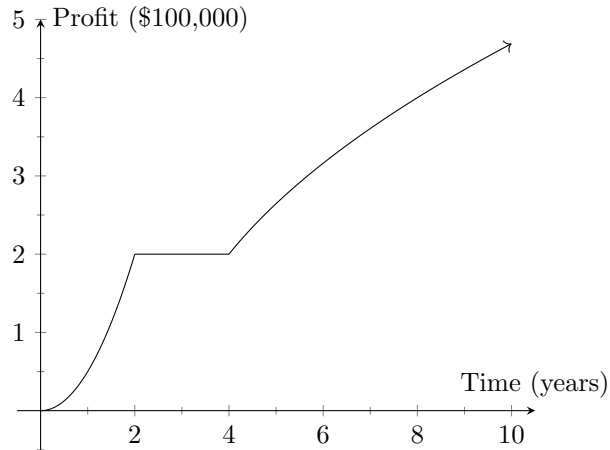# Piecewise Functions: The Analytic View

This section discusses the analytic view of piecewise functions.

Video lecture here:

YouTube link: `https://www.youtube.com/watch?v=YdWP_OZHy_k`

Recall that the idea of piecewise functions is to stitch together several different functions, each one of which applies to a specific segment of the overall relation. Consider the graph from the previous section that depicted profit over time for a new company:

We can clearly break this graph into three pieces as so;
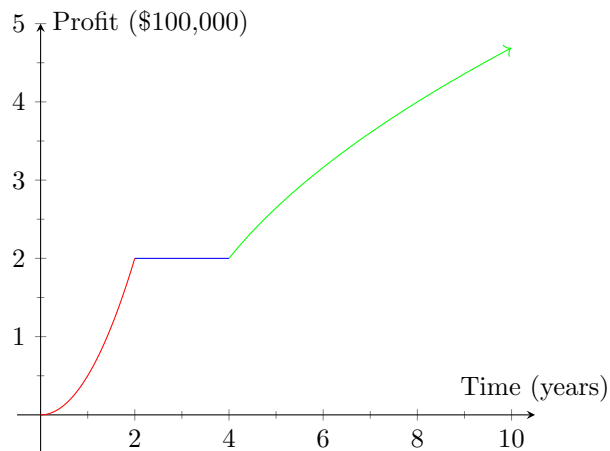
Our goal is to write a (piecewise) function, $P(x)$, that is really a combination of each of these three functions. However, first we need to record the three functions themselves. In our case we have the following:

Red Segment: $f(t) = \dfrac{1}{2}t^2$ is the segment from $t = 0$ to $t = 2$.

Blue Segment: $g(t) = 2$ is the segment from $t = 2$ to $t = 4$.

Green Segment: $h(t) = \sqrt{3t - 8}$ is the segment from $t = 4$ to $t = 10$.

We could simply use this descriptive method to define our piecewise function, but as usual we utilize a more compact form of notation. Consider the following piecewise function notation:

Learning outcomes:

$$P(t) = \begin{cases} \dfrac{1}{2}t^2 & 0 \le t < 2 \\ 2 & 2 \le t \le 4 \\ h(t) = \sqrt{3t-8} & t > 4 \end{cases}$$

This notation is deceptively dense and it is worth breaking down each part of it.

P(x) As normal we name the function as the dependent variable P and the independent variable t as the input.

Large Open Brace The open brace encompasses all the lines that are all part of the piecewise definition. This way, if we have multiple horizontal lines of equations, we know that $P(t)$ is a piecewise function encompassing these three functions, and any other functions are not part of $P(x)$.

Lines with Functions Each line with a function has a left and a right component. The left side, eg "$\dfrac{1}{2}t^2$", "2", or "$h(t) = \sqrt{3t-8}$", each denote a function in a specific segment. The right component, eg "$0 \le t < 2$", "$2 \le t \le 4$", or "$t > 4$" gives the domain of that segment.

So, in particular, the line "$\dfrac{1}{2}t^2 \quad 0 \le t < 2$" is declaring that the function on the section of the graph for t values between 0 and 2, is modeled by the function $\dfrac{1}{2}t^2$.

Something to keep in mind in the above function, is that the domains (the intervals of t for each function) are disjoint, meaning that there *should be no overlap between the intervals*. In particular, we have $0 \le t < 2$ and $2 \le t \le 4$, rather than $0 \le t \le 2$ and $2 \le t \le 4$ which has 2 in both intervals. In general, the chances of having a well defined function with overlapping intervals is almost nil. Having overlapping intervals in the function segments is how a piecewise relation fails the vertical line test. Consider from the geometric view tile the following piecewise relation graph:
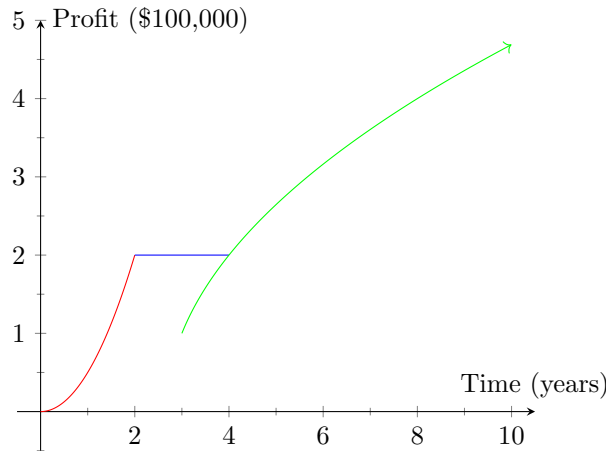
This graph is generated by the following piecewise relation:

$$P(t) = \begin{cases} \dfrac{1}{2}t^2 & 0 \le t < 2 \\ 2 & 2 \le t \le 4 \\ h(t) = \sqrt{3t-8} & t > 3 \end{cases}$$

Notice that the intervals above overlap because the second function is defined on $2 \le t \le 4$ and the third is defined on $t > 3$, which is why the piecewise relation fails the vertical line test on the interval $3 \le t < 4$; ie on the overlap.