

```

In[543]:= MA = k * x / (2 * Sin[k * x / 2])
RA = Exp[I * k * x / 2] * k * x / (2 * Sin[k * x / 2])
GA = k * x / ((H + H^3 / 3 * k^2) * Exp[-I * k * x / 2] * (2 * Sin[k * x / 2]))
FnnA = 0
FnGA = I * k / (1 + H^2 * k^2 / 3)
FGnA = g * H * I * k
FGGA = 0
FmatA = {{FnnA, FnGA}, {FGnA, FGGA}}
wAp = Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2)]
wAm = -Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2)]
Eigenvalues[FmatA]

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$$\text{Out[543]} = \frac{1}{2} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$$

$$\text{Out[544]} = \frac{1}{2} e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$$

$$\text{Out[545]} = \frac{e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right)}$$

$$\text{Out[546]} = 0$$

$$\text{Out[547]} = \frac{i k}{1 + \frac{H^2 k^2}{3}}$$

$$\text{Out[548]} = i g H k$$

$$\text{Out[549]} = 0$$

$$\text{Out[550]} = \left\{ \left\{ 0, \frac{i k}{1 + \frac{H^2 k^2}{3}} \right\}, \{i g H k, 0\} \right\}$$

$$\text{Out[551]} = \sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$$

$$\text{Out[552]} = -\sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$$

$$\text{Out[553]} = \left\{ -\frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2} \right\}$$

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In[554]:= M2 = 1
Series[M2 - MA, {x, 0, 10}]

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$$\text{Out[554]} = 1$$

$$\text{Out[555]} = -\frac{k^2 x^2}{24} - \frac{7 k^4 x^4}{5760} - \frac{31 k^6 x^6}{967680} - \frac{127 k^8 x^8}{154828800} - \frac{73 k^{10} x^{10}}{3503554560} + O[x]^{11}$$

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In[556]:= Rm = (1 + I * Sin[k * x] / 2)
Series[Rm - RA, {x, 0, 10}]
Rp = Exp[I * k * x] * (1 - I * Sin[k * x] / 2)
Series[Rp - RA, {x, 0, 10}]
```

$$\text{Out[556]} = 1 + \frac{1}{2} i \sin[k x]$$

$$\text{Out[557]} = \frac{k^2 x^2}{12} - \frac{1}{12} i k^3 x^3 + \frac{k^4 x^4}{720} + \frac{1}{240} i k^5 x^5 + \frac{k^6 x^6}{30240} - \frac{i k^7 x^7}{10080} + \frac{k^8 x^8}{1209600} + \frac{i k^9 x^9}{725760} + \frac{k^{10} x^{10}}{47900160} + O[x]^{11}$$

$$\text{Out[558]} = e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right)$$

$$\text{Out[559]} = \frac{k^2 x^2}{12} + \frac{1}{6} i k^3 x^3 - \frac{89 k^4 x^4}{720} - \frac{7}{120} i k^5 x^5 + \frac{631 k^6 x^6}{30240} + \frac{31 i k^7 x^7}{5040} - \frac{1889 k^8 x^8}{1209600} - \frac{127 i k^9 x^9}{362880} + \frac{481 k^{10} x^{10}}{6842880} + O[x]^{11}$$

