In[88]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx / 2, xj + dx / 2}] / (dx);
qjnbar = qjbar /. t → tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x → xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx / 2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / 3 * k^2;
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca;
fn1Gca0A = fn1A /. Gca → 0;
fn1eca0A = fn1A /. eca → 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca /. Gh → RA * Gca;
fG1Gca0A = fG1A /. Gca → 0;
fG1eca0A = fG1A /. eca → 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
WfrommatA =
  Simplify[1 - MtA * dt * Eigenvalues[MatA], {k > 0, H > 0, g > 0, dx > 0, dt > 0}];
```

$$wAp = U * k + \frac{\sqrt{3}\ k \sqrt{g\,H\,\left(3 + H^2\,k^2\right)}}{3 + H^2\,k^2};$$

$$wAm = U * k - \frac{\sqrt{3}\ k \sqrt{g\,H\,\left(3 + H^2\,k^2\right)}}{3 + H^2\,k^2};$$

In[124]:=
```
M2 = 1
Series[M2 - MA, {dx, 0, 10}]
```

Out[124]= 1

Out[125]= $$-\frac{k^2\,dx^2}{24} - \frac{7\,k^4\,dx^4}{5760} - \frac{31\,k^6\,dx^6}{967\,680} - \frac{127\,k^8\,dx^8}{154\,828\,800} - \frac{73\,k^{10}\,dx^{10}}{3\,503\,554\,560} + O[dx]^{11}$$

In[126]:=
```
Rm = (1 + I * Sin[k * dx] / 2)
Series[Rm - RA, {dx, 0, 4}]
Rp = Exp[I * k * dx] * (1 - I * Sin[k * dx] / 2)
Series[Rp - RA, {dx, 0, 4}]
```

Out[126]= $$1 + \frac{1}{2}\,i\,\text{Sin}[dx\,k]$$

Out[127]= $$\frac{k^2\,dx^2}{12} - \frac{1}{12}\,i\,k^3\,dx^3 + \frac{k^4\,dx^4}{720} + O[dx]^5$$

Out[128]= $$e^{i\,dx\,k}\left(1 - \frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)$$

Out[129]= $$\frac{k^2\,dx^2}{12} + \frac{1}{6}\,i\,k^3\,dx^3 - \frac{89\,k^4\,dx^4}{720} + O[dx]^5$$

In[130]:=
```
GRHSp1 = -Exp[-I * k * dx / 2] + 2 + 4 * Exp[I * k * dx / 2] +
  Exp[I * k * dx] * (4 * Exp[-I * k * dx / 2] + 2 - Exp[I * k * dx / 2])
GRHSp1 = GRHSp1 / Exp[I * k * dx / 2];
GRHSp1 = Expand[GRHSp1];
GRHSp1 = ExpToTrig[GRHSp1]
GRHSp2 = Exp[-I * k * dx / 2] - 8 + 7 * Exp[I * k * dx / 2] +
  Exp[I * k * dx] * (7 * Exp[-I * k * dx / 2] - 8 + Exp[I * k * dx / 2])
GRHSp2 = GRHSp2 / Exp[I * k * dx / 2];
GRHSp2 = Expand[GRHSp2];
GRHSp2 = ExpToTrig[GRHSp2]
```

Out[130]= $2 - e^{-\frac{1}{2} i\, dx\, k} + 4\, e^{\frac{i\, dx\, k}{2}} + e^{i\, dx\, k} \left(2 + 4\, e^{-\frac{1}{2} i\, dx\, k} - e^{\frac{i\, dx\, k}{2}}\right)$

Out[133]= $8 + 4 \operatorname{Cos}\left[\frac{dx\, k}{2}\right] - 2 \operatorname{Cos}[dx\, k]$

Out[134]= $-8 + e^{-\frac{1}{2} i\, dx\, k} + 7\, e^{\frac{i\, dx\, k}{2}} + e^{i\, dx\, k} \left(-8 + 7\, e^{-\frac{1}{2} i\, dx\, k} + e^{\frac{i\, dx\, k}{2}}\right)$

Out[137]= $14 - 16 \operatorname{Cos}\left[\frac{dx\, k}{2}\right] + 2 \operatorname{Cos}[dx\, k]$

In[138]:=
```
GGLHS = dx / 6 * (Rp + Rm)
GG2 = GGLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2)
Series[GG2, {dx, 0, 3}];
Series[GGA, {dx, 0, 3}];
Series[GG2 - GGA, {dx, 0, 5}]
```

Out[138]= $\frac{1}{6} dx \left(1 + e^{i\, dx\, k} \left(1 - \frac{1}{2} i \operatorname{Sin}[dx\, k]\right) + \frac{1}{2} i \operatorname{Sin}[dx\, k]\right)$

