```
ln[1] = q = q0 * Exp[I * (k * x + w * t)];
     qjn = q0 * Exp[I * (k * xj + w * tn)];
     qjbar = Integrate [q, \{x, xj - dx/2, xj + dx/2\}]/(dx);
     qjnbar = qjbar /. t \rightarrow tn;
     MA = qjn / qjnbar;
     qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
     qjntbar = qntbar /. x \rightarrow xj;
     MtA = qjntbar / qjn;
     qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
     RA = Simplify[MA * qjphn / (qjn)];
     vmultG = H + H^3/3*k^2;
     GnA = -U * RA / vmultG;
     GGA = RA / vmultG;
     GcA = -U * H / vmultG ;
     fn1A = H * vh + U * eh;
     fn1A = fn1A /. vh \rightarrow (GGA*Gca+GnA*eca) /. eh \rightarrow RA*eca;
     fn1Gca0A = fn1A / . Gca \rightarrow 0 ;
     fn1eca0A = fn1A/. eca \rightarrow 0;
     fnnA = Simplify[fn1Gca0A / eca];
     fnGA = fn1eca0A / Gca;
     fncA = H * GcA;
     fG1A = U*Gh + U*H*vh + g*H*eh;
     \texttt{fG1A} = \texttt{fG1A} \ / \ . \ \texttt{vh} \ \rightarrow \ (\texttt{GGA} * \texttt{Gca} + \ \texttt{GnA} * \texttt{eca}) \ / \ . \ \texttt{eh} \ \rightarrow \ \texttt{RA} * \texttt{eca} \ / \ . \ \texttt{Gh} \ \rightarrow \ \texttt{RA} * \texttt{Gca} \ ;
     fG1Gca0A = fG1A / . Gca \rightarrow 0 ;
     fGleca0A = fGlA /. eca \rightarrow 0;
     fGnA = Simplify[fG1Gca0A / eca];
     fGGA = Simplify[fGleca0A / Gca];
     fGcA = U * H * GcA;
     FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
     FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
     FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
     FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;
     MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
```

$$\frac{\mathbf{k}\left(\sqrt{3}\sqrt{\mathbf{g}\,\mathbf{H}\,\left(3+\mathbf{H}^2\,\mathbf{k}^2\right)}\,+\left(3+\mathbf{H}^2\,\mathbf{k}^2\right)\,\mathbf{U}\right)}{\left(3+\mathbf{H}^2\,\mathbf{k}^2\right)}\,\,\rightarrow\,\,-\mathbf{w};$$

wAp =
$$U * k + \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};$$

wAm = $U * k - \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};$

```
ln[38] = M = (26 - 2 * Cos[k * dx]) / 24;
     Merr = Series[M - MA, \{dx, 0, 10\}];
     Rm = (5 - Exp[-I*k*dx] + 2*Exp[I*k*dx]) / 6;
     Rmerr = Series[Rm - RA, \{dx, 0, 10\}];
     Rp = Exp[I*k*dx]*(5 + 2*Exp[-I*k*dx] - Exp[I*k*dx])/6;
     Rperr = Series[Rp - RA, \{dx, 0, 10\}];
     Ru = \left(-Exp[-I*k*dx] + 9*Exp[I*k*dx] - Exp[2*I*k*dx] + 9\right) / 16;
     Ruerr = Series \left[ Ru - Exp \left[ I * k * dx / 2 \right], \left\{ dx, 0, 10 \right\} \right];
     Gold = H - H^3/3 * (32 * Cos[k * dx] - 2 * Cos[2 * k * dx] - 30) / (12 * dx^2);
     GG2 = M * Ru / (Gold);
     GG2err = Series[GG2 - GGA, {dx, 0, 5}];
     Gn2 = -M * Ru * U / (Gold);
     Gn2err = Series[Gn2 - GnA, {dx, 0, 5}];
     Text[Row[{"M ||
                         ", M}]]
     Text[Row[{"M || ", TeXForm[M]}]]
     Text[Row[{"M error || ", TeXForm[Merr]}]]
     Text[Row[{"M error || ", Merr}]]
     Text[" "]
     Text[Row[{"Rm || ", Rm}]]
     Text[Row[{"Rm || ", TeXForm[Rm]}]]
     Text[Row[{"Rm error || ", Rmerr}]]
     Text[Row[{"Rm error || ", TeXForm[Rmerr]}]]
     Text[" "]
     Text[Row[{"Rp ||
                          ", Rp}]]
     Text[Row[{"Rp || ", TeXForm[Rp]}]]
     Text[Row[{"Rp error || ", Rperr}]]
     Text[Row[{"Rp error || ", TeXForm[Rperr]}]]
     Text[" "]
     Text[Row[{"GG2 || ", GG2}]]
     Text[Row[{"GG2 || ", TeXForm[GG2]}]]
     Text[Row[{"GG2 error || ", GG2err}]]
     Text[Row[{"GG2 error || ", TeXForm[GG2err]}]]
     Text[" "]
     Text[Row[{"Gn2 || ", Gn2}]]
     Text[Row[{"Gn2 || ", TeXForm[Gn2]}]]
     Text[Row[{"Gn2 error || ", Gn2err}]]
     Text[Row[{"Gn2 error || ", TeXForm[Gn2err]}]]
Out[51]= M || \frac{1}{24} (26 - 2 \cos[dx \, k])
Out[52]= M || \frac{1}{24} (26-2 \cos (\text{d}x) k)
```

Out[53]= M error |

 $-\frac{3 \text{3 } \text{4 k}^4}{640} + \frac{3 \text{4 k}^6 k^6}{35840} - \frac{149 \text{4 k}^8 k^8}{51609600} + \frac{29 \text{4 k}^10}{10} \frac{10}{13624934400} + O\left(\frac{4x}^{11}\right) + O\left(\frac{4x}{11}\right) + O\left(\frac{4x}{11}$

$$\text{Out}{\text{[54]=}} \ \ M \ error \ \ || \ \ -\frac{3 \, k^4 \, dx^4}{640} + \frac{3 \, k^6 \, dx^6}{35 \, 840} - \frac{149 \, k^8 \, dx^8}{51 \, 609 \, 600} + \frac{29 \, k^{10} \, dx^{10}}{13 \, 624 \, 934 \, 400} + O[dx]^{11}$$

Out[55]=

Out[56]= Rm ||
$$\frac{1}{6} (5 - e^{-i \, dx \, k} + 2 \, e^{i \, dx \, k})$$

Out[57]= Rm || $\frac{1}{6} \left| -e^{-i \cdot text \cdot dx} k \right| + 2 e^{i \cdot text \cdot dx} k + 5 \right|$

$$\text{Out} \text{[58]=} \quad Rm \; error \; \mid \mid \; \; -\frac{1}{12} \, i \, k^3 \; dx^3 \, + \, \frac{k^4 \, dx^4}{120} \, + \, \frac{1}{240} \, i \, k^5 \; dx^5 \, - \, \frac{k^6 \, dx^6}{5040} \, - \, \frac{i \, k^7 \, dx^7}{10080} \, + \, \frac{k^8 \, dx^8}{201600} \, + \, \frac{i \, k^9 \, dx^9}{725760} \, - \, \frac{k^{10} \, dx^{10}}{39916800} \, + \, O[dx]^{11}$$

Out[59]= Rm error ||

 $-\frac{1}{12} i \cdot \frac{dx}^3 k^3 + \frac{dx}^4 k^4}{120} + \frac{1}{240} i \cdot \frac{dx}^5 k^5 - \frac{dx}^6 k^6}{5040} - \frac{i \cdot \frac{dx}^7 k^7}{10080} + \frac{dx}^8 k^8}{201600} + \frac{i \cdot \frac{dx}^7 k^7}{10080} + \frac{dx}^9 k^9}{725760} - \frac{\cot dx}^10} k^10}{39916800} + O\cdot \frac{dx}^11}{ \cdot \frac{dx}^9 k^9}{725760} - \frac{\cot dx}^10} k^10} k^10} + \frac{dx}^9 k^9}{725760} - \frac{\cot dx}^10} k^10} k^10} k^10} + \frac{dx}^9 k^9}{725760} - \frac{\cot dx}^9 k^10}{39916800} + O\cdot \frac{dx}^9 k^9}{11} + \frac{dx}^9$

Out[60]=

Out[61]= Rp ||
$$\frac{1}{6} e^{i \, dx \, k} \left(5 + 2 e^{-i \, dx \, k} - e^{i \, dx \, k} \right)$$

 $\label{eq:continuity} $$\operatorname{Out}(0)= Rp \mid \int \frac{1}{6} e^{i \cdot k} \left(2 e^{i \cdot k} \right) \left(2$

$$\text{Out} \text{[63]=} \quad Rp \; error \; \mid \mid \quad \frac{1}{12} \; \dot{\textit{i}} \; k^3 \; dx^3 \; - \; \frac{3 \, k^4 \, dx^4}{40} \; - \; \frac{3}{80} \; \dot{\textit{i}} \; k^5 \; dx^5 \; + \; \frac{23 \, k^6 \, dx^6}{1680} \; + \; \frac{41 \, \dot{\textit{i}} \; k^7 \, dx^7}{10080} \; - \; \frac{209 \, k^8 \, dx^8}{201600} \; - \; \frac{169 \, \dot{\textit{i}} \; k^9 \, dx^9}{725760} \; + \; \frac{89 \, k^{10} \, dx^{10}}{1900800} \; + \; O[dx]^{11} \; + \; O[dx]^{11} \; dx^4 \; dx^4 \; dx^4 \; + \; O[dx]^{11} \; dx^4 \; dx^4$$

Out[64]= Rp error ||

 $\label{eq:condition} $$ \frac{1}{12} i \text{d}^3 k^3-\frac{3 \ker(dx)^4 k^4}{40}-\frac{3}{80} i \text{d}^5 k^5+\frac{23 \ker(dx)^6 k^6}{1680}+\frac{41 i \det(dx)^7 k^7}{10080}-\frac{209 \det(dx)^8 k^8}{201600}-\frac{169 i \det(dx)^9 k^9}{725760}+\frac{89 \det(dx)^{10} k^{10}}{1900800}+O\left(\frac{dx}^7 1\right)\right) $$$

Out[65]=

$$\text{Out} \text{[G6]=} \quad GG2 \quad \text{[I]} \quad \frac{\left(9 - e^{-i\,dx\,k} + 9\,\,e^{i\,dx\,k} + e^{2\,i\,dx\,k}\right) \left(26 - 2\,\text{Cos}[dx\,k]\right)}{384 \left(H - \frac{H^3\,(-30 + 32\,\text{Cos}[dx\,k] - 2\,\text{Cos}[2\,dx\,k])}{36\,dx^2}\right)}$$

Out[67]= GG2 || $\frac{e^{-i \text{dx} k}+9 e^{i \text{dx} k}-e^{2 i \text{dx} k}+9 \text{right}}{26-2 \cos (\text{dx} k)}}{384 \left(H-\frac{H^3 (32 \cos (\text{dx} k)-2 \cos (2 \text{dx} k)-30)}{36 \text{dx} k}\right)}$

$$\text{Out[68]=} \quad GG2 \; error \; \mid \mid \quad \frac{\left(-243 \, k^4 - 49 \, H^2 \, k^6\right) dx^4}{960 \, H \, (3 + H^2 \, k^2)^2} - \frac{i \, \left(243 \, k^5 + 49 \, H^2 \, k^7\right) dx^5}{1920 \, H \, \left(3 + H^2 \, k^2\right)^2} + O[dx]^6$$

Out[69]= GG2 error ||

 $\label{left(49 H^2 k^6-243 k^4+right)} $$ H^2 k^7+243 k^5\right] + H^2 k^7+243 k^7+244 k^7+244 k^7+244 k^7+244 k^7+24$

Out[70]=

```
\text{Out} [71] = \quad Gn2 \quad || \quad \frac{\left(9 - e^{-i\,dx\,k} + 9\,\,e^{i\,dx\,k} - e^{2\,i\,dx\,k}\right)\,U\,(-26 + 2\,Cos[\,dx\,k])}{384\left(H - \frac{H^3\,(-30 + 32\,Cos[\,dx\,k] - 2\,Cos[\,2\,dx\,k])}{384}\right)}
Outf72|=Gn2 \parallel \frac{U \left(-e^{-i \left(x\right)} k\right)+9 e^{i \left(x\right)} k}{e^{-i \left(x\right)} k}+9 e^{i \left(x\right)} k}+9
                                           k)-26}{384 \left(H-\frac{H^3 (32 \cos (\text{dx} k)-2 \cos (2 \text{dx} k)-30)}{36 \text{dx}^2}\right)}
 \text{Out} \text{[73]= } \text{Gn2 error } \text{[]} \text{ } \frac{ \left( 243\,k^4 + 49\,H^2\,k^6 \right)\,U\,dx^4 }{ 960\,H\,(3 + H^2\,k^2)^2 } + \frac{i\left( 243\,k^5 + 49\,H^2\,k^7 \right)\,U\,dx^5 }{ 1920\,H\,(3 + H^2\,k^2)^2 } + O[dx]^6 
Out[74]= Gn2 error |
                             \frac{d^4 U \left(49 + 2 k^6 + 243 k^4 \right)}{960 H \left(49 + 2 k^2 + 3 \right)}{960 H \left(49 + 2 k^2 + 3 \right)} + \frac{1}{100}
                                          \left(49 \text{ H}^2 \text{ k}^7 + 243 \text{ k}^5\right) \left(1920 \text{ H} \left(4^2 \text{ k}^2 + 3\right)^2\right) + O\left(\frac{40 \text{ k}^2 + 3\right)^2}{1920 \text{ H}^2} + O\left(\frac{40 \text{ k}^2 + 3\right)^2}{1920 \text{ k}^2} + 
  ln[75]:= KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
                      KurFWS = KurF /. ap \rightarrow (U + Sqrt[g * H]) /. am \rightarrow (U - Sqrt[g * H]);
                      KurFWSeta =
                                 KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
                                       qm \rightarrow Rmp * n;
                      KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
                      Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1];
                      KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1];
                      Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
                      KfnG = KfnGp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
                      Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
                      Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 4}];
                      Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                      FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
                      FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 4}];
                      FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                      KurFWSG = KurFWS /. fp \rightarrow (U*Rpp*G + U*H*v + g*H*Rpp*n) /.
                                                  fm \rightarrow (U * Rmp * G + U * H * V + g * H * Rmp * n) /. qp \rightarrow Rpp * G /. qm \rightarrow Rmp * G;
                      KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
                      KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
                      KfGn = KfGnp /. Rpp \rightarrow Rp /. Rmp \rightarrow Rm /. GGp \rightarrow GG2 /. Gnp \rightarrow Gn2;
                      KfGG = KfGGp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
                      FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
                      FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 4}];
                      FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                        fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
                      FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
                      FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 4}];
```

 $FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];$

```
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2/2 + Fmat2.Fmat2.Fmat2/6;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
RKStep =
  Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 + EigvFmat2 * EigvFmat2 * EigvFmat2 / 6]/
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" -Sqrt(gH) < U < Sqrt(gH)"}]]</pre>
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn || ", KfGnp}]]
Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error || ", FGn2TAr}]]
Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG || ", KfGGp}]]
Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error || ", FGG2TAr}]]
Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error || ", RKstepTayr}]]
Text[Row[{"Omega error || ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||
                   ", EA}]]
Text[Row[{"EA || ", TeXForm[EA]}]]
Text[Row[{"Eerr || ", Eerr}]]
Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
```

Out[109]= -Sqrt(gH) < U < Sqrt(gH)

Out[110]=

Out[111]= Fnn ||
$$\frac{1}{2} \left(2 \operatorname{Gnp} H + \operatorname{Rpp} \left(-\sqrt{g H} + U \right) + \operatorname{Rmp} \left(\sqrt{g H} + U \right) \right)$$

 $\label{eq:output} \begin{tabular}{ll} $$\operatorname{Fnn} & \frac{1}{2} \left(\operatorname{Rmp} \left(\operatorname{Rmp} \right) + \operatorname{H-V-ight} \right) + \operatorname{H-V-ight} \right) $$ is the second of the se$

$$\begin{array}{ll} \text{Out} \text{[113]=} & Fnn \; error \; \mid \mid \; \left(-\frac{\left(H^2 \; k^3 \; U \; w \right) dt^2}{2 \left(3 + H^2 \; k^2 \right)} - \frac{i \; H^2 \; k^3 \; U \; w^2 \; dt^3}{6 \left(3 + H^2 \; k^2 \right)} + \frac{H^2 \; k^3 \; U \; w^3 \; dt^4}{24 \left(3 + H^2 \; k^2 \right)} + O[dt]^5 \right) + \\ & \left(-\frac{1}{12} \left(\sqrt{g \; H} \; \; k^4 \right) dt + O[dt]^5 \right) dx^3 + \left(\frac{i \left(45 \; k^5 \; U + 143 \; H^2 \; k^7 \; U + 32 \; H^4 \; k^9 \; U \right) dt}{960 \left(3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \\ \end{array}$$

Out[114]= Fnn error |

 $\left(-\frac{t}{2 \cdot t}\right)^2 \left(-\frac{t}{2 \cdot$ $\left(H^2 k^2+3\right)+\frac{dt}^4 H^2 k^3 U w^3}{24 \left(H^2 k^2 + 3\right)}$

 $k^2+3\left(\frac{1}{1}12\right)\left(\frac{1}12\right)\left(\frac$ \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i}{12 H^4 U k^9+143 H^2 U k^7+45 U $k^5 \right) \left(\frac{dt}{960 \left(\frac{A^2 k^2+3\right)^2}+O\left(\frac{dt}{5}\right) \right)}{1} + O\left(\frac{dt}{5}\right) \right) \left(\frac{dt}{5}\right) \left($

Out[115]=

Out[116]= $FnG \parallel GGp H$

Out[117]= $FnG \parallel \text{text}\{GGp\} H$

$$\text{Out} \\ \text{[118]= } FnG \ error \ || \ \left(-\frac{3 \, (k \, w) \, dt^2}{2 \, (3 + H^2 \, k^2)} - \frac{i \, k \, w^2 \, dt^3}{2 \, (3 + H^2 \, k^2)} + \frac{k \, w^3 \, dt^4}{8 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) \\ + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 +$$

 $\text{text}\{dt\}^3 \text{ k w}^2\}\{2 \left(H^2 \text{ k}^2 + 3\right)\} + \frac{dt}^4 \text{ k w}^3\}\{8 \left(H^2 \text{ k}^2 + 3\right)\}$ $k^2+3\right)+O\left(\frac{d4}{h^2}\right)+\left(\frac{d4}{h^2}\right)$ $\text{dt}{960 \left(H^2 k^2+3\right)^2}+O\left(\left(text{dt}^5\right)\right)+O\left(text{dt}^5\right)+O\left(text{dx}^5\right)+O\left(text{d$

Out[120]=

$$\text{Out} [\text{121}] = \text{ } FGn \text{ } || \text{ } \frac{1}{2} \left(g \text{ } H \text{ } (Rmp + Rpp) + \left(2 \text{ } Gnp \text{ } H + \sqrt{g \text{ } H} \text{ } (Rmp - Rpp) \right) U \right)$$

