A Solow Model Alternative

A Comparison of Simple Neoclassical Variants Accounting for Tax

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TABLE OF CONTENTS

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Introduction

During my studies in macroeconomics at Western Washington University, I've grown a particular interest in Robert Solow's long run growth model. Solow's model is consistently hailed as the foundational building block in economics. The Solow model is distinguished from other models for providing insights on the factors of long-term economic growth through its assumptions on capital accumulation and technological progress with the most notable factor being the concept of technological progress. Solow's concept of growth within technology frustrated me early on in my studies as it was never truly dissected. This was because the original Solow model defined technological growth as an *exogenously* given and constant factor of production. This fundamental theory of growth has been so influential not because of its pinpoint accuracy but rather, its potential to convey long run economic theories due to its simplicity. I have always questioned the legitimacy of this simplistic model and why it is so emphasized in economics regardless of being an exogenous model.

While reflecting on my friction with the exogenous nature of the Solow model, I questioned how this model roots its predictions on growth in capital accumulation and yet doesn't explicitly justify the role of taxation. The Solow model does not account for taxation as it assumes that savings and investment decisions are made independent of the tax system and that taxes do not affect the incentives to work or save. On the contrary, Solow does inexplicitly draw implications on changes in tax through the role of the government and changes in tax policies that can affect a representative agent's savings and investment as well as the economy in the long run.

Differing in my initial doubt in the Solow model, I was captivated by the New Keynesian model and its ability to analyze short-run fluctuations in the economy due to nominal rigidities, such as sticky prices and wages. The New Keynesian can not only be used to interpret the relationships between aggregate income and aggregate spending (Blanchard et al. 2017, 58)[1] but also the contemporary model can be used to interpret a wide variety of topics within economics such as insinuate consumer choices under changes in tax policies and predict long run outcomes. This versatile model created a strong link between my education in mathematics and economics due to its endogenous structure that provided flexible and empirically

[1]Blanchard, Oliver, *Macroeconomics 6th Ed.* (*Pearson, 2013*), 58

grounded framework. From my interpretation, the New Keynesian model is an unsolved differential equations problem that can be built to analyze many different topics in economics.

In Economics 303: *The History of Economic Thought*, we learned about the evolution of economics throughout time and was constantly presented with moments in history where a revolutionary economic theory or model is adopted as fact, only to be disproven and trumped by a new theory or model. However, and more often than the latter, a preexisting model is corrected through an extension of itself. This article will explore two of my current fascinations in economics: what other iterations have the Solow model experienced to explain the role of taxation, and can the Solow model be adapted into an endogenous model that can accurately observes the effects of changes in tax on individuals?

The beginning of my research is a summary of Peter Ireland's "Two Perspectives on Growth and Taxes," in which Ireland compares the capability of the Solow and Knightian model to detect the effects of taxation on overall economic growth. I section my research into three parts. *First*, I incorporate taxes into the Solow model following Ireland's derivations and interpret the issues of the exogenous model's ability to draw implications from changes in tax.

Second, I explore Robert King and Sergio Rebelo's *simple neoclassical model of endogenous growth*, which is their combined attempt to develop a "useful starting point for consideration of the effects of policy on long-term growth" (King and Rebelo 1990, S132-S133)[2]. King and Rebelo's paper alter Solow's model by adopting Frank Knight's early theory of capitalism and economic growth, also known as the Knightian model. The Knightian model is a culmination of works from King and Rebelo, Larry Jones and Rodolfo Manuelli (1990)[3], Robert Barro (1990)[4]. The Knightian model is the synthesis of Solow's model and the theory of Knightian uncertainty. This simple endogenous growth model assumes "an all-encompassing definition of capital that accounts for improvements in land, human capital, and scientific knowledge as well as for physical capital" (Ireland 1994, 1)[5].

Third, I reference Larry Jones and Rodolfo Manuelli's article on developing convex equilibrium models that exhibit diminishing marginal returns while upholding constant long run growth (Jones and Manuelli 1990)[6]. Jones and Manuelli's article exams three different variations of Solow's model to defend their convex model of equilibrium growth. One of which, is a variation produced by Rebelo. The pair also

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[2] King and Rebelo, *Public Policy and Economic Growth: Developing Neoclassical Implications*, S132-S133

[3] Jones, Larry E., and Rodolfo Manuelli, *A Convex Model of Equilibrium Growth: Theory and Policy Implications*, (Journal of Political Economy, vol. 98 October 1990, Part 1), pp. 1008–38.

