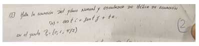
xdxd

sábado, 10 de mayo de 2025 1:22



[2] Halla la ecuación del plano Normal y osculador de Hélice de ecuación

$r(t) = \cos t\, i + \operatorname{sen} t\, j + t k.$

en el punto $P_0 = (0; 1; \pi/2)$

$t \nearrow \frac{\pi}{2}$

1. x_0, y_0, z_0 $r(t)$

2. A, B, C T'

P_N y P_{osc}

$r(t) = \overbrace{\cos(t)}^{x}\check{i} + \overbrace{\operatorname{sen}(t)}^{y}\check{j} + \overbrace{t}^{z}\check{k}$ en $P_0 = (0;\ 1,\ \frac{\pi}{2})$

$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$\cos(t) = 0$

$\operatorname{sen}(t) = 1$

$\boxed{t = \frac{\pi}{2}}$

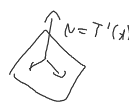
1 $\cos(\frac{\pi}{2}) + \operatorname{sen}(\frac{\pi}{2}) + \frac{\pi}{2} = \overset{x_0}{0}\check{i} + \overset{y_0}{1}\check{j} + \overset{z_0}{\frac{\pi}{2}}\check{k}$

$T(t) = r'(t)$ | $T(t) = \underbrace{-\operatorname{sen}(t)}_{A}\check{i} + \underbrace{\cos(t)}_{B}\check{j} + \underbrace{1}_{C}\check{k}$

$-\operatorname{sen}(\frac{\pi}{2}) \cdot (x - 0) + \cos(\frac{\pi}{2})(y - 1) + 1(z - \frac{\pi}{2}) = 0$

OSCU

REC



$\operatorname{sen}^2(t) + \cos^2(t) = 1$

$1 - \cos^2(t) = \operatorname{sen}^2(t)$

$T'(t) = \underbrace{-\cos(t)}_{A}\check{i} - \underbrace{\operatorname{sen}(t)}_{B}\check{j} + \underbrace{0}_{C}\check{k} = N(t)$

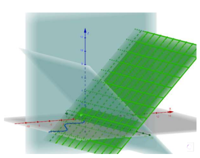
$\boxed{-\cos(\frac{\pi}{2})(x - 0) - \operatorname{sen}(\frac{\pi}{2})(y - 1) = 0}$ REC

$$B(t) = T(t) \times N(t) \longrightarrow \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ -\operatorname{sen}(t) & \cos(t) & 1 \\ -\cos(t) & -\operatorname{sen}(t) & 0 \end{vmatrix} = 0 + \operatorname{sen}(t)\check{i} - [0 + \cos(t)]\check{j} + [+\operatorname{sen}^2(t) - \cos^2(t)]\check{k}$$

(+) (−) (+)

$B(t) = \underbrace{\operatorname{sen}(t)}_{A}\check{i} + \underbrace{\cos(t)}_{B}\check{j} + \underbrace{(\operatorname{sen}^2(t) - \cos^2(t))}_{C}\check{k}$

PLANO OSCULADOR $= \operatorname{sen}(\frac{\pi}{2})(x - 0) + \cos(\frac{\pi}{2})(y - 1) + (\operatorname{sen}^2(\frac{\pi}{2}) - \cos^2(\frac{\pi}{2}))(z - \frac{\pi}{2})$



UlRp 2

viernes, 9 de mayo de 2025 22:51

Hallar Dominio y si existen curvas de nivel M=0, M=1, M=3

$$\frac{1}{\sqrt{4-x^2-y^2}}$$

d) $r(t) = (2sen(t); 5t; 2\cos(t))$ $-10 \le t \le 10$

c) $k(x,y) = xy\sqrt{1-x^2-y^2}$

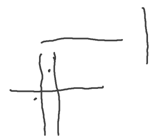
Extremos libre

b) $g(x,y) = \frac{1+x^2}{1+y^2}$ en $(-1,0)$ siendo $u = \left(\frac{1}{\sqrt{3}}, \frac{\sqrt{6}}{3}\right)$

$F(x,y) = \frac{x^2-y^2}{x^2+y^2}$ Hallar curvas de nivel K=1, K=2, K=4

Dom

HOMOGRAFICA



$$F(x,y) = \frac{x^2-y^2}{x^2+y^2}$$

Dom => A.H → 1

A.V = 0 x=0, y=0

Dom = $\mathbb{R} - \{0\}$

$$\frac{x^2-y^2}{x^2+y^2} = K \Rightarrow x^2-y^2 = K\cdot(x^2+y^2)$$

$$x^2-y^2 = Kx^2+Ky^2$$

$$-Kx^2+x^2-y^2 = Ky^2$$

$$-Kx^2+x^2 = Ky^2+y^2$$

$$x^2(-K+1) = y^2(K+1)$$

→ K=1 $x^2(-1+1) = y^2(1+1)$

$0 = 2y^2$

Hallar Dominio y si existen curvas de nivel M=0, M=1, M=3

$$\frac{1}{\sqrt{4-x^2-y^2}}$$

$$\frac{\ln|x^2-2y|}{\sqrt{4-x^2-y^2}}$$

RESTRICCION

$4-x^2-y^2 > 0$

$-x^2-y^2 = -4$

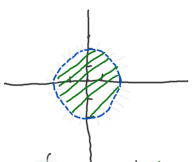
$4 = x^2+y^2$

$r=2$

$x^2-2y > 0$

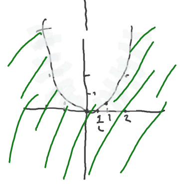
$x^2-2y = 0$

$x^2 = 2y \Rightarrow \frac{x^2}{2} = y$



$P(0,0) = 4-0^2-0^2 > 0$ ✓ V

$P(0,3) = 4-0^2-9 > 0$ X F

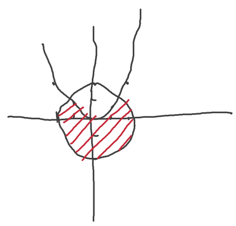


x	$y=\frac{x^2}{2}$
0	0
1	$\frac{1}{2}$
-1	$\frac{1}{2}$
2	2

$\frac{-0}{2\cdot\frac{1}{2}}$

$P(0,1) = 0^2-2\cdot1 > 0$ X

$P(0,-1) = 0^2-2\cdot-1 > 0$ ✓



$Dom = \{F(x,y) \in \mathbb{R}^2 / 4-x^2-y^2>0 \wedge x^2-2y>0\}$

$$\frac{1}{\sqrt{4-x^2-y^2}} = 1 \Rightarrow 1 = 1\cdot\sqrt{4-x^2-y^2}$$

$$1 = 4-x^2-y^2$$

b) $g(x,y) = \frac{2xy}{x^2+y^2}$

A.H

AV =

$$\frac{2xy}{x^2+y^2}$$

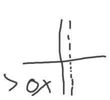
$Dom\ \{F(x,y) \in \mathbb{R}^2 / x \neq 0 \wedge y \neq 0\}$

$$z = \frac{\ln(x-1)}{2\sqrt{2x^2+y}}$$

1° RESTRICCION

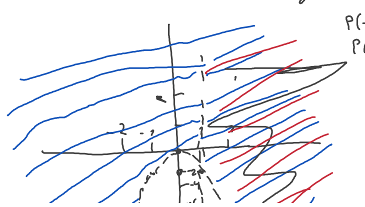
$x-1 > 0$

$x > 1$ $-1-1 > 0$ X



P(-1;0)

P(2;0) → X $2-1 > 0$



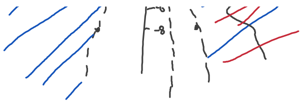
2. $2x^2+y > 0$

$2x^2+y = 0$

$2x^2 = -y$

$V_x = \frac{0}{4a} = 0$

x	$y=-2x^2$
1	-2
2	-8



$-2x^2 = y$

$P(0;-2) = 2\cdot 0^2 - 2 > 0$ ✗ F

$P(0;1) = 2\cdot 0^2 + 1 > 0$ ✓ V

$V_{(x,y,z)} = 5x^2 - 3xy + xyz$

Supongo que V = POTENCIÁL ELECTRICO

A) Encuentre la razon de cambio Potencial en el punto $P(3,4,5)$ en la direccion $\vec{v} = \hat{i} + \hat{j} - \hat{k}$

B) En que direccion CAMBIA V mas rapidamente en el punto P? y menos RAPIDAMENTE?

C) Cual es la mayor derivada en P?

- $W(x,y,z) = 2xyz(3-x)(7-y)(3-z)$

DERIVAR

- $a(t) = \langle t^2; \frac{1}{2}\sqrt{t}; t^3 \rangle$ | Hallar $V(t)$ y $r(t)$ en cualquier t

$V(0) = t\hat{i} + t\hat{k}$

$r(0) = \hat{j}$

$V_{(x,y,z)} = 5x^2 - 3xy + xyz$

Supongo que V = POTENCIÁL ELECTRICO

A) Encuentre la razon de cambio Potencial en el punto $P(3,4,5)$ en la direccion $\vec{v} = \hat{i} + \hat{j} - \hat{k}$

B) En que direccion CAMBIA V mas rapidamente en el punto P? y menos RAPIDAMENTE?

C) Cual es la mayor derivada en P?

$V'_x = 10x - 3y + yz$

$V'_y = -3x + xz$

$V'_z = xy$

$\vec{\nabla} F = (10x - 3y + yz)\hat{i} + (-3x + xz)\hat{j} + (xy)\hat{k}$

$\vec{\nabla} F_{(3;4;5)} = (30 - 12 + 20)\hat{i} + (-9 + 15)\hat{j} + (12)\hat{k}$

$= 38\hat{i} + 6\hat{j} + 12\hat{k}$

$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$ $\quad \hat{i} + \hat{j} - \hat{k}$ $\quad \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\vec{u} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \rangle$

$D_u V_{(x,y,z)} = \vec{\nabla} F_{(P_0)} \cdot \vec{u}$

$D_u V = \langle 38; 6, 12 \rangle \cdot \langle \frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}; \frac{-1}{\sqrt{3}} \rangle$

$= \frac{38}{\sqrt{3}} + \frac{6}{\sqrt{3}} - \frac{12}{\sqrt{3}} = \boxed{\frac{32}{\sqrt{3}}}$

A

B $\vec{\nabla} F_{(P_0)} = 38\hat{i} + 6\hat{j} + 12\hat{k}$ + RAPIDAMENTE

$\vec{\nabla} F_{(P_0)} = -38\hat{i} - 6\hat{j} - 12\hat{k}$ − RAPIDAMENTE

C $|\vec{\nabla} F_{(P_0)}| = \sqrt{(38)^2 + (6)^2 + (12)^2} = \sqrt{1624} \simeq 40{,}29$

$W(x,y,z) = 2xyz(3-x)(7-y)(3-z)$

DERIVAR

$\Rightarrow \underbrace{[x\, 2yz(7-y)(3-z)]}_{u} \cdot \underbrace{(3-x)}_{v} = u'v + u \cdot v'$

$W'_x = [2yz\cancel{(7-y)(3-z)}](3-x) + [(x\, 2yz \cdot (7-y) \cdot (3-z))] \cdot -1$

$= [2yz \cdot \cancel{(7-y) \cdot (3-z)}] \cancel{(3-x)} + [+x\, 2yz \cancel{yz}]$

$= [2y$

$y = K F(x) =$

$K \cdot F'(x)$

$W'_x = 2yz(7-y)(3-z)[x(3-x)]$

$\underbrace{2yz(7-y)(3-z)}_{K}\underbrace{[3x - x^2]}_{F(x)}$

$2yz(7-y)(3-z) \cdot [3 - 2x]$

$W(x,y,z) = 2xyz(3-x)(7-y)(3-z)$

$W'_y = 2xz(3-x)(3-z) \cdot [y \cdot (7-y)]$ $\quad y - y^2$

$$= 2xz(3-x)(3-z)\cdot[1-2y]$$

$$W'_z = 2xy(3-x)(1-y)\cdot[3z-z^2]$$

$$2xy(3-x)(1-y)\cdot[3-2z]$$

$$V = -\check{j} - \check{k}$$

- $a(t) = \langle t^2 ; \frac{1}{2}\sqrt{t} ; t^3 \rangle$ | Hallar $V(t)$ y $r(t)$ en cualquier t

$V(0) = t\check{i} + t\check{k}$

$r(0) = j$

$$V(t) = \int a(t)$$

$$\int t^2\,dt\,\check{i} + \frac{1}{2}\int t^{\frac{1}{2}}dt\,\check{j} + \int t^3\,dt\,\check{k} =$$

$$\frac{t^3}{3}\check{i} + \left(\frac{1}{2}\cdot\frac{3}{2}t^{\frac{3}{2}}\right)\check{j} + \frac{t^4}{4}\check{k} =$$

$$= \frac{t^3}{3}\check{i} + \frac{3}{4}t^{\frac{3}{2}}\check{j} + \frac{t^4}{4}\check{k} + t\check{i} + t\check{k}$$

$$= \frac{t^3}{3}\check{i} + t\check{i} + \frac{3}{4}t^{\frac{3}{2}}\check{j} + \frac{t^4}{4}\check{k} + t\check{k} =$$

$$= \left(\frac{t^3}{3} + t\right)\check{i} + \frac{3}{4}t^{\frac{3}{2}}\check{j} + \left(\frac{t^4}{4} + t\right)\check{k} = V(t)$$

$$V(0) = t\check{i} + t\check{k}$$

$$0^3\check{i} + 0 + 0 + C = t\check{j} + t\check{k}$$

$$r(t) = \int V(t)$$

$$\left[\frac{1}{3}\int t^3\,dt + \int t\,dt\right]\check{i} + \left[\frac{3}{4}\int t^{\frac{3}{2}}\,dt\right]\check{j} + \left[\frac{1}{4}\int t^4\,dt + \int t\,dt\right]\check{k} =$$

$$= \boxed{\left(\frac{t^4}{12} + \frac{t^2}{2} + 1\right)\check{i} + \left(\frac{3}{10}t^{\frac{5}{2}}\right)\check{j} + \left(\frac{t^5}{20} + \frac{t^2}{2}\right)\check{k}}$$

REPASO 22HS

jueves, 8 de mayo de 2025 22:10

5) Encuentre la distancia más corta del punto (1,0,-2) al plan $x + 2y + z = 4$.

$P(1,0,-2)$ al P ⇒ $\boxed{x + 2y + z = 4}$

↘ F ⇒ d   ↘ G (RESTRI)

$\nabla F = (x-1)^2 + (y-0)^2 + (z+2)^2$ $\nabla G \Rightarrow x + 2y + z - 4 = 0$

$$\begin{cases} 2(x-1) = \lambda \cdot 1 \\ 2(y-0) = \lambda \cdot 2 \\ 2(z+2) = \lambda \cdot 1 \\ x + 2y + z - 4 = 0 \end{cases} \Rightarrow \begin{cases} 2x - 2 = \lambda \\ 2y = 2\lambda \Rightarrow y = \lambda \\ 2z + 4 = \lambda \\ x + 2y + z - 4 \end{cases}$$

$2z + 4 = 2x - 2$ | $\boxed{2z + 4 = y}$ | $-\frac{7}{6} + 3 = \frac{11}{6}$ | $2 \cdot \left(\frac{-7}{6}\right) + 4 = \frac{5}{3}$

$2z + 6 = 2x$

$\boxed{z + 3 = x}$

x   y

z)

$(z+3) + 2 \cdot (2z + 4) + z - 4 = 0$

$z + 3 + 4z + 8 + z - 4 = 0$

$6z + 7 = 0$

$6z = -7$

$\boxed{z = -\frac{7}{6}}$

$P_i(\underset{x_0}{1}; \underset{y_0}{0}; \underset{z_0}{-2})$ $P_c\left(\underset{x}{\frac{11}{6}}; \underset{y}{\frac{5}{3}}; \underset{z}{-\frac{7}{6}}\right)$

PASOS

1 Igualo F con λ · RESTRICCION

2 FORMO SISTEMA

3 . Dejo Var. en funcion de UNA

4 . En la RESTRICCION obtengo valor de esa UNA

5 . Reemplazo ese Valor de 4 en 3

$d^2 = \left(\frac{11}{6} - 1\right)^2 + \left(\frac{5}{3} - 0\right)^2 + \left(-\frac{7}{6} + 2\right)^2$

$d^2 \cong 0{,}69 + 2{,}77 + 0{,}69 \cong > d \cong \sqrt{4.15} \cong \boxed{2{,}037}$

7) Emplee el método de multiplicadores de Lagrange para determinar el máximo de $f(x, y) = 9 - x^2 - y^2$ sujeto a x+y=3.

