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Quantitative Relation between Modulational Instability and Several Well-known Nonlinear Excitations

R:关系研究

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We study on explicit relations between modulational instability and several well-known analytical nonlinear excitations in a nonlinear fiber, such as bright soliton, nonlinear continuous wave, Akhmediev breather, Peregrine rogue wave, and Kuznetsov-Ma breather. We present a quantitative correspondence between them based on the dominant frequency and propagation constant of each perturbation on a continuous wave background. Especially, we find rogue wave comes from modulational instability under the "resonance" perturbation with continuous wave background. These results will deepen our realization on rogue wave excitation and could be helpful for controllable nonlinear waves excitations in nonlinear fiber and other nonlinear systems.

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I. INTRODUCTION

Modulation instability (MI) is a fundamental process observed in many nonlinear dispersive systems, associated with the growth of perturbations on a continuous wave background [1]. In the initial evolution phase of MI, the spectral sidebands associated with the instability experience exponential amplification at the expense of the pump, but the subsequent dynamics are more complex and display cyclic energy exchange between multiple spectral modes [2]. It has found important applications in optical amplification of weak signal, material absorption and loss compensation [3, 4], dispersion management, all-optical switching [5], frequency comb for metrology [6], and so on [7–9]. Recently, several analytical nonlinear excitations such as Akhmediev breather(AB) [10], Peregrine rogue wave(RW)[11], and Kuznetsov-Ma breather(K-M) solutions [12] were excited experimentally in nonlinear fiber [13–15], and even high-order RWs in a water wave tank [16–18]. The results indicated that MI can be used to understand the dynamics of these nonlinear excitations [14]. However, most comparisons between the properties of spontaneous MI and the analytic nonlinear excitations have been qualitative rather than quantitative [19]. For example, we just know that RW should come from MI mechanism, but which modes correspond to RW excitation has not been known precisely. The quantitative relation between them is very meaningful for controllable nonlinear excitations, which is an essential step for their applications.

MI@Use
放大

RW-C@Mode
对应什么模式

RW-Co:可控

In this paper, we present a quantitative correspondence between MI and bright soliton(BS), nonlinear continuous wave(CW), RW, AB, and K-M solutions based on the dominant frequency and propagation constant of each perturbation signal. Moreover, we find RW comes from MI under the "resonance" conditions that the dominant frequency and propagation constant of perturbation signal are both equal to the continuous wave background's. The results would be used to find out the potential ways to realize controllable nonlinear wave excitations.

II. THE RELATIONS BETWEEN MODULATION INSTABILITY AND ANALYTIC NONLINEAR EXCITATIONS

In a Kerr nonlinear fiber, the propagation of optical field (pulse duration $> 5\ ps$) can be described by the following nonlinear Schrödinger equation (NLSE) under slowly varying envelope approximation

$$i\Psi_z + \frac{1}{2}\Psi_{tt} + \sigma|\Psi|^2\Psi = 0. \quad (1)$$

DarkS-C@Stab
稳定性非常好

The equation is written in dimensionless form [20]. When the nonlinear coefficient $\sigma < 0$ (corresponding self-defocusing nonlinear fiber), it admits no MI regime on CW background for which dark soliton has been found on CW background [21]. When the nonlinear coefficient $\sigma > 0$ (corresponding self-focusing nonlinear fiber), it admits MI and modulation stability(MS) regime on the MI gain spectrum continuous. Moreover, different types of nonlinear excitations mainly including BS, AB, K-M, and RW have been derived separately [10–12, 22–24], and even high-order ones [25–27]. It has been known that MI can be seen as the mechanism for RW, AB and K-M dynamics. However, the explicit relations between MI and these nonlinear excitations has not been clarified [19]. We intend to study on MI and analytical nonlinear excitations of NLSE described systems, which could be extended to other nonlinear systems similarly. Therefore, we study the NLSE with $\sigma > 0$. It is convenient to set $\sigma = 1$ without losing any nontrivial properties, since there is a trivial scale transformation for different values under $\sigma > 0$. It is noted that the analytical solutions can

