```
In[136]:= MA = k*x / (2*Sin[k*x/2])
          RA = Exp[I*k*x/2]*k*x / (2*Sin[k*x/2])
          GA = k*x/ ((H + H^3/3*k^2) *Exp[-I*k*x/2]*(2*Sin[k*x/2]))
          FnnA = 0
          FnGA = I*k/ (1 + H^2*k^2/3)
          FGnA = g*H*I*k
          FGGA = 0
          FmatA = {{FnnA, FnGA}, {FGnA, FGGA}}
          wAp  =  Sqrt[g*H]*k*Sqrt[3 / (3 + H^2*k^2)]
          wAm  =  -Sqrt[g*H]*k*Sqrt[3 / (3 + H^2*k^2)]
          Eigenvalues[FmatA]
```

Out[136]= $\frac{1}{2} k x \, \mathrm{Csc}\left[\frac{k x}{2}\right]$

Out[137]= $\frac{1}{2} e^{\frac{i k x}{2}} k x \, \mathrm{Csc}\left[\frac{k x}{2}\right]$

Out[138]= $\frac{e^{\frac{i k x}{2}} k x \, \mathrm{Csc}\left[\frac{k x}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right)}$

Out[139]= $0$

Out[140]= $\frac{i k}{1 + \frac{H^2 k^2}{3}}$

Out[141]= $i g H k$

Out[142]= $0$

Out[143]= $\left\{\left\{0, \frac{i k}{1 + \frac{H^2 k^2}{3}}\right\}, \{i g H k, 0\}\right\}$

Out[144]= $\sqrt{3} \sqrt{g H} \, k \sqrt{\frac{1}{3 + H^2 k^2}}$

Out[145]= $-\sqrt{3} \sqrt{g H} \, k \sqrt{\frac{1}{3 + H^2 k^2}}$

Out[146]= $\left\{-\frac{i \sqrt{3} \, k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3} \, k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}\right\}$

```
In[147]:= M = 1
          Series[M - MA, {x, 0, 10}]
```

Out[147]= $1$

Out[148]= $-\frac{k^2 x^2}{24} - \frac{7 k^4 x^4}{5760} - \frac{31 k^6 x^6}{967\,680} - \frac{127 k^8 x^8}{154\,828\,800} - \frac{73 k^{10} x^{10}}{3\,503\,554\,560} + O[x]^{11}$

In[149]:=
```
Rm = 1 + I * Sin[k * x] / 2
Series[Rm - RA, {x, 0, 10}]
Rp = Exp[I * k * x] * (1 - I * Sin[k * x] / 2)
Series[Rp - RA, {x, 0, 10}]
```

Out[149]= $1 + \frac{1}{2} i \, \mathrm{Sin}[k\, x]$

Out[150]= $\frac{k^2 x^2}{12} - \frac{1}{12} i\, k^3 x^3 + \frac{k^4 x^4}{720} + \frac{1}{240} i\, k^5 x^5 + \frac{k^6 x^6}{30\,240} - \frac{i\, k^7 x^7}{10\,080} + \frac{k^8 x^8}{1\,209\,600} + \frac{i\, k^9 x^9}{725\,760} + \frac{k^{10} x^{10}}{47\,900\,160} + O[x]^{11}$

Out[151]= $e^{i\, k\, x} \left(1 - \frac{1}{2} i \, \mathrm{Sin}[k\, x]\right)$

Out[152]= $\frac{k^2 x^2}{12} + \frac{1}{6} i\, k^3 x^3 - \frac{89\, k^4 x^4}{720} - \frac{7}{120} i\, k^5 x^5 + \frac{631\, k^6 x^6}{30\,240} + \frac{31\, i\, k^7 x^7}{5040} - \frac{1889\, k^8 x^8}{1\,209\,600} - \frac{127\, i\, k^9 x^9}{362\,880} + \frac{481\, k^{10} x^{10}}{6\,842\,880} + O[x]^{11}$

```
In[153]:= Ru = (1 + Exp[I*k*x])/2
Series[Ru - Exp[I*k*x/2] , {x, 0, 10}]
Gold = H - H^3/3*(2*Cos[k*x] -2)/x^2
G = Ru / Gold
Series[G, {x, 0, 3} ]
Series[GA, {x, 0, 3} ]
Series[G - GA, {x, 0, 5}]
```