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In[602]:= GLHS = x / 6 * (Rp + Rm)
GRHSp1 = -Exp[-I * k * x / 2] + 2 + 4 * Exp[I * k * x / 2] +
Exp[I * k * x] * (4 * Exp[-I * k * x / 2] + 2 - Exp[I * k * x / 2])
GRHSp1 = GRHSp1 / Exp[I * k * x / 2]
GRHSp1 = Expand[GRHSp1]
GRHSp1 = ExpToTrig[GRHSp1]
GRHSp2 = Exp[-I * k * x / 2] - 8 + 7 * Exp[I * k * x / 2] +
Exp[I * k * x] * (7 * Exp[-I * k * x / 2] - 8 + Exp[I * k * x / 2])
GRHSp2 = GRHSp2 / Exp[I * k * x / 2]
GRHSp2 = Expand[GRHSp2]
GRHSp2 = ExpToTrig[GRHSp2]
G = GLHS / (H * x / 30 * (GRHSp1) + H^3 / (9 * x) * GRHSp2)
Series[G, {x, 0, 3}]
Series[GA, {x, 0, 3}]
Series[G - GA, {x, 0, 5}]
```

$$\text{Out[602]} = \frac{1}{6} x \left(1 + e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right) + \frac{1}{2} i \sin[k x] \right)$$

$$\text{Out[603]} = 2 - e^{-\frac{1}{2} i k x} + 4 e^{\frac{i k x}{2}} + e^{i k x} \left(2 + 4 e^{-\frac{1}{2} i k x} - e^{\frac{i k x}{2}} \right)$$

$$\text{Out[604]} = e^{-\frac{1}{2} i k x} \left(2 - e^{-\frac{1}{2} i k x} + 4 e^{\frac{i k x}{2}} + e^{i k x} \left(2 + 4 e^{-\frac{1}{2} i k x} - e^{\frac{i k x}{2}} \right) \right)$$

$$\text{Out[605]} = 8 + 2 e^{-\frac{1}{2} i k x} + 2 e^{\frac{i k x}{2}} - e^{-i k x} - e^{i k x}$$

$$\text{Out[606]} = 8 + 4 \cos\left[\frac{k x}{2}\right] - 2 \cos[k x]$$

$$\text{Out[607]} = -8 + e^{-\frac{1}{2} i k x} + 7 e^{\frac{i k x}{2}} + e^{i k x} \left(-8 + 7 e^{-\frac{1}{2} i k x} + e^{\frac{i k x}{2}} \right)$$

$$\text{Out[608]} = e^{-\frac{1}{2} i k x} \left(-8 + e^{-\frac{1}{2} i k x} + 7 e^{\frac{i k x}{2}} + e^{i k x} \left(-8 + 7 e^{-\frac{1}{2} i k x} + e^{\frac{i k x}{2}} \right) \right)$$

$$\text{Out[609]} = 14 - 8 e^{-\frac{1}{2} i k x} - 8 e^{\frac{i k x}{2}} + e^{-i k x} + e^{i k x}$$

$$\text{Out[610]} = 14 - 16 \cos\left[\frac{k x}{2}\right] + 2 \cos[k x]$$

$$\text{Out[611]} = \left(x \left(1 + e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right) + \frac{1}{2} i \sin[k x] \right) \right) / \left(6 \left(\frac{1}{30} H x \left(8 + 4 \cos\left[\frac{k x}{2}\right] - 2 \cos[k x] \right) + \frac{H^3 \left(14 - 16 \cos\left[\frac{k x}{2}\right] + 2 \cos[k x] \right)}{9 x} \right) \right)$$

$$\text{Out[612]} = \frac{3}{3 H + H^3 k^2} + \frac{3 i k x}{2 (3 H + H^3 k^2)} + \frac{(-18 k^2 - 5 H^2 k^4) x^2}{40 H (3 + H^2 k^2)^2} + \frac{i (12 k^3 + 5 H^2 k^5) x^3}{80 H (3 + H^2 k^2)^2} + O[x]^4$$

$$\text{Out[613]} = \frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2 \left(H + \frac{H^3 k^2}{3} \right)} - \frac{k^2 x^2}{12 \left(H + \frac{H^3 k^2}{3} \right)} + O[x]^4$$

$$\text{Out[614]} = \frac{(12 k^2 + 5 H^2 k^4) x^2}{40 H (3 + H^2 k^2)^2} + \frac{i (12 k^3 + 5 H^2 k^5) x^3}{80 H (3 + H^2 k^2)^2} + \frac{(-6651 k^4 - 4680 H^2 k^6 - 820 H^4 k^8) x^4}{4800 H (3 + H^2 k^2)^3} - \frac{i (6291 k^5 + 4410 H^2 k^7 + 770 H^4 k^9) x^5}{9600 H (3 + H^2 k^2)^3} + O[x]^6$$

```

In[931]:= fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * G;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;

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Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]

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Fmat = {{Fnn, Fng}, {Fgn, Fgg}}
EigvFmat = Eigenvalues[Fmat];
Simplify[Series[EigvFmat, {x, 0, 5}]]
t = x / (2 * Sqrt[g * H])
RKStep = Log[1 - t * EigvFmat + (t * EigvFmat)^2 / 2] / (I * t);
RKstepTay = Series[RKStep, {x, 0, 5}]
Simplify[RKstepTay, k * H > 0]
Simplify[RKstepTay - {wAp, wAm}, k * H > 0]

