In[1]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];

qjp1n = q0 * Exp[I * (k * (xj + dx) + w * tn)];
qjp1F = Simplify[qjp1n / (qjn)];
qjp2n = q0 * Exp[I * (k * (xj + 2 * dx) + w * tn)];
qjp2F = Simplify[qjp2n / (qjn)];
qjm1n = q0 * Exp[I * (k * (xj - dx) + w * tn)];
qjm1F = Simplify[qjm1n / (qjn)];
qjm2n = q0 * Exp[I * (k * (xj - 2 * dx) + w * tn)];
qjm2F = Simplify[qjm2n / (qjn)];
```

$$\mathtt{wAp} = -U * k - \frac{\sqrt{3}\ k \sqrt{g\,H\,(3 + H^2 k^2)}}{3 + H^2 k^2};$$

$$\mathtt{wAm} = -\ U * k + \frac{\sqrt{3}\ k \sqrt{g\,H\,(3 + H^2 k^2)}}{3 + H^2 k^2};$$

In[13]:=
```
Dx = FullSimplify[(qjp1F - qjm1F) / (2 * dx)];
Dxerr = Series[Dx - (I * k), {dx, 0, 4}];
DxDx = FullSimplify[(qjp1F - 2 + qjm1F) / dx^2];
DxDxerr = Series[DxDx - (-k * k), {dx, 0, 4}];
DxDxDx = FullSimplify[(qjp2F - 2 qjp1F + 2 * qjm1F - qjm2F) / (2 * dx * dx * dx)];
DxDxDxerr = Series[DxDxDx - (-I * k * k * k), {dx, 0, 4}];

Text[Row[{"Dx  ||   ", Dx}]]
Text[Row[{"Dx  ||   ", TeXForm[Dx]}]]
Text[Row[{"Dx error  ||   ", TeXForm[Dxerr]}]]
Text[Row[{"Dx error  ||   ", Dxerr}]]
Text[" "]
Text[Row[{"DxDx  ||   ", DxDx}]]
Text[Row[{"DxDx  ||   ", TeXForm[DxDx]}]]
Text[Row[{"DxDx error  ||   ", TeXForm[DxDxerr]}]]
Text[Row[{"DxDx error  ||   ", DxDxerr}]]
Text[" "]
Text[Row[{"DxDxDx  ||   ", DxDxDx}]]
Text[Row[{"DxDxDx  ||   ", TeXForm[DxDxDx]}]]
Text[Row[{"DxDxDx error  ||   ", TeXForm[DxDxDxerr]}]]
Text[Row[{"DxDxDx error  ||   ", DxDxDxerr}]]
Text[" "]
```

Out[19]= Dx || $\frac{i \,\mathrm{Sin}[dx\ k]}{dx}$

Out[20]= Dx || \frac{i \sin (\text{dx} k)}{\text{dx}}

Out[21]= Dx error || −\frac{1}{6} i \text{dx}^2 k^3+\frac{1}{120} i \text{dx}^4 k^5+O\left(\text{dx}^5\right)

Out[22]= Dx error || $-\frac{1}{6}\, i\, k^3\, dx^2 + \frac{1}{120}\, i\, k^5\, dx^4 + O[dx]^5$

Out[23]=

Out[24]= DxDx || $\frac{2\,(-1+\mathrm{Cos}[dx\, k])}{dx^2}$

Out[25]= DxDx || \frac{2 (\cos (\text{dx} k)−1)}{\text{dx}^2}

Out[26]= DxDx error || \frac{\text{dx}^2 k^4}{12}−\frac{\text{dx}^4 k^6}{360}+O\left(\text{dx}^5\right)

Out[27]= DxDx error || $\frac{k^4\, dx^2}{12} - \frac{k^6\, dx^4}{360} + O[dx]^5$

Out[28]=

Out[29]= DxDxDx || $-\frac{4\, i\, \mathrm{Sin}\left[\frac{dx\, k}{2}\right]^2 \mathrm{Sin}[dx\, k]}{dx^3}$

Out[30]= DxDxDx || −\frac{4 i \sin ^2\left(\frac{\text{dx} k}{2}\right) \sin (\text{dx} k)}{\text{dx}^3}

Out[31]= DxDxDx error || \frac{1}{4} i \text{dx}^2 k^5−\frac{1}{40} i \text{dx}^4 k^7+O\left(\text{dx}^5\right)

Out[32]= DxDxDx error || $\frac{1}{4}\, i\, k^5\, dx^2 - \frac{1}{40}\, i\, k^7\, dx^4 + O[dx]^5$

Out[33]=

```
In[34]:= upsspatderivs = -(g*H*Dx*n + U*H*Dx*v - H^3/3*U*DxDxDx*v);
upsspatderivsLHS =
  H*v - H^3/3*DxDx /. v → 1 /. Cos[dx k] - 1 → -2*Sin[dx k/2]^2;
upsspatderivsu = upsspatderivs /. v → 1 /. n → 0;
upsspatderivsu = Simplify[upsspatderivsu / upsspatderivsLHS ];
upsspatderivsn = upsspatderivs /. n → 1 /. v → 0;
upsspatderivsn = Simplify[upsspatderivsn / upsspatderivsLHS ];
vph = Simplify[((1 + qjp1F) vnp1 + (1 + qjp1F) v) / 4];
vmh = Simplify[((1 + qjm1F) vnp1 + (1 + qjm1F) v) / 4];
hph = n*(qjp1F + 1)/2 - dt/(2*dx)*(H*(qjp1F - 1)*v + U*n*(qjp1F - 1));
hmh = n*(1 + qjm1F )/2 - dt/(2*dx)*(H*(1 - qjm1F )*v + U*n*(1 - qjm1F ));
LWFlux = n - dt / dx*(H*(vph - vmh) + U*(hph - hmh)) /.
   vnp1 → vnm1 + 2*dt*(upsspatderivsu*v + upsspatderivsn*n);