Out[153]= $\frac{1}{2}\left(1+e^{i k x}\right)$

Out[154]= $-\frac{k^2 x^2}{8}-\frac{1}{16} i k^3 x^3+\frac{7 k^4 x^4}{384}+\frac{1}{256} i k^5 x^5-\frac{31 k^6 x^6}{46080}-\frac{i k^7 x^7}{10240}+\frac{127 k^8 x^8}{10321920}+\frac{17 i k^9 x^9}{12386304}-\frac{73 k^{10} x^{10}}{530841600}+O[x]^{11}$

Out[155]= $H-\frac{H^3(-2+2 \operatorname{Cos}[k x])}{3 x^2}$

Out[156]= $\frac{1+e^{i k x}}{2\left(H-\frac{H^3(-2+2 \operatorname{Cos}[k x])}{3 x^2}\right)}$

Out[157]= $\frac{1}{H+\frac{H^3 k^2}{3}}+\frac{i k x}{2\left(H+\frac{H^3 k^2}{3}\right)}+\frac{\left(-9 k^2-2 H^2 k^4\right) x^2}{4 H\left(3+H^2 k^2\right)^2}-\frac{i\left(6 k^3+H^2 k^5\right) x^3}{8 H\left(3+H^2 k^2\right)^2}+O[x]^4$

Out[158]= $\frac{1}{H+\frac{H^3 k^2}{3}}+\frac{i k x}{2\left(H+\frac{H^3 k^2}{3}\right)}-\frac{k^2 x^2}{12\left(H+\frac{H^3 k^2}{3}\right)}+O[x]^4$

Out[159]= $\frac{\left(-6 k^2-H^2 k^4\right) x^2}{4 H\left(3+H^2 k^2\right)^2}-\frac{i\left(6 k^3+H^2 k^5\right) x^3}{8 H\left(3+H^2 k^2\right)^2}+\frac{\left(144 k^4+45 H^2 k^6+4 H^4 k^8\right) x^4}{240 H\left(3+H^2 k^2\right)^3}-\frac{i\left(-54 k^5+H^4 k^9\right) x^5}{480 H\left(3+H^2 k^2\right)^3}+O[x]^6$

In[160]:=
```
fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * G;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;

Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]

Fmat = {{Fnn , Fng}, {Fgn , Fgg}}
EigvFmat = Eigenvalues[Fmat];
 Simplify[Series[EigvFmat , {x, 0, 5}]]
RKStep = Log[ 1 - t * EigvFmat + (t * EigvFmat) ^2 / 2] / (I * t);
RKstepTay = Series[RKStep, {x, 0, 4}, {t, 0, 4}]
Simplify[RKstepTay, k * H > 0 ]
Simplify[RKstepTay - {wAp, wAm}, k * H > 0]
```

Out[164]= $-\frac{1}{2\,x}\left(1-e^{-i\,k\,x}\right)\sqrt{g\,H}\,\left(-1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\mathrm{Sin}[k\,x]\right)-\frac{1}{2}\,i\,\mathrm{Sin}[k\,x]\right)$

Out[165]= $\frac{1}{8}\sqrt{g\,H}\;k^4\,x^3-\frac{1}{48}\left(\sqrt{g\,H}\;k^6\right)x^5+O[x]^6$

Out[166]= $\frac{\left(1-e^{-i\,k\,x}\right)\left(1+e^{i\,k\,x}\right)H}{2\,x\left(H-\frac{H^3\,(-2+2\,\mathrm{Cos}[k\,x])}{3\,x^2}\right)}$

