```
ln[1] = q = q0 * Exp[I * (k * x + w * t)];
     qjn = q0 * Exp[I * (k * xj + w * tn)];
     qjbar = Integrate [q, \{x, xj - dx/2, xj + dx/2\}]/(dx);
     qjnbar = qjbar /. t \rightarrow tn;
     MA = qjn / qjnbar;
     qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
     qjntbar = qntbar /. x \rightarrow xj;
     MtA = qjntbar / qjn;
     qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
     RA = Simplify[MA * qjphn / (qjn)];
     vmultG = H + H^3/3*k^2;
     GnA = -U * RA / vmultG;
     GGA = RA / vmultG;
     GcA = -U * H / vmultG ;
     fn1A = H * vh + U * eh;
     fn1A = fn1A /. vh \rightarrow (GGA * Gca + GnA * eca) /. eh \rightarrow RA * eca;
     fn1Gca0A = fn1A / . Gca \rightarrow 0 ;
     fn1eca0A = fn1A/. eca \rightarrow 0;
     fnnA = Simplify[fn1Gca0A / eca];
     fnGA = fn1eca0A / Gca;
     fncA = H * GcA;
     fG1A = U*Gh + U*H*vh + g*H*eh;
     \texttt{fGlA} = \texttt{fGlA} \ / . \ \texttt{vh} \ \rightarrow \ (\texttt{GGA} \star \texttt{Gca} \ + \ \texttt{GnA} \star \texttt{eca}) \ \ / . \ \ \texttt{eh} \ \rightarrow \ \texttt{RA} \star \texttt{eca} \ / . \ \ \texttt{Gh} \ \rightarrow \ \texttt{RA} \star \texttt{Gca} \ ;
     fG1Gca0A = fG1A / . Gca \rightarrow 0 ;
     fGleca0A = fGlA /. eca \rightarrow 0;
     fGnA = Simplify[fG1Gca0A / eca];
     fGGA = Simplify[fGleca0A / Gca];
     fGcA = U * H * GcA;
     FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
     FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
     FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
     FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;
     MatA = { {FnnA, FnGA}, {FGnA, FGGA} } ;
```

```
EA = Simplify[{{1, 0}, {0, 1}} + MatA];
    wAp = U * k + \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};
    wAm = U * k - \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};
ln[37]:= M = 1;
     Merr = Series[M - MA, \{dx, 0, 5\}];
     Rm = (1 + I * Sin[k * dx] / 2);
     Rmerr = Series[Rm - RA, \{dx, 0, 4\}];
     Rp = Exp[I*k*dx]*(1 - I*Sin[k*dx]/2);
     Rperr = Series[Rp - RA, {dx, 0, 4}];
     GRHSp1 = -\text{Exp}[-\text{I} * \text{k} * \text{dx}/2] + 2 + 4 * \text{Exp}[\text{I} * \text{k} * \text{dx}/2] +
        Exp[I*k*dx]*(4*Exp[-I*k*dx/2]+2-Exp[I*k*dx/2]);
     GRHSp1 = GRHSp1 / Exp[I * k * dx / 2];
     GRHSp1 = Expand[GRHSp1];
     GRHSp1 = ExpToTrig[GRHSp1];
     GRHSp2 = Exp[-I*k*dx/2] - 8 + 7*Exp[I*k*dx/2] +
        \text{Exp}[I*k*dx]*(7*\text{Exp}[-I*k*dx/2] - 8 + \text{Exp}[I*k*dx/2]);
     GRHSp2 = GRHSp2 / Exp[I*k*dx/2];
     GRHSp2 = Expand[GRHSp2];
     GRHSp2 = ExpToTrig[GRHSp2];
     GGLHS = dx / 6 * (Rp + Rm);
     GG2 = GGLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2);
     GG2err = Series[GG2 - GGA, {dx, 0, 5}];
     GnLHS = -U * (dx / 6) * (Rp + Rm);
     Gn2 = GnLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2);
     Gn2err = Series[Gn2 - GnA, {dx, 0, 5}];
     Text[Row[{"M || ", M}]]
     Text[Row[{"M || ", TeXForm[M]}]]
     Text[Row[{"M error || ", TeXForm[Merr]}]]
     Text[Row[{"M error || ", Merr}]]
     Text[" "]
     Text[Row[{"Rm || ", Rm}]]
     Text[Row[{"Rm || ", TeXForm[Rm]}]]
     Text[Row[{"Rm error || ", Rmerr}]]
     Text[Row[{"Rm error || ", TeXForm[Rmerr]}]]
     Text[" "]
     Text[Row[{"Rp || ", Rp}]]
     Text[Row[{"Rp || ", TeXForm[Rp]}]]
```

```
Text[Row[{"Rp error || ", Rperr}]]
                                        Text[Row[{"Rp error || ", TeXForm[Rperr]}]]
                                        Text[" "]
                                          Text[Row[{"GG2 || ", GG2}]]
                                          Text[Row[{"GG2 || ", TeXForm[GG2]}]]
                                        Text[Row[{"GG2 error ||
                                                                                                                                                                                                                                                                 ", GG2err}]]
                                          Text[Row[{"GG2 error || ", TeXForm[GG2err]}]]
                                        Text[" "]
                                        Text[Row[{"Gn2 || ", Gn2}]]
                                          Text[Row[{"Gn2 || ", TeXForm[Gn2]}]]
                                          Text[Row[{"Gn2 error ||
                                                                                                                                                                                                                                                                  ", Gn2err}]]
                                          Text[Row[{"Gn2 error || ", TeXForm[Gn2err]}]]
 Out[57]= M \parallel 1
 Out[58]= M \parallel 1
 \label{eq:outsol} Outsol= Merror \parallel -\frac{k^2}{24}-\frac{4k^2}{5760}+O\left(\frac{k^2}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^2}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(
 \text{Out[60]=} \ \ M \ error \ \ || \ \ -\frac{k^2 \ dx^2}{24} - \frac{7 \ k^4 \ dx^4}{5760} + O[dx]^6
 Out[61]=
 Out[62]= Rm || 1 + \frac{1}{2} i \sin[dx k]
 \label{eq:outf63} \mbox{Outf63]=} \ Rm \ || \ 1 + \mbox{frac}\{1\}\{2\} \ i \ \mbox{$i$ ($$ text{d}x$} \} \ k)
 Out[64]= Rm error || \frac{k^2 dx^2}{12} - \frac{1}{12} i k^3 dx^3 + \frac{k^4 dx^4}{720} + O[dx]^5
 Out[65]= Rm error |
                                                   \label{eq:continuous} $$ \frac{dx}^2 k^2}{12}-\frac{1}{12} i \text{$k^3 k^3+\frac{dx}^4 k^4}{720}+O\left(\frac{dx}^5\right) i \text{$k^3+\frac{dx}^4 k^4} \right) $$
 Out[67]= Rp || e^{i \operatorname{dx} k} \left(1 - \frac{1}{2} i \operatorname{Sin}[\operatorname{dx} k]\right)
 \label{eq:outloss} \begin{tabular}{ll} Outloss=& Rp & || & e^{i \cdot text\{dx\} k} \cdot left(1-\frac{1}{2} i \cdot sin (\cdot text\{dx\} k) \cdot right) \\ \end{tabular}
 Out[69]= Rp error || \frac{k^2 dx^2}{12} + \frac{1}{6} i k^3 dx^3 - \frac{89 k^4 dx^4}{720} + O[dx]^5
 Out[70]= Rp error ||
                                                   \frac{dx}{dx}^2 k^2}{12}+\frac{1}{6} i \cdot \frac{dx}^3 -\frac{g}{4x}^4 k^4}{720}+O\left(\frac{1}{6x}^5\right) i \cdot \frac{dx}^3 -\frac{g}{4x}^4 k^4}{720}+O\left(\frac{1}{6x}^5\right) i \cdot \frac{1}{6x}^5 -\frac{1}{6x}^5 -\frac{1
 Out[71]=
 \text{Out} \text{[72]=} \quad GG2 \quad \text{[I]} \quad \frac{dx \left(1 + e^{i \, dx \, k} \left(1 - \frac{1}{2} \, i \, \text{Sin}[dx \, k]\right) + \frac{1}{2} \, i \, \text{Sin}[dx \, k]\right)}{6 \left(\frac{1}{30} \, dx \, H\left(8 + 4 \, \text{Cos}\left[\frac{dx \, k}{2}\right] - 2 \, \text{Cos}[dx \, k]\right) + \frac{H^3 \left(14 - 16 \, \text{Cos}\left[\frac{dx \, k}{2}\right] + 2 \, \text{Cos}[dx \, k]\right)}{9 \, dx}\right)} \right)
```

 $\label{eq:condition} Out[73]= GG2 || \frac{\text{k} \left(\frac{x} \left(\frac{x} \right) \left(\frac{1}{2} i \sin \left(\frac{x} k\right)\right)}{6 \left(\frac{x} k\right)} \right)}{6 \left(\frac{x} k\right)} || \frac{1}{2} i \sin \left(\frac{x} k\right)}{6 \left(\frac{x} k\right)} || \frac{x} k\right)}{6 \left(\frac{x} k\right)} || \frac{x} k\right$ 

$$\text{Out} \text{[74]= } \text{GG2 error } \text{[} \text{[} \frac{\left(12\,k^2 + 5\,H^2\,k^4\right)dx^2}{40\,H\left(3 + H^2\,k^2\right)^2} + \frac{i\left(12\,k^3 + 5\,H^2\,k^5\right)dx^3}{80\,H\left(3 + H^2\,k^2\right)^2} + \frac{\left(-6651\,k^4 - 4680\,H^2\,k^6 - 820\,H^4\,k^8\right)dx^4}{4800\,H\left(3 + H^2\,k^2\right)^3} - \frac{i\left(6291\,k^5 + 4410\,H^2\,k^7 + 770\,H^4\,k^9\right)dx^5}{9600\,H\left(3 + H^2\,k^2\right)^3} + O[dx]^6$$

Out[75]= GG2 error |

 $\label{left(4x)^2 \left( \frac{k^2 + 1 \ k^2 \right)}{40 \ H \left( \frac{k^2 \ k^2 + 3 \right)^2} + \frac{i \ k^3 \left( \frac{k^3 \ k^4 + 12 \ k^2 + 3 \right)}{40 \ H \left( \frac{k^2 \ k^2 + 3 \right)^2} + \frac{i \ k^4 \ k^3 \left( \frac{k^3 \ k^4 \ k^6 - 6651 \ k^4 \right)}{40 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} - \frac{i \ k^4 \ k^5 \left( \frac{k^3 \ k^4 \ k^6 - 6651 \ k^4 \right)}{40 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} - \frac{i \ k^4 \ k^5 \left( \frac{k^3 \ k^6 - 6651 \ k^4 \right)^3}{40 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} + \frac{k^6 - 6651 \ k^4 \ k^6 + 6291 \ k^5 \right)^3}{4000 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} + \frac{k^6 - 6651 \ k^4 \ k^6 \right)^3}{4000 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} + \frac{k^6 - 6651 \ k^6 + 6291 \ k^6 \right)^3}{4000 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} + \frac{k^6 - 6651 \ k^6 + 6291 \ k^6 + 6291 \ k^6 \right)^3}{4000 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} + \frac{k^6 - 6291 \ k^6 + 6291 \ k^6 + 6291 \ k^6 \right)^3}{4000 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} + \frac{k^6 - 6291 \ k^6 + 6291 \$ 

Out[76]=

$$\text{Out[77]=} \quad Gn2 \quad || \quad - \frac{\text{dx } U \left( 1 + e^{i \cdot \text{dx } k} \left( 1 - \frac{1}{2} \cdot i \cdot \text{Sin}[\text{dx } k] \right) + \frac{1}{2} \cdot i \cdot \text{Sin}[\text{dx } k] \right) }{6 \left( \frac{1}{30} \cdot \text{dx } H \left( 8 + 4 \cdot \text{Cos} \left[ \frac{\text{dx } k}{2} \right] - 2 \cdot \text{Cos}[\text{dx } k] \right) + \frac{H^3 \left( 14 - 16 \cdot \text{Cos} \left[ \frac{\text{dx } k}{2} \right] + 2 \cdot \text{Cos}[\text{dx } k] \right)}{9 \cdot \text{dx}} \right) } \right) }$$

 $\label{eq:condition} Out[78] Gn2 & -\frac{text{dx} U \left(e^{i \left(x_{dx} k\right) k\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin$ 

$$\begin{array}{ll} \text{Out} \text{[79]=} & Gn2 \; error \; || \\ & - \frac{\left(\left(12 \, k^2 + 5 \, H^2 \, k^4\right) \, U\right) \, dx^2}{40 \left(H \, \left(3 + H^2 \, k^2\right)^2\right)} - \frac{i \left(12 \, k^3 + 5 \, H^2 \, k^5\right) \, U \, dx^3}{80 \, H \, \left(3 + H^2 \, k^2\right)^2} + \frac{\left(6651 \, k^4 + 4680 \, H^2 \, k^6 + 820 \, H^4 \, k^8\right) \, U \, dx^4}{4800 \, H \, \left(3 + H^2 \, k^2\right)^3} + \frac{i \left(6291 \, k^5 + 4410 \, H^2 \, k^7 + 770 \, H^4 \, k^9\right) \, U \, dx^5}{9600 \, H \, \left(3 + H^2 \, k^2\right)^3} + O[dx]^6 \\ \end{array}$$

Out[80]= Gn2 error |

 $-\frac{k^2 \cdot k^4+12 \cdot k^2 \cdot k^4+12 \cdot k^2 \cdot k^4+12 \cdot k^2 \cdot k^4+12 \cdot k^2 \cdot k^4+12 \cdot k^4 \cdot k^4+12 \cdot k^4 \cdot k^4+12 \cdot k$ 

```
In[81]= KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (U - Sqrt[g * H]);
KurFWSeta =

KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.

qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
```

```
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
\texttt{KurFWSG} = \texttt{KurFWS} \ / . \ \texttt{fp} \ \rightarrow \ (\texttt{U} * \texttt{Rpp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rpp} * \texttt{n}) \ / .
       \texttt{fm} \rightarrow (\texttt{U} * \texttt{Rmp} * \texttt{G} + \texttt{U} * \texttt{H} * \texttt{v} + \texttt{g} * \texttt{H} * \texttt{Rmp} * \texttt{n}) \ /. \ \texttt{qp} \rightarrow \texttt{Rpp} * \texttt{G} \ /. \ \texttt{qm} \rightarrow \texttt{Rmp} * \texttt{G};
\texttt{KurFWSG} = \texttt{KurFWSG} \; / \; . \; \; \forall \; \rightarrow \; (\texttt{GGp} * \texttt{G} \; + \; \texttt{Gnp} * \texttt{n}) \; ;
KfGn = KfGnp /. Rpp \rightarrow Rp /. Rmp \rightarrow Rm /. GGp \rightarrow GG2 /. Gnp \rightarrow Gn2;
KfGG = KfGGp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" -Sqrt(gH) < U < Sqrt(gH)"}]]</pre>
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn || ", KfGnp}]]
Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error || ", FGn2TAr}]]
```

```
Text[" "]
                                             Text[Row[{"FGG || ", KfGGp}]]
                                              Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
                                             Text[Row[{"FGG error ||
                                                                                                                                                                                                                                                                          ", FGG2TAr}]]
                                             Text[Row[{"FGG error ||
                                                                                                                                                                                                                                                                          ", TeXForm[FGG2TAr]}]]
                                             Text[" "]
                                             Text[" "]
                                             Text[Row[{"Omega error ||
                                                                                                                                                                                                                                                                          ", RKstepTayr}]]
                                                                                                                                                                                                                                                                             ", TeXForm[RKstepTayr]}]]
                                             Text[Row[{"Omega error ||
                                             Text[" "]
                                             Text[Row[{"EA ||
                                                                                                                                                                                                        ", EA}]]
                                              Text[Row[{"EA || ", TeXForm[EA]}]]
                                              Text[Row[{"Eerr || ", Eerr}]]
                                              Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
Out[115]= -Sqrt(gH) < U < Sqrt(gH)
Out[116]=
Out[117]= Fnn \parallel \frac{1}{2} \left( 2 \operatorname{Gnp} H + \operatorname{Rpp} \left( -\sqrt{g H} + U \right) + \operatorname{Rmp} \left( \sqrt{g H} + U \right) \right)
 \label{eq:continuous} Out[118] Fnn \parallel \frac{1}{2} \left( \frac{1}{2} \left( \frac{Rmp} \left( \frac{H}+U\right) + \frac{Rmp}{H} \right) \right) \\
\text{Out} \text{[119]= Fnn error } \text{||} \ \left( -\frac{\left( H^2\,k^3\,U\,w \right)dt^2}{2\left( 3\!+\!H^2\,k^2 \right)} - \frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left( 3\!+\!H^2\,k^2 \right)} + O[dt]^4 \right) + \\
                                                                 \left(-\frac{\imath \left(54 \, k^{3}+45 \, H^{2} \, k^{5}+10 \, H^{4} \, k^{7}\right) \cup dt}{120 \left(3+H^{2} \, k^{2}\right)^{2}}+O[dt]^{4}\right) dx^{2}+\left(-\frac{1}{8} \left(\sqrt{g \, H} \, k^{4}\right) dt+O[dt]^{4}\right) dx^{3}+O[dx]^{4}
 Out[120]= Fnn error ||
                                                        \left(-\frac{t}{k^2 \cdot t}\right) - \frac{1}{k^2 \cdot t} \le \frac{t}{k^2 \cdot t} \le \frac{t}{k^
                                                                               \left(H^2 k^2+3\right)+O\left(t^4 k^7+45 H^2\right)
                                                                               \left(-\frac{1}{8}\right) \left(\frac{1}{8}\right) + O\left(\frac{1}{8}\right) + O\left(\frac{1}
Out[121]=
Out[122]= FnG || GGp H
Out[123]= FnG \parallel \text{text}\{GGp\} H
 \text{Out} [124] = \text{ FnG error } || \left( -\frac{3 (k \text{ w}) \text{ d} t^2}{2 \left(3 + \text{H}^2 \text{ k}^2\right)} - \frac{i \text{ k w}^2 \text{ d} t^3}{2 \left(3 + \text{H}^2 \text{ k}^2\right)} + \text{O}[\text{d} t]^4 \right) + \left( -\frac{i \left(12 \text{ k}^3 + 5 \text{ H}^2 \text{ k}^3\right) \text{ d} t}{40 \left(3 + \text{H}^2 \text{ k}^2\right)^2} + \text{O}[\text{d} t]^4 \right) \text{d} x^2 + \text{O}[\text{d} x]^4
```

Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]

 $\label{eq:continuous} $$ \inf_{0 \le 1/2} FnG error \| \left(-\frac{3 \text{ kw}}{2 \left(k w\right)}{2 \left(k w\right)}{2 \left(k w\right)} - \frac{i \text{ kc}}{3 k w^2}{2 \left(k w^2} - \frac{i \text{ kc}}{3 k w^2}{2 \left(k w\right)}{2 \left(k w\right)} - \frac{i \text{ kc}}{3 k w^2}{2 \left(k w\right)}{2 \left(k w\right)} - \frac{i \text{ kc}}{3 k w^2}{2 k^2} - \frac{i \text{ kc}}{3 k w^2}{2 k^2} - \frac{i \text{ kc}}{3 k w^2} - \frac{i \text{ kc}}{3 k w^2}{2 k w^2}$ 