$\nabla F = 9 - x^2 - y^2$ $\nabla g = x + y = 3 \Rightarrow \boxed{x + y - 3 = 0}$

$$\begin{cases} -2x = \lambda \cdot 1 \\ -2y = \lambda \cdot 1 \\ x + y - 3 = 0 \end{cases}$$

$-2x = -2y$

$x = \frac{\cancel{-2}}{\cancel{-2}} y$

$x = y$

$x + x - 3 = 0$

$2x = 3$

$\boxed{x = \frac{3}{2}}$

$\boxed{\frac{3}{2} = y}$

$P_c\left(\frac{3}{2}; \frac{3}{2}\right)$

$z''_{xx} = -2 \rightarrow (-) \Rightarrow$ es MAX

5) Encontrar los máximos y/o mínimos de las siguientes funciones sujetas a las restricciones dadas.

a) $z = 3x^2 + 4y^2 - xy$ si $2x + y = 21$

b) $u = xyz$ si $x + y + z = 17$

c) $f(x,y,z) = x + z$ si $x^2 + y^2 + z^2 = 1$

A) $z = 3x^2 + 4y^2 - xy$ RESTRICCION $\Rightarrow$ $2x + y - 21 = 0$

$$\begin{cases} 6x - y = \lambda \cdot 2 \\ 8y - x = \lambda \cdot 1 \\ 2x + y - 21 = 0 \end{cases}$$

$6x - y = 2\lambda$

$3x - \frac{y}{2} = \lambda$

$8y - x = 3x - \frac{y}{2}$

$8y = 4x - \frac{y}{2}$

$8y + \frac{y}{2} = 4x$

$\frac{16y + y}{2} = 4x$

$\frac{17}{2} y = 4x$

$\frac{17}{2} y \cdot \frac{1}{4} = x$

$\frac{17}{8} y = x$

$2 \cdot \left(\frac{17}{8} y\right) + y - 21 = 0$

$\frac{34}{8} y + y = 21$

$\frac{21}{4} y = 21$

$y = \cancel{21} \cdot \frac{4}{\cancel{21}}$

$y = 4$

$x = \frac{17}{2\cancel{8}} \cdot \cancel{4} \Rightarrow x = \frac{17}{2}$

$6x - y$

$8y - x$

$z'_{xx} = 6$

$z'_{xy} = -1$

$z'_{yx} = -1$

$z'_{yy} = 8$

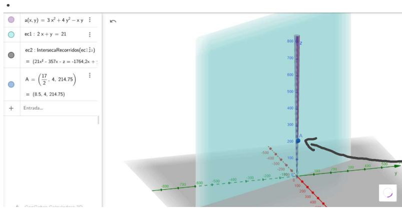
$H = \begin{vmatrix} 6 & -1 \\ -1 & 8 \end{vmatrix} = 47 > 0$ EXTREMO

$z'_{xx} = (+)$ minimo

OBTENGO $\rightarrow$ $z = 3\left(\frac{17}{2}\right)^2 + 2(4)^2 - \left(\frac{17}{2} \cdot 4\right) = \frac{867}{4} + 32 - 34 \cong 214{,}75$

mi minimo en

$P_c = \left(\frac{17}{2} ; 4 ; 214{,}75\right)$



REPASO UNA AM

viernes, 9 de mayo de 2025 1:22

b) $u = xyz$ si $x + y + z = 17$

$\nabla F \quad u = X Y . Z$ RESTRICCION $\nabla g = x + y + z - 17 = 0$

$$\begin{cases} yz = \lambda \cdot 1 \\ xz = \lambda \cdot 1 \\ xy = \lambda \cdot 1 \\ x + y + z - 17 = 0 \end{cases}$$

$YZ = XZ$

$\frac{Y \cdot \cancel{Z}}{\cancel{Z}} = X$

$Y = X$

$X \cdot X = XZ$

$2x = x \cdot z$

$2 = z$

$x + x + 2 - 17 = 0$

$2x = 15$

$x = \frac{15}{2}$; $y = \frac{15}{2}$; $z = 2$

YZ · XZ XY

$P_c\left(\frac{15}{2}; \frac{15}{2}; 2\right)$

$z'_{xx} =$

$z'_{xy} =$

$z'_{yx} =$

$z'_{yy} =$

4) Si $f(t) = (t^3 - 5)i + (4t^2 - t + 1)j + (3t^2 - t)k$, dar la ecuación del plano normal en el punto correspondiente a $t_0 = 2$.

de los 3 planos

$F(2) = (2^3 - 5)\check{i} + (4(2^2) - 2 + 1)\check{j} + (3(2^2) - 2)\check{k}$

$= \underbrace{3\check{i}}_{x_0} + \underbrace{15\check{j}}_{y_0} + \underbrace{10\check{k}}_{z_0}$

$T(t) = \pi'(t)$

$T(t) = (3t^2)\check{i} + (8t - 1)\check{j} + (6t - 1)\check{k}$

$T(2) = \underbrace{12\check{i}}_{A} + \underbrace{15}_{B}\check{j} + \underbrace{11}_{C}\check{k}$

PLANO NORMAL $= \boxed{12 \cdot (x - 3) + 15 \cdot (y - 15) + 11(z - 10)} = 0$

$N(t) = T'(t)$

$N(t) = (6t)\check{i} + 8\check{j} + 6\check{k}$

$N(2) = \underbrace{12}_{A}\check{i} + \underbrace{8}_{B}\check{j} + \underbrace{6}_{C}\check{k}$

PLANO RECTIFICANTE > $\boxed{12(x - 3) + 8(y - 15) + 6(z - 10) = 0}$

$B(t) = T(t) \times N(t)$

$$\begin{vmatrix} \check{i} & \check{j} & \check{k} \\ 3t^2 & (8t-1) & (6t-1) \\ 6t & 8 & 6 \end{vmatrix} = (8t-1) \cdot 6 - [(6t-1) \cdot 8]\check{i} - [(3t^2 \cdot 8) - ((8t-1) \cdot 6t)]\check{j} +$$

$$+ [(3t^2 \cdot 6) - ((6t-1) \cdot 6t)]\check{k}$$

(+) (−) (+)

$= \cancel{48t} - 6 \cancel{- 48t} + 8 \quad \check{i} -$ $\quad 6t \; \check{j}$

$+ (-18t^2) \quad + 6t \; \check{k}$

$$B(t) = 2\check{i} - 6t\check{j} + (-18t^2 + 6t)\check{k}$$

$$B(2) = \underbrace{2}_{A}\check{i} \underbrace{-12}_{B}\check{j} \quad \underbrace{-60}_{C}\check{k}$$

PLANO OSCULADOR $= \boxed{2(x-3) - 12(y-15) - 60(z-10) = 0}$

¿En qué puntos corta la hélice $r(t) = (sen(t); cos(t); t)$ a la esfera $x^2 + y^2 + z^2 = 5$?

$$r(t) = \langle \overbrace{sen(t)}^{x}; \overbrace{cos(t)}^{y}; \overbrace{t}^{z} \rangle$$

$$x^2 + y^2 + z^2 = 5$$

$$(sen(t))^2 + (cos(t))^2 + t^2 = 5$$

$$1 + t^2 = 5$$

$$t^2 = 4$$

$$t = \sqrt{4}$$

$$t = \pm 2$$

$$r(2) = x = sen(2) \qquad y = cos(2) \qquad z = 2$$

$$r(-2) = x = sen(-2) \qquad y = cos(-2) \qquad z = -2$$

$$P_n \Rightarrow P_1(sen\,2; cos(2); 2) \wedge P_2(sen(-2); cos(-2); -2)$$

8) Calcular la longitud de arco para la curva asociada

c) $r(t) = \ln(sen(t))\,i + (t+1)\,j \quad \frac{\pi}{6} \le t \le \frac{\pi}{2}$

$$r(t) = \overbrace{\ln(sen(t))}^{F(t)}\,i + \overbrace{(t+1)}^{g(t)}\,j \qquad \underbrace{\frac{\pi}{6}}_{}^{A} \le t \le \frac{\pi}{2}^{B}$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{[F'(t)]^2 + [g'(t)]^2} =$$

$$F'(t) = \frac{1}{sen(t)} \cdot cos(t) = \frac{cos(t)}{sen(t)}$$

$$g'(t) = 1$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{\frac{cos^2(t)}{sen^2(t)} + 1}$$

$$\int \sqrt{\frac{cos^2(t) + sen^2(t)}{sen^2(t)}} =$$

$$= \int \frac{1}{\operatorname{sen}^2(x)} = \int \sqrt{\operatorname{cosec}^2(x)} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \operatorname{cosec}(x) =$$

$$= -\ln\left|\operatorname{cosec}(x) + \operatorname{cotg}(x)\right| \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= -\ln\left|\operatorname{cosec}\left(\frac{\pi}{2}\right) + \operatorname{cotg}\left(\frac{\pi}{2}\right)\right| - \left[-\ln\left|\operatorname{cosec}\left(\frac{\pi}{6}\right) + \operatorname{cotg}\left(\frac{\pi}{6}\right)\right|\right]$$

$$\cong -4{,}28 - [-5{,}38] \cong \boxed{1{,}1}$$

e) $f(x,y,z) = x^2 + y^2 + 3z^2$ en $(1,1,0)$ con respecto a la dirección $u = i - j + 2k$

$\vec{\nabla} F = f'_x\, \check{i} + f'_y\, \check{j} + f'_z\, \check{k}$ $\quad P(1,1,0) \quad u = i - j + 2k$

$D_u f(x,y,z) = \vec{\nabla} F_{(P_0)} \cdot |\vec{u}|$

$|u| = \sqrt{(1)^2 + (1)^2 + (2)^2} = \sqrt{6}$

$\vec{\nabla} F = 2x\,\check{i} + 2y\,\check{j} + 6z\,\check{k}$

$|\vec{u}| = \left\langle \frac{1}{\sqrt{6}} ; \frac{-1}{\sqrt{6}} ; \frac{2k}{\sqrt{6}} \right\rangle$

$\nabla F(P_0) = 2\check{i} + 2\check{j} + 0\check{k}$

$D_u f(1,1,0) = \langle 2 ; 2 ; 0 \rangle \cdot \left\langle \frac{1}{\sqrt{6}} ; -\frac{1}{\sqrt{6}} ; \frac{2k}{\sqrt{6}} \right\rangle$

$$= \cancel{\frac{2}{\sqrt{6}}} - \cancel{\frac{2}{\sqrt{6}}} + 0 = 0$$

jueves, 8 de mayo de 2025 18:43

$$|\vec{\nabla} F| = \sqrt{(F'_x \, \check{i})^2 + (F'_y \, \check{j})^2}$$

$$= \sqrt{\left[\frac{-4x+8}{\left(\sqrt{(x-2)^2+(y+3)^2}\right)^3}\right]^2 \check{i} + \left[\frac{-4y+12}{\left(\sqrt{(x-2)^2+(y+3)^2}\right)^3}\right]^2 \check{j}} =$$

$$= \sqrt{\frac{(-4x+8)^2}{\sqrt{(x-2)^2+(y+3)^2}^{\,6}} + \frac{(-4y+12)^2}{\sqrt{(x-2)^2+(y+3)^2}^{\,6}}} =$$

$$= \sqrt{\frac{(-4x+8)^2+(-4y+12)^2}{\sqrt{(x-2)^2+(y+3)^2}^{\,6}}} = \sqrt{\frac{-16x \quad -16y+208}{\left(\sqrt{(x-2)^2+(y+3)^2}\right)^6}} =$$

$$\frac{\sqrt{-16x-16y+208}}{\sqrt{\left(\sqrt{(x-2)^2+(y+3)^2}\right)^{\cancel{6}\,4}}} = \frac{\sqrt{-16x-16y+208}}{\left(\sqrt{(x-2)^2+(y+3)^2}\right)^{\cancel{4}\,2}} =$$

$$= \frac{\sqrt{-16x-16y+208}}{((x-2)^2+(y+3)^2)^2} =$$

1)

a) Hallar y graficar el dominio de la función f(x;y): $\Re^2 \to \Re$ definida como:

$$f(x;y) = \frac{1}{\sqrt{4-x^2-y^2}}$$

b) Hallar y graficar (si existen) las curvas de nivel M=0, M=1 y M=3.

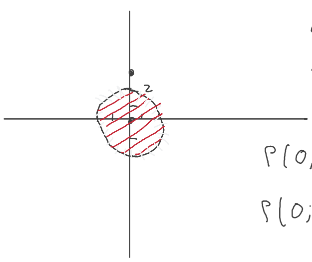
c) Calcular la derivada direcional de f(x;y) en la dirección del vector (2;1) en el punto (1;1).

1) A) DOM $F(x;y) = \dfrac{1}{\sqrt{4-x^2-y^2}}$ C para $M=0$, $M=1$, $M=3$

DOM 1ro → BUSCO RESTRICCION

$$\sqrt{4-x^2-y^2} \Rightarrow 4-x^2-y^2 > 0$$

$$4 = x^2-y^2$$
$$2^2 = x^2-y^2$$



$P(0,0) \Rightarrow 4-0^2-0^2 > 0 \Rightarrow 4>0$ ✓

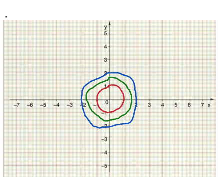
$P(0;3) \Rightarrow 4-0^2-9 > 0 \Rightarrow -5>0$ ✗

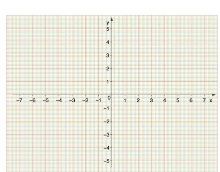
$$\text{Dom} = \{(x,y) \in \mathbb{R}^2 \; / \; 4-x^2-y^2 > 0\}$$

$4-x^2-y^2 = 0 \Rightarrow 4 = x^2-y^2 \Rightarrow R = 2$

$4-x^2-y^2 = 1 \Rightarrow 3 = x^2-y^2 \Rightarrow R = \sqrt{3} \cong 1{,}73$

$4-x^2-y^2 = 3 \Rightarrow 1 = x^2-y^2 \Rightarrow R = 1$





c) Calcular la derivada direcional de f(x;y) en la dirección del vector (2;1) en el punto (1;1).

$$\frac{1}{\sqrt{4-x^2-y^2}}$$

$$D_u F(x,y) = \vec{\nabla} F \cdot \vec{u}$$

$$\vec{\nabla} F = F'_x \, \check{i} + F'_y \, \check{j}$$

$$F'_x = \frac{0 - \left[1 \cdot \frac{1}{2\sqrt{4-x^2-y^2}} \cdot (-2x)\right]}{\left(\sqrt{4-x^2-y^2}\right)^2} =$$

DOY VUELTA

$$= \frac{-x}{\left(\sqrt{4-x^2-y^2}\right)^1} \cdot \frac{1}{\left(\sqrt{4-x^2-y^2}\right)^2} =$$

$$F'x = \frac{-x}{\left(\sqrt{4-x^2-y^2}\right)^3} \qquad \frac{1}{\sqrt{4-x^2-y^2}}$$

$$F'y = \frac{0 - \left[1 \cdot \frac{1}{2\sqrt{4-x^2-y^2}} \cdot -2y\right]}{\left(\sqrt{4-x^2-y^2}\right)^2} =$$

$$F'y = \frac{-y}{\left(\sqrt{4-x^2-y^2}\right)^3}$$

$$\vec{\nabla} F = \frac{x}{\left(\sqrt{4-x^2-y^2}\right)^3} \check{\imath} + \frac{y}{\left(\sqrt{4-x^2-y^2}\right)^3} \check{\jmath}$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{2}{\sqrt{5}}\check{\imath} + \frac{1}{\sqrt{5}}\check{\jmath} = \vec{u} \qquad |\vec{v}| = \sqrt{2^2+1^2} = \sqrt{5}$$

$$D_u F(x,y) = \left\langle \frac{1}{2^{\frac{3}{2}}} ; \frac{1}{2^{\frac{3}{2}}} \right\rangle \cdot \left\langle \frac{2}{\sqrt{5}} ; \frac{1}{\sqrt{5}} \right\rangle =$$

$$= \frac{2}{2^{\frac{3}{2}}\sqrt{5}} + \frac{1}{2^{\frac{3}{2}}\sqrt{5}} = \frac{3}{2^{\frac{3}{2}}\sqrt{5}} \cong 0,4743$$