LWFluxun = FullSimplify[LWFlux /. v → 1 /. vnm1 → 0 /. n → 0];
LWFluxunm1 = FullSimplify[LWFlux /. v → 0 /. vnm1 → 1 /. n → 0];
FullSimplify[ e^(-i dx k) (-1 + e^(2 i dx k))];
FullSimplify[ e^(-i dx k) (-1 + e^(i dx k))^2];
LWFluxn =
  Simplify[LWFlux /. v → 0 /. vnm1 → 0 /. n → 1] /. e^(-i dx k) (-1 + e^(2 i dx k)) →
     2*I*Sin[k*dx] /. e^(-i dx k) (-1 + e^(i dx k))^2 → 2 (-2*Sin[dx k/2]^2);
Emat = {{LWFluxn , LWFluxun }, {2*dt*upsspatderivsn, 2*dt*upsspatderivsu}};
EmatEig = Eigenvalues[Emat + Exp[-I*(wAp)*dt] {{0, LWFluxunm1}, {0, 1}} ];
EmatEig = Series[ wAp - Log[ EmatEig] / (I*dt), {dx, 0, 2}, {dt, 0, 2}];
```

```
In[53]:= Text[Row[{"E00  ||   ", LWFluxn}]]
Text[Row[{"E00  ||   ", TeXForm[LWFluxn]}]]
Text[" "]
Text[Row[{"E01  ||   ", LWFluxun}]]
Text[Row[{"E01  ||   ", TeXForm[LWFluxun]}]]
Text[" "]
Text[Row[{"E03  ||   ", LWFluxunm1}]]
Text[Row[{"E03  ||   ", TeXForm[LWFluxunm1]}]]
Text[" "]
Text[Row[{"E10  ||   ", upsspatderivsn}]]
Text[Row[{"E10  ||   ", TeXForm[upsspatderivsn]}]]
Text[" "]
Text[Row[{"E11  ||   ", upsspatderivsu}]]
Text[Row[{"E11  ||   ", TeXForm[upsspatderivsu]}]]
Text[" "]
Text[Row[{"EmatEig  ||   ", EmatEig }]]
Text[Row[{"EmatEig  ||   ", TeXForm[EmatEig ]}]]
```

Out[53]= $\text{E00} \;||\; \frac{1}{2}\left(2+\text{dt}^2\left(-\frac{4\,U^2\,\text{Sin}\left[\frac{\text{dx}\,k}{2}\right]^2}{\text{dx}^2}+\frac{3\left(-1+e^{2\,i\,\text{dx}\,k}\right)g\,H}{6\,\text{dx}^2+8\,H^2\,\text{Sin}\left[\frac{\text{dx}\,k}{2}\right]^2}\right)-\frac{2\,i\,\text{dt}\,U\,\text{Sin}[\text{dx}\,k]}{\text{dx}}-\frac{3\,i\,\text{dt}^2\,e^{-i\,\text{dx}\,k}\,g\,H\,\text{Sin}[\text{dx}\,k]}{3\,\text{dx}^2+4\,H^2\,\text{Sin}\left[\frac{\text{dx}\,k}{2}\right]^2}\right)$

Out[54]= E00 || \frac{1}{2} \left(\text{dt}^2 \left(−\frac{4 U^2 \sin ^2\left(\frac{\text{dx} k}{2}\right)}{\text{dx}^2}+\frac{3 g H \left(−1+e^{2 i \text{dx} k}\right)}{6 \text{dx}^2+8 H^2 \sin ^2\left(\frac{\text{dx} k}{2}\right)}\right)−\frac{3 i \text{dt}^2 g H e^{−i \text{dx} k} \sin (\text{dx} k)}{3 \text{dx}^2+4 H^2 \sin ^2\left(\frac{\text{dx} k}{2}\right)}−\frac{2 i \text{dt} U \sin (\text{dx} k)}{\text{dx}}+2\right)

Out[55]=

Out[56]= $\text{E01} \;||\; \frac{\text{dt}\,H\,(\text{dt}\,U\,(-3+2\,\text{Cos}[\text{dx}\,k]+\text{Cos}[2\,\text{dx}\,k])-i\,\text{dx}\,\text{Sin}[\text{dx}\,k])}{2\,\text{dx}^2}$

Out[57]= E01 || \frac{\text{dt} H (\text{dt} U (2 \cos (\text{dx} k)+\cos (2 \text{dx} k)−3)−i \text{dx} \sin (\text{dx} k))}{2 \text{dx}^2}

Out[58]=

Out[59]= $\text{E03} \;||\; -\frac{i\,\text{dt}\,H\,\text{Sin}[\text{dx}\,k]}{2\,\text{dx}}$

Out[60]= E03 || −\frac{i \text{dt} H \sin (\text{dx} k)}{2 \text{dx}}

Out[61]=

Out[62]= $\text{E10} \;||\; -\frac{3\,i\,\text{dx}\,g\,\text{Sin}[\text{dx}\,k]}{3\,\text{dx}^2+4\,H^2\,\text{Sin}\left[\frac{\text{dx}\,k}{2}\right]^2}$

Out[63]= E10 || −\frac{3 i \text{dx} g \sin (\text{dx} k)}{3 \text{dx}^2+4 H^2 \sin ^2\left(\frac{\text{dx} k}{2}\right)}

Out[64]=

Out[65]= $\text{E11} \;||\; -\frac{i\,U\,\text{Sin}[\text{dx}\,k]}{\text{dx}}$

Out[66]= E11 || −\frac{i U \sin (\text{dx} k)}{\text{dx}}

Out[67]=

Out[68]=
$$
\begin{aligned}
\text{EmatEig} \;||\; \Bigg\{\Bigg(&-\frac{3\left(\sqrt{3}\,k\sqrt{g\,H\left(3+H^2k^2\right)}+i\sqrt{3}\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\right)}{2\left(3+H^2k^2\right)}+\\