Out[126]=

Out[127]= FGn || 
$$\frac{1}{2} \left( g H \left( Rmp + Rpp \right) + \left( 2 Gnp H + \sqrt{g H} \left( Rmp - Rpp \right) \right) U \right)$$

Out[128]= FGn ||

\frac{1}{2} \left(U \left(\sqrt{g H} (\text{Rmp}\-\text{Rpp}))+2 \text{Gnp} H\right)+g H (\text{Rmp}+\text{Rpp})\right)

$$\begin{array}{ll} \text{Out} \text{[129]=} & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w \right) dt^2}{2 \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^2 \, dt^3}{6 \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \\ & \left( -\frac{i \left( 90 \, g \, H \, k^3 + 60 \, g \, H^3 \, k^5 + 10 \, g \, H^5 \, k^7 - 36 \, k^3 \, U^2 - 15 \, H^2 \, k^5 \, U^2 \right) dt}{120 \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{8} \left( \sqrt{g \, H} \, k^4 \, U \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4 \\ & \left( -\frac{1}{8} \left( \sqrt{g \, H} \, k^4 \, U \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) dx^4 + O[dx]^4 + O$$

Out[130]= FGn error |

 $\label{left-proc} $\left(-\frac{\left(text\{dt\}^2 \left(H^2 k^2+3\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{1}{2 \left(H^2 k^$ 

Out[131]=

Out[132]= FGG || 
$$\frac{1}{2} \left( \sqrt{g H} Rmp - \sqrt{g H} Rpp + (2 GGp H + Rmp + Rpp) U \right)$$

out[133]= FGG || \\frac{1}{2} \\left(\text{Rmp} \\sqrt{g H}\-\text{Rpp} \\sqrt{g H}+U (2 \\text{GGp} H+\\text{Rmp}+\\text{Rmp}+\\text{Rpp})\\right)

$$\begin{array}{ll} \text{Out} \text{[134]=} & FGG \; error \; \parallel \; \left( -\frac{\left( k \left( 6 + H^2 \, k^2 \right) U \, w \right) dt^2}{2 \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 6 + H^2 \, k^2 \right) U \, w^2 \, dt^3}{6 \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( 126 \, k^3 + 75 \, H^2 \, k^5 + 10 \, H^4 \, k^7 \right) U \, dt}{120 \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \\ & \left( -\frac{1}{8} \left( \sqrt{g \, H} \; k^4 \right) dt + O[dt]^4 \right) dx^3 + \left( \frac{i \left( 13311 \, k^5 \, U + 11430 \, H^2 \, k^7 \, U + 3110 \, H^4 \, k^9 \, U + 260 \, H^6 \, k^{11} \, U \right) dt}{4800 \left( 3 + H^2 \, k^2 \right)^3} + O[dt]^4 \right) dx^4 + O[dt]^5 \end{array}$$

Out[135]= FGG error |

 $\label{left-text} $$\left(-\frac{\hat t}^2 \left(U \ \left(U \ \left(H^2 \ k^2+6\right)\right)}{2 \left(H^2 \ k^2+3\right)}-\frac{i \ text{dt}}^3 \ k \ U \ w^2 \left(H^2 \ k^2+6\right)}{6 \left(H^2 \ k^2+3\right)}+O\left(\frac{4}{3}\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}{2 \left(H^2 \ k^2+3\right)}+O\left(\frac{4}{3}\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}{2 \left(H^2 \ k^2+3\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}{2 \left(H^2 \ k^2+3\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}$ 

Out[136]=

Out[137]=

Out[138]= Omega error ||

1

$$\begin{cases} \frac{1}{6(2+H^2k^2)^2}k^3\left(\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\!\left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\!\right)dt^2 + \\ \frac{i\,k^2\left[3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right]^2dt^2}{8(3+H^2k^2)} - \frac{1}{20(3+H^2k^2)^2}\left[k^3\left(\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right]^3dt^2 + Q[dt]^3} + \\ \left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right)dt^4 + Q[dt]^3\right] + \\ \left(\frac{1}{240(3+H^2k^2)^2}k^3\left(20\,H^6k^6\,U^3 + 54\,U^2\left(9\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 10\,U\right)\right) + \\ \frac{1}{480(3+H^2k^2)^2}k^3\left(20\,H^6k^6\,U^3 + 54\,U^2\left(9\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 10\,U\right)\right) + \\ 5\,k^4\,U^2\left(11\,\sqrt{3}\ \sqrt{g\,H^3(3+H^2k^2)} + (216+147\,H^2\,k^2 + 25\,H^4\,k^4)\,U\right) + \\ 3\,k^2\left(15\,\sqrt{3}\ \sqrt{g^2\,11^2\,(3+H^2k^2)} + 109\,\sqrt{3}\ \sqrt{g\,H^3\left(3+H^2k^2\right)}\,U^2 + 180\,H^2\,U^3\right)\right)dt^2 + \\ \frac{1}{480(3+H^2k^2)^2}k^3\left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right) \\ \left(9\,g\,H(14+5\,H^2k^2) + U\left(102\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 180\,U + 20\,H^4k^4\,U + 5\,k^2\left(\sqrt{3}\ \sqrt{g\,H^3(3+H^2k^2)} + 8\,H^2\,U\right)\right)\right) \\ \left(k^2\left(42\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 180\,U + 20\,H^4k^4\,U + 15\,k^2\left(\sqrt{3}\ \sqrt{g\,H^3(3+H^2k^2)} + 8\,H^2\,U\right)\right) \\ \left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 180\,U + 20\,H^4k^4\,U + 15\,k^2\left(\sqrt{3}\ \sqrt{g\,H^3(3+H^2k^2)} + 8\,H^2\,U\right)\right) \\ \left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right)^2\right)dt^4 + O[dt]^5\right)dx^2 + \\ \left(-\frac{i\,k^2\left(2\,g\,H(3+H^2k^2) + \sqrt{3}\ g\,H(3+H^2k^2)}{10\,\sqrt{g\,H(3+H^2k^2)}} + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + \sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)} + 2075\,H^4k^4\right) + 2080\left(9\,\sqrt{g\,H\left(3+H^2k^2\right)} + 6\,k^2\,\sqrt{g\,H^3(3+H^2k^2)} + 46\,k^2\right)U\right)^2 \right)dt^4 \right) \right\} dt^4\right) \right\} dt^4\right) \left(-\left(4$$

$$\left(k^{3}\left[-\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right] + \left(3+H^{2}\,k^{2}\right)U\right)^{3}\left(3\,g\,H + U\left(-2\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right) + \left(3+H^{2}\,k^{2}\right)U\right)\right) \right) \\ dt^{4} + O[dt]^{9} + \\ \left(\frac{1}{240(3+H^{2}\,k^{2})^{2}}k^{3}\left(-42\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right) + 180\,U + 20\,H^{4}\,k^{4}\,U - 15\,k^{2}\left(\sqrt{3}\ \sqrt{g\,H^{5}\left(3+H^{2}\,k^{2}\right)}\right) - 8\,H^{2}\,U\right)\right) + \\ \frac{1}{480(3+H^{2}\,k^{2})^{2}}k^{3}\left(20\,H^{6}\,k^{6}\,U^{3} + 54\,U^{2}\left(-9\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right) + 10\,U\right) + \\ 5\,k^{4}\,U^{2}\left(-11\,\sqrt{3}\ \sqrt{g\,H^{9}\left(3+H^{2}\,k^{2}\right)} + 236\,H^{4}\,U\right) + \\ 6\,g\,H\left(-21\,\sqrt{3}\ \sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + 216\,H^{4}\,H^{2}\,k^{2}\right) + 25\,H^{2}\,k^{3}\right)U\right) - \\ 3\,k^{2}\left(15\,\sqrt{3}\ \sqrt{g\,3}\,H^{7}\left(3+H^{2}\,k^{2}\right) + 109\,\sqrt{3}\ \sqrt{g\,H^{5}\left(3+H^{2}\,k^{2}\right)}\,U^{2} - 180\,H^{2}\,U^{3}\right)\right)dt^{2} + \\ \frac{1}{480(3+H^{2}\,k^{2})^{2}}i^{4}k^{6}\left(3\,g\,H + U\left(-2\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + (3+H^{2}\,k^{2})\,U\right)\right) \\ \left(9\,g\,H\left(14+5\,H^{2}\,k^{2}\right) + U\left(-102\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 180\,U + 20\,H^{4}\,k^{4}\,U - \\ 5\,k^{2}\left(7\,\sqrt{3}\ \sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + 180\,U + 20\,H^{4}\,k^{4}\,U - 15\,k^{2}\left(\sqrt{3}\ \sqrt{g\,H^{5}\left(3+H^{2}\,k^{2}\right)} - 8\,H^{2}\,U\right)\right)\right) \\ \left(3\,g\,H + U\left(-2\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 180\,U + 20\,H^{4}\,k^{4}\,U - 15\,k^{2}\left(\sqrt{3}\ \sqrt{g\,H^{5}\left(3+H^{2}\,k^{2}\right)} - 8\,H^{2}\,U\right)\right)\right) \\ dx^{2} + \left(\frac{1}{16}\,i\,\sqrt{g\,H}\,k^{4}\left(-2+\frac{\sqrt{3}\,U}{\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}} + 3\,\sqrt{3}\,U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + \sqrt{3}\,H^{2}\,U\right)\right)\right)dt^{2} - \\ \frac{1}{12(2+H^{2}\,k^{2})^{3/2}}\left(k^{6}\left(g\left(-6\,H\,\sqrt{g\,H}\,(3+H^{2}\,k^{2}\right) + 15\,\sqrt{3}\,U\,H^{2}\,k^{2}\right) + 4\,H^{2}\,k^{2}\right)U\right)\right)^{2}dt^{4} + O[dt]^{5}\right) \\ dx^{3} + \left(\left(k^{6}\left(2\,g\,H\left(3+H^{2}\,k^{2}\right) + 3\,\sqrt{3}\,U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + \sqrt{3}\,H^{2}\,U\right)\right)\right)\right)\right)dt^{2} - \\ \left(3\,g\,H + U\left(-2\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 3\,\sqrt{3}\ U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + \sqrt{3}\,H^{2}\,U\right)\right)\right)\right)dt^{2} - \\ \frac{1}{12(2+H^{2}\,k^{2})^{3/2}}\left(k^{2}\left(3\,g\,H\left(13+H^{2}\,k^{2}\right) + 3\,\sqrt{3}\,U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + \sqrt{3}\,H^{2}\,U\right)\right)\right)\right)dt^{2} - \\ \frac{1}{12(2+H^{2}\,k^{2})^{3/2}}\left(k^{2}\left(3\,g\,H^{2}\left(3+H^{2}\,k^{2}\right) + 3\,\sqrt{3}\,U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+$$

$$\begin{split} g\,H\,U\left(447\,588\,\sqrt{3}\,\,H^2\,k^2\,U + 16\,705\,\sqrt{3}\,\,H^6\,k^6\,U - 648\left(693\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} - 688\,\sqrt{3}\,\,U\right) - \\ 15\,k^4\left(3408\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)} - 9985\,\sqrt{3}\,\,H^4\,U\right)\right) - \\ 80\left(1836\,\sqrt{g\,H\left(3 + H^2\,k^2\right)}\,\,U^3 + 612\,k^4\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)}\,\,U^3 + 68\,k^6\,\sqrt{g\,H^{13}\left(3 + H^2\,k^2\right)}\,\,U^3 + \\ 9\,k^2\left(421\,\sqrt{g^3\,H^7\left(3 + H^2\,k^2\right)}\,\,U + 204\,\sqrt{g\,H^5\left(3 + H^2\,k^2\right)}\,\,U^3\right)\right)\right)dt^2\right) / \\ \left(230\,400\,\sqrt{g\,H}\,\left(3 + H^2\,k^2\right)^{7/2}\right) - \frac{1}{25\,600\,(3 + H^2\,k^2)}\,i\,k^8\left(6\,g^2\,H^2\left(8046 + 5460\,H^2\,k^2 + 925\,H^4\,k^4\right) + \right. \\ 560\,H^8\,k^8\,U^4 + 432\,U^3\left(-143\,\sqrt{3}\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 105\,U\right) + \\ 5\,k^4\,U^3\left(-4139\,\sqrt{3}\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)} + 6048\,H^4\,U\right) + \\ 5\,k^6\,U^3\left(-461\,\sqrt{3}\,\sqrt{g\,H^{13}\left(3 + H^2\,k^2\right)} + 1344\,H^6\,U\right) - \\ 12\,k^2\left(3585\,\sqrt{3}\,\sqrt{g^3\,H^7\left(3 + H^2\,k^2\right)}\,U + 5161\,\sqrt{3}\,\sqrt{g\,H^5\left(3 + H^2\,k^2\right)}\,U^3 - 5040\,H^2\,U^4\right) + \\ g\,H\,U\left(-63\,720\,\sqrt{3}\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 282\,852\,U + 284\,364\,H^2\,k^2\,U + 10\,640\,H^6\,k^6\,U + \\ 5\,k^4\left(-1451\,\sqrt{3}\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)} + 19\,056\,H^4\,U\right)\right)\right)dt^3 - \frac{1}{460\,800\left(\sqrt{g\,H}\,\left(3 + H^2\,k^2\right)}\right)U}\right) + \\ 108\,k^2\left(2625\,\sqrt{g^5\,H^9\left(3 + H^2\,k^2\right)} - 15\,327\,\sqrt{3}\,g^2\,H^4\,U + 22\,383\,\sqrt{g^3\,H^7\left(3 + H^2\,k^2\right)}\,U^2 - \\ 19\,261\,\sqrt{3}\,g\,H^3\,U^3 + 4640\,\sqrt{g\,H^3\left(3 + H^2\,k^2\right)}\,U^4\right) + \\ g\,H^3\,U^2\left(89\,915\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} - 61\,735\,\sqrt{3}\,g^2\,H^6\,U + 27\,840\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)}\,U^4 + \\ g\,H^3\,U^2\left(89\,915\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} - 115\,737\,\sqrt{3}\,U\right)\right) + 1296\left(323\,\sqrt{g^5\,H^5\left(3 + H^2\,k^2\right)} - \\ 1268\,\sqrt{3}\,g^2\,H^2\,U + 290\,\sqrt{g\,H\left(3 + H^2\,k^2\right)}\,U^4 + g\,H\,U^2\left(1857\,\sqrt{g\,H\left(3 + H^2\,k^2\right)}\,U^3 + \\ g\,H^7\,U\left(-6019\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 15\,454\,\sqrt{3}\,U\right)\right)\right)\right)dt^4 + O[dt]^5\right)dt^4 + O[dt]^5\right)$$

Out[139]= Omega error ||

 $\left(\left(\frac{k^3 \left(\frac{k^2+3\right)} U+\right) + \left(\frac{3} \left(\frac{4}{2} k^2+3\right)\right) + \left(\frac{3} k^2+3\right) \right) + \left(\frac{3} k^2+3\right) \left(\frac{4}{2} k^2+3\right) + \left(\frac{4}$ 

 $\left(\frac{H^2 k^2+3\right}{k^2+3\right} k^4+3\left(\frac{180 H^2 U^3+109 \sqrt{3}}{k^2+3}\right)$  $U^2+15 \sqrt{3} \sqrt{10} U^2 + 15 \sqrt{3} \sqrt{10} U^2 \left( U^2 \right)$  $H \left( \frac{h^2 k^2+3 \right)}{tght} + 6 g H \left( \frac{5 h^4 k^4+147 h^2 k^2+216 \right) U+21 \left( \frac{3}{t} \right)$  $\left( H^2 k^2+3\right)\right)\right)$ g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \left(9 g  $H \left( \frac{5 \text{ H}^2 \text{ k}^2 + 14 \text{ right}}{4 \text{ U k}^4 + 5 \text{ left}(24 \text{ U H}^2 + 7 \text{ sqrt}\{3\} \text{ sqrt}\{g \text{ H}^5 \text{ left}(\text{H}^2 + \text{ left}(\text{H}^2 + \text{ left}) \text{ left$  $k^2+3\left( H \left( H^2 k^2+3\right) \right) \right)$  $\left(H^2 k^2+3\right)^3-\frac{(k^7 \left(H^2 U k^4+15\right)^3}{h^5}\right)^3$  $\left(H^2 k^2+3\right)\right) \$  $H+U \left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{H^2 k^2+3\right)}} \quad U+2 \left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{H^2 k^2+3}} \quad$  $\t x{dt}^4}{960 \left( \frac{h^2 k^2+3\right)^4}{+O\left( \frac{t}{t}^5\right) \right) \left( \frac{k^2+1}{t}^6} \right) }$  $\left(2 g H \left(H^2 k^2+3\right)\right) + \left(H^2 k^2+3\right) \$  $\sqrt{3} U+12 \operatorname{qrt}(g H \left(H^2 k^2+3\right)\right) U^2+g H \left(\frac{4 H^2 k^2+15\right) U+6$  $\label{eq:left} $\operatorname{H}\left(H^2 k^2+3\right)\right)\right) \cdot \left(H^2 k^2+3\right)^{3/2}+\frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2}+\frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2})\right)^{3/2} + \frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2})^{3/2} + \frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2} + \frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2})^{3/2} + \frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2} + \frac{h^2}{2}(32 \left(H$  $g H+U \left(\frac{H^2 k^2+3\right)}{U+2 \left(\frac{H^2 k^2+3\right)}\left(\frac{H^2 k^2+3\right)}{U+2 \left(\frac{H^2 k^2+3\right)}{U+2 \left(\frac{H^$ g H \left( $H^2 k^2+3\right\right)+U \left(\frac{1}{2} K^2+3\right)$  $k^8 \left( 16H(4^2 k^2+3\right) + \left( 16H(4^2 k^2+3\right) \right) + \left( 16H(4^2 k^2+3\right) \left( 16H(4^2 k^2+3\right) \right) + \left( 16H(4^2 k^2+3\right) + \left( 16H(4^2 k^2+3\right) \right) + \left( 16H(4^2 k^2+3\right) + \left( 16H(4^2 k^2 k^2+3\right) + \left( 16H(4^2 k^2 k^2+3\right) + \left( 16H(4^2 k^2+3\right) + \left( 16H(4$  $\label{eq:linear_state} $$ \left( \frac{H^2 k^2+3 \right)^3+O\left( \frac{dt}{5}\right) \operatorname{left}(x)^3+\left( \frac{dt}{5}\right) \operatorname{left}(x)^$ g H \left(2075 H^4 k^4+12180 H^2 k^2+17856\right)+2080 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)})  $k^4+6 \operatorname{qt}(H^2 k^2+3\operatorname{right}) k^2+9 \operatorname{qt}(H^2 k^2+3\operatorname{right}) \operatorname{dt}(H^2 k^2+3\operatorname{qt}(H^2 k^2+3\operatorname{$  $\label{eq:left(sqrt{g H} \left(45 \right)^2 \left(45 \right)^2$ k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6+15 \left(9985 \sqrt{3} U H^4+3408 \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+447588 \sqrt{3} H^2 U k^2+648 \left(688 \sqrt{3} U+693  $k^6 + 612 \sqrt{g H^9 \left( H^2 k^2 + 3\right)} U^3 k^4 + 9 \left( 9 H^5 \right) H^5 \left( H^2 k^2 + 3\right)$  $U^3+421 \cdot g^3 H^7 \cdot (h^2 k^2+3 \cdot g)$  Uright)  $h^2+1836 \cdot g$  H \left( $h^2 k^2+3 \cdot g$ )  $U^3\right) \left( \frac{H^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} -\frac{h^2 k^2}{1$ \left(560 H^8 U^4 k^8+5 U^3 \left(1344 U H^6+461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right)  $k^6+5 U^3 \left(048 U H^4+4139 \right) \left(H^9 \left(H^2 k^2+3\right)\right) k^4+12 \left(5040 H^4+13 \right) k^4+12 \left(141 H^2 k^2+3\right) k^4+12 \left(141 H^2 k$  $H^2 U^4+5161 \sqrt{3} \sqrt{4+5161} \sqrt{3} \sqrt{4+5161}$ k^2+3\right) U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105  $U+143 \cdot \{3\} \cdot \{g \in (19056 \cup 1905) + g \in (19640 \cup$ H^4+1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U+63720  $\left(\left(\frac{H^2 k^2+3\right)}{U+\sqrt{3} \left(\frac{H^2 k^2+3\right)}\right)\right)$  $g H^9+928 \sqrt{14} \left( H^2 k^2+3\right) U\right) h^8+15 U \left( H^2 k^2+3\right) U\right)$ k^2+3\right)} U^3\right) k^6+9 \left(61735 \sqrt{3} g^2 U H^6+g U^2 \left(115737 \sqrt{3} U+89915)  $\$  \\sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+27840 \\sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325  $\$  \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)} \left(k^4+108 \left(15327 \right) g^2 U H^4+19261 \right) sqrt{3} g^5 U H^5+19261

