PRACTICA FINAL

sábado, 6 de julio de 2024 19:07

a) $\int 2x^3 \cdot ln(x^2+1) \cdot dx =$

$$\int 2x^3 \cdot \ln(x^2+1)\, dx$$

$u = \ln(x^2+1)$ $\quad dV = 2x^3\, dx$

$du = \frac{1}{x^2+1} \cdot 2x$ $\quad V = \frac{2}{4} x^4$

$du = \frac{2x}{x^2+1} dx$

$$= \ln(x^2+1) \cdot \frac{2}{4} x^4 - \int \frac{2}{4} x^4 \cdot \frac{2x}{x^2+1}\, dx =$$

$$= \ln(x^2+1) \cdot \frac{2}{4} x^4 - \int \frac{x^5}{x^2+1}\, dx$$

C.A

$$\begin{array}{l|l} x^5+0x^4+0x^3+0x^2+0x+0 & x^2+1 \\ \hline -x^5 \quad +1x^3 & x^3-x \\ 0 \quad -x^3 & C \\ \quad -\,-x^3 \quad -x & \\ \quad\quad 0 \quad +x & \\ \quad\quad\quad R & \end{array}$$

$$= \ln(x^2+1) \cdot \frac{2}{4} x^4 - \int x^3 - x + \frac{x}{x^2+1} =$$

$$= \ln(x^2+1) \cdot \frac{2}{4} x^4 - \left[\int x^3 dx - \int x\, dx - \int \frac{x}{x^2+1}\, dx\right] =$$

C.A

$$\int x \cdot \frac{1}{x^2+1}\, dx$$

$t = x^2 \Rightarrow x = \sqrt{t}$

$dt = 2x\, dx$

$\frac{dt}{2x} = dx$

$$\int \cancel{\sqrt{t}} \cdot \frac{1}{t+1} \frac{dt}{2 \cdot \cancel{\sqrt{t}}} = \int \frac{1}{(t+1) \cdot 2}\, dt = \frac{1}{2} \int \frac{1}{t+1}\, dt =$$

$$= \frac{1}{2} \ln|t+1| = \frac{1}{2} \ln|x^2+1|$$

$$= \ln(x^2+1) \cdot \frac{2}{4} x^4 - \left[\frac{1}{4}x^4 - \frac{1}{2}x^2 - \frac{1}{2}\ln|x^2+1|\right] =$$

$$= \ln|x^2+1| \cdot \frac{2}{4} x^4 - \frac{1}{4}x^4 + \frac{1}{2}x^2 + \frac{1}{2}\ln|x^2+1| + C$$

$$\frac{2x^4 - 4x^2 - 4\ln|x^2+1|}{4 \cdot 2}$$

$F(x) = \frac{4x}{x^2+1}$

$F(x) = \frac{4x}{x^2+1}$

Dom $= \mathbb{R}$

ASINTOTAS

No hay A.V

A.H

$\lim_{x \to \infty} \frac{4x}{x^2+1} = \frac{\infty}{\infty} = \frac{4}{2x} = \frac{4}{\infty} = 0$ $\quad y=0$ es A.H

Cortes Ejes

EJE X

$\frac{4x}{x^2+1} = 0$

$4x = 0$

$x = 0$

$(0;0)$

EJE Y

$F(0) = \frac{4 \cdot 0}{0^2+1} = 0$

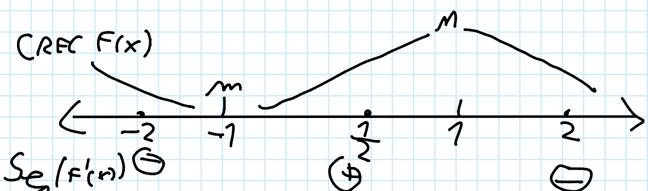
$(0;0)$

Crecimiento / Max / min

$F'(x) = \frac{4 \cdot (x^2+1) - [(4x) \cdot 2x]}{(x^2+1)^2} = \frac{4x^2+4-8x^2}{(x^2+1)^2} = \frac{-4x^2+4}{(x^2+1)^2}$

$\frac{4x}{x^2+1}$

$\frac{4x^2+4}{(x^2+1)^2} = 0 \Rightarrow 4x^2+4 = 0 \rightarrow x_1 = -1$
$\rightarrow x_2 = 1$



$F'(-2) = \frac{-}{+} = -$

$F'(\frac{1}{2})\ \frac{+}{+} = +$

$F'(2)\ \frac{-}{+} = \ominus$

minimo $= (-1; -2)$

Maximo $= (1; 2)$

CONCAVIDAD / P.I

$F'(x) = \frac{4x^2+4}{(x^2+1)^2}$

$F''(x) = \frac{(-8x) \cdot (x^2+1)^2 - [(-4x^2+4) \cdot (2(x^2+1) \cdot 2x)]}{(x^2+1)^4} =$

$= \frac{\cancel{(x^2+1)} \cdot (-8x \cdot (x^2+1) - [(-4x^2+4) \cdot 4x])}{(x^2+1)^{\cancel{4}}} =$

$8x^3 - 24x = 0$

$$= \frac{\cancel{(x^2+1)} \cdot \left(-8x \cdot (x^2+1) - [(-4x^2+4) \cdot 4x]\right)}{(x^2+1)^{\cancel{4}}} =$$

$$= \frac{-8x^3 - 8x \quad +16x^3 - 16x}{(x^2+1)^3} = \frac{8x^3 - 24x}{(x^2+1)^3}$$

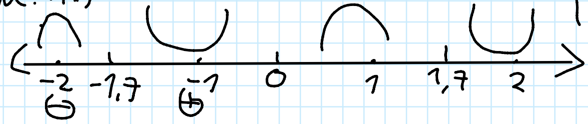
$8x^3 - 24x = 0$

$x_1 = 1,7$

$x_2 = 0$

$x_3 = -1,7$

CONC. F(x)



Sg F''(x)

$F''(-2) = \frac{-}{+} = \ominus$

$F''(-1) = \frac{+}{+} = +$

$F''(1) = \frac{-}{+} = \ominus$

$F''(2) = \frac{+}{+} = \oplus$

P.I (-1,7;

4) Determinar si es posible aplicar el Teorema de Lagrange en la función $f(x) = \frac{x^3}{3} - x$ en el intervalo $[-3;3]$. De ser posible encuentre el punto c que verifica la tesis **y luego GRAFIQUE** mostrando a través del gráfico la conclusión de dicho teorema, **encuentre las ecuaciones de las rectas intervinientes** que necesite para graficar correctamente.