&\frac{1}{2\left(3+H^2k^2\right)^2}i\Big(9\,g\,H\,k^2+3\,g\,H^3k^4+3\,i\,k\sqrt{g\,H\left(3+H^2k^2\right)}\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}-\\
&\quad 6\sqrt{3}\,k^2\sqrt{g\,H\left(3+H^2k^2\right)}\,U-2\sqrt{3}\,H^2k^4\sqrt{g\,H\left(3+H^2k^2\right)}\,U-\\
&\quad 6\,i\sqrt{3}\,k\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U-2\,i\sqrt{3}\,H^2k^3\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U\Big)\text{dt}+\\
&\frac{1}{36\,g\,H\left(3+H^2k^2\right)^2}\Big(54\sqrt{3}\,g^2H^2k^3\sqrt{g\,H\left(3+H^2k^2\right)}+48\,i\sqrt{3}\,g^2H^2k^2\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}-\\
&\quad 324\,g^2H^2k^3\,U-108\,g^2H^4k^5\,U-144\,i\,g\,H\,k^2\sqrt{g\,H\left(3+H^2k^2\right)}\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U+\\
&\quad 108\sqrt{3}\,g\,H\,k^3\sqrt{g\,H\left(3+H^2k^2\right)}\,U^2+36\sqrt{3}\,g\,H^3k^5\sqrt{g\,H\left(3+H^2k^2\right)}\,U^2+\\
&\quad 54\,i\sqrt{3}\,g\,H\,k^2\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U^2+18\,i\sqrt{3}\,g\,H^3k^4\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U^2+
\end{aligned}
$$

$$81\, g\, H\, k^3\, U^3 + 54\, g\, H^3\, k^5\, U^3 + 9\, g\, H^5\, k^7\, U^3 + 3\, i\, k^2 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^3 +$$

$$\left.\left. i\, H^2\, k^4 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^3\right) dt^2 + O[dt]^3\right) +$$

$$\left(\left(i\left(24\sqrt{3}\, g\, H\, k^4 + 6\sqrt{3}\, g\, H^3\, k^6 - 3\, k^4 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U - H^2\, k^6 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U - \right.\right.\right.$$

$$\left.\left. 27\, i\, k^3 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U - 9\, i\, H^2\, k^5 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U\right)\right) \Big/$$

$$\left(9\left(-12 - 4\, H^2\, k^2\right)\sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\right) + \frac{1}{432\left(3+H^2\, k^2\right)^2 \sqrt{-g\, H\, k^2\left(3+H^2\, k^2\right)}}$$

$$\left(-360\, g\, H\, k^5 \sqrt{g\, H\left(3 + H^2\, k^2\right)} - 90\, g\, H^3\, k^7 \sqrt{g\, H\left(3 + H^2\, k^2\right)} - 648\, i\, g\, H\, k^4 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)} - \right.$$

$$162\, i\, g\, H^3\, k^6 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)} + 1548\sqrt{3}\, g\, H\, k^5\, U + 1008\sqrt{3}\, g\, H^3\, k^7\, U +$$

$$164\sqrt{3}\, g\, H^5\, k^9\, U + 216\, i\sqrt{3}\, k^4 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U +$$

$$72\, i\sqrt{3}\, H^2\, k^6 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U - 279\, k^5 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U^2 -$$

$$186\, H^2\, k^7 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U^2 - 31\, H^4\, k^9 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U^2 - 891\, i\, k^4 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}$$

$$\left. U^2 - 594\, i\, H^2\, k^6 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^2 - 99\, i\, H^4\, k^8 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^2\right) dt +$$

$$\frac{1}{11\,664\left(3+H^2\, k^2\right)^3 \sqrt{-g\, H\, k^2\left(3+H^2\, k^2\right)}}\left(57\,024\, i\sqrt{3}\, g^2\, H^2\, k^6 + 33\,264\, i\sqrt{3}\, g^2\, H^4\, k^8 + \right.$$

