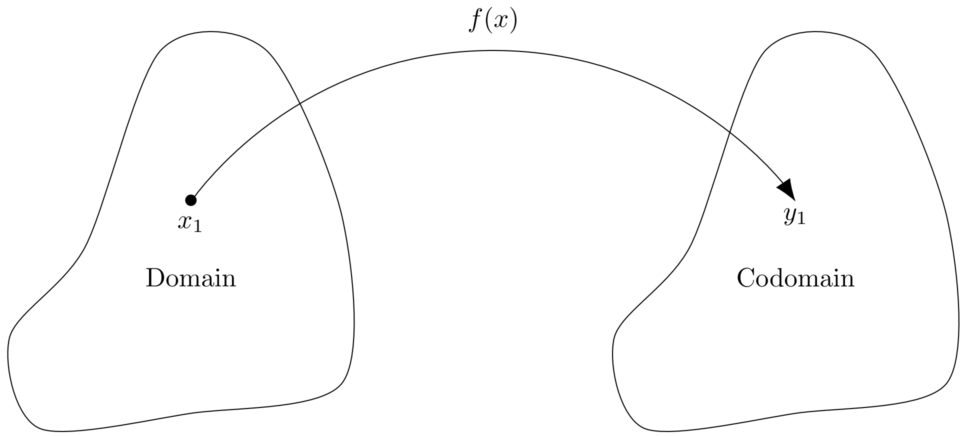
Inverse Functions

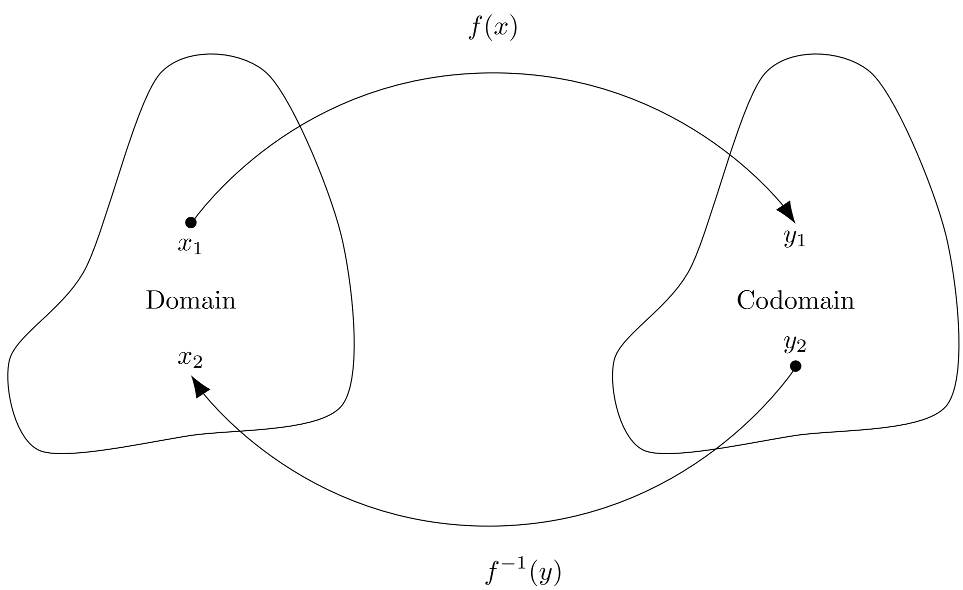
This section introduces the geometric viewpoint of invertability.

Inverse Functions - Geometric View

A recurring perspective as we move toward studying individual functions types will be the idea of *inverting* a function. Remember that a function is a relationship between some domain and a codomain, where it "maps" each domain point to a (single) point in the codomain.



Inverting a function is "merely"[1] the process of reversing the direction of $f(x)$. We will denote the inverse function by $f^{-1}(x)$ and we can see below what this looks like in terms of our domain and codomain.



