# Assignment 3 Answer Sketch

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Due 2017-02-09. Sketch developed 2018-02-10

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|    |   | y(gapminder)<br>y(skimr)  |  |  |  |  |
|    |   | y(broom)  |  |  |  |  |
|    |   | y(leaps)  |  |  |  |  |
|    | •   | y(modelr)   |  |  |  |  |
|    |   | y(tidyverse)  |  |  |  |  |
| sk | im_w  | <pre>ith(numeric = list(hist = NULL),    integer = list(hist = NULL)) # drop histograms</pre> |  |  |  |  |
|    | integer - list(hist - Noll) / # wrop histograms |   |  |  |  |  |

# 1 Question 1

Consider the hbp330 data used in Homeworks 1 and 2. In question 4 of HW2, you built a model for the prediction of body-mass index, considering the following 14 predictors: practice, age, race, eth\_hisp, sex, insurance, income, hsgrad, tobacco, depdiag, sbp, dbp, statin and bpmed. Your task now is to fit a Spearman rho-squared plot to identify the candidate variables on which you might most reasonably try to address non-linearity in a model predicting body-mass index, now making use of as much of the data set that missing data allow. Show the plot, and provide a written explanation of your conclusions about it, and specify the variables that are most appealing for non-linear augmentations, all in complete sentences. Which variables are most appealing candidates to add non-linear evaluations to a linear fit to the complete set of 14 predictors, and why?

#### 1.1 Data Load

```
hbp330 <- read.csv("data/hbp330.csv") %>% tbl_df
```

#### 1.2 Create bmi variable

```
hbp330 <- hbp330 %>%
mutate( bmi = weight / (height*height))
```

```
Full hbp330 data (including Missing Values)
hbp330_full <- hbp330 %>%
    select(subject, bmi, practice, age, race, eth_hisp, sex,
                    insurance, income, hsgrad, tobacco,
                    depdiag, sbp, dbp, statin, bpmed)
hbp330_full %>% skim(-subject)
Skim summary statistics
n obs: 330
n variables: 16
Variable type: factor
  variable missing complete
                               n n_unique
                        330 330
   depdiag
                 0
                                        2
  eth_hisp
                 5
                        325 330
                        330 330
                                        4
 insurance
                 0
                 0
                        330 330
                                        2
  practice
                 2
                                        4
      race
                        328 330
                                        2
       sex
                 0
                        330 330
                 0
                        330 330
                                        3
   tobacco
                            top_counts ordered
             No: 214, Yes: 116, NA: 0
                                         FALSE
              No: 261, Yes: 64, NA: 5
                                         FALSE
 Med: 134, Med: 130, Com: 53, Uni: 13
                                         FALSE
                A: 180, B: 150, NA: 0
                                         FALSE
  Bla: 180, Whi: 131, Asi: 10, Mul: 7
                                         FALSE
                F: 203, M: 127, NA: 0
                                         FALSE
   nev: 140, for: 117, cur: 73, NA: 0
                                         FALSE
Variable type: integer
 variable missing complete
                                                                       p75
                            n
                                    mean
                                               sd
                                                   р0
                                                         p25
                                                              median
                0
                       330 330
                                   55.35
                                            11.53
                                                   23
                                                          48
                                                                57
                                                                        65
      age
                0
                       330 330
                                    0.66
                                             0.48
                                                          0
    bpmed
                                                                 1
                                                                         1
      dbp
                0
                       330 330
                                   74.75
                                            10.2
                                                   41
                                                          68
                                                                74
                                                                        82
   hsgrad
                0
                       330 330
                                   81.51
                                            10.66
                                                  -2
                                                          75
                                                                81
                                                                        89
   income
                0
                       330 330 35243.33 16056.44 100 25600 30600
                                                                     42475
                0
                       330 330
                                                         116
                                                               128.5
                                                                       138
      sbp
                                  128.37
                                            17.3
                                                   84
                0
                       330 330
                                    0.71
                                             0.46
                                                    0
                                                           0
                                                                 1
                                                                         1
   statin
   p100
```

```
77
    1
    106
    100
147400
    194
        1

Variable type: numeric
variable missing complete n mean sd p0 p25 median p75 p100
bmi 0 330 330 34.83 8.03 16.73 29.73 33.92 39.18 64.04
```

There are 5 missing eth\_hisp values and 2 missing race values.

