In[1824]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx / 2, xj + dx / 2}] / (dx);
qjnbar = qjbar /. t → tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x → xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx / 2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / 3 * k^2;
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca;
fn1Gca0A = fn1A /. Gca → 0;
fn1eca0A = fn1A /. eca → 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca /. Gh → RA * Gca;
fG1Gca0A = fG1A /. Gca → 0;
fG1eca0A = fG1A /. eca → 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
```

$$wAp = U * k + \frac{\sqrt{3}\ k \sqrt{g H \left(3 + H^2 k^2\right)}}{3 + H^2 k^2};$$

$$wAm = U * k - \frac{\sqrt{3}\ k \sqrt{g H \left(3 + H^2 k^2\right)}}{3 + H^2 k^2};$$

In[1859]:=
```
M2 = 1
Series[M2 - MA, {dx, 0, 10}]
```

Out[1859]= 1

Out[1860]= $$-\frac{k^2\,dx^2}{24} - \frac{7\,k^4\,dx^4}{5760} - \frac{31\,k^6\,dx^6}{967\,680} - \frac{127\,k^8\,dx^8}{154\,828\,800} - \frac{73\,k^{10}\,dx^{10}}{3\,503\,554\,560} + O[dx]^{11}$$

In[1861]:=
```
Rm = (1 + I * Sin[k * dx] / 2)
Series[Rm - RA, {dx, 0, 10}]
Rp = Exp[I * k * dx] * (1 - I * Sin[k * dx] / 2)
Series[Rp - RA, {dx, 0, 10}]
```

Out[1861]= $$1 + \frac{1}{2}\,i\,\text{Sin}[dx\,k]$$

Out[1862]= $$\frac{k^2\,dx^2}{12} - \frac{1}{12}\,i\,k^3\,dx^3 + \frac{k^4\,dx^4}{720} + \frac{1}{240}\,i\,k^5\,dx^5 + \frac{k^6\,dx^6}{30\,240} - \frac{i\,k^7\,dx^7}{10\,080} + \frac{k^8\,dx^8}{1\,209\,600} + \frac{i\,k^9\,dx^9}{725\,760} + \frac{k^{10}\,dx^{10}}{47\,900\,160} + O[dx]^{11}$$

Out[1863]= $$e^{i\,dx\,k}\left(1 - \frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)$$

Out[1864]= $$\frac{k^2\,dx^2}{12} + \frac{1}{6}\,i\,k^3\,dx^3 - \frac{89\,k^4\,dx^4}{720} - \frac{7}{120}\,i\,k^5\,dx^5 + \frac{631\,k^6\,dx^6}{30\,240} + \frac{31\,i\,k^7\,dx^7}{5040} - \frac{1889\,k^8\,dx^8}{1\,209\,600} - \frac{127\,i\,k^9\,dx^9}{362\,880} + \frac{481\,k^{10}\,dx^{10}}{6\,842\,880} + O[dx]^{11}$$

In[1865]:=
```
Ru = (1 + Exp[I * k * dx]) / 2
Series[Ru - Exp[I * k * dx / 2], {dx, 0, 10}]
Gold = H - H^3 / 3 * (2 * Cos[k * dx] - 2) / dx^2
GG2 = Ru / Gold
Series[GG2 - GGA, {dx, 0, 5}]

Gn2 = -U * Ru / Gold
Series[Gn2 - GnA, {dx, 0, 5}]
```

Out[1865]=
$$\frac{1}{2}\left(1 + e^{i\, dx\, k}\right)$$

Out[1866]=
$$-\frac{k^2\, dx^2}{8} - \frac{1}{16}\, i\, k^3\, dx^3 + \frac{7\, k^4\, dx^4}{384} + \frac{1}{256}\, i\, k^5\, dx^5 - \frac{31\, k^6\, dx^6}{46\,080} - \frac{i\, k^7\, dx^7}{10\,240} + \frac{127\, k^8\, dx^8}{10\,321\,920} + \frac{17\, i\, k^9\, dx^9}{12\,386\,304} - \frac{73\, k^{10}\, dx^{10}}{530\,841\,600} + O[dx]^{11}$$

Out[1867]=
$$H - \frac{H^3\,(-2 + 2\,\text{Cos}[dx\, k])}{3\, dx^2}$$

Out[1868]=
$$\frac{1 + e^{i\, dx\, k}}{2\left(H - \frac{H^3\,(-2+2\,\text{Cos}[dx\, k])}{3\, dx^2}\right)}$$

Out[1869]=
$$\frac{\left(-6\, k^2 - H^2\, k^4\right) dx^2}{4\, H\left(3 + H^2\, k^2\right)^2} - \frac{i\left(6\, k^3 + H^2\, k^5\right) dx^3}{8\, H\left(3 + H^2\, k^2\right)^2} + \frac{\left(144\, k^4 + 45\, H^2\, k^6 + 4\, H^4\, k^8\right) dx^4}{240\, H\left(3 + H^2\, k^2\right)^3} - \frac{i\left(-54\, k^5 + H^4\, k^9\right) dx^5}{480\, H\left(3 + H^2\, k^2\right)^3} + O[dx]^6$$

Out[1870]=
$$-\frac{\left(1 + e^{i\, dx\, k}\right) U}{2\left(H - \frac{H^3\,(-2+2\,\text{Cos}[dx\, k])}{3\, dx^2}\right)}$$

Out[1871]=
$$\frac{\left(6\, k^2 + H^2\, k^4\right) U\, dx^2}{4\, H\left(3 + H^2\, k^2\right)^2} + \frac{i\left(6\, k^3 + H^2\, k^5\right) U\, dx^3}{8\, H\left(3 + H^2\, k^2\right)^2} - \frac{\left(\left(144\, k^4 + 45\, H^2\, k^6 + 4\, H^4\, k^8\right) U\right) dx^4}{240\left(H\left(3 + H^2\, k^2\right)^3\right)} + \frac{i\left(-54\, k^5 + H^4\, k^9\right) U\, dx^5}{480\, H\left(3 + H^2\, k^2\right)^3} + O[dx]^6$$

In[1872]:=
```
Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]]
Text[Row[{"Fnn and FnG "}]]
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnn = FullSimplify[KurFWSeta /. G → 0 /. n → 1]
KfnG = FullSimplify[KurFWSeta /. n → 0 /. G → 1]
Kfnn = Kfnn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]]
KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGn = FullSimplify[KurFWSG /. G → 0 /. n → 1]
KfGG = FullSimplify[KurFWSG /. n → 0 /. G → 1]
KfGn = KfGn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2  + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2^2 / 2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ]
```

Out[1872]= $-\text{Sqrt}[g*H] < U < \text{Sqrt}[g*H]$

Out[1873]= Fnn and FnG

Out[1878]= $\frac{1}{2}\left(2\,\text{Gnp}\,H + \text{Rpp}\left(-\sqrt{g\,H} + U\right) + \text{Rmp}\left(\sqrt{g\,H} + U\right)\right)$

Out[1879]= GGp H

Out[1884]= $\left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{i\left(27\,k^3 + 9\,H^2\,k^5 + H^4\,k^7\right)\,U\,dt}{12\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)\,dx^2 + \left(-\frac{1}{8}\left(\sqrt{g\,H}\;k^4\right)\,dt + O[dt]^4\right)\,dx^3 +$
$\left(\frac{i\left(405\,k^5\,U + 351\,H^2\,k^7\,U + 116\,H^4\,k^9\,U + 13\,H^6\,k^{11}\,U\right)\,dt}{240\left(3+H^2\,k^2\right)^3} + O[dt]^4\right)\,dx^4 + O[dx]^5$

Out[1887]= $\left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(\frac{i\left(6\,k^3 + H^2\,k^5\right)\,dt}{4\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)\,dx^2 + \left(\frac{i\left(-54\,k^5 + H^4\,k^9\right)\,dt}{240\left(3+H^2\,k^2\right)^3} + O[dt]^4\right)\,dx^4 + O[dx]^5$

Out[1888]= FGn and FGG

Out[1891]= $\frac{1}{2}\left(g\,H\,(\text{Rmp}+\text{Rpp}) + \left(2\,\text{Gnp}\,H + \sqrt{g\,H}\;(\text{Rmp}-\text{Rpp})\right)\,U\right)$

Out[1892]= $\frac{1}{2}\left(\sqrt{g\,H}\;\text{Rmp} - \sqrt{g\,H}\;\text{Rpp} + (2\,\text{GGp}\,H + \text{Rmp} + \text{Rpp})\,U\right)$

Out[1897]= $\left(-\frac{\left(k\left(3\,g\,H + g\,H^3\,k^2 - 3\,U^2\right)\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\left(3\,g\,H + g\,H^3\,k^2 - 3\,U^2\right)\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{i\left(9\,g\,H\,k^3 + 6\,g\,H^3\,k^5 + g\,H^5\,k^7 + 18\,k^3\,U^2 + 3\,H^2\,k^5\,U^2\right)\,dt}{12\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)\,dx^2 +$
$\left(-\frac{1}{8}\left(\sqrt{g\,H}\;k^4\,U\right)\,dt + O[dt]^4\right)\,dx^3 +$
