PRACTICA 2

sábado, 13 de julio de 2024 18:59

1) 5) Derivar y reducir $f(x) = \ln\left(\frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}\right)$

2) A) $\int x \cdot \ln(\sqrt{x}) \cdot dx =$

B) $\int \frac{1}{e^{\sqrt{x}}} \cdot dx =$

C) $\int e^{x} \cdot \operatorname{sen}(x) \cdot dx =$

3) Hallar el área encerrada por $f(x) = x^3 + 2$ y la recta tangente a $f(x)$ en $x = 1$.

Graficar dicha área.

4) Determinar si es posible aplicar el Teorema de Lagrange en la función

$f(x) = -x^2 + x + 6$ en el intervalo $[-1\,;3]$. De ser posible encuentre el punto c que

verifica la tesis y luego grafique.

1) $\ln\left(\frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}\right)$

$$F'(x) = \frac{1}{\left(\frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}\right)} \cdot \frac{(\operatorname{sen} x - \cos x)' \cdot (\operatorname{sen} x + \cos x) - [(\operatorname{sen} x - \cos x) \cdot (\operatorname{sen} x + \cos x)']}{(\operatorname{sen} x + \cos x)^2} =$$

C.A

$(\operatorname{sen} x - \cos x)' = \cos x + \operatorname{sen} x$

$(\operatorname{sen} x + \cos x)' = \cos x - \operatorname{sen} x$

$$F'(x) = \frac{\cancel{(\operatorname{sen} x + \cos x)}}{(\operatorname{sen} x - \cos x)} \cdot \frac{(\cos x + \operatorname{sen} x) \cdot (\operatorname{sen} x + \cos x) - [(\operatorname{sen} x - \cos x) \cdot (\cos x - \operatorname{sen} x)]}{(\operatorname{sen} x + \cos x)^{x}} =$$

$$= \frac{1(\cos x \cdot \operatorname{sen} x) + \cos^2 x + \operatorname{sen}^2 x + (\cos x \cdot \operatorname{sen} x) - [(\cos x \cdot \operatorname{sen} x) + (-\operatorname{sen}^2 x) + (-\cos x \cdot -\operatorname{sen} x) + (-\cos^2 x)]}{(\operatorname{sen} x - \cos x) \cdot (\operatorname{sen} x + \cos x)} =$$

$$\frac{\cos^2 x + \operatorname{sen}^2 x + \cancel{2(\cos x \cdot \operatorname{sen} x) - 2 \cdot (\cos x \cdot \operatorname{sen} x)} + (\operatorname{sen}^2 x) + (\cos^2 x)}{(\operatorname{sen} x - \cos x) \cdot (\operatorname{sen} x + \cos x)} =$$

$$= \frac{2 \cos^2 x + 2 \operatorname{sen}^2 x}{(\operatorname{sen} x - \cos x) \cdot (\operatorname{sen} x + \cos x)} =$$

$$= \frac{2\cdot(\cos^2x+\operatorname{sen}^2x)}{(\operatorname{sen}x-\cos x)\cdot(\operatorname{sen}x+\cos x)} =$$

$$= \frac{2}{(\operatorname{sen}x-\cos x)\cdot(\operatorname{sen}x+\cos x)}$$

2) A) $= \int x \cdot \ln(\sqrt{x}) \cdot dx$

$t^2 = x \Rightarrow \sqrt{x} = t$

$2t\,dt = dx$

$\int t^2 \cdot \ln(\sqrt{t^2}) \cdot 2t\,dt =$

$= 2\int t^3 \cdot \ln(t)\, dt =$

$u = \ln(t) \qquad dv = t^3\,dt$

$du = \frac{1}{t}\,dt \qquad v = \frac{1}{4}t^4$

$= \ln(t)\cdot\frac{1}{4}t^4 - \int \frac{1}{4}t^4 \cdot \frac{1}{t}\,dt =$

$= \ln(t)\cdot\frac{1}{4}t^4 - \frac{1}{4}\int t^{4^{3}} \cdot \frac{1}{\cancel{t}}\,dt =$

$= \ln(t)\,\frac{1}{4}t^4 - \frac{1}{4}\int t^3\,dt =$

$= \ln(t)\cdot\frac{1}{4}t^4 - \frac{1}{4}\cdot\frac{1}{4}t^4 + C =$

$= \ln(t)\cdot\frac{1}{4}t^4 - \frac{1}{8}t^4 + C$

$= 2\left[\ln(\sqrt{x})\cdot\frac{1}{4}(\sqrt{x})^4 - \frac{1}{8}(\sqrt{x})^4\right] + C$