$F(x) = \frac{x^3}{3} - x$ en $[-3;3]$

HIPOTESIS

F(x) continua en el intervalo

Dom = $\mathbb{R}$

F(x) es continua en el intervalo $[-3;3]$ porque su ——

$F'(x) = \frac{3x^2 \cdot 3 \boxed{- \cancel{x^2 \cdot 0}}}{9} = \frac{\cancel{9}x^3}{\cancel{9}} = \boxed{x^3 - 1}$

F'(x) es derivable en el intervalo $(\overset{a}{-3};\overset{b}{3})$

Por lo tanto se verifica la tesis:

$$\frac{F(b) - F(a)}{b - a} = F'(c)$$

C.A

$F(3) = \frac{3^3}{3} - 3 = \frac{27}{3} - 3 = 9 - 3 = \boxed{6}$

$F(-3) = -\frac{27}{3} - 3 = -9 - 3 = \boxed{-12}$

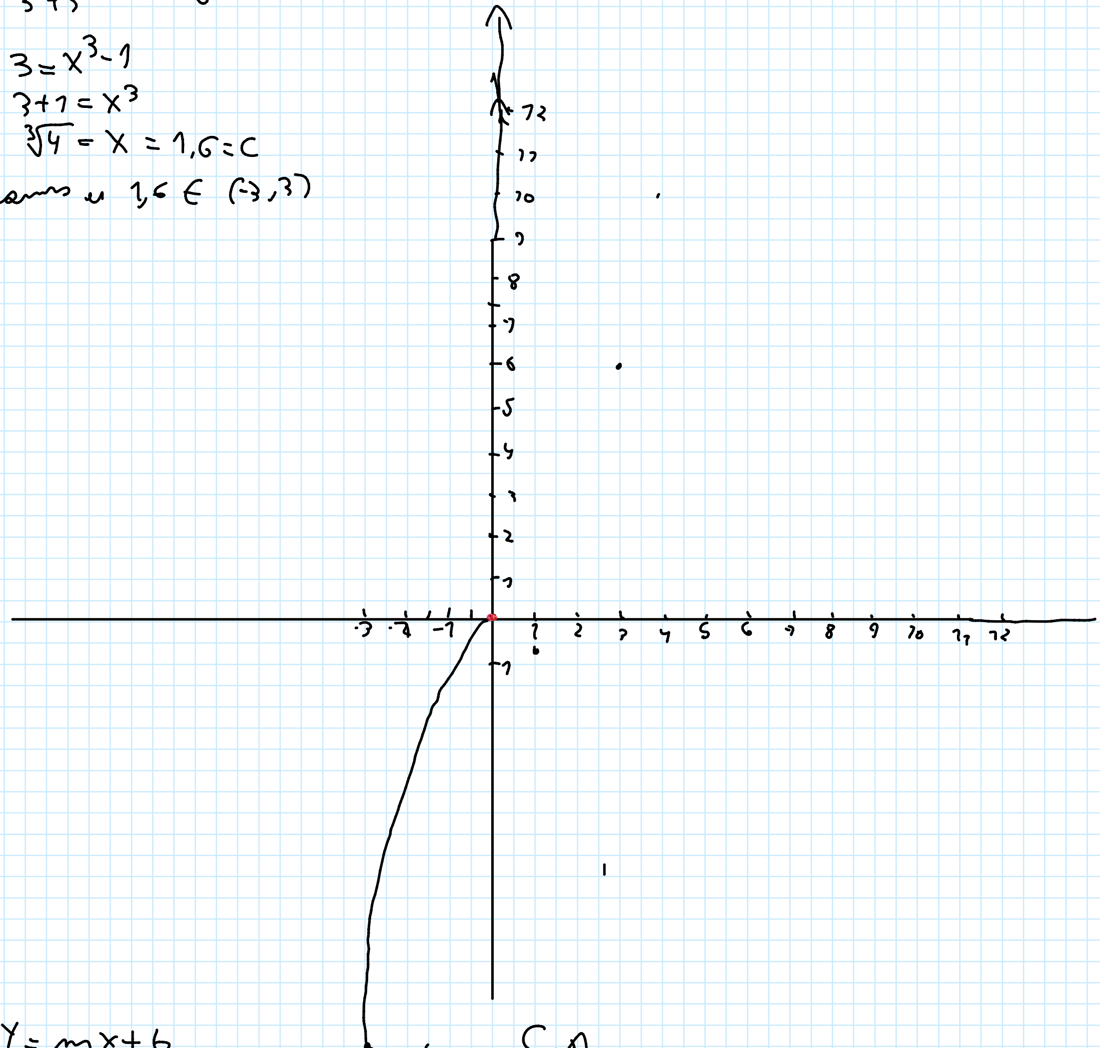
$\frac{6+12}{3+3} = \frac{18}{6} = \boxed{3} = m\ RT\ y\ RS = F'(c)$

$3 = x^3 - 1$

$3+1 = x^3$

$\sqrt[3]{4} = x = 1{,}6 = c$

Como c $1{,}6 \in (-3,3)$



$Y = mx + b$

$-0{,}26 = 3 \cdot 1{,}6 + b$

$-0{,}26 = 4{,}8 + b$

$-5 = b$

$y = 3x - 5$ Ec RT

C.A

$y = f(1{,}6) = \frac{(1{,}6)^3}{3} - 1{,}6 = \frac{4}{3} - 1{,}6 = -\frac{4}{15} \cong -0{,}26$

$F(3) = 6$

RS

$6 = 3 \cdot 3 + b$

$6 = 9 + b$

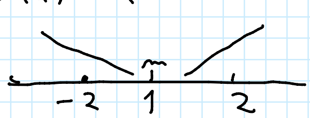
$6 - 9 = b$

$-3 = b$

$y = 3x - 3$ Ec RS

$\frac{x^3}{3} - x$

$F'(x) = x^3 - 1$

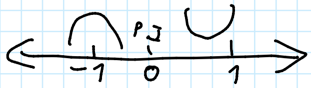


$[-3;3]$

X	Y
-3	-12
-1	0,66
1	-0,66
3	6
-2	-0,66
2	0,66

$(1; -0{,}66)$ ~

$F''(x) = 3x$



$(0;0)$

AREA

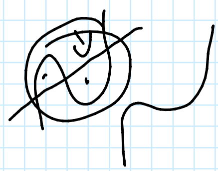
3) Hallar el área encerrada por las curvas $f(x)=x^3$ y $g(x)=3x+2$.

