The Prime Dounts

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It is an ordinary day in room 403. 31,415 donuts are stacked in white Krispy Kreme boxes in every corner. The donuts, sticky with bright, sweet frosting, sit languidly in the arid June sun.

Mr. Kats, a brilliant mathematics teacher, is known for conjuring enthusiastic pupils out of even the most ordinary students. So when he proposed, "I have a challenging problem for you all," the class threw themselves at the challenge. "If we solve it, could we have these donuts?" Michael jokes.

"Sure," Mr. Kats says with a slightly bewildered shrug. Everyone scrambles to their seats as he writes on the board: Prove that there are an infinite number of prime numbers differing by 2.

"This should be a simple proof. After all, the question itself only has eleven words and one number," Michael remarks. There is a buzz in the room as the class frantically scribbles the question down, whispering to each other ecstatically, believing it to be a decisive victory.

"Thank you for the free donuts!" Michael yells.

"Sure thing, Michael," Mr. Kats replies.

As the students shuffle out of the classroom, Mr. Kats walks over to his mejor amigo Mr. Cocoros. "I bought 31,415 donuts for my precalculus class, but they offered to solve the twin prime conjecture first," Mr. Kats said.

"The twin prime conjecture! The one that Terence Tao, 13-year-old IMO medalist, couldn't solve?" Mr. Coco replied in astonishment.

"Of course," Mr. Kats said, "I am sure they will make some progress on it."

Meanwhile, Michael and Ian have stolen the keys to the secret, high-technology computer science lab. They hope to earn these donuts and prove the Twin Prime Conjecture by the power of Java.

"It shouldn't be that hard," Michael said to Ian, "All we have to do is ask the computer to output our prime, differing by two."

"But there are an infinite number of prime numbers! We don't have enough computer power to compute all of these!" Ian replied.

"But we actually do!" Michael points to a heap of tangled wires, "See those right there? I have managed to hack into the main computer server and sync up all the computers in Stuyvesant to my own personal server to compute the prime numbers. While most of the computers are crunching the data, five or six computers would receive the values and analyze them for possible patterns in hopes of a generalization to prove these prime numbers!"

"That brilliant dude! I was working on an AI algorithm a few months back. With your network providing me such data, I know the perfect algorithm to break this code! I can already taste the donuts melting in my mouth!"
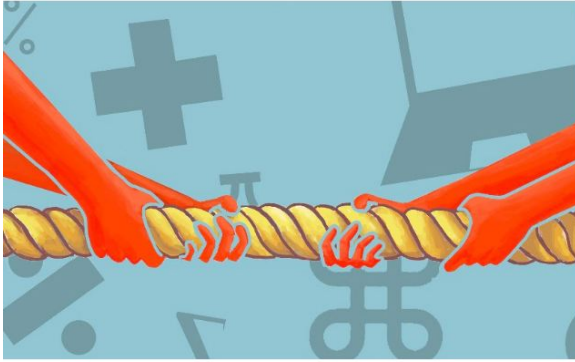
"Hell yea! Dynamic programming is awesome! Teamwork makes dream work!" Michael slapped Ian a high-five, and believed that their success was inevitable.

Fifty yards away in room 405, Theo, Jerry, and Maynard stare at their pile of 314 crumpled-up sheets of paper. 314 failed attempts and miserable sighs. They stare in disbelief that such an innocent problem had turned into a debacle.

"How are you holding over there?" Theo asks frustratingly.

"It is alright. I have already tried direct proof, induction, and proof by contradiction. The most optimistic method right now is proof by contradiction. If I can show that the sum of reciprocal twin primes diverges, I will be able to show that there are an infinite number of twin primes," Jerry said.

"Oh, nice! Right now, I am attempting proof by exhaustion but it is getting increasingly difficult to compute these prime numbers and I am not getting too far with this," Theo replied.

"I don't see a method of solving a problem right now either. I am still experimenting with it and looking for patterns within the numbers. Hopefully, something shows up because if it works, I'm done. If it fails, back to square one. Oh well." Maynard said.

"While you guys work on this, I am going to grab some Fruity Loops from my friend Josh for an enhanced chemiosmosis. Check back in five minutes," Jerry said.

5 hours later...

