432 Class 12 Slides

github.com/THOMASELOVE/432-2018

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Setup

```
library(skimr)
library(pROC)
library(ROCR)
library(rms) # note: also loads Hmisc
library(simputation)
library(broom)
library(tidyverse)
```

Today's Materials

- Logistic Regression and the Framingham Study (conclusion)
- Performing Linear Regression with ols
- Hormone Therapy and Baseline LDL in the HERS trial

The HERS trial is described in Vittinghoff et al., especially Chapter 4.

Logistic Regression and Framingham

Data Ingest, Cleanup (from Class 10)

```
fram <- read.csv("data/fram new.csv") %>% tbl df
set.seed(432001)
fram1 <- fram %>%
    impute_pmm(educ + cigs_day + heart_r ~ age + smoker) %>%
    impute_rlm(bmi + tot_chol ~ sex + age + sbp + heart_r) %>
    impute pmm(bp meds ~ hx htn + bmi + tot chol) %>%
    impute rlm(glucose ~ hx dm + bmi + tot chol + age) %>%
   mutate(ed f = fct recode(factor(educ),
                   "1 Some HS" = "1", "2 HS grad" = "2",
                   "3 Some Col" = "3", "4 Col grad" = "4"))
fram2 <- fram1 %>%
    select(subj, sex, age, smoker, cigs_day, bp_meds,
           hx_stroke, hx_htn, hx_dm, ed_f, tot_chol,
           sbp, dbp, bmi, heart_r, glucose, CHD_10)
```

The Models We've Fit (predicting CHD_10)

```
m 01 \leftarrow glm(CHD 10 \sim hx htn, data = fram2,
             family = binomial)
d <- datadist(fram2)</pre>
options(datadist = "d")
m_01_lrm <- lrm(CHD_10 ~ hx_htn, data = fram2, x = T, y = T)
m_0^2 \leftarrow glm(CHD_1^0 \sim hx_htn + tot_chol,
             data = fram2, family = binomial)
m 02 lrm <- lrm(CHD 10 ~ hx htn + tot chol, data = fram2,
                 x = TRUE, y = TRUE
```

Then We Fit a Kitchen Sink Model (Class 11)

```
m_03 \leftarrow glm(CHD_10 \sim hx_htn + tot_chol + sex + age +
                      smoker + cigs day + bp meds +
                      hx stroke + hx dm + ed f + sbp + dbp +
                      bmi + heart r + glucose,
                 data = fram2, family = binomial)
d <- datadist(fram2)</pre>
options(datadist = "d")
m_03_{lrm} \leftarrow lrm(CHD_10 \sim hx_{htn} + tot_{chol} + sex + age +
                      smoker + cigs_day + bp_meds +
                      hx_stroke + hx_dm + ed_f + sbp + dbp +
                      bmi + heart_r + glucose,
                 data = fram2, x = TRUE, y = TRUE)
```

Comparing the Current Models

Model	C statistic	Nagelkerke R ²
m_01_1rm	0.614	0.051
m_02_1rm	0.640	0.055
m_03_1rm	0.733	0.159

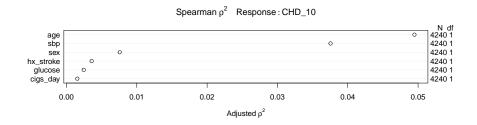
m_03_1rm shows the following predictors as highly significant:

• age, sex, sbp, cigs_day, glucose, and hx_stroke

Fitting a 6-predictor, but still useful model

What looks useful?

By ANOVA on m_03_1rm it looks like age, sex, sbp, cigs_day, glucose, hx_stroke for sure.



New Model 4

```
m 04 \leftarrow glm(CHD 10 \sim rcs(age, 5) + rcs(sbp, 3) + sex +
                     hx_stroke + glucose + cigs_day,
                 data = fram2, family = binomial)
dd <- datadist(fram2)</pre>
options(datadist = "dd")
m 04 1rm < -1rm(CHD 10 \sim rcs(age, 5) + rcs(sbp, 3) + sex +
                     hx stroke + glucose + cigs day,
                 data = fram2, x = TRUE, y = TRUE)
```

