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In[1]:= q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx/2, xj + dx/2}] / (dx);
qjnbar = qjbar /. t -> tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x -> xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / 3 * k^2;
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca;
fn1Gca0A = fn1A /. Gca -> 0;
fn1eca0A = fn1A /. eca -> 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca /. Gh -> RA * Gca;
fG1Gca0A = fG1A /. Gca -> 0;
fG1eca0A = fG1A /. eca -> 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};

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$$\mathbf{wAp} = \mathbf{U} * \mathbf{k} + \frac{\sqrt{3} \, \mathbf{k} \sqrt{\mathbf{g} \, \mathbf{H} \left(3 + \mathbf{H}^2 \, \mathbf{k}^2 \right)}}{3 + \mathbf{H}^2 \, \mathbf{k}^2};$$

$$\mathbf{wAm} = \mathbf{U} * \mathbf{k} - \frac{\sqrt{3} \, \mathbf{k} \sqrt{\mathbf{g} \, \mathbf{H} \left(3 + \mathbf{H}^2 \, \mathbf{k}^2 \right)}}{3 + \mathbf{H}^2 \, \mathbf{k}^2};$$

In[36]:= **M = 1**

Series[M - MA, {dx, 0, 10}]

Out[36]= 1

$$\text{Out[37]} = -\frac{k^2 dx^2}{24} - \frac{7 k^4 dx^4}{5760} - \frac{31 k^6 dx^6}{967680} - \frac{127 k^8 dx^8}{154828800} - \frac{73 k^{10} dx^{10}}{3503554560} + O[dx]^{11}$$

In[38]:= **Rm = 1**

Series[Rm - RA, {dx, 0, 10}]

Rp = Exp[I * k * dx]

Series[Rp - RA, {dx, 0, 10}]

Ru = (1 + Exp[I * k * dx]) / 2

Series[Ru - Exp[I * k * dx / 2], {dx, 0, 10}]

Out[38]= 1

$$\text{Out[39]} = -\frac{1}{2} i k dx + \frac{k^2 dx^2}{12} + \frac{k^4 dx^4}{720} + \frac{k^6 dx^6}{30240} + \frac{k^8 dx^8}{1209600} + \frac{k^{10} dx^{10}}{47900160} + O[dx]^{11}$$

Out[40]= $e^{i dx k}$

$$\text{Out[41]} = \frac{i k dx}{2} - \frac{5 k^2 dx^2}{12} - \frac{1}{6} i k^3 dx^3 + \frac{31 k^4 dx^4}{720} + \frac{1}{120} i k^5 dx^5 - \frac{41 k^6 dx^6}{30240} - \frac{i k^7 dx^7}{5040} + \frac{31 k^8 dx^8}{1209600} + \frac{i k^9 dx^9}{362880} - \frac{61 k^{10} dx^{10}}{239500800} + O[dx]^{11}$$

$$\text{Out[42]} = \frac{1}{2} (1 + e^{i dx k})$$

$$\text{Out[43]} = -\frac{k^2 dx^2}{8} - \frac{1}{16} i k^3 dx^3 + \frac{7 k^4 dx^4}{384} + \frac{1}{256} i k^5 dx^5 - \frac{31 k^6 dx^6}{46080} - \frac{i k^7 dx^7}{10240} + \frac{127 k^8 dx^8}{10321920} + \frac{17 i k^9 dx^9}{12386304} - \frac{73 k^{10} dx^{10}}{530841600} + O[dx]^{11}$$

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In[44]:= Gold = H - H^3 / (3 * (2 * Cos[k * dx] - 2)) / dx^2
GG2 = Ru / Gold
Series[GG2 - GGA, {dx, 0, 5}]
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Gn2 = -U * Ru / Gold
Series[Gn2 - GnA, {dx, 0, 5}]
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$$\text{Out[44]} = H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2}$$

$$\text{Out[45]} = \frac{1 + e^{i dx k}}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2} \right)}$$

$$\text{Out[46]} = \frac{(-6 k^2 - H^2 k^4) dx^2}{4 H (3 + H^2 k^2)^2} - \frac{i (6 k^3 + H^2 k^5) dx^3}{8 H (3 + H^2 k^2)^2} + \frac{(144 k^4 + 45 H^2 k^6 + 4 H^4 k^8) dx^4}{240 H (3 + H^2 k^2)^3} - \frac{i (-54 k^5 + H^4 k^9) dx^5}{480 H (3 + H^2 k^2)^3} + O[dx]^6$$

$$\text{Out[47]} = - \frac{(1 + e^{i dx k}) U}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2} \right)}$$

$$\text{Out[48]} = \frac{(6 k^2 + H^2 k^4) U dx^2}{4 H (3 + H^2 k^2)^2} + \frac{i (6 k^3 + H^2 k^5) U dx^3}{8 H (3 + H^2 k^2)^2} - \frac{((144 k^4 + 45 H^2 k^6 + 4 H^4 k^8) U) dx^4}{240 (H (3 + H^2 k^2)^3)} + \frac{i (-54 k^5 + H^4 k^9) U dx^5}{480 H (3 + H^2 k^2)^3} + O[dx]^6$$

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In[49]:= fnn2 = H*Gn2 + U/2*(Rm + Rp) - (Sqrt[g*H])/(2)*(Rp - Rm);
Fnn2 = -dt*(1 - Exp[-I*k*dx])/dx*fnn2
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 2}];
Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}]
fnG2 = H*GG2;
FnG2 = -dt*(1 - Exp[-I*k*dx])/dx*fnG2
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 2}];
Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}]

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$$\text{Out[50]} = -\frac{1}{dx} dt \left(1 - e^{-i dx k} \right) \left(-\frac{1}{2} (-1 + e^{i dx k}) \sqrt{g H} + \frac{1}{2} (1 + e^{i dx k}) U - \frac{(1 + e^{i dx k}) H U}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2} \right)} \right)$$

$$\begin{aligned} \text{Out[52]} = & \left(-\frac{(H^2 k^3 U w) dt^2}{2 (3 + H^2 k^2)} + O[dt]^3 \right) + \left(-\frac{1}{2} (\sqrt{g H} k^2) dt + O[dt]^3 \right) dx + \\ & \left(\frac{i (9 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3 + H^2 k^2)^2} + O[dt]^3 \right) dx^2 + \left(\frac{1}{24} \sqrt{g H} k^4 dt + O[dt]^3 \right) dx^3 + \\ & \left(-\frac{i k^7 (54 H^2 U + 19 H^4 k^2 U + 2 H^6 k^4 U) dt}{240 (3 + H^2 k^2)^3} + O[dt]^3 \right) dx^4 + O[dx]^5 \end{aligned}$$

$$\text{Out[54]} = -\frac{dt (1 - e^{-i dx k}) (1 + e^{i dx k}) H}{2 dx \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2} \right)}$$

$$\begin{aligned} \text{Out[56]} = & \left(-\frac{3 (k w) dt^2}{2 (3 + H^2 k^2)} + O[dt]^3 \right) + \left(\frac{i (6 k^3 + H^2 k^5) dt}{4 (3 + H^2 k^2)^2} + O[dt]^3 \right) dx^2 + \\ & \left(\frac{i (-54 k^5 + H^4 k^9) dt}{240 (3 + H^2 k^2)^3} + O[dt]^3 \right) dx^4 + O[dx]^5 \end{aligned}$$