Out[167]= $-\frac{i\left(6\,k^3+H^2\,k^5\right)x^2}{4\left(3+H^2\,k^2\right)^2}-\frac{i\left(-54\,k^5+H^4\,k^9\right)x^4}{240\left(3+H^2\,k^2\right)^3}+O[x]^6$

Out[168]= $-\frac{1}{2\,x}\left(1-e^{-i\,k\,x}\right)\sqrt{g\,H}\,\left(-1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\mathrm{Sin}[k\,x]\right)-\frac{1}{2}\,i\,\mathrm{Sin}[k\,x]\right)$

Out[169]= $\frac{1}{8}\sqrt{g\,H}\;k^4\,x^3-\frac{1}{48}\left(\sqrt{g\,H}\;k^6\right)x^5+O[x]^6$

Out[170]= $\frac{\left(1-e^{-i\,k\,x}\right)g\,H\left(1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\mathrm{Sin}[k\,x]\right)+\frac{1}{2}\,i\,\mathrm{Sin}[k\,x]\right)}{2\,x}$

Out[171]= $\frac{1}{12}\,i\,g\,H\,k^3\,x^2-\frac{13}{240}\,i\,g\,H\,k^5\,x^4+O[x]^6$

Out[172]= $$\left\{\left\{-\frac{1}{2x}\left(1-e^{-ikx}\right)\sqrt{gH}\left(-1+e^{ikx}\left(1-\frac{1}{2}i\,\mathrm{Sin}[k\,x]\right)-\frac{1}{2}i\,\mathrm{Sin}[k\,x]\right),\ \frac{\left(1-e^{-ikx}\right)\left(1+e^{ikx}\right)H}{2x\left(H-\frac{H^3(-2+2\,\mathrm{Cos}[k\,x])}{3x^2}\right)}\right\},\ \left\{\frac{\left(1-e^{-ikx}\right)gH\left(1+e^{ikx}\left(1-\frac{1}{2}i\,\mathrm{Sin}[k\,x]\right)+\frac{1}{2}i\,\mathrm{Sin}[k\,x]\right)}{2x},\ -\frac{1}{2x}\left(1-e^{-ikx}\right)\sqrt{gH}\left(-1+e^{ikx}\left(1-\frac{1}{2}i\,\mathrm{Sin}[k\,x]\right)-\frac{1}{2}i\,\mathrm{Sin}[k\,x]\right)\right\}\right\}$$

Out[174]= $$\left\{-\frac{i\sqrt{3}\,gHk}{\sqrt{gH(3+H^2k^2)}}+\frac{i\sqrt{3}\,g^2H^2k^3x^2}{8\left(gH(3+H^2k^2)\right)^{3/2}}+\frac{1}{8}\sqrt{gH}\,k^4x^3+\frac{i\sqrt{3}\,k^5\sqrt{gH(3+H^2k^2)}\,(177+124H^2k^2+20H^4k^4)\,x^4}{640\,(3+H^2k^2)^3}-\frac{1}{48}\left(\sqrt{gH}\,k^6\right)x^5+O[x]^6,\right.$$
$$\left.\frac{i\sqrt{3}\,gHk}{\sqrt{gH(3+H^2k^2)}}-\frac{i\sqrt{3}\,g^2H^2k^3x^2}{8\left(gH(3+H^2k^2)\right)^{3/2}}+\frac{1}{8}\sqrt{gH}\,k^4x^3-\frac{i\sqrt{3}\,k^5\sqrt{gH(3+H^2k^2)}\,(177+124H^2k^2+20H^4k^4)\,x^4}{640\,(3+H^2k^2)^3}-\frac{1}{48}\left(\sqrt{gH}\,k^6\right)x^5+O[x]^6\right\}$$

