Experiment 2.1

Name:

Objectives

To learn to use MATLAB to

1. Generate Polynomials
2. Manipulate Polynomials
3. Generate Transfer Functions
4. Manipulate Transfer Functions
5. Perform Partial-Fraction Expansions

Minimum Required Software Packages

MATLAB and the Control System Toolbox.

Prelab

Problem 1

Calculate the following by hand or with a calculator:

1.a

The roots of $P_1 = s^6 + 7s^5 + 2s^4 + 9s^3 + 10s^2 + 12s + 15$

```
syms s;
P1 = [1 7 2 9 10 12 15];
P1S = poly2sym(P1, s);
roots(P1)
```

```
ans = 6×1 complex
  -6.8731 + 0.0000i
   0.7632 + 1.0822i
   0.7632 - 1.0822i
  -1.0000 + 0.0000i
  -0.3266 + 1.0667i
  -0.3266 - 1.0667i
```

Answer:

$(-6.8731, 0.7632 \pm 1.0822i, -1, -0.3266 \pm 1.0667i)$

1.b

The roots of $P_2 = s^6 + 9s^5 + 8s^4 + 9s^3 + 12s^2 + 15s$

```
P2 = [1 9 8 9 12 15 0];
P2S = poly2sym(P2, s);
roots(P2)
```

```
ans = 6×1 complex
   0.0000 + 0.0000i
  -8.1336 + 0.0000i
  -0.9150 + 0.7038i
  -0.9150 - 0.7038i
   0.4818 + 1.0732i
   0.4818 - 1.0732i
```

Answer:

$(0, -8.1336, -0.9150 \pm 0.7038i, 0.4818 \pm 1.0732i)$

1.c

$P_3 = P_1 + P_2$

$P_4 = P_1 - P_2$

$P_5 = P_1 P_2$

```
P3 = P1 + P2
```

```
P3 = 1×7
     2    16    10    18    22    27    15
```

```
P4 = P1 - P2
```

```
P4 = 1×7
     0    -2    -6     0    -2    -3    15
```

```
P5 = conv(P1, P2)
```

```
P5 = 1×13
     1    16    73    92   182   291   413   459   483   429   360   225     0
```

```
P3S = P1S + P2S
```

P3S = $2\,s^6 + 16\,s^5 + 10\,s^4 + 18\,s^3 + 22\,s^2 + 27\,s + 15$

```
P4S = P1S - P2S
```

P4S = $-2\,s^5 - 6\,s^4 - 2\,s^2 - 3\,s + 15$

```
P5S = expand(P1S * P2S)
```

P5S = $s^{12} + 16\,s^{11} + 73\,s^{10} + 92\,s^9 + 182\,s^8 + 291\,s^7 + 413\,s^6 + 459\,s^5 + 483\,s^4 + 429\,s^3 + 360\,s^2 + 225\,s$

Answer:

$P_3 = 2s^6 + 16s^5 + 10s^4 + 18s^3 + 22s^2 + 27s + 15$

$P_4 = -2s^5 - 6s^4 - 2s^2 - 3s + 15$

$P_5 = s^{12} + 73s^{10} + 92s^9 + 182s^8 + 291s^7 + 413s^6 + 459s^5 + 483s^4 + 429s^3 + 360s^2 + 225s$

Problem 2

Calculate by hand or with a calculator the polynomial

$P_6 = (s + 7)(s + 8)(s + 3)(s + 5)(s + 9)(s + 10)$

```
P6 = conv([1 7], conv([1 8], conv([1 3], conv([1 5], conv([1 9], [1 10])))));
P6S = (s + 7)*(s + 8)*(s + 3)*(s + 5)*(s + 9)*(s + 10);
poly2sym(P6 ,s)
```

ans = $s^6 + 42\,s^5 + 718\,s^4 + 6372\,s^3 + 30817\,s^2 + 76530\,s + 75600$

```
expand(P6S)
```

ans = $s^6 + 42\,s^5 + 718\,s^4 + 6372\,s^3 + 30817\,s^2 + 76530\,s + 75600$

Answer:

$P_6 = s^6 + 42\,s^5 + 718\,s^4 + 6372\,s^3 + 30817\,s^2 + 76530\,s + 75600$

Problem 3

Calculate by hand or with a calculator the following transfer functions:

3.a

$$G_1(s) = \frac{20(s + 2)(s + 3)(s + 6)(s + 8)}{s(s + 7)(s + 9)(s + 10)(s + 15)}$$

*represented as a numerator polynomial divided by a denominator polynomia*l

```
G1S = (20*(s+2)*(s+3)*(s+6)*(s+8))/(s*(s+7)*(s+9)*(s+10)*(s+15));
G1SE = expandSymFraction(G1S, s)
```

G1SE =

$$\frac{20.0\,s^4 + 380.0\,s^3 + 2499.0\,s^2 + 6499.0\,s + 5777.0}{s^5 + 41.0\,s^4 + 613.0\,s^3 + 3999.0\,s^2 + 9466.0\,s}$$

Answer:

$$G_1 = \frac{20\,s^4 + 380\,s^3 + 2480\,s^2 + 6480\,s + 5760}{s^5 + 41\,s^4 + 613\,s^3 + 3975\,s^2 + 9450\,s}$$

3.b

$$G_2 = \frac{s^4 + 17s^3 + 99s^2 + 223s + 140}{s^5 + 32s^4 + 363s^3 + 2092s^2 + 5052s + 4320}$$

expressed as factors in the numerator divided by the factors in the denominator, similar to the form of $G_1(s)$ in Prelab 3a.

```
ns = poly2sym([1 17 99 22331 140], s);
ds = poly2sym([1 32 363 2092 5052 4320], s);
G2S = ns / ds;
G2SF = factorSymFraction(G2S, s)
```

```
G2SF =
```

$$\frac{(s+33.7)\ (s+0.00627)\ (s-8.36-24.3\,\mathrm{i})\ (s-8.36+24.3\,\mathrm{i})}{(s+16.8)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.17\,\mathrm{i})\ (s+5.56+5.17\,\mathrm{i})}$$

Answer:

$$G_2 = \frac{(s+33.7)\ (s+0.00627)\ (s-8.36-24.3\,\mathrm{i})\ (s-8.36+24.3\,\mathrm{i})}{(s+16.8)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.17\,\mathrm{i})\ (s+5.56+5.17\,\mathrm{i})}$$

3.c

$$G_3(s) = G_1(s) + G_2(s)$$

$$G_4(s) = G_1(s) - G_2(s)$$

$$G_5(s) = G_1(s)G_2(s)$$

expressed as factors divided by factors and expressed as polynomials divided by polynomials.

```
G3S = (G1S + G2S);
G4S = (G1S - G2S);
G5S = (G1S * G2S);

G3SE = expandSymFraction(G3S, s)
```

```
G3SE =
```

$$\frac{21.0\,s^9 + 1099.0\,s^8 + 2.33\,10^4\,s^7 + 3.06\,10^5\,s^6 + 3.06\,10^6\,s^5 + 2.4\,10^7\,s^4 + 1.2\,10^8\,s^3 + 2.67\,10^8\,s^2 + 5.84\,10^7\,s + 2.49\,10^7}{s^{10} + 73.0\,s^9 + 2300.0\,s^8 + 4.06\,10^4\,s^7 + 4.5\,10^5\,s^6 + 3.24\,10^6\,s^5 + 1.5\,10^7\,s^4 + 4.25\,10^7\,s^3 + 6.49\,10^7\,s^2 + 4.08\,10^7\,s}$$

```
G3SF = factorSymFraction(G3S, s)
```

G3SF =

$$\frac{21.0\ (s+8.51)\ (s+17.3)\ (s+6.95)\ (s-1.03-9.78\,\mathrm{i})\ (s-1.03+9.78\,\mathrm{i})\ (s+0.0961-0.305\,\mathrm{i})\ (s+0.0961+0.305\,\mathrm{i})}{s\ (s+16.8)\ (s+9.0)\ (s+10.0)\ (s+7.0)\ (s+15.0)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.}$$

```
G4SE = expandSymFraction(G4S, s)
```

G4SE =

$$\frac{19.0\,s^9+962.0\,s^8+2.05\ 10^4\,s^7+2.25\ 10^5\,s^6+9.56\ 10^5\,s^5-4.52\ 10^6\,s^4-6.0\ 10^7\,s^3-1.56\ 10^8\,s^2+5.58\ 10^7\,s+2.49\ 10^7}{s^{10}+73.0\,s^9+2300.0\,s^8+4.06\ 10^4\,s^7+4.5\ 10^5\,s^6+3.24\ 10^6\,s^5+1.5\ 10^7\,s^4+4.25\ 10^7\,s^3+6.49\ 10^7\,s^2+4.08\ 10^7\,s}$$

```
G4SF = factorSymFraction(G4S, s)
```

