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In[2170]:= q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx/2, xj + dx/2}] / (dx);
qjnbar = qjbar /. t -> tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x -> xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / (3 * k^2);
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca;
fn1Gca0A = fn1A /. Gca -> 0;
fn1eca0A = fn1A /. eca -> 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca /. Gh -> RA * Gca;
fG1Gca0A = fG1A /. Gca -> 0;
fG1eca0A = fG1A /. eca -> 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};

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$$\mathbf{wAp} = \mathbf{U} * \mathbf{k} + \frac{\sqrt{3} \, \mathbf{k} \sqrt{\mathbf{g} \, \mathbf{H} \left(3 + \mathbf{H}^2 \, \mathbf{k}^2 \right)}}{3 + \mathbf{H}^2 \, \mathbf{k}^2};$$

$$\mathbf{wAm} = \mathbf{U} * \mathbf{k} - \frac{\sqrt{3} \, \mathbf{k} \sqrt{\mathbf{g} \, \mathbf{H} \left(3 + \mathbf{H}^2 \, \mathbf{k}^2 \right)}}{3 + \mathbf{H}^2 \, \mathbf{k}^2};$$

In[2205]:= **M = 1**

Series[M - MA, {dx, 0, 10}]

Out[2205]= 1

$$\text{Out[2206]} = -\frac{k^2 dx^2}{24} - \frac{7 k^4 dx^4}{5760} - \frac{31 k^6 dx^6}{967680} - \frac{127 k^8 dx^8}{154828800} - \frac{73 k^{10} dx^{10}}{3503554560} + O[dx]^{11}$$

In[2207]:= **Rm = 1**

Series[Rm - RA, {dx, 0, 10}]

Rp = Exp[I * k * dx]

Series[Rp - RA, {dx, 0, 10}]

Ru = (1 + Exp[I * k * dx]) / 2

Series[Ru - Exp[I * k * dx / 2], {dx, 0, 10}]

Out[2207]= 1

$$\text{Out[2208]} = -\frac{1}{2} i k dx + \frac{k^2 dx^2}{12} + \frac{k^4 dx^4}{720} + \frac{k^6 dx^6}{30240} + \frac{k^8 dx^8}{1209600} + \frac{k^{10} dx^{10}}{47900160} + O[dx]^{11}$$

Out[2209]= $e^{i dx k}$

$$\begin{aligned} \text{Out[2210]} = & \frac{i k dx}{2} - \frac{5 k^2 dx^2}{12} - \frac{1}{6} i k^3 dx^3 + \frac{31 k^4 dx^4}{720} + \frac{1}{120} i k^5 dx^5 - \\ & \frac{41 k^6 dx^6}{30240} - \frac{i k^7 dx^7}{5040} + \frac{31 k^8 dx^8}{1209600} + \frac{i k^9 dx^9}{362880} - \frac{61 k^{10} dx^{10}}{239500800} + O[dx]^{11} \end{aligned}$$

$$\text{Out[2211]} = \frac{1}{2} (1 + e^{i dx k})$$

$$\begin{aligned} \text{Out[2212]} = & -\frac{k^2 dx^2}{8} - \frac{1}{16} i k^3 dx^3 + \frac{7 k^4 dx^4}{384} + \frac{1}{256} i k^5 dx^5 - \\ & \frac{31 k^6 dx^6}{46080} - \frac{i k^7 dx^7}{10240} + \frac{127 k^8 dx^8}{10321920} + \frac{17 i k^9 dx^9}{12386304} - \frac{73 k^{10} dx^{10}}{530841600} + O[dx]^{11} \end{aligned}$$

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In[2213]:= Gold = H - H^3 / (3 * (2 * Cos[k * dx] - 2)) / dx^2
GG2 = Ru / Gold
Series[GG2 - GGA, {dx, 0, 5}]
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Gn2 = -U * Ru / Gold
Series[Gn2 - GnA, {dx, 0, 5}]
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$$\text{Out[2213]} = H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2}$$

$$\text{Out[2214]} = \frac{1 + e^{i dx k}}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2} \right)}$$

$$\text{Out[2215]} = \frac{(-6 k^2 - H^2 k^4) dx^2}{4 H (3 + H^2 k^2)^2} - \frac{i (6 k^3 + H^2 k^5) dx^3}{8 H (3 + H^2 k^2)^2} + \frac{(144 k^4 + 45 H^2 k^6 + 4 H^4 k^8) dx^4}{240 H (3 + H^2 k^2)^3} - \frac{i (-54 k^5 + H^4 k^9) dx^5}{480 H (3 + H^2 k^2)^3} + O[dx]^6$$

$$\text{Out[2216]} = - \frac{(1 + e^{i dx k}) U}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2} \right)}$$

$$\text{Out[2217]} = \frac{(6 k^2 + H^2 k^4) U dx^2}{4 H (3 + H^2 k^2)^2} + \frac{i (6 k^3 + H^2 k^5) U dx^3}{8 H (3 + H^2 k^2)^2} - \frac{((144 k^4 + 45 H^2 k^6 + 4 H^4 k^8) U) dx^4}{240 (H (3 + H^2 k^2)^3)} + \frac{i (-54 k^5 + H^4 k^9) U dx^5}{480 H (3 + H^2 k^2)^3} + O[dx]^6$$

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In[2218]:= Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]
Text[Row[{"Fnn and FnG "}]
KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
KurFWS = KurF /. ap -> (U + Sqrt[g*H]) /. am -> (U - Sqrt[g*H]);
KurFWSeta =
  KurFWS /. fp -> (H*v + U*Rpp*n) /. fm -> (H*v + U*Rmp*n) /. qp -> Rpp*n /.
  qm -> Rmp*n;
KurFWSeta = KurFWSeta /. v -> (GGp*G + Gnp*n);
Kfnn = FullSimplify[KurFWSeta /. G -> 0 /. n -> 1]
KfnG = FullSimplify[KurFWSeta /. n -> 0 /. G -> 1]
Kfnn = Kfnn /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfnG = KfnG /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
Fnn2 = -dt*(1 - Exp[-I*k*dx])/dx*Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]
KurFWSG = KurFWS /. fp -> (U*Rpp*G + U*H*v + g*H*Rpp*n) /.
  fm -> (U*Rmp*G + U*H*v + g*H*Rmp*n) /. qp -> Rpp*G /. qm -> Rmp*G;
KurFWSG = KurFWSG /. v -> (GGp*G + Gnp*n);
KfGn = FullSimplify[KurFWSG /. G -> 0 /. n -> 1]
KfGG = FullSimplify[KurFWSG /. n -> 0 /. G -> 1]
KfGn = KfGn /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfGG = KfGG /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;

FGn2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U*H*GG2 + U/2*(Rm + Rp) - (Sqrt[g*H])/2*(Rp - Rm);
FGG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2]/(I*dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}]

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Out[2218]= $-\text{Sqrt}[g*H] < U < \text{Sqrt}[g*H]$

Out[2219]= **Fnn and FnG**

$$\text{Out[2224]} = \frac{1}{2} \left(2 \text{Gnp } H + \text{Rpp} \left(-\sqrt{g H} + U \right) + \text{Rmp} \left(\sqrt{g H} + U \right) \right)$$

Out[2225]= **GGp H**

$$\begin{aligned} \text{Out[2230]} = & \left(-\frac{(H^2 k^3 U w) dt^2}{2 (3 + H^2 k^2)} - \frac{i H^2 k^3 U w^2 dt^3}{6 (3 + H^2 k^2)} + O[dt]^4 \right) + \left(-\frac{1}{2} (\sqrt{g H} k^2) dt + O[dt]^4 \right) dx + \\ & \left(\frac{i (9 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(\frac{1}{24} \sqrt{g H} k^4 dt + O[dt]^4 \right) dx^3 + \\ & \left(-\frac{i k^7 (54 H^2 U + 19 H^4 k^2 U + 2 H^6 k^4 U) dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{aligned}$$

$$\begin{aligned} \text{Out[2233]} = & \left(-\frac{3 (k w) dt^2}{2 (3 + H^2 k^2)} - \frac{i k w^2 dt^3}{2 (3 + H^2 k^2)} + O[dt]^4 \right) + \\ & \left(\frac{i (6 k^3 + H^2 k^5) dt}{4 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(\frac{i (-54 k^5 + H^4 k^9) dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{aligned}$$

Out[2234]= **FGn and FGG**

$$\text{Out[2237]} = \frac{1}{2} \left(g H (\text{Rmp} + \text{Rpp}) + \left(2 \text{Gnp } H + \sqrt{g H} (\text{Rmp} - \text{Rpp}) \right) U \right)$$

$$\text{Out[2238]} = \frac{1}{2} \left(\sqrt{g H} \text{Rmp} - \sqrt{g H} \text{Rpp} + (2 \text{GGp } H + \text{Rmp} + \text{Rpp}) U \right)$$

$$\begin{aligned} \text{Out[2243]} = & \left(-\frac{(k (3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2 (3 + H^2 k^2)} - \frac{i k (3 g H + g H^3 k^2 - 3 U^2) w^2 dt^3}{6 (3 + H^2 k^2)} + O[dt]^4 \right) + \\ & \left(-\frac{1}{2} (\sqrt{g H} k^2 U) dt + O[dt]^4 \right) dx + \\ & \left((i (18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2) dt) / (12 (3 + H^2 k^2)^2) + O[dt]^4 \right) dx^2 + \\ & \left(\frac{1}{24} \sqrt{g H} k^4 U dt + O[dt]^4 \right) dx^3 + \\ & \left(- \left((i (54 g H k^5 + 54 g H^3 k^7 + 18 g H^5 k^9 + 2 g H^7 k^{11} - 54 k^5 U^2 + H^4 k^9 U^2) dt) / \right. \right. \\ & \quad \left. \left. (240 (3 + H^2 k^2)^3) \right) + O[dt]^4 \right) dx^4 + O[dx]^5 \end{aligned}$$

$$\begin{aligned} \text{Out[2247]} = & \left(-\frac{(k (6 + H^2 k^2) U w) dt^2}{2 (3 + H^2 k^2)} - \frac{i k (6 + H^2 k^2) U w^2 dt^3}{6 (3 + H^2 k^2)} + O[dt]^4 \right) + \left(-\frac{1}{2} (\sqrt{g H} k^2) dt + O[dt]^4 \right) dx + \\ & \left(\frac{i (36 k^3 U + 15 H^2 k^5 U + 2 H^4 k^7 U) dt}{12 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(\frac{1}{24} \sqrt{g H} k^4 dt + O[dt]^4 \right) dx^3 + \\ & \left(-\frac{i (108 k^5 U + 54 H^2 k^7 U + 17 H^4 k^9 U + 2 H^6 k^{11} U) dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{aligned}$$

