

```

In[267]:= q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx/2, xj + dx/2}] / (dx);
qjnbar = qjbar /. t -> tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x -> xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / (3 * k^2);
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca;
fn1Gca0A = fn1A /. Gca -> 0;
fn1eca0A = fn1A /. eca -> 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca /. Gh -> RA * Gca;
fG1Gca0A = fG1A /. Gca -> 0;
fG1eca0A = fG1A /. eca -> 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
EA = {{1, 0}, {0, 1}} + MatA;

```

$$wAp = U * k + \frac{\sqrt{3} \, k \sqrt{g \, H \, (3 + H^2 \, k^2)}}{3 + H^2 \, k^2};$$

$$wAm = U * k - \frac{\sqrt{3} \, k \sqrt{g \, H \, (3 + H^2 \, k^2)}}{3 + H^2 \, k^2};$$

```

In[303]:= M = 1;
Merr = Series[M - MA, {dx, 0, 5}];
Rm = (1 + I * Sin[k * dx] / 2);
Rmerr = Series[Rm - RA, {dx, 0, 5}];
Rp = Exp[I * k * dx] * (1 - I * Sin[k * dx] / 2);
Rperr = Series[Rp - RA, {dx, 0, 5}];
Ru = (1 + Exp[I * k * dx]) / 2;
Ruerr = Series[Ru - Exp[I * k * dx / 2], {dx, 0, 5}];
Gold = H - H^3 / 3 * (2 * Cos[k * dx] - 2) / dx^2;
GG2 = Ru / Gold;
GG2err = Series[GG2 - GGA, {dx, 0, 5}];
Gn2 = -U * Ru / Gold;
Gn2err = Series[Gn2 - GnA, {dx, 0, 5}];

Text[Row[{"M  ||  ", M}]]
Text[Row[{"M  ||  ", TeXForm[M]}]]
Text[Row[{"M error  ||  ", TeXForm[Merr]}]]
Text[Row[{"M error  ||  ", Merr}]]
Text[" "]
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Text[Row[{"Gn2  ||  ", Gn2}]]
Text[Row[{"Gn2  ||  ", TeXForm[Gn2]}]]
Text[Row[{"Gn2 error  ||  ", Gn2err}]]
Text[Row[{"Gn2 error  ||  ", TeXForm[Gn2err]}]]

```

Out[316]= M || 1

Out[317]= M || 1

Out[318]= M error || $-\frac{\text{dx}^2 k^2}{24} - \frac{7 \text{dx}^4 k^4}{5760} + O(\text{dx}^6)$

$$\text{Out[319]= M error} \parallel -\frac{k^2 dx^2}{24} - \frac{7 k^4 dx^4}{5760} + O[dx]^6$$

Out[320]=

$$\text{Out[321]= Rm} \parallel 1 + \frac{1}{2} i \sin[dx k]$$

$$\text{Out[322]= Rm} \parallel 1 + \frac{1}{2} i \sin(\text{dx} k)$$

$$\text{Out[323]= Rm error} \parallel \frac{k^2 dx^2}{12} - \frac{1}{12} i k^3 dx^3 + \frac{k^4 dx^4}{720} + \frac{1}{240} i k^5 dx^5 + O[dx]^6$$

$$\text{Out[324]= Rm error} \parallel \frac{\text{dx}^2 k^2}{12} - \frac{1}{12} i \text{dx}^3 k^3 + \frac{\text{dx}^4 k^4}{720} + \frac{1}{240} i \text{dx}^5 k^5 + O(\text{dx}^6)$$

Out[325]=

$$\text{Out[326]= Rp} \parallel e^{i dx k} \left(1 - \frac{1}{2} i \sin[dx k]\right)$$

$$\text{Out[327]= Rp} \parallel e^{i \text{dx} k} \left(1 - \frac{1}{2} i \sin(\text{dx} k)\right)$$

$$\text{Out[328]= Rp error} \parallel \frac{k^2 dx^2}{12} + \frac{1}{6} i k^3 dx^3 - \frac{89 k^4 dx^4}{720} - \frac{7}{120} i k^5 dx^5 + O[dx]^6$$

$$\text{Out[329]= Rp error} \parallel \frac{\text{dx}^2 k^2}{12} + \frac{1}{6} i \text{dx}^3 k^3 - \frac{89 \text{dx}^4 k^4}{720} - \frac{7}{120} i \text{dx}^5 k^5 + O(\text{dx}^6)$$

Out[330]=

$$\text{Out[331]= GG2} \parallel \frac{1 + e^{i dx k}}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2} \right)}$$

$$\text{Out[332]= GG2} \parallel \frac{1 + e^{i \text{dx} k}}{2 \left(H - \frac{H^3 (2 \cos(\text{dx} k) - 2)}{3 \text{dx}^2} \right)}$$

$$\text{Out[333]= GG2 error} \parallel \frac{(-6 k^2 - H^2 k^4) dx^2}{4 H (3 + H^2 k^2)^2} - \frac{i (6 k^3 + H^2 k^5) dx^3}{8 H (3 + H^2 k^2)^2} + \frac{(144 k^4 + 45 H^2 k^6 + 4 H^4 k^8) dx^4}{240 H (3 + H^2 k^2)^3} - \frac{i (-54 k^5 + H^4 k^9) dx^5}{480 H (3 + H^2 k^2)^3} + O[dx]^6$$

$$\text{Out[334]= GG2 error} \parallel \frac{\text{dx}^2 \left(-H^2 k^4 - 6 k^2 \right)}{4 H \left(H^2 k^2 + 3 \right)^2} - \frac{i \text{dx}^3 \left(H^2 k^5 + 6 k^3 \right)}{8 H \left(H^2 k^2 + 3 \right)^2} + \frac{\text{dx}^4 \left(4 H^4 k^8 + 45 H^2 k^6 + 144 k^4 \right)}{240 H \left(H^2 k^2 + 3 \right)^3} - \frac{i \text{dx}^5 \left(H^4 k^9 - 54 k^5 \right)}{480 H \left(H^2 k^2 + 3 \right)^3} + O(\text{dx}^6)$$

Out[335]=

$$\text{Out[336]= Gn2} \parallel -\frac{(1 + e^{i dx k}) U}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2} \right)}$$

$$\text{Out[337]= Gn2} \parallel -\frac{U \left(1 + e^{i \text{dx} k} \right)}{2 \left(H - \frac{H^3 (2 \cos(\text{dx} k) - 2)}{3 \text{dx}^2} \right)}$$

$$\text{Out[338]= Gn2 error} \parallel \frac{(6 k^2 + H^2 k^4) U dx^2}{4 H (3 + H^2 k^2)^2} + \frac{i (6 k^3 + H^2 k^5) U dx^3}{8 H (3 + H^2 k^2)^2} - \frac{((144 k^4 + 45 H^2 k^6 + 4 H^4 k^8) U) dx^4}{240 (H (3 + H^2 k^2)^3)} + \frac{i (-54 k^5 + H^4 k^9) U dx^5}{480 H (3 + H^2 k^2)^3} + O[dx]^6$$

Out[339]= Gn2 error ||

$$\frac{\text{dx}^2 U \left(H^2 k^4 + 6 k^2 \right)}{4 H \left(H^2 k^2 + 3 \right)^2} + \frac{i \text{dx}^3 U \left(H^2 k^5 + 6 k^3 \right)}{8 H \left(H^2 k^2 + 3 \right)^2} - \frac{\text{dx}^4 U \left(4 H^4 k^8 + 45 H^2 k^6 + 144 k^4 \right)}{240 H \left(H^2 k^2 + 3 \right)^3} + \frac{i \text{dx}^5 U \left(H^4 k^9 - 54 k^5 \right)}{480 H \left(H^2 k^2 + 3 \right)^3} + O(\text{dx}^6)$$

```
In[340]:= KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
KurFWS = KurF /. ap -> (U + Sqrt[g*H]) /. am -> (U - Sqrt[g*H]);
KurFWSeta =
  KurFWS /. fp -> (H*v + U*Rpp*n) /. fm -> (H*v + U*Rmp*n) /. qp -> Rpp*n /.
  qm -> Rmp*n;
KurFWSeta = KurFWSeta /. v -> (GGp*G + Gnp*n);
Kfnnp = FullSimplify[KurFWSeta /. G -> 0 /. n -> 1];
KfnGp = FullSimplify[KurFWSeta /. n -> 0 /. G -> 1];
Kfnn = Kfnnp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfnG = KfnGp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
Fnn2 = -dt*(1 - Exp[-I*k*dx])/dx*Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp -> (U*Rpp*G + U*H*v + g*H*Rpp*n) /.
  fm -> (U*Rmp*G + U*H*v + g*H*Rmp*n) /. qp -> Rpp*G /. qm -> Rmp*G;
KurFWSG = KurFWSG /. v -> (GGp*G + Gnp*n);
KfGnp = FullSimplify[KurFWSG /. G -> 0 /. n -> 1];
KfGGp = FullSimplify[KurFWSG /. n -> 0 /. G -> 1];
KfGn = KfGnp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfGG = KfGGp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;

FGn2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U*H*GG2 + U/2*(Rm + Rp) - (Sqrt[g*H])/2*(Rp - Rm);
FGG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2/2;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
```

```

RKStep = Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}];

```

```

Text[Row[{" -Sqrt(gH) < U < Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
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Text[" "]
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Text[Row[{"EA || ", TeXForm[EA]}]]
Text[Row[{"Eerr || ", Eerr}]]
Text[Row[{"Eerr || ", TeXForm[Eerr]}]]

```

Out[374]= $-\text{Sqrt}(gH) < U < \text{Sqrt}(gH)$

Out[375]=

$$\text{Out[376]= Fnn} \parallel \frac{1}{2} \left(2 \text{Gnp} H + \text{Rpp} \left(-\sqrt{g H} + U \right) + \text{Rmp} \left(\sqrt{g H} + U \right) \right)$$

$$\text{Out[377]= Fnn} \parallel \frac{1}{2} \left(\left(\text{Rmp} \left(\sqrt{g H} + U \right) + \text{Rpp} \left(-\sqrt{g H} + U \right) \right) + 2 \text{Gnp} H \right)$$

$$\text{Out[378]= Fnn error} \parallel \left(-\frac{(H^2 k^3 U w) dt^2}{2(3+H^2 k^2)} - \frac{i H^2 k^3 U w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left(-\frac{i(27 k^3 + 9 H^2 k^5 + H^4 k^7) U dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(-\frac{1}{8} \left(\sqrt{g H} k^4 \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4$$

$$\text{Out[379]= Fnn error} \parallel \left(-\frac{dt^2}{2} \left(H^2 k^3 U w \right) \right) \left(H^2 k^2 + 3 \right) - \frac{i dt^3}{6} H^2 k^3 U w^2 \left(H^2 k^2 + 3 \right) + O(dt^4) + dx^2 \left(-\frac{i}{12} \left(H^4 k^7 + 9 H^2 k^5 + 27 k^3 \right) U dt \right) \left(H^2 k^2 + 3 \right)^2 + O(dt^4) + dx^3 \left(-\frac{1}{8} \sqrt{g H} k^4 \right) dt + O(dt^4) + O(dx^4)$$

Out[380]=

$$\text{Out[381]= FnG} \parallel \text{GGp} H$$

$$\text{Out[382]= FnG} \parallel \text{GGp} H$$

$$\text{Out[383]= FnG error} \parallel \left(-\frac{3(k w) dt^2}{2(3+H^2 k^2)} - \frac{i k w^2 dt^3}{2(3+H^2 k^2)} + O[dt]^4 \right) + \left(\frac{i(6 k^3 + H^2 k^5) dt}{4(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + O[dx]^4$$

$$\text{Out[384]= FnG error} \parallel \left(-\frac{3}{2} dt^2 (k w) \right) \left(H^2 k^2 + 3 \right) - \frac{i}{6} dt^3 k w^2 \left(H^2 k^2 + 3 \right) + O(dt^4) + dx^2 \left(\frac{i}{4} \left(H^2 k^5 + 6 k^3 \right) dt \right) \left(H^2 k^2 + 3 \right)^2 + O(dt^4) + O(dx^4)$$

Out[385]=

$$\text{Out[386]= FGn} \parallel \frac{1}{2} \left(g H (\text{Rmp} + \text{Rpp}) + \left(2 \text{Gnp} H + \sqrt{g H} (\text{Rmp} - \text{Rpp}) \right) U \right)$$

$$\text{Out[387]= FGn} \parallel \frac{1}{2} \left(U \left(\sqrt{g H} (\text{Rmp} - \text{Rpp}) + 2 \text{Gnp} H \right) + g H (\text{Rmp} + \text{Rpp}) \right)$$

$$\text{Out[388]= FGn error} \parallel \left(-\frac{(k(3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2(3+H^2 k^2)} - \frac{i k(3 g H + g H^3 k^2 - 3 U^2) w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left(-\frac{i(9 g H k^3 + 6 g H^3 k^5 + g H^5 k^7 + 18 k^3 U^2 + 3 H^2 k^5 U^2) dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(-\frac{1}{8} \left(\sqrt{g H} k^4 U \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4$$

$$\text{Out[389]= FGn error} \parallel \left(-\frac{dt^2}{2} (k w \left(g H^3 k^2 + 3 g H - 3 U^2 \right)) \right) \left(H^2 k^2 + 3 \right) - \frac{i}{6} dt^3 k w^2 \left(g H^3 k^2 + 3 g H - 3 U^2 \right) \left(H^2 k^2 + 3 \right) + O(dt^4) + dx^2 \left(-\frac{i}{12} \left(g H^4 k^7 + 6 g H^3 k^5 + 3 H^2 U^2 k^5 + 18 U^2 k^3 + 9 g H k^3 \right) dt \right) \left(H^2 k^2 + 3 \right)^2 + O(dt^4) + dx^3 \left(-\frac{1}{8} \sqrt{g H} k^4 U \right) dt + O(dt^4) + O(dx^4)$$

Out[390]=

$$\begin{aligned}
& \left. \left(9 U + H^4 k^4 U + 2 k^2 \left(2 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 3 H^2 U \right) \right) dt^4 + O[dt]^5 \right) dx^2 + \\
& \left(-\frac{i k^4 \left(2 g H (3 + H^2 k^2) + \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U \right)}{16 \sqrt{g H} (3 + H^2 k^2)} - \frac{1}{32 (3 + H^2 k^2)^{3/2}} i k^6 \left(g H \left(6 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (15 + 4 H^2 k^2) U \right) + \right. \right. \\
& U^2 \left(12 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U + k^2 \left(2 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \right) dt^2 + \\
& \frac{1}{32 (3 + H^2 k^2)^{5/2}} k^7 \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \\
& \left(2 \sqrt{3} g H (3 + H^2 k^2) + U \left(9 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U + k^2 \left(2 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \right) \right) \\
& dt^3 + \left(i k^8 \left(2 g H (3 + H^2 k^2) + \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U \right) \right. \\
& \left. \left(9 g^2 H^2 + 6 g H U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \right. \\
& \left. U^3 \left(12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + 2 k^2 \left(2 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 3 H^2 U \right) \right) \right) \\
& \left. dt^4 \right) / \left(64 \sqrt{g H} (3 + H^2 k^2)^3 \right) + O[dt]^5 \Big) dx^3 + \\
& \left(-\left(\left(k^5 \left(3 \sqrt{3} g H (177 + 124 H^2 k^2 + 20 H^4 k^4) + 104 \left(9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + \right. \right. \right. \right. \right. \\
& \left. \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \left(1920 \left(\sqrt{g H} (3 + H^2 k^2)^{5/2} \right) \right) \right) - \\
& \left(\left(k^7 \left(27 \sqrt{3} g^2 H^2 (167 + 124 H^2 k^2 + 20 H^4 k^4) + g H U \left(21429 \sqrt{3} H^2 k^2 U + \right. \right. \right. \right. \\
& 764 \sqrt{3} H^6 k^6 U + 81 \left(232 \sqrt{g H (3 + H^2 k^2)} + 267 \sqrt{3} U \right) + \\
& 24 k^4 \left(84 \sqrt{g H^9 (3 + H^2 k^2)} + 293 \sqrt{3} H^4 U \right) \right) + \\
& 16 \left(459 \sqrt{g H (3 + H^2 k^2)} U^3 + 153 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 17 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} \right. \\
& \left. U^3 + 9 k^2 \left(88 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 51 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) dt^2 \Big) / \\
& \left(11520 \left(\sqrt{g H} (3 + H^2 k^2)^{7/2} \right) \right) - \frac{1}{3840 (3 + H^2 k^2)^4} i k^8 \left(54 g^2 H^2 (81 + 62 H^2 k^2 + 10 H^4 k^4) + \right. \\
& 84 H^8 k^8 U^4 + 243 U^3 \left(39 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 28 U \right) + 36 k^4 U^3 \\
& \left(85 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 126 H^4 U \right) + 4 k^6 U^3 \left(83 \sqrt{3} \sqrt{g H^{13} (3 + H^2 k^2)} + 252 H^6 U \right) + \\
& 9 k^2 \left(600 \sqrt{3} \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 1039 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} U^3 + 1008 H^2 U^4 \right) + \\
& 3 g H U \left(13500 H^2 k^2 U + 472 H^6 k^6 U + 27 \left(97 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 504 U \right) + \right. \\
& \left. \left. 4 k^4 \left(71 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 1101 H^4 U \right) \right) \right) dt^3 +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{23040 \sqrt{g H (3+H^2 k^2)^{9/2}}} k^9 \left(81 \sqrt{3} g^3 H^3 (157 + 124 H^2 k^2 + 20 H^4 k^4) + \right. \\
& 6 \sqrt{3} g^2 H^2 (32535 + 32481 H^2 k^2 + 10584 H^4 k^4 + 1124 H^6 k^6) U^2 + \\
& 3 g H U^3 \left(69120 \sqrt{g H (3+H^2 k^2)} + 34155 \sqrt{3} U + 44982 \sqrt{3} H^2 k^2 U + \right. \\
& 396 \sqrt{3} H^8 k^8 U + 27 k^4 \left(816 \sqrt{g H^9 (3+H^2 k^2)} + 821 \sqrt{3} H^4 U \right) + \\
& 28 k^6 \left(84 \sqrt{g H^{13} (3+H^2 k^2)} + 173 \sqrt{3} H^6 U \right) \left. \right) + 8 U \left(348 k^6 \sqrt{g H^{13} (3+H^2 k^2)} U^4 + \right. \\
& 29 k^8 \sqrt{g H^{17} (3+H^2 k^2)} U^4 + 81 \left(130 \sqrt{g^5 H^5 (3+H^2 k^2)} + 29 \sqrt{g H (3+H^2 k^2)} U^4 \right) + \\
& 54 k^2 \left(137 \sqrt{g^5 H^9 (3+H^2 k^2)} + 472 \sqrt{g^3 H^7 (3+H^2 k^2)} U^2 + 58 \sqrt{g H^5 (3+H^2 k^2)} U^4 \right) + \\
& \left. \left. 27 k^4 \left(43 \sqrt{g^5 H^{13} (3+H^2 k^2)} + 58 \sqrt{g H^9 (3+H^2 k^2)} U^4 \right) \right) \right) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5, \\
& \left(\frac{1}{6(3+H^2 k^2)^2} k^3 \left(-\sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \right. \\
& dt^2 + \\
& \frac{i k^4 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right)^2}{8(3+H^2 k^2)^2} dt^3 - \\
& \frac{1}{20(3+H^2 k^2)^4} \\
& \left(k^5 \left(-\sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \right) \\
& dt^4 + O[dt]^5 \Big) + \\
& \left(\frac{k^3 \left(3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 2(3+H^2 k^2)^2 U \right)}{24(3+H^2 k^2)^2} + \frac{1}{48(3+H^2 k^2)^3} k^5 \left(3 g \left(3 \sqrt{3} H \sqrt{g H (3+H^2 k^2)} + 6 H^3 k^2 U + 2 H^5 k^4 U \right) + \right. \\
& U^2 \left(-27 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 54 U + 2 H^6 k^6 U - 3 k^2 \left(7 \sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} - 18 H^2 U \right) - \right. \\
& \left. \left. 2 k^4 \left(2 \sqrt{3} \sqrt{g H^9 (3+H^2 k^2)} - 9 H^4 U \right) \right) \right) dt^2 + \\
& \frac{1}{48(3+H^2 k^2)^3} i k^6 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \\
& \left(-9 g H + U \left(-3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 18 U + 2 H^4 k^4 U - 2 k^2 \left(\sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} - 6 H^2 U \right) \right) \right) \\
& dt^3 - \frac{1}{96(3+H^2 k^2)^4} \\
& \left(k^7 \left(3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 2(3+H^2 k^2)^2 U \right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \right)^2 \Big)
\end{aligned}$$