Out[139]= $\left(dx \left(1 + e^{i\, dx\, k} \left(1 - \frac{1}{2} i \operatorname{Sin}[dx\, k]\right) + \frac{1}{2} i \operatorname{Sin}[dx\, k]\right)\right) \Big/ \left(6 \left(\frac{1}{30} dx\, H \left(8 + 4 \operatorname{Cos}\left[\frac{dx\, k}{2}\right] - 2 \operatorname{Cos}[dx\, k]\right) + \frac{H^3 \left(14 - 16 \operatorname{Cos}\left[\frac{dx\, k}{2}\right] + 2 \operatorname{Cos}[dx\, k]\right)}{9\, dx}\right)\right)$

Out[142]= $\frac{\left(12 k^2 + 5 H^2 k^4\right) dx^2}{40 H \left(3 + H^2 k^2\right)^2} + \frac{i \left(12 k^3 + 5 H^2 k^5\right) dx^3}{80 H \left(3 + H^2 k^2\right)^2} + \frac{\left(-6651 k^4 - 4680 H^2 k^6 - 820 H^4 k^8\right) dx^4}{4800 H \left(3 + H^2 k^2\right)^3} - \frac{i \left(6291 k^5 + 4410 H^2 k^7 + 770 H^4 k^9\right) dx^5}{9600 H \left(3 + H^2 k^2\right)^3} + O[dx]^6$

In[143]:=
```
GnLHS = -U * (dx / 6) * (Rp + Rm)
Gn2 = GnLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2)
Series[Gn2, {dx, 0, 3}];
Series[GnA, {dx, 0, 3}];
Series[Gn2 - GnA, {dx, 0, 5}]
```

Out[143]= $-\frac{1}{6}\,dx\,U\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)$

Out[144]= $-\left(\left(dx\,U\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)\right)\Big/\left(6\left(\frac{1}{30}\,dx\,H\left(8+4\,\text{Cos}\left[\frac{dx\,k}{2}\right]-2\,\text{Cos}[dx\,k]\right)+\frac{H^3\left(14-16\,\text{Cos}\left[\frac{dx\,k}{2}\right]+2\,\text{Cos}[dx\,k]\right)}{9\,dx}\right)\right)\right)$

Out[147]= $-\frac{\left(\left(12\,k^2+5\,H^2\,k^4\right)U\right)dx^2}{40\left(H\left(3+H^2\,k^2\right)^2\right)}-\frac{i\left(12\,k^3+5\,H^2\,k^5\right)U\,dx^3}{80\,H\left(3+H^2\,k^2\right)^2}+\frac{\left(6651\,k^4+4680\,H^2\,k^6+820\,H^4\,k^8\right)U\,dx^4}{4800\,H\left(3+H^2\,k^2\right)^3}+\frac{i\left(6291\,k^5+4410\,H^2\,k^7+770\,H^4\,k^9\right)U\,dx^5}{9600\,H\left(3+H^2\,k^2\right)^3}+O[dx]^6$

In[148]:=
```
GcLHS = -U * H * dx / 3
Gc2 = GcLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2)
Series[Gc2, {dx, 0, 3}]
Series[GcA, {dx, 0, 3}]
Series[Gc2 - GcA, {dx, 0, 5}]
```

Out[148]= $-\frac{1}{3}\,dx\,H\,U$

Out[149]= $-\left((dx\,H\,U)\Big/\left(3\left(\frac{1}{30}\,dx\,H\left(8+4\,\text{Cos}\left[\frac{dx\,k}{2}\right]-2\,\text{Cos}[dx\,k]\right)+\frac{H^3\left(14-16\,\text{Cos}\left[\frac{dx\,k}{2}\right]+2\,\text{Cos}[dx\,k]\right)}{9\,dx}\right)\right)\right)$

Out[150]= $-\frac{3\,U}{3+H^2\,k^2}+\frac{\left(18\,k^2+5\,H^2\,k^4\right)U\,dx^2}{40\left(3+H^2\,k^2\right)^2}+O[dx]^4$

Out[151]= $-\frac{H\,U}{H+\frac{H^3\,k^2}{3}}$

Out[152]= $\frac{\left(18\,k^2+5\,H^2\,k^4\right)U\,dx^2}{40\left(3+H^2\,k^2\right)^2}-\frac{\left(\left(423\,k^4+200\,H^2\,k^6+20\,H^4\,k^8\right)U\right)dx^4}{1600\left(3+H^2\,k^2\right)^3}+O[dx]^6$

In[153]:=
```
fnn2 = H * Gn2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * fnn2
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
fnG2 = H * GG2 ;
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * fnG2
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
```

Out[154]=
