Module 1

1) $$\sum_{k=0}^{\infty} \delta[n-k] = \delta[n] + \delta[n-1] + \delta[n-2] + \dots$$

$\delta[n] = 1$ if $n=0$

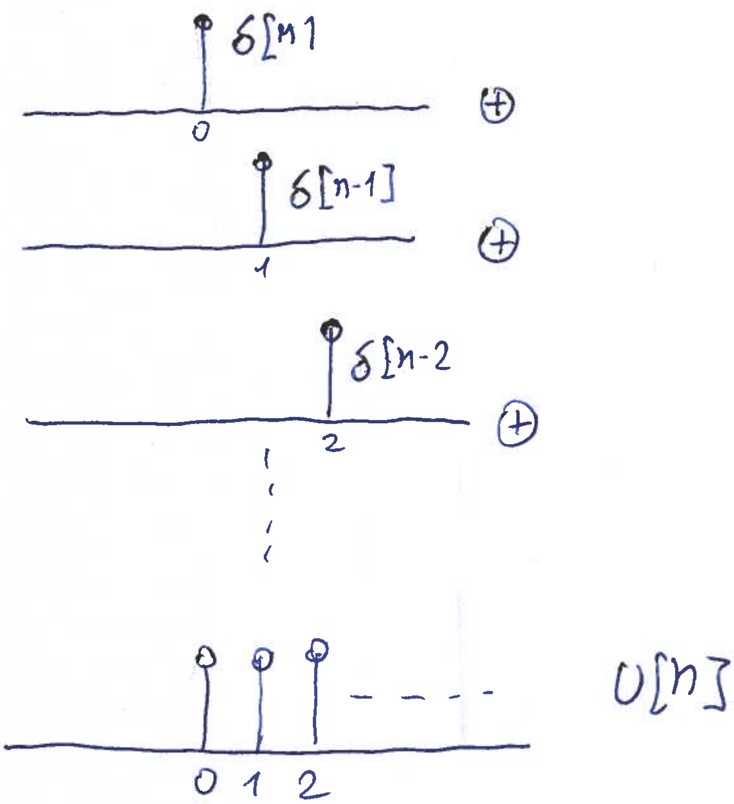
$\delta[n-1] = 1$ if $n=1$ $\qquad \delta[n-k]=0$ if $n \neq k$

$\delta[n-2] = 1$ if $n=2$

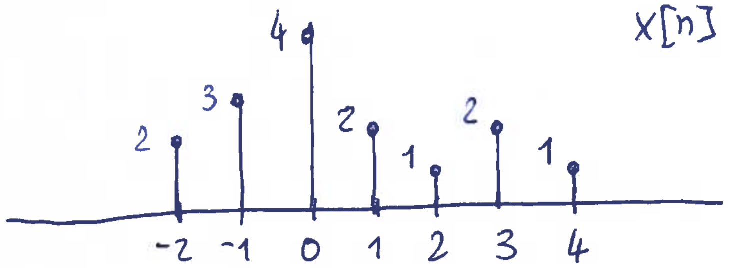
$\vdots$

Since $u[n]=1$ if $n \geq 0 \rightarrow \delta[n] + \delta[n-1] + \delta[n-2] + \dots$

$= u[n]$



c)



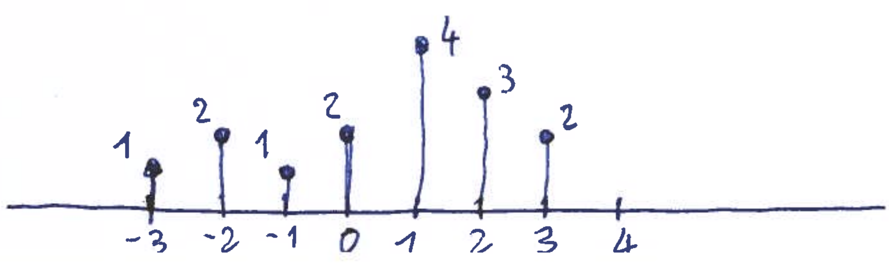
$$x[n] = 2\delta[n+2] + 3\delta[n+1] + 4\delta[n] + 2\delta[n-1] + \delta[n-2] +$$
$$+ 2\delta[n-3] + \delta[n-4].$$

- $$x[n-2] = 2\delta[(n-2)+2] + 3\delta[(n-2)+1] + 4\delta[n-2] + 2\delta[(n-2)-1] +$$
$$+ \delta[(n-2)-2] + 2\delta[(n-2)-3] + \delta[(n-2)-4] =$$
$$= 2\delta[n] + 3\delta[n-1] + 4\delta[n-2] + 2\delta[n-3] + \delta[n-4] +$$
$$+ 2\delta[n-5] + \delta[n-6]$$

