



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

FACULTY OF COMPUTING

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SECI1013 – DISCRETE STRUCTURE

SECTION 3

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ASSIGNMENT 2-CHAPTER 3

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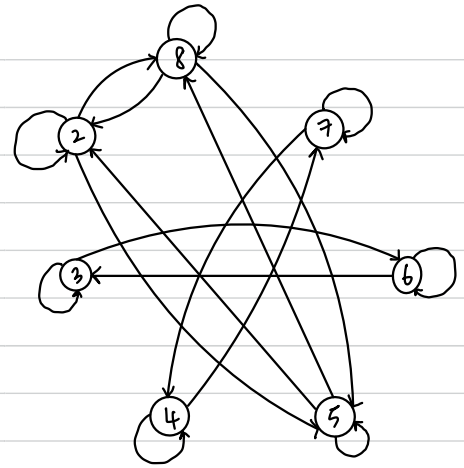
1. Relation

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1. Given $A = \{2, 3, 4, 5, 6, 7, 8\}$ and R a relation over A . Draw the directed graph of R after realising that xRy iff $x - y = 3n$ for some $n \in \mathbb{Z}$. Find all possible equivalence relations for R .

(5 marks)

if $n=0$	if $n=1$	if $n=-1$	$R = \{(2,2), (2,5), (2,8), (3,3), (3,6),$
$3(0)=0$	$3(1)=3$	$3(-1)=-3$	$(4,4), (4,7), (5,2), (5,5), (5,8),$
$2-2=0$	$8-5=3$	$5-8=-3$	$(6,3), (6,6), (7,4), (7,7), (8,2),$
$3-3=0$	$7-4=3$	$4-7=-3$	$(8,5), (8,8)\}$
$4-4=0$	$6-3=3$	$3-6=-3$	
$5-5=0$	$5-2=3$	$2-5=-3$	
if $n=2$	if $n=-2$		
$6-6=0$	$3(2)=6$	$3(-2)=-6$	
$7-7=0$	$8-2=6$	$2-8=-6$	
$8-8=0$			



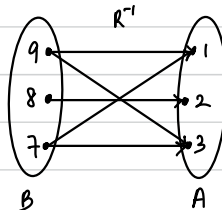
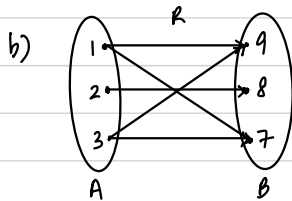
2. Let $A = \{1, 2, 3\}$ and $B = \{9, 8, 7\}$.

Let $R: A$ to B . For all $(a, b) \in A \times B$, and given $a R b \Leftrightarrow a+b$ is an even number,

- Determine R and R^{-1} .
- Draw arrow diagrams for both.
- Describe R^{-1} in words.

(10 marks)

a) $R = \{(1,9), (1,7), (2,8), (3,9), (3,7)\}$
 $R^{-1} = \{(9,1), (7,1), (8,2), (9,3), (7,3)\}$



c) The inverse relation of R is 9 to 1, 7 to 1, 8 to 2, 9 to 3 and 7 to 3

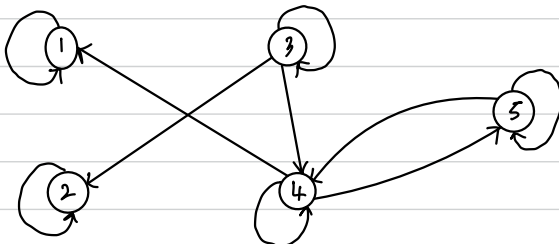
3. Let $A = \{1, 2, 3, 4, 5\}$, and let R be the relation on A that has the matrix (given below)

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 1 & 1 & 0 \\ 4 & 1 & 0 & 0 & 1 & 1 \\ 5 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Construct the digraph of R , and list in-degrees and out-degrees of all vertices.

