```
ln[267] = q = q0 * Exp[I * (k * x + w * t)];
      qjn = q0 * Exp[I * (k * xj + w * tn)];
      qjbar = Integrate [q, \{x, xj - dx/2, xj + dx/2\}]/(dx);
      qjnbar = qjbar /. t \rightarrow tn;
      MA = qjn / qjnbar;
      qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
      qjntbar = qntbar /. x \rightarrow xj;
      MtA = qjntbar / qjn;
      qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
      RA = Simplify[MA * qjphn / (qjn)];
      vmultG = H + H^3/3*k^2;
      GnA = -U * RA / vmultG;
      GGA = RA / vmultG;
      GcA = -U * H / vmultG ;
       fn1A = H * vh + U * eh;
      fn1A = fn1A /. vh \rightarrow (GGA * Gca + GnA * eca) /. eh \rightarrow RA * eca;
      fn1Gca0A = fn1A / . Gca \rightarrow 0 ;
      fn1eca0A = fn1A/. eca \rightarrow 0;
       fnnA = Simplify[fn1Gca0A / eca];
       fnGA = fn1eca0A / Gca;
       fncA = H * GcA;
      fG1A = U*Gh + U*H*vh + g*H*eh;
       \texttt{fG1A} = \texttt{fG1A} \ / \ . \ \texttt{vh} \ \rightarrow \ (\texttt{GGA} * \texttt{Gca} + \texttt{GnA} * \texttt{eca}) \ / \ . \ \texttt{eh} \ \rightarrow \ \texttt{RA} * \texttt{eca} \ / \ . \ \texttt{Gh} \ \rightarrow \ \texttt{RA} * \texttt{Gca} \ ;
       fG1Gca0A = fG1A / . Gca \rightarrow 0 ;
      fGleca0A = fGlA /. eca \rightarrow 0;
       fGnA = Simplify[fG1Gca0A / eca];
       fGGA = Simplify[fGleca0A / Gca];
       fGcA = U * H * GcA;
      FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
      FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
      FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
      FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;
      MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
      EA = \{\{1, 0\}, \{0, 1\}\} + MatA;
```

wAp = 
$$U * k + \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};$$
  
wAm =  $U * k - \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};$ 

```
Ru = (1 + Exp[I * k * dx]) / 2;
      Ruerr = Series \left[ Ru - Exp \left[ I * k * dx / 2 \right], \left\{ dx, 0, 5 \right\} \right];
      Gold = H - H^3/3 * (2 * Cos[k * dx] - 2) / dx^2;
      GG2 = Ru / Gold;
      GG2err = Series[GG2 - GGA, \{dx, 0, 5\}];
      Gn2 = -U * Ru / Gold;
      Gn2err = Series[Gn2 - GnA, \{dx, 0, 5\}];
      Text[Row[{"M ||
                           ", M}]]
      Text[Row[{"M ||
                           ", TeXForm[M]}]]
      Text[Row[{"M error || ", TeXForm[Merr]}]]
      Text[Row[{"M error || ", Merr}]]
      Text[" "]
      Text[Row[{"Rm ||
                            ", Rm}]]
                            ", TeXForm[Rm]}]]
      Text[Row[{"Rm ||
      Text[Row[{"Rm error ||
                                   ", Rmerr}]]
      Text[Row[{"Rm error ||
                                 ", TeXForm[Rmerr]}]]
      Text[" "]
      Text[Row[{"Rp ||
                            ", Rp}]]
      Text[Row[{"Rp || ", TeXForm[Rp]}]]
      Text[Row[{"Rp error ||
                                  ", Rperr}]]
      Text[Row[{"Rp error ||
                                   ", TeXForm[Rperr]}]]
      Text[" "]
      Text[Row[{"GG2 || ", GG2}]]
      Text[Row[{"GG2 || ", TeXForm[GG2]}]]
      Text[Row[{"GG2 error || ", GG2err}]]
      Text[Row[{"GG2 error || ", TeXForm[GG2err]}]]
      Text[" "]
      Text[Row[{"Gn2 || ", Gn2}]]
      Text[Row[{"Gn2 || ", TeXForm[Gn2]}]]
      Text[Row[{"Gn2 error || ", Gn2err}]]
                                    ", TeXForm[Gn2err]}]]
      Text[Row[{"Gn2 error ||
Out[316]= M \parallel 1
Out[317]= M \parallel 1
Out[318] = M error \parallel -\frac{dx}{dx}^2 k^2}{24}-\frac{7 \cdot (4x^4)^4 k^4}{5760} + O(\frac{dx}{6} \cdot (4x^6 - 4x^6)^4 k^4)}
```

ln[303]:= M = 1;

 $Merr = Series[M - MA, \{dx, 0, 5\}];$ 

Rmerr = Series[Rm - RA,  $\{dx, 0, 5\}$ ];

 $\texttt{Rperr = Series[Rp - RA, \{dx, 0, 5\}];}$ 

Rp = Exp[I\*k\*dx]\*(1 - I\*Sin[k\*dx]/2);

Rm = (1 + I \* Sin[k \* dx] / 2);

Out[319]= M error || 
$$-\frac{k^2 dx^2}{24} - \frac{7 k^4 dx^4}{5760} + O[dx]^6$$

Out[320]=

Out[321]= Rm ||  $1 + \frac{1}{2}i \sin[dx k]$ 

 $Out[322] = Rm \mid\mid 1 + \frac{1}{2} i \sin (\text{text}\{dx\} k)$ 

Out[323]= Rm error ||  $\frac{k^2 dx^2}{12} - \frac{1}{12} i k^3 dx^3 + \frac{k^4 dx^4}{720} + \frac{1}{240} i k^5 dx^5 + O[dx]^6$ 

 $\label{eq:continuous} \mbox{Out} \mbox{\cite{continuous}} Rm \mbox{\cite{continuous}} \mbox{\c$ 

 $k^3 + \frac{dx}^4 k^4}{720} + \frac{1}{240} i \cdot \frac{dx}^5 k^5 + O\left(\frac{dx}^6\right)^6\right)$ 

Out[325]=

Out[326]= Rp ||  $e^{i dx k} \left(1 - \frac{1}{2} i Sin[dx k]\right)$ 

 $\label{eq:out_327} \mbox{Out_327]= } Rp \ || \ e^{i \operatorname{k} k} \left(1 - \frac{1}{2} i \sin \left( \operatorname{k} k \right) \right) | \ e^{i \operatorname{k} k} \right) | \ e^{i \operatorname{k} k} | \ e^{i$ 

 $\text{Out} \text{[328]=} \quad Rp \; error \; \mid \mid \quad \frac{k^2 \, dx^2}{12} \, + \, \frac{1}{6} \; \dot{\iota} \; k^3 \; dx^3 \, - \, \frac{89 \, k^4 \, dx^4}{720} \, - \, \frac{7}{120} \; \dot{\iota} \; k^5 \; dx^5 \, + \, O[dx]^6$ 

 $k^3-\frac{89 \text{ } k^4}{720}-\frac{7}{120} i \text{ } k^5+O\left(\frac{4x}^6\right)^6\right)}{120} i \text{ } k^5+O\left(\frac{4x}^6\right)^6\right)$ 

Out[330]=

Out[331]= 
$$GG2 \mid \mid \frac{1 + e^{i dx k}}{2 \left(H - \frac{H^3 (-2 + 2 \cos[dx k])}{3 dx^2}\right)}$$

 $\label{eq:outg32} Outg32 = GG2 \parallel \frac{1+e^{i \text{text}\{dx\} k}}{2 \cdot (4x \cdot (4$ 

$$\text{Out} \text{[333]= } GG2 \ error \ \parallel \ \frac{\left(-6 \, k^2 - H^2 \, k^4\right) dx^2}{4 \, H \, (3 + H^2 \, k^2)^2} - \frac{i \, \left(6 \, k^3 + H^2 \, k^5\right) dx^3}{8 \, H \, (3 + H^2 \, k^2)^2} + \frac{\left(144 \, k^4 + 45 \, H^2 \, k^6 + 4 \, H^4 \, k^8\right) dx^4}{240 \, H \, (3 + H^2 \, k^2)^3} - \frac{i \, \left(-54 \, k^5 + H^4 \, k^9\right) dx^5}{480 \, H \, (3 + H^2 \, k^2)^3} + O[dx]^6$$

Out[334]=  $GG2 error \parallel$ 

 $\label{left} $$ \left( \frac{x^2 + 3 \cdot (H^2 k^2 + 3 \cdot (H^$ 

Out[335]=

$$\text{Out} \text{[336]=} \quad Gn2 \quad || \quad -\frac{\left(1 + e^{i\,dx\,k}\right)U}{2\left(H - \frac{H^3\left(-2 + 2\,Cos\left[dx\,k\right]\right)}{3\,dx^2}\right)}$$

 $Out[337] = Gn2 \parallel -\left\{U \left(1+e^{i \left(t+e^{i \left(t+e^{i \left(t+e^{i \left(t+e^{i}\right)}\right)}\right)}\right\} 2 \left(t+e^{i \left(t+e^{i}\right)}\right)}\right)}}}\right)}\right)}}\right)}\right)}}\right)}}\right)} \right]}$ 

$$\text{Out} \text{[338]= Gn2 error ||} \quad \frac{\left(6\,k^2 + H^2\,k^4\right)\,\text{U}\,dx^2}{4\,H\,(3 + H^2\,k^2)^2} + \\ \frac{i\,\left(6\,k^3 + H^2\,k^5\right)\,\text{U}\,dx^3}{8\,H\,(3 + H^2\,k^2)^2} - \\ \frac{\left(\left(144\,k^4 + 45\,H^2\,k^6 + 4\,H^4\,k^8\right)\,\text{U}\right)dx^4}{240\left(H\,(3 + H^2\,k^2)^3\right)} + \\ \frac{i\,\left(-54\,k^5 + H^4\,k^9\right)\,\text{U}\,dx^5}{480\,H\,(3 + H^2\,k^2)^3} + O[dx]^6 + O[dx]^6$$

```
\frac{\det d^2 k^4+6 k^2\right}{H \left(\frac{k^2 k^4+6 k^2\right)}{H \left(\frac{k^2 k^2+3\right)}^2+\frac{k^3 U \left(\frac{k^2 k^4+6 k^2\right)}{H \left(\frac{k^2 k^2+3\right)}}{H \left(\frac{k^2 k^2+3 k^2 u^2}{H \left(\frac{k^2 k^2 u^2}{H \left(\frac{k^2 u^
                                        k^5+6 k^3\right] \{8 H \left(H^2 k^2+3\right)^2-\frac{d^4 \left(H^4 k^8+45\right)^2}{h^4 \left(H^4 k^8+45\right)^2} = \frac{1}{h^4 k^8+45}
                                        H^2 k^6+144 k^4\right) + \frac{1}{240} \left(H^2 k^2+3\right)^3\right) + \frac{1}{240} \left(H^2 k^2+3\right)^3\left(H^2 k^2+3\right)^3\right) + \frac{1}{240} \left(H^2 k^2+3\right)^3\left(H^2 k^2+3\right)^3\left(H^
                                         U \left( H^4 k^9 - 54 k^5 \right) \left( 480 H \left( H^2 k^2 + 3 \right)^3 + O\left( \left( text \left( dx \right)^6 \right) \right) \right) 
\ln[340] = \text{KurF} = (\text{fm} * \text{ap} - \text{fp} * \text{am} + \text{am} * \text{ap} * (\text{qp} - \text{qm})) / (\text{ap} - \text{am});
                     KurFWS = KurF /. ap \rightarrow (U + Sqrt[g * H]) /. am \rightarrow (U - Sqrt[g * H]);
                     KurFWSeta =
                               KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
                                     qm \rightarrow Rmp * n;
                     KurFWSeta = KurFWSeta /. v \rightarrow (GGp * G + Gnp * n);
                     Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1];
                     KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1];
                     Kfnn = Kfnnp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
                     KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
                     Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
                     Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
                     Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                     FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
                     FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
                     FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                     KurFWSG = KurFWS /. fp \rightarrow (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
                                               fm \rightarrow (U * Rmp * G + U * H * V + g * H * Rmp * n) /. qp \rightarrow Rpp * G /. qm \rightarrow Rmp * G;
                     KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
                     KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
                     KfGGp = FullSimplify[KurFWSG /. n \rightarrow 0 /. G \rightarrow 1];
                     KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
                     KfGG = KfGGp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
                     FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
                     FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
                     FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                      fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g*H]) / (2) * (Rp - Rm);
                     FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
                     FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
                     FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                     Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
                     Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2;
                     Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
                     EigvFmat2 = Eigenvalues[Fmat2];
```

Out[339]= Gn2 error |

```
6 2ndOrder.nb
```

```
RKStep = Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" -Sqrt(gH) < U < Sqrt(gH)"}]]</pre>
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn || ", KfGnp}]]
Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error || ", FGn2TAr}]]
Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG || ", KfGGp}]]
Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error || ", FGG2TAr}]]
Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error || ", RKstepTayr}]]
Text[Row[{"Omega error || ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||
                  ", EA}]]
Text[Row[{"EA || ", TeXForm[EA]}]]
Text[Row[{"Eerr || ", Eerr}]]
Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
```

```
Out[374] = -Sqrt(gH) < U < Sqrt(gH)
```

Out[375]=

Out[376]= Fnn || 
$$\frac{1}{2} \left( 2 \operatorname{Gnp} H + \operatorname{Rpp} \left( -\sqrt{g H} + U \right) + \operatorname{Rmp} \left( \sqrt{g H} + U \right) \right)$$

 $\label{eq:outgazz} \begin{tabular}{ll} $$ Outgazz = Fnn & \frac{1}{2} \left( \frac{1}{2} \left( \frac{Rmp}{ett(sqrt\{g H}+U\right)) + \frac{Rmp}{ett(u-sqrt\{g H}+u)} \right) + \frac{1}{2} \left( \frac{1}{2} \left( \frac{Rmp}{ett(sqrt\{g H}+u)) + \frac{Rmp}{ett(sqrt\{g H}+u)} \right) + \frac{1}{2} \left( \frac{Rmp}{ett(sqrt\{g H}+u)) + \frac{1}{2} \left( \frac{Rmp}{ett(sqrt{g H}+u)} + \frac$ 

Out[378]= Fnn error 
$$\| \left( -\frac{(H^2 k^3 U w) dt^2}{2 (3+H^2 k^2)} - \frac{i H^2 k^3 U w^2 dt^3}{6 (3+H^2 k^2)} + O[dt]^4 \right) + \left( -\frac{i (27 k^3+9 H^2 k^5+H^4 k^7) U dt}{12 (3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{8} \left( \sqrt{g H} k^4 \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) dx^4 + O[dt]^4 dx^4 + O[dt]^$$

Out[379]= Fnn error |

 $\left(-\frac{t}{2 \cdot t}\right)^2 \left(-\frac{t}{2 \cdot t}\right) - \frac{t}{2 \cdot t} \ U \cdot v^2 + 3 \cdot t \ U \cdot v^2 + 3$  $\left(H^2 k^2+3\right)+O\left(\left(t^4 k^7+9 H^2\right)\right)$  $k^5+27 \ k^3 \ U \ \text{text} \ dt \ 12 \ \text{left} \ (4^2 \ k^2+3 \ \text{right})^2 + O \ \text{left} \ (4t)^4 \ \text{right}) \ \text{right}) + \text{text} \ dx \ \text{heave} \ dx \$  $\left(-\frac{1}{8}\right)\left(-\frac$ 

Out[380]=

Out[381]= FnG || GGp H

Out[382]=  $FnG \parallel \text{text}\{GGp\} H$ 

$$\text{Out} \\ \text{383} = \quad FnG \; error \; \mid \mid \; \left( -\frac{3 \; (k \; w) \; dt^2}{2 \left( 3 + H^2 \; k^2 \right)} - \frac{i \; k \; w^2 \; dt^3}{2 \left( 3 + H^2 \; k^2 \right)} + O[dt]^4 \right) \\ + \left( \frac{i \left( 6 \; k^3 + H^2 \; k^5 \right) \; dt}{4 \left( 3 + H^2 \; k^2 \right)^2} + O[dt]^4 \right) dx^2 \\ + O[dx]^4 + O[dx$$

 $Out_{384} = FnG error \parallel \left(-\frac{3 \text{ kext}\{dt\}^2 (k w)}{2 \text{ left}(H^2 k^2 + 3 \text{ right})} - \frac{1}{2 k} k^2 + 3 \text{ right}\right) - \frac{1}{2 k} k^2 + 3 \text{ right}} = \frac{1}{2 k} k^2 + 3 \text{ right}$  $k^2+3\right)+O\left(\frac{4}{2}^2\right)+O\left(\frac{4}{2}^2\right)$  $\label{left} $$ \left(\frac{dt}{4 \left(\frac{h^2 k^2+3\right)}}-2\right)-\left(\frac{dt}{4 \left(\frac{h^2 k^2+3\right)}}-2\right)-2\right)$$ 

Out[385]=

$$\text{Out} [386] = FGn \mid\mid \frac{1}{2} \left( g H \left( Rmp + Rpp \right) + \left( 2 Gnp H + \sqrt{g H} \left( Rmp - Rpp \right) \right) U \right)$$

Out[387]= FGn ||

$$\begin{array}{ll} \text{Out} & \text{388} = & FGn \; error \; || \; \left( -\frac{\left( k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) \, w \right) \, dt^2}{2 \, \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) \, w^2 \, dt^3}{6 \, \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \\ & \left( -\frac{i \left( 9 \, g \, H \, k^3 + 6 \, g \, H^3 \, k^5 + g \, H^5 \, k^7 + 18 \, k^3 \, U^2 + 3 \, H^2 \, k^5 \, U^2 \right) \, dt}{12 \, \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{8} \left( \sqrt{g \, H} \, k^4 \, U \right) \, dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) + O[dt]^4 \right) dx^4 + O[dt]^4 + O[d$$

Out[389]= FGn error ||

 $\text{text}\{dt\}^3 \text{ k w}^2 \left\{ H^3 \text{ k}^2 + 3 \text{ g H} - 3 \text{ U}^2\right\}$  $k^2+3\right)+O\left(\frac{dt}^4\right)+\frac{dx}^2 \left(\frac{dx}^2 \left(\frac{dx}^4\right)\right)+\frac{dx}^2}{dx}$ k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12 \left(H^2  $k^2+3\left(\frac{dt}^4\right)+\left(\frac{dt}^4\right)$ H  $k^4$  Uright)  $\text{text}\{dt\}+O\left(\frac{dt}^4\right)\right)+O\left(\frac{dt}^4\right)+O\left(\frac{dt}^4\right)$ 

Out[390]=

$$\begin{array}{ll} \text{Out} \text{[393]=} & FGG \; error \; || \; \left( -\frac{\left( k \left( 6 + H^2 \, k^2 \right) \text{U} \, w \right) \, dt^2}{2 \, \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 6 + H^2 \, k^2 \right) \text{U} \, w^2 \, dt^3}{6 \, \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( -9 \, k^3 + 3 \, H^2 \, k^5 + H^4 \, k^7 \right) \text{U} \, dt}{12 \, \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \\ & \left( -\frac{1}{8} \left( \sqrt{g \, H} \, k^4 \right) dt + O[dt]^4 \right) dx^3 + \left( \frac{i \left( 297 \, k^5 \, \text{U} + 351 \, H^2 \, k^7 \, \text{U} + 118 \, H^4 \, k^9 \, \text{U} + 13 \, H^6 \, k^{11} \, \text{U} \right) dt}{240 \, \left( 3 + H^2 \, k^2 \right)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{aligned}$$

Out[394]= FGG error |

 $\label{left} $$\left(-\frac{dt}^2 \left( W \right)^2 \left( W \right)^$ 

 $U w^2 \left( H^2 k^2 + 6\right) + O\left( H^2 k^2 + 3\right) + O\left( H^2 k^2 + 3\right)$ 

 $\label{left} $$\left(-\frac{i \left(H^4 k^7+3 H^2 k^5-9 k^3\right) U \left(H^2 k^7+3 H^2 k^5-9 k^3\right) U \right) = 1.00 $$$ 

 $k^2+3\right)^2+O\left(\frac{dt}^4\right)+\frac{dx}^3\left(-\frac{1}{8}\right)^2+O\left(\frac{dt}^4\right)^2+O\left(\frac$ 

H^6 U k^{11}+118 H^4 U k^9+351 H^2 U k^7+297 U k^5\right) \text{dt}}{240

 $\left(H^2 k^2+3\right)^3+O\left(\left(text{dt}^4\right)\right)\right)+O\left(text{dx}^5\right)$ 

