



$$e^{-at} \xleftrightarrow{L} \frac{1}{s+a}$$

$$\mathcal{L}\{f(t)\} = F(s)$$

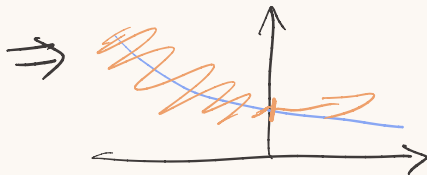
$$= \int_{-\infty}^{\infty} f(t) \cdot e^{-st} dt$$

$$\hookrightarrow F(s) = \int_{-\infty}^{+\infty} \boxed{e^{-at}} e^{-st} dt$$

$$= \int_{-\infty}^{+\infty} e^{-t(a+s)} dt$$

$$= \int_{-\infty}^{+\infty} e^{-u} du \cdot \left( \frac{1}{s+a} \right) \quad \left\{ \begin{array}{l} u = t(s+a) \\ \frac{\partial u}{\partial t} = s+a \end{array} \right.$$

$$= \frac{1}{s+a} \int_{-\infty}^{+\infty} e^{-u} du$$



$$e^{-at} \cdot H(t)$$



$$U(t) = \boxed{\text{step function}} \rightarrow$$

$$\left. \begin{array}{l} \text{step} = 1 \\ \text{amp} = 6V \end{array} \right\} H(t-1) \cdot 6 \xleftrightarrow{L} 6 \cdot \frac{1}{s+1}$$

$$G(z) = \frac{k_0}{1 + \gamma z^2}$$

$$\frac{k_0}{1 + \gamma \cdot z}$$

$$\frac{1}{z+a}$$

↓

$$\frac{k_0/\gamma}{\frac{1}{\gamma} + z} = \frac{k_0}{\gamma} \cdot \frac{1}{\frac{1}{\gamma} + z}$$

↓  
a

↕  $L^{-1}$

$$\frac{k_0}{\gamma} \cdot$$

Reescrita da Resposta

$$\bullet \frac{\Omega_H(z)}{V_p(z)} = G(z) = \frac{K_0}{1 + \gamma z} \Rightarrow p = -\frac{1}{\gamma}$$

$$\Rightarrow \Omega_H(z) = V_p(z) \cdot \frac{K_0/\gamma}{\frac{1}{\gamma} + z}$$

$H(t-1)$

$$\bullet v_p(t) = \cancel{H(t)} \cdot 6 \Leftrightarrow 6 \cdot \frac{e^{-z}}{2} = V_p(z)$$

$$\left( v_p(t) = \cancel{H(t)} \cdot 6 \Leftrightarrow \mathcal{L}\{v_p(t)\} = 6 \cdot \frac{e^{-z}}{2} \right)$$

$$\Rightarrow \Omega_H(z) = \left( \frac{6K_0}{\gamma} \cdot \frac{1}{2} \cdot \frac{1}{\frac{1}{\gamma} + z} \right) e^{-z}$$

$H(t), U(t), u(t),$   
 $1(t),$

Polar:

$$G(z) \rightarrow \infty$$

$$\Rightarrow |G(z)| \rightarrow 0$$

$\Rightarrow p \in \text{zeros dos denominadores}$

• Sabemos que  $\Omega_H(z)$  pode ser reescrito como:

(Teorema X)

$$\Omega_H(z) = \left( \frac{B_1}{z} + \frac{B_2}{\frac{1}{\gamma} + z} \right) e^{-z}, \text{ tg.}$$