- $$x[1-n] = 2\delta[(1-n)+2] + 3\delta[(1-n)+1] + 4\delta[1-n] + 2\delta[(1-n)-1] +$$
$$+ \delta[(1-n)-2] + 2\delta[(1-n)-3] + \delta[(1-n)-4] =$$
$$= 2\delta[3-n] + 3\delta[2-n] + 4\delta[1-n] + 2\delta[-n] +$$
$$+ \delta[-n-1] + 2\delta[-n-2] + \delta[-n-3] =$$

$\delta[-n-x] = \delta[n+x]$

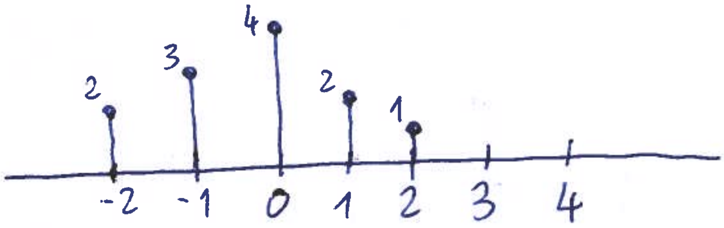
$$= 2\delta[n-3] + 3\delta[n-2] + 4\delta[n-1] + 2\delta[n] +$$
$$+ \delta[n+1] + 2\delta[n+2] + \delta[n+3]$$



- $x[n]\,u[2-n]$

$$u[2-n] = \begin{cases} 1 & 2-n \geq 0 \rightarrow n \leq 2 \\ 0 & 2-n < 0 \rightarrow n > 2 \end{cases}$$

$$x[n]\,u[2-n] = 2\delta[n+2] + 3\delta[n+1] + 4\delta[n] + 2\delta[n-1] + \delta[n-2]$$

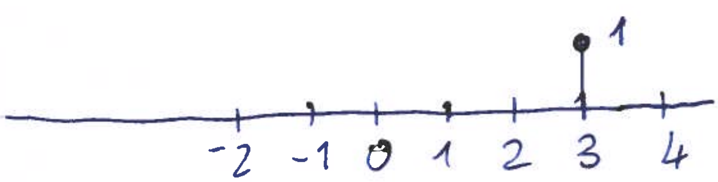


- $x[n-1]\,\delta[n-3]$

$$\delta[n-3] = \begin{cases} 1 & n = 3 \\ 0 & n \neq 3 \end{cases}$$

$$x[n-1] = 2\delta[n+1] + 3\delta[n] + 4\delta[n-1] + 2\delta[n-2] + \delta[n-3] + 2\delta[n-4] + \delta[n-5]$$

$$x[n-1]\,\delta[n-3] = \delta[n-3]$$



3) • $T(x[n]) = e^{x[n]}$

• linearity

Let us consider 2 sequences $x_1[n], x_2[n]$

$$T(x_1[n]) = e^{x_1[n]} \qquad T(x_2[n]) = e^{x_2[n]}$$

We define $\hat{x}[n] = a x_1[n] + b x_2[n]$

$$T(\hat{x}[n]) = e^{a x_1[n] + b x_2[n]} = e^{a x_1[n]} e^{b x_1[n]} \neq$$

$$\neq a T(x_1[n]) + b T(x_2[n])$$

non-linear!

• stability

if $|x[n]| \leq B_x < \infty$

$$|T(x[n])| = |e^{x[n]}| \leq e^{|x[n]|} \leq e^{B_x} < \infty$$

stable system!

• causality

The system does not depend on future values of $x[n]$, and is therefore causal

• time invariance

$$y[n-n_0] = e^{x[n-n_0]}$$

$$T(x[n-n_0]) = e^{x[n-n_0]} = y[n-n_0]$$

time invariant system!

- $T(x[n]) = ax + b, \quad a, b < \infty.$

- linearity

$x_1[n], x_2[n] \rightarrow T(x_1[n]) = a x_1[n] + b,$

$T(x_2[n]) = a x_2[n] + b$

$$c\,T(x_1[n]) + d\,T(x_2[n]) = c(a x_1[n] + b) + d(a x_2[n] + b)$$

Let us define $\tilde{x}[n] = c x_1[n] + d x_2[n]$

$$T(\tilde{x}[n]) = T(c x_1[n] + d x_2[n]) = a(c x_1[n] + d x_2[n]) + b$$
$$\neq c\,T(x_1[n]) + d\,T(x_2[n])$$

not linear!

- causality

Since the system does not depend of future values of $x[n]$, it is causal

- stability

if $|x[n]| \leq B_x < \infty$

$\rightarrow |T(x[n])| = |a x[n] + b| \leq |a| B_x + |b| < \infty$

for $a, b < \infty$

stable system

- time invariance

$y[n - n_0] = a x[n - n_0] + b$

$T(x[n - n_0]) = a x[n - n_0] + b = y[n - n_0]$

time invariant system!