Out[2248]= **W : omega**

$$\begin{aligned}
\text{Out[2253]} = & \left\{ \frac{\left(\mathbf{i} \left(\sqrt{3} \, k \sqrt{g \, H \, (3 + H^2 \, k^2)} + 3 \, k \, U + H^2 \, k^3 \, U \right)^2 \, dt}{2 \, (3 + H^2 \, k^2)^2} - \right. \\
& \frac{1}{3 \, (3 + H^2 \, k^2)^2} \left(k^3 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \right. \\
& \left. \left(3 \, g \, H + U \left(2 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \right) \right) dt^2 - \frac{1}{4 \, (3 + H^2 \, k^2)^3} \\
& \mathbf{i} \, k^4 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \left(3 \, g \left(\sqrt{3} \, H \sqrt{g \, H \, (3 + H^2 \, k^2)} + 9 \, H \, U + 3 \, H^3 \, k^2 \, U \right) + \right. \\
& \left. U^2 \left(H^4 \, k^4 \, U + 9 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + U \right) + 3 \, k^2 \left(\sqrt{3} \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} + 2 \, H^2 \, U \right) \right) \right) dt^3 + \\
& \frac{1}{5 \, (3 + H^2 \, k^2)^3} k^5 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \\
& \left(9 \, g^2 \, H^2 + 6 \, g \, H \, U \left(2 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + 3 \, (3 + H^2 \, k^2) \, U \right) + \right. \\
& \left. U^3 \left(12 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + 9 \, U + H^4 \, k^4 \, U + 2 \, k^2 \left(2 \sqrt{3} \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} + 3 \, H^2 \, U \right) \right) \right) dt^4 + \\
& \left. O[dt]^5 \right) + \left(-\frac{1}{4} \, \mathbf{i} \, k^2 \left(2 \sqrt{g \, H} + \frac{\sqrt{3} \, U}{\sqrt{3 + H^2 \, k^2}} \right) + \left(k^3 \left(2 \sqrt{g \, H \, (3 + H^2 \, k^2)} + \sqrt{3} \, U \right) \right. \right. \\
& \left. \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) dt \right) / \left(4 \, (3 + H^2 \, k^2)^{3/2} + \right. \\
& \left. \left(\mathbf{i} \, k^4 \left(2 \sqrt{g \, H \, (3 + H^2 \, k^2)} + \sqrt{3} \, U \right) \left(3 \, g \, H + U \left(2 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \right) dt^2 \right) / \right. \\
& \left. \left(4 \, (3 + H^2 \, k^2)^{3/2} \right) - \frac{1}{4 \, (3 + H^2 \, k^2)^{5/2}} \right. \\
& \left. \left(k^5 \left(2 \sqrt{g \, H \, (3 + H^2 \, k^2)} + \sqrt{3} \, U \right) \left(3 \, g \left(\sqrt{3} \, H \sqrt{g \, H \, (3 + H^2 \, k^2)} + 9 \, H \, U + 3 \, H^3 \, k^2 \, U \right) + \right. \right. \\
& \left. \left. U^2 \left(H^4 \, k^4 \, U + 9 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + U \right) + 3 \, k^2 \left(\sqrt{3} \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} + 2 \, H^2 \, U \right) \right) \right) \right) \\
& dt^3 - \frac{1}{4 \, (3 + H^2 \, k^2)^{5/2}} \mathbf{i} \, k^6 \left(2 \sqrt{g \, H \, (3 + H^2 \, k^2)} + \sqrt{3} \, U \right) \\
& \left(9 \, g^2 \, H^2 + 6 \, g \, H \, U \left(2 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + 3 \, (3 + H^2 \, k^2) \, U \right) + U^3 \left(12 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + \right. \right. \\
& \left. \left. 9 \, U + H^4 \, k^4 \, U + 2 \, k^2 \left(2 \sqrt{3} \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} + 3 \, H^2 \, U \right) \right) \right) dt^4 + O[dt]^5 \Big) dx + \\
& \left(- \left(\left(k^3 \left(12 \sqrt{3} \, g \, H \, (4 + H^2 \, k^2) + U \left(48 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 9 \sqrt{3} \, U + k^2 \left(16 \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} - \right. \right. \right. \right. \right. \right. \\
& \left. \left. \left. 3 \sqrt{3} \, H^2 \, U \right) \right) \right) \right) / \left(96 \left(\sqrt{g \, H} \, (3 + H^2 \, k^2)^{3/2} \right) \right) - \frac{1}{96 \sqrt{g \, H} \, (3 + H^2 \, k^2)^{5/2}} \\
& \mathbf{i} \, k^4 \left(9 \, U^2 \left(16 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 3 \sqrt{3} \, U \right) + k^4 \, U^2 \left(16 \sqrt{g \, H^9 \, (3 + H^2 \, k^2)} - 3 \sqrt{3} \, H^4 \, U \right) + \right. \\
& 6 \, k^2 \left(18 \sqrt{g^3 \, H^7 \, (3 + H^2 \, k^2)} + 16 \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} \, U^2 - 3 \sqrt{3} \, H^2 \, U^3 \right) + \\
& \left. 4 \, g \, H \left(63 \sqrt{g \, H \, (3 + H^2 \, k^2)} + 99 \sqrt{3} \, U + 63 \sqrt{3} \, H^2 \, k^2 \, U + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& k^4 \left(3 \sqrt{g H^9 (3 + H^2 k^2)} + 10 \sqrt{3} H^4 U \right) \Big) \Big) dt + \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{5/2}} \\
& k^5 \left(12 \sqrt{3} g^2 H^2 (30 + 15 H^2 k^2 + 2 H^4 k^4) + 9 U^3 \left(16 \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} U \right) + \right. \\
& k^4 U^3 \left(16 \sqrt{g H^9 (3 + H^2 k^2)} - 3 \sqrt{3} H^4 U \right) + \\
& 6 k^2 \left(56 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^3 - 3 \sqrt{3} H^2 U^4 \right) + \\
& g H U \left(864 \sqrt{g H (3 + H^2 k^2)} + 675 \sqrt{3} U + 429 \sqrt{3} H^2 k^2 U + \right. \\
& \left. 4 k^4 \left(6 \sqrt{g H^9 (3 + H^2 k^2)} + 17 \sqrt{3} H^4 U \right) \right) \Big) dt^2 + \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{7/2}} \\
& i k^6 \left(9 k^2 \left(84 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 404 \sqrt{3} g^2 H^4 U + 438 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \right. \\
& 314 \sqrt{3} g H^3 U^3 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 9 \sqrt{3} H^2 U^5 \Big) + \\
& 27 \left(52 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 172 \sqrt{3} g^2 H^2 U + 16 \sqrt{g H (3 + H^2 k^2)} U^4 - \right. \\
& 3 \sqrt{3} U^5 + g H U^2 \left(210 \sqrt{g H (3 + H^2 k^2)} + 109 \sqrt{3} U \right) \Big) + \\
& 3 k^4 \left(36 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 304 \sqrt{3} g^2 H^6 U + 48 \sqrt{g H^9 (3 + H^2 k^2)} U^4 - \right. \\
& 9 \sqrt{3} H^4 U^5 + g H^5 U^2 \left(264 \sqrt{g H (3 + H^2 k^2)} + 301 \sqrt{3} U \right) \Big) + k^6 U \left(72 \sqrt{3} g^2 H^8 + 12 g \right. \\
& H^7 U \left(3 \sqrt{g H (3 + H^2 k^2)} + 8 \sqrt{3} U \right) + U^3 \left(16 \sqrt{g H^{13} (3 + H^2 k^2)} - 3 \sqrt{3} H^6 U \right) \Big) \Big) dt^3 - \\
& \frac{1}{96 (\sqrt{g H} (3 + H^2 k^2)^{7/2})} \left(k^7 \left(36 \sqrt{3} g^3 H^3 (48 + 27 H^2 k^2 + 4 H^4 k^4) + 3 \sqrt{3} g^2 H^2 \right. \right. \\
& (4113 + 3075 H^2 k^2 + 712 H^4 k^4 + 48 H^6 k^6) U^2 + 2 g H U^3 \left(5076 \sqrt{g H (3 + H^2 k^2)} + \right. \\
& 1971 \sqrt{3} U + 1872 \sqrt{3} H^2 k^2 U + 3 k^4 \left(216 \sqrt{g H^9 (3 + H^2 k^2)} + 197 \sqrt{3} H^4 U \right) + \\
& 2 k^6 \left(12 \sqrt{g H^{13} (3 + H^2 k^2)} + 31 \sqrt{3} H^6 U \right) \Big) + \\
& U \left(7344 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 432 \sqrt{g H (3 + H^2 k^2)} U^4 - 81 \sqrt{3} U^5 + \right. \\
& k^6 U^4 \left(16 \sqrt{g H^{13} (3 + H^2 k^2)} - 3 \sqrt{3} H^6 U \right) + 9 k^2 \left(400 \sqrt{g^5 H^9 (3 + H^2 k^2)} + \right. \\
& 760 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 9 \sqrt{3} H^2 U^5 \Big) + \\
& 9 k^4 \left(48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 16 \sqrt{g H^9 (3 + H^2 k^2)} U^4 - 3 \sqrt{3} H^4 U^5 \right) \Big) \Big) dt^4 + \\
& O[dt]^5 \Big) dx^2 + \left(\left(i k^4 \left(3 \sqrt{3} (3 + H^2 k^2) U^3 + 4 g H \left(12 \sqrt{g H (3 + H^2 k^2)} + 6 \sqrt{3} U + \right. \right. \right. \right. \\
& k^2 \left(4 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \Big) \Big) \Big) / (384 g H (3 + H^2 k^2)^{3/2}) - \\
& \frac{1}{384 (g H (3 + H^2 k^2)^{5/2})} \left(k^5 \left(8 \sqrt{3} g^2 H^2 (54 + 33 H^2 k^2 + 5 H^4 k^4) + 3 \sqrt{3} (3 + H^2 k^2)^2 U^4 + \right. \right. \\
& 2 g H U \left(81 \left(4 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} U \right) + 24 k^2 \left(7 \sqrt{g H^5 (3 + H^2 k^2)} + 2 \sqrt{3} H^2 U \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& k^4 \left(24 \sqrt{g H^9 (3 + H^2 k^2)} + 7 \sqrt{3} H^4 U \right) \Big) \Big) dt - \frac{1}{384 g H (3 + H^2 k^2)^{5/2}} \\
& i k^6 \left(3 k^2 \left(96 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 380 \sqrt{3} g^2 H^4 U + 224 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \right. \\
& \quad 51 \sqrt{3} g H^3 U^3 + 6 \sqrt{3} H^2 U^5 \Big) + k^4 \left(16 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 168 \sqrt{3} g^2 H^6 U + \right. \\
& \quad 3 \sqrt{3} H^4 U^5 + 8 g H^5 U^2 \left(10 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U \right) \Big) + 9 \left(96 \sqrt{g^5 H^5 (3 + H^2 k^2)} + \right. \\
& \quad 224 \sqrt{3} g^2 H^2 U + 3 \sqrt{3} U^5 + g H U^2 \left(160 \sqrt{g H (3 + H^2 k^2)} + 27 \sqrt{3} U \right) \Big) \Big) dt^2 + \\
& \frac{1}{384 g H (3 + H^2 k^2)^{7/2}} k^7 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left(3 k^2 \left(184 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 608 \sqrt{3} g^2 H^4 U + 334 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \right. \\
& \quad 72 \sqrt{3} g H^3 U^3 - 3 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 6 \sqrt{3} H^2 U^5 \Big) + k^4 \left(48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + \right. \\
& \quad 272 \sqrt{3} g^2 H^6 U + 3 \sqrt{3} H^4 U^5 + 2 g H^5 U^2 \left(56 \sqrt{g H (3 + H^2 k^2)} + 17 \sqrt{3} U \right) \Big) + \\
& \quad 9 \left(160 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 352 \sqrt{3} g^2 H^2 U - 3 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. 3 \sqrt{3} U^5 + 2 g H U^2 \left(121 \sqrt{g H (3 + H^2 k^2)} + 19 \sqrt{3} U \right) \right) \Big) dt^3 + \\
& \frac{1}{384 g H (3 + H^2 k^2)^{9/2}} i k^8 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^2 \\
& \left(3 k^2 \left(304 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 884 \sqrt{3} g^2 H^4 U + 456 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \right. \\
& \quad 95 \sqrt{3} g H^3 U^3 - 6 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 6 \sqrt{3} H^2 U^5 \Big) + k^4 \left(96 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + \right. \\
& \quad 400 \sqrt{3} g^2 H^6 U + 3 \sqrt{3} H^4 U^5 + 4 g H^5 U^2 \left(36 \sqrt{g H (3 + H^2 k^2)} + 11 \sqrt{3} U \right) \Big) + \\
& \quad 27 \left(80 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 168 \sqrt{3} g^2 H^2 U - 2 \sqrt{g H (3 + H^2 k^2)} U^4 + \sqrt{3} U^5 + \right. \\
& \quad \left. g H U^2 \left(112 \sqrt{g H (3 + H^2 k^2)} + 17 \sqrt{3} U \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^3 + \\
& \left(\frac{1}{30720 (g H)^{3/2} (3 + H^2 k^2)^{5/2}} k^5 \left(16 \sqrt{3} g^2 H^2 (144 + 48 H^2 k^2 + 5 H^4 k^4) - 75 \sqrt{3} (3 + H^2 k^2)^2 U^4 + \right. \right. \\
& \quad 8 g H (3 + H^2 k^2) U \left(96 \sqrt{g H (3 + H^2 k^2)} + k^2 \left(32 \sqrt{g H^5 (3 + H^2 k^2)} + 5 \sqrt{3} H^2 U \right) \right) \Big) + \\
& \frac{1}{92160 (g H)^{3/2} (3 + H^2 k^2)^{7/2}} i k^6 \left(9 k^2 \left(8448 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 16160 \sqrt{3} g^2 H^4 U + \right. \right. \\
& \quad 6144 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - 240 \sqrt{3} g H^3 U^3 - 675 \sqrt{3} H^2 U^5 \Big) + \\
& \quad k^6 \left(1920 \sqrt{g^5 H^{17} (3 + H^2 k^2)} + 4368 \sqrt{3} g^2 H^8 U + 2048 g H^7 \sqrt{g H (3 + H^2 k^2)} U^2 - \right. \\
& \quad 225 \sqrt{3} H^6 U^5 \Big) + 27 \left(3968 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 6016 \sqrt{3} g^2 H^2 U - \right. \\
& \quad 225 \sqrt{3} U^5 + 8 g H U^2 \left(256 \sqrt{g H (3 + H^2 k^2)} - 15 \sqrt{3} U \right) \Big) + \\
& \quad \left. 3 k^4 \left(6720 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 14512 \sqrt{3} g^2 H^6 U - 675 \sqrt{3} H^4 U^5 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 24 \, g \, H^5 \, U^2 \left(256 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 5 \sqrt{3} \, U \right) \Big) \Big) \, dt - \frac{1}{92 \, 160 \, (g \, H)^{3/2} \, (3 + H^2 \, k^2)^{7/2}} \\
& \left(k^7 \left(48 \sqrt{3} \, g^3 \, H^3 \, (6192 + 5004 \, H^2 \, k^2 + 1425 \, H^4 \, k^4 + 140 \, H^6 \, k^6) + \right. \right. \\
& \quad 8 \sqrt{3} \, g^2 \, H^2 \, (60 \, 426 + 55 \, 557 \, H^2 \, k^2 + 17 \, 121 \, H^4 \, k^4 + 1772 \, H^6 \, k^6) \, U^2 + \\
& \quad g \, H \, U^3 \left(89 \, 856 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 5265 \sqrt{3} \, U - 4590 \sqrt{3} \, H^2 \, k^2 \, U + \right. \\
& \quad \quad 9 \, k^4 \left(3328 \sqrt{g \, H^9 \, (3 + H^2 \, k^2)} - 145 \sqrt{3} \, H^4 \, U \right) + 8 \, k^6 \left(416 \sqrt{g \, H^{13} \, (3 + H^2 \, k^2)} - \right. \\
& \quad \quad \quad 15 \sqrt{3} \, H^6 \, U \Big) \Big) + 3 \, U \left(81 \left(2816 \sqrt{g^5 \, H^5 \, (3 + H^2 \, k^2)} - 25 \sqrt{3} \, U^5 \right) + \right. \\
& \quad \quad 3 \, k^2 \left(52 \, 352 \sqrt{g^5 \, H^9 \, (3 + H^2 \, k^2)} + 9984 \sqrt{g^3 \, H^7 \, (3 + H^2 \, k^2)} \, U^2 - 675 \sqrt{3} \, H^2 \, U^5 \right) + \\
& \quad \quad 3 \, k^4 \left(11 \, 776 \sqrt{g^5 \, H^{13} \, (3 + H^2 \, k^2)} - 225 \sqrt{3} \, H^4 \, U^5 \right) + 5 \, k^6 \\
& \quad \quad \left. \left. \left(512 \sqrt{g^5 \, H^{17} \, (3 + H^2 \, k^2)} - 15 \sqrt{3} \, H^6 \, U^5 \right) \right) \right) \Big) \Big) \, dt^2 - \frac{1}{30 \, 720 \, (g \, H)^{3/2} \, (3 + H^2 \, k^2)^{9/2}} \\
& i \, k^8 \left(27 \, k^2 \left(20 \, 416 \sqrt{g^7 \, H^{11} \, (3 + H^2 \, k^2)} + 80 \, 544 \sqrt{3} \, g^3 \, H^5 \, U + 82 \, 240 \sqrt{g^5 \, H^9 \, (3 + H^2 \, k^2)} \, U^2 + \right. \right. \\
& \quad 46 \, 960 \sqrt{3} \, g^2 \, H^4 \, U^3 + 6144 \sqrt{g^3 \, H^7 \, (3 + H^2 \, k^2)} \, U^4 - 305 \sqrt{3} \, g \, H^3 \, U^5 - 300 \sqrt{3} \, H^2 \, U^7 \Big) + \\
& \quad 9 \, k^4 \left(19 \, 040 \sqrt{g^7 \, H^{15} \, (3 + H^2 \, k^2)} + 99 \, 856 \sqrt{3} \, g^3 \, H^7 \, U + 88 \, 512 \sqrt{g^5 \, H^{13} \, (3 + H^2 \, k^2)} \, U^2 + \right. \\
& \quad 66 \, 296 \sqrt{3} \, g^2 \, H^6 \, U^3 - 450 \sqrt{3} \, H^4 \, U^7 + 3 \, g \, H^5 \, U^4 \left(3072 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 155 \sqrt{3} \, U \right) \Big) \Big) + \\
& \quad 3 \, k^6 \left(6720 \sqrt{g^7 \, H^{19} \, (3 + H^2 \, k^2)} + 55 \, 216 \sqrt{3} \, g^3 \, H^9 \, U + 39 \, 488 \sqrt{g^5 \, H^{17} \, (3 + H^2 \, k^2)} \, U^2 + \right. \\
& \quad 41 \, 728 \sqrt{3} \, g^2 \, H^8 \, U^3 - 300 \sqrt{3} \, H^6 \, U^7 + 3 \, g \, H^7 \, U^4 \left(2048 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 105 \sqrt{3} \, U \right) \Big) \Big) + \\
& \quad 243 \left(2592 \sqrt{g^7 \, H^7 \, (3 + H^2 \, k^2)} + 8128 \sqrt{3} \, g^3 \, H^3 \, U + 9152 \sqrt{g^5 \, H^5 \, (3 + H^2 \, k^2)} \, U^2 + \right. \\
& \quad 4168 \sqrt{3} \, g^2 \, H^2 \, U^3 - 25 \sqrt{3} \, U^7 + g \, H \, U^4 \left(512 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 25 \sqrt{3} \, U \right) \Big) \Big) + \\
& \quad k^8 \left(480 \sqrt{g^7 \, H^{23} \, (3 + H^2 \, k^2)} + 11 \, 520 \sqrt{3} \, g^3 \, H^{11} \, U + 5760 \sqrt{g^5 \, H^{21} \, (3 + H^2 \, k^2)} \, U^2 + \right. \\
& \quad 9888 \sqrt{3} \, g^2 \, H^{10} \, U^3 - 75 \sqrt{3} \, H^8 \, U^7 + \\
& \quad \left. 16 \, g \, H^9 \, U^4 \left(96 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 5 \sqrt{3} \, U \right) \right) \Big) \Big) \, dt^3 + \frac{1}{92 \, 160 \, (g \, H)^{3/2} \, (3 + H^2 \, k^2)^{9/2}} \\
& k^9 \left(144 \sqrt{3} \, g^4 \, H^4 \, (23 \, 832 + 22 \, 104 \, H^2 \, k^2 + 7395 \, H^4 \, k^4 + 1000 \, H^6 \, k^6 + 40 \, H^8 \, k^8) + \right. \\
& \quad 24 \sqrt{3} \, g^3 \, H^3 \, (893 \, 700 + 953 \, 451 \, H^2 \, k^2 + 376 \, 029 \, H^4 \, k^4 + 64 \, 744 \, H^6 \, k^6 + 4080 \, H^8 \, k^8) \, U^2 + \\
& \quad 3 \sqrt{3} \, g^2 \, H^2 \, (3 + H^2 \, k^2)^2 \, (197 \, 625 + 114 \, 352 \, H^2 \, k^2 + 16 \, 944 \, H^4 \, k^4) \, U^4 + \\
& \quad 2 \, g \, H \, U^5 \left(238 \, 464 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 8505 \sqrt{3} \, U - 13 \, 365 \sqrt{3} \, H^2 \, k^2 \, U + \right. \\
& \quad \quad 27 \, k^4 \left(5888 \sqrt{g \, H^9 \, (3 + H^2 \, k^2)} - 285 \sqrt{3} \, H^4 \, U \right) + 3 \, k^6 \left(11 \, 776 \sqrt{g \, H^{13} \, (3 + H^2 \, k^2)} - \right. \\
& \quad \quad \quad 645 \sqrt{3} \, H^6 \, U \Big) \Big) + 4 \, k^8 \left(736 \sqrt{g \, H^{17} \, (3 + H^2 \, k^2)} - 45 \sqrt{3} \, H^8 \, U \right) \Big) \Big) + \\
& \quad 3 \, U \left(135 \left(33 \, 920 \sqrt{g^7 \, H^7 \, (3 + H^2 \, k^2)} + 39 \, 424 \sqrt{g^5 \, H^5 \, (3 + H^2 \, k^2)} \, U^2 - 45 \sqrt{3} \, U^7 \right) + \right. \\
& \quad \quad 36 \, k^2 \left(105 \, 472 \sqrt{g^7 \, H^{11} \, (3 + H^2 \, k^2)} + 144 \, 512 \sqrt{g^5 \, H^9 \, (3 + H^2 \, k^2)} \, U^2 + \right.
\end{aligned}$$

