

$$Error = Approximation - Exact$$

1 Cell Average to Nodal Values

Variable	Formula	Lowest Order of Error
\mathcal{M}_A	$\frac{2}{k\Delta x} \sin\left(\frac{k\Delta x}{2}\right)$	—
\mathcal{M}_1	1	$\frac{k^2 \Delta x^2}{24}$
\mathcal{M}_2	1	$\frac{k^2 \Delta x^2}{24}$
\mathcal{M}_3	$\frac{24}{26 - 2 \cos(k\Delta x)}$	$\frac{3k^4 \Delta x^4}{640}$

2 Reconstruction

2.1 Reconstructions for η and G

2.1.1 \mathcal{R}^+

Variable	Formula	Lowest Order of Error
\mathcal{R}_A^+	$\exp\left(ik\frac{\Delta x}{2}\right)$	—
\mathcal{R}_1^+	$\exp(ik\Delta x)$	$\frac{1}{2}k\Delta x$
\mathcal{R}_2^+	$\exp(ik\Delta x)\left(1 - \frac{i\sin(k\Delta x)}{2}\right)$	$\frac{1}{8}k^2\Delta x^2$
\mathcal{R}_3^+	$\frac{2\exp(2ik\Delta x) - 10\exp(ik\Delta x) - 4}{\cos(k\Delta x) - 13}$	$\frac{i}{12}k^3\Delta x^3$

2.1.2 \mathcal{R}^-

Variable	Formula	Lowest Order of Error
\mathcal{R}_A^-	$\exp\left(ik\frac{\Delta x}{2}\right)$	—
\mathcal{R}_1^-	1	$\frac{1}{2}k\Delta x$
\mathcal{R}_2^-	$1 + \frac{i\sin(k\Delta x)}{2}$	$\frac{1}{8}k^2\Delta x^2$
\mathcal{R}_3^-	$\frac{2\exp(-ik\Delta x) - 4\exp(ik\Delta x) - 10}{\cos(k\Delta x) - 13}$	$-\frac{i}{12}k^3\Delta x^3$

2.2 Reconstruction for v

2.2.1 \mathcal{R}^u

Variable	Formula	Lowest Order of Error
\mathcal{R}_A^u	$\exp\left(ik\frac{\Delta x}{2}\right)$	—
\mathcal{R}_1^u	$\frac{\exp(ik\Delta x) + 1}{2}$	$-\frac{1}{8}k^2\Delta x^2$
\mathcal{R}_2^u	$\frac{\exp(ik\Delta x) + 1}{2}$	$-\frac{1}{8}k^2\Delta x^2$
\mathcal{R}_3^u	$\frac{-\exp(-ik\Delta x) + 9\exp(ik\Delta x) - \exp(2ik\Delta x) + 9}{16}$	$-\frac{3}{128}k^4\Delta x^4$

3 Elliptic Equation

Variable	Formula	Lowest Order of Error
\mathcal{G}_A	$H + \frac{H^3}{3}k^2$	—
\mathcal{G}_1	$H - \frac{H^3}{3} \frac{2\cos(k\Delta x) - 2}{\Delta x^2}$	$-\frac{H^3}{36}k^4\Delta x^2$
\mathcal{G}_{2FD}	$H - \frac{H^3}{3} \frac{2\cos(k\Delta x) - 2}{\Delta x^2}$	$-\frac{H^3}{36}k^4\Delta x^2$
\mathcal{G}_{2FEM}	*	$-\frac{3H}{40}k^2\Delta x^2 - \frac{H^3}{36}k^4\Delta x^2$
\mathcal{G}_3	$H - \frac{H^3}{3} \frac{32\cos(k\Delta x) - 2\cos(2k\Delta x) - 30}{12\Delta x^2}$	$-\frac{H^3}{270}k^6\Delta x^4$

$$\begin{aligned}
\mathcal{G}_{2FEM} = & \left(\frac{2H^3}{3\Delta x^2} \left(\exp\left(ik\frac{3\Delta x}{2}\right) + 14\exp\left(ik\frac{\Delta x}{2}\right) - 8\exp(ik\Delta x) - 8 + \exp\left(-ik\frac{\Delta x}{2}\right) \right) \right. \\
& + \frac{H}{5} \left(-\exp\left(ik\frac{3\Delta x}{2}\right) + 8\exp\left(ik\frac{\Delta x}{2}\right) + 2\exp(ik\Delta x) + 2 - \exp\left(-ik\frac{\Delta x}{2}\right) \right) \Bigg) \div \\
& \left(-\frac{1}{4}\exp(2i\Delta x k) + \exp(i\Delta x k) + \frac{i}{2}\sin(k\Delta x) + \frac{5}{4} \right) \quad (1)
\end{aligned}$$

4 Conservation Equation

Variable	Exact	Lowest Order Truncation Term			
		FDVM ₁	FDVM ₂	FEVM ₂	FDVM ₃
$\frac{\mathcal{D}}{\mathcal{M}\Delta x}\mathcal{F}^{\eta,\eta}$	0	$\frac{\sqrt{gH}}{2}k^2\Delta x$	$\frac{\sqrt{gH}}{8}k^4\Delta x^3$	$\frac{\sqrt{gH}}{8}k^4\Delta x^3$	$\frac{\sqrt{gH}}{12}k^4\Delta x^3$
$\frac{\mathcal{D}}{\mathcal{M}\Delta x}\mathcal{F}^{\eta,v}$	ikH	$-\frac{iH}{6}k^3\Delta x^2$	$-\frac{iH}{6}k^3\Delta x^2$	$-\frac{iH}{24}k^3\Delta x^2$	$-\frac{9iH}{320}k^5\Delta x^4$
$\frac{\mathcal{D}}{\mathcal{G}\mathcal{M}\Delta x}\mathcal{F}^{G,\eta}$	$\frac{3ikgH}{H^2k^2+3}$	$-\frac{ig\left(H^2k^2+6\right)}{4\left(H^2k^2+3\right)^2}k^3\Delta x^2$	$\frac{ig\left(2H^2k^2+3\right)}{4\left(H^2k^2+3\right)^2}k^3\Delta x^2$	$\frac{ig\left(57+20H^2k^2\right)}{40\left(3+h^2k^2\right)^2}k^3\Delta x^2$	$-\frac{ig\left(9+2H^2k^2\right)}{30\left(3+h^2k^2\right)^2}k^5\Delta x^4$
$\frac{\mathcal{D}}{\mathcal{G}\mathcal{M}\Delta x}\mathcal{F}^{G,v}$	0	$\frac{\sqrt{gH}}{2}k^2\Delta x$	$\frac{\sqrt{gH}}{8}k^4\Delta x^3$	$\frac{\sqrt{gH}}{8}k^4\Delta x^3$	$\frac{\sqrt{gH}}{12}k^4\Delta x^3$