432 Class 4 Slides

github.com/THOMASELOVE/432-2018

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Setup

```
library(skimr)
library(simputation)
library(broom)
library(modelr)
library(tidyverse)

smartcle1 <- read.csv("data/smartcle1.csv")</pre>
```

Today's Materials

- Prediction and Confidence Intervals
- Centering and Rescaling Predictors
- Two-Factor Analysis of Variance
- More to come...

Last time, we built smartcle3 and two models...

```
set.seed(20180123)
smartcle3 <- smartcle1 %>%
  select (SEQNO, bmi, sleephrs, female, alcdays, exerany) %>%
  impute rhd(exerany ~ 1) %>%
  impute_pmm(sleephrs ~ 1) %>%
  impute_rlm(bmi ~ female + sleephrs) %>%
  impute_cart(alcdays ~ .) %>%
  tbl df()
model_int <- lm(bmi ~ female * sleephrs, data = smartcle3)</pre>
model noint <- lm(bmi ~ female + sleephrs, data = smartcle3)
```

Building Predictions for New Data (Individual Subjects)

What do we predict for the bmi of a female subject who gets 10 hours of sleep per night? What if the subject was male, instead?

```
fit lwr upr
1 26.33333 14.13710 38.52955
2 28.35049 16.13121 40.56977
```

Building Predictions for New Data (Average Predictions)

What do we predict for the average bmi of a population of female subjects who sleep for 10 hours? What about the population of male subjects?

```
fit lwr upr
1 26.33333 25.25921 27.40744
2 28.35049 27.04027 29.66071
```

Centering and Rescaling Predictors (See Notes sections 2.13, 2.14 and 4.7)

Centering sleephrs to ease interaction description

```
smartcle3 <- smartcle3 %>%
  mutate(sleep_c = sleephrs - mean(sleephrs))
model_int_c <- lm(bmi ~ female * sleep_c, data = smartcle3)</pre>
model int c
Call:
lm(formula = bmi ~ female * sleep_c, data = smartcle3)
Coefficients:
   (Intercept)
                        female
                                        sleep_c
                                        0.04019
      28.23061
                      -0.67926
female:sleep c
      -0.44857
```

Interpreting Interaction: Centered sleephrs

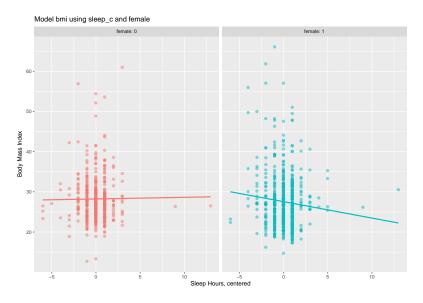
 $\label{eq:bmi} \mbox{bmi} = 28.23 - 0.68 \; \mbox{female} + 0.04 \; \mbox{centered sleep_c} - 0.45 \; \mbox{female} \; \times \\ \mbox{centered sleep_c}$

- Now, 28.23 is the predicted bmi for a male who gets the average amount of sleep (7.02 hours)
- \bullet And 28.23 0.68 = 27.55 is the predicted bmi for a female who gets the average amount of sleep.
- So, the main effect of female is the predictive difference (female male) in bmi for mean sleephrs,
- the product term is the change in the slope of centered sleephrs_c on bmi for a female rather than a male, and
- the residual standard deviation and the R-squared values remain unchanged from the model before centering.

```
glance(model_int_c) %>% round(., 3)
```

r.squared adj.r.squared sigma statistic p.value df 0.009 0.006 6.191 3.08 0.027 4

Plotting bmi on centered sleep_c by female



Rescaling?

Centering helped us interpret the main effects in the regression, but it still leaves a scaling problem.

- The female coefficient estimate is much larger than that of sleephrs, but this is misleading, considering that we are comparing the complete change in one variable (sex = female or not) to a 1-hour change in average sleep.
- Gelman and Hill (2007) recommend all continuous predictors be scaled by dividing by 2 standard deviations
 - A 1-unit change in the rescaled predictor corresponds to a change from 1 standard deviation below the mean, to 1 standard deviation above.
 - An unscaled binary (1/0) predictor with 50% probability of occurring will be exactly comparable