Out[122]= FGn ||

$$\begin{aligned} &\text{Out} \text{[123]=} & FGn \ error \ \parallel \ \left(-\frac{\left(k \left(3 \ g \ H + g \ H^3 \ k^2 - 3 \ U^2 \right) w \right) dt^2}{2 \left(3 + H^2 \ k^2 \right)} - \frac{i \ k \left(3 \ g \ H + g \ H^3 \ k^2 - 3 \ U^2 \right) w^2 \ dt^3}{6 \left(3 + H^2 \ k^2 \right)} + \frac{k \left(3 \ g \ H + g \ H^3 \ k^2 - 3 \ U^2 \right) w^3 \ dt^4}{24 \left(3 + H^2 \ k^2 \right)} + O[dt]^5 \right) + \\ & \left(-\frac{1}{12} \left(\sqrt{g \ H} \ k^4 \ U \right) dt + O[dt]^5 \right) dx^3 + \left(\frac{i \left(288 \ g \ H \ k^5 + 192 \ g \ H^3 \ k^7 + 32 \ g \ H^5 \ k^9 - 243 \ k^5 \ U^2 - 49 \ H^2 \ k^7 \ U^2 \right) dt}{960 \left(3 + H^2 \ k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \end{aligned}$$

Out[124]= FGn error |

 $\label{left-frac} $\left(-\frac{\left(\frac{4t}^2 \left(H^2 k^2+3\right)-\frac{4t}{2} \right)}{2 \left(H^2 k^2+3\right)}-\frac{4t}{3} k^2+3 g H-3 U^2\right)} \\ \left(-\frac{4t}^3 k^2-\frac{4t}^3 k^2+3 g H-3 U^2\right)}{6 \left(H^2 k^2+3\right)}+\frac{4t}^4 k w^3 \left(H^2 k^2+3 g H-3 U^2\right)}{6 \left(H^2 k^2+3\right)}+O\left(\frac{4t}^5\right)+\frac{4t}^4 k w^3 \left(H^2 k^2+3 g H-3 U^2\right)}{24 \left(H^2 k^2+3\right)}+O\left(\frac{4t}^5\right)+\frac{4t}^4 h^3 k^2+3 g H-3 U^2\right)}+\frac{4t}^4 h^3 k^2+3 g H-3 U^2\right)}+\frac{4t}^4 h^3 k^2+3 g H^3 k^4 U\right)+\frac{4t}^4 h^4 U\right)+\frac{4t}^4 U\right)+\frac{4t}{U}$

Out[125]=

Out[126]= FGG ||
$$\frac{1}{2} \left(\sqrt{g H} Rmp - \sqrt{g H} Rpp + (2 GGp H + Rmp + Rpp) U \right)$$

$$\begin{array}{ll} \text{Out} [\text{128}] = & FGG \; error \; \mid \mid \; \left(-\frac{\left(k \left(6 + H^2 \; k^2 \right) U \; w \right) \; dt^2}{2 \left(3 + H^2 \; k^2 \right)} - \frac{i \; k \left(6 + H^2 \; k^2 \right) U \; w^2 \; dt^3}{6 \left(3 + H^2 \; k^2 \right)} + \frac{k \left(6 + H^2 \; k^2 \right) U \; w^3 \; dt^4}{24 \left(3 + H^2 \; k^2 \right)} + O[dt]^5 \right) + \\ & \left(-\frac{1}{12} \left(\sqrt{g \; H} \; \; k^4 \right) dt + O[dt]^5 \right) dx^3 + \left(\frac{i \left(531 \; k^5 \; U + 241 \; H^2 \; k^7 \; U + 32 \; H^4 \; k^9 \; U \right) dt}{960 \left(3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \\ \end{array}$$

Out[129]= FGG error ||

 $\label{left-deft} $$\left(-\frac{\hat U w \left(\frac{4t}^2 k^2+6\right)}{2 \left(\frac{4t}^2 k^2+3\right)}-\frac{i \left(\frac{4t}^3 k U w^2 \left(\frac{4t}^2 k^2+6\right)}{4 k U w^3 \left(\frac{4t}^2 k^2+6\right)}\right)} \right) $$\left(\frac{4t}^2 k^2+6\right)}{4 k U w^3 \left(\frac{4t}^2 k^2+6\right)} \left(\frac{4t}^2 k^2+6\right)} \left(\frac{4t}^2 k^2+6\right)}{24 \left(\frac{4t}^2 k^2+3\right)} +O\left(\frac{4t}^2 k^2+3\right)} +O\left(\frac{4t}^2 k^2+3\right)} \right) $$\left(\frac{4t}^2 k^2+3\right)} +O\left(\frac{4t}^2 k^2+3\right)} +O\left(\frac{4t}^2 k^2+3\right)} \left(\frac{4t}^4 k U k^3+4 U k^3+4 U k^3+24 U k^$

Out[130]=

Out[131]=

Out[132]= Omega error ||

$$\left\{ \left(-\frac{1}{24(3+H^2k^2)^3} i \, k^4 \left(\sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + \left(3 + H^2\,k^2 \right) \, U \right) \left(3 \, g \left(\sqrt{3} \, H \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + 9 \, H \, U + 3 \, H^3 \, k^2 \, U \right) + U^2 \left(H^4 \, k^4 \, U + 9 \left(\sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + U \right) + 3 \, k^2 \left(\sqrt{3} \, \sqrt{g \, H^5 \, (3+H^2\,k^2)} \right. + 2 \, H^2 \, U \right) \right) \right) dt^3 + U^2 \left(H^4 \, k^4 \, U + 9 \left(\sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + \left. (3+H^2\,k^2) \, U \right) \right) + U^3 \left(3 \, H^2 \, k^2 \right) + 2 \, H^2 \, U \right) \right) dt^3 + U^3 \left(12 \, \sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + \left. (3+H^2\,k^2) \, U \right) + U^3 \left(12 \, \sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + 9 \, U + H^4 \, k^4 \, U + 2 \, k^2 \left(2 \, \sqrt{3} \, \sqrt{g \, H^5 \, (3+H^2\,k^2)} \right. + 3 \, H^2 \, U \right) \right) \right) dt^4 + U^3 \left(12 \, \sqrt{g \, H \, (3+H^2\,k^2)} \right) dt^3 + U^3 \, U^$$

$$\begin{array}{c} {}^{3} K^{*}\left(6 \vee g \cdot H^{*}\left(3 + H^{*}\left(K^{*}\right)\right) + V + V \cdot g \cdot H^{*}\left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + \sqrt{3} \cdot H^{9} \cdot U\right) + V \cdot g \cdot H^{9} \cdot H^{9} \cdot \left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + \sqrt{3} \cdot H^{9} \cdot U\right) + V \cdot g \cdot H^{9} \cdot \left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + \sqrt{3} \cdot H^{9} \cdot U\right) + V \cdot g \cdot H^{9} \cdot \left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + \sqrt{3} \cdot H^{9} \cdot U\right) + V \cdot g \cdot H^{9} \cdot \left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + V \cdot \left(63 + 33 \cdot H^{2} \cdot K^{2}\right) \cdot U^{4} + \sqrt{3} \cdot U^{5}\right) + V \cdot g \cdot H^{9} \cdot \left(6 \vee g^{3} \cdot H^{3}\left(3 + H^{2} \cdot K^{2}\right) + 6 \vee g \cdot H^{3}\left(3 + H^{2} \cdot K^{2}\right) \cdot U^{4} + \sqrt{3} \cdot U^{5}\right) + V \cdot g \cdot H^{9} \cdot \left(6 \vee g^{3} \cdot H^{3}\left(3 + H^{2} \cdot K^{2}\right) + 1728 \cdot U + 192 \cdot H^{4} \cdot K^{2} \cdot U^{4} + V \cdot 3 \cdot H^{2} \cdot U^{3}\right)\right) dt^{4} + O(dt)^{5} \cdot dx^{3} + V \cdot g \cdot H^{3} \cdot \left(6 \vee 3 \cdot 4 \cdot H^{2} \cdot K^{2}\right) \cdot U^{2} + 2 \vee 2 \vee 2 \cdot H^{3} \cdot \left(721 \sqrt{3} \cdot g \cdot H^{7} \cdot 4 \cdot H^{2} \cdot K^{2}\right) + 1152 \cdot H^{2} \cdot U\right)\right) + V \cdot g \cdot \left(145 \sqrt{g} \cdot H^{9} \cdot (3 + H^{2} \cdot K^{2}) + 1350 \sqrt{3} \cdot g^{2} \right) + W^{4} \cdot U + 2118 \sqrt{g} \cdot H^{9} \cdot (3 + H^{2} \cdot K^{2}) \cdot U^{2} + 22227 \sqrt{3} \cdot g \cdot H^{3} \cdot U^{3} + 576 \sqrt{g} \cdot H^{3} \cdot (3 + H^{2} \cdot K^{2}) \cdot U^{3}\right) + W^{4} \cdot H^{2} \cdot H^{2}$$

$$O[dil]^{5} + \left[-\frac{ik^{4} \left[2 \pm H(2+H^{2}k^{2}) - \sqrt{3} \sqrt{g \pm H(2+H^{2}k^{2})} \right]}{24 \sqrt{g + H^{2}(3+H^{2}k^{2})}} + \frac{1}{144(3+H^{2}k^{2})} + \sqrt{3} \right] + \frac{1}{144(3+H^{2}k^{2})^{3/2}}$$

$$k^{7} \left(6\sqrt{3} \right) \frac{2^{2}}{g^{2}} H^{2} \left(3 + H^{2}k^{2} \right) + 9 U^{3} \left(-5\sqrt{g H} \left(3 + H^{2}k^{2} \right) + \sqrt{3} \right) U \right) + \frac{1}{144(3+H^{2}k^{2})} \left(-2\sqrt{g H^{9}} \left(3 + H^{2}k^{2} \right) + \sqrt{3} \right) H^{2} U \right) + \frac{1}{3} \frac{1}{3} H^{2} U \left(-21\sqrt{g H} \left(3 + H^{2}k^{2} \right) + \sqrt{3} \right) \left(27 + 15 H^{2}k^{2} + 2 H^{4}k^{4} \right) U \right) - \frac{1}{3} k^{2} \left(6\sqrt{g^{3}} H^{7} \left(3 + H^{2}k^{2} \right) U + 7\sqrt{g H^{3}} \left(3 + H^{2}k^{2} \right) U^{3} - 2\sqrt{3} H^{2}U^{4} \right) \right) dt^{3} + \frac{1}{144(3+H^{2}k^{2})^{2}} k^{2} \left(3\sqrt{3} \right) \frac{g^{2}}{g^{2}} H^{2} \left(27 + 8 H^{2}k^{2} \right) U + k^{2}U^{4} \left(-2\sqrt{g H^{9}} \left(3 + H^{2}k^{2} \right) U + \sqrt{3} H^{4}U \right) + \frac{2}{g H^{12}} \left(-2\sqrt{g^{2}} H^{3} \left(3 + H^{2}k^{2} \right) U + \sqrt{3} H^{2}U^{3} \right) \right) dt^{4} + O[dt]^{3} \right] dt^{3} + \frac{2}{g H^{12}} \left(3 + H^{2}k^{2} \right) U^{2} - 6\sqrt{g H} \left(3 + H^{2}k^{2} \right) U^{4} + \sqrt{3} U^{5} \right) - \frac{1}{34500\sqrt{g H}} \left(3 + H^{2}k^{2} \right) U^{2} + 4\sqrt{g H^{5}} \left(3 + H^{2}k^{2} \right) U^{4} - \sqrt{3} H^{2}U^{3} \right) \right) dt^{4} + O[dt]^{3} \right] dt^{3} + \frac{1}{g^{2}} \left(1 + \frac{1}{3} \left(1 + \frac{1}$$

 $\left(\frac{h^4 \left(\frac{h^2 k^2+3\right)} U+\sqrt{1} u}{1}\right) U+\sqrt{1} u}$ $k^4+3 \left(U H^2+\sqrt{3} \right) \left(U H^2+\sqrt{3} \right)$ $H \left(H^2 k^2 + 3 \right) \right) \right) H \left(H^2 k^2 + 3 \right) \left(H^2 k^2 + 3 \right)$ $U+\sqrt{3} \sqrt{4+2 \left(H^2 k^2+3\right)}\right) \left(H^4 U k^4+2 H^2 U H^2+2 \right)$ $\$ \left(H^2 k^2+3\right)\right) \k^2+9 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right)}\right) U^3+6 g H \left($H^2 k^2+3\right$ \right) U+2 \sqrt{3} \sqrt{g H \left($H^2 k^2+3\right$ \right)\right) U+9 g^2 $H^2\left(\frac{dt}^4}{30 \left(\frac{k^2 + 3 \right)}{3} + O\left(\frac{t}{t}^5 \right) + \left(\frac{k^4 \left(\frac{k^4 \left(\frac{k^4 + k^4 \right)}{3} + O\left(\frac{k^4 \left(\frac{k^4 \left(\frac{k^4 + k^4 \left(\frac{k^4 \left(\frac{k^4 \left(\frac{k^4 + k^4 \left(\frac{k^4 \left(\frac{k^4 + k^4 \left(\frac{k^4 \left(k^4 \left(\frac{k^4 \left(k^4 \left(\frac{k^4 \left(\frac{k^4 \left(\frac{k^4 \left(k^4 \left(\frac{k^4 \left(\frac{k^4 \left(k^4 \left(\frac{k^4 \left(\frac{k^4 \left(k^4 \left(\frac{k^4 \left(k^4 \left(\frac{k^4 \left(k^4 \left(\frac{k^4 \left(k^4 \left(k^4 \left(\frac{k^4 \left(k^4 \left(k^4 \left(k^4 \left(\frac{k^4 \left(k^4 \left(k^4 \left(k^4 \left(k^4 \left(k^4 \left(\frac{k^4 \left(k^4 \left(k^4 \left(k^4 \left(k^4 \left(\frac{k^4 \left(k^4 \right)} \left(k^4 \left(k^4$ $g H \left(H^2 k^2 + 3 \right) + \left(H^2 k^2 + 3 \right) U \left(H^2 k^2 + 3 \right) U$ $\label{eq:k-2+3-right} $$ k^2+3\right-\frac{h^2 \left(k^7 \left(k^7 \left(k^7 \left(k^7 \left(k^4 + 2 \left(k^4 + k^2 + k^4 + k$ $k^4+3 \left(\frac{3}{4^2} \right) U^4+7 \left(\frac{4^2 k^2+3 \right) U^3+6 \left(\frac{3}{4^2} \right) U^3+6 \left(\frac{3}{4^2$ $k^2+3 \right) U \right) U \right) + 2 \left(k^2+3 \right) U \right) + 2 \left(k^2+3 \right) + 2 \left(k^2+3 \right) + 9 \left(k^2+3 \right) + 9$ H \left(H^2 k^2+3\right)\right)+3 g H U \left(\sqrt{3} \left(2 H^4 k^4+15 H^2 k^2+27\right) U+21 $\left(\frac{H^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)}\right)$ \left(4 H^4 k^4+33 H^2 k^2+63\right) U+72 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 \sqrt{3} g^2 H^2 \left(8 H^2 k^2+27\right) U+9 \left(\sqrt{3} U^5+6 \sqrt{g H \left(H^2 k^2+3\right)} U^4+2 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)\right)+6 k^2 \left(\sqrt{3} H^2 U^5+4 \sqrt{g H^5 \left(H^2 $k^2+3\right) U^4+6 \left(y^3 H^7 \left(h^2 k^2+3\right) U^2\right) U^2\right) \left(y^4+6 \left(y^3 H^7 \left(h^2 k^2+3\right) \right) U^2\right) U^2\right)$ $k^2+3\left(-\frac{k^5}{19}+O\left(\frac{19}{2}\right)^{5/2}\right)+O\left(\frac{19}{2}\right)^{5/2}+O\left($ U H^2+145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)\right) k^2+1728 U+531 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} $k^2+3\right)/100$ \sqrt{g H^{13} \left(H^2 k^2+3\right)} U\right) k^6+3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3}) $U+1011 \sqrt{H^2 k^2+3\right} + H^5+576 \sqrt{H^2 k^2+3\right} + U+1011 \sqrt{H^2 k^2+3} + U+1011$ $k^4+9 \left(1350 \right) q^2 U H^4+2227 \right] U^3 H^3+576 \left(H^2 k^2+3\right)$ $U^4+2118 \sqrt{4^2 + 145 \sqrt{g^5 H^9 \left(\frac{h^2 k^2+3\right)}} U^2+145 \sqrt{g^5 H^9 \left(\frac{h^2 k^2+3\right)}\right)}$ $k^2+81 \left(64 \right) U^4+g H\left(65 \right) U^4+g H\left(65 \right) U^4+g H\left(65 \right) U^4+g H\left(65 \right) U^4+g U$ k^2+3\right)\right) U^2+241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)\right)\right)\right) $\text{dt}^3}{34560 \operatorname{gH} \left(\frac{^2 k^2+3\right)^{7/2}}-\frac{(k^9 \left(\frac{9 \operatorname{gH}}{2} \right)^{3/2}}{2} \right)}$ H^2 k^2+531\right) H^3+54 \sqrt{3} g^2 \left(91 H^4 k^4+578 H^2 k^2+915\right) U^2 H^2+g U^3 $\left(913 \right) + 16 U k^6 + 3 \left(2771 \right) + 1732 \left(4^2 + 1732 \right) + 1841$ k^4+25227 \sqrt{3} H^2 U k^2+405 \left(63 \sqrt{3} U+124 \sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right) $H+12 U \left(16 \right) H+12 U \left(16 \right) H-13 \left(16$ U^4 k^4+3 \left(144 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+898 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} $U^2+193 \cdot qrt\{g^5 + h^9 \cdot (h^2 k^2+3 \cdot ght)\} \cdot h^2+432 \cdot qrt\{g + \left(h^2 k^2+3 \cdot ght\right)\} \cdot h^2+2025 \cdot h^2 \cdot h$ $\left(\frac{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \$ $\label{left(H^2 k^2+3\left(H^4 U-\sqrt{3} \right) \left(H^4 U$ $k^4-3 \left(\frac{3} \right) \left(\frac{4^2 + 3^2 + 3^2 + 4^2 +$ $\left(H^2 k^2+3\right) U-3 g H \left(H^2 k^2+3\right) U-3 g$ $k^2+3\left(\frac{t}^3}{24 \left(\frac{t^2}{k^2+3\right)}^3}+\frac{k^5\left(\frac{t^2}{k^5}\right)^3}{k^5\left(\frac{t^2}{k^5}\right)^3} + \frac{k^5\left(\frac{t^2}{k^5}\right)^3}{k^5\left(\frac{t^2}{k^5}\right)^3} + \frac{k^5\left(\frac{t^2}{k^5}\right)^3}{k^5\left(\frac{t^2}{$ $U-\sqrt{3} \sqrt{4 + \left(\frac{H^2 U^4 - 4 \cdot (H^2 U^4 - 4 \cdot (H^4 U^4 - 4 \cdot (H^$ $H^5 \left(H^2 k^2+3\right) \right) + U-12 \left(H^2 k^2+3\right) \right) + U-12 \left(H^2 k^2+3\right) \right) + U-12 \left(H^2 k^2+3\right) +$ TT \1_0/2 \1_0/2TT02 1-02 - 2\...'-1.0 TT - 2 \-....(2) \-....(-TT \1_0/TT02 1-02 - 2\...'-1.0)\...'-1.0 TT - 0 - 02 TT02\...'-1.0