[4] Barro, Robert J. "Government Spending in a Simple Model of Endogenous Growth," Journal of Political Economy, vol. 98 (October 1990, Part 2), pp. S103–25.

[5] Ireland, Peter N., *Two Perspectives on Growth and Taxes* (FRB Richmond Economic Quarterly, vol. 80, no. 1, Winter 1994) pp. 1-17.

[6] See note 4 above.

explain the pitfalls of these attempts to conclude convexity in marginal returns on the factors of production is necessary for more complex optimization problems under changes in fiscal policies. I dissect the legitimacy of Ireland's explanation of the exogenous Solow model, and King and Rebelo's endogenous Knightian model with Jones and Manuelli's article on convex equilibrium models.

Following my research and discussion I conclude that the original Solow model and similar extensions both fall short of being able to precisely analyze the overall impact of taxation on the overall output of an economy. The exogenous growth model is still coveted as the building block of economics not because of its initial accuracy in every topic but rather its versatile ability to branch out and explain rudimentary topics in economics regardless of my conclusion. The Solow model has been subject to many extensions and modifications over the years since its inception. These extensions include but are not limited to the incorporation of government policies and interventions, the role of human capital and education and effects of globalization and trade.

I must reemphasize that the first two sections of this article are not my original work but the work of Peter Ireland's and King and Rebelo's. These sections are purely a summary of their research that I use as a vessel to discuss the viability of modified Solow models when determining long term growth rates of output of an economy when faced with changes in tax.

1. **The Solow Model**

1.1 **Solow and Tax**

In the original Solow model, the economy is characterized by a closed economy with a representative household. Ireland follows the general version of the Cobb-Douglas production function to adapt the Solow model to account for taxation on a representative agent to derive the aggregate growth rate of an economy. So, the version of the Solow model that Ireland follows the following assumptions:

1. *Production function: The economy produces output using capital and labor as inputs. The production function is assumed to be Cobb-Douglas, meaning that output depends on the level of capital and labor inputs, with constant returns to scale.*
2. *Saving and investment: Households save a constant fraction of their income and invest in physical capital. The level of investment is assumed to be proportional to the level of savings.*
3. *Depreciation: Physical capital depreciates over time at a constant rate, reflecting wear and tear and obsolescence.*
4. *Technological progress: The level of technology, which affects the productivity of labor and capital, grows at a constant rate over time.*
5. *Competitive markets: The economy is assumed to be characterized by perfect competition, meaning that firms are price takers and there are no monopolies or market distortions.*
6. *Homogeneous households: Households are assumed to be identical, with the same preferences for consumption and leisure.*
7. *No government or public sector: The model does not include a government sector, which means that there are no taxes or public goods in the economy.*

Theses assumptions give rise to the following production function:

$$Y_t = AK_t^{\alpha}, \qquad [1]^7$$

where Y is output, K is capital, A is technology, α is the capital share of output, and t is the time-period represented with integers starting at 0. Note that labor is not an input in this version of Solow's model, which would violate Solow's first assumption. In this case, Ireland specifically considers the behavior of a single representative agent. This individual output function is equivalent to per-capita output.[8]

"Since $0 < \alpha < 1$, equation [1] indicates the presence of diminishing marginal returns to capital accumulation (MR_K)" (Ireland 1994, 2)[9]. In other words, the additional output that is obtained by investing in additional units of capital is less than the previous. It is important to note the difference between marginal returns to capital R and marginal productivity of capital MP_K. Marginal returns refer to the change in output resulting from a small change in input, while marginal productivity refers to additional output generated by using one more unit of a specific input, while holding other inputs constant. To summarize, marginal returns can technically be either positive or negative while marginal productivity must diminish as more units of input are added. The Solow model exhibits positive and diminishing marginal returns since α is the capital share of output, we find the marginal return R by taking the derivative of Y_t with respects to K_t to produce the following equation: $R = \alpha AK_t^{\alpha-1}$. The marginal return in Solow's model is depicted in figure 1[10].
The Solow model is fabled for accounting for capital accumulation, which allows the model to "predict that output per worker increases in the long run when savings rate increases or when the population rate