Out[176]= $$\left\{\left(\frac{k\sqrt{144gH+48gH^3k^2}}{4\,(3+H^2k^2)}+\frac{\sqrt{3}\,gHk^3\sqrt{gH(3+H^2k^2)}\,t^2}{2\,(3+H^2k^2)^2}-\frac{9\,i\,g^2H^2k^4t^3}{8\,(3+H^2k^2)^2}-\frac{9\left(\sqrt{3}\,g^2H^2k^5\sqrt{gH(3+H^2k^2)}\right)t^4}{20\,(3+H^2k^2)^3}+O[t]^5\right)+\right.$$
$$\left(-\frac{\sqrt{3}\,k^3\sqrt{gH(3+H^2k^2)}}{8\,(3+H^2k^2)^2}-\frac{3\left(\sqrt{3}\,gHk^5\sqrt{gH(3+H^2k^2)}\right)t^2}{16\,(3+H^2k^2)^3}+\frac{9\,i\,g^2H^2k^6t^3}{16\,(3+H^2k^2)^3}+\frac{9\sqrt{3}\,g^2H^2k^7\sqrt{gH(3+H^2k^2)}\,t^4}{32\,(3+H^2k^2)^4}+O[t]^5\right)x^2+$$
$$\left(\frac{1}{8}i\sqrt{gH}\,k^4+\frac{3\,i\,gH\sqrt{gH}\,k^6t^2}{16\,(3+H^2k^2)}+\frac{3\sqrt{3}\,gH\sqrt{gH}\,k^7\sqrt{gH(3+H^2k^2)}\,t^3}{16\,(3+H^2k^2)^2}-\frac{9\,i\,g^2H^2\sqrt{gH}\,k^8t^4}{32\,(3+H^2k^2)^2}+O[t]^5\right)x^3+\left(\frac{-177\sqrt{3}\,gHk^5-124\sqrt{3}\,gH^3k^7-20\sqrt{3}\,gH^5k^9}{640\,(3+H^2k^2)^2\sqrt{gH(3+H^2k^2)}}-\right.$$
$$\frac{3\left(167\sqrt{3}\,g^2H^2k^7+124\sqrt{3}\,g^2H^4k^9+20\sqrt{3}\,g^2H^6k^{11}\right)t^2}{1280\left((3+H^2k^2)^3\sqrt{gH(3+H^2k^2)}\right)}+$$

$$\frac{9\,\mathrm{i}\left(81\,g^2\,H^2\,k^8+62\,g^2\,H^4\,k^{10}+10\,g^2\,H^6\,k^{12}\right)t^3}{640\left(3+H^2\,k^2\right)^4}+$$

$$\left.\frac{9\left(157\sqrt{3}\;g^3\,H^3\,k^9+124\sqrt{3}\;g^3\,H^5\,k^{11}+20\sqrt{3}\;g^3\,H^7\,k^{13}\right)t^4}{2560\left(3+H^2\,k^2\right)^4\sqrt{g\,H\left(3+H^2\,k^2\right)}}+O[t]^5\right)x^4+O[x]^5,$$

$$\left(-\frac{k\sqrt{144\,g\,H+48\,g\,H^3\,k^2}}{4\left(3+H^2\,k^2\right)}-\frac{\left(\sqrt{3}\;g\,H\,k^3\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)t^2}{2\left(3+H^2\,k^2\right)^2}-\frac{9\,\mathrm{i}\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}+\right.$$

$$\left.\frac{9\sqrt{3}\;g^2\,H^2\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\;t^4}{20\left(3+H^2\,k^2\right)^3}+O[t]^5\right)+$$

$$\left(\frac{\sqrt{3}\;k^3\sqrt{g\,H\left(3+H^2\,k^2\right)}}{8\left(3+H^2\,k^2\right)^2}+\frac{3\sqrt{3}\;g\,H\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\;t^2}{16\left(3+H^2\,k^2\right)^3}+\frac{9\,\mathrm{i}\,g^2\,H^2\,k^6\,t^3}{16\left(3+H^2\,k^2\right)^3}-\right.$$

$$\left.\frac{9\left(\sqrt{3}\;g^2\,H^2\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)t^4}{32\left(3+H^2\,k^2\right)^4}+O[t]^5\right)x^2+$$

