# STUYVESANT

# Math Survey

THE SAD BALLAD OF THE JEALOUS CONES

There once were two cones closely plighted
In brotherly love and in bliss
Upon the same axis united
Their vertices joined in a kiss
And then a smooth plane came a flying
(Its smoothness was chief of its charms)
And quickly, without even trying
Siezed one of the cones in its arms.

An element chose at its pleasure-
Then parallel took up its place
While joy settled down without measure
Upon the parabola's face.
The vertical cone found it tragic
For never the gracious smooth plane
Could cut it-not even by magic-
It suffered in agonized pain.

It urged and cajoled without ceasing
And encouraged the plane soon to come
With coaxing and begging and teasing
It promised all comforts of home.
The plane which enjoyed being flattered
Listened and heeded, entranced;
A hyperbola now only mattered
For beauty would thus be enhanced.

Serenely, with great satisfaction,
The plane intersected each cone,
But these were annoyed by its action,
Each jealously deemed it its own.
Each jabbed with its point at its brother
And tore at the unhappy plane
On their axis they turned round each other
And struggled with might and with main.

The plane cried,"Alas cease your railings
Your turning me round and round
And though I despise finite failings
I'll become an ellipse, I'll be bound."
The cones, tho' their struggle was needless
Would cease not, not list to its call;
And they pulled at each other, still heedless
Of how the poor conic grew small.

As the angry cones still madly battled
A terrible shriek rent the air;
The plane thru the vertex had rattled
The section had gone now for fair.
Two surfaces battered and rumpled
Forever and ever apart
A crushed little point badly crumpled-
The tragedy's end breaks my heart!

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## DEDICATION

This issue of the "Math Survey" is dedicated to Mr. Simon L. Berman, chairman of the Mathematics Department of Stuyvesant High School, whose generous and timely advice made our task a much lighter one.

Grateful acknowledgments are also extended to Walter Littman, last term's editor-in-chief, and Herbert Kamowitz, whose valuable assistance aided greatly in the production of this magazine.

# BOOLEAN ALGEBRA

by *Harold Widom*


Thus far in your short mathematical careers, you have undoubtedly come in contact with the term "algebra" any number of times, and each time your mind must have followed this pattern of thought. "Algebra? Isn't that the so'n so subject in which you let x equal whatever you don't know, and then you get all sorts of messy equations and blah, blah.." Well, in a way you are right, however this term "algebra" is not so confined in definition. You, so far, are acquainted with one phase of algebra known as the algebra of numbers. However, believe it or not, there exists another type of algebra known as the algebra of sets or Boolean algebra.

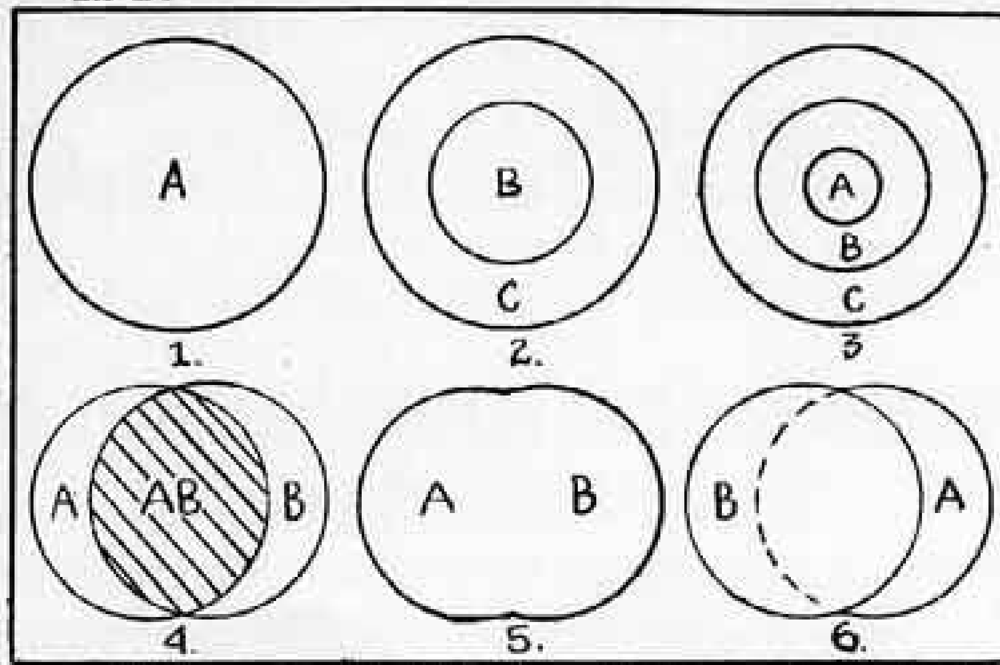
In order to comprehend the power of this field of mathematics we must, as in all other fields, start off with a series of basic definitions. First of all we must all know what is meant by a "set". The latter is defined as a group of objects each having a common property. For example, all the even intigers constitute a set, as they are all multiples of two. A set, A for example, is called a "subset" of another set, which we'll call B, if every element of A is also an element of B. In this manner, the set of the multiples of four is a subset of the even integers. The fact that A is a subset of B is symbolized thus, $A \subset B$ or $B \supset A$.

Now consider the case when $A \subset B$ and $B \subset A$. this means that every element in A is also in B and every element in B is also in A. It is quite obvious that the only time this is possible is when the two sets are identical, is is symbolized as $A = B$.

Another very important rule in Boolean algebra is an analogy from one of Euclid's postulates. It states that if A is a subset of B, and B is a subset of C, then A is a subset of C. Or, if $A \subset B$, and $B \subset C$, then $A \subset C$. It is also important to consider the set that has no elements in it at all. For example, the set of all the people in Australia who have over six legs contains, of course, no elements. This set is called an "empty" or "null" set and is denoted by 0.

In order to simplify your comprehension of Boolean algebra, I have prepared a set of diagrams. Each one is numbered, and each number refers to a basic rule listed below.

1. There exists a set A.
2. $B \subset C$
3. $A \subset B$; $B \subset C$; then $A \subset C$
4. AB, or all the elements which are common to both A and B. For example, the odd integers are common to both the sets of numbers and the set of primes (shaded area in the diagram).
5. A+B, or the set of elements which are common to either A or B.
6. A-B, or the set of all elements in A but not in B.




Next, I would like to list a few facts which are merely corollaries of our basic laws, and I'm sure their reasons are very easily seen

1. If $A \subset B$, then $AB = A$
2. If $A \subset B$, then $A + B = B$
3. $A + 0 = A$
4. $A0 = 0$
5. If $AB = 0$, they have no elements in common.
6. $A - 0 = A$
7. $A + B = B + A$

These rules form the foundation of one of the most powerful tools of mathematical logic known to modern man. For Boolean algebra, as you shall soon see, enables us to break down complicated verbal statements into extremely simple and easily manipulated algebraic forms.

Let us take for example a very simple verbal problem and attempt to solve it using Boolean algebra.

Given: 1) All freshmen are college students.
2) No college students are maniacs.

To Prove: No freshmen are maniacs.