 $H \left(H \right) \left$ $\text{dt}^4{30 \left(H^2 k^2+3\right)}+O\left(\left(text{dt}^5\right)\right)+\left(-\left(text{dt}^6\right)\right)+O\left(text{dt}^6\right)}+O\left(text{dt}^6\right)+O\left(te$ $H \left(H^2 k^2+3\right) - \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) + \left($ $k^2+3\right)+\frac{k^7 \left(U^3 \left(V^3 \left(V^4 U-2 \right)^2 H^9 \left(H^2 k^2+3\right)\right)\right)}{H^4 U-2 \left(H^2 k^2+3\right)}$ $k^4-3 \left(-2 \right) H^2 U^4+7 \left(H^2 k^2+3 \right) U^3+6 U^3+6$ $H \left(H^2 k^2 + 3\right) \right) \ g \ H \ U \left(\frac{3}{4} \left(H^4 k^4 + 15 H^2 k^2 + 27\right) \right) \ U-21$ $\$ \\left(H^2 k^2+3\right)\\right)\\right)\\right)\\right)\\ \text{dt}^3\{144 \\left(H^2 k^2+3\right)^{5/2}}+\\\frac{i}{} $k^8 \left(\frac{4 \left(\frac{4}{1} \right) U^4 + 2 g H}{1} \right) U^4 + 2 g H^6 \left(\frac{4}{1} \right) U^4 + 2 g H^$ $\left(4 + ^4 + ^4 + ^3 + ^2 + ^2 + ^3\right) U - 72 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^4\right) U - 24 \left(4 + ^4 + ^4\right) U - 24 \left(4 + ^4 + ^4\right) U - 24 \left(4 + ^4\right) U$ g^2 H^2 \left(8 H^2 k^2+27\right) U+9 \left(\sqrt{3} U^5-6 \sqrt{g H \left(H^2 k^2+3\right)} U^4-2 $k^2+3\right) U^4+6 \left(y^3 H^7 \left(h^2 k^2+3\right) U^2\right) U^2\right) \left(y^4+6 \left(y^3 H^7 \left(h^2 k^2+3\right) \right) U^2\right) U^2\right)$ $k^2+3\right)^{5/2}+O\left(text{dt}^5\right)\right)$ $k^4+\left(145 \right)^{15} \left(145 \right)^{$ $\left(H^2 k^2+3\right)\right)$ $\sqrt{9} \left(\frac{4^2 h^6+g}{13} \right) \left(\frac{4^2 h^6+g}{13} \right) U-721 \left(\frac{3}{g} \right) H^7\right) h^6-3 U \left(\frac{627 \left(\frac{3}{g} \right) H^6+g}{13} \right)$ U \left(2195 \sqrt{3} U-1011 \sqrt{g H \left(H^2 k^2+3\right)\right) \hat\text{hright)}\right) \hat\text{hright} H^5-576 \sqrt{g H^9 \left(H^2 k^2+3\right)} $k^2+3\right) U^3\right) V^3+9 \left(-1350 \right) 13 Y U^3\right) 13 Y U^3\right)$ $H^5 \left(H^2 k^2+3\right) U^4+2118 \left(H^2 k^2+3\right) U^2+145 \left(H^2 k^2+3\right) U$ $\left(\frac{4 \operatorname{ft}(H^2 k^2+3\right)} \right) h^2+81 \left(\frac{64 \operatorname{ft}(H^2 k^2+3\right)} U^4+g H \right)$ \sqrt{3} g^3 \left(145 H^2 k^2+531\right) H^3+54 \sqrt{3} g^2 \left(91 H^4 k^4+578 H^2 k^2+915\right) U^2 H^2+g U^3 \left(913 \sqrt{3} H^6 U k^6+3 \left(2771 \sqrt{3} H^4 U-1732 \sqrt{g H^9 \left(H^2 $k^2+3\right) \ h^4+25227 \ h^2 U \ h^2+405 \ h^3 \ U-124 \ h^2 U \ h^2$ k^2+3\right)\right)\right) H-12 U \left(16 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^4 k^6+144 \sqrt{g H^9} \left(H^2 k^2+3\right)} U^4 k^4+3 \left(144 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+898 \sqrt{g^3 H^7} $\left(\frac{H^2 k^2+3\right)}{U^2+193 \left(\frac{h^2 k^2+3\right)} L^2+193 \left(\frac{h^2 k^2+3\right)}{L^2+193 \left(\frac{h^2 k^2+3\right)} L^2+193 \left(\frac{h^2 k^2+3\right)}{L^2+193 \left(\frac{h^2 k^2+3\right)}{L^$ $k^2+3\right) U^4+2025 \sqrt{6^5 H^5 \left(H^2 k^2+3\right)}\right) \left(14+2025 \right) 1$ $\left(\frac{4x}{2} + \frac{4x}{4}\right)^{7/2}+O\left(\frac{4x}{5}\right)\right)$

 $\text{Out} [\text{135}] = \ EA \ || \ \left. \left\{ \left\{ \frac{-H^2 \, k^2 \, \left(\left(-1 + e^{i \, dt \, w} \right) k \, U - w \right) + 3 \, w}{\left(3 + H^2 \, k^2 \right) w}, \ - \frac{3 \, \left(-1 + e^{i \, dt \, w} \right) k}{\left(3 + H^2 \, k^2 \right) w} \right\}, \ \left\{ - \frac{\left(-1 + e^{i \, dt \, w} \right) k \left(g \, H \, \left(3 + H^2 \, k^2 \right) - 3 \, U^2 \right)}{\left(3 + H^2 \, k^2 \right) w}, \ 1 \, - \frac{\left(-1 + e^{i \, dt \, w} \right) k \left(6 + H^2 \, k^2 \right) U}{\left(3 + H^2 \, k^2 \right) w} \right\} \right\}$ Out[136]= EA || \left(\begin{array}{cc} $\frac{3 w-H^2 k^2 \left(\left(-1+e^{i \cdot text \cdot dt} w\right)\right)}{\left(-1+e^{i \cdot text \cdot dt} w\right)} k U-w\right)}{\left(-1+e^{i \cdot text \cdot dt} w\right)}$

 $k^2+3\right) w$ & $-\frac{3 \left(1+e^{i \left(t\right) w}\right) k}{\left(t^2 k^2+3\right) w} \$ $-\frac{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}$ \end{array}

\right)

Out[134]=

Out[138]= Eerr || \left(

\begin{array}{cc}

 $\label{left(-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right) text{dt}^2} \\ 2 \left(\frac{H^2 u^2 k^2-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right) + \frac{1}{2} \left(\frac{H^2 u^2 k^2-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right) \\ + \frac{1}{2} \left(\frac{H^2 u^2 k^2-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right) \\ + \frac{1}{2} \left(\frac{H^2 u^2 k^2-H^2 U w k^3+3 U w k^3+3 U^2 k^2-3 g H k^2\right) \\ + \frac{1}{2} \left(\frac{H^2 u^2 k^2-H^2 U w k^3+3 U w k^3+$ \left(H^4 U^3 k^7-3 H^2 U^3 k^5-H^4 U w^2 k^5+9 g H^3 U k^5-18 U^3 k^3-3 H^2 U w^2 k^3+18

 $\label{left} $$k^2+3\right)+O\left(\frac{t}{5}\right)+\left(\frac{1}{12}\left(\frac{1}{12}\right)+\frac{t}{12}\right).$$$ $\$ \\sqrt{g H} \\left(2 H^2 k^7+3 k^5\right) U \\text{dt}^2\{24 \\left(H^2 k^2+3\right)}+\\frac{\\sqrt{g H}} $\left(H^4 U^2 k^{10}\right) + g H^3 k^8 + 3 H^2 U^2 k^8 - 3 U^2 k^6 + 9 g H k^6 \right) \left(t^3\right)$ \left(H^2 k^2+3\right)^2}+O\left(\text{dt}\^5\right)\right)\text{dx}\^3+\left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 H^4 U^2 k^{10}+145 g H^3 k^8+94 H^2 U^2 k^8-486 U^2 k^6+531 g H k^6\right) \text{dt}^2}{1920 \left(H^2 k^2+3\right)^2}-\frac{i k^7 \left(32 H^6 U^3 k^6+47 H^4 U^3 k^4+241 g H^5 U k^4-678 H^2 U^3 k^2+1348 g H^3 U k^2-1593 U^3+1683 g H U\right) \text{dt}^3}{1920 \left(H^2 $\label{left} $$k^2+3\right)^3}+O\left(\frac{dx}^5\right)^3+O\left(\frac{dx}$ $U k^2 + w k (3 h^2) (4 k^2) (4 k^2)$ $w^2 k^3 + 3 g H k^3 - 3 w^2 k \right) \left(\frac{dt}^3}{2 \left(\frac{k^2 + 3 \right)^2}{+ \frac{k^3 - 3 w^2 k \right)^4}} \right) \left(\frac{dt}^4}{8} \right) \left(\frac{k^3 + 3 g H k^3 - 3 w^2 k \right) \left(\frac{k^3 + 3 g H k^3 - 3 w^2 k \right)^4}{2 \left(\frac{k^3 + 3 g H k^3 + 3 w^2 k \right)^4}}}$ $\left(H^2 k^2+3\right)+O\left(\frac{dt}^5\right)+\left(\frac{dt}^5\right)\right)+\left(\frac{dt}^2\right)$ $\label{left(H^2 k^2+3\wedge inght)^2} $$\left(H^2 k^2+3\right)\right) \operatorname{left(H^2 k^2+3\wedge inght)} \operatorname{left(H^2 k^2+3\wedge$ $\left(49 \text{ H}^2 \text{ k}^7 + 243 \text{ k}^5\right) \left(49 \text{ H}^2 \text{ k}^2 + 3\right)^2 + \frac{145 \text{ H}^2}{2} + \frac{145 \text{ H}^2}{2$ k^8+531 k^6\right) U \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(241 H^4 U^2)} k^{11}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7\right) \text{dt}^3}{1920 $\label{left(H^2 k^2+3\right)^3} + O\left(\frac{dt}^5\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^5\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^5\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^4\right) \cdot \left(\frac{dx}^4\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^4\right) \cdot \left(\frac{dx}^4\right) \cdot \left($ $\left(-\frac{h^2}{u^2}\right) \left(-\frac{h^2}{u^2}\right) \left(-\frac$

k^2+3\right)}+\left(\frac{i \left(g H^5 U^2 k^7+g^2 H^4 k^5-3 H^2 U^4 k^5+6 g H^3 U^2 k^5-9 U^4 k^3+3 U^2 k^5-9 U^4 k^3+3 U^2 k^5-9 U^4 k^5+6 U^2 k^5-9 U^4 $g^2 H^2 k^3+6 g H U^2 k^3 \left(\frac{h^2 k^2+3\right)}{2 \left(\frac{h^2 k^2+3\right)}{2}-\frac{h^2 k^3+6 g H U^2 k^3\right)}$ $U^2\left(\frac{h^2 k^2+3\right)}{t} \left(\frac{h^2 k^2+3\right)}{t} \right) + \frac{h^2 k^2+3\right) + \frac{h^2 k^2+3\right) + \frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)}{t}}{t}$ $U^2\right) w^3 \text{-} w^3$ $\left(\frac{H}{k^4} \right) \left(\frac{H^2 \left(\frac{H^2}{k^7 + H^3} \right) \left(\frac{H}{k^7 + H$ g H \sqrt{g H} k^5 \right) \text{dt}^2}{12 \left(\frac{h^2 k^2+3 \right)}{+ \frac{h^4 \left(\frac{h^4 \sqrt{dt}}{2} \right)}{12 \left(\frac{h^2 k^2+3 \right)}{+ \frac{h^4 \sqrt{dt}}{2} \right)}} k^{10}+2 g H^5 \sqrt{g H} U k^{10}+14 g H^3 \sqrt{g H} U k^8-12 \sqrt{g H} U^3 k^6+24 g H $\$ \\ \sqrt{g H} U \\ k^6\\ right) \\ text{dt}^3\{24 \\ left(H^2 k^2+3\\ right)^2\}+O\\ left(\\ text{dt}\^5\\ right)\\ right) \text{dx}^3+\left(\frac{i \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right)\text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 g H^5 U k^{10}-145 H^2 U^3 k^8+384)} g H^3 U k^8-531 U^3 k^6+576 g H U k^6\right) \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i} \left(288 g H^7 U^2 k^{13}+241 g^2 H^6 k^{11}-723 H^4 U^4 k^{11}+2592 g H^5 U^2 k^{11}+1542 g^2 H^4 k^9-4626 H^2 U^4 k^9+7194 g H^3 U^2 k^9-7371 U^4 k^7+2457 g^2 H^2 k^7+5454 g H U² k⁷\right) \text{dt}³{5760 \left(H² k²+3\right)³}+O\left(\text{dt}⁵\right)\right) $\t (dx)^4 + O(\left(\frac{dx}^5 \right)^5 \right) \le \left(\frac{H^2 U^2 k^4 - H^2 U w k^3 - 9 U^2 k^2 - 3 g}{H^2 U^2 k^4 - H^2 U w k^3 - 9 U^2 k^2 - 3 g} \right)$ k^5-H^4 U w^2 k^5+9 g H^3 U k^5+36 U^3 k^3-9 H^2 U w^2 k^3+36 g H U k^3-18 U w^2 k\right) $\label{left} $$k^2+3\right)+O\left(\frac{t}{5}\right)+\left(\frac{1}{12}\left(\frac{1}{12}\right)+\frac{t}{12}\right).$$$ $U^2 \ k^{10} + 15 \ H^2 \ sqrt{g H} \ U^2 \ k^8 + 3 \ g \ H^3 \ sqrt{g H} \ k^8 + 39 \ sqrt{g H} \ U^2 \ k^6 + 9 \ g \ H \ sqrt{g H}$ $\left(32 \text{ H}^4 \text{ U k}^9+241 \text{ H}^2 \text{ U k}^7+531 \text{ U k}^5\right) \left(4t\right) \left($

H⁴ U² k⁶+531 g H³ k⁸+674 H² U² k⁸+1638 U² k⁶+531 g H k⁶right) \text{dt}²{1920 \left(H^2 k^2+3\right)^2}-\frac{i \left(32 H^6 U^3 k^{13}+529 H^4 U^3 k^{11}+241 g H^5 U k^{11}+2406 H^2 U^3 k^9+1736 g H^3 U k^9+3321 U^3 k^7+3231 g H U k^7\right) \text{dt}^3}{1920 $\left(\frac{dx}^4+O\left(\frac{dx}^5\right)\right) \$

```
\end{array}
      \right)
In[139]:= KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
      KurFWS = KurF /. ap \rightarrow (U + Sqrt[g * H]) /. am \rightarrow 0;
      KurFWSeta =
         KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
           qm \rightarrow Rmp * n;
      KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
        Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1]; 
      KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1];
      Kfnn = Kfnnp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
      KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
      Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
      Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 4}];
      Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
      FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
      FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 4}];
      FnG2TAr = Refine[FnG2TA , \{k > 0, U > 0, H > 0, g > 0\}];
      \texttt{KurFWSG} = \texttt{KurFWS} \ / . \ \texttt{fp} \ \rightarrow \ (\texttt{U} * \texttt{Rpp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rpp} * \texttt{n}) \ / .
             fm \rightarrow (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp \rightarrow Rpp * G /. qm \rightarrow Rmp * G;
      KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
      KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
      KfGGp = FullSimplify[KurFWSG /. n \rightarrow 0 /. G \rightarrow 1];
      KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
      KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
      FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
      FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 4}];
      FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
      fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g*H]) / (2) * (Rp - Rm);
      FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
      FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 4}];
      FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
      Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
      Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2/2 + Fmat2.Fmat2.Fmat2/6;
      Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
      EigvFmat2 = Eigenvalues[Fmat2];
```

```
RKStep =
  Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 + EigvFmat2 * EigvFmat2 * EigvFmat2 / 6]/
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" U > -Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn || ", KfGnp}]]
Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error || ", FGn2TAr}]]
Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG || ", KfGGp}]]
Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error || ", FGG2TAr}]]
Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error || ", RKstepTayr}]]
Text[Row[{"Omega error || ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA || ", EA}]]
Text[Row[{"EA || ", TeXForm[EA]}]]
Text[Row[{"Eerr || ", Eerr}]]
Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
```

Out[174]=

Out[175]= Fnn || Gnp H + Rmp U

Out[176]= Fnn || $\text{text}\{Gnp\}\ H+\text{text}\{Rmp\}\ U$

$$\begin{array}{ll} \text{Out} \text{[177]=} & Fnn \; error \; \mid \mid \; \left(-\frac{\left(H^2 \, k^3 \, U \, w \right) \, dt^2}{2 \, \left(3 + H^2 \, k^2 \right)} \, - \, \frac{i \, H^2 \, k^3 \, U \, w^2 \, dt^3}{6 \, \left(3 + H^2 \, k^2 \right)} \, + \, \frac{H^2 \, k^3 \, U \, w^3 \, dt^4}{24 \, \left(3 + H^2 \, k^2 \right)} \, + \, O[dt]^5 \right) + \\ & \left(-\frac{1}{12} \, \left(k^4 \, U \right) dt \, + \, O[dt]^5 \right) dx^3 \, + \left(\frac{i \, \left(45 \, k^5 \, U + 143 \, H^2 \, k^7 \, U + 32 \, H^4 \, k^9 \, U \right) dt}{960 \, \left(3 + H^2 \, k^2 \right)^2} \, + \, O[dt]^5 \right) dx^4 \, + \, O[dx]^5 \\ \end{array}$$

Out[178]= Fnn error |

 $\left(-\frac{t}{2 \cdot t}\right)^2 \left(-\frac{t}{2 \cdot$ $\left(H^2 k^2+3\right)+\frac{dt}^4 H^2 k^3 U w^3}{24 \left(H^2 k^2 + H^2 k^3 U w^3\right)}$ $\label{eq:left} $$ \operatorname{dt}-O\left(\frac{dt}^5\right)^{\frac{dt}{2}} + \operatorname{dt}^4 \operatorname{dt}^4 \left(\frac{32 H^4 U k^9 + 143 H^2 U k^7 + 45 U k^7 + 45 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^9 + 143 H^2 U k^7 + 45 U k^9 + 143 H^2 U k^9 + 144 U k^9 + 143 H^$ $k^5 \right) \left(\frac{d^2 + \sqrt{dt}}{960 \left(\frac{h^2 k^2 + 3\right)}} \right) \left(\frac{h^2 k^2 + 3\right)}{2} + O\left(\frac{h^2 k^2 +$

Out[179]=

Out[180]= FnG || GGp H

Out[181]= $FnG \parallel \text{text}\{GGp\} H$

$$\text{Out} [\text{182}] = \quad FnG \; error \; \mid \mid \; \left(-\frac{3 \; (k \; w) \; dt^2}{2 \left(3 + H^2 \; k^2 \right)} - \frac{i \; k \; w^2 \; dt^3}{2 \left(3 + H^2 \; k^2 \right)} + \frac{k \; w^3 \; dt^4}{8 \left(3 + H^2 \; k^2 \right)} + O[dt]^5 \right) \\ + \left(\frac{i \left(243 \; k^5 + 49 \; H^2 \; k^7 \right) dt}{960 \left(3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5$$

 $Out|183| = FnG error || \left| \left| \frac{1}{2} \left(w \right) \right|^2 \left| \left| \frac{4}{2} \left(w \right) \right|^2 \right|$ $\text{text}\{dt\}^3 \text{ k w}^2\}\{2 \left(H^2 \text{ k}^2+3\right)+\frac{dt}^4 \text{ k w}^3\}\{8 \left(H^2 \text{ k}^2+3\right)+\frac{dt}^4 \text{ k w}^3\}\{8 \right)$ $k^2+3\left(\frac{dt}^5\right)+O\left(\frac{dt}^5\right)+\frac{dt}^5\right)$ $\text{text}_{dt}^{960} \left(\frac{A^2+3\right)^2}+O\left(\frac{dt}{5}\right)+O\left(\frac{A^2+3\right)^2}+O\left(\frac{dt}{5}\right) \right)$

