In[179]:=
```
MA = k * x / (2 * Sin[k * x / 2])
RA = Exp[I * k * x / 2] * k * x / (2 * Sin[k * x / 2])
GA = k * x / ((H + H^3 / 3 * k^2) * Exp[-I * k * x / 2] * (2 * Sin[k * x / 2]))
FnnA = 0
FnGA = I * k / (1 + H^2 * k^2 / 3)
FGnA = g * H * I * k
FGGA = 0
FmatA = {{FnnA, FnGA}, {FGnA, FGGA}}
wAp = Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2)]
wAm = -Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2)]
Eigenvalues[FmatA]
```

Out[179]= $\frac{1}{2} k x \text{Csc}\left[\frac{k x}{2}\right]$

Out[180]= $\frac{1}{2} e^{\frac{i k x}{2}} k x \text{Csc}\left[\frac{k x}{2}\right]$

Out[181]= $\frac{e^{\frac{i k x}{2}} k x \text{Csc}\left[\frac{k x}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right)}$

Out[182]= 0

Out[183]= $\frac{i k}{1 + \frac{H^2 k^2}{3}}$

Out[184]= $i g H k$

Out[185]= 0

Out[186]= $\left\{\left\{0, \frac{i k}{1 + \frac{H^2 k^2}{3}}\right\}, \{i g H k, 0\}\right\}$

Out[187]= $\sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$

Out[188]= $-\sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$

Out[189]= $\left\{-\frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}\right\}$

In[190]:=
```
M = 1
Series[M - MA, {x, 0, 10}]
```

Out[190]= 1

Out[191]= $-\frac{k^2 x^2}{24} - \frac{7 k^4 x^4}{5760} - \frac{31 k^6 x^6}{967\,680} - \frac{127 k^8 x^8}{154\,828\,800} - \frac{73 k^{10} x^{10}}{3\,503\,554\,560} + O[x]^{11}$

In[192]:=
```
Rm = 1
Series[Rm - RA, {x, 0, 10}]
Rp = Exp[I * k * x]
Series[Rp - RA, {x, 0, 10}]
Ru = (1 + Exp[I * k * x]) / 2
Series[Ru - Exp[I * k * x / 2], {x, 0, 10}]
```

Out[192]= 1

Out[193]= $-\frac{1}{2} i k x + \frac{k^2 x^2}{12} + \frac{k^4 x^4}{720} + \frac{k^6 x^6}{30\,240} + \frac{k^8 x^8}{1\,209\,600} + \frac{k^{10} x^{10}}{47\,900\,160} + O[x]^{11}$

Out[194]= $e^{i k x}$

Out[195]= $\frac{i k x}{2} - \frac{5 k^2 x^2}{12} - \frac{1}{6} i k^3 x^3 + \frac{31 k^4 x^4}{720} + \frac{1}{120} i k^5 x^5 - \frac{41 k^6 x^6}{30\,240} - \frac{i k^7 x^7}{5040} + \frac{31 k^8 x^8}{1\,209\,600} + \frac{i k^9 x^9}{362\,880} - \frac{61 k^{10} x^{10}}{239\,500\,800} + O[x]^{11}$

Out[196]= $\frac{1}{2}\left(1 + e^{i k x}\right)$

Out[197]= $-\frac{k^2 x^2}{8} - \frac{1}{16} i k^3 x^3 + \frac{7 k^4 x^4}{384} + \frac{1}{256} i k^5 x^5 - \frac{31 k^6 x^6}{46\,080} - \frac{i k^7 x^7}{10\,240} + \frac{127 k^8 x^8}{10\,321\,920} + \frac{17 i k^9 x^9}{12\,386\,304} - \frac{73 k^{10} x^{10}}{530\,841\,600} + O[x]^{11}$

In[198]:=
```
Gold = H - H^3 / 3 * (2 * Cos[k * x] - 2) / x^2
G = Ru / Gold
Series[G, {x, 0, 3} ]
Series[GA, {x, 0, 3} ]
Series[G - GA, {x, 0, 5}]
```

Out[198]= $H - \frac{H^3 (-2 + 2 \cos[k x])}{3 x^2}$

Out[199]= $\frac{1 + e^{i k x}}{2\left(H - \frac{H^3 (-2+2 \cos[k x])}{3 x^2}\right)}$

Out[200]= $\frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2\left(H + \frac{H^3 k^2}{3}\right)} + \frac{\left(-9 k^2 - 2 H^2 k^4\right) x^2}{4 H \left(3 + H^2 k^2\right)^2} - \frac{i \left(6 k^3 + H^2 k^5\right) x^3}{8 H \left(3 + H^2 k^2\right)^2} + O[x]^4$

Out[201]= $\frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2\left(H + \frac{H^3 k^2}{3}\right)} - \frac{k^2 x^2}{12\left(H + \frac{H^3 k^2}{3}\right)} + O[x]^4$