Proof: Let A represent the set of the college students, let B represent the set of all maniacs, and let C represent the set of all freshmen. Now, from hypothesis (2) we know that $BA = 0$, or that the set of all maniacs and the set of college students have no elements in common. Hypothesis(1) tells us that $C \subset A$, or that the set of freshmen is a subset of the set of college students. However, we saw before that if $C \subset A$, then $AC = C$ (rule 1). Now multiply both sides of the latter by B. Then $BAC = BC$, and referring to hypothesis (2) we find that $0C = BC$. However, since $0C = 0$ (**rule** 4), we substitute and find that $BC = 0$, or in other words, no freshmen are maniacs.

The only thing that one not well acquainted with Boolean algebra must remember, is that the arithmetical signs "+", "-", and "x" have different meanings than those in ordinary algebra. Otherwise the ease of the solution of problems varies directly with the amount of practice. So remember, if at first you don't succeed, try and try again.

# QUIZ KIDS

by *Herbert Kirshman*


In the past, the articles printed in this magazine have dealt mainly with the technical side of mathematics, you know, squaring the circle, circling the square and what not. However there was never one more enticing, more amazing, or harder to believe than the one I shall now pen.

The facts I will attempt to place before you deal not with a phase of mathematics, nor with the lives of great mathematicians. Instead, they deal with the lives of a group of triple-A-number-one morons. Most of them were social outcasts, and you'll soon see why. Practically all of them can be classified psycologically as idiots, and some of them couldn't even pass the Stuyvesant test, yet in a matter of seconds they, without the aid of papyrus and writing implements, could calculate the cube roots of ten digit numbers or square fifteen digit numbers with no trouble at all. Yes, these men were manipulative geniuses, or, as they are sometimes called, "Calculating Prodigies". To be sure, the type of problems they solved were not difficult at all, however they involved the manipulation of tremendous numbers and even with pencil and paper it was quite a tedious solution. Well, by now you are undoubtedly very doubtful as to the existence of such aforementioned individuals, so let's take a look at the facts.

One of the first prodigies of whom we have records is a young boy called Jediah Buxton, who was born around 1707 in Denshire. This lad never learned how to read or write though he was the son of a school teacher. However inspite of his illiteracy, numbers seemed to fascinate him ever since he could recall. No one knows exactly how or when he developed his amazing ability of mental calculating, however when it finally was discovered, it created quite a stir. Once, when he was about eighteen years old he was taken to see a show. When asked if he enjoyed it, it was discovered that he had no idea what the show was about, but was readily able to tell how many words were uttered by the actors, and how many steps were taken by others while dancing. Do you think this is easy? Try it sometimes, I did and.........

But let's get back to our story. When the good people of Denshire learned of Jediah's ability, he became an object of curiosity and enjoyment. Once he was asked by a group of unbelievers, to multiply a 39 digit number by itself, and then to raise this to the ninth power. Believe it or not, though it took him some time (all of two and a half months), he did it mentally. By now you might have sent down to the nearest hardware store for a shovel, but hold on for you might want to order two after you hear this. Mr. Buxton calculated mentally the number of grains of corn that would be needed to fill a bin whose volume is 202,680,000,360 cubic miles. When he did this, he was asked how many human hairs would be needed to fill the same bin, and, although it took him some time he solved it. (Editors note: He could have easily answered the latter with the amount of hair on the head of a Stuyvesantian in need of a hair cut, and have had a very close approximation. But Stuyvesant didn't exist then ........ or did it?).

Such were the abilities of Jediah Buxton, and it truly is a shame that because of his high degree of ignorance, the secret of these amazing abilities went to the grave with him.

Well, now let's explore the life of another calculating prodigy, one Zerah Colburn. He was more fortunate than our last case in that he was able to read and write, and was considered one of the top scholars of his time. And, as we shall soon see, he was more versatile than Mr. Buxton as far as his calculating abilities go. The first time he was interviewed, the interviewers made the grave mistake of underestimating his ability and the results were hilarious For, when they asked him to calculate the numbers 1,2,3,4,.....10 to the twentieth power, he got even with those "fools" by rattling off the answers faster than the secretary could record them. He was able to give the square and cube roots of five digit numbers as quickly as he was given them. He was able to break down ten digit numbers into their prime factors unhesitatingly. Once, however, he did blunder, for when he was asked to factor 36,083 he thought a full two minutes before realizing that it is a prime number.

Before he died, Zerah revealed the method by which he was able to perform his amazing feats. It seems that he used his ability to factor large numbers into relatively smaller ones and then to manipulate them. How he developed the ability to factor large numbers was a mystery to him and is a mystery to us.

The two examples that I presented to you are merely symbolic of the multitudes of calculating prodigies that have existed and will exist through out the ages. Some of them exhibited their extraordinary powers on the stage, others died unknown, and still others suffered a worse fate, they came to Stuyvesant.

Once my math teacher asked me, jokingly, to multiply a 44 digit number by itself and then to multiply the product by a 66 digit number. For a moment I appeared baffled. Then realizing that my teacher was only kidding, I rattled off a two hundred digit number. The teacher then called my bluff and told me to raise that number to the ninth power. I paused for a moment and then called out a 33,000 digit number. My teacher began to laugh and his face turned a deep red, however immediately his complexion turned a bright green and his laugh changed to a shriek as a squeaky voice from the rear of the room stated that my answer was three too much.

# VECTOR ANALYSIS

by *Paul Cohen*


"Number is the basis of modern mathematics. But what is number"? Thus do R. Courant and Herbert Robbins begin their excellent book "What is Mathematics"and with that statement I chose to open this article. Why? Well, I believe that a good understanding of absolute or natural numbers is a must before you tackle the study of vectors, their nature and their use.

For, actually what is "number?" Isn't number merely a figure of the imagination that is correlated with some other fact as soon as it is mentioned? For example, when I say "five men", you immediately picture a specific group of men so that if I would give each a number, 1,2,3,.. the last man would be numbered 5. We say, the "cardinality" of that group is five. Now, if we examine a few more "absolute" or "cardinal" terms, 2 pounds, 5 inches, 7 degrees, 9 cubic yards etc., we find one outstanding fact common to them all, namely, each term tells us not thing more than merely the number of objects in the group be they inches, pounds, or Schmoos.

Now you might ask, "why did you stress the word "merely" in your last statement? If I have six apples, I have six apples, what more must I know?" Well, it's true that for certain groups of objects, knowing their cardinality or magnitude is sufficient. However,there exist other groups in which merely the knowledge of their magnitude is as useless as a 1948 calendar. For example, the term "five pound force" tells you its magnitude, but what can you do with it? If I say that I've got 5 apples and I'm going to get 5 more, I know that I will have 10 apples. However, if I say that I've got a 6 pound-force and I'm going to add to it another 6 pound-force, I don't know what I'm going to have. It may be nothing, it may be a 12 pound-force, it may be a force with a magnitude of $6\sqrt{2}$ pounds, its value varies depending upon the respective directions of the two forces. Ah! There we have it, there's the ambiguity introduced by absolute numbers as far as certain terms are concerned. The whole concept of "direction" was left out. How can we remedy this situation? Simply through the introduction of a new concept, that of vectors.

A vector can be basically defined as a method of notation which includes both magnitude and direction, and is symbolized by a directed line segment. The length of the line segment represents the magnitude of the vector,and the arrow points in its direction. Thus a 3 m.p.h. NNE velocity, which is also a good example of a vector, can be symbolized by a line, 3 units long, and pointing in a north north easterly direction.