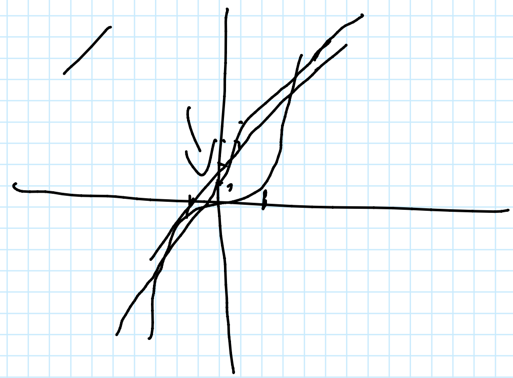
$F(x)=x^3$ $\qquad$ $g(x)=3x+2$

x^3

$F'(\cancel{x}) = 3x^2$

/ 0 /





$F''(x) = 6x$

∩ 0 ∪

$3x+2 = x^3$

$-x^3+3x+2=0 \rightarrow x_1=2$

$\rightarrow x_2=-1$

$\int_{-1}^{2}$

x^3 $\qquad$ $3x+2$

$f(1)$

1 $\qquad$ 5

$\frac{-1}{2}$

$$\int_{-1}^{2} 3x+2-x^3\,dx = -\int x^3 \cancel{dx} \int 3x\,dx + \int 2\,dx = -\frac{1}{4}x^4+\frac{3}{2}x^2+2x\Big]_{-1}^{2}$$

$$-\frac{1}{4}(2)^4+\frac{3}{2}(2)^2+4-\left[-\frac{1}{4}(-1)^4+\frac{3}{2}(-1)^2-2\right]=$$

$$=-4+6+4 \qquad +\frac{1}{4}-\frac{3}{2}+2=\frac{27}{4}u^2$$

$\ln\left(1+\frac{1}{x}\cdot \ln x\right)$

$$F'(x)=\frac{1}{1+\frac{1}{x}\cdot\ln x}\cdot\left(-\frac{1}{x^2}\cdot\frac{1}{x}+\frac{1}{x}\cdot\ln(x)\right)=$$

$$=\frac{1}{1+\frac{1}{x}\cdot\ln x}\cdot\left(-\frac{1}{x^3}+\frac{\ln(x)}{x}\right)=$$

$$=\frac{1}{1+\frac{\ln(x)}{x}}\cdot\left(-\frac{1}{x^3}+\frac{\ln(x)}{x}\right)=$$

$$=\left(\frac{1}{\left(1+\frac{\ln(x)}{x}\right)}\cdot-\frac{1}{x^3}\right)+\left(\frac{1}{\left(1+\frac{\ln(x)}{x}\right)}\cdot\frac{\ln(x)}{x}\right)=$$

$$=-\frac{1}{x^3+x^2\ln(x)}+\frac{1}{\frac{\ln(x)}{x}+\left(\frac{\ln(x)}{x}\right)^2}$$

$$F'(x)=\frac{1}{1+\frac{\ln(x)}{x}}\cdot\frac{\cancel{\frac{1}{x}}\cdot\cancel{x}-(\ln(x)\cdot 1)}{x^2}=$$

$$=\frac{1}{1+\frac{\ln(x)}{x}}-\frac{\ln(x)}{x^2}$$

$$=\frac{x^2-\left(1+\frac{\ln(x)}{x}\right)\cdot\ln(x)}{\left(1+\frac{\ln(x)}{x}\right)\cdot x^2}=$$

$$=\frac{x^2-\left(\ln(x)+\frac{\ln^2(x)}{x}\right)}{x^2+x\ln(x)}=$$

$$=\frac{x^2-\ln(x)-\frac{\ln^2(x)}{x}}{x\cdot(x+\ln(x))}$$

b) $f(x)=\ln\left(1+\frac{1}{x}\cdot\ln x\right)$

c) $f(x)=\operatorname{arctg}\left(\frac{x^2-1}{x^2+1}\right)$

d) $f(x)=\ln\left(\frac{1-\operatorname{sen}x}{1+\operatorname{sen}x}\right)$

$\operatorname{arctg}\left(\frac{x^2-1}{x^2+1}\right)$

$$F'(x)=\frac{1}{\left(\frac{x^2-1}{x^2+1}\right)^2+1}\cdot\frac{2x\cdot(x^2+1)-[(x^2-1)\cdot(2x)]}{(x^2+1)^2}=$$

$$=\frac{1}{\left(\frac{x^2-1}{x^2+1}\right)^2+1}\cdot\frac{2x^3+2x-[2x^3-2x]}{(x^2+1)^2}=$$

$$\frac{1}{\left(\frac{x^2-1}{x^2+1}\right)^2+1}\cdot\frac{\cancel{2x^3}+2x\cancel{-2x^3}+2x}{(x^2+1)^2}=$$

$$=\frac{1}{\left(\frac{x^2-1}{x^2+1}\right)^2+1}\cdot\frac{4x}{(x^2+1)^2}=$$

$$=\frac{4x}{\left(\frac{(x^2-1)^2}{(x^2+1)^2}+1\right)\cdot(x^2+1)^2}=\frac{4x}{\frac{(x^2-1)^2}{\cancel{(x^2+1)^2}}\cancel{(x^2+1)^2}+(x^2+1)^2}=$$

$$\boxed{(a+b)^2=a^2+2ab+b^2}$$

$$=\frac{4x}{(x^2+1)^2+(x^2-1)^2}=\frac{4x}{x^4\cancel{+2x^2}+1+x^4\cancel{-2x^2}+1}=\frac{4x}{2x^4+2}=\frac{\cancel{4}x}{\cancel{2}\cdot(x^4+1)}=\frac{2x}{x^4+1}$$

c) $f(x)=\frac{\ln(x)-1}{\ln^2(x)}$

$F(x)=\frac{\ln(x)-1}{\ldots}$

$$F(x)= \frac{\ln(x)-1}{\ln^2(x)}$$

$$F'(x) = \frac{\frac{1}{x}\cdot \ln^2(x) - \left[(\ln(x)-1)\cdot \frac{2\ln(x)}{x}\right]}{\ln^4(x)} =$$

$$= \frac{\frac{\ln^2(x)}{x} - \left[\frac{2\cdot\ln^2(x)}{x} - \frac{2\ln(x)}{x}\right]}{\ln^4(x)} =$$

$$= \frac{\frac{\ln^2(x)}{x} - \frac{2\cdot\ln^2(x)}{x} + \frac{2\ln(x)}{x}}{\ln^4(x)} =$$

$$= \frac{\cancel{\ln(x)}\cdot\left(\frac{\ln(x)}{x} - \frac{2\ln(x)}{x} + \frac{2}{x}\right)}{\ln^{\cancel{4}\,3}(x)} =$$

$$= \frac{\cancel{\frac{\ln(x)}{x}} - \cancel{\frac{\ln(x)}{x}} - \frac{\ln(x)}{x} + \frac{2}{x}}{\ln^3(x)} =$$