$\left(\frac{i\left(351\,g\,H\,k^5 + 351\,g\,H^3\,k^7 + 117\,g\,H^5\,k^9 + 13\,g\,H^7\,k^{11} + 54\,k^5\,U^2 - H^4\,k^9\,U^2\right)\,dt}{240\left(3+H^2\,k^2\right)^3} + O[dt]^4\right)\,dx^4 +$
$O[dx]^5$

Out[1901]= $\left(-\frac{\left(k\left(6+H^2\,k^2\right)\,U\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\left(6+H^2\,k^2\right)\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{i\left(-9\,k^3 + 3\,H^2\,k^5 + H^4\,k^7\right)\,U\,dt}{12\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)\,dx^2 + \left(-\frac{1}{8}\left(\sqrt{g\,H}\;k^4\right)\,dt + O[dt]^4\right)\,dx^3 +$
$\left(\frac{i\left(297\,k^5\,U + 351\,H^2\,k^7\,U + 118\,H^4\,k^9\,U + 13\,H^6\,k^{11}\,U\right)\,dt}{240\left(3+H^2\,k^2\right)^3} + O[dt]^4\right)\,dx^4 + O[dx]^5$

Out[1902]= W : omega

Out[1907]=

$$\left\{\left(\frac{1}{6\left(3+H^2k^2\right)^2}k^3\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right.\right.$$
$$\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)dt^2+\frac{1}{8\left(3+H^2k^2\right)^3}$$
$$ik^4\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\left(3g\left(\sqrt{3}H\sqrt{gH\left(3+H^2k^2\right)}+9HU+3H^3k^2U\right)+\right.$$
$$\left.U^2\left(H^4k^4U+9\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+U\right)+3k^2\left(\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+2H^2U\right)\right)\right)dt^3-$$
$$\frac{1}{20\left(3+H^2k^2\right)^3}\left(k^5\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right.$$
$$\left(9g^2H^2+6gHU\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+3\left(3+H^2k^2\right)U\right)+\right.$$
$$\left.\left.U^3\left(12\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+9U+H^4k^4U+2k^2\left(2\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+3H^2U\right)\right)\right)\right)$$
$$\left.dt^4+O[dt]^5\right)+\left(\frac{k^3\left(-3\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)^2U\right)}{24\left(3+H^2k^2\right)^2}+\frac{1}{48\left(3+H^2k^2\right)^3}\right.$$
$$k^5\left(g\left(-9\sqrt{3}H\sqrt{gH\left(3+H^2k^2\right)}+18H^3k^2U+6H^5k^4U\right)+\right.$$
$$U^2\left(27\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+54U+2H^6k^6U+3k^2\left(7\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+18H^2U\right)+\right.$$
$$\left.\left.2k^4\left(2\sqrt{3}\sqrt{gH^9\left(3+H^2k^2\right)}+9H^4U\right)\right)\right)dt^2+$$
$$\frac{1}{48\left(3+H^2k^2\right)^3}ik^6\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\left(-9gH+\right.$$
$$\left.U\left(3\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+18U+2H^4k^4U+2k^2\left(\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+6H^2U\right)\right)\right)$$
$$dt^3-\frac{1}{96\left(3+H^2k^2\right)^4}\left(k^7\left(-3\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)^2U\right)\right.$$
$$\left(9g^2H^2+6gHU\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+3\left(3+H^2k^2\right)U\right)+\right.$$
$$\left.\left.U^3\left(12\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+9U+H^4k^4U+2k^2\left(2\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+3H^2U\right)\right)\right)\right)$$
$$\left.dt^4+O[dt]^5\right)dx^2+\left(-\frac{ik^4\left(2gH\left(3+H^2k^2\right)+\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}U\right)}{16\sqrt{gH}\left(3+H^2k^2\right)}-\right.$$
$$\left(ik^6\left(gH\left(6\sqrt{gH\left(3+H^2k^2\right)}+\sqrt{3}\left(15+4H^2k^2\right)U\right)+U^2\left(12\sqrt{gH\left(3+H^2k^2\right)}+\right.\right.\right.$$
$$\left.\left.\left.3\sqrt{3}U+k^2\left(2\sqrt{gH^5\left(3+H^2k^2\right)}+\sqrt{3}H^2U\right)\right)\right)dt^2\right)\Big/\left(32\left(3+H^2k^2\right)^{3/2}\right)+$$
$$\left(k^7\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\left(2\sqrt{3}gH\left(3+H^2k^2\right)+\right.\right.$$
$$\left.\left.U\left(9\sqrt{gH\left(3+H^2k^2\right)}+3\sqrt{3}U+k^2\left(2\sqrt{gH^5\left(3+H^2k^2\right)}+\sqrt{3}H^2U\right)\right)\right)dt^3\right)\Big/$$
$$\left(32\left(3+H^2k^2\right)^{5/2}\right)+\left(ik^8\left(2gH\left(3+H^2k^2\right)+\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}U\right)\right.$$
$$\left(9g^2H^2+6gHU\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+3\left(3+H^2k^2\right)U\right)+\right.$$

$$U^3 \left(12 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 9 U + H^4 k^4 U + 2 k^2 \left(2 \sqrt{3} \sqrt{g H^5 \left(3 + H^2 k^2\right)} + 3 H^2 U\right)\right)\right)$$

$$dt^4\Big) \Big/ \left(64 \sqrt{g H} \left(3 + H^2 k^2\right)^3\right) + O[dt]^5\Bigg) dx^3 +$$

$$\Bigg(-\Bigg(\Big(k^5 \left(3 \sqrt{3} g H \left(177 + 124 H^2 k^2 + 20 H^4 k^4\right) + 104 \left(9 \sqrt{g H \left(3 + H^2 k^2\right)} + 6\right.\right.$$

$$\left.k^2 \sqrt{g H^5 \left(3 + H^2 k^2\right)} + k^4 \sqrt{g H^9 \left(3 + H^2 k^2\right)}\right) U\Big)\Big) \Big/$$

$$\left(1920 \left(\sqrt{g H} \left(3 + H^2 k^2\right)^{5/2}\right)\right)\Bigg) - \frac{1}{11\,520 \left(\sqrt{g H} \left(3 + H^2 k^2\right)^{7/2}\right)}$$

$$\Big(k^7 \left(27 \sqrt{3} g^2 H^2 \left(167 + 124 H^2 k^2 + 20 H^4 k^4\right) +$$

$$g H U \left(21\,429 \sqrt{3} H^2 k^2 U + 764 \sqrt{3} H^6 k^6 U + 81 \left(232 \sqrt{g H \left(3 + H^2 k^2\right)} + 267 \sqrt{3} U\right) +\right.$$

$$24 k^4 \left(84 \sqrt{g H^9 \left(3 + H^2 k^2\right)} + 293 \sqrt{3} H^4 U\right)\Big) +$$

$$16 \left(459 \sqrt{g H \left(3 + H^2 k^2\right)} U^3 + 153 k^4 \sqrt{g H^9 \left(3 + H^2 k^2\right)} U^3 + 17 k^6 \sqrt{g H^{13} \left(3 + H^2 k^2\right)}\right.$$

$$\left.\left.U^3 + 9 k^2 \left(88 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)} U + 51 \sqrt{g H^5 \left(3 + H^2 k^2\right)} U^3\right)\right)\right)\Big) dt^2 -$$

$$\frac{1}{3840 \left(3 + H^2 k^2\right)^4} i k^8 \left(54 g^2 H^2 \left(81 + 62 H^2 k^2 + 10 H^4 k^4\right) + 84 H^8 k^8 U^4 +\right.$$

$$243 U^3 \left(39 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 28 U\right) + 36 k^4 U^3$$

$$\left(85 \sqrt{3} \sqrt{g H^9 \left(3 + H^2 k^2\right)} + 126 H^4 U\right) + 4 k^6 U^3 \left(83 \sqrt{3} \sqrt{g H^{13} \left(3 + H^2 k^2\right)} + 252 H^6 U\right) +$$

$$9 k^2 \left(600 \sqrt{3} \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)} U + 1039 \sqrt{3} \sqrt{g H^5 \left(3 + H^2 k^2\right)} U^3 + 1008 H^2 U^4\right) +$$