Now that we all thoroughly understand what is meant by a vector, let's learn how to use it To do this, we must start off with several very arbitrary definitions.,concerning vectors, which to you may not seem reasonable.However,although they can not be proven rigorously,most of these definitions are assumed because, and only because, actual vectors such as force, velocity, etc. abide by them.

In going over these definitions, the reader should constantly refer to the diagrams below, as, for obvious reasons, they will tend to simplify his comprehension of vectors.

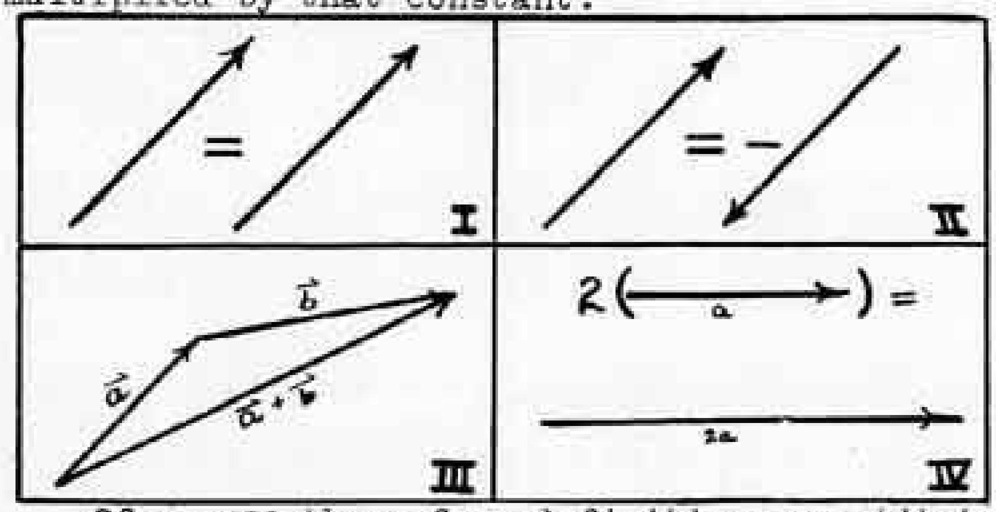
Definition I: Two vectors are equal if they are parallel,extend in the same directions and have equal magnitudes (length).

Definition II: The negative of a vector is the same vector pointing in the opposite direction.

Definition III: The sum of two vectors is the vector produced when the tail(end) of the second is brought to the tip(arrow)of the first and then the tail of the first is joined with the tip of the second by a straight line.

(Remember, vectors can be moved as long as you don't disturb the magnitude, and remain parallel to the original vector. Also remember that we are now dealing with a new concept, and the concept of linear addition is out. So think twice before you deny the last definition).

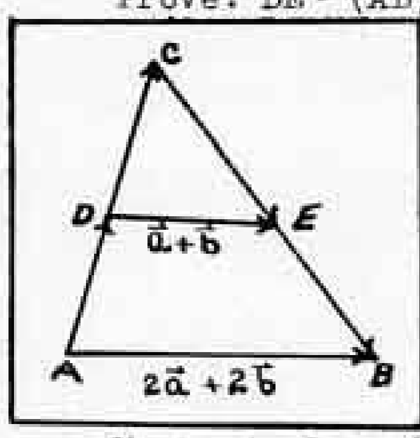
Definition IV: A vector multiplied by a constant is the same vector with the magnitude multiplied by that constant.




Of course these four definitions constitute merely a small fraction of the field of vector analysis, which is covered thoroughly only by many tremendous volumes. However, simplified as it might now seem to you, we can still utilize it to prove certain well known geometrical theorems.

For example, let us prove that the line joining the mid-points of two sides of a triangle is parallel to the third side and equal to one half of it.

Given: Triangle ABC with AD=DC and CE=EB.
Prove: DE = (AB)/2, and $DE \parallel AB$.



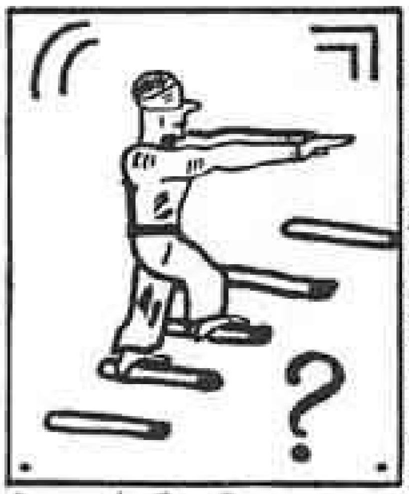

1. Let AD and DC each equal vector $\vec{a}$ ( since they have equal magnitudes and are parallel).
2. For the same reasons let BE and CE equal vector $\vec{b}$.
3. DE = vector $\vec{a}$ + vector $\vec{b}$
4. AB = vector $2\vec{a}$ + vector $2\vec{b}$ = 2(vector $\vec{a}$ + vector $\vec{b}$) by Definition III
5. $\therefore$ DE = (AB)/2
6. $\therefore$ $DE \parallel AB$ (Def. IV)

Much simpler than the geometrical proof, wasn't it?

Well dear and patient reader, in my brief exposition, I have tried to teach you the rudiments of Vector Analysis. I hope I have succeeded, because if I did, I've implanted in you the seeds of one of the most powerful tools the wide field of mathematics has ever seen.

# GUESS AGAIN PAL!

by *Herbert Weber*



In every society there exist certain rules and obligations which must be followed in order that that society should exist. For example, in our "civilized" world there are constitutions and police laws which prevent the people from running amuck. Well, the world of mathematics is no exception to this rule, for, as we all should know, in mathematics there are certain fundemental laws by which we must abide. These basic laws are of course contained in "Euclid's Postulates" which served as the foundation of all geometrical and algebraic work.

There also exists in each society a system of courts and judges whose task it is to interpret our somewhat complicated and versitile laws, and to extract the truth from sometimes very involved situations. And, as I have said before, the field of mathematics also follows suit. For in our mathematical world, we are sometimes faced with certain situations which at first glance seem true but are obviously impossible. These situations are called fallacies for, although the logic behind them seems to hold water, their results are absurd.

Let us examine a very simple fallacy in order to acquaint ourselves with it and its solution.

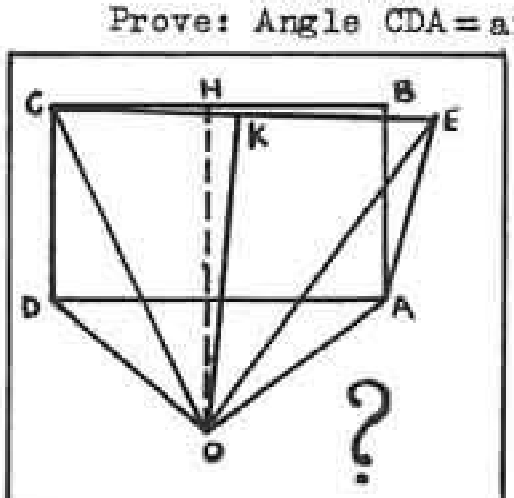
Suppose we let $a = b$, then;

1) $ab = a(a)$
2) $ab - b(b) = a(a) - b(b)$
3) $b(a-b) = (a-b)(a+b)$
4) $b = a + b$
5) $b = 2b$
6) $1 = 2$ ; which is obviously impossible!!!! But why? Didn't we prove it algebraically? Well, the answer is that in our seemingly perfect proof we overlooked the fact that $a-b=0$, and Euclid's axiom clearly states that, "when equals are divided by equals, the results are equal, when the divisor is other than zero."

