Multiple Signals in a Corporate Socially Responsible Equilibrium

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Abstract

In this paper, we examine credence good consumption in a setting with multiple signals by generalizing and extending Calveras and Ganuza [2016]'s Corporate Social Responsibility (CSR) model through their signal reliability parameter. We propose a generalized index for signal unreliability, and then show how this index can be a function of other exogenous reliability and signal richness parameters. We also find an optimal level of signal unreliability that gives a firm the most incentive to invest in CSR, and we evaluate this optimal parameter as a function of signal reliability and richness through examples. We show how this incentive increases in the number of possible signals that consumers receive (information richness), but then decreases. We also find that investment cost certainty is less effective as the number of signals increase. Our results imply that more signals initially provide firms with more incentive to invest in CSR, but with less incentive as the number of signals increases. Policy makers can use our framework to evaluate how multiple signaling affects firm incentive to invest in CSR, and they can use it to optimize firm incentive to invest in CSR through information market regulation.

Keywords: Corporate Social Responsibility, Multiple Signals, Information, Reliability.

JEL classification: C70 - Game Theory and Bargaining Theory, General; D82 - Information, Knowledge, and Uncertainty; L21 - Business Objectives of the Firm; M14 - Corporate Culture, Diversity, Social Responsibility.

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1 Introduction

A company practices Corporate Social Responsibility (CSR) when they voluntarily integrate social and environmental concerns into their business operations, and this practice is generally promoted by regulators in business and international trade; see (European Commission [2001]; U.S. Department of State [2016]).[1] While firms face a trade-off between profits and social/environmental practices under complete information in the long run, this trade-off becomes more involved when consumers cannot accurately observe firms' CSR decisions (e.g., investment in abatement of recycling practices). Without complete information, consumers observe noisy or unreliable signals that can lead to inefficient equilibrium outcomes. For this reason, policy makers have long debated how to get businesses to practice CSR in environments with noisy signaling.

Calveras and Ganuza [2016] investigate signals in a CSR framework, and assume a binary signaling mechanism where consumers only observe whether the firm invested in CSR or not. While this type of signaling helps sustain separating equilibria that convey information to uninformed consumers, they are relatively restrictive, because many real-life signaling mechanisms disseminate signals at various intensity levels. For example, when a news outlet reports about a firm investing in clean technologies, it provides different details about the dollar amount, the type of technology, and how it compares to other, more polluting, technologies. Our model assumes that all of these details can be summarized by an index number, capturing the intensity of the firm's CSR practices, thus allowing for multiple (not necessarily binary) signals. This represents a situation where every individual receives news reports from different outlets, each of them providing a different "perspective" on a firm's CSR practices, and then combines these perspectives into a single index. This type of information gathering is common in many settings, and became more prevalent in recent decades, as the number of information outlets and detailed reporting expanded. The most natural example of this gathering behavior is the increasing usage of social media platforms, where 7 out of 10 US adults said they used some form of social media in 2021 (Pew Research Center [2021]).

We show that the presence of multiple signals first increases a firm's incentive to invest in CSR, and then decreases it until an equilibrium with complete information is reached. This result holds under different probability structures, such as linear conditional probabilities, uniformly distributed signals, and conditional probabilities concentrated in two points, and we evaluate how signal reliability and richness affect our findings. We show that as signal reliability and/or richness increases, a firms' incentive to invest in CSR increases and then decreases. We also show that richer signals reduce the firm's incentives to invest in CSR even when investments become more feasible.

1.1 Related Literature

Calveras and Ganuza [2016] investigate the role of public information in a CSR equilibrium, and develop a perfect bayseian equilibrium (PBE) framework with public information. In their framework, they introduce an exogenous information structure using a "transparency" parameter mapped between zero and one, and this parameter represents signal reliability to consumers and producers. In their benchmark model, they find that the strategic firm's expected profits are increasing (decreasing) in the reliability of the signaling parameter γ for when the firm chooses to practice CSR (or not).

[1] Other concepts closely related to CSR, such as Environmental, Social and Governance (ESG) values (Washington, D.C.: World Bank Group [2017]), have been promoted to expand the philosophical meaning of CSR, but explicitly use the term "CSR" in this paper to be consistent with the previous literature we cite.

We extend Calveras and Ganuza [2016]'s model by proposing a more general framework that facilitates an equilibrium with multiple signals, and we do this through the aforementioned reliability parameter γ. This extension involves generalizing the entire equilibrium framework, and with this generalization, we show that the reliability parameter can be transformed into an "unreliability" parameter and that this parameter can be a function of other parameters that govern the signaling mechanism. With this alteration, we show that there is an optimal unreliability level $\widetilde{\gamma_1}$, in terms of investment uncertainty, that gives a firm the most incentive to invest in CSR. One large contribution to the literature we make is that we provide four different signaling structure examples that support our generalized framework including two examples that introduce a signal richness parameter N, which is dictated by the number of unique signals in the information market.

We show that our generalized model can mimic Calveras and Ganuza [2016]'s benchmark model and results, and we show that as signal richness increases, the strategic firm's difference in expected profits from choosing to invest in CSR are increasing until an optimal richness level, and then decreasing thereafter. We also find that even though the difference in expected profits increases as CSR investment becomes more certain, the intensity of this effect decreases as signal richness increases. These result are important because it gives researchers and policy makers a more general signaling framework to operate in when investigating how public information affects CSR, and it shows how signal richness (i.e. the number of signals in the information market) affects equilibrium efficiency, and ultimately, the behavior of a strategic firm.

2 Model

Consider the setting in Calveras and Ganuza [2016], henceforth CG for compactness, about consumers and firms. In particular, we assume a continuum of consumers (unit mass), each of them with utility $u = v + \alpha g - p$, where v denotes the valuation of the good, $g \in [0, G]$ represents the consumer's valuation of the credence attribute of the good, weighted by $\alpha \in [0, 1]$, which intuitively captures the warm-glow that the consumer experiences from consuming the credence good (Andreoni [1989]), and α is distributed according to a log-concave cumulative distribution function $H(\cdot)$, yielding a reliability function $\overline{H}(\cdot) = 1 - H(\cdot)$. Intuitively, for a given warm-glow α, the reliability function measures the mass of consumers with a warm-glow above α.

Following CG, assume that dirty firms compete a la Bertrand, with marginal cost normalized to zero. One firm, named "the firm" in CG, has the ability to invest in a clean technology, at a fixed cost $F \geq 0$, or to keep its dirty technology at no fixed cost. Technology is denoted as $t \in \{C, D\}$, either clean or dirty. The firm's fixed cost F determines its type: either $F \to +\infty$, which occurs with probability $\frac{1-\theta}{2}$; $F = 0$, which also happens with probability $\frac{1-\theta}{2}$; or $0 < F < +\infty$, which occurs with the remaining probability θ. The firm privately observes its fixed cost F and responds investing in clean technology or not.

Consumers do not observe whether a firm invested in clean or dirty technology, but know the firm has some prior probability of receiving a feasible fixed cost to either invest or not invest in the clean, Corporate Socially Responsible (CSR), technology. These priors change according to the interval at which a feasible fixed cost is realized, yielding two different equilibria: the Corporate Socially Responsible (CSR) and Not Corporate Socially Responsible (NCSR) equilibria. In the CSR equilibrium, the prior probability of a clean firm coincides with that in CG, that is, $\Pr(C) = \frac{(1-\theta)}{2} + \theta = \frac{(1+\theta)}{2}$, and that of the dirty firm is $\Pr(D) = \frac{1-\theta}{2}$. Conversely, the prior probabilities in the NCSR equilibrium are $\Pr(C) = \frac{1-\theta}{2}$ and $\Pr(D) = \frac{(1-\theta)}{2} + \theta = \frac{(1+\theta)}{2}$.