### 1.4 Complete Cases: hbp330 data after deleting cases with NAs

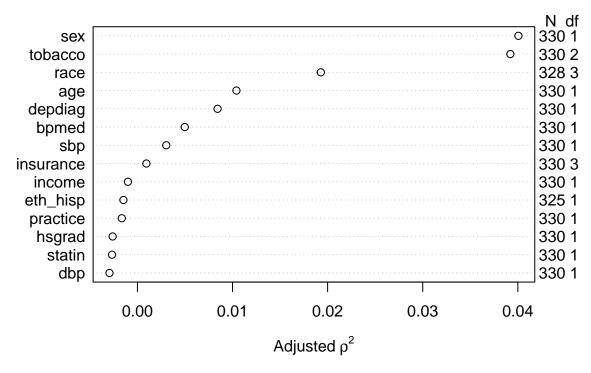
[1] 325 16

We lose a total of five observations (dropping from 330 to 325 subjects) by dropping missing values.

#### 1.5 Spearman rho-squared plot (applied to full data)

You might have chosen to include all observations, and simply allow the Spearman  $\rho^2$  plot to reduce the sample size for the specific variables (race and eth\_hisp) that had missing values. That produces this result.

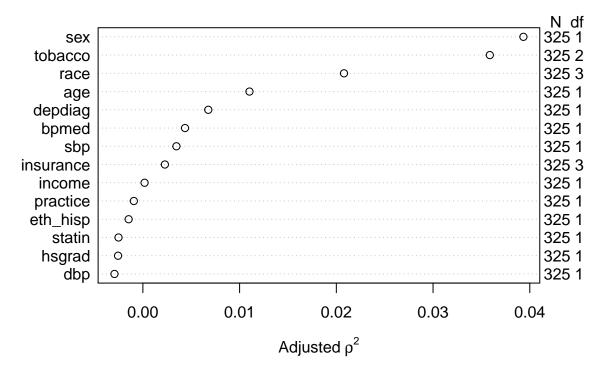




## 1.6 Spearman rho-squared plot (applied to 325 complete cases)

Or, you might have chosen to include only the 325 complete cases, and so the Spearman  $\rho^2$  plot would address only those subjects without missing eth\_hisp or race. That produces this result.

Spearman  $\rho^2$  Response: bmi



In either case, the variables which pack the largest "potential predictive punch" in this setting are, in order, (1) sex and (2) tobacco. Certainly, those are the most appealing variables for which we should consider non-linear augmentations. Since these are categorical variables, the inclusion of interaction terms seems appealing. We might, for instance, include the sex-tobacco interaction or an interaction of sex or tobacco or both with the next two highest variables on the list: race and age.

# 2 Question 2

#### 2.1 The gapminder data

This assignment uses the gapminder library and the gapminder data set within. The data set is the work of Jenny Bryan<sup>1</sup>, and this assignment is also closely related to assignments she has used in her work. I am deeply indebted to her for this.

#### 2.2 Task 1

Bring the gapminder data in by loading the gapminder package. Characterize the data. For instance, how many rows and variables are in the data frame? What do the rows describe?

First, let's glimpse the data to see what we've got:

skim(gapminder)

<sup>&</sup>lt;sup>1</sup>Jenny is a great follow on Twitter @JennyBryan.

```
Skim summary statistics
n obs: 1704
n variables: 6
Variable type: factor
  variable missing complete
                                n n unique
 continent
                        1704 1704
                                          5
   country
                 0
                        1704 1704
                                        142
                              top_counts ordered
 Afr: 624, Asi: 396, Eur: 360, Ame: 300
                                            FALSE
     Afg: 12, Alb: 12, Alg: 12, Ang: 12
                                            FALSE
Variable type: integer
                                                                 p25
 variable missing complete
                                    mean
                                                      p0
                                                                     median
                       1704 1704 3e+07
                                           1.1e+08 60011 2793664
                                                                     7e+06
      pop
     year
                0
                       1704 1704 1979.5 17.27
                                                    1952
                                                             1965.75
                                                                     1979.5
      p75
                p100
 2e+07
             1.3e+09
  1993.25 2007
Variable type: numeric
  variable missing complete
                                                       p0
                                                               p25
                                                sd
                                                                   median
                                n
                                     mean
                        1704 1704 7215.33 9857.45 241.17 1202.06 3531.85
 gdpPercap
                 0
                        1704 1704
                                                             48.2
                                                                     60.71
   lifeExp
                 0
                                    59.47
                                             12.92 23.6
    p75
              p100
9325.46 113523.13
   70.85
             82.6
```