Out[395]=

Out[396]=

Out[397]= Omega error ||

Omega error 
$$\|$$

$$\left\{\left(\frac{1}{6(3+H^2k^2)^2}k^3\left(\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\right)\left(3\,g\,H + U\left(2\,\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\right)\right)dt^2 + \frac{1}{8(3+H^2k^2)^2}k^3\left(\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\right)\right)dt^2 + \frac{1}{8(3+H^2k^2)^2}k^4\left(\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\right)\left(3\,g\left(\sqrt{3} H\sqrt{g\,H\,(3+H^2\,k^2)} + 9\,H\,U + 3\,H^3\,k^2\,U\right) + \frac{1}{20\left(3+H^2\,k^2\right)^2}\left(k^5\left(\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\right)\right)dt^3 - \frac{1}{20\left(3+H^2\,k^2\right)^2}\left(k^5\left(\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\right)\right)dt^3 + 2H^2\,k^2\right)U\right)$$

$$\left(9\,g^2\,H^2 + 6\,g\,H\,U\left(2\,\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 3\,(3+H^2\,k^2)\,U\right) + U^3\left(12\,\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 9\,U + H^4\,k^4\,U + 2\,k^2\left(2\,\sqrt{3} \sqrt{g\,H^5\,(3+H^2\,k^2)} + 3\,H^2\,U\right)\right)\right)dt^4 + O[dt]^5\right) + \frac{1}{48\left(3+H^2\,k^2\right)^2}dt^2\left(2\sqrt{3} \sqrt{g\,H^3\,(3+H^2\,k^2)} + 18\,H^3\,k^2\,U + 6\,H^5\,k^4\,U\right) + \frac{1}{2k^4\left(2\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 54\,U + 2\,H^6\,k^6\,U + 3\,k^2\left(7\,\sqrt{3} \sqrt{g\,H^3\,(3+H^2\,k^2)} + 18\,H^2\,U\right) + 2\,k^4\left(2\sqrt{3} \sqrt{g\,H^3\,(3+H^2\,k^2)} + 54\,U + 2\,H^6\,k^6\,U + 3\,k^2\left(7\sqrt{3} \sqrt{g\,H^3\,(3+H^2\,k^2)} + 18\,H^2\,U\right) + 2\,k^4\left(2\sqrt{3} \sqrt{g\,H^3\,(3+H^2\,k^2)} + 18\,U^2\,U\right)\right)dt^2 + \frac{1}{48\left(3+H^2\,k^2\right)^2}dt^2\left(3\,g\,H + U\left(2\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 18\,U + 2\,H^4\,k^4\,U + 2\,k^2\left(\sqrt{3} \sqrt{g\,H^3\,(3+H^2\,k^2)} + 6\,H^2\,U\right)\right)\right)dt^3 - \frac{1}{96\left(3+H^2\,k^2\right)^2}\left(k^7\left(-3\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 18\,U + 2\,H^4\,k^4\,U + 2\,k^2\left(\sqrt{3} \sqrt{g\,H^3\,(3+H^2\,k^2)} + 6\,H^2\,U\right)\right)\right)dt^3 - \frac{1}{96\left(3+H^2\,k^2\right)^2}\left(k^7\left(-3\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 2\,(3+H^2\,k^2)\,U\right)\right) + U^3\left(12\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 6\,H^2\,U\right)\right)\right)dt^3 - \frac{1}{96\left(3+H^2\,k^2\right)^2}\left(k^7\left(-3\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 2\,(3+H^2\,k^2)\,U\right)\right) + U^3\left(12\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 6\,H^2\,U\right)\right)\right)dt^3 - \frac{1}{96\left(3+H^2\,k^2\right)^2}\left(k^7\left(-3\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 2\,(3+H^2\,k^2)\,U\right)\right) + U^3\left(12\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 6\,H^2\,U\right)\right)\right)dt^3 - \frac{1}{96\left(3+H^2\,k^2\right)^2}\left(k^7\left(-3\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)} + 2\,(3+H^2\,k^2)\,U\right)\right)dt^3 + U^3\left(2\sqrt{3} \sqrt{g\,H\,(3+H^2\,k^2)}\right)dt^3 + U^3\left(2\sqrt{3} \sqrt{g\,H\,(3+$$

$$\frac{1}{234940\sqrt{g \, H} \, (3441842)^{2}} e^{3/2} \left( 81 \sqrt{3} \, g^{2} \, H^{2} \, (157 + 124 \, H^{2} \, k^{2} + 20 \, H^{4} \, k^{4}) + \right. \\ \left. 6 \sqrt{3} \, g^{2} \, H^{2} \, (22535 + 32481 \, H^{2} \, k^{2} + 10584 \, H^{4} \, k^{4} + 1124 \, H^{6} \, k^{6}) \, U^{2} + \\ \left. 3 \, g \, H \, U^{3} \left( 69120 \sqrt{g \, H} \, (3 + H^{2} \, k^{2}) + 34155 \sqrt{3} \, \, U + 44982 \sqrt{3} \, \, H^{2} \, k^{2} \, U + \right. \\ \left. 396 \sqrt{3} \, \, H^{3} \, k^{3} \, U + 27 \, k^{4} \, \left( 816 \sqrt{g \, H^{9} \, (3 + H^{2} \, k^{2})} + 821 \sqrt{3} \, \, H^{4} \, U \right) + \right. \\ \left. 28 \, k^{6} \left( 84 \sqrt{g \, H^{13} \, (3 + H^{2} \, k^{2})} \, U^{4} + 173 \sqrt{3} \, \, H^{6} \, U \right) + 8 \, U \, \left( 348 \, k^{6} \sqrt{g \, H^{13} \, (3 + H^{2} \, k^{2})} \, U^{4} + \right. \\ \left. 29 \, k^{8} \sqrt{g \, H^{17} \, (3 + H^{2} \, k^{2})} \, U^{4} + 81 \, \left( 130 \sqrt{g^{5} \, H^{2} \, k^{2}} \, + 29 \sqrt{g \, H \, (3 + H^{2} \, k^{2})} \, U^{4} \right) + \right. \\ \left. 54 \, k^{2} \left( 137 \sqrt{g^{2} \, H^{9} \, (3 + H^{2} \, k^{2})} \, U^{4} + 172 \sqrt{g^{3} \, H^{7} \, (3 + H^{2} \, k^{2})} \, U^{2} + 58 \sqrt{g \, H^{3} \, (3 + H^{2} \, k^{2})} \, U^{2} + 58 \sqrt{g \, H^{3} \, (3 + H^{2} \, k^{2})} \, U^{2} + 58 \sqrt{g \, H^{3} \, (3 + H^{2} \, k^{2})} \, U^{4} + O (dt)^{5} \right) \right. \\ \left. \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} + (3 + H^{2} \, k^{2}) U \right) \right) \left( 3 \, g \, H + U \left( -2 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} \, + (3 + H^{2} \, k^{2}) U \right) \right) \right. \right. \\ \left. \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} + (3 + H^{2} \, k^{2}) U \right) \right) \right. \right. \\ \left. \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} + (3 + H^{2} \, k^{2}) U \right) \right) \right. \right. \right. \\ \left. \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} + (3 + H^{2} \, k^{2}) U \right) \right) \right. \right. \right. \\ \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} + (3 + H^{2} \, k^{2}) U \right) \right) \right. \right. \\ \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} + (3 + H^{2} \, k^{2}) U \right) \right) \right. \right. \\ \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} + (3 + H^{2} \, k^{2}) U \right) \right) \right. \right. \\ \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} + (3 + H^{2} \, k^{2}) U \right) \right. \right. \\ \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H^{2} \, k^{2})} + (3 + H^{2} \, k^{2}) U \right) \right. \right. \\ \left. \left. \left( 1 + 3 \sqrt{3} \sqrt{g \, H \, (3 + H$$

$$\frac{dt^4 + O[dt]^5}{dt^6} \frac{dx^2 + }{dt^6} = \frac{\sqrt{3} \text{ U}}{\sqrt{g \text{ H}(8 + \text{H}^2 \text{ W}^2)}} + \frac{1}{32(3 + \text{H}^2 \text{ W}^2)^{3/5}} \\ i \text{ $k^6} \left(g \left(-6 \text{ H} \sqrt{g \text{ H}(3 + \text{H}^2 \text{ W}^2)} + 15 \sqrt{3} \text{ H} \text{ U} + 4 \sqrt{3} \text{ H}^2 \text{ k}^2 \text{ U}\right) + \\ U^2 \left(-12 \sqrt{g \text{ H}(3 + \text{H}^2 \text{ k}^2)} + 15 \sqrt{3} \text{ H} \text{ U} + 4 \sqrt{3} \text{ H}^2 \text{ k}^2 \text{ U}\right) + \\ U^2 \left(-12 \sqrt{g \text{ H}(3 + \text{H}^2 \text{ k}^2)} + 13 \sqrt{3} \text{ U} + \text{k}^2 \left(-2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + \sqrt{3} \text{ H}^2 \text{ U}\right)\right)\right) dt^2 - \\ \frac{1}{32(3 + \text{H}^2 \text{ W}^2)^{3/6}} \left[k^2 \left(3 \text{ g} \text{ H} + \text{U} \left(-2 \sqrt{3} \sqrt{g \text{ H}(3 + \text{H}^2 \text{ k}^2)} + (3 + \text{H}^2 \text{ k}^2) \text{ U}\right)\right) \left(2 \sqrt{3} \text{ g} \text{ H}(3 + \text{H}^2 \text{ k}^2) + \text{U}\right)\right)\right) dt^2 - \\ \frac{1}{32(3 + \text{H}^2 \text{ K}^2)^{3/6}} \left[k^2 \left(2 \text{ g} \text{ H}(3 + \text{H}^2 \text{ k}^2) + 3 \sqrt{3} \text{ U} + \text{k}^2 \left(-2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + \sqrt{3} \text{ H}^2 \text{ U}\right)\right)\right)\right) dt^2 - \\ \frac{1}{32(3 + \text{H}^2 \text{ K}^2)^{3/6}} \left[k^2 \left(2 \text{ g} \text{ H}(3 + \text{H}^2 \text{ k}^2)^2 + 3 \sqrt{3} \text{ U} + \text{k}^2 \left(-2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + \sqrt{3} \text{ H}^2 \text{ U}\right)\right)\right)\right) dt^2 - \\ \frac{1}{32(3 + \text{H}^2 \text{ K}^2)^{3/6}} \left[k^2 \left(2 \text{ g} \text{ H}(3 + \text{H}^2 \text{ k}^2)^2 + 3 \sqrt{3} \text{ U} + \text{k}^2 \left(-2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + \sqrt{3} \text{ H}^2 \text{ U}\right)\right)\right)\right) dt^2 - \\ \frac{1}{32(3 + \text{H}^2 \text{ K}^2)^{3/6}} \left[k^2 \left(2 \text{ g} \text{ H}(3 + \text{H}^2 \text{ k}^2)^2 + 3 \sqrt{3} \text{ U} + \text{k}^2 \left(-2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + \sqrt{3} \text{ H}^2 \text{ U}\right)\right)\right)\right)\right) dt^4 dt^4 + 4 \sqrt{3} \left(4 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 3 \sqrt{3} \left(-1 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 3 \sqrt{3} \text{ U} + \text{k}^2 \left(-2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + \sqrt{3} \text{ H}^2 \text{ U}\right)\right)\right)} dt^4 \right) dt^4 dt^4 + 4 \sqrt{3} \left(8 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 4 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + \sqrt{3} \text{ H}^2 \text{ U}\right) + 2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)}} + 2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)}} + 2 \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 2 \sqrt{g \text{ H}^3 (3$$

$$\begin{split} &27 \, k^2 \left(372 \, \sqrt{g^3} \, H^7 \left(3 + H^2 \, k^2\right) \, - 2703 \, \sqrt{3} \, \, g^2 \, H^7 \, U + 4515 \, \sqrt{g^3} \, H^7 \left(3 + H^2 \, k^2\right) \, U^2 \, - \\ &4070 \, \sqrt{3} \, \, g \, H^3 \, U^3 + 928 \, \sqrt{g \, H^5 \left(3 + H^2 \, k^2\right)} \, U^4 \right) + \\ &9 \, k^4 \left(180 \, \sqrt{g^5 \, H^{13} \left(3 + H^2 \, k^2\right)} \, - 2672 \, \sqrt{3} \, \, g^2 \, H^6 \, U + 1392 \, \sqrt{g \, H^9 \left(3 + H^2 \, k^2\right)} \, U^4 + g \, H^5 \right. \\ & \left. U^2 \left(4384 \, \sqrt{g \, H \left(3 + H^2 \, k^2\right)} \, - 5997 \, \sqrt{3} \, \, U\right)\right) + 81 \left(157 \, \sqrt{g^5 \, H^5 \left(3 + H^2 \, k^2\right)} \, - 883 \, \sqrt{3} \right. \\ & \left. g^2 \, H^2 \, U + 232 \, \sqrt{g \, H \left(3 + H^2 \, k^2\right)} \, U^4 + g \, H \, U^2 \left(1527 \, \sqrt{g \, H \left(3 + H^2 \, k^2\right)} \, - 1033 \, \sqrt{3} \, \, U\right)\right) - \\ & 12 \, k^6 \, U \left(213 \, \sqrt{3} \, g^2 \, H^8 - 232 \, \sqrt{g \, H^{13} \left(3 + H^2 \, k^2\right)} \, U^3 \, + \\ & g \, H^7 \, U \left(-349 \, \sqrt{g \, H \left(3 + H^2 \, k^2\right)} \, + 979 \, \sqrt{3} \, \, U\right)\right)\right)\right) dt^4 + O[dt]^5 \right) dx^4 + O[dx]^5 \right\} \end{split}$$

## Out[398]= Omega error ||

\left\\left\\frac{k^3 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U  $\label{left(H^2 k^2+3\rightarrow U+2 \sqrt{3} \sqrt{3} \sqrt{3} \sqrt{4} \frac{H^2 k^2+3\right)} \left(\frac{h^2 k^2+3\right)}{t^2 k^2+3\right)} $$ ight) \cdot text{dt}^2} {6 \left(\frac{h^2 k^2+3\right)}{t^2 k^2+3\right)} $$ ight) \cdot text{dt}^2} {6 \left(\frac{h^2 k^2+3\right)}{t^2 k^2+3\right)} $$ ight)^2 {6 \left(\frac{h^2 k^2+3}{t^2 k^2+3\right)}} $$ ight)^2 {6 \left(\frac{h^2 k^2+3}{t^2 k$  $\label{eq:k-2+3-index} $$k^2+3\right)^2+\frac{k^2+3\left(H^2 k^2+3\right)} U+\sqrt{3} \cdot H\left(H^2 k^2+3\right)^2+\frac{k^2+3\left(H^2 k^2+3\right)}{H^2 k^2+3\right)^2} $$$  $k^2+3\right) H\rightarrow (k^5 \left(\frac{k^5}{k^2+3\right)} H\rightarrow (k^5 \left(\frac{k^5}{k^2+3\right)} + \frac{k^2+3\right) H\rightarrow (k^5 \left(\frac{k^5}{k^2+3\right)} + \frac{k^5}{k^2+3\right) H\rightarrow (k^5 \left(\frac{k^5}{k^2+3\right)} + \frac{k^5}{k^2+3} + \frac$  $U+\sqrt{3} \sqrt{4+2 \left(H^2 k^2+3\right)}\right) \left(H^2 k^2+3\right) \left(H^4 U k^4+2 \left(H^4 U k^4+2 \right)\right)$  $H^5 \left(H^2 k^2+3\right) \right) \left(H^2 k^2+3\right) \left(H^2 k$ \left(3 \left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right)\right)  $\label{eq:left} $$ \left(\frac{dt}^4}{20 \left(\frac{x^2+3\right)^3}+O\left(\frac{dt}^5\right)\right)\right)^2+\left(\frac{x^2+3\right)^3}+O\left(\frac{dt}^5\right)^3\right)^2} $$$  $k^2+3\right)^2 U-3 \left(H^2 k^2+3\right)^2 U-3 \left(H^2 k^2+3\right)^2 +\frac{1}{4} \left(H^2 k^2+3\right)^2 U-3 \left(H^2$  $\left( \frac{H^6 U k^6+2 \left( 9 U H^4+2 \right) }{18} \right)$  $\label{eq:continuity} U H^2+7 \sqrt{3} \sqrt{g} H^5 \left(H^2 k^2+3\right)\right) h^2+5 U+27 \sqrt{3} \sqrt{3} \left(H^2 k^2+3\right)$ k^2+3\right)\right) U^2+g \left(6 k^4 U H^5+18 k^2 U H^3-9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right)\right) \left(U \left(2 H^4 U k^4+2 \left(6 U H^2+\sqrt{3}))  $\sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+18 U+3 \sqrt{3} \sqrt{3} \left(H^2 k^2+3\right)}\right)$ g H\right) \text{dt}^3}{48 \left(H^2 k^2+3\right)^3}-\frac{\\ left(k^7 \\ left(2 \\ left(H^2 k^2+3\right)^2)}  $U-3 \left( H^2 k^2+3\right) \left( H^4 U k^4+2 \left( U k^4+2 \left( U H^2+2 \right) \right) \right)$  $\$  \\ \sqrt{g H^5 \\ left(H^2 k^2+3\right)\\ right) \\  $U^3+6 g H \left( \frac{h^2 k^2+3\right)}{U+2 \left( \frac{h^2 k$  $U+9 g^2 H^2\right) + O\left(\frac{dt}^4}{96 \left(\frac{h^2 k^2+3\right)}{4}+O\left(\frac{dt}^5\right)}\right) + O\left(\frac{h^2 k^2+3\right)}{4} + O\left(\frac{h^2 k^2+3\right)}{4}$  $\text{text}(dx)^2 + \left(-\frac{k^4 \left(\frac{2 \text{ g H } \left(\frac{4^2 \text{ k}^2 + 3\right)}}{k^2 + \frac{3}{k^2 + 3}}\right)} + \frac{3 \text{ kgrt}(g \text{ H } \left(\frac{4^2 \text{ k}^2 + 3\right)}}{k^2 + \frac{3 \text{ kgrt}(g \text{ H } \left(\frac{4^2 \text{ k}^2 + 3\right)}}{k^2 + \frac{3 \text{ kgrt}(g \text{ H } \left(\frac{4^2 \text{ k}^2 + 3\right)}}{k^2 + \frac{3 \text{ kgrt}(g \text{ H } \left(\frac{4^2 \text{ k}^2 + 3\right)}}{k^2 + \frac{3 \text{ kgrt}(g \text{ H } \left(\frac{4^2 \text{ k}^2 + 3\right)}}{k^2 + \frac{3 \text{ kgrt}(g \text{ H } \left(\frac{4^2 \text{ k}^2 + 3\right)}{k^2 + 3}\right)}}$  $\left(H^2 k^2+3\right) \left(H^2 k^2+3\right)$  $\left(\frac{4 h^2 k^2+15\right) U+6 \left(\frac{4 h^2 k^2+15\right)}{U+6 \left(\frac{4 h^2 k^2+3\right)}\right) \left(\frac{4 h^2 k^2+15\right) U+6 \left(\frac{4 h^2 k^2+15\right)}{U+6 \left(\frac{4 h^2$  $\left(H^2 k^2+3\right) + \frac{k^2+3\left(H^2 k^2+3\right)}{3/2} + \frac{k^7 \left(H^2 k^2+3\right) + \frac{k^2+3\left(H^2 k^2+3\right)}{3/2}}{1/2} + \frac{k^2+3\left(H^2 k^2+3\right) + \frac{k^2+3\left(H$  $\$  \sqrt{g H^5 \left(H^2 k^2+3\right)\right) \k^2+3 \sqrt{3} U+9 \sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right  $\text{dt}^3$ {32 \left(H^2 k^2+3\right)^{5/2}}+\frac{i k^8 \cdot left(2 g H \left(H^2 k^2+3\right)+\sqrt{3} \sqrt{g}