$$\begin{aligned}
& 5888 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^4 - 225 \sqrt{3} H^2 U^7 \Big) + \\
& 18 k^4 \Big(60\,608 \sqrt{g^7 H^{15} (3 + H^2 k^2)} + 100\,096 \sqrt{g^5 H^{13} (3 + H^2 k^2)} U^2 - 225 \sqrt{3} H^4 U^7 \Big) + \\
& 12 k^6 \Big(9600 \sqrt{g^7 H^{19} (3 + H^2 k^2)} + 20\,864 \sqrt{g^5 H^{17} (3 + H^2 k^2)} U^2 - 75 \sqrt{3} H^6 U^7 \Big) + \\
& 5 k^8 \Big(384 \sqrt{g^7 H^{23} (3 + H^2 k^2)} + 2048 \sqrt{g^5 H^{21} (3 + H^2 k^2)} U^2 - 15 \sqrt{3} H^8 U^7 \Big) \Big) \Big) dt^4 + \\
& O[dt]^5 \Big) dx^4 + O[dx]^5, \left(\frac{\mathfrak{i} \left(-\sqrt{3} k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^3 U \right)^2 dt}{2 (3 + H^2 k^2)^2} - \right. \\
& \frac{1}{3 (3 + H^2 k^2)^2} \\
& \left(k^3 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \quad \left. \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right) dt^2 - \\
& \frac{1}{4 (3 + H^2 k^2)^3} \mathfrak{i} k^4 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left(3 g H \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \quad \left. U^2 \left(-9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) dt^3 + \\
& \frac{1}{5 (3 + H^2 k^2)^3} k^5 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left(9 g^2 H^2 + 6 g H U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \quad \left. U^3 \left(-12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left(-4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) \\
& dt^4 + O[dt]^5 \Big) + \left(-\frac{1}{4} \mathfrak{i} k^2 \left(2 \sqrt{g H} - \frac{\sqrt{3} U}{\sqrt{3 + H^2 k^2}} \right) + \right. \\
& \left(k^3 \left(2 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} U \right) \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) dt \right) / \\
& \left(4 (3 + H^2 k^2)^{3/2} \right) + \\
& \left(\mathfrak{i} k^4 \left(2 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} U \right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right. \\
& \quad \left. dt^2 \right) / \left(4 (3 + H^2 k^2)^{3/2} \right) - \frac{1}{4 (3 + H^2 k^2)^{5/2}} \\
& \left(k^5 \left(2 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} U \right) \left(3 g H \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \right. \\
& \quad \left. \left. U^2 \left(-9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) \right) \\
& dt^3 - \frac{1}{4 (3 + H^2 k^2)^{5/2}} \mathfrak{i} k^6 \left(2 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} U \right) \\
& \left(9 g^2 H^2 + 6 g H U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& U^3 \left(-12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left(-4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \\
& dt^4 + O[dt]^5 \Big) dx + \left(\left(k^3 \left(12 \sqrt{3} g H (4 + H^2 k^2) - \right. \right. \right. \\
& \quad U \left(48 \sqrt{g H (3 + H^2 k^2)} + 9 \sqrt{3} U + k^2 \left(16 \sqrt{g H^5 (3 + H^2 k^2)} + 3 \sqrt{3} H^2 U \right) \right) \Big) \Big) / \\
& \left(96 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) - \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{5/2}} \\
& i k^4 \left(9 U^2 \left(16 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U \right) + k^4 U^2 \left(16 \sqrt{g H^9 (3 + H^2 k^2)} + 3 \sqrt{3} H^4 U \right) + \right. \\
& \quad 6 k^2 \left(18 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 + 3 \sqrt{3} H^2 U^3 \right) + \\
& \quad 4 g H \left(63 \sqrt{g H (3 + H^2 k^2)} - 99 \sqrt{3} U - 63 \sqrt{3} H^2 k^2 U + \right. \\
& \quad \left. \left. k^4 \left(3 \sqrt{g H^9 (3 + H^2 k^2)} - 10 \sqrt{3} H^4 U \right) \right) \right) dt + \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{5/2}} \\
& k^5 \left(-12 \sqrt{3} g^2 H^2 (30 + 15 H^2 k^2 + 2 H^4 k^4) + 9 U^3 \left(16 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U \right) + \right. \\
& \quad k^4 U^3 \left(16 \sqrt{g H^9 (3 + H^2 k^2)} + 3 \sqrt{3} H^4 U \right) + \\
& \quad 6 k^2 \left(56 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^3 + 3 \sqrt{3} H^2 U^4 \right) + \\
& \quad g H U \left(864 \sqrt{g H (3 + H^2 k^2)} - 675 \sqrt{3} U - 429 \sqrt{3} H^2 k^2 U + \right. \\
& \quad \left. 4 k^4 \left(6 \sqrt{g H^9 (3 + H^2 k^2)} - 17 \sqrt{3} H^4 U \right) \right) \Big) dt^2 + \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{7/2}} \\
& i k^6 \left(9 k^2 \left(84 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 404 \sqrt{3} g^2 H^4 U + 438 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \\
& \quad \left. 314 \sqrt{3} g H^3 U^3 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 9 \sqrt{3} H^2 U^5 \right) + \\
& \quad 3 k^4 \left(36 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 304 \sqrt{3} g^2 H^6 U + 48 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. 9 \sqrt{3} H^4 U^5 + g H^5 U^2 \left(264 \sqrt{g H (3 + H^2 k^2)} - 301 \sqrt{3} U \right) \right) + \\
& \quad 27 \left(52 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 172 \sqrt{3} g^2 H^2 U + 16 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. 3 \sqrt{3} U^5 + g H U^2 \left(210 \sqrt{g H (3 + H^2 k^2)} - 109 \sqrt{3} U \right) \right) - \\
& \quad k^6 U \left(72 \sqrt{3} g^2 H^8 + 12 g H^7 U \left(-3 \sqrt{g H (3 + H^2 k^2)} + 8 \sqrt{3} U \right) - \right. \\
& \quad \left. U^3 \left(16 \sqrt{g H^{13} (3 + H^2 k^2)} + 3 \sqrt{3} H^6 U \right) \right) \Big) dt^3 + \\
& \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{7/2}} k^7 \left(36 \sqrt{3} g^3 H^3 (48 + 27 H^2 k^2 + 4 H^4 k^4) + \right. \\
& \quad 3 \sqrt{3} g^2 H^2 (4113 + 3075 H^2 k^2 + 712 H^4 k^4 + 48 H^6 k^6) U^2 + \\
& \quad 2 g H U^3 \left(-5076 \sqrt{g H (3 + H^2 k^2)} + 1971 \sqrt{3} U + 1872 \sqrt{3} H^2 k^2 U + \right. \\
& \quad \left. 3 k^4 \left(-216 \sqrt{g H^9 (3 + H^2 k^2)} + 197 \sqrt{3} H^4 U \right) + \right. \\
& \quad \left. 2 k^6 \left(-12 \sqrt{g H^{13} (3 + H^2 k^2)} + 31 \sqrt{3} H^6 U \right) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(7344 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 432 \sqrt{g H (3 + H^2 k^2)} U^4 + 81 \sqrt{3} U^5 + \right. \\
& \quad k^6 U^4 \left(16 \sqrt{g H^{13} (3 + H^2 k^2)} + 3 \sqrt{3} H^6 U \right) + 9 k^2 \left(400 \sqrt{g^5 H^9 (3 + H^2 k^2)} + \right. \\
& \quad \left. 760 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 9 \sqrt{3} H^2 U^5 \right) + \\
& \quad \left. 9 k^4 \left(48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 16 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + 3 \sqrt{3} H^4 U^5 \right) \right) dt^4 + O[dt]^5 \Big) \\
& dx^2 + \left(- \left(\left(i k^4 \left(3 \sqrt{3} (3 + H^2 k^2) U^3 + 4 g H \left(-12 \sqrt{g H (3 + H^2 k^2)} + 6 \sqrt{3} U + k^2 \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. - 4 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \right) \right) \right) / \left(384 g H (3 + H^2 k^2)^{3/2} \right) \right) + \\
& \quad \frac{1}{384 g H (3 + H^2 k^2)^{5/2}} k^5 \left(8 \sqrt{3} g^2 H^2 (54 + 33 H^2 k^2 + 5 H^4 k^4) + 3 \sqrt{3} (3 + H^2 k^2)^2 U^4 - \right. \\
& \quad \left. 2 g H U \left(324 \sqrt{g H (3 + H^2 k^2)} - 81 \sqrt{3} U + 24 k^2 \left(7 \sqrt{g H^5 (3 + H^2 k^2)} - 2 \sqrt{3} H^2 U \right) + \right. \right. \\
& \quad \left. \left. k^4 \left(24 \sqrt{g H^9 (3 + H^2 k^2)} - 7 \sqrt{3} H^4 U \right) \right) \right) dt - \frac{1}{384 g H (3 + H^2 k^2)^{5/2}} \\
& i k^6 \left(3 k^2 \left(96 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 380 \sqrt{3} g^2 H^4 U + 224 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \\
& \quad \left. \left. 51 \sqrt{3} g H^3 U^3 - 6 \sqrt{3} H^2 U^5 \right) + 9 \left(96 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 224 \sqrt{3} g^2 H^2 U - \right. \right. \\
& \quad \left. \left. 3 \sqrt{3} U^5 + g H U^2 \left(160 \sqrt{g H (3 + H^2 k^2)} - 27 \sqrt{3} U \right) \right) + k^4 \left(16 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - \right. \right. \\
& \quad \left. \left. 168 \sqrt{3} g^2 H^6 U - 3 \sqrt{3} H^4 U^5 + 8 g H^5 U^2 \left(10 \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} U \right) \right) \right) dt^2 - \\
& \quad \frac{1}{384 (g H (3 + H^2 k^2))^{7/2}} \left(k^7 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \quad \left(-3 k^2 \left(184 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 608 \sqrt{3} g^2 H^4 U + 334 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \\
& \quad \left. \left. 72 \sqrt{3} g H^3 U^3 - 3 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 6 \sqrt{3} H^2 U^5 \right) - \right. \\
& \quad \left. 9 \left(160 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 352 \sqrt{3} g^2 H^2 U - 3 \sqrt{g H (3 + H^2 k^2)} U^4 - 3 \sqrt{3} U^5 + \right. \right. \\
& \quad \left. \left. 2 g H U^2 \left(121 \sqrt{g H (3 + H^2 k^2)} - 19 \sqrt{3} U \right) \right) - k^4 \left(48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - \right. \right. \\
& \quad \left. \left. 272 \sqrt{3} g^2 H^6 U - 3 \sqrt{3} H^4 U^5 + 2 g H^5 U^2 \left(56 \sqrt{g H (3 + H^2 k^2)} - 17 \sqrt{3} U \right) \right) \right) \right) dt^3 - \\
& \quad \frac{1}{384 g H (3 + H^2 k^2)^{9/2}} i k^8 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \\
& \quad \left(-3 k^2 \left(304 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 884 \sqrt{3} g^2 H^4 U + 456 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \\
& \quad \left. \left. 95 \sqrt{3} g H^3 U^3 - 6 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 6 \sqrt{3} H^2 U^5 \right) - \right. \\
& \quad \left. 27 \left(80 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 168 \sqrt{3} g^2 H^2 U - 2 \sqrt{g H (3 + H^2 k^2)} U^4 - \right. \right. \\
& \quad \left. \left. \sqrt{3} U^5 + g H U^2 \left(112 \sqrt{g H (3 + H^2 k^2)} - 17 \sqrt{3} U \right) \right) - \right. \\
& \quad \left. k^4 \left(96 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 400 \sqrt{3} g^2 H^6 U - 3 \sqrt{3} H^4 U^5 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 4 \, g \, H^5 \, U^2 \left(36 \sqrt{g \, H \left(3 + H^2 \, k^2 \right)} - 11 \sqrt{3} \, U \right) \Big) \Big) \, dt^4 + O[dt]^5 \Big) \, dx^3 + \\
& \left(\frac{1}{30 \, 720 \, (g \, H)^{3/2} \left(3 + H^2 \, k^2 \right)^{5/2}} k^5 \left(-16 \sqrt{3} \, g^2 \, H^2 \left(144 + 48 \, H^2 \, k^2 + 5 \, H^4 \, k^4 \right) + \right. \right. \\
& \quad 75 \sqrt{3} \left(3 + H^2 \, k^2 \right)^2 U^4 + \\
& \quad \left. 8 \, g \, H \left(3 + H^2 \, k^2 \right) U \left(96 \sqrt{g \, H \left(3 + H^2 \, k^2 \right)} + k^2 \left(32 \sqrt{g \, H^5 \left(3 + H^2 \, k^2 \right)} - 5 \sqrt{3} \, H^2 \, U \right) \right) \right) \Big) + \\
& \quad \frac{1}{92 \, 160 \, (g \, H)^{3/2} \left(3 + H^2 \, k^2 \right)^{7/2}} i \, k^6 \left(9 \, k^2 \left(8448 \sqrt{g^5 \, H^9 \left(3 + H^2 \, k^2 \right)} - 16 \, 160 \sqrt{3} \, g^2 \, H^4 \, U + \right. \right. \\
& \quad 6144 \sqrt{g^3 \, H^7 \left(3 + H^2 \, k^2 \right)} U^2 + 240 \sqrt{3} \, g \, H^3 \, U^3 + 675 \sqrt{3} \, H^2 \, U^5 \Big) + \\
& \quad k^6 \left(1920 \sqrt{g^5 \, H^{17} \left(3 + H^2 \, k^2 \right)} - 4368 \sqrt{3} \, g^2 \, H^8 \, U + 2048 \, g \, H^7 \sqrt{g \, H \left(3 + H^2 \, k^2 \right)} U^2 + \right. \\
& \quad 225 \sqrt{3} \, H^6 \, U^5 \Big) + 3 \, k^4 \left(6720 \sqrt{g^5 \, H^{13} \left(3 + H^2 \, k^2 \right)} - 14 \, 512 \sqrt{3} \, g^2 \, H^6 \, U + \right. \\
& \quad 675 \sqrt{3} \, H^4 \, U^5 + 24 \, g \, H^5 \, U^2 \left(256 \sqrt{g \, H \left(3 + H^2 \, k^2 \right)} + 5 \sqrt{3} \, U \right) \Big) \Big) + \\
& \quad 27 \left(3968 \sqrt{g^5 \, H^5 \left(3 + H^2 \, k^2 \right)} - 6016 \sqrt{3} \, g^2 \, H^2 \, U + 225 \sqrt{3} \, U^5 + \right. \\
& \quad \left. 8 \, g \, H \, U^2 \left(256 \sqrt{g \, H \left(3 + H^2 \, k^2 \right)} + 15 \sqrt{3} \, U \right) \right) \Big) \Big) \, dt - \frac{1}{92 \, 160 \, (g \, H)^{3/2} \left(3 + H^2 \, k^2 \right)^{7/2}} \\
& \left(k^7 \left(-48 \sqrt{3} \, g^3 \, H^3 \left(6192 + 5004 \, H^2 \, k^2 + 1425 \, H^4 \, k^4 + 140 \, H^6 \, k^6 \right) - \right. \right. \\
& \quad 8 \sqrt{3} \, g^2 \, H^2 \left(60 \, 426 + 55 \, 557 \, H^2 \, k^2 + 17 \, 121 \, H^4 \, k^4 + 1772 \, H^6 \, k^6 \right) U^2 + \\
& \quad g \, H \, U^3 \left(89 \, 856 \sqrt{g \, H \left(3 + H^2 \, k^2 \right)} + 5265 \sqrt{3} \, U + 4590 \sqrt{3} \, H^2 \, k^2 \, U + \right. \\
& \quad 9 \, k^4 \left(3328 \sqrt{g \, H^9 \left(3 + H^2 \, k^2 \right)} + 145 \sqrt{3} \, H^4 \, U \right) + 8 \, k^6 \left(416 \sqrt{g \, H^{13} \left(3 + H^2 \, k^2 \right)} + \right. \\
& \quad \left. 15 \sqrt{3} \, H^6 \, U \right) \Big) + 3 \, U \left(81 \left(2816 \sqrt{g^5 \, H^5 \left(3 + H^2 \, k^2 \right)} + 25 \sqrt{3} \, U^5 \right) + \right. \\
& \quad 3 \, k^2 \left(52 \, 352 \sqrt{g^5 \, H^9 \left(3 + H^2 \, k^2 \right)} + 9984 \sqrt{g^3 \, H^7 \left(3 + H^2 \, k^2 \right)} U^2 + 675 \sqrt{3} \, H^2 \, U^5 \right) + \\
& \quad 3 \, k^4 \left(11 \, 776 \sqrt{g^5 \, H^{13} \left(3 + H^2 \, k^2 \right)} + 225 \sqrt{3} \, H^4 \, U^5 \right) + 5 \, k^6 \\
& \quad \left. \left(512 \sqrt{g^5 \, H^{17} \left(3 + H^2 \, k^2 \right)} + 15 \sqrt{3} \, H^6 \, U^5 \right) \right) \Big) \Big) \, dt^2 - \frac{1}{30 \, 720 \, (g \, H)^{3/2} \left(3 + H^2 \, k^2 \right)^{9/2}} \\
& i \, k^8 \left(27 \, k^2 \left(20 \, 416 \sqrt{g^7 \, H^{11} \left(3 + H^2 \, k^2 \right)} - 80 \, 544 \sqrt{3} \, g^3 \, H^5 \, U + 82 \, 240 \sqrt{g^5 \, H^9 \left(3 + H^2 \, k^2 \right)} U^2 - \right. \right. \\
& \quad 46 \, 960 \sqrt{3} \, g^2 \, H^4 \, U^3 + 6144 \sqrt{g^3 \, H^7 \left(3 + H^2 \, k^2 \right)} U^4 + 305 \sqrt{3} \, g \, H^3 \, U^5 + 300 \sqrt{3} \, H^2 \, U^7 \Big) + \\
& \quad k^8 \left(480 \sqrt{g^7 \, H^{23} \left(3 + H^2 \, k^2 \right)} - 11 \, 520 \sqrt{3} \, g^3 \, H^{11} \, U + 5760 \sqrt{g^5 \, H^{21} \left(3 + H^2 \, k^2 \right)} U^2 - \right. \\
& \quad 9888 \sqrt{3} \, g^2 \, H^{10} \, U^3 + 75 \sqrt{3} \, H^8 \, U^7 + 16 \, g \, H^9 \, U^4 \left(96 \sqrt{g \, H \left(3 + H^2 \, k^2 \right)} + 5 \sqrt{3} \, U \right) \Big) \Big) + \\
& \quad 243 \left(2592 \sqrt{g^7 \, H^7 \left(3 + H^2 \, k^2 \right)} - 8128 \sqrt{3} \, g^3 \, H^3 \, U + 9152 \sqrt{g^5 \, H^5 \left(3 + H^2 \, k^2 \right)} U^2 - \right. \\
& \quad 4168 \sqrt{3} \, g^2 \, H^2 \, U^3 + 25 \sqrt{3} \, U^7 + g \, H \, U^4 \left(512 \sqrt{g \, H \left(3 + H^2 \, k^2 \right)} + 25 \sqrt{3} \, U \right) \Big) \Big) + \\
& \quad 3 \, k^6 \left(6720 \sqrt{g^7 \, H^{19} \left(3 + H^2 \, k^2 \right)} - 55 \, 216 \sqrt{3} \, g^3 \, H^9 \, U + 39 \, 488 \sqrt{g^5 \, H^{17} \left(3 + H^2 \, k^2 \right)} U^2 - \right.
\end{aligned}$$