Other useful descriptions can be obtained through, for example,

- gapminder
- str(gapminder)
- summary(gapminder)
- glimpse(gapminder)
- Hmisc::describe(gapminder)

Since the gapminder data set is part of an R package, it likely has a help file associated with it, which you can see with the command ?gapminder

We could take this in any of many directions – now is a good time to be creative. Some key thoughts:

- Each of the 1704 rows represents the characteristics of a single country in a single year.
- Each row contains a set of 6 variables, which specify:
  - country = the country's name (142 countries are included)
  - continent = the continent in which the country is found (5 continents are included)
  - year = the year in which the last three columns of data are relevant (available years range from 1952 to 2007 in increments of five years)
  - lifeExp = the life expectancy at birth, in years
  - pop = the population of the country
  - gdpPercap = gross domestic product per capita

#### 2.3 Task 2

Pick at least one quantitative and one categorical (factor) variable to explore. What are the possible values? What is a typical value? What is the spread? What is the shape of the distribution, for quantitative variables? What are the levels, for factors?

#### 2.3.1 A "Simple" Approach

Suppose we select the lifeExp variable as our quantitative variable, and the continent as our factor. We might start with some numeric summaries.

```
gapminder %>%
    select(lifeExp, continent) %>%
    Hmisc::describe()
```

2 Variables 1704 Observations
-----lifeExp

| .10  |             |
|------|-------------|
| L.51 |             |
|      |             |
|      |             |
|      | .10<br>1.51 |

lowest: 23.599 28.801 30.000 30.015 30.331, highest: 81.701 81.757 82.000 82.208 82.603

continent

 $\begin{array}{ccc} n & \text{missing distinct} \\ 1704 & 0 & 5 \end{array}$ 

| Value      | Africa A | mericas | Asia  | Europe | Oceania |  |
|------------|----------|---------|-------|--------|---------|--|
| Frequency  | 624      | 300     | 396   | 360    | 24      |  |
| Proportion | 0.366    | 0.176   | 0.232 | 0.211  | 0.014   |  |

-----

The describe function from the Hmisc package provides a useful numeric summary for either quantitative or categorical variables.

- For lifeExp, a quantitative variable, we learn:
  - n, the number of observations, as well as the number of missing observations, and the number of unique (distinct) values.
  - The Mean of the data, which is 59.47 years
  - Several quantiles, including the median (from .50) and the lower and upper quartiles (from .25 and .75, respectively.) For example, half of the life expectancy values fall below 60.71.
  - The smallest five and large five observations, which is a very useful tool for checking variables.
  - The Info, which is related to how continuous the variable is. Values close to 1 indicate that the variable is very continuous, with minimal ties. For more details, see ?Hmisc::describe.
  - The Gmd, which is called the *Gini mean difference*, which is a robust measure of dispersion that is the mean absolute difference between any pairs of observations.
- For a multi-categorical variable (in this case, a factor) like continent, Hmisc::describe shows:
  - The number of observations, missing observations and distinct values.
  - (if there are fewer than 20 distinct values) a frequency table with counts and proportions.

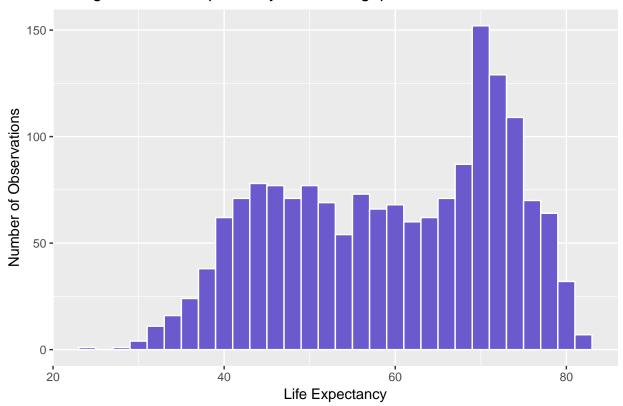
We might also want to get the standard deviation of lifeExp, which turns out to be 12.92 years.

#### 2.3.2 Distribution of your quantitative variable, via ggplot2

I also asked about the shape of the distribution for the quantitative variable (in our case, lifeExp). Let's draw a histogram.

