Topics in Quantitative Finance (FIN 528): Machine Learning for Finance

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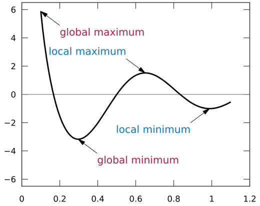
Quantitative finance courses in PHBS

- Y1-M3: Stochastic Finance by Jaehyuk Choi **[required for Qfin MA]**
- Y1-M4: Derivative Pricing by Lei (Jack) Sun
- Y2-M1: Applied Stochastic Processes by Jaehyuk CHOI
  Application, Programming, Course project
- Y2-M3: Topics in Quantitative Finance by Jaehyuk CHOI
  Machine Learning for Finance (Mon-Thurs 1:30 PM)
- Y2-M3: Numerical Methods and Analysis by Jake ZHAO (Mon-Thurs 3:30 PM)
- Y2-M3: Bayesian Statistics by Qian CHEN (Mon-Thurs 10:30 AM)

AlphaGo vs Humans (LEE Sedol, KE Jie)

We have to think again about joseki / dìngshì (定石/ 定式) .
In Go (围棋), a joseki is the studied sequences of moves for which the result is considered balanced for both black and white sides.



LEE Sedol afterwards · · ·

- Became more popular and richer
- Perhaps titled as the last human who beat the machine in Go

ML/AI: Rise of the machines?

Probabily not!

What and why now?

What is ML?

- Prediction based on data (data into knowledge)
- Extended linear/logistic regression
- Pattern recognition

Why now?

- Abundant Data (Big Data)
- Faster computer (Graphics Processing Unit: GPU)
- Advances in research: Geoffrey Hinton (Google), Yann LeCun (Facebook).

Recent applications of ML

- Automated driving system (Google, Apple, etc)
- Suggestion engine (Amazon, Taobao)
- Cancer diagnosis (IBM Watson)
- Digitizing images (Facebook, Google)
- Shopping without checkout (Amazon Go: big data)

ML in finance?

Prediction

- Asset management / investment
- Trading algorithm (alpha)
- Earnings prediction: e.g., Prediction Valley

Cost cut / labor reduction

- Automated accounting / tax
- Automated analyst report
- Chat-bot (trading and sales)
- Data analytics: e.g., Kensho

Softwares to use

Python

- Anaconda (Python distribution + Environment management)
- Python Language Tutorial
- Sci-Kit Learn
- TensorFlow (wrapped by Keras)

Github.com

- Distributed version control system
- Clone or fork a repository to create a local copy
- `https://github.com/PHBS/2016.M3.TQF-ML` (our course)
- `https://github.com/rasbt/python-machine-learning-book` (**PML**)

Other resources for ML

- Coursera ML course (**CML**) by Andrew Ng (Baidu)
- Stanford CS229 Machine Learning: course notes, student projects, etc
- The Elements of Statistical Learning (**ESL**)
- An Introduction to Statistical Learning (with R) (**ISLR**)
- Pattern Recognition and Machine Learning by Bishop (Microsoft)

Homework

- Install Anaconda (Ver 4.3 Python 3.6)
- Create GitHub account, join PHBS (organization) and 2016.M3.TQF-ML (team).
- Fork the two repositories (the **PML** book and our course)
- Watch the first lecture of **CML**

Notations and conventions: vector and matrix

General rules (guess from the context)

- Scalar (non-bold): x, y, X, Y
- Vector (lowercase bold): $\boldsymbol{x} = (x_i), \boldsymbol{y} = (y_i)$
- Matrix (uppercase bold): $\boldsymbol{X} = (X_{ij})$, $\boldsymbol{Y} = (Y_{ij})$
- The (i, j) component of $\boldsymbol{X}$: X_{ij}
- The i-th row vector of $\boldsymbol{X}$: $\boldsymbol{X}_{i*} = (X_{i1}, X_{i2}, \cdots, X_{ip})^T$
- The j-th column vector of $\boldsymbol{X}$: $\boldsymbol{X}_{*j} = (X_{1j}, X_{2j}, \cdots, X_{Nj})$

