

An analysis of international exchange rates using multivariate DLM's

JOSÉ M. QUINTANA & MIKE WEST

Department of Statistics, University of Warwick, Coventry CV4 7AL, U.K.

Abstract. New models for multiple time series are introduced and illustrated in an application to international currency exchange rate data. The models, based on matrix-variate normal extensions of the dynamic linear model (DLM), provide a tractable, sequential procedure for estimation of unknown covariance structure between series. A principal components analysis is carried out providing a basis for easy model assessment. A practically important elaboration of the model incorporates time-variation in covariance matrices.

1 Introduction

Financial investment decisions often depend critically on the relationships over time amongst several similar time series. International currency exchange rate series are a typical case, their contemporaneous variation playing an important role in decisions relating to the spread of risk in investments, as in the design of optimal portfolios (see, e.g. Granger (1972)). In such studies, multivariate time series models are necessary to model and identify the joint structure on which these decisions depend. Extensions of DLM's are used here for estimation of covariance structure across several time series. These models are based on a general class of dynamic, matrix-variate normal DLM's introduced in Quintana (1985). Resulting principal components analyses provide insight into the multivariate structure and also a simple guide to model choice and assessment. A practically important elaboration of the basic model is to incorporate time-variation in covariance matrices. Full details of the data and analyses may be found in Quintana & West (1986).

The exchange rate series appear in Fig. 1. The data, taken from the CSO macro-economic time series data bank, are over the period from January 1975 to August 1984 inclusive, by month. Six series are considered, being the monthly exchange rate in British pounds of the US Dollar, Deutschmark, Yen, French Franc, Lira and Canadian Dollar. These series are plotted after logarithmic transform and, for easy visual comparison, standardised to zero at the beginning of the time period.

2 Dynamic multivariate regression

A dynamic multivariate regression model for a series of q -vector (column vector) observations \mathbf{y}_t , ($t=1, 2, \dots$) is specified by the following equations, sequentially in time.

Observation:

$$\mathbf{y}'_t = \mathbf{x}'_t \boldsymbol{\Theta}_t + \mathbf{e}'_t, \quad \mathbf{e}_t \sim N[0, v_t \boldsymbol{\Sigma}]. \quad (2.1a)$$

Evolution:

$$\boldsymbol{\Theta}_t = G_t \boldsymbol{\Theta}_{t-1} + F_t, \quad F_t \sim N[O, W_t \boldsymbol{\Sigma}]. \quad (2.1b)$$

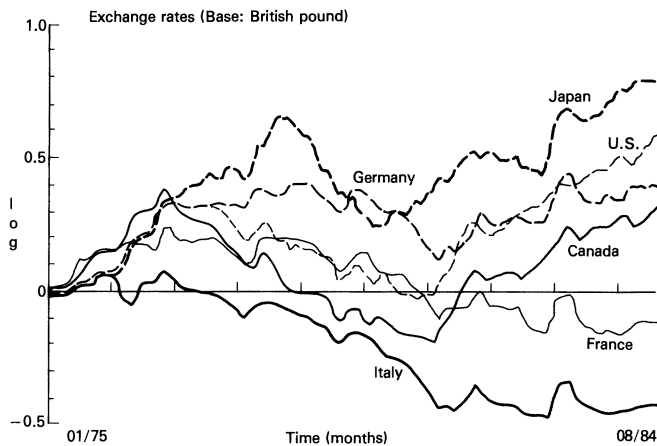


Fig. 1. Exchange rates of the US Dollar, the Deutschmark, the Japanese Yen, the French Franc, the Italian Lira and Canadian Dollar.

Prior information (Given the history of the series up to $t-1$):

$$\Theta_{t-1} \sim N[M_{t-1}, C_{t-1}, \Sigma], \quad \Sigma \sim W^{-1}[S_{t-1}, d_{t-1}]. \quad (2.1c)$$

Here \mathbf{x}_t is a p -vector series of independent variables, Θ_t an unknown $p \times q$ matrix of dynamic regression parameters, \mathbf{e}_t a q -vector of errors, G_t a known $p \times p$ evolution matrix, F_t a $p \times q$ evolution error matrix, v_t a known scalar variance multiplier and W_t a known $p \times p$ variance matrix. These quantities, along with the unknown $q \times q$ variance matrix Σ , define the structural components (2.1a and b) of the model. The matrix normal notation in (2.1b and c) is detailed in Quintana (1985). In (2.1c) a matrix-normal/inverted Wishart prior is specified for Θ_{t-1} and Σ given the history of the series. In addition, \mathbf{e}_t , F_t and Θ_{t-1} are independent when Σ is known.