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$$\text{Out[935]} = -\frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} (-5 + e^{-ikx} - 2e^{ikx}) + \frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) \right) \sqrt{gH}$$

$$\text{Out[936]} = \frac{1}{12} \sqrt{gH} k^4 x^3 - \frac{1}{72} (\sqrt{gH} k^6) x^5 + O[x]^6$$

$$\text{Out[937]} = \frac{(1 - e^{-ikx}) (9 - e^{-ikx} + 9e^{ikx} - e^{2ikx}) H (26 - 2 \cos[kx])}{384 x \left(H - \frac{H^3 (-30 + 32 \cos[kx] - 2 \cos[2kx])}{36 x^2} \right)}$$

$$\text{Out[938]} = -\frac{i (243 k^5 + 49 H^2 k^7) x^4}{960 (3 + H^2 k^2)^2} + O[x]^6$$

$$\text{Out[939]} = -\frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} (-5 + e^{-ikx} - 2e^{ikx}) + \frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) \right) \sqrt{gH}$$

$$\text{Out[940]} = \frac{1}{12} \sqrt{gH} k^4 x^3 - \frac{1}{72} (\sqrt{gH} k^6) x^5 + O[x]^6$$

$$\text{Out[941]} = \frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) + \frac{1}{6} (5 - e^{-ikx} + 2e^{ikx}) \right) gH$$

$$\text{Out[942]} = -\frac{1}{30} i g H k^5 x^4 + O[x]^6$$

$$\text{Out[943]} = \left\{ \left\{ -\frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} (-5 + e^{-ikx} - 2e^{ikx}) + \frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) \right) \sqrt{gH}, \right. \right. \\ \left. \frac{(1 - e^{-ikx}) (9 - e^{-ikx} + 9e^{ikx} - e^{2ikx}) H (26 - 2\cos[kx])}{384x \left(H - \frac{H^3 (-30 + 32\cos[kx] - 2\cos[2kx])}{36x^2} \right)} \right\},$$

$$\left\{ \frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) + \frac{1}{6} (5 - e^{-ikx} + 2e^{ikx}) \right) gH, \right. \\ \left. -\frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} (-5 + e^{-ikx} - 2e^{ikx}) + \frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) \right) \sqrt{gH} \right\}$$

$$\text{Out[945]} = \left\{ -\frac{i\sqrt{3}gHk}{\sqrt{gH(3+H^2k^2)}} + \frac{1}{12}\sqrt{gH}k^4x^3 + \frac{i g^2 H^2 k^5 (531 + 145 H^2 k^2) x^4}{1920 \sqrt{3} (gH (3 + H^2 k^2))^{3/2}} - \frac{1}{72} (\sqrt{gH} k^6) x^5 + O[x]^6, \right. \\ \left. \frac{i\sqrt{3}gHk}{\sqrt{gH(3+H^2k^2)}} + \frac{1}{12}\sqrt{gH}k^4x^3 - \frac{i g^2 H^2 k^5 (531 + 145 H^2 k^2) x^4}{1920 \sqrt{3} (gH (3 + H^2 k^2))^{3/2}} - \frac{1}{72} (\sqrt{gH} k^6) x^5 + O[x]^6 \right\}$$

$$\text{Out[946]} = \frac{x}{2\sqrt{gH}}$$

$$\text{Out[948]} = \left\{ \frac{\sqrt{3}k\sqrt{gH(3+H^2k^2)}}{3+H^2k^2} + \frac{\sqrt{3}k^3\sqrt{gH(3+H^2k^2)}x^2}{8(3+H^2k^2)^2} - \frac{i\sqrt{gH}(-117k^4 - 96H^2k^6 - 16H^4k^8)x^3}{192(3+H^2k^2)^2} - \right. \\ \left. \frac{(gH(1755\sqrt{3}k^5 + 966\sqrt{3}H^2k^7 + 145\sqrt{3}H^4k^9))x^4}{5760((3+H^2k^2)^2\sqrt{gH(3+H^2k^2)})} - \frac{i\sqrt{gH}(3k^6 + 4H^2k^8)x^5}{288(3+H^2k^2)} + O[x]^6, \right. \\ \left. -\frac{\sqrt{3}k\sqrt{gH(3+H^2k^2)}}{3+H^2k^2} - \frac{(\sqrt{3}k^3\sqrt{gH(3+H^2k^2)})x^2}{8(3+H^2k^2)^2} - \right. \\ \left. \frac{i\sqrt{gH}(-117k^4 - 96H^2k^6 - 16H^4k^8)x^3}{192(3+H^2k^2)^2} + \right. \\ \left. \frac{gH(1755\sqrt{3}k^5 + 966\sqrt{3}H^2k^7 + 145\sqrt{3}H^4k^9)x^4}{5760(3+H^2k^2)^2\sqrt{gH(3+H^2k^2)}} - \frac{i\sqrt{gH}(3k^6 + 4H^2k^8)x^5}{288(3+H^2k^2)} + O[x]^6 \right\}$$