Now that we are all acquainted with that mysterious thing called a fallacy, let's explore a few more interesting and more difficult specimens. For example, the famous conjecture that a right angle is equal to an angle greater than a right angle!!!

Given: ABCD is a rectangle: From A draw AE outside the rectangle equal to AB or DC, and making an acute angle with AB.

Prove: Angle CDA = angle DAE, or $90° = 90° +$ BAE




1. Bisect CB at H and draw HO⊥CB.
2. Bisect CE at K and draw KO⊥CE.
3. Join CO, EO, DO, and AO.
4. Now, CO = OE and DO = OA, (because all points on the perpendicular bisector of a line are equidistant from ends of that line).
5. CD = AE, (construction).
6. Therefore △DCO = △OAE, (SSS = SSS)
7. Angle ODA = angle OAD, (if two sides of a triangle are equal, the angles opposite these sides are equal).
8. Therefore angle CDA = angle DAE, or $90° = 90° +$ angle BAE.

There you have it. Preposterous but nevertheless, there are the steps, the reasons, every thing looks alright here. Well, it's true that every step is geometrically true, however, the mistake we made is not one of faulty operation but one of construction. It seems that in the aforedescribed construction, line OE will always fall below point A, thus destroying the essentials of the proof.

This fallacy introduces us to one of the most commonly used flaws in the class of geometrical fallacies, namely that of unconstructability. Look for it first, and you'll rarely sweat over geometrical fallacies.

Now we are equipped with two powerful tools to be used in solving fallacies, they are:
1) Look for violations of Euclid's basic axioms such as division or multiplication by zero, etc.
2) Where geometrical fallacies are concerned, look for impossible constructions.

There is another very important defect to be looked for in algebraic fallacies, and it is best brought out by the following fallacy.

Given: 1) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
2) $\sqrt{a} \times \sqrt{a} = a$

Now:
a) By hypothesis #1, $\sqrt{-1} \times \sqrt{-1} = \sqrt{1}$.
b) By hypothesis #2, $\sqrt{-1} \times \sqrt{-1} = -1$.
c) $\sqrt{1} = 1$
d) However according to statements a and b, $\sqrt{1} = -1$, because quantities equal to the same or equal quantities are equal to each other. Now, substituting 1 for $\sqrt{1}$ in accordance with statement c, we find that $1 = -1$, which is obviously absurd.

In this case our mistake, although simple is very typical. For in our haste we overlooked the fact that $\sqrt{1}$ has two roots, 1 and -1. We took the more obvious of the two and thus were led to a fantastic result. Remember, whenever you are faced with a fallacy which includes raising a number to a power, look for the extraneous roots introduced.

Well, in this article I have introduced you to the fallacy, and three of the many many defects found in them. Do not believe that you are now a master at solving fallacies, for if you were, the class of fallacies would be a narrow one indeed. Also, do not think that the few examples I have presented are typical of the many fallacies in existence, for there are fallacies that can stump even the greatest of mathematicians for a while. However I will consider this article a success, if I have interested you in the realm of mathematical fallacies to the extent that you will further explore this field and derive from it many many hours of enjoyment.

Editor's Note: Those students who are interested in pursuing the realm of fallacies, will find their fill of them in a book called:
Mathematical Recreations
by M. Kraitchik

# PROBLEMS

by *Elias Stein*

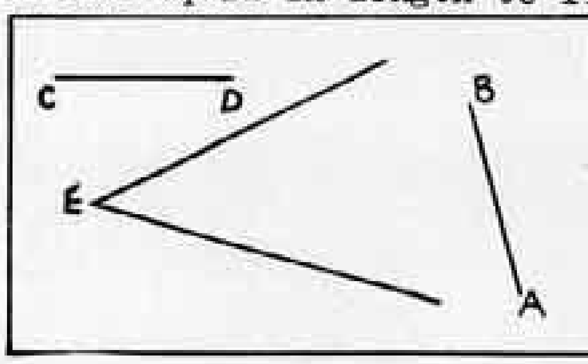
In the great works of the immortal Rudyard Kipling, we find a small verse which modestly portrays the importance of this article. It is:

"I keep six honest serving
men
They taught me all I know:
Their names are What, and
Why, and When,
And How, and Where, andWho."

The following problems are representative of the type of problems which have come to us through the ages, and which have been pondered over since the beginning of number itself.

1. A six digit number has a four for its last digit. If the four is placed at the beginning of the number, the resulting number is four times the original number. Find the original number.

2. Describe the construction of a line ending in the sides of an angle E which is parallel to AB and equal in length to line CD.




3. Two men hold a race. Jones gives Bill a 12 yard head start and wins by 3 seconds. In a second race Jones gave Bill a four second head start and won by nine yards. What is Bill's rate?

4. A lady went into a store to buy 1 1/4 lbs. of meat. She laid the correct amount of money on the counter but the clerk "carelessly" cut 25¢ worth more than she had asked for. Whereupon the customer asked for one half of what he cut and took away ten cents. Find the price of one pound of meat.

5. If you travel at 25 miles an hour for one hour, how fast would you have to travel the next 25 miles in order to average 50 miles an hour for the entire trip?

6. ABCD is a rectangle. If BD is seventeen feet and the area of the figure is 120 square feet, find the sides of the rectangle.

7. The product of two single digit numbers is their sum with the digits reversed. Find these numbers.

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Dear Reader,

There you have them. Some are easy and some are... well, try again maybe you'll still get them. In either case,we have rated these problems according to their difficulty. Here's how to score yourself: Questions,1 and 4 get 5 points. Ques. 2 gets 4 points. Ques. 5 and 7 get 3 points. And 3 gets 2 points. An excellent score is 22. Good is above 15. Average is 12. And below 8.. well,try again maybe you'll still get them.

The Editors.

# SOLUTIONS

1. Since the number is a six digit number, it can be represented by: $10x+4$, where x is some five digit number. When the four is placed at the beginning it can be represented thusly:

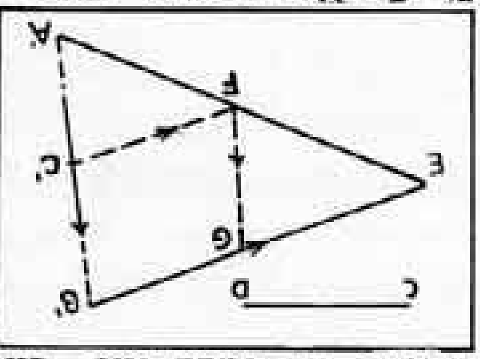
$$400{,}000+x.$$

Now we are told that the second number is four times the first, so setting up that equation:

$$400{,}000+x=4(10x+4)$$

Solving we find that x is equal to 10256 or the original number was 102564.

2. Extend AB so that it intersects the sides of angle E. Call the intersections A'B'. On A'B' lay off B'C' so that it equals CD. Through C' draw a line parallel to EB' and call the intersection with A'E, F. Through F draw a line parallel to A'B' and call the intersection with EB', G. FG is the required construction.




