Lab 5: Transmission Line Measurements

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***Abstract*—In this lab, the transmission line properties, such as γ, Z_0, Z_L and Z_{in}; of a coaxial cable were measured. The effect of six different load impedances: $47\,\Omega$, $50\,\Omega$,$100\,\Omega$, $2.2\,\text{pF}$, 0, and ∞, on these values was measured using a network analyzer. Using fundamental transmission line equations, the theoretical V_0^- reflection of a generated pulse was accurately predicted. Additionally theoretical values for Z_0 and γ for the transmission line were determined used to compare the expected load impedances from the network analyzer's data to the actual loads.**

I. Introduction and Objectives

The purpose of this lab is to explore the properties of transmission lines and observe the effect that a load has on signal reflections. This lab is broken into three parts:

1) Observing Wave Reflection A single, short pulse was generated and sent down a $30\,\text{m}$ long coaxial cable and the voltage at the generator end of the cable was measured using an oscilloscope. The pulse's width and the cable's length were chosen such that the reflected wave would clearly appear at the generator end as a separate pulse a 250 ns after the signal is sent. The reflected wave of the same transmission line with the following terminations were observed:

TABLE I
Terminations

Termination	Z_L
$47\,\Omega$ Resistor	47
$50\,\Omega$ Resistor	50
$100\,\Omega$ Resistor	100
$2.2\,\text{nF}$ Capacitor	$\frac{1}{j\omega 2.2\times 10^{-9}}$
Short Circuit	0
Open Circuit	∞

2) Measuring the Input Impedance A network analyzer was used to measure the input impedance of the transmission line at 1.0 MHz, 10 MHz, 250 MHz, and 500 Mz for each of the six terminations.
3) Deriving Experimental Z_L Values Based on the measured values of γ and Z_0, an experimental Z_L is derived for all 6 terminations and at all four frequencies and compared to the known values.

II. Background and Relevant Theory

1) Voltage Seen at Generator

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \tag{1}$$

2) Complex Propagation Constant

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{2}$$

3) Complex Propagation Constant: Lossless Line

$$\begin{aligned} &R = G = 0 \\ &\gamma = j\omega\sqrt{LC} = \alpha + j\beta \\ &\alpha = 0 \\ &\gamma = j\beta = j\omega\sqrt{LC} \end{aligned} \tag{3}$$

4) Reflection Coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{4}$$

5) Input Impedance

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \tag{5}$$

6) Characteristic Impedance

$$Z_0 = \sqrt{Z_i^{OC} Z_i^{SC}} \tag{6}$$

III. Results and Discussion

A. Observing Wave Reflection

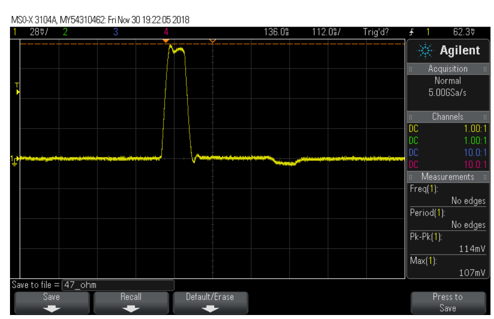

Fig. 1. Voltage Seen at Generator: $47\,\Omega$ Resistor

1) $47\,\Omega$ Resistor: The reflected voltage wave is expected to be an inversion of the input, in the case of a $100\,\text{mV}$ pulse, the reflected signal should be a $-3.1\,\text{mV}$ pulse (Equation 7). The theoretical value accurately predicted the observed reflected wave, the observed reflection voltage was a $-5\,\text{mV}$ signal (Figure 1).

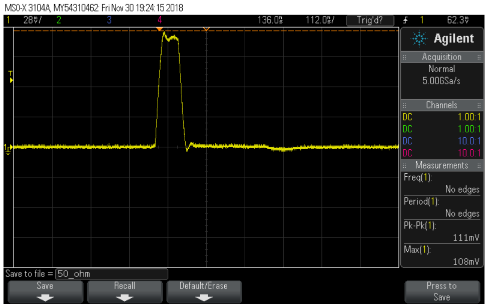

Fig. 2. Voltage Seen at Generator: $50\,\Omega$ Resistor

2) $50\,\Omega$ Resistor: The reflected voltage wave for the $50\,\Omega$ load is expected to be zero regardless of the voltage input (Equation 8). This prediction was accurate, there is almost no reflected wave (Figure 2).