Out[184]=

Out[185]= $FGn \parallel H(gRmp + Gnp U)$

Out[186]= $FGn \parallel H(g \text{text}\{Rmp\}+\text{text}\{Gnp\} U)$

$$\begin{array}{ll} \text{Out} [\text{187}] = & FGn \; error \; \mid \mid \; \left(-\frac{\left(k \left(3 \; g \; H + g \; H^3 \; k^2 - 3 \; U^2 \right) w \right) dt^2}{2 \left(3 + H^2 \; k^2 \right)} - \frac{i \; k \left(3 \; g \; H + g \; H^3 \; k^2 - 3 \; U^2 \right) w^2 \; dt^3}{6 \left(3 + H^2 \; k^2 \right)} + \frac{k \left(3 \; g \; H + g \; H^3 \; k^2 - 3 \; U^2 \right) w^3 \; dt^4}{24 \left(3 + H^2 \; k^2 \right)} + O[dt]^5 \right) + \\ & \left(-\frac{1}{12} \left(g \; H \; k^4 \right) dt + O[dt]^5 \right) dx^3 + \left(\frac{i \left(288 \; g \; H \; k^5 + 192 \; g \; H^3 \; k^7 + 32 \; g \; H^5 \; k^9 - 243 \; k^5 \; U^2 - 49 \; H^2 \; k^7 \; U^2 \right) dt}{960 \left(3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \end{array}$$

Out[188]= FGn error |

 $\label{left} $\left(-\frac{t_4t_4t_5^2 \left(H^2 k^2 + 3 \right)}{2 \left(H^2 k^2 + 3 \right)} - \frac{t_4t_5}{2 \left(H^2 k^2 + 3 \right)} - \frac{t_5}{2 \left(H^2 k^$ $\text{text}(dt)^3 \text{ k w}^2 \left(\frac{H^3 \text{ k}^2+3 \text{ g } H-3 \text{ U}^2\right)} \left(\frac{H^2 \text{ k}^2+3 \text{ right}}{4 \text{ k w}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^2+3 \text{ right}}{4 \text{ k w}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^2+3 \text{ right}}{4 \text{ k w}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^2+3 \text{ right}}{4 \text{ k w}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^2+3 \text{ right}}{4 \text{ k w}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^2+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^2+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^2+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ k}^3} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ k}^3+3 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2 \text{ right}}{4 \text{ right}} \right) + \frac{1}{2} \left(\frac{H^2$ $\label{left} $$\left(g H^3 k^2+3 g H-3 U^2\right)^{24}\left(h^2 k^2+3\right)+O\left(\frac{dt}{5}\right)+O\left(\frac{dt}{5}\right)\right)^{24}. $$$ $\left(-\frac{1}{12} \left(H k^4\right) \right) + \left(\frac{1}{12} \left(H k^4\right) \right)$ \left(\frac{i \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) $\label{left} $$ \operatorname{dt}_{960}\left(\frac{A^2 + 3\right)^2}+O\left(\frac{t}{t}^5\right)\right)+O\left(\frac{t}{t}\right)+O\left(\frac{t}{t}\right)^2. $$$

Out[190]= $FGG \parallel (GGp H + Rmp) U$

Out[191]= $FGG \parallel U (\text{text}\{GGp\} H + \text{text}\{Rmp\})$

$$\begin{array}{ll} \text{Out} \text{[192]=} & FGG \; error \; \mid \mid \; \left(-\frac{\left(k \left(6 + H^2 \; k^2 \right) U \; w \right) \; dt^2}{2 \left(3 + H^2 \; k^2 \right)} - \frac{i \; k \left(6 + H^2 \; k^2 \right) U \; w^2 \; dt^3}{6 \left(3 + H^2 \; k^2 \right)} + \frac{k \left(6 + H^2 \; k^2 \right) U \; w^3 \; dt^4}{24 \left(3 + H^2 \; k^2 \right)} + O[dt]^5 \right) + \\ & \left(-\frac{1}{12} \left(k^4 \; U \right) dt + O[dt]^5 \right) dx^3 + \left(\frac{i \left(531 \; k^5 + 241 \; H^2 \; k^7 + 32 \; H^4 \; k^9 \right) U \; dt}{960 \left(3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \\ \end{array}$$

Out[193]= FGG error ||

 $\left(-\frac{t}{2 \ker\{dt\}^2 \left(H^2 k^2+6\right)}}{2 \ker\{dt\}^2 \left(H^2 k^2+6\right)}}{2 \ker\{dt\}^3 k U w^2 + h^2 \left(H^2 k^2+3\right)}$ $\label{left(H^2 k^2+6\wedge ight)} $$ \left(H^2 k^2+3\right) + \frac{d^4 k U w^3 \left(H^2 k^2+6\right)}{24} \right) $$$ $\left(H^2 k^2+3\right)+O\left(\frac{dt}{5\right)+O\left(\frac{dt}{5\right)}+O\left(\frac{dt}{5\right)}$ $\label{eq:left} $$ \operatorname{left}(\det dt)^5 \right) + \det dt + O(\operatorname{left}(\det dt)^5 \right) + \det dt + O(\operatorname{left}(\det dt)^4 - O(\operatorname{left}(dt)^4 - O(\operatorname$ $k^5 \cdot U \cdot t(dt) = 0 \cdot (text(dt)^5 \cdot U \cdot text(dt)^5 \cdot U \cdot text(dt)^5 \cdot (text(dx)^5 \cdot text(dx)^5 \cdot text(dx)^5$

Out[194]=

Out[195]=

Omega error
$$\| \left\{ -\frac{i \left[\sqrt{3} \ k \sqrt{g \, H \, (3 + H^2 \, k^2)} \, + 3 \, k \, U + H^2 \, k^2 \, U \right]^4 \, dt^3}{24 \, (3 + H^2 \, k^2)^4} + \frac{\left[\sqrt{3} \ k \sqrt{g \, H \, (3 + H^2 \, k^2)} \, + 3 \, k \, U + H^2 \, k^2 \, U \right]^5 \, dt^4}{30 \, (3 + H^2 \, k^2)^3} + O[dt]^5 \right\} + O[dt]^5 + O$$

$$\left(34560 \sqrt{g \, II} \left(3 + II^2 \, k^2\right)^{7/2}\right) - \left(\left(k^9 \left(3 \, g \, II + U \left(2 \, \sqrt{3} \, \sqrt{g \, II} \left(3 + II^2 \, k^2\right) + \left(3 + II^2 \, k^2\right) \, U\right)\right) \right) \right) \\ \left(3 \, \sqrt{3} \, g^2 \, II^2 \left(531 + 1445 \, II^2 \, k^2\right) + g \, II \, U \left(4914 \, \sqrt{g \, II} \left(3 + II^2 \, k^2\right) + \sqrt{3} \, \left(5049 + 3270 \, II^2 \, k^2 + 529 \, II^4 \, k^4\right) U\right) + 6 \left(288 \, \sqrt{g \, II} \, (3 + II^2 \, k^2\right) \, U^3 + 32 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^4 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^2 \, \left(145 \, \sqrt{g \, H^9} \left(3 + II^2 \, k^2\right) \, U^3 + 22 \, k^2 \, U^3 + 22 \, I^2 \, I^2 \, U^3 \right) \right) + 22 \, \left(21 \, \sqrt{3} \, \sqrt{g \, H \, (3 + II^2 \, k^2\right)} + 22 \, I^2 \, I^2 \, U^3 \right) \right) + 22 \, \left(21 \, \sqrt{3} \, \sqrt{g \, H \, (3 + II^2 \, k^2\right)} + 22 \, I^2 \, I^2 \, U^3 \right) \right) + 22 \, \left(21 \, \sqrt{3} \, \sqrt{g \, H \, (3 + II^2 \, k^2\right)} + 22 \, I^2 \, I^2 \, U^3 + 22 \, I^2 \, U^3 \right) \right) + 22 \, \left(21 \, \sqrt{3} \, \sqrt{g \, H \, (3 + II^2 \, k^2\right)} + 22 \, I^2 \, I^2 \, U^3 + 22 \, U^3 \right) \right) + 22 \, \left(21 \, \sqrt{3} \, \sqrt{g \, H \, (3 + II^2 \, k^2\right)} + 22 \, I^2 \, I^2 \, U^3 + 22 \, I^2 \, I^2 \, U^3 \right) \right) + 22 \, \left(21 \, \sqrt{3} \, \sqrt{g \, H \, (3 + II^2 \, k^2\right)} + 22 \, I^2 \, I^2 \, U^3 + 22 \, I^2 \, U^3 \right) \right) + 22 \, \left(21 \, \sqrt{3} \, \sqrt{g \, H \, (3 + II^2 \, k^2\right)} + 22 \, I^2 \, I^2 \, U^3 + 22 \, I^2 \, U^3 \right) \right) + 22 \, \left(21 \, \sqrt{3} \, \sqrt{g \, H \, (3 + II^2 \, k^2\right)} + 22 \, I^2 \, \sqrt{3} \, g \, H^3 \, U^3 + 27 \, I^2 \, U^3 \right) \right)$$

$$\begin{split} &6 \left(288 \, \sqrt{g} \, H \left(3 + H^2 \, k^2\right) \, \, U^3 + 32 \, k^4 \, \sqrt{g} \, H^9 \left(3 + H^2 \, k^2\right) \, \, U^3 \, + \\ & k^2 \left(241 \, \sqrt{g^3 \, H^7 \left(3 + H^2 \, k^2\right)} \, \, U + 192 \, \sqrt{g} \, H^5 \left(3 + H^2 \, k^2\right) \, \, U^3\right) \right) \right) \\ & dt^4 \bigg) \bigg/ \bigg(34 \, 560 \, \sqrt{g \, H} \, \left(3 + H^2 \, k^2\right)^{7/2} \bigg) + O[dt]^5 \bigg) dx^4 + O[dx]^5 \bigg\} \end{split}$$

Out[197]= Omega error |

 $\left(-\frac{1}{\left(-\frac{1}{2} U k^3 + 3 U k + \sqrt{1} k^2 H \left(-\frac{1}{2} k^2 + 3 \right) k \right) k \right) k \left(-\frac{1}{2} U k^3 + 3 U k + \sqrt{1} k \right) k \left(-\frac{1}{2} U k^3 + 3 U k \right) k \left(-\frac{1}{2} U k^3 + 3 U k \right) k \left(-\frac{1}{2} U k^3 + 3 U k \right) k \left(-\frac{1}{2} U k^3 + 3 U k \right) k \left(-\frac{1}{2} U k \right) k \left(-\frac{1}{2}$ $k^2+3\left(H^2 U k^3+3 U k+\sqrt{3} \right) + \left(H^2 U k^3+3 U k+\sqrt{3} \right) + \left(H^2 k^2+3\right) + \left(H^2 U k^3+3 U k+\sqrt{3} \right) + \left(H^2 U k+\sqrt{3$ $\label{left} $$ \left(\frac{dt}^3}{30 \left(\frac{H^2 k^2+3\right)}{5}+O\left(\frac{dt}^5\right)\right)}+\left(\frac{H^2 k^2+3\right)}{5}+O\left(\frac{dt}^5\right)} $$$ $U+\sqrt{3} \sqrt{1} U + \sqrt{3} \sqrt{1} U + \sqrt{1} U + \sqrt{3} \sqrt{1} U + \sqrt{3} \sqrt{1} U + \sqrt{3} \sqrt{1} U + \sqrt{3} \sqrt{3} U + \sqrt{3} \sqrt{3} U + \sqrt{3} U +$ $\label{eq:continuous} $\operatorname{H^5 \left(H^2 k^2+3\right)}\right) \times k^2+18 U+21 \right. $\operatorname{H^6 \left(H^2 k^2+3\right)}\right) \times (h^2 k^2+3 + h^2 k^2+3$ g H \left($9 \left(H^2 \times^2+3\right) U+5 \right) U+5 \left(H^2 \times^2+3\right) U+9 g^2 H^2\right)$ $\text{dt}^3 \{144 \cdot h^2 \cdot h^2 + 3 \cdot h^2 - \frac{i \cdot h^3}{144 \cdot h^2 \cdot h^2 + 3 \cdot h^2} - \frac{i \cdot h^3}{144 \cdot h^2 \cdot h^2 + 3 \cdot h^2} = \frac{144 \cdot h^2 \cdot h^2 + 3 \cdot h^2}{144 \cdot h^2 \cdot h^2 + 3 \cdot h^2} = \frac{144 \cdot h^2 \cdot h^2 + 3 \cdot h^2}{144 \cdot h^2 \cdot h^2 + 3 \cdot h^2} = \frac{144 \cdot h^2 \cdot h^2 \cdot h^2}{144 \cdot h^2 \cdot h^2 + 3 \cdot h^2} = \frac{144 \cdot h^2 \cdot h^2 \cdot h^2}{144 \cdot h^2 \cdot h^2} = \frac{144 \cdot h^2 \cdot h^2 \cdot h^2}{144 \cdot h^2 \cdot h^2} = \frac{144 \cdot h^2 \cdot h^2}{144 \cdot h^2} = \frac{144 \cdot h^2 \cdot h^2}{144 \cdot h^2} = \frac{144 \cdot h^2}{144 \cdot$ $\sqrt{g H \left(\frac{h^2 k^2+3\right)\right)}} \left(\frac{H^5 \left(\frac{H^4 U k^4+\left(12 U H^2+5\right)}{12 U H^2+5}\right)}{12 U H^2+5 \left(\frac{H^5 U H^5}{12 U H^2+5}\right)}\right)}$ $\label{left(H^2 k^2+3\wedge ight)} $$ h^2+18 U+15 \operatorname{sqrt}{g H \left(H^2 k^2+3\wedge ight)\right)} U^2+3 g H \left(H^2 k^2+3\wedge ight\right) U^2+3 g H \left(H^2$ $\left(H^2 k^2+3\right)\right) U+\sqrt{3} \left(H^2 k^2+3\right) \left(H^2$ $k^2+3\right) + O\left(\frac{t}{2} + \frac{dt}{5}\right) \cdot \left(\frac{dt}{5}\right) \cdot \left(\frac{dt}{5}\right)$ $k^2+3\right)\left(U^3\right)\left(U^3\right)\left(U^3\right)$ \sqrt{g H^{13} \left(H^2 k^2+3\right)} U\right) k^6+3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3}) $U+1011 \sqrt{H^2 k^2+3\right} + H^6 +$ $k^4+9 \left(1350 \right) \left(1350 \right$ $U^4+2118 \sqrt{4^2 + 2118 \sqrt{4^2 + 2118}} U^2+145 \sqrt{4^2 + 2118} \sqrt{4^2 + 211$ $k^2+81 \left(64 \right) + 1 \left(64 \right) +$ $\text{dt}^3{34560 } + \text{left}(H^2 k^2+3\right)^{7/2}-\frac{1}{9} = \frac{1}{9} + U \left(\frac{4h^2}{2} + \frac{3}{9} + \frac{1}{9} \right)^{7/2}$ $k^2+3\right) U+2 \sqrt{3} \left(H \left(\frac{445 H^2 k^2+3\right)}{\sinh(445 H^2 k^2+3\right)} \right)$ k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right) U+4914 \sqrt{g} $H \left(H^2 k^2 + 3 \right) H \left(H^2 k^2 + 3 \right) H + 6 \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 + 3 \right) U^3 k^4 + \left(H^2 k^2 k^2 + 3 \right) U^3 k^4$ \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+241 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+288 $\left(H^2 k^2+3\right) U^3\right) \$ $\label{left(left)} $$ k^2+3\right)^{7/2}\right)+O\left(\frac{dt}^5\right)+O\left(\frac$ \left(H^2 U k^3+3 U k-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^4 \text{dt}^3}{24 \left(H^2 k^2+3\right)} $k^2+3\left(H^2 U k^3+3 U k-\sqrt{3} \right) + \left(H^2 U k^3+3 U k-\sqrt{3} \right) + \left(H^2 k^2+3\right) + \left(H^2 U k^3+3 U k-\sqrt{3} \right) + \left(H^2 U k-\sqrt{3$ $\t x_{dt}^3 \left(\frac{dt}^3 \right) \left(\frac{4}{30} \left(\frac{4}{30} \left(\frac{4}{30} \right) \right)^5 + O\left(\frac{4t}^5 \right) \right) \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \right) \right) \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \right) \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \right) \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \right) \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \right) \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \left(\frac{4}{30} \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \left(\frac{4}{30} \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30} \right) \right) + O\left(\frac{4}{30} \left(\frac{4}{30$ $\label{eq:left_hamiltonian} $$ \left(\frac{H^2 k^2+3}{-2 U\right)-\frac{\left(k^7 \left(k^7 \right)^2 H^4 U k^4+\left(12 H^2 U-7 \right)^3 \right)^2 H^6 H^6 L^6 L^6 H^6 L^6 L^6 H^6 L^$ $H^5 \left(H^2 k^2 + 3 \right) \right) h^2 + 18 U - 21 \left(H^2 k^2 + 3 \right) h^2 + 18 U - 21 \right) h^2 H^5 \left(H^2 k^2 + 3 \right) h^2 H^5 \left(H^4 k^2 + 3 \right) h^2 H^5 \left(H^4$ $\left(9 \left(\frac{4^2 k^2+3\right) U-5 \left(\frac{4^2 k^2+3\right) U-5 \left(\frac{4^2 k^2+3\right) U+9 g^2 H^2\left(\frac{4^2 k^2+3\right) U+9 u^2 U$ $\t text{dt}^3}{144 \left(\frac{h^2 k^2 + 3\right)}{h^2 k^2 + 3\right)} - \frac{k^8 \left(\frac{h^2 k^2 + 3\right)}{h^2 k^2 + 3\right)} U - 2}$ $\$ \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(U^2 \left(2 H^4 U k^4+\left(12 H^2 U-5 \sqrt{3}) \) $\label{left(H^2 k^2+3\wedge ight)} $$ \left(H^5 \left(H^2 k^2 + 3\wedge ight \right) \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 \left(H^2 k^2 + 3\wedge ight \right) \ k^2 + 18 U - 15 U - 15$ $\left(H^2 k^2+3\right)^3+O\left(text\left(dt\right)^5\right)\right) \times \left(H^2 k^2+3\right)^3+O\left(text\left(dt\right)^5\right)$

 $k^4+\left(145 \right) k^2 - 1728 U+531 \left(H^2 k^2+3\right) - 1152 H^2 U\right) k^2-1728 U+531 \left(H^2 k^2+3\right) - 1152 H^2 U\right)$ $\sqrt{g H^{13} \left(H^2 k^2 + 3\right)} U - 721 \right) k^{6-3} U \left(H^2 k^2 + 3\right) U - 721 \right) k^{6-3} U \left(H^2 k^2 + 3\right) U - 721 \right)$ U \left(2195 \sqrt{3} U-1011 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5-576 \sqrt{g H^9 \left(H^2 k^2+3\right)} $k^2+3\right) U^3\right) U^3\right) k^4+9\left(-1350 \right) qrt{3} g^2 U H^4-2227 \right] U^3 H^3+576 \right]$ H^5 \left(H^2 k^2+3\right)\\ U^4+2118 \sqrt\\g^3 H^7 \left(H^2 k^2+3\right)\\ U^2+145 \sqrt\\\ g^5 H^9 $\left(\frac{4 \operatorname{ft}(H^2 k^2+3\right)} \right) h^2+81 \left(\frac{64 \operatorname{ft}(H^2 k^2+3\right)} U^4+g H \right)$ $H \left(\frac{H^2 k^2+3 \right)}{-251 \left($ $k^2+3\right) \left(\frac{4t}{3} \frac{34560 \sqrt{g H} \left(\frac{4^2 k^2+3\right)}{7/2} + \frac{4^2 k^9 \left(\frac{4t}{3} \right)}{16t} \right)}{t} \right) \left(\frac{4t}{3} \right) \left(\frac{4t}$ $H+U \left(\left(H^2 k^2+3\right) \right) U-2 \left(H^2 k^2+3\right) \left(H^2 k^2+3\right) \right) H+U \left(H^2 k^2+3\right) \left(H^2 k^2+3\right) H+U \left(H^2$ g^2 \left(145 H^2 k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right) $k^4+\left(192 \right) U^3+241 \left(192 \right) U^3+241$ $\label{left} $$U\rightarrow k^2+288 \left(\frac{H^2 k^2+3\right) U^3\right) \left(\frac{k^2+288 \left(\frac{4t}{4}\right)^4}{34560 \left(\frac{4t}{4}\right)^4} \right) $$$ $\left(H^2 k^2+3\right)^{7/2}+O\left(text\left(dt\right)^5\right)\right) \$