B) $\int \frac{1}{e^{\sqrt{x}}} =$

$t^2 = x$

$2t\,dt = dx$

$= \int \frac{1}{e^{\sqrt{t^2}}}\,2t\,dt = 2\int \frac{1}{e^t}\cdot t\,dt =$

x^2	$\int 2x$
$2x$	$\cancel{2}\frac{x^2}{\cancel{2}}$
sen	$\int \cos$
cos	sen

C) $\int e^x \cdot \operatorname{sen}(x)\cdot dx$

$u = e^x \qquad dv = \operatorname{sen}(x)\,dx$

$du = e^x\,dx \qquad v = -\cos(x)$

$e^x \cdot -\cos(x) - \int -\cos(x)\cdot e^x\,dx =$

$= -e^x\cdot\cos(x) + \int \cos(x)\cdot e^x\,dx =$

$u = e^x \qquad dv = \cos(x)\cdot dx$

$u = e^x \qquad dv = \cos(x) \cdot dx$

$du = e^x dx \qquad v = \operatorname{sen}(x)$

$= -e^x \cdot \cos(x) + e^x \cdot \operatorname{sen}(x) - \int \operatorname{sen}(x) \cdot e^x \, dx = \int e^x \cdot \operatorname{sen}(x) \, dx$

$= -e^x \cdot \cos(x) + e^x \cdot \operatorname{sen}(x) = 2 \int e^x \cdot \operatorname{sen}(x) \, dx =$

$= \frac{-e^x \cdot \cos(x) + e^x \cdot \operatorname{sen}(x)}{2} + C$

b) $\int \frac{1}{e^{\sqrt{x}}} \, dx = \int e^{-\sqrt{x}} \, dx$

$t^2 = x \Rightarrow t = \sqrt{x}$

$2t \, dt = dx$

$= 2 \int e^{-t} \cdot t \, dt =$

$u = t \qquad dv = e^{-t} \, dt$

$du = dt \qquad v = \int e^{-t} \, dt$

$h = -t$

$\frac{dh}{-1} = dt$

$\int e^h \cdot \frac{dh}{-1} = -\int e^h \, dh = -e^h = \boxed{-e^{-t}}$

$= 2 \cdot (t \cdot (-e^{-t}) + \int e^{-t} \, dt) =$

$= 2 (t \cdot (-e^{-t}) - e^{-t}) =$

$= 2 \cdot (\sqrt{x} \cdot (-e^{-\sqrt{x}}) - e^{-\sqrt{x}})$

EXTRA:

$\int \operatorname{arctg}(x) \, dx$

$u = \operatorname{arctg}(x) \qquad dv = dx$

$du = \frac{1}{x^2+1} dx \qquad v = x$

$= \operatorname{arctg}(x) \cdot x - \int \frac{x}{x^2+1} \, dx =$

$t = x^2 + 1$

$dt = 2x \, dx$

$\frac{dt}{2x} = dx$

$= \operatorname{arctg}(x) \cdot x - \int \frac{x}{t} \frac{dt}{2x} =$

$= \operatorname{arctg}(x) \cdot x - \int \frac{1}{2t}\, dt =$

$= \operatorname{arctg}(x) \cdot x - \frac{1}{2} \int \frac{1}{t}\, dt =$

$= \operatorname{arctg}(x) \cdot x - \frac{1}{2} \ln |t| =$

$= x \cdot \operatorname{arctg}(x) - \frac{1}{2} \ln |x^2+1| + C$

3) $F(x) = x^3 + 2$ y RT en $x = 1$.

$F'(x) = 3x^2$

$m_{RT} = F'(1) = 3 \cdot 1^2 = 3$

$F(1) = 1^3 + 2 = 3$

$3 = 3 \cdot 1 + b$

$3 = 3 + b$

$0 = b$

$y = 3x$ es la ecu. de RT

$x^3 + 2 = 3x$

$x^3 - 3x + 2 = \to x_1 = -2,\ x_2 = 1$ } Intervalos

$\frac{1}{2}$ evaluo

$x = 3 \cdot \frac{1}{2} = 1,5$

$F(x)\ \left(\frac{1}{2}\right)^3 + 2 = 2,125$

$F(x)$ es techo

$$\int_{-2}^{1} x^3 - 3x + 2\, dx = \frac{1}{4}x^4 - \frac{3}{2}x^2 + 2x \Big]_{-2}^{1} =$$

$$= \frac{1}{4} - \frac{3}{2} + 2 - \left[\frac{1}{4}(-2)^4 - \frac{3}{2}(-2)^2 + 2 \cdot -2\right] =$$

$$= \frac{3}{4} - [4 - 6 - 4] =$$

$$= \frac{3}{4} + 6 = 6,75\ u^2$$

... del punto 2 y del resto de los ejercicios 1 más correctamente resuelto.
...rio: tener 2 del punto 1 y 1 del punto 2.