g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}  $U^4+g H \left(1202 \right) U+1857 \left(14 H^2 k^2+3\right) U+1857 \left(1$  $H^2 U+323 \left( \frac{g^5 H^5 \left( \frac{40800 \left( + \right)} \right)} \right)} {40800 \left( \frac{40800 \left( + \right)} \right)} {40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( + \right)} {40800 \left( \frac{40800 \left( + \right)} {40800 \left( \frac{40800 \left( + \right)} {40800 \left( +$  $\left(\left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \sqrt{3} \right) + \left(\frac{H^2 k^2+3\right)}\right) \left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \left(\frac{H^2 k^2+3\right)}}\right)$  $k^2+3\right) U-2 \sqrt{3} \sqrt{4} \ln(H^2 k^2+3\right) \int U-2 \sqrt{3} \left( H^2 k^2+3\right) \left($  $\label{eq:k-2+3-right} $$ k^2+3\right)^2+\frac{k^4\left(\frac{3}{4} +U\left(\frac{k^2 k^2+3\right) U-2 \right)U-2 \left(\frac{3}{4} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3}{4$  $\label{left(H^2 k^2+3\right)} $$ \left( H^2 k^2 + 3\right)^2 \left( H^2 k^2 + 3\right)^2 - \frac{h^2}{h^2} \left( H^2 k^2 + 3\right)^2 - \frac{h^2}{h^2} \left( H^2 k^2 + 3\right)^2 + \frac{h^2}{h^2} \left( H^2 k^2 + 3\right)^$  $k^2+3\right\in U-\sqrt{3} \left( U-\sqrt{3} \right) -3 \left( U-\sqrt{3} \right)$  $k^2+3\left( U-2 \right) U-2 \left( H^2 \left( H^2 \right) \right) \right) \left( H^2 \right) \left$  $k^2+3\left(\frac{4t}^5\right)+\left(\frac{4t}^5\right)$  $\left(H^2 k^2+3\right)^2+\frac{h^2 u^2}{h^2 u^2} + \frac{h^2 u^2 \left(H^2 u^2 + \frac{h^2 u^2}{h^2 u^2}\right)}{h^2 u^2 u^2} + \frac{h^2 u^2 u^2}{h^2 u^2} + \frac{h^2 u^2 u^2}{h^2 u^2} + \frac{h^2 u^2}{h^2 u^2} + \frac{$  $\label{left(H^2 k^2+3\wedge left(H^2 k^2+3\wedge left($  $U^2+15 \sqrt{3} \sqrt{6^3 H^7 \left(H^2 k^2+3\right)} \right) + 2 + 4 U^2 \left(10 U-9 \right)$ H^2 k^2+14\right)+U \left(20 H^4 U k^4-5 \left(7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-24 H^2  $U = U + 180 U - 102 \left( \frac{3} \left( \frac{480 e^2 + 3 e^2 + 3 e^2}{180 e^2} \right) \right)$  $k^2 + 3 \cdot (k^7 \cdot (k^7 \cdot k^7 \cdot k^7$  $H^2 U \cdot k^2 + 180 U - 42 \cdot k^2$  $k^2+3\right) U-2 \sqrt{3} \sqrt{g} H \left(\frac{h^2 k^2+3\right)}\right) \left(\frac{h^2 k^2+3\right)}{16h} \left(\frac{h^2 k^2+3\right)}{16h}\right) \left(\frac{h^2 k^2+3\right)}{16h}$  $\left(\frac{4}{2} - \frac{4}{1}(1)^4 + O\left(\frac{4t}{5}\right)\right) \cdot \left(\frac{4t}{2} - \frac{4t}{2}\right)$  $\left(\frac{3}{U}\right) = \frac{1}{2\pi} \left(\frac{3}{U}\right)$  $U-2 \left( H^2 k^2+3\right) + 12 \left( H^2 k^2+3\right) + 12$  $U^2+g \left( \frac{1}{x} k^2 U H^3+15 \right) + H^2(3) U H^6 \left( \frac{1}{x} k^2 + \frac{1}{x} k^2 U H^3 +$  $\label{eq:left} $$ \operatorname{dt}^2_{32}\left(H^2 k^2+3\right)^{3/2}-\frac{(k^7 \left(h^7 \left(h^2 k^2+3\right)^3)^{3/2}\right)^{-1}}{h^7} \right) $$$  $U-2 \sqrt{3} \left( H \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} \right) \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3)} \left( \frac{h^2 k^2+3\right)}{h^2 k^2+3} \left( \frac{h^2 k^2+3}{h^2 k^2+3} \right) \left( \frac{h^2 k^2+3}{h^2 k^2+3} \left( \frac{h^2 k^2+3}{h^2 k^2+3} \right) \left( \frac{h^2 k^2+3}{h^2 k^2+3} \left( \frac{h^2 k^2+3}{h^2 k^2+3} \right) \left( \frac{h^2 k^2+3}{h^2 k^2+3} \left( \frac{h^2 k^2$  $\left(\left(\frac{3}{4}\right) + \frac{3}{4}\right) = \frac{1}{2} \left(\frac{3}{4}\right) + \frac{3}{4}\left(\frac{3}{4}\right) +$ \left(H^2 k^2+3\right)\right)\right)\right)\right)\right)\\text{dt}^3}{32 \left(H^2 k^2+3\right)^{5/2}}+\frac{i k^8 \left(2)}{2}  $\label{eq:linear_line$  $k^2+3\right)^3+O\left(\frac{dt}^5\right)\right)$  $H^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 H^2 k^2 + 1786 H$  $H^5 \left( H^2 k^2 + 3\right) \left( H^2 k^2 + 3\right)$  $\left(H^2 k^2+3\right)^{5/2}+\frac{k^7 \left(45 \right)^2}{9^2 \left(45 \right)^2}$  $H^2+g U \left(16705 \right) -15 \left(14705 \right)$  $H^4 U \cdot k^4 + 447588 \cdot k^4 + 447588 \cdot k^4 + 447588 \cdot k^4 - 447588 \cdot k^4 + 447688 \cdot k^4 + 44768$ \sqrt{3} U\right)\right) H-80 \left(68 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+612 \sqrt{g H^9  $\left(H^2 k^2+3\right) U^3 k^4+9\left(204 \right) 16$  $H^7 \left(H^2 k^2+3\right) U\right) U\right)$ 

\left(1344 H^6 U-461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 H^4  $U-4139 \sqrt{3} \sqrt{4-12 \left( -5040 \text{ H}^2 \text{ U}^4+5161 \right)} \right)$  $\sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \right]$  $k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 U-143 \right)$  $H \left( \frac{h^2 k^2+3\right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 U k^$ H^9 \left(H^2 k^2+3\right)\right) k^4+284364 H^2 U k^2+282852 U-63720 \sqrt{3} \sqrt{g H \left(H^2  $\label{left} $$ k^2+3\right)\right) \right) \left( k^9 \left( k^2 + 3\right) \right) \left( k^9 \left( k^9 \right) \right) \left( k^9 \left( k^9 \right) \right) \right) \left( k^9 \left( k^9 \right) \right) \left( k^9 \left($  $H \left( H^2 k^2 + 3 \right) - \left( H^2 k^2 + 3 \right) - \left( H^2 k^2 + 3 \right) \left( H^2 k^2 + 3 \right) - \left( H$ k^2+3\right)} U-3869 \sqrt{3} g H^9\right) k^8-15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3})  $U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-3 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3} U-6019 U$ k^6+9 \left(-61735 \sqrt{3} g^2 U H^6+g U^2 \left(89915 \sqrt{g H \left(H^2 k^2+3\right)}-115737 \sqrt{3} U\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)\right) k^4+108 \left(-15327 \sqrt{3} g^2 U H^4-19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g}  $H^5 \left(H^2 k^2+3\right) U^4+22383 \left(H^2 k^2+3\right) U^4+22383 \right) U^4+22383 \left(H^2 k^2+3\right) U^4+24383 \left(H^4 k^2+3\right) U^4+24383 \left(H^4 k^2+3\right) U^4+24383 \left(H^4 k^2+3$  $H^9 \left( H^2 k^2 + 3 \right) \right) + H^9 \left( H^2 k^2 + 3 \right) + H^9 \left( H^8 k^2 + 3 \right) + H$ \left(1857 \sqrt{g H \left(H^2 k^2+3\right)}-1202 \sqrt{3} U\right) U^2-1268 \sqrt{3} g^2 H^2 U+323  $\ \left( \frac{g^5 H^5 \left( H^2 k^2 + 3\right)}{\sinh(H^2 k^2 + 3\right)}\right) \$  $k^2+3\right)^{1/2}\right)+O\left(\frac{dt}^5\right)\right)$ 

 $\text{Out} \text{[141]=} \quad EA \text{ ||} \quad \left\{ \left\{ \frac{-H^2 \ k^2 \left( \left( -1 + e^{i \ d \ w} \right) k \ U - w \right) + 3 \ w}{\left( 3 + H^2 \ k^2 \right) w}, \ -\frac{3 \left( -1 + e^{i \ d \ w} \right) k}{\left( 3 + H^2 \ k^2 \right) w} \right\}, \ \left\{ -\frac{\left( -1 + e^{i \ d \ w} \right) k \left( g \ H \left( 3 + H^2 \ k^2 \right) - 3 \ U^2 \right)}{\left( 3 + H^2 \ k^2 \right) w}, \ 1 \ -\frac{\left( -1 + e^{i \ d \ w} \right) k \left( 6 + H^2 \ k^2 \right) U}{\left( 3 + H^2 \ k^2 \right) w} \right\} \right\}$ \begin{array}{cc}  $\frac{3 \text{ w-H}^2 \text{ k}^2 \left(\left(-1+e^{i \text{ text}}\right) \text{ k U-w}\right)}{\left(-1+e^{i \text{ text}}\right) \text{ k U-w}\right)}{\left(-1+e^{i \text{ text}}\right)}$  $k^2+3\right) w$  &  $-\frac{3 \left(1+e^{i \left(t\right) w}\right) k}{\left(t\right) w} \$  $-\frac{1+e^{i \cdot text\{dt\} w}\right| k \left(H^2 k^2+3\right)}{\|h\|^2 k^2+3\right)} \|h\|^2 \|h\|^2$ 

\right)

\end{array}

Out[140]=

$$\begin{aligned} & \sup_{3 \neq |V| \leq |V|} \mathbb{E} \left( \frac{\sqrt{3} |V|}{3 + |V|^2} + \frac{\sqrt{3} |V|}{3 + |V|^2} + \frac{\sqrt{3} |V|}{3 + |V|^2} \right) dt^2 - \frac{1}{24} \left( -\frac{i\sqrt{3} |V|}{3 + |V|^2} + \frac{1}{2} |V|}{3 + |V|^2} \right) dt^2 + O(dt)^2 \right) + \\ & = \frac{1}{6} \left( -\frac{i\sqrt{3} |V|}{3 + |V|^2} + \frac{1}{2} |V|}{3 + |V|^2} \right) dt^2 - \frac{1}{24} \left( -\frac{i\sqrt{3} |V|}{3 + |V|^2} + \frac{1}{2} |V|}{3 + |V|^2} \right) dt^2 + O(dt)^2 \right) dt^2 + O(dt)^2 \right) dt^2 + O(dt)^2 \right) dt^2 + O(dt)^2 dt^2 + O(dt$$

Out[144]= Eerr || \left( \begin{array}{cc}

 $\label{left} $\left(\frac{U + \sqrt{2} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{2} + 3 + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4}}{h^2 k^$  $k^2+\sqrt{3} \operatorname{H}^2 k^2+3\right) U k^2\right) U k^2\right) U k^2\right) U k^2\right) U k^2)$  $\label{eq:thm:linear} U \ k-\left(i \right)^3 \left(i \right)^3$  $\left(-i U k - \frac{i \sqrt{4} \left(H^2 k^2 + 3\right)}{k}{H^2 k^2 + 3\right)} \right)$  $\label{eq:left} $$ \operatorname{dt}^4+O\left(\frac{dt}^5\right)\right)+\left(\frac{10 H^4 k^7+45 H^2 k^5+54 k^3\right) U}{t}$  $\text{dt}_{120 \leq (H^2 k^2+3\right)} = \frac{120 \left(H^2 k^2+3\right)}{120 \left(H^2 k^2+3\right)} = \frac{120 \left(H^2 u^2 k^6+72\right)}{120 \left(H^2 u^2 k^6+72\right)} = \frac{120 \left(H^2 u^2 k^6+72\right)}{120 \left(H^2 u^2 k^6+72\right)}$  $U^2 k^4+126 g H k^4 \right) \left( \frac{4t}^2}{240 \left( \frac{4^2}{2} + \frac{4^2}{16} \right)^2} + O\left( \frac{4t}^5 \right) \right) \left( \frac{4^2}{2} + \frac{4^2}{16} + \frac{4^2}{16} \right) \left( \frac{4^2}{2} + \frac{4^2}{2} + \frac{4^2}{16} + \frac{4^2}{2} + \frac{4^2}{16} \right) \left( \frac{4^2}{2} + \frac{4$  $\t \{dx\}^2 + \left(-\frac{1}{8} \left(\frac{H} k^4\right) + \frac{dt}{+\frac{1}{8} \left(\frac{H^2}{4t}\right)} \right)$  $k^7+3 k^5 \oplus U \text{ (left(H}^2 k^2+3 \text{ right))} + O(\text{left(H}^2 k^2+3 \text{ right)}) + O(\text{left(H}^2 k^2+3 \text{ r$ \text{dx}^3+\left(\frac{i \left(260 H^6 U k^{11}+1570 H^4 U k^9+2610 H^2 U k^7+729 U k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 H^6 U^2 k^{12}+4500 g H^5 H^5)} k^{10}+4140 H^4 U^2 k^{10}+26460 g H^3 k^8-13500 H^2 U^2 k^8-38286 U^2 k^6+38853  $U \right) \left( \frac{dt}^2}{H^2 k^2 + 3} + O \left( \frac{t}^5 \right) \right) \right) \left( -\frac{t}{100} \right)$  $\t (text{dt}^2){40 \left(H^2 k^2+3\right)^2} + O\left(text{dt}^5\right) \right) \t (text{dt}^2-text{dt}^2-text{dx}^2+text{dx}^2-tex$  $i \sqrt{H} k^5 \text{(}dt^2}{8 \left(H^2 k^2+3\right)}+O\left(\text{}t^5\right)\right)$  $\text{text}\{dx\}^3 + \text{left}(\frac{770 \text{ H}^4 \text{ k}^9 + 4410 \text{ H}^2 \text{ k}^7 + 6291 \text{ k}^5} \right) \text{ text}\{dt\} \{4800 \text{ left}(\text{H}^2 \text{ k}^7 + 6291 \text$ k^2+3\right)^3}+\frac{\left(1500 H^4 U k^{10}+8820 H^2 U k^8+12951 U k^6\right) \text{dt}^2}{4800}  $\label{left(H^2 k^2+3\right)^3} + O\left(\frac{dt}^5\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^5\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^5\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^4\right) \cdot \left(\frac{dx}^4\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^4\right) \cdot \left(\frac{dx}^4\right) \cdot \left($  $\left(-\frac{k^2 U}{eft(g k^2 H^3+3 g H-3 U^2\right)}\right)$  $k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3 \right) text{dt}{120 \left( H^2 k^2+3 \right)^2}+\frac{1}{120 \left( H^2 k^2+3 \right)^2}$ g H^5 U k^8+45 H^2 U^3 k^6-120 g H^3 U k^6+126 U^3 k^4-180 g H U k^4\right) \text{dt}^2}{120 U\right) \text{dt}+\frac{i \left(H^2 \sqrt{g H} U^2 k^7+g H^3 \sqrt{g H} k^7+3 g H \sqrt{g H} k^5\right)  $\label{left(H^2 k^2+3\left| (text{dt}^5\right| text{dx}^3+\left| (text{dx}^3+$ g H^7 k^{11}+2340 g H^5 k^9-770 H^4 U^2 k^9+7020 g H^3 k^7-4410 H^2 U^2 k^7-6291 U^2 k^5+7020 g H k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 g H^7 U k^{12}-4500 g H h^7 U k^12}-4500 g H h^7 U k H<sup>4</sup> U<sup>3</sup> k<sup>1</sup> 13140 g H<sup>5</sup> U k<sup>1</sup> 13140 g H<sup>5</sup> U k<sup>1</sup> 10}-26460 H<sup>2</sup> U<sup>3</sup> k<sup>8</sup>+39420 g H<sup>3</sup> U k<sup>8</sup>-38853 U<sup>3</sup>  $k^6+39420 \text{ g H U } k^6 \text{ hight} \text{ $k^6+39420 \text{ g H U } k^6 \text{ hight}} 14400 \left(\frac{h^2 k^2+3 \right)^3}+O\left(\frac{dt}{5}\right)^3$  $\t (\sqrt{dx}^4 + O\left(\frac{dx}^5\right) \cdot \left(\frac{dx}^5\right) \cdot \left(\frac{dx}^6\right) \cdot$ U = U + (1) + ( $\text{dt}^2_{H^2 k^2+3}-\frac{1}{6} \left(i \ U \ -\frac{1}{3} \right) + \frac{1}{6} \left(i \ U \ -\frac{1}{3} \right) + \frac{1}{3} \left(i \ U \ -\frac{1}{3} \right) + \frac{1}{3}$  $k^2+3\right\simeq h^3 \cdot (h^2 - h^2) \cdot$  $k^2+3\right\right)^4 \left(10 H^4 k^7+75 H^2 k^5\right)+\left(10 H^4 k^7+75 H^2 k^5+126 H^2 k^2+3\right)+\left(10 H^4 k^7+75 H^2 k^5+126 H^2 k^5\right)+\left(10 H^4 k^7+75 H^2 k^5\right)+\left(10 H^4 k^5\right)+\left(10$ k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}-\frac{\left(20 H^4 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 H^2 U^2)}  $k^6+432 \text{ U}^2 \text{ k}^4+126 \text{ g H k}^4\right) \text{ (text{dt}}^2{240 \left(\frac{h^2 k^2+3\right)^2}+O\left(\frac{t}{t}\right)^5\right)}$  $\label{left} $$ \left( \frac{1}{8} \left( \frac{1}{8} \right) \left$  $k^2+15\right) U \text{ } U \text{ } t \text{ } t^2} 16 \text{ } \left( H^2 k^2+3\right) + O\left( t \text{ } t^3\right) + O\left( t \text{ } t^3\right) \right) \text{ } t \text{ } t^3+\left( t^3\right) + O\left( t \text{ } t^3\right) + O\left( t \text{ } t^3\right) \right)$ 

```
\left(H^2 k^2+3\right)^3}+\frac{\left(1460 H^6 U^2 k^{12}+4500 g H^5 k^{10}+22140 H^4 U^2)}
                                 k^{10}+26460 g H^3 k^8+92340 H^2 U^2 k^8+117126 U^2 k^6+38853 g H k^6\right) \text{dt}^2\{28800
                                 \left(\frac{dt}^5 \right) \cdot \left(\frac{
                 \end{array}
                 \right)
ln[145] = KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
                 KurFWS = KurF /. ap \rightarrow (U + Sqrt[g * H]) /. am \rightarrow 0;
                 KurFWSeta =
                         \texttt{KurFWS /. fp} \rightarrow (\texttt{H} * \texttt{v} + \texttt{U} * \texttt{Rpp} * \texttt{n}) \ /. \ \texttt{fm} \rightarrow (\texttt{H} * \texttt{v} + \texttt{U} * \texttt{Rmp} * \texttt{n}) \ /. \ \texttt{qp} \rightarrow \texttt{Rpp} * \texttt{n} \ /.
                              qm \rightarrow Rmp * n;
                 KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
                 Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1];
                 KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1];
                 Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
                 KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
                 Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
                 Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
                 Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                 FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
                 FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
                 FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                 KurFWSG = KurFWS /. fp \rightarrow (U*Rpp*G + U*H*v + g*H*Rpp*n) /.
                                     \texttt{fm} \rightarrow (\texttt{U} * \texttt{Rmp} * \texttt{G} + \texttt{U} * \texttt{H} * \texttt{v} + \texttt{g} * \texttt{H} * \texttt{Rmp} * \texttt{n}) \ /. \ qp \rightarrow \texttt{Rpp} * \texttt{G} \ /. \ qm \rightarrow \texttt{Rmp} * \texttt{G};
                 KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
                 KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
                 KfGGp = FullSimplify[KurFWSG /. n \rightarrow 0 /. G \rightarrow 1];
                 KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
                 KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
                 FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
                 FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
                 FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                  fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
                 FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
                 FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
                 FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                 Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
                 Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2;
                 Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], \{dx, 0, 4\}, \{dt, 0, 4\}];
                 EigvFmat2 = Eigenvalues[Fmat2];
```