Consumers receive a unique signal $s_i \in \{s_1, ..., s_N\}$, which is interpreted as information from the media, describing observable decisions by the firm, ultimately helping consumers construct a "score" of how clean the firm is, with s_1 being the highest score (cleanest firm) and s_N denoting the lowest score (dirtiest firm). In this context, $N = 2$ available signals can be understood as a relatively simple news outlet (with a dichotomic presentation of the firm's decisions), whereas $N \geq 2$ represents media outlets describing the firm's decisions in greater detail. Alternatively, $N = 2$ reflects media channels as in CG, which either say that the firm is completely clean or dirty ("radical" news), while $N \geq 2$ allow for less radical news between 1 and N, with more subtleties about the firm's decisions.

2.1 Information Structure

We extend CG's information structure to a setting with $N \geq 2$ signals, where the probability of receiving signal s_i conditional on the firm investing in clean technology is $\Pr(s_1|C) = 1$ and $\Pr(s_i|C) = 0$ for all $i \neq 1$. In contrast, when a firm invests in dirty technology, the probability of receiving signal s_i is $\Pr(s_i|D) = \gamma_i$, where $1 \geq \gamma_i \geq 0$ and $\sum_{i=1}^{N} \gamma_i = 1$. This probability structure allows for several settings, as we illustrate in the next examples.

Example 1 (Two signals and CG) In CG's context with two signals, $N = 2$, $\gamma_1 = 1 - \gamma$ and $\gamma_2 = \gamma$. In summary,

$$\Pr(s_1|C) = 1 \quad \text{and} \quad \Pr(s_1|D) = 1 - \gamma$$

$$\Pr(s_2|C) = 0 \quad \text{and} \quad \Pr(s_2|D) = \gamma,$$

where parameter γ can be understood as the degree of "signal precision,"or as CG defines it more explicitly, as "market transparency." Our probability structure, then, embodies CG's as a special case where consumers can only observe two signals, $N = 2$. Indeed, signal s_2 (dirty, in a two-signal scenario) can only originate from the firm that invested in dirty technology, while signal s_1 (clean) can come from either firm type. For instance, when $\gamma = 0$, consumers only observe s_1 regardless of the firm's investment decision, implying that signals become uninformative; but when $\gamma = 1$, signals perfectly inform consumers about the firm's technology decision, namely, signal s_1 (s_2) only originates from the clean (dirty) firm.

Example 2 (Linear conditional probabilities) If $\Pr(s_i|D)$ is linear, it can take the following form

$$\Pr(s_i|D) = (1 - \gamma) + (i - 1)b$$

where $b > 0$. We now seek to find the exact expression for the slope, b. First, the above conditional probability must satisfy

$$\sum_{i=1}^{N} \big[(1 - \gamma) + (i - 1)b\big] = 1$$

or, after rearranging, and solving for b, yields $b = \frac{2\left[1-(1-\gamma)N\right]}{N(N-1)}$, since $\sum_{i=1}^{N} i = \frac{N(N+1)}{2}$. In addition, its slope b is positive, thus making $\Pr(s_i|D)$ increasing in signal "dirtiness" (higher i) if $\frac{2\left[1-(1-\gamma)N\right]}{N(N-1)} \geq 0$ or, after solving for γ, $\gamma \geq \frac{N-1}{N}$. Otherwise, $\Pr(s_i|D)$ becomes decreasing in the dirtiness of the signal. Intuitively, as the number of signals increases (higher N), condition $\gamma \geq \frac{N-1}{N}$ becomes more stringent, indicating that signals must be more precise if, in a context with more possible signals being received by consumers, we expect dirty signals to be more likely than cleaner ones.

Summarizing, the above conditional probability is

$$\Pr(s_i|D) = (1-\gamma) + (i-1)\frac{2\left[1-(1-\gamma)N\right]}{N(N-1)} \quad \text{where } 1 \geq \gamma \geq 0.$$

Finally, conditional probability $\Pr(s_i|D)$ decreases in N if and only if γ is relatively high, $\gamma \geq \frac{2N-1}{2N+1}$.

Example 3 (Uniform distribution/Infinite signals). If, in the context of Example 2, conditional probabilities are constant, $b = 0$, we obtain that $\Pr(s_i|D) = 1-\gamma$ for all i, which yields $\Pr(s_i|D) = \frac{1}{N}$ since $\gamma = \frac{N-1}{N}$ is the only feasible when solving for $\sum_{i=1}^{N} \Pr(s_i|D) = 1$. This probability structure arises when, for instance, there are infinitely many signals ($N \to +\infty$), since the slope in $\Pr(s_i|D)$ approaches zero.

Example 4 (Extreme cases). In our model, the probability structure in CG could be represented by $\Pr(s_1|C) = 1$ and $\Pr(s_i|C) = 0$ for all $i \neq 1$ when the firm invests in clean technology, and $\gamma_N = \gamma$ while $\gamma_i = \frac{1-\gamma}{N-1}$ for all $i \neq 1$ when the firm invests in dirty technology. Intuitively, consumers receive the cleanest signal, s_1, when the firm invests in clean technology. When the firm invests in dirty technology, consumers receive the dirtiest signal, s_N, with probability γ, while all other (cleaner) signals occur with the same probability, $\gamma_i = \frac{1-\gamma}{N-1}$ for all $i \neq N$.[2] In this setting, the likelihood ratio is $\frac{1}{\gamma_1} = \frac{N-1}{1-\gamma}$; and $\frac{0}{\gamma_i}$=0 for all $i \neq 1$, thus being decreasing in signal s_i and increasing in the number of signals.

2.2 Time structure

The timing of the game coincides with that in CG, but in the context of $N \geq 2$ signals:

1. Nature chooses the firm's type (i.e., its fixed cost F).
2. The firm privately observes F and responds with its technology $t \in \{C, D\}$. Its rivals produce the dirty good.
3. Nature chooses the signal realization, $s_i \in \{s_1, ..., s_N\}$, and all players observe this signal realization.
4. The firm chooses its price p.
5. Each consumer responds buying or not from the firm. If he does not buy, he makes a purchase from the dirty firms.
6. Firm profits are realized.

[2] In this context, γ cannot be readily understood as the information structure's signal precision, as in CG. When $\gamma = 1$, consumers receive the dirtiest signal, s_N, with certainty if the firm invests in dirty technology, and the probability of all other signals collapses to zero, $\gamma_i = 0$ for all $i \neq N$. However, when $\gamma = 0$, consumers never receive the dirtiest signal, s_N, but receive all other signals with the same probability $\gamma_i = \frac{1}{N-1}$ for all $i \neq N$.

2.3 Equilibrium analysis

As usual, we solve by backward induction and find the indifferent consumer ($\bar{\alpha}$) such that $v+\alpha \Pr(C|s)g-p=v$, or $\bar{\alpha}=\frac{p}{\Pr(C|s)g}$. Intuitively, when $\alpha \geq \bar{\alpha}$, he buys the differentiated good since the "warm-glow" effect is sufficiently important for him. Otherwise, he buys the standard/dirty good. Since α is distributed according to $H(\cdot)$, the mass of consumers buying the differentiated good is

$$\alpha = 1 - H\left(\frac{p}{\Pr(C|s)g}\right).$$

Anticipating this consumer demand, firms choose price p to solve the following profit-maximization problem

$$\pi(p,s) = \left(1 - H\left(\frac{p}{P(C|s)G}\right)\right)p.$$

Differentiating with respect to p, and solving for p, yields the following results.