- $T(x[n]) = x[-n]$
  - linearity

    $x_1[n], x_2[n] \rightarrow T(x_1[n]) = x_1[-n]$

    $T(x_2[n]) = x_2[-n]$

    Let us define $\hat{x}[n] = a x_1[n] + b x_2[n]$

    $T(\hat{x}[n]) = T(a x_1[n] + b x_2[n]) = a x_1[-n] + b x_2[-n] =$

    $= a T(x_1[n]) + b T(x_2[n])$

    linear system
  - causality

    if $n < 0$, the system depends on future values of $x[n]$

    $\rightarrow$ non causal
  - stability

    if $|x[n]| \leq B_x < \infty$

    $|T(x[n])| = |x[-n]| \leq B_x$ stable system
  - time invariance

    $y[n-n_0] = x[-n+n_0]$

    $T(x[n-n_0]) = x[-n-n_0] \neq y[n-n_0]$

    non time invariant

- $T(x[n]) = x[n] + u[n+1]$

$$u[n+1] = \begin{cases} 1 & n \geq -1 \\ 0 & n < -1 \end{cases}$$

- linearity

$$x_1[n], x_2[n] \longrightarrow T(x_1[n]) = x_1[n] + u[n+1]$$
$$T(x_2[n]) = x_2[n] + u[n+1]$$

$$aT(x_1[n]) + bT(x_2[n]) = a(x_1[n] + u[n+1]) + b(x_2[n] + u[n+1])$$
$$\tilde{x}[n] = a x_1[n] + b x_2[n]$$
$$T(\tilde{x}[n]) = T(ax_1[n] + bx_2[n]) = ax_1[n] + bx_2[n] + u[n+1]$$
$$\neq aT(x_1[n]) + bT(x_2[n])$$

non linear system!

- causality

The system does not depend on future values of $x[n]$, and is therefore causal

- stability

if $|x[n]| \leq B_x < \infty$

$|T(x[n])| \leq B_x + 1$ if $n \geq -1$ and
$|T(x[n])| \leq B_x$ if $n < -1$ stable system

- time invariance

$$y[n-n_0] = x[n-n_0] + u[n-n_0+1]$$
$$T(x[n-n_0]) = x[n-n_0] + u[n+1] \neq y[n-n_0]$$

non-time invariant!

- $T(x[n]) = \sum_{k=n-1}^{n+1} x[k] = x[n-1] + x[n] + x[n+1]$
  - linearity

    $x_1[n], x_2[n] \longrightarrow T(x_1[n]) = \sum_{k=n-1}^{n+1} x_1[n]$

    $T(x_2[n]) = \sum_{k=n-1}^{n+1} x_2[n]$

    $a\,T(x_1[n]) + b\,T(x_2[n]) = a \sum_{k=n-1}^{n+1} x_1[n] + b \sum_{k=n-1}^{n+1} x_2[n]$

    $\hat{x}[n] = a x_1[n] + b x_2[n]$

    $T(\hat{x}[n]) = \sum_{k=n-1}^{n+1} (a x_1[n] + b x_2[n]) = a \sum_{k=n-1}^{n+1} x_1[n] + b \sum_{k=n-1}^{n+1} x_2[n] =$

    $= a\,T(x_1[n]) + b\,T(x_2[n])$ linear!

  - causality

    the system depends on future values of $x[n]$, and is therefore not causal

  - stability

    if $|x[n]| \leq B_x < \infty$

    $\rightarrow |T(x[n])| = |T(x[n-1] + x[n] + x[n+1])| \leq 3B_x$

    stable system

$\longrightarrow$

- time invariance

$$y[n-n_0] = \sum_{k=n-n_0-1}^{n-n_0+1} x[k]$$

$$T(x[n-n_0]) = \sum_{k=n-1}^{n+1} x[k-n_0] \underset{k-n_0=q}{=} \sum_{q=n-n_0-1}^{n-n_0+1} x[q] = y[n-n_0]$$

time invariant!

- $T(x[n]) = x[n^2]$

  - linearity

    $x_1[n], x_2[n] \longrightarrow T(x_1[n]) = x_1[n^2]$

    $T(x_2[n]) = x_2[n^2]$

    $a T(x_1[n]) + b T(x_2[n]) = a x_1[n^2] + b x_2[n^2]$

    $\tilde{x}[n] = a x_1[n] + b x_2[n]$

    $T(\tilde{x}[n]) = T(a x_1[n] + b x_2[n]) = a x_1[n^2] + b x_2[n^2] =$

    $= a T(x_1[n]) + b T(x_2[n])$

    linear system

  - causality

    The system does depend on future values of $x[n]$, and is therefore non causal.

  - stability

    if $|x[n]| \leq B_x < \infty \Rightarrow |x[n^2]| \leq B_x < \infty$

    stable system

  - time invariance

    $y[n-n_0] = x[(n-n_0)^2] = x[n^2 + n_0^2 - 2n n_0]$

    $T(x[n-n_0]) = x[n^2 - n_0] \neq y[n-n_0]$

    non time-invariant!