Out[202]= $\frac{\left(-6 k^2 - H^2 k^4\right) x^2}{4 H \left(3 + H^2 k^2\right)^2} - \frac{i \left(6 k^3 + H^2 k^5\right) x^3}{8 H \left(3 + H^2 k^2\right)^2} + \frac{\left(144 k^4 + 45 H^2 k^6 + 4 H^4 k^8\right) x^4}{240 H \left(3 + H^2 k^2\right)^3} - \frac{i \left(-54 k^5 + H^4 k^9\right) x^5}{480 H \left(3 + H^2 k^2\right)^3} + O[x]^6$

In[203]:=
```
fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * G;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;

Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]

Fmat = {{Fnn , Fng}, {Fgn , Fgg}}
EigvFmat = Eigenvalues[Fmat];
 Simplify[Series[EigvFmat , {x, 0, 5}]]
RKStep = Log[ 1 - t * EigvFmat] / (I * t);
RKstepTay = Series[RKStep, {x, 0, 4}, {t, 0, 4}]
Simplify[RKstepTay, k * H > 0 ]
Simplify[RKstepTay - {wAp, wAm}, k * H > 0]
```

Out[207]=
$$-\frac{\left(1-e^{-i\,k\,x}\right)\,\left(-1+e^{i\,k\,x}\right)\,\sqrt{g\,H}}{2\,x}$$

Out[208]=
$$\frac{1}{2}\sqrt{g\,H}\;k^2\,x-\frac{1}{24}\left(\sqrt{g\,H}\;k^4\right)\,x^3+\frac{1}{720}\sqrt{g\,H}\;k^6\,x^5+O[x]^6$$

Out[209]=
$$\frac{\left(1-e^{-i\,k\,x}\right)\,\left(1+e^{i\,k\,x}\right)\,H}{2\,x\,\left(H-\frac{H^3\,(-2+2\,\text{Cos}[k\,x])}{3\,x^2}\right)}$$

Out[210]=
$$-\frac{i\,\left(6\,k^3+H^2\,k^5\right)\,x^2}{4\,\left(3+H^2\,k^2\right)^2}-\frac{i\,\left(-54\,k^5+H^4\,k^9\right)\,x^4}{240\,\left(3+H^2\,k^2\right)^3}+O[x]^6$$

Out[211]=
$$-\frac{\left(1-e^{-i\,k\,x}\right)\,\left(-1+e^{i\,k\,x}\right)\,\sqrt{g\,H}}{2\,x}$$

Out[212]=
$$\frac{1}{2}\sqrt{g\,H}\;k^2\,x-\frac{1}{24}\left(\sqrt{g\,H}\;k^4\right)\,x^3+\frac{1}{720}\sqrt{g\,H}\;k^6\,x^5+O[x]^6$$

Out[213]=
$$\frac{\left(1-e^{-i\,k\,x}\right)\,\left(1+e^{i\,k\,x}\right)\,g\,H}{2\,x}$$

Out[214]=
$$-\frac{1}{6}\,i\,g\,H\,k^3\,x^2+\frac{1}{120}\,i\,g\,H\,k^5\,x^4+O[x]^6$$

Out[215]= $$\left\{\left\{-\frac{\left(1-e^{-i k x}\right)\left(-1+e^{i k x}\right)\sqrt{g H}}{2x},\ \frac{\left(1-e^{-i k x}\right)\left(1+e^{i k x}\right)H}{2x\left(H-\frac{H^3(-2+2\cos[k x])}{3x^2}\right)}\right\},\right.$$
$$\left.\left\{\frac{\left(1-e^{-i k x}\right)\left(1+e^{i k x}\right)g H}{2x},\ -\frac{\left(1-e^{-i k x}\right)\left(-1+e^{i k x}\right)\sqrt{g H}}{2x}\right\}\right\}$$

Out[217]= $$\left\{-\frac{i\sqrt{3}\ g H k}{\sqrt{g H\left(3+H^2k^2\right)}}+\frac{1}{2}\sqrt{g H}\ k^2 x+\frac{i\sqrt{3}\ g^2H^2k^3\left(4+H^2k^2\right)x^2}{8\left(g H\left(3+H^2k^2\right)\right)^{3/2}}-\frac{1}{24}\left(\sqrt{g H}\ k^4\right)x^3-\right.$$
$$\frac{i k^5\sqrt{g H\left(3+H^2k^2\right)}\ \left(144+48H^2k^2+5H^4k^4\right)x^4}{640\sqrt{3}\ \left(3+H^2k^2\right)^3}+\frac{1}{720}\sqrt{g H}\ k^6x^5+O[x]^6,$$
$$\frac{i\sqrt{3}\ g H k}{\sqrt{g H\left(3+H^2k^2\right)}}+\frac{1}{2}\sqrt{g H}\ k^2x-\frac{i\sqrt{3}\ g^2H^2k^3\left(4+H^2k^2\right)x^2}{8\left(g H\left(3+H^2k^2\right)\right)^{3/2}}-\frac{1}{24}\left(\sqrt{g H}\ k^4\right)x^3+$$
$$\left.\frac{i k^5\sqrt{g H\left(3+H^2k^2\right)}\ \left(144+48H^2k^2+5H^4k^4\right)x^4}{640\sqrt{3}\ \left(3+H^2k^2\right)^3}+\frac{1}{720}\sqrt{g H}\ k^6x^5+O[x]^6\right\}$$