\left(260 H^6 U k^{11}+3110 H^4 U k^9+11430 H^2 U k^7+13311 U k^5\right) \text{dt}}{4800

```
RKStep = Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2] / (I * dt);
     RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
     RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
     Text[Row[{" U < -Sqrt(gH)"}]]</pre>
     Text[" "]
     Text[Row[{"Fnn || ", Kfnnp}]]
     Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
     Text[Row[{"Fnn error || ", Fnn2TAr}]]
     Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
     Text[" "]
     Text[Row[{"FnG || ", KfnGp}]]
     Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
     Text[Row[{"FnG error || ", FnG2TAr}]]
     Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
     Text[" "]
     Text[Row[{"FGn || ", KfGnp}]]
     Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
     Text[Row[{"FGn error || ", FGn2TAr}]]
     Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
     Text[" "]
     Text[Row[{"FGG || ", KfGGp}]]
     Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
     Text[Row[{"FGG error || ", FGG2TAr}]]
     Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
     Text[" "]
     Text[" "]
     Text[Row[{"Omega error || ", RKstepTayr}]]
     Text[Row[{"Omega error || ", TeXForm[RKstepTayr]}]]
     Text[" "]
     Text[Row[{"EA || ", EA}]]
     Text[Row[{"EA || ", TeXForm[EA]}]]
     Text[Row[{"Eerr || ", Eerr}]]
     Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
Out[179]= U < -Sqrt(gH)
Out[180]=
Out[181]= Fnn || Gnp H + Rmp U
Out[182] = Fnn \parallel \text{text}\{Gnp\} H + \text{text}\{Rmp\} U
```

Out[184]= Fnn error |

 $\label{left-proc} $\left(\frac{dt}^2 \left(\frac{dt}^2 k^3 U \right)_{2 \left(\frac{dt}^2 k^2 + 3\right)} - \frac{i \left(\frac{dt}^3 H^2 k^3 U \right)_{6} \left(\frac{dt}^2 k^2 + 3\right)_{1} + \frac{dt}^4 k^3 U \right)_{2} e^{-t} e^$ 

Out[185]=

Out[186]= FnG || GGp H

Out[187]=  $FnG \parallel \text{text}\{GGp\} H$ 

$$\text{Out[188]=} \quad FnG \ error \ \mid \mid \ \left( -\frac{3 \ (k \ w) \ dt^2}{2 \left( 3 + H^2 \ k^2 \right)} - \frac{i \ k \ w^2 \ dt^3}{2 \left( 3 + H^2 \ k^2 \right)} + O[dt]^4 \right) \\ + \left( -\frac{i \left( 12 \ k^3 + 5 \ H^2 \ k^3 \right) \ dt}{40 \left( 3 + H^2 \ k^2 \right)^2} + O[dt]^4 \right) dx^2 \\ + O[dx]^4 + O[$$

 $\label{eq:continuous} $$\operatorname{FnG\ error} \| \left(-\frac{3 \left(t^2 (k w)}{2 \left(t^2 (k w)}\right)^2 \left(t^2 (k^2 + 3\right)\right) - \frac{1}{k^2 + 3\left(t^2 (k w)}^2 \left(t^2 (k^2 + 3\right)\right)} - \frac{1}{k^2 + 3\left(t^2 (k w)\right)^2} + \frac{1}{k^2 (k w)^2} \left(t^2 (k^2 + 3\right)\right) - \frac{1}{k^2 (k w)^2} - \frac{1}{k^2 (k w)^2} + \frac{1}{k^2 (k w)^2} - \frac{1}{$ 

Out[190]=

Out[191]=  $FGn \parallel H(gRmp + Gnp U)$ 

 $\label{eq:outsign} \mbox{Outsign} \mbox{ outsign} = \mbox{ } FGn \ \, || \ \, H \mbox{ } (g \mbox{ } \mbox{text} \mbox{Rmp} + \mbox{text} \mbox{Gnp} \mbox{ } U)$ 

$$\begin{array}{ll} \text{Out} \text{[193]=} & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w \right) \, dt^2}{2 \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^2 \, dt^3}{6 \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \\ & \left( -\frac{i \left( 90 \, g \, H \, k^3 + 60 \, g \, H^3 \, k^5 + 10 \, g \, H^5 \, k^7 - 36 \, k^3 \, U^2 - 15 \, H^2 \, k^5 \, U^2 \right) dt}{120 \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{8} \left( g \, H \, k^4 \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) dx^4 + O[dt]^4 \right) dx^4 + O[dt]^4 +$$

Out[194]= FGn error |

 $\label{left} $$\left(-\frac{\det(t)^2 \left(t + v \left$ 

Out[195]=

Out[196]=  $FGG \parallel (GGp H + Rmp) U$ 

 $Out[197] = FGG \parallel U (\text{text}{GGp} H + \text{text}{Rmp})$ 

$$\begin{array}{l} \text{Out} \text{[198]=} \ \ FGG \ error \ \parallel \ \left( -\frac{\left( k \left( 6+H^2 \ k^2 \right) U \ w \right) dt^2}{2 \left( 3+H^2 \ k^2 \right)} - \frac{i \ k \left( 6+H^2 \ k^2 \right) U \ w^2 \ dt^3}{6 \left( 3+H^2 \ k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( 126 \ k^3 + 75 \ H^2 \ k^5 + 10 \ H^4 \ k^7 \right) U \ dt}{120 \left( 3+H^2 \ k^2 \right)^2} + O[dt]^4 \right) dx^2 + \\ \left( -\frac{1}{8} \left( k^4 \ U \right) dt + O[dt]^4 \right) dx^3 + \left( \frac{i \left( 13311 \ k^5 + 11430 \ H^2 \ k^7 + 3110 \ H^4 \ k^9 + 260 \ H^6 \ k^{11} \right) U \ dt}{4800 \left( 3+H^2 \ k^2 \right)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{array}$$

Out[199]= FGG error ||

 $\left(-\frac{t}{2 k^2 + 3\right)}-\frac{t}{3 k}$  $U w^2 \left(\frac{h^2 k^2+6\right)}{6\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^2+3\right)}{1+O\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^2+3\right)}{1+O\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^$ \left(-\frac{i \left(10 H^4 k^7+75 H^2 k^5+126 k^3\right) U \text{dt}}{120 \left(H^2  $k^2+3\left(\frac{dt}^4\right)+\left(\frac{dt}^4\right)+\left(\frac{dx}^3\right)=\left(-\frac{1}{8}\right)+\left(\frac{dx}^4\right)$  $U \rightarrow \frac{dt}{-4 \cdot dt} - \frac{dt}{-4 \cdot dt} -$ H^6 k^{11}+3110 H^4 k^9+11430 H^2 k^7+13311 k^5\right) U \text{dt}}{4800  $\left(H^2 k^2+3\right)^3+O\left(\left(text\left(dt\right)^4\right)\right)+O\left(text\left(dx\right)^5\right)$ 

Out[200]=

Out[201]=

Out[202]= Omega error ||

Omega error || 
$$\left\{ \frac{1}{6(3+H^2k^2)^2} k^3 \left( \sqrt{3} \ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \left( 3\,g\,H + U \left( 2\,\sqrt{3} \ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right) dt^2 + \frac{i k^4 \left( 3\,g\,H + U \left( 2\,\sqrt{3} \ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right)^2 dt^3}{8(3+H^2\,k^2)^2} - \frac{1}{20(3+H^2\,k^2)^2} \left( k^5 \left( \sqrt{3} \ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right)^3 + \frac{i k^4 \left( 3\,g\,H + U \left( 2\,\sqrt{3} \ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U \right) \right)^2 dt^3}{8(3+H^2\,k^2)} + 180\,U + 20\,H^4\,k^4\,U + 15\,k^2 \left( \sqrt{3} \ \sqrt{g\,H^5\,(3+H^2\,k^2)} + 8\,H^2\,U \right) \right) + \frac{1}{480(3+H^2\,k^2)^2} k^5 \left( 20\,H^6\,k^6\,U^3 + 54\,U^2 \left( 9\,\sqrt{3} \ \sqrt{g\,H\,(3+H^2\,k^2)} + 10\,U \right) + \frac{1}{480(3+H^2\,k^2)^2} k^5 \left( 20\,H^6\,k^6\,U^3 + 54\,U^2 \left( 9\,\sqrt{3} \ \sqrt{g\,H\,(3+H^2\,k^2)} + 10\,U \right) + \frac{1}{480(3+H^2\,k^2)^2} k^5 \left( 20\,H^6\,k^6\,U^3 + 54\,U^2 \left( 9\,\sqrt{3} \ \sqrt{g\,H\,(3+H^2\,k^2)} + 10\,U \right) + \frac{1}{480(3+H^2\,k^2)^2} k^5 \left( 3\,g\,H\,(3+H^2\,k^2) + 10\,U \right) + \frac{1}{480(3+H^2\,k^2)^2} k^5 \left$$

$$U^{2}\left(15\sqrt{3}\sqrt{g\,H(3+H^{2}\,k^{2})} + 18\,U + 2\,H^{4}\,k^{4}\,U + k^{2}\left(5\sqrt{3}\sqrt{g\,H^{3}(3+H^{2}\,k^{2})} + 12\,H^{2}\,U\right)\right)\right) dt^{2} + \frac{1}{32(2+H^{2}\,k^{2})^{2}}\left(3\,g\,H + U\left(2\sqrt{3}\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 2\left(3+H^{2}\,k^{2}\right)U\right)\right) dt^{3} + \frac{1}{64(2+H^{2}\,k^{2})^{2}}k^{4}k^{8}\left(\sqrt{3}\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 2\left(3+H^{2}\,k^{2}\right)U\right)\right) dt^{3} + \frac{1}{64(2+H^{2}\,k^{2})^{2}}k^{4}k^{8}\left(\sqrt{3}\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 2\left(3+H^{2}\,k^{2}\right)U\right)\right)^{2}dt^{4} + O[dt]^{5}dx^{3} + \frac{1}{64(2+H^{2}\,k^{2})^{2}}k^{3}k^{3}\left(\sqrt{3}\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 2\left(3+H^{2}\,k^{2}\right)U\right)\right)^{2}dt^{4} + O[dt]^{5}dx^{3} + \frac{1}{64(2+H^{2}\,k^{2})^{2}}k^{3}k^{3}\left(\sqrt{3}\sqrt{g\,H^{3}(3+H^{2}\,k^{2})} + 2075\,H^{4}\,k^{4}\right) + 2080\left(9\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 6\,k^{2}\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + \frac{1}{6475}k^{2}\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)^{2}}k^{2}\right)\right) - \frac{1}{6475}k^{4}\sqrt{g\,H^{9}\left(3+H^{2}\,k^{2}\right)}U\right) / \sqrt{38}400\left(\sqrt{g\,H}\left(3+H^{2}\,k^{2}\right)^{5/2}\right)\right) - \frac{1}{6475}k^{4}\sqrt{g\,H^{9}\left(3+H^{2}\,k^{2}\right)}U\right) / \sqrt{38}400\left(\sqrt{g\,H}\left(3+H^{2}\,k^{2}\right)^{5/2}\right)\right) - \frac{1}{16705}k^{4}\left(3408\sqrt{g\,H^{9}\left(3+H^{2}\,k^{2}\right)} + 9985\sqrt{3}\,H^{4}\,U\right) + \frac{1}{1675}k^{4}\left(3408\sqrt{g\,H^{9}\left(3+H^{2}\,k^{2}\right)}U^{3} + 618\,k^{4}\sqrt{g\,H^{9}\left(3+H^{2}\,k^{2}\right)} + 688\,\sqrt{3}\,U\right) + \frac{1}{15}k^{4}\left(3408\sqrt{g\,H^{9}\left(3+H^{2}\,k^{2}\right)}U^{3} + 612\,k^{4}\sqrt{g\,H^{9}\left(3+H^{2}\,k^{2}\right)}U^{3} + 68\,k^{6}\sqrt{g\,H^{13}\left(3+H^{2}\,k^{2}\right)}U^{3}\right) + \frac{1}{12}k^{2}\left(3400\left(\sqrt{g\,H}\left(3+H^{2}\,k^{2}\right)U^{3}\right) + \frac{1}{12}k^{2}\left(3400\left(\sqrt{g\,H}\left(3+H^{2}\,k^{2}\right)U^{3}\right) + \frac{1}{12}k^{2}\left(3400\left(\sqrt{g\,H}\left(3+H^{2}\,k^{2}\right)U^{3}\right) + \frac{1}{12}k^{2}\left(3400\left(\sqrt{g\,H^{13}\left(3+H^{2}\,k^{2}\right)U^{3}\right) + \frac{1}{12}k^{2}\left(3400\left(\sqrt{g\,H^{13}\left(3+H^{2}\,k^{2}\right)U^{3}}\right) + \frac{1}{12}k^{2}\left(3400\left(\sqrt{g\,H^{13}\left(3+H^{2}\,k^$$

$$g H^5 U^2 \left( 89915 \sqrt{g H (3 + H^2 k^2)} + 115737 \sqrt{3} U \right) \right) dt^4 + O[dt]^5 \right) dx^4 + O[dx]^5,$$

$$\left( \frac{1}{6(3 + H^2 k^2)^2} k^3 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) dt^4 + O[dt]^5 dt^4 + O$$

$$\left( \left( k^{5} \left( \sqrt{3} \right. g \, H \left( 17856 + 12180 \, H^{2} \, k^{2} + 2075 \, H^{4} \, k^{4} \right) - \right. \right. \\ \left. \left. \left( \left( k^{5} \left( \sqrt{3} \right. g \, H \left( 17856 + 12180 \, H^{2} \, k^{2} + 2075 \, H^{4} \, k^{4} \right) - \right. \\ \left. \left. \left( \left( k^{5} \left( \sqrt{3} \right. g \, H \left( 17856 + 12180 \, H^{2} \, k^{2} + 2075 \, H^{4} \, k^{4} \right) - \right. \right. \\ \left. \left. \left( \left( 38400 \, \sqrt{g \, H} \, \left( 3 + H^{2} \, k^{2} \right)^{5/2} \right) + \left( k^{2} \left( 45 \, \sqrt{3} \, g^{2} \, H^{2} \left( 3336 + 2268 \, H^{2} \, k^{2} + 385 \, H^{4} \, k^{4} \right) + \right. \\ \left. g \, \Pi \, U \left( 447588 \, \sqrt{3} \, \, 11^{2} \, k^{2} \, U + 16705 \, \sqrt{3} \, \, 11^{6} \, k^{6} \, U - 648 \left( 693 \, \sqrt{g} \, \Pi \left( 3 + H^{2} \, k^{2} \right) - 688 \, \sqrt{3} \, \, U \right) - 15 \, k^{4} \left( 3408 \, \sqrt{g} \, H^{9} \left( 3 + H^{2} \, k^{2} \right) - 9985 \, \sqrt{3} \, \, H^{4} \, U \right) \right) - \\ \left. \left. 80 \left( 1836 \, \sqrt{g} \, H \left( 3 + H^{2} \, k^{2} \right) \, U^{3} + 612 \, k^{4} \, \sqrt{g} \, H^{9} \left( 3 + H^{2} \, k^{2} \right) \, U^{3} + 68 \, k^{6} \, \sqrt{g} \, H^{13} \left( 3 + H^{2} \, k^{2} \right) \, U^{3} + 9 \, k^{2} \left( 421 \, \sqrt{g^{3}} \, H^{7} \left( 3 + H^{2} \, k^{2} \right) \, U + 204 \, \sqrt{g} \, H^{5} \left( 3 + H^{2} \, k^{2} \right) \, U^{3} \right) \right) \right) \right) \right) \right) \right) \right)$$

$$\left( 230400 \, \sqrt{g} \, H \, \left( 3 + H^{2} \, k^{3} \right)^{7/2} \right) - \frac{1}{25600(3418^{2} \, k^{2})^{2}} \, k^{2} \, \left( 6 \, g^{2} \, H^{2} \left( 8046 + 5460 \, H^{2} \, k^{2} + 925 \, H^{4} \, k^{4} \right) + \right. \right. \right. \right.$$

$$\left. 560 \, H^{8} \, k^{8} \, U^{4} + 432 \, U^{2} \left( -143 \, \sqrt{3} \, \sqrt{g} \, H \left( 3 + H^{2} \, k^{2} \right) + 105 \, U \right) + \right. \right.$$

$$\left. 5 \, k^{4} \, U^{3} \left( -461 \, \sqrt{3} \, \sqrt{g} \, H^{13} \, \left( 3 + H^{2} \, k^{2} \right) + 1344 \, H^{6} \, U \right) - \right.$$

$$\left. 12 \, k^{2} \, \left( 3585 \, \sqrt{3} \, \sqrt{g^{3}} \, H^{7} \left( 3 + H^{2} \, k^{2} \right) + 1344 \, H^{6} \, U \right) - \right.$$

$$\left. 12 \, k^{2} \, \left( 3585 \, \sqrt{3} \, \sqrt{g} \, H^{13} \, \left( 3 + H^{2} \, k^{2} \right) + 1344 \, H^{6} \, U \right) \right. \right.$$

$$\left. \left( k^{9} \, \left( \sqrt{3} \, \sqrt{g} \, H^{13} \, H^{2} \, k^{2} \right) - \left( 3 + H^{2} \, k^{2} \right) + 19056 \, H^{4} \, U \right) \right) \right) \right. \right.$$

$$\left. \left( k^{9} \, \left( \sqrt{3} \, \sqrt{g} \, H^{13} \, H^{2} \, k^{2} \right) - \left( 3 + H^{2} \, k^{2} \right) \right. \right) \right. \right. \right. \right. \right. \right. \right.$$

$$\left. \left( k^{9} \, \left( \sqrt{3} \, \sqrt{g} \, H^{13} \, H^{2} \, k^{2} \right) - \left( 3 + H^{2} \, k^{2} \right) \right) \right. \right) \left. \left( \sqrt{3} \, \sqrt{g} \, H$$