3. In the second race, Jones gave Bill one more second and 3 less yards. Therefore 3 yards=1 second, or his rate is three yards per second.

4. Let x equal the cost of meat per pound. Therefore, since cost equals value, our equation is:

$$5/4\ x - 10 = (5/4\ x+25)/2$$

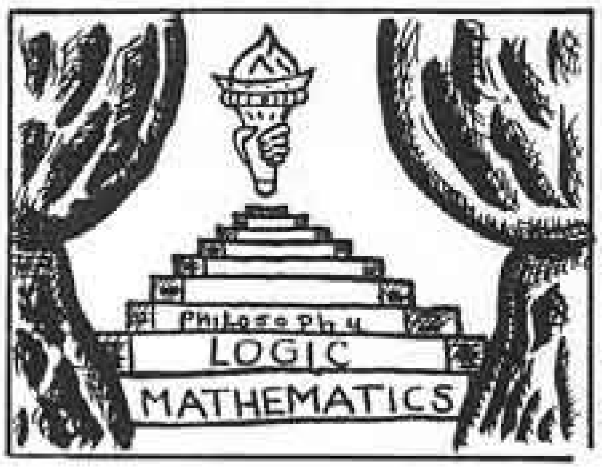
Solving this equation we find that the cost of meat per pound <u>was</u> 36¢.

5. You'll never make it unless you travel the other 25 miles instantaneously. Because, since you have already taken one hour to travel the first 25 miles, the time taken to travel the other 25 miles needed to complete the fifty mile trip would send your total time over one hour, thus making it impossible to average 50 m.p.h. Unless, as I have said,you could do it instantaneously.

6. If the sides are x and y, then $xy=120$, and $x^2+y^2=(17)^2$.This problem therefore is merely the solution of two simultaneous equations, and the answers are 8 and 15.

7. since the sum of any two single digit numbers if a two digit number, can be represented by $10t+u$, where $t=1$, the reverse form of this number can be represented by $10u+t$ or $10u+1$. Therefore our problem breaks down into finding. two numbers whose product ends with the digit 1 and whose sum is more than ten and less than 20 Since the only two numbers less than ten that meet these requirements are 9 and 9, they are the answers.

# AN EDITORIAL




The purpose of this **editorial** is a simple one, namely to bring to light the importance of mathematics in everyday life, and to acquaint the student with the excellent mathematical background available to him through Stuyvesant.

Many people nowadays have the wrong attitude towards mathematics. They think that mathematics is alot of mixed up stuff that's good only for long-haired and absent minded professors to ponder over. They, in their wildest dreams, can not discover the slightest connection between mathematics and their everyday existance.

"Oh it's true that all our machinery and marvelous inventions could not have come about without the aid of mathematics," they might say, "but why should I know it? I'm not going to be an engineer, that's their worry. The only math I need to know is just enough to figure out the amount of change coming to me when I buy something".

Well, that thought has probably run through all our minds at one time or another, however I'm sorry to say that many of us still agree with it. For you see, very few of us have come to think of mathematics, not as a queer method of throwing around fantastic numbers, but as one of the soundest devices of teaching the world what it lacks so much, namely... logic; the science of correct reasoning.

In our math classes we acquire the basic concepts of sound reasoning. There we are taught how to apply certain elementary facts to the solution of more involved situations. There we are introduced to the fundemental tools of logic namely the inverse, converse, counterpositive, and last but not least the concept of a rigorous definition.

No, this is not a lecture in elementary logic, but it is, I hope, an invitation to those who thus far have shied away from mathematics, to take advantage of the wonderful opportunities offered you by the Math Dept. of Stuyvesant. And with the valuable information gained in your studies we'll all be able to make this world a better place to live in.

The following is a list of Math courses given in Stuyvesant with excellent instruction from a fine faculty. Regents courses are denoted by *.

M1-2 ELEMENTARY ALGEBRA
5 Per. Wk. | 1 year | 10 points

A one year course designed to acquaint the student with the fundementals of algebra. It includes an elementary study of trigonometric functions and is a prerequisite to every advanced math course.

M3-4 PLANE GEOMETRY *
5 Per. Wk. | 1 year | 10 point

Two terms dealing with the study of simple figures on a plane, with the intention of sharpenning the students logical senses. It includes a study of length, area, and simple cons struction problems.

M5 INTERMEDIATE ALGEBRA *
5 Per. Wk. | $\frac{1}{2}$ year | 5 points

A one term course picking up where Elementary Algebra left off. It deals with the solution of problems of more than one unknown and of variable degrees. It is a prerequisite to all advanced math work.

M6 SOLID GEOMETRY *
5 Per. Wk. | $\frac{1}{2}$ year | 5 points

A very fascinating and useful course designed for those entering field's of engineering architecture and general college mathematics. It employs the principles set forth in Plane Geometry for the study of solid figures, such as the sphere, pyramid, cone, cylinder, etc.

M7 TRIGONOMETRY *
5 Per. Wk. | $\frac{1}{2}$ year | 5 points

A one term course necessary for those interested in entering the fields of engineering, architecture and surveying. It covers both Plane and Spherical Trigonometry and touches upon the study of navigation. It deals with the solution of triangles when given certain parts.

M8 ADVANCED ALGEBRA *
5 Per. Wk. | $\frac{1}{2}$ year | 5 points

Expanding upon the facts learned in Intermediate Algebra, Advanced Algebra leads the student into a fundemental study of the interesting theory of equations. It also covers problems concerning odds, probability, and combinations, and introduces the student to the beautiful field of Calculus.

M9 THEORY AND PRACTISE OF SURVEYING
10 Per. Wk. | $\frac{1}{2}$ year | $7\frac{1}{2}$ points

Credit for one prepared subject plus one single period shop. Course entails use of transit, level, and other instruments, measurement of elevations, and determination of bounderies and areas. Recommended for those interested in engineering, architecture, and forestry. Recognized by all colleges. M7 is a prerequisite or co-requisite.

M10 PRE-ENGINEERING MATH
10 Per. Wk. | $\frac{1}{2}$ year | $7\frac{1}{2}$ points

Course includes fundemental principles of analytics, Calculus, and Graphics, followed by applied mathematics. Designed for those who

( Continued on page 11 )

# PRIME NUMBERS

by *Stanley Schultz*


Mathematicians are a peculiar bunch of people, (as you have probably already judged from reading this magazine) they refuse to leave things as they are. Take for example the case of I.Q.Math vs. I.M.Prime that has made headline news ever since 300 B.C. For hundreds of years before that time, Mr. Prime and his family were left in peace, so what if they were multiples of only unity and themselves? Ah, but then came the villian, a man called Euclid, it bothered him that such a family should exist. Why should they be different from their neighbors?

As most geometry students know, Euclid was a determined man, and so he set out to explore the facts behind Mr. Prime and his mysterious family. Well, when Euclid finally packed away his pencils and left this fair earth, he had already created such a stir and fuss, that other eager mathematicians followed his steps and delved into the private lives of the Prime family. And let me tell you that it was quite a delving into, for men like Diophantus, Fermat Gauss, and such were not to be easily outdone. But Mr. Prime was a stubborn bird, he took quite a beating and revealed many secrets. However, he refused to open himself up completely, and a few of his secrets have remained as a thorn in the mathematician's side to this very day.

