

```

In[692]:= MA = k * x / (2 * Sin[k * x / 2])
RA = Exp[I * k * x / 2] * k * x / (2 * Sin[k * x / 2])
GA = k * x / ((H + H^3 / 3 * k^2) * Exp[-I * k * x / 2] * (2 * Sin[k * x / 2]))
FnnA = 0
FnGA = I * k / (1 + H^2 * k^2 / 3)
FGnA = g * H * I * k
FGGA = 0
FmatA = {{FnnA, FnGA}, {FGnA, FGGA}}
wAp = Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2) ]
wAm = -Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2) ]
Eigenvalues[FmatA]

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$$\text{Out[692]} = \frac{1}{2} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$$

$$\text{Out[693]} = \frac{1}{2} e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$$

$$\text{Out[694]} = \frac{e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right)}$$

$$\text{Out[695]} = 0$$

$$\text{Out[696]} = \frac{i k}{1 + \frac{H^2 k^2}{3}}$$

$$\text{Out[697]} = i g H k$$

$$\text{Out[698]} = 0$$

$$\text{Out[699]} = \left\{ \left\{ 0, \frac{i k}{1 + \frac{H^2 k^2}{3}} \right\}, \{i g H k, 0\} \right\}$$

$$\text{Out[700]} = \sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$$

$$\text{Out[701]} = -\sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$$

$$\text{Out[702]} = \left\{ -\frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2} \right\}$$

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In[703]:= M = 1
Series[M - MA, {x, 0, 10}]

```

$$\text{Out[703]} = 1$$

$$\text{Out[704]} = -\frac{k^2 x^2}{24} - \frac{7 k^4 x^4}{5760} - \frac{31 k^6 x^6}{967680} - \frac{127 k^8 x^8}{154828800} - \frac{73 k^{10} x^{10}}{3503554560} + O[x]^{11}$$

```

In[705]:= Rm = 1
Series[Rm - RA, {x, 0, 10}]
Rp = Exp[I * k * x]
Series[Rp - RA, {x, 0, 10}]
Ru = (1 + Exp[I * k * x]) / 2
Series[Ru - Exp[I * k * x / 2], {x, 0, 10}]

```

Out[705]= 1

$$\text{Out[706]} = -\frac{1}{2} i k x + \frac{k^2 x^2}{12} + \frac{k^4 x^4}{720} + \frac{k^6 x^6}{30240} + \frac{k^8 x^8}{1209600} + \frac{k^{10} x^{10}}{47900160} + O[x]^{11}$$

Out[707]= $e^{i k x}$

$$\text{Out[708]} = \frac{i k x}{2} - \frac{5 k^2 x^2}{12} - \frac{1}{6} i k^3 x^3 + \frac{31 k^4 x^4}{720} + \frac{1}{120} i k^5 x^5 - \frac{41 k^6 x^6}{30240} - \frac{i k^7 x^7}{5040} + \frac{31 k^8 x^8}{1209600} + \frac{i k^9 x^9}{362880} - \frac{61 k^{10} x^{10}}{239500800} + O[x]^{11}$$

$$\text{Out[709]} = \frac{1}{2} (1 + e^{i k x})$$

$$\text{Out[710]} = -\frac{k^2 x^2}{8} - \frac{1}{16} i k^3 x^3 + \frac{7 k^4 x^4}{384} + \frac{1}{256} i k^5 x^5 - \frac{31 k^6 x^6}{46080} - \frac{i k^7 x^7}{10240} + \frac{127 k^8 x^8}{10321920} + \frac{17 i k^9 x^9}{12386304} - \frac{73 k^{10} x^{10}}{530841600} + O[x]^{11}$$

```

In[711]:= Gold = H - H^3 / 3 * (2 * Cos[k * x] - 2) / x^2
G = Ru / Gold
Series[G, {x, 0, 3}]
Series[GA, {x, 0, 3}]
Series[G - GA, {x, 0, 5}]