$$\text{Out[949]} = \left\{ \frac{\sqrt{3}gHk}{\sqrt{gH(3+H^2k^2)}} + \frac{\sqrt{3}\sqrt{gH}k^3x^2}{8(3+H^2k^2)^{3/2}} + \frac{i\sqrt{gH}k^4(117 + 96H^2k^2 + 16H^4k^4)x^3}{192(3+H^2k^2)^2} - \right. \\ \left. \frac{(\sqrt{gH}k^5(1755 + 966H^2k^2 + 145H^4k^4))x^4}{1920(\sqrt{3}(3+H^2k^2)^{5/2})} - \frac{i\sqrt{gH}k^6(3 + 4H^2k^2)x^5}{288(3+H^2k^2)} + O[x]^6, \right. \\ \left. -\frac{\sqrt{3}gHk}{\sqrt{gH(3+H^2k^2)}} - \frac{(\sqrt{3}\sqrt{gH}k^3)x^2}{8(3+H^2k^2)^{3/2}} + \frac{i\sqrt{gH}k^4(117 + 96H^2k^2 + 16H^4k^4)x^3}{192(3+H^2k^2)^2} + \right. \\ \left. \frac{\sqrt{gH}k^5(1755 + 966H^2k^2 + 145H^4k^4)x^4}{1920\sqrt{3}(3+H^2k^2)^{5/2}} - \frac{i\sqrt{gH}k^6(3 + 4H^2k^2)x^5}{288(3+H^2k^2)} + O[x]^6 \right\}$$

$$\begin{aligned} \text{Out[950]} = & \left\{ \frac{\sqrt{3} \sqrt{g H} k^3 x^2}{8 (3 + H^2 k^2)^{3/2}} + \frac{i \sqrt{g H} k^4 (117 + 96 H^2 k^2 + 16 H^4 k^4) x^3}{192 (3 + H^2 k^2)^2} - \right. \\ & \frac{\left(\sqrt{g H} k^5 (1755 + 966 H^2 k^2 + 145 H^4 k^4) \right) x^4}{1920 \left(\sqrt{3} (3 + H^2 k^2)^{5/2} \right)} - \frac{i \sqrt{g H} k^6 (3 + 4 H^2 k^2) x^5}{288 (3 + H^2 k^2)} + O[x]^6, \\ & - \frac{\left(\sqrt{3} \sqrt{g H} k^3 \right) x^2}{8 (3 + H^2 k^2)^{3/2}} + \frac{i \sqrt{g H} k^4 (117 + 96 H^2 k^2 + 16 H^4 k^4) x^3}{192 (3 + H^2 k^2)^2} + \\ & \left. \frac{\sqrt{g H} k^5 (1755 + 966 H^2 k^2 + 145 H^4 k^4) x^4}{1920 \sqrt{3} (3 + H^2 k^2)^{5/2}} - \frac{i \sqrt{g H} k^6 (3 + 4 H^2 k^2) x^5}{288 (3 + H^2 k^2)} + O[x]^6 \right\} \end{aligned}$$