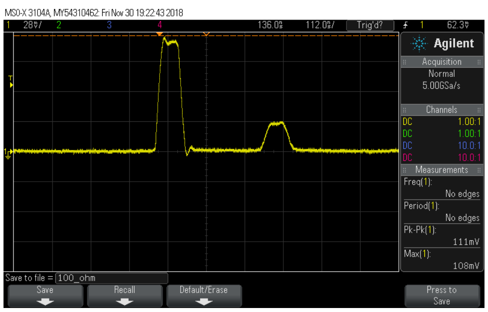

Fig. 3. Voltage Seen at Generator: $100\,\Omega$ Resistor

3) $100\,\Omega$ Resistor: The reflected voltage wave for the $100\,\Omega$ load is expected to be a third of the input voltage (Equation 9). In this case, the same square pulse but with an amplitude of $33.3\,\mathrm{mV}$.

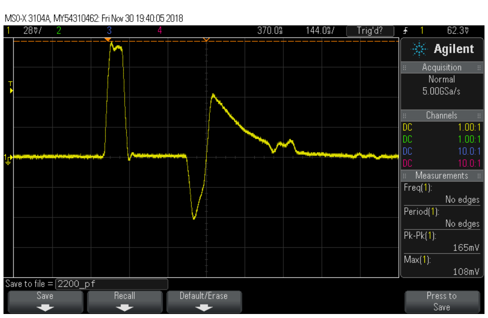

Fig. 4. Voltage Seen at Generator: $22.2\,\mathrm{pF}$ Capacitor

4) $2.2\,\mathrm{pF}$ Capacitor: The reflected voltage wave for the $2.2\,\mathrm{pF}$ load is expected to have a complex V_0^-, meaning the reflected voltage will depend on the frequency of the input voltage. Since a square pulse is used, the reflected wave is not obvious since all frequencies are theoretically contained in the pulse. The reflection observed fits this characteristic since it has a non-square shape and it is has both an inverting and non-inverting peak (Figure 4).

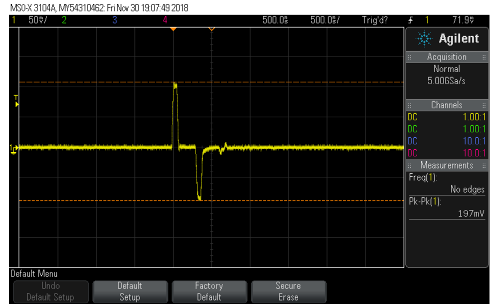

Fig. 5. Voltage Seen at Generator: Short Circuit

5) Short Circuit: The reflected voltage wave of the short circuit is expected to be an inversion of the input voltage, a $-100\,\mathrm{mV}$ pulse (Equation 11). This was observed as a $-90\,\mathrm{mV}$ pulse (Figure 5).

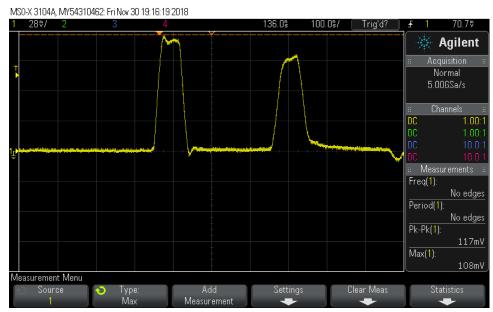

Fig. 6. Voltage Seen at Generator: Open Circuit

6) Open Circuit: The reflected voltage wave of the open circuit is expected to be equal to the input voltage, a $100\,\mathrm{mV}$ pulse (Equation 12). This was observed as a $90\,\mathrm{mV}$ pulse (Figure 6).

B. Measuring the Input Impedance

The input impedance for all terminators were measured using a network analyzer in S_{11} configuration (Appendix B). The Smith chart readings of the input impedances were used to derive experimental values for Z_0 and γ by using the input impedances for an open and short circuit (Appendix C). The experimental values were determined to be $Z_0 = 62.12 + j3.30$ and $\gamma = 4.86 \times 10^{-3} + j2.713 \times 10^{-1}$ (Appendix C).

C. Deriving Experimental Z_L Values

Using the derived Z_0 and γ values, Z_L for each load and at each frequency was able to be determined (Appendix C). The theoretical load impedances were not accurate at predicting the actual load impedances.