 $H \left(H^2 k^2+3\right) \left(H^4 U k^4+2 \left(H^4 U H^2+2 \right) \right)$  $H} \left(\frac{d^2 k^2+3\right)^3}{O\left(\frac{d^3}{3}\right)} + O\left(\frac{d^3}{3}\right) \left(\frac{d^3}{3}\right) \left$  $H \left( 20 \text{ H}^4 \text{ k}^4 + 124 \text{ H}^2 \text{ k}^2 + 177 \right) + 104 \left( \frac{9 \text{ H}^6 \left( \frac{4^2 \text{ k}^2 + 3 \right)} \text{ k}^4 + 6 \right) }{4^2 \text{ k}^2 + 3 \text{$  $H^5 \left( H^2 k^2 + 3\right) k^2 + 9 \left( H^2 k^2 + 3\right) U\right) U\right) \left( H^2 k^2 + 3\right) U\right) U\right) \left( H^2 k^2 + 3\right) U\right) U\right)$ k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6+24 \left(293 \sqrt{3} U H^4+84 \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267 \sqrt{3} U+232 \sqrt{g H \left(H^2  $k^2+3\right) \left( H^2 k^2+3\right) \left( H^2 k^2+3\right) U^3 k^6+153 \right) H^6 \left( H^2 k^2+3\right) U^3 k^6+153 \right) H^6 U^3 k^6+153 \right) H^6 U^3 k^6+153 U^3 k^6+154 U^$  $\left(H^2 k^2+3\right) U^3 k^4+9\left(11 \right) U^3 k^4+9 \left(11 \right) U^3 k^4+9 \left(11 \right) U^3 U^3+88 \left(11 \right) U^3 U^3+88 \left(11 \right) U^3 U^3+88 \left(11 \right) U^3 U^3+10 U^3+10$  $k^2+3\right) U\dot h^2 + 459 \left( H^2 k^2+3\right) U^3\right) U^3\right) text{dt}^2{11520}$  $\left( \frac{4 + 4 \cdot 4 \cdot 4^2 + 3 \cdot 4^2 + 3 \cdot 4^2 + 3 \cdot 4^2 + 3 \cdot 4^2 + 4 \cdot 4^2 \cdot 4^2 + 4 \cdot 4^2 \cdot 4^2 \right)$  $H^6+83 \sqrt{13} \left( H^{13} \left( H^2 k^2+3\right)\right) \right) h^6+36 U^3 \left( 126 U H^4+85 \right) h^6+36 U^3 \right)$  $\$  \sqrt{g H^9 \left(H^2 k^2+3\right)\right) \k^4+9 \left(1008 H^2 U^4+1039 \\ \qrt{3} \\ \qrt{g H^5 \left(H^2 L^2+3\right)} \\ \qrt{1008 H^2 U^4+1039 \\ \qrt{3} \\ \qrt{1008 H^2 U^4+1039 \\ \qrt{10 k^2+3\right)} U^3+600 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U+39 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+3 g H U \left(472 H^6 U k^6+4 \left(1101 U H^4+71 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right)  $k^4 + 13500 \text{ H}^2 \text{ U k}^2 + 27 \left( 504 \text{ U} + 97 \right) \left( 3 \right) \left( 4 \text{ H}^2 \left( 4 \right) \right) \left( 4 \right) \right) \right) \left( 4 \right)$  $\text{dt}^3$ {3840 \left(H^2 k^2+3\right)^4}+\frac{k^9 \left(81 \sqrt{3} g^3 \left(20 H^4 k^4+124 H^2 H^2)} k^2+157\right) H^3+6 \sqrt{3} g^2 \left(1124 H^6 k^6+10584 H^4 k^4+32481 H^2 k^2+32535\right) U^2 H^2+3 g U^3 \left(396 \sqrt{3} H^8 U k^8+28 \left(173 \sqrt{3} U H^6+84 \sqrt{g H^{13} \left(H^2  $k^2+3\right) \ h^6+27 \left(821 \right) \ H^4+816 \ grt{g H^9 \left(H^2 k^2+3\right) \right) \ h^6+27 \ h^6+27 \ h^6+27 \ h^6+27 \ h^6+21 \ h^6+21$  $k^4 + 44982 \sqrt{4} H^2 U k^2 + 34155 \sqrt{3} U + 69120 \sqrt{g H \left(H^2 k^2 + 3\right)} \right) H + 8 U + 69120 \sqrt{g H \left(H^2 k^2 + 3\right)}$  $\left( \frac{9 \left( 4^{17} \left( 4^2 k^2 + 3\right) \right) U^4 k^8 + 348 \right) \left( 4^13 \left( 4^2 k^2 + 3\right) U^4 k^8 + 348 \right) \left( 4^13 \left( 4^2 k^2 + 3\right) U^4 k^8 + 348 \right) \left( 4^13 \left( 4^13 k^2 + 4\right) U^4 k^8 + 348 \right) \left( 4^13 k^2 + 4\right) \left( 4^13 k$  $k^6+27 \left(58 \right) H^9 \left(58 \right) U^4+43 \left(5^2 H^2 \right) U^4+43 \left(5^2 H^2 \right) \left(5^2 H^2 \right) U^4+43 \left(5^2 H^2 \right) U^4+$  $k^4+54 \left(58 \right) U^4+472 \left(5^3 H^7 \left(H^2 k^2+3\right) U^4+472 \right)$  $\label{eq:u^2+137 sqrt} $$U^2 + 137 \operatorname{sqrt}(g^5 H^9 \left(H^2 k^2 + 3\right)\right) + h^2 + 1 \left(19 \operatorname{sqrt}(g H \left(H^2 k^2 + 3\right)\right) + h^2 + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right)\right) + h^2 + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right)\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right)\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right)\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right)\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right)\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right)\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right)\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right) + h^2 \left(19 \operatorname{sqrt}(g H h^2 k^2 + 3\right)\right)$  $U^4+130 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \left(H^2 k^2+3\right)\right)$  $\left(\frac{H^2 k^2+3\right)}{U-\sqrt{H^2 k^2+3\right}} \right) U-\left(\frac{H^2 k^2+3\right)}{U-\sqrt{H^2 k^2+3\right}}$  $k^2+3\right\} \ U-2 \ H\left(H^2 \ k^2+3\right)\right) \ text{dt}^2{6 \left(H^2 \ k^2+3\right)} \ text{dt}^2{6 \left(H^2 \ k^2+3\right)}$  $k^2+3\right)^2+\frac{1}{2}+\frac{1}{2} +\frac{1}{2} +$  $\label{left(H^2 k^2+3\wedge ight)^2} \operatorname{left(H^2 k^2+3\wedge ight)^2}-\operatorname{left(k^5 \left(\left(H^2 k^2+3\right)\right)} \operatorname{left(H^2 k^2+3\wedge ight)^2}-\operatorname{left(h^2 k^2+3\wedge ight)^2}-\operatorname{left(h^2$  $k^2+3\right) U-2 \sqrt{3} \sqrt{4}{20 \left(H^2 k^2+3\right)\right)} \left(H^2 k^2+3\right) \left(H^2 k^2+3\right)$  $k^2+3\right/h^4+O\left(\frac{t}{5\right)}\right)$  $\$  \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}\right)}{24 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(\frac{1}{2} (1-x)^2 + \frac{1}{2} (1-x)^2 + \frac{1}{2  $H^6 U k^6-2 \left( \sqrt{3} \right)^9 \left( H^9 \left( H^2 k^2+3\right) -9 H^4 U \right) k^4-3 \left( \sqrt{3} \right)$  $\$  \sqrt{g H^5 \left(H^2 k^2+3\right)}-18 H^2 U\right) k^2+54 U-27 \sqrt{3} \sqrt{g H \left(H^2 L^2 + 24 L^2 k^2+3\right)\right) U^2+3 g \left(2 k^4 U H^5+6 k^2 U H^3+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}  $H \cdot (H^2 k^2+3\right) \cdot (H^2 k^2+3\right) \cdot (H^2 k^2+3) \cdot (H^2 k^2$ 

 $\left( \frac{3} \right) \left( \frac{4^2 \ln(4^2 k^2 + 3 \right)}{6^2 k^2 + 3 \right) - 6^2 U \right) \left( \frac{3^2 k^2 + 3 \ln(k^2 k^2 + 3 \ln(k$  $k^2+3\right)$  | Hright \text{dt}^3}{48 \left(H^2 k^2+3\right)^3}-\frac{(k^7 \left(K^7 \left(K^7 \right)^3)}{48 \left(K^7 \left(K^7 \right)^3}\right)^3}  $\left(H^2 k^2+3\right)\right) \le H \left(H^2 k^2+3\right) \left(H^2 k^2+3\right) \left(H^2 k^2+3\right) \le H \left(H^2 k^2+3\right) \le$  $k^2+3\right) U-2 \sqrt{3} \sqrt{4}{96} k^2+3\right) 10-2 \sqrt{3} \sqrt{4}{96} k^2+3\right) 10-2 \sqrt{4}{96} k^2+3\right) 10-2 \sqrt{4}$  $\left(\frac{H^2 k^2+3\right)^4}+O\left(\frac{dt}^5\right)\right) \$  $\left(\frac{3}{U}\right) = \frac{1}{2\pi} \left(\frac{3}{U}\right)$  $U-2 \left( H^2 k^2+3\right) + 12 \left( H^2 k^2+3\right) + 12$  $\label{eq:update} $$U^2+g \left( \frac{sqrt{3} k^2 U H^3+15 \left( H-6 \right) H\left( H^2 k^2+3\right) H\left( H\right) H\left( H^2 k^2+3\right) H\left( H\right) H\left( H^2 k^2+3\right) H\left( H\right) H\left( H^2 k^2+3\right) H\left( H^$  $\label{eq:left} $$ \operatorname{dt}^2_{32}\left(H^2 k^2+3\right)^{3/2}\right-\left(h^2 k^7 \left(h^7 \left(h^2 k^2+3\right)\right)^{3/2}\right). $$$  $U-2 \sqrt{3} \left( H \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} \right) \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3)} \left( \frac{h^2 k^2+3\right)}{h^2 k^2+3} \left( \frac{h^2 k^2+3}{h^2 k^2+3} \right) \left( \frac{h^2 k^2+3}{h^2 k^2+3} \left( \frac{h^2 k^2+3}{h^2 k^2+3} \right) \left( \frac{h^2 k^2+3}{h^2 k^2+3} \left( \frac{h^2 k^2+3}{h^2 k^2+3} \right) \left( \frac{h^2 k^2+3}{h^2 k^2+3} \left$  $\label{left(H^2 k^2+3\left| h^2 k^2 + 3\left| h^3 k^2 + 3\left| h^3$  $\left(2 g H \left(\frac{4^2 k^2+3\right)}{sqrt{g H \left(\frac{4^2 k^2+3\right)} \left(\frac{4^2 k^2+3\right)} U\right)} \right) \left(\frac{4 g H}{u}\right)$  $\left(\frac{H^2 k^2+3\left(\frac{4}{64}\right)}{1-2 \sqrt{4}}\right) = \frac{1}{4}\left(\frac{H^2 k^2+3\left(\frac{H^2 k^2+3\left(\frac{4}{64}\right)}{1-4}\right)}{1-2 \sqrt{4}}\right)}$  $\sqrt{3} g H \left( 0 H^4 k^4 + 124 H^2 k^2 + 177 \right) - 104 \left( g H^9 \left( 4^2 k^2 + 3\right) \right)$  $k^4+6 \left( H^2 k^2+3\right) k^2+9 \left( H^2 k^2+3\right) v^2+9 \left$  $k^2+167$  \right)  $H^2+g$  U \left(764 \sqrt{3}  $H^6$  U  $k^6-24$  \left(84 \sqrt{g  $H^9$  \left(H^2  $k^2+3$ \right)}-293  $\sqrt{3} H^4 U \right) k^4+21429 \sqrt{4} L^2 U k^2+81 \left(267 \right) U-232 \right] H^2 U k^2+81 \left(267 \right) L-232 \right]$  $k^2+3\right) \left( H^2 k^2+3\right) \ U^3 k^6+153 \left( H^13 \right) \ U^3 k^6+153 \right) \ U^3 k^6+153 \left( H^2 k^2+3\right) \ U^3 k^6+153 \right) \ U^3 k^6+153 \left( H^3 k^2+3\right) \ U^3 k^4+153 \left( H^3 k^2+153 \left( H$  $H^9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( 51 \right) H^5 \left( H^2 k^2 + 3 \right) U^3 + 8 \left( 51 \right)$  $H^7 \left(H^2 k^2+3\right) U\right) U\right) h^2 + 459 \left(H^2 k^2+3\right) U^3\right) U^3\right)$  $\text{dt}^2_{11520 \ \ H} \left( \frac{h^2 k^2+3\right)^{7/2}}-\frac{i k^8 \left( 4 H^8 U^4 k^8+4 U^3 \right)^{7/2}}{h^2} \right)$ \left(252 H^6 U-83 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+36 U^3 \left(126 H^4  $U-85 \sqrt{3} \sqrt{9 H^2 k^2+3 \sinh k^4-9 \left(-1008 H^2 U^4+1039 \right)}$ k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U-39 \sqrt{3} \sqrt{g H  $\left(H^2 k^2+3\right)\right)$ k^2+3\right)}-1101 H^4 U\right) k^4+13500 H^2 U k^2+27 \left(504 U-97 \sqrt{3} \sqrt{g H  $\left(H^2 k^2+3\right)\right)\right)$ \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}-\left(H^2 k^2+3\right) U\right) \left(4 U^3 \left(58 \sqrt{g}  $H^{17} \left( H^2 k^2 + 3\right) U - 239 \left( H^8 + g \right) R^3 + H^3 \left( H^2 k^2 + 3\right) U \left( H^2 k^2 + 3\right) U \left( H^3 k^2 +$ k^2+3\right)} U^3\right) k^6+9 \left(-2672 \sqrt{3} g^2 U H^6+g U^2 \left(4384 \sqrt{g H \left(H^2  $k^2+3\right)=5997 \cdot H^5+1392 \cdot H^9 \cdot H^2 \cdot H^$  $H^{13} \left( H^2 k^2 + 3\right) \right) + h^4 + 27 \left( -2703 \right) qrt{3} g^2 U H^4 - 4070 \right) qrt{3} g U^3$  $H^3+928 \left( H^5 \left( H^2 k^2+3\right) \right) U^4+4515 \left( H^2 k^2+3\right) U^2+372$  $\left(\frac{9^5 H^9 \left(\frac{4^2 k^2+3\right)}}{v^2+3}\right) k^2+81 \left(\frac{32 \left(\frac{4^2 k^2+3\right)}}{v^2+3}\right) U^4+g^2}$  $H \left(1527 \right) + \left($ 

Out[399]=

$$\begin{aligned} & \text{Out}[400] = & \text{ EA } || & \left. \left\{ \left\{ 1 + \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dx} \, w} \right) \, \text{H}^2 \, \text{k}^3 \, \text{U} \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{6 + 2 \, \text{H}^2 \, \text{k}^2 \right) \, \text{w}}, \, \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, \text{H} \, k \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2 \left( \text{H} + \frac{\text{H}^3 \, k^2}{3} \right) \, \text{w}} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( g \, \text{H} \, \left( 3 + \text{H}^2 \, \, k^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( g \, \text{H} \, \left( 3 + \text{H}^2 \, \, k^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( g \, \text{H} \, \left( 3 + \text{H}^2 \, \, k^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( g \, \text{H} \, \left( 3 + \text{H}^2 \, \, k^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( g \, \text{H} \, \left( 3 + \text{H}^2 \, \, k^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( g \, \text{H} \, \left( 3 + \text{H}^2 \, \, k^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}}{2} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( g \, \text{H} \, \left( 3 + \text{H}^2 \, \, k^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}}{2} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( g \, \text{H} \, \left( 3 + \text{H}^2 \, \, k^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right] \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}}{2} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \,$$

Out[401]=  $EA \parallel \label{eq:entropy} \$ 

\begin{array}{cc}

 $w \rightarrow H^2 U \csc \left( \frac{dx}{k}{2} \right) \\ k^3 \left( H^2 K^2 + 6 \right) \\ w + 1$ &  $\frac{i e^{\left(i e^{\left(k\right) k}}{2}} \left(1-e^{-i \left(k\right) k}\right) \left(1-e^{-i \left(k\right) k}\right) \left(1-e^{-i \left(k\right) k}\right)}{1-e^{-i \left(k\right) k}}$  $\frac{i e^{\frac{i \cdot k}{2}} \left(1 - e^{-i \cdot kxt\{dx\} k}\right)}{\left(1 - e^{-i \cdot kxt\{dx\} k}\right)} \left(1 - e^{-i \cdot kxt\{dx\} k}\right)} \left(1 - e^{-i \cdot kxt\{dx\} k}\right)$  $\label{left(H^2 k^2+3\circ H^2 k^2+6\circ H^2 k^2+$  $w\} \& \frac{i \cdot (i \cdot k}{2}) \left(1 - e^{-i \cdot k} k\right) \left(1$  $k \left( H^2 k^2 + 6 \right) U \csc \left( \frac{dx}{k}{2}\right) \left( H^2 k^2 + 6 \right) w + 1 \left( H^2 k^2 + 6 \right)$ 

\end{array}

\right)

Out[403]= Eerr || \left(

\begin{array}{cc}

 $\left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 + 3 U^2 k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2\right) \left(\frac{-$ H^2 k^3 U w^2 \text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)}+\frac{H^2 k^3 U w^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)}+\frac{H^2 k^3 U w^3 U w^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3  $k^2+3\right)+O\left(\frac{1}{2} +O\left(\frac{1}{2} +O(\frac{1}{2} +O$ \text{dt}}{12 \left(H^2 k^2+3\right)^2}-\frac{\left(2 H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H^4 U^2 k^8+12 U^  $k^4 + \frac{dt}^2 {24 \left(\frac{h^2 k^2 + \sinh(h^2 k^2 + 3\right)}} + O\left(\frac{t}{t}\right) \left(\frac{dt}^5 \right) \left(\frac{dt}^2 + \left(\frac{h^2 k^2 + 3\right)}{2}\right)$  $\left(\frac{H} k^4\right) \left(\frac{H} k^4\right) \left(\frac$  $H^4 U k^9 + 351 H^2 U k^7 + 405 U k^5 + 3 + 405 U k^5 + 3 + 405 U k^5 + 3 + 405 U k^6 +$ H^6 U^2 k^{12}+135 g H^5 k^{10}+387 H^4 U^2 k^{10}+837 g H^3 k^8+297 H^2 U^2 k^8-351 U^2  $k^6+1161 \text{ g H } k^6 \cdot \text{hight} \cdot \text{k}^2 \cdot \text{hight} \cdot \text{k}^2 \cdot \text{hight} \cdot \text{k}^3 + O\left(\frac{dt}^5 \cdot \text{hight}\right) \cdot \text{k}^2 \cdot \text{k}^2 + 3\left(\frac{dt}^5 \cdot \text{hight}\right) \cdot \text{k}^2 \cdot \text{k}^2 \cdot \text{k}^2 + 3\left(\frac{dt}^5 \cdot \text{hight}\right) \cdot \text{k}^2 \cdot \text{k}^2 \cdot \text{k}^2 + 3\left(\frac{dt}^5 \cdot \text{hight}\right) \cdot \text{k}^2 \cdot \text{k}^2 \cdot \text{k}^2 \cdot \text{k}^2 + 3\left(\frac{dt}^5 \cdot \text{hight}\right) \cdot \text{k}^2 \cdot \text{k}^2 \cdot \text{k}^2 \cdot \text{k}^2 + 3\left(\frac{dt}^5 \cdot \text{hight}\right) \cdot \text{k}^2 \cdot \text{k}$  $\label{left} $$ \operatorname{dx}^4+O\left(\frac{dx}^5\right)^s \ \& \left(-\frac{u k^2}{H^2 k^2+3}-\frac{u k^2}{H^2 k^2+3$  $\label{eq:continuity} $$ k^2+3\right) \left( k^2 - \frac{dt}^3}{2 \left( k^2 + 3\right) + \frac{dt}^3}{2 \left( k^2 + 3\right) + \frac{dt}^3}{2 \left( k^2 + 3\right) + \frac{dt}^3} \right) $$$  $\t (text{dt}^4){8 \left( h^2 k^2 + 3\right)} + O\left( text{dt}^5\right) + O\left( text{dt}^5\right) + O\left( text{dt}^6\right) + O\left( text{dt$  $k^2+3\rightight)^2+\frac{1}{2}\left[k^3\frac{1}{2}\left[k^2+3\right]\right]$ 