One of these secrets concerns the distribution of primes among the rest of the number world, and another is the problem of deriving a formula which is capable of producing only primes. Many attempts at the latter have been made, and some of them have fooled even the greatest of mathematicians. However, sooner or later their errors have been discovered. The most famous conjecture to this effect was made by the great mathematician, Fermat. He stated that all numbers of the form

$$f(x)=2^{2^x}+1$$

are primes. He backed this statement by showing that this equation holds true for all values of x up to four, as we shall now see;

$$f(1)=2^2+1=5$$
$$f(2)=2^4+1=17$$
$$f(3)=2^8+1=257$$
$$f(4)=2^{16}+1=65,537$$

however, when x equals 5, f(x) equals 4,294,967,297 which for obvious reasons offers difficulties when one tries to determine whether it is prime or not, and even Fermat was stumped. Well Fermat can console himself in the fact that he was not the only one taken in by this formula, for it was almost one hundred years later before a man by the name of Euler discovered that, 4,294,967,297=641 x 6,700,417, and therefore is not a prime. Now, with the use of many new and very complicated methods, mathematicians have found many of these "Fermat Numbers" to be composite, and there is a question as to whether there are any other prime numbers of the form $f(x)=2^{2^x}+1$ when x is greater than four.

Well, Mr. Prime sure pulled a dirty trick on Fermat!

I said before, that there are many formulae that were offered in the attempt to produce primes, and it seems that some of them are of remarkable simplicity. Take for example the equation;

$$f(n)=n^2-n+41$$

which produces primes for all values of x between 0 and 41, but when x=41 we get;

$$f(41)=(41)^2-41+41=(41)^2$$

which is obviously not prime.

The formula;

$$f(n)=n^2-79n+1601$$

is very similar to the previous one as it also produces primes up to a given number. In this case, when the value of n lies between 0 and 80 f(n) is a prime, however when n=80 the above formula loses its wonderful property.

Needless to say there were more of such formulae offered, but, in the long run, no matter how intricate or simple they were, they all failed to deliver the goods. In fact, mathematicians have conceded that the idea of deriving a formula that would produce only primes, let alone all the primes, is quite futile.

Round one for Mr. Prime!

Another unsolved mystery in the field of primes, concerns the distribution of the primes among the remainder of the number system. When mathematicians gave up all attempts to find a formula capable of producing primes, they also realized the futility of trying to derive a formula which would give the exact number of primes contained in the first N integers. So instead, they turned their interest toward determining the average distribution of prime numbers.

The man who should be given the most credit for aiding the progress of this search, if any one man can be so honored, is Gauss, for his developement of the "Prime Number Theorem". This theorem is one of the greatest discoveries in the field of theory of numbers, and it derivation required a great deal of complicated logic. However, as complicated as its derivation might be, it is very simple to comprehend.

Let the symbol $P_n$ denote the number of primes in the first n integers. Therefore, since there are five primes in the first ten integers (not counting 0), $P_{10}=5$. Verstehen Sie? Now, we define the ratio $P_n/n$ as the thickness or more accurately as the "density" of primes among the first n integers. Therefore, since there are 168 primes between one and one thousand, $P_{1000}/1000=.168$. In that same manner, through the use of tables of primes, the values of $P_n/n$ can be determined for fairly high values of n.

Now we can explore Gauss' theorem. He said that $P_n/n$ tends to equal 1/log n (natural logarithms are used) as n increases. Or, in other words that;

$$\frac{P_n/n}{1/\log n}=1$$

This astounding relation is clearly seen by merely examining the following table.

| $n$ | $P_n/n$ | $1/\log_e n$ | $\frac{P_n/n}{1/\log_e n}$ |
|---|---|---|---|
| $10^3$ | 0.168 | 0.145 | 1.159 |
| $10^6$ | 0.078498 | 0.072382 | 1.084 |
| $10^9$ | 0.050842478 | 0.048254942 | 1.053 |

Although it is no proof, this table shows that as the value of n increases, the values of $P_n/n$ and $1/\log n$ become more and more alike and their ratio gradually approaches one.

The implications of this fact are twofold. Either it is merely an amazing coincidence that two seemingly distant fields should have such a close relationship, or these fields are not as distant as it is commonly thought. In either case it adds fuel to the fire in the mind of a math enthusiast.

There are many more very interesting facts and problems concerning prime numbers, such as the "Goldbach Conjecture" and the "Prime Pair" problem, however time and space prohibit my expanding upon them. Nevertheless, I would like to mention an extremely helpful device which is used in compiling data necessary for exploring the field of Prime numbers.

It seems that the easiest way to collect all the primes between one and N, is by writing down all the integers between one and N and striking out all the multiples of two, three. etc. Until all the composite numbers are removed. This process is known as the "sieve of Eratosthenese" and it is simplified with the use of a factoring machine called "Lehmers Number Sieve". This machine factors large numbers in almost miraculous speed. For example, it took only three seconds to factor;

$$2^{93}+1$$

into

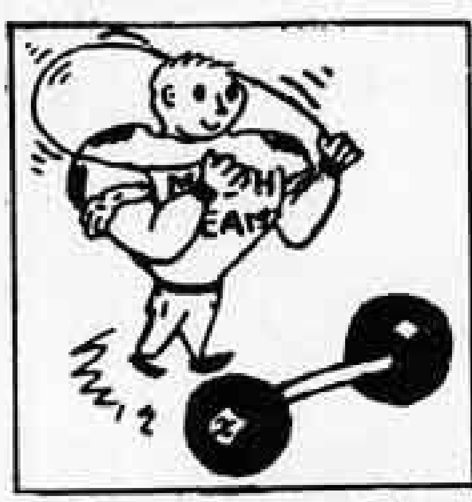
$$3^2 \text{x} 529{,}510{,}939 \text{x} 715{,}827{,}883 \text{x} 2{,}903{,}110{,}321$$

Not bad for an amateur, eh? Further information concerning this machine can be found in the "American Mathematical Monthly" 1933 vol.XI pp. 401-406.

Well, by now I'm pretty sure that you understand why I called Mr.I.M.Prime a stubborn cuss. As hard as mathematicians have tried in the past, they have still only an inkling of the secrets hidden in the family of primes. Whether they will solve any of these problems or uncover any more secrets is anyone's guess. I don't know. Do you?

# MATH TEAM

*by Edward Posner*



Every Stuyvesantian has undoubtedly heard much about our fine athletic teams, however very few are acquainted with the functions of the Math team, not a very athletic team, but nevertheless a team. Some fellows must think that a Math Meet is a pencil throwing contest or another such fantastic pursuit, well, let's try to clear up such thoughts by learning exactly what the Math team is and how it functions.

Our Math team does not exhaust the funds of the G.O., for sneakers and sweatshirts are unnecessary, neither does it require the time of a supervisor. Our Math team is a team built up by its own initiative and glorified by its own satisfaction. The team keeps alive the interests of students who marvel at the wonders of practical mathematics, and above all it is a faction representing the superior Math Dept. of a superior school, Stuyvesant High School.