$$= \frac{2-\ln(x)}{x\cdot\ln^3(x)}$$

b) $f(x) = ln\sqrt{\frac{3-x^2}{3+x^2}}$ (llevar a su mínima expresión)

$$F'(x) = \frac{1}{\sqrt{\frac{3-x^2}{3+x^2}}} \cdot \frac{1}{2\cdot\sqrt{\frac{3-x^2}{3+x^2}}} \cdot \frac{(-2x\cdot(3+x^2)) - ((3-x^2)\cdot 2x)}{(3+x^2)^2} =$$

$$= \frac{1}{2\cdot\left(\sqrt{\frac{3-x^2}{3+x^2}}\right)^{\cancel{2}}} \cdot \frac{-6x-2x^3-[6x-2x^3]}{(3+x^2)^2} =$$

$$= \frac{1}{2\cdot\left(\frac{3-x^2}{3+x^2}\right)} \cdot \frac{-6x\cancel{-2x^3}-6x\cancel{+2x^3}}{(3+x^2)^2} =$$

$$= \frac{-\cancel{12}x}{\cancel{2}\cdot\frac{3-x^2}{\cancel{3+x^2}}\cdot\cancel{(3+x^2)}\cdot(3+x^2)} = \frac{-6x}{(3-x^2)\cdot(3+x^2)} =$$

$$= \frac{-6x}{3^2-(x^2)^2} = \boxed{\frac{-6x}{9-x^4}}$$

a) $\int \frac{3}{e^x-e^{2x}} . dx =$

$$\int \frac{3}{e^x-e^{2x}}\,dx = \int \frac{3}{e^x-(e^x)^2}\,dx$$

$t = e^x$

$dt = e^x\,dx$

$t = e^x$

$dt = e^x\, dx$

$\frac{dt}{e^x} = dx$

$= \int \frac{3}{t - t^2} \cdot \frac{dt}{t} =$

$= \int \frac{3}{-t^3 + t^2}\, dt$

C.A

$\int \frac{3}{-t^3 + t^2} = \int \frac{3}{t^2(t-1)} = \int \frac{A}{t^2} + \frac{B}{t} + \frac{C}{t-1} =$

$t^2 \cdot (-t+1)$

$\downarrow$

0 es raiz doble

$= \int \frac{A \cdot t (t-1) + B \cdot t^2 (t-1) + C \cdot t^2 \cdot t}{t^2 \cdot t \cdot (t-1)}$

si $x = 1$

$3 = \cancel{A \cdot 1 (1-1)} + \cancel{B \cdot 1 (1-1)} + C \cdot 1 \cdot 1$

$3 = C$

b) $\int x^2 \cdot sen\left(ln\, x^3\right) \cdot dx =$

$\int x^2 \cdot sen(ln\, x^3)\, dx =$

$u = x^2$ $\quad dv = sen(ln\, x^3)$

$du = 2x\, dx$ $\quad v =$ $u = sen(ln\, x^3)$ $\quad dv = dx$

$du = cos(ln\, x^3) \cdot \frac{1}{\cancel{x^3}} \cdot 3\cancel{x^2} = \frac{cos(ln\, x^3)}{x}$ $\quad v = x$

$sen(ln\, x^3) \cdot x - \int \frac{\cancel{x} \cdot cos(ln\, x^3)}{\cancel{x}} =$

$u = cos(ln\, x^3)$ $\quad dv = dx$

$du = -sen(ln\, x^3) \cdot \frac{1}{\cancel{x^3}} \cdot 3\cancel{x^2} = \frac{-sen(ln\, x^3)}{x}$ $\quad v = x$

$= sen(ln\, x^3) \cdot x - \left[cos(ln\, x^3) \cdot x - \int \frac{\cancel{x} \cdot -sen(ln\, x^3)}{\cancel{x}} \right] =$

$= sen(ln\, x^3) \cdot x - \left[cos(ln\, x^3) \cdot x + \int sen(ln\, x^3) \right] =$

$= sen(ln\, x^3) \cdot x - cos(ln\, x^3) \cdot x - \int sen(ln\, x^3)\, dx \quad \int sen(ln\, x^3)\, dx$

$= sen(ln\, x^3) \cdot x - cos(ln\, x^3) \cdot x = 2 \cdot \int sen(ln\, x^3)\, dx$

$= \frac{sen(ln\, x^3) \cdot x - cos(ln\, x^3) \cdot x}{2} = v$

$x^2 \cdot \frac{sen(ln\, x^3) \cdot x - cos(ln\, x^3) \cdot x}{2} - \int \frac{sen(ln\, x^3) \cdot x - cos(ln\, x^3) \cdot x}{\cancel{2}} \cdot \cancel{2}x\, dx =$

$= x^2 \cdot \frac{sen(ln\, x^3) \cdot x - cos(ln\, x^3) \cdot x}{2} - \int [sen(ln\, x^3) \cdot x - cos(ln\, x^3) \cdot x] \cdot x\, dx$.

$$= x^2 \cdot \frac{sen(\ln x^3) \cdot x - \cos(\ln x^3) \cdot x}{2} - \int sen(\ln x^3) \cdot x^2 - \cos(\ln x^3) \cdot x^2 \, dx$$

$t = sen(\ln x^3)$

$dt = \cos(\ln x^3) \cdot \frac{1}{x^3} \cdot 3x^2$

c) $\int 3t^2 . sen(2t) . dt$

$= 3 \cdot \int t^2 \cdot sen(t) \, dt =$

$u = t^2 \qquad dv = sen(t) \, dt$

$du = 2t \qquad v = -\cos(t)$

$= 3 \cdot [t^2 \cdot (-\cos(t)) - \int -\cos(t) \cdot 2t] =$

$= 3 \cdot [t^2 \cdot (-\cos(t)) + 2 \int \cos(t) \cdot t \, dt] =$

$u = t \qquad dv = \cos(t) \, dt$

$du = dt \qquad v = sen(t)$

$= 3 \cdot [-t^2 \cdot \cos(t) + 2 \cdot [t \cdot sen(t) - \int sen(t) \, dt]] =$

$= 3 \cdot [-t^2 \cdot \cos(t) + 2 \cdot [t \cdot sen(t) + \cos(t)]] =$

$= 3 \cdot [-t^2 \cdot \cos(t) + 2t \, sen(t) + 2\cos(t)] =$

$= -3t^2 \cos(t) + 6t \, sen(t) + 6\cos t + C$

$$f(x) = \frac{1+2\sqrt{x}}{1-2\sqrt{x}}$$

$$F'(x) = \frac{(1+2\sqrt{x})' \cdot (1-2\sqrt{x}) - (1+2\sqrt{x}) \cdot (1-2\sqrt{x})'}{(1-2\sqrt{x})^2} =$$