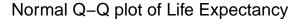
```
ggplot(gapminder, aes(x = lifeExp)) +
   geom_histogram(binwidth = 2, fill = "slateblue", color = "white") +
   labs(x = "Life Expectancy", y = "Number of Observations",
        title = "Histogram of Life Expectancy data from gapminder")
```

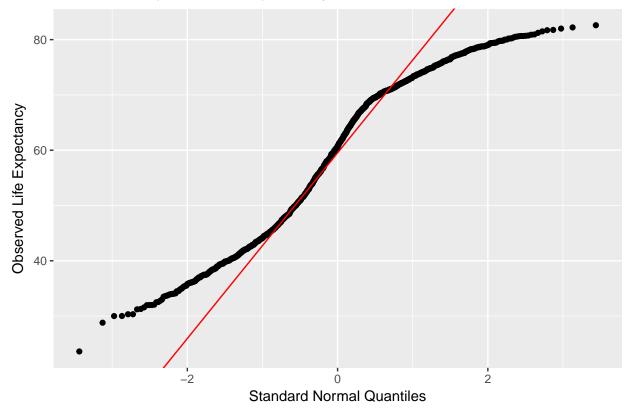
## Histogram of Life Expectancy data from gapminder



We seem to have a mode near 70, but a largely uniform distribution outside that range, with similar counts for most values between 40 and 80. The choice of bandwidth = 2 here simply groups the data into bins of approximately 2 years wide. A different choice would reveal a slightly different picture.

The data certainly aren't perfectly approximated by a Normal distribution, but it's hard to say whether the issue is better described as modest skew or by the more detailed description I gave in the previous paragraph. Here's a Normal Q-Q plot to help us thik about this further, built using the approach modeled in our Slides for Class 6.





The S shape in the Normal Q-Q plot suggests that the data are substantially lighter tailed compared to a Normal distribution.

#### 2.4 Task 3

Build a plot of a quantitative variable, and another plot of a quantitative and a categorical variable, for one or more subsets of the data that interest you. Use dplyr functions to create a subset or two you want to plot. Explore more than one plot type and try to use more than one geom in your work.

We'll start by looking at Life Expectancy (a quantitative variable) across all nations in 1977.

```
gapminder_1977 <-
    gapminder %>%
    filter(year == 1977)
gapminder_1977
```

# # A tibble: 142 x 6

|   | country     | ${\tt continent}$ | year        | lifeExp     | pop         | ${\tt gdpPercap}$ |
|---|-------------|-------------------|-------------|-------------|-------------|-------------------|
|   | <fct></fct> | <fct></fct>       | <int></int> | <dbl></dbl> | <int></int> | <dbl></dbl>       |
| 1 | Afghanistan | Asia              | 1977        | 38.4        | 14880372    | 786               |
| 2 | Albania     | Europe            | 1977        | 68.9        | 2509048     | 3533              |
| 3 | Algeria     | Africa            | 1977        | 58.0        | 17152804    | 4910              |
| 4 | Angola      | Africa            | 1977        | 39.5        | 6162675     | 3009              |
| 5 | Argentina   | Americas          | 1977        | 68.5        | 26983828    | 10079             |
| 6 | Australia   | Oceania           | 1977        | 73.5        | 14074100    | 18334             |

```
7568430
7 Austria
               Europe
                           1977
                                   72.2
                                                      19749
8 Bahrain
               Asia
                           1977
                                   65.6
                                           297410
                                                      19340
9 Bangladesh
                                   46.9 80428306
               Asia
                           1977
                                                        660
10 Belgium
               Europe
                           1977
                                   72.8 9821800
                                                      19118
# ... with 132 more rows
```

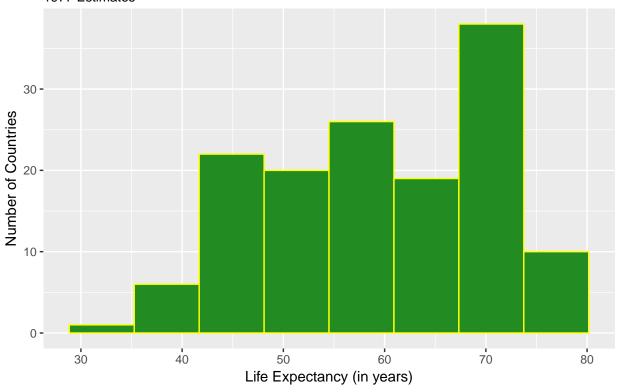
#### 2.4.1 Plotting a Quantitative Variable in a Subgroup of Interest

And now, we'll plot the histogram of these life expectancy estimates. Clearly the default choice of 30 bins is too many in this setting, since the resulting plot with 30 bins isn't smooth at all. The plot below uses just 8 bins.

```
ggplot(gapminder_1977, aes(x = lifeExp)) +
   geom_histogram(bins = 8, col = "yellow", fill = "forestgreen") +
   labs(x = "Life Expectancy (in years)",
        y = "Number of Countries",
        title = "Life Expectancy in Nations of the World",
        subtitle = "1977 Estimates")
```