Out[219]= $$\left\{\left(\frac{k\sqrt{36g H+12g H^3k^2}}{2\left(3+H^2k^2\right)}-\frac{3i g H k^2t}{2\left(3+H^2k^2\right)}-\frac{\sqrt{3}\ g H k^3\sqrt{g H\left(3+H^2k^2\right)}\ t^2}{\left(3+H^2k^2\right)^2}+\right.\right.$$
$$\left.\frac{9i g^2H^2k^4t^3}{4\left(3+H^2k^2\right)^2}+\frac{9\sqrt{3}\ g^2H^2k^5\sqrt{g H\left(3+H^2k^2\right)}\ t^4}{5\left(3+H^2k^2\right)^3}+O[t]^5\right)+$$
$$\left(\frac{1}{2}i\sqrt{g H}\ k^2+\frac{\sqrt{g H}\ k^3\sqrt{36g H+12g H^3k^2}\ t}{4\left(3+H^2k^2\right)}-\frac{3i g H\sqrt{g H}\ k^4t^2}{2\left(3+H^2k^2\right)}-\right.$$
$$\left.\frac{3\left(\sqrt{3}\ g H\sqrt{g H}\ k^5\sqrt{g H\left(3+H^2k^2\right)}\right)t^3}{2\left(3+H^2k^2\right)^2}+\frac{9i g^2H^2\sqrt{g H}\ k^6t^4}{2\left(3+H^2k^2\right)^2}+O[t]^5\right)x+$$
$$\left(-\frac{4\sqrt{3}\ g H k^3+\sqrt{3}\ g H^3k^5}{8\left(\left(3+H^2k^2\right)\sqrt{g H\left(3+H^2k^2\right)}\right)}+\frac{i\left(21g H k^4+9g H^3k^6+g H^5k^8\right)t}{8\left(3+H^2k^2\right)^2}+\right.$$
$$\frac{k^5\left(30\sqrt{3}\ g^2H^2+15\sqrt{3}\ g^2H^4k^2+2\sqrt{3}\ g^2H^6k^4\right)t^2}{8\left(3+H^2k^2\right)^2\sqrt{g H\left(3+H^2k^2\right)}}-\frac{9i g^2H^2k^6\left(13+7H^2k^2+H^4k^4\right)t^3}{8\left(3+H^2k^2\right)^3}-$$
$$\left.\frac{3\left(k^7\left(48\sqrt{3}\ g^3H^3+27\sqrt{3}\ g^3H^5k^2+4\sqrt{3}\ g^3H^7k^4\right)\right)t^4}{8\left(\left(3+H^2k^2\right)^3\sqrt{g H\left(3+H^2k^2\right)}\right)}+O[t]^5\right)x^2+$$
$$\left(-\frac{1}{24}i\sqrt{g H}\ k^4-\frac{\left(\sqrt{g H}\ \left(18\sqrt{3}\ g H k^5+5\sqrt{3}\ g H^3k^7\right)\right)t}{48\left(\left(3+H^2k^2\right)\sqrt{g H\left(3+H^2k^2\right)}\right)}+\right.$$

$$\frac{i\sqrt{g H}\left(54 g H k^6+18 g H^3 k^8+g H^5 k^{10}\right) t^2}{24\left(3+H^2 k^2\right)^2}+$$

$$\left(\sqrt{g H}\left(60\sqrt{3} g^2 H^2 k^7+23\sqrt{3} g^2 H^4 k^9+2\sqrt{3} g^2 H^6 k^{11}\right) t^3\right)\Big/$$

$$\left(16\left(3+H^2 k^2\right)^2\sqrt{g H\left(3+H^2 k^2\right)}\right)-\frac{3 i\sqrt{g H}\left(45 g^2 H^2 k^8+19 g^2 H^4 k^{10}+2 g^2 H^6 k^{12}\right) t^4}{8\left(3+H^2 k^2\right)^3}+$$

$$O[t]^5\Bigg) x^3+\Bigg(\frac{144\sqrt{3} g H k^5+48\sqrt{3} g H^3 k^7+5\sqrt{3} g H^5 k^9}{1920\left(3+H^2 k^2\right)^2\sqrt{g H\left(3+H^2 k^2\right)}}-$$

$$\frac{i\left(558 g H k^6+396 g H^3 k^8+105 g H^5 k^{10}+10 g H^7 k^{12}\right) t}{480\left(3+H^2 k^2\right)^3}+$$

$$\left(\left(-6192\sqrt{3} g^2 H^2 k^7-5004\sqrt{3} g^2 H^4 k^9-1425\sqrt{3} g^2 H^6 k^{11}-140\sqrt{3} g^2 H^8 k^{13}\right) t^2\right)\Big/$$