3) Un insecto se halla en medio de un ambiente tóxico. El nivel de toxicidad está dado por : $T(x,y) = 2x^2 - 4y^2$. El insecto está en (-1,2). ¿En qué dirección y sentido debe moverse para disminuir lo más rápidamente posible la toxicidad

$$\to T(x,y) = 2x^2 - 4y^2 \qquad P(-1;2)$$

$$\Leftarrow \vec{\nabla} F$$

$$F'x = 4x \qquad \vec{\nabla} F = 4x\check{\imath} + -8y\check{\jmath}$$

$$F'y = -8y \qquad \vec{\nabla} F_{(-1,2)} = 4(-1)\check{\imath} + -8 \cdot 2\check{\jmath}$$

$$\vec{\nabla} F = -4\check{\imath} - 16\check{\jmath}$$

$$\overline{(\vec{\nabla} F)} = 4\check{\imath} + 16\check{\jmath}$$

En la dirección del vector $= 4\check{\imath} + 16\check{\jmath}$

4) El vector velocidad del movimiento de una partícula viene dado por

$$v(t) = 3t^2 i + 2tj$$

a) Calcular la velocidad, la aceleración y la rapidez en el instante $t = 1$.

b) Si $r(0) = 0$ calcular $r(t)$

$$V(t) = 3t^2\check{\imath} + 2t\check{\jmath}$$

$$V(t) = r'(t)$$

$$\int V(t) = \boxed{r(t)}$$

A) $V(t)$

$a(t) = V'(t)$ en $t = 1$

$RAP = |V(t)|$

$$V(1) = 3 \cdot 1\check{\imath} + 2 \cdot 1\check{\jmath} = 3\check{\imath} + 2\check{\jmath}$$

$$a(1) =$$

$$a(t) = V'(t) = 6t\check{\imath} + 2\check{\jmath}$$

$$a(1) = 6\check{\imath} + 2\check{\jmath}$$

$$RAP = |V(t)| = |V(1)|$$

$$RAP(1) = \sqrt{(3)^2 + (2)^2} = \sqrt{13} \cong 3,60$$

$$V(t) = 3t^2\check{\imath} + 2t\check{\jmath}$$

$$\int V(t) = 3\int t^2\,dt\,\check{\imath} + 2\int ;$$

$$= \cancel{3}\frac{t^3}{\cancel{3}}\check{\imath} + \cancel{2}\frac{t^2}{\cancel{2}}\check{\jmath}$$

$$= t^3\check{\imath} + t^2\check{\jmath} + C_{(0)}$$

$$r(0) = 0$$

$$= 0^3\check{\imath} + 0^2\check{\jmath} + C = 0$$

$$C = 0$$

$$\boxed{r(t) = t^3\check{\imath} + t^2\check{\jmath}}$$

$M(x) = x'i + x'j$

5) Hallar la ecuación del plano tangente y la recta normal en (-2;1,3) para la superficie:

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$$

$$\boxed{P_T = z - z_0 = z'_{x\,(P_0)}(x - x_0) + z'_{y\,(P_0)}(y - y_0)}$$

$P(-2;1;3)$

$$\boxed{R_N = \frac{x - x_0}{z'_{x\,(P_0)}} = \frac{y - y_0}{z'_{y\,(P_0)}} = \frac{z - z_0}{-1}}$$

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$$

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} - 3 = 0$$

$$z'_x = \frac{-[z'_x]}{z'_z}$$

$$z'_y = \frac{-[z'_y]}{z'_z}$$

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} - 3 = 0$$

$z'_x = \frac{x}{2}$ | $z'_y = 2y$ | $z'_z = \frac{2}{9}z$

$$z'_x = \frac{-\left[\frac{x}{2}\right]}{\frac{2}{9}z} = -\frac{x}{2} \cdot \frac{1}{\frac{2}{9}z} = \boxed{-\frac{x}{\frac{4}{9}z}} \Rightarrow \frac{-(2)}{\frac{4}{9} \cdot 3} = \frac{+2}{\frac{12}{9}} = \boxed{+\frac{18}{12}} \cong$$

$$z'_y = \frac{-[2y]}{\frac{2}{9}z} = -2y \cdot \frac{1}{\frac{2}{9}z} = \boxed{-\frac{18y}{2z}} \Rightarrow -\frac{18 \cdot 1}{2 \cdot 3} = -\frac{18}{6} = \boxed{-3}$$

$P(-2;1;3)$

$$P_T = \frac{18}{12}(x+2) - 3(y-1) = z - 3 \Rightarrow \frac{18}{12}(x+2) - 3(y-1) + 3 = z$$

$$R_N = \frac{x+2}{\frac{18}{12}} = \frac{y-1}{-3} = \frac{z-3}{-1}$$

6) Analizar los extremos de la siguiente función:

$$f(x,y) = xy + \frac{50}{x} + \frac{20}{y}$$

([redacted])

ANALIZAR EXTREMOS $\Rightarrow$ LIBRES $\Leftrightarrow$ NO HAY RESTRICCIÓN

1ro SISTEMA $\begin{cases} z'_x = 0 \\ z'_y = 0 \end{cases}$ RES. SIST. $\longrightarrow$ $P_c = (x_i, y_i)$

2º $H(x,y) = \begin{vmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{vmatrix} \Rightarrow$ EVALUO en c/u P_c

Si > 0 EXTREMO

< 0 NO

$= 0$ PTO ENSILLADURA

3º $z'_{xx} \to (-)$ MAX

$\to (+)$ min

$$F(x,y) = xy + \frac{50}{x} + \frac{20}{y}$$

$$z'_x = y - \frac{50}{x^2}$$

$$z'_y = x - \frac{20}{y^2}$$

$\cancel{0 - \frac{20 \cdot 1}{y^2}}$

$$\begin{cases} y - \frac{50}{x^2} = 0 \\ x - \frac{20}{y^2} = 0 \end{cases}$$

$$\frac{50}{x^2} = +y \Rightarrow x - \frac{20}{\left(\frac{50}{x^2}\right)^2} = 0 \qquad \frac{2500}{x^4}$$

$\cancel{x - \left[20\left(\frac{x^2}{50}\right)\right] = -\left[\frac{20x^3}{50}\right] + x}$

$x - [20 \cdot \cancel{2500}] = x - \cancel{50000} = 0 \; 7 \Rightarrow$

$$x - \left[\cancel{2}\cancel{0} \cdot \frac{x^4}{\cancel{2500}} \right] = x - \frac{x^4}{125} \Rightarrow \frac{x^4}{125}$$

$$\frac{x^{\cancel{4}}}{\cancel{x}} =$$

$$x^3 =$$

$$\boxed{x =}$$

$$\frac{50}{x^2} = y \Rightarrow \frac{50}{25} = y = \boxed{2}$$

P_c en $(5;2)$

$z'_x = y - \left(\frac{50}{x^2}\right)$

$z'_y = x - \frac{20}{y^2}$

$H(x,y)$

$$z_{xx} = -\frac{\left[0 - [50 \cdot 2x]\right]}{x^4} = \frac{100\cancel{x}}{x^{\cancel{4}}} = \frac{100}{x^3}$$

$$z_{xy} = 1 \quad | \quad z_{yx} = 1$$

$$z_{yy} = -\left[0 - \frac{[20 \cdot 2y]}{y^4}\right] = \frac{40\cancel{y}}{y^{\cancel{4}}} = \frac{40}{y^3}$$

$$H(x,y) = \begin{vmatrix} \frac{100}{x^3} & 1 \\ 1 & \frac{40}{y^3} \end{vmatrix} \Rightarrow H(5,2) = \begin{vmatrix} \frac{100}{125} & 1 \\ 1 & \frac{40}{8} \end{vmatrix} = \frac{100}{125} \cdot \frac{40}{8} - 1 \cdot 1 = \frac{4000}{1000} = 4 \cdot$$

Analisis 2 parcial 2

15 June 2025 22:59

Variable separable Ec Diferenciales

1) $\sqrt{y^2+2y+4}\,\sec x\,dx + (6y+6)\cos^2 x\,dy$

(Rta) $\sec x + C = -6\sqrt{y^2+2y+4}$

(2) $y' = \dfrac{\sqrt{4-4y^2}\cdot(2x+1)}{\sqrt{4x^2+4x+1}}$ Hallar la solución particular para $P_0 = (1;0)$

(3) $3y(x^2-3x+2)\,dy - \sqrt{1-y^2}\,dx = 0$

Rta $\ln\frac{x-2}{x-1} + C = -3\sqrt{1-y^2}$

Homogéneas

1) $y\,dx + (2\sqrt{xy} - x)\,dy = 0$

(Rta) $\sqrt{\frac{x}{y}} + \ln y = C$

Lineales 1) $y' - y\tan x = \frac{1}{\cos x}$ Hallar SP para $P_0 = (0,0)$

Rta $y = \frac{1}{\cos x}(x+C)$ SP $y = \frac{x}{\cos x}$

1)RESOLVER LA INTEGRAL CURVILÍNEA: $\oint_{C^+}[(2y+\sqrt{9+x^3})dx + (5x+e^{\arctan y})dy]$ Si $C: x^2+y^2=4$

3)RESOLVER: $(2x^3+y)dx + (x+2y^2)dy = 0$

4)CALCULAR MEDIANTE UNA INTEGRAL TRIPLE, UTILIZANDO COORDENANADAS CILÍNDRICAS, EL VOLUMEN DEL SÓLIDO LIMITADO POR EL PARABOLOIDE: $z = x^2 + y^2$ Y EL PLANO $z=4$

d) $2x^2y\,dx = (3x^3+y^3)dy$ e) $(2xy^2-3)dx + (2x^2y+4)dy = 0$

g) $y' + \frac{2x}{x^2+1}y = x$ e) $e^y sen(2x)dx + cos(x)(e^{2y}-y)dy = 0$

f) $y' + \text{sen}(x) - y = 3\text{sen}(x)$ en $\left(\frac{\pi}{2}, 0\right)$ f) $y' = sen(5x)$

e) Región encerrada por $z= x^2 + y^2$ y el plano $z = 10$

f) Prisma de base triangular limitado por los plano $x=0$, $y=0$, $z=0$, $z=4$, $x+y=1$

g) $f(x;y) = x^2 + y$ $R = \{(x;y) / x^2 \le y \wedge y \le 2x \wedge y \le 1\}$

h) $f(x;y) = y\,x^2$ $R = \{(x;y)/ x \le |y| \wedge x^2 + y^2 \le 1 \wedge 0 \le x \wedge 0 \le y\}$

Calcular el volumen de los siguientes cuerpos:

a) Cuerpo limitado por las superficies $z^2=x^2+y^2$, $2-z=x^2+y^2$, $z \ge 0$ (sugerencia: utilice coordenadas cilíndricas)

Determine el flujo del campo vectorial F(x;y;z), en cada caso:

a) F(x;y;z)= zi +yj +xk a través de la esfera unitaria $1= x^2 + y^2 + z^2$

Determine el trabajo efectuado por el campo de fuerza $F(x;y)=x^2\,i - x\,y\,j$, cuando mueve una partícula a lo largo del cuarto de circulo r(t)= Cos t i+ Sen t j, $0 \le t \le \pi/2$.

c) C: $r(t) = \left(t;\ \frac{t^2}{\sqrt{2}};\ t\right)$, $t \in [0;1]$, $F(x;y;z) = \frac{y}{x+z}$

Determine si el siguiente campo es conservativo o no:

a) F(x;y)= (x-y) i + (x -2) j

Calcular $\int_C f(x;y)\,dl$ con f(x;y) =xy

a) C: círculo de radio 2, con centro en el origen de coordenadas

Calcular el área de la parte del paraboloide $z= x^2 + y^2$ que se ubica bajo el plano z=9.

1) $y\,dx + (2\sqrt{xy} - x)\,dy = 0$

(Rta) $\sqrt{\frac{x}{y}} + \ln y = C$

ESTRUCTURA

$$P(x,y)\,dx + Q(x,y)\,dy = 0$$

$$\underbrace{y}_{P}\,dx + \underbrace{(2\sqrt{xy} - x)}_{Q}\,dy = 0$$

$$F(tx;ty) = t^m \cdot F(x,y)$$

$$P(x;y) = y$$

$$P(tx;ty) = ty \Rightarrow t^1 = \text{GRADO } 1$$

$$Q(x,y) = 2\sqrt{xy} - x$$

$$Q(tx;ty) = 2\sqrt{tx\cdot ty} - tx$$

$$= 2\sqrt{t^2xy} - tx$$

$$= 2\sqrt{t^2}\cdot\sqrt{xy} - tx$$

$$= 2t\cdot\sqrt{xy} - tx$$

$$= t\,(2\sqrt{xy} - x) \Rightarrow t^1 = \text{GRADO } 1$$

Se cumple mismo GRADO

$$y\,dx + (2\sqrt{xy} - x)\,dy = 0$$

$$\frac{y}{x}\,dx + \frac{2\sqrt{xy} - x}{x}\,dy = 0$$

$$\frac{y}{x}\,dx + \left(\frac{2\sqrt{xy}}{x} - \frac{\cancel{x}}{\cancel{x}}\right)dy = 0$$

$$v = \frac{y}{x} \Rightarrow vx = y$$

$$dy = v\,dx + x\,dv$$

$$v\,dx + \left[\frac{2\sqrt{xvx}}{x} - 1\right][v\,dx + x\,dv] = 0$$

$$v\,dx + [2\sqrt{v} - 1]\cdot[v\,dx + x\,dv] = 0$$

C.A

$$\frac{2(xy)^{\frac{1}{2}}}{x}$$

$$\frac{\cdot 2}{x\cdot(xy)^{-\frac{1}{2}}}$$

$$\frac{\sqrt{xvx}}{x} = \frac{\sqrt{x^2}\cdot\sqrt{v}}{x}$$

$$\frac{\cancel{x}\sqrt{v}}{\cancel{x}} = \sqrt{v}$$

$$v\,dx + 2\sqrt{v}\cdot v\,dx + 2\sqrt{v}\,x\,dv - v\,dx - x\,dv = 0$$

$$(v + 2\sqrt{v}\,v - v)\,dx + (2\sqrt{v}\,x - x)\,dv = 0$$

$$(\cancel{v} + 2\sqrt{v}\,v \cancel{-v})\,dx = -(2\sqrt{v}\,x - x)\,dv$$

$$(2\sqrt{v}\,v)\,dx = -x(2\sqrt{v} - 1)\,dv$$

$$\frac{dx}{-x} = \frac{(2\sqrt{v} - 1)}{(2\sqrt{v}\,v)}\,dv$$

$t = 2\sqrt{v}$

$$-\int \frac{1}{x}\,dx = \frac{t - 1}{t\,v}\sqrt{v}\,dt$$

$$-\ln|x| = \frac{t-1}{t\left(\frac{t}{2}\right)^2}\cdot\frac{t}{2}\,dt$$

$$-\ln|x| = \frac{t-1}{t\cdot\frac{t^2}{4}}\cdot\frac{t}{2}\,dt$$

$$-\ln|x| = (t-1)\cdot 4 \cdot t\,dt$$

C.1

$$t = 2\sqrt{v}$$

$$dt = \frac{2}{2}v^{-\frac{1}{2}}\,dv$$

$$dt = \frac{1}{\sqrt{v}}\,dv$$

$$\sqrt{v}\,dt = dv$$

$$t = 2\sqrt{v}$$

$$\frac{t}{2} = \sqrt{v}$$

$t^2 \cdot t \qquad 2$

$$-\ln|x| = \frac{(4t-4)t}{2t^{2}\cdot 2}\,dt$$

$$-\ln|x| = \frac{4t-4}{2t^2}\,dt$$

$$-\ln|x| = \frac{4(t-1)}{2t^2}\,dt$$

$$-\ln|x| = \int 2\frac{(t-1)}{t^2}\,dt$$

$$-\ln|x| = 2\int\frac{(t-1)}{t^2}\,dt$$

$$-\ln|x| = 2\int (t-1)\cdot t^{-2}\,dt$$

$$-\ln|x| = 2\int t^{-1} - t^{-2}\,dt$$

$$-\ln|x| = 2\left[\int \frac{1}{t}\,dt - \int t^{-2}\,dt\right]$$

$$= 2\left[\ln|t| + t^{-1}\right]$$

$$-\ln|x| = 2\ln|t| + \frac{2}{t}$$

$$-\ln|x| = 2\ln|2\sqrt{v}| + \frac{2}{2\sqrt{v}}$$

$$-\ln|x| = 2\ln\left|2\sqrt{\frac{y}{x}}\right| + \frac{1}{\sqrt{\frac{y}{x}}}$$