$$-\frac{1}{dx}dt\left(1-e^{-i\,dx\,k}\right)\left(-\frac{1}{2}\sqrt{g\,H}\left(-1+e^{i\,dx\,k}\left(1-\frac{1}{2}i\,\text{Sin}[dx\,k]\right)-\frac{1}{2}i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}U\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}i\,\text{Sin}[dx\,k]\right)-\frac{dx\,H\,U\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}i\,\text{Sin}[dx\,k]\right)}{6\left(\frac{1}{30}dx\,H\left(8+4\,\text{Cos}\left[\frac{dx\,k}{2}\right]-2\,\text{Cos}[dx\,k]\right)+\frac{H^3\left(14-16\,\text{Cos}\left[\frac{dx\,k}{2}\right]+2\,\text{Cos}[dx\,k]\right)}{9\,dx}\right)}\right)$$

Out[156]=
$$\left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(-\frac{1}{8}\left(\sqrt{g\,H}\,k^4\right)dt+O[dt]^4\right)dx^3+\left(\frac{i\left(729\,k^5\,U+2610\,H^2\,k^7\,U+1570\,H^4\,k^9\,U+260\,H^6\,k^{11}\,U\right)dt}{4800\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)dx^4+O[dx]^5$$

Out[158]=
$$-\frac{dt\left(1-e^{-i\,dx\,k}\right)H\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}i\,\text{Sin}[dx\,k]\right)}{6\left(\frac{1}{30}dx\,H\left(8+4\,\text{Cos}\left[\frac{dx\,k}{2}\right]-2\,\text{Cos}[dx\,k]\right)+\frac{H^3\left(14-16\,\text{Cos}\left[\frac{dx\,k}{2}\right]+2\,\text{Cos}[dx\,k]\right)}{9\,dx}\right)}$$

Out[160]=
$$\left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(12\,k^3+5\,H^2\,k^5\right)dt}{40\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(\frac{i\left(6291\,k^5+4410\,H^2\,k^7+770\,H^4\,k^9\right)dt}{4800\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)dx^4+O[dx]^5$$

In[161]:=
```
fGn2 = U * H * Gn2 + g * H * (Rm + Rp) / 2 + (U * Sqrt[g * H]) / (2) * (Rm - Rp);
FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * fGn2
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * fGG2
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}]
```

Out[162]=
$$-\frac{1}{dx}dt\left(1-e^{-i\,dx\,k}\right)\left(\frac{1}{2}\sqrt{g\,H}\;U\left(1-e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,g\,H\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)-\frac{dx\,H\,U^2\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)}{6\left(\frac{1}{30}\,dx\,H\left(8+4\,\text{Cos}\left[\frac{dx\,k}{2}\right]-2\,\text{Cos}[dx\,k]\right)+\frac{H^3\left(14-16\,\text{Cos}\left[\frac{dx\,k}{2}\right]+2\,\text{Cos}[dx\,k]\right)}{9\,dx}\right)}\right)$$

Out[164]=
$$\left(-\frac{\left(k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(90\,g\,H\,k^3+60\,g\,H^3\,k^5+10\,g\,H^5\,k^7-36\,k^3\,U^2-15\,H^2\,k^5\,U^2\right)dt}{120\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(-\frac{1}{8}\left(\sqrt{g\,H}\;k^4\,U\right)dt+O[dt]^4\right)dx^3+\left(\frac{1}{4800\left(3+H^2\,k^2\right)^3}i\left(7020\,g\,H\,k^5+7020\,g\,H^3\,k^7+2340\,g\,H^5\,k^9+260\,g\,H^7\,k^{11}-6291\,k^5\,U^2-4410\,H^2\,k^7\,U^2-770\,H^4\,k^9\,U^2\right)dt+O[dt]^4\right)dx^4+O[dx]^5$$

Out[166]=
$$-\frac{1}{dx}dt\left(1-e^{-i\,dx\,k}\right)\left(-\frac{1}{2}\sqrt{g\,H}\;\left(-1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,U\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{dx\,H\,U\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)}{6\left(\frac{1}{30}\,dx\,H\left(8+4\,\text{Cos}\left[\frac{dx\,k}{2}\right]-2\,\text{Cos}[dx\,k]\right)+\frac{H^3\left(14-16\,\text{Cos}\left[\frac{dx\,k}{2}\right]+2\,\text{Cos}[dx\,k]\right)}{9\,dx}\right)}\right)$$