$$\left(1920\left(3+H^2 k^2\right)^3\sqrt{g H\left(3+H^2 k^2\right)}\right)+$$

$$\left(i\left(6561 g^2 H^2 k^8+5742 g^2 H^4 k^{10}+1785 g^2 H^6 k^{12}+210 g^2 H^8 k^{14}+5 g^2 H^{10} k^{16}\right) t^3\right)\Big/$$

$$\left(320\left(3+H^2 k^2\right)^4\right)+$$

$$\Bigg(\Big(23832\sqrt{3} g^3 H^3 k^9+22104\sqrt{3} g^3 H^5 k^{11}+7395\sqrt{3} g^3 H^7 k^{13}+1000\sqrt{3} g^3 H^9 k^{15}+$$

$$40\sqrt{3} g^3 H^{11} k^{17}\Big) t^4\Big)\Big/\left(640\left(3+H^2 k^2\right)^4\sqrt{g H\left(3+H^2 k^2\right)}\right)+O[t]^5\Bigg) x^4+O[x]^5,$$

$$\Bigg(-\frac{k\sqrt{36 g H+12 g H^3 k^2}}{2\left(3+H^2 k^2\right)}-\frac{3 i g H k^2 t}{2\left(3+H^2 k^2\right)}+\frac{\sqrt{3} g H k^3\sqrt{g H\left(3+H^2 k^2\right)} t^2}{\left(3+H^2 k^2\right)^2}+$$

$$\frac{9 i g^2 H^2 k^4 t^3}{4\left(3+H^2 k^2\right)^2}-\frac{9\left(\sqrt{3} g^2 H^2 k^5\sqrt{g H\left(3+H^2 k^2\right)}\right) t^4}{5\left(3+H^2 k^2\right)^3}+O[t]^5\Bigg)+$$

$$\Bigg(\frac{1}{2} i\sqrt{g H} k^2-\frac{\left(\sqrt{g H} k^3\sqrt{36 g H+12 g H^3 k^2}\right) t}{4\left(3+H^2 k^2\right)}-\frac{3 i g H\sqrt{g H} k^4 t^2}{2\left(3+H^2 k^2\right)}+$$

$$\frac{3\sqrt{3} g H\sqrt{g H} k^5\sqrt{g H\left(3+H^2 k^2\right)} t^3}{2\left(3+H^2 k^2\right)^2}+\frac{9 i g^2 H^2\sqrt{g H} k^6 t^4}{2\left(3+H^2 k^2\right)^2}+O[t]^5\Bigg) x+$$

$$\Bigg(\frac{4\sqrt{3} g H k^3+\sqrt{3} g H^3 k^5}{8\left(3+H^2 k^2\right)\sqrt{g H\left(3+H^2 k^2\right)}}+\frac{i\left(21 g H k^4+9 g H^3 k^6+g H^5 k^8\right) t}{8\left(3+H^2 k^2\right)^2}-$$

$$\frac{\left(k^5\left(30\sqrt{3} g^2 H^2+15\sqrt{3} g^2 H^4 k^2+2\sqrt{3} g^2 H^6 k^4\right)\right) t^2}{8\left(\left(3+H^2 k^2\right)^2\sqrt{g H\left(3+H^2 k^2\right)}\right)}-$$

$$\frac{9\,i\,g^2\,H^2\,k^6\left(13+7\,H^2\,k^2+H^4\,k^4\right)\,t^3}{8\left(3+H^2\,k^2\right)^3}+$$

$$\left.\frac{3\,k^7\left(48\sqrt{3}\;g^3\,H^3+27\sqrt{3}\;g^3\,H^5\,k^2+4\sqrt{3}\;g^3\,H^7\,k^4\right)\,t^4}{8\left(3+H^2\,k^2\right)^3\sqrt{g\,H\left(3+H^2\,k^2\right)}}+O[t]^5\right)x^2+$$

$$\left(-\frac{1}{24}\,i\,\sqrt{g\,H}\;k^4+\frac{\sqrt{g\,H}\;\left(18\sqrt{3}\;g\,H\,k^5+5\sqrt{3}\;g\,H^3\,k^7\right)\,t}{48\left(3+H^2\,k^2\right)\sqrt{g\,H\left(3+H^2\,k^2\right)}}+\right.$$

$$\frac{i\sqrt{g\,H}\;\left(54\,g\,H\,k^6+18\,g\,H^3\,k^8+g\,H^5\,k^{10}\right)\,t^2}{24\left(3+H^2\,k^2\right)^2}-$$

$$\left(\left(\sqrt{g\,H}\;\left(60\sqrt{3}\;g^2\,H^2\,k^7+23\sqrt{3}\;g^2\,H^4\,k^9+2\sqrt{3}\;g^2\,H^6\,k^{11}\right)\right)\,t^3\right)\Big/$$

$$\left(16\left(\left(3+H^2\,k^2\right)^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)\right)-$$

$$\left.\frac{3\,i\sqrt{g\,H}\;\left(45\,g^2\,H^2\,k^8+19\,g^2\,H^4\,k^{10}+2\,g^2\,H^6\,k^{12}\right)\,t^4}{8\left(3+H^2\,k^2\right)^3}+O[t]^5\right)x^3+$$