1) Resuelve las siguientes integrales indefinidas:

a) $\int \cos(\sqrt{x})\, dx =$

b) $\int \frac{\ln x}{\sqrt[3]{x^2}}\, dx =$

c) $\int \frac{\cos(x)}{e^{\operatorname{sen}(x)} - 1}\, dx =$

d) $\int x \cdot \operatorname{arctg}(x)\, dx =$

e) $\int \frac{\sqrt{x}}{x - \sqrt{x}}\, dx =$

f) $\int \frac{x^2}{x^2 + 2x + 1}\, dx =$

2) Hallar la derivada y llevar a su mínima expresión:

a) $f(x) = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$

b) $f(x) = \ln\left(\frac{\sqrt{x}+3}{\sqrt{x}-3}\right)$

c) $f(x) = \ln \sqrt{\frac{\cos x - \operatorname{sen} x}{\cos x + \operatorname{sen} x}}$

4) Determinar si es posible aplicar el Teorema de Lagrange en la función

$f(x) = -x^2 + x + 6$ en el intervalo $[-1\,;3]$. De ser posible encuentre el punto c que verifica la tesis y luego grafique.

HIPOTESIS

1º Es continua en el intervalo $[-1;3]$ porque es una función polinomia

2º

$F'(x) = -2x+1$

Es derivable en el intervalo $(-1;3)$ por el dominio

TESIS

Entonces podemos verificar la siguiente tesis:

$$\frac{F(b)-F(a)}{b-a} = F'(c)$$

C.A

$F(b) = -3^2+3+6 = -9+9 = 0$

$F(a) = -(-1)^2-1+6 = 4$

$\frac{0-4}{3+1} = \frac{4}{4} = 1 \rightarrow$ m RTg RSec

$-2x+1 = 1$

$-2x = 0$

$x = 0 = C$

GRAFICO

RT $y = mx+b$

$6 = 1 \cdot 0 + b$

$6 = b$

ec RT= $y = x+6$

RSec:

