MATEMÁTICA DISCRETA – PROF. JORGE CAVALCANTI
LISTA 3 – INDUÇÃO / RECURSÃO

10) $2+6+10+\dots+(4n-2)=2n^2$

1) $P(1) = 4(1)-2=2$, $2n^2=2(1)^2=2$ OK
2) $P(K) = 2+6+10+\dots+(4K-2) = 2K^2$
3) $P(K+1) = 2+6+10+\dots+(4K-2)+[4(K+1)-2] = 2(K+1)^2$

USANDO A HIPÓTESE DE INDUÇÃO EM P(K+1):

$P(K+1) = \underbrace{2+6+10+\dots+(4K-2)}_{P(K)}+[4(K+1)-2] = 2(K+1)^2$ PROVAR

⇒ RESOLVENDO O LADO ESQUERDO:
$P(K+1) = 2K^2+[4(K+1)-2] = 2K^2+(4K+4-2)$
$P(K+1) = 2K^2+4K+2 = 2(K^2+2K+1)$
$P(K+1) = 2(K+1)^2$ OK

11) $1+5+9+\dots+(4n-3) = n(2n-1)$
1 – $P(1) = 4(1)-3 = 1$; $1(2(1)-1) = 1$ OK
2 – $P(K) = 1+5+9+\dots+(4K-3) = K(2K-1)$
3 – $P(K+1) = 1+5+9+\dots+(4K-3)+[4(K+1)-3] = (K+1)[2(K+1)-1]$
USANDO A HIPÓTESE P(K) EM P(K+1):
$P(K+1) = \underbrace{1+5+9+\dots+(4K-3)}_{P(K)}+[4(K+1)-3] = (K+1)[2(K+1)-1]$

$P(K+1) = K(2K-1)+[4(K+1)-3] = 2K^2+3K+1$
RESOLVENDO O LADO ESQUERDO:
$P(K+1) = K(2K-1)+[4(K+1)-3]$
$P(K+1) = 2K^2-K+4K+4-3$
$P(K+1) = 2K^2+3K+1$ OK

12) $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

1 - $P(1)=\frac{1}{1.2}=\frac{1}{2}$; $\frac{1}{1+1}=\frac{1}{2}$ =0

2 - $P(K)=\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots+\frac{1}{K(K+1)}=\frac{K}{K+1}$

3 - $P(K+1)=\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots+\frac{1}{K(K+1)}+\frac{1}{(K+1)(K+1+1)}=\frac{(K+1)}{(K+1)+1}$

USANDO A HIPÓTESE P(K) EM P(K+1)

$$P(K+1)=\underbrace{\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\dots+\frac{1}{K(K+1)}}_{P(K)}+\frac{1}{(K+1)(K+1+1)}=\frac{(K+1)}{(K+2)}$$

$$=0\ \frac{K}{(K+1)}+\frac{1}{(K+1)(K+2)}=\frac{K(K+2)+1}{(K+1)(K+2)}=\frac{K^2+2K+1}{(K+1)(K+2)}$$

$$P(K+1)=\frac{(K+1)^2}{(K+1)(K+2)}=\frac{(K+1)}{(K+2)}\ \text{OK}$$

13) $2+6+18+\dots+2.3^{n-1}=3^n-1$

1 - $P(1)=2.3^{1-1}=2$; $3^1-1=2$ OK

2 - $P(K)=2+6+18+\dots+3^{K-1}=3^K-1$

3 - $P(K+1)=\underline{2+6+18+\dots+2.3^{K-1}+2.3^{(K+1)-1}=3^{K+1}-1}$

USANDO P(K) EM P(K+1)

$$3^K-1+2.3^{(K+1-1)}=3^K-1+2.3^K=2.3^K+3^K-1$$
$$=3^K(2+1)-1=(3^K.3)-1=\underline{3^{K+1}-1}\ \text{ok}$$

14) $n^2 \geq 2n+3$, p/ $n \geq 3$

1- $P(3) = 3^2 \geq 2(3)+3 \therefore 9 \geq 9$ OK

2. $P(K) = K^2 \geq 2K+3$

3. $P(K+1) = (K+1)^2 \geq 2(K+1)+3$

$\therefore P(K+1) = (K+1)^2 \geq 2K+5$

$K^2+2K+1 \geq 2K+5$

FAZENDO A HIPÓTESE P(K), $K^2 = 2K+3$ NO LADO ESQ

$(2K+3)+2K+1 \geq 2K+5$

$4K+4 \geq 2K+5$

COMO $(K+1)^2 \geq 4K+4 \geq 2K+5$ $(K \geq 3)$

$(K+1)^2 \geq 2K+5$ OK

15) $n^2 > n+1$ p/ $n > 2$

1- $P(2) = 4 > 3$ OK

2- $P(K) = K^2 > K+1$

3 $P(K+1) = (K+1)^2 > (K+1)+1$

$P(K+1) = (K+1)^2 > K+2$

$K^2+2K+1 > K+2$

USANDO A HIPÓTESE NO LADO ESQUERDO:

$(K+1)+2K+1 > K+2$

$3K+2 > K+2$ OK (COMO $K > 2$)

ENTÃO $(K+1)^2 > K+2$

16) $n! > n^2$, p/ $n \geq 4$

1- $P(4)$: $4! > 4^2 \therefore 24 > 16$ OK

2 $P(K) = K! > K^2$

3 $P(K+1) = (K+1)! > (K+1)^2$

$(K+1)! > (K+1)^2$

$(K+1)(K!) > (K+1)^2$

DIVIDINDO AMBOS O LADOS POR $(K+1)$

$\longrightarrow$ D

CONT 16-

$\Rightarrow$ $K! > K+1$ PROVAR

USANDO A HIPÓTESE $(K! > K^2)$

COMO $K^2 > K+1$ (P/ $K \geq 4$)

$K! > K^2 > K+1$

ENTÃO $\underline{K! > K+1}$

17) $2^n < n!$ p/ $n \geq 4$

1- $P(4) = 2^4 < 4!$ $\therefore$ $16 < 24$ OK

2- $P(K) = 2^K < K!$

3- $P(K+1) = 2^{(K+1)} < (K+1)!$

$2^{K+1} = 2^K \cdot 2^1$

$2^{K+1} < (K+1)!$

$2^K \cdot 2^1 < (K+1)!$

USANDO A HIPÓTESE $2^K < K!$

$K! \cdot 2 < (K+1)!$, COMO $(K+1)! = (K+1)\underbrace{(K)(K-1)\ldots}_{K!}$

$2K! < (K+1)(K)!$ $(\div K!)$

$2 < K+1$

COMO $K \geq 4$, ENTÃO:

$2^{(K+1)} < (K+1)!$

18) $S(1) = 10$

$S(n) = S(n-1) + 10$ p/ $n \geq 2$

$S(2) = S(1) + 10 = 20$, $S(3) = S(2) + 10 = 30$

$S(4) = S(3) + 10 = 40$, $S(5) = S(4) + 10 = 50$

19) $A(1) = 2$

$A(n) = \frac{1}{A(n-1)}$ p/ $n \geq 2$

$A(2) = \frac{1}{A(1)} = \frac{1}{2}$, $A(3) = \frac{1}{A(3)} = \frac{1}{1/2} = 2$, $A(4) = \frac{1}{A(3)} = 2$

$A(5) = \frac{1}{A(4)} = \frac{1}{2}$

CONT LISTA INDUÇÃO RECURSÃO

20) a) $F(n+1) + F(n-2) = 2F(n) \quad n \geq 3$

$F(n+1) = F(n) + F(n-1) \quad (1)$

$F(n) = F(n-2) + F(n-1) \Rightarrow F(n-2) = F(n) - F(n-1) \quad (2)$

De (1) e (2)

$F(n) + F(n-1) + F(n) - F(n-1) = F(n) + F(n) = 2F(n)$

b) $F(n) = 5F(n-4) + 3F(n-5), \quad n \geq 6$

$F(n) = F(n-1) + F(n-2) = F(n-2) + F(n-3) + F(n-3) + F(n-4)$

$F(n) = F(n-3) + F(n-4) + F(n-4) + F(n-5) + F(n-4) - F(n-5) + F(n-4)$

$F(n) = F(n-3) + 4F(n-4) + 2F(n-5)$

$F(n) = F(n-4) + F(n-5) + 4F(n-4) + 2F(n-5)$

$F(n) = 5F(n-4) + 3F(n-5)$

c) $[F(n+1)]^2 = [F(n)]^2 + F(n-1)\,F(n+2) \quad$ P/ $n \geq 2$

$[F(n+1)]^2 = F(n+1) \cdot F(n+1) = [F(n) + F(n-1)]^2$

$= F(n)^2 + 2F(n)F(n-1) + F(n-1)^2 \quad (1)$

$F(n+2) = F(n+1) + F(n) = F(n) + F(n-1) + F(n) = 2F(n) + F(n-1) \quad (2)$

OU $F(n+2) = F(n) + F(n-1) + F(n) = 2F(n) + F(n-1)$

EM (1), COLOCANDO $F(n-1)$ EM EVIDÊNCIA:

$= F(n)^2 + F(n-1)\underbrace{[2F(n) + F(n-1)]}_{F(n+2)} = F(n)^2 + F(n-1) \cdot F(n+2)$

ENTÃO $[F(n+1)]^2 = [F(n)^2 + F(n-1) \cdot F(n+2)]$