Out[168]=
$$\left(-\frac{\left(k\left(6+H^2\,k^2\right)U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(6+H^2\,k^2\right)U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+$$
$$\left(-\frac{i\left(126\,k^3\,U+75\,H^2\,k^5\,U+10\,H^4\,k^7\,U\right)dt}{120\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(-\frac{1}{8}\left(\sqrt{g\,H}\;k^4\right)dt+O[dt]^4\right)dx^3+$$
$$\left(\left(i\left(13\,311\,k^5\,U+11\,430\,H^2\,k^7\,U+3110\,H^4\,k^9\,U+260\,H^6\,k^{11}\,U\right)dt\right)\Big/\left(4800\left(3+H^2\,k^2\right)^3\right)+O[dt]^4\right)$$
$$dx^4+O[dx]^5$$

In[169]:=
```
Fmat2 = {{Fnn2 , FnG2}, {FGn2 , FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];
Simplify[Series[EigvFmat2 , {dx, 0, 5}]];

RKStep = Log[ 1 + EigvFmat2 + (EigvFmat2)^2/2] / (I*dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Refine[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ]
```

Out[174]=
$$\left\{\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)\right.$$
$$\left(3\,g\,H+2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+3\,U^2+H^2\,k^2\,U^2\right)dt^2+\frac{1}{8\left(3+H^2\,k^2\right)^3}$$
$$i\,k^4\left(\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)^2\left(3\,g\,H+2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+3\,U^2+H^2\,k^2\,U^2\right)$$
$$dt^3-\left(\left(k^5\left(\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)^3\right.\right.$$
$$\left.\left.\left(3\,g\,H+2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+3\,U^2+H^2\,k^2\,U^2\right)\right)dt^4\right)\Big/\left(20\left(3+H^2\,k^2\right)^4\right)+O[dt]^5\Bigg)+$$
$$\left(\left(42\sqrt{3}\;k^3\sqrt{g\,H\left(3+H^2\,k^2\right)}+15\sqrt{3}\;H^2\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,k^3\,U+\right.\right.$$
$$\left.120\,H^2\,k^5\,U+20\,H^4\,k^7\,U\right)\Big/\left(240\left(3+H^2\,k^2\right)^2\right)+\frac{1}{480\left(3+H^2\,k^2\right)^3}$$
$$k^5\left(126\sqrt{3}\;g\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}+45\sqrt{3}\;g\,H^3\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}+1296\,g\,H\,U+882\,g\,H^3\right.$$
$$k^2\,U+150\,g\,H^5\,k^4\,U+486\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+327\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+$$
$$\left.55\sqrt{3}\;H^4\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+540\,U^3+540\,H^2\,k^2\,U^3+180\,H^4\,k^4\,U^3+20\,H^6\,k^6\,U^3\right)dt^2+$$
$$\frac{1}{480\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+3\,U^2+H^2\,k^2\,U^2\right)$$
$$\left(126\,g\,H+45\,g\,H^3\,k^2+102\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+35\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+\right.$$
$$\left.180\,U^2+120\,H^2\,k^2\,U^2+20\,H^4\,k^4\,U^2\right)dt^3-\frac{1}{960\left(3+H^2\,k^2\right)^5}$$
$$\left(k^7\left(\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)^2\left(42\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\right.\right.$$