Rescaling to sleep_z and re-fitting the model

```
smartcle3 <- smartcle3 %>%
    mutate(sleep_z = (sleephrs - mean(sleephrs)) /
              (2*sd(sleephrs)))
model_int_z <- lm(bmi ~ female * sleep_z, data = smartcle3)</pre>
model int z
Call:
lm(formula = bmi ~ female * sleep_z, data = smartcle3)
Coefficients:
   (Intercept)
                         female
                                         sleep_z
       28,2306
                       -0.6793
                                          0.1224
female:sleep_z
       -1.3660
```

Comparing our Interaction Models

Original Model

• bmi = 27.95 + 2.47 female + 0.04 sleephrs - 0.45 female \times sleephrs

Centered Model

• $bmi = 28.23 - 0.68 female + 0.04 sleep_c - 0.45 female x sleep_c$

Centered, Rescaled Model

• bmi = 28.23 - 0.68 female + 0.12 sleep_z - 1.37 female x sleep_z

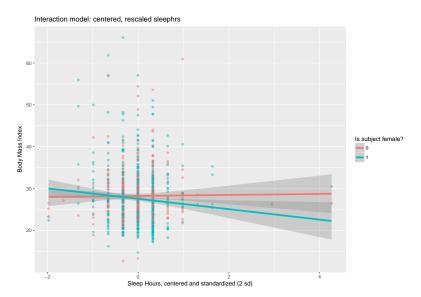
Interpreting the Centered, Rescaled Model

- Main effect of female, -0.68, is still the predictive difference (female male) in bmi with sleephrs at its mean, 7.02 hours,
- Intercept (28.23) is still the predicted bmi for a male who sleeps the mean number of hours, and
- the residual standard deviation and the R-squared values remain unchanged

but now we also have:

- the coefficient of sleep_z is the predictive difference in bmi associated with a change in sleephrs of 2 standard deviations (from one standard deviation below the mean of 7.02 to one standard deviation above 7.02.)
 - Since sd(sleephrs) is 1.52, this corresponds to a change from 5.50 hours per night to 8.54 hours per night.
- the coefficient of the product term (-1.37) corresponds to the change in the coefficient of sleep_z for females as compared to males.

Plotting the Rescaled, Centered Model



Two-Factor Analysis of Variance (see Notes Chapter 3)

How do female and exerany relate to bmi?

```
smart3_sum <- smartcle3 %>%
group_by(female, exerany) %>%
summarize(mean.bmi = mean(bmi), sd.bmi = sd(bmi))
```

Resulting tibble for smart3_sum

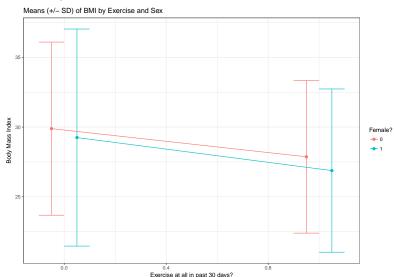
smart3_sum

This would be more useful as a plot.

Building a Means Plot (result on next slide)

```
pd <- position dodge(0.2)
ggplot(smart3_sum, aes(x = exerany, y = mean.bmi,
                       col = factor(female))) +
  geom errorbar(aes(ymin = mean.bmi - sd.bmi,
                    ymax = mean.bmi + sd.bmi),
                width = 0.2, position = pd) +
  geom point(size = 2, position = pd) +
  geom line(aes(group = female), position = pd) +
  scale color discrete(name = "Female?") +
  theme bw() +
  labs(y = "Body Mass Index",
       x = "Exercise at all in past 30 days?",
       title = "Means (+/- SD) of BMI by Exercise and Sex")
```

Means Plot (Do we have a strong interaction effect?)



Two-Way ANOVA model with Interaction

```
model2 <- lm(bmi ~ female * exerany, data = smartcle3)
anova(model2)</pre>
```

