$$\begin{aligned}
&\frac{u_j^{n+1}-u_j^{n-1}}{2\Delta t}+g\frac{h_{j+1}^n-h_{j-1}^n}{2\Delta x}+u_0\frac{u_{j+1}^n-u_{j-1}^n}{2\Delta x}\\
&\qquad-\frac{h_0^2}{3}\left(u_0\frac{-u_{j-2}^n+2u_{j-1}^n-2u_{j+1}^n+u_{j+2}^n}{2\Delta x^3}\right)\\
&\qquad-\frac{h_0^2}{3}\frac{u_{j+1}^{n+1}-2u_j^{n+1}+u_{j-1}^{n+1}-u_{j+1}^{n-1}+2u_j^{n-1}-u_{j-1}^{n-1}}{2\Delta x^2\Delta t}\\
&\qquad\qquad=0 \quad (1)
\end{aligned}$$

$$\begin{aligned}
&\frac{\upsilon_j^{n+1}-\upsilon_j^{n-1}}{2\Delta t}+g\frac{\eta_{j+1}^n-\eta_{j-1}^n}{2\Delta x}+U\frac{\upsilon_{j+1}^n-\upsilon_{j-1}^n}{2\Delta x}\\
&\qquad-\frac{H^2}{3}\left(U\frac{-\upsilon_{j-2}^n+2\upsilon_{j-1}^n-2\upsilon_{j+1}^n+\upsilon_{j+2}^n}{2\Delta x^3}\right)\\
&\qquad-\frac{H^2}{3}\frac{\upsilon_{j+1}^{n+1}-2\upsilon_j^{n+1}+\upsilon_{j-1}^{n+1}-\upsilon_{j+1}^{n-1}+2\upsilon_j^{n-1}-\upsilon_{j-1}^{n-1}}{2\Delta x^2\Delta t}\\
&\qquad\qquad=0 \quad (2)
\end{aligned}$$

$$\begin{aligned}
&\frac{\upsilon_j^{n+1}-\upsilon_j^{n-1}}{2\Delta t}\\
&\qquad-\frac{H^2}{3}\frac{\upsilon_{j+1}^{n+1}-2\upsilon_j^{n+1}+\upsilon_{j-1}^{n+1}-\upsilon_{j+1}^{n-1}+2\upsilon_j^{n-1}-\upsilon_{j-1}^{n-1}}{2\Delta x^2\Delta t}\\
&=-g\frac{\eta_{j+1}^n-\eta_{j-1}^n}{2\Delta x}-U\frac{\upsilon_{j+1}^n-\upsilon_{j-1}^n}{2\Delta x}+\frac{H^2}{3}\left(U\frac{-\upsilon_{j-2}^n+2\upsilon_{j-1}^n-2\upsilon_{j+1}^n+\upsilon_{j+2}^n}{2\Delta x^3}\right)
\end{aligned}$$

(3)

$$
\begin{aligned}
&\upsilon_j^{n+1} - \upsilon_j^{n-1} \\
&\qquad - \frac{H^2}{3}\frac{\upsilon_{j+1}^{n+1} - 2\upsilon_j^{n+1} + \upsilon_{j-1}^{n+1} - \upsilon_{j+1}^{n-1} + 2\upsilon_j^{n-1} - \upsilon_{j-1}^{n-1}}{\Delta x^2} \\
&= \Delta t \left( -g\frac{\eta_{j+1}^n - \eta_{j-1}^n}{\Delta x} - U\frac{\upsilon_{j+1}^n - \upsilon_{j-1}^n}{\Delta x} + \frac{H^2}{3}\left( U\frac{-\upsilon_{j-2}^n + 2\upsilon_{j-1}^n - 2\upsilon_{j+1}^n + \upsilon_{j+2}^n}{\Delta x^3} \right) \right)
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&\upsilon_j^{n+1} - \frac{H^2}{3}\frac{\upsilon_{j+1}^{n+1} - 2\upsilon_j^{n+1} + \upsilon_{j-1}^{n+1}}{\Delta x^2} \\
&\qquad = \upsilon_j^{n-1} - \frac{H^2}{3}\frac{\upsilon_{j+1}^{n-1} - 2\upsilon_j^{n-1} + \upsilon_{j-1}^{n-1}}{\Delta x^2} \\
&+\Delta t \left( -g\frac{\eta_{j+1}^n - \eta_{j-1}^n}{\Delta x} - U\frac{\upsilon_{j+1}^n - \upsilon_{j-1}^n}{\Delta x} + \frac{H^2}{3}\left( U\frac{-\upsilon_{j-2}^n + 2\upsilon_{j-1}^n - 2\upsilon_{j+1}^n + \upsilon_{j+2}^n}{\Delta x^3} \right) \right)
\end{aligned}
\tag{5}
$$