$$\begin{aligned}
& \left. dt^4 + O[dt]^5 \right) dx^2 + \\
& \left(\frac{1}{16} i \sqrt{g H} k^4 \left(-2 + \frac{\sqrt{3} U}{\sqrt{g H (3 + H^2 k^2)}} \right) + \frac{1}{32 (3 + H^2 k^2)^{3/2}} \right. \\
& i k^6 \left(g \left(-6 H \sqrt{g H (3 + H^2 k^2)} + 15 \sqrt{3} H U + 4 \sqrt{3} H^3 k^2 U \right) + \right. \\
& U^2 \left(-12 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U + k^2 \left(-2 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \right) \left. \right) dt^2 - \\
& \frac{1}{32 (3 + H^2 k^2)^{5/2}} \left(k^7 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \left(2 \sqrt{3} g H (3 + H^2 k^2) + \right. \right. \\
& U \left(-9 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U + k^2 \left(-2 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \right) \left. \right) \left. \right) \\
& dt^3 + \left(i k^8 \left(2 g H (3 + H^2 k^2) - \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U \right) \right. \\
& \left. \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right)^2 dt^4 \right) / \left(64 \sqrt{g H} (3 + H^2 k^2)^3 \right) + O[dt]^5 \Bigg) \\
& dx^3 + \left(\left(k^5 \left(3 \sqrt{3} g H (177 + 124 H^2 k^2 + 20 H^4 k^4) - 104 \left(9 \sqrt{g H (3 + H^2 k^2)} + \right. \right. \right. \right. \\
& \left. \left. \left. 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \right. \\
& \left(1920 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \left(k^7 \left(27 \sqrt{3} g^2 H^2 (167 + 124 H^2 k^2 + 20 H^4 k^4) + \right. \right. \\
& g H U \left(21429 \sqrt{3} H^2 k^2 U + 764 \sqrt{3} H^6 k^6 U + 81 \left(-232 \sqrt{g H (3 + H^2 k^2)} + 267 \sqrt{3} U \right) - \right. \\
& 24 k^4 \left(84 \sqrt{g H^9 (3 + H^2 k^2)} - 293 \sqrt{3} H^4 U \right) \left. \right) - \\
& 16 \left(459 \sqrt{g H (3 + H^2 k^2)} U^3 + 153 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 17 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + \right. \\
& \left. 9 k^2 \left(88 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 51 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \left. \right) dt^2 \Bigg) / \\
& \left(11520 \sqrt{g H} (3 + H^2 k^2)^{7/2} \right) - \frac{1}{3840 (3 + H^2 k^2)^4} i k^8 \left(54 g^2 H^2 (81 + 62 H^2 k^2 + 10 H^4 k^4) + 84 H^8 k^8 U^4 + \right. \\
& 243 U^3 \left(-39 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 28 U \right) + 36 k^4 U^3 \left(-85 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 126 H^4 U \right) + \\
& 4 k^6 U^3 \left(-83 \sqrt{3} \sqrt{g H^{13} (3 + H^2 k^2)} + 252 H^6 U \right) - \\
& 9 k^2 \left(600 \sqrt{3} \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 1039 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} U^3 - 1008 H^2 U^4 \right) + \\
& 3 g H U \left(13500 H^2 k^2 U + 472 H^6 k^6 U + 27 \left(-97 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 504 U \right) - \right. \\
& \left. 4 k^4 \left(71 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} - 1101 H^4 U \right) \right) \left. \right) dt^3 - \frac{1}{23040 \left(\sqrt{g H} (3 + H^2 k^2)^{1/2} \right)} \\
& \left(k^9 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right) \left(4 k^8 U^3 \left(-239 \sqrt{3} g H^9 + 58 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \right. \\
& \left. \left. 2512 \left(279 \sqrt{g H^9 (3 + H^2 k^2)} - 2793 \sqrt{3} \sqrt{g H^{13} (3 + H^2 k^2)} + 1515 \sqrt{g H^{17} (3 + H^2 k^2)} \right) U^3 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{2}{k^2} \left(\frac{3}{2} \sqrt{g^2 H^2 (3 + H^2 k^2)} - \frac{2}{03} \sqrt{3} g^2 H^2 U + 4515 \sqrt{g^2 H^2 (3 + H^2 k^2)} U^2 - \right. \\
& \quad \left. 4070 \sqrt{3} g H^3 U^3 + 928 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) + \\
& 9 k^4 \left(180 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 2672 \sqrt{3} g^2 H^6 U + 1392 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + g H^5 \right. \\
& \quad \left. U^2 \left(4384 \sqrt{g H (3 + H^2 k^2)} - 5997 \sqrt{3} U \right) \right) + 81 \left(157 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 883 \sqrt{3} \right. \\
& \quad \left. g^2 H^2 U + 232 \sqrt{g H (3 + H^2 k^2)} U^4 + g H U^2 \left(1527 \sqrt{g H (3 + H^2 k^2)} - 1033 \sqrt{3} U \right) \right) - \\
& 12 k^6 U \left(213 \sqrt{3} g^2 H^8 - 232 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. g H^7 U \left(-349 \sqrt{g H (3 + H^2 k^2)} + 979 \sqrt{3} U \right) \right) \Bigg) dt^4 + O[dt]^5 \Bigg\} dx^4 + O[dx]^5 \Bigg\}
\end{aligned}$$

Out[398]= Omega error ||

$$\begin{aligned}
& \left(\left(\left(\frac{k^3}{\left(\left(H^2 k^2 + 3 \right) U + \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right)} \right) \left(3 g H + U \right) \right. \right. \\
& \quad \left. \left(\left(H^2 k^2 + 3 \right) U + 2 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) \text{dt}^2 \Bigg\} \left(\left(H^2 k^2 + 3 \right) U^2 + \frac{i k^4}{\left(\left(H^2 k^2 + 3 \right) U + \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right)} \right. \right. \\
& \quad \left. \left(\left(H^4 U k^4 + 3 \left(2 U H^2 + \sqrt{3} \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) \right) k^2 + 9 \left(U + \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) \right. \right. \\
& \quad \left. \left(H^2 k^2 + 3 \right) U^2 + 3 g \left(3 k^2 U H^3 + 9 U H + \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) H \right) \text{dt}^3 \Bigg\} \left(\left(H^2 k^2 + 3 \right) U^3 - \frac{\left(k^5 \left(\left(H^2 k^2 + 3 \right) U + \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right)}{\left(\left(H^4 U k^4 + 2 \left(3 U H^2 + 2 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) \right) k^2 + 9 U + 12 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) U^3 + 6 g H \left(3 \left(H^2 k^2 + 3 \right) U + 2 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) U + 9 g^2 H^2 \right) \right) \text{dt}^4 \Bigg\} \left(20 \left(H^2 k^2 + 3 \right) U^3 + O \left(\text{dt}^5 \right) \right) + \left(\frac{k^3}{\left(\left(2 H^2 U k^6 + 2 \left(9 U H^4 + 2 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2 + 3 \right)} \right) \right) k^4 + 3 \left(18 U H^2 + 7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) k^2 + 54 U + 27 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) U^2 + g \left(6 k^4 U H^5 + 18 k^2 U H^3 - 9 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} H \right) \right) \text{dt}^2 \Bigg\} \left(48 \left(H^2 k^2 + 3 \right) U^3 + \frac{i k^6}{\left(3 g H + U \left(\left(H^2 k^2 + 3 \right) U + 2 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right)} \left(U \left(2 H^4 U k^4 + 2 \left(6 U H^2 + \sqrt{3} \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) \right) k^2 + 18 U + 3 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) - 9 g H \right) \text{dt}^3 \Bigg\} \left(48 \left(H^2 k^2 + 3 \right) U^3 - \frac{\left(k^7 \left(2 \left(H^2 k^2 + 3 \right) U - 3 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right)}{\left(\left(H^4 U k^4 + 2 \left(3 U H^2 + 2 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) \right) k^2 + 9 U + 12 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) U^3 + 6 g H \left(3 \left(H^2 k^2 + 3 \right) U + 2 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) U + 9 g^2 H^2 \right) \text{dt}^4 \Bigg\} \left(96 \left(H^2 k^2 + 3 \right) U^4 + O \left(\text{dt}^5 \right) \right) \text{dx}^2 + \left(- \frac{i k^4}{\left(2 g H \left(H^2 k^2 + 3 \right) + \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) U} \right) \left(16 \sqrt{g H \left(H^2 k^2 + 3 \right)} - \frac{i k^6}{\left(\left(\sqrt{3} U H^2 + 2 \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) k^2 + 3 \sqrt{3} U + 12 \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) U^2 + g H \left(\sqrt{3} \left(4 H^2 k^2 + 15 \right) U + 6 \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) \text{dt}^2 \Bigg\} \left(32 \left(H^2 k^2 + 3 \right) U^3 + \frac{k^7}{\left(3 g H + U \left(\left(H^2 k^2 + 3 \right) U + 2 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right)} \left(2 \sqrt{3} g H \left(H^2 k^2 + 3 \right) + U \left(\sqrt{3} U H^2 + 2 \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) k^2 + 3 \sqrt{3} U + 9 \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) \text{dt}^3 \Bigg\} \left(32 \left(H^2 k^2 + 3 \right) U^{5/2} + \frac{i k^8}{\left(2 g H \left(H^2 k^2 + 3 \right) + \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right)} \right) \text{dt}^4 + O \left(\text{dt}^5 \right) \Bigg\}
\end{aligned}$$

$$\begin{aligned}
& H \left(H^2 k^2 + 3 \right) U \right) \left(\left(H^4 U k^4 + 2 \left(3 U H^2 + 2 \sqrt{3} \right) \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) \right) k^2 + 9 U + 12 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} U^3 + 6 g H \left(3 \left(H^2 k^2 + 3 \right) U + 2 \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} U + 9 g^2 H^2 \right) \text{dt}^4 \{ 64 \sqrt{g H} \left(H^2 k^2 + 3 \right)^3 + O \left(\text{dt}^5 \right) \right) \text{dx}^3 + \left(- \frac{k^5 \left(3 \sqrt{3} g H \left(20 H^4 k^4 + 124 H^2 k^2 + 177 \right) + 104 \left(\sqrt{g H^9 \left(H^2 k^2 + 3 \right)} \right) k^4 + 6 \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} k^2 + 9 \sqrt{g H \left(H^2 k^2 + 3 \right)} U \right) \right) \{ 1920 \left(\sqrt{g H} \left(H^2 k^2 + 3 \right) \right)^{5/2} \right) - \frac{\left(k^7 \left(27 \sqrt{3} g^2 \left(20 H^4 k^4 + 124 H^2 k^2 + 167 \right) H^2 + g U \left(764 \sqrt{3} H^6 U k^6 + 24 \left(293 \sqrt{3} U H^4 + 84 \sqrt{g H^9 \left(H^2 k^2 + 3 \right)} \right) k^4 + 21429 \sqrt{3} H^2 U k^2 + 81 \left(267 \sqrt{3} U + 232 \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) H + 16 \left(17 \sqrt{g H^{13} \left(H^2 k^2 + 3 \right)} \right) U^3 k^6 + 153 \sqrt{g H^9 \left(H^2 k^2 + 3 \right)} U^3 k^4 + 9 \left(51 \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) U^3 + 88 \sqrt{g^3 H^7 \left(H^2 k^2 + 3 \right)} U \right) k^2 + 459 \sqrt{g H \left(H^2 k^2 + 3 \right)} U^3 \right) \right) \text{dt}^2 \{ 11520 \left(\sqrt{g H} \left(H^2 k^2 + 3 \right) \right)^{7/2} \right) - \frac{i k^8 \left(84 H^8 U^4 k^8 + 4 U^3 \left(252 U H^6 + 83 \sqrt{3} \right) \sqrt{g H^{13} \left(H^2 k^2 + 3 \right)} \right) k^6 + 36 U^3 \left(126 U H^4 + 85 \sqrt{3} \right) \sqrt{g H^9 \left(H^2 k^2 + 3 \right)} \right) k^4 + 9 \left(1008 H^2 U^4 + 1039 \sqrt{3} \right) \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} U^3 + 600 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2 + 3 \right)} U \right) k^2 + 54 g^2 H^2 \left(10 H^4 k^4 + 62 H^2 k^2 + 81 \right) + 243 U^3 \left(28 U + 39 \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) + 3 g H U \left(472 H^6 U k^6 + 4 \left(1101 U H^4 + 71 \sqrt{3} \right) \sqrt{g H^9 \left(H^2 k^2 + 3 \right)} \right) k^4 + 13500 H^2 U k^2 + 27 \left(504 U + 97 \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) \text{dt}^3 \{ 3840 \left(H^2 k^2 + 3 \right)^4 + \frac{k^9 \left(81 \sqrt{3} g^3 \left(20 H^4 k^4 + 124 H^2 k^2 + 157 \right) H^3 + 6 \sqrt{3} g^2 \left(1124 H^6 k^6 + 10584 H^4 k^4 + 32481 H^2 k^2 + 32535 \right) U^2 H^2 + 3 g U^3 \left(396 \sqrt{3} H^8 U k^8 + 28 \left(173 \sqrt{3} U H^6 + 84 \sqrt{g H^{13} \left(H^2 k^2 + 3 \right)} \right) k^6 + 27 \left(821 \sqrt{3} U H^4 + 816 \sqrt{g H^9 \left(H^2 k^2 + 3 \right)} \right) k^4 + 44982 \sqrt{3} H^2 U k^2 + 34155 \sqrt{3} U + 69120 \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) H + 8 U \left(29 \sqrt{g H^{17} \left(H^2 k^2 + 3 \right)} U^4 k^8 + 348 \sqrt{g H^{13} \left(H^2 k^2 + 3 \right)} U^4 k^6 + 27 \left(58 \sqrt{g H^9 \left(H^2 k^2 + 3 \right)} \right) U^4 + 43 \sqrt{g^5 H^{13} \left(H^2 k^2 + 3 \right)} \right) k^4 + 54 \left(58 \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} \right) U^4 + 472 \sqrt{g^3 H^7 \left(H^2 k^2 + 3 \right)} U^2 + 137 \sqrt{g^5 H^9 \left(H^2 k^2 + 3 \right)} \right) k^2 + 81 \left(29 \sqrt{g H \left(H^2 k^2 + 3 \right)} U^4 + 130 \sqrt{g^5 H^5 \left(H^2 k^2 + 3 \right)} \right) \right) \right) \text{dt}^4 \{ 23040 \sqrt{g H} \left(H^2 k^2 + 3 \right)^{9/2} + O \left(\text{dt}^5 \right) \right) \text{dx}^4 + O \left(\text{dx}^5 \right), \left(\frac{k^3 \left(\left(H^2 k^2 + 3 \right) U - \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \left(3 g H + U \left(\left(H^2 k^2 + 3 \right) U - 2 \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) \text{dt}^2 \{ 6 \left(H^2 k^2 + 3 \right)^2 + \frac{i k^4 \left(3 g H + U \left(\left(H^2 k^2 + 3 \right) U - 2 \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right)^2 \text{dt}^3 \{ 8 \left(H^2 k^2 + 3 \right)^2 - \frac{\left(k^5 \left(\left(H^2 k^2 + 3 \right) U - \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} \right)^3 \left(3 g H + U \left(\left(H^2 k^2 + 3 \right) U - 2 \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) \right) \text{dt}^4 \{ 20 \left(H^2 k^2 + 3 \right)^4 + O \left(\text{dt}^5 \right) \right) + \left(\frac{k^3 \left(2 U \left(H^2 k^2 + 3 \right) \right)^2 + 3 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \{ 24 \left(H^2 k^2 + 3 \right)^2 + \frac{k^5 \left(\left(2 H^6 U k^6 - 2 \left(2 \sqrt{3} \right) \sqrt{g H^9 \left(H^2 k^2 + 3 \right)} \right) - 9 H^4 U \right) k^4 - 3 \left(7 \sqrt{3} \right) \sqrt{g H^5 \left(H^2 k^2 + 3 \right)} - 18 H^2 U \right) k^2 + 54 U - 27 \sqrt{3} \sqrt{g H \left(H^2 k^2 + 3 \right)} U^2 + 3 g \left(2 k^4 U H^5 + 6 k^2 U H^3 + 3 \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} H \right) \text{dt}^2 \{ 48 \left(H^2 k^2 + 3 \right)^3 + \frac{i k^6 \left(3 g H + U \left(\left(H^2 k^2 + 3 \right) U - 2 \sqrt{3} \right) \sqrt{g H \left(H^2 k^2 + 3 \right)} \right) \right) \left(U \left(2 H^4 U k^4 - 2 \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left(\sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} \right) - 6 H^2 U \right) k^2 + 18 U - 3 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right) - 9 g H \right) \text{dt}^3 \{ 48 \left(H^2 k^2 + 3 \right)^3 - \frac{\left(k^7 \left(2 U \right. \right. \\
& \left. \left. \left(H^2 k^2 + 3 \right)^2 + 3 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right) \left(3 g H + U \left(\left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right)^2 \right) \text{dt}^4 \{ 96 \\
& \left(H^2 k^2 + 3 \right)^4 + O \left(\text{dt}^5 \right) \right) \text{dx}^2 + \left(\frac{1}{16} i \sqrt[3]{g H} k^4 \right. \\
& \left. \left(\frac{\sqrt[3]{U}}{\sqrt[3]{g H \left(H^2 k^2 + 3 \right)}} \right) - 2 \right) + \frac{i k^6 \left(\left(\sqrt[3]{H^2} U - 2 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} \right) k^2 + 3 \sqrt[3]{U} - 12 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right. \\
& \left. U^2 + g \left(4 \sqrt[3]{k^2 U H^3} + 15 \sqrt[3]{U H} - 6 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) H \right) \right) \text{dt}^2 \{ 32 \left(H^2 k^2 + 3 \right)^{3/2} - \frac{\left(k^7 \left(3 g H + U \left(\left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right) \right) \left(2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} + U \right. \right. \\
& \left. \left. \left(\left(\sqrt[3]{H^2} U - 2 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} \right) k^2 + 3 \sqrt[3]{U} - 9 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right) \right) \right) \text{dt}^3 \{ 32 \left(H^2 k^2 + 3 \right)^{5/2} + \frac{i k^8}{\left(2 g H \left(H^2 k^2 + 3 \right) - \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} U \right) \left(3 g H + U \left(\left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right)^2} \text{dt}^4 \{ 64 \\
& \sqrt[3]{g H} \left(H^2 k^2 + 3 \right)^3 + O \left(\text{dt}^5 \right) \right) \text{dx}^3 + \left(\frac{k^5 \left(3 \sqrt[3]{g H} \left(20 H^4 k^4 + 124 H^2 k^2 + 177 \right) - 104 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} \right) k^4 + 6 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} k^2 + 9 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} U \right) \{ 1920 \\
& \sqrt[3]{g H} \left(H^2 k^2 + 3 \right)^{5/2} \} + \frac{k^7 \left(27 \sqrt[3]{g^2 \left(20 H^4 k^4 + 124 H^2 k^2 + 167 \right) H^2 + g U \left(764 \sqrt[3]{H^6 U k^6} - 24 \left(84 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} \right) - 293 \sqrt[3]{H^4 U} \right) k^4 + 21429 \sqrt[3]{H^2 U k^2} + 81 \left(267 \sqrt[3]{U} - 232 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right) \right) \right. \\
& \left. H - 16 \left(17 \sqrt[3]{g H^{13}} \left(H^2 k^2 + 3 \right) \right) U^3 k^6 + 153 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} U^3 k^4 + 9 \left(51 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} \right) U^3 + 88 \sqrt[3]{g^3 H^7 \left(H^2 k^2 + 3 \right)} U \right) k^2 + 459 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} U^3 \right) \right) \text{dt}^2 \{ 11520 \sqrt[3]{g H} \left(H^2 k^2 + 3 \right)^{7/2} \} - \frac{i k^8 \left(84 H^8 U^4 k^8 + 4 U^3 \left(252 H^6 U - 83 \sqrt[3]{g H^{13}} \left(H^2 k^2 + 3 \right) \right) k^6 + 36 U^3 \left(126 H^4 U - 85 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} \right) k^4 - 9 \left(-1008 H^2 U^4 + 1039 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} \right) U^3 + 600 \sqrt[3]{g^3 H^7 \left(H^2 k^2 + 3 \right)} U \right) k^2 + 54 g^2 H^2 \left(10 H^4 k^4 + 62 H^2 k^2 + 81 \right) \right) + 243 U^3 \left(28 U - 39 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right) + 3 g H U \left(472 H^6 U k^6 - 4 \left(71 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} \right) - 1101 H^4 U \right) k^4 + 13500 H^2 U k^2 + 27 \left(504 U - 97 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right) \right) \text{dt}^3 \{ 3840 \left(H^2 k^2 + 3 \right)^4 - \frac{\left(k^9 \left(\sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) - \left(H^2 k^2 + 3 \right) U \right) \left(4 U^3 \left(58 \sqrt[3]{g H^{17}} \left(H^2 k^2 + 3 \right) \right) U - 239 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} k^8 - 12 U \left(213 \sqrt[3]{g^2 H^8} + g U \left(979 \sqrt[3]{U} - 349 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) H^7 - 232 \sqrt[3]{g H^{13}} \left(H^2 k^2 + 3 \right) \right) U^3 \right) k^6 + 9 \left(-2672 \sqrt[3]{g^2 U H^6} + g U^2 \left(4384 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) - 5997 \sqrt[3]{U} \right) H^5 + 1392 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} U^4 + 180 \sqrt[3]{g^5 H^{13}} \left(H^2 k^2 + 3 \right) \right) k^4 + 27 \left(-2703 \sqrt[3]{g^2 U H^4} - 4070 \sqrt[3]{g U^3} H^3 + 928 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} U^4 + 4515 \sqrt[3]{g^3 H^7 \left(H^2 k^2 + 3 \right)} U^2 + 372 \sqrt[3]{g^5 H^9 \left(H^2 k^2 + 3 \right)} \right) k^2 + 81 \left(232 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} U^4 + g H \left(1527 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) - 1033 \sqrt[3]{U} \right) U^2 - 883 \sqrt[3]{g^2 H^2 U} + 157 \sqrt[3]{g^5 H^5 \left(H^2 k^2 + 3 \right)} \right) \right) \text{dt}^4 \{ 23040 \left(\sqrt[3]{g H} \left(H^2 k^2 + 3 \right) \right)^{11/2} \} + O \left(\text{dt}^5 \right) \right) \text{dx}^4 + O \left(\text{dx}^5 \right) \right)
\end{aligned}$$