Lemma 1. *The firm sets price $p^*(s)=\alpha^* P(C|s)g$, earning expected profit $\pi^*(s)=P(C|s)(1-H(\alpha^*))g\alpha^*=P(C|s)\Pi$, where $\Pi \equiv (1-H(\alpha^*))g\alpha^*$ denotes expected profits under complete information and $\alpha^* \equiv \frac{p^*}{\Pr(C|s)g}$.*

We examine Perfect Bayesian Equilibria (PBE) where the firm invests in clean technology (denoted in CG as the "CSR Equilibrium") and that where the firm does not invest in clean technology (denoted as the "NCSR Equilibrium"). We focus on how a richer set of signals (increasing N) changes the firm's incentives to invest in clean technology.

3 Corporate Social Responsible Equilibrium

As in CG, in this equilibrium priors beliefs are that the firm invests in the clean technology with probability $\Pr(C)=\frac{1-\theta}{2}+\theta=\frac{1+\theta}{2}$ and in the dirty technology otherwise. For it to be optimal that the firm chooses the clean technology it must be that the expected profits (before realization of the public signal) when choosing the "clean" technology are larger than the expected profits using the "dirty" technology, that is,

$$E[\pi(t=C)] \geq E[\pi(t=D)].$$

Let $\pi^*_{CSR}(s_i) \equiv \Pi \Pr(C|s_i)$ be the firm's profits when, after a public signal s_i is realized, consumers believe that the firm chose a clean technology. Using Bayes' rule we obtain:

$$\pi^*_{CSR}(s_i) = \Pi \frac{\Pr(C)\Pr(s_i|C)}{\Pr(C)\Pr(s_i|C)+\Pr(D)\Pr(s_i|D)}$$

$$\text{which yields, } \pi^*_{CSR}(s_1) = \Pi \frac{(1+\theta)}{(1+\theta)+(1-\theta)\gamma_1} \quad \text{and} \quad \pi^*_{CSR}(s_i)=0 \qquad \text{for all } i \neq 1.$$

We next evaluate the comparative statics for these updated profits.

Lemma 2. *Profit $\pi^*_{CSR}(s_i)$ is weakly decreasing in signal s_i and in γ_1, and weakly increasing in θ.*

While $\pi^*_{CSR}(s_i)$ is not a function of the transparency parameter γ or the number of signals N, γ_1 can be, as shown in Examples 3 and 4. In particular, if γ_1 is decreasing in γ or N, $\pi^*_{CSR}(s_i)$ is then, consequentially, increasing in γ and N. Intuitively, the firm's profits decrease as the signal consumers receive becomes dirtier and when the probability of receiving the cleanest signal, given that the firm has chosen the dirty technology γ_1, increases. γ_1 can be interpreted as signal unreliability because consumers will observe the cleanest signal when, in fact, the firm has chosen the dirty technology. In complete information, consumers theoretically would have no probability of observing this signal if the firm chose to keep the dirty technology.

3.1 CSR Expected Profits

Therefore, the expected profits when a firm chooses to be clean or dirty are

$$E\left[\pi_{CSR}(t=C)\right] = \sum_{i=1}^{N} \Pr(s_i|C)\pi^*_{CSR}(s_i) - \hat{F} = \Pi\frac{(1+\theta)}{(1+\theta)+(1-\theta)\gamma_1} - \hat{F}, \text{ and}$$
$$E\left[\pi_{CSR}(t=D)\right] = \sum_{i=1}^{N} \Pr(s_i|D)\pi^*_{CSR}(s_i) = \Pi\frac{(1+\theta)\gamma_1}{(1+\theta)+(1-\theta)\gamma_1}$$

with the following properties.

Proposition 1. *In a CSR equilibrium, the expected profits of investing in the clean (dirty) technology are decreasing (increasing) in γ_1, and are both are increasing in θ.*

Similar to the results presented in Lemma 2, if γ and N have negative relationship with γ_1, then expected profits when investing in the clean (dirty) technology are instead increasing (decreasing) in market transparency and the number of signals.

Using our above results, we are ready to define that, in a CSR equilibrium, the firm invests in clean technology if and only if expected profits satisfy $E[\pi_{CSR}(t=C)] \geq E[\pi_{CSR}(t=D)]$, which rearranging and solving for $\hat{F}$, yields

$$\begin{aligned}\Delta\pi_{CSR} &= \sum_{i=1}^{n}\big(Pr(s_i|C) - Pr(s_i|D)\big)\,\pi^*_{CSR}(s_i) \geq \hat{F}\\ &= \Pi\frac{(1+\theta)(1-\gamma_1)}{(1+\theta)+(1-\theta)\gamma_1} \geq \hat{F}.\end{aligned}$$

Corollary 1. *In a CSR equilibrium, the difference in expected profits of investing in the clean technology is decreasing in γ_1, but increasing in θ.*

As in Proposition 1, if γ and N have a negative relationship with γ_1, then the difference in expected profits $\Delta\pi_{CSR}$ increases in γ and N. For illustration purposes, Appendix 1 evaluates expected profits $E\left[\pi_{CSR}(t=C)\right]$ and $E\left[\pi_{CSR}(t=D)\right]$ in the special cases of examples 1-4. After characterizing equilibrium behavior in section 4, we ellaborate on these four examples again.

4 Not Corporate Social Responsible Equilibrium

In this equilibrium priors beliefs are that the firm invests in the clean technology with probability $\Pr(C) = \frac{1-\theta}{2}$ and keeps the dirty technology with probability $\Pr(D) = \frac{1-\theta}{2} + \theta = \frac{1+\theta}{2}$. For it to be optimal that the firm does not choose the clean technology it must be that expected profits satisfy

$$E[\pi(t = C)] \leq E[\pi(t = D)].$$

Let $\pi^*_{NCSR}(s_i) \equiv \Pi \Pr(C|s_i)$ be the firm's profits when, after a public signal s_i is realized, consumers believe that the firm chose the clean technology. Using Bayes' rule we obtain:

$$\pi^*_{NCSR}(s_1) = \Pi \frac{(1-\theta)}{(1-\theta) + (1+\theta)\gamma_1} \quad \text{and} \quad \pi^*_{NCSR}(s_i) = 0 \qquad \text{for all } i \neq 1,$$

which produces Lemma 3.

Lemma 3. *Profit $\pi^*_{NCSR}(s_i)$ is weakly decreasing in signal s_i, γ_1, and θ.*

Similar to the CSR equilibrium, profit $\pi^*_{NCSR}(s_i)$ is not directly a function of the transparency parameter γ nor the number of signals N, but it can be indirectly, since γ_1 may depend on γ and N as shown in Examples 3 and 4. All comparative static from Lemma 2 hold in Lemma 3, except for the feasible fixed cost θ, which now produces a decrease in profit $\pi^*_{NCSR}(s_i)$

4.1 NCSR Expected Profits

In this context, the expected profits when a firm chooses to be clean or dirty are

$$\begin{aligned} E\left[\pi_{NCSR}(t = C)\right] &= \sum_{i=1}^{N} \Pr(s_i|C)\pi^*_{NCSR}(s_i) - \hat{F} \\ &= \Pi \frac{(1-\theta)}{(1-\theta) + (1+\theta)\gamma_1} - \hat{F}, \text{ and} \\ E\left[\pi_{NCSR}(t = D)\right] &= \sum_{i=1}^{N} \Pr(s_i|D)\pi^*_{NCSR}(s_i) \\ &= \Pi \frac{(1-\theta)\gamma_1}{(1-\theta) + (1+\theta)\gamma_1}. \end{aligned}$$

These expected profits satisfy the same properties as in the CSR Equilibrium with the exception of θ, which is now decreasing expected profits (see Proposition 1).