$$\left(\frac{-144\sqrt{3}\;g\,H\,k^5-48\sqrt{3}\;g\,H^3\,k^7-5\sqrt{3}\;g\,H^5\,k^9}{1920\left(3+H^2\,k^2\right)^2\sqrt{g\,H\left(3+H^2\,k^2\right)}}-\right.$$

$$\frac{i\left(558\,g\,H\,k^6+396\,g\,H^3\,k^8+105\,g\,H^5\,k^{10}+10\,g\,H^7\,k^{12}\right)\,t}{480\left(3+H^2\,k^2\right)^3}+$$

$$\left(\left(6192\sqrt{3}\;g^2\,H^2\,k^7+5004\sqrt{3}\;g^2\,H^4\,k^9+1425\sqrt{3}\;g^2\,H^6\,k^{11}+140\sqrt{3}\;g^2\,H^8\,k^{13}\right)\,t^2\right)\Big/$$

$$\left(1920\left(3+H^2\,k^2\right)^3\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)+$$

$$\left(i\left(6561\,g^2\,H^2\,k^8+5742\,g^2\,H^4\,k^{10}+1785\,g^2\,H^6\,k^{12}+210\,g^2\,H^8\,k^{14}+5\,g^2\,H^{10}\,k^{16}\right)\,t^3\right)\Big/$$

$$\left(320\left(3+H^2\,k^2\right)^4\right)+$$

$$\left(\left(-23\,832\sqrt{3}\;g^3\,H^3\,k^9-22\,104\sqrt{3}\;g^3\,H^5\,k^{11}-7395\sqrt{3}\;g^3\,H^7\,k^{13}-1000\sqrt{3}\;g^3\,H^9\,k^{15}-\right.\right.$$

$$\left.\left.40\sqrt{3}\;g^3\,H^{11}\,k^{17}\right)\,t^4\right)\Big/\left(640\left(3+H^2\,k^2\right)^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)+O[t]^5\Bigg)\,x^4+O[x]^5\Bigg\}$$

Out[220]=

$$\left\{\left(\frac{\sqrt{3}\;g\,H\,k}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}-\frac{3\,i\,g\,H\,k^2\,t}{6+2\,H^2\,k^2}-\right.\right.$$

$$\left.\sqrt{3}\;k^3\left(\frac{g\,H}{3+H^2\,k^2}\right)^{3/2}t^2+\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{4\left(3+H^2\,k^2\right)^2}+\frac{9}{5}\sqrt{3}\;k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}t^4+O[t]^5\right)+$$

$$\left(\frac{1}{2}\,i\,\sqrt{g\,H}\;k^2+\frac{\sqrt{3}\;g\,H\,k^3\,t}{2\sqrt{3+H^2\,k^2}}-\frac{3\,i\,(g\,H)^{3/2}\,k^4\,t^2}{6+2\,H^2\,k^2}-\frac{3\left(\sqrt{3}\;g^2\,H^2\,k^5\right)\,t^3}{2\left(3+H^2\,k^2\right)^{3/2}}+\frac{9\,i\,(g\,H)^{5/2}\,k^6\,t^4}{2\left(3+H^2\,k^2\right)^2}+\right.$$

$$O[t]^5\Big)\,x+\left(-\frac{\sqrt{3}\,\sqrt{g\,H}\,k^3\left(4+H^2\,k^2\right)}{8\left(3+H^2\,k^2\right)^{3/2}}+\frac{i\,g\,H\,k^4\left(21+9\,H^2\,k^2+H^4\,k^4\right)t}{8\left(3+H^2\,k^2\right)^2}+\right.$$

$$\frac{\sqrt{3}\,(g\,H)^{3/2}\,k^5\left(30+15\,H^2\,k^2+2\,H^4\,k^4\right)t^2}{8\left(3+H^2\,k^2\right)^{5/2}}-\frac{9\,i\,g^2\,H^2\,k^6\left(13+7\,H^2\,k^2+H^4\,k^4\right)t^3}{8\left(3+H^2\,k^2\right)^3}-$$

$$\left.\frac{3\left(\sqrt{3}\,(g\,H)^{5/2}\,k^7\left(48+27\,H^2\,k^2+4\,H^4\,k^4\right)\right)t^4}{8\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\right)x^2+$$

$$\left(-\frac{1}{24}\,i\,\sqrt{g\,H}\,k^4-\frac{\left(g\,H\,k^5\left(18+5\,H^2\,k^2\right)\right)t}{16\left(\sqrt{3}\,\left(3+H^2\,k^2\right)^{3/2}\right)}+\frac{i\,(g\,H)^{3/2}\,k^6\left(54+18\,H^2\,k^2+H^4\,k^4\right)t^2}{24\left(3+H^2\,k^2\right)^2}+\right.$$

$$\frac{\sqrt{3}\,g^2\,H^2\,k^7\left(60+23\,H^2\,k^2+2\,H^4\,k^4\right)t^3}{16\left(3+H^2\,k^2\right)^{5/2}}-\frac{3\,i\,(g\,H)^{5/2}\,k^8\left(45+19\,H^2\,k^2+2\,H^4\,k^4\right)t^4}{8\left(3+H^2\,k^2\right)^3}+O[t]^5\Big)$$