$$4752\, i\sqrt{3}\, g^2\, H^6\, k^{10} - 17\,496\sqrt{3}\, g\, H\, k^5 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)} -$$

$$4374\sqrt{3}\, g\, H^3\, k^7 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)} - 168\,480\, i\, g\, H\, k^6 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U -$$

$$106\,650\, i\, g\, H^3\, k^8 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U - 16\,830\, i\, g\, H^5\, k^{10} \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U +$$

$$170\,586\, g\, H\, k^5 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U + 109\,350\, g\, H^3\, k^7 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U +$$

$$17\,496\, g\, H^5\, k^9 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U + 159\,894\, i\sqrt{3}\, g\, H\, k^6\, U^2 + 161\,676\, i\sqrt{3}\, g\, H^3\, k^8\, U^2 +$$

$$54\,486\, i\sqrt{3}\, g\, H^5\, k^{10}\, U^2 + 6120\, i\sqrt{3}\, g\, H^7\, k^{12}\, U^2 - 52\,488\sqrt{3}\, k^5 \sqrt{g\, H\left(3 + H^2\, k^2\right)}$$

$$\sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^2 - 34\,992\sqrt{3}\, H^2\, k^7 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^2 -$$

$$5832\sqrt{3}\, H^4\, k^9 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^2 - 29\,241\, i\, k^6 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U^3 -$$

$$29\,133\, i\, H^2\, k^8 \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U^3 - 9675\, i\, H^4\, k^{10} \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U^3 -$$

$$1071\, i\, H^6\, k^{12} \sqrt{g\, H\left(3 + H^2\, k^2\right)}\, U^3 + 32\,805\, k^5 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^3 +$$

$$32\,805\, H^2\, k^7 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^3 + 10\,935\, H^4\, k^9 \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^3 +$$

$$1215\, H^6\, k^{11} \sqrt{-g\, H\, k^2\left(3 + H^2\, k^2\right)}\, U^3 - 1296\, i\sqrt{3}\, k^6\, U^4 - 1728\, i\sqrt{3}\, H^2\, k^8\, U^4 -$$

$$\left.\left. 864\, i\sqrt{3}\, H^4\, k^{10}\, U^4 - 192\, i\sqrt{3}\, H^6\, k^{12}\, U^4 - 16\, i\sqrt{3}\, H^8\, k^{14}\, U^4\right) dt^2 + O[dt]^3\right) dx^2 +$$

$$O[dx]^3,\left(-\frac{3\left(\sqrt{3}\,k\sqrt{g\,H\left(3+H^2k^2\right)}-i\sqrt{3}\,\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\right)}{2\left(3+H^2k^2\right)}+\frac{1}{2\left(3+H^2k^2\right)^2}\right.$$

$$i\left(9\,g\,H\,k^2+3\,g\,H^3k^4-3\,i\,k\sqrt{g\,H\left(3+H^2k^2\right)}\,\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,-\right.$$
$$6\sqrt{3}\,k^2\sqrt{g\,H\left(3+H^2k^2\right)}\,U-2\sqrt{3}\,H^2k^4\sqrt{g\,H\left(3+H^2k^2\right)}\,U+$$
$$\left.6\,i\sqrt{3}\,k\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U+2\,i\sqrt{3}\,H^2k^3\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U\right)dt+$$

$$\frac{1}{36\,g\,H\left(3+H^2k^2\right)^2}\left(54\sqrt{3}\,g^2H^2k^3\sqrt{g\,H\left(3+H^2k^2\right)}-48\,i\sqrt{3}\,g^2H^2k^2\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,-\right.$$
$$324\,g^2H^2k^3U-108\,g^2H^4k^5U+144\,i\,g\,H\,k^2\sqrt{g\,H\left(3+H^2k^2\right)}\,\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U+$$
$$108\sqrt{3}\,g\,H\,k^3\sqrt{g\,H\left(3+H^2k^2\right)}\,U^2+36\sqrt{3}\,g\,H^3k^5\sqrt{g\,H\left(3+H^2k^2\right)}\,U^2-$$
$$54\,i\sqrt{3}\,g\,H\,k^2\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U^2-18\,i\sqrt{3}\,g\,H^3k^4\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U^2+$$