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\left(\frac{4t}{2} \times 2+3\right)^2 + O\left(\frac{4t}{2}\right) \cdot \left(\frac{4t}{2}\right) \cdot \left(\frac{4t}{2}
                                                                                                               \left(\frac{4x}{2} - \frac{4x^2+3\right)}+O\left(\frac{4x}{3}-\frac{4x}{3}\right)+O\left(\frac{4x}{3}-\frac{4x}{3}\right)
                                                                                                               \text{dt}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(45 H^4 U k^{10}+279 H^2 U k^8+387 U k^6\right)}
                                                                                                               \t x_{dt}^2_{240 \left(H^2 k^2+3\right)} + O\left(\frac{t}{5}\right) \left(\frac{dt}{5}\right) \left(\frac{dt}{
                                                                \label{left} $\left(-\frac{k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) \operatorname{text}(dt)^2}{2 \left(-\frac{k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right)} - \operatorname{text}(dt)^2}{2 \left(-\frac{k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right)} - \operatorname{text}(dt)^2}
                                                                                                               k \left( \frac{k^2 H^3+3 g H-3 U^2\right) w^2 \left( \frac{k^2 H^3+3 g H-3 U^2\right) + \frac{k^2 H^3+3 g H-3 U^2\right) w^2 \left( \frac{k^2 H^3+3 U^2}{2 U^2}\right) w^2 \left( \frac{k^
                                                                                                               g H-3 U^2 + W^3 \text{ } w^3 \text{ 
                                                                                                               \left( \frac{H^5 k^7 + 6 g H^3 k^5 + 3 H^2 U^2 k^5 + 18 U^2 k^3 + 9 g H k^3 \right) \left( \frac{H^5 k^7 + 6 g H^3 k^5 + 3 H^2 U^2 k^5 + 18 U^2 k^3 + 9 g H k^3 \right) \left( \frac{H^5 k^7 + 6 g H^3 k^5 + 3 H^2 U^2 k^5 + 18 U^2 k^3 + 9 g H k^3 \right) \left( \frac{H^5 k^7 + 6 g H^3 k^5 + 3 H^2 U^2 k^5 + 18 U^2 k^3 + 9 g H k^3 \right) \left( \frac{H^5 k^5 + 3 H^2 U^2 k^5 + 18 U^2 k^5 + 18 U^2 k^3 + 9 g H k^3 \right) \left( \frac{H^5 k^5 + 3 H^2 U^2 k^5 + 18 U^2 k
                                                                                                               k^2+3\right)^2+\frac{1}{2} H^5 U k^8-12 g H^3 U k^6-9 U^3 k^4-18 g H U k^4\right) text{dt}^2}{12}
                                                                                                               \label{left(H^2 k^2+3\left(\frac{1}{8}\left(\frac{1}{8}\right)^2\right)} \operatorname{left(H^2 k^2+\left(\frac{1}{8}\right)^2}+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left(\frac{1}{8}\right)^2+O\left
                                                                                                               U\right) \text{dt}+\frac{i \left(H^2 \sqrt{g H} U^2 k^7+g H^3 \sqrt{g H} k^7+3 g H \sqrt{g H} k^5\right)
                                                                                                               \t (13 g) $$ \left( \frac{dt}^2}{8 \left( \frac{4x^2+3\right)}+O\left( \frac{dt}^5\right)\right)} + O\left( \frac{dt}^5\right) \right) $$
                                                                                                               H^7 k^{11}+117 g H^5 k^9-H^4 U^2 k^9+351 g H^3 k^7+54 U^2 k^5+351 g H k^5 \right]
                                                                                                               \left(H^2 k^2+3\right)^3}+\frac{\left(73 g H^7 U k^{12}-135 H^4 U^3 k^{10}+657 g H^5 U k^{10}-837
                                                                                                               H^2 U^3 k^8+1971 g H^3 U k^8-1161 U^3 k^6+1971 g H U k^6\right) \text{dt}^2\{720 \\left(H^2
                                                                                                               k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)\text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{\left(-H^2
                                                                                                               \label{eq:continuous} $U^2 \ k^4 - H^2 \ U \ w \ k^3 - 9 \ U^2 \ k^2 - 3 \ g \ H \ k^2 - 6 \ U \ w \ k \ | \ text{dt}^2} \ | \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 3 \ right) - \ H^2 \ Left(H^2 \ k^2 + 
                                                                                                                k \left( H^2 k^2 + 6\right) \ U \ w^2 \left( h^2 k^2 + 3\right) + \left( h^2 k^2 + 6\right) \ W^2 \right) 
                                                                                                               U w^3 \text{$t$} (dt)^4 (24 \left( h^2 k^2 + 3\right) + O\left( \text{$t$} (dt)^5 \right) + \left( -\frac{h^2 k^2 + 3}{2} \right) + O\left( h^4 U \right) + O
                                                                                                               k^7+3 H^2 U k^5-9 U k^3\right) \left( \frac{12 \left( H^2 k^2+3\right)}{2} + \frac{12 \left( -2 H^4 U^2 k^8-12 H^2 U k^5-9 U k^3\right) \right) \left( -2 H^4 U^2 k^8-12 H^2 U k^5-9 U k^3\right) \left( -2 H^4 U^2 k^8-12 H^4 U^2 k^8-12 H^4 U k^5-9 U k^3\right) \left( -2 H^4 U^2 k^8-12 H^4 U k^5-9 U k^3\right) \left( -2 H^4 U^2 k^8-12 H^4 U k^5-9 U k^3\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^3\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^5\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^5\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^5\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^5\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^5\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^5\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^5\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^5\right) \left( -2 H^4 U k^5-9 U k^5-9 U k^5\right) \left( -2 H^4 U k^5\right) \left( -2
                                                                                                               H^2 k^2+15 \oplus U \text{ (kext{dt}^2}{16 \left( h^2 k^2+3\right) + O\left( \text{kext{dt}^5 \right) } \right)}
                                                                                                               \text{dx}^3+\left(\frac{i \left(13 H^6 U k^{11}+118 H^4 U k^9+351 H^2 U k^7+297 U k^5\right)
                                                                                                               \text{text}_{dt}^{240 \left( H^2 k^2 + 3\right)^3} + \frac{10}{4000} 
                                                                                                               U^2 k^{10}+837 g H^3 k^8+3645 H^2 U^2 k^8+4293 U^2 k^6+1161 g H k^6\right) \text{dt}^2\{1440
                                                                                                               \end{array}
                                                           \right)
\ln[404] = \text{KurF} = (\text{fm} * \text{ap} - \text{fp} * \text{am} + \text{am} * \text{ap} * (\text{qp} - \text{qm})) / (\text{ap} - \text{am});
                                                            KurFWS = KurF /. ap \rightarrow (U + Sqrt[g * H]) /. am \rightarrow 0;
                                                            KurFWSeta =
                                                                                     KurFWS /. fp \rightarrow (H*v + U*Rpp*n) /. fm \rightarrow (H*v + U*Rmp*n) /. qp \rightarrow Rpp*n /.
                                                                                                     qm \rightarrow Rmp * n;
                                                           KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
                                                            Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1]; 
                                                            KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1]; 
                                                           Kfnn = Kfnnp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
                                                           \texttt{KfnG} \; = \; \texttt{KfnGp} \; / \; . \; \texttt{Rpp} \; \rightarrow \; \texttt{Rp} \; \; / \; . \; \; \texttt{Rmp} \; \rightarrow \; \texttt{Rm} \; \; / \; . \; \; \texttt{GGp} \; \rightarrow \; \texttt{GG2} \; \; / \; . \; \; \texttt{Gnp} \; \rightarrow \; \texttt{Gn2} \; ;
                                                            Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
                                                            Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
                                                            Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
```

```
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA , \{k > 0, U > 0, H > 0, g > 0\}];
\texttt{KurFWSG} = \texttt{KurFWS} \ / . \ \texttt{fp} \ \rightarrow \ (\texttt{U} * \texttt{Rpp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rpp} * \texttt{n}) \ / .
      fm \rightarrow (U*Rmp*G + U*H*v + g*H*Rmp*n) /. qp \rightarrow Rpp*G /. qm \rightarrow Rmp*G;
KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
KfGn = KfGnp /. Rpp \rightarrow Rp /. Rmp \rightarrow Rm /. GGp \rightarrow GG2 /. Gnp \rightarrow Gn2;
KfGG = KfGGp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" U < -Sqrt(gH) < U "}]]</pre>
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn || ", KfGnp}]]
Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
```

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Text[Row[{"FGn error || ", FGn2TAr}]]
                                                          Text[Row[{"FGn error ||
                                                                                                                                                                                                                                                                                                                                                   ", TeXForm[FGn2TAr]}]]
                                                          Text[" "]
                                                           Text[Row[{"FGG ||
                                                                                                                                                                                                                                                                                     ", KfGGp}]]
                                                           Text[Row[{"FGG ||
                                                                                                                                                                                                                                                                                    ", TeXForm[KfGGp] } ] ]
                                                          Text[Row[{"FGG error ||
                                                                                                                                                                                                                                                                                                                                                    ", FGG2TAr}]]
                                                                                                                                                                                                                                                                                                                                                   ", TeXForm[FGG2TAr]}]]
                                                           Text[Row[{"FGG error ||
                                                          Text[" "]
                                                          Text[" "]
                                                           Text[Row[{"Omega error ||
                                                                                                                                                                                                                                                                                                                                                                       ", RKstepTayr}]]
                                                           Text[Row[{"Omega error
                                                                                                                                                                                                                                                                                                                                                       ", TeXForm[RKstepTayr]}]]
                                                                                                                                                                                                                                                                                                        - 11
                                                          Text[" "]
                                                           Text[Row[{"EA ||
                                                                                                                                                                                                                                                               ", EA}]]
                                                           Text[Row[{"EA || ", TeXForm[EA]}]]
                                                           Text[Row[{"Eerr || ", Eerr}]]
                                                           Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
 Out[438]= U < -Sqrt(gH) < U
 Out[439]=
 Out[440]= Fnn || Gnp H + Rmp U
 Out[441] = Fnn \parallel \text{text}\{Gnp\} H + \text{text}\{Rmp\} U
 Out[442]= Fnn error |
                                                                        \left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)\,U\,dt}{12\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(-\frac{1}{8}\left(k^4\,U\right)dt+O[dt]^4\right)dx^3+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O
 Out[443]= Fnn error |
                                                                        \label{left-frac} $\left(\frac{dt}^2 \left(\frac{4t}^2 k^3 U \right)}{2 \left(\frac{4t}^2 k^3 U \right)} - \frac{i \left(\frac{4t}^3 H^2 k^3 U \right)}{2 \left(\frac{4t}^3 U \right)} - \frac{i \left(\frac{4t}^3 U \right)}{2 \left(\frac{4t}^3 U \right)} - \frac{i
                                                                                                     \left(H^2 k^2+3\right)+O\left(\left(t^4 k^7+9 H^2\right)\right)
                                                                                                     k^5+27 \ k^3 \ U \ \text{text} \ dt \ 12 \ \text{left} \ (4x^2+3 \ \text{right})^2 + O \ \text{left} \ (4x)^4 \ \text{right}) \ \text{right}) + \text{text} \ (4x)^3 \ \text{text} \ (4x)^3 \ \text{text} \ (4x)^3 \ \text{text} \ (4x)^4 \ \text{tex
                                                                                                     \left(-\frac{1}{8}\right)\left(\frac{4 \operatorname{c}^4 \operatorname{d}^4\operatorname{c}}{4 \operatorname{c}^4 \operatorname{d}^4\operatorname{c}}\right)\right)
 Out[444]=
 Out[445]= FnG \parallel GGpH
 Out[446]= FnG \parallel \text{text}\{GGp\} H
 \text{Out} [\text{447}] = FnG \ error \ || \ \left( -\frac{3 \ (k \ w) \ dt^2}{2 \left( 3 + H^2 \ k^2 \right)} - \frac{i \ k \ w^2 \ dt^3}{2 \left( 3 + H^2 \ k^2 \right)} + O[dt]^4 \right) + \left( \frac{i \left( 6 \ k^3 + H^2 \ k^5 \right) \ dt}{4 \left( 3 + H^2 \ k^2 \right)^2} + O[dt]^4 \right) dx^2 + O[dx]^4 
 \label{eq:continuous} Out [448] FinG error || \left| \left( -\frac{3 \left( k w \right)}{2 \left( k w \right)} \right. \right. \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left( k w \right) \right| \\ \left. \left( k^2 + 3 \right) -\frac{1}{2} \left(
                                                                                                     \label{left} $$k^2+3\right)+O\left(\frac{t}{4}\right)+O\left(\frac{t}{4}\right)+\left(\frac{dx}^2\right)=0.
                                                                                                     \text{dt}_{4}\left(\frac{4 \left(\frac{4^2 + 3\right)^2}{-4\right)}} + O\left(\frac{4^2 + 3\right)^2}{-4\right)}
```

Out[449]=

Out[450]=  $FGn \parallel H(gRmp + Gnp U)$ 

Out[451]=  $FGn \parallel H(g \text{text}{Rmp}+\text{text}{Gnp} U)$ 

$$\begin{array}{ll} \text{Out} [\text{452}] = & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \; g \; H + g \; H^3 \; k^2 - 3 \; U^2 \right) w \right) \; dt^2}{2 \left( 3 + H^2 \; k^2 \right)} - \frac{i \; k \left( 3 \; g \; H + g \; H^3 \; k^2 - 3 \; U^2 \right) w^2 \; dt^3}{6 \left( 3 + H^2 \; k^2 \right)} + O[dt]^4 \right) + \\ & \left( -\frac{i \left( 9 \; g \; H \; k^3 + 6 \; g \; H^3 \; k^5 + g \; H^5 \; k^7 + 18 \; k^3 \; U^2 + 3 \; H^2 \; k^5 \; U^2 \right) dt}{12 \left( 3 + H^2 \; k^2 \right)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{8} \left( g \; H \; k^4 \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) dx^4 + O[dt]^4 \right) dx^4 + O[dt]^4 + O[dt]^4$$

Out[453]= FGn error |

 $\label{left} $$\left(-\frac{t}^2 \left( H^2 k^2 + 3 g H-3 U^2\right)^{2} \left( H^2 k^2 + 3 g H-3 U^2\right)^{2} \left( H^2 k^2 + 3 g H-3 U^2\right)^{6} \left( H$ 

 $k^2+3\right)+O\left(\frac{dt}^4\right)+\left(\frac{dt}^4\right)+\frac{dt}^2\left(\frac{dt}^4\right)+\frac{dt}^2\left(\frac{dt}^4\right)}{2}\left(\frac{dt}^4\right)$ 

k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12

 $\left(\frac{4}{dt}^4\right) + \left(\frac{4}{3}\right) + \left(\frac{4}{3}\right)$ 

 $\left( H k^4\right) \left( H k^4\right) \left( text{dt} + O\left( text{dt}^4\right) \right) + O\left( text{dx}^4\right) \right)$ 

Out[454]=

Out[455]=  $FGG \parallel (GGp H + Rmp) U$ 

 $\label{eq:out_problem} \mbox{Out}[456] = FGG \ || \ U \ (\text{text}\{GGp\} \ H + \text{text}\{Rmp\})$ 

$$\begin{array}{ll} \text{Out} [457] = & FGG \; error \; || \; \left( -\frac{\left( k \left( 6+H^2 \, k^2 \right) U \, w \right) dt^2}{2 \left( 3+H^2 \, k^2 \right)} - \frac{i \, k \left( 6+H^2 \, k^2 \right) U \, w^2 \, dt^3}{6 \left( 3+H^2 \, k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( -9 \, k^3 + 3 \, H^2 \, k^5 + H^4 \, k^7 \right) U \, dt}{12 \left( 3+H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \\ & \left( -\frac{1}{8} \left( k^4 \, U \right) dt + O[dt]^4 \right) dx^3 + \left( \frac{i \left( 297 \, k^5 + 351 \, H^2 \, k^7 + 118 \, H^4 \, k^9 + 13 \, H^6 \, k^{11} \right) U \, dt}{240 \left( 3+H^2 \, k^2 \right)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{array}$$

Out[458]= FGG error ||

 $\label{left-proc} $\left(-\frac{\hat u^2 \cdot \frac{dt}^2 \left(U \ w \left(H^2 \ k^2+6\right)\right)}{2 \left(H^2 \ k^2+3\right)}-\frac{i \cdot \frac{dt}^3 \ k \ U \ w^2 \left(H^2 \ k^2+6\right)}{6 \left(H^2 \ k^2+3\right)}+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+\frac{dt}^4 \right)}+O\left(\frac{dt}^4 \right)-\frac{dt}^3 \ k \ U \ w^2 \left(H^2 \ k^2+6\right)}{6 \left(H^2 \ k^2+3\right)}+O\left(\frac{dt}^4 \right)+\frac{dt}^4 \right)}+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)}{12 \left(H^2 \ k^2+3\right)}+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)}{12 \left(H^2 \ k^2+3\right)}+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)}{12 \left(H^2 \ k^2+3\right)}+O\left(\frac{dt}^4 \right)+O\left(\frac{dt}^4 \right)+O\left(\frac{d$ 

Out[459]=

Out[460]=

Out[461]= Omega error ||

$$\left\{ \left( \frac{1}{6\left(3+H^{2}\,k^{2}\right)^{2}}k^{3}\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right.\right. \\ \left. + \left(3+H^{2}\,k^{2}\right)U\right) \left(3\,g\,H+U\left(2\,\sqrt{3}\,\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right.\right. \\ \left. + \left(3+H^{2}\,k^{2}\right)U\right) \right)dt^{2} + \left(3+H^{2}\,k^{2}\right)U\right) \left(3\,g\,H+U\left(2\,\sqrt{3}\,\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right)\right. \\ \left. + \left(3+H^{2}\,k^{2}\right)U\right) \left(3\,g\,H+U\left(2\,\sqrt{3}\,\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right)\right] dt^{2} + \left(3+H^{2}\,k^{2}\right)U\right) \left(3\,g\,H+U\left(2\,\sqrt{3}\,\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right)\right) dt^{2} + \left(3+H^{2}\,k^{2}\right)U\right) dt^{2} + \left(3+H^{2}\,k^{2}\right)U$$

$$\frac{i \, k^4 \left(3 \, g \, H + U \left(2 \, \sqrt{3} \, \sqrt{g \, H \, (3 + H^2 \, k^2)} \, + (3 + H^2 \, k^2) \, U\right)\right)^2 \, dt^3}{8 \, (3 + H^2 \, k^2)^2} - \frac{1}{20 \, (3 + H^2 \, k^2)^4} \left(k^5 \left(\sqrt{3} \, \sqrt{g \, H \, \left(3 + H^2 \, k^2\right)} \, + \left(3 + H^2 \, k^2\right) \, U\right)^3 + \left(3 + H^2 \, k^2\right)^2\right)^2 \, dt^3}{4 \, k^2 \, k^2 \, k^2 \, k^2} + \left(3 + H^2 \, k^2\right) \, U \left(3 + H^2 \, k^2\right)^2 + \left(3 + H^2 \,$$

$$\left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U\right)\right)\right) dt^4 + O[dt]^5 \right) + \\ \left(\frac{k^9 \left[-3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} \right]^2}{24 (3 + H^2 k^2)^2} + \frac{1}{48 (3 + H^2 k^2)} + k^5 \left(g \left(-9 \sqrt{3} H \sqrt{g H (3 + H^2 k^2)} + 18 H^3 k^2 U + 6 H^5 k^4 U\right) + U^2 \left(27 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 54 U + 2 H^6 k^6 U + 3 k^2 \left(7 \sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} + 18 H^3 U\right) + 2 k^4 \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 H^4 U\right)\right)\right) dt^2 + \\ \left(\frac{1}{48 (3 + H^2 k^2)^2} i k^6 \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U\right)\right)\right) dt^3 + \\ \left(-9 g H + U \left(3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U + 2 k^2 \left(\sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} + 6 H^2 U\right)\right)\right) dt^3 - \\ \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U + 2 k^2 \left(\sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} + 6 H^2 U\right)\right)\right) dt^3 + \\ \left(-\frac{1}{10} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{3 + H^2 k^2}} + 2 U\right) - \frac{1}{32 (3 + H^2 k^2)} i k^6 \left(3 g H \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 4 (3 + H^2 k^2) U\right) + U^2 \left(15 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U + k^2 \left(5 \sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} + 12 H^2 U\right)\right)\right) dt^2 + \\ \frac{1}{32 (2 + H^2 k^2)^2} k^2 \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 (3 + H^2 k^2) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 \left(3 + H^2 k^2\right) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 \left(3 + H^2 k^2\right) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 \left(3 + H^2 k^2\right) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 \left(3 + H^2 k^2\right) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 \left(3 + H^2 k^2\right) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 \left(3 + H^2 k^2\right) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 \left(3 + H^2 k^2\right) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 \left(3 + H^2 k^2\right) U\right)\right) dt^3 + \\ \frac{1}{34 (2 + H^2 k^2)^2} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 2 \left(3 H^2 k^2\right) U\right) dt^3 + \\ \frac{1}{34 (2$$

