Energy Budget of Wave Disturbances over the Marshall Islands During the Years of 1956 and 1958

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Abstract

Based on the data in the Marshall Islands obtained in 1956 and 1958, vertical velocities and heat source associated with tropical wave disturbances are estimated and the energy transformation of the disturbances are examined. An approximate estimate of the upward transport of sensible and latent heat by small-scale convective motions is also obtained.

Eddy available potential energy is generated by the heat of condensation and is transformed to the eddy kinetic energy in the upper troposphere centered about 300 mb. Energy flux is upward above the level about 300 mb and downward below that level. Other terms of energy transformations such as the conversions from mean kinetic energy and from mean potential energy are one or two order of magnitude smaller than terms of generation and conversion of eddy available potential energy.

The sensible and the latent heat supplied from the sea surface is transported upward by the cumulus convections and strong convective transport corresponds to the upward motion of the large-scale disturbances. A non-dimensional heating parameter $\eta(p)$ is also obtained and is compared with that used in theoretical studies.

1. Introduction

In the past few years a great deal of observational and theoretical studies of large-scale disturbances in the tropics have been performed. The renewed interest in tropical wave disturbances emerged from a timely advance of the basic dynamic theory (*e.g.*, Matsuno, 1966) and the discovery of large-scale stratospheric waves (Yanai and Maruyama, 1966; Wallace and Kousky, 1968). Subsequently many studies of waves in the tropical troposphere have been performed mainly by the use of the spectrum analysis technique (Yanai *et al.*, 1968; Wallace and Chang, 1969; Nitta, 1970 a).

The energy source of the tropical wave disturbances has been a controversial problem. Manabe and Smagorinsky (1967) and also Manabe *et al.* (1970) made an analysis of energetics of the tropical circulation obtained by numerical time integration of moist general circulation models. They showed that kinetic energy of the disturbances in the model tropics is chiefly maintained by the conversion of available potential energy, which is generated by the released heat of condensation. Yamasaki (1969) developed an instability theory of large-scale tropical disturbances, in which the so-called CISK mechanism is the primary cause of the waves. He showed that three types of unstable waves other than the tropical cyclone can exist in the conditionally unstable atmosphere in low latitudes. Hayashi (1970) extended this work to a three-dimensional model and obtained unstable waves which strikingly resemble the observed Yanai-Maruyama waves. On the other hand, Mak (1969) constructed a two-layer model of dry tropical atmosphere and showed that the eddies in the model tropics gain kinetic energy from the lateral pressure work due to middle-latitude disturbances. Nitta and Yanai (1969) examined the possibility of barotropic instability in the trades easterlies. The relative importance of these mechanism must be examined against the observation.

There are very few works which analysed the energy transformation process of the real tropical disturbances. Based on the analysis of the Pacific data of 1962, Nitta (1970 b) showed the importance of pressure interaction with middle latitude disturbances. Recently Nitta (1970 c) comupted the vertical velocity and obtained an estimate of the liberated heat using the upper-air data of the Marshall Islands area during 1956. He showed that a large amount of eddy available potential energy is generated and converted into the eddy kinetic energy in the upper troposphere. Wallace (1971) has reported a similar result based on the data of 1967. It is the primary aim of this study to examine thoroughly the energy cycle of large-scale disturbances in the tropics on the basis of observations for the year 1956 and 1958.

It has been suggested that cumulus convection plays an important role in the generation of eddy available potential energy through the release of latent heat. However, the interaction between the large-scale disturbances and cumulus convection is understood only crudely. In the studies of tropical cyclones, Ooyama (1964) and Charney and Eliassen (1964) assumed that the rate of total heat production by cumulus convection is proportional to the supply of water vapor by the large-scale horizontal convergence in the surface boundary layer. Yamasaki (1969) and Hayashi (1970) applied the same assumption to the analysis of the stability of large-scale wave disturbances with wavelengths ranging from several thousand to ten thousand kilometers. It is a very important problem to find a relation between the heating process in large-scale disturbances and small-scale cumulus convection. In this paper we try to gain some information concerning vertical transports of sensible and latent heat by cumulus convection and to examine the relationship between the activity of cumulus convection and large-scale disturbances.

2. Data and method of analysis

The data of special upper air observations over the Marshall Islands area during two periods are used in this study. One period is from April to July in 1956 and the other periods is from March to July in 1958. Locations of stations are shown in Fig. 1. Almost all stations except Nauru, Rongerik and Utirik were in operation during both periods. Wind, temperature T, relative humidity R and geopotential height ϕ data are available at the constant pressure levels of 1000, 850, 700, 500, 400, 300, 200, 150, 100, 50 and 25 mb in 1956 and at the levels of 1000, 850, 700, 600, 500, 400, 300, 250, 200, 150, 100, 50, and 25 mb in 1958. The original data in 1956 are four times daily upper air observations at 03, 09,

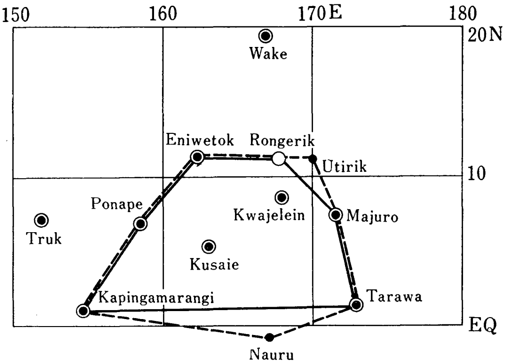


Fig. 1. Upper-wind observation stations. ○ denotes during the period April through July 1956, ● stations during the period March through July 1958 and ⊙ stations during both periods. The area enclosed by the solid lines and the dashed lines are used in calculations of the vertical velocity and the heat source in 1956 and in 1958 respectively.

15 and 21 GMT but we transform these data into twice daily data at 03 and 15 GMT by averaging three serial data weighted by 1:2:1. The data in 1958 are twice daily upper air observations (00 GMT and 12 GMT). We supply the missing data from the linear interpolation of the time series data. The areas enclosed by solid and dashed lines show areas where vertical velocities and heat sources are computed for the 1956 and 1958 data respectively.

We use the cross-spectrum analysis of the time series data to estimate the energy transformation functions. The co-spectrum between two quantities at a certain frequency represents the contribution of oscillations of that frequency to the total variance such as the meridional or the vertical fluxs of heat, momentum and various energy transformation functions. Prior to the spectrum analysis, we use a high-pass filter which eliminates the variations in the period range longer than about 20 days. We apply a maximum lag number 50 (25 days) which gives 51 spectral estimates at a interval of 0.02 cycle per day.