$$81\,g\,H\,k^3U^3+54\,g\,H^3k^5U^3+9\,g\,H^5k^7U^3-3\,i\,k^2\sqrt{g\,H\left(3+H^2k^2\right)}\,\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U^3-$$
$$\left.\left.i\,H^2k^4\sqrt{g\,H\left(3+H^2k^2\right)}\,\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U^3\right)dt^2+O[dt]^3\right)+$$

$$\left(-\left(\left(i\left(24\sqrt{3}\,g\,H\,k^4+6\sqrt{3}\,g\,H^3k^6-3\,k^4\sqrt{g\,H\left(3+H^2k^2\right)}\,U-H^2k^6\sqrt{g\,H\left(3+H^2k^2\right)}\,U+\right.\right.\right.\right.$$
$$\left.\left.27\,i\,k^3\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U+9\,i\,H^2k^5\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U\right)\right)\Big/$$
$$\left.\left(9\left(-12-4\,H^2k^2\right)\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\right)\right)+\frac{1}{432\left(3+H^2k^2\right)^2\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}}$$

$$\left(360\,g\,H\,k^5\sqrt{g\,H\left(3+H^2k^2\right)}+90\,g\,H^3k^7\sqrt{g\,H\left(3+H^2k^2\right)}-648\,i\,g\,H\,k^4\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,-\right.$$
$$162\,i\,g\,H^3k^6\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}-1548\sqrt{3}\,g\,H\,k^5U-1008\sqrt{3}\,g\,H^3k^7U-$$
$$164\sqrt{3}\,g\,H^5k^9U+216\,i\sqrt{3}\,k^4\sqrt{g\,H\left(3+H^2k^2\right)}\,\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U+$$
$$72\,i\sqrt{3}\,H^2k^6\sqrt{g\,H\left(3+H^2k^2\right)}\,\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U+279\,k^5\sqrt{g\,H\left(3+H^2k^2\right)}\,U^2+$$
$$186\,H^2k^7\sqrt{g\,H\left(3+H^2k^2\right)}\,U^2+31\,H^4k^9\sqrt{g\,H\left(3+H^2k^2\right)}\,U^2-891\,i\,k^4\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}$$
$$\left.U^2-594\,i\,H^2k^6\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U^2-99\,i\,H^4k^8\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U^2\right)dt+$$

$$\frac{1}{11\,664\left(3+H^2k^2\right)^3\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}}\left(-57\,024\,i\sqrt{3}\,g^2H^2k^6-33\,264\,i\sqrt{3}\,g^2H^4k^8-\right.$$
$$4752\,i\sqrt{3}\,g^2H^6k^{10}-17\,496\sqrt{3}\,g\,H\,k^5\sqrt{g\,H\left(3+H^2k^2\right)}\,\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,-$$
$$4374\sqrt{3}\,g\,H^3k^7\sqrt{g\,H\left(3+H^2k^2\right)}\,\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}+168\,480\,i\,g\,H\,k^6\sqrt{g\,H\left(3+H^2k^2\right)}\,U+$$
$$106\,650\,i\,g\,H^3k^8\sqrt{g\,H\left(3+H^2k^2\right)}\,U+16\,830\,i\,g\,H^5k^{10}\sqrt{g\,H\left(3+H^2k^2\right)}\,U+$$
$$170\,586\,g\,H\,k^5\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U+109\,350\,g\,H^3k^7\sqrt{-g\,H\,k^2\left(3+H^2k^2\right)}\,U+$$

$17496\,g\,H^5\,k^9\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U-159894\,i\,\sqrt{3}\;g\,H\,k^6\,U^2-161676\,i\,\sqrt{3}\;g\,H^3\,k^8\,U^2-$
$54486\,i\,\sqrt{3}\;g\,H^5\,k^{10}\,U^2-6120\,i\,\sqrt{3}\;g\,H^7\,k^{12}\,U^2-52488\,\sqrt{3}\;k^5\,\sqrt{g\,H\left(3+H^2\,k^2\right)}$
$\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^2-34992\,\sqrt{3}\;H^2\,k^7\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^2-$
$5832\,\sqrt{3}\;H^4\,k^9\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^2+29241\,i\,k^6\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+$
$29133\,i\,H^2\,k^8\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+9675\,i\,H^4\,k^{10}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+$