Out[198]=

$$\text{Out} [\text{199}] = \ EA \ \| \ \left. \left\{ \left\{ \frac{-H^2 \ k^2 \left(\left(-1 + e^{i \ d \ w} \right) k \ U - w \right) + 3 \ w}{\left(3 + H^2 \ k^2 \right) w}, \ - \frac{3 \left(-1 + e^{i \ d \ w} \right) k}{\left(3 + H^2 \ k^2 \right) w} \right\}, \ \left\{ - \frac{\left(-1 + e^{i \ d \ w} \right) k \left(g \ H \left(3 + H^2 \ k^2 \right) - 3 \ U^2 \right)}{\left(3 + H^2 \ k^2 \right) w}, \ 1 \ - \frac{\left(-1 + e^{i \ d \ w} \right) k \left(6 + H^2 \ k^2 \right) U}{\left(3 + H^2 \ k^2 \right) w} \right\} \right\}$$

Out[200]= EA || \left(

\begin{array}{cc}

 $\frac{3 w-H^2 k^2 \left(\left(-1+e^{i \cdot text \cdot dt} w\right)\right)}{\left(-1+e^{i \cdot text \cdot dt} w\right)} k U-w\right)}{\left(-1+e^{i \cdot text \cdot dt} w\right)}$

 $k^2+3\right) w$ & $\frac{1}{e^2+3\right} \left(\frac{1+e^4\left(+e^4\left(\frac{1+e^4\left(\frac{1+e^4\left(\frac{1+e^4\left(\frac{1+e^4\left(\frac{1+e^4\left(\frac{1+e^4\left(\frac{1+e^4\left(\frac{1+e^4\left(\frac{1+e^4\left(+e^4\left(+e^4\left(+e^4\left(+e^4\left(+e^4\left($

 $-\frac{\left(\frac{1+e^{i \cdot k}(1+e^{i \cdot k})}{k}\right)}{\left(\frac{1+e^{i \cdot k}(1+e^{i \cdot k})}{k}\right)} + \frac{1}{k}\left(\frac{1+e^{i \cdot k}(1+e^{i \cdot$

 $w\} \& 1-\frac{\left(-1+e^{i \cdot k^2+3\right)} w}{\left(-1+e^{i \cdot k^2+3\right)} w} \wedge \left(-1+e^{i \cdot k^2+3\right)} w$

\end{array}

\right)

```
Out[201]= Eerr ||
                                                                                                                   \left\{\left\{\left(\frac{\left(-3\,g\,H\,k^{2}+3\,k^{2}\,U^{2}-H^{2}\,k^{4}\,U^{2}-H^{2}\,k^{3}\,U\,w\right)dt^{2}}{2\left(3+H^{2}\,k^{2}\right)}+\frac{1}{6\left(3+H^{2}\,k^{2}\right)^{2}}i\left(18\,g\,H\,k^{3}\,U+9\,g\,H^{3}\,k^{5}\,U-18\,k^{3}\,U^{3}-3\,H^{2}\,k^{5}\,U^{3}+H^{4}\,k^{7}\,U^{3}-18\,k^{2}\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^{2}+4\,u^
                                                                                                                                                                                                                                                                                                   3 \ H^2 \ k^3 \ U \ w^2 - H^4 \ k^5 \ U \ w^2 \Big) \ dt^3 \ + \ \frac{H^2 \ k^3 \ U \ w^3 \ dt^4}{24 \left(3 + H^2 \ k^2\right)} \ + \ O[dt]^5 \Big) \ +
                                                                                                                                                                                      \left(-\tfrac{1}{12}\left(k^4\,\mathrm{U}\right)dt + \tfrac{i\left(3\,g\,H\,k^5 + 2\,H^2\,k^7\,\mathrm{U}^2\right)dt^2}{24\left(3 + H^2\,k^2\right)} + \tfrac{\mathrm{U}\left(15\,g\,H\,k^6 + 6\,g\,H^3\,k^8 - 9\,k^6\,\mathrm{U}^2 + H^4\,k^{10}\,\mathrm{U}^2\right)dt^3}{24\left(3 + H^2\,k^2\right)^2} + \mathrm{O}[dt]^5\right)dx^3 + \\
                                                                                                                                                                                                      \frac{i \left(45 \, k^{5} \, U+143 \, H^{2} \, k^{7} \, U+32 \, H^{4} \, k^{9} \, U\right) dt}{960 \left(3+H^{2} \, k^{2}\right)^{2}} + \frac{\left(531 \, g \, H \, k^{6}+145 \, g \, H^{3} \, k^{8}-486 \, k^{6} \, U^{2}+94 \, H^{2} \, k^{8} \, U^{2}+64 \, H^{4} \, k^{10} \, U^{2}\right) dt^{2}}{1920 \left(3+H^{2} \, k^{2}\right)^{3}} - \frac{1}{1920 \left(3+H^{2} \, k^{2}\right)^{3}}
                                                                                                                                                                                                                                                           i k^7 (1683 g H U + 1348 g H^3 k^2 U + 241 g H^5 k^4 U - 1593 U^3 - 678 H^2 k^2 U^3 + 47 H^4 k^4 U^3 + 32 H^6 k^6 U^3)
                                                                                                                                                                                                                                                                                 dt^3 + O[dt]^5 dx^4 + O[dx]^5,
                                                                                                                                                                \left(-\,\frac{3 \left(2 \, k^{2} \, U+k \, w\right) d t^{2}}{2 \left(3+H^{2} \, k^{2}\right)}\,+\,\frac{i \left(3 \, g \, H \, k^{3}+9 \, k^{3} \, U^{2}+3 \, H^{2} \, k^{5} \, U^{2}-3 \, k \, w^{2}-H^{2} \, k^{3} \, w^{2}\right) d t^{3}}{2 \left(3+H^{2} \, k^{2}\right)^{2}}\,+\,\frac{k \, w^{3} \, d t^{4}}{8 \left(3+H^{2} \, k^{2}\right)}\,+\,O[dt]^{5}\right)+
                                                                                                                                                                                      \left( \tfrac{i\,k^5\,U\,dt^2}{4\,(3+H^2\,k^2)} + \tfrac{\left(g\,H\,k^6+6\,k^6\,U^2+2\,H^2\,k^8\,U^2\right)dt^3}{8\,(3+H^2\,k^2)^2} + O[dt]^5 \right) dx^3 \, + \\
                                                                                                                                                                                        \left(\frac{\mathit{i}\,\underbrace{(243\,k^5+49\,H^2\,k^7)\,dt}}{960\,\big(3+H^2\,k^2\big)^2} + \frac{\big(531\,k^6+145\,H^2\,k^8\big)\,U\,dt^2}{960\,\big(3+H^2\,k^2\big)^2} - \frac{\mathit{i}\,\big(774\,g\,H\,k^7+194\,g\,H^3\,k^9+2457\,k^7\,U^2+1542\,H^2\,k^9\,U^2+241\,H^4\,k^{11}\,U^2\big)\,dt^3}{1920\,\big(3+H^2\,k^2\big)^3} + O[dt]^5\right)dx^4 + O[dt]^5\right)dx^4 + O[dt]^5
                                                                                                                                                                                      O[dx]^5\Big\},\,\,\Big\{\!\Big(\!-\frac{\big(\!\big(3\,g\,H\!+\!g\,H^3\,k^2\!-\!3\,U^2\big)\big(\!\,2\,k^2\,U\!+\!k\,w\big)\!\big)\,dt^2}{2\,\big(\!\,3\!+\!H^2\,k^2\big)}\,\,+\,\,
                                                                                                                                                                                                                                   \left(\frac{1}{2(3+H^2k^2)^2}i\left(3g^2H^2k^3+g^2H^4k^5+6gHk^3U^2+6gH^3k^5U^2+gH^5k^7U^2-9k^3U^4-3H^2k^5U^4\right)-\frac{1}{2(3+H^2k^2)^2}i\left(3g^2H^2k^3+g^2H^4k^5+6gHk^3U^2+6gH^3k^5U^2+gH^5k^7U^2-9k^3U^4-3H^2k^5U^4\right)-\frac{1}{2(3+H^2k^2)^2}i\left(3g^2H^2k^3+g^2H^4k^5+6gHk^3U^2+6gH^3k^5U^2+gH^5k^7U^2-9k^3U^4-3H^2k^5U^4\right)
                                                                                                                                                                                                                                                                                                                 \frac{ \imath \, k \, \left(g \, H \, \left(3 + H^2 \, k^2\right) - 3 \, U^2\right) w^2}{6 \, \left(3 + H^2 \, k^2\right)} \right) dt^3 \, + \, \frac{k \, \left(g \, H \, \left(3 + H^2 \, k^2\right) - 3 \, U^2\right) w^3 \, dt^4}{24 \, \left(3 + H^2 \, k^2\right)} \, + \, O[dt]^5 \bigg) \, + \\
                                                                                                                                                                                        \left(-\,\frac{_{1}}{_{12}}\left(g\,H\,k^{4}\right)dt\,+\,\frac{\mathit{i}\left(6\,g\,H\,k^{5}\,U+2\,g\,H^{3}\,k^{7}\,U-3\,k^{5}\,U^{3}\right)dt^{2}}{_{12}\left(3+H^{2}\,k^{2}\right)}\,+\,\frac{_{1}}{_{24}\left(3+H^{2}\,k^{2}\right)^{2}}\!\left(6\,g^{2}\,H^{2}\,k^{6}\,+\,2\,g^{2}\,H^{4}\,k^{8}\,+\,24\,g\,H\,k^{6}\,U^{2}\,+\,24\,g\,H^{2}\,k^{2}\right)}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)^{2}\left(6\,g^{2}\,H^{2}\,k^{6}\,+\,2\,g^{2}\,H^{4}\,k^{8}\,+\,24\,g\,H^{2}\,k^{6}\,U^{2}\,+\,24\,g\,H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)^{2}\left(6\,g^{2}\,H^{2}\,k^{6}\,+\,2\,g^{2}\,H^{4}\,k^{8}\,+\,24\,g\,H^{2}\,k^{6}\,U^{2}\,+\,24\,g\,H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}}{_{12}}\left(3+H^{2}\,k^{2}\right)dt^{2}+\frac{_{1}
                                                                                                                                                                                                                                                                                                                                18\,g\,H^3\,k^8\,U^2 + 3\,g\,H^5\,k^{10}\,U^2 - 18\,k^6\,U^4 - 6\,H^2\,k^8\,U^4\big)\,dt^3 + O[dt]^5\Big)\,dx^3 + \\
                                                                                                                                                                                                 \frac{i\left(288\,\mathrm{g\,H\,k^5} + 192\,\mathrm{g\,H^3\,k^7} + 32\,\mathrm{g\,H^5\,k^9} - 243\,\mathrm{k^5\,U^2} - 49\,\mathrm{H^2\,k^7\,U^2}\right)\,\mathrm{dt}}{960\left(3+\mathrm{H^2\,k^2}\right)^2} + \frac{\left(576\,\mathrm{g\,H\,k^6\,U} + 384\,\mathrm{g\,H^3\,k^8\,U} + 64\,\mathrm{g\,H^5\,k^{10}\,U} - 531\,\mathrm{k^6\,U^3} - 145\,\mathrm{H^2\,k^8\,U^3}\right)\,\mathrm{dt^2}}{960\left(3+\mathrm{H^2\,k^2}\right)^2} - \frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2
                                                                                                                                                                                                                                                                \frac{1}{5760\,(3+H^2\,k^2)^3}\dot{\imath}\,\left(2457\,g^2\,H^2\,k^7+1542\,g^2\,H^4\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H\,k^7\,U^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H
                                                                                                                                                                                                                                                                                                                                                      H^5 k^{11} U^2 + 288 g H^7 k^{13} U^2 - 7371 k^7 U^4 - 4626 H^2 k^9 U^4 - 723 H^4 k^{11} U^4) dt^3 + O[dt]^5 dx^4 + O[dt]^5 dx^5 + O[dt]^5 dx^4 + O[dt]^5 dx^5 + O[dt]^5 
                                                                                                                                                                                      O[dx]^{5}, \left(\frac{(-3gHk^{2}-9k^{2}U^{2}-H^{2}k^{4}U^{2}-6kUw-H^{2}k^{3}Uw)dt^{2}}{2(3+H^{2}k^{2})} + \frac{1}{6(3+H^{2}k^{2})^{2}}i\left(36gHk^{3}U+9gH^{3}k^{5}U+36k^{3}U^{3}+44k^{2}k^{2}\right)^{2}\right)
                                                                                                                                                                                                                                                                                                            15 \; H^2 \; k^5 \; U^3 \; + \; H^4 \; k^7 \; U^3 \; - \; 18 \; k \; U \; w^2 \; - \; 9 \; H^2 \; k^3 \; U \; w^2 \; - \; H^4 \; k^5 \; U \; w^2 \big) \; dt^3 \; + \; \frac{k \left(6 + H^2 \; k^2\right) U \; w^3 \; dt^4}{24 \left(3 + H^2 \; k^2\right)} \; + \; O[dt]^5 \bigg) \; + \; O[dt]^5 \; + \; O[d
                                                                                                                                                                                      \left(-\frac{1}{12} \left(k^4 \text{ U}\right) dt + \frac{\text{i} \, k^5 \left(3 \, g \, H + 12 \, U^2 + 2 \, H^2 \, k^2 \, U^2\right) dt^2}{24 \left(3 + H^2 \, k^2\right)} + \frac{\left(21 \, g \, H \, k^6 \, U + 6 \, g \, H^3 \, k^8 \, U + 27 \, k^6 \, U^3 + 12 \, H^2 \, k^8 \, U^3 + H^4 \, k^{10} \, U^3\right) dt^3}{24 \left(3 + H^2 \, k^2\right)^2} + O[dt]^5\right) dx^3 + O[dt]^5
                                                                                                                                                                                                   \frac{i \left(531 \, \mathrm{k}^5 + 241 \, \mathrm{H}^2 \, \mathrm{k}^7 + 32 \, \mathrm{H}^4 \, \mathrm{k}^9\right) \, \mathrm{U} \, \mathrm{dt}}{960 \left(3 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \, + \, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^8 \, \mathrm{U}^2 + 64 \, \mathrm{H}^4 \, \mathrm{k}^{10} \, \mathrm{U}^2\right) \, \mathrm{dt}^2}{1920 \left(3 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \, - \, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^8 \, \mathrm{U}^2 + 64 \, \mathrm{H}^4 \, \mathrm{k}^{10} \, \mathrm{U}^2\right) \, \mathrm{dt}^2}{1000 \, \mathrm{g}^2 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \, - \, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^8 \, \mathrm{U}^2 + 64 \, \mathrm{H}^4 \, \mathrm{k}^{10} \, \mathrm{U}^2\right) \, \mathrm{dt}^2}{1000 \, \mathrm{g}^2 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \, - \, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^8 \, \mathrm{U}^2 + 64 \, \mathrm{H}^4 \, \mathrm{k}^{10} \, \mathrm{U}^2\right) \, \mathrm{dt}^2}{1000 \, \mathrm{g}^2 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \, - \, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^2\right)^2}{1000 \, \mathrm{g}^2 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \, - \, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^2\right)^2}{1000 \, \mathrm{g}^2 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \, - \, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^3 \, \mathrm{k}^8\right)^2}{1000 \, \mathrm{g}^2 + \mathrm{H}^3 \, \mathrm{k}^8\right)^2} \, - \, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8\right)^2}{1000 \, \mathrm{g}^2 + \mathrm{H}^3 \, \mathrm{g}^2 + \mathrm{H}^3 \, \mathrm{g}^2} + \mathrm{H}^3 \, \mathrm{g}^2 + \mathrm{H}^3 \, \mathrm{g}^2\right)^2} \, - \, \frac{\left(531 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{g}^2 + \mathrm{H}^3 \, \mathrm{g}^2\right)^2}{1000 \, \mathrm{g}^2 + \mathrm{H}^3 \, \mathrm{g}^2} + \, \frac{1000 \, \mathrm{g}^2}{1000 \, \mathrm
                                                                                                                                                                                                                                                                \frac{_{1920\,(3+H^2\,k^2)^3}}{^{1920\,(3+H^2\,k^2)^3}}i\left(3231\,g\,H\,k^7\,U+1736\,g\,H^3\,k^9\,U+241\,g\,H^5\,k^{11}\,U+3321\,k^7\,U^3+\right.
                                                                                                                                                                                                                                                                                                                                2406 H^2 k^9 U^3 + 529 H^4 k^{11} U^3 + 32 H^6 k^{13} U^3 dt^3 + O[dt]^5 dx^4 + O[dx]^5
```