$$\begin{array}{c} 84\,H^{8}\,k^{8}\,U^{4} + 243\,U^{3}\left(39\,\sqrt{3}\,\sqrt{g\,H\,(3+H^{2}\,k^{2})} + 28\,U\right) + 36\,k^{4}\,U^{3} \\ & \left(85\,\sqrt{3}\,\sqrt{g\,H^{9}\,(3+H^{2}\,k^{2})} + 126\,H^{9}\,U\right) + 4\,k^{6}\,U^{3}\left(83\,\sqrt{3}\,\sqrt{g\,H^{13}\,(3+H^{2}\,k^{2})} + 252\,H^{6}\,U\right) + 9\,k^{2}\left(600\,\sqrt{3}\,\sqrt{g\,^{2}\,H^{7}\,(3+H^{2}\,k^{2})}\,U + 1039\,\sqrt{3}\,\sqrt{g\,H^{5}\,(3+H^{2}\,k^{2})}\,U^{3} + 1008\,H^{2}\,U^{4}\right) + 3\,g\,H\,U\,\left(13\,500\,H^{2}\,k^{2}\,U + 472\,H^{6}\,k^{6}\,U + 27\left(97\,\sqrt{3}\,\sqrt{g\,H\,(3+H^{2}\,k^{2})} + 504\,U\right) + 4\,k^{4}\left(71\,\sqrt{3}\,\sqrt{g\,H^{3}\,(3+H^{2}\,k^{2})} + 1001\,H^{4}\,U\right)\right)\right)d^{2} + \frac{1}{23\,660\,\sqrt{g\,H^{3}\,(3+H^{2}\,k^{2})}} \\ & k^{9}\left(\sqrt{3}\,\sqrt{g\,H\,(3+H^{2}\,k^{2})} + (3+H^{2}\,k^{2})\,U\right)\left(4\,k^{8}\,U^{3}\,(239\,\sqrt{3}\,g\,H^{9} + 58\,\sqrt{g\,H^{17}\,(3+H^{2}\,k^{2})}\,U\right) + 27\,k^{2}\left(372\,\sqrt{g^{3}\,H^{9}\,(3+H^{2}\,k^{2})} + 2703\,\sqrt{3}\,g^{2}\,H^{4}\,U + 45\,15\,\sqrt{g^{3}\,H^{7}\,(3+H^{2}\,k^{2})}\,U^{2} + 4070\,\sqrt{3}\,g\,H^{9}\,U^{3} + 928\,\sqrt{g\,H^{3}\,(3+H^{2}\,k^{2})}\,U^{3} + 10\,H^{4}\,U^{3}\right) + 22\,k^{2}\,U^{2} + 4070\,\sqrt{3}\,g\,H^{9}\,U^{3} + 232\,\sqrt{g\,H^{13}\,(3+H^{2}\,k^{2})}\,U^{3} + g\,H^{7}\,U\,(349\,\sqrt{g\,H\,(3+H^{2}\,k^{2})} + 979\,\sqrt{3}\,U)\right) + 81\left(157\,\sqrt{g^{2}\,H^{3}\,(3+H^{2}\,k^{2})} + 883\,\sqrt{3}\,g^{2}\,H^{2}\,U + 232\,\sqrt{g\,H\,(3+H^{2}\,k^{2})}\,U^{4} + g\,H^{2}\,U^{2}\left(1527\,\sqrt{g\,H\,(3+H^{2}\,k^{2})} + 2672\,\sqrt{3}\,g^{2}\,H^{6}\,U + 1392\,\sqrt{g\,H^{9}\,(3+H^{2}\,k^{2})}\,U^{4} + g\,H^{2}\,U^{2}\right) + 384\,\sqrt{g\,H\,(3+H^{2}\,k^{2})} + 2672\,\sqrt{3}\,g^{2}\,H^{6}\,U + 1392\,\sqrt{g\,H^{9}\,(3+H^{2}\,k^{2})}\,U^{4} + g\,H^{2}\,k^{2}\right) U^{4} + g\,H^{2}\,U^{2}\left(4384\,\sqrt{g\,H\,(3+H^{2}\,k^{2})} + (3+H^{2}\,k^{2})\,U\right)\right) d^{2}\,d^{2} + \frac{i\kappa^{4}\,(3g\,H^{2}\,H^{2}\,k^{2})}{8(3+H^{2}\,k^{2})} + (3+H^{2}\,k^{2})} + (3+H^{2}\,k^{2})\,U\right)\right)}{3(3g\,H + U\left(-2\,\sqrt{3}\,\sqrt{g\,H\,(3+H^{2}\,k^{2})} + (3+H^{2}\,k^{2})\,U\right)\right)} d^{2}\,d^{2}} + \frac{i\kappa^{4}\,(3g\,H^{2}\,H^{2}\,k^{2})}{22(3\pi^{2}\,H^{2}\,H^{2}\,k^{2})} + (3+H^{2}\,k^{2})\,U\right)\right)} d^{2}\,d^{2}} + \frac{i\pi^{2}\,(3H^{2}\,H^{2}\,k^{2})}{8(3+H^{2}\,H^{2}\,k^{2})} + 34\,H^{2}\,k^{2}} + 34\,H^{2}\,k^$$

$$\frac{1}{4(1)+11/2} i^4 k^6 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U\right)\right) \\ \left(-9 g H + U \left(-3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U - 2 k^2 \left(\sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} - 6 H^2 U\right)\right)\right) \\ dt^3 - \frac{1}{96(1+12k^2)^4} \\ \left(k^7 \left(3 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 \left(3 + H^2 k^2\right)^2 U\right) \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U\right)\right)^2\right) \\ dt^4 + O[dt]^5 \right] dx^2 + \\ \left(\frac{1}{16} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{3 + H^2 k^2}} - 2 U\right) - \frac{1}{32(3+12k^2)^2} i k^6 \left(-3 g H \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} - 4 \left(3 + H^2 k^2\right) U\right)\right) + U^2 \left(-15 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U + k^2 \left(-5 \sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} + 12 H^2 U\right)\right)\right) \\ dt^2 + \frac{1}{3(3+12k^2)^2} k^7 \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U\right)\right) dt^3 + \\ \frac{1}{64(3+12k^2)^2} i k^8 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U\right)\right) dt^3 + \\ \frac{1}{64(3+12k^2)^2} i k^8 \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 (3 + H^2 k^2) U\right)\right) \\ dx^3 + \left(\left(k^5 \left(3 \sqrt{3} g H (177 + 124 H^2 k^2 + 20 H^4 k^4) - 10 H^2 k^2\right) + 2 \left(3 + H^2 k^2\right)\right)\right)^2 dt^4 + O[dt]^5\right) \\ dx^3 + \left(\left(k^5 \left(3 \sqrt{3} g H (177 + 124 H^2 k^2 + 20 H^4 k^4) - 10 H^2 k^2\right) + 2 (3 + H^2 k^2)\right)\right)^2 dt^4 + O[dt]^5\right) \\ dx^3 + \left(\left(k^5 \left(3 \sqrt{3} g H (3 + H^2 k^2)\right)^{5/2}\right) + \left(k^2 \left(27 \sqrt{3} g^2 H^2 (167 + 124 H^2 k^2 + 20 H^4 k^4)\right) + 2 (3 + H^2 k^2)\right)U\right)\right)^2 \\ - 24 k^4 \left(84 \sqrt{g H^9 (3 + H^2 k^2)} + 2 (27 \sqrt{3} g^2 H^2 (167 + 124 H^2 k^2 + 20 H^4 k^4)\right) + 2 (4 k^4 \sqrt{g H^9 (3 + H^2 k^2)}\right) \\ - 16 \left(459 \sqrt{g H (3 + H^2 k^2)} + 3 + 153 k^4 \sqrt{g H^9 (3 + H^2 k^2)} + 3 + 17 k^6 \sqrt{g H^{13} (3 + H^2 k^2)}\right) U^3 + 2 (4 k^4 \sqrt{g H^9 (3 + H^2 k^2)} + 3 + 153 k^4 \sqrt{g H^9 (3 + H^2 k^2)}\right) U^3 + 17 k^6 \sqrt{g H^{13} (3 + H^2 k^2)}\right) U^3 + 2 (4 k^4 \sqrt{g H^9 (3 + H^2 k^2)}\right) U^3 + 12 k^6 \left(5 4 g^2 H^2 (81 + 62 H^2 k^2 + 10 H^4 k^4) + 84 H^8 k^8 U^4 + 2 (4 k^2)^3 \left(-83 \sqrt{3} \sqrt{g H (3 + H^2 k^2)}\right) U^3 + 12 k^2 \left(-85 \sqrt{3} \sqrt{g H (3 + H^2 k^2)}\right) U^3 + 12 k^2 \left(-85 \sqrt{3} \sqrt{g H (3 + H^2 k^2)}\right) U^3 + 12 k^2 \left(-85 \sqrt{3} \sqrt{g H (3 + H^2 k^2)}\right) U^3 + 12 k^2 \left(-85 \sqrt{3} \sqrt{g H (3 + H^2 k^2)}\right) U^3 + 12 k^2$$

$$\left(k^9 \left(\sqrt{3} \ \sqrt{g \, H^{\, 7} \, (3 + H^2 \, k^2)} \ - 1101 \, H^4 \, U\right)\right)\right) dt^9 - \frac{\cdot}{23040 \left(\sqrt{g \, H} \, (3 + H^2 \, k^2)^{11/2}\right)}$$
 
$$\left(k^9 \left(\sqrt{3} \ \sqrt{g \, H \, (3 + H^2 \, k^2)} \ - (3 + H^2 \, k^2) \, U\right) \left(4 \, k^8 \, U^3 \left(-239 \, \sqrt{3} \, g \, H^9 + 58 \, \sqrt{g \, H^{17} \, (3 + H^2 \, k^2)} \, U\right) + 27 \, k^2 \left(372 \, \sqrt{g^5 \, H^9 \, (3 + H^2 \, k^2)} \ - 2703 \, \sqrt{3} \, g^2 \, H^4 \, U + 4515 \, \sqrt{g^3 \, H^7 \, (3 + H^2 \, k^2)} \, U^2 - 4070 \, \sqrt{3} \, g \, H^3 \, U^3 + 928 \, \sqrt{g \, H^5 \, (3 + H^2 \, k^2)} \, U^4\right) + 9 \, k^4 \left(180 \, \sqrt{g^5 \, H^{13} \, (3 + H^2 \, k^2)} \ - 2672 \, \sqrt{3} \, g^2 \, H^6 \, U + 1392 \, \sqrt{g \, H^9 \, (3 + H^2 \, k^2)} \, U^4 + g \, H^5 \right)$$
 
$$U^2 \left(4384 \, \sqrt{g \, H \, (3 + H^2 \, k^2)} \ - 5997 \, \sqrt{3} \, U\right)\right) + 81 \left(157 \, \sqrt{g^5 \, H^5 \, (3 + H^2 \, k^2)} \ - 883 \, \sqrt{3} \right)$$
 
$$g^2 \, H^2 \, U + 232 \, \sqrt{g \, H \, (3 + H^2 \, k^2)} \, U^4 + g \, H \, U^2 \left(1527 \, \sqrt{g \, H \, (3 + H^2 \, k^2)} \ - 1033 \, \sqrt{3} \, U\right)\right) - 12 \, k^6 \, U \left(213 \, \sqrt{3} \, g^2 \, H^8 - 232 \, \sqrt{g \, H^{13} \, (3 + H^2 \, k^2)} \, U^3 + g \, H^7 \, U \left(-349 \, \sqrt{g \, H \, (3 + H^2 \, k^2)} \ + 979 \, \sqrt{3} \, U\right)\right)\right) dt^4 + O[dt]^5 \right) dx^4 + O[dx]^5 \right)$$

## Out[462]= Omega error ||

 $\left(\frac{k^3 \left(\frac{k^3 \left(\frac{k^2 + 3\right)} U + \sqrt{3} \left(\frac{k^2 + 3\right)} \right)}{1 + 2 k^2 + 3\right)} \right)$  $\left(\left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3}}\right) + \left(\frac{H^2 k^2+3\right)}{\left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3}}\right)$  $\left(H^2 k^2+3\right)^2+\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$  $\label{left(H^2 k^2+3\left|right(k^5 \left(\frac{H^2 k^2+3\right|right(k^5 \left(\frac{H^2 k^2+3\right|right(k^5 \left(\frac{H^2 k^2+3\right)}{H^2 k^2+3\right)}}{\left(\frac{H^2 k^2+3\right|right(k^5 \left(\frac{H^2 k^2+3\right)}{H^2 k^2+3\right)}{\left(\frac{H^2 k^2+3\right|right(k^5 \left(\frac{H^2 k^2+3\right)}{H^2 k^2+3\right)}{\left(\frac{H^2 k^2+3\right)}{H^2 k^2+3\right)}}}$  $k^2+3\right\in U+\sqrt{3} \operatorname{left}(H^2 k^2+3\right)$  $k^2+3\left( U+2 \right) U+2 \left( H^2 \left( H^2 \left( H^2 \right) \right) \right) \left( H^2 \left( H^2 \right) \right) \\$  $\left(9 U H^4+2 \right) \left(H^9 \left(H^2 k^2+3\right)\right) \right) \$  $\left(6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \left(4 \, H^2 \, k^2 + 3 \right) \, H\right) \, H\right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \right) \, (6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \, (6 \, k^4 \, U \, H^5 - 18 \, k^2 \, U \, H^3 - 9 \, (6 \, k^4 \, U \, H^5 - 18 \, U$  $\left(H^2 k^2+3\right) + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \left(H^2 k^2+3\right) + \frac{1}{3} \left(H^2 k^$  $H \left(H^2 k^2+3\right)\right)\right)$  $\left(H^2 k^2+3\right)\right)$  \quad \(h^2 k^2+3\right) \right) \\right) \  $H \left( H^2 k^2 + 3 \right) \left( H^2 k$  $\label{left} $$k^2+3\right\right)\right) \cdot (k^2+3\right) \cdot (k^2+3) \cdot (k^2+3\right) \cdot (k^2+3) \cdot$  $\left(\frac{4 + 4 U k^4 + left(12 U h^2 + 5 \sqrt{3} \right) + h^5 \left(\frac{4 U k^2 + 3 \right)} \right) + h^2 + h^2 \left(\frac{4 U k^2 + h^2 U h^2 + h^2 U h^2 + h^2 U h^2 U h^2 + h^2 U h$  $\sqrt{3} \right H \left( H^2 k^2 + 3\right) \right) U^2 + 3 g H \left( h^2 k^2 + 3\right) U + \sqrt{3} \right)$ \left(\left(\H^2 k^2+3\right)\ U+2 \sqrt{3} \sqrt{g H \left(\H^2 k^2+3\right)\right)\right)\right) \left(3 g H+U \left(2 k^2+3\right))\right)  $k^2+3\right)^2+\frac{1}{2} \left(k^8 \left(k^6 \left(k^2 + k^2 + 3\right) U + \sqrt{3} \right)^2 + \frac{1}{2} \left(k^6 \left(k^6 + k^6 + k$  $\left(3 g H+U \left(\frac{A^2 k^2+3\right)}{U+2 \left(\frac{3 g H+U \left(\frac{A^2 k^2+3\right)}{U+2 \left(\frac{A^2 k^2+3\right)}{U+2$  $\label{eq:left} $$ \left(\frac{dt}^4}{64 \left(\frac{h^2 k^2+3\right)}^3+O\left(\frac{dt}^5\right)\right)\right) \left(\frac{dt}^4}{64 \left(\frac{h^2 k^2+3\right)}^3+O\left(\frac{dt}^5\right)}\right) \left(\frac{dt}^4\right) \left(\frac{dt$ 

\sqrt{3} g H \left(20 H^4 k^4+124 H^2 k^2+177\right)+104 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)})  $k^4+6 \left( H^2 k^2+3\right) k^2+9 \left( H^2 k^2+3\right) v^2+9 \left$  $\label{left(sqrt{g H} \left( \frac{H^2 k^2+3\right)}{-\frac{1}{2}\left( \frac{H^2 k^2+3\right)}{-\frac$ k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6+24 \left(293 \sqrt{3} U H^4+84  $\sqrt{g} H \left(H^2 k^2+3\right)\right) H+16 \left(17 \right) H+16 \left(17 \right) H+16 \left(17 \right) H+13 \left(H^2 k^2+3\right) U^3$ k^6+153 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(51 \sqrt{g H^5 \left(H^2 k^2+3\right)})  $U^3+88 \sqrt{g^3 H^7 \left(\frac{h^2 k^2+3\right)} U\right)} U\right)$  $U^3\right) \left( H^2 k^2 + \frac{11520 \left( k^8 H^2 k^2 + 3\right)}{1520 \left( k^8 H^2 k^2 + 3\right)} \right) - \frac{1}{1520 \left( k^8 H^2 k^2 + 3\right)} + \frac{1}{1520 \left( k^8 H^2 k^2 + 3\right)} \right) - \frac{1}{1520 \left( k^8 H^2 k^2 + 3\right)} \right) - \frac{1}{1520 \left( k^8 H^2 k^2 + 3\right)} + \frac{1}{1520 \left( k^8 H^2 k^2 + 3\right)} \right) - \frac{1}{1520 \left( k^8 H^2 k^2 + 3\right)} + \frac{1}{1520 \left( k^8 H^2$ \left(84 H^8 U^4 k^8+4 U^3 \left(252 U H^6+83 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right)  $k^6+36 U^3 \left(126 U H^4+85 \right) \left(146 U H^2 k^2+3\right) \$ k^2+3\right) U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U+39) \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right)+3 g H U \left(472 H^6 U k^6+4 \left(1101 U H^4+71 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+13500 H^2 U k^2+27 \left(504 U+97 \sqrt{3})  $\left(\left(\frac{H^2 k^2+3\right)}{U+\sqrt{4 L^2 k^2+3}}\right) + \left(\frac{H^2 k^2+3\right)}{U+\sqrt{4 L^2 k^2+3}}\right)$  $\sqrt{3} g H^9+58 \sqrt{17} \left( H^2 k^2+3\right) U\right) U\right) k^8+12 U \left( 13 \right) g^2$  $H^8+g\ U\ \left(979\ sqrt{3}\ U+349\ sqrt{g}\ H\ \left(H^2\ k^2+3\right)\right)\right) H^7+232\ sqrt{g}\ H^{13}$ \left(H^2 k^2+3\right)\ U^3\right) k^6+9 \left(2672 \sqrt{3}\ g^2 U H^6+g U^2 \left(5997 \sqrt{3}\)  $U+4384 \sqrt{H^2 k^2+3\right} U+4384 \sqrt{H^2 k^2+3\right} U^4+180$  $\sqrt{g^5 H^{13} \left( h^2 k^2 + 3\right)} \right)$ U^3 H^3+928 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+4515 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+372 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)\right) \k^2+81 \left(232 \sqrt{g H \left(H^2 k^2+3\right)})  $U^4+g H \left(1033 \right) + 1527 \left(g H \left(1^2 k^2+3\right)\right) + 1527 \left(g H \left(1^2 k^2+3\right)\right) + 1527 \left(g H \left(1^2 k^2+3\right)\right)$  $H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right) \to \left(H^2 U + 157 \sqrt{g^5 H^5 \left(H^2 k^2 + 3\right)}\right)$  $\left(\left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \sqrt{3} \right) + \left(\frac{H^2 k^2+3\right)}\right) \left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \left(\frac{H^2 k^2+3\right)}}\right)$  $k^2+3\right) U-2 \sqrt{3} \sqrt{9 H \left(H^2 k^2+3\right)}\right) text{dt}^2{6 \left(H^2 k^2+3\right)}\right)$  $k^2+3\right)^2+\frac{1}{2}+\frac{1}{2} +\frac{1}{2} +$  $\label{left(h^2 k^2+3\right)} $$ \left( \frac{h^2 k^2+3\right)^2}{\|h\|^2} \left( \frac{h^2 k^2+3\right)^2}{\|h\|^2} \right) $$$  $k^2+3\right) U-2 \sqrt{3} \sqrt{4}{20 \left(H^2 k^2+3\right)\right)} \left(H^2 k^2+3\right) \left(H^2 k^2+3\right)$  $\label{left} $$k^2+3\right)^4}+O\left(\frac{d^2k^2+3\left(\frac{d^2k^2+3\right)}{k^2+3\left(\frac{d^2k^2+3\left(\frac{d^2k^2+3\right)}{k^2+3\left(\frac{d^2k^2+3\left(\frac{d^2k^2+3\right)}{k^2+3\left(\frac{d^2k^2+3\right)}{k^2+3\left(\frac{d^2k^2+3\left(\frac{d^2k^2+3\right)}{k^2+3\left(\frac{d^2k^2$  $\left(\frac{H^2 k^2+3\right)}{18 H^2 U\right) k^2+54 U-27 \sqrt{3} \sqrt{g H \left(\frac{h^2 k^2+3\right)}\right) right}$  $\label{eq:continuity} $$U^2+3 g \left( k^4 U H^5+6 k^2 U H^3+3 \right) \right] + \left( H^2 k^2+3\right) + H\right) + (H^2 k^2 + 3 + (H^2 k^2 + 4 + (H^2 k^2 + 3 + (H^2 k^2 + 4 + (H^2$  $\label{left} $$ \left(\frac{d^2 k^2+3\right)^3}+\frac{k^6 \left(\frac{3 gH+U \left(\frac{h^2 k^2+3\right)U-2 \left(\frac{3}{4}\right)}{U-2 \left(\frac{3}{4}\right)U-2 \left(\frac{3}$  $\left( H^2 \left( H^2 + 1 \right)\right)\right) \left( H^2 + 1 \right)$  $k^2+3\left(H^2 U\right)-6 H^2 U\right) k^2+18 U-3 \left(H^2 k^2+3\right)-9 g H\left(H^2 k^2+3\right$  $H \left( H^2 k^2 + 3 \right) \right) \left( H^2 k^2 + 3 \right) \left( H^$ 