$$\begin{aligned}
& 41\,728\sqrt{3}\,g^2H^8U^3 + 300\sqrt{3}\,H^6U^7 + 3gH^7U^4 \left(2048\sqrt{gH(3+H^2k^2)} + 105\sqrt{3}\,U \right) \Big) + \\
& 9k^4 \left(19\,040\sqrt{g^7H^{15}(3+H^2k^2)} - 99\,856\sqrt{3}\,g^3H^7U + \right. \\
& 88\,512\sqrt{g^5H^{13}(3+H^2k^2)}\,U^2 - 66\,296\sqrt{3}\,g^2H^6U^3 + 450\sqrt{3}\,H^4U^7 + \\
& \left. 3gH^5U^4 \left(3072\sqrt{gH(3+H^2k^2)} + 155\sqrt{3}\,U \right) \right) \Big) dt^3 + \frac{1}{92\,160\,(gH)^{3/2}(3+H^2k^2)^{9/2}} \\
& k^9 \left(-144\sqrt{3}\,g^4H^4(23\,832 + 22\,104H^2k^2 + 7395H^4k^4 + 1000H^6k^6 + 40H^8k^8) - \right. \\
& 24\sqrt{3}\,g^3H^3(893\,700 + 953\,451H^2k^2 + 376\,029H^4k^4 + 64\,744H^6k^6 + 4080H^8k^8)U^2 - \\
& 3\sqrt{3}\,g^2H^2(3+H^2k^2)^2(197\,625 + 114\,352H^2k^2 + 16\,944H^4k^4)U^4 + \\
& 2gH^5U^5 \left(238\,464\sqrt{gH(3+H^2k^2)} + 8505\sqrt{3}\,U + 13\,365\sqrt{3}\,H^2k^2U + \right. \\
& 27k^4 \left(5888\sqrt{gH^9(3+H^2k^2)} + 285\sqrt{3}\,H^4U \right) + 3k^6 \left(11\,776\sqrt{gH^{13}(3+H^2k^2)} + \right. \\
& \left. 645\sqrt{3}\,H^6U \right) + 4k^8 \left(736\sqrt{gH^{17}(3+H^2k^2)} + 45\sqrt{3}\,H^8U \right) \Big) + \\
& 3U \left(135 \left(33\,920\sqrt{g^7H^7(3+H^2k^2)} + 39\,424\sqrt{g^5H^5(3+H^2k^2)}\,U^2 + 45\sqrt{3}\,U^7 \right) + \right. \\
& 36k^2 \left(105\,472\sqrt{g^7H^{11}(3+H^2k^2)} + 144\,512\sqrt{g^5H^9(3+H^2k^2)}\,U^2 + \right. \\
& \left. 5888\sqrt{g^3H^7(3+H^2k^2)}\,U^4 + 225\sqrt{3}\,H^2U^7 \right) + \\
& 18k^4 \left(60\,608\sqrt{g^7H^{15}(3+H^2k^2)} + 100\,096\sqrt{g^5H^{13}(3+H^2k^2)}\,U^2 + 225\sqrt{3}\,H^4U^7 \right) + \\
& 12k^6 \left(9600\sqrt{g^7H^{19}(3+H^2k^2)} + 20\,864\sqrt{g^5H^{17}(3+H^2k^2)}\,U^2 + 75\sqrt{3}\,H^6U^7 \right) + \\
& 5k^8 \left(384\sqrt{g^7H^{23}(3+H^2k^2)} + 2048\sqrt{g^5H^{21}(3+H^2k^2)}\,U^2 + \right. \\
& \left. \left. 15\sqrt{3}\,H^8U^7 \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5 \}
\end{aligned}$$

```

In[2254]:= Text[Row[{" U > Sqrt[g*H]  " }]]
Text[Row[{"Fnn and FnG " }]]
KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
KurFWS = KurF /. ap -> (U + Sqrt[g*H]) /. am -> (0);
KurFWSeta =
  KurFWS /. fp -> (H*v + U*Rpp*n) /. fm -> (H*v + U*Rmp*n) /. qp -> Rpp*n /.
  qm -> Rmp*n;
KurFWSeta = KurFWSeta /. v -> (GGp*G + Gnp*n);
Kfnm = FullSimplify[KurFWSeta /. G -> 0 /. n -> 1]
KfnG = FullSimplify[KurFWSeta /. n -> 0 /. G -> 1]
Kfnn = Kfnm /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfnG = KfnG /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
Fnn2 = -dt*(1 - Exp[-I*k*dx])/dx*Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG " }]]
KurFWSG = KurFWS /. fp -> (U*Rpp*G + U*H*v + g*H*Rpp*n) /.
  fm -> (U*Rmp*G + U*H*v + g*H*Rmp*n) /. qp -> Rpp*G /. qm -> Rmp*G;
KurFWSG = KurFWSG /. v -> (GGp*G + Gnp*n);
KfGn = FullSimplify[KurFWSG /. G -> 0 /. n -> 1]
KfGG = FullSimplify[KurFWSG /. n -> 0 /. G -> 1]
KfGn = KfGn /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfGG = KfGG /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;

FGn2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U*H*GG2 + U/2*(Rm + Rp) - (Sqrt[g*H])/2*(Rp - Rm);
FGG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega" }]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2]/(I*dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}]

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Out[2254]= U > Sqrt[g*H]

Out[2255]= Fnn and FnG

Out[2260]= Gnp H + Rmp U

Out[2261]= GGp H

$$\text{Out[2266]} = \left(-\frac{(H^2 k^3 U w) dt^2}{2 (3 + H^2 k^2)} - \frac{i H^2 k^3 U w^2 dt^3}{6 (3 + H^2 k^2)} + O[dt]^4 \right) + \left(-\frac{1}{2} (k^2 U) dt + O[dt]^4 \right) dx +$$

$$\left(\frac{i (9 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(\frac{1}{24} k^4 U dt + O[dt]^4 \right) dx^3 +$$

$$\left(-\frac{i (54 H^2 k^7 + 19 H^4 k^9 + 2 H^6 k^{11}) U dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$$

$$\text{Out[2269]} = \left(-\frac{3 (k w) dt^2}{2 (3 + H^2 k^2)} - \frac{i k w^2 dt^3}{2 (3 + H^2 k^2)} + O[dt]^4 \right) +$$

$$\left(\frac{i (6 k^3 + H^2 k^5) dt}{4 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(\frac{i (-54 k^5 + H^4 k^9) dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$$

Out[2270]= FGn and FGG

Out[2273]= H (g Rmp + Gnp U)

Out[2274]= (GGp H + Rmp) U

$$\text{Out[2279]} = \left(-\frac{(k (3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2 (3 + H^2 k^2)} - \frac{i k (3 g H + g H^3 k^2 - 3 U^2) w^2 dt^3}{6 (3 + H^2 k^2)} + O[dt]^4 \right) +$$

$$\left(-\frac{1}{2} (g H k^2) dt + O[dt]^4 \right) dx +$$

$$\left(\left(i (18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2) dt \right) / \left(12 (3 + H^2 k^2)^2 \right) + O[dt]^4 \right) dx^2 +$$

$$\left(\frac{1}{24} g H k^4 dt + O[dt]^4 \right) dx^3 +$$

$$\left(-\left(\left(i (54 g H k^5 + 54 g H^3 k^7 + 18 g H^5 k^9 + 2 g H^7 k^{11} - 54 k^5 U^2 + H^4 k^9 U^2) dt \right) / \right. \right.$$

$$\left. \left. (240 (3 + H^2 k^2)^3) \right) + O[dt]^4 \right) dx^4 + O[dx]^5$$

$$\text{Out[2283]} = \left(-\frac{(k (6 + H^2 k^2) U w) dt^2}{2 (3 + H^2 k^2)} - \frac{i k (6 + H^2 k^2) U w^2 dt^3}{6 (3 + H^2 k^2)} + O[dt]^4 \right) + \left(-\frac{1}{2} (k^2 U) dt + O[dt]^4 \right) dx +$$

$$\left(\frac{i (36 k^3 + 15 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(\frac{1}{24} k^4 U dt + O[dt]^4 \right) dx^3 +$$

$$\left(-\frac{i (108 k^5 + 54 H^2 k^7 + 17 H^4 k^9 + 2 H^6 k^{11}) U dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$$

Out[2284]= W : omega

$$\text{Out[2289]} = \left\{ \frac{i \left(\sqrt{3} k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^3 U \right)^2 dt}{2 (3 + H^2 k^2)^2} - \right.$$

$$\begin{aligned}
& \frac{1}{3 \left(3 + H^2 k^2\right)^2} \left(k^3 \left(\sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U \right) \right. \\
& \quad \left. \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U \right) \right) \right) dt^2 - \frac{1}{4 \left(3 + H^2 k^2\right)^3} \\
& \quad i k^4 \left(\sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U \right) \left(3 g \left(\sqrt{3} H \sqrt{g H \left(3 + H^2 k^2\right)} + 9 H U + 3 H^3 k^2 U \right) + \right. \\
& \quad \left. U^2 \left(H^4 k^4 U + 9 \left(\sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + U \right) + 3 k^2 \left(\sqrt{3} \sqrt{g H^5 \left(3 + H^2 k^2\right)} + 2 H^2 U \right) \right) \right) dt^3 + \\
& \quad \frac{1}{5 \left(3 + H^2 k^2\right)^3} k^5 \left(\sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U \right) \\
& \quad \left(9 g^2 H^2 + 6 g H U \left(2 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 3 \left(3 + H^2 k^2\right) U \right) + \right. \\
& \quad \left. U^3 \left(12 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 9 U + H^4 k^4 U + 2 k^2 \left(2 \sqrt{3} \sqrt{g H^5 \left(3 + H^2 k^2\right)} + 3 H^2 U \right) \right) \right) dt^4 + \\
& \quad O[dt]^5 \Bigg) + \left(-\frac{1}{4} i k^2 \left(\sqrt{3} \sqrt{\frac{g H}{3 + H^2 k^2}} + 2 U \right) + \left(k^3 \left(\sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U \right) \right. \right. \\
& \quad \left. \left(\sqrt{3} g H + 2 \sqrt{g H \left(3 + H^2 k^2\right)} U \right) dt \right) / \left(4 \sqrt{g H} \left(3 + H^2 k^2\right)^{3/2} \right) + \\
& \quad \left(i k^4 \left(\sqrt{3} g H + 2 \sqrt{g H \left(3 + H^2 k^2\right)} U \right) \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U \right) \right) \right. \\
& \quad \left. dt^2 \right) / \left(4 \sqrt{g H} \left(3 + H^2 k^2\right)^{3/2} \right) - \frac{1}{4 \left(\sqrt{g H} \left(3 + H^2 k^2\right)^{5/2} \right)} \\
& \quad \left(k^5 \left(\sqrt{3} g H + 2 \sqrt{g H \left(3 + H^2 k^2\right)} U \right) \left(3 g \left(\sqrt{3} H \sqrt{g H \left(3 + H^2 k^2\right)} + 9 H U + 3 H^3 k^2 U \right) + \right. \right. \\
& \quad \left. \left. U^2 \left(H^4 k^4 U + 9 \left(\sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + U \right) + 3 k^2 \left(\sqrt{3} \sqrt{g H^5 \left(3 + H^2 k^2\right)} + 2 H^2 U \right) \right) \right) \right) \\
& \quad dt^3 - \frac{1}{4 \sqrt{g H} \left(3 + H^2 k^2\right)^{5/2}} i k^6 \left(\sqrt{3} g H + 2 \sqrt{g H \left(3 + H^2 k^2\right)} U \right) \\
& \quad \left(9 g^2 H^2 + 6 g H U \left(2 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 3 \left(3 + H^2 k^2\right) U \right) + U^3 \left(12 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + \right. \right. \\
& \quad \left. \left. 9 U + H^4 k^4 U + 2 k^2 \left(2 \sqrt{3} \sqrt{g H^5 \left(3 + H^2 k^2\right)} + 3 H^2 U \right) \right) \right) dt^4 + O[dt]^5 \Bigg) dx + \\
& \quad \left(- \left(\left(k^3 \left(3 \sqrt{3} g H \left(13 + 3 H^2 k^2 \right) + 16 \left(3 \sqrt{g H \left(3 + H^2 k^2 \right)} + k^2 \sqrt{g H^5 \left(3 + H^2 k^2 \right)} \right) U \right) \right) / \right. \right. \\
& \quad \left. \left(96 \left(\sqrt{g H} \left(3 + H^2 k^2 \right)^{3/2} \right) \right) \right) - \frac{1}{96 \sqrt{g H} \left(3 + H^2 k^2 \right)^{5/2}} \\
& \quad i k^4 \left(g H \left(144 \sqrt{g H \left(3 + H^2 k^2 \right)} + \sqrt{3} \left(369 + 234 H^2 k^2 + 37 H^4 k^4 \right) U \right) + \right. \\
& \quad 4 \left(63 \sqrt{g H \left(3 + H^2 k^2 \right)} U^2 + 7 k^4 \sqrt{g H^9 \left(3 + H^2 k^2 \right)} U^2 + \right. \\
& \quad \left. \left. 3 k^2 \left(3 \sqrt{g^3 H^7 \left(3 + H^2 k^2 \right)} + 14 \sqrt{g H^5 \left(3 + H^2 k^2 \right)} U^2 \right) \right) \right) dt + \\
& \quad \frac{1}{96 \sqrt{g H} \left(3 + H^2 k^2 \right)^{7/2}} k^5 \left(\sqrt{3} \sqrt{g H \left(3 + H^2 k^2 \right)} + \left(3 + H^2 k^2 \right) U \right)
\end{aligned}$$

$$\begin{aligned}
& \left(g H \left(171 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (477 + 306 H^2 k^2 + 49 H^4 k^4) U \right) + \right. \\
& \quad 5 \left(72 \sqrt{g H (3 + H^2 k^2)} U^2 + 8 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \quad \left. \left. 3 k^2 \left(3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt^2 + \\
& \quad \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^6 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^2 \\
& \quad \left(468 \sqrt{g H (3 + H^2 k^2)} U^2 + 52 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad g H \left(198 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (585 + 378 H^2 k^2 + 61 H^4 k^4) U \right) + \\
& \quad \left. 6 k^2 \left(9 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 52 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt^3 - \\
& \quad \frac{1}{96 \left(\sqrt{g H} (3 + H^2 k^2)^{11/2} \right)} \left(k^7 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^3 \right. \\
& \quad \left(576 \sqrt{g H (3 + H^2 k^2)} U^2 + 64 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad g H \left(225 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (693 + 450 H^2 k^2 + 73 H^4 k^4) U \right) + \\
& \quad \left. \left. 3 k^2 \left(21 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 128 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt^4 + O[dt]^5 \Big) dx^2 + \\
& \left(\left(i k^4 \left(\sqrt{3} g H (33 + 7 H^2 k^2) + 16 \left(3 \sqrt{g H (3 + H^2 k^2)} + k^2 \sqrt{g H^5 (3 + H^2 k^2)} \right) U \right) \right) \right) / \\
& \quad \left(384 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) - \frac{1}{128 \left(\sqrt{g H} (3 + H^2 k^2)^{5/2} \right)} \\
& \quad \left(k^5 \left(g H \left(72 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (207 + 126 H^2 k^2 + 19 H^4 k^4) U \right) + \right. \right. \\
& \quad 16 \left(9 \sqrt{g H (3 + H^2 k^2)} U^2 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \quad \left. \left. k^2 \left(\sqrt{g^3 H^7 (3 + H^2 k^2)} + 6 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) \right) dt - \\
& \quad \frac{1}{128 \sqrt{g H} (3 + H^2 k^2)^{5/2}} i k^6 \left(9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \right. \\
& \quad \left(96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) + 32 \left(9 \sqrt{g H (3 + H^2 k^2)} U^3 + \right. \\
& \quad \quad \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left(2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) \\
& \quad dt^2 + \frac{1}{384 \sqrt{g H} (3 + H^2 k^2)^{7/2}} k^7 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \quad \left(24 \sqrt{3} g^2 H^2 (21 + 5 H^2 k^2) + \right. \\
& \quad 3 g H U \left(717 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1011 + 646 H^2 k^2 + 103 H^4 k^4) U \right) + \\
& \quad 5 \left(288 \sqrt{g H (3 + H^2 k^2)} U^3 + 32 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \\
& \quad \quad \left. \left. 3 k^2 \left(39 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 64 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \Big) dt^3 +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{128 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^8 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^2 \\
& \left(5 \sqrt{3} g^2 H^2 (45 + 11 H^2 k^2) + 720 \sqrt{g H (3 + H^2 k^2)} U^3 + 80 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \\
& g H U \left(990 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1449 + 930 H^2 k^2 + 149 H^4 k^4) U \right) + \\
& \left. 2 k^2 \left(137 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 240 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) dt^4 + O[dt]^5 \Bigg) \\
dx^3 + & \left(k^5 \left(3 \sqrt{3} g H (543 + 146 H^2 k^2 + 15 H^4 k^4) + \right. \right. \\
& 256 \left(9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \Bigg) \Bigg/ \\
& \left(30720 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \frac{1}{92160 \sqrt{g H} (3 + H^2 k^2)^{7/2}} \\
& i k^6 \left(3 g H \left(18432 \sqrt{g H (3 + H^2 k^2)} + 51039 \sqrt{3} U + 45735 \sqrt{3} H^2 k^2 U + \right. \right. \\
& 1381 \sqrt{3} H^6 k^6 U + k^4 \left(960 \sqrt{g H^9 (3 + H^2 k^2)} + 13717 \sqrt{3} H^4 U \right) \Bigg) + \\
& 128 \left(837 \sqrt{g H (3 + H^2 k^2)} U^2 + 279 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 31 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^2 + \right. \\
& \left. 27 k^2 \left(7 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 31 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \Bigg) dt - \\
& \frac{1}{92160 \left(\sqrt{g H} (3 + H^2 k^2)^{7/2} \right)} \left(k^7 \left(9 \sqrt{3} g^2 H^2 (12999 + 6258 H^2 k^2 + 775 H^4 k^4) + g H U \right. \right. \\
& \left(617661 \sqrt{3} H^2 k^2 U + 20551 \sqrt{3} H^6 k^6 U + 81 \left(5888 \sqrt{g H (3 + H^2 k^2)} + 8053 \sqrt{3} U \right) + \right. \\
& 3 k^4 \left(12288 \sqrt{g H^9 (3 + H^2 k^2)} + 65021 \sqrt{3} H^4 U \right) \Bigg) + 128 \\
& \left(2322 \sqrt{g H (3 + H^2 k^2)} U^3 + 774 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 86 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} \right. \\
& \left. U^3 + 9 k^2 \left(229 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 258 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Bigg) dt^2 - \\