$$3 g H U \left(13\,500 H^2 k^2 U + 472 H^6 k^6 U + 27 \left(97 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 504 U\right) +\right.$$

$$\left.4 k^4 \left(71 \sqrt{3} \sqrt{g H^9 \left(3 + H^2 k^2\right)} + 1101 H^4 U\right)\right)\Big) dt^3 +$$

$$\frac{1}{23\,040 \sqrt{g H} \left(3 + H^2 k^2\right)^{9/2}} k^9 \left(81 \sqrt{3} g^3 H^3 \left(157 + 124 H^2 k^2 + 20 H^4 k^4\right) +\right.$$

$$6 \sqrt{3} g^2 H^2 \left(32\,535 + 32\,481 H^2 k^2 + 10\,584 H^4 k^4 + 1124 H^6 k^6\right) U^2 +$$

$$3 g H U^3 \left(69\,120 \sqrt{g H \left(3 + H^2 k^2\right)} + 34\,155 \sqrt{3} U + 44\,982 \sqrt{3} H^2 k^2 U +\right.$$

$$396 \sqrt{3} H^8 k^8 U + 27 k^4 \left(816 \sqrt{g H^9 \left(3 + H^2 k^2\right)} + 821 \sqrt{3} H^4 U\right) +$$

$$\left.28 k^6 \left(84 \sqrt{g H^{13} \left(3 + H^2 k^2\right)} + 173 \sqrt{3} H^6 U\right)\right) +$$

$$8 U \left(348 k^6 \sqrt{g H^{13} \left(3 + H^2 k^2\right)} U^4 + 29 k^8 \sqrt{g H^{17} \left(3 + H^2 k^2\right)} U^4 +\right.$$

$$81 \left(130 \sqrt{g^5 H^5 \left(3 + H^2 k^2\right)} + 29 \sqrt{g H \left(3 + H^2 k^2\right)} U^4\right) + 54 k^2$$

$$\left.\left(137 \sqrt{g^5 H^9 \left(3 + H^2 k^2\right)} + 472 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)} U^2 + 58 \sqrt{g H^5 \left(3 + H^2 k^2\right)} U^4\right) +$$

$$27\,k^4\left(43\sqrt{g^5 H^{13}\left(3+H^2 k^2\right)}+58\sqrt{g H^9\left(3+H^2 k^2\right)}\,U^4\right)\Big)\Big)\,dt^4+O[dt]^5\Big)\,dx^4+$$

$$O[dx]^5,\left(\frac{1}{6\left(3+H^2 k^2\right)^2}k^3\left(-\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right)U\right)\right.$$

$$\left(3\,g H+U\left(-2\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right)U\right)\right)$$

$$dt^2+$$

$$\frac{i\,k^4\left(3\,g H+U\left(-2\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right)U\right)\right)^2 dt^3}{8\left(3+H^2 k^2\right)^2}-$$

$$\frac{1}{20\left(3+H^2 k^2\right)^4}$$

$$\left(k^5\left(-\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right)U\right)^3\right.$$

$$\left.\left(3\,g H+U\left(-2\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right)U\right)\right)\right)dt^4+O[dt]^5\Big)+$$

$$\left(\frac{k^3\left(3\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+2\left(3+H^2 k^2\right)^2 U\right)}{24\left(3+H^2 k^2\right)^2}+\frac{1}{48\left(3+H^2 k^2\right)^3}\right.$$

$$k^5\left(3\,g\left(3\sqrt{3}\,H\sqrt{g H\left(3+H^2 k^2\right)}+6\,H^3 k^2 U+2\,H^5 k^4 U\right)+\right.$$

$$U^2\left(-27\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+54\,U+2\,H^6 k^6 U-3\,k^2\right.$$

$$\left.\left.\left.\left(7\sqrt{3}\,\sqrt{g H^5\left(3+H^2 k^2\right)}-18\,H^2 U\right)-2\,k^4\left(2\sqrt{3}\,\sqrt{g H^9\left(3+H^2 k^2\right)}-9\,H^4 U\right)\right)\right)dt^2+$$

$$\frac{1}{48\left(3+H^2 k^2\right)^3}i\,k^6\left(3\,g H+U\left(-2\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right)U\right)\right)\left(-9\,g H+\right.$$

$$\left.U\left(-3\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+18\,U+2\,H^4 k^4 U-2\,k^2\left(\sqrt{3}\,\sqrt{g H^5\left(3+H^2 k^2\right)}-6\,H^2 U\right)\right)\right)$$

$$dt^3-\frac{1}{96\left(3+H^2 k^2\right)^4}\left(k^7\left(3\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+2\left(3+H^2 k^2\right)^2 U\right)\right.$$

$$\left.\left.\left(3\,g H+U\left(-2\sqrt{3}\,\sqrt{g H\left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right)U\right)\right)^2\right)dt^4+O[dt]^5\Big)$$

$$dx^2+\left(\frac{1}{16}i\sqrt{g H}\,k^4\left(-2+\frac{\sqrt{3}\,U}{\sqrt{g H\left(3+H^2 k^2\right)}}\right)+\right.$$

$$\left(i\,k^6\left(g\left(-6\,H\sqrt{g H\left(3+H^2 k^2\right)}+15\sqrt{3}\,H U+4\sqrt{3}\,H^3 k^2 U\right)+U^2\left(-12\sqrt{g H\left(3+H^2 k^2\right)}+\right.\right.\right.$$

$$\left.\left.\left.3\sqrt{3}\,U+k^2\left(-2\sqrt{g H^5\left(3+H^2 k^2\right)}+\sqrt{3}\,H^2 U\right)\right)\right)dt^2\right)\Big/\left(32\left(3+H^2 k^2\right)^{3/2}\right)-$$

$$\left(\left(k^7 \left(3\,g\,H + U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\right)\right)\left(2\sqrt{3}\,g\,H\left(3+H^2\,k^2\right) + U\left(-9\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\sqrt{3}\,U + k^2\left(-2\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + \sqrt{3}\,H^2\,U\right)\right)\right)\right)dt^3\right)\Big/ \left(32\left(3+H^2\,k^2\right)^{5/2}\right) + \left(i\,k^8\left(2\,g\,H\left(3+H^2\,k^2\right) - \sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)\left(3\,g\,H + U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\right)\right)^2 dt^4\right)\Big/ \left(64\sqrt{g\,H}\left(3+H^2\,k^2\right)^3\right) + O[dt]^5\Bigg)\,dx^3 +$$

$$\Bigg(\left(k^5\left(3\sqrt{3}\,g\,H\left(177+124\,H^2\,k^2+20\,H^4\,k^4\right) - 104\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)} + 6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\right)\Big/ \left(1920\sqrt{g\,H}\left(3+H^2\,k^2\right)^{5/2}\right) + \frac{1}{11\,520\sqrt{g\,H}\left(3+H^2\,k^2\right)^{7/2}}$$
$$k^7\left(27\sqrt{3}\,g^2\,H^2\left(167+124\,H^2\,k^2+20\,H^4\,k^4\right) + g\,H\,U\left(21\,429\sqrt{3}\,H^2\,k^2\,U + 764\sqrt{3}\,H^6\,k^6\,U + 81\left(-232\sqrt{g\,H\left(3+H^2\,k^2\right)} + 267\sqrt{3}\,U\right) - 24\,k^4\left(84\sqrt{g\,H^9\left(3+H^2\,k^2\right)} - 293\sqrt{3}\,H^4\,U\right)\right) - 16\left(459\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + 153\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3 + 17\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3 + 9\,k^2\left(88\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U + 51\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\right)dt^2 -$$
$$\frac{1}{3840\left(3+H^2\,k^2\right)^4}i\,k^8\left(54\,g^2\,H^2\left(81+62\,H^2\,k^2+10\,H^4\,k^4\right) + 84\,H^8\,k^8\,U^4 + 243\,U^3\left(-39\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 28\,U\right) + 36\,k^4\,U^3\left(-85\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)} + 126\,H^4\,U\right) + 4\,k^6\,U^3\left(-83\sqrt{3}\,\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)} + 252\,H^6\,U\right) - 9\,k^2\left(600\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U + 1039\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3 - 1008\,H^2\,U^4\right) + 3\,g\,H\,U\left(13\,500\,H^2\,k^2\,U + 472\,H^6\,k^6\,U + 27\left(-97\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 504\,U\right) - 4\,k^4\left(71\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)} - 1101\,H^4\,U\right)\right)\right)dt^3 -$$