m 04 1rm

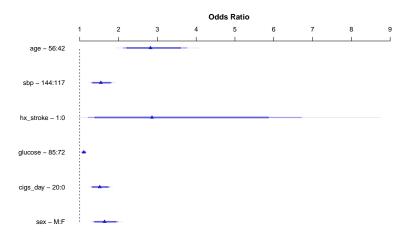
```
> m 04 1rm
Logistic Regression Model
 lrm(formula = CHD 10 \sim rcs(age. 5) + rcs(sbp. 3) + sex + hx stroke +
     glucose + cigs_day, data = fram2, x = TRUE, y = TRUE)
                     Model Likelihood
                                         Discrimination
                                                          Rank Discrim.
                                            Indexes
                                                             Indexes
                        Ratio Test
 0bs
             4240
                     LR chi2
                                401.44
                                         R2
                                                  0.158
                                                                  0.731
  0
             3596
                     d.f.
                                    10
                                                  1.041
                                         a
                                                          Dxv
                                                                  0.461
                     Pr(> chi2) <0.0001
              644
                                         gr
                                                 2.833
                                                          gamma
                                                                  0.461
 max |deriv| 2e-06
                                                  0.120
                                                                  0.119
                                                           tau-a
                                          gp
                                         Brier
                                                  0.115
          Coef
                 S.E.
                        Wald Z Pr(>|Z|)
 Intercept -8.7201 2.4092 -3.62 0.0003
 age
           0.0732 0.0576 1.27 0.2037
 age'
        0.2871 0.3539
                         0.81 \quad 0.4172
 age''
       -1.1057 0.9608 -1.15 0.2498
 age'''
        1.4723 0.9640 1.53 0.1267
 sbp
           0.0147 0.0066 2.23 0.0256
 sbp'
           0.0030 0.0080 0.37 0.7080
           0.4935 0.0973 5.07 < 0.0001
 sex=M
 hx_stroke 1.0514 0.4345 2.42 0.0155
 alucose
           0.0076 0.0016 4.69 < 0.0001
 cigs_day
           0.0208 0.0039 5.39
                               < 0.0001
```

Validating our Summary Statistics

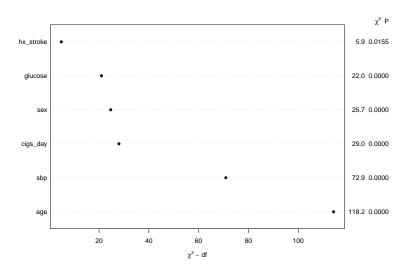
```
set.seed(432329); validate(m_04_lrm)[1:4,]
```

```
index.orig training
                                     test
                                             optimism
Dxy
          0.4611110 0.4672261 0.45678154 0.010444568
R2
          0.1575417 0.1629811 0.15375377 0.009227324
Intercept 0.0000000 0.0000000 -0.04796015 0.047960154
       1.0000000 1.0000000
                               0.96803833 0.031961665
Slope
          index.corrected
Dxy
              0.45066647 40
R.2.
              0.14831438 40
           -0.04796015 40
Intercept
              0.96803833 40
Slope
```

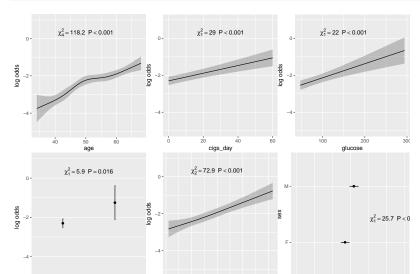
plot(summary(m_04_lrm))



plot(anova(m_04_lrm))

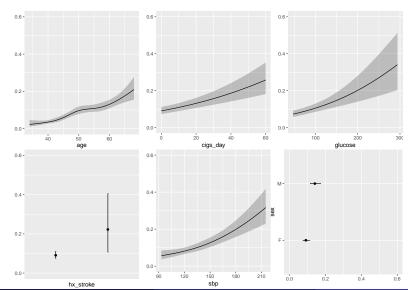


Can we see the prediction results?



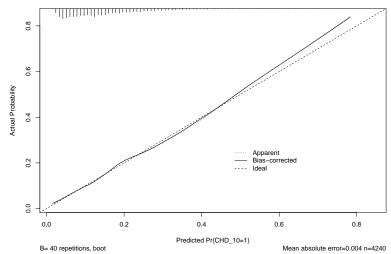
What about on a better scale?

ggplot(Predict(m_04_lrm, fun = plogis))



Calibration of mod_04_1rm

set.seed(432612); plot(calibrate(m_04_lrm))



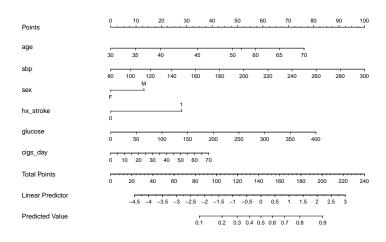
Goodness of fit test?

```
round(residuals(m_04_lrm, type = "gof"),3)
```

```
Sum of squared errors
              488.554
                    SD
                 1.183
                 0.462
```

Expected value | HO 489.423 -0.735

Nomogram of mod_04_1rm



Comparing Models 3 and 4 (which aren't nested)

```
glance(m_03) # kitchen sink but no non-linear terms
```

```
null.deviance df.null logLik AIC BIC

1 3612.209 4239 -1603.407 3242.813 3357.155

deviance df.residual

1 3206.813 4222
```

```
glance(m_04) # six predictors but with non-linear terms
```

```
null.deviance df.null logLik AIC BIC deviance
1    3612.209    4239 -1605.382 3232.764 3302.64 3210.764
    df.residual
1    4229
```

Checking Residuals?