$0 = 1 \cdot 3 + b$

$0 = 3+b$

$-3 = b$

ec Rec = $y = x-3$

C. A

$y = F(c) \Rightarrow F(0) = 6$

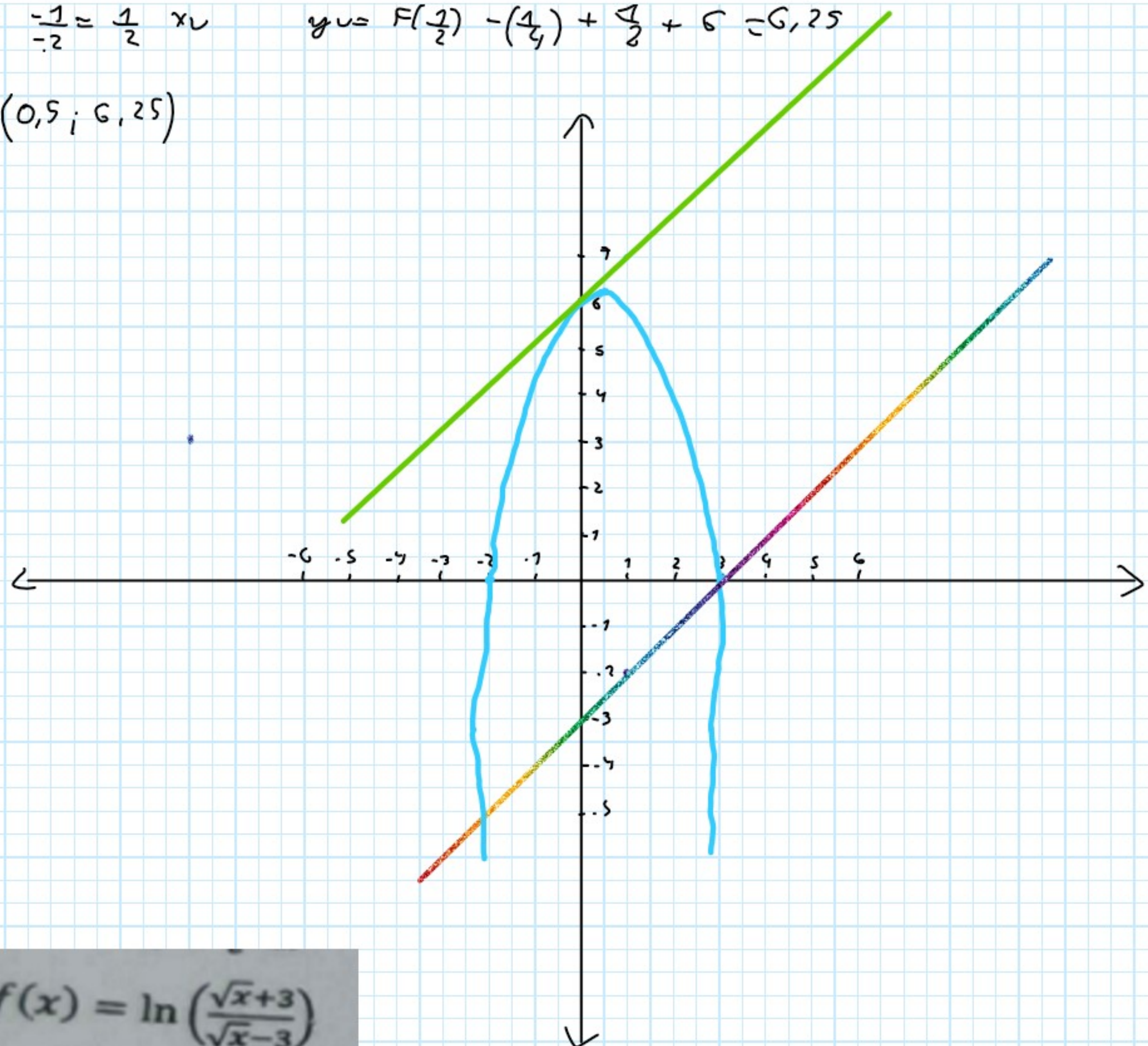
PARABOLA

$-x^2+x+6 = \longrightarrow x_1 = -2$
$x_2 = 3$

$-\frac{b}{2a} = \frac{-1}{-2} = \frac{1}{2}$ x_v $\quad y_v = F(\frac{1}{2})\ -(\frac{1}{4}) + \frac{1}{2} + 6 = 6{,}25$

$-\frac{b}{2a} = \frac{-1}{-2} = \frac{1}{2}$ x_v $y_v = F\left(\frac{1}{2}\right) - \left(\frac{1}{4}\right) + \frac{1}{2} + 6 = 6{,}25$

$V = (0{,}5 \, ; \, 6{,}25)$



b) $f(x) = \ln\left(\frac{\sqrt{x}+3}{\sqrt{x}-3}\right)$

$$F'(x) = \frac{1}{\frac{\sqrt{x}+3}{\sqrt{x}-3}} \cdot \frac{(\sqrt{x}+3)' \cdot (\sqrt{x}-3) - (\sqrt{x}+3) \cdot (\sqrt{x}-3)'}{(\sqrt{x}-3)^2}$$

C.A

$(\sqrt{x}+3)' = \frac{1}{2\sqrt{x}} = (\sqrt{x}-3)'$

$$= \frac{\cancel{\sqrt{x}-3}}{\sqrt{x}+3} \cdot \frac{\frac{1}{2\sqrt{x}} \cdot (\sqrt{x}-3) - (\sqrt{x}+3) \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x}-3)^{\cancel{2}}} \, .$$

$$= \frac{\left[\cancel{\frac{1}{2x}} - \frac{3}{2\sqrt{x}} - \cancel{\frac{1}{2x}} - \frac{3}{2\sqrt{x}}\right]}{(\sqrt{x}+3)(\sqrt{x}-3)} =$$

$$= \frac{-2 \cdot \left(\frac{3}{2\sqrt{x}}\right)}{(\sqrt{x}+3) \cdot (\sqrt{x}-3)} =$$

$$= \frac{\cancel{-2} \cdot \frac{3\sqrt{x}}{\cancel{2}x}}{(\sqrt{x}+3) \cdot (\sqrt{x}-3)} =$$

$$= \frac{\frac{-3\sqrt{x}}{x}}{(\sqrt{x})^2 - (3)^2} =$$

$$= \frac{\frac{-3\sqrt{x}}{x}}{x-9} = \frac{-3\sqrt{x}}{x \cdot (x-9)} = \frac{-3\sqrt{x}}{x^2 - 9x}$$

c) $f(x) = \ln\sqrt{\dfrac{\cos x-\operatorname{sen} x}{\cos x+\operatorname{sen} x}}$

$$F'(x) = \frac{1}{\sqrt{\frac{\cos x-\operatorname{sen} x}{\cos x+\operatorname{sen} x}}} \cdot \frac{1}{2\cdot\sqrt{\frac{\cos x-\operatorname{sen} x}{\cos x+\operatorname{sen} x}}} \cdot \frac{(-\operatorname{sen} x-\cos x)\cdot(\cos x+\operatorname{sen} x)-(\cos x-\operatorname{sen} x)\cdot(-\operatorname{sen} x+\cos x)}{(\cos x+\operatorname{sen} x)^2}$$

$$= \frac{1}{2\cdot\left(\frac{\cos x-\operatorname{sen} x}{\cos x+\operatorname{sen} x}\right)} \cdot \frac{(\cos x\cdot(-\operatorname{sen} x))-\operatorname{sen}^2 x-\cos^2 x+(\cos x\cdot(-\operatorname{sen} x))-\left[(\cos x\cdot(-\operatorname{sen} x))+\cos^2 x+\operatorname{sen}^2 x+(\cos x\cdot(-\operatorname{sen} x))\right]}{(\cos x+\operatorname{sen} x)^2}$$