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In[57]:= fGn2 = U * H * Gn2 + g * H * (Rm + Rp) / 2 + (U * Sqrt[g * H]) / (2) * (Rm - Rp);
FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * fGn2
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 2}];
Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * fGG2
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 2}];
Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}]

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$$\text{Out[58]} = - \frac{dt \left(1 - e^{-i dx k}\right) \left(\frac{1}{2} \left(1 + e^{i dx k}\right) g H + \frac{1}{2} \left(1 - e^{i dx k}\right) \sqrt{g H} U - \frac{(1 + e^{i dx k}) H U^2}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2}\right)}\right)}{dx}$$

$$\begin{aligned} \text{Out[60]} = & \left(- \frac{k \left(3 g H + g H^3 k^2 - 3 U^2\right) w}{2 \left(3 + H^2 k^2\right)} dt^2 + O[dt]^3 \right) + \left(- \frac{1}{2} \left(\sqrt{g H} k^2 U\right) dt + O[dt]^3 \right) dx + \\ & \left(\frac{i \left(18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2\right) dt}{12 \left(3 + H^2 k^2\right)^2} + O[dt]^3 \right) dx^2 + \\ & \left(\frac{1}{24} \sqrt{g H} k^4 U dt + O[dt]^3 \right) dx^3 + \\ & \left(- \frac{i \left(54 g H k^5 + 54 g H^3 k^7 + 18 g H^5 k^9 + 2 g H^7 k^{11} - 54 k^5 U^2 + H^4 k^9 U^2\right) dt}{240 \left(3 + H^2 k^2\right)^3} + O[dt]^3 \right) dx^4 + O[dx]^5 \end{aligned}$$

$$\text{Out[62]} = - \frac{dt \left(1 - e^{-i dx k}\right) \left(-\frac{1}{2} \left(-1 + e^{i dx k}\right) \sqrt{g H} + \frac{1}{2} \left(1 + e^{i dx k}\right) U + \frac{(1 + e^{i dx k}) H U}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2}\right)}\right)}{dx}$$

$$\begin{aligned} \text{Out[64]} = & \left(- \frac{k \left(6 + H^2 k^2\right) U w}{2 \left(3 + H^2 k^2\right)} dt^2 + O[dt]^3 \right) + \left(- \frac{1}{2} \left(\sqrt{g H} k^2\right) dt + O[dt]^3 \right) dx + \\ & \left(\frac{i \left(36 k^3 U + 15 H^2 k^5 U + 2 H^4 k^7 U\right) dt}{12 \left(3 + H^2 k^2\right)^2} + O[dt]^3 \right) dx^2 + \left(\frac{1}{24} \sqrt{g H} k^4 dt + O[dt]^3 \right) dx^3 + \\ & \left(- \frac{i \left(108 k^5 U + 54 H^2 k^7 U + 17 H^4 k^9 U + 2 H^6 k^{11} U\right) dt}{240 \left(3 + H^2 k^2\right)^3} + O[dt]^3 \right) dx^4 + O[dx]^5 \end{aligned}$$

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In[65]:= Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

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RKStep = Log[1 + EigvFmat2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}]

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$$\text{Out[69]} = \left\{ \frac{i \left(\sqrt{3} k \sqrt{g H \left(3 + H^2 k^2\right)} + 3 k U + H^2 k^3 U\right)^2 dt}{2 \left(3 + H^2 k^2\right)^2} - \right.$$