(6 marks)

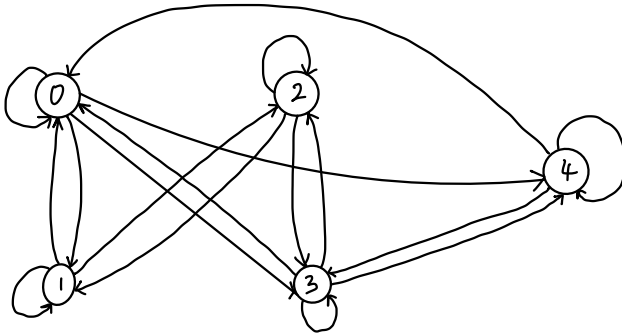
$R = \{(1,1), (2,2), (3,2), (3,3), (3,4), (4,1), (4,4), (4,5), (5,4), (5,5)\}$



	1	2	3	4	5
in-degree	2	2	1	3	2
out-degree	1	1	3	3	2

4. Given $A = \{0, 1, 2, 3, 4\}$, and
 $R = \{(0, 0), (0, 1), (0, 3), (0, 4), (1, 0), (1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 0), (3, 2), (3, 3), (3, 4), (4, 0), (4, 3), (4, 4)\}$. Draw the relation graph and find is R reflexive, symmetric, or transitive?

(12 marks)



$\therefore R$ is reflexive and symmetric but
 not transitive because $(2, 3), (3, 0) \in R$
 but $(2, 0) \notin R$

5. Relation R in the set $A = \{1, 2, 3, \dots, 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$, Determine whether the relation is
 a. Reflexive
 b. Symmetric
 c. Transitive

Support your answer with the reason.

(9 marks)

$$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

- a) Not reflexive because for each element $n \in A$, $(n, n) \notin R$
 b) Not symmetric because $(1, 3) \in R$ but $(3, 1) \notin R$
 c) Not transitive because $(1, 3)$ and $(3, 9) \in R$ but $(1, 9) \notin R$

6. Suppose that the given is a relation matrix for R and S ,

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Using Boolean Arithmetic, Find

- a. RS
 b. SR

(8 marks)

$$b. a) \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$b) \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Q2. Function

7. What is the different between Relation and Function?

(2 Marks)

Relation is a list of connection between things while function is specific connection where each element is linked to only each other.

8. If $A = \{2, 3, 4, 5\}$, then write whether each of the following relations on set A is a function or not. Give reasons also.

- (i) $\{(2, 3), (3, 4), (4, 5), (5, 2)\}$
 (ii) $\{(2, 4), (3, 4), (5, 4), (4, 4)\}$
 (iii) $\{(2, 3), (2, 4), (5, 4)\}$
 (iv) $\{(2, 3), (3, 5), (4, 5)\}$ (v) $\{(2, 2), (2, 3), (4, 4), (4, 5)\}$

(8 marks)

- i) Function because domain of f is equal to A
 ii) Function because $f(u_1) = f(u_2)$ but $u_1 \neq u_2$
 iii) Not function because domain element has 2 corresponding elements
 iv) Not function because the first part is not function as the domain of f is not equal to A , the second part is not function as domain element has 2 corresponding elements. Hence, not function \vee not function, so it is not function.

9. Given the relation of $R = \{(x, y) | y = x + 5, x \text{ is } \mathbb{Z}^+ \text{ less than } 6\}$. Depict this relationship using roster form. Write down the domain and the range.

(3 marks)

$$R = \{(1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$$

$$\text{Domain} = \{1, 2, 3, 4, 5\}$$

$$\text{Range} = \{6, 7, 8, 9, 10\}$$

10. In the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(v) $f = R \rightarrow R, f(x) = 1 - 2x$

(vi) $f = R \rightarrow R, f(x) = 5x^2 - 1$

(vii) $f = R \rightarrow R, f(x) = x^4$

(viii) $f = R \rightarrow R, f(x) = \left(\frac{x-2}{x-3}\right)$

(8 marks)

(v) $f(u_1) = f(u_2)$

$$1 - 2u_1 = 1 - 2u_2$$

$$u_1 = u_2$$

(one-to-one)

Every element in codomain is covered

$$u = \frac{1}{2}(1-y)$$

$$f(u) = 1 - 2\left[\frac{1}{2}(1-y)\right] = y$$

(onto)

\therefore It is bijective

(vi) $f(u_1) = 5u_1^2 - 1$

$$f(-2) = f(2)$$

$$\text{but } -2 \neq 2$$

\therefore Onto function

(vii) $f(u_1) = f(u_2)$

$$u_1^4 = u_2^4$$

$$u_1 = u_2$$

(one-to-one)

$$\text{if } u = \sqrt[4]{y}, f(u) = (u^4)^4 = y$$

(onto function)

\therefore It is bijective

(iv) $f(u) = \left(\frac{u-2}{u-3}\right)$

$$f(u_1) = f(u_2)$$

$$\frac{u_1 - 2}{u_1 - 3} = \frac{u_2 - 2}{u_2 - 3}$$

$$u_1 = u_2$$

(one-to-one)