Analysis of Variance Table

```
Response: bmi
```

```
Df Sum Sq Mean Sq F value Pr(>F)

female 1 118 117.76 3.1288 0.07722 .

exerany 1 947 946.71 25.1530 6.231e-07 ***

female:exerany 1 5 4.97 0.1320 0.71642

Residuals 1032 38843 37.64

---

Signif. codes:
0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Does it seem like we need the interaction term in this case?

Summary of Two-Factor ANOVA with Interaction

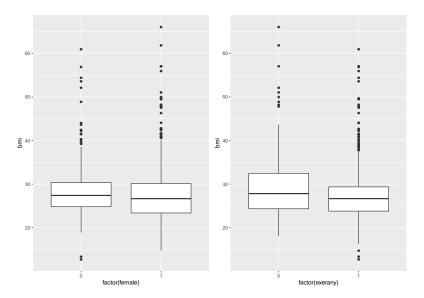
```
> summary(model2)
Call:
lm(formula = bmi ~ female * exerany, data = smartcle3)
Residuals:
   Min 10 Median 30 Max
-15.158 -3.830 -0.763 2.145 36.813
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 29.8887 0.7132 41.909 <2e-16 ***
female -0.6414 0.8514 -0.753 0.4514
exerany -2.0208 0.7870 -2.568 0.0104 *
female:exerany -0.3484 0.9590 -0.363 0.7164
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.135 on 1032 degrees of freedom
Multiple R-squared: 0.0268, Adjusted R-squared: 0.02397
F-statistic: 9.471 on 3 and 1032 DF, p-value: 3.557e-06
```

What if we wanted the model with no interaction?

Here's the key plot, then...

```
p1 <- ggplot(smartcle3, aes(x = factor(female), y = bmi)) +
        geom_boxplot()
p2 <- ggplot(smartcle3, aes(x = factor(exerany), y = bmi)) +
        geom_boxplot()
gridExtra::grid.arrange(p1, p2, nrow = 1)</pre>
```

Key Plot for Two-Way ANOVA, no interaction



Two-Way ANOVA model without Interaction

```
model2_noint <- lm(bmi ~ female + exerany, data = smartcle3)
anova(model2_noint)</pre>
```

Analysis of Variance Table

```
Response: bmi

Df Sum Sq Mean Sq F value Pr(>F)

female 1 118 117.76 3.1314 0.07709 .

exerany 1 947 946.71 25.1742 6.164e-07 ***

Residuals 1033 38848 37.61

---

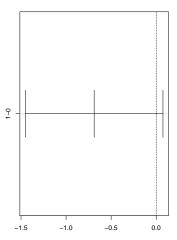
Signif. codes:
0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Summary of Two-Factor No Interaction ANOVA

```
> summary(model2_noint)
Call:
lm(formula = bmi ~ female + exerany, data = smartcle3)
Residuals:
   Min 1Q Median 3Q Max
-15.116 -3.860 -0.736 2.124 36.895
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 30.0814 0.4766 63.119 < 2e-16 ***
female -0.9161 0.3916 -2.339 0.0195 *
exerany -2.2555 0.4495 -5.017 6.16e-07 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.132 on 1033 degrees of freedom
Multiple R-squared: 0.02667, Adjusted R-squared: 0.02479
F-statistic: 14.15 on 2 and 1033 DF, p-value: 8.634e-07
```

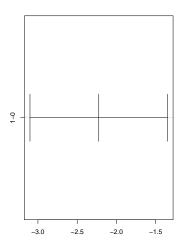
Tukey HSD Comparisons (no interaction)

95% family-wise confidence level



Differences in mean levels of factor(female)

95% family-wise confidence level



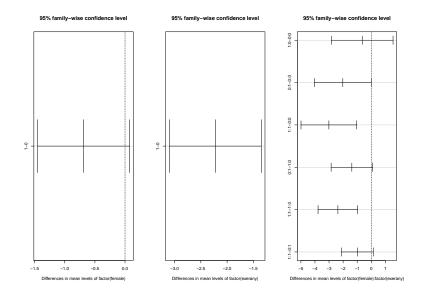
Differences in mean levels of factor(exerany)

Tukey HSD Comparisons (without interaction)

```
Tukey multiple comparisons of means
   95% family-wise confidence level
Fit: aov(formula = bmi ~ factor(female) + factor(exerany), date
$`factor(female)`
             lwr upr p adj
         diff
1-0 -0.6883146 -1.451577 0.07494728 0.0770918
$`factor(exerany)`
        diff lwr upr p adj
```

1-0 -2.225162 -3.101315 -1.349009 7e-07

Tukey HSD comparisons WITH interaction



Tukey HSD comparisons WITH interaction

```
TukeyHSD(aov(bmi ~ factor(female) * factor(exerany), data = smartcle3))
 Tukey multiple comparisons of means
   95% family-wise confidence level
Fit: aov(formula = bmi ~ factor(female) * factor(exerany), data = smartcle3)
$`factor(female)`
         diff lwr upr p adi
1-0 -0.6883146 -1.451898 0.07526902 0.0772162
$`factor(exerany)`
        diff lwr upr p adi
1-0 -2.225162 -3.101685 -1.34864 7e-07
$`factor(female):factor(exerany)`
                       lwr
                                    upr p adj
1:0-0:0 -0.6414435 -2.832366 1.549478791 0.8752356
0:1-0:0 -2.0208224 -4.045876 0.004230988 0.0507142
1:1-0:0 -3.0107133 -4.991656 -1.029770182 0.0005667
0:1-1:0 -1.3793789 -2.850875 0.092117170 0.0754115
1:1-1:0 -2.3692698 -3.779445 -0.959094236 0.0000992
1:1-0:1 -0.9898909 -2.125362 0.145580643 0.1124126
```