$A\ln B = \ln B^A$

$$-\ln|x| = 2\left[\ln|2| + \ln\left|\sqrt{\frac{y}{x}}\right|\right] + \frac{1}{\sqrt{\frac{y}{x}}}$$

$$-\ln|x| = 2\ln|2| + 2\ln\sqrt{\frac{y}{x}} + \frac{1}{\sqrt{\frac{y}{x}}}$$

$$-\ln|x| = \ln|4| + \ln\left|\frac{y}{x}\right| + \frac{1}{\sqrt{\left(\frac{y}{x}\right)}}$$

$t = \sqrt{v}$

$$\frac{dx}{} = \frac{(2\sqrt{v}\,1 - 1)}{}\,dv$$

$t = \sqrt{v}$

$t^2 = v$

$2t\,dt = dv$

$$\frac{dx}{-x} = \frac{(2\sqrt{v} - 1)}{(2\sqrt{v}\, v)}\, dv$$

$$-\int \frac{1}{x}\, dx = \int \frac{2t - 1}{\cancel{2t}\, t^2} \cdot \cancel{2t}\, dt$$

$$-\int \frac{1}{x}\, dx = \int \frac{2t - 1}{t^2}\, dt$$

$$-\int \frac{1}{x}\, dx = \int \frac{2t}{t^2} - \frac{1}{t^2}\, dt$$

$$-\int \frac{1}{x}\, dx = 2\int \frac{1}{t}\, dt - \int t^{-2}\, dt$$

$$-\ln|x| = 2\ln|t| - \frac{t^{-1}}{-1}$$

$$-\ln|x| = 2\ln|t| + t^{-1}$$

$A \ln B = \ln B^A$

$$-\ln|x| = 2\ln|\sqrt{v}| + (\sqrt{v})^{-1}$$

$\ln A + \ln B = \ln(A \cdot B)$

$$-\ln|x| = \ln|v| + \frac{1}{\sqrt{v}}$$

$$-\ln|x| = \ln\left|\frac{y}{x}\right| + \sqrt{\frac{x}{y}}$$

$$0 = \ln\left|\frac{y}{\cancel{x}}\right| + \ln|x| + \sqrt{\frac{x}{y}} + C$$

$$0 = \ln\left|\frac{y}{\cancel{x}} \cdot \cancel{x}\right| + \sqrt{\frac{x}{y}} + C$$

$$\boxed{C = \ln|y| + \sqrt{\frac{x}{y}}}$$

$$\frac{1}{\sqrt{\left(\frac{y}{x}\right)}} = \sqrt{\left(\frac{x}{y}\right)}$$

$$\frac{1}{\frac{\sqrt{y}}{\sqrt{x}}} = \frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$$

Lineales 1) $y' - y \tan x = \frac{1}{\cos x}$ Hallar SP, para $P_0 = (0,0)$

Rta $y = \frac{1}{\cos x}(x + C)$ SP $y = \frac{x}{\cos x}$

$y = u \cdot v$

$\frac{1}{\cos}$

$y = \frac{1}{\cos x}(x+c) \quad SP \quad y = \frac{x}{\cos x}$

$y = u \cdot v$

$y' - y\,\underbrace{\tan(x)}_{P} = \underbrace{\frac{1}{\cos(x)}}_{Q}$

SP $P_0(0;0)$

$y' + P(x)\,y = Q(x)$

$y = u \cdot v \qquad u = e^{-\int P(x)\,dx}$

$v = \int Q(x) \cdot e^{\int P(x)\,dx}\,dx$

$u = e^{-\int \tan(x)\,dx} = e^{\int \tan(x)\,dx} = e^{\ln(\sec(x))} = \sec(x)$

$v = \int \frac{1}{\cos} \cdot e^{\int -\tan(x)\,dx}$

$A \ln B = \ln B^A$

$-1 \ln(B) = \ln B^{-1}$

$-1 \ln(\sec x) = \ln(\sec)^{-1}$

$e^{\ln A} = A$

$-\int \tan(x)\,dx = -\int \frac{\sin}{\cos}\,dx =$

$t = \cos$

$dt = -\sin\,dx$

$\frac{dt}{-\sin} = dx$

$= -\int \frac{\sin}{t}\,\frac{dt}{-\sin} =$

$= \int \frac{1}{t}\,dt = \ln|t| = \ln|\cos|$

$v = \int \frac{1}{\cos} \cdot e^{\ln(\cos)} = \int \frac{1}{\cancel{\cos}} \cdot \cancel{\cos}\,dx = \int 1\,dx = x + C$

$$\boxed{y = \frac{1}{\cos(0)} \cdot (x + c)} \qquad S_p \quad P_0(0;0)$$

$$0 = \frac{1}{\cos(0)} \cdot 0 + c$$

$$0 = \frac{1}{1} \cdot c$$

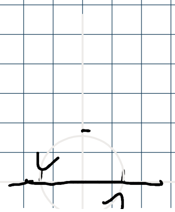
$$0 = c$$

$$\boxed{S_P = \frac{1}{\cos(x)} \cdot x = \frac{x}{\cos(x)}}$$

1)RESOLVER LA INTEGRAL CURVILÍNEA: $\oint_{C^+}[(2y + \sqrt{9 + x^3})dx + (5x + e^{arctag\, y})dy]$ Si $C: x^2+y^2=4$

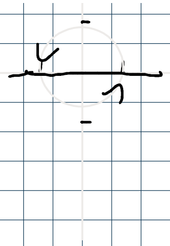
$$\oint_{C^+}\left[(2y + \sqrt{9+x^3})\,dx + (5x + e^{arctg(y)})\,dy\right]$$

$$C: x^2 + y^2 = 4 \rightarrow r = 2$$



$$\oint_{C^+}\left[P(x,y)\,dx + Q(x,y)\,dy\right] =$$

$$\iint_D (Q'_x - P'_y)\,dx\,dy$$



$$\iint_D (Q'_x - P'_y)\,dx\,dy$$

$$Q'_x = 5 \quad \Big| \quad P'_y = 2$$

$$\iint_D 5 - 2 \; dy\,dx$$

$x = r\cos\theta$

$y = r\,\text{sen}\,\theta$

$x^2 + y^2 = r^2$

$r = 2$

$$4\int_0^2\left[\int_0^{\frac{\pi}{2}} 3r\,d\theta\right]dr = 4\int_0^2\left[\;3r\theta\Big|_0^{\frac{\pi}{2}}\right]dr =$$

$$= 4\int_0^2\left[3r\frac{\pi}{2}\right]dr = \frac{\overset{2\cdot 3}{\cancel{4}}\pi}{\cancel{2}}\left[\frac{r^2}{2}\Big|_0^2\right] =$$

$$= 6\pi \cdot \frac{\cancel{4}}{\cancel{2}}^{2} = \boxed{12\pi}$$

3)RESOLVER: $(2x^3 + y)dx + (x + 2y^2)dy = 0$

$$\overbrace{(2x^3 + y)}^{P}\,dx + \overbrace{(x + 2y^2)}^{Q}\,dy = 0$$

$P'_y = 1$

$Q'_x = 1$

= Si cumple simetria

1^{ro} $M(x,y) = \int \rho(x,y)\,dx = \int 2x^3 + y\,dx = 2\int x^3 + y\,dx =$

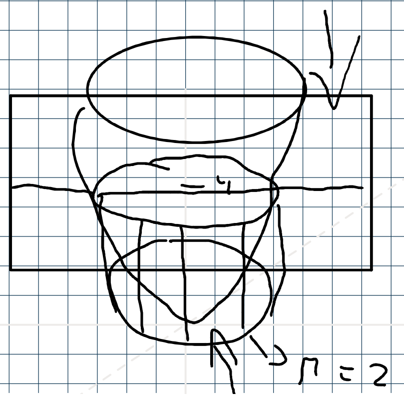
$= 2 \int x^3 dx + \int y\,dx = \cancel{2}\frac{x^4}{\cancel{4}2} + yx = \frac{x^4}{2} + yx + \alpha(y)$

2^{ro} $M(x,y) = \int x + 2y^2\,dy = \int x\,dy + 2\int y^2\,dy =$

$= xy + 2\frac{y^3}{3} + \beta(x)$

$$\boxed{M(x,y) = xy + \frac{x^4}{2} + \frac{2}{3}y^3 = C}$$

4) CALCULAR MEDIANTE UNA INTEGRAL TRIPLE, UTILIZANDO COORDENANADAS CILÍNDRICAS, EL VOLUMEN DEL SÓLIDO LIMITADO POR EL PARABOLOIDE: $z = x^2 + y^2$ Y EL PLANO $z=4$



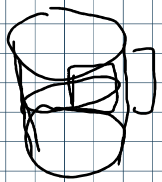
Intersección PARABOLOIDE y $z = 4$ (PLANO)

$4 = x^2 + y^2$ $\qquad z = r^2$

$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \\ x^2 + y^2 = r^2 \end{cases}$$

$r = 2$

$x^2 + y^2 = r^2$



$$V = \int_r \int_\theta \int_{z(r)} r \cdot dz\, d\theta\, dr$$

$$V = 8 \int_0^2 \int_0^{\frac{\pi}{2}} \left[\int_{r^2}^{4} r \cdot dz \right] d\theta\, dr =$$

$$= 8 \int_0^2 \int_0^{\frac{\pi}{2}} \left[rz \Big|_{r^2}^{4} \right] d\theta\, dr =$$

$$(\cdot A \quad 4r - r \cdot r^2 = 4r - r^3$$

$$= 8 \int_0^2 \left[\int_0^{\frac{\pi}{2}} 4r - r^3\, d\theta \right] dr$$

$$= 8 \int_0^2 \left[4r\theta - r^3\theta \Big|_0^{\frac{\pi}{2}} \right] dr =$$

$$8 \left[\cancel{4}\frac{\pi}{2} \frac{r^2}{\cancel{2}} - \frac{\pi}{2} \frac{r^4}{4} \right]$$

$$= 8 \left[\pi r^2 - \frac{\pi r^4}{8} \right]_0^2$$

$$= 8 \left[\pi 4 - \frac{\pi\, 16}{8} \right] =$$

$$= 8\int_0^2 4r\frac{\pi}{2} - r^3\frac{\pi}{2}\,dr$$

$$= 8\left[4\pi - 2\pi\right] = 8\cdot 2\pi = 16\pi$$

$$= \cancel{8}^{4}\,\frac{\pi}{\cancel{2}}\int_0^2 4r - r^3\,dr =$$

$$= 4\pi\left[\frac{\cancel{4}^{2}}{\cancel{2}}r^2 - \frac{r^4}{4}\Big|_0^2\right] =$$

$$= 4\pi\left[2\cdot 4 - \frac{16}{4}\right] = 4\pi\left[8-4\right] = 4\pi\cdot 4 = \boxed{16\pi}$$

$$= \int_0^2\int_0^{2\pi}\left[\int_{r^2}^4 r\,dz\right]d\theta\,dr =$$

$$= \int_0^2\int_0^{2\pi}\left[rz\Big|_{r^2}^4\right]d\theta\,dr =$$

$$= \int_0^2\int_0^{2\pi}\left[4r - r^3\right]d\theta\,dr =$$

$$= \int_0^2 \left[4r\theta - r^3\theta \Big|_0^{2\pi} \right] dr =$$

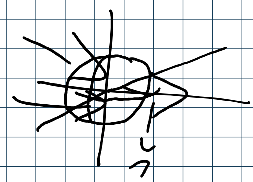
$$= \int_0^2 \left[r8\pi - \cancel{r^3 2\pi} \right] dr$$

$$= 4\cancel{8}\pi \frac{r^2}{\cancel{2}} - \cancel{2}\pi \frac{r^4}{\cancel{4}2} \Big|_0^2 =$$

$$= 4\pi r^2 - \frac{1}{2}\pi r^4 \Big|_0^2 = 4\pi 4 - \frac{1}{\cancel{2}} 8\pi \cancel{16} = 16\pi - 8\pi = 8\pi$$

Determine el flujo del campo vectorial F(x;y;z), en cada caso:

a) F(x;y;z)= zi +yj +xk a través de la esfera unitaria 1= $x^2 + y^2 + z^2$



$$\vec{F}(x, y, z)\ z\hat{i} + y\hat{j} + x\hat{k} \qquad x^2 + y^2 + z^2 = 1$$

1^{ero} PARAMETRIZO SUPERFICIE

COORD. ESF

$\vec{r}$; $(X ; Y ; Z(x,y))$

$\vec{r} = (1 \operatorname{sen}\varphi \cos\theta ; 1 \operatorname{sen}\varphi \operatorname{sen}\theta ; 1 \cos\varphi)$

COORD. ESF.

$$\begin{cases} x = \rho \operatorname{sen}\varphi \cos\theta \\ y = \rho \operatorname{sen}\varphi \operatorname{sen}\theta \\ z = \rho \cos\varphi \\ \rho^2 = x^2 + y^2 + z^2 \end{cases}$$

2^{do} CALCULO $\vec{ds}$

$F(\varphi ; \theta) \neq F(\theta ; \varphi)$

$$\vec{ds} = \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ \cos\theta \cos\varphi & \cos\varphi \operatorname{sen}\theta & -\operatorname{sen}\varphi \\ -\operatorname{sen}\theta \operatorname{sen}\varphi & \cos\theta \operatorname{sen}\varphi & 0 \end{vmatrix} d\varphi \, d\theta =$$

$$= (0 - (-\operatorname{sen}\varphi \cos\theta \operatorname{sen}\varphi)) \check{i} - (0 - [-\operatorname{sen}\varphi (-\operatorname{sen}\theta) \operatorname{sen}\varphi]) \check{j} +$$

$$[(\cos\theta \cos\varphi \cdot \cos\theta \operatorname{sen}\varphi) - (\cos\varphi \operatorname{sen}\theta - \operatorname{sen}\theta \operatorname{sen}\varphi)] \check{k}$$

$$= (\operatorname{sen}^2\varphi \cos\theta)\,\check{i} + (\operatorname{sen}^2\varphi \operatorname{sen}\theta)\,\check{j} + \left[(\cos^2\theta \cos\varphi \operatorname{sen}\varphi) + (\operatorname{sen}^2\theta \cos\varphi \operatorname{sen}\varphi)\right]$$

$$\left[(\cos\varphi \operatorname{sen}\varphi)\cdot\underbrace{(\cos^2\theta + \operatorname{sen}^2\theta)}_{1}\right]$$

$$= (\operatorname{sen}^2\varphi \cos\theta)\,\check{i} + (\operatorname{sen}^2\varphi \operatorname{sen}\theta)\,\check{j} + (\cos\varphi \operatorname{sen}\varphi)\,\check{k}$$