$$x^3+\left(\frac{\sqrt{g\,H}\,k^5\left(144+48\,H^2\,k^2+5\,H^4\,k^4\right)}{640\,\sqrt{3}\,\left(3+H^2\,k^2\right)^{5/2}}-\frac{i\,g\,H\,k^6\left(558+396\,H^2\,k^2+105\,H^4\,k^4+10\,H^6\,k^6\right)t}{480\left(3+H^2\,k^2\right)^3}-\right.$$

$$\frac{\left((g\,H)^{3/2}\,k^7\left(6192+5004\,H^2\,k^2+1425\,H^4\,k^4+140\,H^6\,k^6\right)\right)t^2}{640\left(\sqrt{3}\,\left(3+H^2\,k^2\right)^{7/2}\right)}+$$

$$\frac{i\,g^2\,H^2\,k^8\left(6561+5742\,H^2\,k^2+1785\,H^4\,k^4+210\,H^6\,k^6+5\,H^8\,k^8\right)t^3}{320\left(3+H^2\,k^2\right)^4}+$$

$$\left.\frac{\sqrt{3}\,(g\,H)^{5/2}\,k^9\left(23\,832+22\,104\,H^2\,k^2+7395\,H^4\,k^4+1000\,H^6\,k^6+40\,H^8\,k^8\right)t^4}{640\left(3+H^2\,k^2\right)^{9/2}}+O[t]^5\right)x^4+$$

$$O[x]^5,\left(-\frac{\sqrt{3}\,g\,H\,k}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}-\frac{3\,i\,g\,H\,k^2\,t}{6+2\,H^2\,k^2}+\sqrt{3}\,k^3\left(\frac{g\,H}{3+H^2\,k^2}\right)^{3/2}t^2+\right.$$

$$\left.\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{4\left(3+H^2\,k^2\right)^2}-\frac{9}{5}\left(\sqrt{3}\,k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}\right)t^4+O[t]^5\right)+$$

$$\left(\frac{1}{2}\,i\,\sqrt{g\,H}\,k^2-\frac{\left(\sqrt{3}\,g\,H\,k^3\right)t}{2\,\sqrt{3+H^2\,k^2}}-\frac{3\,i\,(g\,H)^{3/2}\,k^4\,t^2}{6+2\,H^2\,k^2}+\right.$$

$$\left.\frac{3\,\sqrt{3}\,g^2\,H^2\,k^5\,t^3}{2\left(3+H^2\,k^2\right)^{3/2}}+\frac{9\,i\,(g\,H)^{5/2}\,k^6\,t^4}{2\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x+$$

$$\left(\frac{\sqrt{3}\,\sqrt{g\,H}\,k^3\left(4+H^2\,k^2\right)}{8\left(3+H^2\,k^2\right)^{3/2}}+\frac{i\,g\,H\,k^4\left(21+9\,H^2\,k^2+H^4\,k^4\right)t}{8\left(3+H^2\,k^2\right)^2}-\right.$$

$$\frac{\left(\sqrt{3}\,(g\,H)^{3/2}\,k^5\left(30+15\,H^2\,k^2+2\,H^4\,k^4\right)\right)t^2}{8\left(3+H^2\,k^2\right)^{5/2}}-\frac{9\,i\,g^2\,H^2\,k^6\left(13+7\,H^2\,k^2+H^4\,k^4\right)t^3}{8\left(3+H^2\,k^2\right)^3}+$$

$$\frac{3\sqrt{3}\ (g\,H)^{5/2}\,k^7\left(48+27\,H^2\,k^2+4\,H^4\,k^4\right)t^4}{8\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\Bigg)x^2+$$

$$\Bigg(-\frac{1}{24}\,i\,\sqrt{g\,H}\ k^4+\frac{g\,H\,k^5\left(18+5\,H^2\,k^2\right)t}{16\sqrt{3}\ \left(3+H^2\,k^2\right)^{3/2}}+\frac{i\,(g\,H)^{3/2}\,k^6\left(54+18\,H^2\,k^2+H^4\,k^4\right)t^2}{24\left(3+H^2\,k^2\right)^2}-$$

$$\frac{\left(\sqrt{3}\ g^2\,H^2\,k^7\left(60+23\,H^2\,k^2+2\,H^4\,k^4\right)\right)t^3}{16\left(3+H^2\,k^2\right)^{5/2}}-$$

$$\frac{3\,i\,(g\,H)^{5/2}\,k^8\left(45+19\,H^2\,k^2+2\,H^4\,k^4\right)t^4}{8\left(3+H^2\,k^2\right)^3}+O[t]^5\Bigg)x^3+$$

$$\Bigg(-\frac{\sqrt{g\,H}\ k^5\left(144+48\,H^2\,k^2+5\,H^4\,k^4\right)}{640\left(\sqrt{3}\ \left(3+H^2\,k^2\right)^{5/2}\right)}-\frac{i\,g\,H\,k^6\left(558+396\,H^2\,k^2+105\,H^4\,k^4+10\,H^6\,k^6\right)t}{480\left(3+H^2\,k^2\right)^3}+$$

