

Opgave 1

Bestem om der er singulariteter, og hvilket type de har.

$$\frac{z}{z^2(z+1)}$$

$$\frac{1}{1+\cos z}$$

$$z^{-4}e^{z/4}$$

$$\frac{\sin(\pi z)}{z \cos(\pi z)}$$

$$\tan \frac{1}{z}$$

$$\frac{\sin^2 z}{\arctan z}$$

$$\tan z \cdot \cos z$$

$$\csc(3z)$$

$$\frac{2z}{\sin z}$$

$$\frac{1}{e^{2z}}$$

$$\cos(\sin z)$$

$$\sin \frac{(z^2+1)}{(z+i)}$$

Opgave 2

Bestem ved hjælp af l’Hopitals regel

lim_{z -> 0} 2z / sin z =

lim_{z -> 0} (2 sin z - sin 2z) / (z - sin z) =

Opgave 3

Bestem

sum_{k=1}^inf 1/k^2

Brug

cot z = 1 / tan z = cos z / sin z = sum_{n=0}^inf ((-1)^n 2^{2n} B_{2n} / (2n)!) z^{2n-1}

B_0 = 1
B_1 = -1/2
B_2 = 1/6
B_3 = 0
B_4 = -1/30

Poler for f(z):

Laurentrække:

f(z) cot pi z = sum_{n=0}^inf _____ z
= z^{-3} z^{-1} z

Residuum for pol: Res(f(z) cot pi z) = b_1 =

sum_{k=-inf}^inf 1/k^2 = -pi sum_{Poler for f} Res(f(z) cot pi z) = sum_{k=1}^inf 1/k^2 =