G4SF =

$$\frac{19.0\ (s+16.4)\ (s+0.267)\ (s-0.538)\ (s-5.89)\ (s+7.06)\ (s+8.66-0.917\,\mathrm{i})\ (s+8.66+0.917\,\mathrm{i})\ (s+7.99-10.6\,\mathrm{i})}{s\ (s+16.8)\ (s+9.0)\ (s+10.0)\ (s+7.0)\ (s+15.0)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.17\,\mathrm{i})\ (s+}$$

```
G5SE = expandSymFraction(G5S, s)
```

G5SE =

$$\frac{20.0\,s^8+720.0\,s^7+1.09\ 10^4\,s^6+5.33\ 10^5\,s^5+8.85\ 10^6\,s^4+5.62\ 10^7\,s^3+1.46\ 10^8\,s^2+1.3\ 10^8\,s+8.06\ 10^5}{s^{10}+73.0\,s^9+2300.0\,s^8+4.06\ 10^4\,s^7+4.5\ 10^5\,s^6+3.24\ 10^6\,s^5+1.5\ 10^7\,s^4+4.25\ 10^7\,s^3+6.49\ 10^7\,s^2+4.08\ 10^7\,s}$$

```
G5SF = factorSymFraction(G5S, s)
```

G5SF =

$$\frac{20.0\ (s+2.0)\ (s+8.0)\ (s+33.7)\ (s+3.0)\ (s+6.0)\ (s+0.00627)\ (s-8.36-24.3\,\mathrm{i})\ (s-8.36+24.3\,\mathrm{i})}{s\ (s+16.8)\ (s+9.0)\ (s+10.0)\ (s+7.0)\ (s+15.0)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.17\,\mathrm{i})\ (s+5.5}$$

Answers:

$$G_3=\frac{21.0\ (s+8.51)\ (s+17.3)\ (s+6.95)\ (s-1.03-9.78\,\mathrm{i})\ (s-1.03+9.78\,\mathrm{i})\ (s+0.0961-0.305\,\mathrm{i})\ (s+0.0961+0.305\,\mathrm{i})}{s\ (s+16.8)\ (s+9.0)\ (s+10.0)\ (s+7.0)\ (s+15.0)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.}$$
$$=\frac{21.0\,s^9+1099.0\,s^8+2.33\ 10^4\,s^7+3.06\ 10^5\,s^6+3.06\ 10^6\,s^5+2.4\ 10^7\,s^4+1.2\ 10^8\,s^3+2.67\ 10^8\,s^2+5.84\ 10^7\,s+2.49\ 10^7}{s^{10}+73.0\,s^9+2300.0\,s^8+4.06\ 10^4\,s^7+4.5\ 10^5\,s^6+3.24\ 10^6\,s^5+1.5\ 10^7\,s^4+4.25\ 10^7\,s^3+6.49\ 10^7\,s^2+4.08\ 10^7\,s}$$

$$G_4=\frac{19.0\ (s+16.4)\ (s+0.267)\ (s-0.538)\ (s-5.89)\ (s+7.06)\ (s+8.66-0.917\,\mathrm{i})\ (s+8.66+0.917\,\mathrm{i})\ (s+7.99-10.6\,\mathrm{i})}{s\ (s+16.8)\ (s+9.0)\ (s+10.0)\ (s+7.0)\ (s+15.0)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.17\,\mathrm{i})\ (s+}$$
$$=\frac{19.0\,s^9+962.0\,s^8+2.05\ 10^4\,s^7+2.25\ 10^5\,s^6+9.56\ 10^5\,s^5-4.52\ 10^6\,s^4-6.0\ 10^7\,s^3-1.56\ 10^8\,s^2+5.58\ 10^7\,s+2.49\ 10^7}{s^{10}+73.0\,s^9+2300.0\,s^8+4.06\ 10^4\,s^7+4.5\ 10^5\,s^6+3.24\ 10^6\,s^5+1.5\ 10^7\,s^4+4.25\ 10^7\,s^3+6.49\ 10^7\,s^2+4.08\ 10^7\,s}$$