$$\frac{1}{23\,040\left(\sqrt{g\,H}\left(3+H^2\,k^2\right)^{11/2}\right)}\left(k^9\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} - \left(3+H^2\,k^2\right)U\right)\left(4\,k^8\,U^3\left(-239\sqrt{3}\,g\,H^9 + 58\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\,U\right) + 27\,k^2\left(372\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)} - 2703\sqrt{3}\,g^2\,H^4\,U + 4515\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2 - 4070\sqrt{3}\,g\,H^3\,U^3 + 928\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\right) + 9\,k^4\left(180\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)} - 2672\sqrt{3}\,g^2\,H^6\,U + 1392\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^4 + g\,H^5\,U^2\left(4384\sqrt{g\,H\left(3+H^2\,k^2\right)} - 5997\sqrt{3}\,U\right)\right) + 81\left(157\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)} - 883\sqrt{3}\right.$$

$$g^2 H^2 U + 232 \sqrt{g H \left(3 + H^2 k^2\right)}\, U^4 + g H U^2 \left(1527 \sqrt{g H \left(3 + H^2 k^2\right)} - 1033 \sqrt{3}\, U\right)\Big) -$$
$$12 k^6 U \left(213 \sqrt{3}\, g^2 H^8 - 232 \sqrt{g H^{13} \left(3 + H^2 k^2\right)}\, U^3 +\right.$$
$$\left. g H^7 U \left(-349 \sqrt{g H \left(3 + H^2 k^2\right)} + 979 \sqrt{3}\, U\right)\right)\Big)\Big) dt^4 + O[dt]^5\Big) dx^4 + O[dx]^5\Big\}$$

In[1908]:=
```
Text[Row[{" U > Sqrt[g*H]  "}]]
Text[Row[{"Fnn and FnG "}]]
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (0);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnn = FullSimplify[KurFWSeta /. G → 0 /. n → 1]
KfnG = FullSimplify[KurFWSeta /. n → 0 /. G → 1]
Kfnn = Kfnn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]]
KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGn = FullSimplify[KurFWSG /. G → 0 /. n → 1]
KfGG = FullSimplify[KurFWSG /. n → 0 /. G → 1]
KfGn = KfGn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2  + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2^2 / 2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ]
```

Out[1908]= U > Sqrt[g∗H]

Out[1909]= Fnn and FnG

Out[1914]= Gnp H + Rmp U

Out[1915]= GGp H

Out[1920]= $\left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+$
$\left(-\frac{i\,\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)\,U\,dt}{12\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(-\frac{1}{8}\,\left(k^4\,U\right)\,dt+O[dt]^4\right)\,dx^3+$
$\left(\frac{i\,\left(405\,k^5\,U+351\,H^2\,k^7\,U+116\,H^4\,k^9\,U+13\,H^6\,k^{11}\,U\right)\,dt}{240\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$

Out[1923]= $\left(-\frac{3\,(k\,w)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+$
$\left(\frac{i\,\left(6\,k^3+H^2\,k^5\right)\,dt}{4\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{i\,\left(-54\,k^5+H^4\,k^9\right)\,dt}{240\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$

Out[1924]= FGn and FGG

Out[1927]= H (g Rmp + Gnp U)

Out[1928]= (GGp H + Rmp) U

Out[1933]= $\left(-\frac{\left(k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+$
$\left(-\frac{i\,\left(9\,g\,H\,k^3+6\,g\,H^3\,k^5+g\,H^5\,k^7+18\,k^3\,U^2+3\,H^2\,k^5\,U^2\right)\,dt}{12\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+$
$\left(-\frac{1}{8}\,\left(g\,H\,k^4\right)\,dt+O[dt]^4\right)\,dx^3+$
$\left(\left(i\,\left(351\,g\,H\,k^5+351\,g\,H^3\,k^7+117\,g\,H^5\,k^9+13\,g\,H^7\,k^{11}+54\,k^5\,U^2-H^4\,k^9\,U^2\right)\,dt\right)\Big/\left(240\,\left(3+H^2\,k^2\right)^3\right)+O[dt]^4\right)\,dx^4+O[dx]^5$

Out[1937]= $\left(-\frac{\left(k\,\left(6+H^2\,k^2\right)\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(6+H^2\,k^2\right)\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+$
$\left(-\frac{i\,\left(-9\,k^3+3\,H^2\,k^5+H^4\,k^7\right)\,U\,dt}{12\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(-\frac{1}{8}\,\left(k^4\,U\right)\,dt+O[dt]^4\right)\,dx^3+$
$\left(\frac{i\,\left(297\,k^5+351\,H^2\,k^7+118\,H^4\,k^9+13\,H^6\,k^{11}\right)\,U\,dt}{240\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$

Out[1938]= W : omega

Out[1943]= $\left\{\left(\frac{1}{6\,\left(3+H^2\,k^2\right)^2}k^3\,\left(\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)\,U\right)\right.$
$\left.\left(3\,g\,H+U\,\left(2\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)\,U\right)\right)\,dt^2+\right.$

$$\frac{1}{8\left(3+H^2k^2\right)^2} i k^4 \left(3 g H + U\left(2\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + \left(3+H^2k^2\right)U\right)\right)^2 dt^3 -$$

$$\left(\left(k^5\left(\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + \left(3+H^2k^2\right)U\right)^3 \left(3 g H + U\left(2\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + \left(3+H^2k^2\right)U\right)\right)\right) dt^4\right) \Big/ \left(20\left(3+H^2k^2\right)^4\right) +$$

$$O[dt]^5\Bigg) + \Bigg(\frac{k^3\left(-3\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + 2\left(3+H^2k^2\right)^2 U\right)}{24\left(3+H^2k^2\right)^2} +$$

$$\left(k^5\left(g\left(-9\sqrt{3}\,H\sqrt{g H\left(3+H^2k^2\right)} + 18 H^3 k^2 U + 6 H^5 k^4 U\right) + U^2\left(27\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + 54 U + 2 H^6 k^6 U + 3 k^2\left(7\sqrt{3}\sqrt{g H^5\left(3+H^2k^2\right)} + 18 H^2 U\right) + 2 k^4\left(2\sqrt{3}\sqrt{g H^9\left(3+H^2k^2\right)} + 9 H^4 U\right)\right)\right) dt^2\right) \Big/ \left(48\left(3+H^2k^2\right)^3\right) +$$

$$\left(i k^6\left(3 g H + U\left(2\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + \left(3+H^2k^2\right)U\right)\right)\left(-9 g H + U\left(3\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + 18 U + 2 H^4 k^4 U + 2 k^2\left(\sqrt{3}\sqrt{g H^5\left(3+H^2k^2\right)} + 6 H^2 U\right)\right)\right)\right.$$

$$\left. dt^3\right) \Big/ \left(48\left(3+H^2k^2\right)^3\right) - \left(\left(k^7\left(-3\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + 2\left(3+H^2k^2\right)^2 U\right)\left(3 g H + U\left(2\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + \left(3+H^2k^2\right)U\right)\right)^2\right) dt^4\right) \Big/$$