Out[399]=

$$\text{Out[400]= } EA \parallel \left\{ \left\{ 1 + \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) H^2 k^3 U \text{Csc}\left[\frac{dx k}{2}\right]}{(6 + 2 H^2 k^2) w}, \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) H k \text{Csc}\left[\frac{dx k}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right) w} \right\}, \right. \\ \left. \left\{ \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) k (g H (3 + H^2 k^2) - 3 U^2) \text{Csc}\left[\frac{dx k}{2}\right]}{(6 + 2 H^2 k^2) w}, 1 + \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) k (6 + H^2 k^2) U \text{Csc}\left[\frac{dx k}{2}\right]}{(6 + 2 H^2 k^2) w} \right\} \right\}$$

Out[401]= EA \parallel \left(

\begin{array}{cc}

\frac{i e^{\frac{i}{\text{dx}} k}}{2} \left(1 - e^{-i \text{dx} k}\right) \left(-1 + e^{i \text{dt} w}\right) H^2 U \csc\left(\frac{\text{dx} k}{2}\right) k^3 \left\{\left(2 H^2 k^2 + 6\right) w + 1\right. \\

& \frac{i e^{\frac{i}{\text{dx}} k}}{2} \left(1 - e^{-i \text{dx} k}\right) \left(-1 + e^{i \text{dt} w}\right) H k \csc\left(\frac{\text{dx} k}{2}\right) \left\{2 \left(k^2 H^3 + H\right) w\right. \\

\frac{i e^{\frac{i}{\text{dx}} k}}{2} \left(1 - e^{-i \text{dx} k}\right) \left(-1 + e^{i \text{dt} w}\right) k \left(g H \left(3 + H^2 k^2\right) - 3 U^2\right) \csc\left(\frac{\text{dx} k}{2}\right) \left\{\left(H^2 k^2 + 3\right) U^2 \csc\left(\frac{\text{dx} k}{2}\right) \left\{\left(2 H^2 k^2 + 6\right) w + 1\right. \right. \\

\left. \left. \left(2 H^2 k^2 + 6\right) w + 1\right\} \right\} \\

& \frac{i e^{\frac{i}{\text{dx}} k}}{2} \left(1 - e^{-i \text{dx} k}\right) \left(-1 + e^{i \text{dt} w}\right) k \left(6 + H^2 k^2\right) U \csc\left(\frac{\text{dx} k}{2}\right) \left\{\left(2 H^2 k^2 + 6\right) w + 1\right. \\

\left. \left. \left(2 H^2 k^2 + 6\right) w + 1\right\} \right\} \\

\end{array}

\right)

$$\begin{aligned}
\text{Out[402]} = \text{Eerr} \parallel & \left\{ \left(\left(\frac{(-3 \, g \, H \, k^2 + 3 \, k^2 \, U^2 - H^2 \, k^4 \, U^2 - H^2 \, k^3 \, U \, w) \, dt^2}{2 \, (3 + H^2 \, k^2)} - \frac{i \, H^2 \, k^3 \, U \, w^2 \, dt^3}{6 \, (3 + H^2 \, k^2)} + \frac{H^2 \, k^3 \, U \, w^3 \, dt^4}{24 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) + \right. \\
& \left(-\frac{i \, (27 \, k^3 + 9 \, H^2 \, k^5 + H^4 \, k^7) \, U \, dt}{12 \, (3 + H^2 \, k^2)^2} - \frac{(-9 \, g \, H \, k^4 + 36 \, k^4 \, U^2 + 12 \, H^2 \, k^6 \, U^2 + 2 \, H^4 \, k^8 \, U^2) \, dt^2}{24 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^2 + \\
& \left(-\frac{1}{8} \left(\sqrt{g \, H} \, k^4 \right) dt + \frac{i \sqrt{g \, H} \, (3 \, k^5 + 2 \, H^2 \, k^7) \, U \, dt^2}{16 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^3 + \\
& \left(\frac{i \, (405 \, k^5 \, U + 351 \, H^2 \, k^7 \, U + 116 \, H^4 \, k^9 \, U + 13 \, H^6 \, k^{11} \, U) \, dt}{240 \, (3 + H^2 \, k^2)^3} + \frac{1}{1440 \, (3 + H^2 \, k^2)^3} (1161 \, g \, H \, k^6 + 837 \, g \, H^3 \, k^8 + 135 \, g \, H^5 \, k^{10} - \right. \\
& \quad \left. 351 \, k^6 \, U^2 + 297 \, H^2 \, k^8 \, U^2 + 387 \, H^4 \, k^{10} \, U^2 + 73 \, H^6 \, k^{12} \, U^2) \, dt^2 + O[dt]^5 \right) dx^4 + O[dx]^5, \\
& \left(\left(-\frac{3 \, k^2 \, U}{3 + H^2 \, k^2} - \frac{3 \, k \, w}{2 \, (3 + H^2 \, k^2)} \right) dt^2 - \frac{i \, k \, w^2 \, dt^3}{2 \, (3 + H^2 \, k^2)} + \frac{k \, w^3 \, dt^4}{8 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) + \left(\left(\frac{3 \, i \, k^3}{2 \, (3 + H^2 \, k^2)^2} + \frac{i \, H^2 \, k^5}{4 \, (3 + H^2 \, k^2)^2} \right) dt + \frac{3 \, k^4 \, U \, dt^2}{4 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) \\
& \quad dx^2 + \left(\frac{3 \, i \sqrt{g \, H} \, k^5 \, dt^2}{8 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^3 + \left(\frac{i \, (-54 \, k^5 + H^4 \, k^9) \, dt}{240 \, (3 + H^2 \, k^2)^3} + \frac{(387 \, k^6 \, U + 279 \, H^2 \, k^8 \, U + 45 \, H^4 \, k^{10} \, U) \, dt^2}{240 \, (3 + H^2 \, k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5 \Big\}, \\
& \left\{ \left(-\frac{k \, (3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2) \, (2 \, k \, U + w) \, dt^2}{2 \, (3 + H^2 \, k^2)} - \frac{i \, k \, (3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2) \, w^2 \, dt^3}{6 \, (3 + H^2 \, k^2)} + \frac{k \, (3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2) \, w^3 \, dt^4}{24 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) + \right. \\
& \quad \left(-\frac{i \, (9 \, g \, H \, k^3 + 6 \, g \, H^3 \, k^5 + g \, H^5 \, k^7 + 18 \, k^3 \, U^2 + 3 \, H^2 \, k^5 \, U^2) \, dt}{12 \, (3 + H^2 \, k^2)^2} + \frac{(-18 \, g \, H \, k^4 \, U - 12 \, g \, H^3 \, k^6 \, U - 2 \, g \, H^5 \, k^8 \, U - 9 \, k^4 \, U^3) \, dt^2}{12 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^2 + \\
& \quad \left(-\frac{1}{8} \left(\sqrt{g \, H} \, k^4 \, U \right) dt + \frac{i \, (3 \, g \, H \sqrt{g \, H} \, k^5 + g \, H^3 \sqrt{g \, H} \, k^7 + H^2 \sqrt{g \, H} \, k^7 \, U^2) \, dt^2}{8 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^3 + \\
& \quad \left(\frac{i \, (351 \, g \, H \, k^5 + 351 \, g \, H^3 \, k^7 + 117 \, g \, H^5 \, k^9 + 13 \, g \, H^7 \, k^{11} + 54 \, k^5 \, U^2 - H^4 \, k^9 \, U^2) \, dt}{240 \, (3 + H^2 \, k^2)^3} + \frac{1}{720 \, (3 + H^2 \, k^2)^3} (1971 \, g \, H \, k^6 \, U + 1971 \, g \, H^3 \, k^8 \, U + \right. \\
& \quad \left. 657 \, g \, H^5 \, k^{10} \, U + 73 \, g \, H^7 \, k^{12} \, U - 1161 \, k^6 \, U^3 - 837 \, H^2 \, k^8 \, U^3 - 135 \, H^4 \, k^{10} \, U^3) \, dt^2 + O[dt]^5 \right) dx^4 + \\
& \quad O[dx]^5, \left(\frac{(-3 \, g \, H \, k^2 - 9 \, k^2 \, U^2 - H^2 \, k^4 \, U^2 - 6 \, k \, U \, w - H^2 \, k^3 \, U \, w) \, dt^2}{2 \, (3 + H^2 \, k^2)} - \frac{i \, k \, (6 + H^2 \, k^2) \, U \, w^2 \, dt^3}{6 \, (3 + H^2 \, k^2)} + \frac{k \, (6 + H^2 \, k^2) \, U \, w^3 \, dt^4}{24 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) + \\
& \quad \left(-\frac{i \, (-9 \, k^3 \, U + 3 \, H^2 \, k^5 \, U + H^4 \, k^7 \, U) \, dt}{12 \, (3 + H^2 \, k^2)^2} + \frac{(9 \, g \, H \, k^4 - 12 \, H^2 \, k^6 \, U^2 - 2 \, H^4 \, k^8 \, U^2) \, dt^2}{24 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^2 + \\
& \quad \left(-\frac{1}{8} \left(\sqrt{g \, H} \, k^4 \right) dt + \frac{i \sqrt{g \, H} \, k^5 \, (15 + 2 \, H^2 \, k^2) \, U \, dt^2}{16 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^3 + \\
& \quad \left(\frac{i \, (297 \, k^5 \, U + 351 \, H^2 \, k^7 \, U + 118 \, H^4 \, k^9 \, U + 13 \, H^6 \, k^{11} \, U) \, dt}{240 \, (3 + H^2 \, k^2)^3} + \frac{1}{1440 \, (3 + H^2 \, k^2)^3} (1161 \, g \, H \, k^6 + 837 \, g \, H^3 \, k^8 + 135 \, g \, H^5 \, k^{10} + \right. \\
& \quad \left. 4293 \, k^6 \, U^2 + 3645 \, H^2 \, k^8 \, U^2 + 927 \, H^4 \, k^{10} \, U^2 + 73 \, H^6 \, k^{12} \, U^2) \, dt^2 + O[dt]^5 \right) dx^4 + O[dx]^5 \Big\}
\end{aligned}$$

$$\begin{aligned}
\text{Out[403]} = \text{Eerr} \parallel & \text{\texttt{\textbackslash left(}} \\
& \text{\texttt{\textbackslash begin{array}{cc}}} \\
& \text{\texttt{\textbackslash left(\frac{\texttt{\textbackslash left(-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\texttt{\textbackslash right)} \texttt{\textbackslash text{dt}}^2}{2 \texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}}-\frac{i}}}} \\
& \quad \text{\texttt{\textbackslash left(H^2 k^3 U w^2 \texttt{\textbackslash text{dt}}^3)}{6 \texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}}+\frac{H^2 k^3 U w^3 \texttt{\textbackslash text{dt}}^4}{24 \texttt{\textbackslash left(H^2}}}} \\
& \quad \text{\texttt{\textbackslash k^2+3\texttt{\textbackslash right))}}+O\texttt{\textbackslash left(\texttt{\textbackslash text{dt}}^5\texttt{\textbackslash right))\texttt{\textbackslash right)}}+\texttt{\textbackslash left(-\frac{i \texttt{\textbackslash left(H^4 k^7+9 H^2 k^5+27 k^3\texttt{\textbackslash right)} U}}}} \\
& \quad \text{\texttt{\textbackslash text{dt}})}{12 \texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}^2}}-\frac{\texttt{\textbackslash left(2 H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H}}}} \\
& \quad \text{\texttt{\textbackslash k^4\texttt{\textbackslash right)} \texttt{\textbackslash text{dt}}^2)}{24 \texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}^2}}+O\texttt{\textbackslash left(\texttt{\textbackslash text{dt}}^5\texttt{\textbackslash right))\texttt{\textbackslash right)} \texttt{\textbackslash text{dx}}^2+\texttt{\textbackslash left(-\frac{1}{8}}}} \\
& \quad \text{\texttt{\textbackslash left(\sqrt{g H} \texttt{\textbackslash k^4\texttt{\textbackslash right)} \texttt{\textbackslash text{dt}}}+\frac{i \texttt{\textbackslash sqrt{g H} \texttt{\textbackslash left(2 H^2 k^7+3 k^5\texttt{\textbackslash right)} U \texttt{\textbackslash text{dt}}^2)}{16}}}} \\
& \quad \text{\texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}}+O\texttt{\textbackslash left(\texttt{\textbackslash text{dt}}^5\texttt{\textbackslash right))\texttt{\textbackslash right)} \texttt{\textbackslash text{dx}}^3+\texttt{\textbackslash left(\frac{i \texttt{\textbackslash left(13 H^6 U k^{11}+116}}}} \\
& \quad \text{\texttt{\textbackslash H^4 U k^9+351 H^2 U k^7+405 U k^5\texttt{\textbackslash right)} \texttt{\textbackslash text{dt}})}{240 \texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}^3}}+\frac{\texttt{\textbackslash left(73}}}} \\
& \quad \text{\texttt{\textbackslash H^6 U^2 k^{12}+135 g H^5 k^{10}+387 H^4 U^2 k^{10}+837 g H^3 k^8+297 H^2 U^2 k^8-351 U^2}}}} \\
& \quad \text{\texttt{\textbackslash k^6+1161 g H k^6\texttt{\textbackslash right)} \texttt{\textbackslash text{dt}}^2)}{1440 \texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}^3}}+O\texttt{\textbackslash left(\texttt{\textbackslash text{dt}}^5\texttt{\textbackslash right))\texttt{\textbackslash right)} \texttt{\textbackslash text{dx}}^4+O\texttt{\textbackslash left(\texttt{\textbackslash text{dx}}^5\texttt{\textbackslash right)} \& \texttt{\textbackslash left(\texttt{\textbackslash left(-\frac{3 U k^2}{H^2 k^2+3\texttt{\textbackslash right)}}-\frac{3 w k}{2 \texttt{\textbackslash left(H^2}}}}}} \\
& \quad \text{\texttt{\textbackslash k^2+3\texttt{\textbackslash right))}}\texttt{\textbackslash right)} \texttt{\textbackslash text{dt}}^2-\frac{i \, k \, w^2 \, \texttt{\textbackslash text{dt}}^3}{2 \texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}}+\frac{k \, w^3 \, \texttt{\textbackslash text{dt}}^4}{8 \texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}}+O\texttt{\textbackslash left(\texttt{\textbackslash text{dt}}^5\texttt{\textbackslash right))\texttt{\textbackslash right)}}+\texttt{\textbackslash left(\texttt{\textbackslash left(\frac{i H^2 k^5}{4 \texttt{\textbackslash left(H^2}}}}}} \\
& \quad \text{\texttt{\textbackslash k^2+3\texttt{\textbackslash right))}^2}}+\frac{3 \, i \, k^3}{2 \texttt{\textbackslash left(H^2 k^2+3\texttt{\textbackslash right))}^2}\texttt{\textbackslash right)} \texttt{\textbackslash text{dt}}+\frac{3 \, k^4 \, U \, \texttt{\textbackslash text{dt}}^2}{4}
\end{aligned}$$


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\left(H^2 k^2+3\right)^2)+O\left(\text {dt}^5\right)\right) \text {dx}^2+\left(\frac{3 i \sqrt{g H} k^5 \text {dt}^2}{8} \right.
\left(H^2 k^2+3\right))+O\left(\text {dt}^5\right)\right) \text {dx}^3+\left(\frac{i \left(H^4 k^9-54 k^5\right)}{\text {dt}}\right)\{240 \left(H^2 k^2+3\right)^3\}+\frac{\left(45 H^4 U k^{10}+279 H^2 U k^8+387 U k^6\right)}{\text {dt}^2}\{240 \left(H^2 k^2+3\right)^3\}+O\left(\text {dt}^5\right)\right) \text {dx}^4+O\left(\text {dx}^5\right) \backslash
\left(-\frac{\left(k \left(g k^2 H^3+3 g H-3 U^2\right)\left(2 k U+w\right)\right)}{\text {dt}^2}\{2 \left(H^2 k^2+3\right)\}-\frac{i}{k} \left(g k^2 H^3+3 g H-3 U^2\right) w^2 \text {dt}^3\}\{6 \left(H^2 k^2+3\right)\}+\frac{k \left(g k^2 H^3+3 g H-3 U^2\right) w^3 \text {dt}^4}{24 \left(H^2 k^2+3\right)}+O\left(\text {dt}^5\right)\right)+\left(-\frac{i}{\left(g H^5 k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right)} \text {dt}\right)\{12 \left(H^2 k^2+3\right)^2\}+\frac{\left(-2 g H^5 U k^8-12 g H^3 U k^6-9 U^3 k^4-18 g H U k^4\right)}{\text {dt}^2}\{12 \left(H^2 k^2+3\right)^2\}+O\left(\text {dt}^5\right)\right) \text {dx}^2+\left(-\frac{1}{8} \sqrt{g H} k^4 U\right) \text {dt}+\frac{i \sqrt{g H} U^2 k^7+g H^3 \sqrt{g H} k^7+3 g H \sqrt{g H} k^5}{\text {dt}^2}\{8 \left(H^2 k^2+3\right)\}+O\left(\text {dt}^5\right)\right) \text {dx}^3+\left(\frac{i \left(13 g H^7 k^{11}+117 g H^5 k^9-H^4 U^2 k^9+351 g H^3 k^7+54 U^2 k^5+351 g H k^5\right)}{\text {dt}}\right)\{240 \left(H^2 k^2+3\right)^3\}+\frac{\left(73 g H^7 U k^{12}-135 H^4 U^3 k^{10}+657 g H^5 U k^{10}-837 H^2 U^3 k^8+1971 g H^3 U k^8-1161 U^3 k^6+1971 g H U k^6\right)}{\text {dt}^2}\{720 \left(H^2 k^2+3\right)^3\}+O\left(\text {dt}^5\right)\right) \text {dx}^4+O\left(\text {dx}^5\right) \& \left(\frac{\left(-H^2 U^2 k^4-H^2 U w k^3-9 U^2 k^2-3 g H k^2-6 U w k\right)}{\text {dt}^2}\{2 \left(H^2 k^2+3\right)\}-\frac{i}{k} \left(H^2 k^2+6\right) U w^2 \text {dt}^3\}\{6 \left(H^2 k^2+3\right)\}+\frac{k \left(H^2 k^2+6\right) U w^3 \text {dt}^4}{24 \left(H^2 k^2+3\right)}+O\left(\text {dt}^5\right)\right)+\left(-\frac{i \left(H^4 U k^7+3 H^2 U k^5-9 U k^3\right)}{\text {dt}}\right)\{12 \left(H^2 k^2+3\right)^2\}+\frac{\left(-2 H^4 U^2 k^8-12 H^2 U^2 k^6+9 g H k^4\right)}{\text {dt}^2}\{24 \left(H^2 k^2+3\right)^2\}+O\left(\text {dt}^5\right)\right) \text {dx}^2+\left(-\frac{1}{8} \sqrt{g H} k^4\right) \text {dt}+\frac{i \sqrt{g H} k^5 \left(2 H^2 k^2+15\right) U \text {dt}^2}{16 \left(H^2 k^2+3\right)}+O\left(\text {dt}^5\right)\right) \text {dx}^3+\left(\frac{i \left(13 H^6 U k^{11}+118 H^4 U k^9+351 H^2 U k^7+297 U k^5\right)}{\text {dt}}\right)\{240 \left(H^2 k^2+3\right)^3\}+\frac{\left(73 H^6 U^2 k^{12}+135 g H^5 k^{10}+927 H^4 U^2 k^{10}+837 g H^3 k^8+3645 H^2 U^2 k^8+4293 U^2 k^6+1161 g H k^6\right)}{\text {dt}^2}\{1440 \left(H^2 k^2+3\right)^3\}+O\left(\text {dt}^5\right)\right) \text {dx}^4+O\left(\text {dx}^5\right) \backslash
\end{array}
\right)

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```

In[404]:= KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
KurFWS = KurF /. ap -> (U + Sqrt[g*H]) /. am -> 0;
KurFWSeta =
  KurFWS /. fp -> (H*v + U*Rpp*n) /. fm -> (H*v + U*Rmp*n) /. qp -> Rpp*n /.
  qm -> Rmp*n;
KurFWSeta = KurFWSeta /. v -> (GGp*G + Gnp*n);
Kfnnp = FullSimplify[KurFWSeta /. G -> 0 /. n -> 1];
KfnGp = FullSimplify[KurFWSeta /. n -> 0 /. G -> 1];
Kfnn = Kfnnp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfnG = KfnGp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
Fnn2 = -dt*(1 - Exp[-I*k*dx])/dx*Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAref = Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}];

```

```

FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS / . fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) / .
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) / . qp → Rpp * G / . qm → Rmp * G;
KurFWSG = KurFWSG / . v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG / . G → 0 / . n → 1];
KfGGp = FullSimplify[KurFWSG / . n → 0 / . G → 1];
KfGn = KfGnp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
KfGG = KfGGp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2/2;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2/2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}];