 $\left(H^2 k^2+3\right)^2+\frac{1}{2} + \frac{3 + 4 \left(g H^2 k^2+3\right) U+2 \left(g H^2 k^2+3\right) U+2 \right)}{1}$  $\label{left} $$\left(H^2 k^2+3\right)\right]/2\left(H^2 k^2+3\right)^2-\frac{(h^2 k^2+3\right)^2}-\frac{(h^2 k^2+3\right)^2}-\frac{(h^2 k^2+3\right)^2}-\frac{(h^2 k^2+3\right)^2}-\frac{(h^2 k^2+3\right)^2}-\frac{(h^2 k^2+3\right)^2}+\frac{(h^2 k^2+3)^2}+\frac{(h^2 k^$  $k^2+3\right) U+\sqrt{3} \left(H^2 k^2+3\right) \Gamma(H^2 k^2+3\right)$  $k^2+3\right) U+2 \left( H^2 \left( H^2 \right) \right) \left( H^2 \right) \left$  $\label{eq:k-2+3-right} $$k^2+3\right)+O\left(\frac{dt}^5\right)+\left(\frac{dt$ \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 U H^4+11 \sqrt{3} \sqrt{g H^9}  $\left(H^2 k^2+3\right)\right) \ k^4+3 \left(180 H^2 U^3+109 \right) \ \left(H^5 \left(H^2 k^2+3\right)\right)$  $U^2+15 \sqrt{3} \sqrt{9^3 H^7 \left(H^2 k^2+3\right)} k^2+54 U^2 \left(10 U+9 \sqrt{3} \sqrt{10 U+9} \right)$ H \left(H^2 k^2+3\right)\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U+21 \sqrt{3}  $\left( H^2 k^2+3\right)\right)\right)$  $g H+U \left( \frac{h^2 k^2+3\right) U+2 \left( H^2 k^2+3\right) \left( H^2$  $H \left(5 H^2 k^2+14\right)+U \left(5 H^4 U k^4+5 \left(5 H^2 + 14\right)+U \right)$  $k^2+3\left( H \left( H^2 k^2+3\right) \right) \right)$  $\label{left} $$\left(H^2 \times^2 + 3\right)^3 - \frac{k^2 + 3\left(k^7 \cdot H^4 \cup k^4 + 15 \cdot H^4 \cup H^2 + \frac{3}{3}\right) \right] + \frac{1}{3} \cdot H^5 \cdot$  $k^2+3\right)$  ight)  $k^2+180$  U+42  $\sqrt{4}$  sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \left(3 g H+U \left(H^2 Left(H^2 Left(H  $\label{eq:linear_line$  $\$  \sqrt{\frac{g H}{H^2 k^2+3}}\right)-\frac{i k^6 \left| eft(\left(2 H^4 U k^4+\left(12 U H^2+5 \sqrt{3} \sqrt{g}) \right| }  $H^5 \left( H^2 k^2 + 3\right) \right) h^2 + 18 U + 15 \left( H^2 k^2 + 3\right) \right) h^2 + 18 U + 15 \left( H^2 k^2 + 3\right) h^2 \right) h^3 \left( H^2 k^2 + 3\right) h^3 \left( H^2$  $g\ H \left(\frac{4\left(\frac{4^2 k^2+3\right)}{t} U+\sqrt{3} \right) U+\sqrt{3} \left(\frac{4^2 k^2+3\right)}{t} \right) U+\frac{3}{3} \left(\frac{4^2 k^2+3\right)}{t} \left(\frac{4^2 k^2+3\right)}{t} \right) U+\frac{3}{3} \left(\frac{4^2 k^2+3\right)}{t} \left(\frac{4^2 k^2+3}{t} \right) U+\frac{3}{3} \left(\frac{4^2 k^2+3}{t} \right) U+\frac{4^2 k^2+3}{t} U+$  $\label{eq:harmonic} $\left(H^2 k^2+3\right)^2+\frac{k^7 \left(g H+U \left(H^2 k^2+3\right) U+2 \right)}{H^2 k^2+3\right)} = \frac{1}{2} \left(H^2 k^2+3\right)^2 + \frac{1}{2$  $H \left( H^2 k^2 + 3 \right) \right) \left( h^3 \left( H^2 k^2 + 3 \right) \right) \left( h^3 \left( H^2 k^2 + 3 \right) \right) \left( h^3 \left( H^2 k^2 + 3 \right) \right) \right) \left( h^3 h^3 \right) \left($  $\left(H^2 k^2+3\right) U+\sqrt{3} \left(H^2 k^2+3\right)\right)$  $k^2+3\right) U+2 \sqrt{3} \sqrt{4}{64 \left(H^2 k^2+3\right)\right)} \left(H^2 k^2+3\right) \left(H^2 k^2+3\right)$  $k^2+3\right)^3+O\left(\frac{t}{5\right)^3}+O\left(\frac{t}{5\right)^3}+O\left(\frac{t}{5}\right)^3+O\left(\frac{t}{5$  $H^4 k^4 + 12180 H^2 k^2 + 17856 \right) + 2080 \left( \frac{H^9 \left( \frac{K^2 + 3\right)}{k^4 + 6} \right) + 2080 \right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^2 +$  $H^5 \left(H^2 k^2+3\right) k^2+9 \left(H^2 k^2+3\right) U\right) H^5 \left(H^2 k^2+3\right) U\right) H^5 \left(H^2 k^2+3\right) U\right)$  $H} \left(\frac{45 \operatorname{k^2} + 3\operatorname{right}}{5/2}\right) - \left(\frac{45 \operatorname{k^2} + 3\operatorname{right}}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^2} + 3\operatorname{right}}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{$ H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6+15 \left(9985 \sqrt{3} U H^4+3408 \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+447588 \sqrt{3} H^2 U k^2+648 \left(688 \sqrt{3} U+693  $k^6 + 612 \setminus g H^9 \setminus (H^2 k^2 + 3 \cap U^3 k^4 + 9 \setminus g H^5 \setminus (H^2 k^2 + 3 \cap U^3 k^4 + 9 \cap U^3 k^4 + 9$  $U^3+421 \cdot g^3 H^7 \cdot (h^2 k^2+3 \cdot g)$  Uright)  $h^2+1836 \cdot g$  H \left( $h^2 k^2+3 \cdot g$ )  $U^3\right) \left( \frac{H^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} -\frac{h$ \left(560 H^8 U^4 k^8+5 U^3 \left(1344 U H^6+461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 U H^4+4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+12 \left(5040  $H^2 U^4 + 5161 \cdot (3) \cdot (4^2 L^2 + 5161 \cdot (3) \cdot (4^2 L^2 + 16) \cdot (4^2 L^$ k^2+3\right) U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105  $U+143 \sqrt{3} \sqrt{9} H \left(\frac{4^2 k^2+3\right)} H U \left(\frac{10640 H^6 U k^6+5 \left(\frac{19056 U}{4}\right)}{19056 U}\right)$  $H^4+1451 \left( H^2 k^2+3\right) \right) + h^4+1451 \left( H^2 k^2+3\right) \right) + h^4+1451 \left( H^2 k^2+282852 \right) + h^4+1451 \right) + h^4+1451 \left( H^2 k^2+282852 \right) + h^4+1451 \right) + h^4+1451 \left( H^2 k^2+282852 \right) + h^4+1451 \right) + h^4+1451 \left( H^2 k^2+282852 \right) + h^4+1451 \left( H^2 k^2+284864 \right) + h^4+1461 \left( H^2 k^2+284$ 

 $\left(\left(\frac{H^2 k^2+3\right)}{U+\sqrt{3} \left(\frac{H^2 k^2+3\right)}\right)\right)$ g H^9+928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U\right) k^8+15 U \left(4143 \sqrt{3} g^2 H^8+g U k^2+3\right) U^3\right) k^6+9 \left(61735 \sqrt{3} g^2 U H^6+g U^2 \left(115737 \sqrt{3} U+89915  $\$  \\sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+27840 \\sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)\right) k^4+108 \left(15327 \sqrt{3} g^2 U H^4+19261 \sqrt{3}  $g\ U^3\ H^3+4640\ sqrt\{g\ H^5\ left(H^2\ k^2+3\ left(H^$ U^4+g H \left(1202 \sqrt{3} U+1857 \sqrt{g H \left(H^2 k^2+3\right)\right) U^2+1268 \sqrt{3} g^2  $H^2 U+323 \left( \frac{g^5 H^5 \left( \frac{40800 \left( + \right)} \right)} \right)} {40800 \left( \frac{40800 \left( + \right)} \right)} {40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( + \right)} \right)} {40800 \left( \frac{40800 \left( \frac{40800 \left( + \right)} {40800 \left( + \right)} {40800}$  $\left(\left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \sqrt{3} \right) + \left(\frac{H^2 k^2+3\right)}\right) \left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \left(\frac{H^2 k^2+3\right)}}\right)$  $k^2+3\right) U-2 \sqrt{3} \sqrt{4} \ln(H^2 k^2+3\right) \int U-2 \sqrt{3} \left( H^2 k^2+3\right) \left($  $\label{eq:k-2+3-right} $$k^2+\frac{1}{2}+\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$  $k^2+3\left( U-2 \right) U-2 \left( H^2 \left( H^2 \right) \right) \right) \left( H^2 \right) \left$  $\label{left(sqrt{3} holeft(20 H^4 U k^4-15 \left(\sqrt{3} + \sqrt{3} \right)^4)+O(eft(\sqrt{3} + \sqrt{3} + \sqrt{4})^4)} k^2 + O(eft(\sqrt{3} + \sqrt{3} +$  $\sqrt{g H^5 \left(H^2 k^2+3\right)} = H^2 U\right) k^2+180 U-42 \sqrt{3} \sqrt{g H \ell^2}$ k^2+3\right)\right)\right)\{240 \left(H^2 k^2+3\right)^2\}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 H^4 U^5 H^6 U  $H^5 \left(H^2 k^2+3\right) U^2+15 \left(H^2 k^2+3\right) k^2+3 U^2$  $\left(10 U - 9 \right) \right) \ H \left(H^2 k^2 + 3\right) \right) \ H \left(16 U - 9 \right) \ H^2 \ H^4 k^4 + 147 H^2 \ H^$  $k^2+216 \right) U-21 \left( H^2 k^2+3\right) \right) \left( H^2 k^2+3\right) \left( H^2 k^2+3\right)$  $k^2+3\right)^3+\frac{1}{2} k^6 \left(16 k^6 + 16 k^6 + 16$ k^2+3\right)\right)\right)\left(9 g H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4-5 \left(7 \sqrt{3})  $\$  \sqrt{g H^5 \left(H^2 k^2+3\right)}-24 H^2 U\right) k^2+180 U-102 \sqrt{3} \sqrt{g H \left(H^2 L^2 H^2 U\right)}  $k^2+3\right) \right) \cdot k^2+3\right) \cdot k^2+3\left( k^2+3\left( k^2+3\right) \cdot k^2+3\left( k^2+3\left( k^2+3\right) \cdot k^2+3\left( k^2+3\left( k^2+3\right) \cdot k^2+3\left( k^2+3\left( k^2+3\right) \cdot k$  $k^4 - 15 \left( \frac{3} \sqrt{H^5 \left( \frac{4^2 + 3\right)}} - 8 \right) - 8 \left( \frac{4^2 + 3\right)}{8} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{$  $\t x_d^2+\left(\frac{1}{16} i k^4 \left(\frac{3} \right)^2 + \left(\frac{1}{16} i k^4 \right)^2 k^2 + \frac{1}{16} i k^6$  $\label{left} $$\left(U^2 \left(H^4 U k^4 + \left(12 H^2 U - 5 \right) \right)\right) \right] $$\left(H^5 \left(H^2 k^2 + 3\right)\right)\right)$$  $k^2+18 U-15 \sqrt{3} \sqrt{3} \sqrt{4} \ln(H^2 k^2+3\right)/19$  $\label{left} $$k^2+3\right)^4\left(\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3}+\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac$ \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right)  $\left(3 g H+U \left(2 \left(4^2 k^2+3\right)\right) U-3 \right) + \left(4^2 k^2+3\right) \left(4^2 k^2+3\right) \right)$  $\t (dt)^3 {32 \left( H^2 k^2 + 3\right) U - \left( h^2 k^2 + 3$  $\left(H^2 k^2+3\right)\right)$  $\label{eq:k-2+3-right} $$ k^2+3\right)^2 \text{text}(dt)^4{64 \left(\frac{h^2 k^2+3\right)^3}+O\left(\frac{dt}{5\right)^5}\right)^2} $$$  $\text{text}_{dx}^3 + \text{left}_{fac}_{k^5} \left[ \frac{3}{9} H \left( \frac{2075 \text{ H}^4 \text{ k}^4 + 12180 \text{ H}^2 \text{ k}^2 + 17856} \right) - 2080 \right]$  $\left( \frac{H^2}{h^2} \right) \left( \frac{H^2}{$  $H \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} U\right) = H \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)}$ \1a\frac{1}{1} \alpha\frac{1}{2} \alpha\frac{1}{