 $\label{eq:kappa} $$\kappa'^2+3\left(\frac{4}{9}\left(\frac{K'^2+3\right)}{4}+O\left(\frac{K'^2+3}{1}\right)^4+O\left(\frac{K'^2+3}{$  $\t x_d^2+\left(\frac{1}{16} i k^4 \left(\frac{3} \right)^2 + \left(\frac{1}{16} i k^4 \right)^2 k^2 + \frac{1}{16} i k^6$  $\label{left} $$\left(U^2 \left(H^4 U k^4 + \left(12 H^2 U - 5 \right) \right) \right) \left(H^5 \left(H^2 k^2 + 3\right)\right)\right) right)$$  $k^2+18 U-15 \sqrt{3} \sqrt{3} \sqrt{4} \ln(H^2 k^2+3\right)/19$  $\label{left} $$k^2+3\right)^4\left(\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3}+\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac$  $\label{left} $$\left(\frac{3 g H+U \left(\frac{h^2 k^2+3\right)} U-2 \right) + \left(\frac{3 g H+U \left(\frac{h^2 k^2+3\right)}{right}} U-2 \right)} $$$  $\left(3 g H+U \left(2 \left(4^2 k^2+3\right)\right) U-3 \right) + \left(4^2 k^2+3\right) \left(4^2 k^2+3\right) \right)$  $\t (dt)^3 {32 \left( H^2 k^2 + 3\right) U - \left( h^2 k^2 + 3$  $\label{eq:k-2+3-right} $$ k^2+3\right)^2 \text{text}(dt)^4{64 \left(\frac{h^2 k^2+3\right)^3}+O\left(\frac{dt}{5\right)^5}\right)^2} $$$  $\text{dx}^3+\left(\frac{4x}{3}+\frac{4x^2+177}{190}-104\right)$  $H^9 \left(H^2 k^2+3\right) k^4+6 \left(H^2 k^2+3\right) k^2+9 \left(H^2 k^2+3\right) k^2+9$  $\label{lem:lem:hard-prop} $$ k^2+3\right) \left( H^2 k^2+3\right)^{5/2}+\frac{k^7 \left( R^7 \left( R^7 \right)^{3} + \frac{k^2+3\right)^{5/2}}{R^7 \left( R^7 \right)^{5/2}} \right)^{5/2} + \frac{k^2+3\left( R^7 \right)^{5/2}}{R^7 \left( R^7 \right)^{5/2}} + \frac{k^2+3\left( R^7 \right)^{5/2}}{R^7$ g^2 \left(20 H^4 k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6-24 \left(84 \sqrt{g} H^9 \left(H^2 k^2+3\right)\-293 \sqrt{3} H^4 U\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267  $\sqrt{3} U-232 \right(H \le H^2 + 1)$ k^2+3\right)} U^3 k^6+153 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(51 \sqrt{g H^5 \left(H^2 k^2+3\right)}) k^2+3\right)} U^3+88 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+459 \sqrt{g H \left(H^2 k^2+3\right)}  $k^2+3\right) U^3\right) \text{dt}^2{11520 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}-\frac{i k^8}{2}$  $\left(84 \text{ H}^8 \text{ U}^4 \text{ k}^8 + 4 \text{ U}^3 \left(52 \text{ H}^6 \text{ U} - 83 \right)\right)\right)$  $H^2 U^4+1039 \sqrt{3} \sqrt{3} \sqrt{4+1039 \sqrt{3}} \sqrt{4+1039 \sqrt{$ k^2+3\right) U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U-39)  $\sqrt{3} \right \| H \left( H^2 k^2 + 3\right) \right) \$  $H^9 \left( H^2 k^2 + 3 \right) - 1101 H^4 U \right) + 13500 H^2 U k^2 + 27 \left( 504 U - 97 \right) \right)$  $H \left( H^2 k^2 + 3\right) \right) \right) \left( H^2 k^2 + 3\right) \right) \left( H^2 k^2 + 3\right) \right) \left( H^2 k^2 + 3\right) \left( H^2 k^2 + 3\right) \right) \left( H^2 k^2 + 3\right) \right) \left( H^2 k^2 + 3\right) \left$  $\left(\frac{3} \right) \left(\frac{4 U^2 k^2+3\right)}{\left(\frac{4 U^3 \left(\frac{4 U^3 \left(\frac{4 U^3 k^2+3\right)}{16 U^3 k^2+3}\right)}{16 U^3 k^2+3 u^3 u^3 k^2+3 u^3 k^2+3 u^3 k^2+3 u^3 k^2+3 u^3 k^2+3 u^3 k^2+3 u^3 u^3 k^2+3 u^3 k^2+3 u^3 k^2+3 u^3 k^2+3 u^3 k^2+3 u^3 k^2+3 u^3 u^3 k^2+4 u^3 k^2+4 u^3 k^2+4 u^3 k^2+4 u^3 k^2+4 u^3 k^2+4 u^3$  $H^{17} \left( H^2 k^2 + 3\right) U - 239 \left( H^9\right) k^8 - 12 U \left( 13 \right) g^2 H^8 + g^2$  $U \left( \frac{979 \sqrt{3} U - 349 \sqrt{g H \left( \frac{4^2 k^2 + 3\right)}} \right) H^7 - 232 \right) + H^{13} \left( \frac{4^2 k^2 + 3\right)}{16}$ k^2+3\right)} U^3\right) k^6+9 \left(-2672 \sqrt{3} g^2 U H^6+g U^2 \left(4384 \sqrt{g H \left(H^2  $k^2+3\right)=5997 \cdot qrt{3} U\right)+5997 \cdot qrt{3} U\right)+5997 \cdot qrt{9} +199 \cdot qrt{9} \cdot qrt{9} \cdot qrt{9} \cdot qrt{9}$  $H^{13} \left( H^2 k^2 + 3\right) \right) + h^4 + 27 \left( -2703 \right) qrt{3} g^2 U H^4 - 4070 \right) qrt{3} g U^3$  $H^3+928 \left( H^5 \left( H^2 k^2+3\right) \right) U^4+4515 \left( H^2 k^2+3\right) U^2+372$  $\left(\frac{9^5 H^9 \left(\frac{4^2 k^2+3\right)}}{v^2+3}\right) k^2+81 \left(\frac{32 \left(\frac{4^2 k^2+3\right)}}{v^2+3}\right) U^4+g^2}$  $H \left(1527 \right) + \left($  $\$  \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{23040 \left(\sqrt{g H} \left(H^2 k^2+3\right))\right)\right} 

$$\begin{aligned} & \text{Out}[464] = & \text{ EA } || & \left\{ \left\{ 1 + \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, \text{H}^2 \, \text{k}^3 \, \text{U} \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{6 + 2 \, \text{H}^2 \, \text{k}^2 \, \text{w}}, \, \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, \text{H} \, k \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2 \left( \text{H} + \frac{\text{H}^3 \, k^2}{3} \right) \, \text{w}} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( g \, H \, \left( 3 + H^2 \, k^2 \right) - 3 \, U^2 \right) \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{2} \right\}, \, 1 \, + \, \frac{i \, e^{\frac{i \, \text{dx} \, k}{2}} \left( 1 - e^{-i \, \text{dx} \, k} \right) \left( -1 + e^{i \, \text{dt} \, w} \right) \, k \, \left( 6 + H^2 \, k^2 \right) \, U \, \text{Csc} \left[ \frac{\text{dx} \, k}{2} \right]}{6 + 2 \, H^2 \, k^2 \right) \, w} \right\} \right\} \end{aligned}$$

Out[465]= EA || \left(

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 $\frac{i e^{\frac{1}{2}} \left(1-e^{-i \text{text}dx} k\right) \left(1-e^{i \cdot text}dx\right) \left(1$  $w \rightarrow H^2 U \csc \left( \frac{dx}{k}{2} \right) \\ k^3 \left( H^2 K^2 + 6 \right) \\ w \rightarrow H^2 U \csc \left( \frac{dx}{k} \right) \\ k^3 \left( H^2 K^2 + 6 \right) \\ w \rightarrow H^2 U \end{aligned}$ &  $\frac{i e^{\frac{i \cdot k}{2}} \left(1-e^{-i \cdot k} k}{i \cdot k}\right) \left(1-e^{-i \cdot k} k\right) \left(1-e^{-i \cdot k} k\right)$  $w\} right) H k \csc \left( \frac{dx}{k}^2 \right) {2 \left( \frac{k^2 H^3}{3} + H \right) w}$ 

 $\label{eq:linear_energy} $$ \frac{i \cdot x_{dx} k}{2}} \left(1-e^{-i \cdot x_{dx} k}\right) \left(1 \left(H^2 k^2+3\right)-3 U^2\right) \cos \left(\frac{4x}{k}^2\right) \left(H^2 k^2+6\right)$  $w\} \& \frac{i e^{\frac{i \cdot k}{2}} \left(1-e^{-i \cdot k} k}{i \cdot k}\right) \left(1-e^{i \cdot k} k\right) \left(1-e^{i \cdot$  $k \left( \frac{h^2 k^2+6\right)}{\left( \frac{h^$ 

\end{array}

\right)

$$\begin{array}{l} \text{Eerr} \, \| & \left\{ \left( \frac{(-3 \, \mathrm{g} \, \mathrm{H} \, k^2 \, \mathrm{H}^2 \, k^2 \, \mathrm{U}^2 - \mathrm{H}^2 \, k^2 \, \mathrm{U} \, \mathrm{w}}{2 \, (3 \, \mathrm{H}^2 \, k^2)} - \frac{i \, \mathrm{H}^2 \, k^2 \, \mathrm{U} \, \mathrm{w}^2 \, \mathrm{d}^4}{24 \, (3 \, \mathrm{H}^2 \, k^2)} + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) + \\ & \left( -\frac{i \, (27 \, k^3 \, \mathrm{y} \, \mathrm{H}^2 \, k^3 \, \mathrm{H}^2 \, k^3 \, \mathrm{U}^2 \, \mathrm{H}^2 \, k^3 \, \mathrm{U}^2 \, \mathrm{H}^2 \, k^3 \, \mathrm{U}^2 \, \mathrm{H}^2 \, k^2 \, \mathrm{U}^2 \, \mathrm{H}^2 \, k^2 \, \mathrm{U}^2 \, \mathrm{d}^2}{24 \, (3 \, \mathrm{H}^2 \, k^2)} + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) \mathrm{d} x^2 + \\ & \left( -\frac{1}{8} \, \left( k^4 \, \mathrm{U} \right) \, \mathrm{d} \, \mathrm{t} + \left( \frac{3 \, i \, g \, \mathrm{H}^4 \, k^3 \, k^2 \, \mathrm{U}^2 \, \mathrm{H}^2 \, k^2 \, \mathrm{U}^2 \, \mathrm{d}^2}{8 \, (3 \, \mathrm{H}^2 \, k^2)} \right) \, \mathrm{d} t^2 + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) \mathrm{d} x^3 + \\ & \left( \frac{i \, (405 \, k^3 \, \mathrm{U} \, + 35 \, \mathrm{H}^2 \, k^2 \, \mathrm{U}^2 \, \mathrm{H}^2 \, k^2 \, \mathrm{U}^2 \, \mathrm{d}^2}{8 \, (3 \, \mathrm{H}^2 \, k^2)^3} \right) \, \mathrm{d} t^2 + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) \mathrm{d} x^3 + \\ & \left( \frac{i \, (405 \, k^3 \, \mathrm{U} \, + 35 \, \mathrm{H}^2 \, k^2 \, \mathrm{U}^2 \, \mathrm{H}^2 \, k^2 \, \mathrm{U}^2 \, \mathrm{d}^2}{16 \, (3 \, \mathrm{H}^2 \, k^2)^3} \right) \, \mathrm{d} t^2 + 2 \, 38 \, \mathrm{H}^4 \, k^{10} \, \mathrm{U}^2 + 73 \, \mathrm{H}^6 \, k^{12} \, \mathrm{U}^2 \right) \, \mathrm{d} t^2 + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) \, \mathrm{d} x^4 + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) \, \mathrm{d} x^3 + \left( \frac{1 \, k \, w^3 \, \mathrm{d}^4}{8 \, (3 \, \mathrm{H}^2 \, k^2)^3} \right) + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) \, \mathrm{d} x^4 + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) \, \mathrm{d} x^3 + \left( \frac{1 \, k \, w^3 \, \mathrm{d}^4}{8 \, (3 \, \mathrm{H}^2 \, k^2)^3} \right) + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) \, \mathrm{d} x^3 + \left( \frac{1 \, k \, w^3 \, \mathrm{d}^4}{4 \, (3 \, \mathrm{H}^2 \, k^2)^3} \right) + \mathrm{O}[\mathrm{d} \, \mathrm{I}]^5 \right) \, \mathrm{d} x^3 + \frac{1 \, \mathrm{I}^2 \, k^3 \, \mathrm{U}^2 \, \mathrm{U$$

## \begin{array}{cc}

 $\left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2 \right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2 \right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2 \right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2 \right) \left(\frac{-H^2 U w k^3 + 3 U w k^3 + 3 U^2 k^2 - 3 g H k^2 \right) \left(\frac{-H^2 U w k^3 + 3 U w k^3$  $H^2 k^3 U w^2 \text{text} dt^3$ {6 \left( $H^2 k^2 + 3 \text{right}$ )+\frac{ $H^2 k^3 U w^3 \text{text} dt^4$ {24 \left( $H^2 k^2 + 3 \text{right}$ )}  $k^2+3\right)+O\left(\frac{t}{x}^5\right)+O\left(\frac{t}{x}^5\right)+O\left(\frac{t}{x}^5\right)$  $U \text{text}\{dt\}\{12 \left(H^2 k^2+3\right)^2-\frac{1}{4}U^2 k^8+12 H^2 U^2 k^6+36 U^2\right)$  $k^4-9$  g H  $k^4$ right) \text{dt}^2}{24 \left(H^2 k^2+3\right)^2}+O\left(H(\text{text}(dt)^5\right)\right)  $\label{left} $$ \left( -\frac{1}{8} \left( -\frac{1}{8} \right) \cdot \left( -\frac{1}{8} \right) \cdot \left( -\frac{1}{8} \right) \cdot \left( -\frac{1}{8} \cdot \frac{1}{8} \cdot \frac$  $k^2+3\right)+\frac{3}{g}H k^5}{16\left(H^2 k^2+3\right)\right)+\frac{3}{g}H k^5}{16\left(H^2 k^2+3\right)}\right)$ \text{dx}^3+\left(\frac{i \left(13 H^6 U k^{11}+116 H^4 U k^9+351 H^2 U k^7+405 U k^5\right)  $\label{eq:local_condition} $$ \left( H^2 k^2 + \sinh^3 + \frac{135 g H^5 k^{10}}{3} + \frac{135 g H^5 k^{10}}{3} \right) = 1.5 h^6 U^2 k^{12} + 135 g H^5 k^{10} + 387 H^4 U^2 k^{12} + 135 g H^5 k^{10} + 387 H^4 U^2 k^{10} + 136 g H^5 k^{10}$  $U^2 k^{10}+837 g H^3 k^8+297 H^2 U^2 k^8-351 U^2 k^6+1161 g H k^6 ight) \text{ } text{dt}^2{1440}$  $\left(\frac{dt}{5}\right) \cdot \left(\frac{dt}{5}\right) \cdot \left(\frac{$ &  $\left(\left(\frac{3 U k^2}{H^2 k^2+3}-\frac{3 w k}{2 \left(H^2 k^2+3\right)\right)\right)\right)$  $\text{dt}^2-\left(k w^2 \left(t\right)^3\right)^2 \left(H^2 k^2+3\right)^2+\left(k w^3 \left(t\right)^4\right)^8$  $k^2+3\right)^2+\frac{3 i k^3}{2 \left( \frac{4t}{^2} \right)^2}$  $\label{left} $$\left(H^2 k^2+3\right)^2+O\left(text\{dt\}^5\right)\right) \text{ text}\{dx\}^2+\left(t^2 k^2+t^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text{dt}^2\right)^2+O\left(text{dt}^2\right)^2+O\left(text{dt}^2\right)^2+O\left(text{dt}^2\right)^2+$  $\left(H^2 k^2+3\right) + O\left(\frac{dt}^5\right) + O\left($ \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(45 H^4 U k^{10}+279 H^2 U k^8+387 U k^6\right)}  $\t x_{dt}^2_{240 \left(H^2 k^2+3\right)} + O\left(t_{t}^2_{t}^5\right) \right) \t x_{dx}^4 + O\left(t_{t}^2_{t}^5\right) \right) \t x_{dx}^4 + O\left(t_{t}^2_{t}^5\right) \t x_$  $\left(-\frac{k^2 H^3+3 g H^3 U^2\right)}{(2 k U+w)\right) + \left(\frac{k^2 H^3+3 g H^3 U^2\right)}{(2 k U+w)\right) + \left(\frac{k^2 H^3}{(2 k^2 H^3+3 g H^3 U^2)}\right) + \left(\frac{k^2 H^3}{(2 k^2 H^3 H^3 U^2)}$ k \left(g k^2 H^3+3 g H-3 U^2\right) w^2 \text{dt}\^3\{6 \left(H^2 k^2+3\right)}+\frac{k \left(g k^2 H^3+3 g H-3 U^2\right)}  $g H-3 U^2\left(t\right)^4\left(t\right)$ \left(g H^5 k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}{12 \left(H^2 k^2+3\right)^2}+\frac{\left(-2 g H^5 U k^8-12 g H^3 U k^6-9 U^3 k^4-18 g H U k^4\right)}  $\label{left} $$ \operatorname{dt}^2_{12 \left( h^2 k^2 + 3\right)^2} + \operatorname{left}(\operatorname{dt}^5\right) \operatorname{dt}(\operatorname{dt}^5\right) \operatorname{dt}(\operatorname{dt}^2)^2 + \operatorname{left}(-\frac{1}{8})^2 + \operatorname{lef$  $\left(g + k^4\right) \left(g + k^4\right) \left(dt + \frac{dt}{+ \frac{dt}{2}} + \frac{dt}{2}\right) \left(g + k^5 - g + U + k^5\right) \left(dt + \frac{dt}{2}\right) \left(g + k^6 - g + U + k^5\right) \left(dt + k^6\right) \left$  $\left(\frac{41}{11}+117\right)$ g H^5 k^9-H^4 U^2 k^9+351 g H^3 k^7+54 U^2 k^5+351 g H k^5\right) \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73 g H^7 U k^{12}-135 H^4 U^3 k^{10}+657 g H^5 U k^{10}-837 H^2 U^3 k^8+1971 g H^3 U k^8-1161 U^3 k^6+1971 g H U k^6\right) \text{dt}^2\{720 \left(H^2  $k \left( H^2 k^2 + \frac{1}{2} \right) + \frac{1}{2} \left( H^2 k^2 + \frac{1}{2} \right) + \frac{1}{2} \left( H^2 k^2 + \frac{1}{2} \right) + \frac{1}{2} \left( H^2 k^2 + \frac{1}{2} \right)$  $\label{eq:continuity} U w^3 \text{ } \text{$$ \text{$dt}^4}{24 \left(\frac{h^2 k^2+3\right)}+O\left(\frac{t}{h^2 k^2}\right)\right)} + O\left(\frac{h^4 U}{h^2 k^2}\right) + O\left$  $H^2 U^2 k^6 + 9 g H k^4 + g H k^4$  $\text{dx}^2+\left(-\frac{1}{8}\right) \cdot \left(\frac{4}{4}\right) \cdot \left$  $U^2+3$  g H\right) \text{dt}^2\{16 \left(H^2 k^2+3\right)\}+O\\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(13 H^6 k^{11}+118 H^4 k^9+351 H^2 k^7+297 k^5\right) U  $\text{text}_{dt}_{240 \left( H^2 k^2 + 3\right)^3} + \frac{10}{400}$ U^2 k^{10}+837 g H^3 k^8+3645 H^2 U^2 k^8+4293 U^2 k^6+1161 g H k^6\right) \text{dt}^2\{1440  $\left(\frac{dt}^5 \right) \cdot \left(\frac{dt}^5 \right) \cdot \left(\frac{$ 