Text[Row[{" U < -Sqrt(gH) < U "}] ]
Text[" "]
Text[Row[{"Fnn  ||  ", Kfnnp}]]
Text[Row[{"Fnn  ||  ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||  ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||  ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||  ", KfnGp}]]
Text[Row[{"FnG  ||  ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error  ||  ", FnG2TAr}]]
Text[Row[{"FnG error  ||  ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||  ", KfGnp}]]
Text[Row[{"FGn  ||  ", TeXForm[KfGnp]}]]

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Text[Row[{"FGn error  ||  ", FGn2TAr}]]
Text[Row[{"FGn error  ||  ", TeXForm[FGn2TAr]}]]]
Text[" "]
Text[Row[{"FGG  ||  ", KfGGp}]]
Text[Row[{"FGG  ||  ", TeXForm[KfGGp]}]]]
Text[Row[{"FGG error  ||  ", FGG2TAr}]]
Text[Row[{"FGG error  ||  ", TeXForm[FGG2TAr]}]]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||  ", RKstepTayr}]]
Text[Row[{"Omega error  ||  ", TeXForm[RKstepTayr]}]]]
Text[" "]
Text[Row[{"EA  ||  ", EA}]]
Text[Row[{"EA  ||  ", TeXForm[EA]}]]]
Text[Row[{"Eerr  ||  ", Eerr}]]
Text[Row[{"Eerr  ||  ", TeXForm[Eerr]}]]]

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Out[438]= $U < -\text{Sqrt}(gH) < U$

Out[439]=

Out[440]= $F_{nn} \parallel G_{np} H + R_{mp} U$

Out[441]= $F_{nn} \parallel \text{\texttt{Gnp}} H + \text{\texttt{Rmp}} U$

Out[442]= $F_{nn} \text{ error } \parallel$

$$\left(-\frac{H^2 k^3 U w}{2(3+H^2 k^2)} \frac{dt^2}{dt} - \frac{i H^2 k^3 U w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4\right) + \left(-\frac{i(27 k^3 + 9 H^2 k^5 + H^4 k^7) U dt}{12(3+H^2 k^2)^2} + O[dt]^4\right) dx^2 + \left(-\frac{1}{8}(k^4 U) dt + O[dt]^4\right) dx^3 + O[dx]^4$$

Out[443]= $F_{nn} \text{ error } \parallel$

$$\text{\texttt{left}}(-\frac{\text{\texttt{left}}(\text{\texttt{dt}}^2 \text{\texttt{left}}(H^2 k^3 U w \text{\texttt{right}}))}{2} \text{\texttt{left}}(H^2 k^2 + 3 \text{\texttt{right}})) - \frac{\text{\texttt{left}}(i \text{\texttt{left}}(\text{\texttt{dt}}^3 H^2 k^3 U w^2 \text{\texttt{right}}))}{6} \text{\texttt{left}}(H^2 k^2 + 3 \text{\texttt{right}}) + O(\text{\texttt{left}}(\text{\texttt{dt}}^4 \text{\texttt{right}}) \text{\texttt{right}}) + \text{\texttt{left}}(dx)^2 \text{\texttt{left}}(-\frac{\text{\texttt{left}}(H^4 k^7 + 9 H^2 k^5 + 27 k^3 \text{\texttt{right}}) U \text{\texttt{left}}(\text{\texttt{dt}})}{12} \text{\texttt{left}}(H^2 k^2 + 3 \text{\texttt{right}})^2 + O(\text{\texttt{left}}(\text{\texttt{dt}}^4 \text{\texttt{right}}) \text{\texttt{right}}) + \text{\texttt{left}}(dx)^3 \text{\texttt{left}}(-\frac{1}{8} \text{\texttt{left}}(k^4 U \text{\texttt{right}}) \text{\texttt{left}}(\text{\texttt{dt}}) + O(\text{\texttt{left}}(\text{\texttt{dt}}^4 \text{\texttt{right}}) \text{\texttt{right}}) + O(\text{\texttt{left}}(\text{\texttt{dx}}^4 \text{\texttt{right}})$$

Out[444]=

Out[445]= $F_{nG} \parallel G_{Gp} H$

Out[446]= $F_{nG} \parallel \text{\texttt{GGp}} H$

Out[447]= $F_{nG} \text{ error } \parallel$

$$\left(-\frac{3(k w) dt^2}{2(3+H^2 k^2)} - \frac{i k w^2 dt^3}{2(3+H^2 k^2)} + O[dt]^4\right) + \left(\frac{i(6 k^3 + H^2 k^5) dt}{4(3+H^2 k^2)^2} + O[dt]^4\right) dx^2 + O[dx]^4$$

Out[448]= $F_{nG} \text{ error } \parallel$

$$\text{\texttt{left}}(-\frac{\text{\texttt{left}}(3 \text{\texttt{left}}(\text{\texttt{dt}}^2 (k w) \text{\texttt{right}}))}{2} \text{\texttt{left}}(H^2 k^2 + 3 \text{\texttt{right}})) - \frac{\text{\texttt{left}}(i \text{\texttt{left}}(\text{\texttt{dt}}^3 k w^2 \text{\texttt{right}}))}{2} \text{\texttt{left}}(H^2 k^2 + 3 \text{\texttt{right}}) + O(\text{\texttt{left}}(\text{\texttt{dt}}^4 \text{\texttt{right}}) \text{\texttt{right}}) + \text{\texttt{left}}(dx)^2 \text{\texttt{left}}(\frac{\text{\texttt{left}}(H^2 k^5 + 6 k^3 \text{\texttt{right}}) \text{\texttt{left}}(\text{\texttt{dt}})}{4} \text{\texttt{left}}(H^2 k^2 + 3 \text{\texttt{right}})^2 + O(\text{\texttt{left}}(\text{\texttt{dt}}^4 \text{\texttt{right}}) \text{\texttt{right}}) + O(\text{\texttt{left}}(\text{\texttt{dx}}^4 \text{\texttt{right}})$$

Out[449]=

Out[450]= FGn || H (g Rmp + Gnp U)

Out[451]= FGn || H (g \text{Rmp}+\text{Gnp} U)

Out[452]= FGn error || $\left(-\frac{(k(3gH+gH^3k^2-3U^2)w)dt^2}{2(3+H^2k^2)} - \frac{ik(3gH+gH^3k^2-3U^2)w^2dt^3}{6(3+H^2k^2)} + O[dt]^4 \right) +$
 $\left(-\frac{i(9gHk^3+6gH^3k^5+gH^5k^7+18k^3U^2+3H^2k^5U^2)dt}{12(3+H^2k^2)^2} + O[dt]^4 \right) dx^2 + \left(-\frac{1}{8}(gHk^4)dt + O[dt]^4 \right) dx^3 + O[dx]^4$

Out[453]= FGn error ||

$\left(-\frac{\text{dt}^2 \left(k w \left(g H^3 k^2 + 3 g H - 3 U^2 \right) \right)}{2 \left(H^2 k^2 + 3 \right)} \right) - \frac{i \text{dt}^3 k w^2 \left(g H^3 k^2 + 3 g H - 3 U^2 \right)}{6 \left(H^2 k^2 + 3 \right)} + O \left(\text{dt}^4 \right) + \text{dx}^2 \left(-\frac{i \left(g H^5 k^7 + 6 g H^3 k^5 + 3 H^2 U^2 k^5 + 18 U^2 k^3 + 9 g H k^3 \right) \text{dt}}{12 \left(H^2 k^2 + 3 \right)^2} + O \left(\text{dt}^4 \right) \right) + \text{dx}^3 \left(-\frac{1}{8} \left(g H k^4 \right) \text{dt} + O \left(\text{dt}^4 \right) \right) + O \left(\text{dx}^4 \right)$

Out[454]=

Out[455]= FGG || (GGp H + Rmp) U

Out[456]= FGG || U (\text{GGp} H+\text{Rmp})

Out[457]= FGG error || $\left(-\frac{(k(6+H^2k^2)Uw)dt^2}{2(3+H^2k^2)} - \frac{ik(6+H^2k^2)Uw^2dt^3}{6(3+H^2k^2)} + O[dt]^4 \right) + \left(-\frac{i(-9k^3+3H^2k^5+H^4k^7)Udt}{12(3+H^2k^2)^2} + O[dt]^4 \right) dx^2 +$
 $\left(-\frac{1}{8}(k^4U)dt + O[dt]^4 \right) dx^3 + \left(\frac{i(297k^5+351H^2k^7+118H^4k^9+13H^6k^{11})Udt}{240(3+H^2k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$

Out[458]= FGG error ||

$\left(-\frac{\text{dt}^2 \left(k U w \left(H^2 k^2 + 6 \right) \right)}{2 \left(H^2 k^2 + 3 \right)} \right) - \frac{i \text{dt}^3 k U w^2 \left(H^2 k^2 + 6 \right)}{6 \left(H^2 k^2 + 3 \right)} + O \left(\text{dt}^4 \right) + \text{dx}^2 \left(-\frac{i \left(H^4 k^7 + 3 H^2 k^5 - 9 k^3 \right) U \text{dt}}{12 \left(H^2 k^2 + 3 \right)^2} + O \left(\text{dt}^4 \right) \right) + \text{dx}^3 \left(-\frac{1}{8} \left(k^4 U \right) \text{dt} + O \left(\text{dt}^4 \right) \right) + \text{dx}^4 \left(\frac{i \left(13 H^6 k^{11} + 118 H^4 k^9 + 351 H^2 k^7 + 297 k^5 \right) U \text{dt}}{240 \left(H^2 k^2 + 3 \right)^3} + O \left(\text{dt}^4 \right) \right) + O \left(\text{dx}^5 \right)$

Out[459]=

Out[460]=

Out[461]= Omega error ||

$\left\{ \frac{1}{6(3+H^2k^2)^2} k^3 \left(\sqrt{3} \sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U \right) \left(3gH+U \left(2\sqrt{3} \sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U \right) \right) dt^2 + \right.$
 $\frac{ik^4 \left(3gH+U \left(2\sqrt{3} \sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U \right) \right)^2 dt^3}{8(3+H^2k^2)^2} - \frac{1}{20(3+H^2k^2)^4} \left(k^5 \left(\sqrt{3} \sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U \right)^3 \right.$
 $\left. \left. \right) \right\}$

$$\begin{aligned}
& \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) dt^4 + O[dt]^5 \Bigg) + \\
& \left(\frac{k^3 \left(-3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2)^2 U \right)}{24 (3 + H^2 k^2)^2} + \frac{1}{48 (3 + H^2 k^2)^3} k^5 \left(g \left(-9 \sqrt{3} H \sqrt{g H (3 + H^2 k^2)} + 18 H^3 k^2 U + 6 H^5 k^4 U \right) + \right. \right. \\
& U^2 \left(27 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 54 U + 2 H^6 k^6 U + 3 k^2 \left(7 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 18 H^2 U \right) + \right. \\
& \left. \left. 2 k^4 \left(2 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 9 H^4 U \right) \right) \right) dt^2 + \\
& \frac{1}{48 (3 + H^2 k^2)^3} i k^6 \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \\
& \left(-9 g H + U \left(3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U + 2 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) \\
& dt^3 - \frac{1}{96 (3 + H^2 k^2)^4} \left(k^7 \left(-3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2)^2 U \right) \right. \\
& \left. \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right)^2 \right) dt^4 + O[dt]^5 \Bigg) dx^2 + \\
& \left(-\frac{1}{16} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{3 + H^2 k^2}} + 2 U \right) - \frac{1}{32 (3 + H^2 k^2)^2} i k^6 \left(3 g H \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 4 (3 + H^2 k^2) U \right) + \right. \right. \\
& U^2 \left(15 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U + k^2 \left(5 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 12 H^2 U \right) \right) \Bigg) dt^2 + \\
& \frac{1}{32 (3 + H^2 k^2)^2} k^7 \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \\
& \left(3 g H + U \left(3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U \right) \right) dt^3 + \\
& \frac{1}{64 (3 + H^2 k^2)^3} i k^8 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U \right) \\
& \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right)^2 dt^4 + O[dt]^5 \Bigg) dx^3 + \\
& \left(-\left(\left(k^5 \left(3 \sqrt{3} g H (177 + 124 H^2 k^2 + 20 H^4 k^4) + 104 \left(9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + \right. \right. \right. \right. \right. \\
& \left. \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \left(1920 \left(\sqrt{g H (3 + H^2 k^2)}^{5/2} \right) \right) \right) - \\
& \left(\left(k^7 \left(27 \sqrt{3} g^2 H^2 (167 + 124 H^2 k^2 + 20 H^4 k^4) + g H U \left(21 429 \sqrt{3} H^2 k^2 U + \right. \right. \right. \right. \\
& 764 \sqrt{3} H^6 k^6 U + 81 \left(232 \sqrt{g H (3 + H^2 k^2)} + 267 \sqrt{3} U \right) + \\
& 24 k^4 \left(84 \sqrt{g H^9 (3 + H^2 k^2)} + 293 \sqrt{3} H^4 U \right) \right) + \\
& 16 \left(459 \sqrt{g H (3 + H^2 k^2)} U^3 + 153 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 17 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} \right. \\
& \left. \left. U^3 + 9 k^2 \left(88 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 51 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) dt^2 \Bigg) / \\
& \left(11 520 \left(\sqrt{g H (3 + H^2 k^2)}^{7/2} \right) \right) - \frac{1}{96 (3 + H^2 k^2)^4} i k^8 \left(54 g^2 H^2 (81 + 62 H^2 k^2 + 10 H^4 k^4) + \right.
\end{aligned}$$

$$\begin{aligned}
& 84 H^8 k^8 U^4 + 243 U^3 \left(39 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 28 U \right) + 36 k^4 U^3 \\
& \left(85 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 126 H^4 U \right) + 4 k^6 U^3 \left(83 \sqrt{3} \sqrt{g H^{13} (3 + H^2 k^2)} + 252 H^6 U \right) + \\
& 9 k^2 \left(600 \sqrt{3} \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 1039 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} U^3 + 1008 H^2 U^4 \right) + \\
& 3 g H U \left(13 500 H^2 k^2 U + 472 H^6 k^6 U + 27 \left(97 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 504 U \right) + \right. \\
& \left. 4 k^4 \left(71 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 1101 H^4 U \right) \right) dt^3 + \frac{1}{23 040 \sqrt{g H (3 + H^2 k^2)^{11/2}}} \\
& k^9 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left(4 k^8 U^3 \left(239 \sqrt{3} \sqrt{g H^9} + 58 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& 27 k^2 \left(372 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 2703 \sqrt{3} \sqrt{g^2 H^4 U} + 4515 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \\
& 4070 \sqrt{3} \sqrt{g H^3 U^3} + 928 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \left. \right) + 12 k^6 U \\
& \left(213 \sqrt{3} \sqrt{g^2 H^8} + 232 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + g H^7 U \left(349 \sqrt{g H (3 + H^2 k^2)} + 979 \sqrt{3} U \right) \right) + \\
& 81 \left(157 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 883 \sqrt{3} \sqrt{g^2 H^2 U} + 232 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& g H U^2 \left(1527 \sqrt{g H (3 + H^2 k^2)} + 1033 \sqrt{3} U \right) \left. \right) + \\
& 9 k^4 \left(180 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 2672 \sqrt{3} \sqrt{g^2 H^6 U} + 1392 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + \right. \\
& g H^5 U^2 \left(4384 \sqrt{g H (3 + H^2 k^2)} + 5997 \sqrt{3} U \right) \left. \right) dt^4 + O[dt]^5 dx^4 + O[dx]^5, \\
& \left(\frac{1}{6(3+H^2 k^2)^2} k^3 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right. \\
& dt^2 + \\
& \frac{i k^4 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right)^2 dt^3}{8 (3 + H^2 k^2)^2} - \frac{1}{20 (3 + H^2 k^2)^4} \\
& \left(k^5 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right. \\
& dt^4 + O[dt]^5 \left. \right) + \\
& \left(\frac{k^3 \left(3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2)^2 U \right)}{24 (3 + H^2 k^2)^2} + \frac{1}{48 (3 + H^2 k^2)^3} k^5 \left(3 g \left(3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 6 H^3 k^2 U + 2 H^5 k^4 U \right) + \right. \\
& U^2 \left(-27 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 54 U + 2 H^6 k^6 U - 3 k^2 \left(7 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 18 H^2 U \right) - \right. \\
& \left. \left. 2 k^4 \left(2 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} - 9 H^4 U \right) \right) \right) dt^2 +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{48(3+H^2 k^2)^3} i k^6 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \\
& \left(-9 g H + U \left(-3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U - 2 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 6 H^2 U \right) \right) \right) \\
& dt^3 - \frac{1}{96(3+H^2 k^2)^4} \\
& \left(k^7 \left(3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2)^2 U \right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right)^2 \right) \\
& dt^4 + O[dt]^5 \Bigg) dx^2 + \\
& \left(\frac{1}{16} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{3+H^2 k^2}} - 2 U \right) - \frac{1}{32(3+H^2 k^2)^2} i k^6 \left(-3 g H \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - 4 (3 + H^2 k^2) U \right) + \right. \right. \\
& \quad \left. U^2 \left(-15 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U + k^2 \left(-5 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 12 H^2 U \right) \right) \right) \\
& \quad dt^2 + \frac{1}{32(3+H^2 k^2)^2} k^7 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \\
& \quad \left(3 g H + U \left(-3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U \right) \right) dt^3 + \\
& \quad \frac{1}{64(3+H^2 k^2)^3} i k^8 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U \right) \\
& \quad \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right)^2 dt^4 + O[dt]^5 \Bigg) \\
& dx^3 + \left(\left(k^5 \left(3 \sqrt{3} g H (177 + 124 H^2 k^2 + 20 H^4 k^4) - \right. \right. \right. \\
& \quad \left. \left. 104 \left(9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \\
& \quad \left(1920 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \left(k^7 \left(27 \sqrt{3} g^2 H^2 (167 + 124 H^2 k^2 + 20 H^4 k^4) + \right. \right. \\
& \quad g H U \left(21429 \sqrt{3} H^2 k^2 U + 764 \sqrt{3} H^6 k^6 U + 81 \left(-232 \sqrt{g H (3 + H^2 k^2)} + 267 \sqrt{3} U \right) - \right. \\
& \quad \left. 24 k^4 \left(84 \sqrt{g H^9 (3 + H^2 k^2)} - 293 \sqrt{3} H^4 U \right) \right) - \\
& \quad 16 \left(459 \sqrt{g H (3 + H^2 k^2)} U^3 + 153 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 17 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. 9 k^2 \left(88 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 51 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Bigg) dt^2 \Bigg) / \\
& \left(11520 \sqrt{g H} (3 + H^2 k^2)^{7/2} \right) - \frac{1}{3840(3+H^2 k^2)^4} i k^8 \left(54 g^2 H^2 (81 + 62 H^2 k^2 + 10 H^4 k^4) + 84 H^8 k^8 U^4 + \right. \\
& \quad 243 U^3 \left(-39 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 28 U \right) + 36 k^4 U^3 \left(-85 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 126 H^4 U \right) + \\
& \quad 4 k^6 U^3 \left(-83 \sqrt{3} \sqrt{g H^{13} (3 + H^2 k^2)} + 252 H^6 U \right) - \\
& \quad 9 k^2 \left(600 \sqrt{3} \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 1039 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} U^3 - 1008 H^2 U^4 \right) + \\
& \quad 3 g H U \left(13500 H^2 k^2 U + 472 H^6 k^6 U + 27 \left(-97 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 504 U \right) - \right. \\
& \quad \left. \left. \left. \dots \right) \right) \right) \Bigg) dt^2 \Bigg)
\end{aligned}$$