3ro MODULO <u>NO</u>

4to PARAMETRIZO el CAMPO VECTORIAL

$$\vec{F}(x,y,z)\; z\check{i} + 3\check{j} + x\check{k}$$

$$\vec{n} = \langle 1\cdot\cos\varphi\,;\; 1\cdot\operatorname{sen}\varphi\cdot\operatorname{sen}\theta\,;\; 1\cdot\operatorname{sen}\varphi\cos\theta \rangle$$

Cio

$5^{\underline{o}}$

$$\langle (1 \cdot \cos\varphi); (1 \cdot \operatorname{sen}\varphi \cdot \operatorname{sen}\theta); (1 \cdot \operatorname{sen}\varphi \cos\theta) \rangle$$

$$\langle (\operatorname{sen}^2\varphi \cos\theta); (\operatorname{sen}^2\varphi \operatorname{sen}\theta); (\cos\varphi \operatorname{sen}\varphi) \rangle$$

$$= \operatorname{sen}^2\varphi \cos\theta \cos\varphi + \operatorname{sen}^3\varphi \operatorname{sen}^2\theta + \operatorname{sen}^2\varphi \cos\varphi \cos\theta =$$

$$= 2(\operatorname{sen}^2\varphi \cos\theta \cos\varphi) + \operatorname{sen}^3\varphi \operatorname{sen}^2\theta$$

$$\int_\theta \int_\varphi 2(\operatorname{sen}^2\varphi \cos\theta \cos\varphi) + \operatorname{sen}^3\varphi \operatorname{sen}^2\theta \; d\varphi \, d\theta$$

$$= 8 \cdot \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} 2(\operatorname{sen}^2\varphi \cos\theta \cos\varphi) + \operatorname{sen}^3\varphi \operatorname{sen}^2\theta \; d\varphi \, d\theta$$

$$= 8 \int_0^{\frac{\pi}{2}} 2 \cos\theta$$

$\not{t}$

QUE LO HAGA DIOS

TEOREMA DE
LA DIVERGENCIA

P2_3

18 June 2025 00:12

QUE VA A TOMAR EL PROFE: 

ECU DIF
- HOMOGENEA
- LINEALES
- EDTE
- SEPARABLES

4. Resolver:

$$(2xy - y + 2x)\,dx + (x^2 - x)\,dy = 0$$

5. Dada la ecuación: $y' + \text{sen}(x)y = 2x\,e^{\cos(x)}$, halla la solución para P: (0 ; e)

1) $\sqrt{y^2+2y+4}\;\text{Sen}\,x\,dx + (6y+6)\cos^2 x\,dy$

2) $y' = \dfrac{\sqrt{4-4y^2}\,(2x+1)}{\sqrt{4x^2+4x+1}}$ ∧ Solución Particular en $P_0 = (1,0)$

3) $3y(x^2-3x+2)\,dy - \sqrt{1-y^2}\,dx = 0$

4) $y\,dx + (2\sqrt{xy} - x)\,dy = 0$

5) $y' - y\,\text{Tag}(x) = \dfrac{1}{\cos x}$ ∧ Solución Particular en $P_0 = (0,0)$

6) $y' + \text{Sec}(x)\,y = 2\,e^{\cos x}$ ∧ Solución Particular en $P_0 = (0,e)$

7) $(2x^3+y)\,dx + (x+2y^2)\,dy = 0$

8) $(2xy - y + 2x)\,dx + (x^2 - x)\,dy = 0$

INTEGRALES DE LINEA (TEOREMA)

3. Hallar el volumen del solido limitado por la superficie $Z = x^2 + y^2$, siendo el dominio del solido el círculo de radio 2. Utilizar coordenadas polares (tener en cuenta $Z = 4$)

3) Calcular mediante una Integral Triple el Volumen del PRISMA limitado por los planos coordenados ($x=0$ ∧ $y=0$ ∧ $z=0$) (Primer Octante) y los planos

$3x + 2z = 12$ ∧ $y = 2$

4)

$$\int_C [P(x,y)\,dx + Q(x,y)\,dy]$$

Calcular la integral:

$$\int_C [4xy\,dx + (2x^2 - 3xy)\,dy]$$

Sobre la curva $C = C_1 \cup C_2$

Ecu recta: $\boxed{y - y_0 = \dots (x - x_0)}$

C_1 = Recta que une $\begin{cases} P_0 = (-3;-2) \\ P_1 = (1;0) \\ -3 \le x \le 1 \end{cases}$

$C_2 = \begin{cases} x^2 + y^2 = 1 \\ x \ge 0 \wedge y \ge 0 \text{ (Primer Cuadrante)} \end{cases}$

5)

Aplicando T.G.G calcular:

$$\oint_C \left[\frac{dx}{y} + \frac{dy}{x}\right] \text{ a lo largo de } C = C_1 \cup C_2 \cup C_3$$

$C_1 = \begin{cases} y = 1 \\ 1 \le x \le 4 \end{cases}$ | $C_2 = \begin{cases} x = 4 \\ 1 \le y \le 2 \end{cases}$ | $C_3 = \begin{cases} y = \sqrt{x} \\ 1 \le x \le 4 \end{cases}$

Sentido: de (1,1) a (4,1) a (4,2)

9) **Calcular**

$$\oint_c (3x^2y + 3x^2)dx + (x^3 - 2y)dy \;\; a\ lo\ largo\ de\ C = c_1 \cup c_2$$

$c_2 = \begin{cases} y = 2x+1 \\ 1 \le x \le 2 \end{cases}$; $c_1 = \begin{cases} y = x^2+1 \\ 1 \le x \le 2 \end{cases}$;

4) **Aplicando el Teorema de Gauss-Green. Calcular**

$$\oint_c \frac{1}{y}dx + \frac{1}{x}dy \;\; a\ lo\ largo\ de\ C = c_1 \cup c_2 \cup c_3$$

$c_1 = \begin{cases} y = 1 \\ 1 \le x \le 4 \end{cases}$; $c_2 = \begin{cases} x = 4 \\ 1 \le y \le 2 \end{cases}$; $c_1 = \begin{cases} y = \sqrt{x} \\ 1 \le x \le 4 \end{cases}$

INTEGRALES DE SUPERFICIE
- CAMPOS VECTORIALES — ABIERTO $\vec{F} \cdot \vec{ds}$
- CAMPOS ESCALARES $F \cdot ds$

Determine el trabajo efectuado por el ~~campo de fuerza F(x;y)=x² i~~ – x y j, cuando mueve una partícula a lo largo ~~del cu~~arto de circulo r(t)= Cos t i+ Sen t j, 0 ≤ t ≤π/2.

Calcular Integral de SUPERFICIE de $z = 4 - x^2 - y^2$

$\iint F(x,y) \cdot |ds|$

$\iint \vec{F}(x,y) \cdot \vec{ds}$

10. Calcular $\iint_s f(x;y;z)ds$, para:

a) $f(x;y;z) = 2x + 4/3\,y + z$, siendo s la parte del plano de ecuación $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ ubicada en el primer octante

b) donde s es la esfera unitaria $x^2 + y^2 + z^2 = 1$

Dado $F(x,y,z) = xz$ donde S es la porción del plano $x + 2y + 3z = 6$ en el primer Octante calcular la Integral de SUPERFICIE

Calcular FLUJO Φ a traves de la semiesfera de $r = 2$ con centro en el origen

1) $\dfrac{dy}{dx} - y^2 = -9$

$\dfrac{dy}{dx} = y^2 - 9$

$\dfrac{dy}{y^2 - 9} = dx$

$\displaystyle\int \frac{1}{y^2 - 9}\,dy = \int dx$

b) $\dfrac{dx}{dy} = 4(x^2 + 1)$

$$\frac{dx}{dy} = 4(x^2+1)$$

$$dx = 4(x^2+1)\,dy$$

$$\frac{dx}{(x^2+1)} = 4\,dy$$

$$\int \frac{1}{x^2+1}\,dx = \int 4\,dy \qquad P\left(\frac{\pi}{4};\ 1\right) \qquad \boxed{\operatorname{arctg}(x)+3{,}33 = 4y}$$

si $y\left(\frac{\pi}{4}\right) = 1$

$$\operatorname{arctg}(x)+C = 4y$$

$$\operatorname{arctg}\left(\frac{\pi}{4}\right)+C = 4\cdot 1 \Rightarrow 0{,}6657 + C = 4 \Rightarrow 3{,}33 = C$$

4. Resolver:

$$(2xy - y + 2x)\,dx + (x^2 - x)\,dy = 0$$

$$(\overbrace{2xy - y + 2x}^{P})\,dx + (\overbrace{x^2 - x}^{Q})\,dy = 0 \quad \begin{cases} \text{EDTE} \\ \text{HOMOGENEA} \end{cases}$$

COMPROBAR SIMETRIA

$$\boxed{P'_y = Q'_x}$$

$$P'_y = 2x - 1$$
$$=$$
$$Q'_x = 2x - 1$$

$$\mu(x,y) = \int 2xy - y + 2x\,dx \quad\Bigg|\quad \mu(x,y) = \int x^2 - x\,dy$$

$$= \cancel{2}y\frac{x^2}{\cancel{2}} - yx + \cancel{2}\frac{x^2}{\cancel{2}} + \alpha(y) \quad\Bigg|\quad = x^2y - xy + \beta(x)$$

"COMPARO"

$$\mu(x,y) = \boxed{x^2y - xy + x^2 = C}$$

5. Dada la ecuación: $y' + \overbrace{\operatorname{sen}(x)}^{P(x)}y = \overbrace{2x\,e^{\cos(x)}}^{Q(x)}$, **halla la solución para P: (0 ; e)**

$$y' + P(x)\,y = Q(x) \quad\Bigg|\quad y = u\cdot v \quad\Bigg|\quad u = e^{-\int P(x)dx}, \quad v = \int Q(x)\cdot e^{\int P(x)dx}\,dx$$

$$u = e^{-\int \operatorname{sen}(x)dx} = e^{\cos(x)}$$

Siempre con la "v"

$$v = \int 2x\,e^{\cos(x)}\cdot e^{\int \operatorname{sen}(x)dx}\,dx + C$$

$$= \int 2x\, e^{\cos(x)} \cdot e^{-\cos(x)} dx = \int 2x\, e^{0} = \int 2x\, dx = \cancel{2} \frac{x^2}{\cancel{2}} = x^2$$

$$y = e^{\cos(x)} \cdot [x^2 + C] \qquad P_0\ (\overset{x}{0} ; \overset{y}{e})$$

$$e = e^{1} \cdot [0 + C]$$

$$e = eC$$

$$\frac{e}{e} = C$$

$$1 = C$$

SP: $\boxed{y = e^{\cos(x)} \cdot [x^2 + 1]}$

$$y' + \text{sen}(x)(-y) = 3\,\text{sen}(x) \qquad P(\tfrac{\pi}{2} ; 0)$$

$$y' - y = 3\,\text{sen}(x) - \text{sen}(x)$$

$$y' - 1y = 2\,\text{sen}(x)$$

$$P(x) = -1$$

$$Q(x) = 2\,\text{sen}(x)$$

$$u = e^{-\int -1\, dx} = e^{x}$$

* ILPET

$$v = \int 2\,\text{sen}(x) \cdot e^{\int -1\, dx} dx = 2 \int \text{sen}(x) \cdot e^{-x} dx$$

$u = \text{sen}(x) \qquad dv = e^{-x} dx$

$du = \cos(x) dx \qquad v = e^{-x} \cdot -1 = -e^{-x}$

$$= \text{sen}(x) \cdot -e^{-x} - \int -e^{-x} \cos(x)\, dx$$

$$= \text{sen}(x) \cdot -e^{-x} + \int e^{-x} \cos(x)\, dx$$

$u = \cos(x) \qquad dv = e^{-x} dx$

$du = -\text{sen}(x) dx \qquad v = -e^{-x}$

$$= \text{sen}(x) \cdot -e^{-x} + \cos(x) \cdot (-e^{-x}) - \int e^{-x}\, \text{sen}(x)\, dx = \int e^{-x}\, \text{sen}(x)\, dx$$

$$= \text{sen}(x)(-e^{-x}) + \cos(x) \cdot (-e^{-x}) = \int e^{-x}\, \text{sen}(x)\, dx + \int e^{-x}\, \text{sen}(x)\, dx$$

$$= \text{sen}(x)(-e^{-x}) + \cos(x) \cdot (-e^{-x}) = 2 \int e^{-x}\, \text{sen}(x)\, dx$$

$$= 2\left[\frac{\operatorname{sen}(x)(-e^{-x}) + \cos(x)\cdot(-e^{-x})}{2}\right] = 2\left[\int e^{-x}\operatorname{sen}(x)\,dx\right]$$

$$= e^{-x}(-\operatorname{sen}(x) - \cos(x))$$

$$y = e^{x}\cdot\left[\, e^{-x}(-\operatorname{sen}(x) - \cos(x)) + C \,\right]$$

$$y = e^{x-x}\cdot(-\operatorname{sen}(x) - \cos(x)) + C\cdot e^{x} = \boxed{-\operatorname{sen}(x) - \cos(x) + C\cdot e^{x}} = y$$

$P(\frac{\pi}{2} ; 0)$

$$0 = -\operatorname{sen}(\tfrac{\pi}{2}) - \cos(\tfrac{\pi}{2}) + C\cdot e^{\frac{\pi}{2}}$$

$$0 = -1 - 0 + C\cdot 4{,}81$$

$$\frac{1}{4{,}81} \cong C$$

$$0{,}20 \cong C$$

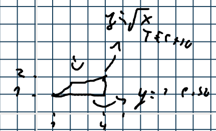
$$\boxed{-\operatorname{sen}(x) - \cos(x) + 0{,}20\cdot e^{x} = y}$$

4) Aplicando el Teorema de Gauss-Green. Calcular

$$\oint_c \frac{1}{y}dx + \frac{1}{x}dy \ \ a\ lo\ largo\ de\ C = c_1 \cup c_2 \cup c_3$$

$$c_1 = \begin{cases} y = 1 \\ 1 \le x \le 4 \end{cases} ; \quad c_2 = \begin{cases} x = 4 \\ 1 \le y \le 2 \end{cases} ; \quad c_1 = \begin{cases} y = \sqrt{x} \\ 1 \le x \le 4 \end{cases}$$

x	y
$\sqrt{1}$	1
$\sqrt{2}$	1,41
$\sqrt{4}$	2



$$\oint_c \frac{1}{y}dx + \frac{1}{x}dy \qquad C = C_1 \cup C_2 \cup C_3$$

$$\oint_c P\,dx + Q\,dy = \iint_D (Q'_x - P'_y)\,dx\,dy$$

$$Q'_x = -\frac{1}{x^2}$$

$$P'_y = -\frac{1}{y^2}$$

$$\int_1^4\left[\int_1^{\sqrt{x}} \left(-\frac{1}{x^2}\right) - \left[-\frac{1}{y^2}\right] dy\right]dx =$$

$$= \int_1^4\left[\int_1^{\sqrt{x}} -\frac{1}{x^2} + \frac{1}{y^2}\,dy\right]dx =$$

$$= \int_1^4\left[-\frac{y}{x^2} - y^{-1}\Big|_1^{\sqrt{x}}\right]dx$$

C.A

$$-\frac{\sqrt{x}}{x^2} - \frac{1}{\sqrt{x}} - \left[-\frac{1}{x^2} - 1\right] =$$

$$= -\frac{1}{\underbrace{x^2 \cdot x^{-\frac{1}{2}}}_{x^{3/2}}} - \frac{1}{\sqrt{x}} + \frac{1}{x^2} + 1 = -x^{-\frac{3}{2}} - x^{-\frac{1}{2}} + x^{-2} + 1$$

$$\int_1^4 -x^{-\frac{3}{2}} - x^{-\frac{1}{2}} + x^{-2} + 1 \, dx =$$

$$= +2x^{-\frac{1}{2}} - 2x^{\frac{1}{2}} - x^{-1} + x \Big|_1^4 =$$

$$= 2\left(4^{-\frac{1}{2}}\right) - 2(2) - \frac{1}{4} + 4 - \Big[\underbrace{\cancel{2-2-1+1}}_{0} \Big]$$