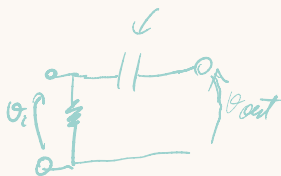
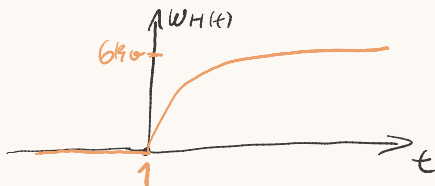
$$B_1 = \Omega_H(z) \cdot \left( z \right) \Big|_{z=0} = \frac{6K_0}{\gamma} \cdot \frac{1}{\frac{1}{\gamma} + z} \cdot \frac{1}{z} \Big|_{z=0}$$

$$B_2 = \Omega_H(z) \cdot \left( \frac{1}{\gamma} + z \right) \Big|_{z=-\frac{1}{\gamma}} = \frac{6K_0}{\gamma} \cdot \frac{1}{\frac{1}{\gamma} + z} \Big|_{z=-\frac{1}{\gamma}} = \frac{6K_0}{\gamma} \cdot \frac{1}{\frac{1}{\gamma} - \frac{1}{\gamma}} = \frac{6K_0}{\gamma} \cdot \frac{1}{0} = 6K_0$$

$$\Rightarrow \Omega_H(s) = \left( \frac{6K_0}{2} + \frac{-6K_0}{\frac{1}{\tau} + 2} \right) \cdot e^{-2}$$

$\downarrow \mathcal{L}^{-1}$

$$\begin{aligned} \Rightarrow w_H(t) &= 6K_0 \cdot \cancel{H(t-1)} \cdot 1 - 6K_0 \cdot \cancel{H(t-1)} \cdot e^{-\frac{1}{\tau} \cdot (t-1)} \\ &= \cancel{H(t)} \cdot 6K_0 (1 - e^{-\frac{1}{\tau} (t-1)}) \end{aligned}$$



• achse  $K_0 \Rightarrow t \rightarrow \infty$

• achse  $\tau \Rightarrow w_H(t) = 63,2\% \rightarrow t = \tau$   
 $\hookrightarrow$  all temporal

$$\left( e^{-\frac{1}{\tau} \cdot t} \xrightarrow{t=\tau} e^{-\frac{1}{\tau} \cdot \tau} = e^{-1} = \frac{1}{e} \right)$$

$$3.2.1) \ddot{\theta} = \frac{1}{J_p} \left( \bar{F}_0 \cdot D_t - D_p \dot{\theta} - M_B \cdot g \cdot D_m \cdot \sin(\theta) \right)$$

• Linearizing:

$$\frac{\partial}{\partial t} \dot{\theta} = \left[ \left. \frac{\partial \ddot{\theta}}{\partial \theta} \right|_{\bar{\theta}=0} \quad \left. \frac{\partial \ddot{\theta}}{\partial \dot{\theta}} \right|_{\bar{\theta}=0} \right] \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \left. \frac{\partial \ddot{\theta}}{\partial \bar{F}_0} \right|_{\bar{F}_0=\bar{F}_0} \cdot \bar{F}_0$$

$$\frac{\partial \ddot{\theta}}{\partial t} = \frac{1}{J_p} \left( \begin{bmatrix} -M_B \cdot g \cdot D_m & -D_p \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + D_t \cdot \bar{F}_0 \right)$$

↳ certeza que vai ser citada, então já bota label

3.2.2)

Add no início:

$$\cancel{\ddot{\theta}(t)} \cdot \cancel{J_p} = \cancel{\bar{F}_0(t)} \cdot D_t - D_p \cancel{\dot{\theta}(t)} - M_B \cdot g \cdot D_m \cdot \sin(\cancel{\theta(t)})$$

( $\ddot{\theta}$  e  $\dot{\theta}$  não mudam por estarem "steady-state")

### 3.2.2)

- If  $F_0 = 0$  and  $\sin(\theta) \approx \theta$ :

$$\ddot{\theta} J_p = -D_p \dot{\theta} - M_b \cdot g \cdot I_m \cdot \theta \quad (\text{eq})$$

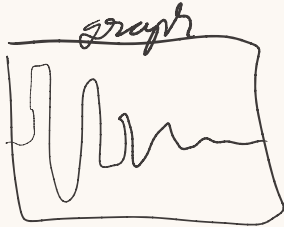
$$\Rightarrow \ddot{\theta} + \frac{D_p}{J_p} \dot{\theta} + \frac{M_b \cdot g \cdot I_m}{J_p} \cdot \theta = 0 \quad (\text{eq})$$