$$\begin{aligned}
& \sqrt{3} \, g \, H \left(20 \, H^4 \, k^4 + 124 \, H^2 \, k^2 + 177 \right) + 104 \sqrt{g \, H^9 \left(H^2 \, k^2 + 3 \right)} \\
& k^4 + 6 \sqrt{g \, H^5 \left(H^2 \, k^2 + 3 \right)} \, k^2 + 9 \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \, U \right) \{ 1920 \\
& \left(\sqrt{g \, H} \left(H^2 \, k^2 + 3 \right)^{5/2} \right) - \frac{\left(k^7 \left(27 \sqrt{3} \, g^2 \left(20 \, H^4 \right. \right. \right. \\
& k^4 + 124 \, H^2 \, k^2 + 167 \right) H^2 + g \, U \left(764 \sqrt{3} \, H^6 \, U \, k^6 + 24 \left(293 \sqrt{3} \, U \, H^4 + 84 \right. \right. \\
& \sqrt{g \, H^9 \left(H^2 \, k^2 + 3 \right)} \right) \, k^4 + 21429 \sqrt{3} \, H^2 \, U \, k^2 + 81 \left(267 \sqrt{3} \, U + 232 \right. \\
& \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \, H + 16 \left(17 \sqrt{g \, H^{13}} \left(H^2 \, k^2 + 3 \right) \right) \, U^3 \\
& k^6 + 153 \sqrt{g \, H^9 \left(H^2 \, k^2 + 3 \right)} \, U^3 \, k^4 + 9 \left(51 \sqrt{g \, H^5 \left(H^2 \, k^2 + 3 \right)} \right) \\
& U^3 + 88 \sqrt{g^3 \, H^7 \left(H^2 \, k^2 + 3 \right)} \, U \right) \, k^2 + 459 \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \\
& U^3 \right) \, U \right) \, \text{dt}^2 \{ 11520 \left(\sqrt{g \, H} \left(H^2 \, k^2 + 3 \right)^{7/2} \right) - \frac{i \, k^8 \\
& \left(84 \, H^8 \, U^4 \, k^8 + 4 \, U^3 \left(252 \, U \, H^6 + 83 \sqrt{3} \sqrt{g \, H^{13}} \left(H^2 \, k^2 + 3 \right) \right) \right) \, U \\
& k^6 + 36 \, U^3 \left(126 \, U \, H^4 + 85 \sqrt{3} \sqrt{g \, H^9 \left(H^2 \, k^2 + 3 \right)} \right) \, k^4 + 9 \left(1008 \right. \\
& H^2 \, U^4 + 1039 \sqrt{3} \sqrt{g \, H^5 \left(H^2 \, k^2 + 3 \right)} \, U^3 + 600 \sqrt{3} \sqrt{g^3 \, H^7 \left(H^2 \right.} \\
& k^2 + 3 \right) \, U \right) \, k^2 + 54 \, g^2 \, H^2 \left(10 \, H^4 \, k^4 + 62 \, H^2 \, k^2 + 81 \right) + 243 \, U^3 \left(28 \, U + 39 \right. \\
& \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \, U \right) + 3 \, g \, H \, U \left(472 \, H^6 \, U \, k^6 + 4 \left(1101 \, U \, H^4 + 71 \right. \right. \\
& \sqrt{3} \sqrt{g \, H^9 \left(H^2 \, k^2 + 3 \right)} \right) \, k^4 + 13500 \, H^2 \, U \, k^2 + 27 \left(504 \, U + 97 \sqrt{3} \right. \\
& \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \, U \right) \, U \right) \, \text{dt}^3 \{ 3840 \left(H^2 \, k^2 + 3 \right)^4 + \frac{k^9 \\
& \left(\left(H^2 \, k^2 + 3 \right) \, U + \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \left(4 \, U^3 \left(239 \right. \right. \\
& \sqrt{3} \sqrt{g \, H^9 + 58 \sqrt{g \, H^{17}} \left(H^2 \, k^2 + 3 \right)} \, U \right) \, k^8 + 12 \, U \left(213 \sqrt{3} \, g^2 \right. \\
& H^8 + g \, U \left(979 \sqrt{3} \, U + 349 \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \, H^7 + 232 \sqrt{g \, H^{13}} \\
& \left(H^2 \, k^2 + 3 \right) \, U^3 \right) \, k^6 + 9 \left(2672 \sqrt{3} \, g^2 \, U \, H^6 + g \, U^2 \left(5997 \sqrt{3} \right. \right. \\
& U + 4384 \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \, H^5 + 1392 \sqrt{g \, H^9 \left(H^2 \, k^2 + 3 \right)} \, U^4 + 180 \\
& \sqrt{g^5 \, H^{13}} \left(H^2 \, k^2 + 3 \right) \, U \right) \, k^4 + 27 \left(2703 \sqrt{3} \, g^2 \, U \, H^4 + 4070 \sqrt{3} \, g \right. \\
& U^3 \, H^3 + 928 \sqrt{g \, H^5 \left(H^2 \, k^2 + 3 \right)} \, U^4 + 4515 \sqrt{g^3 \, H^7 \left(H^2 \, k^2 + 3 \right)} \\
& U^2 + 372 \sqrt{g^5 \, H^9 \left(H^2 \, k^2 + 3 \right)} \, U \right) \, k^2 + 81 \left(232 \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \\
& U^4 + g \, H \left(1033 \sqrt{3} \, U + 1527 \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \, U^2 + 883 \sqrt{3} \, g^2 \\
& H^2 \, U + 157 \sqrt{g^5 \, H^5 \left(H^2 \, k^2 + 3 \right)} \, U \right) \, \text{dt}^4 \{ 23040 \sqrt{g \, H} \left(H^2 \right. \\
& k^2 + 3 \right)^{11/2} + O \left(\text{dt}^5 \right) \, \text{dx}^4 + O \left(\text{dx}^5 \right), \left(\frac{k^3}{\left(\left(H^2 \, k^2 + 3 \right) \, U - \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right)} \right) \left(3 \, g \, H + U \left(\left(H^2 \right. \right. \right. \\
& k^2 + 3 \right) \, U - 2 \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \, \text{dt}^2 \{ 6 \left(H^2 \right. \\
& k^2 + 3 \right)^2 + \frac{i \, k^4}{\left(3 \, g \, H + U \left(\left(H^2 \, k^2 + 3 \right) \, U - 2 \sqrt{3} \sqrt{g \, H \left(H^2 \right.} \right.} \\
& k^2 + 3 \right) \, U \right) \, \text{dt}^3 \{ 8 \left(H^2 \, k^2 + 3 \right)^2 - \frac{\left(k^5 \left(\left(H^2 \right. \right. \right. \\
& k^2 + 3 \right) \, U - \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right)^3 \left(3 \, g \, H + U \left(\left(H^2 \right. \right. \right. \\
& k^2 + 3 \right) \, U - 2 \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \, U \right) \, \text{dt}^4 \{ 20 \left(H^2 \right. \\
& k^2 + 3 \right)^4 + O \left(\text{dt}^5 \right) \, U \left(\frac{k^3}{\left(2 \, U \left(H^2 \, k^2 + 3 \right) \right)^2 + 3} \right. \\
& \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \{ 24 \left(H^2 \, k^2 + 3 \right)^2 + \frac{k^5}{\left(2 \, H^6 \, U \right.} \\
& k^6 - 2 \left(2 \sqrt{3} \sqrt{g \, H^9 \left(H^2 \, k^2 + 3 \right)} \right) - 9 \, H^4 \, U \right) \, k^4 - 3 \left(7 \sqrt{3} \sqrt{g \, H^5} \right. \\
& \left(H^2 \, k^2 + 3 \right) \right) - 18 \, H^2 \, U \right) \, k^2 + 54 \, U - 27 \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \, U \\
& U^2 + 3 \, g \left(2 \, k^4 \, U \, H^5 + 6 \, k^2 \, U \, H^3 + 3 \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \, H \right) \, U \\
& \text{dt}^2 \{ 48 \left(H^2 \, k^2 + 3 \right)^3 + \frac{i \, k^6}{\left(3 \, g \, H + U \left(\left(H^2 \, k^2 + 3 \right) \, U - 2 \sqrt{3} \sqrt{g \, H \left(H^2 \right.} \right.} \right.} \\
& k^2 + 3 \right) \, U \right) \, U \left(2 \, H^4 \, U \, k^4 - 2 \sqrt{3} \sqrt{g \, H^5 \left(H^2 \, k^2 + 3 \right)} \right) - 6 \, H^2 \, U \right) \\
& k^2 + 18 \, U - 3 \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \, U - 9 \, g \, H \right) \, \text{dt}^3 \{ 48 \left(H^2 \, k^2 + 3 \right)^3 - \frac{\left(k^7 \left(2 \, U \left(H^2 \, k^2 + 3 \right) \right)^2 + 3 \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \left(3 \, g \, H + U \left(\left(H^2 \, k^2 + 3 \right) \, U - 2 \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \right)}{\left(3 \, g \, H + U \left(\left(H^2 \, k^2 + 3 \right) \, U - 2 \sqrt{3} \sqrt{g \, H \left(H^2 \, k^2 + 3 \right)} \right) \right)} \}
\end{aligned}$$

$$\begin{aligned}
& k^2+3\right)\right)\right)^2\right)\text{dt}^4\}\sqrt[96]{\left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \\
& \text{dx}^2+\left(\frac{1}{16} i k^4 \sqrt[3]{\sqrt[3]{g H}\left(H^2 k^2+3\right)}-2 U\right)-\frac{i k^6}{\left(U^2 \sqrt[3]{\left(2 H^4 U k^4+\left(12 H^2 U-5 \sqrt[3]{\sqrt[3]{g H^5 \left(H^2 k^2+3\right)}\right)}\right)}\right)} \\
& k^2+18 U-15 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}-3 g H \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}-4 \sqrt[3]{\left(H^2 k^2+3\right)} U\right)\right)\text{dt}^2\}\sqrt[32]{\left(H^2 k^2+3\right)^2}+\frac{k^7}{\left(\left(3 g H+U \sqrt[3]{\left(H^2 k^2+3\right)} U-2 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)\right)} \\
& \sqrt[3]{\left(3 g H+U \sqrt[3]{\left(2 \sqrt[3]{\left(H^2 k^2+3\right)} U-3 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}\right)}\right)\text{dt}^3\}\sqrt[32]{\left(H^2 k^2+3\right)^2}+\frac{i k^8 \sqrt[3]{\left(2 \sqrt[3]{\left(H^2 k^2+3\right)} U-\sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} \sqrt[3]{\left(3 g H+U \sqrt[3]{\left(H^2 k^2+3\right)} U-2 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}\right)}^2 \text{dt}^4\}\sqrt[64]{\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \\
& \text{dx}^3+\left(\frac{k^5 \sqrt[3]{\left(3 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(20 H^4 k^4+124 H^2 k^2+177\right)}}-104 \sqrt[3]{\sqrt[3]{g H^9 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} k^4+6 \sqrt[3]{g H^5 \sqrt[3]{\left(H^2 k^2+3\right)}} k^2+9 \sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}\right)} U\right)\}\sqrt[1920]{\sqrt[3]{g H} \sqrt[3]{\left(H^2 k^2+3\right)}^{5/2}}+\frac{k^7 \sqrt[3]{\left(27 \sqrt[3]{\sqrt[3]{g^2 \sqrt[3]{\left(20 H^4 k^4+124 H^2 k^2+167\right)}} H^2+g U \sqrt[3]{\left(764 \sqrt[3]{\sqrt[3]{H^6 U k^6-24 \sqrt[3]{\left(84 \sqrt[3]{g H^9 \sqrt[3]{\left(H^2 k^2+3\right)}}-293 \sqrt[3]{\sqrt[3]{H^4 U}\right)} k^4+21429 \sqrt[3]{\sqrt[3]{H^2 U k^2+81 \sqrt[3]{\left(267 \sqrt[3]{\sqrt[3]{U-232 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} H-16 \sqrt[3]{\left(17 \sqrt[3]{\sqrt[3]{g H^{13}} \sqrt[3]{\left(H^2 k^2+3\right)}}\right)} U^3 k^6+153 \sqrt[3]{\sqrt[3]{g H^9 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U^3 k^4+9 \sqrt[3]{\left(51 \sqrt[3]{\sqrt[3]{g H^5 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U^3+88 \sqrt[3]{\sqrt[3]{g^3 H^7 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U\right)} k^2+459 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U^3\right)\right)\text{dt}^2\}\sqrt[11520]{\sqrt[3]{g H} \sqrt[3]{\left(H^2 k^2+3\right)}^{7/2}}-\frac{k^8}{\left(\left(84 H^8 U^4 k^8+4 U^3 \sqrt[3]{\left(252 H^6 U-83 \sqrt[3]{\sqrt[3]{g H^{13}} \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}\right)} k^6+36 U^3 \sqrt[3]{\left(126 H^4 U-85 \sqrt[3]{\sqrt[3]{g H^9 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}\right)} k^4-9 \sqrt[3]{\left(-1008 H^2 U^4+1039 \sqrt[3]{\sqrt[3]{g H^5 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U^3+600 \sqrt[3]{\sqrt[3]{g^3 H^7 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U\right)} k^2+54 g^2 H^2 \sqrt[3]{\left(10 H^4 k^4+62 H^2 k^2+81\right)}+243 U^3 \sqrt[3]{\left(28 U-39 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}+3 g H U \sqrt[3]{\left(472 H^6 U k^6-4 \sqrt[3]{\left(71 \sqrt[3]{\sqrt[3]{g H^9 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}-1101 H^4 U\right)} k^4+13500 H^2 U k^2+27 \sqrt[3]{\left(504 U-97 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}\right)}\right)\right)\text{dt}^3\}\sqrt[3840]{\left(H^2 k^2+3\right)^4}-\frac{\sqrt[3]{\left(k^9 \sqrt[3]{\sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}-\sqrt[3]{\left(H^2 k^2+3\right)} U\right)} \sqrt[3]{\left(4 U^3 \sqrt[3]{\left(58 \sqrt[3]{\sqrt[3]{g H^{17} \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U-239 \sqrt[3]{\sqrt[3]{g H^9 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} k^8-12 U \sqrt[3]{\left(213 \sqrt[3]{\sqrt[3]{g^2 H^8+g U \sqrt[3]{\left(979 \sqrt[3]{\sqrt[3]{U-349 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} H^7-232 \sqrt[3]{\sqrt[3]{g H^{13}} \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U^3\right)} k^6+9 \sqrt[3]{\left(-2672 \sqrt[3]{\sqrt[3]{g^2 U H^6+g U^2 \sqrt[3]{\left(4384 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}-5997 \sqrt[3]{\sqrt[3]{U\right)} H^5+1392 \sqrt[3]{\sqrt[3]{g H^9 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U^4+180 \sqrt[3]{\sqrt[3]{g^5 H^{13} \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}\right)} k^4+27 \sqrt[3]{\left(-2703 \sqrt[3]{\sqrt[3]{g^2 U H^4-4070 \sqrt[3]{\sqrt[3]{g U^3 H^3+928 \sqrt[3]{\sqrt[3]{g H^5 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U^4+4515 \sqrt[3]{\sqrt[3]{g^3 H^7 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U^2+372 \sqrt[3]{\sqrt[3]{g^5 H^9 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}\right)} k^2+81 \sqrt[3]{\left(232 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)} U^4+g H \sqrt[3]{\left(1527 \sqrt[3]{\sqrt[3]{g H \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}-1033 \sqrt[3]{\sqrt[3]{U\right)} U^2-883 \sqrt[3]{\sqrt[3]{g^2 H^2 U+157 \sqrt[3]{\sqrt[3]{g^5 H^5 \sqrt[3]{\left(H^2 k^2+3\right)}}}\right)}\right)}\right)\text{dt}^4\}\sqrt[23040]{\sqrt[3]{g H} \sqrt[3]{\left(H^2 k^2+3\right)}^{11/2}}\right)}+O\left(\text{dt}^5\right)\right)\text{dx}^4+O\left(\text{dx}^5\right)\right)
\end{aligned}$$

Out[463]=

$$\text{Out[464]= EA} \parallel \left\{ \left\{ 1 + \frac{i e^{\frac{i \, dx \, k}{2}} (1 - e^{-i \, dx \, k}) (-1 + e^{i \, dt \, w}) H^2 k^3 \, U \, \text{Csc} \left[\frac{dx \, k}{2} \right]}{(6 + 2 H^2 k^2) w}, \frac{i e^{\frac{i \, dx \, k}{2}} (1 - e^{-i \, dx \, k}) (-1 + e^{i \, dt \, w}) H k \, \text{Csc} \left[\frac{dx \, k}{2} \right]}{2 \left(H + \frac{H^3 k^2}{3} \right) w} \right\}, \right. \\ \left. \left\{ \frac{i e^{\frac{i \, dx \, k}{2}} (1 - e^{-i \, dx \, k}) (-1 + e^{i \, dt \, w}) k (g H (3 + H^2 k^2) - 3 U^2) \, \text{Csc} \left[\frac{dx \, k}{2} \right]}{(6 + 2 H^2 k^2) w}, 1 + \frac{i e^{\frac{i \, dx \, k}{2}} (1 - e^{-i \, dx \, k}) (-1 + e^{i \, dt \, w}) k (6 + H^2 k^2) \, U \, \text{Csc} \left[\frac{dx \, k}{2} \right]}{(6 + 2 H^2 k^2) w} \right\} \right\}$$

$$\begin{aligned} & \frac{e^{\frac{i}{\text{dx}} k \{2\}} \left(1 - e^{-i \text{dx} k}\right) \left(-1 + e^{i \text{dt} w}\right) H^2 U \csc \left(\frac{\text{dx} k \{2\}}{\text{right}}\right) k^3}{\left(2 H^2 k^2 + 6\right) w} + 1 \\ & \& \frac{e^{\frac{i}{\text{dx}} k \{2\}} \left(1 - e^{-i \text{dx} k}\right) \left(-1 + e^{i \text{dt} w}\right) w}{H k \csc \left(\frac{\text{dx} k \{2\}}{\text{right}}\right) \{2 \left(\frac{k^2 H^3}{3} + H\right) w} \\ & \frac{e^{\frac{i}{\text{dx}} k \{2\}} \left(1 - e^{-i \text{dx} k}\right) \left(-1 + e^{i \text{dt} w}\right) k}{\left(H^2 k^2 + 3\right) - 3 U^2} \csc \left(\frac{\text{dx} k \{2\}}{\text{right}}\right) \left(2 H^2 k^2 + 6\right) w \\ & \& \frac{e^{\frac{i}{\text{dx}} k \{2\}} \left(1 - e^{-i \text{dx} k}\right) \left(-1 + e^{i \text{dt} w}\right) w}{k \left(H^2 k^2 + 6\right) U} \csc \left(\frac{\text{dx} k \{2\}}{\text{right}}\right) \left(2 H^2 k^2 + 6\right) w + 1 \end{aligned}$$