Examples

- Dot product: $\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \boldsymbol{x}^T \boldsymbol{y}$
- Vector norm: $|\boldsymbol{x}| = \sqrt{\boldsymbol{x}^T \boldsymbol{x}}$
- Matrix multiplication: $\boldsymbol{Z} = \boldsymbol{X}\boldsymbol{Y} \rightarrow Z_{ij} = \boldsymbol{X}_{i*}\boldsymbol{Y}_{j*}$

Notation and conventions: variables and observations

General rules

- Generic (or representative) variables (uppercase non-bold): X (input), Y (output), G (classification output)
- The predictions: $\hat{Y}$, $\hat{G}$
- X (input) may be p-dimensional (features): X_j ($j \le p$), row vector
- Y (output) may be K-dimensional (responses): Y_k ($k \le K$), row vector.
- The N observations of X or Y is stacked as rows: $\boldsymbol{X}$ ($N \times p$), $\boldsymbol{Y}$ ($N \times K$)
- The i-th observation set: $\boldsymbol{X}_{i*}$ ($1 \times p$)
- All observation of j-th feature X_j: $\boldsymbol{X}_{*j}$ ($N \times 1$)
- $\boldsymbol{X} = (\boldsymbol{X}_{*1} \cdots \boldsymbol{X}_{*p})$ (column-wise concatenation)
- The weight vector: $\boldsymbol{\beta}$ or $\boldsymbol{w}$ used interchangeably.

Simple Linear Regression (Ordinary Least Square)

For scalar predictor (X) and response (Y),

$$Y \approx \beta_0 + \beta_1 X \quad \longrightarrow \quad \hat{\boldsymbol{y}} = \beta_0 + \beta_1 \boldsymbol{x}.$$

For N observations $(x_1, y_1), \cdots, (x_N, y_N)$, the set of $(\hat\beta_0, \hat\beta_1)$ to minimize the residual sum of squares (RSS):

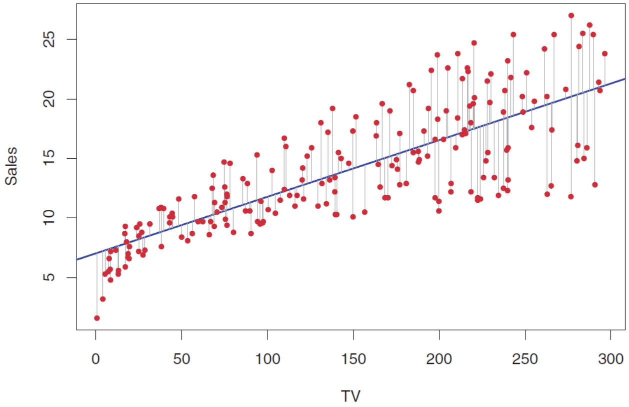
$$\text{RSS}(\beta) = \sum_{i=1}^{N} (y_i - \beta_0 - \beta_1 x_i)^2 = (\boldsymbol{y} - \beta_0 - \beta_1 \boldsymbol{x})^T (\boldsymbol{y} - \beta_0 - \beta_1 \boldsymbol{x})$$

is given as

$$\hat\beta_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{\text{Cov}(X, Y)}{\text{Var}(X)},$$
$$\hat\beta_0 = \bar{y} - \beta_1 \bar{x}$$

for $\bar{x} = \sum x_i / N$ and $\bar{y} = \sum y_i / N$.

Figure 3.1 (p62) from **ISLR**



Multi-dimensional Linear Regression

For $(p+1)$-vector predictor (X) and scalar response (Y),

$$Y \approx X\beta \quad \longrightarrow \quad \hat{\boldsymbol{y}} = \boldsymbol{X}\beta,$$

where $X_0 = 1$ ($\boldsymbol{X}_{*0} = \mathbf{1}$) and β is a $(p+1)$-column vector.

$$\mathrm{RSS}(\beta) = \frac{1}{2}(\boldsymbol{y} - \boldsymbol{X}\beta)^T(\boldsymbol{y} - \boldsymbol{X}\beta)$$

$$\frac{\partial}{\partial\beta}\mathrm{RSS}(\beta) = -\boldsymbol{X}^T(\boldsymbol{y} - \boldsymbol{X}\beta) \quad \Rightarrow \quad \hat{\beta} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}$$