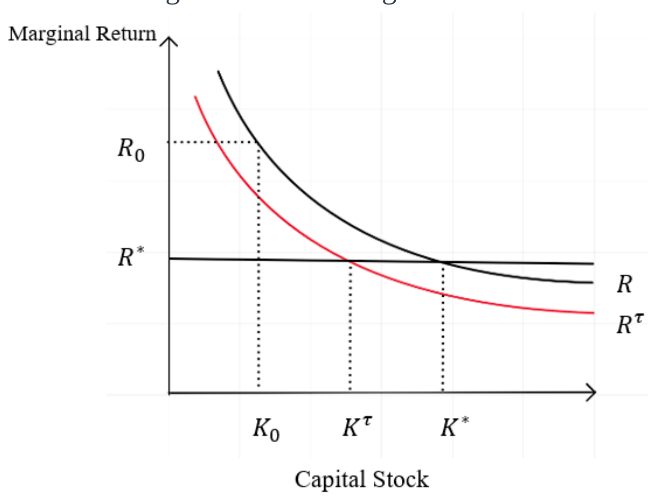

Figure 1: Solow and Taxes

[7] Ireland, *Two Perspectives on Growth and Taxes*, 2.

[8] This function mentioned on page 2 is the Solow per capita output function: $Y/_L = A(K/_L)^{(1-\alpha)} = Ak^{\alpha}$.

[9] See note 8 above.

[10] Figure 1 is a replica of Ireland's graph *Taxes in the Solow Model*.
Ireland, *Two Perspectives on Growth and Taxes*, 3.

decreases" (Williamson 2018, 276)[11]. The original Solow model's savings function is $S = sY$, where S is the level of savings, Y is the level of output or income, and s is the savings rate. The savings rate represents the ratio of income that the representative agent chooses to save rather than consume. Again, Ireland uses the neoclassical production per capita function. So, to assess the effects of taxation within the Solow model, Ireland uses the following savings function:

$$S_t = S(R - R^*) \text{ if } R > R^* \qquad [2]$$

where S is a function of the marginal return on capital R (Ireland 1994, 2)[12] In this case, R^* represents the marginal return on capital where the agent's incentive to save is zero. Therefore, S is an increasing function for any $R > R^*$

The representative agent's output or per capita output is represented with equation [1] in the Solow model. Say the government imposes a flat-rate income tax τ, the new post-tax income is the rather simplistic function $Y^\tau = (1-\tau)\alpha AK_t^\alpha$. Using this function, the derivative in respects to the capital, we obtain the marginal return on capital function:

$$R^\tau = (1-\tau)\alpha AK_t^{\alpha-1}. \qquad [3]$$

In Figure 1., at K_0, the representative agent has the marginal return on capital R_0. The agent moves along the curve R as he increases his capital stock until reaching K^*, at which point he loses incentive to save. After an increase in taxes, the marginal returns on capital shift down according to equation [3]. This indicates that the agent will remain at the same lack of incentive to save at a lower stock of capital K^τ. It is also easy to interpret that in the event of a flat-rate tax in Solow's model, the representative agent will be worse off permanently in the long run if the tax does not decrease in the future.

So, implementing taxes in the Solow model produces overall lower marginal returns on capital. This ultimately lowers the incentive to save, which lowers individual capital stock, which lowers aggregate output. To summarize mathematically, $\mathrm{A}(K_t^\tau)^\alpha < \mathrm{A}(K_t^*)^\alpha$ for any integer $t > 0$. Recall Solow assumed that diminishing marginal returns implied that the economy would eventually halt permanently in growth. Incorporating an income tax in the Solow model provides no substantial observations that aren't already predicted within Solow's assumptions.

