Curvature and Graphing

This section shows and explains graphical examples of function curvature.

Another common aspect of the graph of a function, is the graphs *curvature*. In general, curvature (like local extrema) is difficult to determine without tools from calculus and as such, it is a major area of study in calculus. However, for this class we will restrict ourselves to a description of what different types of curvature 'look like' on a graph so that we can identify curvature types visually.

One typically refers to the curvature as a combination of whether the function is increasing or decreasing, and whether it is concave up or concave down. We will describe each of these terms/features by themselves and then give graphical representations of their combinations which may be more helpful to understanding.[1]

Increasing: A function is increasing on an interval (a, b) if, for *every single pair of values* x and w such that $a < x < w < b$, we have $f(x) \leq f(w)$.
In 'human speak' a function is increasing on an interval (a, b), if larger numbers in that interval *always* get sent to larger values by f.

Decreasing: A function is decreasing on an interval (a, b) if, for *every single pair of values* x and w such that $a < x < w < b$, we have $f(x) \geq f(w)$.
In 'human speak' a function is decreasing on an interval (a, b), if larger numbers in that interval *always* get sent to smaller values by f.

Concave Up: A function is concave up on an interval (a, b) if, for *every single pair of values* x and w such that $a < x < w < b$, the inequality $\dfrac{f(a) - f(x)}{a - x} \leq \dfrac{f(w) - f(b)}{w - b}$ is true.
In 'human speak' a function is concave up if the line connecting x to w is *always above* the graph of f for every value between x and w. Alternatively, one can say an interval (a, b) is concave up if the graph 'bends upward' as you graph it from left to right.

Concave Down: A function is concave down on an interval (a, b) if, for *every single pair of values* x and w such that $a < x < w < b$, the inequality $\dfrac{f(a) - f(x)}{a - x} \geq \dfrac{f(w) - f(b)}{w - b}$ is true.
In 'human speak' a function is concave down if the line connecting x

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Learning outcomes:

[1]The graphical representation tends to be easier to understand than the description, so if you don't understand the description immediately try looking at the graphs and then return to the analytic descriptions to see if you can figure out what they are saying.

to w is *always below* the graph of f for every value between x and w. Alternatively, one can say an interval (a, b) is concave down if the graph 'bends downward' as you graph it from left to right.

The following are graphical representations of various combinations of increasing/decreasing and concave up/down;
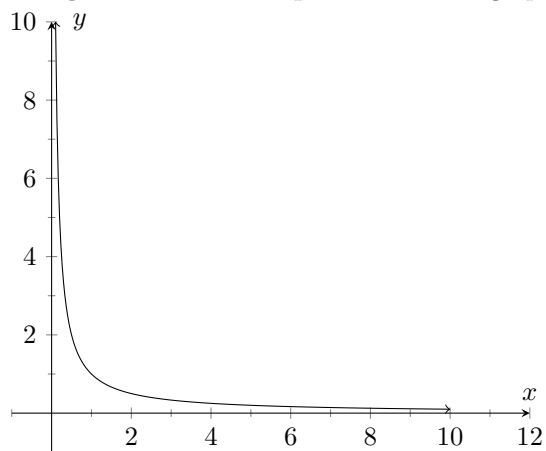
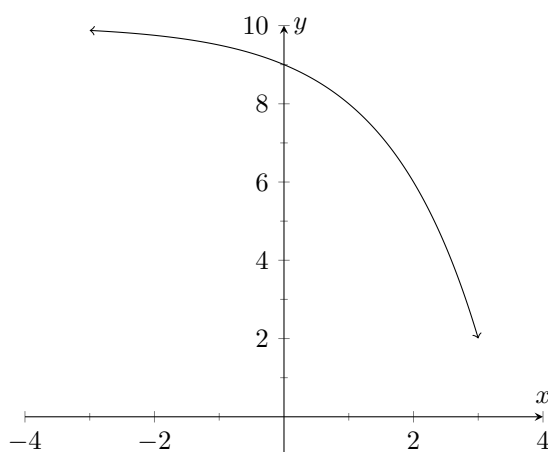**Increasing and Concave Up:** Consider the graph for $f(x)$
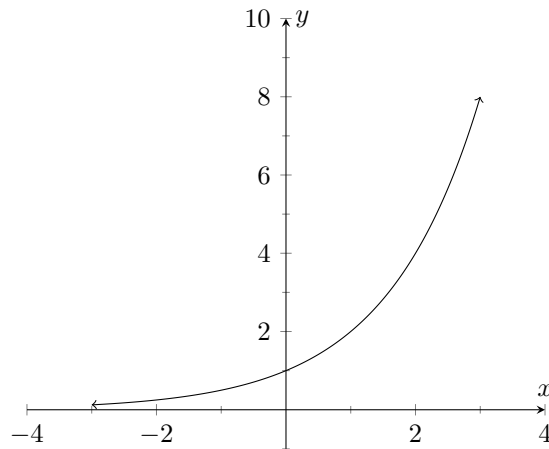
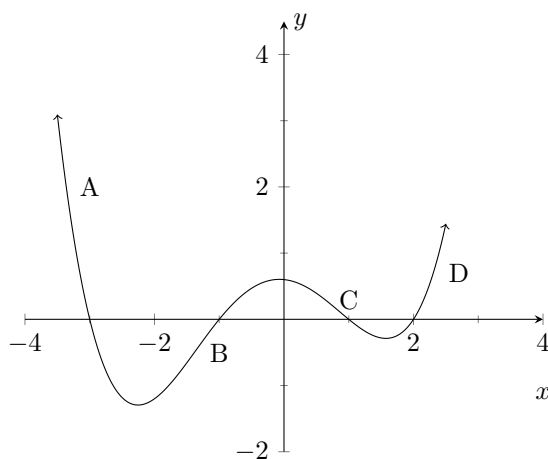**Increasing and Concave down:** Consider the graph for $f(x)$
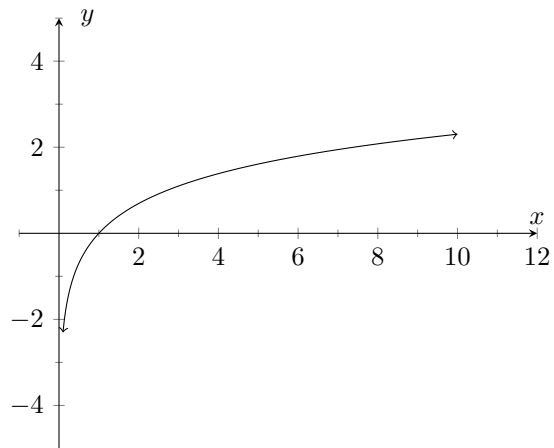
Decreasing and Concave Up: Consider the graph for $f(x)$

Decreasing and Concave Down: Consider the graph for $f(x)$

Combination of each of the above in the same graph: Consider the graph of the polynomial $f(x)$

Key:

  A: Decreasing and Concave Up

  B: Increasing and Concave Down

  C: Decreasing and Concave Down

  D: Increasing and Concave Up.

Thus we have all four combinations in one polynomial, which is not unusual for higher degree polynomials.