We assemble daily at the start of the 7th. period in room 412, and informal practice is held until the 8th.period. During this practice session, Captain Elias Stein, a very capable mathematician, hands out problems typical of those given at meets, and, after everyone has had a chance at their solution, he goes over them, bringing out the tricky points as only he could do it. As Stein is graduating this term he will be succeeded by "Handsome Harry" Widom, also a student well versed in mathematics.

Well, now that we're all acquainted with the practice session, let's see what is meant by a Math Meet.

Competition is arranged by the Interscholastic Mathematics Association whose secretary school is Boy's High. The purpose of the secretary school is to inform every school of its score and rank periodically. Each school contends with identical problems on the same day, usually Friday, however, for obvious reasons, two schools meet at the same place. We are paired with the Bronx High School of Science. Three sets of two problems each, are given at every meet, with a specific time alloted for each set. As five members from each team compete, the maximum score for a team is 30 points.

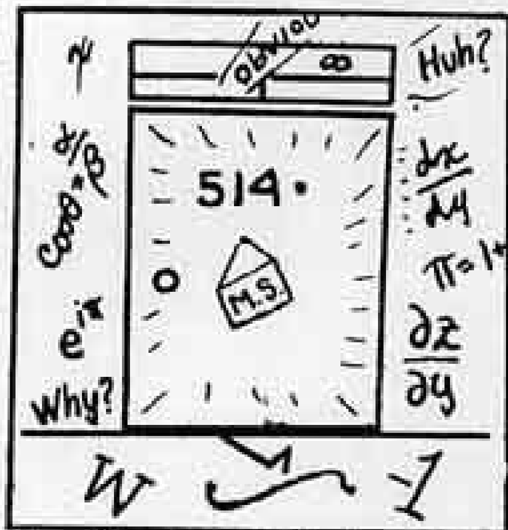
Unfortunately, due to certain unavoidable circumstances among which is the inavailability of "Truncated" Paul Cohen, one of Stuyvesant's best mathematicians, our scores this season have not been as high as expected. However we are working and planning for next term when, unless fate is against us, we are a good bet to cinch the City Championship.

On our gallant squad this term we have had the assistance of Messrs. Elias Stein, Martin Brilliant, and Maurice Silberman who receive their diplomas this term. And remaining to carry the Stuyvesant banner into future battles next term, are Co-captain Widom, (soon to be Captain), Elihu Lubkin, Edward Posner, Paul Cohen, Peter Meyers, Connie Catzelis, Kennith Orski, Murray Weiner and Ben Tissenbaum.

Any Stuyvesant genius who recieved more than a $\pi$ rating in Math on his report card, is encouraged to try out for next term's championship team.

# MATH SOCIETY

by *Kennith Orski*




Dear Reader,

Under the leadership of president Elias Stein, the Math Society of Stuyvesant High School has had one of its most successful seasons in its long history.

Having in mind the purpose of this organization, which is to advance the interests of pure and applied mathematics in Stuyvesant H. S., the president with the collaboration of the officers of the society, Harold Widom, vice-president, and Kenneth Orski secretary, started a series of excellent lectures covering a few of the interesting aspects of the wide field of mathematics.

Following the traditional opening talk by Mr. Berman, chairman of the Math Department, which explained the purpose and activities of the Math Society, and its official publication, the Math Survey, the Math Society officially opened its Fall term activities.

Edward Posner, also a member of the Math Team, opened the season with an interesting discussion of the Nine Point Circle. Speaking before a rather disappointing audience of only twelve members, Posner opened his talk with a statement that will be long remembered by his audience, namely, "the Nine Point Circle is a circle having more than nine points".

President Stein was the next speaker, and the topic he chose was one of both theoretical and practical importance, that of Conic Sections. Among the points covered in this talk were the properties of the parabola, hyperbola, and the circle, as well as the fundementals of Analytic Geometry.

Harold Widom, who was one of the more active speakers during the term, addressed the society on the "Limit of Functions", and on another occasion he proved that the number of points on a 1 inch line is equal to the number of points on an infinitely long line, much to the delight of the entire assembly.

Vector Analysis, Navigation and Boolean Algebra were among the other topics discussed at the Math Society meetings this term.

Finally, to end the term with a grand performance, the Math Society held a dance and quiz with Hunter High School which proved to be a great success.

Hoping to see you next term in the Math Society, we remain.

The officers and members of the Math Society of Stuyvesant High School.

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( Continued from page 8 )

have completed or are completing M5, M7, and a ten period drafting course. It is very advisable for those entering any division of engineering. Given only during the 7th. & 8th. periods.

M11 CALCULUS

10 Per. Wk. — $\frac{1}{2}$ year — $7\frac{1}{2}$ points

One period each morning plus two afternoon meetings weekly. Course includes a study of Analytic Geometry and blends into a consice study of Differential and Integral Calculus stressing both practical and theoretical views. Important for Math, Science, and Engineering college courses. This course is unique for a High School syllabus, and is given one year math credit in most colleges. M7 is prerequisite or co-requisite.