$$\left.\left.15\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+120\,H^2\,k^2\,U+20\,H^4\,k^4\,U\right)\right.$$

$$\left(3\,g\,H+2\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)\Big)\,dt^4+O[dt]^5\Big)\,dx^2+$$

$$\Bigg(i\left(-\frac{1}{8}\sqrt{g\,H}\,k^4-\frac{\sqrt{3}\,k^4\,U}{16\sqrt{3+H^2\,k^2}}\right)-\frac{1}{32\left(3+H^2\,k^2\right)\sqrt{g\,H\left(3+H^2\,k^2\right)}}$$

$$i\left(6\,g^2\,H^2\,k^6\sqrt{3+H^2\,k^2}+15\sqrt{3}\,g\,H\sqrt{g\,H}\,k^6\,U+4\sqrt{3}\,g\,H^3\sqrt{g\,H}\,k^8\,U+12\,g\,H\,k^6\right.$$

$$\left.\sqrt{3+H^2\,k^2}\,U^2+2\,g\,H^3\,k^8\sqrt{3+H^2\,k^2}\,U^2+3\sqrt{3}\,\sqrt{g\,H}\,k^6\,U^3+\sqrt{3}\,H^2\sqrt{g\,H}\,k^8\,U^3\right)dt^2+$$

$$\frac{1}{32\left(3+H^2\,k^2\right)^2\sqrt{g\,H\left(3+H^2\,k^2\right)}}k^7\left(3\,g\,H+2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)$$

$$\left(6\sqrt{3}\,g\,H\sqrt{g\,H}+2\sqrt{3}\,g\,H^3\sqrt{g\,H}\,k^2+9\,g\,H\sqrt{3+H^2\,k^2}\,U+\right.$$

$$\left.2\,g\,H^3\,k^2\sqrt{3+H^2\,k^2}\,U+3\sqrt{3}\,\sqrt{g\,H}\,U^2+\sqrt{3}\,H^2\sqrt{g\,H}\,k^2\,U^2\right)dt^3+$$

$$\frac{1}{64\,g\,H\left(3+H^2\,k^2\right)^4}i\,k^8\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)^2$$

$$\left(6\,g\,H\sqrt{g\,H}+2\,g\,H^3\sqrt{g\,H}\,k^2+\sqrt{3}\,g\,H\sqrt{3+H^2\,k^2}\,U\right)$$

$$\left(3\,g\,H+2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)dt^4+O[dt]^5\Big)\,dx^3+$$

$$\Bigg(-\Bigg(\Big(17\,856\sqrt{3}\,g\,H\,k^5+12\,180\sqrt{3}\,g\,H^3\,k^7+2075\sqrt{3}\,g\,H^5\,k^9+18\,720\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+$$

$$12\,480\,H^2\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+2080\,H^4\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\Big)\Big/$$

$$\left(38\,400\left(\left(3+H^2\,k^2\right)^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)\right)\Bigg)-$$

$$\Bigg(\Big(k^7\left(150\,120\sqrt{3}\,g^2\,H^2+102\,060\sqrt{3}\,g^2\,H^4\,k^2+17\,325\sqrt{3}\,g^2\,H^6\,k^4+\right.$$

$$449\,064\,g\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+303\,120\,g\,H^3\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+$$

$$51\,120\,g\,H^5\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+445\,824\sqrt{3}\,g\,H\,U^2+$$

$$447\,588\sqrt{3}\,g\,H^3\,k^2\,U^2+149\,775\sqrt{3}\,g\,H^5\,k^4\,U^2+16\,705\sqrt{3}\,g\,H^7\,k^6\,U^2+$$

$$146\,880\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+146\,880\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+$$

$$\left.48\,960\,H^4\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+5440\,H^6\,k^6\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3\right)\Big)\,dt^2\Big)\Big/$$

$$\left(230\,400\left(\left(3+H^2\,k^2\right)^3\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)\right)+\frac{1}{25\,600\left(3+H^2\,k^2\right)^4}$$

$$i\left(-48\,276\,g^2\,H^2\,k^8-32\,760\,g^2\,H^4\,k^{10}-5550\,g^2\,H^6\,k^{12}-63\,720\sqrt{3}\,g\,H\,k^8\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U-\right.$$

$$43\,020\sqrt{3}\,g\,H^3\,k^{10}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U-7255\sqrt{3}\,g\,H^5\,k^{12}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U-$$

