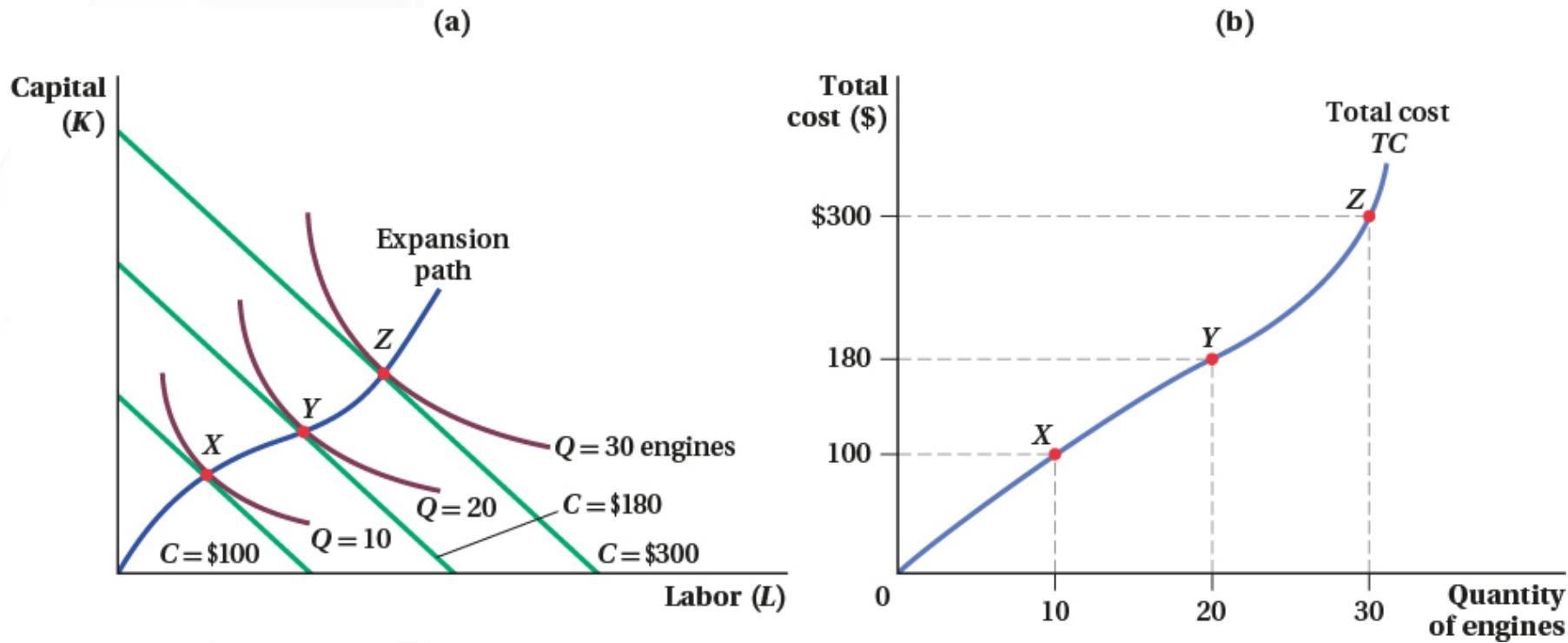
- We can use the cost minimization approach to describe how capital and labor change as output increases.

An **expansion path** is a curve that illustrates how the optimal mix of inputs varies with total output.

This allows construction of the **total cost curve**, which shows a firm's cost of producing particular quantities.

# The Firm's Expansion Path and Total Cost Curve (2/2)

Figure 6.14 The Expansion Path and the Total Cost Curve



# Conclusion (1/1)

This chapter looked closely at how firms make decisions.

- Firms are assumed to minimize costs at every level of production.
- The cost-minimizing combination of inputs occurs where the marginal rate of technical substitution is equal to the slope of the isocost line.

In **Chapter 7**, we delve deeper into the different costs facing firms, and how they change with the level of production.