The End

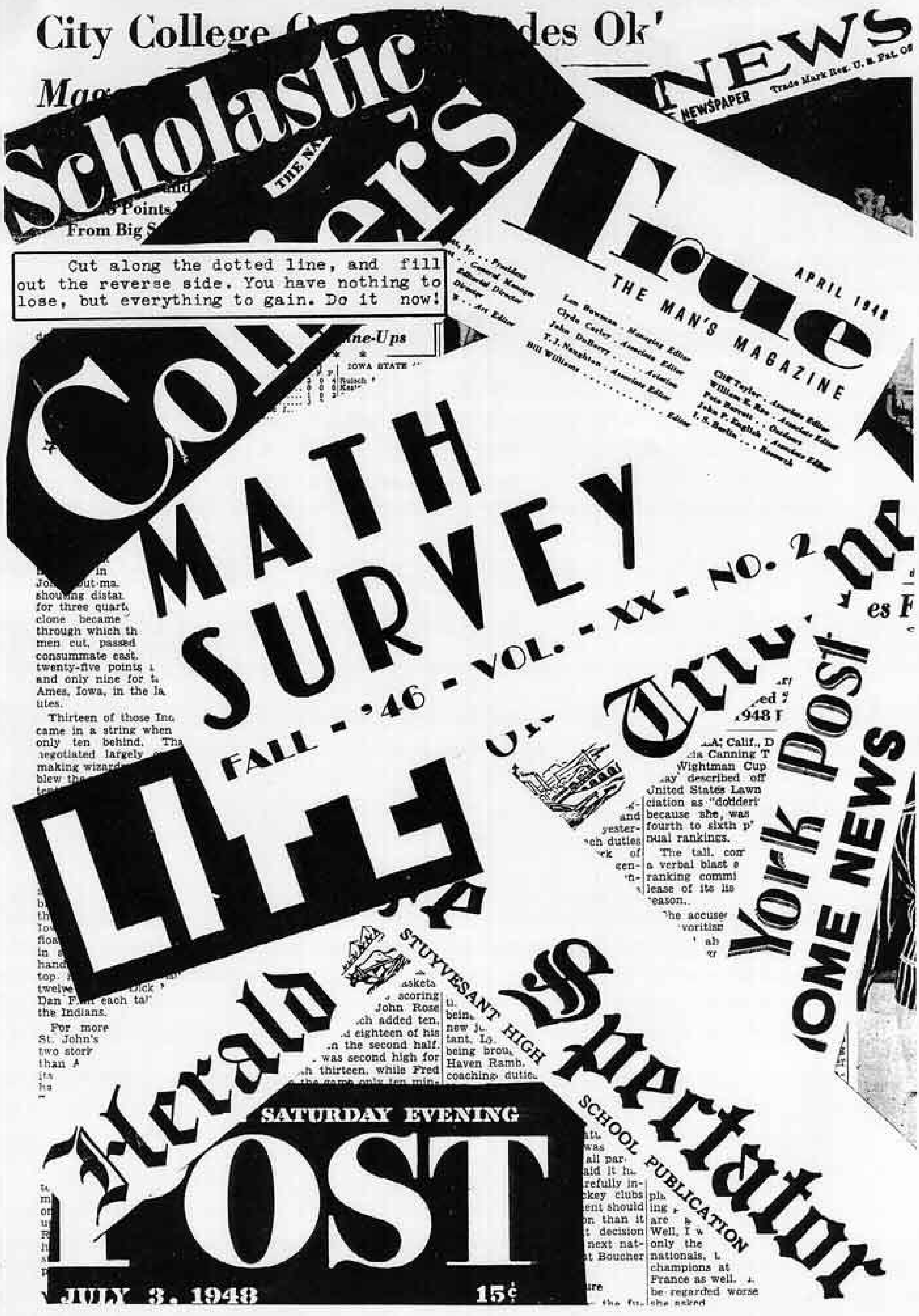
# WE NEED *you!*

The MATH SOCIETY and the MATH TEAM are confined, as far as their activities are concerned, by a lack of student support. Lecturers can not be obtained unless our membership is greatly increased. Socials, projects, etc. are equally hampered.

You will be doing both us, and yourselves a great favor if you make it a point to join both of these organizations, NEXT TERM.

Notices as to place and time of meetings will be posted early in the Spring Term.

Thank you.




Cut along the dotted line, and fill out the reverse side. You have nothing to lose, but everything to gain. Do it now!

Application for a Position on the Math Survey
the
Official Publication of the Mathematical Society
of
Stuyvesant High School

Positions on the staff of the Math Survey, with good chances for advancement, are now open.

Simply fill out the form below, tear it out, and bring it to the Math Office (Room 3k0) by the second week of the Spring 1949 term.

Everyone is invited to try. Let us be the judges of your abilities.

Fill it out now.

Name (print) ______________________ Sp. '49 Off.Cl.____ Rm.____

Fill in the following table giving all the information you can. If you have not taken any course, simply make a dash in the corresponding space

| Course | Teacher (opt.) | Final Mk. | Regents | Remarks |
|---|---|---|---|---|
| El. Algebra | | | | |
| Plane Geo. | | | | |
| Int. Algebra | | | | |
| Trigonometry | | | | |
| English | | | | |
| (any 3 terms | | | | |
| if possible) | | | | |
| Drawing | | | | |
| Mech. Drawing | | | | |
| Other Art. | | | | |
| | | | | |
| typing | | | | |

Have you ever written for, or been a member of another magazine? If so give names,

______________________

______________________

Can you type?____ Can you draw?____ Can you write? (proof read, etc.) ____

Do you have any connections through which you can obtain ADDS for the Math Survey? If so give specific examples, ______________________

______________________

Are you a member of Jr. or Sr. Arista? If so, which, ______________________

Do you belong to any other school organizations? If so specify, ______________________

______________________

______________________

# BRAIN TEASERS

edited by *Jonas Schultz*

SCRAMBLED WORDS PUZZLE

Most of you have seen at one time or another, a "Scrambled Word Puzzle", and know how to handle them. However for those that do not, I will explain. You are given a meaningless assortment of letters, and it is your task to rearrange them in such a way that they assume their proper meanings. For example:

Sticathemam→Mathematics

Now that you all know how to unscramble words, try some of these. Perhaps it would make your job easier if I tell you that all of the following terms are connected with Geometry and Intermediate Algebra.

| | |
|---|---|
| 1. Stance | 6. Cunnoift |
| 2. Stainmas | 7. Nadira |
| 3. Norgatypeha | 8. So Nice |
| 4. Mathgolir | 9. Quartdan |
| 5. Itraceschartic | 10. Alengrit |

HIDDEN MATHEMATICIANS

Hidden in the following ten sentences are the names of ten famous mathematicians. Let's see if you can uncover all ten, it's not quite as easy as you think.

Example: On her lap, lace of the finest design lay.

Answer: Laplace

Get it?

1. His horse was new; to name it was the next problem.
2. There was I with a broken arch, I. Medes and Persians how I suffered.
3. Frank accepted the offer; Matthew refused it.
4. He heard the bell tinkle in the hall.
5. I hate cant or blah-blah.
6. I say Edna, pie reveals your culinary talent.
7. I gave him back four, i.e. returned all but one.
8. It was as fine a tulip as California could produce.
9. They say that his garden wall is something to behold.
10. It was a Mercedes car, Tess.

MOVE ALONG!!!

In not more than 26 moves, rearrange the letters in figure I to the position shown in Figure II. The letters can be moved only to the vacant space left by each move. Hint: Cut out 8 pieces of cardboard and print the letters on them.

figure I

| | | |
|---|---|---|
| G | E | F |
| H | B | A |
| D | C | |

figure II

| | | |
|---|---|---|
| A | B | C |
| D | E | F |
| | G | H |

Solutions are inverted in the next column.

. SCRAMBLED WORDS PUZZLE

Pretty tough weren't they? Well here are the correct answers.

| | |
|---|---|
| 1. Secant | 6. Function |
| 2. Mantissa | 7. Radian |
| 3. Pythagorean | 8. Cosine |
| 4. Logaritm | 9. Quadrant |
| 5. Characteristic | 10. Triangle |

HIDDEN MATHEMATICIANS

Well, most of you probably didn't uncover all of the hidden mathematicians, because some of them are not very well known. Therefore I have listed the correct answers below, along with their contributions to the field of mathematics, so that their importance can be estimated.

1. Newton, Issac, who developed the Binomial theorem.
2. Archimedes, developed certain aspects of Geometry and the study of Levers.
3. Fermat, Pierre, advanced the Theory of Numbers.
4. Klein, Felix, aided in the developement of Non-Euclidean Geometry.
5. Cantor, Georg, helped advance the theory of numbers
6. Napier, developed many theories in the field of Spherical Trigonometry.
7. Fourier, formulated many theories in the Calculus.
8. Pascal, developed many aspects of Projective Geometry.
9. Wallis, formulated many theories in the field of Calculus.
10. Descartes, known to all Advanced Algebra students for his theory of signs.

MOVE ALONG!!!

In order to perform the required pperation you should move each letter in the order given below to the vacant space next to it; C,D,H,G,E,F,A,B,D,H,G,D,F,A,B,C,H,G,D,E,A,B,C,F,E,D.

* * *

DID YOU KNOW?

1. Spherical trigonometry was developed earlier than plane trigonometry because of its use in astronomy and navigation.

2. Logarithms were used before exponents. Now in the teaching of logarithms we derive it from exponents. This derivation was first brought to our attention by Leonard Euler more than a century later.

4. Solution of the third and forth degree equations preceded the use of the equal sign!!!

4. Integral calculus was used many years before differential calculus. Now we are taught integral calculus as the inverse process of differential calculus.