IV. Conclusion

This lab was successful in observing the characteristics of a transmission line. The predicted reflection waves for the generated pulse matched the observed waveform, the intrinsic values of the transmission line were able to be calculated accurately while the theoretical load impedances differed from the actual values. This lab can be improved in the future by using a calibrated network analyzer in order to get more accurate theoretical load impedances.

References

[1] F. Farahmand, "Introduction to Transmission Lines, Part II", University of California, Davis, 2018.

[2] William H. Hayt, Jr. and John A. Buck, *Engineering Electromagnetics*, 8th ed., McGraw Hill (2012).

Appendix A
V_0^- Predictions

Using Equation 4 and the fact that the characteristic impedance of the coaxial cable is $50\,\Omega$, the following equation can be derived for the theoretical reflected wave V_0^-:

$$V_0^- = V_0^+ \frac{Z_L - Z_0}{Z_0 + Z_L}$$

A. $47\,\Omega$ Resistor

$$\begin{aligned} V_0^- &= V_0^+ \tfrac{47-50}{50+47} \\ V_0^- &= V_0^+ \tfrac{-3}{97} \\ V_0^- &= -0.031 V_0^+ \end{aligned} \tag{7}$$

B. $50\,\Omega$ Resistor

$$\begin{aligned} V_0^- &= V_0^+ \tfrac{50-50}{50+50} \\ V_0^- &= 0 \end{aligned} \tag{8}$$

C. $100\,\Omega$ Resistor

$$\begin{aligned} V_0^- &= V_0^+ \tfrac{100-50}{50+100} \\ V_0^- &= V_0^+ \tfrac{50}{150} \\ V_0^- &= 0.33 V_0^+ \end{aligned} \tag{9}$$

D. $2.2\,\mathrm{pF}$ Capacitor

$$\begin{aligned} V_0^- &= V_0^+ \frac{\frac{1}{j\omega 2.2\times 10^{-9}} - 50}{50 + \frac{1}{j\omega 2.2\times 10^{-9}}} \\ V_0^- &= \tfrac{-(\omega^2 - 8.26e13)}{\omega^2 8.26e13} - j\tfrac{1.82e7\omega}{\omega^2 + 8.26e13} \\ V_0^- &= \Re\left(\tfrac{-(\omega^2 - 8.26e13)}{\omega^2 8.26e13} - j\tfrac{1.82e7\omega}{\omega^2 + 8.26e13} e^{j\omega t}\right) \\ V_0^- &= \Re\left(\left(\tfrac{-(\omega^2 - 8.26e13)}{\omega^2 8.26e13} - j\tfrac{1.82e7\omega}{\omega^2 + 8.26e13}\right) e^{j\omega t}\right) \\ V_0^- &= \tfrac{2.076\times 10^{17}\cos\omega t}{(((4.54\times 10^8/\omega)^2 + 2500)\omega^2} \end{aligned} \tag{10}$$

E. Short Circuit

$$\begin{aligned} V_0^- &= V_0^+ \tfrac{0-50}{50+0} \\ V_0^- &= -V_0^+ \end{aligned} \tag{11}$$

F. Open Circuit

$$\begin{aligned} V_0^- &= \lim_{x\to\infty} V_0^+ \tfrac{x-50}{50+x} \\ V_0^- &= \lim_{x\to\infty} V_0^+ \tfrac{1-0}{0+1} \\ V_0^- &= V_0^+ \end{aligned} \tag{12}$$