```

$$\text{Out[711]} = H - \frac{H^3 (-2 + 2 \cos[k x])}{3 x^2}$$

$$\text{Out[712]} = \frac{1 + e^{i k x}}{2 \left(H - \frac{H^3 (-2 + 2 \cos[k x])}{3 x^2} \right)}$$

$$\text{Out[713]} = \frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2 \left(H + \frac{H^3 k^2}{3} \right)} + \frac{(-9 k^2 - 2 H^2 k^4) x^2}{4 H (3 + H^2 k^2)^2} - \frac{i (6 k^3 + H^2 k^5) x^3}{8 H (3 + H^2 k^2)^2} + O[x]^4$$

$$\text{Out[714]} = \frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2 \left(H + \frac{H^3 k^2}{3} \right)} - \frac{k^2 x^2}{12 \left(H + \frac{H^3 k^2}{3} \right)} + O[x]^4$$

$$\text{Out[715]} = \frac{(-6 k^2 - H^2 k^4) x^2}{4 H (3 + H^2 k^2)^2} - \frac{i (6 k^3 + H^2 k^5) x^3}{8 H (3 + H^2 k^2)^2} + \frac{(144 k^4 + 45 H^2 k^6 + 4 H^4 k^8) x^4}{240 H (3 + H^2 k^2)^3} - \frac{i (-54 k^5 + H^4 k^9) x^5}{480 H (3 + H^2 k^2)^3} + O[x]^6$$

```
In[997]:= fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * G;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;
```

```
Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]
```

```
Fmat = {{Fnn, Fng}, {Fgn, Fgg}}
EigvFmat = Eigenvalues[Fmat];
Simplify[Series[EigvFmat, {x, 0, 5}]]
t = x / (2 * Sqrt[g * H])
RKStep = Log[1 - t * EigvFmat] / (I * t);
RKstepTay = Series[RKStep, {x, 0, 5}]
Simplify[RKstepTay, k * H > 0]
Simplify[RKstepTay - {wAp, wAm}, k * H > 0]
```

$$\text{Out[1001]} = -\frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} (-5 + e^{-ikx} - 2e^{ikx}) + \frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) \right) \sqrt{gH}$$

$$\text{Out[1002]} = \frac{1}{12} \sqrt{gH} k^4 x^3 - \frac{1}{72} (\sqrt{gH} k^6) x^5 + O[x]^6$$

$$\text{Out[1003]} = \frac{(1 - e^{-ikx}) (9 - e^{-ikx} + 9e^{ikx} - e^{2ikx}) H (26 - 2 \cos[kx])}{384 x \left(H - \frac{H^3 (-30 + 32 \cos[kx] - 2 \cos[2kx])}{36 x^2} \right)}$$

$$\text{Out[1004]} = -\frac{i (243 k^5 + 49 H^2 k^7) x^4}{960 (3 + H^2 k^2)^2} + O[x]^6$$

$$\text{Out[1005]} = -\frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} (-5 + e^{-ikx} - 2e^{ikx}) + \frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) \right) \sqrt{gH}$$

$$\text{Out[1006]} = \frac{1}{12} \sqrt{gH} k^4 x^3 - \frac{1}{72} (\sqrt{gH} k^6) x^5 + O[x]^6$$

$$\text{Out[1007]} = \frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) + \frac{1}{6} (5 - e^{-ikx} + 2e^{ikx}) \right) gH$$

$$\text{Out[1008]} = -\frac{1}{30} i g H k^5 x^4 + O[x]^6$$

$$\text{Out}[1009]= \left\{ \left\{ -\frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} (-5 + e^{-ikx} - 2e^{ikx}) + \frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) \right) \sqrt{gH}, \right. \right. \\ \left. \frac{(1 - e^{-ikx}) (9 - e^{-ikx} + 9e^{ikx} - e^{2ikx}) H (26 - 2\cos[kx])}{384x \left(H - \frac{H^3 (-30 + 32\cos[kx] - 2\cos[2kx])}{36x^2} \right)} \right\}, \\ \left\{ \frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) + \frac{1}{6} (5 - e^{-ikx} + 2e^{ikx}) \right) gH, \right. \\ \left. -\frac{1}{2x} (1 - e^{-ikx}) \left(\frac{1}{6} (-5 + e^{-ikx} - 2e^{ikx}) + \frac{1}{6} e^{ikx} (5 + 2e^{-ikx} - e^{ikx}) \right) \sqrt{gH} \right\} \right\}$$