 $k^6-15 \left(3408 \right) k^4+447588 \left(4^2 + 3\right) -9985 \left(4^4 + 447588 \right) k^6-15 \left(4^6 + 447588 \right) k^6-15 \left(4^6 + 47588 \right) k^6-15 \left(4^$ H^2 U k^2-648 \left(693 \sqrt{g H \left(H^2 k^2+3\right)}-688 \sqrt{3} U\right)\right) H-80 \left(68  $k^2+3\right/f(1344 + 6 - 461 \cdot 6^{14})^{7/2}}-\frac{k^8 \left(1344 + 6 - 461 \cdot 6^{14}\right)}{1344 + 6 - 461 \cdot 6^{14}}$ k^2+3\right)\right) k^4-12 \left(-5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 U \left(10640 H^6 U k^6+5 \left(19056 H^4 U-1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U-63720 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right)  $\text{dt}^3{25600 \left(H^2 k^2+3\right)^4}-\text{dt}^3}{25600 \left(H^2 k^2+3\right)^4}-\text{dt}^3} \right. \$  $k^2+3\right)-\left(H^2 k^2+3\right)-\left(H^2 k^2+3\right)$ U-3869 \sqrt{3} g H^9\right) k^8-15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U-6019  $\left( H^2 k^2+3\right) H^7-3712 \left( H^13 \left( h^2 k^2+3\right) \right) U^3\right)$ k^6+9 \left(-61735 \sqrt{3} g^2 U H^6+g U^2 \left(89915 \sqrt{g H \left(H^2 k^2+3\right)}-115737 \sqrt{3} U\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2  $k^2+3\left( h^4-108 \left( -15327 \right) \right) + h^4-19261 \left( h^4-19261 \right) + h^4-19261 \left( h^4-1926$  $H^5 \left(H^2 k^2+3\right) U^4+22383 \left(H^2 k^2+3\right) U^2+2625 \left(H^2 k^2+3\right) U^4+22383 \left(H^2 k^2+3\right) U^4+22383 \left(H^2 k^2+3\right) U^4+22383 U^4+22384 U^4+2444 U^4+2444 U^4+244 U^4+244 U^4+244 U^4+244 U^4+244 U^4+244 U^4+244 U^4+244$  $H^9 \left(H^2 k^2+3\right) \right) + (H^2 k^2+3\right) + (H^2 k^2+3) + (H^2$  $\left(1857 \right) + \left(1857 \right) + \left(18$ 

```
Out[204]=
\text{Out} \text{[205]=} \quad EA \text{ ||} \quad \left\{ \left\{ \frac{-H^2 \, k^2 \, \left( \left( -1 + e^{i \, d \, w} \right) k \, U - w \right) + 3 \, w}{(3 + H^2 \, k^2) \, w}, \, -\frac{3 \, \left( -1 + e^{i \, d \, w} \right) k}{(3 + H^2 \, k^2) \, w} \right\}, \, \left\{ -\frac{\left( -1 + e^{i \, d \, w} \right) k \, \left( g \, H \, \left( 3 + H^2 \, k^2 \right) - 3 \, U^2 \right)}{(3 + H^2 \, k^2) \, w}, \, 1 \, -\frac{\left( -1 + e^{i \, d \, w} \right) k \, \left( 6 + H^2 \, k^2 \right) U}{(3 + H^2 \, k^2) \, w} \right\} \right\}
Out[206]= EA || \left(
                                                           \begin{array}{cc}
                                                                \frac{3 \text{ w-H}^2 \text{ k}^2 \left(\left(-1+e^{i \text{ text}}\right) \text{ k U-w}\right)}{\left(-1+e^{i \text{ text}}\right) \text{ k U-w}\right)}{\left(-1+e^{i \text{ text}}\right)}
                                                                                                          k^2+3\right) w & -\frac{3 \left(1+e^{i \left(t\right) w}\right) k}{\left(t\right) w} \
                                                                 -\frac{1+e^{i \cdot text}}{dt}  w\right) k \left(g H \left(H^2 k^2+3\right)-3 U^2\right)\{\left(H^2 k^2+3\right)-3 U^2\right)\} \left(H^2 k^2+3\right)}
                                                                                                           w\} \& 1-\frac{\left(-1+e^{i \cdot k^2+6 \cdot k^2+6 \cdot k^2+6 \cdot k^2+6 \cdot k^2+3 \cdot k^2+6 \cdot k^2+3 \cdot k^2+6 \cdot k^2+3 \cdot k^2+6 
                                                           \end{array}
                                                           \right)
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$$\begin{aligned} & \frac{1}{3 + \text{IP} \cdot \text{EP}} \left\{ \frac{\left( \sqrt{3} \cdot \text{k} \sqrt{\text{gH}(1 + \text{IP} \cdot \text{k})} + \text{k} \cdot \text{U} \right) dt}{3 + \text{IP} \cdot \text{k}^2} + \frac{\left( \sqrt{3} \cdot \text{k} \sqrt{\text{gH}(1 + \text{IP} \cdot \text{k})} \right) dt^2 - \frac{1}{24} \left( -\frac{i \sqrt{3} \cdot \text{k} \sqrt{\text{gH}(1 + \text{IP} \cdot \text{k})}}{3 + \text{IP} \cdot \text{k}^2} - i \cdot \text{k} \cdot \text{U} \right)^4 dt^4 + O[dt]^3 \right) + \\ & - \frac{i \left( -\frac{i \left( 5 + \text{k}^2 \cdot + \text{k} \cdot \text{IP} \cdot \text{k}^2 \cdot \text{H}(1 + \text{IP} \cdot \text{k})}{3 + \text{IP} \cdot \text{k}^2} \right) - i \cdot \text{k} \cdot \text{U} \right)^3 dt^2 - \frac{1}{24} \left( -\frac{i \sqrt{3} \cdot \text{k} \sqrt{\text{gH}(1 + \text{IP} \cdot \text{k})}}{3 + \text{IP} \cdot \text{k}^2} - i \cdot \text{k} \cdot \text{U} \right)^4 dt^4 + O[dt]^3 \right) dx^2 + \\ & - \left( -\frac{i \left( 5 + \text{k}^2 \cdot + \text{k} \cdot \text{IP} \cdot \text{k}^2 \cdot \text{H}(1 + \text{IP} \cdot \text{k}^2)} \right) + \frac{1}{2 + \text{IP} \cdot \text{k}^2 \cdot \text{U}^2} + O[dt]^3 \right) dx^2 + \\ & - \left( -\frac{1}{8} \left( \text{k}^4 \cdot \text{U} \right) dt + \left( -\frac{1}{16 \cdot \text{k}^2 \cdot \text{H}^2} \right) + \frac{1}{2 \cdot \text{k}^2 \cdot \text{IP} \cdot \text{k}^2 \cdot \text{U}^2} \right) dt^2 + O[dt]^3 \right) dx^2 + \\ & - \left( -\frac{1}{8} \left( \text{k}^4 \cdot \text{U} \right) dt + \left( -\frac{1}{16 \cdot \text{k}^2 \cdot \text{H}^2} \right) + \frac{1}{2 \cdot \text{k}^2 \cdot \text{IP} \cdot \text{k}^2 \cdot \text{U}^2} \right) dt^2 + O[dt]^3 \right) dx^2 + \\ & - \left( \frac{1}{12 \cdot \text{k}^2 \cdot \text{k}^2 \cdot \text{H}^2} + \frac{1}{2 \cdot \text{k}^2 \cdot \text{k}^2} \right) dt^2 + O[dt]^3 \right) dx^2 + O[dt]^5 \right)$$

Out[208]= Eerr || \left(

\begin{array}{cc}

 $\label{left} $$\left(\frac{1 \cdot \left(\frac{1}{4} \cdot \frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}$ 

 $\label{eq:thm:linear} U \ k-\frac{i \sqrt{h^2 k^2+3}\right}^3 \text{ } text{dt}^3-\frac{1}{24}$  $\left(-i U k - \frac{1}{sqrt}3\right) \left(H^2 k^2 + 3\right) k}{H^2 k^2 + 3\right)^4}$  $\text{text}(dt)^4 + O(\text{text}(dt)^5) + \left(10 \text{ H}^4 \text{ k}^7 + 45 \text{ H}^2 \text{ k}^5 + 54 \text{ k}^3\right) U$  $\label{eq:local_hamiltonian} $$ \operatorname{light}^2 e^2+ \sinh^2 -\frac{120 e^2}{120 e^2} = \frac{120 e^2}{120 e^2} e^2 + \frac{120 e^2}{120 e^2}$  $U^2 k^4 + 126 g H k^4 + 126$  $\t (-\frac{1}{8} \left( -\frac{1}{8} \right) \times (-\frac{1}{8} \left( -\frac{1}{8} \left( -\frac{1}{8} \right) \times (-\frac{1}{8} \left( -\frac{1}{8} \left( -\frac{1}{8} \right) \times (-\frac{1}{8} \right) \times (-\frac{1}{8} \left( -\frac{1}{8} \left( -\frac{1}{8} \right) \times (-\frac{1}{8} \left( -\frac{1}{$  $k^2+3\right)+\frac{3}{g}H k^5}{16 \left(H^2 k^2+3\right)\right)+\frac{3}{g}H k^5}{16 \left(H^2 k^2+3\right)}\right)}$ \text{dx}^3+\left(\frac{i \left(260 H^6 U k^{11}+1570 H^4 U k^9+2610 H^2 U k^7+729 U  $k^5 \righti \left(1460 H^6 U^2 k^{12} + 4500 g H^5 \right)$ k^{10}+4140 H^4 U^2 k^{10}+26460 g H^3 k^8-13500 H^2 U^2 k^8-38286 U^2 k^6+38853 g H k^6\right) \text{dt}^2\{28800 \left(H^2 k^2+3\right)^3\}+O\left(\text{dt}^5\right)\right)  $\t (-\frac{3 i k \text{d}}{H^2 k^2+3}-\frac{3 i k \text{d}}{$  $U \right) \left( \frac{dt}^2}{H^2 k^2 + 3} + O\left( \frac{dt}^5 \right) \right) + \left( -\frac{1}{4} \right) + O\left( \frac{dt}^5 \right) \right)$  $k^5+12 k^3\right) \left( \frac{40}{40} \left( \frac{40}{40} \right)^2 \right) - \frac{3\left( \frac{40}{100} \right)^2}{100} \right)$  $k^4\right) U\right) \perp (t_{dt}^2){40 \left(H^2 k^2+3\right)^2} + O\left(t_{dt}^5\right)\right)$  $\t (4x)^2 + \left( \frac{3 i k^5 U \left( \frac{4t}^2}{8 \left( \frac{H^2 k^2 + 3\right)}{+ O\left( \frac{4t}^5 \right)} \right)} \right)$ \text{dx}^3+\left(\frac{i \left(770 H^4 k^9+4410 H^2 k^7+6291 k^5\right) \text{dt}}{4800 \left(H^2  $k^2+3\right)^3+\frac{1500 H^4 U k^{10}+8820 H^2 U k^8+12951 U k^6\right) \text{ } text{dt}^2}{4800 U k^6}$  $\left(\frac{dx}^4+O\left(\frac{dx}^5\right)^3\right)+O\left(\frac{dx}^5\right)\right)$  $\left(-\frac{k^2 U}{e^2 h^3+3 g H-3 U^2\right) \left(-\frac{k^2 U}{e^2 h^3+3 U^2}\right) \left($  $U^2\right) \left(\frac{dt}^2}{H^2 k^2+3} + O\left(\frac{dt}^5\right)\right) \left(\frac{dt}^6 - \frac{dt}^6}{H^2 k^2+3} + O\left(\frac{dt}^5\right)\right) \left(\frac{dt}^6 - \frac{dt}^6}{H^2 k^2+3}\right) \left(\frac{dt}^6$  $k^5 - 15 H^2 U^2 k^5 - 36 U^2 k^3 + 90 g H k^3 \right) \left( \frac{120 \left( \frac{4}{2} k^2 + 3\right)}{120 \left( \frac{4}{2} k^2 + 3\right)} \right)^2 + \frac{1}{20 k^2 + 3k^2 + 3k^2$ g H^5 U k^8+45 H^2 U^3 k^6-120 g H^3 U k^6+126 U^3 k^4-180 g H U k^4\right) \text{dt}^2\{120  $\label{left} $$\left(H^2 k^2+3\right)^2+O\left(\frac{dt}^5\right)\right) \left(H^2 k^2+\left(\frac{1}{8}\right)^2\right)$$ k^4\right) \text{dt}+\frac{i \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \text{dt}}^2\{8 \left(H^2  $\label{left} $$ k^2+3\right)+O\left(\frac{dt}^5\right)+O\left(\frac{dt}^5\right) \cdot \left(\frac{dt}^3+\left(\frac{dt}^3+\left(\frac{dt}^3\right)+O\left(\frac{dt}^3\right)}{1}+2340g\right)\right)$$ H<sup>5</sup> k<sup>9</sup>-770 H<sup>4</sup> U<sup>2</sup> k<sup>9</sup>+7020 g H<sup>3</sup> k<sup>7</sup>-4410 H<sup>2</sup> U<sup>2</sup> k<sup>7</sup>-6291 U<sup>2</sup> k<sup>5</sup>+7020 g H k<sup>5</sup>right)  $\label{eq:left} $$ \operatorname{dt}_{4800 \left( h^2 k^2 + 3\right)^3} + \frac{10}{1400 g h^7 U k^{12} - 4500 h^4 U^3 k^{10} + 13140 h^2 U^3 k^{10} + 13140 h^2$ g H<sup>5</sup> U k<sup>10</sup>-26460 H<sup>2</sup> U<sup>3</sup> k<sup>8</sup>+39420 g H<sup>3</sup> U k<sup>8</sup>-38853 U<sup>3</sup> k<sup>6</sup>+39420 g H U k<sup>6</sup>right) &  $\left(\frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \left(\frac{1}{frac} \right)\right) + \frac{1}{frac} \left(\frac{1}{frac} \left(\frac$  $k^2+3+\frac{1}{k^2} + \frac{1}{k^2} + \frac{1}{k^2}$  $k^2+3-\sqrt{1}\{6\} \left(-i \ U \ k-\sqrt{3} \right) + \left(-i \ U \ k^2+3\right) + \left(-i \ U \$  $\label{eq:left} $\operatorname{dt}^3-\frac{1}{24} \left(i \ U \ k-\frac{3} \right) \ \left(i \ h^2 \ k^2+3\right) \ k}{H^2 \ k^2+3} \right)^4 $$  $\left(\frac{dt}^4+O\left(\frac{dt}^5\right)\right)+\left(\frac{10 H^4 k^7+75 H^2 k^5+126 k^3\right)}{10 H^4 k^7+75 H^2 k^5+126 k^3\right)}$  $\text{text}_{dt}_{120 \text{ heft}(H^2 k^2+3\right)^2}-\frac{(100 \text{ h}^4 U^2 k^8+45 \text{ g} H^3 k^6+210 H^2 U^2 k^6+432)}{(100 \text{ h}^4 U^2 k^8+45 \text{ g} H^3 k^6+210 H^2 U^2 k^6+432)}$  $U^2 k^4 + 126 g H k^4 \right) \left( \frac{1}{2} (4t)^2 \right) \left( \frac{k^2 + 3 \right) + O\left( \frac{t}{2} + O\left( \frac{t}{2} \right) \right) \right)$  $\t (dx)^2 + \left( -\frac{1}{8} \right) \left( -\frac{1$  $H\left(\frac{1}{2}\left(\frac{1}{2}\right) + O\left(\frac{1}{2}\right) + O\left(\frac{1}{2}$  $\left(4800 + 6 k^{11} + 3110 + 4 k^9 + 11430 + 2 k^7 + 13311 k^5 \right)$  \\ \text{dt}{4800 \\ left(H^2 \)  $k^2+3\right)^3+\frac{10}{2400}$ g H^3 k^8+92340 H^2 U^2 k^8+117126 U^2 k^6+38853 g H k^6\right) \text{dt}^2\{28800  $\left(\frac{dt}^5 \right) \cdot \left(\frac{dt}^5 \right) \cdot \left(\frac{$ 

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\enu{array}
       \right)
\ln[209] = \text{KurF} = (\text{fm} * \text{ap} - \text{fp} * \text{am} + \text{am} * \text{ap} * (\text{qp} - \text{qm})) / (\text{ap} - \text{am});
       KurFWS = KurF /. ap \rightarrow 0 /. am \rightarrow (U - Sqrt[g * H]);
       KurFWSeta =
          KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
            qm \rightarrow Rmp * n;
       KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
       Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1];
        KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1]; 
       Kfnn = Kfnnp / Rpp \rightarrow Rp / Rmp \rightarrow Rm / GGp \rightarrow GG2 / Gnp \rightarrow Gn2;
       KfnG = KfnGp / Rpp \rightarrow Rp / Rmp \rightarrow Rm / GGp \rightarrow GG2 / Gnp \rightarrow Gn2;
       Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
       Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
       Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
       FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
       FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
       FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
       \texttt{KurFWSG} = \texttt{KurFWS} \ / . \ \texttt{fp} \ \rightarrow \ (\texttt{U} * \texttt{Rpp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rpp} * \texttt{n}) \ / .
               \texttt{fm} \ \rightarrow \ (\texttt{U} * \texttt{Rmp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rmp} * \texttt{n}) \ / . \ \texttt{qp} \ \rightarrow \ \texttt{Rpp} * \texttt{G} \ / . \ \texttt{qm} \ \rightarrow \ \texttt{Rmp} * \texttt{G};
       KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
       KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
       KfGGp = FullSimplify[KurFWSG /. n \rightarrow 0 /. G \rightarrow 1];
       KfGn = KfGnp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
       KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
       FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
       FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
       FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
       fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
       FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
       FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
       FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
       Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
       Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2;
       Eerr = Series [Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], \{dx, 0, 4\}, \{dt, 0, 4\}];
       EigvFmat2 = Eigenvalues[Fmat2];
       RKStep = Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2] / (I * dt);
       RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
       RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
```

```
Text[Row[{" U < -Sqrt(gH)"}]]</pre>
      Text[" "]
       Text[Row[{"Fnn || ", Kfnnp}]]
      Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
      Text[Row[{"Fnn error ||
                                        ", Fnn2TAr}]]
       Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
      Text[" "]
      Text[Row[{"FnG || ", KfnGp}]]
       Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
       Text[Row[{"FnG error || ", FnG2TAr}]]
      Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
      Text[" "]
       Text[Row[{"FGn || ", KfGnp}]]
      Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
       Text[Row[{"FGn error || ", FGn2TAr}]]
       Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
      Text[" "]
      Text[Row[{"FGG || ", KfGGp}]]
       Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
      Text[Row[{"FGG error || ", FGG2TAr}]]
      Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
      Text[" "]
      Text[" "]
      Text[Row[{"Omega error || ", RKstepTayr}]]
       Text[Row[{"Omega error || ", TeXForm[RKstepTayr]}]]
      Text[" "]
      Text[Row[{"EA || ", EA}]]
       Text[Row[{"EA || ", TeXForm[EA]}]]
      Text[Row[{"Eerr || ", Eerr}]]
      Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
Out[243]= U < -Sqrt(gH)
Out[244]=
Out[245]= Fnn \parallel Gnp H + Rpp U
Out[246]= Fnn || \text{text}\{Gnp\}\ H+\text{text}\{Rpp\}\ U
Out[247]= Fnn error |
        \left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(\frac{1}{8}\,k^4\,U\,dt+O[dt]^4\right)dx^3+O[dx]^4
```

 $\label{left-frac} $\left(\frac{t}^2 \left(\frac{t}^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \left(\frac{t}^2 \right)^2 \left(\frac{t}^2 \right)^$ 

Out[249]=

Out[250]= FnG || GGp H

Out[251]=  $FnG \parallel \text{text}\{GGp\} H$ 

$$\text{Out} \text{[252]= } FnG \ error \ || \ \left( -\frac{3 \ (k \ w) \ dt^2}{2 \left( 3 + H^2 \ k^2 \right)} - \frac{i \ k \ w^2 \ dt^3}{2 \left( 3 + H^2 \ k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( 12 \ k^3 + 5 \ H^2 \ k^3 \right) \ dt}{40 \left( 3 + H^2 \ k^2 \right)^2} + O[dt]^4 \right) dx^2 + O[dx]^4$$

 $\label{eq:continuous} $$\operatorname{FnG error} \| \left(-\frac{3 \text{dt}}^2 (k w)}{2 \left(\frac{4 w}{2 k^2 + 3\right)} - \frac{i \text{dt}}^3 k w^2}{2 \left(\frac{4 w}{2 k^2 + 3\right)} + O\left(\frac{4 w}{4 k^2 + 3\right)} + O\left(\frac{4 w}{$ 

Out[254]=

Out[255]=  $FGn \parallel H(gRpp + Gnp U)$ 

 $\label{eq:output} \mbox{Out}[256] = FGn \mid\mid H(g \text{$\texttt{q}$}+\text{$\texttt{qp}$}+\text{$\texttt{Qnp}$})$ 

$$\begin{array}{ll} \text{Out} \text{[257]=} & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w \right) dt^2}{2 \, \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^2 \, dt^3}{6 \, \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \\ & \left( -\frac{i \left( 90 \, g \, H \, k^3 + 60 \, g \, H^3 \, k^5 + 10 \, g \, H^5 \, k^7 - 36 \, k^3 \, U^2 - 15 \, H^2 \, k^5 \, U^2 \right) dt}{120 \, \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \left( \frac{1}{8} \, g \, H \, k^4 \, dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) dx^4 + O[dt]^4 + O[dt]^4$$

Out[258]= FGn error ||

 $\label{left-frac} $\left(-\frac{\left(\frac{4t}^2 \left(\frac{4t}^2 \right)^2 \left(\frac{4t}^2 \right)^2$ 

Out[259]=

Out[260]=  $FGG \parallel (GGp H + Rpp) U$ 

Out[261]=  $FGG \parallel U (\text{text}\{GGp\} H+\text{text}\{Rpp\})$ 

 $\label{left-frac} $\left(\frac{text{dt}^2 \left(text{dt}^2 \left(text{dt}^2 \right)^2 \left(text{dt}^2 \left(text{dt}^3 \left(text{dt}^3 \right)\right)^2 \left(text{dt}^4 \right)^2 \left(text{dt}^4 \left(text{dt}^4 \right)^2 \left(text{dt}^4 \left(text{dt}^4 \right)\right)^2 \left(text{dt}^4 \left(text{dt}^4 \right)^2 \left(text{dt}^4 \left(text{dt}^4 \right)\right)^2 \right)^2 \left(text{dt}^4 \left(text{dt}^4 \left(text{dt}^4 \right)\right)^2 \left(text{dt}^4 \left(text{dt}^4 \left(text{dt}^4 \right)\right)^2 \left(text{dt}^4 \left(text{dt}^4$ 