3. Computation of vertical velocity and heat source

The mothod of obtaining the large-scale vertical motion and heat source are the same as those described in Nitta (1970 c). The vertical p-velocity $\omega \equiv \frac{dp}{dt}$ is computed by the use of the continuity relation

$$\nabla \cdot V + \frac{\partial \omega}{\partial p} = 0 \tag{1}$$

which was integrated over the area enclosed by thick solid lines for the 1956 data and thick dashed lines for the 1958 data as shown in Fig. 1. We assume that ω vanishes at 1000 mb and at 50 mb for the 1956 series. Because of substantial number of missing data of winds above the 50 mb level, we assume $\omega(100\,\text{mb})=0$ for the 1958 series. In order to satisfy the assumption $\omega=0$ at the top level, we use modified values of $\nabla \cdot V$ defined by

$$(\nabla \cdot V)_{md} \equiv \nabla \cdot V + \frac{\int_{P_0}^{P_T} \nabla \cdot V dp}{p_0 - p_T} \tag{2}$$

where $(\nabla \cdot V)_{md}$ is the modified value of divergence, $\nabla \cdot V$ the computed value of divergence from the real data, $p_0 = 1000\,\text{mb}$ and p_T the pressure of the top level. We compute the vertical velocity using $(\nabla \cdot V)_{md}$ in (2) and then $\omega_{p=P_T}=0$ is satisfied. We computed also ω at 100 mb for the 1958 series by the use of an adiabatic method to check the varidity of this assumption. The order of the maximum value of ω at 100 mb obtained by the adiabatic method was about 10^{-1} mb/hour so that the assumption of $\omega(100\,\text{mb})=0$ can be justified. In addition to the mean horizontal divergence and vertical velocity we compute the mean vorticity of the same area.

We compute also the liberated heat of condensation associated with disturbances as residuals of the thermodynamic equation and the moisture continuity equation. These equations are written as follows

$$Q_\theta = \frac{c_p}{\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}}} \left\{ \frac{\partial \theta}{\partial t} + \nabla \cdot (\theta V) + \frac{\partial}{\partial p}(\theta \omega) \right\} - Q_R \tag{3}$$

$$Q_q = -L \left\{ \frac{\partial q}{\partial t} + \nabla \cdot (q V) + \frac{\partial}{\partial p}(q \omega) \right\} \tag{4}$$

where Q_θ is the time diabatic heating rate except the heating rate due to radiation Q_R, Q_q the time moisture sink, θ the potential temperature, q the mixing ratio, c_p the specific heat of air under constant pressure, $\gamma = c_p/c_v$, c_v the specific heat of air under constant volume and L the latent heat of condensation. If we take the average of equations (3) and (4) over the area under consideration, we obtain

$$[Q_\theta] = a\left\{ \left[\frac{\partial \theta}{\partial t}\right] + [\nabla \cdot \theta V] + \frac{\partial}{\partial p}([\theta][\omega]) + \frac{\partial}{\partial p}[\theta^* \omega^*] \right\} - [Q_R] \tag{5}$$

$$[Q_q] = -b\left\{ \left[\frac{\partial q}{\partial t}\right] + [\nabla \cdot q V] + \frac{\partial}{\partial p}([q][\omega]) + \frac{\partial}{\partial p}[q^* \omega^*] \right\} \tag{6}$$

where $a \equiv c_p/(p_0/p)^{1-1/\gamma}$, $b \equiv L$, the bracket denotes areal mean and the asterisk the deviation from it. $[\theta^*\omega^*]$ and $[q^*\omega^*]$ are the vertical eddy transfer of the sensible heat and the moisture due to eddies of the subgrid size containing both eddies in the boundary layer and cumulus convections. When we estimate the heat source using the data at stations as shown in Fig. 1, we calculate the following terms in (5) and (6)

$$Q_1 = a\left\{ \left[\frac{\partial \theta}{\partial t}\right] + [\nabla \cdot \theta V] + \frac{\partial}{\partial p}([\theta][\omega]) \right\} \tag{7}$$

$$Q_2 = -b\left\{ \left[\frac{\partial q}{\partial t}\right] + [\nabla \cdot q V] + \frac{\partial}{\partial p}([q][\omega]) \right\} \tag{8}$$

We use ω (modified vertical velocity) obtained from (1) as $[\omega]$ and averaged values of θ, q, $\partial\theta/\partial t$ and $\partial q/\partial t$ in the analysed area as $[\theta]$, $[q]$, $[\partial\theta/\partial t]$ and $[\partial q/\partial t]$. When we calculate $[\nabla \cdot \theta V]$ and $[\nabla \cdot q V]$ we use the modified wind velocity whose horizontal divergence equals to the modified divergence defined in (2). We consider that effects of the cumulus-scale motions are negligible in terms of $[\nabla \cdot \theta V]$ and $[\nabla \cdot q V]$. Computed results show that the order of $[\partial\theta/\partial t]$ is about 10^{-1}°k/day, $[\nabla \cdot \theta V]$ about 10^{1}°k/day and $\partial/\partial p([\theta][\omega])$ about 10^{1}°k/day. $[\nabla \cdot \theta V]$ and $\partial/\partial p([\theta][\omega])$ cancel each other and residuals are about 10^{0}°k/day. Since the