$$\text{Out}[1011]= \left\{ -\frac{i\sqrt{3}gHk}{\sqrt{gH(3+H^2k^2)}} + \frac{1}{12}\sqrt{gH}k^4x^3 + \frac{i g^2 H^2 k^5 (531 + 145 H^2 k^2) x^4}{1920 \sqrt{3} (gH(3+H^2k^2))^{3/2}} - \frac{1}{72}(\sqrt{gH}k^6)x^5 + O[x]^6, \right. \\ \left. \frac{i\sqrt{3}gHk}{\sqrt{gH(3+H^2k^2)}} + \frac{1}{12}\sqrt{gH}k^4x^3 - \frac{i g^2 H^2 k^5 (531 + 145 H^2 k^2) x^4}{1920 \sqrt{3} (gH(3+H^2k^2))^{3/2}} - \frac{1}{72}(\sqrt{gH}k^6)x^5 + O[x]^6 \right\}$$

$$\text{Out}[1012]= \frac{x}{2\sqrt{gH}}$$

$$\text{Out}[1014]= \left\{ \frac{\sqrt{3}k\sqrt{gH(3+H^2k^2)}}{3+H^2k^2} - \frac{3i\sqrt{gH}k^2x}{4(3+H^2k^2)} - \frac{\left(\sqrt{3}k^3\sqrt{gH(3+H^2k^2)}\right)x^2}{4(3+H^2k^2)^2} - \right. \\ \frac{i\sqrt{gH}(-99k^4 - 48H^2k^6 - 8H^4k^8)x^3}{96(3+H^2k^2)^2} + \frac{gH(1215\sqrt{3}k^5 + 474\sqrt{3}H^2k^7 + 95\sqrt{3}H^4k^9)x^4}{5760(3+H^2k^2)^2\sqrt{gH(3+H^2k^2)}} - \\ \frac{i\sqrt{gH}(7641k^6 + 5742H^2k^8 + 1725H^4k^{10} + 160H^6k^{12})x^5}{11520(3+H^2k^2)^3} + O[x]^6, \\ \left. -\frac{\sqrt{3}k\sqrt{gH(3+H^2k^2)}}{3+H^2k^2} - \frac{3i\sqrt{gH}k^2x}{4(3+H^2k^2)} + \frac{\sqrt{3}k^3\sqrt{gH(3+H^2k^2)}x^2}{4(3+H^2k^2)^2} - \right. \\ \frac{i\sqrt{gH}(-99k^4 - 48H^2k^6 - 8H^4k^8)x^3}{96(3+H^2k^2)^2} - \frac{\left(gH(1215\sqrt{3}k^5 + 474\sqrt{3}H^2k^7 + 95\sqrt{3}H^4k^9)\right)x^4}{5760\left((3+H^2k^2)^2\sqrt{gH(3+H^2k^2)}\right)} - \\ \left. \frac{i\sqrt{gH}(7641k^6 + 5742H^2k^8 + 1725H^4k^{10} + 160H^6k^{12})x^5}{11520(3+H^2k^2)^3} + O[x]^6 \right\}$$