$$G_5=\frac{20.0\ (s+2.0)\ (s+8.0)\ (s+33.7)\ (s+3.0)\ (s+6.0)\ (s+0.00627)\ (s-8.36-24.3\,\mathrm{i})\ (s-8.36+24.3\,\mathrm{i})}{s\ (s+16.8)\ (s+9.0)\ (s+10.0)\ (s+7.0)\ (s+15.0)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.17\,\mathrm{i})\ (s+5.56}$$
$$=\frac{20.0\,s^8+720.0\,s^7+1.09\ 10^4\,s^6+5.33\ 10^5\,s^5+8.85\ 10^6\,s^4+5.62\ 10^7\,s^3+1.46\ 10^8\,s^2+1.3\ 10^8\,s+8.06\ 10^5}{s^{10}+73.0\,s^9+2300.0\,s^8+4.06\ 10^4\,s^7+4.5\ 10^5\,s^6+3.24\ 10^6\,s^5+1.5\ 10^7\,s^4+4.25\ 10^7\,s^3+6.49\ 10^7\,s^2+4.08\ 10^7\,s}$$

Problem 4

Calculate by hand or with a calculator the partial-fraction expansion of the following transfer functions:

4.a

$$G_6 = \frac{5(s+2)}{s(s^2+8s+15)}$$

```
G6S = 5*(s+2) / s*(s^2+8*s+15);
[ns, ds] = numden(G6S)
```

ns = $5\ (s+2)\ (s^2+8\,s+15)$

ds = s

```
n = sym2poly(ns);
d = sym2poly(ds);

[r,p,k]=residue(n,d)
```

```
r = 150
p = 0
k = 1×3
     5    50   155
```

Answer:

$$G_6 = \frac{-1.5}{s-5} + \frac{0.8333}{s-3} + \frac{0.6667}{s}$$

4.b

$$G_7 = \frac{5(s+2)}{s(s^2+6s+9)}$$

```
ns = 5*(s+2);
ds = s*(s^2 + 6*s + 9);
n = sym2poly(ns);
d = sym2poly(ds);

[r, p, k] = residue(n,d)
```

```
r = 3×1
   -1.1111
    1.6667
    1.1111
p = 3×1
    -3
    -3
     0
k =
```

```
        []
```

Answer:

$$G_7 = \frac{-1.1111}{s-3} + \frac{1.6667}{s-3} + \frac{1.1111}{s}$$

4.c

$$G_8 = \frac{5(s+2)}{s(s^2+6s+34)}$$

```
ns = 5*(s+2);
ds = s*(s^2 + 6*s + 34);
n = sym2poly(ns);
d = sym2poly(ds);

[r, p, k] = residue(n,d)
```

```
r = 3×1 complex
  -0.1471 - 0.4118i
  -0.1471 + 0.4118i
   0.2941 + 0.0000i
p = 3×1 complex
  -3.0000 + 5.0000i
  -3.0000 - 5.0000i
   0.0000 + 0.0000i
k =

        []
```

Answer:

$$G_8 = \frac{-0.1471 \pm 0.4118i}{s-3 \pm 5i} + \frac{0.2941}{s}$$

Lab

Problem 1

Use MATLAB to find P_3, P_4, and P_5 in Prelab 1

```
syms s
% poly example
P1 = [1 7 2 9 10 12 15]
```

```
P1 = 1×7
```

```
     1     7     2     9    10    12    15
```

```
P2 = [1 9 8 9 12 15 0]
```

```
P2 = 1×7
     1     9     8     9    12    15     0
```

```
P3 = P1 + P2
```

```
P3 = 1×7
     2    16    10    18    22    27    15
```

```
P4 = P1 - P2
```

```
P4 = 1×7
     0    -2    -6     0    -2    -3    15
```

```
P5 = conv(P1, P2)
```

```
P5 = 1×13
     1    16    73    92   182   291   413   459   483   429   360   225     0
```

```
% symbolic example
P1S = poly2sym(P1,s)
```

P1S = $s^6 + 7\,s^5 + 2\,s^4 + 9\,s^3 + 10\,s^2 + 12\,s + 15$

```
P2S = poly2sym(P2,s)
```

P2S = $s^6 + 9\,s^5 + 8\,s^4 + 9\,s^3 + 12\,s^2 + 15\,s$

```
P3S = P1S + P2S
```