The Great Depression of the 1930's has been interpreted as confirming Solow's predictions on stagnation were correct (Ireland 1990, 3)[13]. However, the U.S. economic rebound and sustained growth observed from the Great Depression caused Solow to relax his assumptions on constant technological

[11] Williamson Stephen D and Pearson. 2018. "Economic Growth: Malthus and Solow," 276. *Macroeconomics* Sixth edition. Global ed. Harlow: Pearson.
[12] See note 8 above.
[13] Ireland, *Two Perspectives on Growth and Taxes*, 3.

progress. Solow augmented this assumption by changing the parameters of A to be steadily increasing over time at the rate μ. Adding this characteristic to the parameter, then produces the following output at time t:

$$Y_t = A_t K_t^{\alpha}, \quad [4]$$

where

$$A_{t-1} = \mu A_t. \quad [5]$$

Equations [4] and [5] describe the effects of constant technological progress. With the parameter A_t described as a steadily increasing constant, the representative agent can now yield output or income with the same amount of capital stock.

[14]

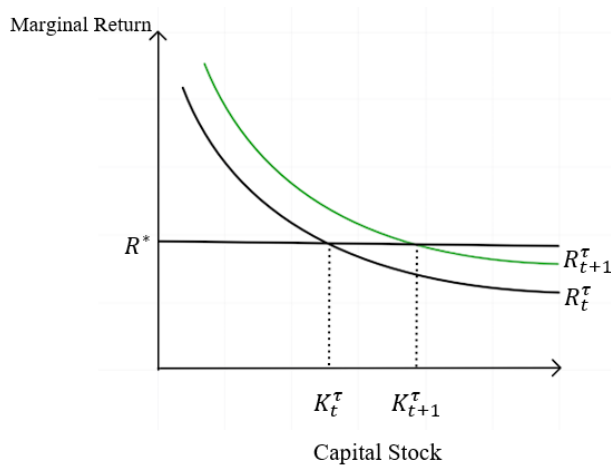

Figure 2: Technology and Solow

Now, suppose the government imposes the tax τ. Our per capita production function at time $t+1$ is now $Y_{t+1} = \mu A_{t+1} K_{t+1}^{\alpha}$ with its marginal returns on capital being $R_{t+1}^{\tau} = (1-\tau)\mu A_{t+1} K_{t+1}^{\alpha}$. The marginal return on capital is a downward shift where, at the end of time t, the capital stock K_t^{τ} reaches the level in which it is equivalent to the minimum marginal return on capital R^*. Recall equation [2], when $R_t^{\tau} \leq R^*$, the representative consumer's savings at time t is 0. This lack of capital accumulation produces the same predictions as Solow's previous model where he assumes A is an exogenous constant. However, as time t increases to $t+1$, the function [4] describing the parameter of technological progress becomes $A_t = \mu A_{t+1}$. The increasing parameter A causes the marginal return to rise above R^* and capital accumulation or savings begins again and the capital stock K_t^{τ} increases to K_{t+1}^{τ}. The new parameter A now constantly offsets the effects of diminishing marginal returns, which generates sustained growth in the augmented Solow model and is illustrated in Figure 2 (Ireland 1994, 5)[15].

[14] Figure 2 is a reproduction of Ireland's graph *Technological Change in the Solow Model*. Ireland, *Two Perspectives on Growth and Taxes*, 4.

[15] Ireland, *Two Perspectives on Growth and Taxes*, 5.

It is important to note that even though the augmented Solow model exhibits sustained growth, μ is still considered an exogenous factor. Thus, taxes have level effects but not growth effects in the Solow model (Ireland 1994, 5)[16]. After obtaining the tax adjusted Solow model for the representative consumer, the role of the government must be studied to produce the long run growth rate of output.

1.2 **The Governmental Role**

In the Solow model, the government uses its tax revenue to provide the representative agent with a lump-sum transfer of G_t units of output at time t (Ireland 1994, 5)[17]. The importance of understanding the differences in a flat-rate income tax and a lump-sum transfer must be stressed. A flat-rate income tax effectively lowers the representative agent's marginal return on capital, which disincentives the agent's desire to save and inadvertently lowers his output. The lump-sum transfer is obtained by the agent regardless of how much he saves and so the lump-sum transfer G_t has no effect on the representative agent's incentives.