Out[202]= Eerr || \left(

\begin{array}{cc}

 $\left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U w k^3+3 U w k^3+3$ \left(H^4 U^3 k^7-3 H^2 U^3 k^5-H^4 U w^2 k^5+9 g H^3 U k^5-18 U^3 k^3-3 H^2 U w^2 k^3+18 $\label{left} $$k^2+3\right)+O\left(\frac{dt}^5\right)+\left(\frac{dt}^5\right)+\left(\frac{dt}^4U\right)+\frac{dt}^4}{h^2}.$ U^2 k^7+3 g H k^5\right) \text{dt}^2\{24 \\left(H^2 k^2+3\right)}+\\frac{U \\left(H^4 U^2 k^{10}+6 g H^3 + 10)}{2} \\left(H^2 k^2+3\right)\} $k^8 - 9 \ U^2 \ k^6 + 15 \ g \ H \ k^6 \wedge (text{dt}^3){24 \left(h^2 \ k^2 + 3 \right)} + O\left(text{dt}^5 \right) + O\left($ \text{dx}^3+\left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}\{960 \left(H^2

k^2+3\right)^2}+\frac{\left(64 H^4 U^2 k^{10}+145 g H^3 k^8+94 H^2 U^2 k^8-486 U^2 k^6+531 H^2 U^2 k^8-486 U^2 k^6+531 H^2 U^2 k^8-486 U^2 k^8+145 g H^3 k^8+145 g H^2 U^2 k^2 g H k^6\right) \text{dt}^2}{1920 \left(H^2 k^2+3\right)^2}-\frac{i k^7 \left(32 H^6 U^3 k^6+47 H^4 U^3 k^4+241 g H^5 U k^4-678 H^2 U^3 k^2+1348 g H^3 U k^2-1593 U^3+1683 g H U\right) $\label{left(dt)^3} $\{1920 \left(H^2 k^2+3\right)^3\}+O\left(text\{dt\}^5\right)\right) \text\{dx\}^4+O\left(text\{dx\}^5\right)^3\} $$ & $\left(-\frac{3 \left(U k^2+w \right)^2}{2 \left(U k^2+w k\right)^2}\right) + \frac{3 \left(U k^2+w k\right)^2}{2 \left(U k^2+w k\right)^2}$ $k^5+9 U^2 k^3-H^2 w^2 k^3+3 g H k^3-3 w^2 k \right) \left(\frac{1}{2} \left(\frac{4}{3} \right) \left(\frac{4}{3} \right) \left(\frac{4}{3} \right) \right) \left(\frac{4}{3} \right) \left(\frac{4}{3$ $w^3 \text{$\eft(H^2 k^2+3\right)} + O\left(\frac{dt}^5\right) + O\left(\frac{dt$ \text{dt}^2}{4 \left(H^2 k^2+3\right)}+\frac{\left(2 H^2 U^2 k^8+6 U^2 k^6+g H k^6\right)} $\t (t_3)_{8 \left(t_4^2 + \frac{dt}^3}_{8 \left(t_4^2 + \frac{dt}^3}\right) \right) \\$ $\left(49 \text{ H}^2 \text{ k}^7 + 243 \text{ k}^5\right) \left(49 \text{ H}^2 \text{ k}^2 + 3\right)^2 + \frac{145 \text{ H}^2}{2} + \frac{145 \text{ H}^2}{2$ k^8+531 k^6\right) U \text{dt}^2\{960 \left(H^2 k^2+3\right)^2\}-\frac\{i \left(241 H^4 U^2 \) k^{11}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7\right) \text{dt}^3}{1920 $\left(H^2 k^2+3\right)^3+O\left(\frac{dt}^5\right)\right) \left(\frac{dt}^3+O\left(\frac{dt}^5\right)\right)$ $\label{left(-frac} \left(\left(\frac{k^2 H^3 + 3 g H - 3 U^2 \right) \left(U k^2 + w \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) \\ = \frac{1}{2} \left$ k^2+3\right)}+\left(\frac{i \left(g H^5 U^2 k^7+g^2 H^4 k^5-3 H^2 U^4 k^5+6 g H^3 U^2 k^5-9 U^4 k^3+3 U^2 k^5-9 U^4 k^3+3 U^2 k^5-9 U^4 k^5+6 U^2 k^5-9 U^4 $g^2 H^2 k^3+6 g H U^2 k^3 \left(\frac{h^2 k^2+3\right)}{2 \left(\frac{h^2 k^2+3\right)}{2}-\frac{h^2 k^3+6 g H U^2 k^3\right)}$ $U^2\right\} w^3 \text{$t$ xt{dt}^4}{24 \left(\frac{h^2 k^2+3\right)}+O\left(\frac{dt}{5}\right) \right)} + O\left(\frac{h^2 k^2+3\right)}{12}$ $\left(g H k^4\right) \left(g H k^4\right) \left(g H k^4\right) \left(g H k^5\right) \left(g$ \text{dt}^2}{12 \left(H^2 k^2+3\right)}+\frac{\left(3 g H^5 U^2 k^{10}+2 g^2 H^4 k^8-6 H^2)} U⁴ k⁸+18 g H³ U² k⁸-18 U⁴ k⁶+6 g² H² k⁶+24 g H U² k⁶right) \text{dt}³{24 $\left(H^2 k^2+3\right)^2+O\left(\frac{dt}{3}\right) \left(\frac{dt}{3}\right)$ k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\\left(64 g H^5 U k^{10}-145 H^2 U^3 k^8+384 g H^3 U k^8-531 U^3 k^6+576 g H U k^6\right) \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(288 g H^7 U^2 k^{13}+241 g^2)} H^6 k^{11}-723 H^4 U^4 k^{11}+2592 g H^5 U^2 k^{11}+1542 g^2 H^4 k^9-4626 H^2 U^4 k^9+7194 g H^3 U^2 k^9-7371 U^4 k^7+2457 g^2 H^2 k^7+5454 g H U^2 k^7\right) \text{dt}^3}{5760 \left(H^2 $\label{left(dextdet} $$k^2+3\right)^3$+O\left(\frac{dt}^5\right)^3\right) \cdot \left(\frac{dt}{dt}^5\right)^3+O\left(\frac{dt}{dt}^5\right)^3+O\left(\frac{dt}{dt}^6\right)$ $U^2 k^4 - H^2 U w k^3 - 9 U^2 k^2 - 3 g H k^2 - 6 U w k \right) + \frac{1}{2} \left[2 \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right] + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right] + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right] + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) + \frac{1}{2} \left(\frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 3 \right) +$ \left(H^4 U^3 k^7+15 H^2 U^3 k^5-H^4 U w^2 k^5+9 g H^3 U k^5+36 U^3 k^3-9 H^2 U w^2 k^3+36 g H U k^3-18 U w^2 k\right) \text{dt}^3}{6 \left(H^2 k^2+3\right)^2} + \frac{k \left(H^2 k^2+6\right) U k^3-18 U w^2 k}{18 U k^3-18 U w^2 k} \right) = 0 $w^3 \text{$text(dt)^4$} \{24 \left(\frac{h^2 k^2 + 3 \right) + O\left(\frac{dt}{5} \right) \right) + \left(\frac{h^2 k^2 + 3 \right) + O\left(\frac{dt}{5} \right) + O\left(\frac{dt}{5} \right) + O\left(\frac{h^2 k^2 + 3 \right) + O\left(\frac{dt}{5} \right) + O\left(\frac{$ U\right) \text{dt}+\frac{i k^5 \left(2 H^2 k^2 U^2+12 U^2+3 g H\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)}+\frac{\left(H^4 U^3 k^{10}+12 H^2 U^3 k^8+6 g H^3 U k^8+27 U^3 k^6+21 g H U $k^6 + k^6 + k^3 + 24 \left(\frac{dt}^3}{24 \left(\frac{dt}^3}{24 \left(\frac{dt}^5\right)} \right) \left(\frac{dt}^3 + \frac{dt}^3}{24 \left(\frac{dt}^5\right)} \right) \left(\frac{dt}^5 + \frac{dt}^5}{24 \left(\frac{dt}^5\right)} \right) \left(\frac{dt}^5 + \frac{dt}^5$ $\left(32 \text{ H}^4 \text{ k}^9 + 241 \text{ H}^2 \text{ k}^7 + 531 \text{ k}^5\right) \text{ U} \left(4t\right) \left(960 \left(\frac{4}{2} \text{ k}^2 + 3\right)\right)^2 + \frac{64}{2} \left(64 \text{ k}^2 + 3\right)^2 + \frac{64}{2} \left(164 \text{ k}^2$ H^4 U^2 k^{10}+145 g H^3 k^8+674 H^2 U^2 k^8+1638 U^2 k^6+531 g H k^6\right) \text{dt}^2\{1920} \left(H^2 k^2+3\right)^2}-\frac{i \left(32 H^6 U^3 k^{13}+529 H^4 U^3 k^{11}+241 g H^5 U k^{11}+2406 H^2 U^3 k^9+1736 g H^3 U k^9+3321 U^3 k^7+3231 g H U k^7\right) \text{dt}^3\{1920} $\left(H^2 k^2+3\right)^3+O\left(\frac{dt}^5\right)\right) \left(\frac{dt}^3+O\left(\frac{dt}^5\right)\right)$

\end{array}

\right)

```
\ln[203] = \text{KurF} = (\text{fm} * \text{ap} - \text{fp} * \text{am} + \text{am} * \text{ap} * (\text{qp} - \text{qm})) / (\text{ap} - \text{am});
       KurFWS = KurF /. ap \rightarrow 0 /. am \rightarrow (U - Sqrt[g * H]);
       KurFWSeta =
          KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
            qm \rightarrow Rmp * n;
       KurFWSeta = KurFWSeta /. v \rightarrow (GGp * G + Gnp * n);
       Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1];
       KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1];
       \texttt{Kfnn} \; = \; \texttt{Kfnnp} \; / \; . \; \mathsf{Rpp} \; \rightarrow \; \mathsf{Rp} \; \; / \; . \; \; \mathsf{Rmp} \; \rightarrow \; \mathsf{Rm} \; \; / \; . \; \; \mathsf{GGp} \; \rightarrow \; \mathsf{GG2} \; \; / \; . \; \; \mathsf{Gnp} \; \rightarrow \; \mathsf{Gn2} \; ;
       KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
       Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
       Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 4}];
       Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
       FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
       FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 4}];
       FnG2TAr = Refine[FnG2TA , \{k > 0, U > 0, H > 0, g > 0\}];
       \texttt{KurFWSG} = \texttt{KurFWS} \ / . \ \texttt{fp} \ \rightarrow \ (\texttt{U} * \texttt{Rpp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rpp} * \texttt{n}) \ / .
               fm \rightarrow (U * Rmp * G + U * H * V + g * H * Rmp * n) / . qp \rightarrow Rpp * G / . qm \rightarrow Rmp * G;
       KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
       KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
       KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
       \texttt{KfGG} \; = \; \texttt{KfGGp} \; / \; . \; \mathsf{Rpp} \; \rightarrow \; \mathsf{Rp} \; \; / \; . \; \; \mathsf{Rmp} \; \rightarrow \; \mathsf{Rm} \; \; / \; . \; \; \mathsf{GGp} \; \rightarrow \; \mathsf{GG2} \; \; / \; . \; \; \mathsf{Gnp} \; \rightarrow \; \mathsf{Gn2} \; ;
       FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
       FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 4}];
       FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
       fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
       FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
       FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 4}];
       FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
       Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
       Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 + Fmat2.Fmat2.Fmat2 / 6;
       Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
       EigvFmat2 = Eigenvalues[Fmat2];
       RKStep =
          Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 + EigvFmat2 * EigvFmat2 * EigvFmat2 / 6]/
       RKstepTay = Series[RKStep, \{dx, 0, 4\}, \{dt, 0, 4\}];
       RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
```

```
Text[Row[{"U < -Sqrt(gH) < U"}]]</pre>
     Text[" "]
     Text[Row[{"Fnn || ", Kfnnp}]]
     Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
     Text[Row[{"Fnn error ||
                                 ", Fnn2TAr}]]
     Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
     Text[" "]
     Text[Row[{"FnG || ", KfnGp}]]
     Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
     Text[Row[{"FnG error ||
                                 ", FnG2TAr}]]
     Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
     Text[" "]
     Text[Row[{"FGn ||
                           ", KfGnp}]]
     Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
     Text[Row[{"FGn error ||
                                 ", FGn2TAr}]]
     Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
     Text[" "]
     Text[Row[{"FGG || ", KfGGp}]]
     Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
     Text[Row[{"FGG error || ", FGG2TAr}]]
     Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
     Text[" "]
     Text[" "]
     Text[Row[{"Omega error || ", RKstepTayr}]]
     Text[Row[{"Omega error ||
                                   ", TeXForm[RKstepTayr]}]]
     Text[" "]
     Text[Row[{"EA ||
                         ", EA}]]
     Text[Row[{"EA || ", TeXForm[EA]}]]
     Text[Row[{"Eerr || ", Eerr}]]
     Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
Out[237]= U < -Sqrt(gH) < U
Out[238]=
Out[239]= Fnn \parallel Gnp H + Rpp U
Out[240] = Fnn \parallel \text{text}\{Gnp\} H + \text{text}\{Rpp\} U
```

$$\begin{array}{ll} \text{Out} [241] = & Fnn \; error \; \mid \mid \; \left(-\frac{\left(H^2 \, k^3 \, U \, w \right) dt^2}{2 \, \left(3 + H^2 \, k^2 \right)} \, - \, \frac{i \, H^2 \, k^3 \, U \, w^2 \, dt^3}{6 \, \left(3 + H^2 \, k^2 \right)} \, + \, \frac{H^2 \, k^3 \, U \, w^3 \, dt^4}{24 \, \left(3 + H^2 \, k^2 \right)} \, + \, O[dt]^5 \right) + \\ & \left(\frac{1}{12} \, k^4 \, U \, dt \, + \, O[dt]^5 \right) dx^3 \, + \left(\frac{i \, \left(45 \, k^5 \, U + 143 \, H^2 \, k^7 \, U + 32 \, H^4 \, k^9 \, U \right) dt}{960 \, \left(3 + H^2 \, k^2 \right)^2} \, + \, O[dt]^5 \right) dx^4 \, + \, O[dx]^5 \\ \end{array}$$

Out[242]= Fnn error ||

 $\label{left} $\left(-\frac{t_4t_4t_5^2 \left(t_4^2 k^3 U \right)}{2 \left(t_4^2 k^2 + 3\right)} - \frac{t_4t_5}{2 k^3 U k^3 U k^2 t_5} \right) $$$ $\left(H^2 k^2+3\right)+\frac{dt}^4 H^2 k^3 U w^3}{24 \left(H^2 k^2+3\right)}$ $k^2+3\right)+O\left(\frac{dt}^5\right)+\frac{dt}^3\left(\frac{dt}^5\right)+\frac{dt}^3\left(\frac{dt}^3\right)+\frac{dt}^3\left(\frac{dt}^3\right)}{2}$ \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i} \left(32 H^4 U k^9+143 H^2 U k^7+45 U $k^5 + \frac{dt}{960 \left(\frac{H^2 k^2 + 3\right)^2} + O\left(\frac{dt}{5}\right) + O\left(\frac{dt}{5}\right) + O\left(\frac{dt}{5}\right)} + O\left(\frac{dt}{5}\right) + O\left(\frac$

Out[243]=

Out[244]= FnG || GGp H

Out[245]= $FnG \parallel \text{text}\{GGp\} H$

$$\text{Out} \text{[246]=} \quad FnG \; error \; \mid \mid \; \left(-\frac{3 \, (k \, w) \, dt^2}{2 \, (3 + H^2 \, k^2)} - \frac{i \, k \, w^2 \, dt^3}{2 \, (3 + H^2 \, k^2)} + \frac{k \, w^3 \, dt^4}{8 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) + \left(\frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 + O[dx]^5$$

 $Out[247] = FnG error || \left(-\frac{3 \text{ } \text{text}(dt)^2 (k w)}{2 \text{ } \text{left}(H^2 k^2 + 3 \text{ } \text{right})} - \frac{i}{2} \right) - \frac{i}{2} || (1 + i)^2 || (1 + i)$ $\text{dt}^3 \text{ k w}^2$ {2 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k w^3}{8 \left(H^2 k^2+3)}+\frac{\text{dt}^4 k $k^2+3\right)+O\left(\frac{dt}^5\right)+\frac{dt}^5\right)$ $\label{left} $$ \operatorname{dt}_{960}\left(\frac{A^2 + 3\right)^2}+O\left(\frac{t}{t}^5\right)\right)+O\left(\frac{t}{t}\right)+O\left(\frac{t}{t}\right)^2. $$$

Out[248]=

Out[249]= $FGn \parallel H(gRpp + GnpU)$

 $Out[250] = FGn \parallel H(g \text{text}{Rpp}+\text{text}{Gnp} U)$

$$\begin{array}{ll} \text{Out} [251] = & FGn \; error \; \mid \mid \; \left(-\frac{\left(k \left(3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w \right) dt^2}{2 \left(3 + H^2 \, k^2 \right)} - \frac{i \, k \left(3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^2 \, dt^3}{6 \left(3 + H^2 \, k^2 \right)} + \frac{k \left(3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^3 \, dt^4}{24 \left(3 + H^2 \, k^2 \right)} + O[dt]^5 \right) + \\ & \left(\frac{1}{12} \, g \, H \, k^4 \, dt + O[dt]^5 \right) dx^3 + \left(\frac{i \left(288 \, g \, H \, k^5 + 192 \, g \, H^3 \, k^7 + 32 \, g \, H^5 \, k^9 - 243 \, k^5 \, U^2 - 49 \, H^2 \, k^7 \, U^2 \right) dt}{960 \left(3 + H^2 \, k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \end{array}$$

Out[252]= FGn error ||

 $\text{text}_dt^3 k w^2 \left(\frac{H^3 k^2+3 g H-3 U^2\left(\frac{H^2 k^2+3\right)}{4 k w^3} \right) + \frac{1}{4 k w^3}$ $\left(\frac{H^3 k^2+3 g H-3 U^2\right)}{24 \left(\frac{h^2 k^2+3\right)}+O\left(\frac{t}{t}\right)^5\right)}$ $\left(\frac{1}{12} g H k^4 \left(\frac{dt}{+0}\right)\right)\right) + \left(\frac{dt}{4}\right) + \left(\frac{dt}{4}\right)^5\right)$ \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) $\label{left} $$ \operatorname{dt}_{960}\left(\frac{A^2 k^2+3\right)^2}+O\left(\frac{t}{t}^5\right)\right)^2+O\left(\frac{t}{t}^5\right)^2\right)^2$$

Out[253]=

Out[254]= $FGG \parallel (GGp H + Rpp) U$

Out[255]= $FGG \parallel U (\text{text}\{GGp\} H + \text{text}\{Rpp\})$

$$\begin{array}{ll} \text{Out} \text{[256]=} & FGG \; error \; \mid \mid \; \left(-\frac{\left(k \left(6 + H^2 \; k^2 \right) \text{U} \; w \right) \, dt^2}{2 \left(3 + H^2 \; k^2 \right)} - \frac{i \; k \left(6 + H^2 \; k^2 \right) \text{U} \; w^2 \; dt^3}{6 \left(3 + H^2 \; k^2 \right)} + \frac{k \left(6 + H^2 \; k^2 \right) \text{U} \; w^3 \; dt^4}{24 \left(3 + H^2 \; k^2 \right)} + O[dt]^5 \right) + \\ & \left(\frac{1}{12} \; k^4 \; \text{U} \; dt + O[dt]^5 \right) dx^3 + \left(\frac{i \left(531 \; k^5 + 241 \; H^2 \; k^7 + 32 \; H^4 \; k^9 \right) \text{U} \; dt}{960 \left(3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \\ \end{array}$$

Out[257]= FGG error ||

 $\label{left} $\left(-\frac{d^2 \ker d^2 \cdot (H^2 k^2+6\right)}{2 \left(H^2 k^2+6\right)}^2 \left(H^2 k^2+3\right)^{-\frac{1}{2}} \right) -\frac{1}{2} \left(H^2 k^2+3\right)^{-\frac{1}{2}} \left(H^2 k^2+3\right)^{-\frac{$ $k U w^2 \left(\frac{h^2 k^2 + 6 \right)}{6 \left(\frac{h^2 k^2 + 3 \right)} + \frac{dt}{4} k U w^3 \left(\frac{h^2 k^2 + 3 \right)}{6 \left(\frac{h^2 k^2 + 3 \right$ $k^2 + 6 \cdot (h^2 k^2 + 3 \cdot (t^3 k^2$ $k^5 + U \text{ } U$

