Transformation

MVP分别代指三种变换（Model, View, Projection）

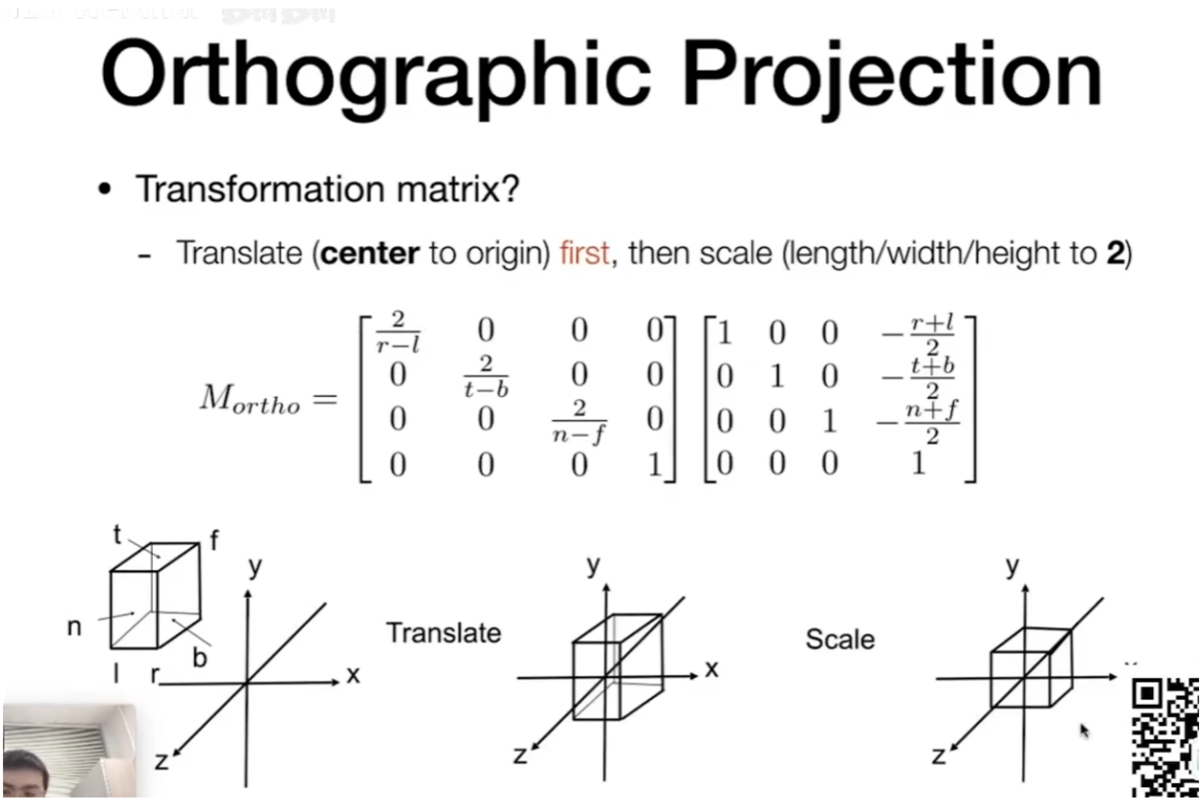
1. Model transformation(模型变换)