$$\left.\left(96\left(3+H^2k^2\right)^4\right) + O[dt]^5\right) dx^2 + \left(-\frac{1}{16} i k^4\left(\sqrt{3}\sqrt{\frac{g H}{3+H^2k^2}} + 2U\right) -$$

$$\left(i k^6\left(3 g H\left(\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + 4\left(3+H^2k^2\right)U\right) + U^2\left(15\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + 18 U + 2 H^4 k^4 U + k^2\left(5\sqrt{3}\sqrt{g H^5\left(3+H^2k^2\right)} + 12 H^2 U\right)\right)\right) dt^2\right) \Big/$$

$$\left(32\left(3+H^2k^2\right)^2\right) + \left(k^7\left(3 g H + U\left(2\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + \left(3+H^2k^2\right)U\right)\right)\left(3 g H + U\left(3\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + 2\left(3+H^2k^2\right)U\right)\right) dt^3\right) \Big/$$

$$\left(32\left(3+H^2k^2\right)^2\right) + \left(i k^8\left(\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + 2\left(3+H^2k^2\right)U\right)\left(3 g H + U\left(2\sqrt{3}\sqrt{g H\left(3+H^2k^2\right)} + \left(3+H^2k^2\right)U\right)\right)^2 dt^4\right) \Big/ \left(64\left(3+H^2k^2\right)^3\right) + O[dt]^5\Bigg)$$

$$dx^3 + \Bigg(-\left(\left(k^5\left(3\sqrt{3}\,g H\left(177 + 124 H^2 k^2 + 20 H^4 k^4\right) + 104\left(9\sqrt{g H\left(3+H^2k^2\right)} + 6 k^2\sqrt{g H^5\left(3+H^2k^2\right)} + k^4\sqrt{g H^9\left(3+H^2k^2\right)}\right)U\right)\right) \Big/ \left(1920\left(\sqrt{g H}\,\left(3+H^2k^2\right)^{5/2}\right)\right)\right) - \frac{1}{11\,520\left(\sqrt{g H}\,\left(3+H^2k^2\right)^{7/2}\right)}$$

$$\left(k^7\left(27\sqrt{3}\,g^2 H^2\left(167 + 124 H^2 k^2 + 20 H^4 k^4\right) +\right.\right.$$

$$
\begin{aligned}
&g\,H\,U\left(21\,429\sqrt{3}\,H^2\,k^2\,U+764\sqrt{3}\,H^6\,k^6\,U+81\left(232\sqrt{g\,H\left(3+H^2\,k^2\right)}+267\sqrt{3}\,U\right)+\right.\\
&\left.24\,k^4\left(84\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+293\sqrt{3}\,H^4\,U\right)\right)+\\
&16\left(459\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+153\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+17\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+\right.\\
&\left.\left.9\,k^2\left(88\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+51\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\right)\Big)\,dt^2-\\
&\frac{1}{3840\left(3+H^2\,k^2\right)^4}i\,k^8\left(54\,g^2\,H^2\left(81+62\,H^2\,k^2+10\,H^4\,k^4\right)+84\,H^8\,k^8\,U^4+\right.\\
&243\,U^3\left(39\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+28\,U\right)+36\,k^4\,U^3\\
&\left(85\sqrt{3}\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+126\,H^4\,U\right)+4\,k^6\,U^3\left(83\sqrt{3}\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}+252\,H^6\,U\right)+\\
&9\,k^2\left(600\sqrt{3}\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+1039\sqrt{3}\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3+1008\,H^2\,U^4\right)+\\
&3\,g\,H\,U\left(13\,500\,H^2\,k^2\,U+472\,H^6\,k^6\,U+27\left(97\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+504\,U\right)+\right.\\
&\left.4\,k^4\left(71\sqrt{3}\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+1101\,H^4\,U\right)\right)\Big)\,dt^3+\\
&\frac{1}{23\,040\sqrt{g\,H}\left(3+H^2\,k^2\right)^{11/2}}k^9\left(\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\\
&\left(4\,k^8\,U^3\left(239\sqrt{3}\,g\,H^9+58\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\,U\right)+\right.\\
&27\,k^2\left(372\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}+2703\sqrt{3}\,g^2\,H^4\,U+4515\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2+\right.\\
&\left.4070\sqrt{3}\,g\,H^3\,U^3+928\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\right)+12\,k^6\,U\left(213\sqrt{3}\,g^2\,H^8+\right.\\
&\left.232\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+g\,H^7\,U\left(349\sqrt{g\,H\left(3+H^2\,k^2\right)}+979\sqrt{3}\,U\right)\right)+\\
&81\left(157\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+883\sqrt{3}\,g^2\,H^2\,U+232\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+\right.\\
&\left.g\,H\,U^2\left(1527\sqrt{g\,H\left(3+H^2\,k^2\right)}+1033\sqrt{3}\,U\right)\right)+\\
&9\,k^4\left(180\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}+2672\sqrt{3}\,g^2\,H^6\,U+1392\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^4+\right.\\
&\left.\left.g\,H^5\,U^2\left(4384\sqrt{g\,H\left(3+H^2\,k^2\right)}+5997\sqrt{3}\,U\right)\right)\right)\Big)\,dt^4+O[dt]^5\Bigg)\,dx^4+\\
&O[dx]^5,\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(-\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right.\\
&\left.\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.\\
&\left.dt^2+\frac{1}{8\left(3+H^2\,k^2\right)^2}\right.\\
&\left.i\,k^4\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^3-\right.\\
&\left.\left(\left(k^5\left(-\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\right.\right.\right.\\
&\left.\left.\left.\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)dt^4\right)\Big/\left(20\left(3+H^2\,k^2\right)^4\right)+
\end{aligned}
$$

$$O[dt]^5\Big) + \Bigg(\frac{k^3\left(3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2\,U\right)}{24\left(3+H^2\,k^2\right)^2} +$$

$$\Big(k^5\Big(3\,g\Big(3\sqrt{3}\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,H^3\,k^2\,U+2\,H^5\,k^4\,U\Big)+$$

$$U^2\Big(-27\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+54\,U+2\,H^6\,k^6\,U-3\,k^2\Big(7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-$$

$$18\,H^2\,U\Big)-2\,k^4\Big(2\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}-9\,H^4\,U\Big)\Big)\Big)\,dt^2\Big)\Big/\Big(48\left(3+H^2\,k^2\right)^3\Big)+$$

$$\Big(i\,k^6\Big(3\,g\,H+U\Big(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\Big)\Big)\Big(-9\,g\,H+$$

$$U\Big(-3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U-2\,k^2\Big(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-6\,H^2\,U\Big)\Big)\Big)$$

$$dt^3\Big)\Big/\Big(48\left(3+H^2\,k^2\right)^3\Big)-\Big(\Big(k^7\Big(3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2\,U\Big)$$

