Terminology To Know

These are important terms and notations for this section.

Terminology

Definition 1 (Monomial)**.** *A term of the form ax^n for some constant a and some non-negative integer n. From "mono" meaning "one" and "nomen" meaning "name".*

Definition 2 (Binomial)**.** *An expression that is the sum or difference of two monomials. From "bi" meaning "two".*

Definition 3 (Polynomial)**.** *A function or expression that is entirely composed of the sum or differences of monomials. From "poly" meaning "many".*

Definition 4 (Leading Term (of a polynomial))**.** *The leading term of a polynomial is the term with the largest exponent, along with its coefficient. Another way to describe it (which is where this term gets its name) is that; if we arrange the polynomial from highest to lowest power, than the first term is the so-called 'leading term'.*
For Example: *For the polynomial $p(x) = x^2 - 13x^3 + 4x - 1$ we could rewrite it in descending order of exponents, to get $p(x) = -13x^3 + x^2 + 4x - 1$ which makes clear that $-13x^3$ as the 'leading term' of $p(x)$.*

Definition 5 ((Complex) Conjugates)**.** *A pair of complex numbers whose real parts are the same, and whose imaginary parts differ only by a negative sign are called complex conjugates.*
Note: *We often ask for 'the complex conjugate to' a complex number, in which case we are asking for the associated number in the pair.*
For Example: *The numbers $5 + 3i$ and $5 - 3i$ are complex conjugates. If one were to ask 'what is the complex conjugate of $5 - 3i$ the answer would be the other number of the complex conjugate pair, ie $5 + 3i$.*

Definition 6 (Curvature)**.** *Curvature refers to monotonicity (increasing/decreasing) and the concavity (bending up or down) of a curve.*

Definition 7 (Irreducible Polynomial)**.** *A polynomial that cannot be factored any further. We will often specify under what type of numbers we are factoring the polynomial; eg real numbers or complex numbers. This indicates whether all numbers in the factored form must be real or complex numbers (respectively).*
For Example: *$(x^2 + 1)$ is irreducible under the real numbers because there is no way to factor this quadratic with real numbers only.* ***However*** *$x^2 + 1$ is not irreducible under the complex numbers, as we can write $x^2 + 1 = (x + i)(x - i)$.*

Definition 8 (Root (of a polynomial))**.** *A root of a polynomial is an irreducible polynomial that is a factor of the given polynomial.*

For Example: *The polynomial $x+1$ is a root of the polynomial x^2-1 because $(x+1)(x-1) = x^2-1$. In comparison x^2-1 is not a root of the polynomial x^4-1, even though $(x^2-1)(x^2+1) = x^4-1$ because x^2-1 is not irreducible.*

Definition 9 (Multiplicity (of a value/zero))**.** *The multiplicity of a value is a count of how many times that value occurs. This is most often used in reference to the 'multiplicity of a zero' or 'multiplicity of a root'.*
For Example: *Let's say we have factored a polynomial into the form:*

$$p(x) = (x+1)^3(x-1)^2(x+5)(x+17)$$

We would say that "the root $(x+1)$ has multiplicity 3", because the term $(x+1)$ occurs 3 times (hence the power of 3). Similarly the root $(x-1)$ has multiplicity 2, and the roots $(x+5)$ and $(x+17)$ both have multiplicity 1. This can be even easier to see if we re-write $p(x)$ without using exponents;

$$p(x) = (x+1)(x+1)(x+1) \cdot (x-1)(x-1) \cdot (x+5)(x+17)$$

NOTE: *When we say that a polynomial has n roots "up to multiplicity" what we mean is that if we add all the multiplicity numbers together of all the roots, we would get n. So in the case of $p(x)$ we would say $p(x)$ has 7 roots "up to multiplicity" since there are 4 unique "roots", but two of them occur more than once, so there are a total of 7 roots if you account for repeats.*

Notation

Definition 10 (Notation for an arbitrary polynomial)**.** *The standard notation for a polynomial ($p(x)$) of degree n and with coefficients $c_0, c_1, \ldots, c_n \in \mathbb{R}$ is as follows:*

$$p(x) = c_n x^n + x_{n-1}x^{n-1} + c_{n-2}x^{n-2} + \cdots + c_2x^2 + c_1x + c_0$$

This notation should be explained however. The polynomial $p(x)$, is degree n, so its highest power is n, and each of the coefficients "$\in \mathbb{R}$" means that all the coefficients are real numbers. The fact that there are $c_0, c_1, \ldots c_n$ and the polynomial is degree n is not a coincidence; the subscript on the coefficient and the degree of the polynomial are the same n. Moreover, looking at the definition of $p(x)$ above you can see that each term is of the form

$$c_{\textit{(some value)}}x^{\textit{(the same value)}}$$

meaning that the power of x and the subscript on the coefficient match. This is also not an accident, this is how we tell which coefficient goes with which term. For example; if we wanted to know the coefficient of x^{17}, we would immediately know that is c_{17} because that's how they are named.

Finally, notice that the last term in the standard notation is just c_0, however, that is better written as c_0x^0, meaning that it conforms to all the other terms.

It's just easier to simplify the $x^0 = 1$ *and then omit writing it, so we only write* c_0. *But notice that the pattern holds for every term, including the last one, despite the fact that we don't write the* x^0 *piece of the last term.*