$$= \frac{1}{2}\cdot\frac{\cancel{(\cos x+\operatorname{sen} x)}}{(\cos x-\operatorname{sen} x)}\cdot\frac{\cancel{(\cos x\cdot(-\operatorname{sen} x))^2}-\operatorname{sen}^2 x-\cos^2 x-\cancel{(\cos x\cdot(-\operatorname{sen} x))^2}-\cos^2 x-\operatorname{sen}^2 x}{(\cos x+\operatorname{sen} x)^{\cancel{2}}}$$

$$= \frac{1}{2}\cdot\frac{1}{(\cos x-\operatorname{sen} x)}\cdot\frac{-2\operatorname{sen}^2 x-2\cos^2 x}{(\cos x+\operatorname{sen} x)} =$$

$$= \frac{1}{\cancel{2}}\cdot\frac{1}{(\cos x-\operatorname{sen} x)}\cdot\frac{-\cancel{2}\cdot\overbrace{(\operatorname{sen}^2 x+\cos^2 x)}^{1}}{(\cos x+\operatorname{sen} x)}$$

$$= \frac{-1}{(\cos x-\operatorname{sen} x)\cdot(\cos x+\operatorname{sen} x)} \qquad a^2-b^2=(a+b)\cdot(a-b)$$

$$= \frac{-1}{\cos^2 x-\operatorname{sen}^2 x}$$

...ntegrales inde

a) $\int \cos(\sqrt{x})\,dx =$

b) $\int \frac{\ln x}{\sqrt[3]{x^2}}\,dx =$

c) $\int \frac{\cos(x)}{e^{\operatorname{sen}(x)}-1}\,dx =$

a) $\int \cos(\sqrt{x})\,dx$

$u = \cos(\sqrt{x}) \qquad dv = dx$

$du = -\operatorname{sen}(\sqrt{x})\cdot\frac{1}{2\sqrt{x}} \qquad v = x$

$= \frac{-\operatorname{sen}(\sqrt{x})}{2\sqrt{x}}$

$\cos(\sqrt{x})\cdot x - \int x\cdot\frac{-\operatorname{sen}(\sqrt{x})}{2\sqrt{x}}\,dx$

$t^2 = x \Rightarrow t = \sqrt{x}$

$2t\,dt = dx$

$= \cos(\sqrt{x})\cdot x - \int t^2\cdot\frac{-\operatorname{sen}(\sqrt{t^2})}{2\sqrt{t^2}}\cdot 2t\,dt =$

$= \cos(\sqrt{x}) \cdot x - \int t^2 \cdot \frac{-sen(\sqrt{t^2})}{2\sqrt{t^2}} \cdot 2t \, dt =$

$= \cos(\sqrt{x}) \cdot x - \int t^3 \cdot \frac{-sen(t)}{t} \, dt =$

$= \cos(\sqrt{x}) \cdot x + \int t^2 \cdot sen(t) \, dt =$

$u = t^2 \qquad dv = sen(t) \, dt$

$du = 2t \, dt \qquad v = -\cos(t)$

$= \cos(\sqrt{x}) \cdot x + t \cdot -\cos(t) + \int \cos(t) \cdot 2t \, dt =$

$= \cos(\sqrt{x}) \cdot x + t \cdot -\cos(t) + 2 \int \cos(t) \cdot t \, dt =$

$u = t \qquad dv = \cos(t) \, dt$

$du = dt \qquad v = sen(t)$

$= \cos(\sqrt{x}) \cdot x - t \cdot \cos(t) + 2 \cdot [t \cdot (sen(t) + \cos(t)] =$

$= \cos(\sqrt{x}) \cdot x - \sqrt{x} \cdot \cos(\sqrt{x}) + 2 \cdot [\sqrt{x} \cdot sen(\sqrt{x}) + \cos(\sqrt{x}))] + C$

$= \int \cos(\sqrt{x}) \, dx$

$t^2 = x$

$2t \, dt = dx$

$= \int \cos(\sqrt{t^2}) \, 2t \, dt =$

$= 2 \int \cos(t) \cdot t \, dt =$

$u = t \qquad dv = \cos(t) \, dt$

$du = dt \qquad v = sen(t)$

$2 \cdot (t \cdot sen(t) + \cos(t)) =$

$2 \cdot (\sqrt{x} \cdot sen(\sqrt{x}) + \cos(\sqrt{x})) + C$

b) $\int \frac{\ln x}{\sqrt[3]{x^2}} \, dx =$

$= \int \ln x \cdot \frac{1}{x^{\frac{2}{3}}} \, dx = \int \ln x \cdot x^{-\frac{2}{3}} \, dx$