$$\begin{aligned}
\text{Out}[1015] = & \left\{ \frac{\sqrt{3} \, g \, H \, k}{\sqrt{g \, H \, (3 + H^2 \, k^2)}} - \frac{3 \, i \, \sqrt{g \, H} \, k^2 \, x}{4 \, (3 + H^2 \, k^2)} - \frac{(\sqrt{3} \, \sqrt{g \, H} \, k^3) \, x^2}{4 \, (3 + H^2 \, k^2)^{3/2}} + \right. \\
& \frac{i \, \sqrt{g \, H} \, k^4 \, (99 + 48 \, H^2 \, k^2 + 8 \, H^4 \, k^4) \, x^3}{96 \, (3 + H^2 \, k^2)^2} + \frac{\sqrt{g \, H} \, k^5 \, (1215 + 474 \, H^2 \, k^2 + 95 \, H^4 \, k^4) \, x^4}{1920 \, \sqrt{3} \, (3 + H^2 \, k^2)^{5/2}} - \\
& \frac{i \, \sqrt{g \, H} \, k^6 \, (7641 + 5742 \, H^2 \, k^2 + 1725 \, H^4 \, k^4 + 160 \, H^6 \, k^6) \, x^5}{11520 \, (3 + H^2 \, k^2)^3} + O[x]^6, \\
& - \frac{\sqrt{3} \, g \, H \, k}{\sqrt{g \, H \, (3 + H^2 \, k^2)}} - \frac{3 \, i \, \sqrt{g \, H} \, k^2 \, x}{4 \, (3 + H^2 \, k^2)} + \frac{\sqrt{3} \, \sqrt{g \, H} \, k^3 \, x^2}{4 \, (3 + H^2 \, k^2)^{3/2}} + \\
& \frac{i \, \sqrt{g \, H} \, k^4 \, (99 + 48 \, H^2 \, k^2 + 8 \, H^4 \, k^4) \, x^3}{96 \, (3 + H^2 \, k^2)^2} - \frac{(\sqrt{g \, H} \, k^5 \, (1215 + 474 \, H^2 \, k^2 + 95 \, H^4 \, k^4)) \, x^4}{1920 \, (\sqrt{3} \, (3 + H^2 \, k^2)^{5/2})} - \\
& \frac{i \, \sqrt{g \, H} \, k^6 \, (7641 + 5742 \, H^2 \, k^2 + 1725 \, H^4 \, k^4 + 160 \, H^6 \, k^6) \, x^5}{11520 \, (3 + H^2 \, k^2)^3} + O[x]^6 \} \\
\text{Out}[1016] = & \left\{ - \frac{3 \, i \, \sqrt{g \, H} \, k^2 \, x}{4 \, (3 + H^2 \, k^2)} - \frac{(\sqrt{3} \, \sqrt{g \, H} \, k^3) \, x^2}{4 \, (3 + H^2 \, k^2)^{3/2}} + \right. \\
& \frac{i \, \sqrt{g \, H} \, k^4 \, (99 + 48 \, H^2 \, k^2 + 8 \, H^4 \, k^4) \, x^3}{96 \, (3 + H^2 \, k^2)^2} + \frac{\sqrt{g \, H} \, k^5 \, (1215 + 474 \, H^2 \, k^2 + 95 \, H^4 \, k^4) \, x^4}{1920 \, \sqrt{3} \, (3 + H^2 \, k^2)^{5/2}} - \\
& \frac{i \, \sqrt{g \, H} \, k^6 \, (7641 + 5742 \, H^2 \, k^2 + 1725 \, H^4 \, k^4 + 160 \, H^6 \, k^6) \, x^5}{11520 \, (3 + H^2 \, k^2)^3} + O[x]^6, \\
& - \frac{3 \, i \, \sqrt{g \, H} \, k^2 \, x}{4 \, (3 + H^2 \, k^2)} + \frac{\sqrt{3} \, \sqrt{g \, H} \, k^3 \, x^2}{4 \, (3 + H^2 \, k^2)^{3/2}} + \frac{i \, \sqrt{g \, H} \, k^4 \, (99 + 48 \, H^2 \, k^2 + 8 \, H^4 \, k^4) \, x^3}{96 \, (3 + H^2 \, k^2)^2} - \\
& \frac{(\sqrt{g \, H} \, k^5 \, (1215 + 474 \, H^2 \, k^2 + 95 \, H^4 \, k^4)) \, x^4}{1920 \, (\sqrt{3} \, (3 + H^2 \, k^2)^{5/2})} - \\
& \frac{i \, \sqrt{g \, H} \, k^6 \, (7641 + 5742 \, H^2 \, k^2 + 1725 \, H^4 \, k^4 + 160 \, H^6 \, k^6) \, x^5}{11520 \, (3 + H^2 \, k^2)^3} + O[x]^6 \}
\end{aligned}$$