Appendix B
Smith Charts

A. $50\,\Omega$ Resistor

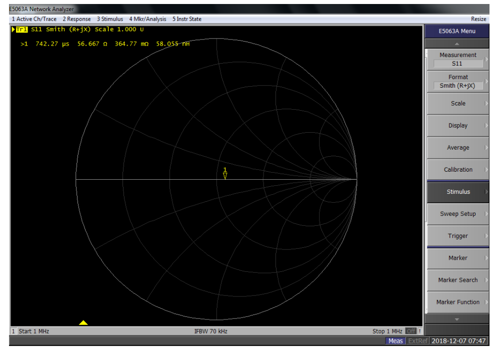

Fig. 7. Smith Chart: $50\,\Omega$ Resistor, 1 MHz

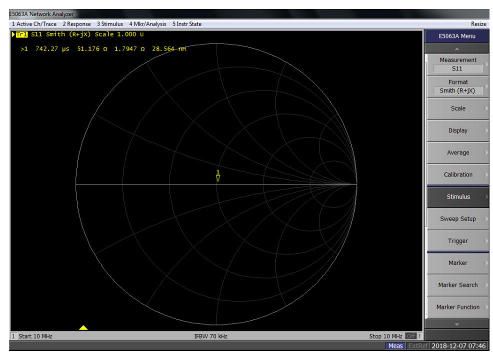

Fig. 8. Smith Chart: $50\,\Omega$ Resistor, 10 MHz

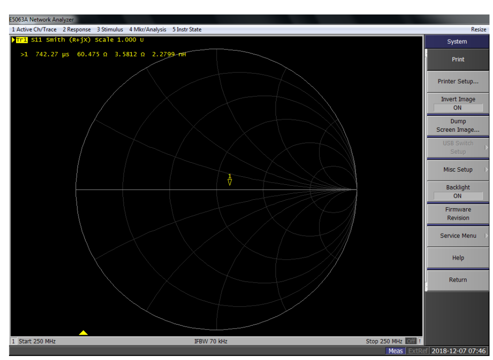

Fig. 9. Smith Chart: $50\,\Omega$ Resistor, 250 MHz

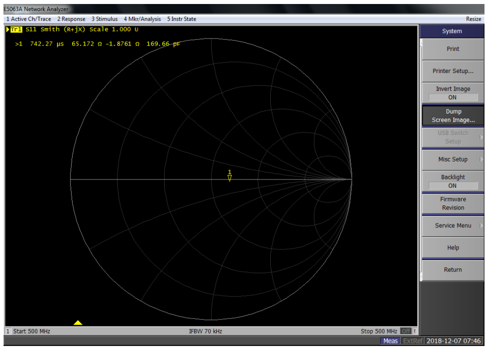

Fig. 10. Smith Chart: 50 Ω Resistor, 500 MHz

B. 100 Ω Resistor

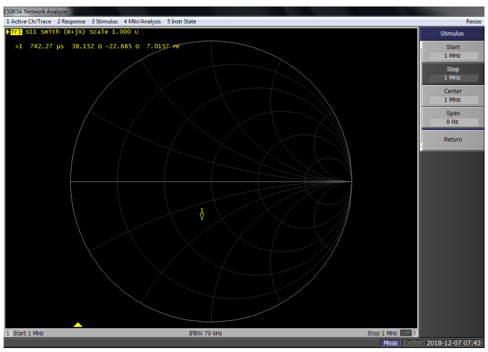

Fig. 11. Smith Chart: 100 Ω Resistor, 1 MHz

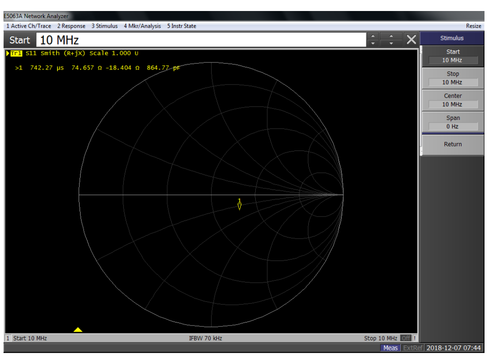

Fig. 12. Smith Chart: 100 Ω Resistor, 10 MHz

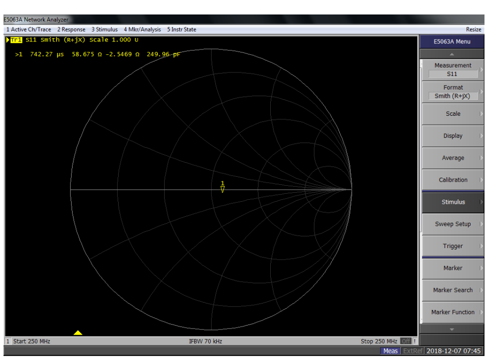

Fig. 13. Smith Chart: 100 Ω Resistor, 250 MHz

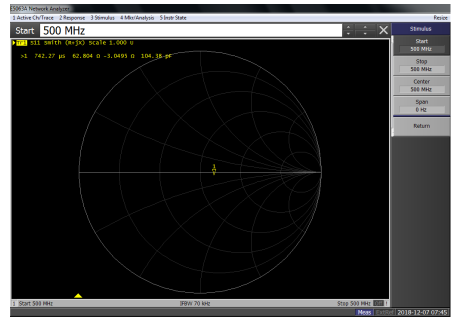