$$\begin{aligned} \text{Err} \parallel & \left\{ \left(\frac{(-3 \text{ g H k}^2 + 3 \text{ k}^2 \text{ U}^2 - \text{H}^2 \text{ k}^3 \text{ U w}) \text{ dt}^2}{2(3 + \text{H}^2 \text{ k}^2)} - \frac{i \text{H}^2 \text{ k}^3 \text{ U w}^2 \text{ dt}^2}{6(3 + \text{H}^2 \text{ k}^2)} + \frac{\text{H}^2 \text{ k}^3 \text{ U w}^3 \text{ dt}^2}{24(3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) + \right. \\ & \left(-\frac{i(27 \text{ k}^3 + 9 \text{ H}^2 \text{ k}^5 + \text{H}^4 \text{ k}^7) \text{ U dt}}{12(3 + \text{H}^2 \text{ k}^2)^2} - \frac{(-9 \text{ g H k}^4 + 36 \text{ k}^4 \text{ U}^2 + 12 \text{ H}^2 \text{ k}^6 \text{ U}^2 + 2 \text{ H}^4 \text{ k}^8 \text{ U}^2) \text{ dt}^2}{24(3 + \text{H}^2 \text{ k}^2)^2} + \text{O}[\text{dt}]^5 \right) \text{ dx}^2 + \\ & \left(-\frac{1}{8} (\text{k}^4 \text{ U}) \text{ dt} + \left(\frac{3 i \text{ g H k}^5}{16(3 + \text{H}^2 \text{ k}^2)} + \frac{i \text{H}^2 \text{ k}^7 \text{ U}^2}{8(3 + \text{H}^2 \text{ k}^2)} \right) \text{ dt}^2 + \text{O}[\text{dt}]^5 \right) \text{ dx}^3 + \\ & \left(\frac{i(405 \text{ k}^5 \text{ U} + 351 \text{ H}^2 \text{ k}^7 \text{ U} + 116 \text{ H}^4 \text{ k}^9 \text{ U} + 13 \text{ H}^6 \text{ k}^{11} \text{ U}) \text{ dt}}{240(3 + \text{H}^2 \text{ k}^2)^3} + \frac{1}{1440(3 + \text{H}^2 \text{ k}^2)^3} (1161 \text{ g H k}^6 + 837 \text{ g H}^3 \text{ k}^8 + 135 \text{ g H}^5 \text{ k}^{10} - \right. \\ & \left. 351 \text{ k}^6 \text{ U}^2 + 297 \text{ H}^2 \text{ k}^8 \text{ U}^2 + 387 \text{ H}^4 \text{ k}^{10} \text{ U}^2 + 73 \text{ H}^6 \text{ k}^{12} \text{ U}^2) \text{ dt}^2 + \text{O}[\text{dt}]^5 \right) \text{ dx}^4 + \text{O}[\text{dx}]^5, \\ & \left(\left(-\frac{3 \text{ k}^2 \text{ U}}{3 + \text{H}^2 \text{ k}^2} - \frac{3 \text{ k w}}{2(3 + \text{H}^2 \text{ k}^2)} \right) \text{ dt}^2 - \frac{i \text{ k w}^2 \text{ dt}^3}{2(3 + \text{H}^2 \text{ k}^2)} + \frac{\text{k w}^3 \text{ dt}^4}{8(3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) + \left(\left(\frac{3 i \text{ k}^3}{2(3 + \text{H}^2 \text{ k}^2)^2} + \frac{i \text{H}^2 \text{ k}^5}{4(3 + \text{H}^2 \text{ k}^2)^2} \right) \text{ dt} + \frac{3 \text{ k}^4 \text{ U dt}^2}{4(3 + \text{H}^2 \text{ k}^2)^2} + \text{O}[\text{dt}]^5 \right) \\ & \left. \text{ dx}^2 + \left(\frac{3 i \text{ k}^5 \text{ U dt}^2}{8(3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) \text{ dx}^3 + \left(\frac{i(-54 \text{ k}^5 + \text{H}^4 \text{ k}^9) \text{ dt}}{240(3 + \text{H}^2 \text{ k}^2)^3} + \frac{(387 \text{ k}^6 \text{ U} + 279 \text{ H}^2 \text{ k}^8 \text{ U} + 45 \text{ H}^4 \text{ k}^{10} \text{ U}) \text{ dt}^2}{240(3 + \text{H}^2 \text{ k}^2)^3} + \text{O}[\text{dt}]^5 \right) \text{ dx}^4 + \text{O}[\text{dx}]^5 \right\}, \\ & \left\{ \left(-\frac{(\text{k}(3 \text{ g H g H}^3 \text{ k}^2 - 3 \text{ U}^2)(2 \text{ k U} + \text{w})) \text{ dt}^2}{2(3 + \text{H}^2 \text{ k}^2)} - \frac{i(\text{k}(3 \text{ g H g H}^3 \text{ k}^2 - 3 \text{ U}^2) \text{ w}^2 \text{ dt}^3}{6(3 + \text{H}^2 \text{ k}^2)} + \frac{\text{k}(3 \text{ g H g H}^3 \text{ k}^2 - 3 \text{ U}^2) \text{ w}^3 \text{ dt}^4}{24(3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) + \right. \\ & \left(-\frac{i(9 \text{ g H k}^3 + 6 \text{ g H}^3 \text{ k}^5 + \text{g H}^5 \text{ k}^7 + 18 \text{ k}^3 \text{ U}^2 + 3 \text{ H}^2 \text{ k}^5 \text{ U}^2) \text{ dt}}{12(3 + \text{H}^2 \text{ k}^2)^2} + \frac{(-18 \text{ g H k}^4 \text{ U} - 12 \text{ g H}^3 \text{ k}^6 \text{ U} - 2 \text{ g H}^5 \text{ k}^8 \text{ U} - 9 \text{ k}^4 \text{ U}^3) \text{ dt}^2}{12(3 + \text{H}^2 \text{ k}^2)^2} + \text{O}[\text{dt}]^5 \right) \text{ dx}^2 + \\ & \left(-\frac{1}{8} (\text{g H k}^4) \text{ dt} + \frac{i(6 \text{ g H k}^5 \text{ U} + 2 \text{ g H}^3 \text{ k}^7 \text{ U} - 3 \text{ k}^5 \text{ U}^3) \text{ dt}^2}{8(3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) \text{ dx}^3 + \\ & \left(\frac{i(351 \text{ g H k}^5 + 351 \text{ g H}^3 \text{ k}^7 + 117 \text{ g H}^5 \text{ k}^9 + 13 \text{ g H}^7 \text{ k}^{11} + 54 \text{ k}^5 \text{ U}^2 - \text{H}^4 \text{ k}^9 \text{ U}) \text{ dt}}{240(3 + \text{H}^2 \text{ k}^2)^3} + \frac{1}{720(3 + \text{H}^2 \text{ k}^2)^3} (1971 \text{ g H k}^6 \text{ U} + 1971 \text{ g H}^3 \text{ k}^8 \text{ U} + \right. \\ & \left. 657 \text{ g H}^5 \text{ k}^{10} \text{ U} + 73 \text{ g H}^7 \text{ k}^{12} \text{ U} - 1161 \text{ k}^6 \text{ U}^3 - 837 \text{ H}^2 \text{ k}^8 \text{ U}^3 - 135 \text{ H}^4 \text{ k}^{10} \text{ U}^3) \text{ dt}^2 + \text{O}[\text{dt}]^5 \right) \text{ dx}^4 + \\ & \text{O}[\text{dx}]^5, \left(\frac{(-3 \text{ g H k}^2 - 9 \text{ k}^2 \text{ U}^2 - \text{H}^2 \text{ k}^4 \text{ U}^2 - 6 \text{ k U w} - \text{H}^2 \text{ k}^3 \text{ U w}) \text{ dt}^2}{2(3 + \text{H}^2 \text{ k}^2)} - \frac{i \text{ k}(6 + \text{H}^2 \text{ k}^2) \text{ U w}^2 \text{ dt}^3}{6(3 + \text{H}^2 \text{ k}^2)} + \frac{\text{k}(6 + \text{H}^2 \text{ k}^2) \text{ U w}^3 \text{ dt}^4}{24(3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) + \\ & \left(-\frac{i(-9 \text{ k}^3 \text{ U} + 3 \text{ H}^2 \text{ k}^5 \text{ U} + \text{H}^4 \text{ k}^7 \text{ U}) \text{ dt}}{12(3 + \text{H}^2 \text{ k}^2)^2} + \frac{(9 \text{ g H k}^4 - 12 \text{ H}^2 \text{ k}^6 \text{ U}^2 - 2 \text{ H}^4 \text{ k}^8 \text{ U}^2) \text{ dt}^2}{24(3 + \text{H}^2 \text{ k}^2)^2} + \text{O}[\text{dt}]^5 \right) \text{ dx}^2 + \\ & \left(-\frac{1}{8} (\text{k}^4 \text{ U}) \text{ dt} + \frac{i \text{ k}^5 (3 \text{ g H} + 12 \text{ U}^2 + 2 \text{ H}^2 \text{ k}^2 \text{ U}^2) \text{ dt}^2}{16(3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) \text{ dx}^3 + \\ & \left(\frac{i(297 \text{ k}^5 + 351 \text{ H}^2 \text{ k}^7 + 118 \text{ H}^4 \text{ k}^9 + 13 \text{ H}^6 \text{ k}^{11}) \text{ U dt}}{240(3 + \text{H}^2 \text{ k}^2)^3} + \frac{1}{1440(3 + \text{H}^2 \text{ k}^2)^3} (1161 \text{ g H k}^6 + 837 \text{ g H}^3 \text{ k}^8 + 135 \text{ g H}^5 \text{ k}^{10} + \right. \\ & \left. 4293 \text{ k}^6 \text{ U}^2 + 3645 \text{ H}^2 \text{ k}^8 \text{ U}^2 + 927 \text{ H}^4 \text{ k}^{10} \text{ U}^2 + 73 \text{ H}^6 \text{ k}^{12} \text{ U}^2) \text{ dt}^2 + \text{O}[\text{dt}]^5 \right) \text{ dx}^4 + \text{O}[\text{dx}]^5 \Big\} \end{aligned}$$

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Out[467]= Eerr || \left(
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\begin{array}{cc}
\left(\frac{\left(-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right)}{\text{dt}^2}\right)^2\left(H^2 k^2+3\right)-\frac{i}{H^2 k^3 U w^2} \frac{\left(H^2 k^2+3\right)}{\text{dt}^3}\left\{6 \left(H^2 k^2+3\right)+\frac{H^2 k^3 U w^3}{\text{dt}^4}\right\} 24 \left(H^2 k^2+3\right)+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right)+\left(-\frac{i}{\left(H^4 k^7+9 H^2 k^5+27 k^3\right)} U \frac{\text{dt}}{\text{dt}}\right)\left\{12 \left(H^2 k^2+3\right)^2-\frac{\left(2 H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H k^4\right)}{\text{dt}^2}\right\} 24 \left(H^2 k^2+3\right)^2+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \\
\frac{\text{dx}^2}{\text{dt}^2}+\left(-\frac{1}{8}\right) \frac{\left(k^4 U\right)}{\text{dt}} \frac{\text{dt}}{\text{dt}}+\frac{i}{\left(H^2 U^2 k^7\right)} 8 \left(H^2 k^2+3\right)+\frac{3 i}{g H k^5}\left\{16 \left(H^2 k^2+3\right)\right\} \frac{\text{dt}}{\text{dt}}+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^3}{\text{dt}^3}+\frac{i}{\left(13 H^6 U k^{11}+116 H^4 U k^9+351 H^2 U k^7+405 U k^5\right)} \frac{\text{dt}}{\text{dt}}\left\{240 \left(H^2 k^2+3\right)^3+\frac{\left(73 H^6 U^2 k^{12}+135 g H^5 k^{10}+387 H^4 U^2 k^{10}+837 g H^3 k^8+297 H^2 U^2 k^8-351 U^2 k^6+1161 g H k^6\right)}{\text{dt}^2}\right\} 1440 \\
\left(H^2 k^2+3\right)^3+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^4}{\text{dt}^4}+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right) \\
& \frac{\left(-\frac{3 U k^2}{H^2 k^2+3}-\frac{3 w k}{2 \left(H^2 k^2+3\right)}\right)}{\text{dt}^2}-\frac{i}{k w^2} \frac{\text{dt}^3}{\text{dt}^3}\left\{2 \left(H^2 k^2+3\right)+\frac{k w^3}{\text{dt}^4}\right\} 8 \\
\left(H^2 k^2+3\right)+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right)+\frac{\left(H^2 k^5\right)}{4 \left(H^2 k^2+3\right)^2}+\frac{3 i}{k^3}\left\{2 \left(H^2 k^2+3\right)^2\right\} \frac{\text{dt}}{\text{dt}}+\frac{3 k^4 U}{\left(H^2 k^2+3\right)^2}+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^2}{\text{dt}^2}+\frac{3 i}{\left(H^2 k^2+3\right)} U \frac{\text{dt}^2}{\text{dt}^2}\left\{8 \left(H^2 k^2+3\right)+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^3}{\text{dt}^3}+\frac{i}{\left(H^4 k^9-54 k^5\right)} \frac{\text{dt}}{\text{dt}}\left\{240 \left(H^2 k^2+3\right)^3+\frac{\left(45 H^4 U k^{10}+279 H^2 U k^8+387 U k^6\right)}{\text{dt}^2}\right\} 240 \left(H^2 k^2+3\right)^3+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^4}{\text{dt}^4}+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right) \\
\left(-\frac{\left(k \left(g k^2 H^3+3 g H-3 U^2\right)\right)}{\left(2 k U+w\right)}\right) \frac{\text{dt}^2}{\text{dt}^2}\left(H^2 k^2+3\right)-\frac{i}{k \left(g k^2 H^3+3 g H-3 U^2\right)} w^2 \frac{\text{dt}^3}{\text{dt}^3}\left\{6 \left(H^2 k^2+3\right)+\frac{k \left(g k^2 H^3+3 g H-3 U^2\right)}{w^3} \frac{\text{dt}^4}{\text{dt}^4}\right\} 24 \left(H^2 k^2+3\right)+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right)+\left(-\frac{i}{\left(g H^5 k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right)} \frac{\text{dt}}{\text{dt}}\right)\left\{12 \left(H^2 k^2+3\right)^2+\frac{\left(-2 g H^5 U k^8-12 g H^3 U k^6-9 U^3 k^4-18 g H U k^4\right)}{\text{dt}^2}\right\} 12 \left(H^2 k^2+3\right)^2+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^2}{\text{dt}^2}+\left(-\frac{1}{8}\right) \frac{\left(g H k^4\right)}{\text{dt}} \frac{\text{dt}}{\text{dt}}+\frac{i}{\left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right)} \frac{\text{dt}^2}{\text{dt}^2}\left\{8 \left(H^2 k^2+3\right)+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^3}{\text{dt}^3}+\frac{i}{\left(13 g H^7 k^{11}+117 g H^5 k^9-H^4 U^2 k^9+351 g H^3 k^7+54 U^2 k^5+351 g H k^5\right)} \frac{\text{dt}}{\text{dt}}\left\{240 \left(H^2 k^2+3\right)^3+\frac{\left(73 g H^7 U k^{12}-135 H^4 U^3 k^{10}+657 g H^5 U k^{10}-837 H^2 U^3 k^8+1971 g H^3 U k^8-1161 U^3 k^6+1971 g H U k^6\right)}{\text{dt}^2}\right\} 720 \left(H^2 k^2+3\right)^3+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^4}{\text{dt}^4}+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right) \\
& \frac{\left(-H^2 U^2 k^4-H^2 U w k^3-9 U^2 k^2-3 g H k^2-6 U w k\right)}{\text{dt}^2}\right)^2\left(H^2 k^2+3\right)-\frac{i}{k \left(H^2 k^2+6\right)} U w^2 \frac{\text{dt}^3}{\text{dt}^3}\left\{6 \left(H^2 k^2+3\right)+\frac{k \left(H^2 k^2+6\right)}{U w^3} \frac{\text{dt}^4}{\text{dt}^4}\right\} 24 \left(H^2 k^2+3\right)+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right)+\left(-\frac{i}{\left(H^4 U k^7+3 H^2 U k^5-9 U k^3\right)} \frac{\text{dt}}{\text{dt}}\right)\left\{12 \left(H^2 k^2+3\right)^2+\frac{\left(-2 H^4 U^2 k^8-12 H^2 U^2 k^6+9 g H k^4\right)}{\text{dt}^2}\right\} 24 \left(H^2 k^2+3\right)^2+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^2}{\text{dt}^2}+\left(-\frac{1}{8}\right) \frac{\left(k^4 U\right)}{\text{dt}} \frac{\text{dt}}{\text{dt}}+\frac{i}{\left(2 H^2 k^2 U^2+12 U^2+3 g H\right)} \frac{\text{dt}^2}{\text{dt}^2}\left\{16 \left(H^2 k^2+3\right)+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^3}{\text{dt}^3}+\frac{i}{\left(13 H^6 k^{11}+118 H^4 k^9+351 H^2 k^7+297 k^5\right)} U \frac{\text{dt}}{\text{dt}}\left\{240 \left(H^2 k^2+3\right)^3+\frac{\left(73 H^6 U^2 k^{12}+135 g H^5 k^{10}+927 H^4 U^2 k^{10}+837 g H^3 k^8+3645 H^2 U^2 k^8+4293 U^2 k^6+1161 g H k^6\right)}{\text{dt}^2}\right\} 1440 \\
\left(H^2 k^2+3\right)^3+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right)\right) \frac{\text{dx}^4}{\text{dt}^4}+O\left(\frac{\text{dt}^5}{\text{dt}^5}\right) \\

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\end{array}

\right)

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In[468]:= KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
KurFWS = KurF /. ap → 0 /. am → (U - Sqrt[g*H]);
KurFWSeta =
  KurFWS /. fp → (H*v + U*Rpp*n) /. fm → (H*v + U*Rmp*n) /. qp → Rpp*n /.
  qm → Rmp*n;
KurFWSeta = KurFWSeta /. v → (GGp*G + Gnp*n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt*(1 - Exp[-I*k*dx])/dx*Kfnnp;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U*Rpp*G + U*H*v + g*H*Rpp*n) /.
  fm → (U*Rmp*G + U*H*v + g*H*Rmp*n) /. qp → Rpp*G /. qm → Rmp*G;
KurFWSG = KurFWSG /. v → (GGp*G + Gnp*n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U*H*GG2 + U/2*(Rm + Rp) - (Sqrt[g*H])/(2)*(Rp - Rm);
FGG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2/2;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2 + EigvFmat2*EigvFmat2/2]/(I*dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}];

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Text[Row[{" U > Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
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Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
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Text[" "]
Text[" "]
Text[Row[{"Omega error || ", RKstepTayr}]]
Text[Row[{"Omega error || ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA || ", EA}]]
Text[Row[{"EA || ", TeXForm[EA]}]]
Text[Row[{"Eerr || ", Eerr}]]
Text[Row[{"Eerr || ", TeXForm[Eerr]}]]

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Out[502]= $U > \text{Sqrt}(gH)$

Out[503]=

Out[504]= $Fnn \parallel Gnp H + Rpp U$

Out[505]= $Fnn \parallel \text{\texttt{Gnp}} H + \text{\texttt{Rpp}} U$

Out[506]= $Fnn \text{ error } \parallel$

$$\left(-\frac{(H^2 k^3 U w) dt^2}{2(3+H^2 k^2)} - \frac{i H^2 k^3 U w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left(-\frac{i(27 k^3 + 9 H^2 k^5 + H^4 k^7) U dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(\frac{1}{8} k^4 U dt + O[dt]^4 \right) dx^3 + O[dx]^4$$

Out[507]= Fnn error ||

$$\begin{aligned} & \left(-\frac{\text{dt}^2}{2} \left(H^2 k^3 U w \right) \right) \left\{ 2 \left(H^2 k^2 + 3 \right) \right\} - \frac{i}{6} \text{dt}^3 H^2 k^3 U w^2 \left\{ 6 \right. \\ & \left. \left(H^2 k^2 + 3 \right) \right\} + O\left(\text{dt}^4 \right) \right\} + \text{dx}^2 \left(-\frac{i}{12} \left(H^4 k^7 + 9 H^2 \right. \right. \\ & \left. \left. k^5 + 27 k^3 \right) U \text{dt} \right\} \left\{ 12 \left(H^2 k^2 + 3 \right)^2 \right\} + O\left(\text{dt}^4 \right) \right\} + \text{dx}^3 \\ & \left(\frac{1}{8} k^4 U \text{dt} + O\left(\text{dt}^4 \right) \right) + O\left(\text{dx}^4 \right) \end{aligned}$$

Out[508]=

Out[509]= FnG || GGp H

Out[510]= FnG || \text{GGp} H

Out[511]= FnG error || $\left(-\frac{3(k w) dt^2}{2(3+H^2 k^2)} - \frac{i k w^2 dt^3}{2(3+H^2 k^2)} + O[dt]^4 \right) + \left(\frac{i(6 k^3 + H^2 k^5) dt}{4(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + O[dx]^4$

Out[512]= FnG error || $\left(-\frac{3}{2} \text{dt}^2 (k w) \right) \left\{ 2 \left(H^2 k^2 + 3 \right) \right\} - \frac{i}{6} \text{dt}^3 k w^2 \left\{ 2 \left(H^2 k^2 + 3 \right) \right\} + O\left(\text{dt}^4 \right) \right\} + \text{dx}^2 \left(\frac{i}{12} \left(H^2 k^5 + 6 k^3 \right) \right. \right.$
 $\left. \left. \text{dt} \right\} \left\{ 4 \left(H^2 k^2 + 3 \right)^2 \right\} + O\left(\text{dt}^4 \right) \right\} + O\left(\text{dx}^4 \right)$

Out[513]=

Out[514]= FGn || H (g Rpp + Gnp U)

Out[515]= FGn || H (g \text{Rpp} + \text{Gnp} U)

Out[516]= FGn error || $\left(-\frac{(k(3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2(3+H^2 k^2)} - \frac{i k(3 g H + g H^3 k^2 - 3 U^2) w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) +$
 $\left(-\frac{i(9 g H k^3 + 6 g H^3 k^5 + g H^5 k^7 + 18 k^3 U^2 + 3 H^2 k^5 U^2) dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left(\frac{1}{8} g H k^4 dt + O[dt]^4 \right) dx^3 + O[dx]^4$

Out[517]= FGn error ||

$$\begin{aligned} & \left(-\frac{\text{dt}^2}{2} \left(k w \left(g H^3 k^2 + 3 g H - 3 U^2 \right) \right) \right) \left\{ 2 \left(H^2 k^2 + 3 \right) \right\} - \frac{i}{6} \text{dt}^3 k w^2 \left(g H^3 k^2 + 3 g H - 3 U^2 \right) \left\{ 6 \left(H^2 \right. \right. \\ & \left. \left. k^2 + 3 \right) \right\} + O\left(\text{dt}^4 \right) \right\} + \text{dx}^2 \left(-\frac{i}{12} \left(g H^5 \right. \right. \\ & \left. \left. k^7 + 6 g H^3 k^5 + 3 H^2 U^2 k^5 + 18 U^2 k^3 + 9 g H k^3 \right) \text{dt} \right\} \left\{ 12 \right. \\ & \left. \left(H^2 k^2 + 3 \right)^2 \right\} + O\left(\text{dt}^4 \right) \right\} + \text{dx}^3 \left(\frac{1}{8} g H k^4 \right. \\ & \left. \text{dt} + O\left(\text{dt}^4 \right) \right) + O\left(\text{dx}^4 \right) \end{aligned}$$

Out[518]=

Out[519]= FGG || (GGp H + Rpp) U

Out[520]= FGG || U (\text{GGp} H + \text{Rpp})

Out[521]= FGG error || $\left(-\frac{(k(6+H^2 k^2) U w) dt^2}{2(3+H^2 k^2)} - \frac{i k(6+H^2 k^2) U w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left(-\frac{i(-9 k^3 + 3 H^2 k^5 + H^4 k^7) U dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 +$
 $\left(\frac{1}{8} k^4 U dt + O[dt]^4 \right) dx^3 + \left(\frac{i(297 k^5 + 351 H^2 k^7 + 118 H^4 k^9 + 13 H^6 k^{11}) U dt}{240(3+H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$

Out[522]= FGG error ||

$$\begin{aligned} & \left(-\frac{\text{dt}^2}{2} \left(k U w \left(H^2 k^2 + 6 \right) \right) \right) \left(2 \left(H^2 k^2 + 3 \right) \right) - \frac{i}{3} \text{dt}^3 k \\ & U w^2 \left(H^2 k^2 + 6 \right) \left(6 \left(H^2 k^2 + 3 \right) \right) + O \left(\text{dt}^4 \right) + \text{dx}^2 \\ & \left(-\frac{i}{12} \left(H^4 k^7 + 3 H^2 k^5 - 9 k^3 \right) U \text{dt} \right) \left(12 \left(H^2 \right. \right. \\ & \left. \left. k^2 + 3 \right)^2 \right) + O \left(\text{dt}^4 \right) + \text{dx}^3 \left(\frac{1}{8} k^4 \right. \\ & U \text{dt} + O \left(\text{dt}^4 \right) \right) + \text{dx}^4 \left(\frac{i}{13} H^6 \right. \\ & \left. k^{11} + 118 H^4 k^9 + 351 H^2 k^7 + 297 k^5 \right) U \text{dt} \left(240 \left(H^2 \right. \right. \\ & \left. \left. k^2 + 3 \right)^3 \right) + O \left(\text{dt}^4 \right) + O \left(\text{dx}^5 \right) \end{aligned}$$

