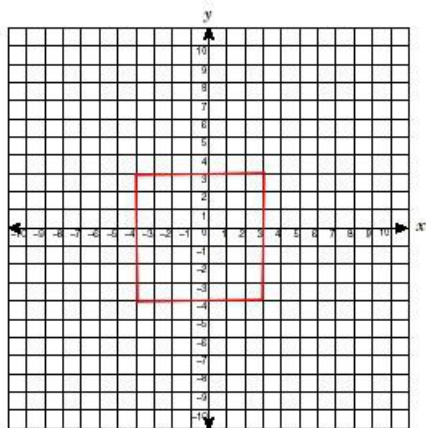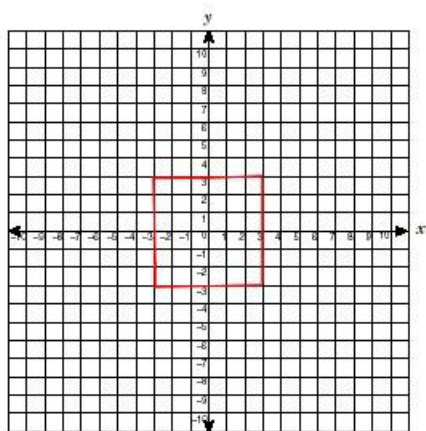# Steps on The Coordinate Plane

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Rules

1. A *n-step*, where n is a positive integer, is a line segment of length n and slope 0 or *undefined*, or a line segment of length $n\sqrt{2}$ and slope of 1 or -1 . The endpoints of a *n-step* must be on a lattice point. For example, you may construct a *2-step* segment with length of $2\sqrt{2}$ and endpoints on $(2, 2)$ and $(4, 4)$.

2. Your *nth* construction must be an *n-step*. For example, your *1st* construction must be a *1-step*. Your *2nd* construction must be a *2-step*. Your *14th* construction must be a *14-step*.

3. For every $n \geq 2$, a *n-step* must share an endpoint with a *(n-1)-step*.

4. Two *n-steps* may only intersect at their endpoints.

5. Every lattice point may contain at most two *n-steps*.

Tasks

1. On the graph, construct a figure that passes through all lattice points (a, b) where $-3 \leq a \leq 3$ and $-3 \leq b \leq 3$.

2. On the graph, construct a figure that passes through all lattice points (a, b) where $-4 \leq a \leq 3$ and $-4 \leq b \leq 3$.

3. Construct a figure that passes through all lattice points (a, b) where $0 \leq a \leq x$ and $0 \leq b \leq y$.

(a) What is the minimum number of lattice points (including the endpoints of line segments) a figure passes through, in terms of the given coordinates?

(b) How many ways are there to construct such a figure?