In[1]:=
```
MA = k*x / (2*Sin[k*x/2])
RA = Exp[I*k*x/2]*k*x / (2*Sin[k*x/2])
GA = k*x/ ((H + H^3/3*k^2) *Exp[-I*k*x/2]*(2*Sin[k*x/2]))
FnnA = 0
FnGA = I*k/ (1 + H^2*k^2/3)
FGnA = g*H*I*k
FGGA = 0
FmatA = {{FnnA, FnGA}, {FGnA, FGGA}}
Eigenvalues[FmatA]
```

Out[1]= $\frac{1}{2} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$

Out[2]= $\frac{1}{2} e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$

Out[3]= $\frac{e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]}{2\left(H + \frac{H^3 k^2}{3}\right)}$

Out[4]= 0

Out[5]= $\frac{i k}{1 + \frac{H^2 k^2}{3}}$

Out[6]= $i g H k$

Out[7]= 0

Out[8]= $\left\{\left\{0, \frac{i k}{1 + \frac{H^2 k^2}{3}}\right\}, \{i g H k, 0\}\right\}$

Out[9]= $\left\{-\frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}\right\}$

In[10]:=
```
M = 1
Series[M - MA, {x, 0, 10}]
```

Out[10]= 1

Out[11]= $-\frac{k^2 x^2}{24} - \frac{7 k^4 x^4}{5760} - \frac{31 k^6 x^6}{967\,680} - \frac{127 k^8 x^8}{154\,828\,800} - \frac{73 k^{10} x^{10}}{3\,503\,554\,560} + O[x]^{11}$

In[12]:=
```
Rm = 1
Series[Rm - RA, {x, 0, 10}]
Rp = Exp[I*k*x]
Series[Rp - RA, {x, 0, 10}]
Ru = (1 + Exp[I*k*x])/2
Series[Ru - Exp[I*k*x/2] , {x, 0, 10}]
```

Out[12]= 1

Out[13]= $-\frac{1}{2}\, i\, k\, x + \frac{k^2 x^2}{12} + \frac{k^4 x^4}{720} + \frac{k^6 x^6}{30240} + \frac{k^8 x^8}{1209600} + \frac{k^{10} x^{10}}{47900160} + O[x]^{11}$

Out[14]= $e^{i k x}$

Out[15]= $\frac{i k x}{2} - \frac{5 k^2 x^2}{12} - \frac{1}{6}\, i\, k^3 x^3 + \frac{31 k^4 x^4}{720} + \frac{1}{120}\, i\, k^5 x^5 - \frac{41 k^6 x^6}{30240} - \frac{i k^7 x^7}{5040} + \frac{31 k^8 x^8}{1209600} + \frac{i k^9 x^9}{362880} - \frac{61 k^{10} x^{10}}{239500800} + O[x]^{11}$

Out[16]= $\frac{1}{2}\left(1 + e^{i k x}\right)$

Out[17]= $-\frac{k^2 x^2}{8} - \frac{1}{16}\, i\, k^3 x^3 + \frac{7 k^4 x^4}{384} + \frac{1}{256}\, i\, k^5 x^5 - \frac{31 k^6 x^6}{46080} - \frac{i k^7 x^7}{10240} + \frac{127 k^8 x^8}{10321920} + \frac{17\, i\, k^9 x^9}{12386304} - \frac{73 k^{10} x^{10}}{530841600} + O[x]^{11}$

In[18]:=
```
Gold = H - H^3/3*(2*Cos[k*x] -2)/x^2
G = Ru / Gold
Series[G, {x, 0, 3} ]
Series[GA, {x, 0, 3} ]
Series[G - GA, {x, 0, 5}]
```

Out[18]= $H - \frac{H^3 \left(-2 + 2 \operatorname{Cos}[k\, x]\right)}{3 x^2}$

Out[19]= $\frac{1 + e^{i k x}}{2\left(H - \frac{H^3 (-2+2 \operatorname{Cos}[k x])}{3 x^2}\right)}$

Out[20]= $\frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2\left(H + \frac{H^3 k^2}{3}\right)} + \frac{\left(-9 k^2 - 2 H^2 k^4\right) x^2}{4 H \left(3 + H^2 k^2\right)^2} - \frac{i \left(6 k^3 + H^2 k^5\right) x^3}{8 H \left(3 + H^2 k^2\right)^2} + O[x]^4$

Out[21]= $\frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2\left(H + \frac{H^3 k^2}{3}\right)} - \frac{k^2 x^2}{12\left(H + \frac{H^3 k^2}{3}\right)} + O[x]^4$

Out[22]= $\frac{\left(-6 k^2 - H^2 k^4\right) x^2}{4 H \left(3 + H^2 k^2\right)^2} - \frac{i \left(6 k^3 + H^2 k^5\right) x^3}{8 H \left(3 + H^2 k^2\right)^2} + \frac{\left(144 k^4 + 45 H^2 k^6 + 4 H^4 k^8\right) x^4}{240 H \left(3 + H^2 k^2\right)^3} - \frac{i \left(-54 k^5 + H^4 k^9\right) x^5}{480 H \left(3 + H^2 k^2\right)^3} + O[x]^6$

In[23]:=
```
fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * G;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;

Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]
```

Out[27]= $$-\frac{\left(1-e^{-i k x}\right)\left(-1+e^{i k x}\right)\sqrt{g H}}{2 x}$$

Out[28]= $$\frac{1}{2}\sqrt{g H}\, k^2 x-\frac{1}{24}\left(\sqrt{g H}\, k^4\right) x^3+\frac{1}{720}\sqrt{g H}\, k^6 x^5+O[x]^6$$

Out[29]= $$\frac{\left(1-e^{-i k x}\right)\left(1+e^{i k x}\right) H}{2 x\left(H-\frac{H^3(-2+2 \operatorname{Cos}[k x])}{3 x^2}\right)}$$

Out[30]= $$-\frac{i\left(6 k^3+H^2 k^5\right) x^2}{4\left(3+H^2 k^2\right)^2}-\frac{i\left(-54 k^5+H^4 k^9\right) x^4}{240\left(3+H^2 k^2\right)^3}+O[x]^6$$

Out[31]= $$-\frac{\left(1-e^{-i k x}\right)\left(-1+e^{i k x}\right)\sqrt{g H}}{2 x}$$

Out[32]= $$\frac{1}{2}\sqrt{g H}\, k^2 x-\frac{1}{24}\left(\sqrt{g H}\, k^4\right) x^3+\frac{1}{720}\sqrt{g H}\, k^6 x^5+O[x]^6$$

Out[33]= $$\frac{\left(1-e^{-i k x}\right)\left(1+e^{i k x}\right) g H}{2 x}$$

Out[34]= $$-\frac{1}{6} i g H k^3 x^2+\frac{1}{120} i g H k^5 x^4+O[x]^6$$