Proposition 2. *In a NCSR equilibrium, the expected profits of investing in the clean (dirty) technology are decreasing (increasing) in γ_1, and both are decreasing in θ.*

Similar to the results presented in Lemma 3, if γ and N have a negative relationship with γ_1, then expected profits when investing in the clean (dirty) technology are instead increasing (decreasing) in market transparency and the number of signals.

Using our above results, we are ready to define that, in a NCSR equilibrium, the firm does not invest in clean technology if and only if expected profits satisfy $E[\pi_{CSR}(t = C)] \leq E[\pi_{CSR}(t = D)]$. Rearranging and solving for $\hat{F}$ yields

$$\begin{aligned}\Delta\pi_{NCSR} &= \sum_{i=1}^{n} \big(Pr(s_i|C) - Pr(s_i|D)\big)\, \pi^*_{NCSR}(s_i) \leq \hat{F} \\ &= \Pi \frac{(1-\theta)(1-\gamma_1)}{(1-\theta) + (1+\theta)\gamma_1} \leq \hat{F}.\end{aligned}$$

Corollary 2. *In a NCSR equilibrium, the difference in expected profits of investing in the clean technology is decreasing in both γ_1 and θ.*

As in Proposition 2, if γ and N have a negative relationship with γ_1, then the difference in expected profits $\Delta\pi_{NCSR}$ increases in γ and N. As an illustration, Appendix 1 evaluates expected profits $E\left[\pi_{NCSR}(t = C)\right]$ and $E\left[\pi_{NCSR}(t = D)\right]$ in examples 1-4.

5 Equilibrium Conditions

Using the results from Sections 3.1 and 4.1, the firm adopts the clean technology if and only if the $\widehat{F}$ satisfies $\Delta\pi_{CSR} \geq \widehat{F} \geq \Delta\pi_{NCSR}$ or, alternatively, solving for the unreliability parameter $\widehat{\gamma_1}$,

$$\underbrace{\frac{(\Pi - \widehat{F})(1+\theta)}{\Pi(1+\theta) + \widehat{F}(1-\theta)}}_{\overline{\gamma}_1} \geq \widehat{\gamma_1} \geq \underbrace{\frac{(\Pi - \widehat{F})(1-\theta)}{\Pi(1-\theta) + \widehat{F}(1+\theta)}}_{\underline{\gamma}_1},$$

where $\overline{\gamma}_1$ and $\underline{\gamma}_1$ represent the upper and lower bounds for signal unreliability, respectively. Cutoffs $\overline{\gamma}_1$ and $\underline{\gamma}_1$ are both increasing in Π and decreasing in $\widehat{F}$, but $\overline{\gamma}_1$ ($\underline{\gamma}_1$) is increasing (decreasing, respectively) in θ.

From the first inequality, we can find the difference in profit gains $\Delta\pi = \Delta\pi_{CSR} - \Delta\pi_{NCSR}$,

$$\begin{aligned}\Delta\pi &= \Delta\pi_{CSR} - \Delta\pi_{NCSR} \\ &= \sum_{i=1}^{n} \big(Pr(s_i|C) - Pr(s_i|D)\big)\, (\pi^*_{CSR}(s_i) - \pi^*_{NCSR}(s_i)) \geq \widehat{F} \\ &= \Pi \frac{\gamma_1(1-\gamma_1)\left[(1+\theta)^2 - (1-\theta)^2\right]}{\left[(1+\theta) + (1-\theta)\gamma_1\right]\left[(1-\theta) + (1+\theta)\gamma_1\right]} \geq \widehat{F}.\end{aligned}$$

Corollary 3. *The difference in profit gains, $\Delta\pi$, is increasing in γ_1 if and only if $\gamma_1 < \widetilde{\gamma_1}$, where cutoff $\widetilde{\gamma_1} \equiv \frac{\theta^2 + 2\sqrt{1-\theta^2} - 1}{3+\theta^2}$ is decreasing in θ. Furthermore, $\Delta\pi$ is unambiguously increasing in θ, but in Example 4 $\frac{\partial\Delta\pi}{\partial\theta}$ is decreasing in the number of signals N.*

Intuitively, a higher profit gain expands the region of $\widehat{F}$ where the CSR equilibrium can be sustained. In terms of the unreliability parameter γ_1, Corollary 3 indicates that, when γ_1 is relatively low (reliable signals) the equilibrium cannot be supported; nor can when γ_1 is relatively high (extremely unreliable signals). The equilibrium, however, can be sustained under the largest set of parameter values when signals are relatively reliable (intermediate values of γ_1), as captured by the $\Delta\pi$ reaching its highest point at $\widetilde{\gamma}_1$. This intuition holds when γ_1 is specified as a function of other exogenous parameters, such as signal reliability γ and signal richness N. For example, when operating in the framework of Example 4, we find that $\Delta\pi$ is increasing in N if and only if $N < \left\lfloor \widetilde{N} \right\rfloor$, where $\left\lfloor \widetilde{N} \right\rfloor$ is the smallest integer solving $\gamma_1 < \widetilde{\gamma}_1$. Additionally in this setting, the difference in profit gains $\Delta\pi$ is unambiguously increasing in θ, making the CSR more likely to be sustained, but this effect is decreasing in signal richness N, implying that equilibrium sustainability becomes less sensitive to investment cost uncertainty θ when signals become richer. We elaborate on the intuition of Corollary 3 and provide details on how signal reliability γ and richness N affect equilibrium conditions in our numerical simulations below.

5.1 Equilibrium conditions - Examples

Example 1 (Two signals and CG). As expected, when considering the probability structure in Example 1, we have

$$\Delta\pi = \Pi\frac{\gamma(1-\gamma)[(1+\theta)^2-(1-\theta)^2]}{[(1+\theta)+(1-\theta)(1-\gamma)][(1-\theta)+(1+\theta)(1-\gamma)]} \geq \widehat{F},$$

which is increasing in both signal reliability γ and fix cost feasibility θ. As in CG, we solve for γ, finding the conditions on γ that support this equilibrium

$$\underbrace{\frac{2\widehat{F}}{\Pi(1-\theta)+\widehat{F}(1+\theta)}}_{\overline{\gamma}} \geq \gamma \geq \underbrace{\frac{2\widehat{F}}{\Pi(1+\theta)+\widehat{F}(1-\theta)}}_{\underline{\gamma}}.$$

Figure 1 (a) separately depicts expected profit gains $\Delta\pi_{CSR}$ and $\Delta\pi_{NCSR}$, implying that their vertical difference measures the profit gain $\Delta\pi$ where, for simplicity, we assume $\Pi = 1$ and $\theta = 1/2$. The figure illustrates that for any increase in signal reliability γ, we have an increase in profit gain, in both the CSR and NCSR equilibrium, where the profit gain is weakly higher in the CSR equilibrium. Figure 1 (b) plots the difference in profit gains $\Delta\pi$, which is always positive, but increases and then decreases in signal reliability γ.

Example 2 (Linear conditional probabilities). Assuming the same probability structure from Example 2 in Section 2.1, our results are analogous to that of Example 1 with the exception of $N-2$ other profits equating to zero factor into the equation.

