Experiment 3.1

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Objectives

To learn to use MATLAB to

1. Generate an LTI State-Space representation of a system
2. Convert an LTI State-Space representation of a system to an LTI transfer function.

Minimum Required Software Packages

MATLAB and the Control System Toolbox.

Prelab

Problem 1

Derive the state-space representation of the translational mechanical system shown in Skill-Assesment Exercise 3.2 and provided below if you have not already done so. Consider the output ro be $x_3(t)$.

Skill-Assessment Exercise 3.2

PROBLEM: Represent the translational mechanical system shown in Figure 3.9 in state space, where $x_3(t)$ is the output.

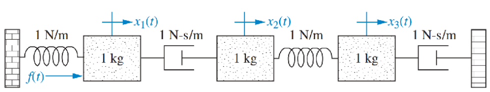
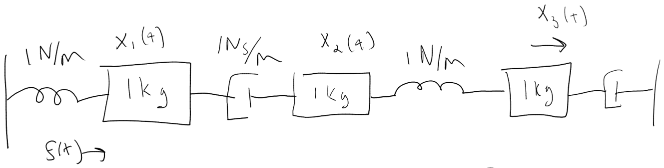


FIGURE 3.9 Translational mechanical system for Skill-Assessment Exercise 3.2

<u>Answer:</u>



$$(s^2+s+1)X_1(s) - sX_2(s) = F(s)$$
$$-sX_1(s) + (s^2+s+1)X_2(s) - X_3(s) = 0$$
$$-X_2(s) + (s^2+s+1)X_3(s) = 0$$

$$z_1 = x_1 \qquad z_3 = x_2 \qquad z_5 = x_3$$
$$z_2 = \dot{x}_1 \qquad z_4 = \dot{x}_2 \qquad z_6 = \dot{x}_3$$

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = \ddot{x}_1 = -\dot{x}_1 - x_1 + \dot{x}_2 + f = -z_2 - z_1 + z_4 + f$$
$$\dot{z}_3 = \dot{x}_2 = z_4$$
$$\dot{z}_4 = \ddot{x}_2 = z_2 - z_4 - z_3 + z_5$$
$$\dot{z}_5 = z_6$$
$$\dot{z}_6 = -z_6 - z_5 + z_3$$

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix} z + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} f(t) \qquad x_3(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} z$$

Problem 2

Derive the transfer function, $\frac{X_3(s)}{F(s)}$, from the equations of motion for the translational mechanical system shown above from Skill-Assessment Exercise 3.2.

Answer:

Lab

Problem 1

Use MATLAB to generate the LTI state-space representation derived in Prelab 1.

```
A=[0 1 0 0 0 0;-1 -1 0 1 0 0;0 0 0 1 0 0;0 1 -1 -1 0;0 0 0 0 0 1;0 0 1 0 -1 -1]
```

```
A = 6×6
     0     1     0     0     0     0
    -1    -1     0     1     0     0
     0     0     0     1     0     0
     0     1    -1    -1     1     0
     0     0     0     0     0     1
     0     0     1     0    -1    -1
```

```
B=[0;1;0;0;0;0]
```

```
B = 6×1
     0
     1
     0
     0
     0
     0
```

```
C=[0 0 0 0 1 0]
```

```
C = 1×6
     0     0     0     0     1     0
```

```
D=0
```

```
D = 0
```

```
sys=ss(A,B,C,D)
```

```
sys =

  A =
       x1  x2  x3  x4  x5  x6
   x1   0   1   0   0   0   0
   x2  -1  -1   0   1   0   0
   x3   0   0   0   1   0   0
   x4   0   1  -1  -1   1   0
   x5   0   0   0   0   0   1
   x6   0   0   1   0  -1  -1

  B =
       u1
   x1   0
   x2   1
   x3   0
   x4   0
   x5   0
   x6   0

  C =
       x1  x2  x3  x4  x5  x6
   y1   0   0   0   0   1   0

  D =
       u1
   y1   0
```

```
Continuous-time state-space model.
ans =

                      s + 1.11e-16
  -----------------------------------------------------
  s^6 + 3 s^5 + 5 s^4 + 6 s^3 + 4 s^2 + 2 s + 6.177e-16

Continuous-time transfer function.
```

Problem 2

Use MATLAB to convert the LTI state-space representation found in Lab 1 to the LTI transfer function found in Prelab 2.

```
tf(sys)
```

```
ans =

                      s + 1.11e-16
  -----------------------------------------------------
  s^6 + 3 s^5 + 5 s^4 + 6 s^3 + 4 s^2 + 2 s + 6.177e-16

Continuous-time transfer function.
```

Postlab

Problem 1

Compare your transfer functions as found from Prelab 2 and Lab 2.

The transfer functions matched.

Problem 2

Discuss the use of MATLAB to create LTI state-space representations and the use of MATLAB to convert these representations to transfer functions.

Creating LTI state-space representation in MATLAB is really easy using the `ss` function and then converting state-space's to transfer functions is as easy as passing the `ss` to `tf`.