$u = \ln x \qquad dv = x^{-\frac{2}{3}}$

$du = \frac{1}{x} \, dx \qquad v = 3x^{\frac{1}{3}}$

$= \ln x \cdot 3x^{\frac{1}{3}} - 3 \int x^{-\frac{2}{3}} \, dx =$

$= \ln x \cdot 3x^{\frac{7}{3}} - 9x^{\frac{7}{3}} + C$

$$\int \frac{\cos(x)}{e^{\operatorname{sen}(x)} - 1}\,dx =$$

$\int \frac{\cos(x)}{e^{\operatorname{sen}(x)} - 1}\,dx$

$t = \operatorname{sen}(x)$

$dt = \cos(x)\,dx$

$\frac{dt}{\cos(x)} = dx$

$\int \frac{\cancel{\cos(x)}}{e^{t} - 1} \cdot \frac{dt}{\cancel{\cos(x)}} = \int \frac{1}{e^{t} - 1}\,dt$

$h = e^{t} - 1 \Rightarrow e^{t} = h + 1$

$dh = e^{t}\,dt$

$\frac{dh}{e^{t}} = dt$

$= \int \frac{1}{h} \frac{dh}{e^{t}} = \int \frac{1}{h} \cdot \frac{1}{h+1}\,dh =$

$= \int \frac{1}{h \cdot (h+1)}\,dh$

C. A

$\int \frac{1}{h \cdot (h+1)} = \int \frac{A}{h} + \frac{B}{(h+1)} = \int \frac{A \cdot (h+1) + B \cdot h}{h \cdot (h+1)} =$

si $h = -1$

$1 = \cancel{A \cdot (-1+1)} + B \cdot (-1)$

$1 = -B$

$-1 = B$

si $h = 0$

$1 = A \cdot (0+1) + \cancel{B \cdot 0}$

$1 = A \cdot 1$

$1 = A$

$= \int \frac{1}{h} - \frac{1}{h+1}\,dh = \int \frac{1}{h}\,dh - \int \frac{1}{h+1}\,dh =$

$= \ln|h| - \ln|h-1| =$

$= \ln|e^{t} - 1| - \ln|e^{t}| =$

$= \ln|e^{\operatorname{sen} x} - 1| - \ln|e^{\operatorname{sen} x}| + C$

$= \int \operatorname{arctg}(\sqrt{x})\,dx =$

$t^2 = x \Rightarrow t = \sqrt{x}$

$2t\,dt = dx$

$= \int \operatorname{arctg}(\sqrt{t^2})\cdot 2t\,dt$

$= 2\int \operatorname{arctg}(t)\cdot t\,dt =$

$u = \operatorname{arctg}(t) \qquad dv = t\,dt$

$du = \frac{1}{t^2+1}\,dt \qquad v = \frac{1}{2}t^2$

$2(\operatorname{arctg}(t)\cdot \frac{1}{2}t^2 - \int \frac{1}{2}t^2\cdot\frac{1}{t^2+1}\,dt =$

$= 2\cdot\left[\operatorname{arctg}\cdot\frac{1}{2}t^2 - \frac{1}{2}\int \cdot\frac{t^2}{t^2+1}\,dt\right]$

C. 1

$t^2+0t+0 \;\lfloor\, t^2+1$

$-\; t^2+0+1 \qquad 1$

$0 \;\; 0 \;\; -1$

$C + \frac{R}{D}$

$= 2\cdot\left[\operatorname{arctg}x\cdot\frac{1}{2}t^2 - \frac{1}{2}\int 1 - \frac{1}{t^2+1}\,dt\right] =$

$= 2\cdot\left[\operatorname{arctg}x\cdot\frac{1}{2}t^2 - \frac{1}{2}\cdot\left[\int 1\,dt - \int\frac{1}{t^2+1}\,dt\right]\right]$

$= 2\cdot\left[\operatorname{arctg}x\cdot\frac{1}{2}t^2 - \frac{1}{2}\cdot\left[t - \operatorname{arctg}(t)\right]\right] =$

$= 2\cdot\left[\operatorname{arctg}(\sqrt{x})\cdot\frac{1}{2}(\sqrt{x})^2 - \frac{1}{2}\cdot\left[\sqrt{x} - \operatorname{arctg}(\sqrt{x})\right]\right] + C =$

$= 2\cdot\left[\frac{\operatorname{arctg}(\sqrt{x})\cdot x}{2} - \frac{\sqrt{x} - \operatorname{arctg}(\sqrt{x})}{2}\right] + C$

4) Determinar si es posible aplicar el Teorema de Lagrange en la función $f(x)=\frac{x^3}{3}-x$ en el intervalo $[-3;3]$. De ser posible encuentre el punto c que verifica la tesis y **luego GRAFIQUE** mostrando a través del gráfico la conclusión de dicho teorema, **encuentre las ecuaciones de las rectas intervinientes** que necesite para graficar correctamente.