# Life Expectancy in Nations of the World





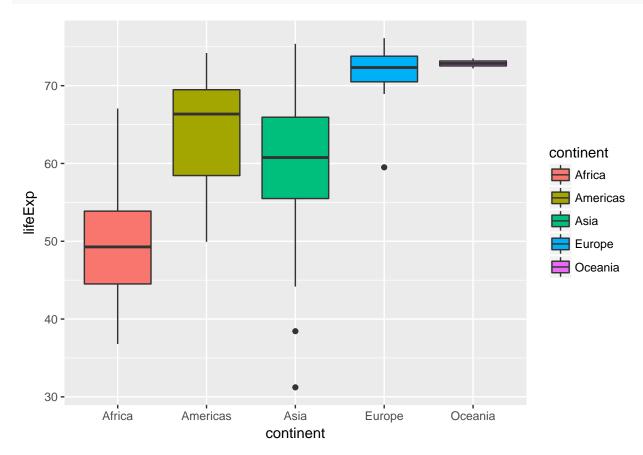
#### 2.4.2 Plotting a Quantitative Variable and a Categorical Variable in a Subgroup of Interest

Let's build a plot of lifeExp (quantitative) stratified by continent (categorical) using, again, the 1977 data. First, build the data we'll need, using dplyr operations:

```
gapminder_1977_a <- gapminder %>%
  filter(year == 1977) %>%
  select(year, country, continent, lifeExp)
```

And now, we plot the results in two ways...

#### 2.4.3 A boxplot

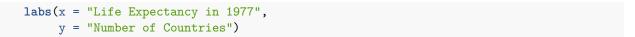


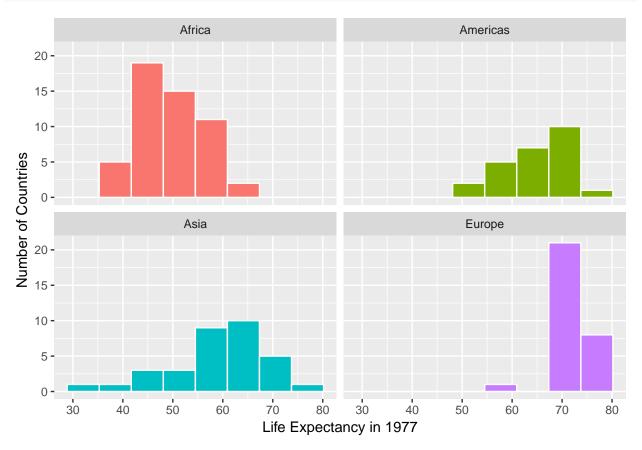
### 2.4.4 Using Facets to build Multiple Histograms in One Plot

We'll drop Oceania here, since it's only a couple of countries, and so a histogram (like the boxplot above) seems silly. We'll also drop the legend, since it's repetitive.

```
gapminder_1977_b <- filter(gapminder_1977_a, continent != "Oceania")

ggplot(gapminder_1977_b, aes(x = lifeExp, fill = continent)) +
    geom_histogram(bins = 8, color = "white") +
    guides(fill = FALSE) + ## leaves out the legend
    facet_wrap(~ continent, nrow = 2) +</pre>
```





#### 2.5 Task 4

Make a scatterplot which shows the relationship between two quantitative variables, either for a clearly specified subset of interest, or for the data as a whole. Include a regression line in the plot. Make an active choice as to which variable should be the predictor and which the outcome.

We'll use per-capita gross domestic product (gdpPercap) as a predictor of an outcome, lifeExp, here, for all countries in the continents of Africa and the Americas, in the most recent available data period (2007).

