Exponents and Extrema: An Example

This section contains a demonstration of how odd versus even powers can effect extrema.

YouTube link: https://www.youtube.com/watch?v=LBV7xacNcGo

In this section we want to explore how the leading term effects global extrema. To this end, let's initially consider two monomials; $p(x) = x^3$ and $q(x) = x^4$. If we plug in some positive numbers, they seem similar (although $q(x)$ does increase much faster), but the negative values are where the more noteworthy differences lay. Consider the following table of values;

x-values	$p(x) = x^3$	$q(x) = x^4$
$x = -3$	$p(-3) = -27$	$q(-3) = 81$
$x = -2$	$p(-2) = -8$	$q(-2) = 16$
$x = -1$	$p(-1) = -1$	$q(-1) = 1$
$x = 0$	$p(0) = 0$	$q(0) = 0$
$x = 1$	$p(1) = 1$	$q(1) = 1$
$x = 2$	$p(2) = 8$	$q(2) = 16$
$x = 3$	$p(3) = 27$	$q(3) = 81$

Notice that $p(x)$ has both positive and negative values, whereas $q(x)$ has only positive values. After some consideration one can probably see that it's because the even power of x in $q(x)$ is eliminating the negative sign for any negative imput, whereas the odd power of $p(x)$ does not. Specifically, odd powers preserve the negatives, whereas even powers annihilate them.

So for $p(x) = x^3$, plugging in a large positive x value yields a large (and still positive) output. On the other hand, if we were to use a large negative x value, we would get a large (and negative) y value as the output. But this means that, no matter what value we think of, a big enough positive or negative input will yield a more positive or more negative output. In other words, $p(x)$ won't have a global maximum *or* minimum, because we can always just take a larger positive number to overcome any proposed maximum number, or larger negative numbers to overcome any proposed minimum number.

However, for $q(x) = x^4$ we can see that both large positive and large negative numbers x values will yield large positive y value outputs. This means that, on the one hand there is no maximum value, but on the other hand this *also* means there *must* be a minimum somewhere, because the y value output will never get large and negative.

Learning outcomes:

We should also recall that, if we use a negative coefficient, it flips the overall function over the x axis; so maximums become minimums and minimums become maximums. Thus $p(x) = -x^3$ still doesn't have a max or min, but $q(x) = -x^4$ would have a maximum somewhere.

The general result is that a polynomial (with a domain of all real numbers) whose *leading term* has an odd power can't have any global max or min, but if the leading term has an even power, then it has a global minimum if the leading coefficient is positive, and a global maximum if the leading coefficient is negative.

Problem 1 *Which of the following have absolute extrema over the domain of all real numbers? (Select all that apply)*

Select All Correct Answers:

(a) $p(x) = 13x^5 - 12x^3 + x^6 - 13$ ✓

(b) $p(x) = 2x^2 - x^3 + 14x^7$

(c) $p(x) = 5x^4 - 2x^3 + x$ ✓

(d) $p(x) = 18$ ✓

Feedback(`attempt`): *Remember to select all the polynomials that apply. A polynomial has absolute extrema only if it's degree is even, but the degree is based off the largest degree, not necessarily the first term to be written down.*