C.A

$(1+2\sqrt{x})' = \cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x}} = \frac{1}{\sqrt{x}}$

$(1-2\sqrt{x})' = -\cancel{2} \cdot \frac{1}{\cancel{2}\sqrt{x}} = -\frac{1}{\sqrt{x}}$

$$= \frac{\left(\frac{1}{\sqrt{x}}\right) \cdot (1-2\sqrt{x}) - \left((1+2\sqrt{x}) \cdot \left(-\frac{1}{\sqrt{x}}\right)\right)}{(1-2\sqrt{x})^2} =$$

$$= \frac{\frac{1}{\sqrt{x}} - \frac{2\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}} - \left[-\frac{1}{\sqrt{x}} - \frac{2\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}}\right]}{(1-2\sqrt{x})^2} =$$

$$= \frac{\frac{1}{\sqrt{x}} \cancel{-2} + \frac{1}{\sqrt{x}} \cancel{+2}}{(1-2\sqrt{x})^2} =$$

$$= \frac{2 \cdot \left(\frac{1}{\sqrt{x}}\right)}{(1-2\sqrt{x})^2} = \frac{2}{\sqrt{x}} \cdot \frac{1}{(1-2\sqrt{x})^2} = \frac{2}{\sqrt{x} \cdot (1-2\sqrt{x})^2}$$

5) Hallar el área encerrada por la función $f(x)=x^3-x^2$ y la recta que pasa por el origen del sistema cartesiano y que es paralela a la recta tangente a la función en el punto de abscisa igual a 1.

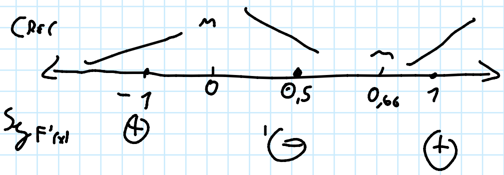
$F(x) = x^3 - x^2$ y Recta pasa por $(0;0)$ paralela a la RT en $x=1$

$x^3-x^2=\quad x_1=0 \quad x_2=1$

$F'(x)=3x^2-2x \longrightarrow F'(1)=3\cdot 1^2-2\cdot 1 = 3-2=1$

$3x^2-2x=0$

$x_1=0 \quad x_2=0{,}66$



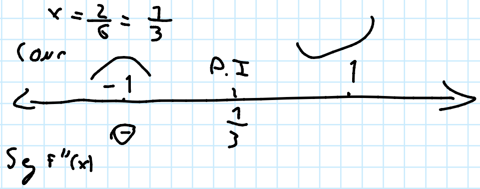
$(0,0)$ es M

$(0{,}66;-0{,}1)$ es m

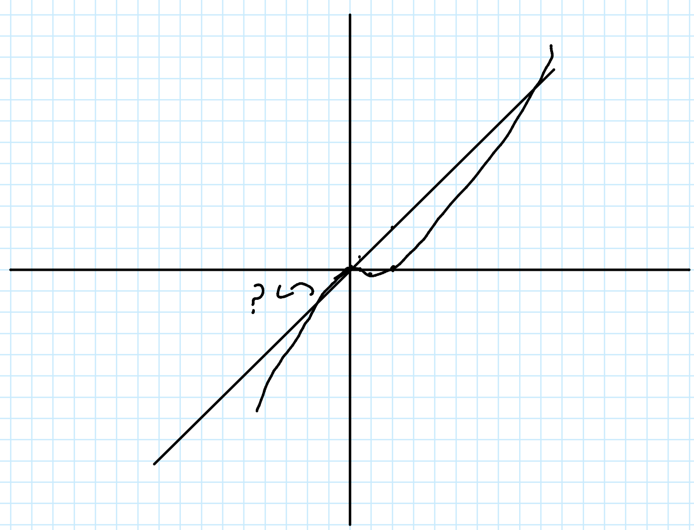
$F''(x)=6x-2$

$6x=2$

$x=\frac{2}{6}=\frac{1}{3}$



$(0{,}33;0)$ es P.I



$x^3-x^2=x$

$x^3-x^2-x \rightarrow x_1=1{,}6$

$\rightarrow x_2=0$

$\rightarrow x_3=-0{,}6$

2 AREAS → una desde $-0{,}6$ a 0 → X es techo

y otra desde 0 a $1{,}6$ → X es piso

1era $\int_{-0,6}^{0} X-[x^3-x^2] = \int_{-0,6}^{0} -x^3+x^2+x \, dx = -\frac{1}{4}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2 \Big]_{-0,6}^{0}$

$-\frac{1}{4}0^4+\frac{1}{3}0^3+\frac{1}{2}0^2-\left[-\frac{1}{4}(-0{,}6)^4+\frac{1}{3}(-0{,}6)^3+\frac{1}{2}(-0{,}6)^2\right] \cong 0{,}09\, u^2$

2da $\int_{0}^{1,6} x^3-x^2-x = \frac{1}{4}x^4-\frac{1}{3}x^3-\frac{1}{2}x^2 \Big]_{0}^{1,6}$:

$$= \frac{1}{4}(1{,}6)^4 - \frac{1}{3}(1{,}6)^3 - \frac{1}{2}(1{,}6)^2 - 0 \approx 1\,u^2$$

$$A^1 + A^2 = 1{,}07\,u^2$$

c) $f(x) = \frac{3 \cdot \ln(x)}{2 \cdot \sqrt{x}}$

$$F'(x) = \frac{(3\ln x)' \cdot (2\sqrt{x}) - (3\ln x)\cdot(2\sqrt{x})'}{(2\sqrt{x})^2}$$

$$= \frac{\frac{3}{x} \cdot 2\sqrt{x} - \left[3\ln x \cdot \frac{1}{\sqrt{x}}\right]}{(2\sqrt{x})^2} =$$

$$= \frac{\frac{6\sqrt{x}}{x} - \frac{3\ln x}{\sqrt{x}}}{(2\sqrt{x})^2} =$$

$$= \frac{\frac{6\sqrt{x}}{x} - \frac{3\ln x\sqrt{x}}{x}}{(2\sqrt{x})^2} =$$

$$= \frac{6\sqrt{x} - 3\ln x\sqrt{x}}{x}$$

$$\frac{3}{x} \cdot 2\sqrt{x} - \left[3\ln x \cdot \frac{2}{2\sqrt{x}}\right]$$

$$\frac{3}{x} \cdot 2\sqrt{x} - \left[3\ln x \cdot \frac{2\sqrt{x}}{2x}\right]$$

$$= \frac{2\sqrt{x} \cdot \left[\frac{3}{x} - \frac{3\ln x}{2x}\right]}{(2\sqrt{x})^2} =$$