Now, the representative agent's income after a flat-rate income tax τ is now $Y_t^\tau = (1-\tau)Y_t + G_t$. Solow's original income expenditure identity holds as an equilibrium condition for households is where $Y = C + I$ meaning the agent divides his income between consumption and investment and faces the following budget constraint:

$$(1-\tau)Y_t + G_t = C_t + I_t \qquad [6]$$

The natural log of the representative agent's consumption is taken to derive his utility. This simplifies further computations since to observe the agent's lifetime utility, we must take a summation of the agent's consumption:

$$\sum_{t=0}^{\infty} \beta^t \ln(C_t). \qquad [7]$$

"Where $0 < \beta < 1$ is the factor that discounts utility in future periods relative to utility in the current period. By investing I_t at time t, the agent adds to his capital stock at time $t+1$," (Ireland 1994, 5)[18]. So, the agents capital stock in the future time period $t+1$ is as follows

$$K_{t+1} = (1-\delta)K_t + I_t \qquad [8]$$

where δ is the depreciation rate.

The representative agent's lifetime utility function [7] is maximized with the constraints of equations [4]-[8]. The solution to this maximization problem suggests that consumption and capital accumulation will eventually grow at the same rate γ. That is,

$$\lim_{t\to\infty} {C_{t+1}}/{C_t} = \lim_{t\to\infty} {K_{t+1}}/{K_t} = \gamma. \qquad [9]$$

Note below that the limit of consumption and capital ratio also converges to a growth rate of ξ.

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[16] See note 18 above.
[17] See note 18 above.
[18] See note 18 above.

$$\lim_{t\to\infty} {}^{C_t}/_{K_t} = \xi \qquad [10]$$

The government faces constraints much like the representative agent where consumption and investment equal output. The government's output or income is the total portion of taxed income from the infinitely many representative agents. This constraint is as follows:

$$\tau Y_t = G_t \qquad [11]$$

The economy's aggregate resource constraint (output equals consumption plus investment) is an assembly of equations [4], [6], [8], and [11] and produces:

$$A_t K_t^{\alpha} = Y_t = C_t + I_t = C_t + K_{t+1} - (1-\delta)K_t \qquad [12]$$

Since equation [12] is the aggregate resource constraint, dividing it by K_t^{α} and taking the limit provides insight on the long run capital accumulation and consumption ratio.

$$\lim_{t=\infty} \frac{A_t}{K_t^{1-\alpha}} = \xi + \gamma - (1-\delta) \qquad [13]$$

therefore, μ must be equal to $\gamma^{\frac{1}{1-\alpha}}$ and respectively γ must be

$$\gamma = \mu^{1-\alpha} \qquad [14]$$

Finally, the long run growth rate using equations [3] and [4] becomes

$$\lim_{t\to\infty} {}^{Y_{t+1}}/_{Y_t} = \lim_{t\to\infty} ({}^{A_{t+1}}/_{A_t})({}^{K_{t+1}}/_{K_t}) = \mu\mu^{\frac{\alpha}{1-\alpha}} = \mu^{\frac{1}{1-\alpha}} \qquad [15]$$

The long run growth rate function indicates that the growth rate is dependent on the rate of technological progress μ. Notice that the imposed flat rate income tax τ is not present in [14]. This must mean that tax related fiscal policies are only capable of level effects and not growth effects within the Solow model. This model does not define how μ is determined, which leaves broad variation observed in long run growth rate between countries over time and within countries over time unexplained (Ireland 1990, 10)[19]. The exogenous assumptions on technological progress make the Solow model no merely a learning tool for introductory concepts in long run growth in an economy and makes it insufficient in describing growth effects of a change in income tax.

[19] Ireland *Two Perspectives on Growth*, 10.

2. **A Simple Endogenous Growth Model:** *The Knightian Model*

2.1 **Knightian Capital**

In the previous section, I used the work of Peter Ireland to mathematically justify why the Solow model is short from predicting long run aggregate growth rates in the event of changes in tax. This section focuses King and Rebelo's attempt to incorporate changes in tax in Solow's model to better explain constant economic growth without having annual adjustments through technological progress. To reiterate from the introduction, the Knightian model is the synthesis of Solow and Knightian uncertainty. The main difference between the Solow model and the Knightian model comes down to their assumptions on the factors of production.

The Knightian model assumes the factors of production to be categorized between land, labor, and physical capital. This model describes land as permanently fixed, labor as slightly adjustable, and physical capital as completely flexible. Aside from Knight's assumption of land as being permanently fixed, he argues that the quality of the land can me improved on infinitely. This allows for the productivity of land to increase over time (Ireland 1990, 8)[20].