Out[258]=

Outgoon- Omega error
$$\| \left\{ -\frac{i \left[\sqrt{3} \ k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^2 U \right]^4 dt^4}{24 (3 + H^2 k^2)^4} + \frac{\left[\sqrt{3} \ k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^2 U \right]^3 dt^4}{30 (3 + H^2 k^2)} + O[d I]^5 \right\} + O[d I]^5 + O$$

$$\frac{k^{2}\left(241\sqrt{g^{2}H^{2}(3+H^{2}k^{2})} \ U+192\sqrt{g\,H^{5}(3+H^{2}k^{2})} \ U^{2}\right)\right)\right)}{dt^{4}} + \left(\frac{1}{34560\sqrt{g\,H}} \left(3+H^{2}k^{2}\right)^{72}\right) + O[du]^{5}\right)dx^{4} + O[dx]^{5},$$

$$-\frac{i\left[-\sqrt{3}\ k\sqrt{g\,H(3+H^{2}k^{2})^{-3}\,k\,U+H^{2}k^{2})}\right]^{4}dt^{3}}{24(3+H^{2}k^{2})} + \frac{\left[-\sqrt{3}\ k\sqrt{g\,H(3+H^{2}k^{2})^{-3}\,k\,U+H^{2}k^{2})}\right]^{3}dt^{4}}{30(3+H^{2}k^{2})} + \frac{1}{3}$$

$$O[di]^{5}\right) +$$

$$O[di]^{5}\right) +$$

$$U^{3}\left(-21\sqrt{3}\ \sqrt{g\,H(3+H^{2}k^{2})^{-2}} \ U\right) + \frac{1}{144(3+H^{2}k^{2})} + 9\left(3+H^{2}k^{2}\right)U\right) +$$

$$U^{3}\left(-21\sqrt{3}\ \sqrt{g\,H(3+H^{2}k^{2})^{-2}} \ H^{2}k^{2}\right) + 18\,U + 2\,H^{4}k^{4}\,U + k^{2}\left(-7\,\sqrt{3}\ \sqrt{g\,H(3+H^{2}k^{2})} + 12\,H^{2}U\right)\right)\right)$$

$$dt^{3} + \frac{1}{144(3+H^{2}k^{2})^{-2}}k^{2}\left(3\,g\,H + U\left(-2\,\sqrt{3}\ \sqrt{g\,H(3+H^{2}k^{2})} + (3+H^{2}k^{2})\,U\right)\right)$$

$$\left(-3\,g\,H\left(\sqrt{3}\ \sqrt{g\,H(3+H^{2}k^{2})} - 4\left(3+H^{2}k^{2}\right)U\right) + U^{2}\left(-15\,\sqrt{3}\ \sqrt{g\,H(3+H^{2}k^{2})} + 12\,H^{2}U\right)\right)\right)dt^{4} + O[di]^{5}\right)dx^{3} +$$

$$18\,U + 2\,H^{4}\,k^{4}\,U + k^{2}\left(-5\,\sqrt{3}\ \sqrt{g\,H^{5}(3+H^{2}k^{2})} + 12\,H^{2}U\right)\right)dt^{4} + O[di]^{5}\right)dx^{3} +$$

$$\left(\frac{1}{3760(3+H^{2}k^{2})^{5}}k^{5}\left(531\sqrt{3}\ \sqrt{g\,H^{2}(3+H^{2}k^{2})} - 1728\,U - 192\,H^{4}\,k^{4}\,U +$$

$$k^{2}\left(145\,\sqrt{g^{5}\,H^{3}(3+H^{2}k^{2})} - 1350\,\sqrt{3}$$

$$g^{2}\,H^{4}\,U + 2118\,\sqrt{g^{3}\,H^{7}(3+H^{2}k^{2})} U^{2} - 2227\,\sqrt{3}\,g\,H^{3}\,U^{3} + 576\,\sqrt{g\,H^{3}(3+H^{2}k^{2})} U^{4}\right) +$$

$$81\left(59\,\sqrt{g^{2}\,H^{3}(3+H^{2}k^{2})} - 241\,\sqrt{3}\,g^{2}\,H^{2}\,U + 64\,\sqrt{g\,H(3+H^{2}k^{2})}\,U^{4} +$$

$$g\,H\,U^{2}\left(369\,\sqrt{g\,H(3+H^{2}k^{2})} - 241\,\sqrt{3}\,g^{2}\,H^{2}\,U + 64\,\sqrt{g\,H(3+H^{2}k^{2})}\,U^{4} +$$

$$g\,H\,U^{2}\left(369\,\sqrt{g\,H(3+H^{2}k^{2})} - 241\,\sqrt{3}\,g^{2}\,H^{2}\,U + 64\,\sqrt{g\,H(3+H^{2}k^{2})}\,U^{4} +$$

$$g\,H\,U^{2}\left(369\,\sqrt{g\,H(3+H^{2}k^{2})} - 343\,H^{2}\,k^{2}\right) + 3320\,H^{2}\,k^{2}\,U^{3}\right)$$

$$dt^{3} + \left(628\,\sqrt{g\,H(3+H^{2}k^{2})} + 3322\,k^{4}\,\sqrt{g\,H^{9}(3+H^{2}k^{2})}\,U^{3}\right) +$$

$$6\left(288\,\sqrt{g\,H(3+H^{2}k^{2})} \,U^{3} + 322\,k^{4}\,\sqrt{g\,H^{9}(3+H^{2}k^{2})}\,U^{3}\right)$$

$$dt^{4} \right) \left(34560\,\sqrt{g\,H}\left(3+H^{2}k^{2}\right) \,U^{3} + 9192\,U\,H^{3}\right) + O[dt]^{5}\right)dx^{4} + O[dt]^{5}\right)$$

Out[261]= Omega error |

 $\left(-\frac{1}{\left(-\frac{1}{2} U k^3 + 3 U k + \sqrt{1} k^2 H \left(-\frac{1}{2} k^2 + 3 \right) k \right) k \right) k \left(-\frac{1}{2} U k^3 + 3 U k + \sqrt{1} k \right) k \left(-\frac{1}{2} U k^3 + 3 U k \right) k \left(-\frac{1}{2} U k^3 + 3 U k \right) k \left(-\frac{1}{2} U k^3 + 3 U k \right) k \left(-\frac{1}{2} U k^3 + 3 U k \right) k \left(-\frac{1}{2} U k \right) k \left(-\frac{1}{2}$ $k^2+3\right)^4+\frac{1}{2} k^2+3\right)^4+\frac{1}{2} k^2+3\left(\frac{1}{2} k^2+3\right)^5$ $\t (t_3)^4{30 \left(t_2^2 + t_3\right)} + t_4^2 (t_3)^5 + O\left(t_4^3 - t_4\right)^5 +$ $U+\sqrt{3} \sqrt{1} U +\sqrt{3} \sqrt{1} U +\sqrt{3} \sqrt{1} U +\sqrt{3} \sqrt{1} U +\sqrt{3} U$ $\sqrt{g H^5 \left(H^2 k^2+3\right)}\right) U^3+3$ $g \ H \left(\frac{4^2 k^2+3\right)}{U+5 \left(\frac{4^2 k^2+3$ $\label{eq:linear_transform} $$ \left(\frac{dt}^3}{144 \left(\frac{H^2 k^2 + 3\right)^2}{+ \frac{k^2 + 3\left(\frac{k^8 \left(\frac{H}{2 g} H + U \left(\frac{H^2 k^2 + 3\right)}{H + U} \right) + \frac{k^2 H + U}{H + U} \right)}{H + U} \right) } \right) $$$ $\$ \\ \quad H \\ \left(H^2 k^2+3\\ \right)\\ \right)\\ \left(\\ \left(2 H^4 U k^4+\\ \left(12 U H^2+5 \\ \right)\\ \right)\\ \right)\\ \quad H^5 \\ \quad \quad H^5 \\ \quad $\left(H^2 k^2+3\right) \ k^2+18 U+15 \ \sqrt{15} \ \sqrt{15}$ $\label{left(H^2 k^2+3\left| h(H^2 k^2+3\right|)\left| h(H^$ $\label{eq:k-2+3-right} $$ k^2+3\right) + O\left(\frac{t}{t}^5\right) \left(\frac{dt}{5}\right) \left(\frac{$ U H^2+145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)\right) k^2+1728 U+531 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} $k^2+3\right)\left(U^3\right)\left(U^3\right)\left(U^3\right)$ $U+1011 \sqrt{H^2 k^2+3\right} + H^5+576 \sqrt{H^2 k^2+3\right} + U+1011 \sqrt{H^2 k^2+3} + U+1011$ k^4+9 \left(1350 \sqrt{3} g^2 U H^4+2227 \sqrt{3} g U^3 H^3+576 \sqrt{g H^5 \left(H^2 k^2+3\right)} $\label{eq:conditional} $$ U^4+2118 \left(g^3 H^7 \left(H^2 k^2+3\right) U^2+145 \left(g^5 H^9 \left(H^2 k^2+3\right)\right)\right)\right) $$$ $k^2 + 81 \left(\frac{64 \operatorname{sqrt}\{g \ H \left(\frac{4^2 \ k^2 + 3\right)} U^4 + g \ H \left(\frac{51 \ \operatorname{sqrt}\{3\} \ U + 369 \ \operatorname{sqrt}\{g \ H \left(\frac{4^2 \ k^2 + 3\right)} \right) }{1 + 369 \ \operatorname{sqrt}\{g \ H \left(\frac{4^2 \ k^2 + 3\right)} \right) } \right)$ k^2+3\right)\right) U^2+241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)\right)\right)\right) $\t (4t)^3 {34560 } g H+U \left(h^2 k^2+3\right)^{7/2} - \frac{(6+6)^2 H+U \left(h^2 k^2+3\right)^{7/2}}{h^2} \right)$ $k^2+3\right\} U+2 \sqrt{3} \sqrt{4} H \left(H^2 k^2+3\right)\right) \left(H^2 k^2+3\right) \left$ k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right) U+4914 \sqrt{g $H \left(\frac{4^2 k^2+3\right)}{h^2 k^2+3\right) + h^6 \left(\frac{4^2 k^2+3\right) + h^6 \left(\frac{$ \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+241 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+288 $\left(H^2 \cup k^3 + 3 \cup k - \sqrt{4}\right) k - \left(H^2 \cup k^3 + 3 \cup k^3 + 3$ $k^2+3\right)^4+\frac{h^2 U k^3+3 U k-\sqrt{3}}{gH \left(\frac{h^2 U k^3+3 U k-\sqrt{3}}{gH \left(\frac{h^2 U k^2+3\right)}{gH \left(\frac{h^2 U k^3+3 U k-\sqrt{3}}{gH \left(\frac{h^2 U k-\sqrt{3$ $\label{eq:left} $$ \operatorname{dt}^4{30 \left(\frac{1}{24} i k^2 + 3\right)} + \operatorname{left}(\operatorname{dt}^5\right) + \operatorname{left}(-\frac{1}{24} i k^4 \left(\frac{3}{24} i k^4 \right)) + \operatorname{left}(-\frac{1}{24} i k^4 \left(\frac{3}{24} i k^4 i$ $\$ \sqrt{\frac{g H}{H^2 k^2+3}}-2 U\right)+\frac{k^7 \\left(\left(2 H^4 U k^4+\\left(12 H^2 U-7 \\sqrt{3} \\sqrt{g})} $H^5 \left(H^2 k^2+3\right) \left(H^2 k^2+$ $H \left(\frac{h^2 k^2+3\right) U-5 \left(H^2 k^2+3\right) U-5 \left(H^2 k^2+3\right) U-9 g^2 H^2\right)$ $\label{eq:left} $$ \left(\frac{dt}^3}{144 \left(\frac{H^2 k^2 + 3\right)}{144 eft(H^2 k^2 + 3\right)} U - 2 \right) $$ (4.5) $$ \left(\frac{H^2 k^2 + 3\right) U - 2 + \frac{H^2 k^2 + 3}{144 eft(H^2 k^2 + 3\right)} U - 2 \right) $$ (4.5)$ $\sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+18 U-15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \right)}$ $g\ H \left(\frac{1}{4} \right) - \left(\frac{1}$ $\left(H^2 k^2+3\right)^3+O\left(t_{4}^5\right)\right) \times \left(H^2 k^2+3\right)^3+O\left(t_{4}^5\right)$ k^4+\left(145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-1152 H^2 U\right) k^2-1728 U+531 \sqrt{3} $\label{eq:left(H^2 k^2+3\wedge ight)} $$\left(H^2 k^2+3 \right)^2 + \frac{k^2+3 \cdot (U^3 \left(H^2 k^2+3 \right)^2}{h^2} + \frac{k^8 \left(U^3 \left(H^2 k^2+3 \right)^2}{h^2} \right)^2 + \frac{k^3 \left(H^2 k^2 + 1 \right)^2}{h^2} +$ $\left(H^{13} \left(H^2 k^2 + 3\right) U - 721 \right) H^7\right) h^2 \left(H^2 k^2 + 3\right) U \left(H^2 k^2 + 3\right) U \left(H^2 k^2 + 3\right) H^6 + g H^7\right) h^2 \left(H^2 k^2 + 3\right) H^6 + g H^7\right) H^7$ $U \left(\frac{195 \sqrt{3} U - 1011 \sqrt{4} H^2 k^2 + 3 \right) H^5 - 576 \sqrt{4} }{U - 1011 \sqrt{4} k^2 + 3 \right)$ $k^2+3\right) U^3\right) U^3\right) k^4+9\left(-1350 \right) 3 q^2 U H^4-2227 \right] 4 U^3 H^3+576 \right]$

H^5 \left(H^2 k^2+3\right)\} U^4+2118 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)\} U^2+145 \sqrt{g^5 H^9} $\label{left} $\left(H^2 k^2+3\right)\right] \ h^2+81 \left(H^2 k^2+3\right) \ h^2+81 \left(H^2 k^2+3\right) \ h^2+9 \ h^2+9$ H \left(H^2 k^2+3\right)\-251 \sqrt{3} U\right) U^2-241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2 $k^2+3\right) \left(\frac{dt}^3}{34560 \cdot \frac{H} \left(\frac{h^2 k^2+3\right)}{172}} + \frac{h^2 k^9 \left(\frac{dt}^3}{34560 \cdot \frac{h^2 k^2+3\right)}} \right) \right)$ $H+U \left(\frac{h^2 k^2+3\right)}{U-2 \sqrt{3} \sqrt{3} \left(\frac{h^2 k^2+3\right)} \right) \left(\frac{h^2 k^2+3\right)}{U-2 \sqrt{3} \sqrt{3} \sqrt{3}}$ g^2 \left(145 H^2 k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right) $U-4914 \sqrt{H^2 k^2+3\right} H-6 \left(32 \sqrt{H^9 \left(H^2 k^2+3\right)}\right) U-3$ $k^4+\left(192 \right) \left(192 \right) U^3+241 \right) U^3+241$ $U\left(h^2 + 288 \right) + U\left(h^2 + 24\right) + U^3\right) + U^3) + U^3) + U^3$ + UV^3) + U^3) + U^3) + U^3) + U^3 + UV^3) + U^3) + U^3) + U^3 + UV^3) + UV^3 + UV^3) + UV^3 + UV^3) + UV^3 $\left(H^2 k^2+3\right)^{7/2}+O\left(\frac{dt}^5\right)\right)$

Out[262]=

$$\text{Out} [263] = \quad EA \mid \mid \quad \left\{ \left\{ \frac{-H^2 \ k^2 \left(\left(-1 + e^{i \ d \ w} \right) k \ U - w \right) + 3 \ w}{\left(3 + H^2 \ k^2 \right) w}, \quad -\frac{3 \left(-1 + e^{i \ d \ w} \right) k}{\left(3 + H^2 \ k^2 \right) w} \right\}, \quad \left\{ -\frac{\left(-1 + e^{i \ d \ t \ w} \right) k \left(g \ H \left(3 + H^2 \ k^2 \right) - 3 \ U^2 \right)}{\left(3 + H^2 \ k^2 \right) w}, \quad 1 \quad -\frac{\left(-1 + e^{i \ d \ t \ w} \right) k \left(6 + H^2 \ k^2 \right) U}{\left(3 + H^2 \ k^2 \right) w} \right\} \right\}$$

Out[264]= EA || \left(

\begin{array}{cc}

 $\frac{3 w-H^2 k^2 \left(\left(-1+e^{i \cdot text \cdot dt} w\right)\right)}{\left(-1+e^{i \cdot text \cdot dt} w\right)} k U-w\right)}{\left(-1+e^{i \cdot text \cdot dt} w\right)}$ $k^2+3\right) w$ & $-\frac{3 \left(1+e^{i \left(t\right) w}\right) k}{\left(t^2 k^2+3\right) w} \$ $-\frac{1+e^{i \cdot text\{dt\} w}\right| k \left(H^2 k^2+3\right)}{\|h\|^2 k^2+3\right)} \|h\|^2 \|h\|^2$ \end{array}