$$= \frac{\frac{3 \cdot 2x - 3\ln x\, x}{2x^2}}{2\sqrt{x}} =$$

$$= \frac{6x - 3x\ln(x)}{2x^2} \cdot \frac{1}{2\sqrt{x}} =$$

$$= \frac{6x - 3x\ln(x)}{4x^2\sqrt{x}} =$$

$$= \frac{\cancel{x} \cdot (6 - 3\ln(x))}{4x^{\cancel{2}}\sqrt{x}} =$$

$$= \frac{6 - 3\ln(x)}{4x\sqrt{x}} = \frac{6 - 3\ln(x)}{4x^{\frac{3}{2}}}$$

b) $\int \frac{\ln(\ln x)}{x.\ln x} . dx$

$t = \ln x$

$dt = \frac{1}{x} dx$

$dt \cdot x = dx$

$$\int \frac{\ln(t)}{\cancel{x} \cdot t} \cdot dt \cdot \cancel{x} = \int \frac{\ln(t)}{t} dt = \int \ln(t) \cdot \frac{1}{t}\, dt =$$

$u = \ln(t)$ $\quad dv = \frac{1}{t} dt$

$du = \frac{1}{t} dt$ $\quad v = \ln(t)$

$$\ln(t) \cdot \ln(t) - \int \frac{\ln(t)}{t} dt = \int \frac{\ln(t)}{t} dt$$

$$\ln^2(t) = 2\int \frac{\ln(t)}{t} dt$$

$$\frac{\ln^2(t)}{2} = \int \frac{\ln(t)}{t} dt =$$

$$= \frac{\ln^2(\ln x)}{2} + C$$

6) Determinar si es posible aplicar el Teorema de Lagrange en la función $f(x) = -x^3 + x$ en el intervalo $[-2;1]$. De ser posible encuentre el punto c que verifica la tesis y luego grafique mostrando a través del gráfico la conclusión de dicho teorema, encuentre las ecuaciones de las rectas que sean necesarias para graficar correctamente.

Lagrange $F(x) = -X^3 + X$ en $[-2;1]$

Hips.

1° F(x) es continua en el intervalo $[-2;1]$ ya que ninguno de los valores pertenecientes al dominio se encuentran excluidos

2° $F'(x) = -3x^2 + 1$

F(x) es derivable en el intervalo $[-2;1]$ ya que sus valores se encuentran incluidos en el dominio de la derivada

Al cumplirse la hipótesis podemos verificar la siguiente tesis:

$$\frac{F(b) - F(a)}{b - a} = F'(c)$$

$-x^3 + x$

$[-2;1]$

$F(b) = -(1)^3 + 1 = 0$

$F(a) = -(-2)^3 - 2 = 8 - 2 = 6$

$$\frac{0 - 6}{1 + 2} = -\frac{6}{3} = -2 \qquad = m R_T \; y \; R_{sec}$$

$-2 = -3x^2 + 1$

$-2 - 1 = -3x^2$

$\frac{-3}{-3} = x^2$

$\sqrt{1} = X \rightarrow 1$

$\rightarrow -1 \rightarrow$ Es el perteneciente al intervalo $(-2;1)$

GRAFICO

$F(x) = -X^3 + x$

Rectas

$R_T - y = mx + b$

$0 = -2 \cdot -1 + b$

$0 = 2 + b$

$-2 = b$

$y = -2x - 2$

$F(-1) = -(-1)^3 - 1$

$= 1 - 1$

$= 0$

R_{sec}: $y = mx + b$

$0 = -2 \cdot 1 + b$

$0 = -2 + b$

$2 = b$

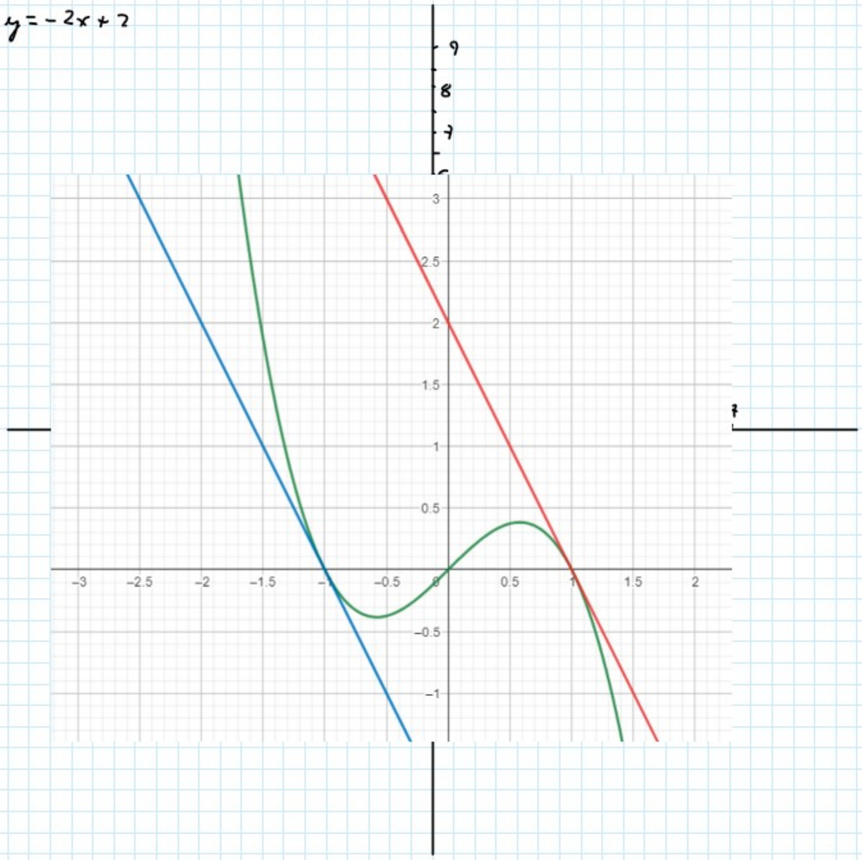
$-x^3 + x = -2x + 2$

$-x^3 + 3x - 2 = \rightarrow x_1 = -2$

$\rightarrow x_2 = 1$

X	y
-2	$-(-2)^3 - 2 = 6$
-1	
0	$0 + 0 = 0$
1	$-(-1)^3 - 1 = 0$

$y = -2x + 2$



d) $\int 16^x . \ln(4^x) . dx$

e) $\int \cos^3(x) . \cos(senx) . dx$

f) $\int arc.sen(x) . dx$

d) $\int 16^x \cdot \ln(4^x)$

$u = \ln(4^x)$ $\qquad$ $dv = 16^x \, dx$

$du = \frac{1}{4^x} \cdot 4^x \cdot \ln(4) = \ln(4)\, dx$ $\qquad$ $v = \frac{16^x}{\ln(16)}$