$1071\,i\,H^6\,k^{12}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+32805\,k^5\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3+$
$32805\,H^2\,k^7\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3+10935\,H^4\,k^9\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3+$
$1215\,H^6\,k^{11}\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3+1296\,i\,\sqrt{3}\;k^6\,U^4+1728\,i\,\sqrt{3}\;H^2\,k^8\,U^4+$
$864\,i\,\sqrt{3}\;H^4\,k^{10}\,U^4+192\,i\,\sqrt{3}\;H^6\,k^{12}\,U^4+16\,i\,\sqrt{3}\;H^8\,k^{14}\,U^4\Big)\,dt^2+O[dt]^3\Bigg)\,dx^2+O[dx]^3\Big\}$

Out[69]= EmatEig ||

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\left\{\left(-\frac{3 \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k+i \sqrt{3} \sqrt{-g H k^2 \left(H^2 k^2+3\right)}\right)}{2 \left(H^2 k^2+3\right)}+\frac{i \left(3 g H^3 k^4-2 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^4-2 i \sqrt{3} H^2 \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U k^3+9 g H k^2-6 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2-6 i \sqrt{3} \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U k+3 i \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{-g H k^2 \left(H^2 k^2+3\right)} k\right) \text{dt}}{2 \left(H^2 k^2+3\right)^2}+\frac{\left(9 g H^5 U^3 k^7+54 g H^3 U^3 k^5+36 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5-108 g^2 H^4 U k^5+i H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U^3 k^4+18 i \sqrt{3} g H^3 \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U^2 k^4+81 g H U^3 k^3+108 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^3-324 g^2 H^2 U k^3+54 \sqrt{3} g^2 H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^3+3 i \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U^3 k^2+54 i \sqrt{3} g H \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U^2 k^2-144 i g H \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U k^2+48 i \sqrt{3} g^2 H^2 \sqrt{-g H k^2 \left(H^2 k^2+3\right)} k^2\right) \text{dt}^2}{36 g H \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^3\right)\right)+\left(\frac{i \left(6 \sqrt{3} g H^3 k^6-H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^6-9 i H^2 \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U k^5+24 \sqrt{3} g H k^4-3 \sqrt{g H \left(H^2 k^2+3\right)} U k^4-27 i \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U k^3\right)}{9 \left(-4 H^2 k^2-12\right) \sqrt{-g H k^2 \left(H^2 k^2+3\right)}}+\frac{\left(-31 H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^9+164 \sqrt{3} g H^5 U k^9-99 i H^4 \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U^2 k^8-186 H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^7+1008 \sqrt{3} g H^3 U k^7-90 g H^3 \sqrt{g H \left(H^2 k^2+3\right)} k^7-594 i H^2 \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U^2 k^6+72 i \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U k^6-162 i g H^3 \sqrt{-g H k^2 \left(H^2 k^2+3\right)} k^6-279 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5+1548 \sqrt{3} g H U k^5-360 g H \sqrt{g H \left(H^2 k^2+3\right)} k^5-891 i \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U^2 k^4+216 i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U k^4-648 i g H \sqrt{-g H k^2 \left(H^2 k^2+3\right)} k^4\right) \text{dt}}{432 \left(H^2 k^2+3\right)^2 \sqrt{-g H k^2 \left(H^2 k^2+3\right)}}+\frac{\left(-16 i \sqrt{3} H^8 U^4 k^{14}-192 i \sqrt{3} H^6 U^4 k^{12}-1071 i H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^{12}+6120 i \sqrt{3} g H^7 U^2 k^{12}+1215 H^6 \sqrt{-g H k^2 \left(H^2 k^2+3\right)} U^3 k^{11}+4752 i \sqrt{3} g^2 H^6 k^{10}-864 i \sqrt{3} H^4 U^4 k^{10}-9675 i H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^{10}+54486 i \sqrt{3} g H^5 U^2