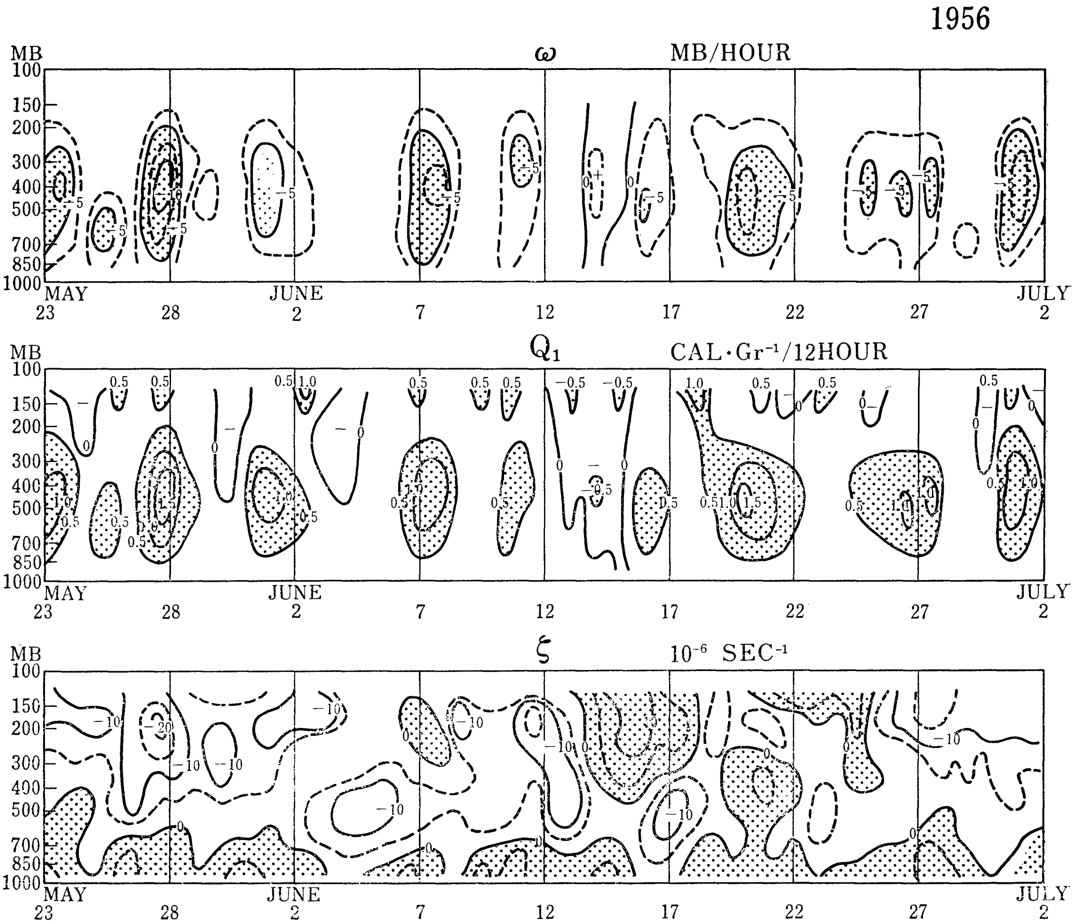


Fig. 2. Vertical-time sections of the vertical velocity, the heat source and the relative vorticity from 23 May to 2 July 1956.

horizontal gradient of the temperature is small, $[\nabla\cdot\theta V]\simeq[\theta][\nabla\cdot V]$ so that we obtain

$$Q_1\simeq a\left\{[\nabla\cdot\theta V]+\frac{\partial}{\partial p}([\theta][\omega])\right\}$$
$$\simeq a[\omega]\partial/\partial p[\theta] \tag{9}$$

(9) means that the diabatic heating nearly balances with the adiabatic cooling.

Comparing (7) and (8) with (5) and (6), we find that computed heat sources Q_1 and Q_2 do not include the terms of radiation heating, the vertical transport of the sensible and the latent heat due to the cumulus convection and the vertical eddy flux due to the diffusion process in the boundary layer. Since the non-adiabatic heating except the heating of the radiation is considered to be mainly due to the latent heat in the tropical troposphere, we can set $[Q_\theta]=[Q_q]\equiv Q_c$ and then Q_1 and Q_2 are rewritten as follows

$$Q_1=Q_c-a\frac{\partial}{\partial p}[\theta^*\omega^*]+[Q_R] \tag{10}$$

$$Q_2=Q_c+b\frac{\partial}{\partial p}[q^*\omega^*] \tag{11}$$

Fig. 2 illustrates vertical time sections of ω, Q_1 and ζ (relative vorticity) from 23 May to 2 July in 1956. Both ω and Q_1 vary in a similar manner and a typical periodicity of about 5 days is clearly seen. The maximum value of Q_1 is about 1.5 $\mathrm{Cal}\cdot g_r^{-1}/12$ hour. There exists positive relative vorticity in the lower troposphere below 700 mb and strong positive vorticities generally correspond to strong upward motions.

In Fig. 3 thick solid lines and thick dashed lines show vertical distributions of the time mean and the standard deviation of ω for the two sampling periods. The mean vertical motion is

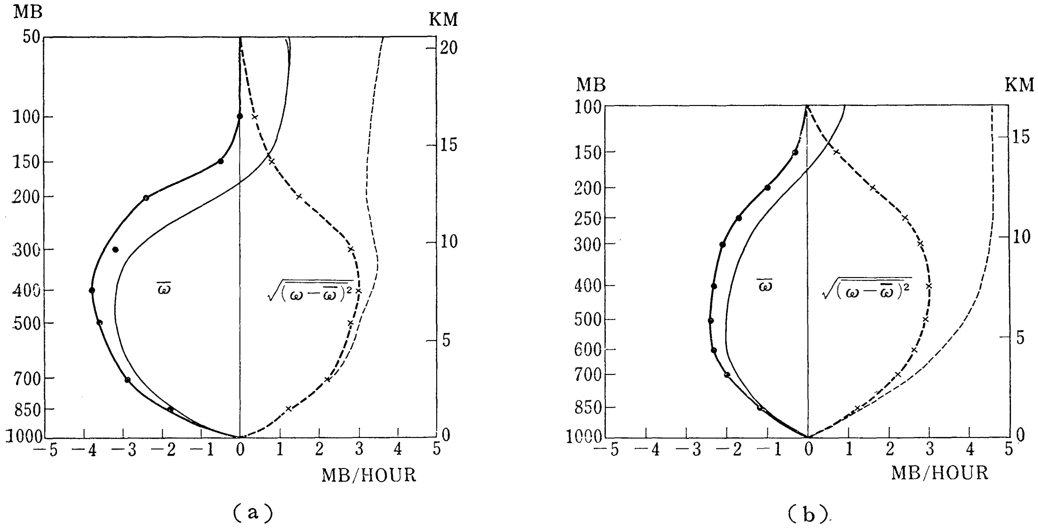


Fig. 3. Vertical distributions of mean vertical velocities and their standard deviations for (a) 1956 and (b) 1958. Thick lines denote results when $\omega_{TOP}=0$ is assumed and thin lines denote results when $\omega_{TOP}=0$ in not assumed.

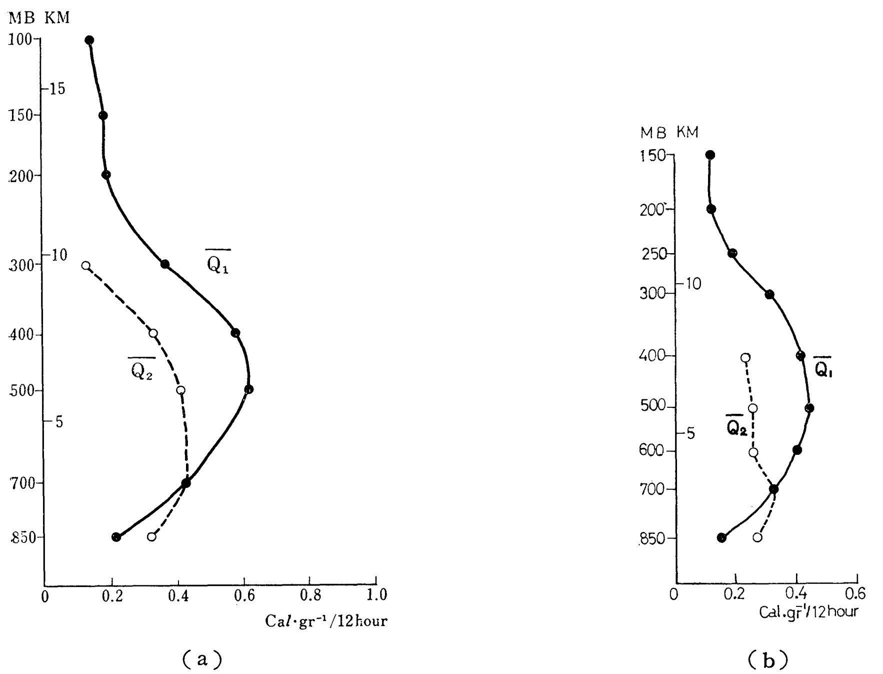


Fig. 4. Vertical distributions of mean values of Q_1 and Q_2 for (a) 1956 and (b) 1958.