$$\left(\frac{1}{8}\,\mathrm{i}\sqrt{g\,H}\;k^4+\frac{3\,\mathrm{i}\,g\,H\sqrt{g\,H}\;k^6\,t^2}{16\left(3+H^2\,k^2\right)}-\frac{3\left(\sqrt{3}\;g\,H\sqrt{g\,H}\;k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)t^3}{16\left(3+H^2\,k^2\right)^2}-\right.$$

$$\left.\frac{9\,\mathrm{i}\,g^2\,H^2\sqrt{g\,H}\;k^8\,t^4}{32\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+\left(\frac{177\sqrt{3}\;g\,H\,k^5+124\sqrt{3}\;g\,H^3\,k^7+20\sqrt{3}\;g\,H^5\,k^9}{640\left(3+H^2\,k^2\right)^2\sqrt{g\,H\left(3+H^2\,k^2\right)}}+\right.$$

$$\frac{3\left(167\sqrt{3}\;g^2\,H^2\,k^7+124\sqrt{3}\;g^2\,H^4\,k^9+20\sqrt{3}\;g^2\,H^6\,k^{11}\right)t^2}{1280\left(3+H^2\,k^2\right)^3\sqrt{g\,H\left(3+H^2\,k^2\right)}}+$$

$$\frac{9\,\mathrm{i}\left(81\,g^2\,H^2\,k^8+62\,g^2\,H^4\,k^{10}+10\,g^2\,H^6\,k^{12}\right)t^3}{640\left(3+H^2\,k^2\right)^4}-$$

$$\left.\frac{9\left(157\sqrt{3}\;g^3\,H^3\,k^9+124\sqrt{3}\;g^3\,H^5\,k^{11}+20\sqrt{3}\;g^3\,H^7\,k^{13}\right)t^4}{2560\left(\left(3+H^2\,k^2\right)^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)}+O[t]^5\right)x^4+O[x]^5\}$$

Out[177]=

$$\left\{\left(\frac{\sqrt{3}\, g\, H\, k}{\sqrt{g\, H\left(3+H^2 k^2\right)}}+\frac{1}{2}\sqrt{3}\, k^3\left(\frac{g\, H}{3+H^2 k^2}\right)^{3/2} t^2-\frac{9\, i\, g^2 H^2 k^4 t^3}{8\left(3+H^2 k^2\right)^2}-\frac{9}{20}\left(\sqrt{3}\, k^5\left(\frac{g\, H}{3+H^2 k^2}\right)^{5/2}\right) t^4+O[t]^5\right)+\right.$$

$$\left(-\frac{\sqrt{3}\sqrt{g\, H}\, k^3}{8\left(3+H^2 k^2\right)^{3/2}}-\frac{3\left(\sqrt{3}\,(g\, H)^{3/2} k^5\right) t^2}{16\left(3+H^2 k^2\right)^{5/2}}+\frac{9\, i\, g^2 H^2 k^6 t^3}{16\left(3+H^2 k^2\right)^3}+\frac{9\sqrt{3}\,(g\, H)^{5/2} k^7 t^4}{32\left(3+H^2 k^2\right)^{7/2}}+O[t]^5\right) x^2+$$

$$\left(\frac{1}{8}\, i\sqrt{g\, H}\, k^4+\frac{3\, i\,(g\, H)^{3/2} k^6 t^2}{16\left(3+H^2 k^2\right)}+\frac{3\sqrt{3}\, g^2 H^2 k^7 t^3}{16\left(3+H^2 k^2\right)^{3/2}}-\frac{9\, i\,(g\, H)^{5/2} k^8 t^4}{32\left(3+H^2 k^2\right)^2}+O[t]^5\right) x^3+$$