- Yes/No outcomes contain less information than quantitative outcomes
- Residuals cannot be observed predicted
 - There are several different types of residuals defined
- Assumptions of logistic regression are different
 - Model is deliberately non-linear
 - Error variance is a function of the mean, so it isn't constant
 - Errors aren't assumed to follow a Normal distribution
 - Only thing that's the same: leverage and influence

So, plot 5 (residuals/leverage/influence) can be a little useful, but that's it.

• We'll need better diagnostic tools for generalized linear models.

Any observations particularly influential on Model 4?

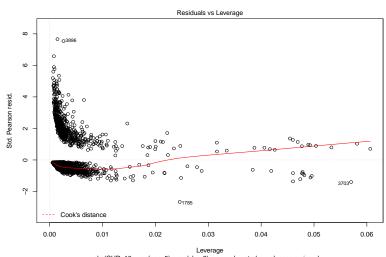
```
which.influence(m_04_lrm, cutoff = 0.3)
$Intercept
```

```
$glucose
[1] "1785" "3703"
```

[1] "3896"

Influence and Model 4?

$$plot(m_04, which = 5)$$



 $\label{eq:glm} \mbox{glm(CHD_10 \sim $rcs(age, 5)$ + $rcs(sbp, 3)$ + $sex + hx_stroke + glucose + $cigs_d$...}$

The HERS Trial

Hormone Therapy and Baseline LDL in the HERS Trial

HERS clinical trial of hormone therapy (HT). We're excluding the women with diabetes here.

The Data

head(hers1)

```
# A tibble: 6 \times 10
 subject LDL HT age smoking drinkany
                                      SBP physact
   <int> <dbl> <fct> <int> <fct>
                             <fct> <int> <fct>
      1 122 place~ 70 no
                                      138 much m~
                             no
      2 242 place~ 62 no no 118 much 1~
3
      4 116 place~ 64 yes yes 152 much 1~
4
      5
         151 place~ 65 no
                             no
                                    175 somewh~
5
      6 138 hormo~ 68 no
                             yes 174 about ~
6
         121 hormo~ 69 no
                                  178 much m~
                             no
 ... with 2 more variables: BMI <dbl>, diabetes <fct>
```

The Codebook (n = 2,032 women without diabetes)

Variable	Description	Missing?
subject	subject code	0
LDL	LDL cholesterol in mg/dl	7
HT	factor: hormone therapy or placebo	0
age	age in years	0
smoking	yes or no	0
drinkany	yes or no	2
SBP	systolic BP in mm Hg	0
physact	5-level factor	0
BMI	body-mass index in kg/m ²	2
diabetes	yes or no (all of these are no)	0

Our Modeling Goal

Predict LDL using

- age
- smoking
- drinkany
- SBP
- physact
- BMI
- the interaction of smoking and BMI

Details on physact variable

```
hers1 %>% count(physact)
```

```
# A tibble: 5 x 2
physact n
<fct> cint>
1 about as active 674
2 much less active 107
3 much more active 252
4 somewhat less active 322
5 somewhat more active 677
```

Skim?

hers1 %>% select(-subject) %>% skim()

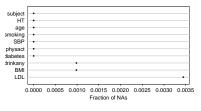
```
> hers1 %>% select(-subject) %>% skim()
Skim summary statistics
n obs: 2032
n variables: 9
Variable type: factor
 variable missing complete n n_unique
                                                           top_counts ordered
diabetes
                                           no: 2032, yes: 0, NA: 0
                  2032 2032
                                                                      FALSE
 drinkany 2 2030 2032
                                              no: 1135, yes: 895, NA: 2
                                                                      FALSE
     HT 0 2032 2032
                                            pla: 1031, hor: 1001, NA: 0
                                                                      FALSE
         0 2032 2032 5 som: 677, abo: 674, som: 322, muc: 252
 physact
                                                                      FALSE
 smokina
                                              no: 1733. ves: 299. NA: 0
                  2032 2032
                                                                      FALSE
Variable type: integer
 variable missing complete n mean sd p0 p25 median p75 p100
     age
             0 2032 2032 66.89 6.75 44 62 67 72 79
     SBP
                  2032 2032 133.38 18.47 83 120 132 145 197
Variable type: numeric
 variable missing complete n
                                    sd
                                         pO p25 median
                                                         p75
                                                              p100
                                                                      hist
                             mean
             2 2030 2032 27.67 5.14 15.21 24.2 26.89
                                                       30.27 54.13
     BMI
                  2025 2032 145.65 37.07 36.8 120.6 141.4 166
                                                             351.2
     LDL
```

Missingness pattern?