Fig. 14. Smith Chart: 100 Ω Resistor, 500 MHz

C. 2.2 pF Capacitor

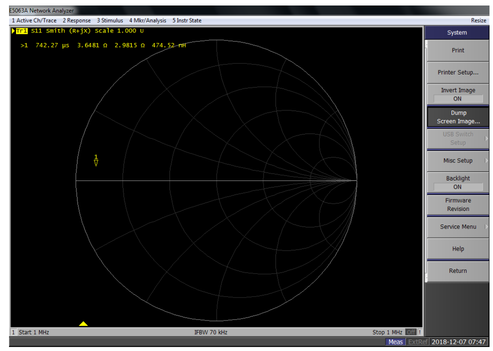

Fig. 15. Smith Chart: 2.2 pF Capacitor, 1 MHz

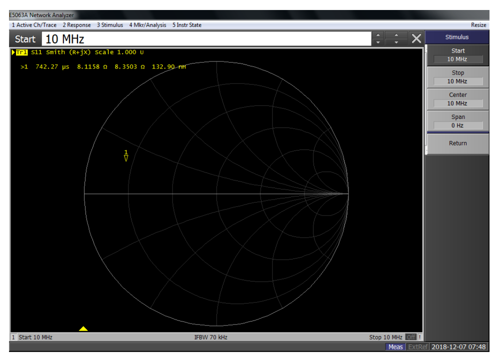

Fig. 16. Smith Chart: 2.2 pF Capacitor, 10 MHz

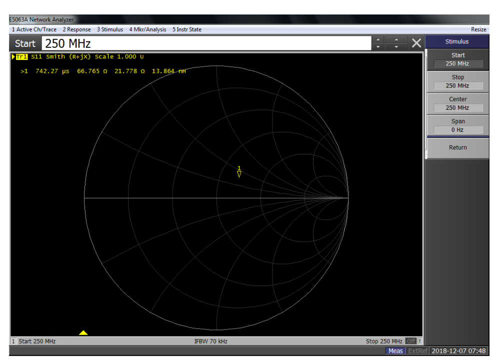

Fig. 17. Smith Chart: 2.2 pF Capacitor, 250 MHz

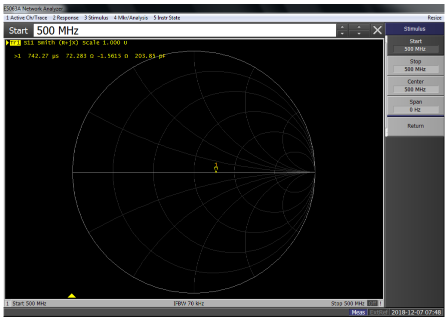

Fig. 18. Smith Chart: 2.2 pF Capacitor, 500 MHz

D. Short Circuit

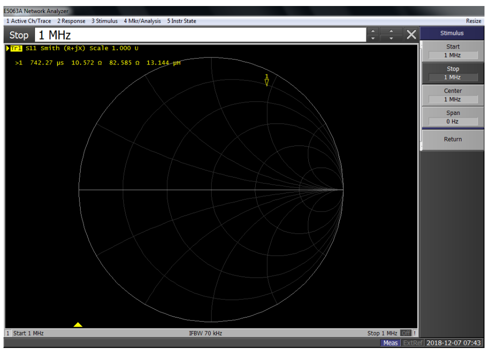

Fig. 19. Smith Chart: Short Circuit, 1 MHz

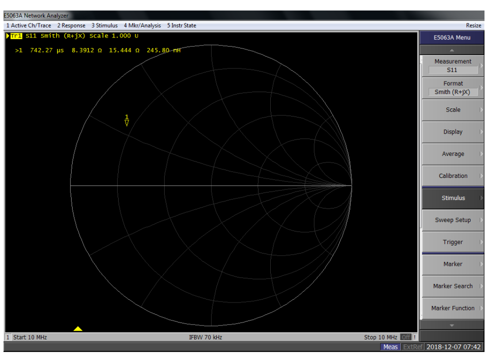

Fig. 20. Smith Chart: Short Circuit, 10 MHz

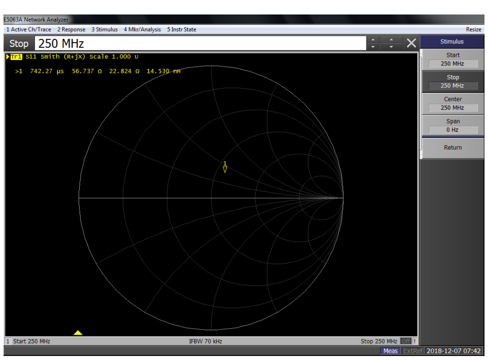

Fig. 21. Smith Chart: Short Circuit, 250 MHz

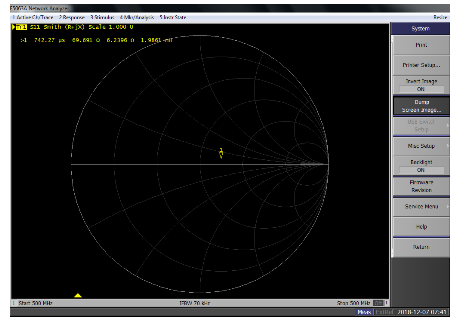