$$\Big(3\,g\,H+U\Big(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\Big)\Big)^2\Big)\,dt^4\Big)\Big/$$

$$\Big(96\left(3+H^2\,k^2\right)^4\Big)+O[dt]^5\Bigg)\,dx^2+\Bigg(\frac{1}{16}\,i\,k^4\Big(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}}-2\,U\Big)-$$

$$\Big(i\,k^6\Big(-3\,g\,H\Big(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-4\left(3+H^2\,k^2\right)U\Big)+U^2\Big(-15\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+$$

$$18\,U+2\,H^4\,k^4\,U+k^2\Big(-5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\Big)\Big)\Big)\,dt^2\Big)\Big/$$

$$\Big(32\left(3+H^2\,k^2\right)^2\Big)+\Big(k^7\Big(3\,g\,H+U\Big(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\Big)\Big)$$

$$\Big(3\,g\,H+U\Big(-3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\Big)\Big)\,dt^3\Big)\Big/$$

$$\Big(32\left(3+H^2\,k^2\right)^2\Big)+\Big(i\,k^8\Big(-\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\Big)$$

$$\Big(3\,g\,H+U\Big(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\Big)\Big)^2\,dt^4\Big)\Big/\Big(64\left(3+H^2\,k^2\right)^3\Big)+O[dt]^5\Bigg)$$

$$dx^3+\Bigg(\Big(k^5\Big(3\sqrt{3}\,g\,H\left(177+124\,H^2\,k^2+20\,H^4\,k^4\right)-$$

$$104\Big(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\Big)\,U\Big)\Big)\Big/$$

$$\Big(1920\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{5/2}\Big)+\frac{1}{11\,520\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}}$$

$$k^7\Big(27\sqrt{3}\,g^2\,H^2\left(167+124\,H^2\,k^2+20\,H^4\,k^4\right)+$$

$$g\,H\,U\Big(21\,429\sqrt{3}\,H^2\,k^2\,U+764\sqrt{3}\,H^6\,k^6\,U+81\Big(-232\sqrt{g\,H\left(3+H^2\,k^2\right)}+267\sqrt{3}\,U\Big)-$$

$$24\,k^4\Big(84\sqrt{g\,H^9\left(3+H^2\,k^2\right)}-293\sqrt{3}\,H^4\,U\Big)\Big)-$$

$$16\Big(459\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+153\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+17\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+$$

$$9\,k^2\Big(88\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+51\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\Big)\Big)\Big)\,dt^2-$$

$$\frac{1}{3840\left(3+H^2k^2\right)^4}i\,k^8\left(54\,g^2H^2\left(81+62\,H^2k^2+10\,H^4k^4\right)+84\,H^8k^8U^4+243\,U^3\right.$$
$$\left(-39\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+28\,U\right)+36\,k^4U^3\left(-85\sqrt{3}\,\sqrt{gH^9\left(3+H^2k^2\right)}+126\,H^4U\right)+$$
$$4\,k^6U^3\left(-83\sqrt{3}\,\sqrt{gH^{13}\left(3+H^2k^2\right)}+252\,H^6U\right)-$$
$$9\,k^2\left(600\sqrt{3}\,\sqrt{g^3H^7\left(3+H^2k^2\right)}\,U+1039\sqrt{3}\,\sqrt{gH^5\left(3+H^2k^2\right)}\,U^3-1008\,H^2U^4\right)+$$
$$3\,gHU\left(13\,500\,H^2k^2U+472\,H^6k^6U+27\left(-97\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+504\,U\right)-\right.$$
$$\left.\left.4\,k^4\left(71\sqrt{3}\,\sqrt{gH^9\left(3+H^2k^2\right)}-1101\,H^4U\right)\right)\right)dt^3-$$
$$\frac{1}{23\,040\left(\sqrt{gH}\,\left(3+H^2k^2\right)^{11/2}\right)}\left(k^9\left(\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}-\left(3+H^2k^2\right)U\right)\right.$$
$$\left(4\,k^8U^3\left(-239\sqrt{3}\,gH^9+58\sqrt{gH^{17}\left(3+H^2k^2\right)}\,U\right)+\right.$$
$$27\,k^2\left(372\sqrt{g^5H^9\left(3+H^2k^2\right)}-2703\sqrt{3}\,g^2H^4U+\right.$$
$$\left.4515\sqrt{g^3H^7\left(3+H^2k^2\right)}\,U^2-4070\sqrt{3}\,gH^3U^3+928\sqrt{gH^5\left(3+H^2k^2\right)}\,U^4\right)+$$
$$9\,k^4\left(180\sqrt{g^5H^{13}\left(3+H^2k^2\right)}-2672\sqrt{3}\,g^2H^6U+1392\sqrt{gH^9\left(3+H^2k^2\right)}\,U^4+gH^5U^2\right.$$
$$\left.\left(4384\sqrt{gH\left(3+H^2k^2\right)}-5997\sqrt{3}\,U\right)\right)+81\left(157\sqrt{g^5H^5\left(3+H^2k^2\right)}-883\sqrt{3}\right.$$
$$\left.g^2H^2U+232\sqrt{gH\left(3+H^2k^2\right)}\,U^4+gHU^2\left(1527\sqrt{gH\left(3+H^2k^2\right)}-1033\sqrt{3}\,U\right)\right)-$$
$$12\,k^6U\left(213\sqrt{3}\,g^2H^8-232\sqrt{gH^{13}\left(3+H^2k^2\right)}\,U^3+\right.$$
$$\left.\left.\left.\left.gH^7U\left(-349\sqrt{gH\left(3+H^2k^2\right)}+979\sqrt{3}\,U\right)\right)\right)\right)dt^4+O[dt]^5\right)dx^4+O[dx]^5\}$$

In[1944]:=
```
Text[Row[{" U < -Sqrt[g*H]  "}]]
Text[Row[{"Fnn and FnG "}]]
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (0) /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnn = FullSimplify[KurFWSeta /. G → 0 /. n → 1]
KfnG = FullSimplify[KurFWSeta /. n → 0 /. G → 1]
Kfnn = Kfnn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]]
KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGn = FullSimplify[KurFWSG /. G → 0 /. n → 1]
KfGG = FullSimplify[KurFWSG /. n → 0 /. G → 1]
KfGn = KfGn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2 + EigvFmat2^2 / 2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}]
```

Out[1944]= $U < -\text{Sqrt}[g*H]$

Out[1945]= Fnn and FnG

Out[1950]= Gnp H + Rpp U

Out[1951]= GGp H

Out[1956]= $$\left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)\,U\,dt}{12\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{1}{8}\,k^4\,U\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(405\,k^5\,U+351\,H^2\,k^7\,U+116\,H^4\,k^9\,U+13\,H^6\,k^{11}\,U\right)\,dt}{240\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1959]= $$\left(-\frac{3\,(k\,w)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(\frac{i\,\left(6\,k^3+H^2\,k^5\right)\,dt}{4\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{i\,\left(-54\,k^5+H^4\,k^9\right)\,dt}{240\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1960]= FGn and FGG

Out[1963]= H (g Rpp + Gnp U)

Out[1964]= (GGp H + Rpp) U

Out[1969]= $$\left(-\frac{\left(k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(9\,g\,H\,k^3+6\,g\,H^3\,k^5+g\,H^5\,k^7+18\,k^3\,U^2+3\,H^2\,k^5\,U^2\right)\,dt}{12\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{1}{8}\,g\,H\,k^4\,dt+O[dt]^4\right)\,dx^3+\left(\left(i\,\left(351\,g\,H\,k^5+351\,g\,H^3\,k^7+117\,g\,H^5\,k^9+13\,g\,H^7\,k^{11}+54\,k^5\,U^2-H^4\,k^9\,U^2\right)\,dt\right)\Big/\left(240\,\left(3+H^2\,k^2\right)^3\right)+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1973]= $$\left(-\frac{\left(k\,\left(6+H^2\,k^2\right)\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(6+H^2\,k^2\right)\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(-9\,k^3+3\,H^2\,k^5+H^4\,k^7\right)\,U\,dt}{12\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{1}{8}\,k^4\,U\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(297\,k^5+351\,H^2\,k^7+118\,H^4\,k^9+13\,H^6\,k^{11}\right)\,U\,dt}{240\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1974]= W : omega

