MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.
   (a) The difference of issuing bonds and printing money is that issuing bonds does not change the total amount of money while printing money does, where printing money may cause the money declines in value and cause inflation.

   (b) For example, the U.S. 2 Year Treasury rate is 2.2 % while the U.S. 20 Year Treasury rate is only 2.5 %, which means the long-term part (2 - 20 years) of the yield curve almost flatten because investors expect that future inflation will decrease or expect Federal Reserve will raise the federal funds rate in the near term, as investors expect the rise of interest rate is more likely to happen in short-term instead of long-term, long-term bonds less attractive to investors and the yield curve becomes eventually flatten.

   (c) Quantitative easing is a monetary policy by central banks to increasing the money supply and stimulate economic, for example, on the March 15, 2020, the U.S. Fed purchased Treasury securities,MBS, CMBS to sustain the functioning of markets for these securities.

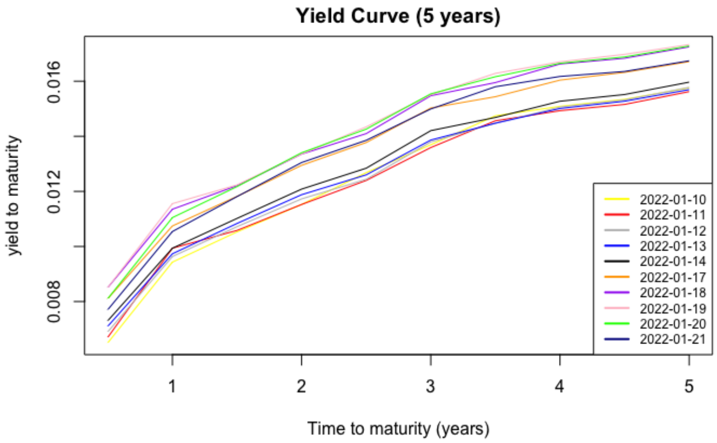
2. To construct a “0 - 5 years” yield and spot curves, I will find 10 bonds to perform this task. And the bonds I am choosing are: “CAN 0.25 Jul 31”, “CAN 0.25 Jan 31”, “CAN 0.25 Jul 31” , “CAN 0.75 Jan 31”, “CAN 1.5 Aug 31”, “CAN 1.25 Feb 28”, “CAN 0.5 Aug 31”, “CAN 0.25 Feb 28”, “CAN 1.0 Aug 31”, ”CAN 0.625 Feb 28”. Here I selected these 10 bonds with the interval gap 0.5 years, since the these bonds issued by Canada all with semi-annual coupon. One thing should be noted is that when selecting the bonds with certain maturity date we want, it happens that there could be multiple selections. I will choose the one listed first based on the original data I collected, since by observing the original data, the issue date of first listed bonds are very similar that the yield to maturity should also be similar for the 10 bonds I selected.

3. If we have the data in several stochastic processes for which each process represents a unique point along a stochastic curve, we can use PCA to convert certain signal from such stochastic data into fundamental frequencies by using the convenient eigenvalues and eigenvectors calculated from the covariance matrix of those stochastic processes. And PCA is a method finding combinations of variables extract maximum information that maximizing the variance of principal components. Furthering, the first PCA can be calculated by the first or largest eigenvalue times the corresponding eigenvector, which is the largest value that can capture most of the motion of the stochastic process. The first PCA of yield curve can happened to represent the parallel shift in the curve, and will be very similar to the weighted average of all bonds in the market.

Empirical Questions - 75 points

4.

(a) Here for the yield curve, I calculated the yield to bond by calculating the unique root satisfying: Price $= (\sum_{t=1}^{T}$ coupon payment $\times e^{-ytm\times\frac{1}{2}t}) +$ notional payment $\times e^{-r\times\frac{1}{2}T}$. One thing should be noted here is that the data we have for the maturity date gap forth bond and the fifth bond selected is 7 months which is more than exact 6 months. However, since the period of time between the known interest rate we are interpolating from is only 1 month, which is relatively small, according to article **How to Interpolate Interest Rates**, estimating using the linear interpolation can also be relatively accurate. Hence, here I use the linear interpolation for the period that not exact 6 months.



