Exponents and Extrema 2: Local Extrema

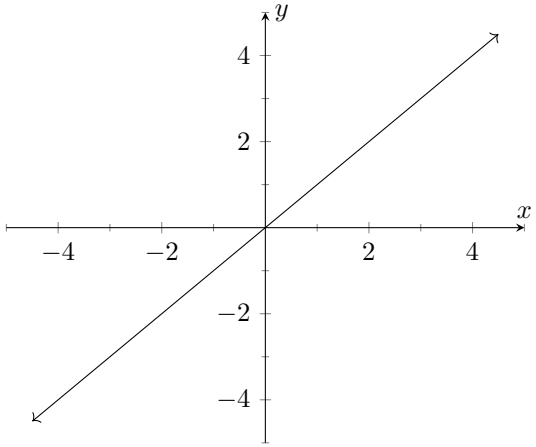
This section contains information on how exponents effect local extrema

Local extrema

YouTube link: https://www.youtube.com/watch?v=2yeDIOF7Qw8

Although we lack the analytic methods to tackle local extrema properly[1] there is some information we can gather just from the equation itself. Consider the following three graphs and notice what happens as we increase the degree of the leading term;

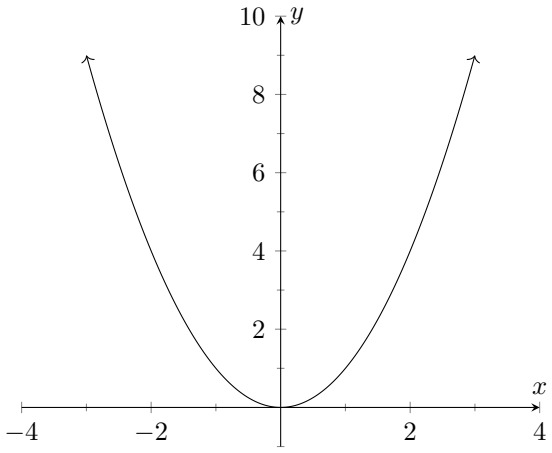
The graph of $y = x$;



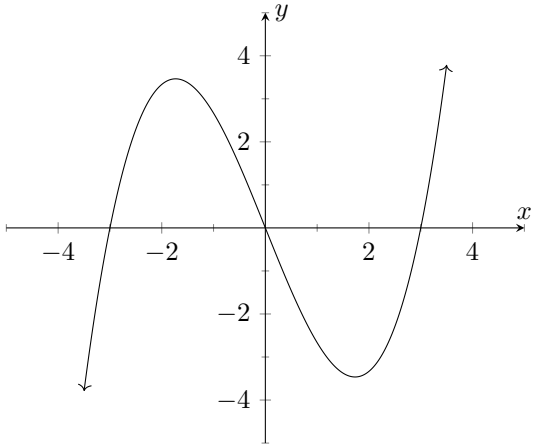
Learning outcomes:

[1] Indeed, this is a major area of study in the first calculus class, comprising about a third of the course

The graph of $y = x^2$;



The graph of $y = \frac{1}{3}x^3 - 3x$



Each time we increase the exponent we get another 'bend' in the graph. But this need not always happen, after all the graph of x^3 doesn't have the same number of bends as the graph of $\frac{1}{3}x^3 - 3x$ above. Nonetheless, we know that a parabola will always have one bend, and a line will never have any. Continuing this pattern we can eventually come to the following observation; the number of bends in the graph of a polynomial will be, at most, one less than the degree of the polynomial. This is important because each bend is *also* a local extrema, so a more precise (or mathematical) way to state our observation is;

Lemma 1. *For any polynomial of degree n, there are **at most** $n-1$ local extrema on the graph of $p(x)$.*

For example, based on our lemma, if we have a fifth degree polynomial, we know that it will have at most four local extrema, even if we don't know anything at all about what the graph actually looks like.

Problem 1 *Consider the polynomial $p(x) = 3x^2 - 12x^4 + x^3 - 2x + 1$. What can be said about the extrema of $p(x)$? (Select all that apply)*

Select All Correct Answers:

(a) *$p(x)$ has an absolute extrema.* ✓

(b) *$p(x)$ has no absolute extrema.*

(c) *$p(x)$ has at most one local extrema.*

(d) *$p(x)$ has at most four local extrema.*

(e) *$p(x)$ has, at most, three local extrema.* ✓

(f) $p(x)$ *has no local extrema.*

(g) $p(x)$ *has exactly one local extrema.*

Feedback(attempt): *Remember that a polynomial has absolute extrema if it is even degree, and it has* ***at most*** *the degree minus 1 local extrema.*

Feedback(correct): *Since the polynomial is even degree (it's largest degree term is* $-12x^4$ *which has degree 4) it has an absolute extrema. Moreover, since it has degree 4, it has at most* $4 - 1 = 3$ *local extrema.*