Every element in codomain is covered

(onto)

\therefore It is bijective

11. Given the following functions, find the function $f(g(x))$ and

find the value of the function if $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1; g(x) = x^2 - 1$

(x) $f(x) = x^2; g(x) = 5x - 6$

(xi) $f(x) = x - 1; g(x) = x^3 + 1$

(9 marks)

$$\begin{aligned} \text{(ix)} \quad f(g(x)) &= 3(x^2 - 1) - 1 \\ &= 3x^2 - 3 - 1 \\ &= 3x^2 - 4 \end{aligned}$$

$$f(g(0)) = 3(0)^2 - 4 = -4$$

$$f(g(1)) = 3(1)^2 - 4 = -1$$

$$f(g(2)) = 3(2)^2 - 4 = 8$$

$$f(g(3)) = 3(3)^2 - 4 = 23$$

$$\begin{aligned} \text{(x)} \quad f(g(x)) &= (5x - 6)^2 \\ &= 25x^2 - 60x + 36 \end{aligned}$$

$$f(g(0)) = 25(0)^2 - 60(0) + 36 = 36$$

$$f(g(1)) = 25(1)^2 - 60(1) + 36 = 1$$

$$f(g(2)) = 25(2)^2 - 60(2) + 36 = 16$$

$$f(g(3)) = 25(3)^2 - 60(3) + 36 = 81$$

$$\begin{aligned} \text{(xi)} \quad f(g(x)) &= (x^3 + 1) - 1 \\ &= x^3 \end{aligned}$$

$$f(g(0)) = 0^3 = 0$$

$$f(g(1)) = 1^3 = 1$$

$$f(g(2)) = 2^3 = 8$$

$$f(g(3)) = 3^3 = 27$$

Q3. Recurrence Relation

12. Solve the recurrence relation given;

(xii) $a_n = 6a_{n-1} - 9a_{n-2}$; initial conditions $a_0 = 1$ and $a_1 = 6$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; initial conditions $a_0 = 2, a_1 = 5$ and $a_2 = 15$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$; initial conditions $a_0 = 1, a_1 = -2$ and $a_2 = -1$

(12 marks)

(xii) $a_0 = 1, a_1 = 6$

$$a_2 = 6(a_1) - 9(a_0) = 6(6) - 9(1) = 27$$

$$a_3 = 6(a_2) - 9(a_1) = 6(27) - 9(6) = 108$$

$$a_4 = 6(a_3) - 9(a_2) = 6(108) - 9(27) = 405$$

$$a_5 = 6(a_4) - 9(a_3) = 6(405) - 9(108) = 1458$$

$$1, 6, 27, 108, 405, 1458, \dots$$

(xiii) $a_0 = 2, a_1 = 5, a_2 = 15$

$$a_3 = 6(a_2) - 11(a_1) + 6(a_0) = 6(15) - 11(5) + 6(2) = 47$$

$$a_4 = 6(a_3) - 11(a_2) + 6(a_1) = 6(47) - 11(15) + 6(5) = 147$$

$$a_5 = 6(a_4) - 11(a_3) + 6(a_2) = 6(147) - 11(47) + 6(15) = 455$$

$$2, 5, 15, 47, 147, 455, \dots$$

(xiv) $a_0 = 1, a_1 = -2, a_2 = -1$

$$a_3 = -3(a_2) - 3(a_1) + a_0 = -3(-1) - 3(-2) + 1 = 10$$

$$a_4 = -3(a_3) - 3(a_2) + a_1 = -3(10) - 3(-1) + (-2) = -29$$

$$a_5 = -3(a_4) - 3(a_3) + a_2 = -3(-29) - 3(10) + (-1) = 56$$

$$1, -2, -1, 10, -29, 56, \dots$$

13. A sequence $a_1, a_2, a_3, a_4, \dots$ is given by

$$a_{n+1} = 5a_n - 3; a_1 = k$$

where k is a non-zero constant.

(i) Find the value of a_4 in terms of k .

(ii) Given that $a_4 = 7$, determine the value of k .

(8 marks)

i) $a_1 = k$

$$a_2 = 5k - 3$$

$$a_3 = 5(5k - 3) - 3$$

$$= 25k - 15 - 3$$

$$= 25k - 18$$

$$a_4 = 5(25k - 18) - 3$$

$$= 125k - 93$$

ii) $a_4 = 7$

$$125k - 93 = 7$$

$$125k = 100$$

$$k = 0.8$$