$$\begin{aligned}
& \frac{1}{3 (3 + H^2 k^2)^2} \left(k^3 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \quad \left. \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right) dt^2 - \frac{1}{4 (3 + H^2 k^2)^3} \\
& \quad i k^4 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left(3 g \left(\sqrt{3} H \sqrt{g H (3 + H^2 k^2)} + 9 H U + 3 H^3 k^2 U \right) + \right. \\
& \quad \left. U^2 \left(H^4 k^4 U + 9 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + U \right) + 3 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 2 H^2 U \right) \right) \right) dt^3 + \\
& \quad \frac{1}{5 (3 + H^2 k^2)^3} k^5 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \quad \left(9 g^2 H^2 + 6 g H U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \quad \left. U^3 \left(12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + 2 k^2 \left(2 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 3 H^2 U \right) \right) \right) dt^4 + \\
& \quad O[dt]^5 \Bigg) + \left(-\frac{1}{4} i k^2 \left(2 \sqrt{g H} + \frac{\sqrt{3} U}{\sqrt{3 + H^2 k^2}} \right) + \frac{1}{4 (3 + H^2 k^2)^{3/2}} \right. \\
& \quad k^3 \left(2 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} U \right) \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) dt + \\
& \quad \frac{1}{4 (3 + H^2 k^2)^{3/2}} i k^4 \left(2 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} U \right) \\
& \quad \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) dt^2 - \frac{1}{4 (3 + H^2 k^2)^{5/2}} \\
& \quad \left(k^5 \left(2 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} U \right) \left(3 g \left(\sqrt{3} H \sqrt{g H (3 + H^2 k^2)} + 9 H U + 3 H^3 k^2 U \right) + \right. \right. \\
& \quad \left. \left. U^2 \left(H^4 k^4 U + 9 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + U \right) + 3 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 2 H^2 U \right) \right) \right) \right) \\
& \quad dt^3 - \frac{1}{4 (3 + H^2 k^2)^{5/2}} i k^6 \left(2 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} U \right) \\
& \quad \left(9 g^2 H^2 + 6 g H U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + U^3 \left(12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + \right. \right. \\
& \quad \left. \left. 9 U + H^4 k^4 U + 2 k^2 \left(2 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 3 H^2 U \right) \right) \right) dt^4 + O[dt]^5 \Bigg) dx + \\
& \quad \left(- \left(\left(k^3 \left(12 \sqrt{3} g H (4 + H^2 k^2) + U \left(48 \sqrt{g H (3 + H^2 k^2)} - 9 \sqrt{3} U + k^2 \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left(16 \sqrt{g H^5 (3 + H^2 k^2)} - 3 \sqrt{3} H^2 U \right) \right) \right) \right) / \left(96 \left(\sqrt{g H} (3 + H^2 k^2)^{3/2} \right) \right) \right) - \\
& \quad \left(i k^4 \left(9 U^2 \left(16 \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} U \right) + k^4 U^2 \left(16 \sqrt{g H^9 (3 + H^2 k^2)} - 3 \sqrt{3} H^4 U \right) + \right. \right. \\
& \quad \left. \left. 6 k^2 \left(18 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 - 3 \sqrt{3} H^2 U^3 \right) + \right. \right. \\
& \quad \left. \left. 4 g H \left(63 \sqrt{g H (3 + H^2 k^2)} + 99 \sqrt{3} U + 63 \sqrt{3} H^2 k^2 U + \right. \right. \right. \\
& \quad \left. \left. \left. k^4 \left(3 \sqrt{g H^9 (3 + H^2 k^2)} + 10 \sqrt{3} H^4 U \right) \right) \right) \right) / \left(96 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{5/2}} k^5 \left(12 \sqrt{3} g^2 H^2 (30 + 15 H^2 k^2 + 2 H^4 k^4) + \right. \\
& 9 U^3 \left(16 \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} U \right) + k^4 U^3 \left(16 \sqrt{g H^9 (3 + H^2 k^2)} - 3 \sqrt{3} H^4 U \right) + \\
& 6 k^2 \left(56 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^3 - 3 \sqrt{3} H^2 U^4 \right) + \\
& g H U \left(864 \sqrt{g H (3 + H^2 k^2)} + 675 \sqrt{3} U + 429 \sqrt{3} H^2 k^2 U + \right. \\
& \left. 4 k^4 \left(6 \sqrt{g H^9 (3 + H^2 k^2)} + 17 \sqrt{3} H^4 U \right) \right) \Big) dt^2 + \frac{1}{96 \sqrt{g H} (3 + H^2 k^2)^{7/2}} \\
& i k^6 \left(9 k^2 \left(84 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 404 \sqrt{3} g^2 H^4 U + 438 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \right. \\
& \left. 314 \sqrt{3} g H^3 U^3 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 9 \sqrt{3} H^2 U^5 \right) + \\
& 27 \left(52 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 172 \sqrt{3} g^2 H^2 U + 16 \sqrt{g H (3 + H^2 k^2)} U^4 - \right. \\
& \left. 3 \sqrt{3} U^5 + g H U^2 \left(210 \sqrt{g H (3 + H^2 k^2)} + 109 \sqrt{3} U \right) \right) + \\
& 3 k^4 \left(36 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 304 \sqrt{3} g^2 H^6 U + 48 \sqrt{g H^9 (3 + H^2 k^2)} U^4 - \right. \\
& 9 \sqrt{3} H^4 U^5 + g H^5 U^2 \left(264 \sqrt{g H (3 + H^2 k^2)} + 301 \sqrt{3} U \right) \Big) + k^6 U \left(72 \sqrt{3} g^2 H^8 + 12 g \right. \\
& \left. H^7 U \left(3 \sqrt{g H (3 + H^2 k^2)} + 8 \sqrt{3} U \right) + U^3 \left(16 \sqrt{g H^{13} (3 + H^2 k^2)} - 3 \sqrt{3} H^6 U \right) \right) \Big) dt^3 - \\
& \frac{1}{96 \left(\sqrt{g H} (3 + H^2 k^2)^{7/2} \right)} \left(k^7 \left(36 \sqrt{3} g^3 H^3 (48 + 27 H^2 k^2 + 4 H^4 k^4) + 3 \sqrt{3} g^2 H^2 \right. \right. \\
