

CMSC 401 SPRING 2022
STUDY PROBLEMS FOR Test #2

Below are the study problems. The questions in the test will be similar to these.

STUDY PROBLEM 1

What are the advantages and disadvantages of Adjacency List vs Adjacency Matrix for sparse, and for dense graphs?

STUDY PROBLEM 2

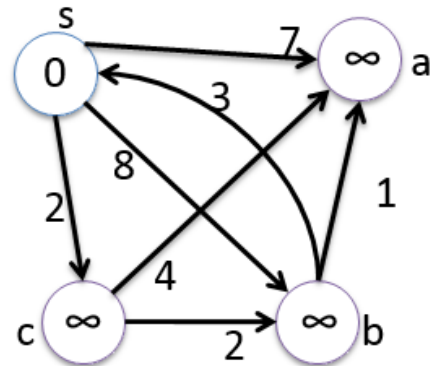
Given an adjacency-list representation of a directed graph, how long does it take to compute the out-degree of a given vertex (number of edges going out of the vertex)? How long does it take to compute the in-degree (number of edges going into the vertex)?

STUDY PROBLEM 3

Given the pseudocodes, analyze the computational complexity of the adjacency list version of the Dijkstra algorithm, and the adjacency matrix of the Dijkstra algorithm. For a given type of the graph (given in terms of the number of edges w.r.t. to number of vertices, e.g. $|E|=5|V|$) decide which one is (asymptotically) faster.

STUDY PROBLEM 4

Draw iterations of Dijkstra shortest path method for the graph on the right, with vertex *s* as the source. What are the temporary distances to each node AFTER each iterations? What is the content of the queue AFTER each iterations? Which nodes are final AFTER each iterations? What are the resulting shortest paths?



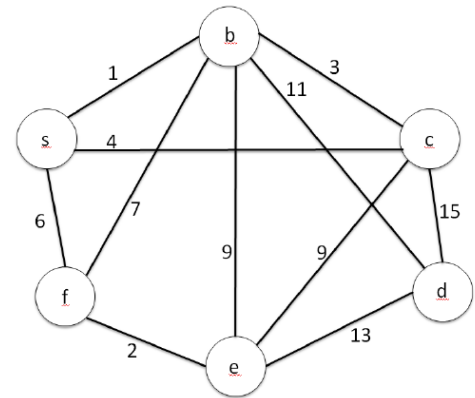
STUDY PROBLEM 5

Draw steps of Breadth-First Search method and Depth-First Search method for the graph from the previous Problem (ignoring the weights), with vertex *s* as the starting vertex.

STUDY PROBLEM 6

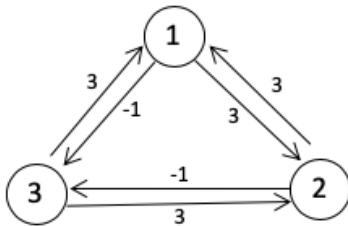
Describe what a minimal spanning tree is.

Draw a minimal spanning tree for the following graph. In which order the Prim/Kruskal's method would analyze edges, assuming the starting vertex is s (for Prim)?



STUDY PROBLEM 7

What is the basic idea behind Floyd-Warshall method? Show how it works on the example graph below by filling out the D matrix that shows the distances after introduction of each intermediate vertex. $D^{(i)}$ shows the matrix of distances (rows show “from” node, columns show “to” node) after introduction of vertex i (so the allowed intermediate nodes become vertices from 1 to i).



$$D^{(0)} =$$

	1	2	3
1			
2			
3			

$$D^{(1)} =$$

	1	2	3
1			
2			
3			

$$D^{(2)} =$$

	1	2	3
1			
2			
3			

$$D^{(3)} =$$

	1	2	3
1			
2			
3			

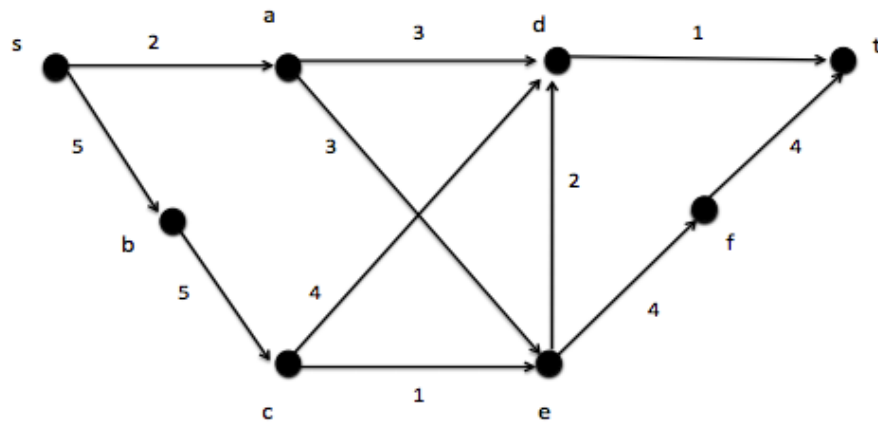
STUDY PROBLEM 8

For a given real-world problem like the one below, which algorithm (from lectures) will you use to solve it? How would you map the input information into the input required by the algorithm? What computational complexity the chosen algorithm has?