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In[915]:= t = x / (4 * Sqrt[g * H])
RKStep = Log[1 - t * EigvFmat] / (1 * t);
RKstepTay = Series[RKStep, {x, 0, 5}]
Simplify[RKstepTay, k * H > 0]
Simplify[RKstepTay - {wAp, wAm}, k * H > 0]

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$$\text{Out}[915] = \frac{x}{4 \sqrt{g \, H}}$$

$$\begin{aligned}
\text{Out[917]} = & \left\{ \frac{\sqrt{3} \, k \sqrt{g \, H \, (3 + H^2 \, k^2)}}{3 + H^2 \, k^2} - \frac{3 \, i \sqrt{g \, H} \, k^2 \, x}{8 \, (3 + H^2 \, k^2)} - \frac{\left(\sqrt{3} \, k^3 \sqrt{g \, H \, (3 + H^2 \, k^2)} \right) x^2}{16 \, (3 + H^2 \, k^2)^2} - \right. \\
& i \sqrt{g \, H} \left(-\frac{k^4}{12} - \frac{9 \, k^4}{256 \, (3 + H^2 \, k^2)^2} \right) x^3 - \frac{\left(g \, H \left(945 \sqrt{3} \, k^5 + 492 \sqrt{3} \, H^2 \, k^7 + 50 \sqrt{3} \, H^4 \, k^9 \right) \right) x^4}{11 \, 520 \left((3 + H^2 \, k^2)^2 \sqrt{g \, H \, (3 + H^2 \, k^2)} \right)} - \\
& \frac{i \sqrt{g \, H} \, (28 \, 809 \, k^6 + 31 \, 608 \, H^2 \, k^8 + 11 \, 220 \, H^4 \, k^{10} + 1280 \, H^6 \, k^{12}) x^5}{92 \, 160 \, (3 + H^2 \, k^2)^3} + O[x]^6, \\
& - \frac{\sqrt{3} \, k \sqrt{g \, H \, (3 + H^2 \, k^2)}}{3 + H^2 \, k^2} - \frac{3 \, i \sqrt{g \, H} \, k^2 \, x}{8 \, (3 + H^2 \, k^2)} + \frac{\sqrt{3} \, k^3 \sqrt{g \, H \, (3 + H^2 \, k^2)} x^2}{16 \, (3 + H^2 \, k^2)^2} - \\
& i \sqrt{g \, H} \left(-\frac{k^4}{12} - \frac{9 \, k^4}{256 \, (3 + H^2 \, k^2)^2} \right) x^3 + \frac{g \, H \left(945 \sqrt{3} \, k^5 + 492 \sqrt{3} \, H^2 \, k^7 + 50 \sqrt{3} \, H^4 \, k^9 \right) x^4}{11 \, 520 \, (3 + H^2 \, k^2)^2 \sqrt{g \, H \, (3 + H^2 \, k^2)}} - \\
& \frac{i \sqrt{g \, H} \, (28 \, 809 \, k^6 + 31 \, 608 \, H^2 \, k^8 + 11 \, 220 \, H^4 \, k^{10} + 1280 \, H^6 \, k^{12}) x^5}{92 \, 160 \, (3 + H^2 \, k^2)^3} + O[x]^6 \} \\
\text{Out[918]} = & \left\{ \frac{\sqrt{3} \, g \, H \, k}{\sqrt{g \, H \, (3 + H^2 \, k^2)}} - \frac{3 \, i \sqrt{g \, H} \, k^2 \, x}{8 \, (3 + H^2 \, k^2)} - \frac{\left(\sqrt{3} \, \sqrt{g \, H} \, k^3 \right) x^2}{16 \, (3 + H^2 \, k^2)^{3/2}} - \right. \\
& \frac{1}{768} i \sqrt{g \, H} \, k^4 \left(-64 - \frac{27}{(3 + H^2 \, k^2)^2} \right) x^3 - \frac{\left(\sqrt{g \, H} \, k^5 \left(945 + 492 \, H^2 \, k^2 + 50 \, H^4 \, k^4 \right) \right) x^4}{3840 \, \left(\sqrt{3} \, (3 + H^2 \, k^2)^{5/2} \right)} - \\
& \frac{i \sqrt{g \, H} \, k^6 \left(28 \, 809 + 31 \, 608 \, H^2 \, k^2 + 11 \, 220 \, H^4 \, k^4 + 1280 \, H^6 \, k^6 \right) x^5}{92 \, 160 \, (3 + H^2 \, k^2)^3} + O[x]^6, \\
& - \frac{\sqrt{3} \, g \, H \, k}{\sqrt{g \, H \, (3 + H^2 \, k^2)}} - \frac{3 \, i \sqrt{g \, H} \, k^2 \, x}{8 \, (3 + H^2 \, k^2)} + \frac{\sqrt{3} \, \sqrt{g \, H} \, k^3 \, x^2}{16 \, (3 + H^2 \, k^2)^{3/2}} - \\
& \frac{1}{768} i \sqrt{g \, H} \, k^4 \left(-64 - \frac{27}{(3 + H^2 \, k^2)^2} \right) x^3 + \frac{\sqrt{g \, H} \, k^5 \left(945 + 492 \, H^2 \, k^2 + 50 \, H^4 \, k^4 \right) x^4}{3840 \, \sqrt{3} \, (3 + H^2 \, k^2)^{5/2}} - \\
& \frac{i \sqrt{g \, H} \, k^6 \left(28 \, 809 + 31 \, 608 \, H^2 \, k^2 + 11 \, 220 \, H^4 \, k^4 + 1280 \, H^6 \, k^6 \right) x^5}{92 \, 160 \, (3 + H^2 \, k^2)^3} + O[x]^6 \}
\end{aligned}$$