& \left(4113 + 3075 H^2 k^2 + 712 H^4 k^4 + 48 H^6 k^6 \right) U^2 + 2 g H U^3 \left(5076 \sqrt{g H (3 + H^2 k^2)} + \right. \\
& 1971 \sqrt{3} U + 1872 \sqrt{3} H^2 k^2 U + 3 k^4 \left(216 \sqrt{g H^9 (3 + H^2 k^2)} + 197 \sqrt{3} H^4 U \right) + \\
& 2 k^6 \left(12 \sqrt{g H^{13} (3 + H^2 k^2)} + 31 \sqrt{3} H^6 U \right) \Big) + \\
& U \left(7344 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 432 \sqrt{g H (3 + H^2 k^2)} U^4 - 81 \sqrt{3} U^5 + \right. \\
& k^6 U^4 \left(16 \sqrt{g H^{13} (3 + H^2 k^2)} - 3 \sqrt{3} H^6 U \right) + 9 k^2 \left(400 \sqrt{g^5 H^9 (3 + H^2 k^2)} + \right. \\
& 760 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 9 \sqrt{3} H^2 U^5 \Big) + \\
& \left. \left. 9 k^4 \left(48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 16 \sqrt{g H^9 (3 + H^2 k^2)} U^4 - 3 \sqrt{3} H^4 U^5 \right) \right) \right) dt^4 + \\
& O[dt]^5 \Big) dx^2 + \left(\left(i k^4 \left(3 \sqrt{3} (3 + H^2 k^2) U^3 + 4 g H \left(12 \sqrt{g H (3 + H^2 k^2)} + 6 \sqrt{3} U + \right. \right. \right. \right. \\
& \left. \left. \left. k^2 \left(4 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \right) \right) \right) \Big) / \left(384 g H (3 + H^2 k^2)^{3/2} \right) - \\
& \left(\left(k^5 \left(8 \sqrt{3} g^2 H^2 (54 + 33 H^2 k^2 + 5 H^4 k^4) + 3 \sqrt{3} (3 + H^2 k^2)^2 U^4 + \right. \right. \right. \\
& 2 g H U \left(81 \left(4 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} U \right) + 24 k^2 \left(7 \sqrt{g H^5 (3 + H^2 k^2)} + 2 \sqrt{3} H^2 U \right) + \right. \\
& \left. k^4 \left(24 \sqrt{g H^9 (3 + H^2 k^2)} + 7 \sqrt{3} H^4 U \right) \right) \Big) dt \Big) / \left(384 (g H (3 + H^2 k^2)^{5/2}) \right) - \\
& \frac{1}{384 g H (3 + H^2 k^2)^{5/2}} i k^6 \left(3 k^2 \left(96 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 380 \sqrt{3} g^2 H^4 U + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 224 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 51 \sqrt{3} g H^3 U^3 + 6 \sqrt{3} H^2 U^5 \Big) + \\
& k^4 \Big(16 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 168 \sqrt{3} g^2 H^6 U + 3 \sqrt{3} H^4 U^5 + \\
& 8 g H^5 U^2 \Big(10 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U \Big) \Big) + 9 \Big(96 \sqrt{g^5 H^5 (3 + H^2 k^2)} + \\
& 224 \sqrt{3} g^2 H^2 U + 3 \sqrt{3} U^5 + g H U^2 \Big(160 \sqrt{g H (3 + H^2 k^2)} + 27 \sqrt{3} U \Big) \Big) \Big) dt^2 + \\
& \frac{1}{384 g H (3 + H^2 k^2)^{7/2}} k^7 \Big(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \Big) \\
& \Big(3 k^2 \Big(184 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 608 \sqrt{3} g^2 H^4 U + 334 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \\
& 72 \sqrt{3} g H^3 U^3 - 3 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 6 \sqrt{3} H^2 U^5 \Big) + k^4 \Big(48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + \\
& 272 \sqrt{3} g^2 H^6 U + 3 \sqrt{3} H^4 U^5 + 2 g H^5 U^2 \Big(56 \sqrt{g H (3 + H^2 k^2)} + 17 \sqrt{3} U \Big) \Big) \Big) + \\
& 9 \Big(160 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 352 \sqrt{3} g^2 H^2 U - 3 \sqrt{g H (3 + H^2 k^2)} U^4 + \\
& 3 \sqrt{3} U^5 + 2 g H U^2 \Big(121 \sqrt{g H (3 + H^2 k^2)} + 19 \sqrt{3} U \Big) \Big) \Big) dt^3 + \\
& \frac{1}{384 g H (3 + H^2 k^2)^{9/2}} i k^8 \Big(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \Big)^2 \\
& \Big(3 k^2 \Big(304 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 884 \sqrt{3} g^2 H^4 U + 456 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \\
& 95 \sqrt{3} g H^3 U^3 - 6 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 6 \sqrt{3} H^2 U^5 \Big) + k^4 \Big(96 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + \\
& 400 \sqrt{3} g^2 H^6 U + 3 \sqrt{3} H^4 U^5 + 4 g H^5 U^2 \Big(36 \sqrt{g H (3 + H^2 k^2)} + 11 \sqrt{3} U \Big) \Big) \Big) + \\
& 27 \Big(80 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 168 \sqrt{3} g^2 H^2 U - 2 \sqrt{g H (3 + H^2 k^2)} U^4 + \sqrt{3} U^5 + \\
& g H U^2 \Big(112 \sqrt{g H (3 + H^2 k^2)} + 17 \sqrt{3} U \Big) \Big) \Big) dt^4 + O[dt]^5 \Big) dx^3 + \\
& \Big(\Big(k^5 \Big(16 \sqrt{3} g^2 H^2 (144 + 48 H^2 k^2 + 5 H^4 k^4) - 75 \sqrt{3} (3 + H^2 k^2)^2 U^4 + \\
& 8 g H (3 + H^2 k^2) U \Big(96 \sqrt{g H (3 + H^2 k^2)} + k^2 \Big(32 \sqrt{g H^5 (3 + H^2 k^2)} + 5 \sqrt{3} H^2 U \Big) \Big) \Big) \Big) \Big) / \\
& \Big(30720 (g H)^{3/2} (3 + H^2 k^2)^{5/2} \Big) + \frac{1}{92160 (g H)^{3/2} (3 + H^2 k^2)^{7/2}} \\
& i k^6 \Big(9 k^2 \Big(8448 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 16160 \sqrt{3} g^2 H^4 U + 6144 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \\
& 240 \sqrt{3} g H^3 U^3 - 675 \sqrt{3} H^2 U^5 \Big) + k^6 \Big(1920 \sqrt{g^5 H^{17} (3 + H^2 k^2)} + 4368 \sqrt{3} g^2 H^8 U + \\
& 2048 g H^7 \sqrt{g H (3 + H^2 k^2)} U^2 - 225 \sqrt{3} H^6 U^5 \Big) + 27 \Big(3968 \sqrt{g^5 H^5 (3 + H^2 k^2)} + \\
& 6016 \sqrt{3} g^2 H^2 U - 225 \sqrt{3} U^5 + 8 g H U^2 \Big(256 \sqrt{g H (3 + H^2 k^2)} - 15 \sqrt{3} U \Big) \Big) \Big) + \\
& 3 k^4 \Big(6720 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 14512 \sqrt{3} g^2 H^6 U - 675 \sqrt{3} H^4 U^5 + \\
& 24 g H^5 U^2 \Big(256 \sqrt{g H (3 + H^2 k^2)} - 5 \sqrt{3} U \Big) \Big) \Big) dt - \frac{1}{92160 ((g H)^{3/2} (3 + H^2 k^2)^{7/2})}
\end{aligned}$$