Soliton-C@Inst
存在不稳定区,对应丰富的现象

RW-N@MI-R@NE
MI和NE之间的关系还没有澄清

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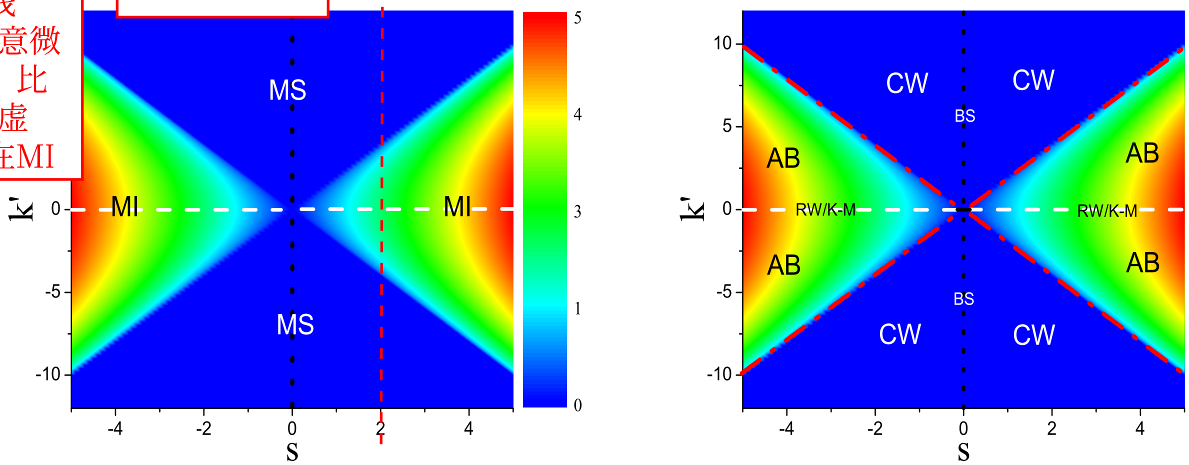
实验上Noise难以控制，找不到对应MI和NE的关系

不同区域动力学过程不一

白线共振最大，黑线s=0，任意微扰稳定，比如s=2红虚线，存在MI



(b)

R@Bilinear则没有这种功能

DT-N@Superposition DT解析解可以视作微扰叠加

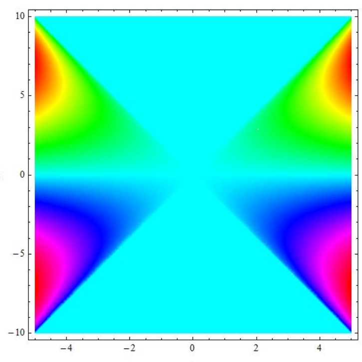


FIG. 1: (Color c … gain spectrum distributed on the continuous wave background amplitude s and perturbat … ” denote modulational instability and modulational stability respectively. The white dash … e black dotted line is another special line for which all perturbations are stable. (b) The … odulational instability gain spectrum plane. “CW”, “AB”, “BS”, “RW”, and “K-M” den … ediev breather, bright soliton, Peregrine rogue wave, and Kuznetsov-Ma breather respect … ves of NLS are all placed clearly on the MI plane. It should be noted that RW and K-M ca … the plane, but their differences can be clarified by propagation constants of their correspo …

be written in the form of nonlinear continuous wave plus a perturbation term, which can be compared with MI analyze conveniently. This provides possibilities to clarify their explicit relation. Nextly, we study the MI property of NLSE firstly to discuss the quantitative relations.

It has been known widely that MI analysis can present us a approximate characterization on the stability of perturbations on CW background [1, 28]. Linear stability analysis can present us a good description on … ation of each spectral mode [6, 29]. We … standard linear instability analyze on CW … $\Psi_0 = s \exp[is^2 z]$ in the system, namely, add small-amplitude Fourier modes on the continuous wave background as $\Psi = \Psi_0(1 + f_+ \exp[i\, k'(t - \Omega z)] + f_-^* \exp[-i\, k'(t - \Omega^* z)])$ (where f_+, f_- are small amplitudes of the Fourier modes) [30]. Substituting them to Eq. (1) and after linearizing the equations, one can get the following dispersion relation $\Omega' = k'\Omega$ where $\Omega = \frac{1}{2}\sqrt{k'^2 - 4s^2}$. The MI form $\Omega' = k'\Omega$ can not demonstrate the MI gain value reasonably for the perturbation frequencies near the line $k' = 0$ [19], since the factor k' can eliminate the MI growth rate near the line. The MI growth effect mainly comes from the factor $Im(\Omega)$. Therefore, we choose the form $Im(\Omega)$ to discuss its MI property, as done in [31]. Similar forms for baseband MI have been chosen to explain the existence of rogue wave reasonably in defocusing-defocusing coupled NLSEs [32]. We can demonstrate the MI gain $Im(\Omega)$ on the perturbation frequency k' vs amplitude of the backgr… in Fig. 1(a). It is seen that there are two dist…