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\end{array}
      \right)
\ln[468] = \text{KurF} = (\text{fm} * \text{ap} - \text{fp} * \text{am} + \text{am} * \text{ap} * (\text{qp} - \text{qm})) / (\text{ap} - \text{am});
      KurFWS = KurF /. ap \rightarrow 0 /. am \rightarrow (U - Sqrt[g * H]);
      KurFWSeta =
         KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
          qm \rightarrow Rmp * n;
      KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
      Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1];
      \texttt{Kfnn} \; = \; \texttt{Kfnnp} \; / \; . \; \mathsf{Rpp} \; \rightarrow \; \mathsf{Rp} \; \; / \; . \; \; \mathsf{Rmp} \; \rightarrow \; \mathsf{Rm} \; \; / \; . \; \; \mathsf{GGp} \; \rightarrow \; \mathsf{GG2} \; \; / \; . \; \; \mathsf{Gnp} \; \rightarrow \; \mathsf{Gn2} \; ;
      KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
      Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
      Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
      Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
      FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
      FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
      FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
      KurFWSG = KurFWS / . fp \rightarrow (U * Rpp * G + U * H * v + g * H * Rpp * n) / .
             fm \rightarrow (U*Rmp*G + U*H*v + g*H*Rmp*n) /. qp \rightarrow Rpp*G /. qm \rightarrow Rmp*G;
      KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
      KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
      KfGGp = FullSimplify[KurFWSG /. n \rightarrow 0 /. G \rightarrow 1];
      KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
      KfGG = KfGGp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
      FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
      FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
      FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
      fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g*H]) / (2) * (Rp - Rm);
      FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
      FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
      FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
      Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
      Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2;
      Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
      EigvFmat2 = Eigenvalues[Fmat2];
      RKStep = Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2] / (I * dt);
      RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
      RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
```