$$\begin{aligned}
& \left(k^7 \left(48 \sqrt{3} g^3 H^3 (6192 + 5004 H^2 k^2 + 1425 H^4 k^4 + 140 H^6 k^6) + \right. \right. \\
& \quad 8 \sqrt{3} g^2 H^2 (60426 + 55557 H^2 k^2 + 17121 H^4 k^4 + 1772 H^6 k^6) U^2 + \\
& \quad g H U^3 \left(89856 \sqrt{g H (3 + H^2 k^2)} - 5265 \sqrt{3} U - 4590 \sqrt{3} H^2 k^2 U + \right. \\
& \quad \quad 9 k^4 \left(3328 \sqrt{g H^9 (3 + H^2 k^2)} - 145 \sqrt{3} H^4 U \right) + 8 k^6 \left(416 \sqrt{g H^{13} (3 + H^2 k^2)} - \right. \\
& \quad \quad \quad 15 \sqrt{3} H^6 U \left. \right) \left. \right) + 3 U \left(81 \left(2816 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 25 \sqrt{3} U^5 \right) + \right. \\
& \quad \quad 3 k^2 \left(52352 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 9984 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - 675 \sqrt{3} H^2 U^5 \right) + \\
& \quad \quad 3 k^4 \left(11776 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 225 \sqrt{3} H^4 U^5 \right) + 5 k^6 \\
& \quad \quad \left. \left. \left(512 \sqrt{g^5 H^{17} (3 + H^2 k^2)} - 15 \sqrt{3} H^6 U^5 \right) \right) \right) dt^2 - \frac{1}{30720 (g H)^{3/2} (3 + H^2 k^2)^{9/2}} \\
& \quad i k^8 \left(27 k^2 \left(20416 \sqrt{g^7 H^{11} (3 + H^2 k^2)} + 80544 \sqrt{3} g^3 H^5 U + 82240 \sqrt{g^5 H^9 (3 + H^2 k^2)} U^2 + \right. \right. \\
& \quad \quad 46960 \sqrt{3} g^2 H^4 U^3 + 6144 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^4 - 305 \sqrt{3} g H^3 U^5 - 300 \sqrt{3} H^2 U^7 \left. \right) + \\
& \quad \quad 9 k^4 \left(19040 \sqrt{g^7 H^{15} (3 + H^2 k^2)} + 99856 \sqrt{3} g^3 H^7 U + 88512 \sqrt{g^5 H^{13} (3 + H^2 k^2)} U^2 + \right. \\
& \quad \quad 66296 \sqrt{3} g^2 H^6 U^3 - 450 \sqrt{3} H^4 U^7 + 3 g H^5 U^4 \left(3072 \sqrt{g H (3 + H^2 k^2)} - 155 \sqrt{3} U \right) \left. \right) + \\
& \quad \quad 3 k^6 \left(6720 \sqrt{g^7 H^{19} (3 + H^2 k^2)} + 55216 \sqrt{3} g^3 H^9 U + 39488 \sqrt{g^5 H^{17} (3 + H^2 k^2)} U^2 + \right. \\
& \quad \quad 41728 \sqrt{3} g^2 H^8 U^3 - 300 \sqrt{3} H^6 U^7 + 3 g H^7 U^4 \left(2048 \sqrt{g H (3 + H^2 k^2)} - 105 \sqrt{3} U \right) \left. \right) + \\
& \quad \quad 243 \left(2592 \sqrt{g^7 H^7 (3 + H^2 k^2)} + 8128 \sqrt{3} g^3 H^3 U + 9152 \sqrt{g^5 H^5 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \quad 4168 \sqrt{3} g^2 H^2 U^3 - 25 \sqrt{3} U^7 + g H U^4 \left(512 \sqrt{g H (3 + H^2 k^2)} - 25 \sqrt{3} U \right) \left. \right) + \\
& \quad \quad k^8 \left(480 \sqrt{g^7 H^{23} (3 + H^2 k^2)} + 11520 \sqrt{3} g^3 H^{11} U + 5760 \sqrt{g^5 H^{21} (3 + H^2 k^2)} U^2 + \right. \\
& \quad \quad 9888 \sqrt{3} g^2 H^{10} U^3 - 75 \sqrt{3} H^8 U^7 + \\
& \quad \quad \left. \left. 16 g H^9 U^4 \left(96 \sqrt{g H (3 + H^2 k^2)} - 5 \sqrt{3} U \right) \right) \right) dt^3 + \frac{1}{92160 (g H)^{3/2} (3 + H^2 k^2)^{9/2}} \\
& \quad k^9 \left(144 \sqrt{3} g^4 H^4 (23832 + 22104 H^2 k^2 + 7395 H^4 k^4 + 1000 H^6 k^6 + 40 H^8 k^8) + \right. \\
& \quad \quad 24 \sqrt{3} g^3 H^3 (893700 + 953451 H^2 k^2 + 376029 H^4 k^4 + 64744 H^6 k^6 + 4080 H^8 k^8) U^2 + \\
& \quad \quad 3 \sqrt{3} g^2 H^2 (3 + H^2 k^2)^2 (197625 + 114352 H^2 k^2 + 16944 H^4 k^4) U^4 + \\
& \quad \quad 2 g H U^5 \left(238464 \sqrt{g H (3 + H^2 k^2)} - 8505 \sqrt{3} U - 13365 \sqrt{3} H^2 k^2 U + \right. \\
& \quad \quad 27 k^4 \left(5888 \sqrt{g H^9 (3 + H^2 k^2)} - 285 \sqrt{3} H^4 U \right) + 3 k^6 \left(11776 \sqrt{g H^{13} (3 + H^2 k^2)} - \right. \\
& \quad \quad \quad 645 \sqrt{3} H^6 U \left. \right) + 4 k^8 \left(736 \sqrt{g H^{17} (3 + H^2 k^2)} - 45 \sqrt{3} H^8 U \right) \left. \right) + \\
& \quad \quad 3 U \left(135 \left(33920 \sqrt{g^7 H^7 (3 + H^2 k^2)} + 39424 \sqrt{g^5 H^5 (3 + H^2 k^2)} U^2 - 45 \sqrt{3} U^7 \right) + \right. \\
& \quad \quad 36 k^2 \left(105472 \sqrt{g^7 H^{11} (3 + H^2 k^2)} + 144512 \sqrt{g^5 H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \quad 5888 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^4 - 225 \sqrt{3} H^2 U^7 \left. \right) + \\
& \quad \quad \left. 18 k^4 \left(60608 \sqrt{g^7 H^{15} (3 + H^2 k^2)} + 100096 \sqrt{g^5 H^{13} (3 + H^2 k^2)} U^2 - 225 \sqrt{3} H^4 U^7 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 12 k^6 \left(9600 \sqrt{g^7 H^{19} (3 + H^2 k^2)} + 20864 \sqrt{g^5 H^{17} (3 + H^2 k^2)} U^2 - 75 \sqrt{3} H^6 U^7 \right) + \\
& 5 k^8 \left(384 \sqrt{g^7 H^{23} (3 + H^2 k^2)} + 2048 \sqrt{g^5 H^{21} (3 + H^2 k^2)} U^2 - 15 \sqrt{3} H^8 U^7 \right) \Big) dt^4 + \\
& O[dt]^5 \Big) dx^4 + O[dx]^5, \left(\frac{i \left(-\sqrt{3} k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^3 U \right)^2 dt}{2 (3 + H^2 k^2)^2} - \right. \\
& \frac{1}{3 (3 + H^2 k^2)^2} \\
& \left(k^3 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \left. \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right) dt^2 - \\
& \frac{1}{4 (3 + H^2 k^2)^3} i k^4 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left(3 g H \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^2 \left(-9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) dt^3 + \\
& \frac{1}{5 (3 + H^2 k^2)^3} k^5 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left(9 g^2 H^2 + 6 g H U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^3 \left(-12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left(-4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) \\
& dt^4 + O[dt]^5 \Big) + \left(-\frac{1}{4} i k^2 \left(2 \sqrt{g H} - \frac{\sqrt{3} U}{\sqrt{3 + H^2 k^2}} \right) + \right. \\
& \frac{1}{4 (3 + H^2 k^2)^{3/2}} \\
& k^3 \left(2 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} U \right) \\
& \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) dt + \\
& \frac{1}{4 (3 + H^2 k^2)^{3/2}} i k^4 \left(2 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} U \right) \\
& \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \\
& dt^2 - \frac{1}{4 (3 + H^2 k^2)^{5/2}} \\
& \left(k^5 \left(2 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} U \right) \left(3 g H \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \right. \\
& \left. U^2 \left(-9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) \Big) \\
& dt^3 - \frac{1}{4 (3 + H^2 k^2)^{5/2}} i k^6 \left(2 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} U \right)
\end{aligned}$$