P3S = $2\,s^6 + 16\,s^5 + 10\,s^4 + 18\,s^3 + 22\,s^2 + 27\,s + 15$

```
P4S = P1S - P2S
```

P4S = $-2\,s^5 - 6\,s^4 - 2\,s^2 - 3\,s + 15$

```
P5S = expand(P1S * P2S)
```

P5S = $s^{12} + 16\,s^{11} + 73\,s^{10} + 92\,s^9 + 182\,s^8 + 291\,s^7 + 413\,s^6 + 459\,s^5 + 483\,s^4 + 429\,s^3 + 360\,s^2 + 225\,s$

$\mathrm{P}_3(s) = 2\,s^6 + 16\,s^5 + 10\,s^4 + 18\,s^3 + 22\,s^2 + 27\,s + 15$

$P_4(s) = -2\,s^5 - 6\,s^4 - 2\,s^2 - 3\,s + 15$

$P_5(s) = s^{12} + 16\,s^{11} + 73\,s^{10} + 92\,s^9 + 182\,s^8 + 291\,s^7 + 413\,s^6 + 459\,s^5 + 483\,s^4 + 429\,s^3 + 360\,s^2 + 225\,s$

Problem 2

Use only one MATLAB command to find P_6 in Prelab 2.

```
P6S = (s + 7)*(s + 8)*(s + 3)*(s + 5)*(s + 9)*(s + 10);
P6Se = expand(P6S)
```

P6Se = $s^6 + 42\,s^5 + 718\,s^4 + 6372\,s^3 + 30817\,s^2 + 76530\,s + 75600$

$$P_6(s) = s^6 + 42\,s^5 + 718\,s^4 + 6372\,s^3 + 30817\,s^2 + 76530\,s + 75600$$

Problem 3

Use only two MATLAB commands to find $G_1(s)$ in Prelab 3a represented as a polynomial divided by a polynomial.

```
G1SE = expand((20*(s+2)*(s+3)*(s+6)*(s+8)))/expand((s*(s+7)*(s+9)*(s+10)*(s+15)))
```

G1SE =

$$\frac{20\,s^4 + 380\,s^3 + 2480\,s^2 + 6480\,s + 5760}{s^5 + 41\,s^4 + 613\,s^3 + 3975\,s^2 + 9450\,s}$$

$$G_1(s) = \frac{20\,s^4 + 380\,s^3 + 2480\,s^2 + 6480\,s + 5760}{s^5 + 41\,s^4 + 613\,s^3 + 3975\,s^2 + 9450\,s}$$

Problem 4

Use only two MATLAB commands to find $G_2(s)$ expressed as factors in the numerator divided by factors in the denominator.

```
[n, d] = numden(G2S);
nf = prod(factor(n, 'FactorMode', 'complex'));
df= prod(factor(d, 'FactorMode', 'complex'));
G2SF = vpa(nf / df, 3)
```

G2SF =

$$\frac{(s + 33.7)\ (s + 0.00627)\ (s - 8.36 - 24.3\,\mathrm{i})\ (s - 8.36 + 24.3\,\mathrm{i})}{(s + 16.8)\ (s + 2.05 - 0.522\,\mathrm{i})\ (s + 2.05 + 0.522\,\mathrm{i})\ (s + 5.56 - 5.17\,\mathrm{i})\ (s + 5.56 + 5.17\,\mathrm{i})}$$

$$G_2(s) = \frac{(s + 33.7)\ (s + 0.00627)\ (s - 8.36 - 24.3\,\mathrm{i})\ (s - 8.36 + 24.3\,\mathrm{i})}{(s + 16.8)\ (s + 2.05 - 0.522\,\mathrm{i})\ (s + 2.05 + 0.522\,\mathrm{i})\ (s + 5.56 - 5.17\,\mathrm{i})\ (s + 5.56 + 5.17\,\mathrm{i})}$$

Problem 5

Using various combinations of $G_1(s)$ *and* $G_2(s)$*, find* $G_3(s)$, $G_4(s)$*, and* $G_5(s)$*. Various combinations implies mixing and matching* $G_1(s)$ *and* $G_2(s)$ *expressed as factors and polynomials. For example, in finding* $G_3(s)$, $G_1(s)$ *can be expressed in a factored form and* $G_2(s)$ *can be expressed in polynomial form. Another combination is* $G_1(s)$ *and* $G_2(s)$ *both expressed as polynomials. Still another combination is* $G_1(s)$ *and* $G_2(s)$ *both expressed in factored form.*

```
[G3SN, G3SD] = numden(G3S);
G3SEF = vpa(expand(G3SN) / prod(factor(G3SD, s, 'FactorMode','complex')), 3)
```