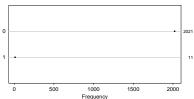
```
na.pattern(hers1) # from Hmisc
pattern
2021
names(hers1)
 [1] "subject" "LDL"
                       "HT"
                                 "age"
                                           "smoking"
 [6] "drinkany" "SBP"
                                           "diabetes"
                       "physact" "BMI"
Next slide
par(mfrow = c(2,2))
naplot(naclus(hers1))
par(mfrow = c(1,1))
```

naplot(naclus(hers1))

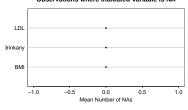
Fraction of NAs in each Variable

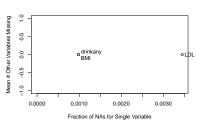


Number of Missing Variables Per Observation



Mean Number of Other Variables Missing for Observations where Indicated Variable is NA





Simple Imputation into hers2

Simple Imputation for drinkany, BMI and LDL

Since drinkany is a factor, we have to do some extra work to impute.

```
set.seed(432092)
hers2 <- hers1 %>%
    mutate(drinkany n =
               ifelse(drinkany == "yes", 1, 0)) %>%
    impute_pmm(drinkany_n ~ age + smoking) %>%
    mutate(drinkany =
               ifelse(drinkany_n == 1, "yes", "no")) %>%
    impute_rlm(BMI ~ age + smoking + SBP) %>%
    impute_rlm(LDL ~ age + smoking + SBP + BMI)
```

Now, check missingness...

```
na.pattern(hers2)
```

pattern 0000000000 2032

names(hers2)

```
"subject"
                  "LDL"
                                "HT"
                                              "age"
[5]
   "smoking"
                                              "physact"
                  "drinkany"
                                "SBP"
                  "diabetes"
[9]
   "BMI"
                                "drinkany_n"
```

Multiple Imputation with aregImpute

Multiple Imputation using aregImpute from Hmisc

Model to predict all missing values of any variables, using additive regression bootstrapping and predictive mean matching.

Steps are:

- aregImpute draws a sample with replacement from the observations where the target variable is observed, not missing.
- ② It then fits a flexible additive model to predict this target variable while finding the optimum transformation of it.
- It then uses this fitted flexible model to predict the target variable in all of the original observations.
- Finally, it imputes each missing value of the target variable with the observed value whose predicted transformed value is closest to the predicted transformed value of the missing value.

Fitting a Multiple Imputation Model

```
Iteration 1
Iteration 2
Iteration 3
Iteration 4
Iteration 5
Iteration 6
Iteration 7
Iteration 8
Iteration 9
```

Multiple Imputation using aregImpute from Hmisc

aregImpute requires specifications of all variables, and several other details:

- n.impute = number of imputations, we'll run 20
- nk = number of knots to describe level of complexity, with our choice
 nk = c(0, 3:5) we'll fit both linear models and models with
 restricted cubic splines with 3, 4, and 5 knots
- tlinear = FALSE allows the target variable to have a non-linear transformation when nk is 3 or more
- B = 10 specifies 10 bootstrap samples will be used
- data specifies the source of the variables

aregImpute Imputation Results (1 of 3)

fit3

```
> fit3
Multiple Imputation using Bootstrap and PMM
aregImpute(formula = ~LDL + age + smoking + drinkanv + SBP +
    physact + BMI, data = hers1, n.impute = 5, nk = c(0, 3:5),
    tlinear = FALSE, B = 10)
n: 2032 p: 7 Imputations: 5 <u>nk: 0</u>
Number of NAs:
             age smoking drinkany SBP
                                             physact
    LDL
                                                         \mathsf{BMI}
        type d.f.
LDL
age
smoking
drinkany c
                1
SRP
physact c
                4
\mathsf{BMI}
```