$$\begin{aligned}
& \left(9 g^2 H^2 + 6 g H U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \quad \left. U^3 \left(-12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left(-4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) \\
& dt^4 + O[dt]^5 \Big) dx + \left(\left(k^3 \left(12 \sqrt{3} g H (4 + H^2 k^2) - U \left(48 \sqrt{g H (3 + H^2 k^2)} + 9 \sqrt{3} U + \right. \right. \right. \right. \\
& \quad \left. \left. \left. k^2 \left(16 \sqrt{g H^5 (3 + H^2 k^2)} + 3 \sqrt{3} H^2 U \right) \right) \right) \right) / \left(96 \sqrt{g H (3 + H^2 k^2)}^{3/2} \right) - \\
& \left(i k^4 \left(9 U^2 \left(16 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U \right) + k^4 U^2 \left(16 \sqrt{g H^9 (3 + H^2 k^2)} + 3 \sqrt{3} H^4 U \right) + \right. \right. \\
& \quad \left. \left. 6 k^2 \left(18 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 + 3 \sqrt{3} H^2 U^3 \right) + \right. \right. \\
& \quad \left. \left. 4 g H \left(63 \sqrt{g H (3 + H^2 k^2)} - 99 \sqrt{3} U - 63 \sqrt{3} H^2 k^2 U + \right. \right. \right. \\
& \quad \left. \left. \left. k^4 \left(3 \sqrt{g H^9 (3 + H^2 k^2)} - 10 \sqrt{3} H^4 U \right) \right) \right) \right) dt \Big) / \left(96 \sqrt{g H (3 + H^2 k^2)}^{5/2} \right) + \\
& \frac{1}{96 \sqrt{g H (3 + H^2 k^2)}^{5/2}} k^5 \left(-12 \sqrt{3} g^2 H^2 (30 + 15 H^2 k^2 + 2 H^4 k^4) + \right. \\
& \quad \left. 9 U^3 \left(16 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U \right) + k^4 U^3 \left(16 \sqrt{g H^9 (3 + H^2 k^2)} + 3 \sqrt{3} H^4 U \right) + \right. \\
& \quad \left. 6 k^2 \left(56 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^3 + 3 \sqrt{3} H^2 U^4 \right) + \right. \\
& \quad \left. g H U \left(864 \sqrt{g H (3 + H^2 k^2)} - 675 \sqrt{3} U - 429 \sqrt{3} H^2 k^2 U + \right. \right. \\
& \quad \left. \left. 4 k^4 \left(6 \sqrt{g H^9 (3 + H^2 k^2)} - 17 \sqrt{3} H^4 U \right) \right) \right) dt^2 + \frac{1}{96 \sqrt{g H (3 + H^2 k^2)}^{7/2}} \\
& i k^6 \left(9 k^2 \left(84 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 404 \sqrt{3} g^2 H^4 U + 438 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \\
& \quad \left. \left. 314 \sqrt{3} g H^3 U^3 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 9 \sqrt{3} H^2 U^5 \right) + \right. \\
& \quad \left. 3 k^4 \left(36 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 304 \sqrt{3} g^2 H^6 U + 48 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + \right. \right. \\
& \quad \left. \left. 9 \sqrt{3} H^4 U^5 + g H^5 U^2 \left(264 \sqrt{g H (3 + H^2 k^2)} - 301 \sqrt{3} U \right) \right) + \right. \\
& \quad \left. 27 \left(52 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 172 \sqrt{3} g^2 H^2 U + 16 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \right. \\
& \quad \left. \left. 3 \sqrt{3} U^5 + g H U^2 \left(210 \sqrt{g H (3 + H^2 k^2)} - 109 \sqrt{3} U \right) \right) - \right. \\
& \quad \left. k^6 U \left(72 \sqrt{3} g^2 H^8 + 12 g H^7 U \left(-3 \sqrt{g H (3 + H^2 k^2)} + 8 \sqrt{3} U \right) - \right. \right. \\
& \quad \left. \left. U^3 \left(16 \sqrt{g H^{13} (3 + H^2 k^2)} + 3 \sqrt{3} H^6 U \right) \right) \right) dt^3 + \\
& \frac{1}{96 \sqrt{g H (3 + H^2 k^2)}^{7/2}} k^7 \left(36 \sqrt{3} g^3 H^3 (48 + 27 H^2 k^2 + 4 H^4 k^4) + \right. \\
& \quad \left. 3 \sqrt{3} g^2 H^2 (4113 + 3075 H^2 k^2 + 712 H^4 k^4 + 48 H^6 k^6) U^2 + \right. \\
& \quad \left. 2 g H U^3 \left(-5076 \sqrt{g H (3 + H^2 k^2)} + 1971 \sqrt{3} U + 1872 \sqrt{3} H^2 k^2 U + \right. \right. \\
& \quad \left. \left. 3 k^4 \left(-216 \sqrt{g H^9 (3 + H^2 k^2)} + 197 \sqrt{3} H^4 U \right) + \right. \right. \\
& \quad \left. \left. 2 k^6 \left(-12 \sqrt{g H^{13} (3 + H^2 k^2)} + 31 \sqrt{3} H^6 U \right) \right) - \right. \\
& \quad \left. U \left(7344 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 432 \sqrt{g H (3 + H^2 k^2)} U^4 + 81 \sqrt{3} U^5 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& k^6 U^4 \left(16 \sqrt{g H^{13} (3 + H^2 k^2)} + 3 \sqrt{3} H^6 U \right) + 9 k^2 \left(400 \sqrt{g^5 H^9 (3 + H^2 k^2)} + \right. \\
& \quad \left. 760 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 9 \sqrt{3} H^2 U^5 \right) + \\
& \quad \left. 9 k^4 \left(48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 16 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + 3 \sqrt{3} H^4 U^5 \right) \right) dt^4 + O[dt]^5 \Big) \\
& dx^2 + \left(- \left(\left(i k^4 \left(3 \sqrt{3} (3 + H^2 k^2) U^3 + 4 g H \left(-12 \sqrt{g H (3 + H^2 k^2)} + 6 \sqrt{3} U + k^2 \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. -4 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \right) \right) \right) \right) / \left(384 g H (3 + H^2 k^2)^{3/2} \right) \right) + \\
