CMSC401 – Spring 2022

THEORY HOMEWORK 1

Due: Monday, 2/14, 5pm

Submit a PDF (preferred) or Word file with the solution though Canvas

PROBLEM 1 Use Master Method to find solution

(T(n) = Θ (?)) for the following recurrence equations, if possible. If not possible, indicate why.

a) T(n) = 16T(n/4) + n^2

A = 16, b = 4, f(n) = n^2 >= 0 for all n

N^2 = n^($\log_4$(16))=n^2

T(n) = Θ((n^2)(log n))

b) T(n) = 4T(n/4) + n

A = 4, b = 4, f(n) = n >=0 for all n >= 0

N = n^($\log_4$ 4) = n

T(n) = Θ((n)(log n))

c) T(n) = T(n-2) + 2n2

We cannot use master method because the equation gives n-2 instead of n/b. The given recurrence is not of the correct form aT(n/b) + f(n), so we cannot apply the Master Method.

PROBLEM 2 Use the substitution method to show that for the recurrence equation:

T(1) = 8

T(n) = T(n-1) + 4n

the solution is T(n) = O(n^2)

For n=1, T(n) <=cn^2

For all n>1, if T(n-1) <= c(n-1)^2, then T(n) <= cn^2

Part I, BASE CASE:

Since T(1) = 8, T(1) <= c if c >= 8

Part II, INDUCTIVE STEP:

$T(n) = T(n-1) + 4n$

$T(n-1) <= c(n-1)^2$

$T(n) <= c(n-1)^2 + 4n = cn^2-2cn+c+4n$

$T(n) <= cn^2-2cn+c+4n$

For some c, $T(n) <= cn^2-2cn+c+4n <= cn^2$

$Cn^2-2cn+c+4n<=cn^2$

$-2cn+c+4n <= 0$

$-2cn+c+4n <= -2cn+cn +4n = n(-2c+c+4) = n(4-c)$

$(4-c)n <= 0$

For any c >= 4, we have $-2cn +c +4n <= 0$, $cn^2-2cn+c+4n <= cn^2$, **AND** $T(n) <=cn^2$

Together from Parts I and II, we have that for any c >= 4, for any n >= 1, $T(n) <= cn^2$