The Knightian definition of labor and physical capital are somewhat analogous. Although labor is limited and considered fixed in the short run, the quality of the work force can also be improved on infinitely. A worker investing time and income towards education and skills can yield higher levels of output in the future. Physical capital is flexible to changes, investing in the quality of a workforce yields a higher human capital and physical capital over time. A firm investing assets today can produce more, yielding higher levels of output in the future.

Another key difference between Knightian and Solow model is that Solow was able to directly implement in his model was the technology factor but considers it to grow at an exogenous and constant rate. King proposes "a model of economic growth in which a comprehensive measure of 'technical progress' is made endogenous" (King 1990, S147)[21].

2.1 **Knightian Model and Tax**

King and Rebelo use the most simplifies the Solow model to apply Knight's assumptions on a generalized definition of capital. I forgo some of King and Rebelo's computations and summarize their findings through the most basic form of their model's marginal returns in the following section as I believe it is sufficient at describing the effects of taxation in this endogenous model. The Knightian production function of an individual after incorporating taxation is

$$Y_t = (1-\tau)AK_t \text{ where } A > 0. \qquad [1.1]$$

[20] Ireland, *Two Perspectives on Growth and Taxes*, 9.

[21] King and Rebelo *Public Policy and Economic Growth*, S147

Initially, this representative agent's production function is difficult to differentiate between Solow's. However, function [1.1] diverges greatly from Solow's as the parameter A is merely a fixed and exogeneous constant, no longer reflects technological progress, and is not dependent on time t. King and Rebelo point out that in the basic neoclassical model, it "assumes that technical progress is labor-augmenting to ensure that steady-state growth is feasible" (King and Rebelo 1990, S129)[22]. Recall, the role of technology is held as an endogenous variable within physical stock K_t in the Knightian model. Another vast difference in models is in [1.1], the exponent α is discarded. This implies that marginal return is

$$R = A \qquad [2.1]$$

$$R^{\tau} = (1-\tau)A. \qquad [2.2]$$

Under Knightian assumptions in equation [2.1], "there may be diminishing returns to accumulating one type of capital alone, but there is no tendency for their marginal product to fall as all types are increased together. Under Knight's broad definition of capital, therefore, production features constant, rather than diminishing, returns" (Ireland 1990, 9)[23].

Equation [2.1] implies that the individual is unrestricted by the system of equations for savings in [2] for when $R \leq R^*$ because R cannot dip below the critical value R^*. Mathematically, this relationship between R and R^* is

$$R = A > R.^* \qquad [2.3]$$

Therefore, capital accumulation and long run growth continues indefinitely. It appears the economy grows linearly while holding all other variables constant due to the broad definitions of capital in this model.

When comparing the marginal return before and after tax, [2.1] and [2.2], a flat-rate income tax would shift an individual's marginal return down, much like in the basic neoclassical model. This also suggests that due to relation [2.1], an increase in the tax rate would additionally alter the slope of the Knightian production function permanently. This means that despite the Knightian model being endogenous, it is in ways an extension and simplification of Solow's model that not only exhibits level effects but also growth effects due to changes in tax policies. Figure 3 overlays the Knightian production function, over its marginal return graphs to illustrate the level and growth effects depicted for when $R^{\tau} > R^* = A$ where Y_t^{τ} is the post tax Knightian output at time t.

[22] King and Rebelo, *Public Policy and Economic Growth: Developing Neoclassical Implications*, S129
[23] Ireland, *Two Perspectives on Growth and Taxes*, 9.