O@NonlinearCW Fourier->零频

W@LSA note,k'Omega是正常的增长率

Th@DispR: 代进去得到BdG方程，由于不含非线性项，所以可以获得解析的色散关系，本文结果表明k'=0处色散关系描述NE分布失效

Th@Omega:可以正确描述NE分布

regimes, namely, MI and MS. We can qualitatively know that perturbations in MI can be amplified exponentially and unstable, and the ones in MS are stable and do no grow up. Especially, there is a maximum growth rate value line ($k' = 0$) in the MI regime, which corresponds to the perturbation frequency equal to the CW's (write the perturbations on CW as $\Psi = \Psi_0 + f_{pert}$ form). Therefore, it can be called a resonant line (white dashed line in Fig. 1(a)). There is another special line $s = 0$ in MS regime (black dotted line in Fig. 1 (a)), on which any perturbations is all stable. The perturbations in different regimes should demonstrate different dynamical behaviors. Recent numerical simulations showed that different dynamical processes can be described well by nonlinear wave solutions [19]. However, the noise include many uncontrollable different spectral modes. It is hard to know which modes correspond to each type nonlinear wave excitations. Nextly, we discuss the quantitative relations between MI and the solutions, based on their spectrum analysis. Furthermore, we derive a generalized solution which can be reduced to all these well-known solutions [10–12, 22, 23], and specify the relations between these solutions.

The seed solution $\Psi_0' = s \exp[ikt + i(s^2 - k^2/2)z]$ with arbitrary frequency k can be transformed to be Ψ_0 by Galilei transformation without losing any nontrivial dynamical characters. Therefore, we present a generalized nonlinear wave solution from the seed solution Ψ_0 as follows,

DT种子解

R@Galilei:简单地伽利略变换，对于KdV可以实现W型孤子

详细有推导，DT有两个重要的东西，一是LaxPair，二是对应方程的DT变换

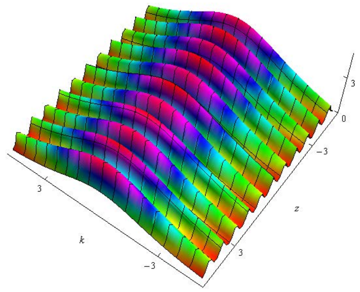
Fourier变换后，频率k`是b与s的函数，b在这里是Lax谱参数

Omega增长率的信息反映在振幅上

$$\Psi = \left[s - 2\frac{(b^2-s^2)\cos(2bz\sqrt{b^2-s^2}) + ib\sqrt{b^2-s^2}\sin(2bz\sqrt{b^2-s^2})}{b\cosh(2t\sqrt{b^2-s^2}) - s\cos(2bz\sqrt{b^2-s^2})} \right] e^{is^2 z}. \quad (2)$$

第一项背景波，第二项微扰项

KM主频为0，周期无穷大，和CW一致，共振放大



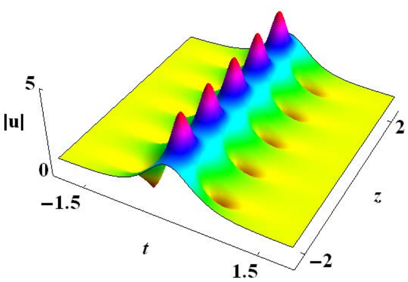
差算法^^
有限积分
解析法

The parameter b determines the init... ized wave's shape, and s is the backgr... localized waves. In the following, we d... the nonlinear wave solution according... ters. It should be noted that the solut... terms. The first term is the CW ba... other term corresponds to perturbatic... above linear instability analysis. Th... be varied to be many different type nonlinear excitations which can be analyzed exactly by Fourier transformation. The frequency spectrum admits a distribution. But there is a "dominant" one among various frequencies, and the dominant one plays essential role in the perturbation evolution(see analyze on AB and K-M). This can be used to specify the relations between MI and analytical solutions.