Indicator Variables

What if I used (1 = yes, 2 = no) instead of (1 = yes, 0 = no) for exerany? What if I tell R that exerany is a factor?

```
smartcle3 <- smartcle3 %>%
  mutate(exer_12 = 2 - exerany,
        exer_yn = fct_recode(factor(exerany), Y = "1", N = "0"))
smartcle3 %>% count(exerany, exer_12, exer_yn)
```

Two-Predictor model with exerany (1 = yes, 0 = no)

```
lm(bmi ~ exerany * alcdays, data = smartcle3)
Call:
lm(formula = bmi ~ exerany * alcdays, data = smartcle3)
Coefficients:
    (Intercept)
                                           alcdays
                         exerany
       29.79211
                        -2.10499
                                          -0.10141
exerany:alcdays
        0.02546
```

Two-Predictor model with exer_12 (1 = yes, 2 = no)

```
lm(bmi ~ exer_12 * alcdays, data = smartcle3)
```

```
Call:
lm(formula = bmi ~ exer 12 * alcdays, data = smartcle3)
Coefficients:
    (Intercept)
                          exer 12
                                           alcdays
       25.58214
                          2.10499
                                          -0.05049
exer_12:alcdays
       -0.02546
```

Compare to

(Intercept) exerany alcdays exerany:alcdays 29.79211 -2.10499 -0.10141 0.02546

Two-Predictor model with exer_yn (factor)

```
lm(bmi ~ exer_yn * alcdays, data = smartcle3)
```

Compare to

```
(Intercept) exerany alcdays exerany:alcdays 29.79211 -2.10499 -0.10141 0.02546
```

Fitting Linear Regressions, and then Validating Them

A Linear Regression for bmi from smartcle3

```
(Intercept) female sleephrs alcdays exerany 32.32 -1.19 -0.24 -0.10 -2.15
```

```
glance(mod_ks)
```

tidy(mod_ks)

```
estimate std.error statistic
        term
  (Intercept) 32.32268299 1.00240361 32.245178
2
       female -1.18547540 0.39507596 -3.000626
3
     sleephrs -0.24394812 0.12430035 -1.962570
4
     alcdays -0.09690421 0.02446772 -3.960492
5
      exerany -2.14510628 0.44678217 -4.801235
        p.value
1 2.601681e-158
2 2.759053e-03
3 4.996479e-02
4 7.993482e-05
5 1.809953e-06
```

ANOVA for sequential testing of predictors

```
anova(mod_ks)
```

Analysis of Variance Table

```
Response: bmi

Df Sum Sq Mean Sq F value Pr(>F)

female 1 118 117.76 3.1828 0.07471 .

sleephrs 1 119 119.37 3.2263 0.07276 .

alcdays 1 675 675.22 18.2494 2.117e-05 ***

exerany 1 853 852.91 23.0519 1.810e-06 ***

Residuals 1031 38147 37.00

---

Signif. codes:

0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Different order but the same model?

Analysis of Variance Table

```
Response: bmi

Df Sum Sq Mean Sq F value Pr(>F)

exerany 1 859 858.67 23.2075 1.672e-06 ***

alcdays 1 425 425.22 11.4926 0.0007252 ***

female 1 339 338.87 9.1586 0.0025369 **

sleephrs 1 143 142.51 3.8517 0.0499648 *

Residuals 1031 38147 37.00

---

Signif. codes:
0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Does Order Matter? Comparing Slopes

model_ks Order	Estimate
Intercept	32.32
female	-1.19
sleephrs	-0.24
alcdays	-0.10
exerany	-2.15

Revised Order	Estimate
Intercept	32.32
exerany	-2.15
alcdays	-0.10
female	-1.19
sleephrs	-0.24