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H^4 U^4 k^{10}−9675 i H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^{10}+34486 i \sqrt{3} g H^5 U^2
k^{10}−16830 i g H^5 \sqrt{g H \left(H^2 k^2+3\right)} U k^{10}+10935 H^4 \sqrt{−g H k^2 \left(H^2
k^2+3\right)} U^3 k^9−5832 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2
k^2+3\right)} U^2 k^9+17496 g H^5 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^9+33264 i \sqrt{3} g^2
H^4 k^8−1728 i \sqrt{3} H^2 U^4 k^8−29133 i H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^8+161676
i \sqrt{3} g H^3 U^2 k^8−106650 i g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U k^8+32805 H^2 \sqrt{−g
H k^2 \left(H^2 k^2+3\right)} U^3 k^7−34992 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H
k^2 \left(H^2 k^2+3\right)} U^2 k^7+109350 g H^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^7−4374
\sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^7−1296 i \sqrt{3}
U^4 k^6−29241 i \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6+57024 i \sqrt{3} g^2 H^2 k^6+159894
i \sqrt{3} g H U^2 k^6−168480 i g H \sqrt{g H \left(H^2 k^2+3\right)} U k^6+32805 \sqrt{−g H k^2
\left(H^2 k^2+3\right)} U^3 k^5−52488 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2
k^2+3\right)} U^2 k^5+170586 g H \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^5−17496 \sqrt{3} g H
\sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^5\right) \text{dt}^2}{11664
\left(H^2 k^2+3\right)^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)}}+O\left(\text{dt}^3\right)\right)
\text{dx}^2+O\left(\text{dx}^3\right),\left(−\frac{3 \left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}−i
\sqrt{3} \sqrt{−g H k^2 \left(H^2 k^2+3\right)}\right)}{2 \left(H^2 k^2+3\right)}+\frac{i \left(3 g H^3
k^4−2 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^4+2 i \sqrt{3} H^2 \sqrt{−g H k^2 \left(H^2
k^2+3\right)} U k^3+9 g H k^2−6 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2+6 i \sqrt{3} \sqrt{−g
H k^2 \left(H^2 k^2+3\right)} U k−3 i \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2
k^2+3\right)} k\right) \text{dt}}{2 \left(H^2 k^2+3\right)^2}+\frac{\left(9 g H^5 U^3 k^7+54 g H^3 U^3
k^5+36 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5−108 g^2 H^4 U k^5−i H^2 \sqrt{g H
\left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^4−18 i \sqrt{3} g H^3 \sqrt{−g H
k^2 \left(H^2 k^2+3\right)} U^2 k^4+81 g H U^3 k^3+108 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)}
U^2 k^3−324 g^2 H^2 U k^3+54 \sqrt{3} g^2 H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^3−3 i \sqrt{g H
\left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^2−54 i \sqrt{3} g H \sqrt{−g H k^2
\left(H^2 k^2+3\right)} U^2 k^2+144 i g H \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2