Example 3 (Uniform distribution / Infinite signals). If, in the context of Example 2, conditional probabilities are constant, we obtain that $\Pr(s_i|D) = \frac{1}{N}$ for all i, so that,

$$\Delta\pi = \Pi\frac{(N-1)[(1+\theta)^2-(1-\theta)^2]}{[(1+\theta)N+(1-\theta)][(1-\theta)N+(1+\theta)]} \geq \widehat{F},$$

and when the number of signals is limited to $N = 2$, the difference in profit gains $\Delta\pi$ coincide with that in CG when $\gamma = \frac{1}{2}$.

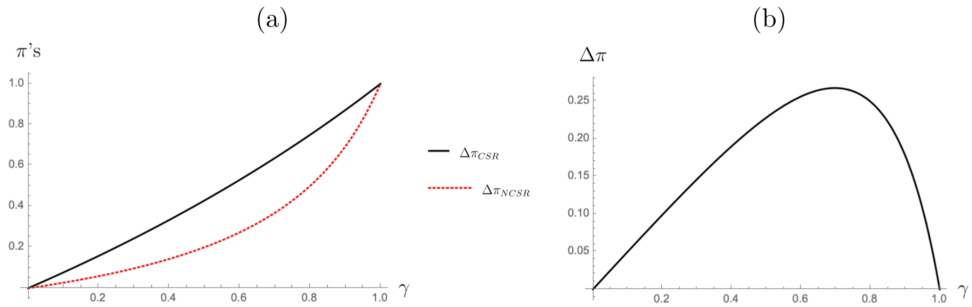

Figure 1: Profit Gains (a) and Different in Profit Gains (b) in CG and Example 2

Figure 2 (a) shows the difference in expected profits in the CSR and NCSR equilibriums over a range of signal richness values N. At each N there is difference in expected profits ($\Delta\pi_{CSR}$ and $\Delta\pi_{NCSR}$, respectively) for each equilibrium, and for the separating equilibrium to exist, the fixed cost $\widehat{F}$ of the firm must be between these upper and lower bounds. In the extreme case when signal richness becomes unbounded ($N \to +\infty$), both bounds converge to those under complete information ($\Delta\pi_{CSR} = \Pi$ and $\Delta\pi_{NCSR} = \Pi$, respectively) because consumers' posterior beliefs match their priors. Figure 2 (b) shows the difference between these expected differences $\Delta\pi$ where initially there is an increase in the difference before $N = 3$, and then a decrease as signal richness becomes unbounded.

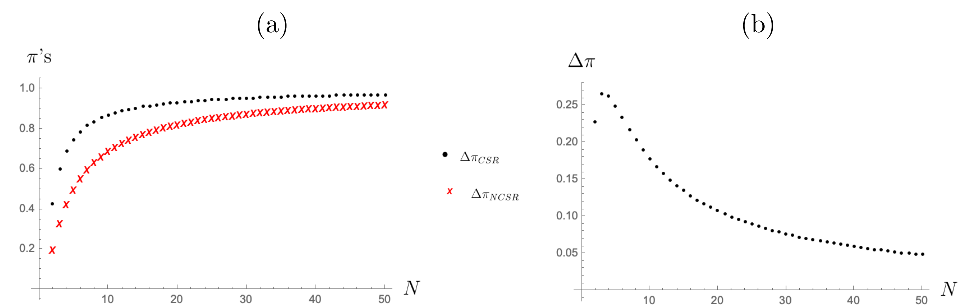

Figure 2: Profit Gains (a) and Different in Profit Gains (b) in Example 3

Example 4 (Extreme cases). In in the setting of Example 4, we have $\gamma_N = \gamma$ and $\gamma_i = \frac{1-\gamma}{N-1}$ for all $i \neq N$ when the firm invests in dirty technology, and under this information structure we have that

$$\Delta\pi = \Pi \frac{(N+\gamma)(1-\gamma)[(1+\theta)^2 - (1-\theta)^2]}{[(1+\theta)(N-1) + (1-\theta)(1-\gamma)][(1-\theta)(N-1) + (1+\theta)(1-\gamma)]} \geq \widehat{F},$$

which first increases, and then decreases, as signals become more reliable (higher γ) or richer (higher N). Figure 3 (a) shows this effect where, similar to CG's results in Figure 1 (a), the expected profit

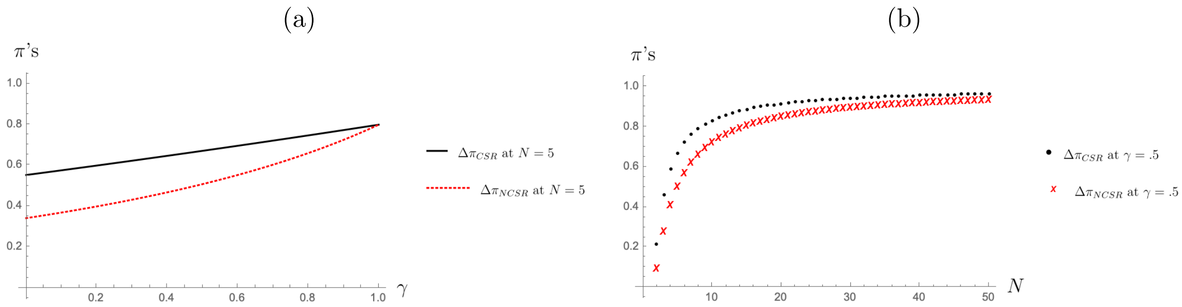

Figure 3: Profit Gains over Signal Reliability γ (a) and over Signal Richness N (b) in Example 4

gain in each equilibrium ($\Delta\pi_{CSR}$ and $\Delta\pi_{NCSR}$) are plotted over the feasible range of γ values given signal richness is $N = 5$. Expected profit gains increase in both equilibriums, and these expected profit gains act as upper and lower bound for the firm's fixed costs $\widehat{F}$ at a given level of signal reliability γ and richness N. Figure 3 (b) shows the same effect as Figure 3 (a), but over signal richness values N and assuming signal reliability is fixed at $\gamma = .5$. Expected profit gains increase in both equilibriums in this case too. With that said, the difference in the expected profit gains for each equilibrium $\Delta\pi$ initially increases in signal reliability γ and richness N, but then decreases. We show this effect in Figure 4 where this difference increases initially, but then decreases over a range of values for signal reliability γ. Similarity, this difference is initially larger for small values of signal richness N, and then decreases as signal richness increases.