$$\frac{(g\,H)^{3/2}\,k^7\left(6192+5004\,H^2\,k^2+1425\,H^4\,k^4+140\,H^6\,k^6\right)t^2}{640\sqrt{3}\ \left(3+H^2\,k^2\right)^{7/2}}+$$

$$\frac{i\,g^2\,H^2\,k^8\left(6561+5742\,H^2\,k^2+1785\,H^4\,k^4+210\,H^6\,k^6+5\,H^8\,k^8\right)t^3}{320\left(3+H^2\,k^2\right)^4}-$$

$$\frac{\left(\sqrt{3}\ (g\,H)^{5/2}\,k^9\left(23\,832+22\,104\,H^2\,k^2+7395\,H^4\,k^4+1000\,H^6\,k^6+40\,H^8\,k^8\right)\right)t^4}{640\left(3+H^2\,k^2\right)^{9/2}}+$$

$$O[t]^5\Bigg)x^4+O[x]^5\Big\}$$

Out[221]=

$$\Bigg\{\Bigg(-\frac{3\,i\,g\,H\,k^2\,t}{6+2\,H^2\,k^2}-\sqrt{3}\ k^3\left(\frac{g\,H}{3+H^2\,k^2}\right)^{3/2}t^2+\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{4\left(3+H^2\,k^2\right)^2}+\frac{9}{5}\sqrt{3}\ k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}t^4+O[t]^5\Bigg)+$$

$$\Bigg(\frac{1}{2}\,i\,\sqrt{g\,H}\ k^2+\frac{\sqrt{3}\ g\,H\,k^3\,t}{2\sqrt{3+H^2\,k^2}}-\frac{3\,i\,(g\,H)^{3/2}\,k^4\,t^2}{6+2\,H^2\,k^2}-\frac{3\left(\sqrt{3}\ g^2\,H^2\,k^5\right)t^3}{2\left(3+H^2\,k^2\right)^{3/2}}+\frac{9\,i\,(g\,H)^{5/2}\,k^6\,t^4}{2\left(3+H^2\,k^2\right)^2}+$$

$$O[t]^5\Bigg)x+\Bigg(-\frac{\sqrt{3}\ \sqrt{g\,H}\ k^3\left(4+H^2\,k^2\right)}{8\left(3+H^2\,k^2\right)^{3/2}}+\frac{i\,g\,H\,k^4\left(21+9\,H^2\,k^2+H^4\,k^4\right)t}{8\left(3+H^2\,k^2\right)^2}+$$

$$\frac{\sqrt{3}\ (g\,H)^{3/2}\,k^5\left(30+15\,H^2\,k^2+2\,H^4\,k^4\right)t^2}{8\left(3+H^2\,k^2\right)^{5/2}}-\frac{9\,i\,g^2\,H^2\,k^6\left(13+7\,H^2\,k^2+H^4\,k^4\right)t^3}{8\left(3+H^2\,k^2\right)^3}-$$

$$\frac{3\left(\sqrt{3}\ (g\,H)^{5/2}\,k^7\left(48+27\,H^2\,k^2+4\,H^4\,k^4\right)\right)t^4}{8\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\Bigg)x^2+$$

$$\Bigg(-\frac{1}{24}\,i\,\sqrt{g\,H}\ k^4-\frac{\left(g\,H\,k^5\left(18+5\,H^2\,k^2\right)\right)t}{16\left(\sqrt{3}\ \left(3+H^2\,k^2\right)^{3/2}\right)}+\frac{i\,(g\,H)^{3/2}\,k^6\left(54+18\,H^2\,k^2+H^4\,k^4\right)t^2}{24\left(3+H^2\,k^2\right)^2}+$$

$$\frac{\sqrt{3}\ g^2 H^2 k^7 \left(60+23 H^2 k^2+2 H^4 k^4\right) t^3}{16 \left(3+H^2 k^2\right)^{5/2}}-\frac{3 i\ (g H)^{5/2} k^8 \left(45+19 H^2 k^2+2 H^4 k^4\right) t^4}{8 \left(3+H^2 k^2\right)^3}+O[t]^5\Bigg)$$

$$x^3+\Bigg(\frac{\sqrt{g H}\ k^5 \left(144+48 H^2 k^2+5 H^4 k^4\right)}{640 \sqrt{3}\ \left(3+H^2 k^2\right)^{5/2}}-\frac{i g H k^6 \left(558+396 H^2 k^2+105 H^4 k^4+10 H^6 k^6\right) t}{480 \left(3+H^2 k^2\right)^3}-$$

$$\frac{\left((g H)^{3/2} k^7 \left(6192+5004 H^2 k^2+1425 H^4 k^4+140 H^6 k^6\right)\right) t^2}{640 \left(\sqrt{3}\ \left(3+H^2 k^2\right)^{7/2}\right)}+$$