Nonlinear CW

Obviously, when $b = 0$, the solution will become the nonlinear CW solution. When $|b| > |s|$ and $s \neq 0$, ...o K-M solution [12]. Per-...ion ... K-... ca... ...an... ...e frequency $k' = 0$, which is

V@Func: cosh和cos的差别，cos周期有限，cosh周期无穷大



'dominant frequency ... that the temporal ... ility analyze results, we ...uency of perturbation is ...plified rate is maximum ...-M excitation is on the ... the point $(s = 0, k' = 0)$ plane (see white dashed line in Fig. 1(b)).

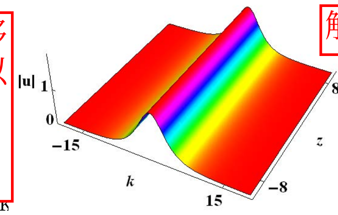
When $s = 0$ and $b \neq 0$, the K-M solution can be reduced to be a generalized bright soliton solution [22] as

$$\Psi = 2b\,\mathrm{sech}(2bt)e^{i2b^2 z}. \quad (3)$$

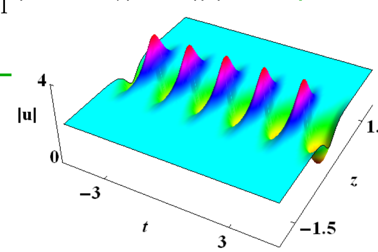
It is kno... point o... is stable... practica... types of... fiber. T... the MS spectrum (see black...

额外微扰可以移动主频，BS可以稳定存在，而KM就消失了

解析法



When $|b| < |s|$, the $\sqrt{b^2 - s^2}$ in Eq. (2) will become $i\sqrt{s^2 - b^2}$, and the solution will ... AB solution [10] as

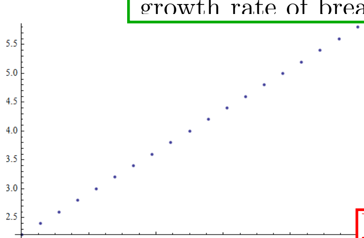


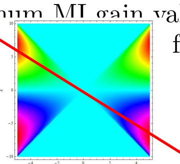
$$\Psi = \left[s - 2\frac{(b^2-s^2)\cosh(2bz\sqrt{s^2-b^2}) - ib\sqrt{s^2-b^2}\sinh(2bz\sqrt{s^2-b^2})}{b\cos(2t\sqrt{s^2-b^2}) - s\cosh(2bz\sqrt{s^2-b^2})} \right]$$

无法fourier，主频通过cos看出~主频随着b改变而改变

... perturbation's frequency can be varied arbitrarily in the certain regime $|b| < |s|$. Spectral analysis of the AB could tell us that the dominant mode is $2\sqrt{s^2 - b^2}$, and there are some other much weaker nonzero frequency-multiplication modes of the perturbation. This can be proved by exact Fourier transformation [33]. The temporal period of AB is indeed determined by the difference between dominant frequency and CW background's. This partly means that our analyze based on dominant frequency is reasonable. Varying the parameter b, the perturbation's dominant mode can be changed, and the growth rate of breather changes correspondingly. This can be used to verify which mode admits maximum MI gain value. It is found that with dominant frequencies are in the regime $|k'| < \sqrt{2}s$, the growth rate of breathers are all larger than the ones on ...uld have "maximum MI gain value" ... form $k' Im(\Omega)$ ... form

AB主频和CW零频有差距，形成时间周期





k`=2sqrt(s2-b2),s/sqrt(2)<b<s,b=s时，k1=0，增益最大

$Im(\Omega)$ does not have this contradictory. When $b = s$, the mode is on the resonance line, the AB excitation will become RW excitation [13]. Namely, the maximum peak and growth rate emerge on the resonance line. Therefore, we call it as resonance excitation. The MI amplification rate will become smaller with decreasing the value of b. When $b = 0$, the mode will become the maximum frequencies $k' = \pm 2s$ for AB excitations, they correspond to the point on the red dash-dotted line in Fig. 1(b). Namely, AB excitation is in the regimes between the white dashed line and red dash-dotted line in the MI continuous. Moreover, the breathing behavior for AB comes from the frequency difference between perturbation signal's dominant one and CW background's. It has been demonstrated that the spectral dynamics of MI can be described exactly by AB solution [34]. Based on the this, we can know that if the weak perturbation modes are in the zero MI band, the perturbation will be stable, and

CW,稳定，无非线性激发，远离共振频率

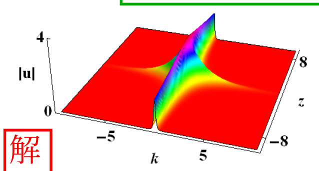
there is no significant excitations on CW background. The state of system can be still seen as CW. Therefore, we denote these regimes as CW phase on MI continuous (see Fig. 1(b)). This point agrees well with resonant vibration in classical mechanics for which large vibrations can not emerge when driving frequency is far from resonant one.