- So, it is possible to correlate

$$2 \gamma \omega_0 = \frac{D_p}{J_p}$$

$$\text{and } \omega_0^2 = \frac{M_b \cdot g \cdot I_m}{J_p}$$

- (no matter: • **pegar o T** (circulo e marque no gráfico)
- Escolhi  $k=4$  para pegar um intervalo maior do que T porém ainda não distorcido pela não-linearidade)



• calcular  $\eta$

• calcular  $\xi$  pelo  $\eta$

$$\left(\frac{\eta}{\pi}\right)^2 = \frac{\xi^2}{1 - \xi^2} \Rightarrow \left(\frac{\eta}{\pi}\right)^2 - \left(\frac{\eta}{\pi}\right)^2 \cdot \xi^2 = \xi^2$$

$$\left(\frac{\eta}{\pi}\right)^2 = \xi^2 \left(1 + \left(\frac{\eta}{\pi}\right)^2\right)$$

$$\xi = \frac{2/\pi}{\sqrt{1 + \eta/\pi}}$$

• calcular  $\omega_0$  pelo  $T$

$$\omega_0 = \frac{2\pi}{T \sqrt{1 - \xi^2}}$$

$$\bullet F_p = \frac{M_a \cdot g \cdot D_m}{\omega_0^2}$$

$$\bullet D_p = 2 \cdot \xi \cdot \omega_0 \cdot F_p$$



3.2.5) A partir da equação linearizada (famosa):

$$\frac{\partial \ddot{\theta}}{\partial t} = \frac{1}{J_p} \left( \begin{bmatrix} -M_B \cdot g \cdot D_m & -D_p \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + D_t \cdot F_o \right)$$

$$\begin{cases} \frac{\partial}{\partial t} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-M_B \cdot g \cdot D_m}{J_p} & -\frac{D_p}{J_p} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{D_t}{J_p} \end{bmatrix} F_o \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \end{cases}$$

→ mas também vai precisar de label 2

4.2.1)

$$F_o(t) = K_t \cdot W_H(t)$$

⇒ Substituindo na label 2

$$\begin{cases} \frac{\partial}{\partial t} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-M_B \cdot g \cdot D_m}{J_p} & -\frac{D_p}{J_p} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{D_t \cdot K_t}{J_p} \end{bmatrix} W_H \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \end{cases}$$

$$\Rightarrow A = \begin{bmatrix} 0 & 1 \\ \frac{-M_B \cdot g \cdot D_m}{J_p} & -\frac{D_p}{J_p} \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{D_t \cdot K_t}{J_p} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

4.2.2.)

(montado em LATEX)

4.2.3)

Vai precisar rodar de novo quando  
tiver o  $K_z$  e o  $FO$  a correr

4.2.4)

$$\begin{cases} \frac{d}{dt} \begin{bmatrix} \theta \\ \dot{\theta} \\ z_i \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-M_b g \cdot \Delta m}{J_p} & -\frac{D_p}{J_p} & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ z_i \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{D_t \cdot K_t}{J_p} \\ 0 \end{bmatrix} w_H + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \theta_f + B' \cdot d \\ y = [1 \ 0 \ 0] \begin{bmatrix} \theta \\ \dot{\theta} \\ z_i \end{bmatrix} \end{cases}$$

6.2.1)

$$\bullet \hat{x} = \begin{bmatrix} \hat{\theta} \\ \hat{\theta} \end{bmatrix} \text{ and } \hat{y} = C \hat{x}$$

$$\bullet \dot{\hat{x}} = A \cdot \hat{x} + B \cdot u + L(y - \hat{y})$$

$$\Rightarrow \dot{\hat{x}} = A \cdot \hat{x} + B \cdot u + L y - L C \hat{x}$$

$$\boxed{\dot{\hat{x}} = (A - LC) \hat{x} + B \cdot u + L y}$$