The component series of \mathbf{y}_t follow dynamic linear models linked via the correlation structure determined by Σ . As a particular special case, note that if $v_t=1$, $G_t=I$ and $W_t=0$, then the model is a standard, static (i.e. $\Theta_t=\Theta$ for all t) multivariate regression. Generalisations to cases in which \mathbf{y}_t becomes a matrix of observations are studied in Quintana (1985). Also detailed in that paper are the components of the analysis of the model, which parallel the standard DLM theory. Observing \mathbf{y}_t provides information about Θ_t and Σ that is used to update (2.1c) to analogous equations with index $t-1$ replaced by t . This updating uses matrix analogues of the Kalman filter recurrence relationships for updating Θ_{t-1} and related, simple sequential updating equations for Σ .

3 Exchange rate models

We consider the 6 series, on the shifted logarithmic scale appearing in Fig. 1. Now $\mathbf{y}_t = (y_{1t}, \dots, y_{6t})'$ with y_{jt} being the monthly observation on the j th series in month t . The class of models used assumes that each series follows a univariate linear growth DLM with time-varying level and growth parameters. At time t , the level and growth parameters for y_{jt} are the elements of the j th column of the 2×6 matrix Θ_t , and

$$\mathbf{x}_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad G_t = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

for all t . All joint variations across series is now determined by Σ .

With this relatively general framework we have several specific cases of interest based on particular choices of the driving parameters v_t and W_t , and a vague-like prior parameters

$$M_0 = 0, \quad C_0 = \begin{bmatrix} \varepsilon^{-1} & 0 \\ 0 & \alpha \end{bmatrix}, \quad S_0 = \varepsilon I, \quad d_0 = \varepsilon = 10^{-5}.$$

A standard, static linear trend (SLT) model for each component series corresponds to the above model with $\alpha = \varepsilon^{-1}$, $v_t = 1$ and $W_t = 0$, for all t . A multivariate random walk (RW) model obtains, when $\alpha = 0$, $v_t = 0$ and

$$W_t = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

for all t . Thus, within the dynamic linear growth model, it is possible to assess the relative merits of a variety of models, including these two specific cases, that are of interest, in view of the trend-random walk controversy amongst financial time series analysts. Some further discussion of this appears in later sections.

4 Principal components

Principal components analysis of Σ requires, in principle, posterior distributions at each t for the eigen-values/vectors of Σ . When Σ has an inverted Wishart distribution, these posteriors are extremely difficult to work with. We restrict ourselves to consideration of point estimates over time in which the loss in estimating is the Kullback-Leibler (K-L) directed divergence of the estimated likelihood from the actual likelihood. It is easy to verify that optimal estimates minimise the K-L directed divergence of the estimated likelihood from the predictive density, and therefore our estimators coincide with those found in Amaral & Dunsmore (1980). On observing y_t at time t , the estimate $\hat{\Sigma}$ is easily updated in terms of C_t , S_t and d_t .

A very convenient property of this estimate is the invariance under one-to-one parameter transformations. Therefore, to estimate principal components, the value of Σ may be substituted into the defining equations

$$\Sigma = P \Lambda P' \text{ and } P P' = I$$

in order to deduce estimates of the eigen-values and eigen-vectors.