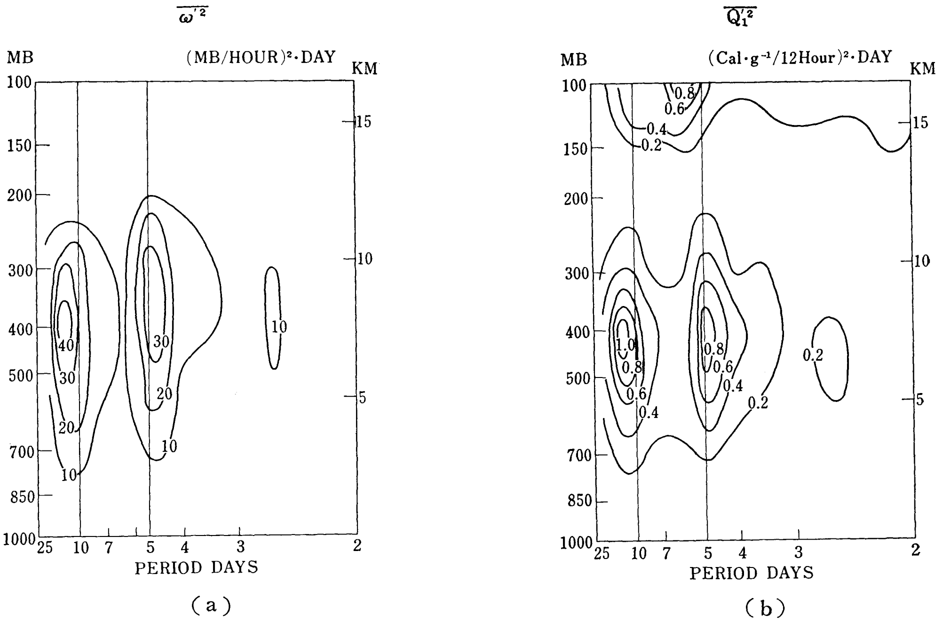


Fig. 5. Vertical distributions of (a) the ω-spectra and (b) the Q_1-spectra for 1956.

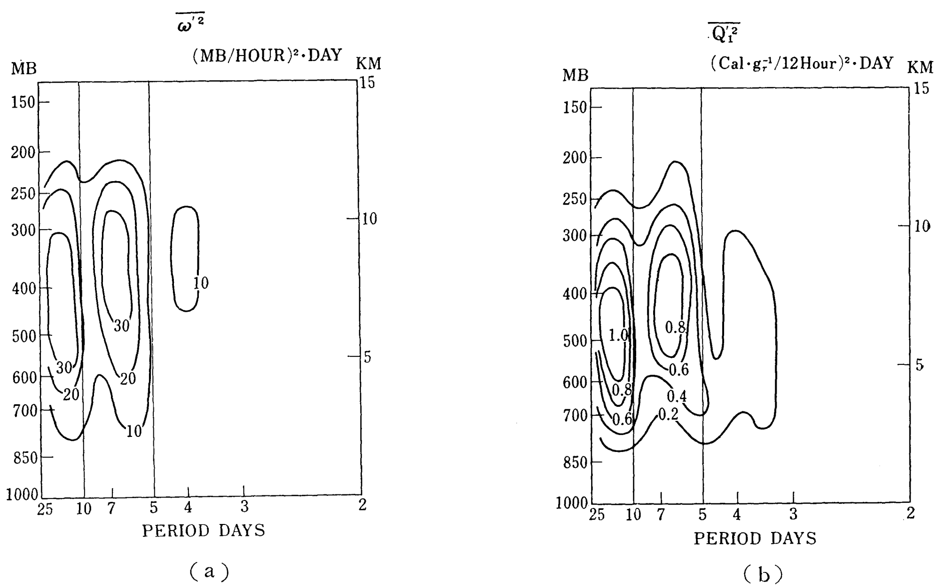


Fig. 6. The same as Fig. 5 except for 1958.

upward in both periods, showing that inter tropical convergence zone may be situated over the area under consideration. $\bar{\omega}$ for 1958 is smaller than that for 1956. Thin solid lines and thin dashed lines respectively show vertical distributions of the time mean and the standard deviation of ω which were not adjusted to the upper boundary conditions. In this case variances of ω become large in the upper troposphere.

Fig. 4 illustrates vertical distributions of time-mean values of Q_1 and Q_2. Maximum values of $\bar{Q}_1$ occur at 500 mb but those of $\bar{Q}_2$ occur at 700 mb. $\bar{Q}_2$ is larger than $\bar{Q}_1$ at levels below 700 mb and smaller at levels above 700 mb. As shown in equations (9) and (10), the difference between $\bar{Q}_1$ and $\bar{Q}_2$ may be due to upward transports of sensible and latent heat by cumulus convections and eddy motions in the boundary layer and the radiation cooling. This point will be discussed in section 5. Both $\bar{Q}_1$ and $\bar{Q}_2$ for 1958 are smaller than those for 1956.

Fig. 5 shows height-frequency distributions of the power spectrum density of ω and that of Q_1 for 1956. There are two distinct maximum of the power density centered at 5-day and 12.5-day periods both in the ω- and the Q_1-spectra. Both the ω- and the Q_1-spectral density attain their maximum values at about 400 mb. The power spectra of the moisture sink Q_2 obtained from the moisture budget (not shown) have also two distinct maxima centered at the same periods but the maximum values of Q_2-spectra occur at a level lower than the level of maxumum Q_1-spectral density. Fig. 6 shows vertical distributions of the power spectrum of ω and that of Q_1 for 1958. In 1958 there are two distinct maxima of the power density centered at about 6-day and 12.5-day periods. Maximum values of the ω-spectral density and those of the Q_1-spectral density in 1958 are almost of the same order of magnitude as those in 1956.