$$\left(-\frac{\sqrt{3}\sqrt{g\, H}\, k^5\left(177+124 H^2 k^2+20 H^4 k^4\right)}{640\left(3+H^2 k^2\right)^{5/2}}-\frac{3\left(\sqrt{3}\,(g\, H)^{3/2} k^7\left(167+124 H^2 k^2+20 H^4 k^4\right)\right) t^2}{1280\left(3+H^2 k^2\right)^{7/2}}+\frac{9\, i\, g^2 H^2 k^8\left(81+62 H^2 k^2+10 H^4 k^4\right) t^3}{640\left(3+H^2 k^2\right)^4}+\frac{9\sqrt{3}\,(g\, H)^{5/2} k^9\left(157+124 H^2 k^2+20 H^4 k^4\right) t^4}{2560\left(3+H^2 k^2\right)^{9/2}}+O[t]^5\right) x^4+O[x]^5,$$

$$\left(-\frac{\sqrt{3}\, g\, H\, k}{\sqrt{g\, H\left(3+H^2 k^2\right)}}-\frac{1}{2}\left(\sqrt{3}\, k^3\left(\frac{g\, H}{3+H^2 k^2}\right)^{3/2}\right) t^2-\frac{9\, i\, g^2 H^2 k^4 t^3}{8\left(3+H^2 k^2\right)^2}+\frac{9}{20}\sqrt{3}\, k^5\left(\frac{g\, H}{3+H^2 k^2}\right)^{5/2} t^4+O[t]^5\right)+$$

$$\left(\frac{\sqrt{3}\sqrt{g\, H}\, k^3}{8\left(3+H^2 k^2\right)^{3/2}}+\frac{3\sqrt{3}\,(g\, H)^{3/2} k^5 t^2}{16\left(3+H^2 k^2\right)^{5/2}}+\frac{9\, i\, g^2 H^2 k^6 t^3}{16\left(3+H^2 k^2\right)^3}-\frac{9\left(\sqrt{3}\,(g\, H)^{5/2} k^7\right) t^4}{32\left(3+H^2 k^2\right)^{7/2}}+O[t]^5\right) x^2+$$

$$\left(\frac{1}{8}\, i\sqrt{g\, H}\, k^4+\frac{3\, i\,(g\, H)^{3/2} k^6 t^2}{16\left(3+H^2 k^2\right)}-\frac{3\left(\sqrt{3}\, g^2 H^2 k^7\right) t^3}{16\left(3+H^2 k^2\right)^{3/2}}-\frac{9\, i\,(g\, H)^{5/2} k^8 t^4}{32\left(3+H^2 k^2\right)^2}+O[t]^5\right) x^3+$$

$$\left.\left(\frac{\sqrt{3}\sqrt{g\, H}\, k^5\left(177+124 H^2 k^2+20 H^4 k^4\right)}{640\left(3+H^2 k^2\right)^{5/2}}+\frac{3\sqrt{3}\,(g\, H)^{3/2} k^7\left(167+124 H^2 k^2+20 H^4 k^4\right) t^2}{1280\left(3+H^2 k^2\right)^{7/2}}+\frac{9\, i\, g^2 H^2 k^8\left(81+62 H^2 k^2+10 H^4 k^4\right) t^3}{640\left(3+H^2 k^2\right)^4}-\frac{9\left(\sqrt{3}\,(g\, H)^{5/2} k^9\left(157+124 H^2 k^2+20 H^4 k^4\right)\right) t^4}{2560\left(3+H^2 k^2\right)^{9/2}}+O[t]^5\right) x^4+O[x]^5\right\}$$

Out[178]=

$$\Bigg\{\left(\frac{1}{2}\sqrt{3}\,k^3\left(\frac{g\,H}{3+H^2\,k^2}\right)^{3/2}t^2-\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}-\frac{9}{20}\left(\sqrt{3}\,k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}\right)t^4+O[t]^5\right)+$$

$$\left(-\frac{\sqrt{3}\,\sqrt{g\,H}\,k^3}{8\left(3+H^2\,k^2\right)^{3/2}}-\frac{3\left(\sqrt{3}\,(g\,H)^{3/2}\,k^5\right)t^2}{16\left(3+H^2\,k^2\right)^{5/2}}+\frac{9\,i\,g^2\,H^2\,k^6\,t^3}{16\left(3+H^2\,k^2\right)^3}+\frac{9\sqrt{3}\,(g\,H)^{5/2}\,k^7\,t^4}{32\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\right)x^2+$$