& \frac{1}{30720 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^8 \left(k^8 U^3 \left(21253 \sqrt{3} g H^9 + 7776 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& 243 \left(832 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 4423 \sqrt{3} g^2 H^2 U + 2592 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& g H U^2 \left(8832 \sqrt{g H (3 + H^2 k^2)} + 7823 \sqrt{3} U \right) \Bigg) + 3 k^6 U \left(9181 \sqrt{3} g^2 H^8 + \right. \\
& 31104 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + 4 g H^7 U \left(5312 \sqrt{g H (3 + H^2 k^2)} + 21787 \sqrt{3} U \right) \Bigg) + \\
& 27 k^2 \left(35319 \sqrt{3} g^2 H^4 U + 91580 \sqrt{3} g H^3 U^3 + 64 \right. \\
& \left(59 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 1155 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 486 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) \Bigg) + \\
& 9 k^4 \left(31231 \sqrt{3} g^2 H^6 U + 2 g H^5 U^2 \left(34336 \sqrt{g H (3 + H^2 k^2)} + 67003 \sqrt{3} U \right) + \right. \\
& \left. 288 \left(5 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 162 \sqrt{g H^9 (3 + H^2 k^2)} U^4 \right) \right) \Bigg) dt^3 +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{92\,160\,\sqrt{g\,H}\,(3+H^2\,k^2)^{11/2}} k^9 \left(\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \\
& \left(k^8\,U^3 \left(111\,559\,\sqrt{3}\,g\,H^9 + 42\,368\,\sqrt{g\,H^{17}\,(3+H^2\,k^2)}\,U \right) + \right. \\
& \quad 27\,k^2 \left(18\,078\,\sqrt{g^5\,H^9\,(3+H^2\,k^2)} + 172\,047\,\sqrt{3}\,g^2\,H^4\,U + 372\,075\,\sqrt{g^3\,H^7\,(3+H^2\,k^2)}\,U^2 + \right. \\
& \quad \left. 477\,940\,\sqrt{3}\,g\,H^3\,U^3 + 169\,472\,\sqrt{g\,H^5\,(3+H^2\,k^2)}\,U^4 \right) + \\
& \quad 81 \left(11\,603\,\sqrt{g^5\,H^5\,(3+H^2\,k^2)} + 63\,917\,\sqrt{3}\,g^2\,H^2\,U + 42\,368\,\sqrt{g\,H\,(3+H^2\,k^2)}\,U^4 + \right. \\
& \quad \left. g\,H\,U^2 \left(132\,513\,\sqrt{g\,H\,(3+H^2\,k^2)} + 122\,207\,\sqrt{3}\,U \right) \right) + 3\,k^6\,U \left(45\,573\,\sqrt{3}\,g^2\,H^8 + \right. \\
& \quad \left. 169\,472\,\sqrt{g\,H^{13}\,(3+H^2\,k^2)}\,U^3 + g\,H^7\,U \left(108\,089\,\sqrt{g\,H\,(3+H^2\,k^2)} + 456\,644\,\sqrt{3}\,U \right) \right) + \\
& \quad 9\,k^4 \left(7035\,\sqrt{g^5\,H^{13}\,(3+H^2\,k^2)} + 153\,703\,\sqrt{3}\,g^2\,H^6\,U + 254\,208\,\sqrt{g\,H^9\,(3+H^2\,k^2)}\,U^4 + \right. \\
& \quad \left. g\,H^5\,U^2 \left(347\,651\,\sqrt{g\,H\,(3+H^2\,k^2)} + 700\,818\,\sqrt{3}\,U \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^4 + \\
& O[dx]^5, \left(\frac{i \left(-\sqrt{3}\,k\,\sqrt{g\,H\,(3+H^2\,k^2)} + 3\,k\,U + H^2\,k^3\,U \right)^2 dt}{2\,(3+H^2\,k^2)^2} - \right. \\
& \frac{1}{3\,(3+H^2\,k^2)^2} \left(k^3 \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right. \\
& \quad \left. \left(3\,g\,H + U \left(-2\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right) \right) dt^2 - \\
& \frac{1}{4\,(3+H^2\,k^2)^3} i\,k^4 \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \\
& \quad \left(3\,g\,H \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + 3\,(3+H^2\,k^2)\,U \right) + \right. \\
& \quad \left. U^2 \left(-9\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + 9\,U + H^4\,k^4\,U - 3\,k^2 \left(\sqrt{3}\,\sqrt{g\,H^5\,(3+H^2\,k^2)} - 2\,H^2\,U \right) \right) \right) dt^3 + \\
& \frac{1}{5\,(3+H^2\,k^2)^3} k^5 \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \\
& \quad \left(9\,g^2\,H^2 + 6\,g\,H\,U \left(-2\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + 3\,(3+H^2\,k^2)\,U \right) + \right. \\
& \quad \left. U^3 \left(-12\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + 9\,U + H^4\,k^4\,U + k^2 \left(-4\,\sqrt{3}\,\sqrt{g\,H^5\,(3+H^2\,k^2)} + 6\,H^2\,U \right) \right) \right) \\
& \quad \left. dt^4 + O[dt]^5 \right) + \left(\frac{1}{4}\,i\,k^2 \left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}} - 2\,U \right) - \right. \\
& \quad \left(\left(k^3 \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \left(\sqrt{3}\,g\,H - 2\,\sqrt{g\,H\,(3+H^2\,k^2)}\,U \right) \right) dt \right) / \\
& \quad \left(4\,\left(\sqrt{g\,H}\,(3+H^2\,k^2)^{3/2} \right) \right) - \\
& \quad \left(i\,k^4 \left(\sqrt{3}\,g\,H - 2\,\sqrt{g\,H\,(3+H^2\,k^2)}\,U \right) \left(3\,g\,H + U \left(-2\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& dt^2 \Big) / \left(4 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) + \frac{1}{4 \sqrt{g H} (3 + H^2 k^2)^{5/2}} \\
& k^5 \left(\sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left(3 g H \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \quad \left. U^2 \left(-9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) dt^3 + \\
& \frac{1}{4 \sqrt{g H} (3 + H^2 k^2)^{5/2}} i k^6 \left(\sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \\
& \left(9 g^2 H^2 + 6 g H U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \quad \left. U^3 \left(-12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + \right. \right. \\
& \quad \left. \left. k^2 \left(-4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) dt^4 + O[dt]^5 \Big) dx + \\
& \left(\left(k^3 \left(3 \sqrt{3} g H (13 + 3 H^2 k^2) - 16 \left(3 \sqrt{g H (3 + H^2 k^2)} U + k^2 \sqrt{g H^5 (3 + H^2 k^2)} U \right) \right) \right) / \right. \\
& \quad \left(96 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) + \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{5/2}} \\
& \quad i k^4 \left(g H \left(-144 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (369 + 234 H^2 k^2 + 37 H^4 k^4) U \right) - \right. \\
& \quad \left. 4 \left(63 \sqrt{g H (3 + H^2 k^2)} U^2 + 7 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \right. \\
& \quad \left. \left. 3 k^2 \left(3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 14 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt - \\
& \frac{1}{96 \left(\sqrt{g H} (3 + H^2 k^2)^{7/2} \right)} \left(k^5 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right) \right. \\
& \quad \left(g H \left(171 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (477 + 306 H^2 k^2 + 49 H^4 k^4) U \right) + \right. \\
& \quad \left. 5 \left(72 \sqrt{g H (3 + H^2 k^2)} U^2 + 8 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \right. \\
& \quad \left. \left. 3 k^2 \left(3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt^2 + \\
& \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^6 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \\
& \left(468 \sqrt{g H (3 + H^2 k^2)} U^2 + 52 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad g H \left(198 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (585 + 378 H^2 k^2 + 61 H^4 k^4) U \right) + \\
& \quad \left. 6 k^2 \left(9 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 52 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt^3 + \\
& \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{11/2}} k^7 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^3 \\
& \left(576 \sqrt{g H (3 + H^2 k^2)} U^2 + 64 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. g H \left(225 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (693 + 450 H^2 k^2 + 73 H^4 k^4) U \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& 3 k^2 \left(21 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 128 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) dt^4 + O[dt]^5 \Big) dx^2 + \\
& \left(\left(i k^4 \left(-\sqrt{3} g H (33 + 7 H^2 k^2) + 16 \left(3 \sqrt{g H (3 + H^2 k^2)} + k^2 \sqrt{g H^5 (3 + H^2 k^2)} \right) U \right) \right) / \right. \\
& \quad \left(384 \sqrt{g H (3 + H^2 k^2)}^{3/2} \right) + \frac{1}{128 \sqrt{g H (3 + H^2 k^2)}^{5/2}} \\
& k^5 \left(g H \left(-72 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (207 + 126 H^2 k^2 + 19 H^4 k^4) U \right) - \right. \\
& \quad 16 \left(9 \sqrt{g H (3 + H^2 k^2)} U^2 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. k^2 \left(\sqrt{g^3 H^7 (3 + H^2 k^2)} + 6 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \Big) dt + \\
& \frac{1}{128 \sqrt{g H (3 + H^2 k^2)}^{5/2}} i k^6 \left(9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \right. \\
& \quad \left(-96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) - 32 \left(9 \sqrt{g H (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left(2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) \\
& dt^2 - \frac{1}{384 \left(\sqrt{g H (3 + H^2 k^2)}^{7/2} \right)} \left(k^7 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \quad \left(24 \sqrt{3} g^2 H^2 (21 + 5 H^2 k^2) + 3 g H U \left(-717 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1011 + 646 H^2 k^2 + \right. \right. \\
& \quad \left. \left. 103 H^4 k^4) U \right) - 5 \left(288 \sqrt{g H (3 + H^2 k^2)} U^3 + 32 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \right. \\
& \quad \left. \left. 3 k^2 \left(39 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 64 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \Big) dt^3 - \\
& \frac{1}{128 \sqrt{g H (3 + H^2 k^2)}^{9/2}} i k^8 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \\
& \left(5 \sqrt{3} g^2 H^2 (45 + 11 H^2 k^2) + \right. \\
& \quad g H U \left(-990 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1449 + 930 H^2 k^2 + 149 H^4 k^4) U \right) - \\
& \quad 2 \left(360 \sqrt{g H (3 + H^2 k^2)} U^3 + 40 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. k^2 \left(137 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 240 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^3 + \\
& \left(\left(k^5 \left(-3 \sqrt{3} g H (543 + 146 H^2 k^2 + 15 H^4 k^4) + 256 \left(9 \sqrt{g H (3 + H^2 k^2)} + \right. \right. \right. \right. \\
& \quad \left. \left. \left. 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \right. \\
& \quad \left(30720 \sqrt{g H (3 + H^2 k^2)}^{5/2} \right) + \frac{1}{92160 \sqrt{g H (3 + H^2 k^2)}^{7/2}} \\
& i k^6 \left(-3 g H \left(-18432 \sqrt{g H (3 + H^2 k^2)} + 51039 \sqrt{3} U + 45735 \sqrt{3} H^2 k^2 U + \right. \right. \\
& \quad \left. \left. 1381 \sqrt{3} H^6 k^6 U + k^4 \left(-960 \sqrt{g H^9 (3 + H^2 k^2)} + 13717 \sqrt{3} H^4 U \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 128 \left(837 \sqrt{g H (3 + H^2 k^2)} U^2 + 279 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 31 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. 27 k^2 \left(7 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 31 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt + \\
& \frac{1}{92160 \sqrt{g H} (3 + H^2 k^2)^{7/2}} k^7 \left(9 \sqrt{3} g^2 H^2 (12999 + 6258 H^2 k^2 + 775 H^4 k^4) + g H \right. \\
& \quad U \left(617661 \sqrt{3} H^2 k^2 U + 20551 \sqrt{3} H^6 k^6 U + 81 \left(-5888 \sqrt{g H (3 + H^2 k^2)} + 8053 \sqrt{3} U \right) - \right. \\
& \quad \left. 3 k^4 \left(12288 \sqrt{g H^9 (3 + H^2 k^2)} - 65021 \sqrt{3} H^4 U \right) \right) - \\
& 128 \left(2322 \sqrt{g H (3 + H^2 k^2)} U^3 + 774 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 86 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} \right. \\
& \quad \left. U^3 + 9 k^2 \left(229 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 258 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) dt^2 - \\
& \frac{1}{30720 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^8 \left(k^8 U^3 \left(-21253 \sqrt{3} g H^9 + 7776 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& \quad 243 \left(832 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 4423 \sqrt{3} g^2 H^2 U + 2592 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. g H U^2 \left(8832 \sqrt{g H (3 + H^2 k^2)} - 7823 \sqrt{3} U \right) \right) - 3 k^6 U \left(9181 \sqrt{3} g^2 H^8 - \right. \\
& \quad \left. 31104 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + 4 g H^7 U \left(-5312 \sqrt{g H (3 + H^2 k^2)} + 21787 \sqrt{3} U \right) \right) + \\
& \quad 27 k^2 \left(-35319 \sqrt{3} g^2 H^4 U - 91580 \sqrt{3} g H^3 U^3 + 64 \right. \\
& \quad \left. \left(59 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 1155 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 486 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) \right) + \\
& \quad 9 k^4 \left(-31231 \sqrt{3} g^2 H^6 U + 2 g H^5 U^2 \left(34336 \sqrt{g H (3 + H^2 k^2)} - 67003 \sqrt{3} U \right) + \right. \\
& \quad \left. 288 \left(5 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 162 \sqrt{g H^9 (3 + H^2 k^2)} U^4 \right) \right) dt^3 - \\
& \frac{1}{92160 \left(\sqrt{g H} (3 + H^2 k^2) \right)^{11/2}} \left(k^9 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right) \right. \\
& \quad \left(k^8 U^3 \left(-111559 \sqrt{3} g H^9 + 42368 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& \quad 27 k^2 \left(18078 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 172047 \sqrt{3} g^2 H^4 U + 372075 \right. \\
& \quad \left. \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - 477940 \sqrt{3} g H^3 U^3 + 169472 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) + \\
& \quad 9 k^4 \left(7035 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 153703 \sqrt{3} g^2 H^6 U + 254208 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. g H^5 U^2 \left(347651 \sqrt{g H (3 + H^2 k^2)} - 700818 \sqrt{3} U \right) \right) + \\
& \quad 81 \left(11603 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 63917 \sqrt{3} g^2 H^2 U + 42368 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. g H U^2 \left(132513 \sqrt{g H (3 + H^2 k^2)} - 122207 \sqrt{3} U \right) \right) - \\
& \quad 3 k^6 U \left(45573 \sqrt{3} g^2 H^8 - 169472 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + g H^7 U \right. \\
& \quad \left. \left. \left(-108089 \sqrt{g H (3 + H^2 k^2)} + 456644 \sqrt{3} U \right) \right) \right) dt^4 + O[dt]^5 \} dx^4 + O[dx]^5 \}
\end{aligned}$$

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In[2290]:= Text[Row[{" U< -Sqrt[g*H] " }]]
Text[Row[{"Fnn and FnG " }]]
KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
KurFWS = KurF /. ap -> (0) /. am -> (U - Sqrt[g*H]);
KurFWSeta =
  KurFWS /. fp -> (H*v + U*Rpp*n) /. fm -> (H*v + U*Rmp*n) /. qp -> Rpp*n /.
  qm -> Rmp*n;
KurFWSeta = KurFWSeta /. v -> (GGp*G + Gnp*n);
Kfnn = FullSimplify[KurFWSeta /. G -> 0 /. n -> 1]
KfnG = FullSimplify[KurFWSeta /. n -> 0 /. G -> 1]
Kfnn = Kfnn /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfnG = KfnG /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
Fnn2 = -dt*(1 - Exp[-I*k*dx])/dx*Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG " }]]
KurFWSG = KurFWS /. fp -> (U*Rpp*G + U*H*v + g*H*Rpp*n) /.
  fm -> (U*Rmp*G + U*H*v + g*H*Rmp*n) /. qp -> Rpp*G /. qm -> Rmp*G;
KurFWSG = KurFWSG /. v -> (GGp*G + Gnp*n);
KfGn = FullSimplify[KurFWSG /. G -> 0 /. n -> 1]
KfGG = FullSimplify[KurFWSG /. n -> 0 /. G -> 1]
KfGn = KfGn /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfGG = KfGG /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;

FGn2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U*H*GG2 + U/2*(Rm + Rp) - (Sqrt[g*H])/2*(Rp - Rm);
FGG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega" }]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2]/(I*dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}]