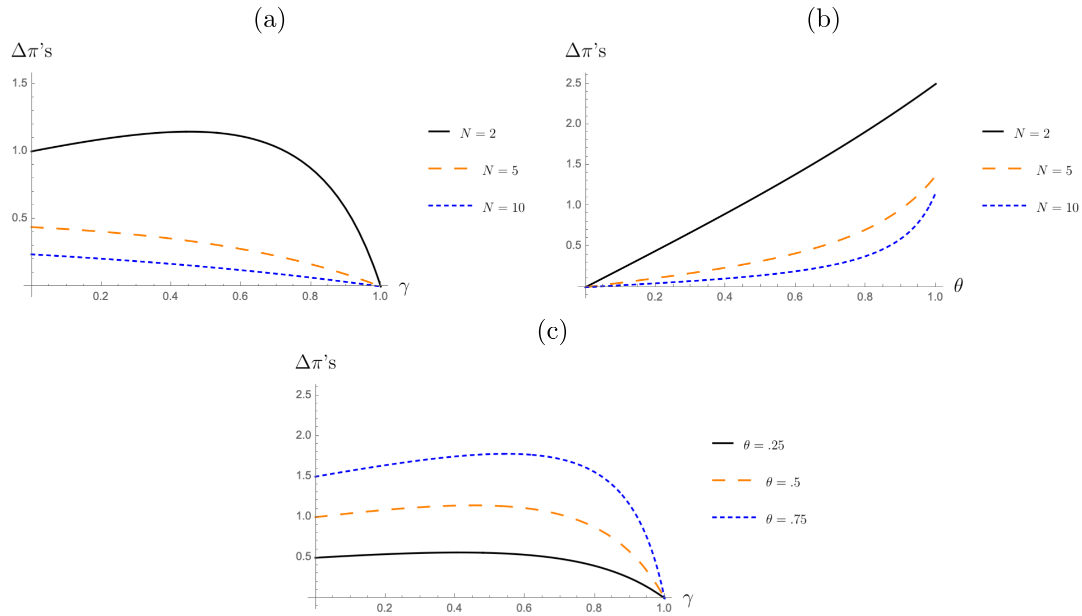

Figure 4: Simulated Comparative Statics with All Parameters in Example 4

6 Discussion

We extend CG's binary signaling setting by introducing a framework that allows for more signals. In this setting, we can still support the equilibrium where it is profitable for a firm to invest in CSR when fixed costs are feasible (i.e. $\Delta\pi \geq \widehat{F} \geq 0$). This result informs policy makers that firms still have incentives to invest in CSR, even when operating in an environment with multiple signals. With this new framework, we find an optimal level of signal unreliability $\widetilde{\gamma}_1$, in terms of investment cost uncertainty θ, that maximizes profit gains $\Delta\pi$ and gives a firm the most incentive to invest in CSR. This result is important because $\widetilde{\gamma}_1$ can be specified as a function of other exogenous parameters, like signal reliability γ and richness N, and therefore, there exists an optimal reliability parameter $\widetilde{\gamma}$ and richness parameter $\widetilde{N}$ that maximizes CSR profit gains. This finding entails that there is an optimal level of signal reliability and number of signals that provide the firm with the most incentive to invest in CSR. This is important to policy makers because it allows them to evaluate how the number of unique signals influences a firms CSR investment decision, and how if an increases or decreases in signals would give a firm more or less incentive to invest in CSR. Lastly, we find that profit gains from practicing CSR are increasing as fixed costs become more feasible (higher θ), but also find that as the number of signals N increases, the effect is diminished. Our result, then, entails that even though more feasible fixed costs induces firms to invest in CSR, the number of signals implicitly dampens this effect, ultimately, reducing firm's incentive to invest in CSR.

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7 Appendix 1 - Parametric examples

Example 1 (Two signals and CG). As expected, when we limit the number of signals to $N = 2$, $\gamma_1 = 1 - \gamma$, and $\gamma_2 = \gamma$, profit $\pi^*_{CSR}(s_i)$ coincides with that in CG, that is,

$$\pi^*_{CSR}(s_1) = \Pi \frac{1+\theta}{2-\gamma(1-\theta)} \text{ and } \pi^*_{CSR}(s_2) = 0;$$

and so do expected profits

$$E\left[\pi_{CSR}(t = C)\right] = \Pi \frac{1+\theta}{2-\gamma(1-\theta)} - \widehat{F} \text{ and } E\left[\pi_{CSR}(t = D)\right] = \Pi \frac{(1-\gamma)(1+\theta)}{2-\gamma(1-\theta)}.$$

Example 2 (Linear conditional probabilities). Assuming the same probability structure from Example 2 in Section 2.1, profit $\pi^*_{CSR}(s_i)$ coincides with that in CG with the exception of having $N - 2$ more profits that equate to zero, that is,

$$\pi^*_{CSR}(s_1) = \Pi \frac{1+\theta}{2-\gamma(1-\theta)} \text{ and } \pi^*_{CSR}(s_i) = 0 \quad \text{for all } i \neq 1,$$

and expected profits become

$$E\left[\pi_{CSR}(t = C)\right] = \Pi \frac{1+\theta}{2-\gamma(1-\theta)} - \widehat{F} \text{ and } E\left[\pi_{CSR}(t = D)\right] = \Pi \frac{(1-\gamma)(1+\theta)}{2-\gamma(1-\theta)}.$$

Example 3 (Uniform distribution / Infinite signals). If, in the context of Example 2, conditional probabilities are constant, we obtain that $\Pr(s_i|D) = \frac{1}{N}$ for all i, we have that,

$$\pi^*_{CSR}(s_1) = \Pi \frac{N(1+\theta)}{(N+1)+\theta(N-1)} \text{ and } \pi^*_{CSR}(s_i) = 0 \quad \text{for all } i \neq 1,$$

and when the number of signals is limited to $N = 2$, profits $\pi^*_{CSR}(s_i)$ also coincide with that in CG when $\gamma = \frac{1}{2}$. That is, $\pi^*_{CSR}(s_1) = \Pi \frac{2(1+\theta)}{3+\theta}$ and $\pi^*_{CSR}(s_2) = 0$. In this case, consumers and firms update their beliefs based on a signal s_i that can be drawn from N different levels with equal probability, and since each additional signal decreases the probability of the firm receiving the cleanest signal, this actually increases profits $\pi^*_{CSR}(s_i)$. In the extreme case when signals are extremely rich ($N \to +\infty$), profits converge to those under complete information $\pi^*_{CSR}(s_1) = \Pi$ because consumers' posterior beliefs match their priors. In this context, expected profits become

$$E\left[\pi_{CSR}(t = C)\right] = \Pi \frac{N(1+\theta)}{(N+1)+\theta(N-1)} - \widehat{F} \text{ and } E\left[\pi_{CSR}(t = D)\right] = \frac{\Pi}{N} \frac{N(1+\theta)}{(N+1)+\theta(N-1)}.$$

Intuitively, as signals become richer (higher N), expected profits, when choosing either the clean or dirty technology, increase and decrease, respectively.

Example 4 (Extreme cases). In the case non-uniform signaling, we have $\gamma_N = \gamma$ and $\gamma_i = \frac{1-\gamma}{N-1}$ for all $i \neq N$ when the firm invests in dirty technology. Under this information structure we have that

$$\pi^*_{CSR}(s_1) = \Pi \frac{(1+\theta)(N-1)}{(1+\theta)(N-1)+(1-\theta)(1-\gamma)} \quad \text{and} \quad \pi^*_{CSR}(s_i) = 0 \qquad \text{for all } i \neq 1$$

which increases as market transparency (γ) and signal richness (N) increases. Therefore, expected profits are

$$E\left[\pi_{CSR}(t=C)\right] = \Pi \frac{(1+\theta)(N-1)}{(1+\theta)(N-1)+(1-\theta)(1-\gamma)} - \widehat{F}, \text{ and}$$
$$E\left[\pi_{CSR}(t=D)\right] = \Pi \frac{(1-\gamma)}{N-1} \frac{(1+\theta)(N-1)}{(1+\theta)(N-1)+(1-\theta)(1-\gamma)}.$$

Similar to Example 3, expected profits when the firm chooses the clean technology are increasing in market transparency and signal richness, and expected profits when the firm chooses the dirty technology are decreasing. As expected, when we limit the number of signals to $N = 2$, expected profits coincide with that in CG.