Out[1979]= $$\left\{\left(\frac{1}{6\,\left(3+H^2\,k^2\right)^2}k^3\,\left(\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)\,U\right)\left(3\,g\,H+U\,\left(2\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)\,U\right)\right)\,dt^2+\right.$$

$$\frac{1}{8\left(3+H^2k^2\right)^2}ik^4\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2dt^3-$$
$$\left(\left(k^5\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)^3\right.\right.$$
$$\left.\left.\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right)dt^4\right)\Big/\left(20\left(3+H^2k^2\right)^4\right)+$$
$$O[dt]^5\Bigg)+\Bigg(\frac{k^3\left(-3\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)^2U\right)}{24\left(3+H^2k^2\right)^2}+$$
$$\left(k^5\left(g\left(-9\sqrt{3}H\sqrt{gH\left(3+H^2k^2\right)}+18H^3k^2U+6H^5k^4U\right)+\right.\right.$$
$$U^2\left(27\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+54U+2H^6k^6U+3k^2\left(7\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+\right.\right.$$
$$\left.\left.\left.\left.18H^2U\right)+2k^4\left(2\sqrt{3}\sqrt{gH^9\left(3+H^2k^2\right)}+9H^4U\right)\right)\right)dt^2\right)\Big/\left(48\left(3+H^2k^2\right)^3\right)+$$
$$\left(ik^6\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\left(-9gH+\right.\right.$$
$$\left.\left.U\left(3\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+18U+2H^4k^4U+2k^2\left(\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+6H^2U\right)\right)\right)$$
$$dt^3\Big)\Big/\left(48\left(3+H^2k^2\right)^3\right)-\left(\left(k^7\left(-3\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)^2U\right)\right.\right.$$
$$\left.\left.\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2\right)dt^4\Big)\Big/$$
$$\left.\left(96\left(3+H^2k^2\right)^4\right)+O[dt]^5\right)dx^2+\left(\frac{1}{16}ik^4\left(\sqrt{3}\sqrt{\frac{gH}{3+H^2k^2}}+2U\right)+\right.$$
$$\left(ik^6\left(3gH\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+4\left(3+H^2k^2\right)U\right)+U^2\left(15\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\right.\right.\right.$$
$$\left.\left.\left.18U+2H^4k^4U+k^2\left(5\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+12H^2U\right)\right)\right)dt^2\right)\Big/$$
$$\left(32\left(3+H^2k^2\right)^2\right)-\left(\left(k^7\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right.\right.$$
$$\left.\left.\left(3gH+U\left(3\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)U\right)\right)\right)dt^3\Big)\Big/$$
$$\left(32\left(3+H^2k^2\right)^2\right)-\left(ik^8\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)U\right)\right.$$
$$\left.\left.\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2dt^4\right)\Big/\left(64\left(3+H^2k^2\right)^3\right)+O[dt]^5\Bigg)$$
$$dx^3+\Bigg(-\left(\left(k^5\left(3\sqrt{3}gH\left(177+124H^2k^2+20H^4k^4\right)+\right.\right.\right.$$
$$\left.\left.\left.104\left(9\sqrt{gH\left(3+H^2k^2\right)}+6k^2\sqrt{gH^5\left(3+H^2k^2\right)}+k^4\sqrt{gH^9\left(3+H^2k^2\right)}\right)U\right)\right)\Big/$$
$$\left.\left(1920\left(\sqrt{gH}\left(3+H^2k^2\right)^{5/2}\right)\right)\right)-\frac{1}{11\,520\left(\sqrt{gH}\left(3+H^2k^2\right)^{7/2}\right)}$$
$$\left(k^7\left(27\sqrt{3}g^2H^2\left(167+124H^2k^2+20H^4k^4\right)+\right.\right.$$

$$
\begin{aligned}
&g\,H\,U\left(21\,429\sqrt{3}\,H^2\,k^2\,U+764\sqrt{3}\,H^6\,k^6\,U+81\left(232\sqrt{g\,H\left(3+H^2\,k^2\right)}+267\sqrt{3}\,U\right)+\right.\\
&\left.24\,k^4\left(84\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+293\sqrt{3}\,H^4\,U\right)\right)+\\
&16\left(459\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+153\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+17\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+\right.\\
&\left.\left.\left.9\,k^2\left(88\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+51\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\right)\right)\,dt^2-\\
&\frac{1}{3840\left(3+H^2\,k^2\right)^4}i\,k^8\left(54\,g^2\,H^2\left(81+62\,H^2\,k^2+10\,H^4\,k^4\right)+84\,H^8\,k^8\,U^4+\right.\\
&243\,U^3\left(39\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+28\,U\right)+36\,k^4\,U^3\\
&\left(85\sqrt{3}\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+126\,H^4\,U\right)+4\,k^6\,U^3\left(83\sqrt{3}\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}+252\,H^6\,U\right)+\\
&9\,k^2\left(600\sqrt{3}\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+1039\sqrt{3}\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3+1008\,H^2\,U^4\right)+\\
&3\,g\,H\,U\left(13\,500\,H^2\,k^2\,U+472\,H^6\,k^6\,U+27\left(97\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+504\,U\right)+\right.\\
&\left.\left.4\,k^4\left(71\sqrt{3}\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+1101\,H^4\,U\right)\right)\right)\,dt^3+\\
&\frac{1}{23\,040\sqrt{g\,H}\left(3+H^2\,k^2\right)^{11/2}}k^9\left(\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\\
&\left(4\,k^8\,U^3\left(239\sqrt{3}\,g\,H^9+58\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\,U\right)+\right.\\
&27\,k^2\left(372\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}+2703\sqrt{3}\,g^2\,H^4\,U+4515\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2+\right.\\
&\left.4070\sqrt{3}\,g\,H^3\,U^3+928\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\right)+12\,k^6\,U\left(213\sqrt{3}\,g^2\,H^8+\right.\\
&\left.232\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+g\,H^7\,U\left(349\sqrt{g\,H\left(3+H^2\,k^2\right)}+979\sqrt{3}\,U\right)\right)+\\
&81\left(157\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+883\sqrt{3}\,g^2\,H^2\,U+232\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+\right.\\
&\left.g\,H\,U^2\left(1527\sqrt{g\,H\left(3+H^2\,k^2\right)}+1033\sqrt{3}\,U\right)\right)+\\