```

Out[2290]= U< -Sqrt[g*H]

Out[2291]= **Fnn and FnG**

Out[2296]= **Gnp H + Rpp U**

Out[2297]= **GGp H**

$$\text{Out[2302]} = \left(-\frac{(H^2 k^3 U w) dt^2}{2 (3 + H^2 k^2)} - \frac{i H^2 k^3 U w^2 dt^3}{6 (3 + H^2 k^2)} + O[dt]^4 \right) + \left(\frac{1}{2} k^2 U dt + O[dt]^4 \right) dx +$$

$$\left(\frac{i (9 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(-\frac{1}{24} (k^4 U) dt + O[dt]^4 \right) dx^3 +$$

$$\left(-\frac{i k^7 (54 H^2 U + 19 H^4 k^2 U + 2 H^6 k^4 U) dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$$

$$\text{Out[2305]} = \left(-\frac{3 (k w) dt^2}{2 (3 + H^2 k^2)} - \frac{i k w^2 dt^3}{2 (3 + H^2 k^2)} + O[dt]^4 \right) +$$

$$\left(\frac{i (6 k^3 + H^2 k^5) dt}{4 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(\frac{i (-54 k^5 + H^4 k^9) dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$$

Out[2306]= **FGn and FGG**

Out[2309]= **H (g Rpp + Gnp U)**

Out[2310]= **(GGp H + Rpp) U**

$$\text{Out[2315]} = \left(-\frac{(k (3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2 (3 + H^2 k^2)} - \frac{i k (3 g H + g H^3 k^2 - 3 U^2) w^2 dt^3}{6 (3 + H^2 k^2)} + O[dt]^4 \right) +$$

$$\left(\frac{1}{2} g H k^2 dt + O[dt]^4 \right) dx +$$

$$\left(\frac{i (18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2) dt}{12 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 +$$

$$\left(-\frac{1}{24} (g H k^4) dt + O[dt]^4 \right) dx^3 +$$

$$\left(-\frac{i (54 g H k^5 + 54 g H^3 k^7 + 18 g H^5 k^9 + 2 g H^7 k^{11} - 54 k^5 U^2 + H^4 k^9 U^2) dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$$

$$\text{Out[2319]} = \left(-\frac{(k (6 + H^2 k^2) U w) dt^2}{2 (3 + H^2 k^2)} - \frac{i k (6 + H^2 k^2) U w^2 dt^3}{6 (3 + H^2 k^2)} + O[dt]^4 \right) + \left(\frac{1}{2} k^2 U dt + O[dt]^4 \right) dx +$$

$$\left(\frac{i (36 k^3 + 15 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3 + H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(-\frac{1}{24} (k^4 U) dt + O[dt]^4 \right) dx^3 +$$

$$\left(-\frac{i (108 k^5 + 54 H^2 k^7 + 17 H^4 k^9 + 2 H^6 k^{11}) U dt}{240 (3 + H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$$