4. Energy budget of wave disturbances

In this section we shall study the energy cycle associated with wave disturbances, including the generation and conversion of eddy available potential energy. We use the same notations as in Nitta (1970 b). We assume that the wavy perturbations with a certain periodicity propagates in the east-west direction without changing amplitude in a basic state whose variations in time and space are slow. From linearized equations of motion and the first law of thermodynamics we can derive the following energy equations for the wave disturbances.

$$\frac{\partial \overline{k'}}{\partial t}+\frac{\partial}{\partial y}\overline{(\phi' v')}+\frac{\partial}{\partial p}\overline{(\phi'\omega')}=-\frac{\partial U}{\partial y}\overline{u'v'}-\frac{\partial U}{\partial p}\overline{u'\omega'}-\overline{\alpha'\omega'} \qquad (12)$$

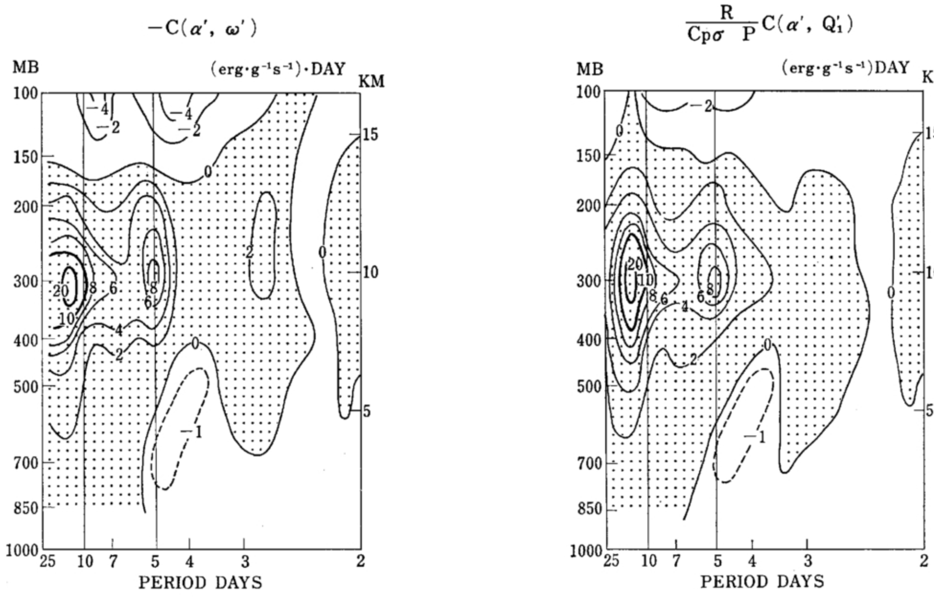
$$\frac{\partial \overline{e'}}{\partial t}=-\frac{\frac{\partial \bar{\alpha}}{\partial y}}{\sigma}\overline{v'\alpha'}+\overline{\alpha'\omega'}+\frac{R}{c_p\sigma p}\overline{\alpha' Q'} \qquad (13)$$

where $k'\equiv\frac{1}{2}(u'^2+v'^2)$ and $e'\equiv\frac{1}{2\sigma}\,\alpha'^2$ are the eddy kinetic energy and the eddy available potential energy respectively, U the basic mean zonal current, $\bar{\alpha}$ the mean specific volume and σ the mean static stability defined by $\sigma=-\frac{1}{\rho\bar{\theta}}\times\frac{\partial\bar{\theta}}{\partial p}$. Over-bars denote means in the longitudinal direction and primes denote quantities of disturbances. From the above assumption that variations of a basic state are slow in time and space and that wave disturbances propagates in the east-west direction without changing amplitudes, we can consider that over-bars denote time means and primes denote quantities of disturbances deviated from the time mean. $\frac{\partial}{\partial y}\overline{(\phi'v')}$ and $\frac{\partial}{\partial p}\overline{(\phi'\omega')}$ in (12) denote the divergence of wave energy flux, $(K_Z, K_E)_H\equiv-\partial U/\partial y\;\overline{u'v'}$ and $(K_Z, K_E)_V\equiv-\partial U/\partial p\;\overline{u'\omega'}$ the energy conversions from the mean zonal wind and $(A_E, K_E)\equiv-\overline{\alpha'\omega'}$ the energy conversion from eddy available potential evergy to eddy kinetic energy. $(A_Z, A_E)\equiv-(\partial\bar{\alpha}/\partial y)/\sigma\;\overline{v'\alpha'}$ in (13) denotes the conversion from mean potential energy to eddy available potential energy and $G(A_E)\equiv-R/(c_p\sigma p)\;\overline{\alpha'Q'}$ the generaion of eddy available potential energy.

$\overline{u'v'}$, $\overline{\alpha'\omega'}$, $\overline{\alpha'Q'}$, …… *etc.* are estimated from the covariance between u' and v', α' and ω', α' and Q',…… *etc.*. We integrate the co-spectra between various parameters in the period range from 1 day to 20 days but the contribution of diurnal variations is excluded. We take Q_1 as Q and estimate the term of $G(A_E)$. When we estimate covariances between ω or Q_1 and other parameters, we average other parameter in the analysed area becuase only one value of ω and that of Q_1 are obtained at each observation time respectively. When we estimate covariances which do not involve ω or Q_1 such as $\overline{u'v'}$, we first compute the variance at the individual stations and then take an average for the analysed area.

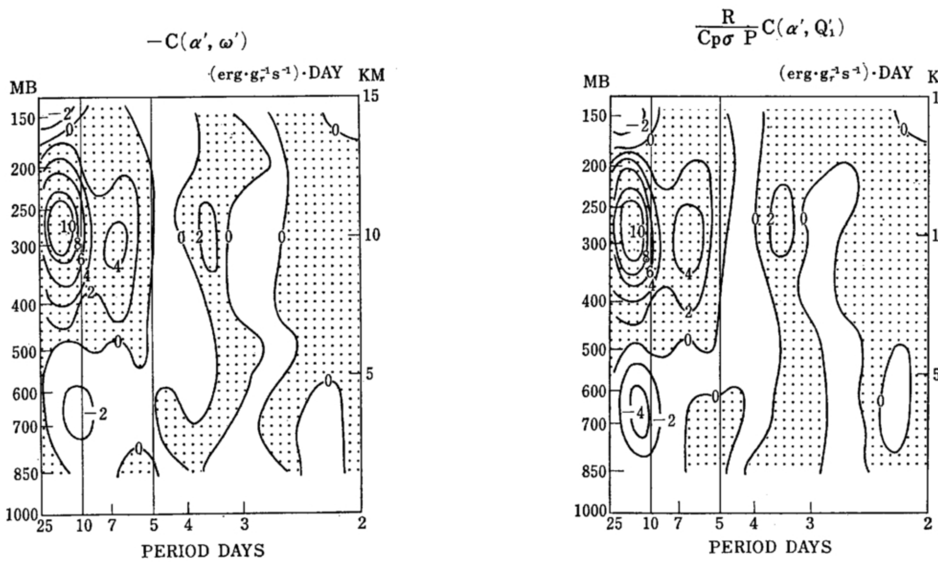
4.1. *Conversion and generation of eddy available potential energy*

Fig. 7 and Fig. 8 respectively illustrate vertical distributions of spectral density of conversion and that of generation of eddy available potential energy for the 1956 and the 1958 observation period. We find that a large conversion from eddy available potential energy



(a) (b)

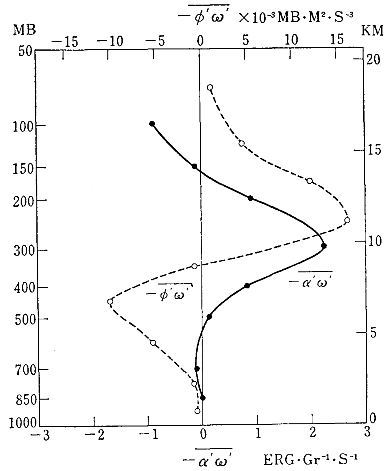
Fig. 7. Vertical distributions of the co-spectra representing (a) the conversion of eddy available potential energy and (b) the generation of eddy available potential energy for 1956.