& \left(k^5 \left(8 \sqrt{3} g^2 H^2 (54 + 33 H^2 k^2 + 5 H^4 k^4) + 3 \sqrt{3} (3 + H^2 k^2)^2 U^4 - \right. \right. \\
& \quad \left. 2 g H U \left(324 \sqrt{g H (3 + H^2 k^2)} - 81 \sqrt{3} U + 24 k^2 \left(7 \sqrt{g H^5 (3 + H^2 k^2)} - 2 \sqrt{3} H^2 U \right) + \right. \right. \\
& \quad \left. \left. k^4 \left(24 \sqrt{g H^9 (3 + H^2 k^2)} - 7 \sqrt{3} H^4 U \right) \right) \right) dt \Big) / \left(384 g H (3 + H^2 k^2)^{5/2} \right) - \\
& \frac{1}{384 g H (3 + H^2 k^2)^{5/2}} i k^6 \left(3 k^2 \left(96 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 380 \sqrt{3} g^2 H^4 U + \right. \right. \\
& \quad \left. 224 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - 51 \sqrt{3} g H^3 U^3 - 6 \sqrt{3} H^2 U^5 \right) + 9 \left(96 \sqrt{g^5 H^5 (3 + H^2 k^2)} - \right. \\
& \quad \left. 224 \sqrt{3} g^2 H^2 U - 3 \sqrt{3} U^5 + g H U^2 \left(160 \sqrt{g H (3 + H^2 k^2)} - 27 \sqrt{3} U \right) \right) + \\
& \quad \left. k^4 \left(16 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 168 \sqrt{3} g^2 H^6 U - 3 \sqrt{3} H^4 U^5 + \right. \right. \\
& \quad \left. \left. 8 g H^5 U^2 \left(10 \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} U \right) \right) \right) dt^2 - \\
& \frac{1}{384 \left(g H (3 + H^2 k^2) \right)^{7/2}} \left(k^7 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \quad \left(-3 k^2 \left(184 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 608 \sqrt{3} g^2 H^4 U + 334 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \\
& \quad \left. 72 \sqrt{3} g H^3 U^3 - 3 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 6 \sqrt{3} H^2 U^5 \right) - \\
& \quad 9 \left(160 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 352 \sqrt{3} g^2 H^2 U - 3 \sqrt{g H (3 + H^2 k^2)} U^4 - 3 \sqrt{3} U^5 + \right. \\
& \quad \left. 2 g H U^2 \left(121 \sqrt{g H (3 + H^2 k^2)} - 19 \sqrt{3} U \right) \right) - k^4 \left(48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - \right. \\
& \quad \left. 272 \sqrt{3} g^2 H^6 U - 3 \sqrt{3} H^4 U^5 + 2 g H^5 U^2 \left(56 \sqrt{g H (3 + H^2 k^2)} - 17 \sqrt{3} U \right) \right) \Big) dt^3 - \\
& \frac{1}{384 g H (3 + H^2 k^2)^{9/2}} i k^8 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \\
& \left(-3 k^2 \left(304 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 884 \sqrt{3} g^2 H^4 U + 456 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \\
& \quad \left. 95 \sqrt{3} g H^3 U^3 - 6 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 6 \sqrt{3} H^2 U^5 \right) - \\
& \quad 27 \left(80 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 168 \sqrt{3} g^2 H^2 U - 2 \sqrt{g H (3 + H^2 k^2)} U^4 - \right. \\
& \quad \left. \sqrt{3} U^5 + g H U^2 \left(112 \sqrt{g H (3 + H^2 k^2)} - 17 \sqrt{3} U \right) \right) - \\
& \quad \left. k^4 \left(96 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 400 \sqrt{3} g^2 H^6 U - 3 \sqrt{3} H^4 U^5 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 4 \, g \, H^5 \, U^2 \left(36 \sqrt{g \, H \, (3 + H^2 \, k^2)} - 11 \sqrt{3} \, U \right) \Big) \Big) \, dt^4 + O[dt]^5 \Big) \, dx^3 + \\
& \left(k^5 \left(-16 \sqrt{3} \, g^2 \, H^2 \, (144 + 48 \, H^2 \, k^2 + 5 \, H^4 \, k^4) + 75 \sqrt{3} \, (3 + H^2 \, k^2)^2 \, U^4 + \right. \right. \\
& \quad \left. \left. 8 \, g \, H \, (3 + H^2 \, k^2) \, U \left(96 \sqrt{g \, H \, (3 + H^2 \, k^2)} + k^2 \left(32 \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} - 5 \sqrt{3} \, H^2 \, U \right) \right) \right) \right) \Big) \Big) / \\
& \quad \left(30 \, 720 \, (g \, H)^{3/2} \, (3 + H^2 \, k^2)^{5/2} \right) + \frac{1}{92 \, 160 \, (g \, H)^{3/2} \, (3 + H^2 \, k^2)^{7/2}} \\
& \, i \, k^6 \left(9 \, k^2 \left(8448 \sqrt{g^5 \, H^9 \, (3 + H^2 \, k^2)} - 16 \, 160 \sqrt{3} \, g^2 \, H^4 \, U + \right. \right. \\
& \quad \left. \left. 6144 \sqrt{g^3 \, H^7 \, (3 + H^2 \, k^2)} \, U^2 + 240 \sqrt{3} \, g \, H^3 \, U^3 + 675 \sqrt{3} \, H^2 \, U^5 \right) + \right. \\
& \quad k^6 \left(1920 \sqrt{g^5 \, H^{17} \, (3 + H^2 \, k^2)} - 4368 \sqrt{3} \, g^2 \, H^8 \, U + 2048 \, g \, H^7 \sqrt{g \, H \, (3 + H^2 \, k^2)} \, U^2 + \right. \\
& \quad \left. 225 \sqrt{3} \, H^6 \, U^5 \right) + 3 \, k^4 \left(6720 \sqrt{g^5 \, H^{13} \, (3 + H^2 \, k^2)} - 14 \, 512 \sqrt{3} \, g^2 \, H^6 \, U + \right. \\
& \quad \left. 675 \sqrt{3} \, H^4 \, U^5 + 24 \, g \, H^5 \, U^2 \left(256 \sqrt{g \, H \, (3 + H^2 \, k^2)} + 5 \sqrt{3} \, U \right) \right) \Big) + \\
& \quad 27 \left(3968 \sqrt{g^5 \, H^5 \, (3 + H^2 \, k^2)} - 6016 \sqrt{3} \, g^2 \, H^2 \, U + 225 \sqrt{3} \, U^5 + \right. \\
& \quad \left. 8 \, g \, H \, U^2 \left(256 \sqrt{g \, H \, (3 + H^2 \, k^2)} + 15 \sqrt{3} \, U \right) \right) \Big) \Big) \, dt - \frac{1}{92 \, 160 \, (g \, H)^{3/2} \, (3 + H^2 \, k^2)^{7/2}} \\