$$282\,852\,g\,H\,k^8\,U^2-284\,364\,g\,H^3\,k^{10}\,U^2-95\,280\,g\,H^5\,k^{12}\,U^2-10\,640\,g\,H^7\,k^{14}\,U^2-$$

$$61\,776\sqrt{3}\,k^8\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3-61\,932\sqrt{3}\,H^2\,k^{10}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3-$$

$$20\,695\sqrt{3}\,H^4\,k^{12}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3-2305\sqrt{3}\,H^6\,k^{14}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3-$$

$$\left.45\,360\,k^8\,U^4-60\,480\,H^2\,k^{10}\,U^4-30\,240\,H^4\,k^{12}\,U^4-6720\,H^6\,k^{14}\,U^4-560\,H^8\,k^{16}\,U^4\right)dt^3+$$

$$\left(\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)\left(418\,608\,g^2\,H^2\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}+283\,500\,g^2\,H^4\,k^{11}\sqrt{g\,H\left(3+H^2\,k^2\right)}+47\,925\,g^2\,H^6\,k^{13}\sqrt{g\,H\left(3+H^2\,k^2\right)}+1\,643\,328\sqrt{3}\,g^2\,H^2\,k^9\,U+1\,655\,316\sqrt{3}\,g^2\,H^4\,k^{11}\,U+555\,615\sqrt{3}\,g^2\,H^6\,k^{13}\,U+62\,145\sqrt{3}\,g^2\,H^8\,k^{15}\,U+2\,406\,672\,g\,H\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+2\,417\,364\,g\,H^3\,k^{11}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+809\,235\,g\,H^5\,k^{13}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+90\,285\,g\,H^7\,k^{15}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+1\,557\,792\sqrt{3}\,g\,H\,k^9\,U^3+2\,080\,188\sqrt{3}\,g\,H^3\,k^{11}\,U^3+1\,041\,633\sqrt{3}\,g\,H^5\,k^{13}\,U^3+231\,810\sqrt{3}\,g\,H^7\,k^{15}\,U^3+19\,345\sqrt{3}\,g\,H^9\,k^{17}\,U^3+375\,840\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+501\,120\,H^2\,k^{11}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+250\,560\,H^4\,k^{13}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+55\,680\,H^6\,k^{15}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+4640\,H^8\,k^{17}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4\right)\,dt^4\Big)\Big/\left(460\,800\left(3+H^2\,k^2\right)^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)+O[dt]^5\Big)\,dx^4+O[dx]^5,$$

$$\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)\left(3\,g\,H-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)\,dt^2+\frac{1}{8\left(3+H^2\,k^2\right)^3}i\,k^4\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)^2\left(3\,g\,H-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)\,dt^3-\left(\left(k^5\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)^3\left(3\,g\,H-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)\right)\,dt^4\right)\Big/\left(20\left(3+H^2\,k^2\right)^4\right)+O[dt]^5\Big)+\left(-\left(\left(\left(42\sqrt{3}\,k^3\sqrt{g\,H\left(3+H^2\,k^2\right)}+15\sqrt{3}\,H^2\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}-180\,k^3\,U-120\,H^2\,k^5\,U-20\,H^4\,k^7\,U\right)\Big/\left(240\left(3+H^2\,k^2\right)^2\right)\right)+\frac{1}{480\left(3+H^2\,k^2\right)^3}k^5\left(-126\sqrt{3}\,g\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}-45\sqrt{3}\,g\,H^3\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}+1296\,g\,H\,U+882\,g\,H^3\,k^2\,U+150\,g\,H^5\,k^4\,U