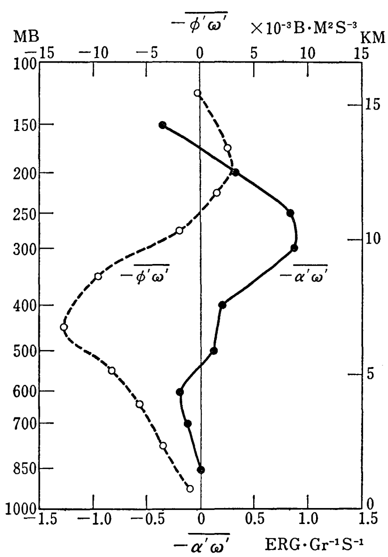
(a) (b)

Fig. 8. The same as Fig. 7. except for 1958.

to eddy kinetic energy occurs at levels between 400 mb and 200 mb in both the periods. In 1956, disturbances with periods longer than 10 days and those with period near 5 days mainly contributed to the energy conversion, while in 1958, disturbances with periods longer than 10 days and those with period near 6 days mainly contributed to the conversion. In the lower troposphere there are weak negative energy conversion in a period range near 4 day in 1956



(a)



(b)

Fig. 9. Vertical distributions of energy conversion of eddy available potential energy and the vertical flux of energy integrated from 1-day to 20-day periods for (a) 1956 and (b) 1958.

and in a period range near 10 days in 1958. Vertical distributions of the spectral density of generation of eddy available potential energy are nearly identical with those of the energy conversion. This is an obvious result because the diabatic heating Q_1 nearly balances with the adiabatic cooling as described in (9). We obtain the following relation from (9)

$$-\overline{\alpha'\omega'} \simeq \frac{R}{c_p \sigma p} \overline{\alpha' Q'} \tag{14}$$

Fig. 9 show vertical distributions of the total conversion of eddy available potential energy and the total vertical eddy flux of energy due to all disturbances with periods shorter than 20 days excluding the contribution due to diurnal variations. For the both analysed periods, a large positive energy conversion takes place in the upper troposphere centered around the 300 mb level and a small negative energy conversion occurs in the lower troposphere. The maximum value of energy conversion in 1956 is two times larger than that in 1958. The direction of the vertical energy flux is upward above the 300 mb level and is downward below in agreement with the results of Yanai and Hayashi (1969). Upward energy flux in 1956 is also larger than that in 1958.

Results of the above estimates of generation and conversion of eddy available potential energy and the vertical energy flux show that eddy available potential energy is generated by the heating in the upper troposphere centered at about 300 mb, and it is converted into eddy kinetic energy. The energy is also redistributed upward and downward from the level of maximum generation. The vertical distributions of energy conversion and vertical energy flux in this study are qualitatively similar to those obtained numerically by Manabe *et al.* (1970) in their model tropics including moist convection. But the magnitudes of conversion and those of vertical energy flux in this study are a few times smaller than those of Manabe *et al.* (1970). This discrepancy may be due to the facts that energy conversion of Manabe *et al.* (1970) contains that of standing eddies or that the heating estimated from the parameterization of moist convection is larger than that in the real atmosphere, but further works will be needed to clear these points. Recently Wallace (1971) has computed the vertical motion and has estimated in the same manner as that in this study for the data of 1967 and has obtained very similar results.

4.2. Energy budget of wave disturbances

In this subsection we shall study the whole energy cycle associated with wave disturbances. The estimates of various energy trasformation functions described in equations (12) and (13)

Table 1. Energy transformations due to the disturbances with periods from 1 days to 20 days. Unit is 10^{-1} erg·gr^{-1}·s^{-1}.

(a) 1956.

MB	$-U_y\ \overline{u'v'}$	$-U_p\ \overline{u'\omega'}$	$-\frac{\partial}{\partial p}\overline{(\phi'\omega')}$	$-\overline{\alpha'\omega'}$	$\frac{R}{c_p\sigma p}\ \overline{\alpha'Q'}$	$-\frac{\bar{\alpha}_y}{\sigma}\overline{v'\alpha'}$
50	−0.4					−0.2
100	0.0	−0.5	6.6	−8.8	−5.3	−0.2
150	0.1	0.1	15.0	−1.1	−0.8	−0.0
200	−0.1	0.3	5.2	8.9	6.5	−0.2
300	−1.0	−0.3	−16.0	22.3	23.3	−0.1
400	−0.4	−0.1	−9.3	8.4	8.7	0.0
500	−0.2	0.5	3.1	1.3	1.9	0.0
700	0.0	0.3	2.5	−0.9	−0.7	−0.0
850	0.0	−0.2	0.2	0.0	−0.0	0.0
1000	0.1					−0.0

(b) 1958

MB	$-U_y\ \overline{u'v'}$	$-U_p\ \overline{u'\omega'}$	$-\frac{\partial}{\partial p}\overline{(\phi'\omega')}$	$-\overline{\alpha'\omega'}$	$\frac{R}{c_p\sigma p}\ \overline{\alpha'Q'}$	$-\frac{\bar{\alpha}_y}{\sigma}\overline{v'\alpha'}$
100	−0.1					−0.0
150	1.2	−0.3	5.4	−3.7	−3.2	−0.0
200	3.2	−0.1	−1.9	3.2	4.1	0.1
250	1.2	−0.2	−7.4	8.6	8.6	0.1
300	−0.5	−0.1	−10.0	8.8	9.9	0.1
400	−0.3	0.1	−3.1	2.0	2.4	−0.0
500	0.1	0.1	4.4	1.3	−1.5	−0.0
600	−0.1	0.0	2.8	−2.0	−2.5	−0.0
700	−0.2	0.1	2.9	−1.3	−1.6	−0.0
850	−0.0	−0.0	4.4	0.0	0.2	0.0
1000	0.3					−0.2