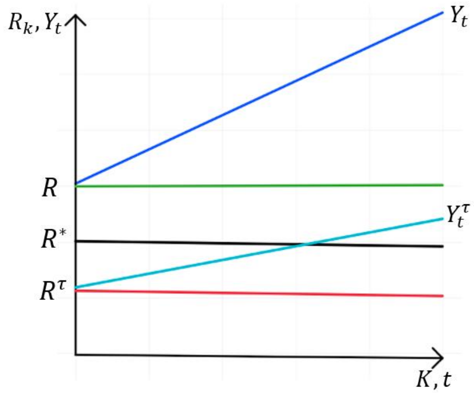

Figure 3: Taxed Knightian Marginal Return and Output

3. A Comparison of Literature

3.1 The Technological Role

Jones and Manuelli compare the evolution of economic theories and models early in their article to explain their approach on constructing endogenous and convex equilibrium growth functions as a means to effectively draw implications on policy changes. From comparing the two articles, Ireland's exposition of the Solow model was accurate and effective at relaying the concepts of the model. Neoclassical models have "the property that the only potential sources of growth are sustained exogenous increases in factor supplies (e.g. population growth) and exogenously given technological change. Thus, *except for the possibility of exogenous technological change*, these models of growth lead one to the startling conclusion that there is no growth in per capita terms," (Jones and Manuelli 1990, 1009)[24]. Jones and Manuelli have little criticism to the Solow model with an augmented technological progress input. Rather, they focus on the nuances in the more simplistic endogenous variations of the neoclassical model.

Jones and Manuelli refer to the Knightian model as a variation *on the more standard (i.e. Solowian) model of capital theory*[25] due to the egress from the neoclassical assumption of technology, its reliance on nonconvex production variables and in some cases, the absence of fixed factors. I agreed with this point as I found it difficult to believe the applicability of an endogenous growth model that violates the law of diminishing marginal returns. It is important to note that prior to King and Rebelo's paper on a simplistic neoclassical growth model, Rebelo had analyzed a convex model of endogenous growth. Rebelo concentrated on different forms of preferences and technologies to create optimal paths that also exhibit constant growth rates. Rebelo specifically focused on the role of changes in policy that would influence

[24] Jones and Manuelli, *Equilibrium Growth*, 1009.
[25] Jones and Manuelli, *Equilibrium Growth*, 1010.

long run growth (Jones and Manuelli 1990, 1010)[26]. The article by Rebelo and King was a simplification of Rebelo's previous work on a more complex endogenous model.

3.2 **Effects of Taxation**

Jones and Manuelli maintain a 'simplistic' definition of tax within their model much like the definitions of tax in the neoclassical models discussed above. This is not only to remove complexities from their model, but it also succeeds at emphasizing the impact of capital income taxation in long run growth in an economy (Jones and Manuelli 1990, 1023)[27]. Due to convexity and regardless of its simplistic tax, Jones and Manuelli's growth model can analyze more complex fiscal policies.

The Knightian model can give rudimentary insight on simple tax policies but is assumed to have constant marginal returns, making it incapable of drawing realistic predictions on more complex policies. Although the Knightian model can demonstrate growth and level effects, it still leaves the transitionary growth rates from one growth rate to another. Instead, the Knightian model jumps from one growth rate to another without being able to observe anything in between. This is due to the linear form of the production function. The Solow model demonstrates diminish marginal returns on capital, but technological progress is considered to grow at a constant and exogenous rate. Unfortunately, we have shown that Solow's model can only reveal level changes in growth rates and may not be able to generate accurate long run assumptions on growth.

4. Conclusion

To summarize my initial findings for this paper, I conclude the Solow growth model but ultimately falls short of accurately estimating outcomes due to its assumptions on technology and its omission of underlying variables/parameters. Despite this, Solow's assumption on technology is both a pro and a con of the model as it makes for an excellent medium for education as it upholds the law of diminishing returns and constant growth of an economy over time.

On the other hand, the Knightian model is capable of analyzing both level and growth effects due to the endogenous assumptions on technology and its broad and all-encompassing definition of capital. The Knightian model is not well known despite being a simple endogenous growth model, capable of exhibiting both level and growth effects of an economy. I had originally investigated the Knightian model to offer an alternative to the Solow model. Unfortunately, I pinpointed the detrimental faults of this model through my derivations and emphasized its ability to exhibit both level and growth effects through a linear production function and constant marginal returns. This makes it impossible to gather any implications on long run growth without reasonable doubt. In other words, this endogenous model is a more complex version of the

[26] Jones and Manuelli, *Equilibrium Growth*, 1010.
[27] See note 26.

Solow model, but its complexity is what leads the model to be inefficient in predicting changes in growth from an increase or decrease of tax. Thus, the doubt I once had for the Solow model has transitioned into a conviction. I recognize that my interpretation of the Solow model was incorrect. Instead of seeing its simplicity as a fault, I now recognize the simplicity as an elegant and excellent medium for explaining long run aggregate growth.

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