8 Appendix 2

8.1 Proof of Lemma 1

Differentiating with respect to p, and solving for p, we obtain

$$\frac{p^*}{\Pr(C|s)g} = \frac{1 - H\left(\frac{p^*}{\Pr(C|s)g}\right)}{h\left(\frac{p^*}{\Pr(C|s)g}\right)}.$$

Since $\bar{\alpha} \equiv \frac{p}{\Pr(C|s)g}$, let us now define $\alpha^* \equiv \frac{p^*}{\Pr(C|s)g}$, so we can rewrite the above equilibrium condition as

$$\alpha^* = \frac{1 - H\left(\alpha^*\right)}{h\left(\alpha^*\right)}.$$

Using this condition, we find the firm's optimal price, where in order to be competitive with the other firms in a Bertrand competition, will only identify the consumers for which price p satisfies

$$v + \alpha^* \Pr(C|s)g - p = v.$$

Therefore, the firm sets a price $p^*(s) = \alpha^* \Pr(C|s)g$, earning expected profits

$$\pi^*(s) = \Pr(C|s)\Pi = \Pr(C|s)(1 - H(\alpha^*))g\alpha^*$$

where $\Pi \equiv (1 - H(\alpha^*))g\alpha^*$ denotes expected profits under complete information.

8.2 Proof of Lemma 2

Expected Profits of the clean firm $\pi^*_{CSR}(s_i)$ is weakly decreasing in i. We know $\pi^*_{CSR}(s_1) \geq \pi^*_{CSR}(s_i)$ $i \neq 1$, implying that

$$\Pi \frac{1+\theta}{(1+\theta)+(1-\theta)\gamma_1} \geq 0 \qquad i \neq 1$$

which holds weakly for all $j < i$ where $i, j \in \{1, \ldots, N\}$.

Profit $\pi^*_{CSR}(s_i)$ is weakly decreasing in γ_1 since

$$\frac{\partial \pi^*_{CSR}(s_i)}{\partial \gamma_1} = -\frac{\pi(1-\theta)(1+\theta)}{[(1+\theta)+(1-\theta)\gamma_1]^2} \leq 0$$

and profit $\pi^*_{CSR}(s_i)$ is increasing in θ because

$$\frac{\partial \pi^*_{CSR}(s_i)}{\partial \theta} = \Pi \frac{2\gamma_i}{((1+\theta)+\gamma_i(1-\theta))^2},$$

which is unambiguously positive for $i = 1$, and $\frac{\partial \pi^*_{CSR}(s_i)}{\partial \theta} = 0$ for all $i \neq 1$.

8.3 Proof of Proposition 1

The expected profits when a firm chooses to be clean or dirty are

$$\begin{aligned}
E\left[\pi_{CSR}(t=C)\right] &= \sum_{i=1}^{N} \Pr(s_i|C)\pi^*_{CSR}(s_i) - \hat{F} \\
&= \Pi \frac{(1+\theta)}{(1+\theta)+(1-\theta)\gamma_1} - \hat{F}, \text{ and} \\
E\left[\pi_{CSR}(t=D)\right] &= \sum_{i=1}^{N} \Pr(s_i|D)\pi^*_{CSR}(s_i) \\
&= \Pi \frac{(1+\theta)\gamma_1}{(1+\theta)+(1-\theta)\gamma_1}.
\end{aligned}$$

The expected profit for the clean firm $E\left[\pi_{CSR}(t=C)\right]$ is weakly decreasing in γ_1 since

$$\frac{\partial E\left[\pi_{CSR}(t=C)\right]}{\partial \gamma_1} = -\Pi \frac{(1-\theta)(1+\theta)}{[(1+\theta)+(1-\theta)\gamma_1]^2} \leq 0.$$

The expected profit for the dirty firm $E\left[\pi_{CSR}(t=D)\right]$ is weakly increasing in γ_1 given that

$$\frac{\partial E\left[\pi_{CSR}(t=D)\right]}{\partial \gamma_1} = \Pi \frac{(1+\theta)^2}{[(1+\theta)+(1-\theta)\gamma_1]^2} \geq 0.$$

The expected profit $E\left[\pi_{CSR}(t=C)\right]$ is increasing in θ because

$$\frac{\partial E\left[\pi_{CSR}(t=C)\right]}{\partial\theta} = \Pi\frac{2\gamma_1}{((1+\theta)+\gamma_1(1-\theta))^2} > 0 \text{ for } i=1,$$

which is unambiguously positive, and $\frac{\partial E\left[\pi_{CSR}(t=C)\right]}{\partial\theta} = 0$ for all $i \neq 1$.

Finally, the expected profit $E\left[\pi_{CSR}(t=D)\right]$ is increasing in θ since

$$\frac{\partial E\left[\pi_{CSR}(t=D)\right]}{\partial\theta} = \Pi\frac{2\gamma_1^2}{((1+\theta)+\gamma_1(1-\theta))^2} > 0 \text{ for } i=1,$$

which is unambiguously positive, and $\frac{\partial E\left[\pi_{CSR}(t=D)\right]}{\partial\theta} = 0$ for all $i \neq 1$.

8.4 Proof of Corollary 1

$\Delta\pi_{CSR}$ *is decreasing in* γ_1. Let $\gamma_i' > \gamma_i$, and we seek to show that expected profit when choosing the clean technology does not satisfy $\Delta\pi_{CSR}' > \Delta\pi_{CSR}$. This entails that

$$\frac{\partial\Delta\pi_{CSR}}{\partial\gamma_1} = -\Pi\frac{2(1+\theta)}{[(1+\theta)+(1-\theta)\gamma_1]^2} \leq 0.$$

$\Delta\pi_{CSR}$ *is increasing in* θ. Differentiating $\Delta\pi_{CSR}$ with respect to θ we obtain

$$\frac{\partial\Delta\pi_{CSR}}{\partial\theta} = \Pi\frac{2\gamma_1(1-\gamma_1)}{((1+\theta)+\gamma_1(1-\theta))^2} > 0 \text{ for } i=1,$$

which is unambiguously positive, and $\frac{\partial\Delta\pi_{CSR}}{\partial\theta} = 0$ for all $i \neq 1$.

8.5 Proof of Lemma 3

We first check that profit $\pi^*_{NCSR}(s_i)$ is weakly decreasing in i. We know that $\pi^*_{NCSR}(s_1) \geq \pi^*_{NCSR}(s_i)$ $i \neq 1$, which implies

$$\Pi\frac{1-\theta}{(1-\theta)+(1+\theta)\gamma_1} \geq 0 \quad \text{for all} \quad i \neq 1$$

which holds weakly for all $j < i$ where $i, j \in \{1, \ldots, N\}$.

In addition, profit $\pi^*_{NCSR}(s_i)$ is weakly decreasing in γ_1, since

$$\frac{\partial\pi^*_{NCSR}(s_i)}{\partial\gamma_1} = -\Pi\frac{(1-\theta)(1+\theta)}{[(1-\theta)+(1+\theta)\gamma_1]^2} \leq 0.$$

Finally, profit $\pi^*_{NCSR}(s_i)$ is decreasing in θ because

$$\frac{\partial\pi^*_{NCSR}(s_i)}{\partial\theta} = -\Pi\frac{2\gamma_i}{((1-\theta)+\gamma_i(1+\theta))^2} < 0 \text{ for } i=1,$$

which is unambiguously negative, and $\frac{\partial\pi^*_{CSR}(s_i)}{\partial\theta} = 0$ for all $i \neq 1$.

