Experiment 2.1

Name:

Objectives

To learn to use MATLAB to

1. Generate Polynomials
2. Manipulate Polynomials
3. Generate Transfer Functions
4. Manipulate Transfer Functions
5. Perform Partial-Fraction Expansions

Minimum Required Software Packages

MATLAB and the Control System Toolbox.

Prelab

Problem 1

Calculate the following by hand or with a calculator:

1.a

The roots of $P_1 = s^6 + 7s^5 + 2s^4 + 9s^3 + 10s^2 + 12s + 15$

Answer:

```
syms s;
P1 = [1 7 2 9 10 12 15];
P1S = poly2sym(P1, s);
roots(P1)
```

```
ans = 6×1 complex
  -6.8731 + 0.0000i
   0.7632 + 1.0822i
   0.7632 - 1.0822i
  -1.0000 + 0.0000i
  -0.3266 + 1.0667i
  -0.3266 - 1.0667i
```

1.b

The roots of $P_2 = s^6 + 9s^5 + 8s^4 + 9s^3 + 12s^2 + 15s$

Answer:

```
P2 = [1 9 8 9 12 15 0];
P2S = poly2sym(P2, s);
roots(P2)
```

```
ans = 6×1 complex
   0.0000 + 0.0000i
  -8.1336 + 0.0000i
  -0.9150 + 0.7038i
  -0.9150 - 0.7038i
   0.4818 + 1.0732i
   0.4818 - 1.0732i
```

1.c

$P_3 = P_1 + P_2$

$P_4 = P_1 - P_2$

$P_5 = P_1 P_2$

Answer:

```
P3 = P1 + P2
```

```
P3 = 1×7
     2    16    10    18    22    27    15
```

```
P4 = P1 - P2
```

```
P4 = 1×7
     0    -2    -6     0    -2    -3    15
```

```
P5 = conv(P1, P2)
```

```
P5 = 1×13
     1    16    73    92   182   291   413   459   483   429   360   225     0
```

```
P3S = P1S + P2S
```

P3S = $2\,s^6 + 16\,s^5 + 10\,s^4 + 18\,s^3 + 22\,s^2 + 27\,s + 15$

```
P4S = P1S - P1S
```

P4S = 0

```
P5S = P1S * P2S
```

P5S = $(s^6 + 9\,s^5 + 8\,s^4 + 9\,s^3 + 12\,s^2 + 15\,s)\;(s^6 + 7\,s^5 + 2\,s^4 + 9\,s^3 + 10\,s^2 + 12\,s + 15)$

Problem 2

Calculate by hand or with a calculator the polynomial

$$P_6 = (s+7)(s+8)(s+3)(s+5)(s+9)(s+10)$$

Answer:

```
P6 = conv([1 7], conv([1 8], conv([1 3], conv([1 5], conv([1 9], [1 10])))));
P6S = (s + 7)*(s + 8)*(s + 3)*(s + 5)*(s + 9)*(s + 10);
poly2sym(P6 ,s)
```

ans = $s^6 + 42\,s^5 + 718\,s^4 + 6372\,s^3 + 30817\,s^2 + 76530\,s + 75600$

```
expand(P6S)
```

ans = $s^6 + 42\,s^5 + 718\,s^4 + 6372\,s^3 + 30817\,s^2 + 76530\,s + 75600$

Problem 3

Calculate by hand or with a calculator the following transfer functions:

3.a

$$G_1(s) = \frac{20(s+2)(s+3)(s+6)(s+8)}{s(s+7)(s+9)(s+10)(s+15)}$$

represented as a numerator polynomial divided by a denominator polynomial

Answer:

```
G1S = expand((20*(s+2)*(s+3)*(s+6)*(s+8)))/expand((s*(s+7)*(s+9)*(s+10)*(s+15)))
```

G1S =

$$\frac{20\,s^4 + 380\,s^3 + 2480\,s^2 + 6480\,s + 5760}{s^5 + 41\,s^4 + 613\,s^3 + 3975\,s^2 + 9450\,s}$$

3.b

$$G_2 = \frac{s^4 + 17s^3 + 99s^2 + 223s + 140}{s^5 + 32s^4 + 363s^3 + 2092s^2 + 5052s + 4320}$$

expressed as factors in the numerator divided by the factors in the denominator, similar to the form of $G_1(s)$ *in* Prelab 3a.