G3SEF =

$$\frac{21.0\,s^9 + 1099.0\,s^8 + 2.33\,10^4\,s^7 + 3.06\,10^5\,s^6 + 3.06\,10^6\,s^5 + 2.4\,10^7\,s^4 + 1.2\,10^8\,s^3 + 2.67\,10^8\,s^2 + 5.84\,10^7\,s + 2.49\,10}{s\,(s+16.8)\,(s+9.0)\,(s+10.0)\,(s+7.0)\,(s+15.0)\,(s+2.05-0.522\,\mathrm{i})\,(s+2.05+0.522\,\mathrm{i})\,(s+5.56-5.17\,\mathrm{i})\,(s+5.56-}$$

```
G3SFE = vpa(prod(factor(G3SN, s, 'FactorMode','complex')) / expand(G3SD), 3)
```

G3SFE =

$$\frac{21.0\,(s+8.51)\,(s+17.3)\,(s+6.95)\,(s-1.03-9.78\,\mathrm{i})\,(s-1.03+9.78\,\mathrm{i})\,(s+0.0961-0.305\,\mathrm{i})\,(s+0.0961+0.305\,\mathrm{i})\,(s}{s^{10} + 73.0\,s^9 + 2300.0\,s^8 + 4.06\,10^4\,s^7 + 4.5\,10^5\,s^6 + 3.24\,10^6\,s^5 + 1.5\,10^7\,s^4 + 4.25\,10^7\,s^3 + 6.49\,10^7\,s^2}$$

```
[G4SN, G4SD] = numden(G4S);
G4SEF = vpa(expand(G4SN) / prod(factor(G4SD, s, 'FactorMode','complex')), 3)
```

G4SEF =

$$\frac{19.0\,s^9 + 962.0\,s^8 + 2.05\,10^4\,s^7 + 2.25\,10^5\,s^6 + 9.56\,10^5\,s^5 - 4.52\,10^6\,s^4 - 6.0\,10^7\,s^3 - 1.56\,10^8\,s^2 + 5.58\,10^7\,s + 2.49\,10}{s\,(s+16.8)\,(s+9.0)\,(s+10.0)\,(s+7.0)\,(s+15.0)\,(s+2.05-0.522\,\mathrm{i})\,(s+2.05+0.522\,\mathrm{i})\,(s+5.56-5.17\,\mathrm{i})\,(s+5.56-}$$

```
G4SFE = vpa(prod(factor(G4SN, s, 'FactorMode','complex')) / expand(G4SD), 3)
```

G4SFE =

$$\frac{19.0\,(s+16.4)\,(s+0.267)\,(s-0.538)\,(s-5.89)\,(s+7.06)\,(s+8.66-0.917\,\mathrm{i})\,(s+8.66+0.917\,\mathrm{i})\,(s+7.99-10.6\,\mathrm{i})\,(s}{s^{10} + 73.0\,s^9 + 2300.0\,s^8 + 4.06\,10^4\,s^7 + 4.5\,10^5\,s^6 + 3.24\,10^6\,s^5 + 1.5\,10^7\,s^4 + 4.25\,10^7\,s^3 + 6.49\,10^7\,s^2 + 4.08\,10}$$

```
[G5SN, G5SD] = numden(G5S);
G5SEF = vpa(expand(G5SN) / prod(factor(G5SD, s, 'FactorMode','complex')), 3)
```

G5SEF =

$$\frac{20.0\,s^8 + 720.0\,s^7 + 1.09\,10^4\,s^6 + 5.33\,10^5\,s^5 + 8.85\,10^6\,s^4 + 5.62\,10^7\,s^3 + 1.46\,10^8\,s^2 + 1.3\,10^8\,s + 8.06\,10^5}{s\,(s+16.8)\,(s+9.0)\,(s+10.0)\,(s+7.0)\,(s+15.0)\,(s+2.05-0.522\,\mathrm{i})\,(s+2.05+0.522\,\mathrm{i})\,(s+5.56-5.17\,\mathrm{i})\,(s+5.56-}$$

```
G5SFE = vpa(prod(factor(G5SN, s, 'FactorMode','complex')) / expand(G5SD), 3)
```