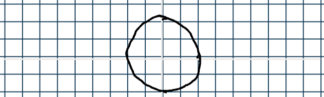
$$= 1 \cancel{-4} - \frac{1}{4} \cancel{+4} = \boxed{\frac{3}{4}}$$

2|

3. Hallar el volumen del solido limitado por la superficie $Z = x^2 + y^2$, siendo el dominio del solido el círculo de radio 2. Utilizar coordenadas polares (tener en cuenta $Z = 4$)

$\underset{(Z)}{f(x,y)} = x^2 + y^2 \;\Rightarrow\; 2^2 = 4 = x^2 + y^2$

RADIO 2



POLARES

$x = r \cos\theta$

$y = r \operatorname{sen}\theta$

$x^2 + y^2 = r^2$

$$4 \int_0^2 \int_0^{\frac{\pi}{2}} r^2 \cdot r \, d\theta \, dr =$$

$$= 4 \int_0^2 r^3 \theta \Big|_0^{\frac{\pi}{2}} dr$$

$$= 4 \int_0^2 r^3 \frac{\pi}{2} \, dr$$

$$= 2\pi \left[\frac{r^4}{4} \Big|_0^2 \right] = 2\pi \cdot \left[\frac{2^4}{4} \right] = 2\pi \cdot 4 = \boxed{8\pi}$$

$$4 \int_0^2 \int_0^{\frac{\pi}{2}} (r^2 - 4) \, r \, d\theta \, dr$$

$$4 \int_0^2 \int_0^{\frac{\pi}{2}} r^3 - 4r \, d\theta \, dr = 4 \int_0^2 r^3\theta - 4r\theta \Big|_0^{\frac{\pi}{2}} dr$$

$$4 \int_0^2 r^3 \frac{\pi}{2} - \overset{2}{\cancel{4}} r \frac{\pi}{\cancel{2}} \, dr = 4 \left[\frac{\pi}{2} \cdot \frac{r^4}{4} - 2\pi \frac{r^2}{\cancel{2}} \Big|_0^2 \right]$$

$$4\cdot\left[\pi\cdot\frac{2^4}{8}-\pi\cdot 4\right]=4\left[2\pi-4\pi\right]=-8\pi$$

$$V=\iint_D(\text{superior}-\text{inferior})\,dA$$

Me dio negativo porque le erre aca, es superior menos inferior(el superior el plano z=4)

3) Calcular mediante una Integral Triple el Volumen del PRISMA limitado por los planos coordenados ($x=0 \wedge y=0 \wedge z=0$) (Primer octante) y los planos

$3x+2z=12 \wedge y=2$

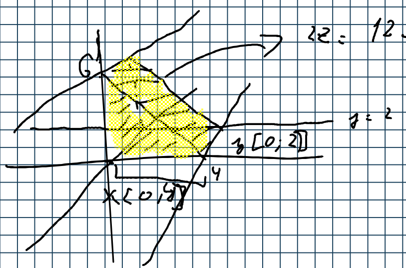
$3x+2z=12$

$x=0$

$z=\frac{12}{2}=6$ corta en $6=z$

$z=0$

$x=\frac{12}{3}=4$ Corta en la $4=x$



$2z=12-3x \Rightarrow z=\frac{12-3x}{2}$

$$V=\int_x\int_y\int_{z=f(x,y)}1\cdot dz\,dy\,dx=\int_0^4\int_0^2\int_0^{\frac{12-3x}{2}}1\cdot dz\,dy\,dx$$

$$=\int_0^4\int_0^2 z\Big|_0^{\frac{12-3x}{2}}dy\,dx=\int_0^4\int_0^2\frac{12-3x}{2}\,dy\,dx$$

$$=\int_0^4\frac{12-3x}{2}\cdot y\Big|_0^2\,dx=\int_0^4\frac{12-3x}{\cancel{2}}\cdot\cancel{2}\,dx=$$

$$=12x-\frac{3x^2}{2}\Big|_0^4=12\cdot 4-\frac{3}{2}\cdot 16=48-24=\boxed{24}$$

9) Calcular

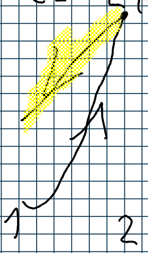
$$\oint_c(3x^2y+3x^2)dx+(x^3-2y)dy\ \ a\ lo\ largo\ de\ C=c_1\cup c_2$$

$$c_2=\begin{cases}y=2x+1\\1\le x\le 2\end{cases};\quad c_1=\begin{cases}y=x^2+1\\1\le x\le 2\end{cases};$$

$\oint_c (3x^2y+3x^2)\,dx + (x^3-2y)\,dy \rightarrow C = C_1 \cup C_2$

$C_2 = \begin{cases} y = 2x+1 \\ 1 \le x \le 2 \end{cases}$; $C_1 = \begin{cases} y = x^2+1 \\ 1 \le x \le 2 \end{cases}$

$C_2 = [(2;5):(1;3)]$



$C_1 : [(1;2):(2;5)]$

$\oint_c (3x^2y+3x^2)\,dx + (x^3-2y)\,dy$

$C_1 : \quad y = x^2+1 \Rightarrow \sqrt{y-1} = x$

$\int_1^2 3\underbrace{x^2(x^2+1)+3x^2}_{3x^4+3x^2}\,dx + \int_2^5 (\sqrt{y-1})^2\cdot(\sqrt{y-1}) - 2y\,dy$

1ra $\int$

$3\int_1^2 x^4 dx + 3\int x^2\,dx + 3\int x^2\,dx = \frac{3}{5}x^5 + x^3 + x^3 \Big|_1^2$

$= \left[\frac{3}{5}2^5 + 8 + 8\right] - \left[\frac{3}{5} + 1 + 1\right]$

$\frac{176}{5} - \frac{13}{5} = \boxed{\frac{163}{5}} \approx 32,6$

2da $\int$

$\int_2^5 (y-1)\cdot(\sqrt{y-1}) - 2y\,dy$

$\int_2^5 (y-1)\cdot(y-1)^{\frac{1}{2}}dy - 2\int_2^5 y\,dy$

$\left[\int_2^5 (y-1)^{\frac{3}{2}}\,dy\right] - \cancel{2}\frac{y^2}{\cancel{2}}$

$t = y-1$

$dt = dy$

$\left[\int_2^5 (t)^{\frac{3}{2}}\,dt\right] - y^2 \Big|_2^5$

$$\frac{2}{5}x^{\frac{5}{2}}\Big|_2^5 - y^2\Big|_2^5$$

$$= \frac{2}{5}(y-1)^{\frac{5}{2}}\Big|_2^5 - y^2\Big|_2^5 =$$

$$= \left[\frac{2}{5}(4)^{\frac{5}{2}} - \frac{2}{5}\right] - \left[25 - 4\right]$$

$$\frac{62}{5} - 21 = -\frac{43}{5}$$

$$C_1 = \underset{\text{1er término}}{\frac{163}{5}} - \underset{\text{2do término}}{\frac{43}{5}} = \frac{120}{5} = \boxed{24}$$

C2) $\oint_c (3x^2y+3x^2)\,dx + (x^3-2y)\,dy$ $\quad c_2 = \begin{cases} y = 2x+1 \\ 1 \le x \le 2 \end{cases}$ $\quad C_2 = [(2;5):(1;3)]$

$$\int_2^1 (3x^2(2x+1)+3x^2 \quad dx$$

$$\int_5^3 \left(\frac{y-1}{2}\right)^3 - 2y \; dy$$

$$y = 2x+1$$

$$x = \frac{y-1}{2}$$

1er lo x

C.A

$$(2x+1)\cdot(2x+1) = 4x^2+4x+1$$

$$\int_2^1 (3x^2(2x+1)+3(2x+1)^2\,dx$$

$$\int_2^1 (3x^2(2x+1)+3x^2 \quad dx$$

$$\int_2^1 6x^3+3x^2+3x^2\,dx = \frac{6}{4}x^4 + x^3 + \frac{\cancel{3}}{\cancel{3}}x^3\Big|_2^1$$

$$= \left[\left(\frac{6}{4}+1+1\right) - \left[\frac{6}{4}\cdot 16 + 8 + 8\right] = \frac{14}{4} - 40 = \boxed{-\frac{73}{2}}\right.$$

2do lo de y

$$\int_5^3 \left(\frac{y-1}{2}\right)^3 - 2y \, dy$$

$$\int_5^3 \left(\frac{y-1}{2}\right)^3 dy - 2\int y \, dy =$$

$$t = \frac{y-1}{2}$$

$$dt = \frac{(1)\cdot 2 - [(y-1)\cdot 0]}{(2)^2} = \frac{2}{4} = \frac{1}{2} dy$$

$$2dt = dy$$

$$2\int_5^3 t^3 \, dt - 2\int y \, dy = \cancel{2}\,\frac{t^4}{\cancel{4}\,2}\Big|_5^3 - \cancel{2}\,\frac{y^2}{\cancel{2}}\Big|_5^3 =$$

$$= \left[\left(\frac{y-1}{2}\right)^4\right]\frac{1}{2}\Big|_5^3 - y^2\Big|_5^3 =$$

$$\left[\left(\frac{1}{2} - 8\right) - \left(9 - 25\right)\right] = -\frac{15}{2} + 16 = \frac{17}{2}$$

$$c_2 = -\frac{73}{2} + \frac{17}{2} = \boxed{-28}$$

$$\boxed{c_1 + c_2 = 24 - 28 = \boxed{-4}}$$

1) $< \frac{x}{2} ; \frac{y}{3} ; \overline{4-2x-\frac{4}{3}y} >$

10. Calcular $\iint_s f(x;y;z)ds$, para:

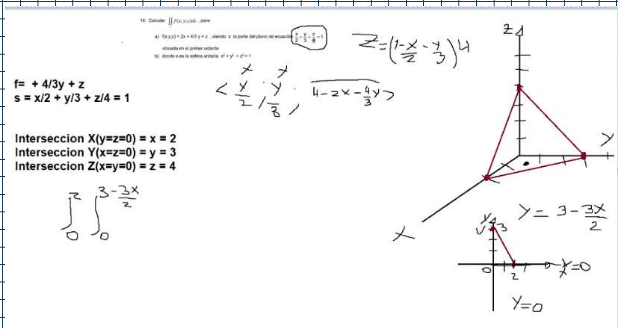
a) f(x;y;z) = 2x + 4/3 y + z , siendo s la parte del plano de ecuación $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$ ubicada en el primer octante

$$< x ; y ; 4-2x-\frac{4}{3}y > \quad \begin{vmatrix} \breve{i} & \breve{j} & \breve{k} \\ 1 & 0 & -2 \\ 0 & 1 & -\frac{4}{3} \end{vmatrix} dx\,dy =$$

$$= (0-(-2))\breve{i} - \left(-\frac{4}{3}-0\right)\breve{j} + (1-0)\breve{k} =$$

$$= 2\breve{i} + \frac{4}{3}\breve{j} + 1\breve{k} = d\vec{s}$$

3) $|d\vec{s}| = \sqrt{4+\left(\frac{4}{3}\right)^2+1} = \sqrt{\frac{61}{9}} = \frac{\sqrt{61}}{3}$



4) f(x;y;z) = 2x + 4/3 y + z Parametrizo

$$z = -2x - \frac{4}{3}y$$

$$F = 2x + \frac{4}{3}y \left(-2x - \frac{4}{3}y\right)$$

$$z = \left(1 - \frac{x}{2} - \frac{y}{3}\right)4 \Rightarrow z = 4 - 2x - \frac{2}{3}y$$

$$\int_0^2\int_0^{3-\frac{3}{2}x} \left[\cancel{2x} + \cancel{\frac{4}{3}y} + \left(4 - \cancel{2x} - \cancel{\frac{4}{3}y}\right)\right] \cdot \frac{\sqrt{61}}{3}\, dy\, dx =$$

$$\int_0^2\int_0^{3-\frac{3}{2}x} 4\frac{\sqrt{61}}{3}\, dy\, dx$$

$$\int_0^2 \left[4\frac{\sqrt{61}}{3}\cdot\left(3 - \frac{3}{2}x\right)\right] dx$$

$$\int_0^2 4\frac{\sqrt{61}}{\cancel{3}}\cancel{3} - \frac{\cancel{3}}{\cancel{2}}\,{}^{2}4\frac{\sqrt{61}}{\cancel{3}}\, x\, dx =$$

$$\left[4\sqrt{61}\cdot x \Big|_0^2 - \cancel{2}\sqrt{61}\,\frac{x^2}{\cancel{2}} \Big|_0^2 \right] =$$

$$4\sqrt{61}\cdot 2 - 4\sqrt{61}$$

$$= \left[8\sqrt{61} - 4\sqrt{61}\right] \cong \boxed{31,2}$$

Determine el trabajo efectuado por el campo de fuerza $F(x;y)=x^2\, i - x\, y\, j$, cuando mueve una partícula a lo largo del cuarto de circulo $r(t)=$ Cos t i+ Sen t j, $0 \leq t \leq \pi/2$.

$\vec{F}(x,y) = x^2\check{i} - xy\check{j}$ $\quad S = r(t) = \cos t\,\check{i} + \operatorname{sen} t\,\check{j} \quad 0 \leq t \leq \pi/2$

1) PARAMETRIZAR SUPERFICIE

$r(t) = \langle \cos t ; \operatorname{sen} t \rangle \Rightarrow x^2 + y^2 = 1$

$\hookrightarrow r(\theta) = \langle r\cos\theta ; r\operatorname{sen}\theta \rangle$

2do) $d\vec{s}$

$$\begin{vmatrix} \check{i} & \check{j} & \check{k} \\ \cos\theta & \operatorname{sen}\theta & 0 \\ -r\operatorname{sen}\theta & r\cos\theta & 0 \end{vmatrix} \quad dr\,d\theta = [0]\check{i} - [0]\check{j} + [r\cos^2\theta + r\operatorname{sen}^2\theta)]$$

$$= 0\check{i} + 0\check{j} + r(\overbrace{\cos^2\theta + \operatorname{sen}^2\theta}^{1})\check{k} = 0\check{i} + 0\check{j} + r\check{k}$$

$$= d\vec{s}$$

3er) NO MODULO

4to) PARAMETRIZO el CAMPO

$$\vec{F}(x,y) = x^2\check{i} - xy\check{j} \Rightarrow \langle (r\cos\theta)^2 ; -(r\cos\theta)\cdot(r\operatorname{sen}\theta) \rangle$$

$$\langle (r\cos\theta)^2 ; -r^2\cos\theta\cdot\operatorname{sen}\theta \rangle$$

$$\vec{F}\cdot d\vec{s} = \langle (r\cos\theta)^2 ; -r^2\cos\theta\cdot\operatorname{sen}\theta \rangle$$

NO LO VIMOS $\Rightarrow$ SUPERFICIE en $\mathbb{R}^2$ (4 CIRCULO)

NO es SUPERFICIE / NO TIENE $d\vec{s}$

① $\sqrt{y^2 + 2y + 4}\ \operatorname{Sen} x\, dx + (6y + 6)\cos^2 x \cdot dxy = 0$

$$\underbrace{\sqrt{y^2+2y+4}\ \operatorname{sen} x}_{P(x,y)}\, dx + \underbrace{(6y+6)\cos^2 x}_{Q(x,y)}\, dy = 0$$

EDTE

$P'_y = Q'_x \quad \Big| \quad P'_y = \operatorname{sen} x \cdot \dfrac{1}{2\sqrt{\quad}} \cdot 2y + 2$

EDTF

$P'_y = Q'_x$ | $P'_y = \text{sen}\,x \cdot \frac{1}{2\sqrt{y^2+2y+4}} \cdot 2y+2$