"Screw this problem! This stupid program concluded that there are an infinite number of prime numbers differing by a gap of between 2 and 70 million," Michael yelled. He pounded on the table with such force that the keyboard smashed into the wall of the CS Dojo room.

"Chill bro. I updated the AI program which I nicknamed as Yitang v. 2013 to better analyze the frequency and distribution of prime's gap value using the input you gave me," Ian said. Pointing at another program called "Polymath Project," he said, "Using binary search, I tweaked the existing divide-and-conquer paradigm to function collectively as one unit, enabling me millions of more possible combinations. Thus, I have been able to lower the prime gap value to 4,680."

"That is much less than my 70 million. It reminds me almost of a quantum computer in the fashion that your program was able to check for so many more results. But 4,680 is still nowhere close to our desired gap of 2. Perhaps we should update our string pattern matching to better analyze a uniform pattern," Michael remarked.

"That's probably a good idea. In the meantime, let's go get some ice cream." Ian said.

"Bet. You are paying though!" Michael laughed.

Room 403

"Ugh. This series converges.

$$\sum_{p,\,p+2\in primes}\left(\frac{1}{p}+\frac{1}{p+2}\right)=\left(\frac{1}{3}+\frac{1}{5}\right)+\left(\frac{1}{5}+\frac{1}{7}\right)+\left(\frac{1}{9}+\frac{1}{11}\right)+\cdots\approx 1.902166$$

Bummer." Jerry said.

At this point, you should call that the Brun's constant for the brown Nutella smudge next to your equation." Theo laughed.

"I got a result but you might not like it," Maynard said. "What is it?" Theo and Jerry cried in unison. The paper said, "$\lim_{n\to\infty}\inf(p_{n+1}-p_n)\le 600$."

"You proved that there is an infinite number of primes differing by at most 600. That is incredible! How did you do it?" asked Jerry.

"I altered the existing GPY sieve to 'show that for each k, the prime k-tuples conjecture holds for a positive proportion of admissible k-tuples.'" I am not sure how to reduce the bound to any less than 600 though."

Looking through the stacks of proofs and lemmas, Theo said, "Wow! You have a very good engineer's induction..."

"But if you assume the Elliott–Halberstam conjecture, then you can optimize the distribution of the prime numbers," Jerry said in realization. Frantically scribbling the prime counting function and Euler's totient function, he wrote the error function of Dirichlet's theorem of primes in arithmetic progression onto Maynard's paper.

"Hmm. If the conjecture is actually true, then the gap value differs by no more than 6," Theo replied.

"That is true but unfortunately, we cannot use this result because we do not know if Elliott–Halberstam conjecture is valid," Maynard said.

"Like what my friend Rishabh says, 'what a loser,'" Jerry sighed. "Well, you know what people say. It is better to give up than die trying," Theo said.

"Haha!" Maynard laughed, "I have mentally died already so I am out boys."

"I am exhausted. Let's call it a day. My brain is fried right now." Jerry said.

"Yeah me too. Hopefully, Mr. Kats will give us the solution tomorrow. Bye!" Theo replied.

Next day

"Have you made any progress on the problem?" Mr. Kats asked, looking around. "Ian and I were able to reduce the difference between consecutive primes from 70 million to 4680 to finally 246 using sieving techniques and an AI program Ian built," Michael replied.

"Sadly, we were unable to reduce that to the value of two that was necessary for this problem," Ian said.

"Yea we weren't able to do it either. We found an upper bound of 600 but we didn't really know what to do next. Then, we realized that if we assumed Elliott–Halberstam conjecture, we would get a small gap of 6 between the primes numbers. Obviously, that doesn't guarantee a value of 6, but even if it does, it still doesn't reach 2." Jerry said.

Even Mr. Kats was amazed at hearing these results from his students. He passed around the Krispy Kreme donuts and cheerfully said, "That is great progress. In truth, there is no solution. But, in order to become a real mathematician, you must solve unanswered questions."

Citations:

1. "Twin Prime Conjecture." From Wolfram MathWorld, mathworld.wolfram.com/TwinPrimeConjecture.html.

2. Ho, Kwan-Hung. "On the Prime Twins Conjecture and Almost-Prime k-Tuples." doi:10.5353/th_b2976842.