Out[523]=

Out[524]=

Out[525]= Omega error ||

$$\begin{aligned} & \left\{ \frac{1}{6(3+H^2 k^2)^2} k^3 \left(\sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) dt^2 + \right. \\ & \frac{i k^4 \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right)^2}{8(3+H^2 k^2)^2} dt^3 - \frac{1}{20(3+H^2 k^2)^4} \left(k^5 \left(\sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right)^3 \right. \\ & \left. \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \right) dt^4 + O[dt]^5 \Bigg\} + \\ & \left(\frac{k^3 \left(-3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 2(3+H^2 k^2)^2 U \right)}{24(3+H^2 k^2)^2} + \frac{1}{48(3+H^2 k^2)^3} k^5 \left(g \left(-9 \sqrt{3} H \sqrt{g H (3+H^2 k^2)} + 18 H^3 k^2 U + 6 H^5 k^4 U \right) + \right. \right. \\ & U^2 \left(27 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 54 U + 2 H^6 k^6 U + 3 k^2 \left(7 \sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} + 18 H^2 U \right) + \right. \\ & \left. \left. 2 k^4 \left(2 \sqrt{3} \sqrt{g H^9 (3+H^2 k^2)} + 9 H^4 U \right) \right) \right) dt^2 + \\ & \frac{1}{48(3+H^2 k^2)^3} i k^6 \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \\ & \left(-9 g H + U \left(3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 18 U + 2 H^4 k^4 U + 2 k^2 \left(\sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} + 6 H^2 U \right) \right) \right) \\ & dt^3 - \frac{1}{96(3+H^2 k^2)^4} \left(k^7 \left(-3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 2(3+H^2 k^2)^2 U \right) \right. \\ & \left. \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right)^2 \right) dt^4 + O[dt]^5 \Bigg\} dx^2 + \\ & \left(\frac{1}{16} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{3+H^2 k^2}} + 2 U \right) + \frac{1}{32(3+H^2 k^2)^2} i k^6 \left(3 g H \left(\sqrt{3} \sqrt{g H (3+H^2 k^2)} + 4(3+H^2 k^2) U \right) + \right. \right. \\ & \left. \left. 18 H^3 k^2 U + 6 H^5 k^4 U \right) + \frac{1}{48(3+H^2 k^2)^3} i k^6 \left(3 g H \left(\sqrt{3} \sqrt{g H (3+H^2 k^2)} + 4(3+H^2 k^2) U \right) + \right. \right. \\ & \left. \left. 18 H^3 k^2 U + 6 H^5 k^4 U \right) + \frac{1}{96(3+H^2 k^2)^4} i k^7 \left(-3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 2(3+H^2 k^2)^2 U \right) \right) dx^3 + \end{aligned}$$

$$\begin{aligned}
& U^2 \left(15 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^2 k^2 U + k^2 \left(5 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 12 H^2 U \right) \right) dt^2 - \\
& \frac{1}{32 (3 + H^2 k^2)^2} \left(k^7 \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right. \\
& \quad \left. \left(3 g H + U \left(3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U \right) \right) \right) dt^3 - \\
& \frac{1}{64 (3 + H^2 k^2)^3} i k^8 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U \right) \\
& \quad \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right)^2 dt^4 + O[dt]^5 \Big) dx^3 + \\
& \left(- \left(\left(k^5 \left(3 \sqrt{3} g H (177 + 124 H^2 k^2 + 20 H^4 k^4) + 104 \left(9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \left(1920 \left(\sqrt{g H (3 + H^2 k^2)}^{5/2} \right) \right) \right) - \\
& \left(\left(k^7 \left(27 \sqrt{3} g^2 H^2 (167 + 124 H^2 k^2 + 20 H^4 k^4) + g H U \left(21 429 \sqrt{3} H^2 k^2 U + \right. \right. \right. \right. \\
& \quad 764 \sqrt{3} H^6 k^6 U + 81 \left(232 \sqrt{g H (3 + H^2 k^2)} + 267 \sqrt{3} U \right) + \\
& \quad 24 k^4 \left(84 \sqrt{g H^9 (3 + H^2 k^2)} + 293 \sqrt{3} H^4 U \right) \right) + \\
& \quad 16 \left(459 \sqrt{g H (3 + H^2 k^2)} U^3 + 153 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 17 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} \right. \\
& \quad \left. U^3 + 9 k^2 \left(88 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 51 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) dt^2 \Big) / \\
& \left(11 520 \left(\sqrt{g H (3 + H^2 k^2)}^{7/2} \right) \right) - \frac{1}{3840 (3 + H^2 k^2)^4} i k^8 \left(54 g^2 H^2 (81 + 62 H^2 k^2 + 10 H^4 k^4) + \right. \\
& \quad 84 H^8 k^8 U^4 + 243 U^3 \left(39 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 28 U \right) + 36 k^4 U^3 \\
& \quad \left(85 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 126 H^4 U \right) + 4 k^6 U^3 \left(83 \sqrt{3} \sqrt{g H^{13} (3 + H^2 k^2)} + 252 H^6 U \right) + \\
& \quad 9 k^2 \left(600 \sqrt{3} \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 1039 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} U^3 + 1008 H^2 U^4 \right) + \\
& \quad 3 g H U \left(13 500 H^2 k^2 U + 472 H^6 k^6 U + 27 \left(97 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 504 U \right) + \right. \\
& \quad \left. 4 k^4 \left(71 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 1101 H^4 U \right) \right) \Big) dt^3 + \frac{1}{23 040 \sqrt{g H (3 + H^2 k^2)}^{11/2}} \\
& k^9 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left(4 k^8 U^3 \left(239 \sqrt{3} g H^9 + 58 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& \quad 27 k^2 \left(372 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 2703 \sqrt{3} g^2 H^4 U + 4515 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. 4070 \sqrt{3} g H^3 U^3 + 928 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) + 12 k^6 U \\
& \quad \left(213 \sqrt{3} g^2 H^8 + 232 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + g H^7 U \left(349 \sqrt{g H (3 + H^2 k^2)} + 979 \sqrt{3} U \right) \right) + \\
& \quad 81 \left(157 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 883 \sqrt{3} g^2 H^2 U + 232 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. g H U^2 \left(1527 \sqrt{g H (3 + H^2 k^2)} + 1033 \sqrt{3} U \right) \right) + \\
& \quad 9 k^4 \left(180 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 2672 \sqrt{3} g^2 H^6 U + 1392 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. g H^5 U^2 \left(4384 \sqrt{g H (3 + H^2 k^2)} + 5997 \sqrt{3} U \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5,
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{6(3+H^2 k^2)^2} k^3 \left(-\sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \right. \\
& \quad dt^2 + \\
& \quad \frac{i k^4 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right)^2}{8(3+H^2 k^2)^2} - \frac{1}{20(3+H^2 k^2)^4} \\
& \quad \left(k^5 \left(-\sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right)^3 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \right) \\
& \quad \left. dt^4 + O[dt]^5 \right) + \\
& \left(\frac{k^3 \left(3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 2(3+H^2 k^2)^2 U \right)}{24(3+H^2 k^2)^2} + \frac{1}{48(3+H^2 k^2)^3} k^5 \left(3 g \left(3 \sqrt{3} H \sqrt{g H (3+H^2 k^2)} + 6 H^3 k^2 U + 2 H^5 k^4 U \right) + \right. \\
& \quad U^2 \left(-27 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 54 U + 2 H^6 k^6 U - 3 k^2 \left(7 \sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} - 18 H^2 U \right) - \right. \\
& \quad \left. \left. 2 k^4 \left(2 \sqrt{3} \sqrt{g H^9 (3+H^2 k^2)} - 9 H^4 U \right) \right) \right) dt^2 + \\
& \quad \frac{1}{48(3+H^2 k^2)^3} i k^6 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \\
& \quad \left(-9 g H + U \left(-3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 18 U + 2 H^4 k^4 U - 2 k^2 \left(\sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} - 6 H^2 U \right) \right) \right) \\
& \quad dt^3 - \frac{1}{96(3+H^2 k^2)^4} \\
& \quad \left(k^7 \left(3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 2(3+H^2 k^2)^2 U \right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right)^2 \right) \\
& \quad \left. dt^4 + O[dt]^5 \right) dx^2 + \\
& \left(-\frac{1}{16} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{3+H^2 k^2}} - 2 U \right) + \frac{1}{32(3+H^2 k^2)^2} i k^6 \left(-3 g H \left(\sqrt{3} \sqrt{g H (3+H^2 k^2)} - 4(3+H^2 k^2) U \right) + \right. \\
& \quad U^2 \left(-15 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 18 U + 2 H^4 k^4 U + k^2 \left(-5 \sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} + 12 H^2 U \right) \right) \right) \\
& \quad dt^2 - \frac{1}{32(3+H^2 k^2)^2} \left(k^7 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \right. \\
& \quad \left. \left(3 g H + U \left(-3 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 2(3+H^2 k^2) U \right) \right) \right) dt^3 - \\
& \quad \frac{1}{64(3+H^2 k^2)^3} i k^8 \left(-\sqrt{3} \sqrt{g H (3+H^2 k^2)} + 2(3+H^2 k^2) U \right) \\
& \quad \left. \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right)^2 dt^4 + O[dt]^5 \right)
\end{aligned}$$

$$\begin{aligned}
& dx^3 + \left(\left(k^5 \left(3 \sqrt{3} g H (177 + 124 H^2 k^2 + 20 H^4 k^4) - \right. \right. \right. \\
& \quad \left. \left. \left. 104 \left(9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \right. \\
& \quad \left(1920 \sqrt{g H (3 + H^2 k^2)^{5/2}} \right) + \left(k^7 \left(27 \sqrt{3} g^2 H^2 (167 + 124 H^2 k^2 + 20 H^4 k^4) + \right. \right. \\
& \quad \left. \left. g H U \left(21429 \sqrt{3} H^2 k^2 U + 764 \sqrt{3} H^6 k^6 U + 81 \left(-232 \sqrt{g H (3 + H^2 k^2)} + 267 \sqrt{3} U \right) - \right. \right. \right. \\
& \quad \left. \left. \left. 24 k^4 \left(84 \sqrt{g H^9 (3 + H^2 k^2)} - 293 \sqrt{3} H^4 U \right) \right) \right) - \right. \\
& \quad \left. 16 \left(459 \sqrt{g H (3 + H^2 k^2)} U^3 + 153 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 17 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + \right. \right. \\
& \quad \left. \left. 9 k^2 \left(88 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 51 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) dt^2 \Big/ \\
& \quad \left(11520 \sqrt{g H (3 + H^2 k^2)^{7/2}} \right) - \frac{1}{3840 (3 + H^2 k^2)^4} i k^8 \left(54 g^2 H^2 (81 + 62 H^2 k^2 + 10 H^4 k^4) + 84 H^8 k^8 U^4 + \right. \\
& \quad \left. 243 U^3 \left(-39 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 28 U \right) + 36 k^4 U^3 \left(-85 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} + 126 H^4 U \right) + \right. \\
& \quad \left. 4 k^6 U^3 \left(-83 \sqrt{3} \sqrt{g H^{13} (3 + H^2 k^2)} + 252 H^6 U \right) - \right. \\
& \quad \left. 9 k^2 \left(600 \sqrt{3} \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 1039 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} U^3 - 1008 H^2 U^4 \right) + \right. \\
& \quad \left. 3 g H U \left(13500 H^2 k^2 U + 472 H^6 k^6 U + 27 \left(-97 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 504 U \right) - \right. \right. \\
& \quad \left. \left. 4 k^4 \left(71 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} - 1101 H^4 U \right) \right) \right) dt^3 - \frac{1}{23040 \left(\sqrt{g H (3 + H^2 k^2)} \right)^{11/2}} \\
& \quad \left(k^9 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right) \left(4 k^8 U^3 \left(-239 \sqrt{3} g H^9 + 58 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \right. \\
& \quad \left. \left. 27 k^2 \left(372 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 2703 \sqrt{3} g^2 H^4 U + 4515 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \right. \\
& \quad \left. \left. \left. 4070 \sqrt{3} g H^3 U^3 + 928 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) + \right. \right. \\
& \quad \left. \left. 9 k^4 \left(180 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 2672 \sqrt{3} g^2 H^6 U + 1392 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + g H^5 \right. \right. \right. \\
& \quad \left. \left. \left. U^2 \left(4384 \sqrt{g H (3 + H^2 k^2)} - 5997 \sqrt{3} U \right) \right) + 81 \left(157 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 883 \sqrt{3} \right. \right. \right. \\
& \quad \left. \left. \left. g^2 H^2 U + 232 \sqrt{g H (3 + H^2 k^2)} U^4 + g H U^2 \left(1527 \sqrt{g H (3 + H^2 k^2)} - 1033 \sqrt{3} U \right) \right) \right) - \right. \\
& \quad \left. 12 k^6 U \left(213 \sqrt{3} g^2 H^8 - 232 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + \right. \right. \\
& \quad \left. \left. g H^7 U \left(-349 \sqrt{g H (3 + H^2 k^2)} + 979 \sqrt{3} U \right) \right) \right) dt^4 + O[dt]^5 \Big\} dx^4 + O[dx]^5 \Big\}
\end{aligned}$$

Out[526]= Ω error ||

$$\begin{aligned}
& \left(\left(\left(\frac{k^3}{\sqrt{3}} \sqrt{g H (H^2 k^2 + 3)} U + \sqrt{3} \sqrt{g H (H^2 k^2 + 3)} \right) \sqrt{3} g H + U \right. \right. \\
& \quad \left. \left(\left(\left(H^2 k^2 + 3 \right) U + 2 \sqrt{3} \sqrt{g H (H^2 k^2 + 3)} \right) \sqrt{3} \sqrt{g H (H^2 k^2 + 3)} \right) \right) dt^2 \Big/ \left(6 \right. \\
& \quad \left(H^2 k^2 + 3 \right)^2 + \frac{i k^4}{\sqrt{3}} \left(3 g H + U \sqrt{3} \sqrt{g H (H^2 k^2 + 3)} U + 2 \sqrt{3} \sqrt{g H (H^2 k^2 + 3)} \right) \right) dt^3 \Big\} \\
& \quad \left(8 \left(H^2 k^2 + 3 \right)^2 - \frac{k^5}{\sqrt{3}} \sqrt{g H (H^2 k^2 + 3)} U + \sqrt{3} \sqrt{g H (H^2 k^2 + 3)} \right) dt^4 + O[dt]^5 \Big\} dx^4 + O[dx]^5 \Big\}
\end{aligned}$$

$$\begin{aligned}
& k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right) \text {dt}^4\{20 \left(H^2\right. \\
& k^2+3\right)^4\}+O\left(\text {dt}^5\right)\right)+\left(\frac{k^3 \left(2 \left(H^2 k^2+3\right)^2 U-3 \sqrt{3}\right. \right. \\
& \left.\left.\sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right)\{24 \left(H^2 k^2+3\right)^2\}+\frac{k^5 \left(\left(2 H^6 U k^6+2\right. \right. \\
& \left.\left.9 U H^4+2 \sqrt{3}\right) \sqrt{g H^9 \left(H^2 k^2+3\right)\right)\right) k^4+3 \left(18 U H^2+7 \sqrt{3}\right) \sqrt{g} \\
& H^5 \left(H^2 k^2+3\right)\right) k^2+54 U+27 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right) U^2+g \\
& \left(6 k^4 U H^5+18 k^2 U H^3-9 \sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right) H}\right)\right) \text {dt}^2\{48 \\
& \left(H^2 k^2+3\right)^3\}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3}\right) \sqrt{g} \right. \\
& H \left(H^2 k^2+3\right)\right)\right)\right) \left(U \left(2 H^4 U k^4+2 \left(6 U H^2+\sqrt{3}\right) \sqrt{g H^5} \right. \right. \\
& \left.\left.\left(H^2 k^2+3\right)\right)\right) k^2+18 U+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right)\right)-9 g H}\right) \\
& \text {dt}^3\{48 \left(H^2 k^2+3\right)^3\}-\frac{\left(k^7 \left(2 \left(H^2 k^2+3\right)^2 U-3 \sqrt{3}\right) \sqrt{g} \right. \\
& H \left(H^2 k^2+3\right)\right)\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3}\right) \sqrt{g H \left(H^2\right.} \right. \\
& \left.\left.k^2+3\right)\right)\right)\right)^2\right) \text {dt}^4\{96 \left(H^2 k^2+3\right)^4\}+O\left(\text {dt}^5\right)\right) \\
& \text {dx}^2+\left(\frac{1}{16}\right) i k^4 \left(2 U+\sqrt{3}\right) \sqrt{\frac{g H}{H^2 k^2+3}}\right)+\frac{i k^6}{\left(2 H^4 U k^4+\left(12 U H^2+5 \sqrt{3}\right) \sqrt{g H^5} \left(H^2 k^2+3\right)\right)} \\
& k^2+18 U+15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right) U^2+3 g H \left(4 \left(H^2 k^2+3\right) U+\sqrt{3}\right) \sqrt{g} \\
& H \left(H^2 k^2+3\right)\right)\right) \text {dt}^2\{32 \left(H^2 k^2+3\right)^2\}-\frac{\left(k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right) \left(3 g H+U \left(2 \left(H^2 k^2+3\right) U+3 \sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right)} \\
& \text {dt}^3\{32 \left(H^2 k^2+3\right)^2\}-\frac{i k^8 \left(2 \left(H^2 k^2+3\right) U+\sqrt{3}\right) \sqrt{g H} \left(H^2 k^2+3\right)\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right)\right)\right)^2 \text {dt}^4\{64 \left(H^2 k^2+3\right)^3\}+O\left(\text {dt}^5\right)\right) \\
& \text {dx}^3+\left(-\frac{k^5}{\sqrt{3}}\right) g H \left(20 H^4 k^4+124 H^2 k^2+177\right)+104 \sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)} U\right)\}\{1920 \sqrt{g H} \left(H^2 k^2+3\right)^{5/2}\right)\}-\frac{\left(k^7 \left(27 \sqrt{3}\right) g^2 \left(20 H^4 k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3}\right) H^6 U k^6+24 \left(293 \sqrt{3}\right) U H^4+84 \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267 \sqrt{3}\right) U+232 \sqrt{g H \left(H^2 k^2+3\right)\right) H+16 \left(17 \sqrt{g H^{13}} \left(H^2 k^2+3\right) U^3 k^6+153 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(51 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+88 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+459 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text {dt}^2\{11520 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)\}-\frac{i k^8 \left(84 H^8 U^4 k^8+4 U^3 \left(252 U H^6+83 \sqrt{3}\right) \sqrt{g H^{13}} \left(H^2 k^2+3\right)\right) k^6+36 U^3 \left(126 U H^4+85 \sqrt{3}\right) \sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+9 \left(1008 H^2 U^4+1039 \sqrt{3}\right) \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+600 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U+39 \sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right)\right)+3 g H U \left(472 H^6 U k^6+4 \left(1101 U H^4+71 \sqrt{3}\right) \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+13500 H^2 U k^2+27 \left(504 U+97 \sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right)\right)\right) \text {dt}^3\{3840 \left(H^2 k^2+3\right)^4\}+\frac{k^9 \left(\left(H^2 k^2+3\right) U+\sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right)\right) \left(4 U^3 \left(239 \sqrt{3}\right) g H^9+58 \sqrt{g H^{17}} \left(H^2 k^2+3\right) U\right) k^8+12 U \left(213 \sqrt{3}\right) g^2 H^8+g U \left(979 \sqrt{3}\right) U+349 \sqrt{g H \left(H^2 k^2+3\right)\right) H^7+232 \sqrt{g H^{13}} \left(H^2 k^2+3\right) U^3\right) k^6+9 \left(2672 \sqrt{3}\right) g^2 U H^6+g U^2 \left(5997 \sqrt{3}\right) U+4384 \sqrt{g H \left(H^2 k^2+3\right)\right) H^5+1392 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+180 \sqrt{g^5 H^{13}} \left(H^2 k^2+3\right)\right) k^4+27 \left(2703 \sqrt{3}\right) g^2 U H^4+4070 \sqrt{3} g}
\end{aligned}$$