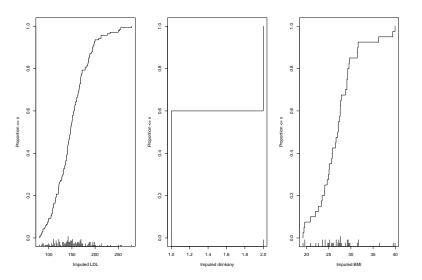
aregImpute Imputation Results (2 of 3)

```
R-squares for Predicting Non-Missing Values for Each Variable
Using Last Imputations of Predictors
    LDL drinkanv
                      BMT
  0.019
           0.029
                    0.093
Resampling results for determining the complexity of imputation models
Variable being imputed: LDL
                                          nk=0 nk=3 nk=4
                                                                   nk=5
Bootstrap bias-corrected R^2
                                        0.0151 0.0149 0.0101 0.00976
10-fold cross-validated R^2
                                        0.0154 0.0224 0.0129 0.01913
Bootstrap bias-corrected mean
                               lerror 28.2956 42.5462 43.9458 39.56345
10-fold cross-validated mean
                               lerror 145.8339 43.2577 44.8477 45.07517
Bootstrap bias-corrected median |error| 22.7110 35.0460 38.9420 32.95988
10-fold cross-validated median |error| 142.1896 35.3638 38.3446 38.08496
```

aregImpute Imputation Results (3 of 3)

```
Variable being imputed: drinkany
                                         nk=0
                                                nk=3 nk=4
                                                              nk=5
Bootstrap bias-corrected R^2
                                       0.0138 0.0110 0.0131 0.0104
10-fold cross-validated R^2
                                       0.0191 0.0184 0.0182 0.0122
Bootstrap bias-corrected mean
                                |error| 0.4535 0.4529 0.4551 0.4567
10-fold cross-validated
                        mean
                                error 0.4482 0.4499 0.4382 0.4644
Bootstrap bias-corrected median |error| 0.0000 0.0000 0.0000 0.0000
10-fold cross-validated median |error| 0.2000 0.3000 0.0000 0.2000
Variable being imputed: BMI
                                          nk=0
                                                 nk=3 nk=4
                                                               nk=5
                                        0.0851 0.0878 0.0858 0.0892
Bootstrap bias-corrected R^2
10-fold cross-validated
                        R\Lambda 2
                                        0.0880 0.0940 0.0930 0.0923
Bootstrap bias-corrected mean
                                lerror 3.8316 4.8352 4.9466 5.1806
10-fold cross-validated
                        mean
                                error 27.6718 4.8115 4.9504 4.9765
Bootstrap bias-corrected median
                                lerrorl
                                        2.9944 4.0176 4.0021 4.2517
10-fold cross-validated
                        median
                                error
                                       27.0170 4.0261 3.9930 4.0836
```

A plot of the imputed values... (results)



A plot of the imputed values... (code)

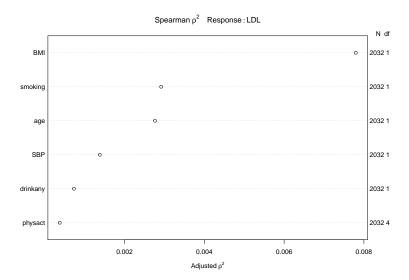
```
par(mfrow = c(1,3))
plot(fit3)
par(mfrow = c(1,1))
```

- For LDL, we imputed most of the 7 missing subjects in most of the 20 imputation runs to values within a range of around 120 through 200, but occasionally, we imputed values that were substantially lower than 100.
- For drinkany we imputed about 70% no and 30% yes.
- For BMI, we imputed values ranging from about 23 to 27 in many cases, and up near 40 in other cases.
- This method never imputes a value for a variable that doesn't already exist in the data.

Spearman ρ^2 **Plot**

We've already decided to include a BMI*smoking product term, but how should we prioritize the degrees of freedom we spend on non-linearity otherwise?

Spearman ρ^2 **Plot** Result



Fitting a Linear Regression with ols

Model we'll fit

Fitting a model to predict LDL using

- BMI with a restricted cubic spline, 5 knots
- age with a quadratic polynomial
- SBP as a linear term
- drinkany indicator
- physact factor
- smoking indicator and its interaction with BMI

We could fit this to the data

- restricted to complete cases (hers1, effectively)
- after simple imputation (hers2)
- after our multiple imputation (fit3)

Fitting the model after simple imputation

where %ia% identifies the linear interaction alone.