Does Order Matter? Comparing t test and CI results

• t tests in summary and tidy test value as "last predictor in"

model_ks Order	Estimate	t test p	95% CI
Intercept	32.32	< 2e-16	(30.3, 34.3)
female	-1.19	0.0028	(-2.0, -0.4)
sleephrs	-0.24	0.0499	(-0.5, -0.0)
alcdays	-0.10	7.9e-05	(-0.14, -0.05)
exerany	-2.15	1.8e-06	(-3.0, -1.2)

Revised Order	Estimate	t test p	95% CI
Intercept	32.32	< 2e-16	(30.3, 34.3)
exerany	-2.15	1.8e-06	(-2.0, -0.4)
alcdays	-0.10	7.9e-05	(-0.5, -0.0)
female	-1.19	0.0028	(-0.14, -0.05)
sleephrs	-0.24	0.0499	(-3.0, -1.2)

Does Order Matter? Comparing Slopes and *p* values

- t tests in summary and tidy test value as "last predictor in"
- anova tests of a single 1m consider predictive value "in sequence"

model_ks Order	Estimate	t test p	ANOVA p
Intercept	32.32	< 2e-16	-
female	-1.19	0.0028	0.075
sleephrs	-0.24	0.0499	0.073
alcdays	-0.10	7.9e-05	2.1e-05
exerany	-2.15	1.8e-06	1.8e-06

Revised Order	Estimate	t test p	ANOVA p
Intercept	32.32	< 2e-16	_
exerany	-2.15	1.8e-06	1.7e-06
alcdays	-0.10	7.9e-05	0.0007
female	-1.19	0.0028	0.0025
sleephrs	-0.24	0.0499	0.0499
	400 01 44		

Do we need all of those variables in mod_ks? (Sections 7-8)

Stepwise Regression (backwards elimination)

step(mod_ks)

```
Start: AIC=3745.89
bmi ~ female + sleephrs + alcdays + exerany
          Df Sum of Sq RSS AIC
                      38147 3745.9
<none>
- sleephrs 1 142.51 38289 3747.8
- female 1 333.14 38480 3752.9
- alcdays 1 580.36 38727 3759.5
- exerany 1 852.91 39000 3766.8
```

```
Call:
```

lm(formula = bmi ~ female + sleephrs + alcdays + exerany, data

Coefficients:

Stepwise Regression (forwards selection)

```
with(smartcle3,
    step(lm(bmi ~ 1),
    scope = (~ exerany + alcdays + female + sleephrs),
    direction = "forward"))
```

Forward Selection Stepwise Regression, Results: 1

Start: AIC=3784.76 bmi ~ 1

```
Df Sum of Sq RSS AIC
+ exerany 1 858.67 39053 3764.2
+ alcdays 1 528.88 39383 3772.9
+ sleephrs 1 124.34 39788 3783.5
+ female 1 117.76 39794 3783.7
<none> 39912 3784.8
```

Step: AIC=3764.23 bmi ~ exerany

	Df	Sum of Sq	RSS	AIC
+ alcdays	1	425.22	38628	3754.9
+ female	1	205.80	38848	3760.8
+ sleephrs	1	126.97	38926	3762.9
<none></none>			39053	3764 2

Forward Selection Stepwise Regression, Results: 2

```
Step: AIC=3754.88
bmi ~ exerany + alcdays
         Df Sum of Sq RSS AIC
+ female 1 338.87 38289 3747.8
+ sleephrs 1 148.24 38480 3752.9
        38628 3754.9
<none>
Step: AIC=3747.76
bmi ~ exerany + alcdays + female
         Df Sum of Sq RSS AIC
+ sleephrs 1 142.51 38147 3745.9
             38289 3747.8
<none>
```

Forward Selection Stepwise Regression, Results: 3

```
Step: AIC=3745.89
bmi ~ exerany + alcdays + female + sleephrs

Call:
lm(formula = bmi ~ exerany + alcdays + female + sleephrs)

Coefficients:
(Intercept) exerany alcdays female sleephrs
32.3227 -2.1451 -0.0969 -1.1855 -0.2439
```

Conclusions?

- Forward selection and backwards elimination show the same model, which is also the kitchen sink model.
 - Does that mean that the model is right?
 - Does that mean that the model is good?
 - Does that mean that the model is the best possible combination of these predictors?
- Should we feel substantially more confident about the above statements when the forward selection result = the backwards elimination result, as in our model for bmi using smartcle3?

Conclusions?