-486\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2-327\sqrt{3}\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2-55\sqrt{3}\,H^4\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+540\,U^3+540\,H^2\,k^2\,U^3+180\,H^4\,k^4\,U^3+20\,H^6\,k^6\,U^3\right)\,dt^2+\frac{1}{480\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)\left(126\,g\,H+45\,g\,H^3\,k^2-102\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\,U-35\sqrt{3}\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+180\,U^2+120\,H^2\,k^2\,U^2+20\,H^4\,k^4\,U^2\right)\,dt^3-\frac{1}{960\left(3+H^2\,k^2\right)^5}\left(k^7\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)^2\left(-42\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}-15\sqrt{3}\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+120\,H^2\,k^2\,U+20\,H^4\,k^4\,U\right)$$

$$\left(3\,g\,H-2\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)\Big)\,dt^4+O[dt]^5\Big)\,dx^2+$$

$$\Bigg(i\left(-\frac{1}{8}\sqrt{g\,H}\,k^4+\frac{\sqrt{3}\,k^4\,U}{16\sqrt{3+H^2\,k^2}}\right)+\frac{1}{32\left(3+H^2\,k^2\right)\sqrt{g\,H\,\left(3+H^2\,k^2\right)}}$$

$$i\left(-6\,g^2\,H^2\,k^6\sqrt{3+H^2\,k^2}+15\sqrt{3}\,g\,H\sqrt{g\,H}\,k^6\,U+4\sqrt{3}\,g\,H^3\sqrt{g\,H}\,k^8\,U-\right.$$

$$\left.12\,g\,H\,k^6\sqrt{3+H^2\,k^2}\,U^2-2\,g\,H^3\,k^8\sqrt{3+H^2\,k^2}\,U^2+3\sqrt{3}\,\sqrt{g\,H}\,k^6\,U^3+\sqrt{3}\,H^2\sqrt{g\,H}\,k^8\,U^3\right)$$

$$dt^2-\Bigg(\Big(k^7\left(-3\,g\,H+2\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U-3\,U^2-H^2\,k^2\,U^2\right)$$

$$\left(-6\sqrt{3}\,g\,H\sqrt{g\,H}-2\sqrt{3}\,g\,H^3\sqrt{g\,H}\,k^2+9\,g\,H\sqrt{3+H^2\,k^2}\,U+2\,g\,H^3\,k^2\sqrt{3+H^2\,k^2}\,U-\right.$$

$$\left.3\sqrt{3}\,\sqrt{g\,H}\,U^2-\sqrt{3}\,H^2\sqrt{g\,H}\,k^2\,U^2\right)\Big)\,dt^3\Bigg)\Big/\left(32\left(\left(3+H^2\,k^2\right)^2\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\right)\right)+$$

$$\frac{1}{64\,g\,H\left(3+H^2\,k^2\right)^4}i\,k^8\left(-\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\right)^2$$

$$\left(6\,g\,H\sqrt{g\,H}+2\,g\,H^3\sqrt{g\,H}\,k^2-\sqrt{3}\,g\,H\sqrt{3+H^2\,k^2}\,U\right)$$

$$\left.\left(3\,g\,H-2\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)dt^4+O[dt]^5\right)dx^3+$$

$$\Bigg(\Big(17\,856\,\sqrt{3}\,g\,H\,k^5+12\,180\,\sqrt{3}\,g\,H^3\,k^7+2075\,\sqrt{3}\,g\,H^5\,k^9-18\,720\,k^5\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U-$$

$$12\,480\,H^2\,k^7\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U-2080\,H^4\,k^9\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U\Big)\Big/$$

$$\left(38\,400\,\left(3+H^2\,k^2\right)^2\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\right)-$$

$$\Bigg(\Big(k^7\left(-150\,120\,\sqrt{3}\,g^2\,H^2-102\,060\,\sqrt{3}\,g^2\,H^4\,k^2-17\,325\,\sqrt{3}\,g^2\,H^6\,k^4+\right.$$

$$449\,064\,g\,H\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U+303\,120\,g\,H^3\,k^2\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U+$$

$$51\,120\,g\,H^5\,k^4\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U-445\,824\,\sqrt{3}\,g\,H\,U^2-$$