m2 results (slide 1 of 2)

```
> m2
Linear Regression Model
ols(formula = LDL \sim rcs(BMI, 5) + pol(age, 2) + SBP + drinkany +
    physact + smoking + smoking %ia% BMI, data = hers2, x = TRUE,
    V = TRUE
              Model Likelihood Discrimination
                Ratio Test
                                  Indexes
0bs
    2032 LR chi2 53.14
                               R2 0.026
sigma36.6503 d.f. 14
                                R2 adi 0.019
d.f. 2017 Pr(> chi2) 0.0000
                                g 6.631
Residuals
    Min 1Q Median 3Q Max
 -113.379 -24.326 -3.835 20.832 197.097
```

m2 results (slide 2 of 2)

	Coef	S.E.	t	Pr(> t)
Intercept	120.2662	67.6113	1.78	0.0754
BMI	1.5508	1.0071	1.54	0.1237
BMI'	-8.4486	9.0978	-0.93	0.3532
BMI''	39.6413	37.1378	1.07	0.2859
BMI'''	-54.8924	44.2677	-1.24	0.2151
age	-0.5249	1.9490	-0.27	0.7877
age^2	0.0014	0.0148	0.10	0.9233
SBP	0.1209	0.0451	2.68	0.0074
drinkany=yes	-3.7023	1.6544	-2.24	0.0253
physact=much less active	-4.7408	3.8621	-1.23	0.2198
physact=much more active	-0.2635	2.7391	-0.10	0.9234
physact=somewhat less active	0.0130	2.5101	0.01	0.9959
physact=somewhat more active		2.0193	1.88	0.0598
smoking=yes		12.0196	-0.57	0.5662
smoking=yes * BMI	0.4892	0.4375	1.12	0.2636
3 7				

Validation of summary statistics

validate(m2)

```
index.orig
                     training
                                  test optimism
R-square
             0.0258
                       0.0307
                                0.0188
                                         0.0119
MSE
          1333.3300 1320.0677 1342.9027 -22.8350
             6.6306 7.1548
                                5.8726 1.2821
g
             0.0000 0.0000 26.2153 -26.2153
Intercept
Slope
             1.0000 1.0000
                                0.8208 0.1792
         index.corrected
R-square
                  0.0139 40
               1356.1650 40
MSE.
g
                  5.3485 40
                 26.2153 40
Intercept
Slope
                  0.8208 40
```

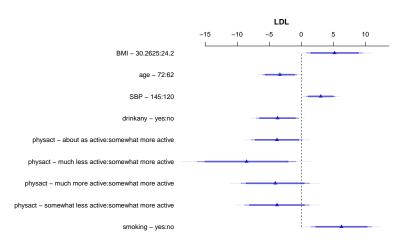
anova(m2) results

> anova(m2)	
Analysis of Variance	Response: LDL
Factor	d.f. Partial SS MS F P
BMI (Factor+Higher Order Factors)	5 2.758824e+04 5517.64861 4.11 0.0010
All Interactions	1 1.679813e+03 1679.81344 1.25 0.2636
Nonlinear	3 9.735452e+03 3245.15068 2.42 0.0647
age	2 9.175762e+03 4587.88077 3.42 0.0330
Nonlinear	1 1.244351e+01 12.44351 0.01 0.9233
SBP	1 9.657476e+03 9657.47569 7.19 0.0074
drinkany	1 6.726918e+03 6726.91809 5.01 0.0253
physact	4 9.709992e+03 2427.49791 1.81 0.1247
smoking (Factor+Higher Order Factors)	2 1.085405e+04 5427.02463 4.04 0.0177
All Interactions	1 1.679813e+03 1679.81344 1.25 0.2636
smoking * BMI (Factor+Higher Order Factors)	1 1.679813e+03 1679.81344 1.25 0.2636
TOTAL NONLINEAR	4 9.738807e+03 2434.70175 1.81 0.1237
TOTAL NONLINEAR + INTERACTION	5 1.171134e+04 2342.26845 1.74 0.1214
REGRESSION	14 7.178905e+04 5127.78931 3.82 <.0001
ERROR	2017 2.709327e+06 1343.24569