$$\begin{aligned}
\text{Out[919]} = & \left\{ -\frac{3 \, i \, \sqrt{g \, H} \, k^2 \, x}{8 \, (3 + H^2 \, k^2)} - \frac{(\sqrt{3} \, \sqrt{g \, H} \, k^3) \, x^2}{16 \, (3 + H^2 \, k^2)^{3/2}} - \right. \\
& \frac{1}{768} \, i \, \sqrt{g \, H} \, k^4 \left(-64 - \frac{27}{(3 + H^2 \, k^2)^2} \right) x^3 - \frac{(\sqrt{g \, H} \, k^5 (945 + 492 \, H^2 \, k^2 + 50 \, H^4 \, k^4)) \, x^4}{3840 \, (\sqrt{3} \, (3 + H^2 \, k^2)^{5/2})} - \\
& \frac{i \, \sqrt{g \, H} \, k^6 (28809 + 31608 \, H^2 \, k^2 + 11220 \, H^4 \, k^4 + 1280 \, H^6 \, k^6) \, x^5}{92160 \, (3 + H^2 \, k^2)^3} + O[x]^6, \\
& -\frac{3 \, i \, \sqrt{g \, H} \, k^2 \, x}{8 \, (3 + H^2 \, k^2)} + \frac{\sqrt{3} \, \sqrt{g \, H} \, k^3 \, x^2}{16 \, (3 + H^2 \, k^2)^{3/2}} - \frac{1}{768} \, i \, \sqrt{g \, H} \, k^4 \left(-64 - \frac{27}{(3 + H^2 \, k^2)^2} \right) x^3 + \\
& \frac{\sqrt{g \, H} \, k^5 (945 + 492 \, H^2 \, k^2 + 50 \, H^4 \, k^4) \, x^4}{3840 \, \sqrt{3} \, (3 + H^2 \, k^2)^{5/2}} - \\
& \left. \frac{i \, \sqrt{g \, H} \, k^6 (28809 + 31608 \, H^2 \, k^2 + 11220 \, H^4 \, k^4 + 1280 \, H^6 \, k^6) \, x^5}{92160 \, (3 + H^2 \, k^2)^3} + O[x]^6 \right\}
\end{aligned}$$