$F(x) = \frac{x^3}{3} - x \qquad [-3;3]$

1ro es continua

2do es derivable $\quad F'(x) = \frac{1}{3}\cdot x^3 - x = \quad \frac{\cancel{3}}{\cancel{3}}x^2 - 1$

$F'(x) = x^2 - 1$

3) Hallar el área encerrada por las curvas $f(x)=x^3$ y $g(x)=3x+2$.

$F(x) = x^3 \quad$ y $\quad G(x) = 3x+2 \qquad$ 1ro busco las inter

3) Hallar el área encerrada por las curvas $f(x)=x^3$ y $g(x)=3x+2$.

$F(x)=x^3$ y $g(x)=3x+2$ 1ro busco las inter

$x^3=3x+2$

$x^3-3x-2=0 \longrightarrow x_1=2$
$\quad \hookrightarrow x_2=-1$

1

$f(1)=x^3=1 \qquad g(1)=3\cdot 1+2=5 \rightarrow$ en techo

$$\int_{-1}^{2} 3x+2-x^3=\int_{-1}^{2}(-x^3+3x+2)\,dx=\boxed{-\frac{1}{4}x^4+\frac{3}{2}x^2+2x}\Bigg]_{-1}^{2}=$$

$$=-\frac{1}{4}\cdot(2)^4+\frac{3}{2}(2)^2+2\cdot 2-\left[-\frac{1}{4}(-1)^4+\frac{3}{2}\cdot(-1)^2+2\cdot -1\right]=$$

$$=-4+6+4+\frac{1}{4}-\frac{3}{2}+2=6{,}75\,u^2$$

$f(x)=\frac{e^{2x}+e^{-2x}}{e^{2x}-e^{-2x}}$

$$F'(x)=\frac{(e^{2x}+e^{-2x})'\cdot(e^{2x}-e^{-2x})-(e^{2x}+e^{-2x})\cdot(e^{2x}-e^{-2x})'}{(e^{2x}-e^{-2x})^2}$$

C.A

$(e^{2x}+e^{-2x})=e^{2x}\cdot 2+e^{-2x}\cdot -2$

$(e^{2x}-e^{-2x})=e^{2x}\cdot 2-e^{-2x}\cdot -2=e^{2x}\cdot 2+2\cdot e^{-2x}$

$$=\frac{(e^{2x}\cdot 2+e^{-2x}\cdot -2)\cdot(e^{2x}-e^{-2x})-(e^{2x}+e^{-2x})\cdot(e^{2x}\cdot 2+e^{-2x}\cdot 2)}{(e^{2x}-e^{-2x})^2}$$

$$=\frac{2e^{4x}-2-2+2e^{-4x}-[2e^{4x}+2+2+2e^{-4x}]}{(e^{2x}-e^{-2x})^2}=$$

$$=\frac{\cancel{2e^{4x}}-4+\cancel{2e^{-4x}}-\cancel{2e^{4x}}-4-\cancel{2e^{-4x}}}{(e^{2x}-e^{-2x})^2}=\frac{-8}{(e^{2x}-e^{-2x})^2}$$

c) $\int x^3\cdot cos(x^2)\cdot dx=$

$\int x^3\cdot\cos(x^2)\,dx=\int x^2\cdot x\cdot\cos(x^2)\,dx$

$t=x^2$

$dt=2x\,dx$

$\frac{dt}{2x}=dx$

$\int t\cdot\cancel{x}\cdot\cos(t)\frac{dt}{2\cancel{x}}=\frac{1}{2}\int t\cdot\cos(t)\,dt=$

$u=t \qquad dv=\cos(t)\cdot dt$

$du=dt \qquad v=\operatorname{sen}(t)$

$$= \frac{1}{2} \cdot [t \cdot sen(t) - \int sen(t)\ dt] =$$

$$= \frac{1}{2} \cdot [t \cdot sen(t) + \cos(t)] =$$

$$= \frac{x^2 \cdot sen(x^2)}{2} + \frac{\cos(x^2)}{2} + C$$

a) $\int \frac{1}{x+2\sqrt{x}-8} \cdot dx =$

$$\int \frac{1}{x+2\sqrt{x}-8} \cdot dx$$

$t^2 = x \Rightarrow t = \sqrt{x}$

$2t\,dt = dx$

$$\int \frac{1}{t^2+2\sqrt{t^2}-8}\, 2t\ dt = 2\int \frac{t}{t^2+2t-8}\ dt =$$

C.A

$$\frac{t}{t^2+2t-8} = \frac{t}{(t-2)\cdot(t+4)} = \frac{A}{(t-2)} + \frac{B}{(t+4)} = \frac{A(t+4)+B(t-2)}{(t-2)(t+4)} =$$

