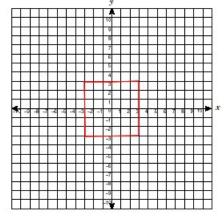
Steps on The Coordinate Plane

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- 1. A n-step, where n is a positive integer, is a line segment of length n and slope 0 or undefined, or a line segment of length $n\sqrt{2}$ and slope of 1 or -1. The endpoints of a n-step must be on a lattice point. For example, you may construct a 2-step segment with length of $2\sqrt{2}$ and endpoints on (2,2) and (4,4).
- 2. Your *nth* construction must be an *n-step*. For example, your *1st* construction must be a *1-step*. Your *2nd* construction must be a *2-step*. Your *14th* construction must be a *14-step*.
- 3. For every $n \geq 2$, a *n*-step must share an endpoint with a (n-1)-step.
- 4. Two *n-steps* may only intersect at their endpoints.
- 5. Every lattice point may contain at most two *n-steps*.

Tasks

- 1. On the graph, construct a figure that passes through all lattice points (a, b) where $-3 \le a \le 3$ and $-3 \le b \le 3$.
- 2. On the graph, construct a figure that passes through all lattice points (a, b) where $-4 \le a \le 3$ and $-4 \le b \le 3$.
- 3. Construct a figure that passes through all lattice points (a,b) where $0 \le a \le x$ and $0 \le b \le y$.
- (a) What is the minimum number of lattice points (including the endpoints of line segments) a figure passes through, in terms of the given coordinates?
- (b) How many ways are there to construct such a figure?