Algorithm 1 Spot rate calculation.

(b) i. Initialize a dictionary for storing the spot rate calculated by each date.

ii. For $m = 1$ to 10:

A. Initialize a vector storing spot rate for each bond on the m date.

B. let P be the vector of price of the m date price for 10 bonds selected.

C. For $i = 1$ to 10:

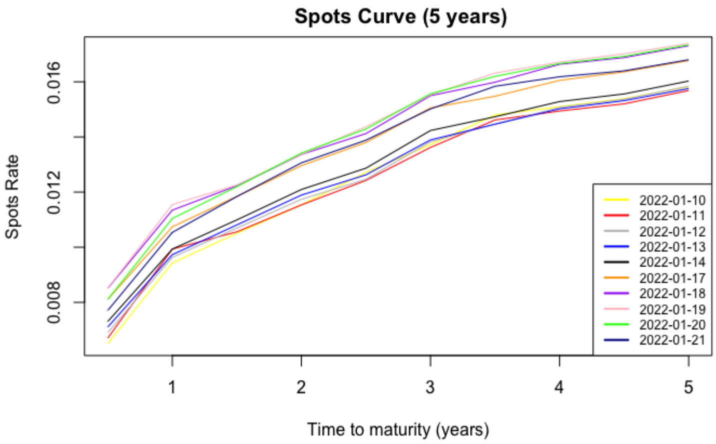
A. **if i = 1,** i.e., it is the first bond with time to maturity date exact 0.5 years as we selected, **Then,** $r = \frac{log\frac{Price}{\text{cpn payment + notional value}}}{T}$.

B. **Else,** time to maturity date $>$ 6 months, dirty price = accrued interest + clean price. Here we can calaulate accrued interest $= \sum_{t=1}^{j}$ cpn $\times e^{-r(\frac{1}{2}t)}$, spot rate $r = \frac{log(\frac{\text{Price - accrued interest}}{\text{cpn + notional value}})}{T}$. Here t refer the t period instead of the t year.

C. Store the spots rate calculated in the Spots data frame.

More precisely, price we refer is the dirty price, and the formula of spot rate is

$$r = \frac{log(\frac{\text{clean price}}{\text{coupon payment of last period + notional value}})}{T} = \frac{log(\frac{\text{Price - accrued interest}}{\text{coupon payment of last period + notional value}})}{T}$$



Algorithm 2 forward rate calculation.

(c) i. Initialize a dictionary for storing the forward rate calculated by each date.

ii. For $j = 1$ to 10:

A. spot refer to vector of the spots rate of the j date.

B. r_1 = the first year's spot rate. let P be the vector of price of the m date price for 10 bonds selected.

C. Initialize a vector storing forward rate for each bond on the j date.

D. For $i = 2$ to 5:

A. forward rate $f_{1,i} = \frac{r_i \times T_i - r_1 \times 1}{T_i - 1}$. Here r_i refers to the spots rates of i^{th} year.

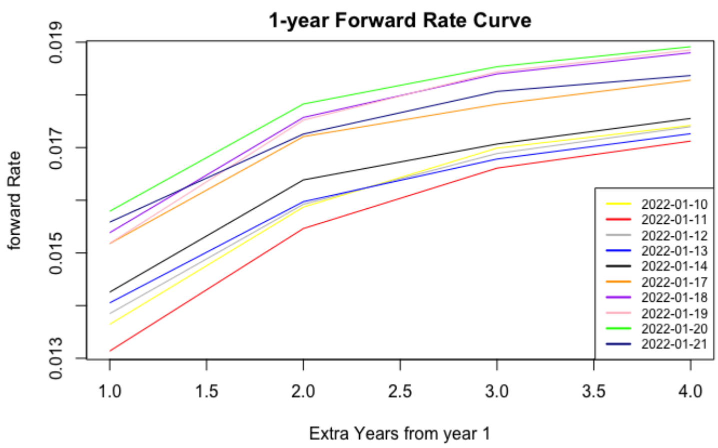
B. Store the forward rate calculated in the Forward data frame.