Answer:

```
n = poly2sym([1 17 99 22331 140],s);
d = poly2sym([1 32 363 2092 5052 4320],s);
G2S = n/d
```

G2S =

$$\frac{s^4 + 17\, s^3 + 99\, s^2 + 22331\, s + 140}{s^5 + 32\, s^4 + 363\, s^3 + 2092\, s^2 + 5052\, s + 4320}$$

3.c

$G_3(s) = G_1(s) + G_2(s)$

$G_4(s) = G_1(s) - G_2(s)$

$G_5(s) = G_1(s)G_2(s)$

expressed as factors divided by factors and expressed as polynomials divided by polynomials.

<u>Answers:</u>

```
G3S = G1S - G1S
```

G3S = 0

```
G4S = G1S - G2S
```

G4S =

$$\frac{20\, s^4 + 380\, s^3 + 2480\, s^2 + 6480\, s + 5760}{s^5 + 41\, s^4 + 613\, s^3 + 3975\, s^2 + 9450\, s} - \frac{s^4 + 17\, s^3 + 99\, s^2 + 22331\, s + 140}{s^5 + 32\, s^4 + 363\, s^3 + 2092\, s^2 + 5052\, s + 4320}$$

```
G5S = G1S * G2S
```

G5S =

$$\frac{(s^4 + 17\, s^3 + 99\, s^2 + 22331\, s + 140)\;(20\, s^4 + 380\, s^3 + 2480\, s^2 + 6480\, s + 5760)}{(s^5 + 41\, s^4 + 613\, s^3 + 3975\, s^2 + 9450\, s)\;(s^5 + 32\, s^4 + 363\, s^3 + 2092\, s^2 + 5052\, s + 4320)}$$

Problem 4

Calculate by hand or with a calculator the partial-fraction expansion of the following transfer functions:

4.a

$$G_6 = \frac{5(s+2)}{s(s^2+8s+15)}$$

<u>Answer:</u>

```
ns = 5*(s+2);
ds = s*(s^2+8*s+15);
n = sym2poly(ns);
d = sym2poly(ds);

[r,p,k]=residue(n,d)
```

```
r = 3×1
   -1.5000
    0.8333
    0.6667
p = 3×1
    -5
    -3
     0
k =

     []
```

$$G_6 = \frac{-1.5}{s-5} + \frac{0.8333}{s-3} + \frac{0.6667}{s}$$

4.b

$$G_7 = \frac{5(s+2)}{s(s^2+6s+9)}$$

Answer:

```
ns = 5*(s+2);
ds = s*(s^2 + 6*s + 9);
n = sym2poly(ns);
d = sym2poly(ds);

[r, p, k] = residue(n,d)
```

```
r = 3×1
   -1.1111
    1.6667
    1.1111
p = 3×1
    -3
    -3
     0
k =

     []
```

$$G_7 = \frac{-1.1111}{s-3} + \frac{1.6667}{s-3} + \frac{1.1111}{s}$$

4.c

$$G_8 = \frac{5(s+2)}{s(s^2+6s+34)}$$

Answer:

```
ns = 5*(s+2);
ds = s*(s^2 + 6*s + 34);
n = sym2poly(ns);
d = sym2poly(ds);

[r, p, k] = residue(n,d)
```

```
r = 3×1 complex
  -0.1471 - 0.4118i
  -0.1471 + 0.4118i
   0.2941 + 0.0000i
p = 3×1 complex
  -3.0000 + 5.0000i
  -3.0000 - 5.0000i
   0.0000 + 0.0000i
k =

     []
```

$$G_8 = \frac{-0.1471 - j0.4118}{s-3+j5} + \frac{-0.1471 + j0.4118}{s-3-j5} + \frac{0.2941}{s}$$

Lab

Problem 1

Use MATLAB to find P_3, P_4, and P_5 in Prelab 1

```
syms s
% poly example
P1 = [1 7 2 9 10 12 15]
```

```
P1 = 1×7
     1     7     2     9    10    12    15
```

```
P2 = [1 9 8 9 12 15 0]
```

```
P2 = 1×7
     1     9     8     9    12    15     0
```

```
P3 = P1 + P2
```

```
P3 = 1×7
```

```
     2    16    10    18    22    27    15
```

```
P4 = P1 - P2
```

```
P4 = 1×7
     0    -2    -6     0    -2    -3    15
```

```
P5 = conv(P1, P2)
```

```
P5 = 1×13
     1    16    73    92   182   291   413   459   483   429   360   225     0
```

```
% symbolic example
P1S = poly2sym(P1,s)
```