$$\begin{aligned} & U^3 H^3 + 928 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} U^4 + 4515 \sqrt[3]{g^3 H^7 \left(H^2 k^2 + 3 \right)} \\ & U^2 + 372 \sqrt[3]{g^5 H^9 \left(H^2 k^2 + 3 \right)} U + 1527 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} U^2 + 883 \sqrt[3]{g^2} \\ & H^2 U + 157 \sqrt[3]{g^5 H^5 \left(H^2 k^2 + 3 \right)} \text{dt}^4 + 23040 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \\ & \text{dx}^{11/2} + O \left(\text{dt}^5 \right) \text{dx}^4 + O \left(\text{dx}^5 \right) \left(\frac{k^3}{\left(H^2 k^2 + 3 \right) U - \sqrt[3]{g H \left(H^2 k^2 + 3 \right)}} \right) \left(3 g H + U \left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \text{dt}^2 + 6 \left(H^2 k^2 + 3 \right)^2 + \frac{i k^4}{\left(3 g H + U \left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right)} \text{dt}^3 + 8 \left(H^2 k^2 + 3 \right)^2 - \frac{k^5}{\left(H^2 k^2 + 3 \right) U - \sqrt[3]{g H \left(H^2 k^2 + 3 \right)}} \left(3 g H + U \left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \text{dt}^4 + 20 \left(H^2 k^2 + 3 \right)^4 + O \left(\text{dt}^5 \right) + \left(\frac{k^3}{2 U \left(H^2 k^2 + 3 \right)^2 + 3 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)}} \right) \left(24 \left(H^2 k^2 + 3 \right)^2 + \frac{k^5}{\left(2 H^6 U k^6 - 2 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} - 9 H^4 U \right) k^4 - 3 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} - 18 H^2 U} \right) k^2 + 54 U - 27 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} U^2 + 3 g \left(2 k^4 U H^5 + 6 k^2 U H^3 + 3 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} H \right) \text{dt}^2 + 48 \left(H^2 k^2 + 3 \right)^3 + \frac{i k^6}{\left(3 g H + U \left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right)} \left(U \left(2 H^4 U k^4 - 2 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} - 6 H^2 U \right) k^2 + 18 U - 3 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} - 9 g H \right) \text{dt}^3 + 48 \left(H^2 k^2 + 3 \right)^3 - \frac{\left(k^7 \left(2 U \left(H^2 k^2 + 3 \right)^2 + 3 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \left(3 g H + U \left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right)^2 \right)}{\text{dt}^4} + 96 \left(H^2 k^2 + 3 \right)^4 + O \left(\text{dt}^5 \right) \text{dx}^2 + \left(-\frac{1}{16} i k^4 \sqrt[3]{\frac{g H}{H^2 k^2 + 3}} - 2 U \right) + \frac{i k^6}{\left(U^2 \left(2 H^4 U k^4 + \left(12 H^2 U - 5 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} \right) k^2 + 18 U - 15 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} - 3 g H \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} - 4 \left(H^2 k^2 + 3 \right) U \right) \text{dt}^2} + 32 \left(H^2 k^2 + 3 \right)^2 - \frac{\left(k^7 \left(3 g H + U \left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \left(3 g H + U \left(2 \left(H^2 k^2 + 3 \right) U - 3 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \right) \text{dt}^3}{32 \left(H^2 k^2 + 3 \right)^2} - \frac{i k^8}{\left(2 \left(H^2 k^2 + 3 \right) U - \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right) \left(3 g H + U \left(H^2 k^2 + 3 \right) U - 2 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} \right)^2 \text{dt}^4} + 64 \left(H^2 k^2 + 3 \right)^3 + O \left(\text{dt}^5 \right) \text{dx}^3 + \left(\frac{k^5}{\left(20 H^4 k^4 + 124 H^2 k^2 + 177 \right) - 104 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)}} k^4 + 6 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} k^2 + 9 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} U \right) \left(1920 \sqrt[3]{g H} \left(H^2 k^2 + 3 \right)^{5/2} + \frac{k^7}{\left(27 \sqrt[3]{g^2 \left(20 H^4 k^4 + 124 H^2 k^2 + 167 \right) H^2 + g U \left(764 \sqrt[3]{H^6 U k^6 - 24 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)}} - 293 \sqrt[3]{H^4 U} \right) k^4 + 21429 \sqrt[3]{H^2 U k^2 + 81 \left(267 \sqrt[3]{U - 232 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)}} H - 16 \sqrt[3]{17 \sqrt[3]{g H^{13}} \left(H^2 k^2 + 3 \right)} \right) U^3 k^6 + 153 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} U^3 k^4 + 9 \sqrt[3]{51 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} U^3 + 88 \sqrt[3]{g^3 H^7 \left(H^2 k^2 + 3 \right)} U} \right) k^2 + 459 \sqrt[3]{g H \left(H^2 k^2 + 3 \right)} U^3 \right) \text{dt}^2 + 11520 \sqrt[3]{g H} \left(H^2 k^2 + 3 \right)^{7/2} - \frac{i k^8}{\left(84 H^8 U^4 k^8 + 4 U^3 \left(252 H^6 U - 83 \sqrt[3]{g H^{13}} \left(H^2 k^2 + 3 \right) \right) k^6 + 36 U^3 \left(126 H^4 U - 85 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} \right) k^4 - 9 \left(-1008 H^2 U^4 + 1039 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} U^2 + 600 \sqrt[3]{g^2 H^7 \left(H^2 k^2 + 3 \right)} U \right) \right)} \left(84 H^8 U^4 k^8 + 4 U^3 \left(252 H^6 U - 83 \sqrt[3]{g H^{13}} \left(H^2 k^2 + 3 \right) \right) k^6 + 36 U^3 \left(126 H^4 U - 85 \sqrt[3]{g H^9 \left(H^2 k^2 + 3 \right)} \right) k^4 - 9 \left(-1008 H^2 U^4 + 1039 \sqrt[3]{g H^5 \left(H^2 k^2 + 3 \right)} U^2 + 600 \sqrt[3]{g^2 H^7 \left(H^2 k^2 + 3 \right)} U \right) \right) \text{dt}^3 + O \left(\text{dt}^4 \right) \end{aligned}$$

$$\begin{aligned} \text{Err} \parallel & \left\{ \left(\frac{(-3 \text{ g H k}^2 + 3 \text{ k}^2 \text{ U}^2 - \text{H}^2 \text{ k}^3 \text{ U w}) \text{ dt}^2}{2 (3 + \text{H}^2 \text{ k}^2)} - \frac{i \text{H}^2 \text{ k}^3 \text{ U w}^2 \text{ dt}^3}{6 (3 + \text{H}^2 \text{ k}^2)} + \frac{\text{H}^2 \text{ k}^3 \text{ U w}^3 \text{ dt}^4}{24 (3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) + \right. \\ & \left(-\frac{i (27 \text{ k}^3 + 9 \text{ H}^2 \text{ k}^5 + \text{H}^4 \text{ k}^7) \text{ U dt}}{12 (3 + \text{H}^2 \text{ k}^2)^2} - \frac{(-9 \text{ g H k}^4 + 36 \text{ k}^4 \text{ U}^2 + 12 \text{ H}^2 \text{ k}^6 \text{ U}^2 + 2 \text{ H}^4 \text{ k}^8 \text{ U}^2) \text{ dt}^2}{24 (3 + \text{H}^2 \text{ k}^2)^2} + \text{O}[\text{dt}]^5 \right) \text{ dx}^2 + \\ & \left(\frac{1}{8} \text{ k}^4 \text{ U dt} + \left(-\frac{3 i \text{ g H k}^5}{16 (3 + \text{H}^2 \text{ k}^2)} - \frac{i \text{H}^2 \text{ k}^7 \text{ U}^2}{8 (3 + \text{H}^2 \text{ k}^2)} \right) \text{ dt}^2 + \text{O}[\text{dt}]^5 \right) \text{ dx}^3 + \\ & \left(\frac{i (405 \text{ k}^5 \text{ U} + 351 \text{ H}^2 \text{ k}^7 \text{ U} + 116 \text{ H}^4 \text{ k}^9 \text{ U} + 13 \text{ H}^6 \text{ k}^{11} \text{ U}) \text{ dt}}{240 (3 + \text{H}^2 \text{ k}^2)^3} + \frac{1}{1440 (3 + \text{H}^2 \text{ k}^2)^3} (1161 \text{ g H k}^6 + 837 \text{ g H}^3 \text{ k}^8 + 135 \text{ g H}^5 \text{ k}^{10} - \right. \\ & \left. 351 \text{ k}^6 \text{ U}^2 + 297 \text{ H}^2 \text{ k}^8 \text{ U}^2 + 387 \text{ H}^4 \text{ k}^{10} \text{ U}^2 + 73 \text{ H}^6 \text{ k}^{12} \text{ U}^2) \text{ dt}^2 + \text{O}[\text{dt}]^5 \right) \text{ dx}^4 + \text{O}[\text{dx}]^5, \\ & \left(\left(-\frac{3 \text{ k}^2 \text{ U}}{3 + \text{H}^2 \text{ k}^2} - \frac{3 \text{ k w}}{2 (3 + \text{H}^2 \text{ k}^2)} \right) \text{ dt}^2 - \frac{i \text{ k w}^2 \text{ dt}^3}{2 (3 + \text{H}^2 \text{ k}^2)} + \frac{\text{ k w}^3 \text{ dt}^4}{8 (3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) + \left(\left(\frac{3 i \text{ k}^3}{2 (3 + \text{H}^2 \text{ k}^2)^2} + \frac{i \text{H}^2 \text{ k}^5}{4 (3 + \text{H}^2 \text{ k}^2)^2} \right) \text{ dt} + \frac{3 \text{ k}^4 \text{ U dt}^2}{4 (3 + \text{H}^2 \text{ k}^2)^2} + \text{O}[\text{dt}]^5 \right) \\ & \text{ dx}^2 + \left(-\frac{3 i \text{ k}^5 \text{ U dt}^2}{8 (3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) \text{ dx}^3 + \left(\frac{i (-54 \text{ k}^5 + \text{H}^4 \text{ k}^9) \text{ dt}}{240 (3 + \text{H}^2 \text{ k}^2)^3} + \frac{(387 \text{ k}^6 \text{ U} + 279 \text{ H}^2 \text{ k}^8 \text{ U} + 45 \text{ H}^4 \text{ k}^{10} \text{ U}) \text{ dt}^2}{240 (3 + \text{H}^2 \text{ k}^2)^3} + \text{O}[\text{dt}]^5 \right) \text{ dx}^4 + \text{O}[\text{dx}]^5 \Big\}, \\ & \left\{ \left(-\frac{(\text{ k (3 g H + g H}^3 \text{ k}^2 - 3 \text{ U}^2) (2 \text{ k U + w}) \text{ dt}^2}{2 (3 + \text{H}^2 \text{ k}^2)} - \frac{i \text{ k (3 g H + g H}^3 \text{ k}^2 - 3 \text{ U}^2) \text{ w}^2 \text{ dt}^3}{6 (3 + \text{H}^2 \text{ k}^2)} + \frac{\text{ k (3 g H + g H}^3 \text{ k}^2 - 3 \text{ U}^2) \text{ w}^3 \text{ dt}^4}{24 (3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) + \right. \\ & \left(-\frac{i (9 \text{ g H k}^3 + 6 \text{ g H}^3 \text{ k}^5 + \text{g H}^5 \text{ k}^7 + 18 \text{ k}^3 \text{ U}^2 + 3 \text{ H}^2 \text{ k}^5 \text{ U}^2) \text{ dt}}{12 (3 + \text{H}^2 \text{ k}^2)^2} + \frac{(-18 \text{ g H k}^4 \text{ U} - 12 \text{ g H}^3 \text{ k}^6 \text{ U} - 2 \text{ g H}^5 \text{ k}^8 \text{ U} - 9 \text{ k}^4 \text{ U}^3) \text{ dt}^2}{12 (3 + \text{H}^2 \text{ k}^2)^2} + \text{O}[\text{dt}]^5 \right) \text{ dx}^2 + \\ & \left(\frac{1}{8} \text{ g H k}^4 \text{ dt} - \frac{i (6 \text{ g H k}^5 \text{ U} + 2 \text{ g H}^3 \text{ k}^7 \text{ U} - 3 \text{ k}^5 \text{ U}^3) \text{ dt}^2}{8 (3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) \text{ dx}^3 + \\ & \left(\frac{i (351 \text{ g H k}^5 + 351 \text{ g H}^3 \text{ k}^7 + 117 \text{ g H}^5 \text{ k}^9 + 13 \text{ g H}^7 \text{ k}^{11} + 54 \text{ k}^5 \text{ U}^2 - \text{H}^4 \text{ k}^9 \text{ U}^2) \text{ dt}}{240 (3 + \text{H}^2 \text{ k}^2)^3} + \frac{1}{720 (3 + \text{H}^2 \text{ k}^2)^3} (1971 \text{ g H k}^6 \text{ U} + 1971 \text{ g H}^3 \text{ k}^8 \text{ U} + \right. \\ & \left. 657 \text{ g H}^5 \text{ k}^{10} \text{ U} + 73 \text{ g H}^7 \text{ k}^{12} \text{ U} - 1161 \text{ k}^6 \text{ U}^3 - 837 \text{ H}^2 \text{ k}^8 \text{ U}^3 - 135 \text{ H}^4 \text{ k}^{10} \text{ U}^3) \text{ dt}^2 + \text{O}[\text{dt}]^5 \right) \text{ dx}^4 + \\ & \text{O}[\text{dx}]^5, \left(\frac{(-3 \text{ g H k}^2 - 9 \text{ k}^2 \text{ U}^2 - \text{H}^2 \text{ k}^4 \text{ U}^2 - 6 \text{ k U w} - \text{H}^2 \text{ k}^3 \text{ U w}) \text{ dt}^2}{2 (3 + \text{H}^2 \text{ k}^2)} - \frac{i \text{ k (6 + H}^2 \text{ k}^2) \text{ U w}^2 \text{ dt}^3}{6 (3 + \text{H}^2 \text{ k}^2)} + \frac{\text{ k (6 + H}^2 \text{ k}^2) \text{ U w}^3 \text{ dt}^4}{24 (3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) + \\ & \left(-\frac{i (-9 \text{ k}^3 \text{ U} + 3 \text{ H}^2 \text{ k}^5 \text{ U} + \text{H}^4 \text{ k}^7 \text{ U}) \text{ dt}}{12 (3 + \text{H}^2 \text{ k}^2)^2} + \frac{(9 \text{ g H k}^4 - 12 \text{ H}^2 \text{ k}^6 \text{ U}^2 - 2 \text{ H}^4 \text{ k}^8 \text{ U}^2) \text{ dt}^2}{24 (3 + \text{H}^2 \text{ k}^2)^2} + \text{O}[\text{dt}]^5 \right) \text{ dx}^2 + \\ & \left(\frac{1}{8} \text{ k}^4 \text{ U dt} - \frac{i \text{ k}^5 (3 \text{ g H} + 12 \text{ U}^2 + 2 \text{ H}^2 \text{ k}^2 \text{ U}^2) \text{ dt}^2}{16 (3 + \text{H}^2 \text{ k}^2)} + \text{O}[\text{dt}]^5 \right) \text{ dx}^3 + \\ & \left(\frac{i (297 \text{ k}^5 + 351 \text{ H}^2 \text{ k}^7 + 118 \text{ H}^4 \text{ k}^9 + 13 \text{ H}^6 \text{ k}^{11}) \text{ U dt}}{240 (3 + \text{H}^2 \text{ k}^2)^3} + \frac{1}{1440 (3 + \text{H}^2 \text{ k}^2)^3} (1161 \text{ g H k}^6 + 837 \text{ g H}^3 \text{ k}^8 + 135 \text{ g H}^5 \text{ k}^{10} + \right. \\ & \left. 4293 \text{ k}^6 \text{ U}^2 + 3645 \text{ H}^2 \text{ k}^8 \text{ U}^2 + 927 \text{ H}^4 \text{ k}^{10} \text{ U}^2 + 73 \text{ H}^6 \text{ k}^{12} \text{ U}^2) \text{ dt}^2 + \text{O}[\text{dt}]^5 \right) \text{ dx}^4 + \text{O}[\text{dx}]^5 \Big\} \end{aligned}$$

$$\begin{aligned} & \left(\frac{\left(-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2 \right) \text{dt}^2}{2 \left(H^2 k^2 + 3 \right)} - \frac{H^2 k^3 U w^2 \text{dt}^3}{6 \left(H^2 k^2 + 3 \right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3 \right)} + O\left(\text{dt}^5 \right) \right) \\ & + \left(-\frac{i \left(H^4 k^7 + 9 H^2 k^5 + 27 k^3 \right) U \text{dt}}{12 \left(H^2 k^2 + 3 \right)^2} - \frac{\left(2 H^4 U^2 k^8 + 12 H^2 U^2 k^6 + 36 U^2 k^4 - 9 g H k^4 \right) \text{dt}^2}{24 \left(H^2 k^2 + 3 \right)^2} + O\left(\text{dt}^5 \right) \right) \\ & + \left(\text{dx}^2 + \frac{\left(1 \right) \{ 8 \} k^4 U \text{dt}}{8 \left(H^2 k^2 + 3 \right)} - \frac{i g H k^5}{16 \left(H^2 k^2 + 3 \right)} \right) \text{dt}^2 + O\left(\text{dt}^5 \right) \\ & + \left(\text{dx}^3 + \frac{i \left(13 H^6 U k^{11} + 116 H^4 U k^9 + 351 H^2 U k^7 + 405 U k^5 \right) \text{dt}}{240 \left(H^2 k^2 + 3 \right)^3} + \frac{\left(73 H^6 U^2 k^{12} + 135 g H^5 k^{10} + 387 H^4 U^2 k^{10} + 837 g H^3 k^8 + 297 H^2 U^2 k^8 - 351 U^2 k^6 + 1161 g H k^6 \right) \text{dt}^2}{1440 \left(H^2 k^2 + 3 \right)^3} + O\left(\text{dt}^5 \right) \right) \\ & + \left(\text{dx}^4 + O\left(\text{dx}^5 \right) \right) \\ & + \left(\left(-\frac{3 U k^2}{H^2 k^2 + 3} - \frac{3 w k}{2 \left(H^2 k^2 + 3 \right)} \right) \text{dt}^2 - \frac{i k w^2 \text{dt}^3}{2 \left(H^2 k^2 + 3 \right)} + \frac{k w^3 \text{dt}^4}{8 \left(H^2 k^2 + 3 \right)} + O\left(\text{dt}^5 \right) \right) \\ & + \left(\frac{i H^2 k^5}{4 \left(H^2 k^2 + 3 \right)^2} + \frac{3 i k^3}{2 \left(H^2 k^2 + 3 \right)^2} \right) \text{dt} + \frac{3 k^4 U \text{dt}^2}{4 \left(H^2 k^2 + 3 \right)^2} + O\left(\text{dt}^5 \right) \\ & + \left(\text{dx}^2 + \left(-\frac{3 i k^5 U \text{dt}^2}{8 \left(H^2 k^2 + 3 \right)^2} + \frac{3 k^4 U \text{dt}^2}{4 \left(H^2 k^2 + 3 \right)^2} + O\left(\text{dt}^5 \right) \right) \right) \end{aligned}$$

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\left(H^2 k^2+3\right))+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{1}{240} \left(H^4 k^9-54 k^5\right) \right.
\text{dt}\left.\right)\left\{240 \left(H^2 k^2+3\right)^3+\frac{\left(45 H^4 U k^{10}+279 H^2 U k^8+387 U k^6\right)}{\text{dt}^2}\right\} 240 \left(H^2 k^2+3\right)^3+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dt}^5\right) \backslash
\left(-\frac{\left(k \left(g k^2 H^3+3 g H-3 U^2\right)\left(2 k U+w\right)\right)}{\text{dt}^2}\left\{2 \left(H^2 k^2+3\right)\right\}-\frac{i}{k} \left(g k^2 H^3+3 g H-3 U^2\right) w^2 \text{dt}^3\right)\left\{6 \left(H^2 k^2+3\right)+\frac{k \left(g k^2 H^3+3 g H-3 U^2\right) w^3}{\text{dt}^4}\right\} 24 \left(H^2 k^2+3\right)+O\left(\text{dt}^5\right)\right) \text{dx}^5+\left(-\frac{i}{12} \left(H^2 k^2+3\right)^2+\frac{\left(-2 g H^5 U k^8-12 g H^3 U k^6-9 U^3 k^4-18 g H U k^4\right)}{\text{dt}^2}\right)\left\{12 \left(H^2 k^2+3\right)^2+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{8}\right)
g H k^4 \text{dt}-\frac{i}{8} \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \text{dt}^2\right)\left\{8 \left(H^2 k^2+3\right)+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i}{12} \left(13 g H^7 k^{11}+117 g H^5 k^9-H^4 U^2 k^9+351 g H^3 k^7+54 U^2 k^5+351 g H k^5\right) \text{dt}\right)\left\{240 \left(H^2 k^2+3\right)^3+\frac{\left(73 g H^7 U k^{12}-135 H^4 U^3 k^{10}+657 g H^5 U k^{10}-837 H^2 U^3 k^8+1971 g H^3 U k^8-1161 U^3 k^6+1971 g H U k^6\right)}{\text{dt}^2}\right\} 720 \left(H^2 k^2+3\right)^3+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dt}^5\right) \& \left(\frac{1}{24} \left(-H^2 U^2 k^4-H^2 U w k^3-9 U^2 k^2-3 g H k^2-6 U w k\right) \text{dt}^2\right)\left\{2 \left(H^2 k^2+3\right)\right\}-\frac{i}{k} \left(H^2 k^2+6\right) U w^2 \text{dt}^3\right)\left\{6 \left(H^2 k^2+3\right)+\frac{k \left(H^2 k^2+6\right) U w^3}{\text{dt}^4}\right\} 24 \left(H^2 k^2+3\right)+O\left(\text{dt}^5\right)\right) \text{dx}^5+\left(-\frac{i}{12} \left(H^4 U k^7+3 H^2 U k^5-9 U k^3\right) \text{dt}\right)\left\{12 \left(H^2 k^2+3\right)^2+\frac{\left(-2 H^4 U^2 k^8-12 H^2 U^2 k^6+9 g H k^4\right)}{\text{dt}^2}\right\} 24 \left(H^2 k^2+3\right)^2+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{8}\right) k^4 U \text{dt}-\frac{i}{8} \left(2 H^2 k^2 U^2+12 U^2+3 g H\right) \text{dt}^2\right)\left\{16 \left(H^2 k^2+3\right)+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i}{240} \left(13 H^6 k^{11}+118 H^4 k^9+351 H^2 k^7+297 k^5\right) U \text{dt}\right)\left\{240 \left(H^2 k^2+3\right)^3+\frac{\left(73 H^6 U^2 k^{12}+135 g H^5 k^{10}+927 H^4 U^2 k^{10}+837 g H^3 k^8+3645 H^2 U^2 k^8+4293 U^2 k^6+1161 g H k^6\right)}{\text{dt}^2}\right\} 1440 \left(H^2 k^2+3\right)^3+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dt}^5\right) \backslash
\end{array}
\right)

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