

$$\text{In[112]:= FGn1FDdt} = - \frac{(k (3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2 (3 + H^2 k^2)};$$

$$- \frac{dt^2 k (gH (3 + H^2 k^2) - 3 U^2) w}{2 (3 + H^2 k^2)};$$

$$\text{FGn1FDdtRed} = -dt^2 k / 2 w \left( gH - \frac{3 U^2}{(3 + H^2 k^2)} \right)$$

$$\text{FGn1FDdxdt} = - \frac{1}{2} \left( \sqrt{g H} k^2 U \right) dt * dx$$

$$\text{FGn1FDdx} = 0;$$

$$\text{Out[114]=} - \frac{1}{2} dt^2 k \left( gH - \frac{3 U^2}{3 + H^2 k^2} \right) w$$

$$\text{Out[115]=} - \frac{1}{2} dt dx \sqrt{g H} k^2 U$$

$$\text{In[117]:= FGn2FDdt} = - \frac{(k (3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2 (3 + H^2 k^2)};$$

$$\text{FGn2FDdtRed} = -dt^2 k / 2 w \left( gH - \frac{3 U^2}{(3 + H^2 k^2)} \right)$$

$$\text{FGn2FDdxdt} = - \frac{i (9 g H k^3 + 6 g H^3 k^5 + g H^5 k^7 + 18 k^3 U^2 + 3 H^2 k^5 U^2) dt}{12 (3 + H^2 k^2)^2} dx^2;$$

$$\text{Expand}[(3 + H^2 k^2)^2];$$

$$- \frac{i (g H k^3 (3 + H^2 k^2)^2 + 18 k^3 U^2 + 3 H^2 k^5 U^2) dt}{12 (3 + H^2 k^2)^2} dx^2;$$

$$- \frac{i (g H k^3 (3 + H^2 k^2)^2 + k^3 U^2 (18 + 3 H^2 k^2)) dt}{12 (3 + H^2 k^2)^2} dx^2;$$

$$\text{FGn2FDdxdtRed} = - (i * dt * k^3 * dx^2 / 12) * \left( gH + \frac{U^2 (18 + 3 H^2 k^2)}{(3 + H^2 k^2)^2} \right)$$

$$\text{FGn2FDdx} = 0;$$

$$\text{Out[118]=} - \frac{1}{2} dt^2 k \left( gH - \frac{3 U^2}{3 + H^2 k^2} \right) w$$

$$\text{Out[123]=} - \frac{1}{12} dt dx^2 i k^3 \left( gH + \frac{(18 + 3 H^2 k^2) U^2}{(3 + H^2 k^2)^2} \right)$$

$$\begin{aligned}
\text{In[125]:= } \mathbf{FGn2FEMdt} &= - \frac{\left( \mathbf{k} \left( 3 \mathbf{g} \mathbf{H} + \mathbf{g} \mathbf{H}^3 \mathbf{k}^2 - 3 \mathbf{U}^2 \right) \mathbf{w} \right) \mathbf{dt}^2}{2 \left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)}; \\
&- \frac{\mathbf{dt}^2 \mathbf{k} \left( \mathbf{g} * \mathbf{H} * \left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right) - 3 \mathbf{U}^2 \right) \mathbf{w}}{2 \left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)}; \\
\mathbf{FGn2FEMdtRed} &= - \mathbf{dt}^2 \mathbf{k} / 2 \mathbf{w} \left( \mathbf{gH} - \frac{3 \mathbf{U}^2}{\left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)} \right) \\
\mathbf{FGn2FEMdxdt} &= - \frac{\mathbf{i} \left( 90 \mathbf{g} \mathbf{H} \mathbf{k}^3 + 60 \mathbf{g} \mathbf{H}^3 \mathbf{k}^5 + 10 \mathbf{g} \mathbf{H}^5 \mathbf{k}^7 - 36 \mathbf{k}^3 \mathbf{U}^2 - 15 \mathbf{H}^2 \mathbf{k}^5 \mathbf{U}^2 \right) \mathbf{dt}}{120 \left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)^2} \mathbf{dx}^2; \\
&- \frac{\mathbf{i} \mathbf{dt} \mathbf{dx}^2 \left( 10 \mathbf{g} \mathbf{H} \mathbf{k}^3 \left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)^2 - \mathbf{k}^3 \mathbf{U}^2 \left( 36 + 15 \mathbf{H}^2 \mathbf{k}^2 \right) \right)}{120 \left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)^2}; \\
\mathbf{FGn2FEMdxdtRed} &= - \left( \mathbf{i} * \mathbf{dt} * \mathbf{k}^3 * \mathbf{dx}^2 / 12 \right) * \left( \mathbf{gH} - \frac{\mathbf{U}^2 \left( 36 + 15 \mathbf{H}^2 \mathbf{k}^2 \right)}{10 \left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)^2} \right) \\
\mathbf{FGn2FEMdx} &= 0; \\
\text{Out[127]= } &- \frac{1}{2} \mathbf{dt}^2 \mathbf{k} \left( \mathbf{gH} - \frac{3 \mathbf{U}^2}{3 + \mathbf{H}^2 \mathbf{k}^2} \right) \mathbf{w} \\
\text{Out[130]= } &- \frac{1}{12} \mathbf{dt} \mathbf{dx}^2 \mathbf{i} \mathbf{k}^3 \left( \mathbf{gH} - \frac{\left( 36 + 15 \mathbf{H}^2 \mathbf{k}^2 \right) \mathbf{U}^2}{10 \left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)^2} \right) \\
\text{In[132]:= } \mathbf{FGn3FDdt} &= - \frac{\left( \mathbf{k} \left( 3 \mathbf{g} \mathbf{H} + \mathbf{g} \mathbf{H}^3 \mathbf{k}^2 - 3 \mathbf{U}^2 \right) \mathbf{w} \right) \mathbf{dt}^2}{2 \left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)}; \\
\mathbf{FGn3FDdtRed} &= - \mathbf{dt}^2 \mathbf{k} / 2 \mathbf{w} \left( \mathbf{gH} - \frac{3 \mathbf{U}^2}{\left( 3 + \mathbf{H}^2 \mathbf{k}^2 \right)} \right) \\
\mathbf{FGn3FDdxdt} &= - \frac{1}{12} \left( \sqrt{\mathbf{g} \mathbf{H}} \mathbf{k}^4 \mathbf{U} \right) \mathbf{dt} \mathbf{dx}^3 \\
\mathbf{FGn3FDdx} &= 0; \\
\text{Out[133]= } &- \frac{1}{2} \mathbf{dt}^2 \mathbf{k} \left( \mathbf{gH} - \frac{3 \mathbf{U}^2}{3 + \mathbf{H}^2 \mathbf{k}^2} \right) \mathbf{w} \\
\text{Out[134]= } &- \frac{1}{12} \mathbf{dt} \mathbf{dx}^3 \sqrt{\mathbf{g} \mathbf{H}} \mathbf{k}^4 \mathbf{U}
\end{aligned}$$