k^2+3\right)} U k^2−48 i \sqrt{3} g^2 H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^2\right) \text{dt}^2}{36
g H \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^3\right)\right)+\left(−\frac{i \left(6 \sqrt{3} g H^3 k^6−H^2
\sqrt{g H \left(H^2 k^2+3\right)} U k^6+9 i H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^5+24
\sqrt{3} g H k^4−3 \sqrt{g H \left(H^2 k^2+3\right)} U k^4+27 i \sqrt{−g H k^2 \left(H^2 k^2+3\right)}
U k^3\right)}{9 \left(−4 H^2 k^2−12\right) \sqrt{−g H k^2 \left(H^2 k^2+3\right)}}+\frac{\left(31 H^4
\sqrt{g H \left(H^2 k^2+3\right)} U^2 k^9−164 \sqrt{3} g H^5 U k^9−99 i H^4 \sqrt{−g H k^2 \left(H^2
k^2+3\right)} U^2 k^8+186 H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^7−1008 \sqrt{3} g H^3 U
k^7+90 g H^3 \sqrt{g H \left(H^2 k^2+3\right)} k^7−594 i H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)}
U^2 k^6+72 i \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)}
U k^6−162 i g H^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^6+279 \sqrt{g H \left(H^2 k^2+3\right)}
U^2 k^5−1548 \sqrt{3} g H U k^5+360 g H \sqrt{g H \left(H^2 k^2+3\right)} k^5−891 i \sqrt{−g H
k^2 \left(H^2 k^2+3\right)} U^2 k^4+216 i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2
\left(H^2 k^2+3\right)} U k^4−648 i g H \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^4\right) \text{dt}}{432
\left(H^2 k^2+3\right)^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)}}+\frac{\left(16 i \sqrt{3} H^8 U^4
k^{14}+192 i \sqrt{3} H^6 U^4 k^{12}+1071 i H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^{12}−6120
i \sqrt{3} g H^7 U^2 k^{12}+1215 H^6 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^{11}−4752 i
\sqrt{3} g^2 H^6 k^{10}+864 i \sqrt{3} H^4 U^4 k^{10}+9675 i H^4 \sqrt{g H \left(H^2 k^2+3\right)}

[illegible]
U^3 k^{10}−54486 i \sqrt{3} g H^5 U^2 k^{10}+16830 i g H^5 \sqrt{g H \left(H^2 k^2+3\right)} U
k^{10}+10935 H^4 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^9−5832 \sqrt{3} H^4 \sqrt{g H \left(H^2
k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^9+17496 g H^5 \sqrt{−g H k^2 \left(H^2
k^2+3\right)} U k^9−33264 i \sqrt{3} g^2 H^4 k^8+1728 i \sqrt{3} H^2 U^4 k^8+29133 i H^2 \sqrt{g H
\left(H^2 k^2+3\right)} U^3 k^8−161676 i \sqrt{3} g H^3 U^2 k^8+106650 i g H^3 \sqrt{g H \left(H^2
k^2+3\right)} U k^8+32805 H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^7−34992 \sqrt{3}
H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^7+109350 g H^3
\sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^7−4374 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)}
\sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^7+1296 i \sqrt{3} U^4 k^6+29241 i \sqrt{g H \left(H^2
k^2+3\right)} U^3 k^6−57024 i \sqrt{3} g^2 H^2 k^6−159894 i \sqrt{3} g H U^2 k^6+168480 i g H
\sqrt{g H \left(H^2 k^2+3\right)} U k^6+32805 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^5−52488
\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^5+170586 g
H \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^5−17496 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)}
\sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^5\right) \text{dt}^2}{11664 \left(H^2 k^2+3\right)^3 \sqrt{−g H
k^2 \left(H^2 k^2+3\right)}}+O\left(\text{dt}^3\right)\right) \text{dx}^2+O\left(\text{dx}^3\right)\right\}