si $t=2$

$2 = A\cdot(2+4) + \cancel{B(2-2)}$

$2 = A \cdot 6$

$\frac{2}{6} = A = \frac{1}{3}$

si $t=-4$

$-4 = \cancel{A(-4+4)} + B(-4-2)$

$-4 = B(-6)$

$\frac{-4}{-6} = B = \frac{2}{3}$

$$2 \cdot \int \frac{\frac{1}{3}}{(t-2)} + \frac{\frac{2}{3}}{(t+4)} = 2 \cdot \left[\int \frac{1}{3(t-2)}\ dt + \int \frac{2}{3(t+4)}\ dt\right] =$$

$$= 2 \cdot \left[\frac{1}{3}\int \frac{1}{t-2}\ dt + \frac{2}{3}\int \frac{1}{t+4}\ dt\right] =$$

$$= 2 \cdot \left[\frac{1}{3} \ln|t-2| + \frac{2}{3} \ln|t+4|\right]$$

$$= \frac{2}{3} \ln|\sqrt{x}-2| + \frac{4}{3} \ln|\sqrt{x}+4|$$

2) Dada la función $f(x) = \frac{x^2}{x-1}$ hallar: cortes con los ejes, intervalos de crecimiento, extremos, intervalos de concavidad, puntos de inflexión y graficar.

$Dom = \mathbb{R} - \{1\}$

A.V

$$\lim_{x\to 1} \frac{x^2}{x-1} = \frac{1}{\to 0} = \infty \Rightarrow x=1 \text{ es A.V}$$

A.H

$$\lim_{x\to\infty} \frac{x^2}{x-1} = \frac{\infty}{\infty} = \frac{2x}{1} = \frac{\infty}{1} = \infty \quad \text{no tiene}$$

$m = F(x) \cdot \frac{1}{x}$

$b = F(x) - mx$

A.O

$$m = \lim_{x\to\infty} \frac{x^2}{x-1} \cdot \frac{1}{x} = \frac{x^2}{x^2-x} = \frac{\infty}{\infty} = \frac{2x}{2x-1} = \frac{\infty}{\infty} = \frac{2}{2} = 1$$

$m = 1$

$$b = \frac{x^2}{x-1} - \frac{x}{1} = \frac{x^2 - x\cdot(x-1)}{(x-1)\cdot 1} = \lim_{x\to\infty} \frac{\cancel{x^2} \cancel{-x^2} + x}{(x-1)} = \frac{x}{x-1} = \frac{\infty}{\infty} = \frac{1}{1} = 1$$

$y = x+1$ es A.O

CORTES

EJE X

$\frac{x^2}{x+1} = 0$

$x^2 = 0$

$x = 0$

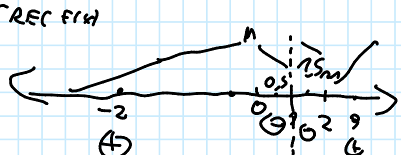
EJE Y

$F(0) = \frac{0}{0+1} = \frac{0}{1} = 0$

CREC. max min

$$F'(x) = \frac{2x\cdot(x-1) - [(x^2)\cdot 1]}{(x+1)^2} = \frac{2x^2 - 2x \cancel{-x^2}}{(x+1)^2} = \frac{-2x}{(x+1)^2}$$

CREC F(x)



$x^2 - 2x = 0 \to 0, \to 2$

Sg F'(x)

$F'(-2) = \frac{+}{+} = +$ $\quad F'(0{,}5) = \frac{-}{+} = -$

$F'(1{,}5) = \frac{-}{+} = -$ $\quad F'(3) = \frac{+}{+} = (+)$

Crece desde $(-\infty; 0) \cup (2; +\infty)$

Decrece desde $(0; 1) \cup (1; 2)$

M en $(0;0)$

m en $(2;4)$

CONCAVIDAD . P.I $\quad \frac{x^2-2x}{(x-1)^2} = F'(x)$

$$F''(x) = \frac{(2x-2)\cdot(x-1)^2 - [(x^2-2x)\cdot 2(x-1)\cdot 1]}{(x-1)^4} =$$

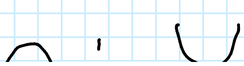
$$= \frac{\cancel{(x-1)}\cdot[(2x-2)\cdot(x-1) - [(x^2-2x)\cdot 2]]}{(x-1)^{\cancel{4}3}} =$$

$$= \frac{2x^2 \cancel{-2x} \cancel{-2x} + 2 - 2x^2 \cancel{+4x}}{(x-1)^3} =$$

$$= \frac{4x^2+2}{(x-1)^3}$$

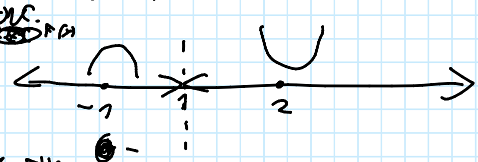
$4x^2+2=0 \to$ No tiene raíces reales

Conc. F(x)



$\frac{}{(x-1)^3}$

Conc. $f''(x)$



Sg $f''(x)$ -

$F(-1) = \frac{+}{-} = -$ $\quad$ $F(2) = \frac{+}{+} = +$