There are a few subtle and key observations that can be made from this seemingly simple diagram however. The most obvious,[2] observation we can make is that the codomain of the original function becomes the domain of the inverse function, and the domain of the original function becomes the codomain of the inverse function. That is to say; the role of domain and codomain *switch* for the inverse function.

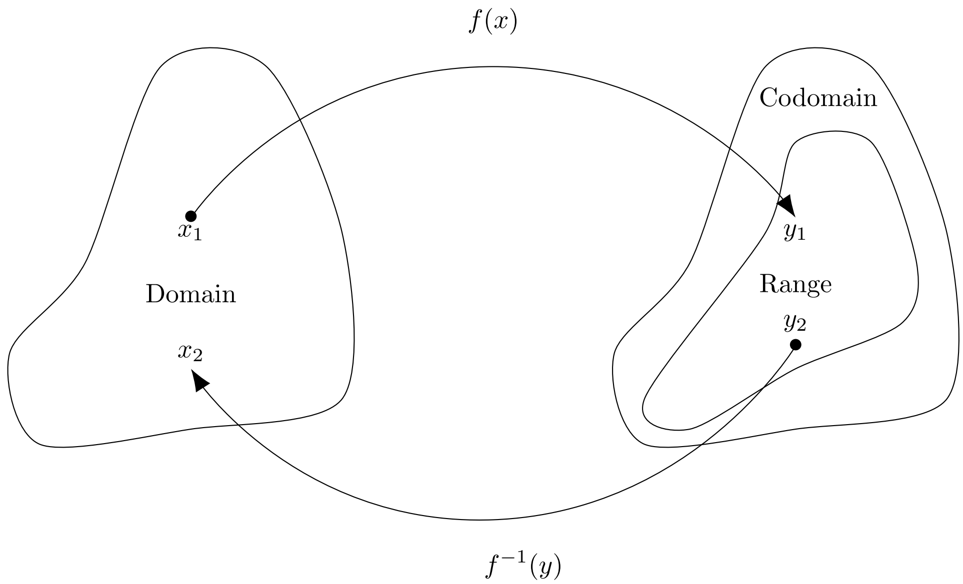
Learning outcomes:

[1] Like most things, "merely" is entirely misleading here... in fact this tends to be the hard part, and doesn't always work, as we'll see

[2] and most important it turns out

This is a lot more important than it might initially seem, for two reasons. First, the inverse function taking an *entire codomain* as it's domain could be rather problematic. Take, for example, the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = e^x$. The inverse function for $f(x)$ would be $f^{-1}(x) = \ln(x)$ (you can just take this on faith for now, we'll cover this later). But if we try to use the entire codomain (ie $\mathbb{R}$) as the domain for the inverse, then we would have a problem because the domain of $\ln(x)$ is $\mathbb{R}^+$ not $\mathbb{R}$. It turns out though that the *range* of e^x is actually $\mathbb{R}^+$, not $\mathbb{R}$.

So, it is more helpful to take the *range* of the function as the domain of it's inverse rather than the codomain. With this adjustment our picture would look like:



Problem 1 *What is the difference between the codomain and the range of a function?*

Multiple Choice:

(a) *The codomain is the type of thing that the output is, whereas the range is the actual achieveable output.* ✓

(b) *The range is the type of thing that the output is, whereas the codomain is the actual achieveable output.*

(c) *The codomain and the range are the same, so there is no difference.*

(d) *The codomain is the input, and the range is the output of a function.*

(e) *The codomain is the achieveable output of a function, and the range is the input of the inverse function.*

By using the range of $f(x)$ as the domain of $f^{-1}(y)$, we make sure that every point in the domain of $f^{-1}(y)$ is defined. Another way to say this is that we only consider the 'points that actually came from some x-value' when we reverse the relationship to make the inverse relation.

Here is a video with more!

YouTube link: `https://www.youtube.com/watch?v=tgEUPCJ-SKk`