First, I'll create the data set with the pieces I need.

```
gap_4 <- gapminder %>%
    filter(year == 2007) %>%
    filter(continent %in% c("Africa", "Americas")) %>%
    select(country, year, continent, gdpPercap, lifeExp)
gap_4
```

```
# A tibble: 77 x 5
```

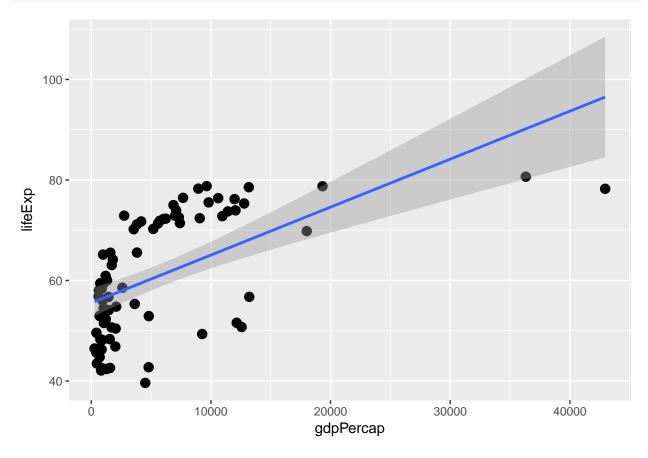
|   | country     | year        | ${\tt continent}$ | ${\tt gdpPercap}$ | lifeExp     |
|---|-------------|-------------|-------------------|-------------------|-------------|
|   | <fct></fct> | <int></int> | <fct></fct>       | <dbl></dbl>       | <dbl></dbl> |
| 1 | Algeria     | 2007        | Africa            | 6223              | 72.3        |
| 2 | Angola      | 2007        | Africa            | 4797              | 42.7        |
| 3 | Argentina   | 2007        | Americas          | 12779             | 75.3        |

```
2007 Africa
                                              56.7
 4 Benin
                                      1441
 5 Bolivia
                 2007 Americas
                                      3822
                                               65.6
                 2007 Africa
                                              50.7
6 Botswana
                                     12570
7 Brazil
                 2007 Americas
                                      9066
                                              72.4
8 Burkina Faso
                 2007 Africa
                                      1217
                                              52.3
9 Burundi
                 2007 Africa
                                       430
                                               49.6
10 Cameroon
                 2007 Africa
                                      2042
                                               50.4
# ... with 67 more rows
```

#### 2.5.1 A scatterplot ignoring the continent information

Now, I'll build a plot using the gap\_4 data, and ignoring the continent information.

```
ggplot(gap_4, aes(x = gdpPercap, y = lifeExp)) +
   geom_point(size = 3) +
   geom_smooth(method = "lm")
```



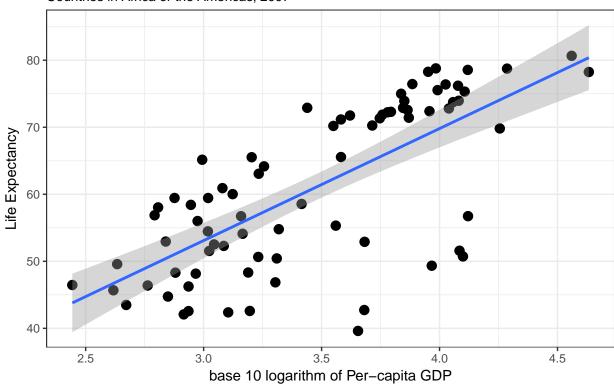
Typically, gdpPercap is represented in Rosling's gapminder presentations on the log scale. Let's try a base 10 logarithm here for our predictor, and add some titles, while switching themes.

```
ggplot(gap_4, aes(x = log10(gdpPercap), y = lifeExp)) +
    geom_point(size = 3) +
    geom_smooth(method = "lm") +
    theme_bw() +
    labs(x = "base 10 logarithm of Per-capita GDP",
        y = "Life Expectancy",
```

```
title = "log of per-capita GDP and Life Expectancy",
subtitle = "Countries in Africa or the Americas, 2007")
```

# log of per-capita GDP and Life Expectancy

Countries in Africa or the Americas, 2007



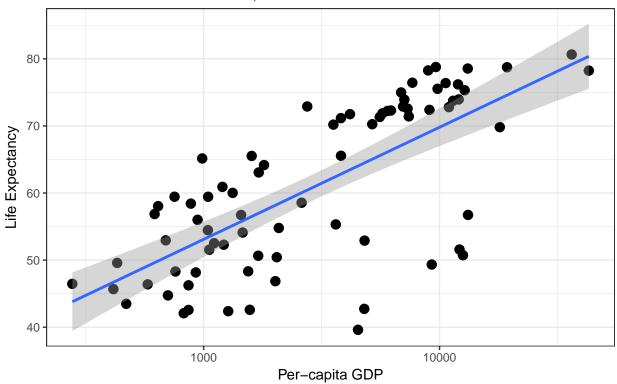
#### 2.5.2 Rescaling the axis to the log