Blizzard has come over the whole state, covering all highways with 40 inches of snow. To keep the distribution of food, roads between major cities need to be cleared, so that each major city can be reached from each other. But getting rid of the snow from a highway is expensive - \$1000 per mile. You, member of the Emergency Services, need to devise a plan that keeps all major cities connected through cleared roads, with the cost of clearing as small as possible. You are given the list of distances (in miles) of direct connections (i.e. not going through any other major city) between the major cities. You have to do the math, and choose which of those connections should be cleared.

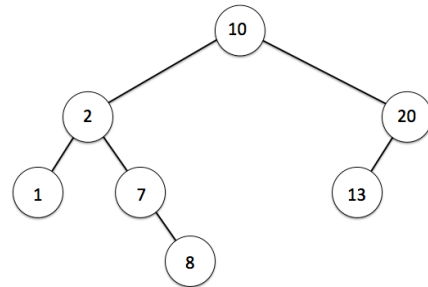
STUDY PROBLEM 9

Apply the Ford-Fulkerson algorithm to find the maximum flow in the following graph (from s to t)? Show the augmenting path and residual network at each step.



STUDY PROBLEM 10

What would the following Binary Search Tree look like after inserting a key “4”. Then, after deleting “2”? Then, after deleting “10”?:



STUDY PROBLEM 11

- What are the differences in functionality between BST and hash table?
- Imagine you have two tasks:

Task A:

Build a database of employees of a large company, with a functionality to quickly search for employee record based on his/her phone number.

Task B:

Build a database of friends, with a functionality to find a friend by his/her birthday, and once the friend is found, having the ability to find who among friends has his/her birthday next.

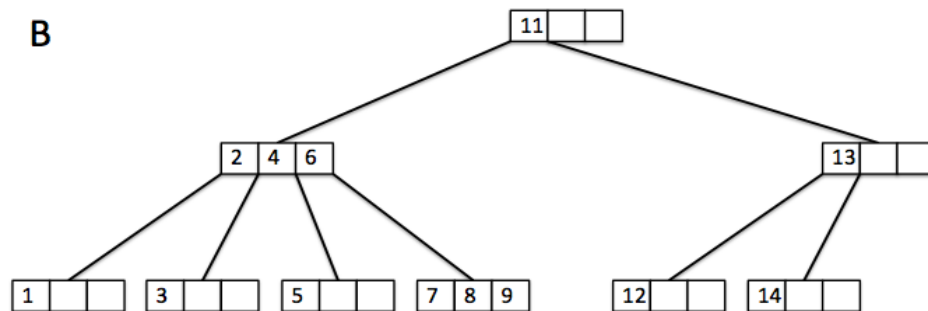
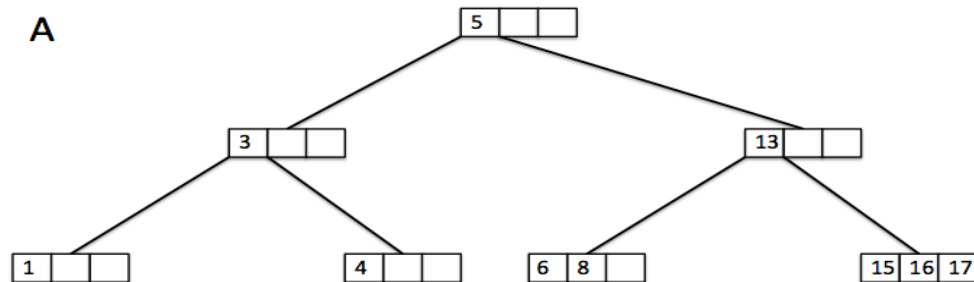
What data structure would you use for Task A? What for Task B? Justify your choice.

STUDY PROBLEM 12

In what situations B-Tree is better than Binary Search Tree? Describe why.

STUDY PROBLEM 13

Draw subsequent steps during Insert of a key “10” into the following B-trees, using the procedure with preemptive node splits. In both trees, the minimum degree $t=2$.



STUDY PROBLEM 14 (True/False)

- DFS can be used to find the connected components of a directed graph.
- In an undirected graph G with distinct nonnegative edge weights, if we increase all the edge weights by 1, all shortest paths will stay the same.
- The complexity of running Dijkstra’s algorithm with adjacency list representation is $O(V \log E)$.
- The complexity of searching an item in a B-Tree can be the same as the complexity of searching in a BST with a careful design.
- The B-Tree on the right is a valid B-tree for both $t=2$ and $t=3$
- If we order the following algorithms in terms of their complexity, it would be $\text{BFS} < \text{Dijkstra} \leq \text{Floyd-Warshall}$
- For this pre-order traversal/walk “12, 5, 3, 7, 45, 70” of a BST, there can be only one such BST drawn.