$$\left(\frac{1}{8}\,i\sqrt{g\,H}\,k^4+\frac{3\,i\,(g\,H)^{3/2}\,k^6\,t^2}{16\left(3+H^2\,k^2\right)}+\frac{3\sqrt{3}\,g^2\,H^2\,k^7\,t^3}{16\left(3+H^2\,k^2\right)^{3/2}}-\frac{9\,i\,(g\,H)^{5/2}\,k^8\,t^4}{32\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+$$

$$\left(-\frac{\sqrt{3}\,\sqrt{g\,H}\,k^5\left(177+124\,H^2\,k^2+20\,H^4\,k^4\right)}{640\left(3+H^2\,k^2\right)^{5/2}}-\frac{3\left(\sqrt{3}\,(g\,H)^{3/2}\,k^7\left(167+124\,H^2\,k^2+20\,H^4\,k^4\right)\right)t^2}{1280\left(3+H^2\,k^2\right)^{7/2}}+\frac{9\,i\,g^2\,H^2\,k^8\left(81+62\,H^2\,k^2+10\,H^4\,k^4\right)t^3}{640\left(3+H^2\,k^2\right)^4}+\frac{9\sqrt{3}\,(g\,H)^{5/2}\,k^9\left(157+124\,H^2\,k^2+20\,H^4\,k^4\right)t^4}{2560\left(3+H^2\,k^2\right)^{9/2}}+O[t]^5\right)x^4+O[x]^5,$$

$$\left(-\frac{1}{2}\left(\sqrt{3}\,k^3\left(\frac{g\,H}{3+H^2\,k^2}\right)^{3/2}\right)t^2-\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}+\frac{9}{20}\sqrt{3}\,k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}t^4+O[t]^5\right)+$$

$$\left(\frac{\sqrt{3}\,\sqrt{g\,H}\,k^3}{8\left(3+H^2\,k^2\right)^{3/2}}+\frac{3\sqrt{3}\,(g\,H)^{3/2}\,k^5\,t^2}{16\left(3+H^2\,k^2\right)^{5/2}}+\frac{9\,i\,g^2\,H^2\,k^6\,t^3}{16\left(3+H^2\,k^2\right)^3}-\frac{9\left(\sqrt{3}\,(g\,H)^{5/2}\,k^7\right)t^4}{32\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\right)x^2+$$

$$\left(\frac{1}{8}\,i\sqrt{g\,H}\,k^4+\frac{3\,i\,(g\,H)^{3/2}\,k^6\,t^2}{16\left(3+H^2\,k^2\right)}-\frac{3\left(\sqrt{3}\,g^2\,H^2\,k^7\right)t^3}{16\left(3+H^2\,k^2\right)^{3/2}}-\frac{9\,i\,(g\,H)^{5/2}\,k^8\,t^4}{32\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+$$

$$\left(\frac{\sqrt{3}\,\sqrt{g\,H}\,k^5\left(177+124\,H^2\,k^2+20\,H^4\,k^4\right)}{640\left(3+H^2\,k^2\right)^{5/2}}+\frac{3\sqrt{3}\,(g\,H)^{3/2}\,k^7\left(167+124\,H^2\,k^2+20\,H^4\,k^4\right)t^2}{1280\left(3+H^2\,k^2\right)^{7/2}}+\frac{9\,i\,g^2\,H^2\,k^8\left(81+62\,H^2\,k^2+10\,H^4\,k^4\right)t^3}{640\left(3+H^2\,k^2\right)^4}-\frac{9\left(\sqrt{3}\,(g\,H)^{5/2}\,k^9\left(157+124\,H^2\,k^2+20\,H^4\,k^4\right)\right)t^4}{2560\left(3+H^2\,k^2\right)^{9/2}}+O[t]^5\right)x^4+O[x]^5\Bigg\}$$