Out[2320]= **W : omega**

$$\begin{aligned}
\text{Out[2325]} = & \left\{ \frac{\left(\mathfrak{i} \left(\sqrt{3} \, k \sqrt{g \, H \, (3 + H^2 \, k^2)} + 3 \, k \, U + H^2 \, k^3 \, U \right)^2 \, dt}{2 \, (3 + H^2 \, k^2)^2} - \right. \\
& \frac{1}{3 \, (3 + H^2 \, k^2)^2} \left(k^3 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \right. \\
& \left. \left(3 \, g \, H + U \left(2 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \right) \right) dt^2 - \frac{1}{4 \, (3 + H^2 \, k^2)^3} \\
& \mathfrak{i} \, k^4 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \left(3 \, g \left(\sqrt{3} \, H \sqrt{g \, H \, (3 + H^2 \, k^2)} + 9 \, H \, U + 3 \, H^3 \, k^2 \, U \right) + \right. \\
& \left. U^2 \left(H^4 \, k^4 \, U + 9 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + U \right) + 3 \, k^2 \left(\sqrt{3} \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} + 2 \, H^2 \, U \right) \right) \right) dt^3 + \\
& \frac{1}{5 \, (3 + H^2 \, k^2)^3} k^5 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \\
& \left(9 \, g^2 \, H^2 + 6 \, g \, H \, U \left(2 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + 3 \, (3 + H^2 \, k^2) \, U \right) + \right. \\
& \left. U^3 \left(12 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + 9 \, U + H^4 \, k^4 \, U + 2 \, k^2 \left(2 \sqrt{3} \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} + 3 \, H^2 \, U \right) \right) \right) \\
& dt^4 + O[dt]^5 \Bigg) + \left(\frac{1}{4} \mathfrak{i} \, k^2 \left(\sqrt{3} \sqrt{\frac{g \, H}{3 + H^2 \, k^2}} + 2 \, U \right) - \right. \\
& \left(\left(k^3 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \left(\sqrt{3} \, g \, H + 2 \sqrt{g \, H \, (3 + H^2 \, k^2)} \, U \right) \right) dt \right) / \\
& \left(4 \left(\sqrt{g \, H} \, (3 + H^2 \, k^2)^{3/2} \right) \right) - \\
& \left(\mathfrak{i} \, k^4 \left(\sqrt{3} \, g \, H + 2 \sqrt{g \, H \, (3 + H^2 \, k^2)} \, U \right) \left(3 \, g \, H + U \left(2 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + (3 + H^2 \, k^2) \, U \right) \right) \right. \\
& \left. dt^2 \right) / \left(4 \sqrt{g \, H} \, (3 + H^2 \, k^2)^{3/2} \right) + \frac{1}{4 \sqrt{g \, H} \, (3 + H^2 \, k^2)^{5/2}} \\
& k^5 \left(\sqrt{3} \, g \, H + 2 \sqrt{g \, H \, (3 + H^2 \, k^2)} \, U \right) \left(3 \, g \left(\sqrt{3} \, H \sqrt{g \, H \, (3 + H^2 \, k^2)} + 9 \, H \, U + 3 \, H^3 \, k^2 \, U \right) + \right. \\
& \left. U^2 \left(H^4 \, k^4 \, U + 9 \left(\sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + U \right) + 3 \, k^2 \left(\sqrt{3} \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} + 2 \, H^2 \, U \right) \right) \right) dt^3 + \\
& \frac{1}{4 \sqrt{g \, H} \, (3 + H^2 \, k^2)^{5/2}} \mathfrak{i} \, k^6 \left(\sqrt{3} \, g \, H + 2 \sqrt{g \, H \, (3 + H^2 \, k^2)} \, U \right) \\
& \left(9 \, g^2 \, H^2 + 6 \, g \, H \, U \left(2 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + 3 \, (3 + H^2 \, k^2) \, U \right) + U^3 \left(12 \sqrt{3} \sqrt{g \, H \, (3 + H^2 \, k^2)} + \right. \right. \\
& \left. \left. 9 \, U + H^4 \, k^4 \, U + 2 \, k^2 \left(2 \sqrt{3} \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} + 3 \, H^2 \, U \right) \right) \right) dt^4 + O[dt]^5 \Bigg) dx + \\
& \left(- \left(\left(k^3 \left(3 \sqrt{3} \, g \, H \, (13 + 3 \, H^2 \, k^2) + 16 \left(3 \sqrt{g \, H \, (3 + H^2 \, k^2)} + k^2 \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} \right) \, U \right) \right) / \right. \right. \\
& \left. \left(96 \left(\sqrt{g \, H} \, (3 + H^2 \, k^2)^{3/2} \right) \right) \right) - \frac{1}{96 \sqrt{g \, H} \, (3 + H^2 \, k^2)^{5/2}} \\
& \mathfrak{i} \, k^4 \left(g \, H \left(144 \sqrt{g \, H \, (3 + H^2 \, k^2)} + \sqrt{3} \, (369 + 234 \, H^2 \, k^2 + 37 \, H^4 \, k^4) \, U \right) + \right. \\
& \left. 4 \left(63 \sqrt{g \, H \, (3 + H^2 \, k^2)} \, U^2 + 7 \, k^4 \sqrt{g \, H^9 \, (3 + H^2 \, k^2)} \, U^2 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. \left(3 k^2 \left(3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 14 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt + \\
& \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{7/2}} k^5 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left(g H \left(171 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (477 + 306 H^2 k^2 + 49 H^4 k^4) U \right) + \right. \\
& \quad \left. 5 \left(72 \sqrt{g H (3 + H^2 k^2)} U^2 + 8 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \right. \\
& \quad \left. \left. 3 k^2 \left(3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt^2 + \\
& \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^6 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^2 \\
& \left(468 \sqrt{g H (3 + H^2 k^2)} U^2 + 52 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad g H \left(198 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (585 + 378 H^2 k^2 + 61 H^4 k^4) U \right) + \\
& \quad \left. 6 k^2 \left(9 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 52 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt^3 - \\
& \frac{1}{96 \left(\sqrt{g H} (3 + H^2 k^2)^{11/2} \right)} \left(k^7 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^3 \right. \\
& \quad \left(576 \sqrt{g H (3 + H^2 k^2)} U^2 + 64 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. g H \left(225 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (693 + 450 H^2 k^2 + 73 H^4 k^4) U \right) + \right. \\
& \quad \left. \left. 3 k^2 \left(21 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 128 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt^4 + O[dt]^5 \Big) dx^2 + \\
& \left(- \left(\left(i k^4 \left(\sqrt{3} g H (33 + 7 H^2 k^2) + 16 \left(3 \sqrt{g H (3 + H^2 k^2)} + k^2 \sqrt{g H^5 (3 + H^2 k^2)} \right) U \right) \right) \right) / \right. \\
& \quad \left. \left(384 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) \right) + \frac{1}{128 \sqrt{g H} (3 + H^2 k^2)^{5/2}} k^5 \\
& \left(g H \left(72 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (207 + 126 H^2 k^2 + 19 H^4 k^4) U \right) + 16 \left(9 \sqrt{g H (3 + H^2 k^2)} U^2 + \right. \right. \\
& \quad \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + k^2 \left(\sqrt{g^3 H^7 (3 + H^2 k^2)} + 6 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt + \\
& \frac{1}{128 \sqrt{g H} (3 + H^2 k^2)^{5/2}} i k^6 \left(9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \right. \\
& \quad \left(96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) + 32 \left(9 \sqrt{g H (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left(2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \\
& dt^2 - \frac{1}{384 \left(\sqrt{g H} (3 + H^2 k^2)^{7/2} \right)} \left(k^7 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left(24 \sqrt{3} g^2 H^2 \right. \right. \\
& \quad \left. \left(21 + 5 H^2 k^2 \right) + 3 g H U \left(717 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1011 + 646 H^2 k^2 + 103 H^4 k^4) U \right) + \right. \\
& \quad \left. 5 \left(288 \sqrt{g H (3 + H^2 k^2)} U^3 + 32 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \right. \\
& \quad \left. \left. 3 k^2 \left(39 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 64 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \right) dt^3 -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{128 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^8 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^2 \\
& \left(5 \sqrt{3} g^2 H^2 (45 + 11 H^2 k^2) + 720 \sqrt{g H (3 + H^2 k^2)} U^3 + 80 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \\
& g H U \left(990 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1449 + 930 H^2 k^2 + 149 H^4 k^4) U \right) + \\
& \left. 2 k^2 \left(137 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 240 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) dt^4 + O[dt]^5 \Bigg) \\
& dx^3 + \left(k^5 \left(3 \sqrt{3} g H (543 + 146 H^2 k^2 + 15 H^4 k^4) + \right. \right. \\
& \left. 256 \left(9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \Bigg) / \\
& \left(30720 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \frac{1}{92160 \sqrt{g H} (3 + H^2 k^2)^{7/2}} \\
& i k^6 \left(3 g H \left(18432 \sqrt{g H (3 + H^2 k^2)} + 51039 \sqrt{3} U + 45735 \sqrt{3} H^2 k^2 U + \right. \right. \\
& 1381 \sqrt{3} H^6 k^6 U + k^4 \left(960 \sqrt{g H^9 (3 + H^2 k^2)} + 13717 \sqrt{3} H^4 U \right) \Bigg) + \\
& 128 \left(837 \sqrt{g H (3 + H^2 k^2)} U^2 + 279 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 31 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^2 + \right. \\
& \left. 27 k^2 \left(7 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 31 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \Bigg) dt - \\
& \frac{1}{92160 \left(\sqrt{g H} (3 + H^2 k^2)^{7/2} \right)} \left(k^7 \left(9 \sqrt{3} g^2 H^2 (12999 + 6258 H^2 k^2 + 775 H^4 k^4) + g H U \right. \right. \\
& \left(617661 \sqrt{3} H^2 k^2 U + 20551 \sqrt{3} H^6 k^6 U + 81 \left(5888 \sqrt{g H (3 + H^2 k^2)} + 8053 \sqrt{3} U \right) + \right. \\
& \left. 3 k^4 \left(12288 \sqrt{g H^9 (3 + H^2 k^2)} + 65021 \sqrt{3} H^4 U \right) \right) + 128 \\
& \left(2322 \sqrt{g H (3 + H^2 k^2)} U^3 + 774 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 86 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} \right. \\
& \left. U^3 + 9 k^2 \left(229 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 258 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Bigg) dt^2 - \\
& \frac{1}{30720 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^8 \left(k^8 U^3 \left(21253 \sqrt{3} g H^9 + 7776 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& 243 \left(832 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 4423 \sqrt{3} g^2 H^2 U + 2592 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& g H U^2 \left(8832 \sqrt{g H (3 + H^2 k^2)} + 7823 \sqrt{3} U \right) \Bigg) + 3 k^6 U \left(9181 \sqrt{3} g^2 H^8 + \right. \\
& 31104 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + 4 g H^7 U \left(5312 \sqrt{g H (3 + H^2 k^2)} + 21787 \sqrt{3} U \right) \Bigg) + \\
& 27 k^2 \left(35319 \sqrt{3} g^2 H^4 U + 91580 \sqrt{3} g H^3 U^3 + 64 \right. \\
& \left(59 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 1155 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 486 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) \Bigg) + \\
& 9 k^4 \left(31231 \sqrt{3} g^2 H^6 U + 2 g H^5 U^2 \left(34336 \sqrt{g H (3 + H^2 k^2)} + 67003 \sqrt{3} U \right) + \right. \\
& \left. 288 \left(5 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 162 \sqrt{g H^9 (3 + H^2 k^2)} U^4 \right) \right) \Bigg) dt^3 +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{92\,160\,\sqrt{g\,H}\,(3+H^2\,k^2)^{11/2}} k^9 \left(\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \\
& \left(k^8\,U^3 \left(111\,559\,\sqrt{3}\,g\,H^9 + 42\,368\,\sqrt{g\,H^{17}\,(3+H^2\,k^2)}\,U \right) + \right. \\
& \quad 27\,k^2 \left(18\,078\,\sqrt{g^5\,H^9\,(3+H^2\,k^2)} + 172\,047\,\sqrt{3}\,g^2\,H^4\,U + 372\,075\,\sqrt{g^3\,H^7\,(3+H^2\,k^2)}\,U^2 + \right. \\
& \quad \left. 477\,940\,\sqrt{3}\,g\,H^3\,U^3 + 169\,472\,\sqrt{g\,H^5\,(3+H^2\,k^2)}\,U^4 \right) + \\
& \quad 81 \left(11\,603\,\sqrt{g^5\,H^5\,(3+H^2\,k^2)} + 63\,917\,\sqrt{3}\,g^2\,H^2\,U + 42\,368\,\sqrt{g\,H\,(3+H^2\,k^2)}\,U^4 + \right. \\
& \quad \left. g\,H\,U^2 \left(132\,513\,\sqrt{g\,H\,(3+H^2\,k^2)} + 122\,207\,\sqrt{3}\,U \right) \right) + 3\,k^6\,U \left(45\,573\,\sqrt{3}\,g^2\,H^8 + \right. \\
& \quad \left. 169\,472\,\sqrt{g\,H^{13}\,(3+H^2\,k^2)}\,U^3 + g\,H^7\,U \left(108\,089\,\sqrt{g\,H\,(3+H^2\,k^2)} + 456\,644\,\sqrt{3}\,U \right) \right) + \\