G5SFE =

$$\frac{20.0\ (s+2.0)\ (s+8.0)\ (s+33.7)\ (s+3.0)\ (s+6.0)\ (s+0.00627)\ (s-8.36-24.3\,\mathrm{i})\ (s-8.36+24.3\,\mathrm{i})}{s^{10}+73.0\,s^9+2300.0\,s^8+4.06\,10^4\,s^7+4.5\,10^5\,s^6+3.24\,10^6\,s^5+1.5\,10^7\,s^4+4.25\,10^7\,s^3+6.49\,10^7\,s^2+4.08\,10^7\,s}$$

$$G_3(s)=\frac{21.0\,s^9+1099.0\,s^8+2.33\,10^4\,s^7+3.06\,10^5\,s^6+3.06\,10^6\,s^5+2.4\,10^7\,s^4+1.2\,10^8\,s^3+2.67\,10^8\,s^2+5.84\,10^7\,s+2.49}{s\ (s+16.8)\ (s+9.0)\ (s+10.0)\ (s+7.0)\ (s+15.0)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.17\,\mathrm{i})\ (s+5.5}$$

$$G_4(s)=\frac{19.0\,s^9+962.0\,s^8+2.05\,10^4\,s^7+2.25\,10^5\,s^6+9.56\,10^5\,s^5-4.52\,10^6\,s^4-6.0\,10^7\,s^3-1.56\,10^8\,s^2+5.58\,10^7\,s+2.49}{s\ (s+16.8)\ (s+9.0)\ (s+10.0)\ (s+7.0)\ (s+15.0)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.17\,\mathrm{i})\ (s+5.5}$$

$$G_5(s)=\frac{20.0\,s^8+720.0\,s^7+1.09\,10^4\,s^6+5.33\,10^5\,s^5+8.85\,10^6\,s^4+5.62\,10^7\,s^3+1.46\,10^8\,s^2+1.3\,10^8\,s+8.06\,10^5}{s\ (s+16.8)\ (s+9.0)\ (s+10.0)\ (s+7.0)\ (s+15.0)\ (s+2.05-0.522\,\mathrm{i})\ (s+2.05+0.522\,\mathrm{i})\ (s+5.56-5.17\,\mathrm{i})\ (s+5.5}$$

Problem 6

Use MATLAB to evaluate the partial fraction expansions shown in Prelab 4.

```
G6S = 5*(s+2) / s*(s^2+8*s+15);
[ns, ds] = numden(G6S)
```

ns = $5\ (s+2)\ (s^2+8\,s+15)$

ds = s

```
n = sym2poly(ns);
d = sym2poly(ds);

[r,p,k]=residue(n,d)
```

```
r = 150
p = 0
k = 1×3
     5    50   155
```

Answer:

$$G_6=\frac{-1.5}{s-5}+\frac{0.8333}{s-3}+\frac{0.6667}{s}$$

```
function factored = factorSymFraction(sym, var)
    [n, d] = numden(sym);
    nf = prod(factor(n, var, 'FactorMode', 'complex'));
    df = prod(factor(d, var, 'FactorMode', 'complex'));
    factored = vpa(nf/df, 3);
end

function expanded = expandSymFraction(sym, var)
    [n, d] = numden(sym);
    ne = expand(n);
    de = expand(d);
```

```
    expanded = vpa(ne/de, 3);

end
```

Postlab

Problem 1

Discuss your findings for Lab Problem 5. What can you conclude?

Getting MATLAB to output polynomials in a factored numerator and denominator takes some manipulation. I eneded up making my own functions to factor the numerator and denominator of the transfer functions to make the formatting a little easier, but there isn't a simple solution for formatting polynomial fractions.

Problem 2

Discuss the use of MATLAB to manipulate transfer functions and polynomials. Discuss any shortcomings in using MATLAB to evaluate partial fraction expansions.

Additional functions are needed to manipulate transfer function and polynomials to print them out in the format desired. For instance, you have to factor the numerator and denominator seperatly, then combine them inorder to get a factored form of a transfer function. Also, partial fraction expansion is easily calculated using the `residue` function, but getting the results from the vector form into the polynomial form is an additional step.