For $(p+1)$-vector predictor (X) and K-vector response (Y), the result is similarly given as

$$\hat{\boldsymbol{Y}} = \boldsymbol{X}\boldsymbol{B} \quad \text{where} \quad \hat{\boldsymbol{B}} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y},$$

which is the independent regressions on Y_j ($\boldsymbol{Y}_{*j}$) combined together,

$$\hat{\boldsymbol{B}}_{*j} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y}_{*j}$$

Newton's method

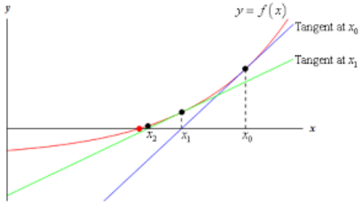
The root x satisfying $f(x) = 0$ can be found by the following iteration:

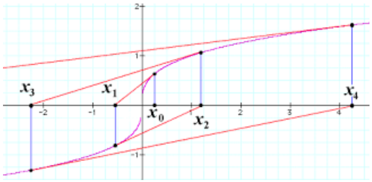
$$x_{n+1} = x_n - \eta \frac{f(x_n)}{f'(x_n)}$$

In multi-dimensional problems, the gradient is used instead of the differentiation:

$$\boldsymbol{x}_{n+1} = \boldsymbol{x}_n - \eta \frac{f(\boldsymbol{x}_n)\nabla f(\boldsymbol{x}_n)}{|\nabla f(\boldsymbol{x}_n)|^2}$$

Typically we use $0 < \eta < 1$ to avoid *overshooting*.





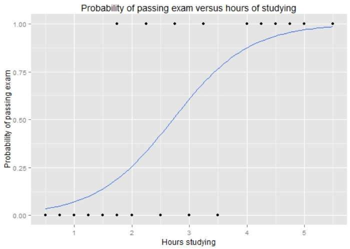
Logistic Regression (Classification)

- Qualitative (categorical) response (binary dependent variable, $Y \in \{0,1\}$)
- Multiple categories: how to give order?
- Linear regression (quantitative) is not proper
- Logistic (sigmoid) function: $\sigma(\text{logit}) = \text{quantile}$

$$p = \phi(t) = \frac{e^t}{1+e^t} = \frac{1}{1+e^{-t}} \quad \text{for} \quad t = X\beta \; (X_0 = 1)$$

- Logit function (the inverse): log odds

$$\phi^{-1}(p) = \log\left(\frac{p}{1-p}\right) = \log(p) - \log(1-p)$$



Fitting of logistic regression

Likelihood function

- For a given the prediction model, measures the likelihood of a data set.
- The best prediction model/weight is the one that maximizes the likelihood of the dataset.

For a data set $(\mathbf{X}, \mathbf{y})$ where $y_i \in \{0, 1\}$),

$$L(\beta) = \prod_i P(y_i = \hat{y}_i) = \prod_{i:y_i=1} \phi(\mathbf{X}_{i*}\beta) \prod_{i:y_i=0} \left(1 - \phi(\mathbf{X}_{i*}\beta)\right)$$

$$= \prod_i \phi(\mathbf{X}_{i*}\beta)^{y_i} \left(1 - \phi(\mathbf{X}_{i*}\beta)\right)^{1-y_i}$$

$$\log L(\beta) = \sum_i y_i \log \phi(\mathbf{X}_{i*}\beta) + (1 - y_i) \log \left(1 - \phi(\mathbf{X}_{i*}\beta)\right)$$

The cost function (to minimize) is $J(\beta) = -\log L(\beta)$

Updating weights

After some algebra,

$$\boldsymbol{w} := \boldsymbol{w} + \Delta \boldsymbol{w}, \quad \Delta \boldsymbol{w} = -\eta \boldsymbol{\nabla} J(\boldsymbol{w}) \tag{1}$$

$$\frac{\partial J(\boldsymbol{w})}{\partial w_j} = -\sum_i (y_i - \phi(\boldsymbol{X}_{i*} w)) X_{ij} = -\boldsymbol{X}_{*j}^T (\boldsymbol{y} - \phi(\boldsymbol{X} w)) \tag{2}$$

$$\text{or} \quad \boldsymbol{\nabla} J(\boldsymbol{w}) = -\boldsymbol{X}^T (\boldsymbol{y} - \phi(\boldsymbol{X} w)). \tag{3}$$

We get a similar weight updating rule as that of linear regression and Adaline! It gives a basis for Perceptron's updating rule.