2. View/Camera transformation(视图变换)

保证相机和物体的相对位置

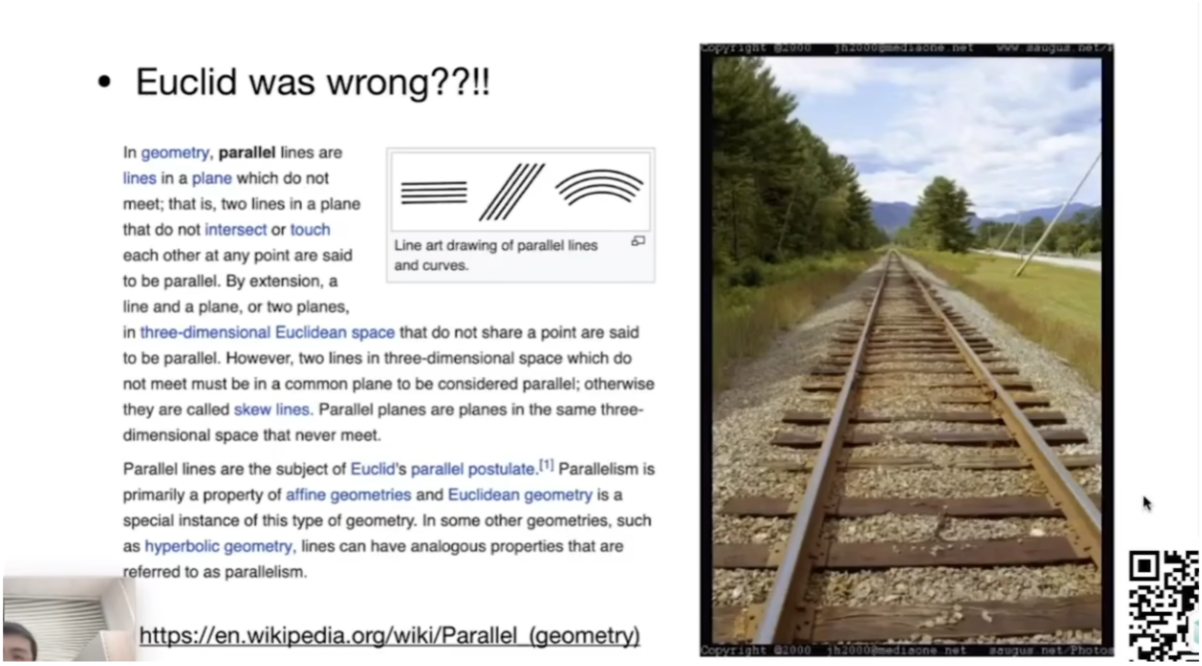
3. Projection transformation(投影变换)