are summarized in Table 1. Generation of eddy available potential energy $G(A_E)$, conversion from eddy available potential energy (A_E, K_E) and vertical energy flux convergence are the most important in the energy balance of disturbances in the tropics. Though a large energy conversion from the kinetic energy of the mean zonal wind occurs at 200 mb in 1958, magnitudes of energy convesions from the mean zonal wind and the mean potential energy are generally one or two order smaller than (A_E, K_E). As described in *4.1.*, magnitudes of $G(A_E)$ and (A_E, K_E) in 1956 are about two times larger than those in 1958. Values of $-\frac{\partial}{\partial y}\overline{(\phi'v')}$ have the order of magnitude which is comparable to that of (A_E, K_E) at some upper tropospheric levels, but the estimates of $-\frac{\partial}{\partial y}\overline{(\phi'v')}$ are not so reliable because the estimated values of $\overline{\phi'v'}$ are scattered from station to station. This is contrasted to the year of 1962 for which the lateral flux of energy was more systematically obtained (Nitta, 1970 b). This irregularity of $\overline{\phi'v'}$ may be partly due to the inaccuracy of the geopotential height data. The value of $\overline{\phi'v'}$ for 1962 estimated by Nitta (1970 b) was a third of the value of $-\overline{\alpha'\omega'}$ in this study for 1956 and we consider that $\overline{\phi'v'}$ also plays important role on energy balance in the tropics and will be estimated more precisely.

Table 2. Heating rate by net radiative process averaged in the western Pacific region between 0°N and 10°N. (After Katayama, 1967). Unit is °C/day.

MB	$[Q_R]$
100	−0.5
200	−0.8
300	−1.5
400	−2.0
500	−1.6
600	−0.9
700	−0.6
850	−0.5
900	−0.7

5. Vertical transport of the sensible and the latent heat by cumulus convections and non-dimensional parameter of heating

We cannot directly obtain the sensible and the latent heat transports by small-scale convective motions, but we can indirectly infer them from the difference between the estimates of Q_1 and Q_2. If we subtract (10) from (11), we obtain

$$Q_2-Q_1=a\frac{\partial}{\partial p}[\theta^*\omega^*]+b\frac{\partial}{\partial p}[q^*\omega^*]-[Q_R] \tag{15}$$

The climatorogical value of the radiation cooling in the low latitudes has maximum at about 400 mb and its value is 2°C/day which corresponds to $[Q_R]\sim -0.2\mathrm{Cal\cdot gr^{-1}}/12$ hour. In the troposphere lower than 500 mb where the radiation cooling is small, the difference between Q_1 and Q_2 implies the effects of the small-scale convective motion. If we ignore the pressure dependence of a and integrate (15) from the surface to some pressure level p, we obtain

$$a[\theta^*\omega^*]_p+b[q^*\omega^*]_p\simeq a[\theta^*\omega^*]_s+b[q^*\omega^*]_s+\int_{p_s}^{p}\{Q_2-Q_1+[Q_R]\}dp \tag{16}$$

where $a[\theta^*\omega^*]_s$ and $b[q^*\omega^*]_s$ are the sensible and the latent heat flux from the sea surface due to eddy diffusion. If we calculate the terms on the right hand side in (15), we can obtain the sensible and the latent heat flux due to cumulus convections.

We use $\bar{Q}_1$, $\bar{Q}_2$ (time means) and the climatorogical value of $[Q_R]$ in the western Pacific in July estimated by Katayama (1967) as listed in Table 2. Since the latent heat flux from the surface is much larger than the sensible heat flux over the tropical oceans, we calculate only $b[q^*\omega^*]_s$ by the Jacob's method,

$$b[q^*\omega^*]_s=-\rho gLCe(q_s-q_a)V \tag{17}$$

where ρ is the density of air at the surface Ce the dimensionless exchange coefficients for latent heat, q_s the saturated mixing ratio corresponding to the sea surface temperature, q_a the mixing ratio of surface air temperature and V the surface wind speed. We assume $Ce=Cd$ where Cd is the darg coefficient. We take $Cd=1.0\times10^{-3}$ in this study because the surface wind speed is small. We calculate the latent heat flux from the data of Kusaie because of the centered station in the analysed area. For 1956, $V=3.5$ m/s and $q_s-q_a=6.3$ g/kg and then $b[q^*\omega^*]_s=-77$ Cal·gr^{-1}·MB/12 hour. For 1958, $V=4.9$ m/s

Table 3. Water balance over the analysed area (mm/day).

	$\frac{1}{g}\int_{p_s}^{200}\frac{dq}{dt}dp$	precip.	evapor.
1956	7.5	8.5	2.6
1958	6.4	9.6	3.1

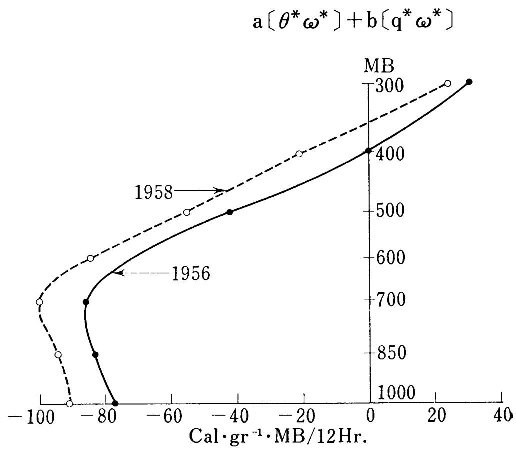


Fig. 10. The sum of the vertical transports of the sensible and latent heat for 1956 (solid lines) and 1958 (dashed lines) due to cumulus convections and the turbulance in the surface boundary layer.

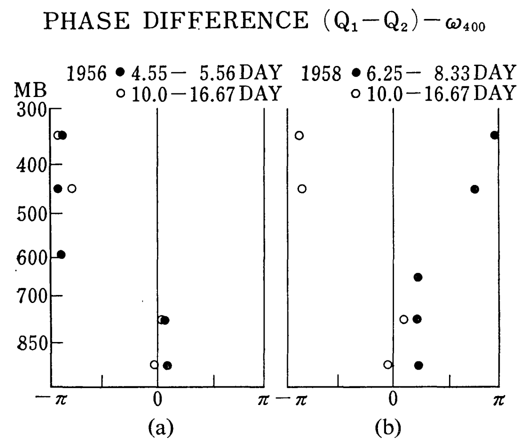


Fig. 11. Vertical distributions of the phase difference between the activity of the small-scale convective motions and the vertical velocity at 400 mb for (a) the 4.55～5.56 day period (full circles) and 10.0～16. 67-day period (open circles) in 1956 and (b) the 6.25 ～8.33-day period (full circles) and 10.0～16.67-day period (open circles) in 1958.