Regularization

We avoid $\boldsymbol{w}$ being too big.

$$J(\beta) = -\log L(\boldsymbol{w}) + \frac{\lambda}{2} |\boldsymbol{w}|^2 \tag{4}$$

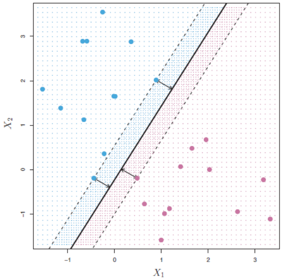
$$J(\beta) = -C \log L(\boldsymbol{w}) + \frac{1}{2} |\boldsymbol{w}|^2 \quad (C = \frac{1}{\lambda}, \text{SciKit-Learn}) \tag{5}$$

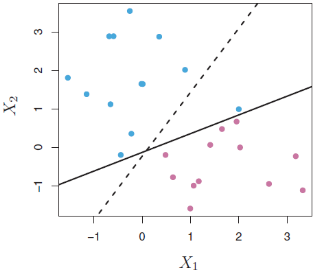
Maximal margin classifier

For $y_i \in \{-1, 1\}$, maximize the margin of the separating hyperplane M,

$$y_i(w_0 + \textstyle\sum_{j=1}^{p} X_{ij} w_j) = y_i(w_0 + \boldsymbol{X}_{i*}\boldsymbol{w}) \geq M > 0 \text{ for all } i, \text{ with } |\boldsymbol{w}| = 1$$

Maximal margin classifier only works for the separable data set and is sensitive to the change in the *support vectors*.





Support vector classifier

We make maximal margin classifier flexible: maximize the margin of the separating hyperplane M with $|\boldsymbol{w}| = 1$,

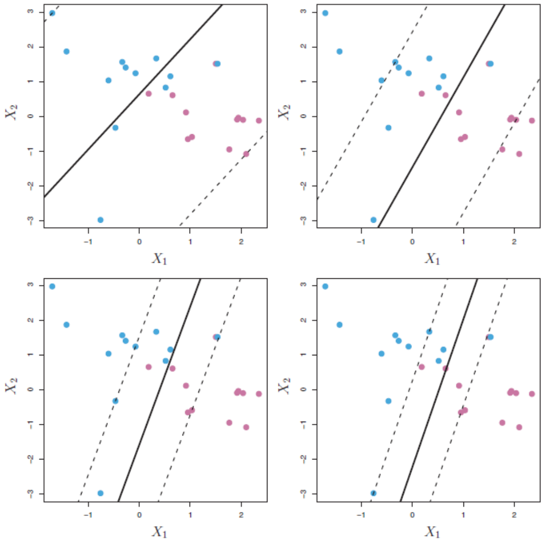
$$y_i(w_0 + \boldsymbol{X}_{i*}\boldsymbol{w}) \geq M(1 - \epsilon_i) \text{ for all } i, \quad \textstyle\sum_{i=1}^n \epsilon_i \leq C,$$

where $\epsilon_i \geq 0$ is *slack variable* indicating the degree of violation ($\varepsilon_i = 0$: no violation, $\varepsilon_i < 1$: margin violation, $\varepsilon_i > 1$: classification violation) and C is a *budget* for the amount of violations by all observations.
Alternatively (in PML), we minimize

$$\frac{1}{2}|\boldsymbol{w}|^2 + C' \sum_{i=1}^{n} \xi_i,$$

where $\{\xi_i \geq 0\}$ satisfies $y_i(w_0 + \boldsymbol{X}_{i*}\boldsymbol{w}) \geq 1 - \xi_i$ for all i. The model converges to maximal margin classifier if C' is very large, so the role of C' is opposite to that of C in the original formulation.

