Opgave 1

Bestem om der er singulariteter, og hvilket type de har.

$$\frac{z}{z^2(z+1)} \qquad \frac{1}{1+\cos z} \qquad z^{-4}e^{z/4} \qquad \frac{\sin(\pi z)}{z\cos(\pi z)}$$

$$\tan\frac{1}{z} \qquad \frac{\sin^2 z}{\arctan z} \qquad \tan z \cdot \cos z \qquad \csc(3z)$$

$$\frac{2z}{\sin z} \qquad \frac{1}{e^{2z}} \qquad \cos(\sin z) \qquad \sin\frac{(z^2+1)}{(z+i)}$$

Opgave 2

Bestem ved hjælp af l'Hopitals regel

$$\lim_{z\to 0} \frac{2z}{\sin z} =$$

$$\lim_{z\to 0} \frac{2\sin z - \sin 2z}{z - \sin z} =$$

Opgave 3

Bestem $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$

Brug $\displaystyle\cot z = \frac{1}{\tan z} = \frac{\cos z}{\sin z} = \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} B_{2n}}{(2n)!} z^{2n-1}$

$$B_0 = 1$$
$$B_1 = -1/2$$
$$B_2 = 1/6$$
$$B_3 = 0$$
$$B_4 = -1/30$$

Poler for $f(z)$:

Laurentrække: $\displaystyle f(z)\cot \pi z = \sum_{n=0}^{\infty} \frac{\qquad}{\qquad} z$

$$= \qquad z^{-3} \qquad z^{-1} \qquad z$$

Residuum for pol: $\displaystyle\operatorname*{Res}_{z=}\big(f(z)\cot \pi z\big) = b_1 =$

$$\sum_{k=-\infty}^{\infty} \frac{1}{k^2} = -\pi \sum_{\substack{\text{Poler} \\ \text{for } f}} \operatorname{Res}\big(f(z)\cot \pi z\big) = \qquad \sum_{k=1}^{\infty} \frac{1}{k^2} =$$