EconS 305: Intermediate Microeconomics w/o Calculus

Homework 3:
Market Analysis, Monopoly and Perfect Competition

Due: Friday, June 5th, 2020 at 5:00pm via Blackboard

- Please submit all homework solutions in the order the questions are presented and as **one .PDF**.

- Please **show all calculations** as these exercises are meant to refine your quantitative tool set. If I can not follow your calculations or it seems as you just "copy and pasted" answers from the internet, I will be deducting half the points from that solution.

1. **A Market Welfare Analysis of a Tax using Linear Demand and Supply Curves**

Consider a demand curve for rice $Q^D = 22 - 2P$ and a supply curve for rice $Q^S = 3P - 23$, where both quantities are measured in pounds.

(a) Find the before tax market equilibrium (i.e. Q^* and P^*).

(b) Find the pre-tax consumer surplus.

(c) Find the pre-tax producer surplus.

(d) What is the total social welfare in the market of rice?

(e) Now, assume that we are going to apply a \$.50 tax to each pound of rice sold, which means that each rice producer will have pay \$.50 to the government for every pound sold. This means that the price the buyer will pay is $P_b = P_s + \$.50$. What is the equilibrium price received by the buyer (P_b) and producer (P_s), respectively? (Hint: the new demand curve is $Q^D = 22 - 2(P_b)$ and the new supply curve for rice $Q^S = 3P_s - 23$. Solve for P_s first, then plug in for P_b.)

(f) What is the new quantity demanded and supplied in the new equilibrium with the tax? Show that they are equivalent using the prices you just found.

(g) What is the Consumer Surplus after the tax?

(h) What is the Producer Surplus after the tax?

(i) What is the Total Social Welfare after the tax? What is the loss in Social Welfare because of the tax?

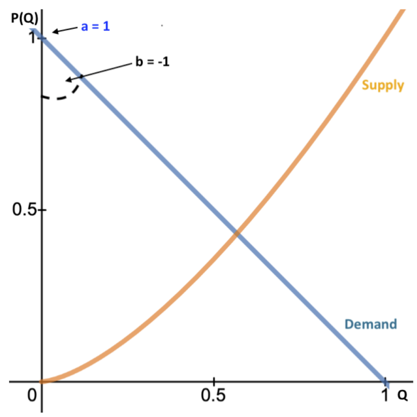
(j) What is the Government Revenues?

(k) Calculate the Dead Weight Loss (DWL) from the tax.

2. **The Basic Case of a Monopoly with Fixed Costs**

Consider a monopolist facing a linear inverse demand function of $p(Q) = a - b(Q)$, where $a > c$, and a total cost function of $TC(Q) = cQ + F$. We interpret a as the intercept, or the choke price consumers are willing to pay for $Q = 0$, of the inverse demand curve, and b as the slope of the inverse demand curve. Graphically, it can be represented as

Figure 1: The Linear Demand Curve



We can interpret the fixed cost (F) as perhaps some "entry" fee, and we interpret c as the marginal cost the firm has to pay according to how much output they produce. We can represent the Profit Maximization Problem (PMP) for firm as:

$$\max_{Q \geq 0} \ \pi = p(Q)Q - (cQ + F)$$

$$\implies \max_{Q \geq 0} \ \pi = [a - bQ]\, Q - (cQ + F)$$

CALCULUS PART:

From here, we can take our derivatives and set them equal to zero

$$\frac{\partial \pi(Q)}{\partial Q} \quad = \quad a - 2bQ - c = 0 \tag{1}$$

where we now have one equation ((1)), and one choice variable (Q) to solve for.

CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.

(a) Find the firm's optimal allocation of production (Q) to maximize its profit in equilibrium (i.e. find Q^*).

(b) What is the equilibrium price the firm will receive (i.e. find $p(Q^*)$?

(c) What is the optimal profit function of the firm (i.e. $\pi^*(Q^*)$)?

(d) What is the level of fixed cost in which the firm will choose to continue to operate?

(e) Will the firm produce if $a < c$? Careful when answering this question.

3. **A Cournot Game of Competing in Quantities w/ Fixed Costs**

Consider two firms competing a la Cournot in a market with an inverse demand function of $p(Q) = a - b(Q)$ where $Q = q_i + q_j$ and $a > c$, and total cost function of $TC_i(q_i) = F + c_i q_i$. Notice that each firm has the same fixed cost (F) but their marginal costs (c_i) are not equal to each other (i.e. $c_i \neq c_j$). This means these homogeneous product producing firms have asymmetric costs, and we can represent the Profit Maximization Problem (PMP_i) for firm i as:

CALCULUS PART:

$$\max_{q_i \geq 0} \quad \pi_i = [a - b(q_i + q_j)]\, q_i - (F + c_i q_i)$$

$$\frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = a - 2bq_i - bq_j - c_i = 0 \tag{2}$$

And through symmetry we know that firm j's PMP is

$$\max_{q_j \geq 0} \quad \pi_j = [a - b(q_i + q_j)]\, q_j - (F + c_j q_j)$$

$$\frac{\partial \pi_j(q_i, q_j)}{\partial q_j} = a - 2bq_j - bq_i - c_j = 0 \tag{3}$$

where we now have two equations ((2) and (3)), and two choice variables (q_i and q_j) to solve for.

CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.

(a) Before you solve for the optimal equilibrium allocations, find the Best Response Functions ($BRFs$) for each firm (i.e. find $q_i(q_j)$ and $q_j(q_i)$). How does the firm respond in their own quantities with respect to an increase in a, b, c_i, and q_j?

(b) Find the optimal equilibrium allocation for each firm when they are competing a la Cournot. That is, find q_i^* and q_j^*. How does firm i's equilibrium allocation change with respect to an increase in their own marginal costs (c_i) and their opponents marginal cost (c_j)? Which increase is larger in absolute magnitude?

(c) Now, consider that the firm's have symmetric costs (i.e. $c_i = c_j = c$) in the competitive equilibrium and for all analyses from here on out. Find the competitive equilibrium quantities (i.e. find q_i^* and q_j^*).

(d) Find the equilibrium price (i.e. $p(Q^*) = a - b(Q^*)$).

(e) Find the equilibrium profits (i.e. π^*).

4. **A Cournot Game with N Firms Competing in Quantities w/ Fixed Costs**

Consider N firms competing a la Cournot in a market with an inverse demand function of $p(Q) = a - b(Q)$, where $Q = \sum_{i=1}^{N} q_i$ and $a > c$, and total cost function of $TC_i(q_i) = F + cq_i$. Notice that each firm has the same fixed cost (F) and, for simplicity, their marginal costs (c) are equal to each other (i.e. $c_i = c_j = \cdots = c_N = c$). This means these homogeneous product producing firms have symmetric costs, and we can represent the Profit Maximization Problem (PMP_i) for firm i as:

CALCULUS PART:

$$\max_{q_i \geq 0} \ \pi_i = \left[a - b\left(\sum_{i=1}^{N} q_i\right)\right] q_i - (F + cq_i)$$

$$\implies \max_{q_i \geq 0} \ \pi_i = \left[a - b\left(q_i + \sum_{i \neq j}^{N} q_j\right)\right] q_i - (F + cq_i)$$

$$\frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = a - 2bq_i - b\sum_{i \neq j}^{N} q_j - c = 0 \tag{4}$$

where we now have a symmetric equation ((4)) and one choice variable for each firm i (q_i) to solve for.

CALCULUS PART FINISHED. YOUR CALCULATIONS START HERE.

(a) Before you solve for the optimal equilibrium allocation for firm i, find the Best Response Function ($BRFs$) for firm i (i.e. find $q_i\left(\sum_{i \neq j}^{N} q_j\right)$). How does the firm respond in their own quantity with respect to an increase in a, b, c and all other quantities (i.e. q_j)?

(b) Find the optimal equilibrium allocation for each firm i when they are competing a la Cournot. That is, find q_i^* for all $i \in \{1, 2, \ldots, N\}$. To do this, please invoke the assumption that the firms are symmetric in output (i.e. $q_i = q_j$), and that the sum of a constant is equal to the multiplying by the number of constants in the sum (i.e. $\sum_{i=1}^{N} q_i = Nq_i$ when $q_i = q_j$).

(c) Find the Aggregate Quantity Demanded (i.e. $Q^* = \sum_{i=1}^{N} q_i^*$)

(d) Find the equilibrium price (i.e. $P(Q^*)$)

(e) Find the equilibrium profits for each firm (i.e. π^*).

(f) Assuming that we are operating in a perfectly competitive equilibrium (i.e. set $\pi^* = 0$, find the optimal number of firms in the industry (i.e. solve for N^*). Does the equilibrium number of firms increase or decrease as the demand curve becomes more inealastic?

5. **Comparing Outputs and Profits across Market Structures**

- Please assume $a > c$ throughout the analysis.

(a) Take each optimal quantity produced from Questions 2-4, and compare them mathematically (i.e. $q_i^{Monopoly}$ ($<$ or $>$) $q_i^{Duopoly}$ ($<$ or $>$) $q_i^{Perfect\ Competition}$). Please rank them in terms of highest quantities to lowest, assuming that $N \geq 3$. What happens as the number of firms increases?

(b) Take each optimal price you found from Questions 2-4, and compare them mathematically (i.e. $p(Q^*)^{Monopoly}$ ($<$ or $>$) $p(Q^*)^{Duopoly}$ ($<$ or $>$) $p(Q^*)^{Perfect\ Competition}$). Please rank them in terms of highest prices to lowest, assuming that $N \geq 3$. Which price is the greatest and which is the least? Is this different than the quantities ranking? If so, why is this?

(c) Take each profit you found from Questions 2-4, and compare them mathematically (i.e. $\pi^{Monopoly}$ ($<$ or $>$) $\pi^{Duopoly}$ ($<$ or $>$) $\pi^{Perfect\ Competition}$). Please rank them in terms of highest profits to lowest, assuming that $N \geq 3$.