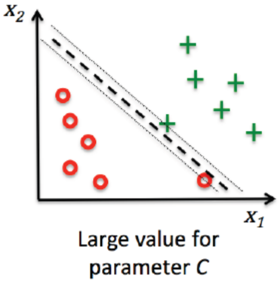
Support vector classifier: role of C

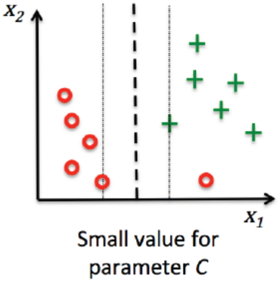


The value of C decreasing from the largest value on top left.

Support vector classifier: role of C'

Large C' (left) vs small C' (right)





Support vector machines (SVM)

How can we deal with non-linear decision boundary?

Enlarging feature space

Including high-order terms, $1, X_j, \cdots, X_j^2, \cdots, X_i X_j, \cdots$, can be helpful, but the computation becomes very heavy.

Kernel

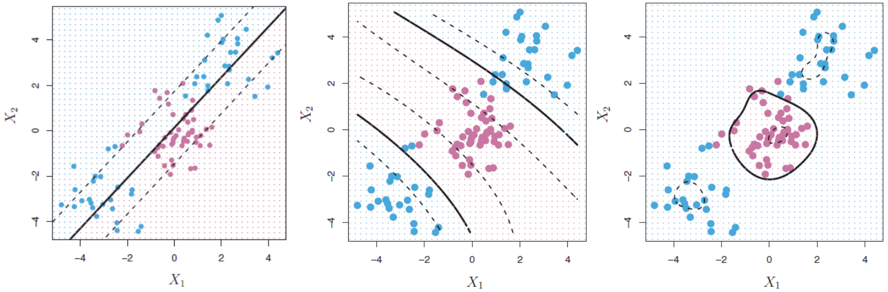
Instead we introduce kernel function, as a generalization of dot product in hyperplane:

- Linear: $K(\boldsymbol{X}_{i*}, \boldsymbol{X}_{i'*}) = \boldsymbol{X}_{i*} \boldsymbol{X}_{i'*}^T$
- Polynomial: $K(\boldsymbol{X}_{i*}, \boldsymbol{X}_{i'*}) = (1 + \boldsymbol{X}_{i*} \boldsymbol{X}_{i'*}^T)^d$
- Radial basis: $K(\boldsymbol{X}_{i*}, \boldsymbol{X}_{i'*}) = \exp(-\gamma |\boldsymbol{X}_{i*} - \boldsymbol{X}_{i'*}|^2)$

Kernel $K(\boldsymbol{X}_{i*}, \boldsymbol{X}_{i'*})$ can be understood as a *distance* between two observations: $\boldsymbol{X}_{i*}$ and $\boldsymbol{X}_{i'*}$ are similar if the kernel value is high (low) whereas they are different if low (high).

SVM: non-linear decision boundary

SVM classification with linear kernel (left) polynomial kernel of degree 3 (middle) and radial basis kernel (right)



K Nearest Neighbor (KNN)

If $N_K(x)$ is the set of the K nearest neighbors around x,

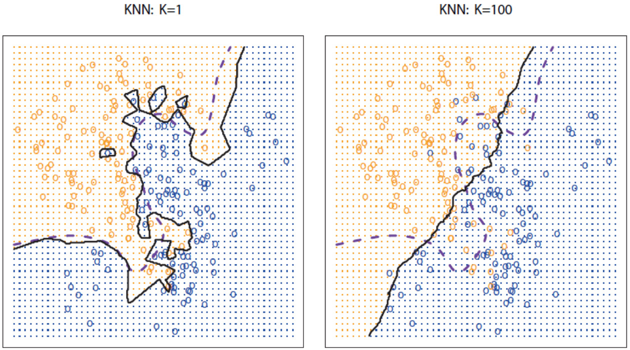
$$\text{Regression:} \hat{y} = f(x) = \frac{1}{K} \sum_{x_i \in N_K(x)} y_i$$

$$\text{Classifier: } \hat{y} = \text{majority of } \{y_i\} \text{ for } x_i \in N_K(x)$$

$$\text{Prob}(y = j|x) = \frac{1}{K} \sum_{x_i \in N_K(x)} I(y_i = j)$$

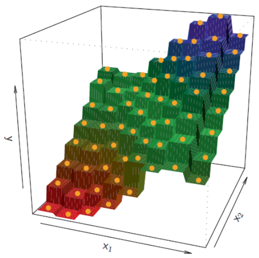
- Parametric vs Non-parametric model
- Learning step is not required, but KNN is intensive in both computation and storage (*memorize* training data set for prediction)