When $|b| = |s|$, the K-M and AB solution can be both reduced to be RW solution [11] as

设d=\sqrt(s2-b2),小量相消

$$\Psi = s\left[1 - \frac{4(2is^2z+1)}{4s^4z^2+4s^2t^2+1}\right]e^{is^2z}. \quad (5)$$

The spectrum analysis on the RW demonstrate that the dominant frequency of RW is on the resonant line except



, $k' = 0$) on the MI plane (white dashed). It is should be noted that K-M is on en, how to distinct RW and K-M? analytical expression of K-M, we can s propagation constant is different from $bz\sqrt{b^2-s^2}$) terms in Eq. (2)). With $b \neq s$, the perturbation signal admits many propagation modes such as $2\ nb\sqrt{b^2-s^2}+s^2$ $(n = \pm1, \pm2, \pm3, \cdots)$, with writing the K-M solution as $\Psi = \Psi_0 + f_{pert}$ form. The dominant one is $\pm 2b\sqrt{b^2-s^2}+s^2$. And the oscillating period is indeed determined by the the dominant one. Therefore, the breathing behavior for K-M comes from the propagation constant difference between perturbation signal's dominant one and CW background's. With $b \to s$, the dominant perturbation propagation constant will be equal to the CW's. Namely, RW also corresponds to the resonant excitation for perturbation propagation constant. This could be used to understand the mathematical process for RW derivation, for which the spectra of Lax-pair should be equal with each other for rational solution. Therefore, the degeneration of Lax-pair spectra corresponds to that the frequency and propagation con- nt of perturbation signal are both equal to the ones CW seed. This result also stands for other coupled S systems [32, 35–39], and NLS with high-order effects –45], since the RW solutions are all derived under the generations of Lax-pair spectra.

解析

KM传播常数

周期差

RW传播常数与RW一致

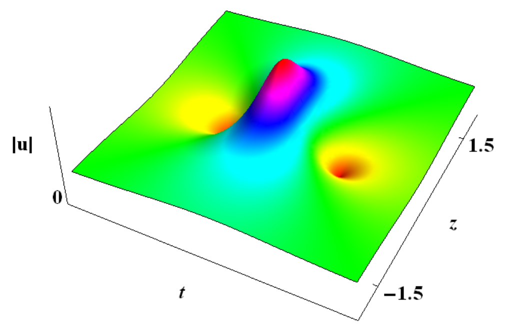
LaxPair谱，当微扰频率和传播常数等价于CW，解是退化的，也因此是共振最大的

III. CONCLUSION AND DISCUSSION

We demonstrate that BS, CW, RW, AB, and K-M ex- tions can be located quantitatively on the MI gain

RW的产生机理

spectrum plane, shown in Fig. 1(b), based on the dominant frequency and propagation constant of each perturbation form. The results provide many possibilities to realize controllable nonlinear excitations, and are helpful to understand further on the numerical simulation results in [19]. The numerical studies suggested that MI yields a series of high-contrast peaks in the evolving wave field seeded from noise on continuous wave back-



a self-focusing elocity disper- n be described solutions [19]. y different fre- trate different tensity modu- ll in nonlinear to realize con- ced in the near alysis just can be used to understand the growth process of perturbations for nonlinear localized waves, but it can not explain the whole dynamics process of them.

Moreover, we find the breathing behavior of AB or K-M comes from the frequency or propagation constant mode difference between the dominant ones of perturbation signal and CW background's. Especially, RW comes from MI under "resonance" perturbations for which both dominant frequency and propagation constant are equal to continuous wave background's. It is well known that the degeneration of Lax-pair spectra can be used to derive rational solution which can be used to describe RW in many cases [25–27, 35–39]. It could be expected that it should be related with some resonance things. But this has not been known explicitly before. Here we find out that the degenerations correspond to the dominant frequency and propagation constant of the perturbation signal are both equal to the ones of CW seed. This will deepen our realization on RW dynamics in many different physical systems, such as nonlinear fiber, Bose-Einstein condensate, water wave tank, plasma, and even financial systems [46].

Acknowledgments

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