Example 1

For initial illustration we report some features of the analysis in which $\alpha = 0$, $v_t = 1$ and

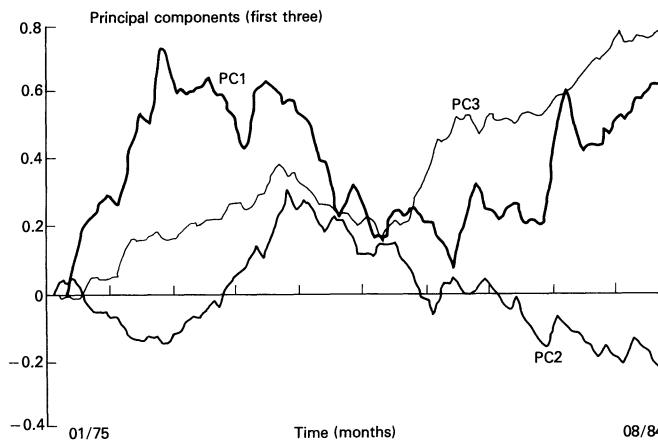
$$W_t = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

This model allows for time variation in both level and growth parameters of the linear trend component models, with magnitudes of such changes determined by W_t . Some features of the estimate of Σ at times 116 are evident in the first three principal components, shown in Table 1.

Table 1. Weights of currencies in first three principal components

	Principal components		
	(1)	(2)	(3)
USA	0.364	-0.555	0.010
Germany	0.398	0.210	-0.367
Japan	0.443	0.417	0.783
France	0.454	0.241	-0.256
Italy	0.387	0.189	-0.408
Canada	0.397	-0.622	0.094
% Variation	65.6	14.6	12.0

The first, dominant, component gives roughly equal weights to the six currencies, thus represents an average performance index relative to the British pound. In fact, after a renormalisation, it is very similar to the index of the pound used by the Bank of England. The second component weights the EEC countries roughly equally, adds in the yen at about twice the weight, and contrasts this EEC/Japan aggregate with an average USA/Canada index. The third, still significant, component drops out the North American countries, contrasting Japan with an average EEC index. Time variation in the relative well-being of the three American/European/Japanese sectors can be seen from the plot of the first three principal components, (using the final estimates in Table 1) over time. This is Fig. 2.

**Fig. 2.** The first three principal components based on the initial model.

5 Model assessment

Formal assessment of predictive performance of models using the cumulative product of observed one-step predictive densities can be applied to the entire multivariate model. For simplicity and economy, however, we assess models on the basis of

forecasting performance on the univariate series obtained using the first principal component which accounts for over 65% total variation. Predictive performance on this series should carry through to the original series. From (2.1) it is not difficult to see that, with \mathbf{x}_t and G_t as defined in Section 3, any linear combination of the components series follows a simple univariate linear trend DLM.

Initially, the (SLT) and (RW) models, both as described in Section 3, may be compared using the Bayes' factor—just the ratio of predictive densities from the two models (Harrison & West, 1986). The SLT model is rapidly rejected in favour of RW which performs consistently better yielding a final cumulative Bayes' factor with value of $\exp(189.2)$. A similar study on the entire multivariate series confirms this message coming from the first principal component.

A further class of models of interest are those in which $v_t = v$,

$$W_t = \begin{bmatrix} w_{11} & 0 \\ 0 & w_{22} \end{bmatrix}$$

and $v + w_{11} + w_{22} = 1.02$ for all t . The latter constraint is for identifiability; a common factor in v and W is absorbed in Σ . Using the first principal component series above, it turns out that predictive performance is optimised by RW-like models ($v=0$), and in particular with models close to that with $v=0$ and $w_{11}=1$, $w_{22}=0.02$.

6 Time-dependent variance matrix

There has been considerable discussion amongst financial time series modellers about the suggestion that series such as ours have variances that are essentially infinite. This has led, for example, to the use of stable distributions as alternatives to normality (Fama, 1965, for example). It is absolutely clear, however, that such contentions depend entirely on the models used and within which the variances have meaning. Use of an inflexible, static time series or regression model can easily lead to persistent over-estimation of observational variances. If the structural form of the model is generally adequate, large variance estimates may derive from the assumption that observational variances are constant whereas, in fact, they are subject to change over time in a deterministic and/or stochastic manner. Such time dependent variances are not uncommon in economic and commercial applications (Harrison & West, 1986; Granger, 1972). In many cases, purely stochastic variation is evident and, often, a model allowing for slow, random changes in variances can adequately capture the important features of any structural changes. A simple and natural approach to such modelling is used here in a straight forward multivariate generalisation of the discount method as in Harrison & West (1986). Basically, the estimation of Σ is discounted