$= \ln(4^x) \cdot \frac{16^x}{\ln(16)} - \int \frac{16^x}{\ln(16)} \cdot \ln(4)\, dx =$

$= \ln(4^x) \cdot \frac{16^x}{\ln(16)} - \frac{\ln(4)}{\ln(16)} \cdot \int 16^x \, dx$

$= \ln(4^x) \cdot \frac{16^x}{\ln(16)} - \frac{\ln(4)}{\ln(16)} \cdot \frac{16^x}{\ln(16)} + C$

d) $\int 16^x . \ln(4^x) . dx$

e) $\int \cos^3(x) . \cos(senx) . dx$

f) $\int arc.sen(x) . dx$

$\int \cos^3(x) \cdot \cos(sen x)\, dx =$

$\int 1 - sen^2(x) \cdot \cos(x) \cdot \cos(sen(x))$

$t = sen(x)$

$dt = \cos(x)\, dx$

$sen^2(x) + \cos^2(x) = 1$

$\cos^2 = 1 - sen^2(x)$

$\cos^2 = 1 - sen^2(x)$

$t = sen(x)$

$dt = \cos(x)\, dx$

$\frac{dt}{\cos(x)} = dx$

$\int (1 - t^2) \cdot \cancel{\cos(x)} \cdot \cos(t) \cdot \frac{dt}{\cos(x)} =$

$= \int (1 - t^2) \cdot \cos(t) \quad dt$

$u = 1 - t^2 \qquad dv = \cos(t) \quad dt$

$du = -2t \; dt \qquad v = sen(t)$

$= (1 - t^2) \cdot sen(t) - \int sen(t) \cdot -2t \; dt =$

$= (1 - t^2) \cdot sen(t) + 2 \int sen(t) \cdot t \; dt$

$u = t \qquad dv = sen(t)$

$du = dt \qquad v = -\cos(t)$

$= (1 - t^2)\, sen(t) + 2 \cdot \left[t \cdot -\cos(t) + \int \cos(t)\, dt \right] =$

$= (1 - t)^2 \, sen(t) + 2 \cdot [t - \cos(t) + sen(t)] =$

$= (1 - sen(x))^2 \cdot sen(sen x) + 2 \cdot [sen(x) - \cos(sen(x)) + sen(sen(x))] + C$

F) $= \int \arcsen(x) \, dx$

a) $\int \frac{(e^x - 1)^2}{e^x + 2} \cdot dx$

$\int \frac{(e^x - 1)^2}{e^x + 2} \cdot dx$

$t = e^x$

$dt = e^x \, dx$

$\frac{dt}{e^x} = dx$

$= \int \frac{(t-1)^2}{t+2} \cdot \frac{dt}{t} = \int \frac{(t-1)^2}{t^2 + 2t} \, dt =$

$(t-1) \cdot (t-1)$

$t^2 - t - t + 1$

$= \int \frac{t^2 - 2t + 1}{t^2 + 2t} \quad dt$

C . A

$$\begin{array}{r|l} t^2 - 2t + 1 & t^2 + 2t \\ \hline -t^2 + 2t & 1 \\ \hline 0 \;\; -4t \;\; 1 & \end{array}$$

$C + \frac{R}{D}$

$= \int 1 + \frac{1}{t^2 + 2t} \, dt$

$= \int 1 \, dt + \int \frac{1}{t^2 + 2t} \quad dx$

$= t + \int \frac{1}{t^2 + 2t} \, dt$

C . A

$$\int \frac{1}{t^2+2t} = \frac{1}{x\cdot(x+2)} = \frac{A}{x} + \frac{B}{(x+2)} = \frac{A\cdot(x+2)+B\cdot x}{x\cdot(x+2)}$$

Si $x=-2$

$1 = \cancel{A\cdot(-2+2)} + B\cdot(-2)$

$-\frac{1}{2} = B$

Si $x=0$

$1 = A\cdot(0+2) + \cancel{B\,0}$

$1 = A\cdot 2$

$\frac{1}{2} = A$

$$= \int \frac{\frac{1}{2}}{t} + \left(\frac{-\frac{1}{2}}{(t+2)}\right) dt =$$

$$= \int \frac{1}{2t} - \frac{1}{2(t+2)}\, dt =$$

$$= \frac{1}{2}\int \frac{1}{t}\, dt - \frac{1}{2}\int \frac{1}{t+2}\, dt =$$

$$= \frac{1}{2}\ln|t| - \frac{1}{2}\ln|t+2| + C =$$

$$e^x + \frac{1}{2}\ln|e^x| - \frac{1}{2}\ln|e^x+2| + C$$

) Hallar el área encerrada por las siguientes funciones:

- $f(x) = x^2 - 3x$
- $g(x) = -x^2 + 2x - 2$

aficar dicha área, para esto es preciso analizar ambas funciones.

$F(x) = x^2 - 3x$

$g(x) = -x^2 + 2x - 2$

$-x^2 + 2x - 2 = x^2 - 3x$

$-2x^2 + 5x - 2 = 0 \Rightarrow x_1 = 0{,}5$, $x_2 = 2$

en $x = 1$

$F(1) = 1^2 - 3\cdot 1 = 1 - 3 = -2$

$g(1) = -1^2 + 2\cdot 1 - 2 = 1 + 2 - 2 = 1$

$g(x)$ es techo y $F(x)$ es piso

$$A = \int_{0,5}^{2} -2x^2 + 5x - 2\, dt = \frac{-2}{3}x^3 + \frac{5}{2}x^2 - 2x \Big|_{0,5}^{2}$$

$$= -\frac{2}{3}(2)^3 + \frac{5}{2}(2)^2 - 2\cdot 2 - \left[-\frac{2}{3}(0{,}5)^3 + \frac{5}{2}(0{,}5)^2 - 2\cdot(0{,}5)\right] =$$

$$= -\frac{16}{3} + 10 - 4 - \left[-\frac{1}{12} + \frac{5}{8} - 1\right] =$$

$$= -\frac{16}{3} + 10 - 4 + \frac{1}{12} - \frac{5}{8} + 1 = \frac{9}{8}\, u^2$$

$F(x) = x^2 - 3x$

$g(x) = -x^2 + 2x - 2$

$g(x) = -x^2 + 2x - 2$

f(1)
cocrd. orig. = G

$F(1) = -2 \quad V = (1;-2)$

$g(x) = -x^2 + 2x - 2$
Coord Orig $= -2$
$x_v = \frac{-b}{2A} = \frac{-2}{2 \cdot -1} = \frac{-2}{-2} = 1$