Or, instead of plotting the log, we could do something better still, and just plot the data on a log (base 10) scale, using the scale\_x\_log10 function, which results in a plot that is far easier to interpret. Let's do that for the x axis here.

```
ggplot(gap_4, aes(x = gdpPercap, y = lifeExp)) +
    geom_point(size = 3) +
    geom_smooth(method = "lm") +
    scale_x_log10() +
    theme_bw() +
    labs(x = "Per-capita GDP",
        y = "Life Expectancy",
        title = "Per-capita GDP and Life Expectancy",
        subtitle = "Countries in Africa or the Americas, 2007")
```

# Per-capita GDP and Life Expectancy

Countries in Africa or the Americas, 2007



#### 2.6 Task 5

Specify the regression equation fitted for Task 4, and evaluate it using summary statistics like  $R^2$  or the residual standard deviation, as well as through assessments of the direction and size of the coefficient estimates.

The equation for Task 4 involves one predictor (per-capita GDP, or actually its base 10 logarithm looks better) and one outcome (life expectancy). The model is:

```
mod4 <- lm(lifeExp ~ log10(gdpPercap), data = gap_4)
summary(mod4)</pre>
```

```
lm(formula = lifeExp ~ log10(gdpPercap), data = gap_4)
Residuals:
   Min
            1Q
                Median
                            ЗQ
                                   Max
-24.418 -4.254
                 2.730
                         6.117 12.466
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)
                   2.963
                              6.694
                                      0.443
                                               0.659
log10(gdpPercap)
                  16.710
                              1.910
                                      8.750 4.45e-13 ***
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 8.586 on 75 degrees of freedom Multiple R-squared: 0.5052, Adjusted R-squared: 0.4986 F-statistic: 76.57 on 1 and 75 DF, p-value: 4.447e-13

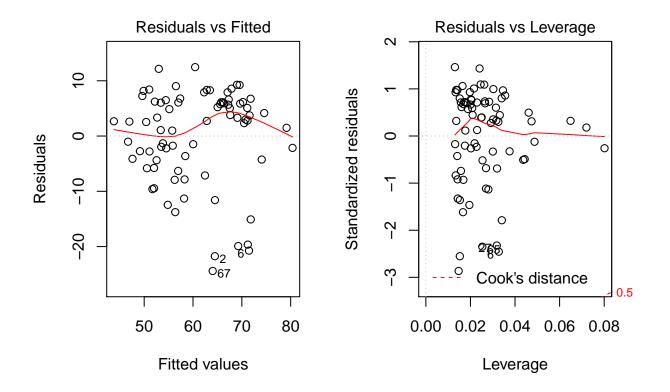
If you ran the model on gdpPercap without the logarithm, you would have to increase the number of digits in our model's display to get anything useful in the gdpPercap variables.

A strong assessment here would say something about:

- the direction of the relationship (an increasing relationship between log of gdpPercap and lifeExp,)
- the fact that both predictors in this model are statistically significantly different from 0 at the 5% level,
- the summary statistics  $R^2$  which is 0.51 which is at best moderately strong, and the residual SD,  $\hat{\sigma}$ , which is 8.6, so that a typical error in predicting life Expectancy should be less than 2(8.6) = 17.2 years, which doesn't actually sound all that great.

You could look at residual plots here. That wasn't something we saw as vital. The log scale/transformation for gdpPercap definitely seems to help produce a more linear relationship, reigning in some outliers.