And the formula forward rate is conducted by: after given the price $P(t, T_1 = 1), P(t, T_i)$, $i = 2, 3, 4$.

$$f(t, T_1, T_i) = \frac{log(P(t, T_i)) - log(P(t, T_1))}{T_i - T_1}$$

Here, $P(t, T_1 = 1) = e^{r_1 \times T_1} = e^{r_1 \times 1}$, $P(t, T_i) = e^{r_i \times T_i}$. Hence, the forward rate formula will lastly be:

$$f(t, T_1 = 1, T_i) = f_{1,i} = \frac{r_i \times T_i - r_1 \times 1}{T_i - 1}$$



5. Covariance matrix for the time series of daily log-return of **yield**:

$$\begin{bmatrix} 0.0020597 & 0.0008245 & 0.0007755 & 0.0007537 & 0.0007230 \\ 0.0008245 & 0.0006882 & 0.0006559 & 0.0005885 & 0.0005649 \\ 0.0007755 & 0.0006559 & 0.0006626 & 0.0005963 & 0.0005771 \\ 0.0007537 & 0.0005885 & 0.0005963 & 0.0005665 & 0.0005429 \\ 0.0007230 & 0.0005649 & 0.0005771 & 0.0005429 & 0.0005236 \end{bmatrix}$$

Covariance matrix for the time series of daily log-return of **forward rate**:

$$\begin{bmatrix} 0.0010833 & 0.0007757 & 0.0005786 & 0.0005286 \\ 0.0007757 & 0.0007026 & 0.0005769 & 0.0005497 \\ 0.0005786 & 0.0005769 & 0.0005330 & 0.0005056 \\ 0.0005286 & 0.0005497 & 0.0005056 & 0.0004862 \end{bmatrix}$$

6. Eigenvalues for **yield covariance** matrix:
$\lambda_1 = 3.768 \times 10^{-03}$, $\lambda_2 = 6.729 \times 10^{-04}$, $\lambda_3 = 4.872 \times 10^{-05}$, $\lambda_4 = 1.016 \times 10^{-05}$, $\lambda_5 = 9.653 \times 10^{-07}$.
The corresponding eigenvectors are:
$e_1 = (-0.6695880488, -0.3933447, -0.38382195, -0.3601528, -0.34626941)^T$;
$e_2 = (0.7418639440, -0.3361004, -0.38685354, -0.3099972, -0.30153022)^T$:
$e_3 = (-0.0005562983, -0.7547413, -0.07401551, 0.4664469, 0.45531777)^T$:
$e_4 = (-0.0348310775, 0.3847581, -0.79831153, 0.4605888, 0.03611946)^T$:
$e_5 = (-0.0087304161, 0.1210092, -0.24544947, -0.5869216, 0.76194366)^T$:
The first eigenvalue is the largest eigenvalue, the corresponding eigenvectors multiple with the eigenvalue is the largest PCA which can capture most of the motion and variation of the yield curve, since it explained 83.71% of the total variance of the yield curve.

Eigenvalues for **forward covariance** matrix:
$\lambda_1 = 2.515363 \times 10^{-03}$, $\lambda_2 = 2.701074 \times 10^{-04}$, $\lambda_3 = 1.783833 \times 10^{-05}$, $\lambda_4 = 1.657474 \times 10^{-06}$.
The corresponding eigenvectors are:
$e_1 = (-0.6093561, -0.5227831, -0.4341568, -0.4085227)^T$;
$e_2 = (0.7403384, -0.1088452, -0.4457765, -0.4912586)^T$;
$e_3 = (-0.2608899, 0.8137379, -0.5068244, -0.1135611)^T$;
$e_4 = (0.1118958, -0.2295238, -0.5965905, 0.7608403)^T$
The first eigenvalue is the largest eigenvalue, the corresponding eigenvectors multiple with the eigenvalue is the largest PCA which can capture most of the motion and variation of the forward rate curve, since it explained 89.68% of the total variance of the forward rate curve.

References and GitHub Link to Code

1. **How to Interpolate Interest Rates**. https://www.sapling.com/8396129/interpolate-interest-rates
2. **GitHub Link**. https://github.com/JOANNE724/APM466-Assignment-1