Out[264]=

Out[265]=

Out[266]= Omega error ||

Omega error || 
$$\left\{ \frac{1}{6(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + (3+H^2k^2) \, U \right) \left( 3 \, g \, H + U \left( 2 \, \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + (3+H^2k^2) \, U \right) \right) dt^2 + \frac{1}{8(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + (3+H^2k^2) \, U \right) \right) dt^2 + \frac{1}{8(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + (3+H^2k^2) \, U \right) \right) dt^2 + \frac{1}{20(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + (3+H^2k^2) \, U \right) \right) dt^4 + O[dt]^5 \right) + \\ \left( \frac{1}{240(3+H^2k^2)^3} k^3 \left( \sqrt{2} \ \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + (3+H^2k^2) \, U \right) \right) dt^4 + O[dt]^5 \right) + \frac{1}{480(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 34 \, U^2 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 10 \, U \right) + \frac{1}{480(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 36 \, H^4 \, U \right) + \frac{1}{480(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + (216+147\, H^2k^2 + 25\, H^4k^4) \, U \right) + \frac{1}{480(3+H^2k^2)^3} k^6 \left( 3 \, g \, H + U \left( \sqrt{2} \ \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + (3+H^2k^2) \, U \right) \right) dt^2 + \frac{1}{480(3+H^2k^2)^3} k^6 \left( 3 \, g \, H + U \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 24 \, H^2 \, U \right) \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) dt^3 - \frac{1}{960(3+H^2k^2)^3} k^3 \left( \sqrt{3} \ \sqrt{g \, H \, (3+H^2k^2)} \ + 8 \, H^2 \, U \right) dt$$

$$gH^5U^2\Big(89915\sqrt{gH(3+H^2k^2)} + 115737\sqrt{3}U\Big)\Big)\Big)dt^4 + O[dU^3\Big)dx^4 + O[dx]^5,$$

$$\left(\frac{1}{\sigma(3+H^2k^2)^2}k^3\Big(-\sqrt{3}\sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U\Big)\Big(3gH + U\Big(-2\sqrt{3}\sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U\Big)\Big)$$

$$dt^2 + \frac{1k^4\Big(3gH^4U\Big(-2\sqrt{3}\sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U\Big)\Big)^2dt^3}{8(3gH^4U\Big(-2\sqrt{3}\sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U\Big)\Big)}dt^3 - \frac{1}{20(3+H^2k^2)^4}\Big(k^2\Big(-\sqrt{3}\sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U\Big)\Big)^3\Big(3gH + U\Big(-2\sqrt{3}\sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U\Big)\Big)\Big)$$

$$dt^4 + O[dU^5\Big) + \frac{1}{400(3+H^2k^2)^4}k^5\Big(2014^6k^6U^3 + 54U^2\Big(-9\sqrt{3}\sqrt{gH(3+H^2k^2)} + 10U\Big) + \frac{1}{340(3+H^2k^2)^4}k^5\Big(2014^6k^6U^3 + 54U^2\Big(-9\sqrt{3}\sqrt{gH(3+H^2k^2)} + 10U\Big) + \frac{1}{340(3+H^2k^2)^4}k^5\Big(3gH + U\Big(-2\sqrt{3}\sqrt{gH(3+H^2k^2)} + 251H^4k^4\Big)U\Big) - \frac{3k^2\Big(15\sqrt{3}\sqrt{gH^3}(3+H^2k^2) + (216+147H^2k^2 + 251H^4k^4\Big)U\Big) - \frac{3k^2\Big(15\sqrt{3}\sqrt{gH^3}(3+H^2k^2) + (216+147H^2k^2 + 251H^4k^4\Big)U\Big) - \frac{1}{3k^2\Big(15\sqrt{3}\sqrt{gH^3}(3+H^2k^2) + (109\sqrt{3}\sqrt{gH(3+H^2k^2)} + 180U + 20H^4k^4U - 5k^2\Big(\sqrt{3}\sqrt{gH^3}(3+H^2k^2) + 180U + 20H^4k^4U - 5k^2\Big(\sqrt{3}\sqrt{gH^3}(3+H^2k^2) + 180U + 20H^4k^4U - 5k^2\Big(\sqrt{3}\sqrt{gH^3}(3+H^2k^2) + 180U + 20H^4k^4U - 15k^2\Big(\sqrt{3}\sqrt{gH^3}(3+H^2k^2) - 8H^2U\Big)\Big)$$

$$\Big(8gH + U\Big(-2\sqrt{3}\sqrt{gH(3+H^2k^2)} + 18U + 20H^4k^4U - 15k^2\Big(\sqrt{3}\sqrt{gH^3}(3+H^2k^2) - 8H^2U\Big)\Big) - \frac{1}{32(3+H^2k^2)}h^2\Big(3gH + U\Big(-2\sqrt{3}\sqrt{gH(3+H^2k^2)} + (3+H^2k^2)U\Big)\Big)^2\Big)dt^4 + O[dU^5\Big)dx^2 + \frac{1}{32(3+H^2k^2)}k^2\Big(-3gH(3+H^2k^2) + 18U + 2H^4k^2U + k^2\Big(-5\sqrt{3}\sqrt{gH^3}(3+H^2k^2) + 12H^2U\Big)\Big)\Big)$$

$$dt^2 - \frac{1}{32(3+H^2k^2)}k^2\Big(-3\sqrt{gH(3+H^2k^2)} + 18U + 2H^4k^2U + k^2\Big(-5\sqrt{3}\sqrt{gH^3}(3+H^2k^2) + 12H^2U\Big)\Big)\Big)$$

$$dt^2 - \frac{1}{32(3+H^2k^2)}k^2\Big(-3\sqrt{gH(3+H^2k^2)} + 2(3+H^2k^2)U\Big)\Big)^2dt^4 + O[dU^5\Big)dx^2 + 12H^2U\Big)\Big)\Big)$$

$$dt^2 - \frac{1}{32(3+H^2k^2)}k^2\Big(-3\sqrt{gH(3+H^2k^2)} + 2(3+H^2k^2)U\Big)\Big)^2dt^4 + O[dU^5\Big)dx^3 + \frac{1}{3}(4+H^2k^2)U\Big)\Big)$$

$$dt^2 - \frac{1}{32(3+H^2k^2)}k^2\Big(-3\sqrt{gH(3+H^2k^2)} + 2(3+H^2k^2)U\Big)\Big)^2dt^4 + O[dU^5\Big)dx^3 + \frac{1}{3}(4+H^2k^2)U\Big)\Big)$$

$$dt^2 - \frac{1}{32(3+H^2k^2)}k^2\Big(-3\sqrt{gH(3+H^2k^2)} + 2(3+H^2k^2)U\Big)\Big)^2dt^4 + O[dU^5\Big)dx^3 + \frac{1}{3}(4+H^2k^2)U\Big)\Big)$$

$$dt^2 - \frac{1}{32(3+H^2k^2)}k^2\Big(-3\sqrt{gH(3+H^2k^2)} + 2(3+H^2k^2)U\Big)\Big)^2dt^4 + O[$$

$$\left( \left( k^{5} \left( \sqrt{3} \right. g \, H \left( 17856 + 12180 \, H^{2} \, k^{2} + 2075 \, H^{4} \, k^{4} \right) - \right. \right. \\ \left. \left. \left( \left( k^{5} \left( \sqrt{3} \right. g \, H \left( 17856 + 12180 \, H^{2} \, k^{2} + 2075 \, H^{4} \, k^{4} \right) - \right. \\ \left. \left. \left( \left( k^{5} \left( \sqrt{3} \right. g \, H \left( 17856 + 12180 \, H^{2} \, k^{2} + 2075 \, H^{4} \, k^{4} \right) - \right. \right. \\ \left. \left. \left( \left( 38400 \, \sqrt{g \, H} \, \left( 3 + H^{2} \, k^{2} \right)^{5/2} \right) + \left( k^{2} \left( 45 \, \sqrt{3} \, g^{2} \, H^{2} \left( 3336 + 2268 \, H^{2} \, k^{2} + 385 \, H^{4} \, k^{4} \right) + \right. \\ \left. g \, \Pi \, U \left( 447588 \, \sqrt{3} \, \, 11^{2} \, k^{2} \, U + 16705 \, \sqrt{3} \, \, 11^{6} \, k^{6} \, U - 648 \left( 693 \, \sqrt{g} \, \Pi \left( 3 + H^{2} \, k^{2} \right) - 688 \, \sqrt{3} \, \, U \right) - 15 \, k^{4} \left( 3408 \, \sqrt{g} \, H^{9} \left( 3 + H^{2} \, k^{2} \right) - 9985 \, \sqrt{3} \, \, H^{4} \, U \right) \right) - \\ \left. \left. 80 \left( 1836 \, \sqrt{g} \, H \left( 3 + H^{2} \, k^{2} \right) \, U^{3} + 612 \, k^{4} \, \sqrt{g} \, H^{9} \left( 3 + H^{2} \, k^{2} \right) \, U^{3} + 68 \, k^{6} \, \sqrt{g} \, H^{13} \left( 3 + H^{2} \, k^{2} \right) \, U^{3} + 9 \, k^{2} \left( 421 \, \sqrt{g^{3}} \, H^{7} \left( 3 + H^{2} \, k^{2} \right) \, U + 204 \, \sqrt{g} \, H^{5} \left( 3 + H^{2} \, k^{2} \right) \, U^{3} \right) \right) \right) \right) \right) \right) \right) \right)$$

$$\left( 230400 \, \sqrt{g} \, H \, \left( 3 + H^{2} \, k^{3} \right)^{7/2} \right) - \frac{1}{25600(3418^{2} \, k^{2})^{2}} \, k^{2} \, \left( 6 \, g^{2} \, H^{2} \left( 8046 + 5460 \, H^{2} \, k^{2} + 925 \, H^{4} \, k^{4} \right) + \right. \right. \right. \right.$$

$$\left. 560 \, H^{8} \, k^{8} \, U^{4} + 432 \, U^{2} \left( -143 \, \sqrt{3} \, \sqrt{g} \, H \left( 3 + H^{2} \, k^{2} \right) + 105 \, U \right) + \right. \right.$$

$$\left. 5 \, k^{4} \, U^{3} \left( -461 \, \sqrt{3} \, \sqrt{g} \, H^{13} \, \left( 3 + H^{2} \, k^{2} \right) + 1344 \, H^{6} \, U \right) - \right.$$

$$\left. 12 \, k^{2} \, \left( 3585 \, \sqrt{3} \, \sqrt{g^{3}} \, H^{7} \left( 3 + H^{2} \, k^{2} \right) + 1344 \, H^{6} \, U \right) - \right.$$

$$\left. 12 \, k^{2} \, \left( 3585 \, \sqrt{3} \, \sqrt{g} \, H^{13} \, \left( 3 + H^{2} \, k^{2} \right) + 1344 \, H^{6} \, U \right) \right. \right.$$

$$\left. \left( k^{9} \, \left( \sqrt{3} \, \sqrt{g} \, H^{13} \, H^{2} \, k^{2} \right) - \left( 3 + H^{2} \, k^{2} \right) + 19056 \, H^{4} \, U \right) \right) \right) \right. \right.$$

$$\left. \left( k^{9} \, \left( \sqrt{3} \, \sqrt{g} \, H^{13} \, H^{2} \, k^{2} \right) - \left( 3 + H^{2} \, k^{2} \right) \right. \right) \right. \right. \right. \right. \right. \right. \right.$$

$$\left. \left( k^{9} \, \left( \sqrt{3} \, \sqrt{g} \, H^{13} \, H^{2} \, k^{2} \right) - \left( 3 + H^{2} \, k^{2} \right) \right) \right. \right) \left. \left( \sqrt{3} \, \sqrt{g} \, H$$

 $\left(H^2 k^2+3\right)^2+\frac{1}{2} + \frac{3 + 4 \left(g H^2 k^2+3\right) U+2 \left(g H^2 k^2+3\right) U+2 \right)}{1}$  $\label{left(H^2 k^2+3\wedge right)} $$\left(H^2 k^2+3\wedge right\right)^2\right]-\frac{h^2 k^2+3\wedge right}^2-\frac{h^2 k^2+3\wedge right}^2-\frac{h^2 k^2+3\wedge right}^2-\frac{h^2 k^2+3\wedge right}^2}{h^2 k^2+3\wedge right}$  $k^2+3\right) U+\sqrt{3} \left(H^2 k^2+3\right) \Gamma(H^2 k^2+3\right)$  $k^2+3\right) U+2 \left( H^2 \left( H^2 \right) \right) \left( H^2 \right) \left$  $\label{eq:k-2+3-right} $$k^2+3\right)+O\left(\frac{dt}^5\right)+\left(\frac{dt$ \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 U H^4+11 \sqrt{3} \sqrt{g H^9}  $\left(H^2 k^2+3\right)\right) \ k^4+3 \left(H^2 U^3+109 \right) \ \left(H^5 \left(H^2 k^2+3\right)\right)$  $U^2+15 \sqrt{3} \sqrt{9^3 H^7 \left(H^2 k^2+3\right)} k^2+54 U^2 \left(10 U+9 \sqrt{3} \sqrt{10 U+9} \right)$ H \left(H^2 k^2+3\right)\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U+21 \sqrt{3}  $\left( H^2 k^2+3\right)\right)\right)$  $g H+U \left( \frac{h^2 k^2+3\right) U+2 \left( H^2 k^2+3\right) \left( H^2$  $H \left(5 H^2 k^2+14\right)+U \left(5 H^4 U k^4+5 \left(5 H^2 + 14\right)+U \right)$  $k^2+3\left( H \left( H^2 k^2+3\right) \right) \right)$  $\left( H^2 k^2 + 3\right) - \left( H^2 k^2$  $k^2+3\right)$  ight)  $k^2+180$  U+42  $\sqrt{4}$  sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \left(3 g H+U \left(H^2 Left(H^2 Left(H  $\label{eq:linear_line$  $\label{left} $$ k^2+3\right)^4+O\left(\frac{t}{5}\right)\right) \cdot \left(\frac{1}{16} i k^4 \cdot \frac{1}{16} i$  $\$  \sqrt{\frac{g H}{H^2 k^2+3}}\right)+\frac{i k^6 \left(H^4 U k^4+\left(12 U H^2+5 \right)\right)} \  $H^5 \left( H^2 k^2 + 3\right) \right) h^2 + 18 U + 15 \left( H^2 k^2 + 3\right) \right) h^2 + 18 U + 15 \left( H^2 k^2 + 3\right) h^2 \right) h^3 \left( H^2 k^2 + 3\right) h^3 \left( H^2$  $g\ H \left(\frac{4\left(\frac{4^2 k^2+3\right)}{t} U+\sqrt{3} \right) U+\sqrt{3} \left(\frac{4^2 k^2+3\right)}{t} \right) U+\frac{3}{3} \left(\frac{4^2 k^2+3\right)}{t} \left(\frac{4^2 k^2+3\right)}{t} \right) U+\frac{3}{3} \left(\frac{4^2 k^2+3\right)}{t} \left(\frac{4^2 k^2$  $\left( H^2 k^2 + 3 \right) U+2 \left( h^7 \left( h^2 k^2 + 3 \right) U+2 \right) U+2 \left( h^2 k^2 + 3 \right) U+2 \left( h^2$  $H \left( H^2 k^2 + 3\right) \right) \left( 3 H + U \left( 4 k^2 + 3\right) U + 3 \right) \left( 3 H + U \left( 4 k^2 + 3\right) U + 3 \right) \right)$  $H \left(\frac{h^2 k^2+3 \right)}{right}\right) \left(\frac{h^2 k^2+3 \right)}{right}$  $\left(H^2 k^2+3\right) U+\sqrt{3} \left(H^2 k^2+3\right)\right)$  $k^2+3\right) U+2 \sqrt{3} \sqrt{4}{64 \left(H^2 k^2+3\right)\right)} \left(H^2 k^2+3\right) \left(H^2 k^2+3\right)$  $k^2+3\right)^3+O\left(\frac{t}{5\right)^3}+O\left(\frac{t}{5\right)^3}+O\left(\frac{t}{5}\right)^3+O\left(\frac{t}{5$  $H^4 k^4 + 12180 H^2 k^2 + 17856 \right) + 2080 \left( \frac{H^9 \left( \frac{K^2 + 3\right)}{k^4 + 6} \right) + 2080 \right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^2 +$  $H^5 \left(H^2 k^2+3\right) k^2+9 \left(H^2 k^2+3\right) U\right) H^5 \left(H^2 k^2+3\right) U\right) H^5 \left(H^2 k^2+3\right) U\right)$  $H} \left(\frac{45 \operatorname{k^2} + 3\operatorname{right}}{5/2}\right) - \left(\frac{45 \operatorname{k^2} + 3\operatorname{right}}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^2} + 3\operatorname{right}}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{3} g^2 \right) + \frac{4 \operatorname{k^4} + 268}{3} g^2 \left(\frac{45 \operatorname{k^4} + 268}{$ H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6+15 \left(9985 \sqrt{3} U H^4+3408 \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+447588 \sqrt{3} H^2 U k^2+648 \left(688 \sqrt{3} U+693  $k^6 + 612 \setminus g H^9 \setminus (H^2 k^2 + 3 \cap U^3 k^4 + 9 \setminus g H^5 \setminus (H^2 k^2 + 3 \cap U^3 k^4 + 9 \cap U^3 k^4 + 9$  $U^3+421 \cdot g^3 H^7 \cdot (h^2 k^2+3 \cdot g)$  Uright)  $h^2+1836 \cdot g$  H \left( $h^2 k^2+3 \cdot g$ )  $U^3\right) \left( \frac{H^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} -\frac{h^2 k^2}{1} \right) \left( \frac{h^2 k^2+3\right)}{1} -\frac{h^2 k^2}{1} -\frac{h$ \left(560 H^8 U^4 k^8+5 U^3 \left(1344 U H^6+461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 U H^4+4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+12 \left(5040  $H^2 U^4 + 5161 \cdot (3) \cdot (4^2 L^2 + 5161 \cdot (3) \cdot (4^2 L^2 + 16) \cdot (4^2 L^$ k^2+3\right) U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 U+143 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+g H U \left(10640 H^6 U k^6+5 \left(19056 U  $H^4+1451 \left( H^2 k^2+3\right) \right) + h^4+1451 \left( H^2 k^2+3\right) \right) + h^4+1451 \left( H^2 k^2+282852 \right) + h^4+1451 \right) + h^4+1451 \left( H^2 k^2+282852 \right) + h^4+1451 \right) + h^4+1451 \left( H^2 k^2+282852 \right) + h^4+1451 \right) + h^4+1451 \left( H^2 k^2+282852 \right) + h^4+1451 \left( H^2 k^2+284864 \right) + h^4+1461 \left( H^2 k^2+284$ 