$Q'_x = \cos^2 x \cdot (6)$

NO SE CUMPLE SIMETRIA

HOMOGENEA BUSCO GRADO

$P(tx+ty) = \sqrt{t y^2 + 2ty + 4} \cdot \text{sen}(tx) =$ BARDO

SEPARABLES

$$\sqrt{y^2+2y+4}\,\text{sen}\,x\,dx + (6y+6)\cos^2 x\,dy = 0$$

$$\sqrt{y^2+2y+4}\,\text{sen}\,x\,dx = -(6y+6)\cos^2 x\,dy$$

$$\frac{\sqrt{y^2+2y+4}\,\text{sen}\,x\,dx}{\cos^2 x} = (6y+6)dy$$

$$\sqrt{y^2+2y+4} \cdot \text{sen}\,x \cdot \frac{1}{\cos^2 x}dx = -(6y+6)\,dy$$

$$\text{sen}\,x \cdot \frac{1}{\cos^2 x}dx = \frac{-(6y+6)}{\sqrt{y^2+2y+4}}\,dy$$

$$\int \text{sen}\,x \cdot \frac{1}{\cos^2 x}\,dx = -\int \frac{(6y+6)}{\sqrt{y^2+2y+4}}\,dy$$

$t = \cos x$

$dt = -\text{sen}\,x\,dx$

$\frac{dt}{-\text{sen}\,x} = dx$

$$\int \cancel{\text{sen}\,x} \cdot \frac{1}{t^2} \cdot \frac{dt}{-\cancel{\text{sen}\,x}} = -\int t^{-2}\,dt = -\left[\frac{t^{-1}}{-1}\right] = t^{-1} =$$

$$\Rightarrow \cos^{-1} x = \boxed{\frac{1}{\cos x}}$$

$$-\int \frac{(6y+6)}{\sqrt{y^2+2y+4}}\,dy$$

$$-\int \frac{(6y+6)}{\sqrt{y^2+2y+4}}\, dy$$

$t = y^2+2y+4$

$dt = 2y+2\, dx$

$$\frac{dt}{2y+2} \Rightarrow -\int \frac{6y+6}{\sqrt{t}} \cdot \frac{dt}{2y+2} = -\int \frac{6\cdot\cancel{(y+1)}}{\sqrt{t}\cdot 2\cancel{(y+1)}}\, dt = -3\int t^{-\frac{1}{2}}\, dt =$$

$$= -3\left[2t^{\frac{1}{2}}\right] \Rightarrow -\left[6\,(y^2+2y+4)^{\frac{1}{2}}\right]$$

$$\frac{1}{\cos(x)} + C = -6\sqrt{y^2+2y+4}$$

EJERCICIOS DEL PROFE

3) Calcular mediante una Integral Triple el Volumen del PRISMA limitado por los planos coordenados $(x=0 \wedge y=0 \wedge z=0)$ (Primer Octante) y los planos

$$3x + 2z = 12 \wedge y = 2$$

4)

$$\int_C [P(x,y)\,dx + Q(x,y)\,dy]$$

Calcular la integral:

$$\int_C [4xy\,dx + (2x^2 - 3xy)\,dy]$$

Sobre la curva $C = C_1 \cup C_2$

Ecu Recta $\boxed{y - y_0 = y_1 - y_0 (x - x_0)}$

C_1 = Recta que une $\begin{cases} P_0 = (-3; -2) \\ P_1 = (1; 0) \\ -3 \leq x \leq 1 \end{cases}$

$$C_2 = \begin{cases} x^2 + y^2 = 1 \\ x \geq 0 \wedge y \geq 0 \text{ (Primer Cuadrante)} \end{cases}$$

5)

Aplicando T.G.G calcular:

$$\oint_C \left[\frac{dx}{y} + \frac{dy}{x}\right] \text{ a lo largo de } C = C_1 \cup C_2 \cup C_3$$

$C_1 = \begin{cases} y = 1 \\ 1 \leq x \leq 4 \end{cases}$ $C_2 = \begin{cases} x = 4 \\ 1 \leq y \leq 2 \end{cases}$ $C_3 = \begin{cases} y = \sqrt{x} \\ 1 \leq x \leq 4 \end{cases}$

Sentido: de $(1,1)$ a $(4,1)$ a $(4,2)$

6) T.G.G

Aplicar T.G.G

$$\int_C (x + 2y)\,dx + y^2\,dy$$

$C = f(t) = \langle \cos t; \operatorname{sen} t \rangle \rightarrow$ Circu $r = 1$

$0 \leq t \leq 2\pi$

7) Teorema Fundamental de las INTEGRALES CURVILÍNEAS $(Q'_x = P'_y)$

$$\oint_{C^+} [P\,dx + Q\,dy] = \int_A^B du = U\Big|_A^B = U_B - U_A$$

Dado el campo vectorial

$$\vec{F}(x) = (2xy^3; 3x^2y^2)$$

Calcular

$$\int_C [2xy^3\,dx + 3x^2y^2\,dy]$$

Considerando que la curva C es

$y = x^2 + 1$ | $A: (0; 1)$ Pini:

Considerando que la curva C es

$y = x^2 + 1$ | $A: (0;1)$ P_{ini}
$0 \le x \le 2$ | $B = (2;5)$ P_{fin}

8) Integral Curvilínea sobre una curva en el espacio (LABIMA?)

TEOREMA FUNDAMENTAL en $\mathbb{R}^3$

$$\boxed{P'_y = Q'_x} \equiv \boxed{\text{rot } \vec{F} = 0}$$

$$\int_{C^+} [P\,dx + Q\,dy + R\,dz] = U_B - U_A$$

Dado el campo Vectorial

$$\vec{F}(\vec{x}) = \langle \overset{P}{2xy + z^3} ; \overset{Q}{x^2} ; \overset{R}{3xz^2} \rangle$$

$A = (1; -2; 1)$ y $B = (3; 1; 4)$

$U = \int 2xy + z^2\,dx$ | $U = \int x^2\,dy$ | $U = \int 3xz^2\,dz$

1) $\sqrt{y^2+2y+4}\ \text{Sen}\,x\,dx + (6y+6)\cos^2 x \cdot dxy$

2) $y' = \dfrac{\sqrt{4-4y^2}\cdot(2x+1)}{\sqrt{4x^2+4x+1}}$ ∧ Solución Particular en $P_0 = (1,0)$

3) $3y(x^2-3x+2)\,dy - \sqrt{1-y^2}\,dx = 0$

4) $y\,dx + (2\sqrt{xy} - x)\,dy = 0$

5) $y' - y\,\text{Tag}(x) = \dfrac{1}{\cos x}$ ∧ Solución Particular en $P_0 = (0,0)$

6) $y' + \text{Sen}(x)\cdot y = 2\cdot e^{\cos x}$ ∧ Solución Particular en $P_0 = (0, e)$

7) $(2x^3+y)\,dx + (x+2y^2)\,dy = 0$

8) $(2xy - y + 2x)\,dx + (x^2 - x)\,dy = 0$

Calcular el volumen de los siguientes cuerpos:

a) Cuerpo limitado por las superficies $z^2=x^2+y^2$, $2-z=x^2+y^2$, $z \ge 0$ (sugerencia: utilice coordenadas cilíndricas)

Determine el trabajo efectuado por el campo de fuerza $F(x;y)=x^2\,i - x\,y\,j$, cuando mueve una partícula a lo largo del cuarto de circulo $r(t)= \cos t\,i + \text{Sen}\,t\,j$, $0 \le t \le \pi/2$.

Calcular volumen de una esfera de radio 3

3. Hallar el volumen del solido limitado por la superficie $Z = x^2 + y^2$, siendo el dominio del solido el círculo de radio 2. Utilizar coordenadas polares (tener en cuenta $Z = 4$)

4. Resolver:

$$(2xy - y + 2x)\,dx + (x^2 - x)\,dy = 0$$

5. Dada la ecuación: $y' + \text{sen}(x)y = 2x\,e^{\cos(x)}$, halla la solución para P: (0 ; e)

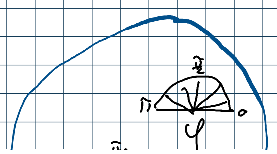
Calcular volumen de una esfera de radio 3

$$\boxed{3^2 = x^2 + y^2 + z^2}$$

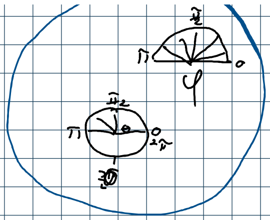
COORDENADAS ESFERICAS

$\rho = 3$

$$V = \int\int\int \rho^2 \,\text{sen}\,\varphi \; d\varphi \; d\theta \; d\rho$$



$$V = \int_\rho \int_\theta \int_\varphi \rho^2 \operatorname{sen}\varphi \, d\varphi \, d\theta \, d\rho$$



$$\boxed{V = 8\int_0^3 \int_0^{\frac{\pi}{2}} \left[\int_0^{\frac{\pi}{2}} \rho^2 \operatorname{sen}\varphi\right] d\varphi \, d\theta \, d\rho}$$

$$8\int_0^3 \int_0^{\frac{\pi}{2}} \left[-\rho^2 \cos\varphi \Big|_0^{\frac{\pi}{2}}\right] d\theta \, d\rho$$

C.A

$$\left[-\rho^2 \cos\left(\frac{\pi}{2}\right) - \left[-\rho^2 \cos(0)\right]\right] =$$

$$\rho^2 \cdot 0 \qquad -[-\rho^2 \cdot 1] = \rho^2$$

$$8\int_0^3 \left[\int_0^{\frac{\pi}{2}} \rho^2 \, d\theta\right] d\rho =$$

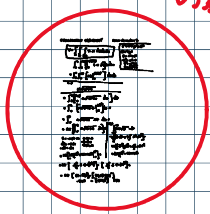
$$8\int_0^3 \left[\rho^2 \theta \Big|_0^{\frac{\pi}{2}}\right] d\rho =$$

$$8\left(\int_0^3 \rho^2 \frac{\pi}{2} \, d\rho\right) =$$

$$= \cancel{8}^{4} \frac{\pi}{\cancel{2}} \left[\frac{\rho^3}{3}\Big|_0^3\right] = 4\pi \left[\frac{\cancel{27}^{9}}{\cancel{3}}\right] = 4\pi \cdot 9 = \boxed{36\pi}$$

CILINDRICAS

NO CONVIENE



4)

$\int_C [P(x,y)\,dx + Q(x,y)\,dy]$

Calcular la integral:

$\int_C [4xy\,dx + (2x^2 - 3xy)\,dy]$

Sobre la curva $C = C_1 \cup C_2$

Ecu recta

$y - y_1 = y_1 - y_0 (x_1 - x_1)$

C_1 = Recta que une $\begin{cases} P_0 = (-3; -2) \\ P_1 = (1; 0) \\ -3 \le x \le 1 \end{cases}$

$C_2 = \begin{cases} x^2 + y^2 = 1 \\ x \ge 0 \wedge y \ge 0 \text{ (Primer Cuadrante)} \end{cases}$

C_1)

Ecu. RECTA 2 PUNTOS

$m = (y_2 - y_1)/(x_2 - x_1)$

$y - y_1 = m(x - x_1)$

$m = (-3 - 0)/(-3 - 1)$

$m = \frac{-2}{-4} = \frac{1}{2}$

$y - 0 = \frac{1}{2}(x - 1)$

$x \geq 0 \wedge y \geq 0$ (Primer Cuadrante)

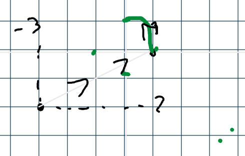
$C_1 = y = \frac{1}{2}x - \frac{1}{2} \quad (-3 \leq x \leq 1)$

$C_2 = x^2 + y^2 = 1 \quad (x \geq 0 \wedge y \geq 0)$

$y - 0 = \frac{1}{2}(x-1)$

$C_1 = y = \frac{1}{2}(x-1) = \boxed{y = \frac{1}{2}x - \frac{1}{2}}$

$(-3 \leq x \leq 1)$



$y = \frac{1}{2}(-3) - \frac{1}{2} = -2$

$y = \frac{1}{2} \cdot 1 - \frac{1}{2} = 0$

$y = \frac{1}{2}x - \frac{1}{2}$

$y + \frac{1}{2} = \frac{1}{2}x$

$\left(y + \frac{1}{2}\right) \cdot 2 = x$

$\boxed{2y + 1 = x}$

$$\int_C [4xy\,dx + (2x^2 - 3xy)\,dy]$$

$C_1 = \int_{-3}^{1} \left[4x\left(\frac{1}{2}x - \frac{1}{2}\right)dx + \left(2(2y+1)^2 - 3(2y+1)y\right]\right)dy$

$\int_{-3}^{1} [2x^2 - 2x]dx + \int_{-2}^{0} [8y^2 + 8y + 2 - 6y^2 - 3y]\,dy$

$\int_{-3}^{1} [2x^2 - 2x]dx + \int_{-2}^{0} 2y^2 + 5y + 2\,dy$

$\left[2\frac{x^3}{3} - 2\frac{x^2}{2}\Big|_{-3}^{1}\right] + \left[2\frac{y^3}{3} + \frac{5y^2}{2} + 2y\Big|_{-2}^{0}\right]$

C. A

$(2y+1)\cdot(2y+1)$

$4y^2 + (2y + 2y) + 1$

$4y^2 + 4y + 1$

C. A

1er Termino

$\left[\frac{2}{3} - 1\right] - \left[\frac{2}{3}(-3)^3 - (-3)^2\right] = -\frac{1}{3} - \left[\frac{-18 - 9}{-27}\right] = \frac{80}{3}$

2do Termino

$[0] - \left[\frac{2}{3}(-2)^3 + \frac{5}{2}(-2)^2 + 2\cdot(-2)\right]$

$$\left[\ 0\ \right] - \left[\frac{2}{3}(-2)^3 + \frac{5}{2}(-2)^2 + 2\cdot(-2)\right]$$

$$-\left[-\frac{16}{3} + 10 - 4\right] = -\frac{2}{3}$$

$$\left[\frac{80}{3} - \frac{2}{3}\right] = 26$$

POLARES

$$\begin{cases} x = r\cos\theta \\ y = r \operatorname{sen}\theta \\ x^2 + y^2 = r^2 \end{cases}$$

$x = \sqrt{1-y^2}$

$y = \sqrt{1-x^2}$

$C_2)\quad c_2 = x^2 + y^2 = 1 \quad (x \geq 0 \wedge y \geq 0)$

$$\int_C \left[4xy\,dx + (2x^2 - 3xy)\,dy\right]$$

$$\int_0^1 4x\sqrt{1-x^2}\,dx + \int_0^1 2(\sqrt{1-y^2})^2 - 3y(\sqrt{1-y^2})\,dy$$

1er TERMINO

$$4\int_0^1 x\sqrt{t^2}\,\frac{t\,dt}{-\cancel{x}} = -4\int_0^1 t^2\,dt = -4\left[\frac{t^3}{3}\Big|_1^0\right] =$$

$t^2 = 1 - x^2$

$2t\,dt = -2x\,dx$

$\frac{\cancel{2}t\,dt}{-2x} = dx$

$\frac{t\,dt}{-x} = dx$

$$= -4\left[\frac{(1-x^2)^3}{3}\Big|_1^0\right] =$$

$$= -4\left[\left[\frac{(1-0)^3}{3}\right] - 0\right]$$

$$-4\left[\frac{1}{3}\right] = \boxed{-\frac{4}{3}}$$

2do TERMINO

$$\int_0^1 2(\sqrt{1-y^2})^2 - 3y(\sqrt{1-y^2})\,dy$$

$$\int_0^1 2 - 2y^2 - 3y(\sqrt{1-y^2})\,dy$$

$$= \int_0^1 2\,dy - 2\int_0^1 y^2\,dy - 3\int y\sqrt{1-y^2}\,dy$$

$$= 2y\Big|_0^1 - 2\left[\frac{y^3}{3}\Big|_0^1\right] + 3\left[\int \cancel{y}\sqrt{t^2}\,\frac{t\,dt}{-\cancel{y}}\right]$$

t^2

$t^2 = 1 - y^2$

$2t\,dt = -2y\,dy$

$\frac{2t\,dt}{-2y} = dy$

$= [2] - 2\left[\frac{1}{3}\right] + 3\int\left[t^2\,dt\right] =$

$= 2 - \frac{2}{3} + 3\left[\frac{t^3}{3}\Big|_0^1\right] =$

$+ 3\left[\frac{(1-y^2)^3}{3}\Big|_0^1\right] =$

$+ \cancel{3}\left[0 - \left[\frac{1}{\cancel{3}}\right]\right] =$

$2 - \frac{2}{3} - 1 = \boxed{\frac{1}{3}}$

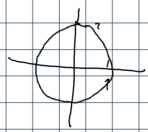
$-\frac{4}{3} + \frac{1}{3} = \boxed{-1}$