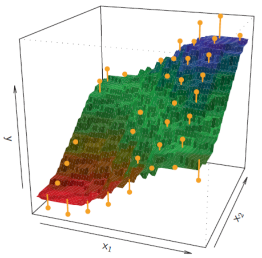
KNN: Classfier example

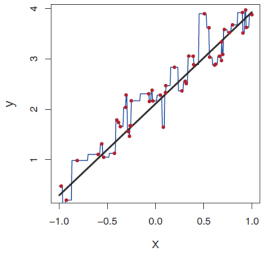


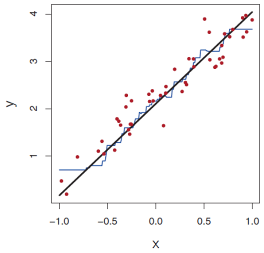
KNN: Regression example

$K = 1$ (left) vs $K = 9$ (right)

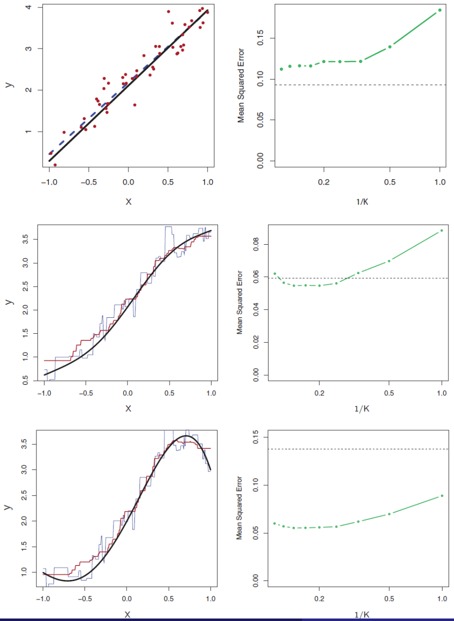








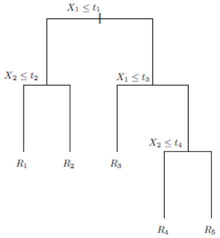
KNN: Parametric vs Non-parametric

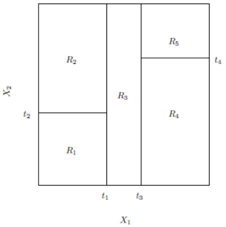


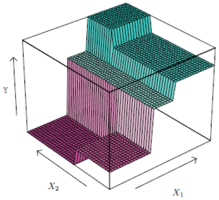
Non-parametric regression works better as the true function deviates from the basis function (linear function in this example).

Decision Tree

- Regression/classification is made on a series of conditions on input variables
- Final conclusion on each *terminal node* or *leaf* of the (upside down) tree.
- The predictor space is broken down to boxes
- Regression: the average value is assigned to each box
- Classification: the majority class is assigned to each box







Growing Tree

We want to minimize the error in each leaf. For a given leaf of tree, t, Regression Error:

$$RSS(t) = \sum_{i \in t} (y_i - \hat{y}_t)^2$$

Classification Error (measure of impurity):

- Gini index: $I_G(t) = \sum_{k=1}^K p(k|t)\left(\boxed{1} - p(k|t)\right) = 1 - \sum_{k=1}^K p^2(k|t)$
- (Cross-)Entropy: $I_H(t) = -\sum_{i=1}^K p(k|t) \log_2 p(k|t)$
- Classification Error: $I_E(t) = 1 - \max_{1 \le k \le K} p(k|t)$
- $I_G(t) = I_H(t) = I_E(t) = 0$ if the composition is pure, i.e., $p(k|t) = 1$ for some k. Otherwise > 0.
- I_E is less sensitive for branching options, so not recommended for growing tree.

Ideally, we can grow tree such that each leaf contains one sample point, but it is over-fitting. Thus we need to regularize the number of leaves or the maximum level of branching.