summary(m2) results

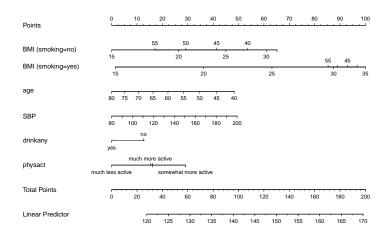
```
summary(m2)
            Effects
                                  Response : LDL
Factor
                                                           High
                                                                           Effect S.E.
                                                                                          Lower 0.95 Upper 0.95
 RMT
                                                            30.263
                                                                   6.0625
                                                                            5.1862 2.2217
                                                                                            0.82921
                                                                                                       9.54330
                                                      62.0 72.000 10.0000 -3.3412 1.3450
                                                                                           -5.97890
                                                                                                     -0.70357
 age
 SRP
                                                     120.0 145.000 25.0000
                                                                            3.0218 1.1270
                                                                                            0.81165
                                                                                                       5.23190
drinkany - yes:no
                                                       1.0
                                                             2.000
                                                                        NA -3.7023 1.6544
                                                                                            -6.94690
                                                                                                      -0.45779
physact - about as active:somewhat more active
                                                       5.0
                                                             1.000
                                                                        NA -3.8031 2.0193
                                                                                                      0.15695
                                                                                           -7.76310
physact - much less active:somewhat more active
                                                       5.0
                                                             2.000
                                                                        NA -8.5439 3.9035 -16.19900
                                                                                                     -0.88862
physact - much more active:somewhat more active
                                                       5.0
                                                             3.000
                                                                        NA -4.0666 2.7125
                                                                                           -9.38630
                                                                                                       1.25310
physact - somewhat less active:somewhat more active
                                                       5.0
                                                             4.000
                                                                        NA -3.7901 2.5633
                                                                                           -8.81720
                                                                                                       1.23690
smoking - yes:no
                                                       1.0
                                                             2.000
                                                                        NA 6.2635 2.4009
                                                                                            1.55500
                                                                                                     10.97200
Adjusted to: BMI=26.9 smoking=no
```

plot(summary(m2)) results



Adjusted to:BMI=26.9 smoking=no

plot(nomogram(m2))



Making Predictions for an Individual

Suppose now that we want to use R to get a prediction for a new individual subject with BMI = 30, age = 50, smoking = yes and physact = about as active, drinkany= yes and SBP of 150.

```
$linear.predictors $lower $upper
160.9399 88.48615 233.3936
```

Making Predictions for a Long-Run Mean

The other kind of prediction we might wish to make is for the mean of a series of subjects whose BMI = 30, age = 50, smoking = yes and physact = about as active, drinkany= yes and SBP of 150.

```
$linear.predictors $lower $upper
160.9399 151.8119 170.0679
```

Of course, the confidence interval will always be narrower than the prediction interval given the same predictor values.

Influential Points?

```
which.influence(m2, cutoff = 0.4)
$Intercept
[1] 1135
$age
[1] 1135
$smoking
[1] 132
```

Fitting the model to the complete cases

where %ia% identifies the linear interaction alone.

Putting it Together

What have we got?

• An imputation model fit3

A prediction model

Now we put them together

Linear Regression & Imputation Model

Variance Inflation Factors Due to Imputation:

Intercept	BMI
1.00	1.00
BMI'	BMI''
1.00	1.00
BMI'''	age
1.00	1.00
age^2	SBP

m3imp results (1 of 2)

```
> m3imp
Linear Regression Model
fit.mult.impute(formula = LDL ~ rcs(BMI, 5) + pol(age, 2) + SBP +
    drinkany + physact + smoking + smoking %ia% BMI, fitter = ols,
    xtrans = fit3, data = hers1)
              Model Likelihood
                                Discrimination
                 Ratio Test
                                  Indexes
0bs
    2032 LR chi2 53.30
                                R2 0.026
sigma36.7128 d.f. 14
                                R2 adj 0.019
d.f. 2017 Pr(> chi2) 0.0000
                                 6.652
Residuals
    Min 10 Median 30
                               Max
 -113.10 -24.46 -3.81 20.92 197.42
```

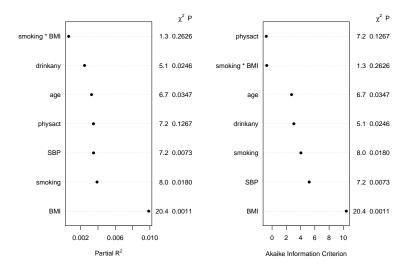
m3imp results (2 of 2)

	Coef	S.E.	t	Pr(> t)
Intercept	121.1499	67.7998	1.79	0.0741
BMI	1.5445	1.0097	1.53	0.1263
BMI'	-8.2945	9.1027	-0.91	0.3623
BMI''	39.0890	37.3055	1.05	0.2949
BMI'''	-54.2119	44.4779	-1.22	0.2230
age	-0.5521	1.9547	-0.28	0.7776
age^2	0.0016	0.0148	0.11	0.9119
SBP	0.1216	0.0453	2.69	0.0073
drinkany=yes	-3.7404	1.6625	-2.25	0.0246
	-4.7426	3.8692	-1.23	0.2204
physact=much more active	-0.2665	2.7455	-0.10	0.9227
physact=somewhat less active	0.0313	2.5214	0.01	0.9901
physact=somewhat more active		2.0257	1.88	0.0604
smoking=yes		12.0472		
smoking=yes * BMI	0.4917	0.4388	1.12	0.2626