&9\,k^4\left(180\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}+2672\sqrt{3}\,g^2\,H^6\,U+1392\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^4+\right.\\
&\left.\left.\left.g\,H^5\,U^2\left(4384\sqrt{g\,H\left(3+H^2\,k^2\right)}+5997\sqrt{3}\,U\right)\right)\right)\,dt^4+O[dt]^5\right)\,dx^4+\\
&O[dx]^5,\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(-\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right.\\
&\left.\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.\\
&\left.dt^2+\frac{1}{8\left(3+H^2\,k^2\right)^2}\right.\\
&i\,k^4\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\,dt^3-\\
&\left(\left(k^5\left(-\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\right.\right.\\
&\left.\left.\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)dt^4\right)\Big/\left(20\left(3+H^2\,k^2\right)^4\right)+
\end{aligned}
$$

$$O[dt]^5\Big) + \Bigg(\frac{k^3 \left(3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 2\left(3+H^2\,k^2\right)^2 U\right)}{24\left(3+H^2\,k^2\right)^2} +$$

$$\Big(k^5\Big(3\,g\Big(3\sqrt{3}\,H\sqrt{g\,H\left(3+H^2\,k^2\right)} + 6\,H^3\,k^2\,U + 2\,H^5\,k^4\,U\Big) +$$

$$U^2\Big(-27\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 54\,U + 2\,H^6\,k^6\,U - 3\,k^2\Big(7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} -$$

$$18\,H^2\,U\Big) - 2\,k^4\Big(2\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)} - 9\,H^4\,U\Big)\Big)\Big)\,dt^2\Big) \Big/ \Big(48\left(3+H^2\,k^2\right)^3\Big) +$$

$$\Big(i\,k^6\Big(3\,g\,H + U\Big(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\Big)\Big)\Big(-9\,g\,H +$$

$$U\Big(-3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 18\,U + 2\,H^4\,k^4\,U - 2\,k^2\Big(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} - 6\,H^2\,U\Big)\Big)\Big)$$

$$dt^3\Big) \Big/ \Big(48\left(3+H^2\,k^2\right)^3\Big) - \Big(\Big(k^7\Big(3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 2\left(3+H^2\,k^2\right)^2 U\Big)$$

$$\Big(3\,g\,H + U\Big(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\Big)\Big)^2\Big)\,dt^4\Big) \Big/$$

$$\Big(96\left(3+H^2\,k^2\right)^4\Big) + O[dt]^5\Bigg)\,dx^2 + \Bigg(-\frac{1}{16}\,i\,k^4\Big(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}} - 2\,U\Big) +$$

$$\Big(i\,k^6\Big(-3\,g\,H\Big(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} - 4\left(3+H^2\,k^2\right)U\Big) + U^2\Big(-15\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} +$$

$$18\,U + 2\,H^4\,k^4\,U + k^2\Big(-5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + 12\,H^2\,U\Big)\Big)\Big)\,dt^2\Big) \Big/$$

$$\Big(32\left(3+H^2\,k^2\right)^2\Big) - \Big(\Big(k^7\Big(3\,g\,H + U\Big(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\Big)\Big)$$

$$\Big(3\,g\,H + U\Big(-3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 2\left(3+H^2\,k^2\right)U\Big)\Big)\Big)\,dt^3\Big) \Big/$$

$$\Big(32\left(3+H^2\,k^2\right)^2\Big) - \Big(i\,k^8\Big(-\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 2\left(3+H^2\,k^2\right)U\Big)$$

$$\Big(3\,g\,H + U\Big(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\Big)\Big)^2\,dt^4\Big) \Big/ \Big(64\left(3+H^2\,k^2\right)^3\Big) + O[dt]^5\Bigg)$$

$$dx^3 + \Bigg(\Big(k^5\Big(3\sqrt{3}\,g\,H\left(177 + 124\,H^2\,k^2 + 20\,H^4\,k^4\right) -$$

$$104\Big(9\sqrt{g\,H\left(3+H^2\,k^2\right)} + 6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\Big)U\Big)\Big) \Big/$$

$$\Big(1920\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{5/2}\Big) + \frac{1}{11\,520\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}}$$

$$k^7\Big(27\sqrt{3}\,g^2\,H^2\left(167 + 124\,H^2\,k^2 + 20\,H^4\,k^4\right) +$$

$$g\,H\,U\Big(21\,429\sqrt{3}\,H^2\,k^2\,U + 764\sqrt{3}\,H^6\,k^6\,U + 81\Big(-232\sqrt{g\,H\left(3+H^2\,k^2\right)} + 267\sqrt{3}\,U\Big) -$$

$$24\,k^4\Big(84\sqrt{g\,H^9\left(3+H^2\,k^2\right)} - 293\sqrt{3}\,H^4\,U\Big)\Big) -$$

$$16\Big(459\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + 153\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3 + 17\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3 +$$

$$9\,k^2\Big(88\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U + 51\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\Big)\Big)\Big)\,dt^2 -$$

$$\frac{1}{3840\left(3+H^2k^2\right)^4}i\,k^8\left(54\,g^2H^2\left(81+62\,H^2k^2+10\,H^4k^4\right)+84\,H^8k^8U^4+243\,U^3\right.$$
$$\left(-39\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+28\,U\right)+36\,k^4U^3\left(-85\sqrt{3}\,\sqrt{gH^9\left(3+H^2k^2\right)}+126\,H^4U\right)+$$
$$4\,k^6U^3\left(-83\sqrt{3}\,\sqrt{gH^{13}\left(3+H^2k^2\right)}+252\,H^6U\right)-$$
$$9\,k^2\left(600\sqrt{3}\,\sqrt{g^3H^7\left(3+H^2k^2\right)}\,U+1039\sqrt{3}\,\sqrt{gH^5\left(3+H^2k^2\right)}\,U^3-1008\,H^2U^4\right)+$$
$$3\,gHU\left(13\,500\,H^2k^2U+472\,H^6k^6U+27\left(-97\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+504\,U\right)-\right.$$
$$\left.\left.4\,k^4\left(71\sqrt{3}\,\sqrt{gH^9\left(3+H^2k^2\right)}-1101\,H^4U\right)\right)\right)dt^3-$$
$$\frac{1}{23\,040\left(\sqrt{gH}\,\left(3+H^2k^2\right)^{11/2}\right)}\left(k^9\left(\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}-\left(3+H^2k^2\right)U\right)\right.$$
$$\left(4\,k^8U^3\left(-239\sqrt{3}\,gH^9+58\sqrt{gH^{17}\left(3+H^2k^2\right)}\,U\right)+\right.$$
$$27\,k^2\left(372\sqrt{g^5H^9\left(3+H^2k^2\right)}-2703\sqrt{3}\,g^2H^4U+\right.$$
$$\left.4515\sqrt{g^3H^7\left(3+H^2k^2\right)}\,U^2-4070\sqrt{3}\,gH^3U^3+928\sqrt{gH^5\left(3+H^2k^2\right)}\,U^4\right)+$$
$$9\,k^4\left(180\sqrt{g^5H^{13}\left(3+H^2k^2\right)}-2672\sqrt{3}\,g^2H^6U+1392\sqrt{gH^9\left(3+H^2k^2\right)}\,U^4+gH^5U^2\right.$$
$$\left.\left(4384\sqrt{gH\left(3+H^2k^2\right)}-5997\sqrt{3}\,U\right)\right)+81\left(157\sqrt{g^5H^5\left(3+H^2k^2\right)}-883\sqrt{3}\right.$$
$$\left.g^2H^2U+232\sqrt{gH\left(3+H^2k^2\right)}\,U^4+gHU^2\left(1527\sqrt{gH\left(3+H^2k^2\right)}-1033\sqrt{3}\,U\right)\right)-$$
$$12\,k^6U\left(213\sqrt{3}\,g^2H^8-232\sqrt{gH^{13}\left(3+H^2k^2\right)}\,U^3+\right.$$
$$\left.\left.\left.\left.gH^7U\left(-349\sqrt{gH\left(3+H^2k^2\right)}+979\sqrt{3}\,U\right)\right)\right)\right)dt^4+O[dt]^5\right)dx^4+O[dx]^5\}$$