Discrete time systems and z-transform

Exercises module 5

1.

A stable LTI system is characterized by the following difference equation:

$$\frac{1}{2}y[n-1]-\frac{9}{4}y[n]+y[n+1]=x[n]$$

a. Calculate the system function $H(z)$.
b. Calculate the impulse response $h[n]$.

2.

The output of a LTI system is $y[n]=6\left(\frac{1}{2}\right)^n u[n]-6\left(\frac{3}{4}\right)^n u[n]$, when the input is

$x[n]=\left(\frac{1}{2}\right)^n u[n]+(2)^n u[-n-1]$.

a. Calculate the system function $H(z)$ and its ROC.
b. Find the system impulse response $h[n]$.
c. Write the difference equation that characterizes the system.
d. Is the system stable and/or causal? Motivate your response.

3.

When the input is $x[n]=-\frac{1}{3}\left(\frac{1}{2}\right)^n u[n]-\frac{4}{3}(2)^n u[-n-1]$, the z-transform of the output signal of an LTI system is $Y(z)=\frac{1-z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)(1-2z^{-1})}$. Calculate the impulse response $h[n]$ of the system.

4.

Given the following system function

$$H(z)=\frac{1-3z^{-1}}{(1-z^{-1})(1-2z^{-1})}, \qquad |z|>2$$

Determine the impulse response $h_i(n)$ of the inverse system.

5.

Determine the impulse response $h_i(n)$ of an unstable LTI system whose system function is given by

$$H(z)=\frac{21}{\left(1-\frac{1}{2}z^{-1}\right)(1-2z^{-1})(1-4z^{-1})}$$

Is there a single possible $h_i(n)$?