$$447\,588\,\sqrt{3}\,g\,H^3\,k^2\,U^2-149\,775\,\sqrt{3}\,g\,H^5\,k^4\,U^2-16\,705\,\sqrt{3}\,g\,H^7\,k^6\,U^2+$$

$$146\,880\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U^3+146\,880\,H^2\,k^2\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U^3+$$

$$\left.48\,960\,H^4\,k^4\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U^3+5440\,H^6\,k^6\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U^3\right)\Big)\,dt^2\Bigg)\Big/$$

$$\left(230\,400\,\left(\left(3+H^2\,k^2\right)^3\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\right)\right)+\frac{1}{25\,600\,\left(3+H^2\,k^2\right)^4}$$

$$i\left(-48\,276\,g^2\,H^2\,k^8-32\,760\,g^2\,H^4\,k^{10}-5550\,g^2\,H^6\,k^{12}+63\,720\,\sqrt{3}\,g\,H\,k^8\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U+\right.$$

$$43\,020\,\sqrt{3}\,g\,H^3\,k^{10}\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U+7255\,\sqrt{3}\,g\,H^5\,k^{12}\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U-$$

$$282\,852\,g\,H\,k^8\,U^2-284\,364\,g\,H^3\,k^{10}\,U^2-95\,280\,g\,H^5\,k^{12}\,U^2-10\,640\,g\,H^7\,k^{14}\,U^2+$$

$$61\,776\,\sqrt{3}\,k^8\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U^3+61\,932\,\sqrt{3}\,H^2\,k^{10}\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U^3+$$

$$20\,695\,\sqrt{3}\,H^4\,k^{12}\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U^3+2305\,\sqrt{3}\,H^6\,k^{14}\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\,U^3-$$

$$\left.45\,360\,k^8\,U^4-60\,480\,H^2\,k^{10}\,U^4-30\,240\,H^4\,k^{12}\,U^4-6720\,H^6\,k^{14}\,U^4-560\,H^8\,k^{16}\,U^4\right)\,dt^3-$$

$$\Bigg(\Big(\Big(\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}-3\,U-H^2\,k^2\,U\Big)\,\Big(418\,608\,g^2\,H^2\,k^9\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+283\,500\,g^2\,H^4$$

$$k^{11}\sqrt{g\,H\left(3+H^2\,k^2\right)}+47\,925\,g^2\,H^6\,k^{13}\sqrt{g\,H\left(3+H^2\,k^2\right)}-1\,643\,328\sqrt{3}\,g^2\,H^2\,k^9\,U-1\,655\,316\sqrt{3}\,g^2\,H^4\,k^{11}\,U-555\,615\sqrt{3}\,g^2\,H^6\,k^{13}\,U-62\,145\sqrt{3}\,g^2\,H^8\,k^{15}\,U+2\,406\,672\,g\,H\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+2\,417\,364\,g\,H^3\,k^{11}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+809\,235\,g\,H^5\,k^{13}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+90\,285\,g\,H^7\,k^{15}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2-1\,557\,792\sqrt{3}\,g\,H\,k^9\,U^3-2\,080\,188\sqrt{3}\,g\,H^3\,k^{11}\,U^3-1\,041\,633\sqrt{3}\,g\,H^5\,k^{13}\,U^3-231\,810\sqrt{3}\,g\,H^7\,k^{15}\,U^3-19\,345\sqrt{3}\,g\,H^9\,k^{17}\,U^3+375\,840\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+501\,120\,H^2\,k^{11}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+250\,560\,H^4\,k^{13}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+55\,680\,H^6\,k^{15}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+4640\,H^8\,k^{17}\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4\Big)\Big)\,dt^4\Big)\Big/\left(460\,800\left(\left(3+H^2\,k^2\right)^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)+O[dt]^5\right)\,dx^4+O[dx]^5\Big\}$$