 $\left(\left(\frac{H^2 k^2+3\right)}{U+\sqrt{3} \left(\frac{H^2 k^2+3\right)}\right)\right)$ g H^9+928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U\right) k^8+15 U \left(4143 \sqrt{3} g^2 H^8+g U k^2+3\right) U^3\right) k^6+9 \left(61735 \sqrt{3} g^2 U H^6+g U^2 \left(115737 \sqrt{3} U+89915  $\$  \\sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+27840 \\sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)\right) k^4+108 \left(15327 \sqrt{3} g^2 U H^4+19261 \sqrt{3}  $g\ U^3\ H^3+4640\ sqrt\{g\ H^5\ left(H^2\ k^2+3\ left(H^$ U^4+g H \left(1202 \sqrt{3} U+1857 \sqrt{g H \left(H^2 k^2+3\right)\right) U^2+1268 \sqrt{3} g^2  $H^2 U+323 \left( \frac{g^5 H^5 \left( \frac{40800 \left( + \right)} \right)} \right)} {40800 \left( \frac{40800 \left( + \right)} \right)} {40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( + \right)} \right)} {40800 \left( \frac{40800 \left( \frac{40800 \left( + \right)} {40800 \left( + \right)} {40800}$  $\left(\left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \sqrt{3} \right) + \left(\frac{H^2 k^2+3\right)}\right) \left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \left(\frac{H^2 k^2+3\right)}}\right)$  $k^2+3\right) U-2 \sqrt{3} \sqrt{4} \ln(H^2 k^2+3\right) \int U-2 \sqrt{3} \left( H^2 k^2+3\right) \left($  $\label{eq:k-2+3-right} $$k^2+\frac{1}{2}+\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$  $k^2+3\left( U-2 \right) U-2 \left( H^2 \left( H^2 \right) \right) \right) \left( H^2 \right) \left$  $\label{left(sqrt{3} holeft(20 H^4 U k^4-15 \left(\sqrt{3} + \sqrt{3} \right)^4)+O(eft(\sqrt{3} + \sqrt{3} + \sqrt{4})^4)} k^2 + O(eft(\sqrt{3} + \sqrt{3} +$  $\sqrt{g H^5 \left(H^2 k^2+3\right)} = H^2 U\right) k^2+180 U-42 \sqrt{3} \sqrt{g H \ell^2}$ k^2+3\right)\right)\right)\{240 \left(H^2 k^2+3\right)^2\}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 H^4 U^5 H^6 U  $H^5 \left(H^2 k^2+3\right) U^2+15 \left(H^2 k^2+3\right) k^2+3 U^2$  $\left(10 U - 9 \right) \right) \ H \left(H^2 k^2 + 3\right) \right) \ H \left(16 U - 9 \right) \ H^2 \ H^4 k^4 + 147 H^2 \ H^$  $k^2+216 \right) U-21 \left( H^2 k^2+3\right) \right) \left( H^2 k^2+3\right) \left( H^2 k^2+3\right)$  $k^2+3\right)^3+\frac{1}{2} k^6 \left(16 k^6 + 16 k^6 + 16$ k^2+3\right)\right)\right)\left(9 g H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4-5 \left(7 \sqrt{3})  $\$  \sqrt{g H^5 \left(H^2 k^2+3\right)}-24 H^2 U\right) k^2+180 U-102 \sqrt{3} \sqrt{g H \left(H^2 L^2 H^2 U\right)}  $k^2+3\right) \right) \cdot k^2+3\right) \cdot k^2+3\left( k^2+3\left( k^2+3\right) \cdot k^2+3\left( k^2+3\left( k^2+3\right) \cdot k^2+3\left( k^2+3\left( k^2+3\right) \cdot k^2+3\left( k^2+3\left( k^2+3\right) \cdot k$  $k^4 - 15 \left( \frac{3} \sqrt{H^5 \left( \frac{4^2 + 3\right)}} - 8 \right) - 8 \left( \frac{4^2 + 3\right)}{8} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{4^2 + 3\left( \frac{4^2 + 3\left( \frac{4^2 + 3\right)}{16}}{16} \right) - 8 \left( \frac{4^2 + 3\left( \frac{$  $k^2+3\right) \right) \left( k^2+3\right) \right) \left( k^2+3\right) \left( k^2$  $\text{dx}^2+\left(-\frac{1}{16} i k^4 \left(\frac{3} \right) \right) + \frac{4 k^2 k^2+3}{-2 k^2+3}$  $\label{left} $\left(U^2 \left(H^4 \cup k^4 + \left(12 + 2 \cup 5 \right) \right) \right) \ \\ \left(H^5 \left(H^2 \cdot k^2 + 3 \right) \right) \ \\ \left(H^2 \cdot k^2 + 3 \right) \ \\ \left(H^2$  $U-15 \sqrt{3} \sqrt{g H \left(\frac{H^2 k^2+3\right)}{\sinh(\theta)}} g H \left(\frac{3} \sqrt{g H \left(\frac{H^2 k^2+3\right)}} - 4 \right)$  $\label{left(H^2 k^2+3) right) U right) right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^2}-\frac{1}{4} Right)^2}-\frac{1}{4} Right)^2}-\frac{1}{4} Right)^2}{1} Right)^2}{1} Right)^2}-\frac{1}{4} Right)^2}-\frac{1}{4} Right)^2}{1} Right)^2}-\frac{1}{4} Right)^2}{1} Right)^2}-\frac{1}{4} Right)^2}{1} Right)^2}-\frac{1}{4} Right)^2}{1} Right)^2}-\frac{1}{4} Right)^2}{1} Right)^2}-\frac{1}{4} Right)^2}-\frac{1}{4} Right)^2}{1} Right)^2}-\frac{1}{4} Right)^2}{1} Right)^2}-\frac{1}{4} Right)^2}-\frac{1}{4} Right)^2}-\frac{1}{4} Right)^2}{1} Right)^2}-\frac{1}{4} Right)^2}$  $g H+U \left( \frac{H^2 k^2+3\right)}{U-2 \left( H^2 k^2+3\right)} \right) U-2 \left( H^2 k^2+3\right) \left( H^2 k^2+3\right) \right) U-2 \left( H^2 k^2+3\right) U-2$  $g H+U \left( \frac{h^2 k^2+3\right) U-3 \left( H \left( \frac{h^2 k^2+3\right) v_1}{H^2 k^2+3\right) } \right)$  $\t (dt)^3 {32 \left( H^2 k^2 + 3\right) U - \left( h^2 k^2 + 3$  $\left(H^2 k^2+3\right)\right)$  $\label{eq:k-2+3-right} $$ k^2+3\right)^2 \text{text}(dt)^4{64 \left(\frac{h^2 k^2+3\right)^3}+O\left(\frac{dt}{5\right)^5}\right)^2} $$$  $\text{text}_{dx}^3 + \text{left}_{fac}_{k^5} \left[ \frac{3}{9} H \left( \frac{2075 \text{ H}^4 \text{ k}^4 + 12180 \text{ H}^2 \text{ k}^2 + 17856} \right) - 2080 \right]$  $\left( \frac{H^2}{h^2} \right) \left( \frac{H^2}{$  $H \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} U\right) = H \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)}$ \1a\frac{1}{1} \alpha\frac{1}{2} \alpha\frac{1}{

 $k^6-15 \left(3408 \right) k^4+447588 \left(4^2 + 3\right) -9985 \left(4^4 + 447588 \right) k^6-15 \left(4^6 + 447588 \right) k^6-15 \left(4^6 + 47588 \right) k^6-15 \left(4^$ H^2 U k^2-648 \left(693 \sqrt{g H \left(H^2 k^2+3\right)}-688 \sqrt{3} U\right)\right) H-80 \left(68  $k^2+3\right/f(1344 + 6 - 461 \cdot 6^{14})^{7/2}}-\frac{k^8 \left(1344 + 6 - 461 \cdot 6^{14}\right)}{1344 + 6 - 461 \cdot 6^{14}}$ k^2+3\right)\right) k^4-12 \left(-5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 U \left(10640 H^6 U k^6+5 \left(19056 H^4 U-1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U-63720 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right)  $\text{dt}^3{25600 \left(H^2 k^2+3\right)^4}-\text{dt}^3}{25600 \left(H^2 k^2+3\right)^4}-\text{dt}^3} \right. \$  $k^2+3\right)-\left(H^2 k^2+3\right)-\left(H^2 k^2+3\right)$ U-3869 \sqrt{3} g H^9\right) k^8-15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U-6019  $\left( H^2 k^2+3\right) H^7-3712 \left( H^13 \left( h^2 k^2+3\right) \right) U^3\right)$ k^6+9 \left(-61735 \sqrt{3} g^2 U H^6+g U^2 \left(89915 \sqrt{g H \left(H^2 k^2+3\right)}-115737 \sqrt{3} U\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2  $k^2+3\left( h^4-108 \left( -15327 \right) \right) + h^4-19261 \left( h^4-19261 \right) + h^4-19261 \left( h^4-1926$  $H^5 \left(H^2 k^2+3\right) U^4+22383 \left(H^2 k^2+3\right) U^2+2625 \left(H^2 k^2+3\right) U^4+22383 \left(H^2 k^2+3\right) U^4+22383 \left(H^2 k^2+3\right) U^4+22383 U^4+22384 U^4+2444 U^4+2444 U^4+244 U^4+244 U^4+244 U^4+244 U^4+244 U^4+244 U^4+244 U^4+244$  $H^9 \left(H^2 k^2+3\right) \right) + (H^2 k^2+3\right) + (H^2 k^2+3) + (H^2$  $\left(1857 \right) + \left(1857 \right) + \left(18$ 

Out[268]=  $\text{Out} \text{[269]=} \quad EA \text{ ||} \quad \left\{ \left\{ \frac{-H^2 \, k^2 \, \left( \left( -1 + e^{i \, d \, w} \right) k \, U - w \right) + 3 \, w}{(3 + H^2 \, k^2) \, w}, \, \, -\frac{3 \, \left( -1 + e^{i \, d \, w} \right) k}{(3 + H^2 \, k^2) \, w} \right\}, \, \, \left\{ -\frac{\left( -1 + e^{i \, d \, w} \right) k \, \left( g \, H \, \left( 3 + H^2 \, k^2 \right) - 3 \, U^2 \right)}{(3 + H^2 \, k^2) \, w}, \, \, 1 \, - \, \frac{\left( -1 + e^{i \, d \, w} \right) k \, \left( 6 + H^2 \, k^2 \right) U}{(3 + H^2 \, k^2) \, w} \right\} \right\}$ Out[270]= EA || \left( \begin{array}{cc}  $\frac{3 \text{ w-H}^2 \text{ k}^2 \left(\left(-1+e^{i \text{ text}}\right) \text{ k U-w}\right)}{\left(-1+e^{i \text{ text}}\right) \text{ k U-w}\right)}{\left(-1+e^{i \text{ text}}\right)}$  $k^2+3\right) w$  &  $-\frac{3 \left(1+e^{i \left(t\right) w}\right) k}{\left(t\right) w} \$  $-\frac{1+e^{i \cdot text}}{dt}$  w\right) k \left(g H \left(H^2 k^2+3\right)-3 U^2\right)\{\left(H^2 k^2+3\right)-3 U^2\right)\} \left(H^2 k^2+3\right)}  $w\} \& 1-\frac{\left(-1+e^{i \cdot k^2+6 \cdot k^2+6 \cdot k^2+6 \cdot k^2+6 \cdot k^2+3 \cdot k^2+6 \cdot k^2+3 \cdot k^2+6 \cdot k^2+3 \cdot k^2+6 \cdot k^2+6$ \end{array} \right)

Out[272]= Eerr || \left(

\begin{array}{cc}

 $\label{left(3 U k+sqrt{3} sqrt{g H left(H^2 k^2+3\right)} k\right) \text{dt}}{H^2 k^2+3}+\frac{3 V^2+3}{H^2 k^2+3}-\frac{1}{6}$ 

 $\t \{dt\}^3 - \{1\}\{24\} \cdot \{-i \ U \ k - \{i \ sqrt\{3\} \ sqrt\{g \ H \ left(H^2 \ k^2 + 3 \ right)\} \ k\}\{H^2 \ left(H^2 \ k^2 + 3 \ right)\}$  $k^2+3\right\right)$   $text(dt)^4+O\left(text(dt)^5\right)$   $text(dt)^4+O\left(text(dt)^5\right)$  $H^2 k^5+54 k^3\right) U \text{ } U \text{ } left(H^2 k^2+3\right)^2}-\frac{120 \left(H^2 k^2+3\right)^2}{H^2 k^5+54 k^3\right) U \text{ } left(H^2 k^2+3\right)^2}-\frac{120 \left(H^2 k^2+3\right)^2}{H^2 k^5+54 k^3\right)^2}$ k^8+45 g H^3 k^6+30 H^2 U^2 k^6-72 U^2 k^4+126 g H k^4\right) \text{dt}^2}{240 \left(H^2  $\label{left} $$k^2+3\right)^2}+O\left(\frac{dt}{5\right)^5\right) \cdot \left(\frac{dt}{5\right)^5} + O\left(\frac{dt}{5\right)^5}\right) \cdot \left(\frac{dt}{5\right)^5} + O\left(\frac{dt}{5\right)^5} + O\left(\frac{dt}{5}\right)^5 + O$  $H^2 U^2 k^7$ {8 \left( $H^2 k^2 + 3 \right) - \frac{3 i g H k^5}{16 \left(H^2 k^2 + 3 \right) \cdot \frac{16 \left(H^2 k^2 + 3 \right)}{16 \left(H^2 k^2 + 3 \right) \cdot \frac{16 \left(H^2 k^2 + 3 \right) \cdot \frac{16 \left(H^2 k^2 + 3 \right)}{16 \left(H^2 k^2 + 3 \right) \cdot \frac{16 \left$  $\label{eq:linear_continuous_con$ U^2 k^{12}+4500 g H^5 k^{10}+4140 H^4 U^2 k^{10}+26460 g H^3 k^8-13500 H^2 U^2 k^8-38286  $U^2 \ k^6 + 38853 \ g \ H \ k^6 \right) \ | \ text{dt}^2}{28800 \ (H^2 \ k^2 + 3\right)} + O\left( text{dt}^5 \right) \ | \ text{dt}^5 \right) | \ text{dt}^5 \right) | \ text{dt}^5 |$  $\t (-\frac{3 i k \text{d}}{H^2 k^2+3}-\frac{3 i k \text{d}}{H^2 k^2+3}-\frac{3 i k^2}{H^2 k^2+3}-\frac{3 i k^2$  $U \right) \left( \frac{dt}^2}{H^2 k^2 + 3} + O\left( \frac{dt}^5 \right) \right) + \left( -\frac{1}{4} \right) + O\left( \frac{dt}^5 \right) \right)$  $k^5+12 k^3\right) \left( \frac{dt}{40 \left( \frac{H^2 k^2+3\right)}} \right) - \frac{3 \left( \frac{H^2 k^6+14}{40 \left( \frac{H^2 k^2+3\right)}} \right)}{h^2} \right) + \frac{h^2 k^3}{h^2} \left( \frac{H^2 k^6+14}{h^2} \right) + \frac{h^2 k^6+14}{h^2} \left( \frac{H^2 k^6+14}{h^2} \right) + \frac{h^2 k^6$  $k^4\right) U\right) \perp (t_{dt}^2){40 \left(H^2 k^2+3\right)^2} + O\left(t_{dt}^5\right)\right)$  $\label{eq:left} $$ \operatorname{dx}^2+\left(-\frac{3 i k^5 U \left(\frac{dt}^2}{8 \left(\frac{h^2 k^2+3\right)}+O\left(\frac{dt}{h^2}\right)}\right)} + O\left(\frac{h^2 k^2}{h^2 k^2}\right) \right) $$$ \text{dx}^3+\left(\frac{i \left(770 H^4 k^9+4410 H^2 k^7+6291 k^5\right) \text{dt}}{4800 \left(H^2  $k^2+3\right)^3+\frac{1500 H^4 U k^{10}+8820 H^2 U k^8+12951 U k^6\right) \text{ } text{dt}^2}{4800 U k^6}$  $\left(\frac{dx}^4+O\left(\frac{dx}^5\right)^3\right)+O\left(\frac{dx}^5\right)\right)$  $\left(-\frac{k^2 U}{e^2 u^3}\right)$ g H-3 U $^2$ right) \text{dt} $^2$ {H $^2$  k $^2$ +3}+O\left(\text{dt} $^5$ \right)\right)+\left( $-\frac{1}{9}$ H^5 k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}}{120 \left(H^2 k^2+3\right)^2}+\frac{\left(-20 g H^5 U k^8+45 H^2 U^3 k^6-120 g H^3 U k^6+126 U^3  $\t x_{dx}^2 + \left( \frac{1}{8} g H k^4 \left( \frac{dt}{-\frac{1}{8}} g H k^4 \right) - \frac{dt}{2} H^3 U k^7 - 3 U^3 k^5 + 6 g H U k^5 \right)$ g H^7 k^{11}+2340 g H^5 k^9-770 H^4 U^2 k^9+7020 g H^3 k^7-4410 H^2 U^2 k^7-6291 U^2 k^5+7020 g H k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 g H^7 U k^{12}-4500 g H^7 U k^12}+\frac{\left(1460 g H^7 U k^12}+\frac{\ H^4 U^3 k^{10}+13140 g H^5 U k^{10}-26460 H^2 U^3 k^8+39420 g H^3 U k^8-38853 U^3  $k^6+39420 \text{ g H U } k^6 \cdot \text{light} \text{ $k^6+39420 g H U } k^6 \cdot \text{light} \cdot \text{light} \text{ $k^2+3\right)} + O\left(\frac{k^2}{k^2}+3\right) \cdot \text{light} \cdot$  $\text{text}(dx)^4 + O\left(\frac{dx}^5\right) \times \left(\frac{dx}^5\right) \times \left(\frac{dx}^5\right) \times \left(\frac{dx}^6\right) \times \left(\frac{dx}^6\right)$ k U\right) \text{dt}}{H^2 k^2+3}+ \frac{\left( \sqrt{3} k^2 \sqrt{4} H \left( \sqrt{4} k^2 + 3 \right) U-3 k^2 \right)} U-3 k^2}  $U^2\left(\frac{1}{6}\left(-i U - \frac{1}{6}\right) \left(-i U - \frac{3}{3}\right) \left(-i U - \frac{3}{3}\right$  $k\H^2 k^2+3\right\right) ight)^3 \left(t^3-\frac{1}{24}\right) \left(i U k-\frac{i V k-\frac{3}{3}}{1 v^2}\right) + i U k^2 \left(i V k-\frac{3}{3}\right) \left(i V k-\frac{3}{3}\right) + i V k^2 \left(i V k-\frac{$  $k^2+3\right) k^{H^2 k^2+3}\right) + k^{2+3}\right) + k^{2+3}\left( k^{2+3}\left( k^{2+3}\right) + k^{2+3}\left( k^{2+$  $H^4 k^7 + 75 H^2 k^5 + 126 k^3 \oplus U \text{ text} dt} 120 \left( H^2 k^2 + 3 \right)^2 - \frac{126 H^4 k^7 + 75 H^2 k^5 + 126 k^3 \oplus U \text{ text} dt}{120 H^4 k^7 + 75 H^2 k^5 + 126 k^3 \oplus U \text{ text} dt} \right)$  $U^2 k^8 + 45 g H^3 k^6 + 210 H^2 U^2 k^6 + 432 U^2 k^4 + 126 g H k^4 \right) \text{ text} dt^2 \left[ 40 \left( \frac{1}{2} \right) + \frac{1}{2} \left( \frac{1}{2} \right) \right] + \frac{1}{2} \left( \frac{1}{2} \right) + \frac{1}{2} \left( \frac{$  $\label{left} $$k^2+3\right)^2}+O\left(\frac{t}{\sqrt{t}}^5\right)\right) \cdot \left(\frac{dx}^2+\left(\frac{1}{8} k^4 U \right)^2}+O\left(\frac{t}{\sqrt{t}}^5\right)\right)^2}$  $H^2 k^2 U^2+12 U^2+3 g H\right) \left(\frac{dt}^2}{16 \left(\frac{h^2 k^2+3\right)}+O\left(\frac{dt}^5\right)}\right)$ \text{dx}^3+\left(\frac{i \left(260 H^6 k^{11}+3110 H^4 k^9+11430 H^2 k^7+13311 k^5\right) U \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 H^6 U^2 k^{12}+4500 g H^5 k^{10}+22140 H^4 U^2 k^{10}+26460 g H^3 k^8+92340 H^2 U^2 k^8+117126 U^2 k^6+38853 g H k^6\right) \text{dt}^2\{28800  $\left(\frac{dx}^4+O\left(\frac{dx}^5\right)^3\right)+O\left(\frac{dx}^5\right)\right)$ 

 $\left(-i U k - \frac{i \sqrt{3} \right)^3} \left(-i U k - \frac{i \sqrt{3} \right)^3}$ 

\enu{array} \right)