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In[1022]:= t = x / (4 * Sqrt[g * H])
RKStep = Log[1 - t * EigvFmat + (t * EigvFmat) ^ 2 / 2] / (1 * t);
RKstepTay = Series[RKStep, {x, 0, 5}]
Simplify[RKstepTay, k * H > 0]
Simplify[RKstepTay - {wAp, wAm}, k * H > 0]
```

$$\text{Out[1022]} = \frac{x}{4 \sqrt{g H}}$$

$$\begin{aligned} \text{Out[1024]} = & \left\{ \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2} + \frac{\sqrt{3} k^3 \sqrt{g H (3 + H^2 k^2)} x^2}{32 (3 + H^2 k^2)^2} - i \sqrt{g H} \left(-\frac{k^4}{12} + \frac{9 k^4}{512 (3 + H^2 k^2)^2} \right) x^3 - \right. \\ & \frac{\left(g H (12825 \sqrt{3} k^5 + 7728 \sqrt{3} H^2 k^7 + 1160 \sqrt{3} H^4 k^9) \right) x^4}{46080 \left((3 + H^2 k^2)^2 \sqrt{g H (3 + H^2 k^2)} \right)} - \\ & \frac{i \sqrt{g H} (39 k^6 + 16 H^2 k^8) x^5}{1152 (3 + H^2 k^2)} + O[x]^6, \\ & - \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2} - \frac{\left(\sqrt{3} k^3 \sqrt{g H (3 + H^2 k^2)} \right) x^2}{32 (3 + H^2 k^2)^2} - i \sqrt{g H} \left(-\frac{k^4}{12} + \frac{9 k^4}{512 (3 + H^2 k^2)^2} \right) x^3 + \\ & \left. \frac{g H (12825 \sqrt{3} k^5 + 7728 \sqrt{3} H^2 k^7 + 1160 \sqrt{3} H^4 k^9) x^4}{46080 (3 + H^2 k^2)^2 \sqrt{g H (3 + H^2 k^2)}} - \frac{i \sqrt{g H} (39 k^6 + 16 H^2 k^8) x^5}{1152 (3 + H^2 k^2)} + O[x]^6 \right\} \end{aligned}$$

$$\begin{aligned}
\text{Out}[1025] = & \left\{ \frac{\sqrt{3} \, g \, H \, k}{\sqrt{g \, H \, (3 + H^2 \, k^2)}} + \frac{\sqrt{3} \, \sqrt{g \, H} \, k^3 \, x^2}{32 \, (3 + H^2 \, k^2)^{3/2}} - \frac{i \, \sqrt{g \, H} \, k^4 \left(-128 + \frac{27}{(3 + H^2 \, k^2)^2} \right) x^3}{1536} - \right. \\
& \frac{\left(\sqrt{g \, H} \, k^5 \left(12825 + 7728 \, H^2 \, k^2 + 1160 \, H^4 \, k^4 \right) \right) x^4}{15360 \, \left(\sqrt{3} \, (3 + H^2 \, k^2)^{5/2} \right)} - \frac{i \, \sqrt{g \, H} \, k^6 \left(39 + 16 \, H^2 \, k^2 \right) x^5}{1152 \, (3 + H^2 \, k^2)} + O[x]^6, \\
& - \frac{\sqrt{3} \, g \, H \, k}{\sqrt{g \, H \, (3 + H^2 \, k^2)}} - \frac{\left(\sqrt{3} \, \sqrt{g \, H} \, k^3 \right) x^2}{32 \, (3 + H^2 \, k^2)^{3/2}} - \frac{i \, \sqrt{g \, H} \, k^4 \left(-128 + \frac{27}{(3 + H^2 \, k^2)^2} \right) x^3}{1536} + \\
& \frac{\sqrt{g \, H} \, k^5 \left(12825 + 7728 \, H^2 \, k^2 + 1160 \, H^4 \, k^4 \right) x^4}{15360 \, \sqrt{3} \, (3 + H^2 \, k^2)^{5/2}} - \frac{i \, \sqrt{g \, H} \, k^6 \left(39 + 16 \, H^2 \, k^2 \right) x^5}{1152 \, (3 + H^2 \, k^2)} + O[x]^6 \} \\
\text{Out}[1026] = & \left\{ \frac{\sqrt{3} \, \sqrt{g \, H} \, k^3 \, x^2}{32 \, (3 + H^2 \, k^2)^{3/2}} - \frac{i \, \sqrt{g \, H} \, k^4 \left(-128 + \frac{27}{(3 + H^2 \, k^2)^2} \right) x^3}{1536} - \right. \\
& \frac{\left(\sqrt{g \, H} \, k^5 \left(12825 + 7728 \, H^2 \, k^2 + 1160 \, H^4 \, k^4 \right) \right) x^4}{15360 \, \left(\sqrt{3} \, (3 + H^2 \, k^2)^{5/2} \right)} - \frac{i \, \sqrt{g \, H} \, k^6 \left(39 + 16 \, H^2 \, k^2 \right) x^5}{1152 \, (3 + H^2 \, k^2)} + O[x]^6, \\
& - \frac{\left(\sqrt{3} \, \sqrt{g \, H} \, k^3 \right) x^2}{32 \, (3 + H^2 \, k^2)^{3/2}} - \frac{i \, \sqrt{g \, H} \, k^4 \left(-128 + \frac{27}{(3 + H^2 \, k^2)^2} \right) x^3}{1536} + \\
& \frac{\sqrt{g \, H} \, k^5 \left(12825 + 7728 \, H^2 \, k^2 + 1160 \, H^4 \, k^4 \right) x^4}{15360 \, \sqrt{3} \, (3 + H^2 \, k^2)^{5/2}} - \frac{i \, \sqrt{g \, H} \, k^6 \left(39 + 16 \, H^2 \, k^2 \right) x^5}{1152 \, (3 + H^2 \, k^2)} + O[x]^6 \}
\end{aligned}$$