P1S = $s^6+7\,s^5+2\,s^4+9\,s^3+10\,s^2+12\,s+15$

```
P2S = poly2sym(P2,s)
```

P2S = $s^6+9\,s^5+8\,s^4+9\,s^3+12\,s^2+15\,s$

```
P3S = P1S + P2S
```

P3S = $2\,s^6+16\,s^5+10\,s^4+18\,s^3+22\,s^2+27\,s+15$

```
P4S = P1S - P1S
```

P4S = 0

```
P5S = P1S * P2S
```

P5S = $(s^6+9\,s^5+8\,s^4+9\,s^3+12\,s^2+15\,s)\ (s^6+7\,s^5+2\,s^4+9\,s^3+10\,s^2+12\,s+15)$

Problem 2

Use only one MATLAB command to find P_6 in Prelab 2.

```
P6S = (s + 7)*(s + 8)*(s + 3)*(s + 5)*(s + 9)*(s + 10)
```

P6S = $(s+3)\ (s+5)\ (s+7)\ (s+8)\ (s+9)\ (s+10)$

```
expand(P6S)
```

ans = $s^6+42\,s^5+718\,s^4+6372\,s^3+30817\,s^2+76530\,s+75600$

Problem 3

Use only two MATLAB commands to find $G_1(s)$ in Prelab 3a represented as a polynomial divided by a polynomial.

```
G1S = expand((20*(s+2)*(s+3)*(s+6)*(s+8)))/expand((s*(s+7)*(s+9)*(s+10)*(s+15)))
```

```
G1S =
```

$$\frac{20\,s^4+380\,s^3+2480\,s^2+6480\,s+5760}{s^5+41\,s^4+613\,s^3+3975\,s^2+9450\,s}$$

Problem 4

Use only two MATLAB commands to find $G_2(s)$ expressed as factors in the numerator divided by factors in the denominator.

```
n = [1 17 99 22331 140]
```

```
n = 1×5
           1          17          99       22331         140
```

```
d = [1 32 363 2092 5052 4320]
```

```
d = 1×6
           1          32         363        2092        5052        4320
```

```
rn = roots(n)
```

```
rn = 4×1 complex
 -33.7104 + 0.0000i
   8.3583 +24.3425i
   8.3583 -24.3425i
  -0.0063 + 0.0000i
```

```
rd = roots(d)
```

```
rd = 5×1 complex
 -16.7851 + 0.0000i
  -5.5591 + 5.1669i
  -5.5591 - 5.1669i
  -2.0483 + 0.5221i
  -2.0483 - 0.5221i
```

```
ns = poly2sym(n,s)
```

ns = $s^4+17\,s^3+99\,s^2+22331\,s+140$

```
ds = poly2sym(d,s)
```

```
ds = 
```
$$s^5 + 32\,s^4 + 363\,s^3 + 2092\,s^2 + 5052\,s + 4320$$

```
G2S = ns/ds
```

```
G2S =
```
$$\frac{s^4 + 17\,s^3 + 99\,s^2 + 22331\,s + 140}{s^5 + 32\,s^4 + 363\,s^3 + 2092\,s^2 + 5052\,s + 4320}$$

```
[a, b ] = deconv(rn, rd)
```

```
a = 0
b = 4×1 complex
 -33.7104 + 0.0000i
   8.3583 +24.3425i
   8.3583 -24.3425i
  -0.0063 + 0.0000i
```

Problem 5

Using various combinations of $G_1(s)$ and $G_2(s)$, find $G_3(s)$, $G_4(s)$, and $G_5(s)$. Various combinations implies mixing and matching $G_1(s)$ and $G_2(s)$ expressed as factors and polynomials. For example, in finding $G_3(s)$, $G_1(s)$ can be expressed in a factored form and $G_2(s)$ can be expressed in polynomial form. Another combination is $G_1(s)$ and $G_2(s)$ both expressed as polynomials. Still another combination is $G_1(s)$ and $G_2(s)$ both expressed in factored form.

```
% Insert your code here
```

Problem 6

Use MATLAB to evaluate the partial fraction expansions shown in Prelab 4.

```
% Insert your code here
```

Postlab

Problem 1

Discuss your findings for Lab Problem 5. What can you conclude?

Problem 2

Discuss the use of MATLAB to manipulate transfer functions and polynomials. Discuss any shortcomings in using MATLAB to evaluate partial fraction expansions.