anova(m3imp)

```
anova(m3imp)
               Analysis of Variance
                                              Response: LDL
                                             d.f. Partial SS
Factor
                                                                MS
BMI (Factor+Higher Order Factors)
                                                     27514.6406 5502.9281 4.08 0.0011
 All Interactions
                                                     1692 6044 1692 6044 1 26 0 2626
 Nonlinear
                                                     9741.6194 3247.2065 2.41 0.0653
                                                     9078.9851 4539.4926 3.37 0.0347
age
 Nonlinear
                                                        16.5032
                                                                  16.5032 0.01 0.9119
SBP
                                                     9721.1667 9721.1667 7.21 0.0073
drinkany
                                                     6822.3861 6822.3861 5.06 0.0246
physact
                                                     9690.3632 2422.5908 1.80 0.1267
smoking (Factor+Higher Order Factors)
                                                     10845.6127 5422.8063 4.02 0.0180
 All Interactions
                                                     1692.6044 1692.6044 1.26 0.2626
smoking * BMI (Factor+Higher Order Factors)
                                                     1692.6044 1692.6044 1.26 0.2626
TOTAL NONLINEAR
                                                     9747.0966 2436.7741 1.81 0.1246
TOTAL NONLINEAR + INTERACTION
                                                     11717.3715 2343.4743 1.74 0.1225
REGRESSTON
                                                     71571.1297 5112.2236 3.79 < .0001
ERROR
                                              2017 2718570.0412 1347.8285
```

Evaluation via Partial R² and AIC (result)



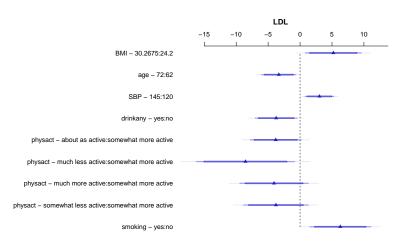
Evaluation via Partial R² and AIC (code)

```
par(mfrow = c(1,2))
plot(anova(m3imp), what="partial R2")
plot(anova(m3imp), what="aic")
par(mfrow = c(1,1))
```

summary(m3imp)

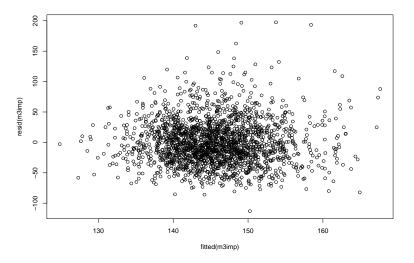
```
summary(m3imp)
            Effects
                                  Response : LDL
Factor
                                                            Hiah
                                                                    Diff.
                                                                            Effect S.F.
                                                                                           Lower 0.95 Upper 0.95
                                                     Low
 RMT
                                                       24.2
                                                             30.268
                                                                     6.0675 5.2165 2.2287
                                                                                             0.84565
                                                                                                        9.58730
 age
                                                            72,000 10,0000 -3,3219 1,3498
                                                                                             -5.96910
                                                                                                       -0.67463
SBP
                                                      120.0 145.000 25.0000
                                                                             3.0394 1.1317
                                                                                             0.81989
                                                                                                        5.25880
drinkany - ves:no
                                                              2,000
                                                                         NA -3.7404 1.6625
                                                                                            -7.00080
                                                                                                       -0.47996
                                                        1.0
physact - about as active:somewhat more active
                                                        5.0
                                                              1,000
                                                                         NA -3.8060 2.0257
                                                                                             -7.77860
                                                                                                        0.16663
physact - much less active:somewhat more active
                                                        5.0
                                                              2.000
                                                                         NA -8.5486 3.9114 -16.21900
                                                                                                       -0.87779
physact - much more active:somewhat more active
                                                        5.0
                                                              3.000
                                                                         NA -4.0724 2.7198
                                                                                            -9.40640
                                                                                                        1.26160
physact - somewhat less active:somewhat more active
                                                        5.0
                                                              4.000
                                                                         NA -3.7746 2.5773
                                                                                            -8.82900
                                                                                                        1.27980
                                                                            6.3043 2.4196
smoking - yes:no
                                                        1.0
                                                              2.000
                                                                                             1.55900
                                                                                                      11.05000
Adjusted to: BMI=26.895 smoking=no
```

plot(summary(m3imp))



Adjusted to:BMI=26.895 smoking=no

plot(resid(m1imp) ~ fitted(m1imp))



plot(nomogram(m3imp))