Fig. 22. Smith Chart: Short Circuit, 500 MHz

E. Open Circuit

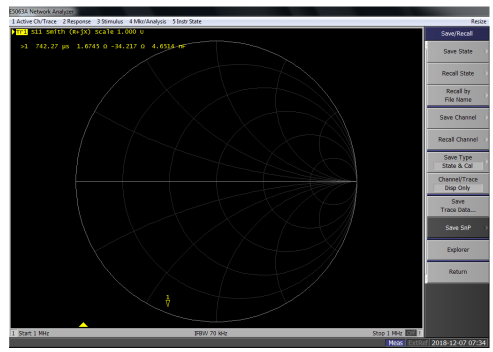

Fig. 23. Smith Chart: Open Circuit, 1 MHz

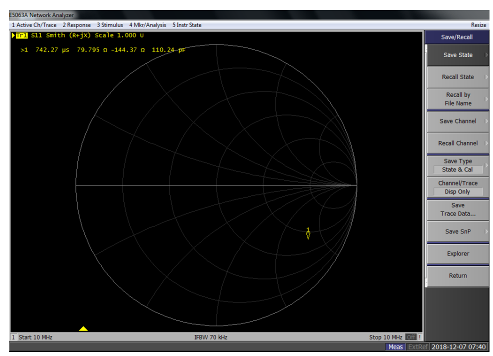

Fig. 24. Smith Chart: Open Circuit, 10 MHz

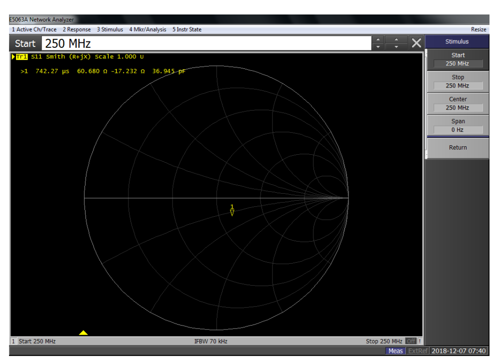

Fig. 25. Smith Chart: Open Circuit, 250 MHz

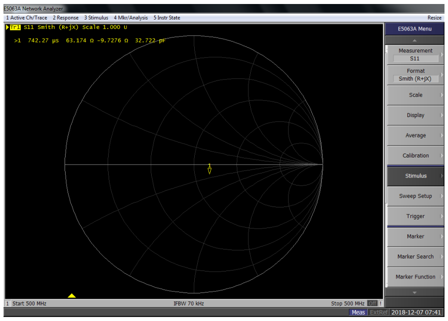

Fig. 26. Smith Chart: Open Circuit, 500 MHz

Appendix C
Z_0 Calculations

Using Equation 6, Equation 5 and the network analyzer values for the input impedance of the open and short circuits for the transmission line, it is possible to calculate Z_0 and γ.

$$\begin{aligned} &\gamma = \frac{\operatorname{arctanh}\left(\frac{Z_0(Z_{in}-Z_L)}{Z_0^2-Z_{in}Z_L}\right)+j\pi n}{l} \\ &n \in \mathbb{Z} \end{aligned}$$