8.6 Proof of Proposition 2

The expected profits when a firm chooses to be clean or dirty are

$$\begin{aligned} E\left[\pi_{NCSR}(t=C)\right] &= \sum_{i=1}^{N} \Pr(s_i|C)\pi^*_{NCSR}(s_i) - \hat{F} \\ &= \Pi \frac{(1-\theta)}{(1-\theta)+(1+\theta)\gamma_1} - \hat{F}, \text{ and} \\ E\left[\pi_{NCSR}(t=D)\right] &= \sum_{i=1}^{N} \Pr(s_i|D)\pi^*_{NCSR}(s_i) \\ &= \Pi \frac{(1-\theta)\gamma_1}{(1-\theta)+(1+\theta)\gamma_1}. \end{aligned}$$

In addition, the expected profit for the clean firm $E\left[\pi_{NCSR}(t=C)\right]$ is weakly decreasing in γ_1 since

$$\frac{\partial E\left[\pi_{NCSR}(t=C)\right]}{\partial \gamma_1} = -\Pi \frac{(1-\theta)(1+\theta)}{[(1-\theta)+(1+\theta)\gamma_1]^2} \leq 0$$

and the expected profit for the dirty firm $E\left[\pi_{NCSR}(t=D)\right]$ is weakly increasing in γ_1 because

$$\frac{\partial E\left[\pi_{NCSR}(t=D)\right]}{\partial \gamma_1} = \Pi \frac{(1-\theta)^2}{[(1+\theta)+(1-\theta)\gamma_1]^2} \geq 0.$$

Expected profit $E\left[\pi_{NCSR}(t=C)\right]$ is decreasing in θ since

$$\frac{\partial E\left[\pi_{NCSR}(t=C)\right]}{\partial \theta} = -\Pi \frac{2\gamma_1}{((1-\theta)+\gamma_1(1+\theta))^2} < 0 \text{ for } i=1,$$

which is unambiguously negative, and $\frac{\partial E\left[\pi_{NCSR}(t=C)\right]}{\partial \theta} = 0$ for all $i \neq 1$.

Finally, expected profit $E\left[\pi_{NCSR}(t=D)\right]$ is decreasing in θ because

$$\frac{\partial E\left[\pi_{NCSR}(t=D)\right]}{\partial \theta} = -\Pi \frac{2\gamma_1^2}{((1-\theta)+\gamma_1(1+\theta))^2} < 0 \text{ for } i=1,$$

which is unambiguously negative, and $\frac{\partial E\left[\pi_{CSR}(t=D)\right]}{\partial \theta} = 0$ for all $i \neq 1$.

8.7 Proof of Corollary 2

$\Delta\pi_{NCSR}$ is decreasing in γ_1 since

$$\frac{\partial \Delta\pi_{NCSR}}{\partial \gamma_1} = -\Pi \frac{2(1-\theta)}{[(1+\theta)+(1-\theta)\gamma_1]^2} \leq 0.$$

and decreasing in θ because

$$\frac{\partial \Delta\pi_{NCSR}}{\partial \theta} = -\Pi \frac{2\gamma_1(1-\gamma_1)}{((1-\theta)+\gamma_1(1+\theta))^2} < 0 \text{ for } i=1,$$

which is unambiguously negative, and $\frac{\partial \Delta\pi_{NCSR}}{\partial\theta} = 0$ for all $i \neq 1$.

8.8 Proof of Corollary 3

Differentiating $\Delta\pi$ with respect to γ_1 we obtain

$$\frac{\partial \Delta\pi}{\partial \gamma_1} = \Pi \frac{4\theta(1-\gamma_1)^2\theta^2 + (1-\gamma_1)(1-3\gamma_1)}{[(1+\gamma_1)^2 - (1-\gamma_1)^2\theta^2]^2}$$

where the denominator is unambiguously positive, while the numerator is positive if and only if $\gamma_1 < \widetilde{\gamma_1}$ where

$$\widetilde{\gamma_1} = \frac{\theta^2 + 2\sqrt{1-\theta^2} - 1}{3+\theta^2}.$$

Evaluating this derivative at $\gamma_1 = 0$ yields

$$\left.\frac{\partial \Delta\pi}{\partial \gamma_1}\right|_{\gamma_1=0} = \Pi \frac{4\theta^3 + 1}{(1-\theta^2)^2}$$

which is positive, implying that starting from $\gamma_1 = 0$, $\Delta\pi$ is always positive until $\gamma_1 = \widetilde{\gamma_1}$. Differentiating cutoff $\widetilde{\gamma_1}$ with respect to θ we find

$$\frac{\partial \widetilde{\gamma_1}}{\partial \theta} = \frac{2\theta(\theta^2 + 4\sqrt{1-\theta^2} - 5)}{\sqrt{1-\theta^2}(3+\theta^2)^2} \leq 0,$$

because $\theta^2 + 4\sqrt{1-\theta^2} < 5$ since $-\theta^2(6+\theta^2) < 9$.

Differentiating $\Delta\pi$ with respect to N we obtain $\frac{\partial \Delta\pi}{\partial N} = \frac{\partial \Delta\pi}{\partial \gamma_1}\frac{\partial \gamma_1}{\partial N}$, where γ_1 is decreasing in N and, as shown above, $\frac{\partial \Delta\pi}{\partial \gamma_1} > 0$ for all $\gamma_1 < \widetilde{\gamma_1}$. Therefore, there exist a value of N, $\widetilde{N}$, that solves $\gamma_1 = \widetilde{\gamma_1}$, so that $N > \widetilde{N}$ entails $\gamma_1 < \widetilde{\gamma_1}$; while $N \leq \widetilde{N}$ implies $\gamma_1 \geq \widetilde{\gamma_1}$. Overall, we can conclude that $\frac{\partial \Delta\pi}{\partial N} > 0$ for all $N \leq \widetilde{N}$, but $\frac{\partial \Delta\pi}{\partial N} < 0$ otherwise.

Differentiating $\Delta\pi$ with respect to θ we obtain

$$\frac{\partial \Delta\pi}{\partial \theta} = \Pi \frac{2\gamma_1(1-\gamma_1)}{((1+\theta)+\gamma_1(1-\theta))^2} + \Pi \frac{2\gamma_1(1-\gamma_1)}{((1-\theta)+\gamma_1(1+\theta))^2} > 0 \text{ for } i = 1,$$

which is unambiguously positive, and $\frac{\partial \Delta\pi_{NCSR}}{\partial\theta} = 0$ for all $i \neq 1$.

Finally, comparing $\frac{\partial \Delta\pi}{\partial \theta}$ evaluated at N, and $\frac{\partial \Delta\pi}{\partial \theta}$ evaluated at $N+1$, we find that

$$\left.\frac{\partial \Delta\pi}{\partial \theta}\right|_{N+1} \leq \left.\frac{\partial \Delta\pi}{\partial \theta}\right|_{N}$$

or, after expanding,

$$
\begin{aligned}
& \frac{1}{\left[(N+1)-\gamma+\theta(N+\gamma-2)\right]^4}+\frac{1}{\left[\gamma-(N+1)+\theta(N+\gamma-2)\right]^4} \\
\leq \quad & \frac{1}{\left[N-\gamma+\theta(N+\gamma-2)\right]^4}+\frac{1}{\left[\gamma-N+\theta(N+\gamma-2)\right]^4}
\end{aligned}
$$

which can be numerically proven to hold for all values of parameters θ, γ, and N. Therefore, $\frac{\partial \Delta\pi}{\partial\theta}$ decreases as the number of signals, N, increases; indicating that $\Delta\pi$, while increasing in θ, becomes less sensitive to θ, as the number of signals increases.