& \left(k^7 \left(-48 \sqrt{3} \, g^3 \, H^3 \, (6192 + 5004 \, H^2 \, k^2 + 1425 \, H^4 \, k^4 + 140 \, H^6 \, k^6) - \right. \right. \\
& \quad \left. \left. 8 \sqrt{3} \, g^2 \, H^2 \, (60 \, 426 + 55 \, 557 \, H^2 \, k^2 + 17 \, 121 \, H^4 \, k^4 + 1772 \, H^6 \, k^6) \, U^2 + \right. \right. \\
& \quad \left. \left. g \, H \, U^3 \left(89 \, 856 \sqrt{g \, H \, (3 + H^2 \, k^2)} + 5265 \sqrt{3} \, U + 4590 \sqrt{3} \, H^2 \, k^2 \, U + \right. \right. \right. \\
& \quad \left. \left. 9 \, k^4 \left(3328 \sqrt{g \, H^9 \, (3 + H^2 \, k^2)} + 145 \sqrt{3} \, H^4 \, U \right) + 8 \, k^6 \left(416 \sqrt{g \, H^{13} \, (3 + H^2 \, k^2)} + \right. \right. \right. \\
& \quad \left. \left. 15 \sqrt{3} \, H^6 \, U \right) \right) + 3 \, U \left(81 \left(2816 \sqrt{g^5 \, H^5 \, (3 + H^2 \, k^2)} + 25 \sqrt{3} \, U^5 \right) + \right. \\
& \quad \left. 3 \, k^2 \left(52 \, 352 \sqrt{g^5 \, H^9 \, (3 + H^2 \, k^2)} + 9984 \sqrt{g^3 \, H^7 \, (3 + H^2 \, k^2)} \, U^2 + 675 \sqrt{3} \, H^2 \, U^5 \right) + \right. \\
& \quad \left. 3 \, k^4 \left(11 \, 776 \sqrt{g^5 \, H^{13} \, (3 + H^2 \, k^2)} + 225 \sqrt{3} \, H^4 \, U^5 \right) + 5 \, k^6 \right. \\
& \quad \left. \left(512 \sqrt{g^5 \, H^{17} \, (3 + H^2 \, k^2)} + 15 \sqrt{3} \, H^6 \, U^5 \right) \right) \Big) \Big) \, dt^2 - \frac{1}{30 \, 720 \, (g \, H)^{3/2} \, (3 + H^2 \, k^2)^{9/2}} \\
& \, i \, k^8 \left(27 \, k^2 \left(20 \, 416 \sqrt{g^7 \, H^{11} \, (3 + H^2 \, k^2)} - 80 \, 544 \sqrt{3} \, g^3 \, H^5 \, U + 82 \, 240 \sqrt{g^5 \, H^9 \, (3 + H^2 \, k^2)} \, U^2 - \right. \right. \\
& \quad \left. \left. 46 \, 960 \sqrt{3} \, g^2 \, H^4 \, U^3 + 6144 \sqrt{g^3 \, H^7 \, (3 + H^2 \, k^2)} \, U^4 + 305 \sqrt{3} \, g \, H^3 \, U^5 + 300 \sqrt{3} \, H^2 \, U^7 \right) + \right. \\
& \quad \left. k^8 \left(480 \sqrt{g^7 \, H^{23} \, (3 + H^2 \, k^2)} - 11 \, 520 \sqrt{3} \, g^3 \, H^{11} \, U + 5760 \sqrt{g^5 \, H^{21} \, (3 + H^2 \, k^2)} \, U^2 - \right. \right. \\
& \quad \left. \left. 9888 \sqrt{3} \, g^2 \, H^{10} \, U^3 + 75 \sqrt{3} \, H^8 \, U^7 + 16 \, g \, H^9 \, U^4 \left(96 \sqrt{g \, H \, (3 + H^2 \, k^2)} + 5 \sqrt{3} \, U \right) \right) + \right. \\
& \quad \left. 243 \left(2592 \sqrt{g^7 \, H^7 \, (3 + H^2 \, k^2)} - 8128 \sqrt{3} \, g^3 \, H^3 \, U + 9152 \sqrt{g^5 \, H^5 \, (3 + H^2 \, k^2)} \, U^2 - \right. \right. \\
& \quad \left. \left. 4168 \sqrt{3} \, g^2 \, H^2 \, U^3 + 25 \sqrt{3} \, U^7 + g \, H \, U^4 \left(512 \sqrt{g \, H \, (3 + H^2 \, k^2)} + 25 \sqrt{3} \, U \right) \right) + \right. \\
& \quad \left. 3 \, k^6 \left(6720 \sqrt{g^7 \, H^{19} \, (3 + H^2 \, k^2)} - 55 \, 216 \sqrt{3} \, g^3 \, H^9 \, U + 39 \, 488 \sqrt{g^5 \, H^{17} \, (3 + H^2 \, k^2)} \, U^2 - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 41\,728\sqrt{3}\,g^2H^8U^3 + 300\sqrt{3}\,H^6U^7 + 3\,gH^7U^4 \left(2048\sqrt{gH(3+H^2k^2)} + 105\sqrt{3}\,U \right) \Big) + \\
& 9\,k^4 \left(19\,040\sqrt{g^7H^{15}(3+H^2k^2)} - 99\,856\sqrt{3}\,g^3H^7U + \right. \\
& 88\,512\sqrt{g^5H^{13}(3+H^2k^2)}\,U^2 - 66\,296\sqrt{3}\,g^2H^6U^3 + 450\sqrt{3}\,H^4U^7 + \\
& \left. 3\,gH^5U^4 \left(3072\sqrt{gH(3+H^2k^2)} + 155\sqrt{3}\,U \right) \right) \Big) dt^3 + \frac{1}{92\,160\,(gH)^{3/2}(3+H^2k^2)^{9/2}} \\
& k^9 \left(-144\sqrt{3}\,g^4H^4(23\,832 + 22\,104H^2k^2 + 7395H^4k^4 + 1000H^6k^6 + 40H^8k^8) - \right. \\
& 24\sqrt{3}\,g^3H^3(893\,700 + 953\,451H^2k^2 + 376\,029H^4k^4 + 64\,744H^6k^6 + 4080H^8k^8)U^2 - \\
& 3\sqrt{3}\,g^2H^2(3+H^2k^2)^2(197\,625 + 114\,352H^2k^2 + 16\,944H^4k^4)U^4 + \\
& 2\,gH^5U^5 \left(238\,464\sqrt{gH(3+H^2k^2)} + 8505\sqrt{3}\,U + 13\,365\sqrt{3}\,H^2k^2U + \right. \\
& 27\,k^4 \left(5888\sqrt{gH^9(3+H^2k^2)} + 285\sqrt{3}\,H^4U \right) + 3\,k^6 \left(11\,776\sqrt{gH^{13}(3+H^2k^2)} + \right. \\
& \left. 645\sqrt{3}\,H^6U \right) + 4\,k^8 \left(736\sqrt{gH^{17}(3+H^2k^2)} + 45\sqrt{3}\,H^8U \right) \Big) + \\
& 3\,U \left(135 \left(33\,920\sqrt{g^7H^7(3+H^2k^2)} + 39\,424\sqrt{g^5H^5(3+H^2k^2)}\,U^2 + 45\sqrt{3}\,U^7 \right) + \right. \\
& 36\,k^2 \left(105\,472\sqrt{g^7H^{11}(3+H^2k^2)} + 144\,512\sqrt{g^5H^9(3+H^2k^2)}\,U^2 + \right. \\
& \left. 5888\sqrt{g^3H^7(3+H^2k^2)}\,U^4 + 225\sqrt{3}\,H^2U^7 \right) + \\
& 18\,k^4 \left(60\,608\sqrt{g^7H^{15}(3+H^2k^2)} + 100\,096\sqrt{g^5H^{13}(3+H^2k^2)}\,U^2 + 225\sqrt{3}\,H^4U^7 \right) + \\
& 12\,k^6 \left(9600\sqrt{g^7H^{19}(3+H^2k^2)} + 20\,864\sqrt{g^5H^{17}(3+H^2k^2)}\,U^2 + 75\sqrt{3}\,H^6U^7 \right) + \\
& 5\,k^8 \left(384\sqrt{g^7H^{23}(3+H^2k^2)} + 2048\sqrt{g^5H^{21}(3+H^2k^2)}\,U^2 + \right. \\
& \left. \left. 15\sqrt{3}\,H^8U^7 \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5 \}
\end{aligned}$$