- Forward selection and backwards elimination show the same model, which is also the kitchen sink model.
 - Does that mean that the model is right?
 - Does that mean that the model is good?
 - Does that mean that the model is the best possible combination of these predictors?
- Should we feel substantially more confident about the above statements when the forward selection result = the backwards elimination result, as in our model for bmi using smartcle3?
- No.

Validating the Model (See Section 6)

Training and Test Samples (as in 431)

Suppose we want to evaluate whether our model_ks predicts effectively in new data.

One approach (used, for instance, in 431) would be to split our sample into a separate training (perhaps 70% of the data) and test (perhaps 30% of the data) samples, and then:

- 1 fit the model in the training sample,
- use the resulting model to make predictions for bmi in the test sample, and
- evaluate the quality of those predictions, perhaps by comparing the results to what we'd get using a different model.

But there are problems with this approach, especially if n is small.

What else could we do?

Suppose we're afraid that our model building and testing will be hampered by a small sample size.

- A potential solution is the idea of cross-validation, which involves
 partitioning our data into a series of training-test subsets, multiple
 times, and then combining the results.
- So, in the next slides, I'll show you how to do something called 10-fold cross validation using some tools from the modelr package, which is a non-core part of the tidyverse.

10-fold cross validation: The idea

- Split the 1,036 observations in our smartcle3 data frame into a partition of about 90% (so about 932 observations) for a training sample, leaving the remaining 10% (about 104 observations) for a test sample. Label the test sample .id = 1 in R.
- Refit our model (here, kitchen sink) to the training sample, and use it to predict our outcome (bmi) in the test sample.
- Store the prediction results for the subjects in the test sample.
- Split the observations again, ensuring that a completely new 10% gets held out for the test sample, labeling this new test sample .id = 2 in R. Then redo parts 2 and 3. Now you have prediction results for 20% of the subjects in the original data.
- Repeat the process (10x in total) until you have prediction results for all 100% of the subjects in the original data. Thus, each observation is used 9 times in the training sample, and once in the test sample.

10-fold cross-validation of mod_ks

```
set.seed(432021)
sink models <- smartcle3 %>%
    crossv kfold(k = 10) %>%
    mutate(model = map(train, ~
                         lm(bmi ~ female + sleephrs +
                              alcdays + exerany, data = .)))
sink predictions <- sink models %>%
    unnest(map2(model, test, ~ augment(.x, newdata = .y)))
```

The first few cross-validated predictions

```
# A tibble: 3 \times 13
 .id
           SEQNO bmi sleephrs female alcdays exerany
 <chr>
           <dbl> <dbl>
                         <int> <int> <dbl> <dbl>
1 01 2016000003 26.9
                            8
                                       4.00
2 01 2016000025 21.0
                            8
                                         1.00
3 01 2016000028 21.2
                            7
                                       4.00 1.00
# ... with 6 more variables: sleep c <dbl>, sleep z <dbl>,
   exer 12 <dbl>, exer yn <fct>, .fitted <dbl>, .se.fit
#
```

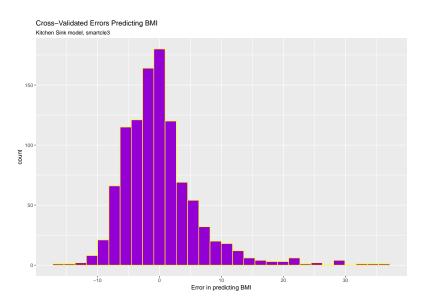
<dbl>

#

head(sink predictions, 3)

Graphing the Cross-Validated Prediction Errors of bmi (code)

Cross-Validated Prediction Errors of bmi



Summary Statistics Based on Cross-Validated Prediction Errors

We'll look at the **root mean squared prediction error** or RMSE, and the **mean absolute error**, too.

Comparison to a Model with the Intercept Only (predict mean BMI)?

```
sink_predictions %>%
summarize(RMSE_sink = sqrt(mean((bmi - .fitted) ^2)),
    RMSE_intercept = sqrt(mean((bmi - mean(bmi))^2)),
    MAE_sink = mean(abs(bmi - .fitted)),
    MAE_intercept = mean(abs(bmi - mean(bmi)))) %>%
round(., 3)
```

Next Week

- Homework 1 discussion in class Tuesday
- Stepwise Regression via the Allen-Cady Procedure
- Best Subsets approaches to Variable Selection
- Making Decisions about Non-Linearity in Y or in the Xs