2) Estudiar si se cumplen las hipótesis del Teorema de Lagrange. De cumplirse hallar el valor de c y graficar para mostrar la interpretación geométrica. Incluir cálculo de las rectas y análisis de la función.

$$f(x) = -\frac{5}{2}x^2 - \frac{5}{2}x + 5 \; en \; [-3;1]$$

$f(x) - \frac{5}{2}x^2 - \frac{5}{2}x + 5$ en $[-3;1]$

Hipótesis
Es continua en en $[-3;1]$ por dominio polinomico
$F'(x) = -\frac{5 \cdot 2}{2}x - \frac{5}{2} = -5x - \frac{5}{2}$

Es ederivable en $(-3;1)$ por dominio
Tesis
Por lo tanto podemos verificar que:
$\frac{F(b) - F(a)}{b - a} = F'(c)$

$F(b) = -\frac{5}{2} - \frac{5}{2} + 5 = 0$

$F(a) = -\frac{5}{2} \cdot (-3)^2 - \frac{5}{2} \cdot (-3) + 5 = -10$

$\frac{0-(-10)}{1-(-3)} = \frac{10}{4} = \frac{5}{2} = m$ de R_T y R_S

$\frac{5}{2} = -5x - \frac{5}{2}$

$\frac{5}{2} + \frac{5}{2} = -5x$

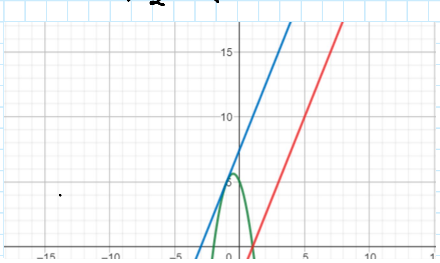
$\frac{5}{-5} = x$

$-1 = x = c$

Grafico
$R_T = y = mx + b$
$5 = \frac{5}{2}(-1) + b$
$5 = -\frac{5}{2} + b$
$5 + \frac{5}{2} = b$
$\frac{15}{2} = b$

$y = \frac{5}{2}x + \frac{15}{2}$

$y = F(-1) = -\frac{5}{2} + \frac{5}{2} + 5 = 5$



$y = \frac{5}{2}x + \frac{15}{2}$

$R_{sec}: \; y = mx + b$

$0 = \frac{5}{2} \cdot 1 + b$

$-\frac{5}{2} = b$

$y = \frac{5}{2}x - \frac{5}{2}$

) Hallar los intervalos de concavidad de la función

$$f(x) = \frac{2}{x^3 + 2x^2}$$

$F(x) = \frac{2}{x^3 + 2x^2}$

$x^3 + 2x^2 \to x = 0$, $\to x = -2$ } Están afuera del dominio

Dom = $\mathbb{R} - \{0, -2\}$

Asíntotas

A.V

$\lim_{x \to 0} \frac{2}{x^3 + 2x^2} = \frac{2}{\to 0} = \infty$

$\lim_{x \to -2} \frac{2}{(-2)^3 + 2 \cdot (-2)^2} = \frac{2}{-8 + 2 \cdot 4} = \frac{2}{\to 0} = \infty$

$x = 0$ y $x = -2$ son A.V

A.H

$\lim_{x \to \infty} \frac{2}{x^3 + 2x^2} = \frac{2}{\infty} = 0$

$y = 0$ es A.H

Cortes ejes:

eje X

$\frac{2}{x^3 + 2x^2} = 0$

$2 = 0 \cdot x^3 + 2x^3$

$2 = 0$ ABSURDO $\Rightarrow$ No HAY

eje Y

$F(0) = \frac{2}{x^3 + 2x^2} =$ No hay

$F'(x) = \frac{\cancel{0 \cdot (x^3 + 2x^2)} - [2) \cdot (3x^2 + 4x)}{(x^3 + 2x^2)^2} =$

$= \frac{-6x^2 - 8x}{(x^3 + 2x^2)^2}$

$F''(x) = \frac{(-6x^2 - 8x)' \cdot (x^3 + 2x^2)^2 - [(-6x^2 - 8x) \cdot ((x^3 + 2x^2)^2)']}{((x^3 + 2x^2)^2)^2}$

$= \frac{(-12x - 8) \cdot (x^3 + 2x^2)^2 - [(-6x^2 - 8x) \cdot 2(x^3 + 2x^2) \cdot (3x^2 + 4x)]}{((x^3 + 2x^2)^2)^2}$

C. A

$(-6x^2 - 8x)' =$

$= -6 \cdot 2x - 8$

$\boxed{-12x - 8}$

$2(x^3 + 2x^2) \cdot (3x^2 + 4x)$

$$((x^3+2x^2)^2)^2$$

$$= \frac{(-12x-8)\cdot(x^3+2x^2)^2 - [(-6x^2-8x)\cdot 2(x^3+2x^2)\cdot(3x^2+4x)]}{((x^3+2x^2)^2)^2}$$

$$= \frac{\cancel{(x^3+2x^2)}\cdot[(-12x-8)\cdot(x^3+2x^2) - [(-6x^2-8x)\cdot(6x^2+8x)]]}{(x^3+2x^2)^{4\,3}}$$

$$= \frac{-12x^4-24x^3-8x^3-16x^2 \quad +36x^4+48x^3+48x^3+64x^2}{(x^3+2x^2)^3} =$$

$$= \frac{24x^4+72x^3+40x^2}{(x^3+2x^2)^3} =$$

$$24x^4+72x^3+40x^2=0$$

$$x\cdot(24x^3+72x^2+40x)$$

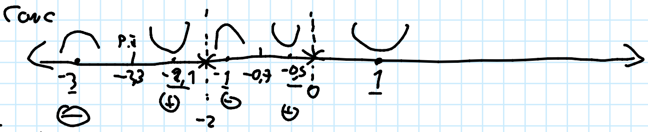
$x_1=0$

$\to x_2=0$

$\to x_?=-2,3$

$\to x_3=-0,7$

Conc



Sg $F''(x)$

$F''(-3) = \frac{+}{-} = -$ $\qquad$ $F''(-2,1) = \frac{-}{-} = +$ $\qquad$ $F''(-1) = \frac{-}{+} = -$

$F''(-0,5) = \frac{+}{+} = +$ $\qquad$ $F''(1) = \frac{+}{+} = +$

La f(x) es concava negativa de $(-\infty;-2,3) \cup (-2;-0,7)$

y es concava positiva de $(-2,3;-2) \cup (-0,7;0) \cup (0;+\infty)$