& \quad 9\,k^4 \left(7035\,\sqrt{g^5\,H^{13}\,(3+H^2\,k^2)} + 153\,703\,\sqrt{3}\,g^2\,H^6\,U + 254\,208\,\sqrt{g\,H^9\,(3+H^2\,k^2)}\,U^4 + \right. \\
& \quad \left. g\,H^5\,U^2 \left(347\,651\,\sqrt{g\,H\,(3+H^2\,k^2)} + 700\,818\,\sqrt{3}\,U \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^4 + \\
& O[dx]^5, \left(\frac{i \left(-\sqrt{3}\,k\,\sqrt{g\,H\,(3+H^2\,k^2)} + 3\,k\,U + H^2\,k^3\,U \right)^2 dt}{2\,(3+H^2\,k^2)^2} - \right. \\
& \frac{1}{3\,(3+H^2\,k^2)^2} \left(k^3 \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right. \\
& \quad \left. \left(3\,g\,H + U \left(-2\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right) \right) dt^2 - \\
& \frac{1}{4\,(3+H^2\,k^2)^3} i\,k^4 \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \\
& \left(3\,g\,H \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + 3\,(3+H^2\,k^2)\,U \right) + \right. \\
& \quad U^2 \left(-9\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + 9\,U + H^4\,k^4\,U - 3\,k^2 \left(\sqrt{3}\,\sqrt{g\,H^5\,(3+H^2\,k^2)} - 2\,H^2\,U \right) \right) \Big) dt^3 + \\
& \frac{1}{5\,(3+H^2\,k^2)^3} k^5 \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \\
& \left(9\,g^2\,H^2 + 6\,g\,H\,U \left(-2\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + 3\,(3+H^2\,k^2)\,U \right) + \right. \\
& \quad U^3 \left(-12\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + 9\,U + H^4\,k^4\,U + k^2 \left(-4\,\sqrt{3}\,\sqrt{g\,H^5\,(3+H^2\,k^2)} + 6\,H^2\,U \right) \right) \Big) \\
& dt^4 + O[dt]^5 \Big) + \left(-\frac{1}{4}\,i\,k^2 \left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}} - 2\,U \right) + \right. \\
& \left(k^3 \left(-\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \left(\sqrt{3}\,g\,H - 2\,\sqrt{g\,H\,(3+H^2\,k^2)}\,U \right) dt \right) / \\
& \left(4\,\sqrt{g\,H}\,(3+H^2\,k^2)^{3/2} \right) + \\
& \left(i\,k^4 \left(\sqrt{3}\,g\,H - 2\,\sqrt{g\,H\,(3+H^2\,k^2)}\,U \right) \left(3\,g\,H + U \left(-2\,\sqrt{3}\,\sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& dt^2 \Bigg) \Bigg/ \left(4 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) - \frac{1}{4 \left(\sqrt{g H} (3 + H^2 k^2)^{5/2} \right)} \\
& \left(k^5 \left(\sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left(3 g H \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \right. \\
& \quad \left. U^2 \left(-9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) \Bigg) \\
& dt^3 - \frac{1}{4 \sqrt{g H} (3 + H^2 k^2)^{5/2}} i k^6 \left(\sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \\
& \left(9 g^2 H^2 + 6 g H U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \quad \left. U^3 \left(-12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + \right. \right. \\
& \quad \left. \left. k^2 \left(-4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) dt^4 + O[dt]^5 \Bigg) dx + \\
& \left(\left(k^3 \left(3 \sqrt{3} g H (13 + 3 H^2 k^2) - 16 \left(3 \sqrt{g H (3 + H^2 k^2)} U + k^2 \sqrt{g H^5 (3 + H^2 k^2)} U \right) \right) \right) \Bigg/ \right. \\
& \quad \left(96 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) + \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{5/2}} \\
& \quad i k^4 \left(g H \left(-144 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (369 + 234 H^2 k^2 + 37 H^4 k^4) U \right) - \right. \\
& \quad 4 \left(63 \sqrt{g H (3 + H^2 k^2)} U^2 + 7 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. \left. 3 k^2 \left(3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 14 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt - \\
& \quad \frac{1}{96 \left(\sqrt{g H} (3 + H^2 k^2)^{7/2} \right)} \left(k^5 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right) \right. \\
& \quad \left(g H \left(171 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (477 + 306 H^2 k^2 + 49 H^4 k^4) U \right) + \right. \\
& \quad 5 \left(72 \sqrt{g H (3 + H^2 k^2)} U^2 + 8 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. \left. 3 k^2 \left(3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt^2 + \\
& \quad \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^6 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \\
& \quad \left(468 \sqrt{g H (3 + H^2 k^2)} U^2 + 52 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad g H \left(198 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (585 + 378 H^2 k^2 + 61 H^4 k^4) U \right) + \\
& \quad \left. 6 k^2 \left(9 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 52 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt^3 + \\
& \quad \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{11/2}} k^7 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^3 \\
& \quad \left(576 \sqrt{g H (3 + H^2 k^2)} U^2 + 64 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. g H \left(225 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (693 + 450 H^2 k^2 + 73 H^4 k^4) U \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 3 k^2 \left(21 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 128 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) dt^4 + O[dt]^5 \Big) dx^2 + \\
& \left(\left(i k^4 \left(\sqrt{3} g H (33 + 7 H^2 k^2) - 16 \left(3 \sqrt{g H (3 + H^2 k^2)} U + k^2 \sqrt{g H^5 (3 + H^2 k^2)} U \right) \right) \right) / \right. \\
& \quad \left(384 \sqrt{g H (3 + H^2 k^2)}^{3/2} \right) + \frac{1}{128 \sqrt{g H (3 + H^2 k^2)}^{5/2}} k^5 \\
& \quad \left(g H \left(72 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (207 + 126 H^2 k^2 + 19 H^4 k^4) U \right) + 16 \left(9 \sqrt{g H (3 + H^2 k^2)} U^2 + \right. \right. \\
& \quad \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + k^2 \left(\sqrt{g^3 H^7 (3 + H^2 k^2)} + 6 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt + \\
& \quad \frac{1}{128 \sqrt{g H (3 + H^2 k^2)}^{5/2}} i k^6 \left(-9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) - 5 g H U \right. \\
& \quad \left(-96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) + 32 \left(9 \sqrt{g H (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left(2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) \\
& dt^2 + \frac{1}{384 \sqrt{g H (3 + H^2 k^2)}^{7/2}} k^7 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left(24 \sqrt{3} g^2 H^2 (21 + 5 H^2 k^2) + \right. \\
& \quad 3 g H U \left(-717 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1011 + 646 H^2 k^2 + 103 H^4 k^4) U \right) - \\
& \quad 5 \left(288 \sqrt{g H (3 + H^2 k^2)} U^3 + 32 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. 3 k^2 \left(39 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 64 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) dt^3 + \\
& \quad \frac{1}{128 \sqrt{g H (3 + H^2 k^2)}^{9/2}} i k^8 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \\
& \left(5 \sqrt{3} g^2 H^2 (45 + 11 H^2 k^2) + \right. \\
& \quad g H U \left(-990 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1449 + 930 H^2 k^2 + 149 H^4 k^4) U \right) - \\
& \quad 2 \left(360 \sqrt{g H (3 + H^2 k^2)} U^3 + 40 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. k^2 \left(137 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 240 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^3 + \\
& \left(\left(k^5 \left(-3 \sqrt{3} g H (543 + 146 H^2 k^2 + 15 H^4 k^4) + 256 \left(9 \sqrt{g H (3 + H^2 k^2)} + \right. \right. \right. \right. \\
& \quad \left. \left. 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \right. \\
& \quad \left(30720 \sqrt{g H (3 + H^2 k^2)}^{5/2} \right) + \frac{1}{92160 \sqrt{g H (3 + H^2 k^2)}^{7/2}} \\
& \quad i k^6 \left(-3 g H \left(-18432 \sqrt{g H (3 + H^2 k^2)} + 51039 \sqrt{3} U + 45735 \sqrt{3} H^2 k^2 U + \right. \right. \\
& \quad \left. \left. 1381 \sqrt{3} H^6 k^6 U + k^4 \left(-960 \sqrt{g H^9 (3 + H^2 k^2)} + 13717 \sqrt{3} H^4 U \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned} & \left(128 \left(837 \sqrt{g H (3 + H^2 k^2)} U^2 + 279 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 31 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^2 + \right. \right. \\ & \quad \left. \left. 27 k^2 \left(7 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 31 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt + \\ & \frac{1}{92\,160 \sqrt{g H} (3 + H^2 k^2)^{7/2}} k^7 \left(9 \sqrt{3} g^2 H^2 (12\,999 + 6258 H^2 k^2 + 775 H^4 k^4) + g H \right. \\ & \quad U \left(617\,661 \sqrt{3} H^2 k^2 U + 20\,551 \sqrt{3} H^6 k^6 U + 81 \left(-5888 \sqrt{g H (3 + H^2 k^2)} + 8053 \sqrt{3} U \right) - \right. \\ & \quad \left. 3 k^4 \left(12\,288 \sqrt{g H^9 (3 + H^2 k^2)} - 65\,021 \sqrt{3} H^4 U \right) \right) - \\ & \quad 128 \left(2322 \sqrt{g H (3 + H^2 k^2)} U^3 + 774 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 86 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} \right. \\ & \quad \left. U^3 + 9 k^2 \left(229 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 258 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) dt^2 - \\ & \frac{1}{30\,720 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^8 \left(k^8 U^3 \left(-21\,253 \sqrt{3} g H^9 + 7776 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\ & \quad 243 \left(832 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 4423 \sqrt{3} g^2 H^2 U + 2592 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\ & \quad g H U^2 \left(8832 \sqrt{g H (3 + H^2 k^2)} - 7823 \sqrt{3} U \right) \Big) - 3 k^6 U \left(9181 \sqrt{3} g^2 H^8 - \right. \\ & \quad \left. 31\,104 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + 4 g H^7 U \left(-5312 \sqrt{g H (3 + H^2 k^2)} + 21\,787 \sqrt{3} U \right) \right) \Big) + \\ & \quad 27 k^2 \left(-35\,319 \sqrt{3} g^2 H^4 U - 91\,580 \sqrt{3} g H^3 U^3 + 64 \right. \\ & \quad \left(59 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 1155 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 486 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) \Big) + \\ & \quad 9 k^4 \left(-31\,231 \sqrt{3} g^2 H^6 U + 2 g H^5 U^2 \left(34\,336 \sqrt{g H (3 + H^2 k^2)} - 67\,003 \sqrt{3} U \right) + \right. \\ & \quad \left. 288 \left(5 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 162 \sqrt{g H^9 (3 + H^2 k^2)} U^4 \right) \right) \Big) dt^3 - \\ & \frac{1}{92\,160 (\sqrt{g H} (3 + H^2 k^2))^{11/2}} \left(k^9 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right) \right. \\ & \quad \left(k^8 U^3 \left(-111\,559 \sqrt{3} g H^9 + 42\,368 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\ & \quad 27 k^2 \left(18\,078 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 172\,047 \sqrt{3} g^2 H^4 U + 372\,075 \right. \\ & \quad \left. \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - 477\,940 \sqrt{3} g H^3 U^3 + 169\,472 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) + \\ & \quad 9 k^4 \left(7035 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 153\,703 \sqrt{3} g^2 H^6 U + 254\,208 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + \right. \\ & \quad \left. g H^5 U^2 \left(347\,651 \sqrt{g H (3 + H^2 k^2)} - 700\,818 \sqrt{3} U \right) \right) \Big) + \\ & \quad 81 \left(11\,603 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 63\,917 \sqrt{3} g^2 H^2 U + 42\,368 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\ & \quad \left. g H U^2 \left(132\,513 \sqrt{g H (3 + H^2 k^2)} - 122\,207 \sqrt{3} U \right) \right) - \\ & \quad 3 k^6 U \left(45\,573 \sqrt{3} g^2 H^8 - 169\,472 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + g H^7 U \right. \\ & \quad \left. \left. \left. \left. \left. (-108\,089 \sqrt{g H (3 + H^2 k^2)} + 456\,644 \sqrt{3} U) \right) \right) \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5 \Big\} \end{aligned}$$