3.1 Orthographic Projection(正交投影)

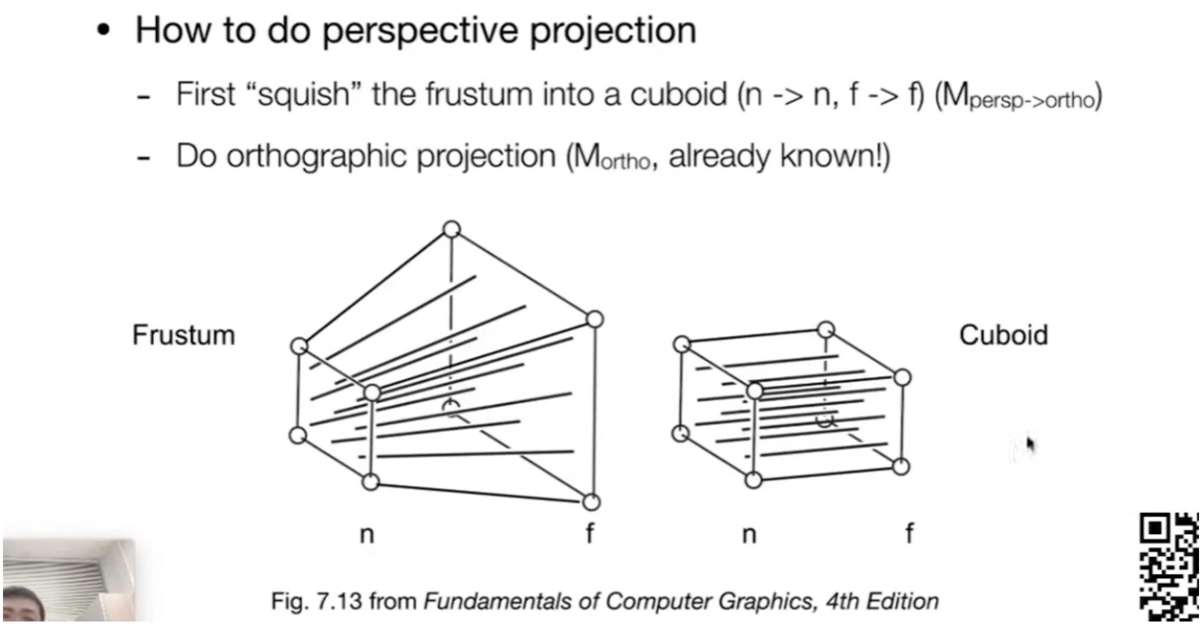


3.2 Perspective Projection(透视投影)

3.2.1 推理



透视投影需要先将Frustum(截锥体)变成Cuboid(正方体)，再做正交投影

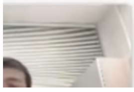


转换方法是使用相似三角形

- In order to find a transformation
  - Find the relationship between transformed points (x', y', z') and the original points (x, y, z)

$$y' = \frac{n}{z}y \qquad x' = \frac{n}{z}x \text{ (similar to y')}$$

- In homogeneous coordinates,



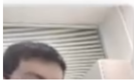
$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} nx/z \\ ny/z \\ \text{unknown} \\ 1 \end{pmatrix} \overset{\text{mult. by z}}{==} \begin{pmatrix} nx \\ ny \\ \text{still unknown} \\ z \end{pmatrix}$$

可以得到除z轴外，透视投影矩阵所有的信息

- So the “squish” (persp to ortho) projection does this

$$M^{(4\times4)}_{persp\to ortho}\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} nx \\ ny \\ \text{unknown} \\ z \end{pmatrix}$$

- Already good enough to figure out part of $M_{persp\text{->}ortho}$



$$M_{persp\to ortho} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ ? & ? & ? & ? \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

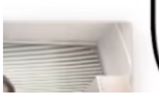
WHY?

z轴满足 z = n 或 z = f 时没有任何变化，得到如下关系

- What do we have now?

$$\begin{pmatrix}0 & 0 & A & B\end{pmatrix}\begin{pmatrix}x\\y\\n\\1\end{pmatrix} = n^2 \quad \Rightarrow \quad An + B = n^2$$

- Any point's z on the far plane will not change



$$\begin{pmatrix}0\\0\\f\\1\end{pmatrix} \Rightarrow \begin{pmatrix}0\\0\\f\\1\end{pmatrix} == \begin{pmatrix}0\\0\\f^2\\f\end{pmatrix} \quad \Rightarrow \quad Af + B = f^2$$

- Solve for A and B

$$\begin{aligned}An + B &= n^2\\ Af + B &= f^2\end{aligned} \quad \Rightarrow \quad \begin{aligned}A &= n + f\\ B &= -nf\end{aligned}$$

- Finally, every entry in $M_{persp\text{->}ortho}$ is known!

- What's next?
  - Do orthographic projection (M_{ortho}) to finish
  - $M_{persp} = M_{ortho} M_{persp \rightarrow ortho}$

得到：

$$M_{persp \rightarrow ortho} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -nf \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

最终的透视投影矩阵为：

$$M_{persp} = M_{ortho} M_{persp \rightarrow ortho}$$

Tip:

经过 $M_{persp \rightarrow ortho}$ 后 n 与 f 平面之间的点会被**往 n 平面推**，推导如下：

选取中间平面任意一点：

$$\begin{pmatrix} x \\ y \\ \frac{n+f}{2} \\ 1 \end{pmatrix}$$

左乘 $M_{persp \to ortho}$ 得到如下结果:

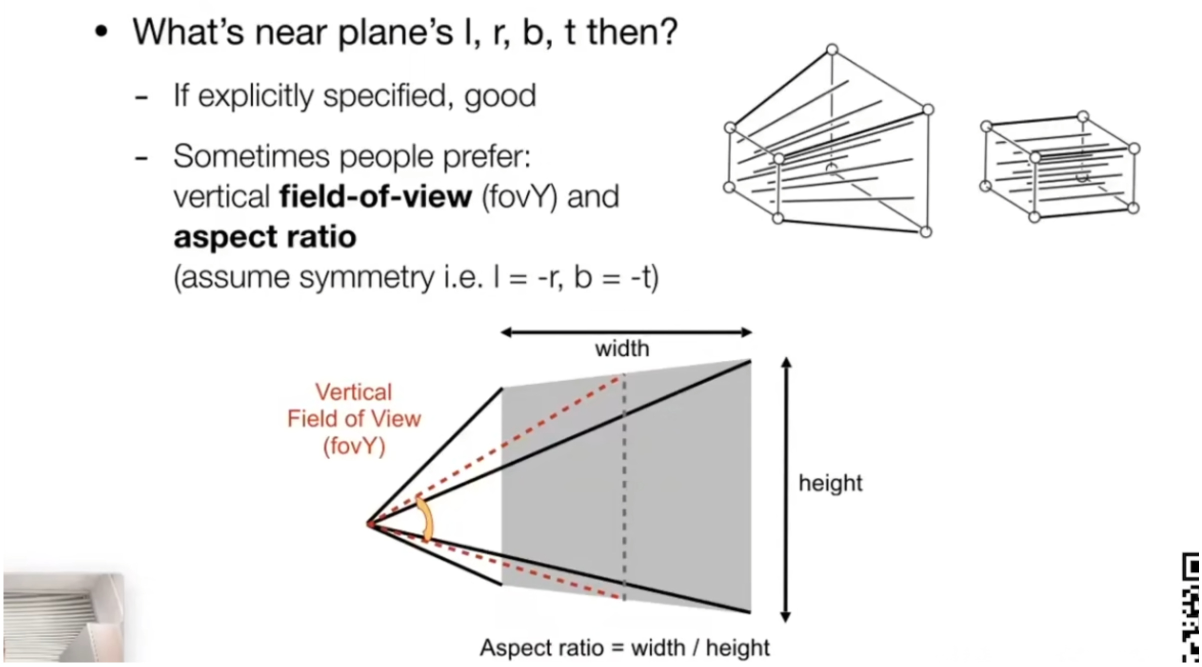
$$\begin{pmatrix} nx \\ ny \\ \frac{(n+f)^2}{2} - nf \\ \frac{n+f}{2} \end{pmatrix} == \begin{pmatrix} \frac{2nx}{n+f} \\ \frac{2ny}{n+f} \\ n+f-\frac{2nf}{n+f} \\ 1 \end{pmatrix}$$

其中新的Z轴坐标 $n+f-\frac{2nf}{n+f} == \frac{2n^2+2f^2}{2(n+f)}$ ，而原Z轴坐标为 $\frac{n+f}{2} == \frac{n^2+f^2+2nf}{2(n+f)}$，所以只需要判断 $2n^2+2f^2$ 和 n^2+f^2+2nf 大小即可。

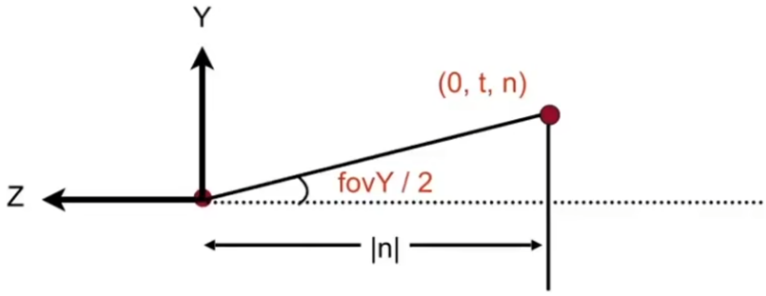
两边同减 n^2+f^2+2nf 得到 $(n-f)^2$ 和 0，所以新的Z轴更大，而在右手坐标系中，n > f，所以**比之前更接近n平面**

3.2.2 Fov and Aspect ratio

FovY(垂直视场角)，Aspect ratio(宽高比)



- How to convert from fovY and aspect to l, r, b, t?
  - Trivial



$$\tan\frac{fovY}{2} = \frac{t}{|n|}$$

$$aspect = \frac{r}{t}$$