Table 2. Weights of currencies in first three principal components at $t=116$: dynamic variance model

	Principal components		
	(1)	(2)	(3)
USA	0.228	-0.656	-0.122
Germany	0.457	0.269	-0.323
Japan	0.483	0.016	0.870
France	0.517	0.211	-0.273
Italy	0.433	0.135	-0.180
Canada	0.226	-0.658	-0.115
% Variation	62.8	23.6	9.7

using a discount factor β , ($0 < \beta < 1$), simulating a random walk type of evolution for the variance matrix which will thus be indexed by t . The net effect of the discount in the updating recurrence at time t is that the values of S_{t-1} and d_{t-1} must be replaced by βS_{t-1} and βd_{t-1} .

The factor β multiplying the prior parameters S_{t-1} and d_{t-1} before updating represents a loss of $100(\beta^{-1}-1)\%$ of the information about Σ_{t-1} in evolving to Σ_t . Notice that, in the degenerate case $\beta=1$, $\Sigma_t=\Sigma$ and the recurrence formula is unchanged. This special case can be assessed, as can other values of β , using Bayes' factors as usual.

Example 2

The model chosen in Section 5 as optional for the exchange rate series with constant Σ is re-examined and compared with an alternative in which the only difference is the use of $\beta=0.95$ rather than $\beta=1$. The overwhelming weight of evidence in favour of the dynamic variance model is evident in the Bayes' factor for $\beta=0.95$ versus $\beta=1$, calculated over time and plotted, on a logarithmic scale in Fig. 3. Note that this is based on predictive densities for the full multivariate series. The estimated $\hat{\Sigma}_{116}$ from the dynamic variance model has first three principal components given in Table 2.

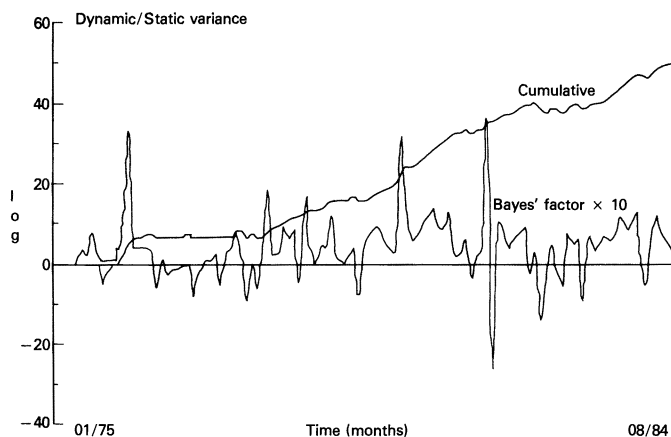


Fig. 3. Bayes' factor and cumulative Bayes' factor for the 'optimal' model with dynamic variance relative to its static counterpart.

The general features are similar to those in Example 1. There are, however, clear changes reflecting the need for a time varying Σ_t to adapt to changing economic relationships over the period of the data. The time variant nature of Σ is noticeable in the sequential estimates, at 6 month intervals, of the first three eigenvalues plotted in Fig. 4.

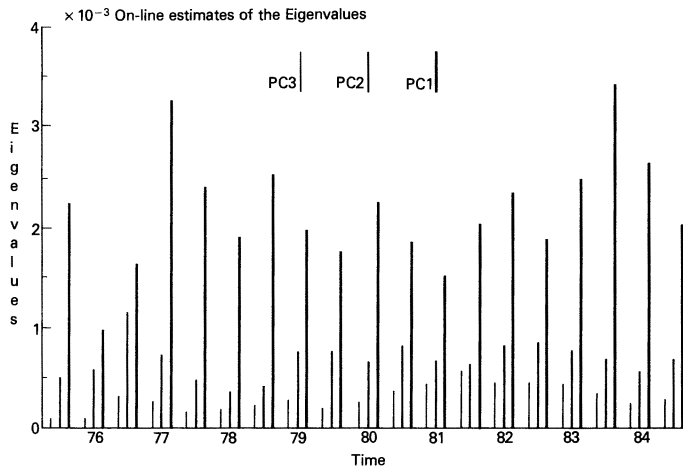


Fig. 4. On line estimates of the eigenvalues of Σ_t .

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