A. 1 MHz

$$\begin{aligned} &Z_{in}^{OC} = 10.574 + j82.585 \\ &Z_{in}^{SC} = 1.6745 - j34.217 \\ &Z_0 = \sqrt{(10.574 + j82.585)(1.6745 - j34.217)} \\ &Z_0 = 53.4068 - j2.09 \\ &\gamma = 1.33 \times 10^{-3} + j2.952 \times 10^{-1} \end{aligned} \tag{13}$$

(Figures 19 and 23))

B. 10 MHz

$$\begin{aligned} &Z_{in}^{OC} = 8.3912 + j15.444 \\ &Z_{in}^{SC} = 79.795 - j144.37 \\ &Z_0 = \sqrt{(8.3912 + j15.444)(79.795.37 - 144.37)} \\ &Z_0 = 53.8452 + j0.1942 \\ &\gamma = 4.86 \times 10^{-3} + j2.713 \times 10^{-1} \end{aligned} \tag{14}$$

(Figures 20 and 24)

C. 250 MHz

$$\begin{aligned} &Z_{in}^{OC} = 56.737 + j22.8524 \\ &Z_{in}^{SC} = 60.680 - j17.232 \\ &Z_0 = \sqrt{(56.737 + j22.8524)(60.680 - j17.232)} \\ &Z_0 = 62.12 + j3.30 \\ &\gamma = 4.86 \times 10^{-3} + j2.713 \times 10^{-1} \end{aligned} \tag{15}$$

(Figures 21 and 25)

D. 500 MHz

$$\begin{aligned} &Z_{in}^{OC} = 69.691 + j6.2396 \\ &Z_{in}^{SC} = 63.174 - j9.7276 \\ &Z_0 = \sqrt{(69.691 + j6.2396)(63.174 - j9.7276)} \\ &Z_0 = 66.84 - j2.12 \\ &\gamma = 4.86 \times 10^{-3} + j2.713 \times 10^{-1} \end{aligned} \tag{16}$$

(Figures 22 and 26)

Appendix D
Z_L Calculations

Using the theoretical Z_0 and γ values, as well as the measured Z_{in}, a theoretical Z_L can be calculated using Equation 5 for each transmission line.

$$Z_L = \frac{-(e^{2\gamma l}(Z_0 - Z_{in})Z0}{e^{2\gamma l}(Z_0 - Z_{in}) + Z_0 + Z_{in}}$$

A. 50 Ω Resistor

Sample Calculation, 50 Ω at 1 MHz (Figure 7).

$$\begin{aligned} &Z_L = \frac{-(e^{2(4.86 \times 10^{-3}+j2.713 \times 10^{-1})(30)}((53.4068-j2.09)-(56.67+j364.77))(53.4068-j2.09)}{e^{2(4.86 \times 10^{-3}+j2.713 \times 10^{-1})(30)}((53.4068-j2.09)-(56.67+j364.77))+(53.4068-j2.09)+(56.67+j364.77)} \\ &Z_L = -9.215 - j2.355 \end{aligned}$$

50 Ω at 10 MHz (Figure 8).

$$Z_L = -9.215 - j2.355$$

50 Ω at 250 MHz (Figure 9).

$$Z_L = -9.215 - j2.355$$

50 Ω at 500 MHz (Figure 10).

$$Z_L = -9.215 - j2.355$$

B. 100 Ω Resistor

100 Ω at 1 MHz (Figure 11).

$$Z_L = 37.2 + j48.9$$

100 Ω at 10 MHz (Figure 12).

$$Z_L = 37.2 + j48.9$$

100 Ω at 250 MHz (Figure 13).

$$Z_L = 58.8 - j2.55$$

100 Ω at 500 MHz (Figure 14).

$$Z_L = 62.8 - j3.05$$

C. $2\,\mathrm{F}$ *Capacitor*

$2\,\mathrm{pF}$ at 1 MHz (Figure 15).

$$Z_L = 3.65 + j2.98$$

$2\,\mathrm{pF}$ at 10 MHz (Figure 16).

$$Z_L = 37.2 + j48.9$$

$2\,\mathrm{pF}$ at 250 MHz (Figure 17).

$$Z_L = 66.765 + j2.18$$

$2\,\mathrm{pF}$ at 500 MHz (Figure 18).

$$Z_L = 72.283 - j1.56$$