

CARA MENCARI MATRIKS INVERS

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MATRIKS INVERS (A^{-1})

Jika : $AB = BA = I \rightarrow$ maka A adalah invers dari B, dan B adalah invers dari A (saling invers).

CARA MENCARI INVERS

1. CARA OBE :

$$[A | I] \xrightarrow{\text{OBE}} [I | A^{-1}] \rightarrow \text{RUMUSNYA}$$

Contoh :

Carilah invers dari matriks : $A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Jawab :

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{b_2 + 1b_1 \\ b_4 - 1b_1}} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 4 & -1 & 0 & 1 \end{array} \right] \xrightarrow{\substack{b_1 - 1b_2 \\ b_3 - 1b_2}} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 4 & -2 & -1 & 1 \end{array} \right] \xrightarrow{b \cdot (1/4)}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & -1/4 & 1/4 \end{array} \right] \xrightarrow{b_1 + 1b_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/2 & -5/4 & 1/4 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & -1/4 & 1/4 \end{array} \right]$$

Jadi $A^{-1} = \begin{bmatrix} -1/2 & -5/4 & 1/4 \\ 1 & 1 & 0 \\ -1/2 & -1/4 & 1/4 \end{bmatrix}$

2.CARA KOFAKTOR :

$$A^{-1} = \frac{1}{|A|} \text{adjoint } A \rightarrow \text{RUMUSNYA}$$

Carilah invers dari matriks : $A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Jawab:

$$|A| = (0 + 1 + 2) - (0 - 3 + 2) = 3 - (-1) = 4$$

Mencari kofaktor: $k_{ij} = (-1)^{i+j} |M_{ij}|$

$$k_{11} = (-1)^2 \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = -2 \quad ; \quad k_{12} = (-1)^3 \begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} = 4 \quad ; \quad k_{13} = (-1)^4 \begin{vmatrix} -1 & 0 \\ 1 & 2 \end{vmatrix} = -2$$

$$k_{21} = (-1)^3 \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = -5 \quad ; \quad k_{22} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} = 4 \quad ; \quad k_{23} = (-1)^5 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = -1$$

$$k_{31} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1 \quad ; \quad k_{32} = (-1)^5 \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} = 0 \quad ; \quad k_{33} = (-1)^6 \begin{vmatrix} 1 & 1 \\ -1 & 0 \end{vmatrix} = 1$$

$$\text{Matriks kofaktornya : } K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} -2 & 4 & -2 \\ -5 & 4 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\text{Adjoint } A = K^t = \begin{bmatrix} -2 & 4 & -2 \\ -5 & 4 & -1 \\ 1 & 0 & 1 \end{bmatrix}^t = \begin{bmatrix} -2 & -5 & 1 \\ 4 & 4 & 0 \\ -2 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} -2 & -5 & 1 \\ 4 & 4 & 0 \\ -2 & -1 & 1 \end{bmatrix} = \begin{bmatrix} -1/2 & -5/4 & 1/4 \\ 1 & 1 & 0 \\ -1/2 & -1/4 & 1/4 \end{bmatrix}$$

SOAL UNTUK DICOBASENDIRI:

Carilah invers dari matriks :

$$K = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & 1 & 2 & 0 \\ 2 & 2 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{bmatrix}$$