$$\frac{i g^2 H^2 k^8 \left(6561+5742 H^2 k^2+1785 H^4 k^4+210 H^6 k^6+5 H^8 k^8\right) t^3}{320 \left(3+H^2 k^2\right)^4}+$$

$$\frac{\sqrt{3}\ (g H)^{5/2} k^9 \left(23\,832+22\,104 H^2 k^2+7395 H^4 k^4+1000 H^6 k^6+40 H^8 k^8\right) t^4}{640 \left(3+H^2 k^2\right)^{9/2}}+O[t]^5\Bigg)\ x^4+$$

$$O[x]^5,\ \Bigg(-\frac{3 i g H k^2 t}{6+2 H^2 k^2}+\sqrt{3}\ k^3 \left(\frac{g H}{3+H^2 k^2}\right)^{3/2} t^2+\frac{9 i g^2 H^2 k^4 t^3}{4 \left(3+H^2 k^2\right)^2}-$$

$$\frac{9}{5}\left(\sqrt{3}\ k^5 \left(\frac{g H}{3+H^2 k^2}\right)^{5/2}\right) t^4+O[t]^5\Bigg)+$$

$$\Bigg(\frac{1}{2} i \sqrt{g H}\ k^2-\frac{\left(\sqrt{3}\ g H k^3\right) t}{2 \sqrt{3+H^2 k^2}}-\frac{3 i\ (g H)^{3/2} k^4 t^2}{6+2 H^2 k^2}+$$

$$\frac{3 \sqrt{3}\ g^2 H^2 k^5 t^3}{2 \left(3+H^2 k^2\right)^{3/2}}+\frac{9 i\ (g H)^{5/2} k^6 t^4}{2 \left(3+H^2 k^2\right)^2}+O[t]^5\Bigg)\ x+$$

$$\Bigg(\frac{\sqrt{3}\ \sqrt{g H}\ k^3 \left(4+H^2 k^2\right)}{8 \left(3+H^2 k^2\right)^{3/2}}+\frac{i g H k^4 \left(21+9 H^2 k^2+H^4 k^4\right) t}{8 \left(3+H^2 k^2\right)^2}-$$

$$\frac{\left(\sqrt{3}\ (g H)^{3/2} k^5 \left(30+15 H^2 k^2+2 H^4 k^4\right)\right) t^2}{8 \left(3+H^2 k^2\right)^{5/2}}-\frac{9 i g^2 H^2 k^6 \left(13+7 H^2 k^2+H^4 k^4\right) t^3}{8 \left(3+H^2 k^2\right)^3}+$$

$$\frac{3 \sqrt{3}\ (g H)^{5/2} k^7 \left(48+27 H^2 k^2+4 H^4 k^4\right) t^4}{8 \left(3+H^2 k^2\right)^{7/2}}+O[t]^5\Bigg)\ x^2+$$

$$\Bigg(-\frac{1}{24} i \sqrt{g H}\ k^4+\frac{g H k^5 \left(18+5 H^2 k^2\right) t}{16 \sqrt{3}\ \left(3+H^2 k^2\right)^{3/2}}+\frac{i\ (g H)^{3/2} k^6 \left(54+18 H^2 k^2+H^4 k^4\right) t^2}{24 \left(3+H^2 k^2\right)^2}-$$

$$\frac{\left(\sqrt{3}\ g^2 H^2 k^7 \left(60+23 H^2 k^2+2 H^4 k^4\right)\right) t^3}{16 \left(3+H^2 k^2\right)^{5/2}}-$$

$$\frac{3 i\ (g H)^{5/2} k^8 \left(45+19 H^2 k^2+2 H^4 k^4\right) t^4}{8 \left(3+H^2 k^2\right)^3}+O[t]^5\Bigg)\ x^3+$$

$$\Bigg(-\frac{\sqrt{g H}\ k^5 \left(144+48 H^2 k^2+5 H^4 k^4\right)}{640 \left(\sqrt{3}\ \left(3+H^2 k^2\right)^{5/2}\right)}-\frac{i g H k^6 \left(558+396 H^2 k^2+105 H^4 k^4+10 H^6 k^6\right) t}{480 \left(3+H^2 k^2\right)^3}+$$

$$\frac{(g\,H)^{3/2}\,k^7\left(6192+5004\,H^2\,k^2+1425\,H^4\,k^4+140\,H^6\,k^6\right)t^2}{640\sqrt{3}\,\left(3+H^2\,k^2\right)^{7/2}}+$$

$$\frac{i\,g^2\,H^2\,k^8\left(6561+5742\,H^2\,k^2+1785\,H^4\,k^4+210\,H^6\,k^6+5\,H^8\,k^8\right)t^3}{320\,\left(3+H^2\,k^2\right)^4}-$$

$$\frac{\left(\sqrt{3}\,(g\,H)^{5/2}\,k^9\left(23\,832+22\,104\,H^2\,k^2+7395\,H^4\,k^4+1000\,H^6\,k^6+40\,H^8\,k^8\right)\right)t^4}{640\,\left(3+H^2\,k^2\right)^{9/2}}+$$

$$O[t]^5\Bigg)\,x^4+O[x]^5\Big\}$$