Growing Tree

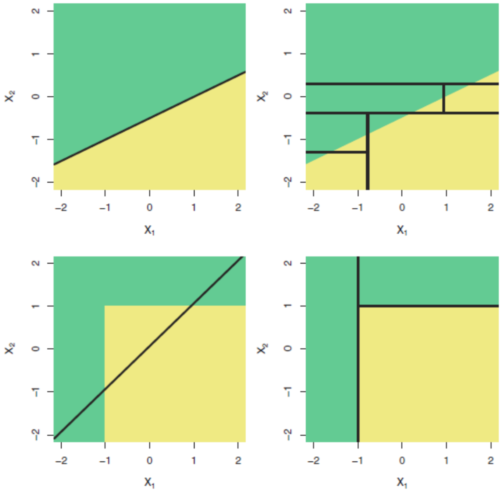
We decide the binary split condition (and feature) in such a way that maximize the information gain (or increase purity):

$$\mathrm{IG}(D_P) = I(D_P) - \frac{N_L}{N_p} I(D_L) - \frac{N_R}{N_P} I(D_R)$$

where D_P, D_L and D_R are the parent, left and right data set and N is the number of the samples in the corresponding sets.

$$I(D) = \sum_{t \in D} I(t) \quad \text{where} \quad I = I_E, I_H \text{ or } I_G$$

Tree vs Linear model



Two classification problems (top vs bottom) approached by linear model (left) and decision tree (right)
Linear model outperforms decision tree on the top problem, whereas

Decision Tree

Pros

- Intuitive and easy to explain. (Even easier than linear regression)
- Closely mirror human decision making
- Can be displayed graphically and easily interpreted by non-experts

Cons

- Prediction is less accurate than other ML methods
- Model variance is high-variant (sensitive to input samples)
  $\longrightarrow$ Bagging, Random Forests, Boosting

Bagging, Random Forest, etc (Ensemble Learning, Ch 7)

Build an ensemble of different classifiers/regressors: the result is interpreted as majority vote/average.

$$\hat{f}_E(X) = \mathrm{Avg}_{e\in E}\{\hat{f}_e(X)\} \quad \text{or} \quad \hat{f}_E(X) = \mathrm{Majority}_{e\in E}\{\hat{f}_e(X)\}$$

Bagging

- Build different trees out of subsets (*bags*) of training set.
- Increase prediction accuracy and reduce model variance

Random Forest (RF)

- Build different trees out of subsets of training set.
- At each split, randomly select m ($\approx \sqrt{p}$) out of p features.
- By random selection of features, RF reduces the correlation between the trained trees.

Both bagging and RF can be applied to any ML methods.

Data Pre-processing (Ch 4)

Handling missing data

- Remove the sample
- Fill in `NaN` with the mean of the average of the feature

Normalization

$$X'_{ij} = \frac{X_{ij} - \min(\boldsymbol{X}_{*j})}{\max(\boldsymbol{X}_{*j}) - \min(\boldsymbol{X}_{*j})}$$

Standardization

$$X'_{ij} = \frac{X_{ij} - E(\boldsymbol{X}_{*j})}{\sigma(\boldsymbol{X}_{*j})}$$

Regularization L-1 vs L-2

Give a penalty for complexity or over-fitting. The cost function to minimize:

$$J(\boldsymbol{w}) = J_0(\boldsymbol{w}) + \lambda R(\boldsymbol{w}) \quad (= C\, J_0(\boldsymbol{w}) + R(\boldsymbol{w})),$$

where $J_0(\boldsymbol{w})$ is the un-regularized cost function, e.g., log-maximum likelihood (logistic) or RSS/SEE (linear).

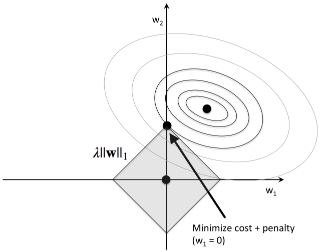
L-2 Regularization

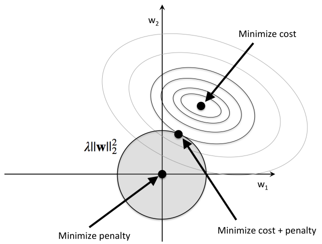
- $R(\boldsymbol{w}) = ||\boldsymbol{w}||_2^2 = \sum_j w_j^2$
- N-sphere boundary (e.g., circle or sphere). Easy to solve the minimum.