$C = C_1 + C_2 = 26 - 1 = \boxed{25}$

6) T.6.6

Aplicar T.6.6

$$\int_C (x+2y)\,dx + y^2\,dy$$



$C = f(t) = \langle \cos t\,;\ \operatorname{sen} t \rangle \rightarrow$ Circu $r=1$

$0 \leq t \leq 2\pi$

$$\int_C [P(x,y,z)\,dx + Q(x,y,z)\,dy] = \iint_D (Q'x - P'y)\,dy\,dx$$

$$\int_C \underbrace{(x+2y)}_{P}\,dx + \underbrace{y^2}_{Q}\,dy$$

$Q'x = 0$

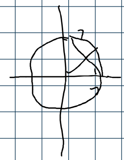
$P'y = 2$

POLARES

$x = r\cos\theta$

$y = r\operatorname{sen}\theta$

$r^2 = x^2 + y^2$



Si uso POLARES en GAUSS GREEN $\Rightarrow$ USO JACOBIANO

$$4\int_0^1\int_0^{\frac{\pi}{2}} -2r\,d\theta\,dr$$

$$-4\int_0^1 \cancel{2}r\,\frac{\pi}{\cancel{2}}\,dr$$

$$-4\pi\int_0^1 r\,dr = -4\cdot\left[\frac{1}{2}\right]\pi = \boxed{-2\pi}$$

"PARCIAL"

1)

A) (4) $y\,dx + (2\sqrt{xy} - x)\,dy = 0$

B) (5) $y' - y\,\operatorname{Tang}(x) = \dfrac{1}{\cos x}$ ∧ Solución Particular en $P_0 : (0,0)$

2) 7) Teorema Fundamental de las Integrales Curvilíneas $(Q'x = P'y)$

$\oint_{C^+} [P\,dx + Q\,dy] = \int_A^B du = U_A^B = U_B - U_A$

Dado el campo vectorial

$\vec{F}(x) = (2xy^3\,;\ 3x^2y^2)$

Calcular

$\int_C [2xy^3\,dx + 3x^2y^2\,dy]$

Considerando que la curva C es

$y = x^2 + 1$, $0 \leq x \leq 2$ | A: (0;1) P_{ini}, B: (2;5) P_{fin}

3) Calcular el V de una esfera de radio 4, con Integral Doble y Triple

4) a) $f(x;y;z) = 2x + 4/3\,y + z$,

donde s es la esfera unitaria $x^2 + y^2 + z^2 = 1$

1)

A) (4) $y\,dx + (2\sqrt{xy} - x)\,dy = 0$

1)

A) ④ $\underbrace{y}_{P}\,dx + (\underbrace{2\sqrt{xy}-x}_{Q})\,dy = 0$

$\boxed{P'_y = Q'_x}$

$P'_y = 1$

$\neq$

$Q'_x = \cancel{2}\,\frac{1}{2\sqrt{xy}}\cdot y - 1$

HOMOGENEA

$F(tx, ty)$

t^1 = GRADO 1

$ty\,dt =$

$2\sqrt{tx\,ty} - tx$

$2\sqrt{t^2}\cdot\sqrt{xy} - tx$

t^1 = GRADO 1

$2t\sqrt{xy} - tx \Rightarrow t\cdot 2\sqrt{xy} - x$

AMBAS DE GRADO 1 / HOMOGENEAS

$y\,dx + (2\sqrt{xy} - x)\,dy = 0$

$\frac{y}{x}\,dx + \frac{2\sqrt{xy} - x}{x}\,dy = 0$

$\frac{y}{x}\,dx + \frac{2\sqrt{xy}}{x} - \frac{\cancel{x}}{\cancel{x}}\,dy = 0$

$\frac{y}{x}\,dx + \frac{2\sqrt{xy}}{x} - 1\;dy = 0$

$v = \frac{y}{x} \Rightarrow v\,x = y$

$dy = (v\,dx + x\,dv)$

$v\;dx + \left(\frac{2\sqrt{xvx}}{x} - 1\right)(v\,dx + x\,dv) = 0$

$v\,dx + \left(\frac{2\sqrt{x^2}\sqrt{v}}{\cancel{x}} - 1\right)\cdot(v\,dx + x\,dv) = 0$

$v\,dx + (2\sqrt{v} - 1)\cdot(v\,dx + x\,dv) = 0$

$\cancel{v\,dx} + 2\sqrt{v}\,v\,dx + 2\sqrt{v}\,x\,dv - \cancel{v\,dx} - x\,dv = 0$

$2\sqrt{v}\,(v\,dx + x\,dv) = x\,dv$

$v\,dx + x\,dv = \frac{x\,dv}{2\sqrt{v}}$

$\frac{v\,dx + x\,dv}{x} = \frac{dv}{2\sqrt{v}}$

$\frac{v\,dx}{x} + \frac{\cancel{x}\,dv}{\cancel{x}} = \frac{dv}{2\sqrt{v}}$

$$\frac{v\,dx}{x} + \frac{\cancel{x}\,dv}{\cancel{x}} = \frac{dv}{2\sqrt{v}}$$

$$\frac{v\,dx}{x} = \frac{dv}{2\sqrt{v}} - 1dv$$

$$\frac{v\,dx}{x} = \left(\frac{1}{2\sqrt{v}} - 1\right)dv$$

$$\frac{dx}{x} = \frac{\left(\frac{1}{2\sqrt{v}} - 1\right)dv}{v}$$

$$\frac{1}{x}\,dx = \left(\frac{1}{2\sqrt{v}} - 1\right)\cdot\frac{1}{v}\,dv$$

$$\int\frac{1}{x}\,dx = \int\frac{1}{2\sqrt{v}\,v} - \frac{1}{v}\,dv$$

$$\ln|x| = \int\frac{1}{2\sqrt{v}\,v} - \int\frac{1}{v}\,dv$$

$$\ln|x| = \frac{1}{2}\int\frac{1}{(v)^{\frac{1}{2}}\cdot v^{1}} - \ln|v|$$

$$\ln|x| = \frac{1}{2}\int v^{-\frac{3}{2}}\,dv - \ln|v|$$

$$\ln|x| = \frac{1}{\cancel{2}}\cdot\left[-\cancel{2}v^{-\frac{1}{2}}\right] - \ln|v|$$

$$\ln|x| = -v^{-\frac{1}{2}} - \ln|v|$$

$$\boxed{\ln|x| + C = -\left(\frac{y}{x}\right)^{-\frac{1}{3}} - \ln\left|\frac{y}{x}\right|}$$

5) $y' - y\,\text{Tag}(x) = \frac{1}{\cos x}$ ∧ Solución Particular en $P_0 : (0,0)$

$P(x) = -\tan(x)$

$Q(x) = \frac{1}{\cos(x)}$

$$y = u.v \qquad u = e^{-\int P(x)\,dx} \qquad v = \int Q(x).\,e^{\int P(x)\,dx} \to + C$$

$$u = e^{+\int \tan(x)} = e^{\ln|\sec|} = \sec = \frac{1}{\cos(x)}$$

$$v = \int\frac{1}{\cos(x)}\cdot e^{-\int \tan(x)} = \int\frac{1}{\cos(x)}\cdot e^{-\ln|\sec|} =$$

$= \int \frac{1}{\cos(x)} \cdot \sec(x) = \int \frac{1}{\cos(x)} \cdot \frac{1}{\sec(x)} = \int \frac{1}{\cos(x)} \cdot \frac{1}{\frac{1}{\cos(x)}} = \int dx = x$

$y = \frac{1}{\cos(x)} \cdot |x + C|$ $\quad P(0;0)$

$0 = \frac{1}{\cos(0)} \cdot (0 + C) \Rightarrow 0 = 1 \cdot C \Rightarrow C = 0$

SP $P_0(0;0)$

$y = \frac{1}{\cos(x)} \cdot |x + 0|$

7) Teorema Fundamental de las Integrales Curvilíneas $(Q'_x = P'_y)$

$\oint_{C^+} [P\,dx + Q\,dy] = \int_A^B du = U_A^B = U_B - U_A$

Dado el campo vectorial

$\vec{F}(x) = (2xy^3; 3x^2y^2)$

Calcular

$\int_C [2xy^3\,dx + 3x^2y^2\,dy]$

Considerando que la curva C es

$y = x^2 + 1$, $0 \le x \le 2$ | A: $(0;1)$ P ini; B: $(2;5)$ P fin

$\int_C [\underbrace{2xy^2}_{P}\,dx + \underbrace{3x^2y^2}_{Q}\,dy]$

$\int_C [2xy^3\,dx + 3x^2y^2\,dy] = \int_A^B du = U_B - U_A$

$y = x^2 + 1$

$0 \le x \le 2$

$\vec{F}(x) = (2xy^2; 3x^2y^2)$

$\int 2xy^2\,dx = \cancel{2}y^3\frac{x^2}{\cancel{2}} = y^3x^2$

$U = x^2y^3$

$\int 3x^2y^2\,dy = \cancel{3}x^2\frac{y^3}{\cancel{3}} = x^2y^3$

$U_B - U_A = [2^2 \cdot 5^3] - [0^2 \cdot 1^3] = [4 \cdot 125] - 1 = 500 - 1 = \boxed{499}$

3) Calcular el V de una esfera de radio 4, con Integral Doble y Triple

ESFERICAS

$$4^2 = x^2 + y^2 + z^2.$$

ESFÉRICAS

ESFÉRICAS

$x = \rho . sen\varphi cos\theta$

$y = \rho . sen\varphi sen\theta$

$z = \rho . cos\varphi$

$\rho^2 = x^2 + y^2 + z^2$

INTEGRAL DOBLE

$$\underset{\varphi}{\int_0^{2\pi}} \underset{\theta}{\int_0^{2\pi}} 4^2 \cdot 4^2 \, sen\varphi \, d\theta \, d\varphi$$

$$\int_0^{2\pi} \left[256 \int_0^{2\pi} sen\varphi \, d\theta \right] d\varphi$$

$$\int_0^{2\pi} \left[256 \cdot \left[sen\varphi \, 2\pi \right] \right] d\varphi$$

$$\int_0^{2\pi} 512\pi \, sen\varphi \, d\varphi$$

$$512\pi \cdot \left[-cos\varphi \Big|_0^{2\pi} \right] =$$

$$= 512\pi \left[\left(-cos(2\pi) \right) - \left[-cos(0) \right] \right] = 512\pi \cdot \left[-1 - [-1] \right]$$

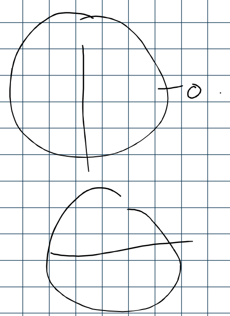
$$\underset{\rho}{\int} \underset{\theta}{\int} \underset{\varphi}{\int} 1 \, d\varphi \, d\theta \, d\rho$$

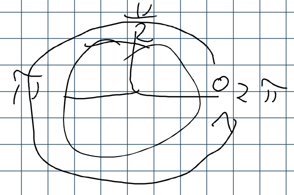
$$\int_0^4 \int_0^{2\pi} \int_0^{\pi} 1 \cdot \rho^2 sen\varphi \, d\varphi \, d\theta \, d\rho$$

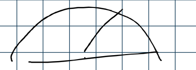
$$\int_0^4 \int_0^{2\pi} -\rho^2 \cdot cos\varphi \Big|_0^{\pi} d\theta \, d\rho$$

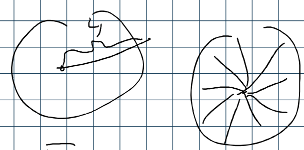
$$\int_0^4 \int_0^{2\pi} -\rho^2 \cdot 0 - \left[-\rho^2 \cdot 1 \right] d\theta \, d\rho$$

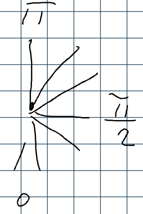
$$\int_0^4 \int_0^{2\pi} \rho^2 \, d\theta \, d\rho$$











$$\int_0 \int_0$$

$$\int_0^4 \rho^2 \cdot \theta \Big|_0^{2\pi} \, d\rho$$

$$2\pi \int_0^4 \rho^2 \, d\rho$$

$$2\pi \cdot \left[\frac{\rho^3}{3} \Big|_0^4 \right] = 2\pi = \frac{64}{3} \cdot 2\pi = \boxed{\frac{128}{3}\pi}$$

4)

a) f(x;y;z) = 2x + 4/3 y + z ,
donde s es la esfera unitaria $x^2 + y^2 + z^2 = 1$

$F(x;y;z) = 2x + \frac{4}{3} + z \rightarrow$ CAMPO ESCALAR

Superficie: $x^2 + y^2 + z^2 = 1$

ESFÉRICAS
$x = \rho.sen\varphi cos\theta$
$y = \rho.sen\varphi sen\theta$
$z = \rho.cos\varphi$
$\rho^2 = x^2 + y^2 + z^2$

$JACOBIANO = \rho^2 sen\varphi$

1^er PARAMETRIZO Sen

$\langle 1 \cdot sen\varphi cos\theta ; 1 sen\varphi sen\theta ; 1 \cdot cos\varphi \rangle$

2^do ds

$$\begin{array}{c} \\ \varphi \\ \theta \end{array} \begin{vmatrix} \check{i} & \check{j} & \check{k} \\ -cos\theta cos\varphi & -sen\theta cos\varphi & sen\varphi \\ -sen\varphi sen\theta & sen\varphi cos\theta & 0 \end{vmatrix} d\varphi \, d\theta$$

$= (0 - (sen^2\varphi \cos\theta))\check{i} - [0 + [sen^2\varphi \, sen\theta)\check{j} + [$

$+ [-cos^2\theta \, sen\varphi cos\varphi - (sen^2\theta \, sen\varphi cos\varphi)] \check{k}$

$= -sen^2\varphi \cos\theta\, \check{i} - sen^2\varphi\, sen\theta\, \check{j} - sen\varphi \cos\varphi\, \check{k}$

3] MODULO

* CUANDO HAGO el $|ds|$ ME CONVIENE PONER LOS NEGATIVOS COMO POSITIVOS

$$\sqrt{(-sen^2\varphi\cos\theta)^2 + (sen^2\varphi\, sen\theta)^2 + (-sen\varphi\cos\varphi)^2}$$

$$\sqrt{-sen^4\varphi\cos^2\theta \quad \cdot sen^4\varphi\, sen^2\theta \quad \cdot sen^2\varphi\cos^2\varphi}$$

$$\sqrt{+sen^4\varphi \cdot (\cos^2\theta + sen^2\theta) + sen^2\varphi\cos^2\varphi}$$

$$\sqrt{+sen^4\varphi + sen^2\varphi\cos^2\varphi}$$

$$\sqrt{+sen^2\varphi \cdot (\overbrace{sen^2\varphi + \cos^2\varphi}^{1})}$$

$$\sqrt{+sen^2\varphi} = sen\varphi = |ds|$$

4TO] PARAMETRIZO LA FUNCION $\langle 1 \cdot sen\varphi\cos\theta;\ 1 sen\varphi\, sen\theta;\ 1 \cdot \cos\varphi \rangle$

$$F(x;y;z) = 2x + \frac{4}{3}y + z$$

$$r = \left(2 sen\varphi\cos\theta + \frac{4}{3} sen\varphi\, sen\theta + \cos\varphi\right) \cdot sen\varphi$$

$$\int_{0}^{\pi}\int_{0}^{2\pi} 2 sen^2\varphi\cos\theta + \frac{4}{3} sen^2\varphi\, sen\theta + \cos\varphi\, sen\varphi \; d\theta\, d\varphi = 0$$

$\varphi \quad \theta$