\right)

```
Out[265]= Eerr |
                                                                                                                                                \left\{\left\{\left(\frac{\left(-3\,g\,H\,k^2+3\,k^2\,U^2-H^2\,k^4\,U^2-H^2\,k^3\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}+\right.\\ \left.\frac{1}{6\,\left(3+H^2\,k^2\right)^2}i\left(18\,g\,H\,k^3\,U+9\,g\,H^3\,k^5\,U-18\,k^3\,U^3-3\,H^2\,k^5\,U^3+H^4\,k^7\,U^3-18\,k^3\,U^3+H^2\,k^2\right)\right\}\right\} + \frac{1}{6\,\left(3+H^2\,k^2\right)^2}i\left(18\,g\,H\,k^3\,U+9\,g\,H^3\,k^5\,U-18\,k^3\,U^3-3\,H^2\,k^5\,U^3+H^4\,k^7\,U^3-18\,k^3\,U^3+H^2\,k^2\right)\right\}
                                                                                                                                                                                                                                                                                                                                                                              3\;H^2\;k^3\;U\;w^2-H^4\;k^5\;U\;w^2\big)\,dt^3\,+\, \frac{H^2\,k^3\,U\,w^3\,dt^4}{24\,(3+H^2\,k^2)}\,+\,O[dt]^5\Big)\,+
                                                                                                                                                                                                                                   \left(\frac{1}{12}\,k^4\,U\,dt - \frac{\it i\,(3\,g\,H\,k^5 + 2\,H^2\,k^7\,U^2)\,dt^2}{24\,(3 + H^2\,k^2)} - \frac{\left(U\,(15\,g\,H\,k^6 + 6\,g\,H^3\,k^8 - 9\,k^6\,U^2 + H^4\,k^{10}\,U^2)\right)dt^3}{24\,(3 + H^2\,k^2)^2} + O[dt]^5\right)dx^3 + \\ \left(\frac{\it i\,(45\,k^5\,U + 143\,H^2\,k^7\,U + 32\,H^4\,k^9\,U)\,dt}{960\,(3 + H^2\,k^2)^2} + \frac{\left(531\,g\,H\,k^6 + 145\,g\,H^3\,k^8 - 486\,k^6\,U^2 + 94\,H^2\,k^8\,U^2 + 64\,H^4\,k^{10}\,U^2\right)dt^2}{1920\,(3 + H^2\,k^2)^2} - \frac{1}{1920\,(3 + H^2\,k^2)^3}\right) + \frac{1}{1920\,(3 + H^2\,k^2)^2}
                                                                                                                                                                                                                                                                                                                         i\,k^{7}\left(1683\,g\,H\,U\,+\,1348\,g\,H^{3}\,k^{2}\,U\,+\,241\,g\,H^{5}\,k^{4}\,U\,-\,1593\,U^{3}\,-\,678\,H^{2}\,k^{2}\,U^{3}\,+\,47\,H^{4}\,k^{4}\,U^{3}\,+\,32\,H^{6}\,k^{6}\,U^{3}\right)
                                                                                                                                                                                                                                                                                                                                                     dt^3 + O[dt]^5 dx^4 + O[dx]^5,
                                                                                                                                                                                                       \left(-\frac{3 \left(2 \, k^2 \, U+k \, w\right) dt^2}{2 \left(3+H^2 \, k^2\right)}+\frac{i \left(3 \, g \, H \, k^3+9 \, k^3 \, U^2+3 \, H^2 \, k^5 \, U^2-3 \, k \, w^2-H^2 \, k^3 \, w^2\right) dt^3}{2 \left(3+H^2 \, k^2\right)^2}+\frac{k \, w^3 \, dt^4}{8 \left(3+H^2 \, k^2\right)}+O[dt]^5\right)+\frac{k \, w^3 \, dt^4}{2 \left(3+H^2 \, k^2\right)^2}+O[dt]^5
                                                                                                                                                                                                                                   \left(-\tfrac{i\,k^5\,U\,dt^2}{4\,(3+H^2\,k^2)}+\tfrac{\left(-g\,H\,k^6-6\,k^6\,U^2-2\,H^2\,k^8\,U^2\right)dt^3}{8\,(3+H^2\,k^2)^2}+O[dt]^5\right)dx^3\,+
                                                                                                                                                                                                                                \left(\frac{\imath \left(243 \, k^5 + 49 \, H^2 \, k^7\right) dt}{960 \left(3 + H^2 \, k^2\right)^2} + \frac{\left(531 \, k^6 + 145 \, H^2 \, k^8\right) U \, dt^2}{960 \left(3 + H^2 \, k^2\right)^2} - \frac{\imath \left(774 \, g \, H \, k^7 + 194 \, g \, H^3 \, k^9 + 2457 \, k^7 \, U^2 + 1542 \, H^2 \, k^9 \, U^2 + 241 \, H^4 \, k^{11} \, U^2\right) dt^3}{1920 \left(3 + H^2 \, k^2\right)^3} + O[dt]^5\right) dx^4 + O[dt]^5
                                                                                                                                                                                                                                   O[dx]^5\Big\},\,\,\Big\{\!\Big(\!-\frac{\big(\!\big(3\,g\,H+g\,H^3\,k^2-3\,U^2\big)\big(2\,k^2\,U+k\,w\big)\!\big)\,dt^2}{2\,\big(3+H^2\,k^2\big)}\,+\,
                                                                                                                                                                                                                                                                                           \left(\frac{1}{2\left(3+H^{2}\,k^{2}\right)^{2}}\emph{i}\left(3\,g^{2}\,H^{2}\,k^{3}+g^{2}\,H^{4}\,k^{5}+6\,g\,H\,k^{3}\,U^{2}+6\,g\,H^{3}\,k^{5}\,U^{2}+g\,H^{5}\,k^{7}\,U^{2}-9\,k^{3}\,U^{4}-3\,H^{2}\,k^{5}\,U^{4}\right)-\right.
                                                                                                                                                                                                                                                                                                                                                                                            \frac{i \, k \, (g \, H \, (3 + H^2 \, k^2) - 3 \, U^2) \, w^2}{6 \, (3 + H^2 \, k^2)} \right) dt^3 + \frac{k \, (g \, H \, (3 + H^2 \, k^2) - 3 \, U^2) \, w^3 \, dt^4}{24 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) +
                                                                                                                                                                                                                                   \Big( \frac{_{12}}{_{12}} g H k^4 dt - \frac{_{\it i} (6 \, g \, H \, k^5 \, U + 2 \, g \, H^3 \, k^7 \, U - 3 \, k^5 \, U^3) \, dt^2}{_{12} (3 + H^2 \, k^2)} - \frac{_{\it 1}}{_{\it 24} (3 + H^2 \, k^2)^2} \Big( 6 \, g^2 \, H^2 \, k^6 + 2 \, g^2 \, H^4 \, k^8 + 24 \, g \, H \, k^6 \, U^2 + 2 \, g^2 \, H^4 \, k^6 + 2 \, g^2 \, H^4 \, k^8 + 24 \, g \, H \, k^6 \, U^2 + 2 \, g^2 \, H^4 \, k^6 + 2 \, g^2 \, H^4 \, k^8 + 24 \, g \, H \, k^6 \, U^2 + 2 \, g^2 \, H^4 \, k^6 + 2 \, g^2 \, H
                                                                                                                                                                                                                                                                                                                                                                                                    18 g H^3 k^8 U^2 + 3 g H^5 k^{10} U^2 - 18 k^6 U^4 - 6 H^2 k^8 U^4) dt^3 + O[dt]^5 dx^3 +
                                                                                                                                                                                                                                                 \frac{i\left(288\,\mathrm{g\,H\,k^5} + 192\,\mathrm{g\,H^3\,k^7} + 32\,\mathrm{g\,H^5\,k^9} - 243\,\mathrm{k^5\,U^2} - 49\,\mathrm{H^2\,k^7\,U^2}\right)\,\mathrm{dt}}{960\,(3 + \mathrm{H^2\,k^2})^2} + \frac{\left(576\,\mathrm{g\,H\,k^6\,U} + 384\,\mathrm{g\,H^3\,k^8\,U} + 64\,\mathrm{g\,H^5\,k^{10}\,U} - 531\,\mathrm{k^6\,U^3} - 145\,\mathrm{H^2\,k^8\,U^3}\right)\,\mathrm{dt^2}}{960\,(3 + \mathrm{H^2\,k^2})^2} - \frac{1}{120\,\mathrm{g\,H^3\,k^7} + 32\,\mathrm{g\,H^3\,k^7} + 32\,\mathrm{
                                                                                                                                                                                                                                                                                                                               \frac{1}{5760\,(3+H^2\,k^2)^3}\dot{\imath}\,\left(2457\,g^2\,H^2\,k^7+1542\,g^2\,H^4\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H\,k^7\,U^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H^2\,k^2+7194\,g\,H^3\,k^9\,U^2+2592\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H^2\,k^9+241\,g^2\,H
                                                                                                                                                                                                                                                                                                                                                                                                                                              H^5 k^{11} U^2 + 288 g H^7 k^{13} U^2 - 7371 k^7 U^4 - 4626 H^2 k^9 U^4 - 723 H^4 k^{11} U^4) dt^3 + O[dt]^5 dx^4 + O[dt]^5 dx^4 + O[dt]^5 dt^3 + O[dt]^5 dx^4 + O[dt]^5 dx^5 + O[dt]^5 dx^4 + O[dt]^5 dx^5 + O[dt]^5 
                                                                                                                                                                                                                                   O[dx]^5, \left(\frac{\left(-3\,g\,H\,k^2-9\,k^2\,U^2-H^2\,k^4\,U^2-6\,k\,U\,w-H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)} + \frac{1}{6\left(3+H^2\,k^2\right)^2}i\left(36\,g\,H\,k^3\,U + 9\,g\,H^3\,k^5\,U + 36\,k^3\,U^3 + 46\,k^3\,U^3 + 46\,k^3\,U^3
                                                                                                                                                                                                                                                                                                                                                                                   15 H^2 k^5 U^3 + H^4 k^7 U^3 - 18 k U w^2 - 9 H^2 k^3 U w^2 - H^4 k^5 U w^2 dt^3 + \frac{k (6 + H^2 k^2) U w^3 dt^4}{24 (3 + H^2 k^2)} + O[dt]^5 + O
                                                                                                                                                                                                                                   \left(\frac{1}{12}\,k^4\,U\,dt - \frac{\imath\,k^5\left(3\,g\,H + 12\,U^2 + 2\,H^2\,k^2\,U^2\right)dt^2}{24\left(3 + H^2\,k^2\right)} - \frac{\left(21\,g\,H\,k^6\,U + 6\,g\,H^3\,k^8\,U + 27\,k^6\,U^3 + 12\,H^2\,k^8\,U^3 + H^4\,k^{10}\,U^3\right)dt^3}{24\left(3 + H^2\,k^2\right)^2} + O[dt]^5\right)dx^3 + O[dt]^{-1}\left(\frac{1}{12}\,k^4\,U\,dt - \frac{\imath\,k^5\left(3\,g\,H + 12\,U^2 + 2\,H^2\,k^2\,U^2\right)dt^2}{24\left(3 + H^2\,k^2\right)^2} - \frac{\left(21\,g\,H\,k^6\,U + 6\,g\,H^3\,k^8\,U + 27\,k^6\,U^3 + 12\,H^2\,k^8\,U^3 + H^4\,k^{10}\,U^3\right)dt^3}{24\left(3 + H^2\,k^2\right)^2} + O[dt]^{-1}\left(\frac{1}{12}\,k^4\,U\,dt - \frac{3}{12}\,k^4\,U\,dt - \frac{3}
                                                                                                                                                                                                                                                      \frac{i \left(531 \, \mathrm{k}^5 + 241 \, \mathrm{H}^2 \, \mathrm{k}^7 + 32 \, \mathrm{H}^4 \, \mathrm{k}^9\right) \, \mathrm{U} \, \mathrm{dt}}{960 \left(3 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \,\, + \,\, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^8 \, \mathrm{U}^2 + 64 \, \mathrm{H}^4 \, \mathrm{k}^{10} \, \mathrm{U}^2\right) \, \mathrm{dt}^2}{1920 \left(3 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \,\, - \,\, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^8 \, \mathrm{U}^2 + 64 \, \mathrm{H}^4 \, \mathrm{k}^{10} \, \mathrm{U}^2\right) \, \mathrm{dt}^2}{1920 \left(3 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \,\, - \,\, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^8 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^8 \, \mathrm{U}^2 + 64 \, \mathrm{H}^4 \, \mathrm{k}^{10} \, \mathrm{U}^2\right) \, \mathrm{dt}^2}{1920 \left(3 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \,\, - \,\, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^6 + 1638 \, \mathrm{k}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^2 \, \mathrm{k}^8 \, \mathrm{U}^2 + 64 \, \mathrm{H}^4 \, \mathrm{k}^{10} \, \mathrm{U}^2\right) \, \mathrm{dt}^2}{1920 \left(3 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \,\, - \,\, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^6 + 1638 \, \mathrm{h}^6 \, \mathrm{U}^2 + 674 \, \mathrm{H}^4 \, \mathrm{k}^{10} \, \mathrm{U}^2\right) \, \mathrm{dt}^2}{1920 \left(3 + \mathrm{H}^2 \, \mathrm{k}^2\right)^2} \,\, - \,\, \frac{\left(531 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{k}^6 + 145 \, \mathrm{g} \, \mathrm{H}^3 \, \mathrm{H}^4 \, \mathrm{H}
                                                                                                                                                                                                                                                                                                                               \frac{1}{1920(3+H^2k^2)^3}i\left(3231 \text{ g H k}^7 \text{ U} + 1736 \text{ g H}^3 \text{ k}^9 \text{ U} + 241 \text{ g H}^5 \text{ k}^{11} \text{ U} + 3321 \text{ k}^7 \text{ U}^3 + 411 \text{ U}^3 + 411
                                                                                                                                                                                                                                                                                                                                                                                                                  2406 H^2 k^9 U^3 + 529 H^4 k^{11} U^3 + 32 H^6 k^{13} U^3 dt^3 + O[dt]^5 dx^4 + O[dx]^5
```

Out[266]= Eerr || \left(

\begin{array}{cc}

 $\left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{t c^2} \left(\frac{-H^2 U w k^3+3 U w k^3+3$ \left(H^4 U^3 k^7-3 H^2 U^3 k^5-H^4 U w^2 k^5+9 g H^3 U k^5-18 U^3 k^3-3 H^2 U w^2 k^3+18 $k^2+3\left(\frac{1}{12} k^4 U \left(\frac{1}{11} k^4 U \right)\right) + 0 k^2 H^2 U^2 k^7+3 H^2$ $U^2 k^6 + 15 g H k^6 \left(\frac{t}^3}{24 \left(\frac{K^2 + 3\left(\frac{k^2 + 3\left(\frac{t}^2 + \frac{t}{k^2} \right)}{k^2 + \frac{t}{k^2}} \right)}{k^2 + \frac{t}{k^2}} \right)} + O\left(\frac{t}{k^2} + \frac{t}{k^2} \right) + O\left(\frac{t}{k^2} + \frac{t}{k^2$ \text{dx}^3+\left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}}\960 \left(H^2

k^2+3\right)^2}+\frac{\left(64 H^4 U^2 k^{10}+145 g H^3 k^8+94 H^2 U^2 k^8-486 U^2 k^6+531 H^2 U^2 k^8-486 U^2 k^6+531 H^2 U^2 k^8-486 U^2 k^8+145 g H^3 k^8+145 g H^2 U^2 k^2 g H k^6\right)\text{dt}^2\{1920 \left(H^2 k^2+3\right)^2}-\frac{i k^7 \left(32 H^6 U^3 k^6+47 H^4 U^3 k^4+241 g H^5 U k^4-678 H^2 U^3 k^2+1348 g H^3 U k^2-1593 U^3+1683 g H U\right) $\label{left(dt)^3} $\{1920 \left(H^2 k^2+3\right)^3\}+O\left(text\{dt\}^5\right)\right) \text\{dx\}^4+O\left(text\{dx\}^5\right)^3\} + O\left(text\{dx\}^6\right) \text\{dx\}^6\right) \text\{dx\}^6 + O\left(text\{dx\}^6\right) \text\{dx\}^6\right) \text\{dx\}^6 + O\left(text\{dx\}^6\right) \text\{dx\}^6\right) \text{}$ & $\left(\frac{1}{2} \left(\frac{U k^2+w \left(\frac{dt}{2}\right)^2}{2 \left(\frac{H^2 k^2+w \left(\frac{dt}{2}\right)^2}{2 (\frac{H^2 k^$ $k^5+9 U^2 k^3-H^2 w^2 k^3+3 g H k^3-3 w^2 k \right) \left(\frac{1}{2} \left(\frac{4}{3} \right) \left(\frac{4}{3} \right) \left(\frac{4}{3} \right) \right) \left(\frac{4}{3} \right) \left(\frac{4}{3$ $w^3 \text{$\ensuremath{\wedge}} = k^2 + 3 \text{$\ensuremath{$\wedge$}} + O\left(\frac{dt}^5 \right) + O\left$ \text{dt}^2}{4 \left(H^2 k^2+3\right)}+\frac{\left(-2 H^2 U^2 k^8-6 U^2 k^6-g H k^6\right)} $\t (t_3)_{8 \left(t_4^2 + \frac{dt}^3}_{8 \left(t_4^2 + \frac{dt}^3}\right) \right) \\$ $\left(49 \text{ H}^2 \text{ k}^7 + 243 \text{ k}^5\right) \left(49 \text{ H}^2 \text{ k}^2 + 3\right)^2 + \frac{145 \text{ H}^2}{2} + \frac{145 \text{ H}^2}{2$ k^8+531 k^6\right) U \text{dt}^2\{960 \left(H^2 k^2+3\right)^2\}-\frac\{i \left(241 H^4 U^2 \) k^{11}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7\right) \text{dt}^3}{1920 $\left(H^2 k^2+3\right)^3+O\left(\frac{dt}^5\right)\right) \left(\frac{dt}^3+O\left(\frac{dt}^5\right)\right)$ $\label{left(-frac} \left(\left(\frac{k^2 H^3 + 3 g H - 3 U^2 \right) \left(U k^2 + w \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \\ = \frac{1}{2} \left(\frac{1}{2} \left(\frac{1}{2} \right) \right) \\ = \frac{1}{2} \left$ k^2+3\right)}+\left(\frac{i \left(g H^5 U^2 k^7+g^2 H^4 k^5-3 H^2 U^4 k^5+6 g H^3 U^2 k^5-9 U^4 k^3+3 U^2 k^5-9 U^4 k^3+3 U^2 k^5-9 U^4 k^5+6 U^2 k^5-9 U^4 $g^2 H^2 k^3+6 g H U^2 k^3 \left(\frac{h^2 k^2+3\right)}{2 \left(\frac{h^2 k^2+3\right)}{2}-\frac{k^2 k^3+6 g H U^2 k^3\right)}$ $U^2\left(t\right) w^3 \left(t\right)^4\left(t\right)^4\left(t\right)^2 k^2+3\left(t\right)^4\left$ g H k⁴ \text{dt}-\frac{i \left(2 g H³ U k⁷-3 U³ k⁵+6 g H U k⁵\right) \text{dt}²{12 $\left(H^2 k^2+3\right) - \left(g H^5 U^2 k^{10}+2 g^2 H^4 k^8-6 H^2 U^4 k^8+18\right)$ g H^3 U^2 k^8-18 U^4 k^6+6 g^2 H^2 k^6+24 g H U^2 k^6\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(32 g H^5 k^9+192 g H^3 k^7-49 $H^2 U^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right] \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 - 243 U^2 k^5 + 288 g H k^5 \right) \left(\frac{d^2 k^7 + 288 g H k^7 \right) \left(\frac{d^2 k^7 + 288 g H k^7 \right) \left(\frac{d^2 k^7 + 288 g H k^7 \right) \left(\frac{d^2 k^7 + 288 g H k^7 \right) \left(\frac$ U^2 k^{11}+1542 g^2 H^4 k^9-4626 H^2 U^4 k^9+7194 g H^3 U^2 k^9-7371 U^4 k^7+2457 g^2 H^2 k^7+5454 g H U² k⁷\right) \text{dt}³{5760 \left(H² k²+3\right)³}+O\left(\text{dt}⁵\right)\right) $\label{eq:left} $$ \left(\frac{dx}^4+O\left(\frac{dx}^5\right) & \left(\frac{-H^2 U^2 k^4-H^2 U w k^3-9 U^2 k^2-3 u^2 k^4-H^2 U w k^3-9 U^2 k^4-H^2 U w k^3-1 U w$ $g \ H \ k^2 - 6 \ U \ w \ k \ | \ text{dt}^2{2} \ left(H^2 \ k^2 + 3 \ right)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)} + \ frac{i \ left(H^4 \ U^3 \ k^7 + 15 \ H^2)}{i \ left(H^4 \ U^3 \ k^7$ U³ k⁵-H⁴ U w² k⁵+9 g H³ U k⁵+36 U³ k³-9 H² U w² k³+36 g H U k³-18 U w² $$ \left(\frac{d^3}{6 \left(\frac{4}{2} \right)^2} + \frac{k \left(\frac{4}{2} \right)^2}{16 \left(\frac{4}{2} \right)^2} + \frac{k \left(\frac{4}{2} \right)^2}{16 \left(\frac{4}{2} \right)^2} \right) } $$ $\label{left} $$\left(H^2 k^2+3\right)+O\left(\frac{dt}^5\right)+$ U^3 k^{10}+12 H^2 U^3 k^8+6 g H^3 U k^8+27 U^3 k^6+21 g H U k^6\right) \text{dt}^3}{24 $\left(H^2 k^2+3\right)^2+O\left(text\left(dt\right)^5\right)\right) \left(text\left(dx\right)^3+\left(text\left(dx\right)^3+C\left(dx\right)^3\right)\right)$ $k^9+241 H^2 k^7+531 k^5 right) U \text{ } (64 H^4 k^9+24) right)^2+\frac{h^4 k^9+241 H^2 k^7+531 k^5 right)}{1}$ U^2 k^{10}+145 g H^3 k^8+674 H^2 U^2 k^8+1638 U^2 k^6+531 g H k^6\right) \text{dt}^2\{1920 \left(H^2 k^2+3\right)^2}-\frac{i \left(32 H^6 U^3 k^{13}+529 H^4 U^3 k^{11}+241 g H^5 U k^{11}+2406 H^2 U^3 k^9+1736 g H^3 U k^9+3321 U^3 k^7+3231 g H U k^7\right) \text{dt}^3}{1920 $\left(H^2 k^2+3\right)^3+O\left(\frac{dt}^5\right) \left(\frac{dt}^3+O\left(\frac{dt}^3\right)\right)$

\end{array}

\right)

In[267]:=