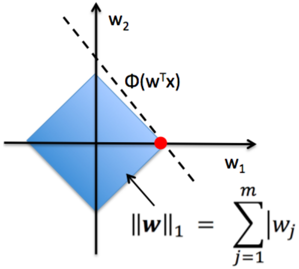
L-1 Regularization

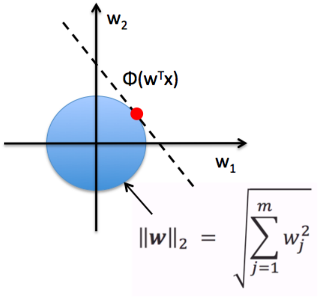
- $R(\boldsymbol{w}) = ||\boldsymbol{w}||_1 = \sum_j |w_j|$
- 'Diamond' boundary: leads to sparse vector (many zero components)
- Effectively works as feature selection

Regularization L-1 vs L-2





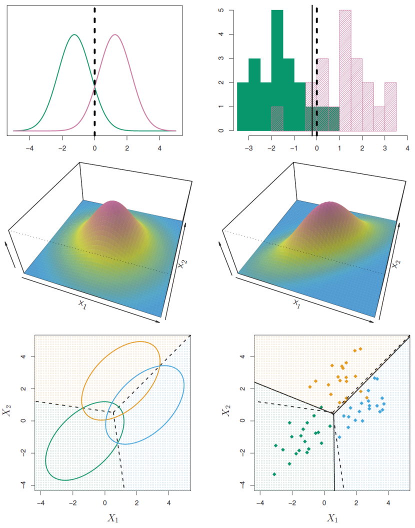




Feature selection

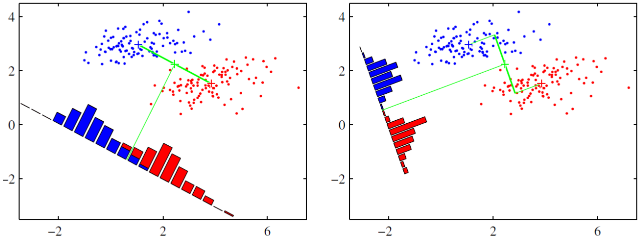
Linear Discriminant Analysis (LDA) as a classifier

- Assume the samples in each class follow normal (Gaussian) distribution.
- Estimate mean $\hat{\boldsymbol{\mu}}_k$ and variance $\hat{\boldsymbol{\Sigma}}_k$ of class k:
- Obtain multi-variate normal PDF: $f_k(\boldsymbol{x}) = n(\boldsymbol{x}|\hat{\boldsymbol{\mu}}_k, \hat{\boldsymbol{\Sigma}}_k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$
- LDA if $\boldsymbol{\Sigma}_W = \sum_{k=1}^K \boldsymbol{\Sigma}_k$ (within covariance)is used for all $\boldsymbol{\Sigma}_k$.
- QDA if $\boldsymbol{\Sigma}_k$ is estimated for each class k
- A test sample $\boldsymbol{x}$ is classified to the class k for which $f_k(\boldsymbol{x})$ is largest.



LDA as a dimensionality reduction

- Given the LDA assumptions, which direction $\boldsymbol{w}$ best separates the feature?
- $\boldsymbol{w} \approx \boldsymbol{\mu}_2 - \boldsymbol{\mu}_1$? Probably not the best
- Want to minimize $J(\boldsymbol{w}) = \frac{(\mu_2-\mu_1)^2}{\sigma_1^2+\sigma_2^2}$, where (μ_1, σ_1^2) and (μ_2, σ_2^2) are mean variance of the samples (1-D) projected on $\boldsymbol{w}$.
- The best directions $\boldsymbol{w}$ are the first eigenvectors of $\Sigma_W^{-1}\Sigma_B$, where Σ_W and Σ_B are within and between covariance matrices.
- The transformation $\boldsymbol{z} = \boldsymbol{x}\boldsymbol{W}$ is the extracted factors with the best separability, which can be used for other ML methods.


4
2
0
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6