```
par(mfrow = c(1,2))
plot(mod4, which = c(1,5))
```



2.7

par(mfrow = c(1,1))

Task 6

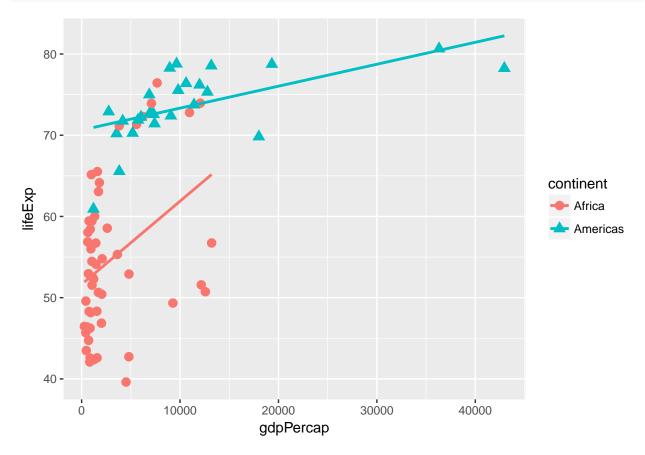
Augment your scatterplot from Task 4 by incorporating a categorical variable as well as a predictor. Show the new predictor in a useful way as part of the plot, and show the regression

model incorporating the predictor as part of the plot.

We'll use per-capita gross domestic product (gdpPercap) as a predictor of an outcome, lifeExp, here, for all countries in the continents of Africa and the Americas, in the most recent available data period (2007), and use the continent as the other predictor.

I'll use the same data set I created in Task 4, called gap\_4. Here's my first attempt at a plot.

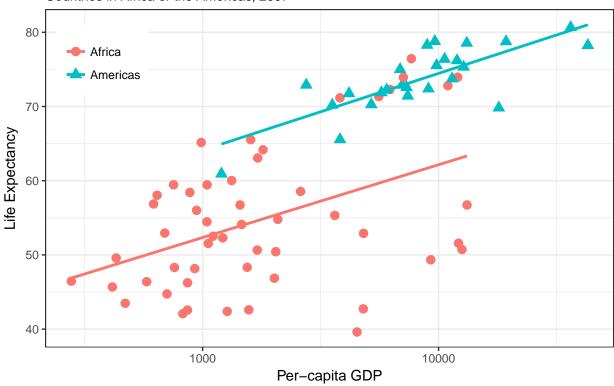
```
ggplot(gap_4, aes(x = gdpPercap, y = lifeExp, color = continent, shape = continent)) +
    geom_point(size = 3) +
    geom_smooth(method = "lm", se = FALSE)
```



As we've seen, gdpPercap is represented in Rosling's gapminder presentations on the log scale. Here, we plot the data on a log (base 10) scale, using the scale\_x\_log10 function, which results in a plot that is far easier to interpret. Let's do that for the x axis here, and also move the legend to the top left of the plot area, while dropping its title (continent).

# Per-capita GDP and Life Expectancy

Countries in Africa or the Americas, 2007



#### 2.8 Task 7

Specify the regression equation from Task 6, and describe the nature of the relationship between the predictors, and their impact on your outcome of interest through useful summaries.

The equation for Task 6 involves two predictors (continent, and base-10 log of per-capita GDP) and one outcome (life expectancy). The model is:

```
mod6 <- lm(lifeExp ~ log10(gdpPercap) * continent, data = gap_4)
summary(mod6)</pre>
```

#### Call:

```
lm(formula = lifeExp ~ log10(gdpPercap) * continent, data = gap_4)
```

#### Residuals:

```
Min 1Q Median 3Q Max -19.1483 -4.0185 -0.0751 4.2449 15.4213
```

#### Coefficients:

|   | Estimate | Sta. Error | t value | Pr(> t ) |
|---|----------|------------|---------|----------|
| (Intercept)                                   | 22.9068  | 7.6253     | 3.004   | 0.00365  |
| log10(gdpPercap)                              | 9.8110   | 2.3242     | 4.221   | 6.9e-05  |
| continentAmericas                             | 10.1583  | 19.2143    | 0.529   | 0.59863  |
| <pre>log10(gdpPercap):continentAmericas</pre> | 0.5369   | 5.0521     | 0.106   | 0.91565  |

So the models are:

| Continent | Model for lifeExp  |
|-----------|--|
|           | 22.91 + 9.81 log_10_(gdpPercap) (22.91 + 10.16) + (9.81 + 0.54) log_{10}(gdpPercap), which is $33.07 + 10.35 \log_{10}(gdpPercap)$ |

A strong assessment here would say something about:

- the shift between the two continents, with the Americas living an additional 10 years (comparing the intercepts, since the slopes are similar) as compared to African residents with the same value of Per-Capita GDP, on average,
- the direction of the relationships in the two continents (both show an increasing relationship between gdpPercap and lifeExp, with a slightly larger slope in the Americas,)
- the fact that the two main effects are statistically significant at the 5% level, but not the interaction term,
- the summary statistics:  $R^2$  which is 0.61 and seems fairly strong, and the residual SD,  $\hat{\sigma}$ , which is 7.7, so that a typical error in predicting life Expectancy should be less than 15.4 years.

#### 2.9 Task 8

Now, report your process. Write a brief essay (at least 75 words but probably not much more than 150) reflecting on what was hard/easy, problems you solved, helpful tutorials you read, etc. What things were hard, even though you saw them in class? What was easy (or sort of easy) even though we haven't done it in class?\*

A good essay (and this is the part we'll be reviewing most closely) will address the questions well. We don't write answer sketches for essay questions.