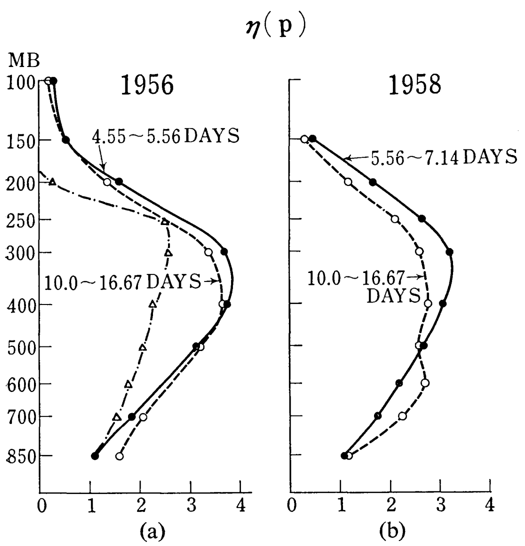


Fig. 12. Vertical distributions of the heat parameter $\eta(p)$ for (a) 4.55 ～5.56-day period (full circles) and 10.0～16.67-day period (open circles) in 1956 and (b) 5.56～7.14-day period (full circles) and 10.0～16.67-day period (open circles) in 1958. Triangles denote $\eta(p)$ assumed by Hayashi (1970) in his theoretical study.

and $q_s-q_a=5.2$ g/kg and then $b[q^*\omega^*]_s=-91$ Cal·gr^{-1}·MB/12 hour. In order to check the estimate of the latent heat flux from the sea surface, we examine the balance of the water vapor, *i.e.*, the balance between precipitation P, evaporation from the surface E and moisture change in the atmosphere. Table 3 shows values of each terms averaged for analysed area and period for 1956 and 1958. When we calculate moisture change, we assume $dq/dt=0$ at 200 mb and integrate from 1000 mb to 200 mb. The balance between three terms is good for 1958, while $P-E$ is larger than the moisture change for 1956. Estimation of evaporation from the surface can be said to be reasonable on the whole.

Fig. 10 shows the vertical profiles of upward transports of sensible and latent heat obtained from (16). The sensible and the latent heat supplied from the surface is transported upward by the cumulus convection. The level where the vertical transports are zero is about 400 mb and may correspond to the top level of the cumulus. Since we use $\bar{Q}_1$, $\bar{Q}_2$, the climatorogical value of $[Q_R]$ and $b[q^*\omega^*]_s$ averaged in the analysed periods, the estimated height of the cumulus indicates mean height of the cumulus in the analysed area and periods.

As described in (15), Q_2-Q_1 shows the activity of the convective motions. Fig. 11 shows the phase relation between Q_1-Q_2 and ω at 400 mb for the both analysed periods. Q_1-Q_2 is in phase with ω_{400} below 600 mb and is out of phase above that level. This phase reversal is caused by the reversal of the sign of Q_1-Q_2 at about 700 mb as shown in Fig. 4. The above phase relation between Q_1-Q_2 and ω_{400} may indicate that strong convective transport of heat corresponds to the strong upward motions of the large-scale disturbances.

Yamasaki (1969) and Hayashi (1970) assumed that the release of latent heat responds to the large-scale horizontal convergence of water vapor in the subcloud layer and examined the instability of large-scale wave motions in low latitudes. They parameterized the heating by

$$\frac{R}{c_p p}Q'=-\eta(p)\sigma\omega'_{p=900\text{mb}} \tag{18}$$

where $\eta(p)$ is a non-dimensional parameter expressing the vertical distribution of heating. They assumed certain distributions of $\eta(p)$ and obtained unstable disturbances. It has been said that vertical distributions of $\eta(p)$ is impotant for characteristic features of large-scale disturbances but vertical distributions of $\eta(p)$ have never been estimated observationally in the real atmosphere. Fig. 12 shows vertical distributions of $\eta(p)$ obtained from data for 1956 and 1958 which is defined by (18). We use the vertical velocity at 850 mb and take Q_1 as the heating. We pay attention on wave disturbances which have large power density and estimate Q' and $\omega'_{p=850\mathrm{mb}}$ from the power spectra of Q_1 and $\omega_{p=850\mathrm{mb}}$. Vertical distributions of $\eta(p)$ for two disturbances for 1956 are similar to each other. $\eta(p)$ has maximum at levels from 400 mb to 300 mb and decreases rapidly above 150 mb. $\eta(p)$ of the disturbance with period about 6 days for 1958 shows maximum values at 300 mb but that of the disturbance with longer period has maximum at levels from 600 mb to 300 mb. Comparing $\eta(p)$ in this study with $\eta(p)$ assumed by Yamasaki (1969) and Hayashi (1970), values of the latter are smaller than those of the former and the level of the maximum value of the latter is higher than that of the former. But it can be said that their assumption of $\eta(p)$ is almost suitable for representation of the heating of the large-scale disturbances in the tropic.

6. Summary and concluding remarks

We have examined the energy budget of the wave disturbances in the tropical troposphere observed in 1956 and 1958 by a spectral method. The generation of eddy available potential energy $G(A_E)$ due to the heating by the small-scale convective motions and the conversion from eddy available potential energy to eddy kinetic energy $C(A_E, K_E)$ play to the most important role in the energy balace of the disturbances. Other energy conversions such as the conversions from mean kinetic energy and from mean potential energy are one or two order smaller than terms of $G(A_E)$ and $C(A_E, K_E)$. We cannot obtain reliable values of energy flux convergence from higher latitudes by pressure interaction because of the inaccuracy of the geopotential height data. It will be desirable to gain reliable value of energy flux convergence and to compare these values with and $C(A_E, K_E)$ in future. Results about the energy budget in this study agree with the numerical study by Manabe *et al.* (1970) and the theoretical study by Hayashi (1970).

Then we studied the vertical transport of heat and moisture due to small-scale convective motions. The sensible and the latent heat are transported upward by the small-scale motions and the activity of the convective motions relates to the large-scale mean upward motions. We obtained the non-dimensional parameter expressing heating $\eta(p)$. Results of $\eta(p)$ show that the assumption of $\eta(p)$ of Yamasaki (1969) and Hayashi (1970) is suitable on the role for representation of the heating of the large-scale disturbances in the tropics.

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1956年および1958年のマーシャル諸島における熱帯擾乱のエネルギー収支

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1956年と1958年のマーシャル諸島におけるデータを使い，熱帯擾乱による上昇流，発熱量を見積り，擾乱のエネルギー変換について調べた．さらに積雲対流によって上方へ輸送される顕熱および潜熱の量を求めた．

上部対流圏の 300 mb 付近で凝結熱により擾乱の有効位置エネルギーが生成され擾乱の運動エネルギーへ変換される．このエネルギーは 300 mb 付近から上層および下層に流れていく．他のエネルギー変換量——平均場の運動エネルギーおよび有効位置エネルギーからの変換量——は，擾乱の有効位置エネルギーの生成量，変換量に比較して１～２オーダー小さい．

海面から供給された顕熱，潜熱は積雲対流により上方へ輸送され，対流による強い上方輸送は大規模擾乱の上昇流に対応している．発熱量の無次元パラメータ $\eta(p)$ を求め，これまで理論研究で仮定されていた $\eta(p)$ の値と比較した．