```
Text[Row[{" U > Sqrt(gH)"}]]
       Text[" "]
       Text[Row[{"Fnn || ", Kfnnp}]]
       Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
       Text[Row[{"Fnn error ||
                                         ", Fnn2TAr}]]
       Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
       Text[" "]
       Text[Row[{"FnG || ", KfnGp}]]
       Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
       Text[Row[{"FnG error ||
                                         ", FnG2TAr}]]
       Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
       Text[" "]
       Text[Row[{"FGn || ", KfGnp}]]
       Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
       Text[Row[{"FGn error || ", FGn2TAr}]]
       Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
       Text[" "]
       Text[Row[{"FGG || ", KfGGp}]]
       Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
       Text[Row[{"FGG error || ", FGG2TAr}]]
       Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
       Text[" "]
       Text[" "]
       Text[Row[{"Omega error || ", RKstepTayr}]]
       Text[Row[{"Omega error ||
                                           ", TeXForm[RKstepTayr]}]]
       Text[" "]
       Text[Row[{"EA ||
                               ", EA}]]
       Text[Row[{"EA || ", TeXForm[EA]}]]
       Text[Row[{"Eerr || ", Eerr}]]
       Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
Out[502]= U > Sqrt(gH)
Out[503]=
Out[504]= Fnn \parallel Gnp H + Rpp U
Out[505] = Fnn \parallel \text{text}\{Gnp\} H + \text{text}\{Rpp\} U
Out[506]= Fnn error |
        \left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)}\,-\,\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}\,+\,O[dt]^4\right)\,+\,\left(-\,\frac{i\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)\,U\,dt}{12\left(3+H^2\,k^2\right)^2}\,+\,O[dt]^4\right)dx^2\,+\,\left(\frac{1}{8}\,k^4\,U\,dt\,+\,O[dt]^4\right)dx^3\,+\,O[dx]^4
```

Out[507]= Fnn error |

 $\left(-\frac{t}{2 \left(t\right)^2 \left($  $k^5+27 \ k^3 \ U \ \text{text} \ dt \ 12 \ \text{left} \ (4^2 \ k^2+3 \ \text{right})^2 + O \ \text{left} \ (4t)^4 \ \text{right}) \ \text{right}) + \text{text} \ dx \ \text{head} \ \text{text} \ \text{head} \ \text{text} \ \text{head} \ \text{text} \ \text{head} \ \text{he$  $\left(\frac{1}{8} k^4 U \left(\frac{dt}{+O\left(\frac{dt}{4}\right)\right)\right)} + O\left(\frac{dt}{4}\right) + O\left(\frac{dt}{4}\right)$ 

Out[508]=

Out[509]= FnG || GGp H

Out[510]= FnG || \text{GGp} H

$$\text{Out} \\ \text{[511]} = \quad FnG \; error \; \mid \mid \; \left( -\frac{3 \; (k \; w) \; dt^2}{2 \; (3 + H^2 \; k^2)} - \frac{i \; k \; w^2 \; dt^3}{2 \; (3 + H^2 \; k^2)} + O[dt]^4 \right) \\ + \left( \frac{i \; (6 \; k^3 + H^2 \; k^5) \; dt}{4 \; (3 + H^2 \; k^2)^2} + O[dt]^4 \right) dx^2 \\ + O[dx]^4 +$$

 $\label{left-frac} $\operatorname{Im}_{\sigma} = \operatorname{FnG}_{\sigma} = \operatorname{FnG}_{\sigma} \left( \frac{dt}^2 (k w)}{2 \left( \frac{k^2 + 3\right)}{-\frac{dt}{2}}} \right) - \frac{dt}{2} \left( \frac{dt}{2} (k w)^2 \right)$  $\label{eq:continuity} $$ k^2+3\right)+O\left(\frac{t}{4}\right)+\left(\frac{dx}^2\right)=0. $$ k^2+3\right)+O\left(\frac{t}{4}\right)+O\left(\frac{t}{4$  $\label{left} $$ \left(\frac{dt}{4 \left(\frac{h^2 k^2+3\right)}}-2\right)-\left(\frac{dt}{4 \left(\frac{h^2 k^2+3\right)}}-2\right)-2\right)$$ 

Out[513]=

Out[514]=  $FGn \parallel H(gRpp + Gnp U)$ 

Out[515]=  $FGn \parallel H(g \text{text}{Rpp}+\text{text}{Gnp} U)$ 

$$\begin{array}{ll} \text{Out} [\text{516}] = & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \; g \; H + g \; H^{3} \; k^{2} - 3 \; U^{2} \right) w \right) \; dt^{2}}{2 \left( 3 + H^{2} \; k^{2} \right)} - \frac{i \; k \left( 3 \; g \; H + g \; H^{3} \; k^{2} - 3 \; U^{2} \right) w^{2} \; dt^{3}}{6 \left( 3 + H^{2} \; k^{2} \right)} + O[dt]^{4} \right) + \\ & \left( -\frac{i \left( 9 \; g \; H \; k^{3} + 6 \; g \; H^{3} \; k^{5} + g \; H^{5} \; k^{7} + 18 \; k^{3} \; U^{2} + 3 \; H^{2} \; k^{5} \; U^{2} \right) dt}{12 \left( 3 + H^{2} \; k^{2} \right)^{2}} + O[dt]^{4} \right) dx^{2} + \left( \frac{1}{8} \; g \; H \; k^{4} \; dt + O[dt]^{4} \right) dx^{3} + O[dx]^{4} \\ \end{array}$$

Out[517]= FGn error ||

 $\label{left} $\left(-\frac{t_4t_4t_5^2 \left( H^2 k^2 + 3 \right)}{2 \left( H^2 k^2 + 3 \right)} - \frac{t_4t_5}{2 \left( H^2 k^2 + 3 \right)} - \frac{t_5}{2 \left( H^2 k^$  $\text{text}\{dt\}^3 \text{ k w}^2 \left\{ H^3 \text{ k}^2 + 3 \text{ g H} - 3 \text{ U}^2\right\}$  $k^2+3\right)+O\left(\frac{dt}^4\right)+\frac{dx}^2\left(\frac{dx}^2\right)+C\left(\frac{dt}^4\right)$ k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12  $\left(\frac{H^2 k^2+3\right)^2}+O\left(\frac{dt}^4\right)\right)+\left(\frac{dt}^3 \left(\frac{1}{8}\right)\right)}{2}+O\left(\frac{dt}^4\right)$ g H  $k^4 \text{text}dt}+O\left(\frac{dt}^4\right)\right)$ 

Out[518]=

Out[519]=  $FGG \parallel (GGp H + Rpp) U$ 

Out[520]=  $FGG \parallel U (\text{text}\{GGp\} H+\text{text}\{Rpp\})$ 

$$\begin{array}{ll} \text{Out} \text{[S21]=} & FGG \; error \; \mid \mid \; \left( -\frac{\left( k \left( 6 + H^2 \; k^2 \right) U \; w \right) \; dt^2}{2 \left( 3 + H^2 \; k^2 \right)} - \frac{i \; k \left( 6 + H^2 \; k^2 \right) U \; w^2 \; dt^3}{6 \left( 3 + H^2 \; k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( -9 \; k^3 + 3 \; H^2 \; k^5 + H^4 \; k^7 \right) U \; dt}{12 \left( 3 + H^2 \; k^2 \right)^2} + O[dt]^4 \right) dx^2 + \\ & \left( \frac{1}{8} \; k^4 \; U \; dt + O[dt]^4 \right) dx^3 + \left( \frac{i \left( 297 \; k^5 + 351 \; H^2 \; k^7 + 118 \; H^4 \; k^9 + 13 \; H^6 \; k^{11} \right) U \; dt}{240 \left( 3 + H^2 \; k^2 \right)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{array}$$

Out[522]= FGG error ||

 $\left(-\frac{t}{2 k^2 + 3\right)}-\frac{t}{3 k}$ 

 $U w^2 \left(\frac{h^2 k^2+6\right)}{6\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^2+3\right)}{1+O\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^2+3\right)}{1+O\left(\frac{h^2 k^2+3\right)}+O\left(\frac{h^2 k^$ 

 $\left(-\frac{i \left(H^4 k^7+3 H^2 k^5-9 k^3\right) U \left(H^2 k^5\right)}{12 \left(H^2 k^5-9 k^3\right)}\right)$ 

 $k^2+3\right)^2+O\left(\frac{dt}^4\right)+\left(\frac{dt}^4\right)+\left(\frac{dt}^4\right)^2+O\left(\frac{dt}^4\right)^2+$ 

 $U \text{$t$} + O\left(\frac{dt}^4\right) + \left(\frac{dt}^4\right) + \left($ 

k^{11}+118 H^4 k^9+351 H^2 k^7+297 k^5\right) U \text{dt}}{240 \left(H^2

 $k^2+3\right)^3}+O\left(\frac{dt}^4\right)+O\left(\frac{dx}^5\right)$ 

Out[523]=

Out[524]=

Out[525]= Omega error ||

Omega error || 
$$\left\{ \frac{1}{6(3+H^2k^2)^2} k^3 \left( \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + (3+H^2\,k^2) \, U \right) \left( 3 \, g \, H \, + \, U \left( 2 \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + (3+H^2\,k^2) \, U \right) \right) dt^2 \, + \\ \frac{i \, k^4 \left( 3 \, g \, H + U \left( 2 \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + (3+H^2\,k^2) \, U \right) \right)^2 dt^3}{8(3+H^2\,k^2)^2} \, - \, \frac{1}{20(3+H^2\,k^2)^4} \left( k^5 \left( \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + (3+H^2\,k^2) \, U \right) \right) dt^4 \, + \, O[dt]^5 \, + \\ \left( \frac{k^3 \left( -3 \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + (3+H^2\,k^2) \, U \right) \right)}{24(3+H^2\,k^2)^2} \, + \, \frac{1}{48(3+H^2\,k^2)^2} \, k^5 \left( g \left( -9 \sqrt{3} \, H \, \sqrt{g \, H \, (3+H^2\,k^2)} \, + 18 \, H^3 \, k^2 \, U \, + 6 \, H^5 \, k^4 \, U \right) + \\ U^2 \left( 27 \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + 24 \, U \, + 2 \, H^6 \, k^6 \, U \, + 3 \, k^2 \left( 7 \sqrt{3} \sqrt{g \, H^3 \, (3+H^2\,k^2)} \, + 18 \, H^2 \, U \right) + \\ 2 \, k^4 \left( 2 \sqrt{3} \sqrt{g \, H^9 \, (3+H^2\,k^2)} \, + 54 \, U \, + 2 \, H^6 \, k^6 \, U \, + 3 \, k^2 \left( 7 \sqrt{3} \sqrt{g \, H^3 \, (3+H^2\,k^2)} \, + 18 \, H^2 \, U \right) + \\ \frac{1}{48(3+H^2\,k^2)^2} \, k^6 \left( 3 \, g \, H \, + \, U \left( 2 \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + 9 \, H^4 \, U \right) \right) \right) dt^2 \, + \\ \frac{1}{496(3+H^2\,k^2)^2} \, k^6 \left( 3 \, g \, H \, + \, U \left( 2 \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + 18 \, U \, + 2 \, H^2 \, k^2 \right) U \right) \\ \left( -9 \, g \, H \, + \, U \left( 3 \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + 18 \, U \, + 2 \, H^4 \, k^4 \, U \, + 2 \, k^2 \left( \sqrt{3} \sqrt{g \, H^5 \, (3+H^2\,k^2)} \, + 6 \, H^2 \, U \right) \right) \right) \right) dt^3 \, - \\ \left( \frac{1}{16} \, i \, k^4 \left( \sqrt{3} \sqrt{\frac{g \, H \, (3+H^2\,k^2)}{3 \, H^2\,k^2}} \, + 2 \, U \right) \, + \frac{1}{32(3+H^2\,k^2)^2} \, i \, k^6 \left( 3 \, g \, H \left( \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + 4 \, (3+H^2\,k^2) \, U \right) \right) \right) dt^4 \, + O[dt]^5 \right) dt^2 \, + \\ \left( \frac{1}{16} \, i \, k^4 \left( \sqrt{3} \sqrt{\frac{g \, H \, (3+H^2\,k^2)}{3 \, H^2\,k^2}} \, + 2 \, U \right) \, + \frac{1}{32(3+H^2\,k^2)^2} \, i \, k^6 \left( 3 \, g \, H \left( \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + 4 \, (3+H^2\,k^2) \, U \right) \right) \right) dt^4 \, + O[dt]^5 \right) dt^2 \, + \\ \left( \frac{1}{16} \, i \, k^4 \left( \sqrt{3} \sqrt{\frac{g \, H \, (3+H^2\,k^2)}{3 \, H^2\,k^2}} \, + 2 \, U \right) \, + \frac{1}{32(3+H^2\,k^2)^2} \, i \, k^6 \left( 3 \, g \, H \left( \sqrt{3} \sqrt{g \, H \, (3+H^2\,k^2)} \, + 4 \, (3+H^2\,k^2) \, U \right) \right) \right) dt^4 \, + O[dt]^5 \right) dt^4 \, + O[dt]^5 \, dt^4 \, + O[dt]^5 \, dt^4$$

$$\begin{array}{c} U^{-}\left[15\, \sqrt{3}\, \sqrt{g}\, H(3+H^{+}K^{-}) + 18\, U + 2\, H^{+}K^{-}\, U + K^{-}\left[3\, \sqrt{3}\, \sqrt{g}\, H^{+}(3+H^{+}K^{-}) + 12\, H^{+}\, U\right]\right)\right) dt^{-} \\ = \frac{1}{32\, (3+H^{+}K^{-})^{2}} \left[k^{8}\left(\sqrt{3}\, \sqrt{g}\, H(3+H^{2}\,k^{2}) + 2\, (3+H^{2}\,k^{2})\, U\right)\right)\right) dt^{3} - \\ = \frac{1}{64\, (3+H^{+}K^{-})^{2}} i^{2}\, k^{8}\left(\sqrt{3}\, \sqrt{g}\, H(3+H^{2}\,k^{2}) + 2\, (3+H^{2}\,k^{2})\, U\right)\right)\right) dt^{3} - \\ = \frac{1}{64\, (3+H^{+}K^{-})^{2}} i^{2}\, k^{8}\left(\sqrt{3}\, \sqrt{g}\, H(3+H^{2}\,k^{2}) + 2\, (3+H^{2}\,k^{2})\, U\right)\right) dt^{3} + O(dt)^{5}\, dx^{3} + \\ \left(-\left(\left(k^{5}\left(3\, \sqrt{3}\, g\, H(177+124\, H^{2}\,k^{2} + 20\, H^{4}\,k^{4}\right) + 104\, \left(9\, \sqrt{g}\, H(3+H^{2}\,k^{2}) + 6\, k^{2}\, \sqrt{g}\, H^{5}\, (3+H^{2}\,k^{2}) + 6\, k^{2}\, \sqrt{g}\, H^{5}\, (3+H^{2}\,k^{2}) + 6\, k^{2}\, \sqrt{g}\, H^{5}\, (3+H^{2}\,k^{2}) + 20\, H^{4}\, k^{4} + g\, H\, U\, \left(21\, 429\, \sqrt{3}\, H^{2}\, k^{2}\, U + 764\, \sqrt{3}\, H^{6}\, k^{6}\, U + 81\, \left(232\, \sqrt{g}\, H(3+H^{2}\,k^{2}) + 267\, \sqrt{3}\, U\right) + \\ 24\, k^{4}\, \left(84\, \sqrt{g}\, H^{9}\, (3+H^{2}\,k^{2}) + 293\, \sqrt{3}\, H^{4}\, U\right) + \\ 16\, \left(459\, \sqrt{g}\, H\, (3+H^{2}\,k^{2}) \, U^{3} + 153\, k^{2}\, \sqrt{g}\, H^{9}\, (3+H^{2}\,k^{2}) \, U^{3} + 17\, k^{6}\, \sqrt{g}\, H^{13}\, (3+H^{2}\,k^{2}) + 29\, k^{2}\, \left(88\, \sqrt{g}\, H^{7}\, (3+H^{2}\,k^{2}) \, U^{3} + 17\, k^{6}\, \sqrt{g}\, H^{13}\, (3+H^{2}\,k^{2}) \right) \right) \right) \right) dt^{2} \right] / \\ \left(11\, 520\, \left(\sqrt{g}\, H\, (3+H^{2}\,k^{2})^{2}\, U^{3} + 16\, k^{2}\, H^{2}\, k^{2}\right) \, U^{3} + 10\, H^{4}\, k^{4}\right) + \\ 84\, H^{8}\, k^{8}\, U^{4} + 243\, U^{3}\, \left(39\, \sqrt{3}\, \sqrt{g}\, H^{3}\, (3+H^{2}\,k^{2}) \, U + 28\, U\right) + 36\, k^{4}\, U^{3} \right) \\ \left(85\, \sqrt{3}\, \sqrt{g}\, H^{9}\, (3+H^{2}\,k^{2}) + 126\, H^{4}\, U\right) + 4\, k^{6}\, U^{3}\, \left(83\, \sqrt{3}\, \sqrt{g}\, H^{3}\, (3+H^{2}\,k^{2}) \, U^{3} + 1008\, H^{2}\, U^{4}\right) + \\ 3\, g\, H\, U\, \left(13\, 500\, H^{2}\, k^{2}\, U + 472\, H^{6}\, k^{6}\, U + 27\, \left(97\, \sqrt{3}\, \sqrt{g}\, H(3+H^{2}\,k^{2}) \, U^{3} + 1008\, H^{2}\, U^{4}\right) + \\ 4\, k^{4}\, \left(71\, \sqrt{3}\, \sqrt{g}\, H^{9}\, (3+H^{2}\,k^{2}) + 270\, \sqrt{3}\, \sqrt{g}\, H^{9}\, (3+H^{2}\,k^{2}) + 594\, U\right) + \\ 27\, k^{2}\, \left(372\, \sqrt{g}\, H^{9}\, (3+H^{2}\,k^{2}) + 270\, \sqrt{3}\, \sqrt{g}\, H^{9}\, (3+H^{2}\,k^{2}) \, U^{3} + 1008\, H^{2}\, U^{3}\right) + \\ 27\, k^{2}\, \left(372\, \sqrt{g}\, H^{9}\, (3+H^{2}\,k^{2}) + 270\, \sqrt{3}\, \sqrt{g}\, H^{9}\, (3+H$$

$$\begin{cases} \frac{1}{6(3+H^2k_1^2)^2}k^3\Big[-\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\Big)\Big(3\,g\,H + U\Big(-2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\Big)\Big) \\ dt^2 + \\ \frac{1k^4}{3^2g\,H^4U}\Big[-2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\Big)\Big]^2dt^4}{8(2+H^2\,k^2)} - \frac{1}{20(3+H^2\,k^2)^2} \\ \left(k^5\Big(-\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\Big)^3\Big(3\,g\,H + U\Big(-2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\Big)\Big)\Big) \\ dt^4 + O[dt]^5\Big] + \\ \frac{\left(k^6\Big(3\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 2(3+H^2\,k^2)\,U\Big)^3\Big(3\,g\,H + U\Big(-2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 6\,H^3\,k^2\,U + 2\,H^5\,k^4\,U\Big) + 2\,H^5\,k^4\,U\Big) + \\ U^2\Big(-27\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 54\,U + 2\,H^6\,k^6\,U - 3\,k^2\Big(7\,\sqrt{3}\ \sqrt{g\,H^6\,(3+H^2\,k^2)} - 18\,H^2\,U\Big) - \\ 2\,k^4\Big(2\,\sqrt{3}\ \sqrt{g\,H^2\,(3+H^2\,k^2)} + 54\,U + 2\,H^6\,k^6\,U - 3\,k^2\Big(7\,\sqrt{3}\ \sqrt{g\,H^6\,(3+H^2\,k^2)} - 18\,H^2\,U\Big) - \\ -2\,k^4\Big(2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 9\,H^4\,U\Big)\Big)\Big)dt^2 + \\ \frac{1}{48(3+H^2\,k^2)}k^3k^6\Big(3\,g\,H + U\Big(-2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\Big)\Big) \\ \Big(-9\,g\,H + U\Big(-3\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 2\,(3+H^2\,k^2)} + 18\,U + 2\,H^4\,k^4\,U - 2\,k^2\Big(\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\Big)\Big)\Big) \\ dt^3 - \frac{1}{36(3+H^2\,k^2)}k^3\Big(3\,k^2 + 2\,(3+H^2\,k^2)^2\,U\Big)\Big(3\,g\,H + U\Big(-2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\Big)\Big) + \\ U^2\Big(-15\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 2\,(3+H^2\,k^2)^2\,U\Big)\Big(3\,g\,H + U\Big(-2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 4\,(3+H^2\,k^2)\,U\Big)\Big) + \\ dt^2 - \frac{1}{32(3+H^2\,k^2)}k^3\Big(3\,g\,H + U\Big(-2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 2\,(3+H^2\,k^2)\,U\Big)\Big)\Big) \\ dt^2 - \frac{1}{32(3+H^2\,k^2)}k^3\Big(3\,g\,H + U\Big(-2\,\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 2\,(3+H^2\,k^2)\,U\Big)\Big)\Big) dt^3 - \\ \frac{1}{64(3+H^2\,k^2)}k^3\Big(-\sqrt{3}\ \sqrt{g\,H\,(3+H^2\,k^2)} + 2\,(3+H^2\,k^2)\,U\Big)\Big)\Big)^2 dt^4 + O[dt]^5\Big)$$

$$dx^3 + \left( \left( k^5 \left( 3 \sqrt{3} \ g \, H \, (177 + 124 \, H^2 \, k^2 + 20 \, H^4 \, k^4 \right) - \right. \\ \left. 104 \left( 9 \sqrt{g \, H} \, \left( 3 + H^2 \, k^2 \right) + 6 \, k^2 \sqrt{g \, H^3} \, \left( 3 + H^2 \, k^2 \right) + k^4 \sqrt{g \, H^9} \, \left( 3 + H^2 \, k^2 \right) \right) U \right) \right) \right/ \\ \left( 1920 \sqrt{g \, H} \, \left( 3 + H^2 \, k^2 \right)^{5/2} \right) + \left( k^7 \left( 27 \sqrt{3} \, g^2 \, H^2 \, (167 + 124 \, H^3 \, k^2 + 20 \, H^4 \, k^4 \right) + g \, H \, U \left( 21429 \sqrt{3} \, H^2 \, k^2 \, U + 764 \sqrt{3} \, H^6 \, k^6 \, U + 81 \left( -232 \sqrt{g \, H} \, (3 + H^2 \, k^2 \right) + 267 \sqrt{3} \, \, U \right) - 24 \, k^4 \left( 84 \sqrt{g \, H^9} \, \left( 3 + H^2 \, k^2 \right) - 293 \sqrt{3} \, H^4 \, U \right) \right) - \\ \left( 16 \left( 459 \sqrt{g \, H} \, \left( 3 + H^2 \, k^2 \right) \, U^3 + 153 \, k^4 \sqrt{g \, H^9} \, \left( 3 + H^2 \, k^2 \right) \, U^3 + 17 \, k^6 \sqrt{g \, H^{13}} \, \left( 3 + H^2 \, k^2 \right) \, U^3 + 9 \, k^2 \left( 88 \sqrt{g^3 \, H^7} \, \left( 3 + H^2 \, k^2 \right) \, U + 51 \sqrt{g \, H^5} \, \left( 3 + H^2 \, k^2 \right) \, U^3 \right) \right) \right) dt^2 \right) / \\ \left( 11520 \sqrt{g \, H} \, \left( 3 + H^2 \, k^2 \right)^{7/2} \right) - \frac{1}{3840[54 H^2 \, k^2)} \, t^2 \, k^8 \left( 54 \, g^2 \, H^2 \, (81 + 62 \, H^2 \, k^2 + 10 \, H^4 \, k^4 \right) + 84 \, H^8 \, k^3 \, U^4 + 24 \, U^4 \right) \right) + 4 \, k^6 \, U^3 \left( -39 \sqrt{3} \, \sqrt{g \, H} \, \left( 3 + H^2 \, k^2 \right) + 28 \, U \right) + 36 \, k^4 \, U^3 \left( -85 \sqrt{3} \, \sqrt{g \, H^9} \, \left( 3 + H^2 \, k^2 \right) + 126 \, H^4 \, U \right) \right) + 4 \, k^6 \, U^3 \left( -83 \sqrt{3} \, \sqrt{g \, H^{13}} \, \left( 3 + H^2 \, k^2 \right) + 252 \, H^6 \, U \right) - \\ \left( 9 \, k^2 \left( 600 \sqrt{3} \, \sqrt{g \, H^3} \, \left( 3 + H^2 \, k^2 \right) + 126 \, H^4 \, U \right) \right) \right) dt^3 - \frac{1}{23040} \left( \sqrt{g \, H^3} \, \left( 3 + H^2 \, k^2 \right) + 126 \, H^4 \, U \right) \right) + 4 \, k^4 \left( 71 \sqrt{3} \, \sqrt{g \, H^9} \, \left( 3 + H^2 \, k^2 \right) - 1101 \, H^4 \, U \right) \right) \right) dt^3 - \frac{1}{23040} \left( \sqrt{g \, H^3} \, \left( 3 + H^2 \, k^2 \right) + 126 \, H^2 \, U \right) \right) + 2 \, 27 \, k^2 \left( 372 \sqrt{g^5 \, H^9} \, \left( 3 + H^2 \, k^2 \right) - 2703 \sqrt{3} \, g^2 \, H^4 \, U + 4515 \sqrt{g^3 \, H^7} \, \left( 3 + H^2 \, k^2 \right) \, U \right) + 2 \, 27 \, k^2 \left( 372 \sqrt{g^5 \, H^9} \, \left( 3 + H^2 \, k^2 \right) - 2703 \sqrt{3} \, g^2 \, H^4 \, U + 4515 \sqrt{g^3 \, H^7} \, \left( 3 + H^2 \, k^2 \right) \, U^4 + g \, H^3 \right) \right) \right) dt^4 + 2 \, U^4 + 232 \, \sqrt{g \, H^4 \, \left( 3 + H^2 \, k^2 \right)} \, U^4 + g \, H^3 \, U^4 + 2 \, U^4 + 232 \, U^4 +$$

## Out[526]= Omega error ||

 $\left(\frac{h^2 k^3 \left(\frac{h^2 k^2+3\right)} U+\sqrt{h^2 k^2+3\right)} U+\sqrt{h^2 k^2+3\right)} \right)$  $\label{left(H^2 k^2+3) right) U+2 sqrt{3} \operatorname{H \left(H^2 k^2+3\right)}\right) \operatorname{left(H^2 k^2+3\right)} \operatorname{left(H^2 k^2+3)} \operatorname{left(H^$  $\left(H^2 k^2+3\right) + \frac{k^4 \left(g H+U\right)}{g H+U} \left(H^2 k^2+3\right) + \frac{k^4 \left(g H+U\right)}{g H+U} \right)$  $\left(\frac{4^2 k^2+3\right)}{right}\right)^2 \left(\frac{4^2 k^2+3\right)^2} \left(\frac{4^2 k^2+3\right)^2}{right}\right)^2 \left(\frac{4^2 k^2+3\right)^2}{right}$  $k^2+3\right\in U+\sqrt{3} \operatorname{left}(H^2 k^2+3\right)$ 

 $k^2+3\left( U+2 \right) +2 \left( U+2 \right)$  $\label{eq:k-2+3-right} $$k^2+3\right+O\left(\frac{t}{5}\right)+\left(\frac{k^3 \left(\frac{k^3 \left(k^3 \left(\frac{k^3 \left(\frac{k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(\frac{k^3 \left(\frac{k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(k^3 \left(k^3 \left(k^3 \left(\frac{k^3 \left(k^3 \left(k^{3} \left(k^3 \left(k^4 \left(k^3 \left(k^{k^3 \left(k^3 \left(k^3 \left(k^3 \left(k^3 \left(k^4 \left(k^3 \left(k^{k^3 \left(k^{k^3 \left(k^{k^3 \left(k^{k^3 \left(k^{k^3 \left(k^{k^3 \left(k^3 \left(k^{k^3 \left(k^{k^3 \left(k^{k} \left(k^{k^3 \left(k^{k^3 \left(k^{k^3 \left(k^{k} \left(k^{$  $\left( \frac{H^2 k^2+3\right)}{24\left( \frac{H^2 k^2+3\right)}}{24\left( \frac{H^2 k^2+3\right)}{24\left( \frac{H^2 k^2+3\right)}}{24\left( \frac{H^2 k^2+3\right)}{24\left( \frac{H^2 k^2+3\right)}{24\left($  $\left(9 U H^4+2 \right) \left(H^9 \left(H^2 k^2+3\right)\right) \right) \left(9 U H^4+2 \right) \left(18 U H^2+7 \right) \left(18$  $H^5 \left(H^2 k^2+3\right)\right) + h^2+54 U+27 \left(H^2 k^2+3\right) + h^2 \left(H^2 k^2+$  $\left(6 \, k^4 \, U \, H^5 + 18 \, k^2 \, U \, H^3 - 9 \, \right) \left(1 \, H^2 \, k^2 + 3 \right) \, H\right) \, \left(1 \, k^2 \, U \, H^3 - 9 \, \right) \, \left(1 \, H^2 \, k^2 + 3 \right) \, H\right) \, H\right) \, H$  $\label{left(H^2 k^2+3\rightarrow hight)^3} + \frac{i k^6 \left(3 g H+U \left(\frac{h^2 k^2+3\right) U+2 \sqrt{3} \right)}{1 + \frac{h^2 k^2+3 \pi ght}{2}}$  $H \left(H^2 k^2+3\right)\right) \left(U \left(H^4 U k^4+2 \left(H^2 + H^4 U H^2+\right)\right)\right)$  $\left(H^2 k^2+3\right) \ k^2+18 U+3 \ \ \ H\left(H^2 k^2+3\right) \ \ h^2+3 \ \ h^$  $\t (dt)^3 {48 \left( \frac{h^2 k^2+3 \right)^3}-\frac{(k^7 \left( \frac{h^2 k^2+3 \right)^3}{48 \left( \frac{h^2 k^2+3 \right)^3}-\frac{h^2 k^2 + \frac{h^2 k^2 + h^2 + \frac{h^2 k^2 + h^2 h^2 + \frac{h^2 k^2 + \frac{h^2 k^2 + \frac{h^2 k^2 + h^2 + h^2 + h^2 h^2 + h^2 h^2 + h^2 + h^2 h^2 + h^2 h^2 + h^2 h^2 + h^2 h^2 +$  $H \left( H^2 k^2 + 3 \right) \left( H^2 k$  $\label{eq:k-2+3-ight} $$ k^2+3\right)\right) $$ k^2+3\right) $$ k^2+3\right) $$ (H^2 k^2+3\right) $$ k^2+3\left( k^2+3\right) $$ k^2+3\left$  $\label{left} $\left( \frac{H^4 U k^4+\left(12 U H^2+5 \right) + (14 U k^4+U k^4+U$  $\$  \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g H \left(4 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g}  $H \left( H^2 k^2 + 3 \right) \right) \left( H^2 k^2 + 3 \right) \left( H^$  $g H+U \left( \frac{h^2 k^2+3\right)}{U+2 \left( \frac{h^2 k^2+3$  $g \ H+U \ left(2 \ h^2 \ k^2+3 \ U+3 \ yqrt{3} \ sqrt{g} \ H \ left(H^2 \ k^2+3 \ h)\ right) \ right) \ right)$  $\t text{dt}^3}{32 \left(H^2 k^2+3\right)U+\sqrt{k^2+3\right)} - \frac{k^8 \left(1 k^8 \left(H^2 k^2+3\right)U+\sqrt{3}\right)U+\sqrt{3}}{2} \right)}$ \left(H^2 k^2+3\right)\right)\left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right) U+2 \sqrt{3} \q  $\label{left} $$k^2+3\right)\right)\right) + (k^2+3\right) + (k^2+3)\right] + (k^2+3)$  $H^9 \left(H^2 k^2+3\right) k^4+6 \left(H^2 k^2+3\right) k^2+9 \left(H^2 k^2+3\right) k^2+9$  $k^2+3\right) \ U\left( \frac{H^2 k^2+3\right)}{1920 \left( \frac{H^2$ \left(27 \sqrt{3} g^2 \left(20 H^4 k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6+24 \left(293 \sqrt{3} U H^4+84 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+21429 \sqrt{3} H^2  $\label{left} U \ k^2+81 \left( 267 \right) \ H+16 \left( 17 \right) \ H+16 \left($  $H^{13} \left( H^2 k^2 + 3 \right) U^3 k^6 + 153 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( H^2 k^$  $\sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+88 \left(g^3 H^7 \left(H^2 k^2+3\right)\right) U\right) U\right)$  $H^{13} \left( H^2 k^2 + 3 \right) \right) + h^6 + 36 U^3 \left( 126 U H^4 + 85 \right) + grt{3} \left( H^9 \left( H^2 H^4 + 85 \right) \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + 36 U^3 \left( H^4 + 85 \right) + h^6 + h^$  $k^2+3\right) k^4+9\left(1008 H^2 U^4+1039 \right) kqrt{g H^5 \left(H^2 k^2+3\right) U^3+600}$ \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2)  $k^2+81\right)+243 U^3 \left(28 U+39\right)+31\left(28 U+39\right)$ H^6 U k^6+4 \left(1101 U H^4+71 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+13500 H^2 U k^2+27 \left(504 U+97 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\r  $k^2+3\left(\frac{4^2 + 1}{1} + \frac{4^2 + 2^2 + 1}{1$  $\$  \sqrt{3} g^2 H^8+g U \left(979 \sqrt{3} U+349 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7+232 \sqrt{g} H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(2672 \sqrt{3} g^2 U H^6+g U^2 \left(5997 \sqrt{3})  $U+4384 \sqrt{g H \left(\frac{h^2 k^2+3\right)} U^4+180} = U+4384 \sqrt{g H} \left(\frac{h^2 k^2+3\right)} U^4+180 = U+4484 \sqrt{g H} \left(\frac{h^2 k^2+3\right)} U^4+$ 

 $U^3 H^3 + 928 \sqrt{(H^2 k^2 + 3 + 15)} U^4 + 4515 \sqrt{(H^2 k^2 + 3 +$  $U^2+372 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)} \ln k^2+81 \left(H^2 k^2+3\right)$  $U^4+g \ H \left(1033 \right) - 1527 \left(H^2 \ H^2 \ H^2 \ H^2 \ H^2 \ H^2 \right) - 1527 \left(H^2 \ H^2 \ H^2 \ H^2 \ H^2 \ H^2 \right) - 1527 \left(H^2 \ H^2 \right) - 1527 \left(H^2 \ H^2 \$  $H^2 U+157 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \left(H^2 U+157 \right)$ \left(\left(H^2 k^2+3\right) \U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right) \left(3 g H+U \left(H^2 k^2+3\right)) \left(3 g H+U \left(H^2 k^2+3\righ  $k^2+3\left( U-2 \right) \left( U-2 \right$  $\label{eq:k-2+3-right} $$k^2+\frac{1}{2}+\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$  $\label{left} $$k^2+3\right)\right] \cdot (H^2 k^2+3\right) - \frac{h^2}{k^2+3\right)} \left( H^2 k^2+3\right) - \frac{h^2}{k^2+3\right)} - \frac{h^2}{k^2+3\right) - \frac{h^2}{k^2+3} - \frac{h^2}{k^2+3}$  $k^2+3\right\in U-\sqrt{3} \operatorname{H-U-\left(H^2 k^2+3\right)}\right$  $k^2+3\right/4+O\left(\frac{4t}^5\right)\right)$  $\$  \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}\right)}{24 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(\left(2 k^2+3))}{24 \left(H^2 k^2+3)\right)^2}+\frac{k^5 \left(H^2 k^2+3)}{24 \left(H^2 k^2+3)\right)^2}+\frac{k^5  $H^6 U k^6-2 \left(2 \sqrt{3} \right) + H^6 U k^6-2 \left(2$ k^2+3\right)\right) U^2+3 g \left(2 k^4 U H^5+6 k^2 U H^3+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}  $k^2+3\right) U-2 \left( H^4 U \right) \left( H^2 k^2+3\right) \left( H^4 U \right) \left$  $\left( \frac{3} \right) \left( \frac{4^2 \ln(4^2 k^2 + 3 + 18 U - 18 U \label{eq:heft} $$\left(H^2 \ ^2+3\right)^2+3\right)^2+3\right) + H\left(H^2 \ ^2+3\right)^2+3\left(H^2 \ H^2 \ H^2\right)^2+3\left(H^2 \ H^2 \ H^2\right)^2+3\left(H^2 \ H^2 \ H^2\right)^2+3\left(H^2 \ H^2\right)^2+3\left(H^2$  $k^2+3\left( U-2 \right) + U-2 \left( H^2 k^2+3\right) \right) + U-2 \left( H^2 k^2+3\right) + U-2 \left($  $H_{H^2 k^2+3}=2 U\right)+\frac{k^6 \left(U^2 \left(U^2 + 4 U k^4 + \left(12 H^2 U^5 \right) + 3\right)}{k^6 U^2 U^5 U^5}$  $H^5 \left(H^2 k^2+3\right) + L-15 \left(H$  $H \left( \frac{3} \left( \frac{4}{2} \right) - \frac{4\left( \frac{4}{2} \right)}{4} \right) - 4\left( \frac{4}{2} \right) - 4\left$  $\left(H^2 k^2+3\right)^2-\frac{1}{3}$  $\left( H^2 k^2+3\right)\right)$  $\left(\frac{H^2 k^2+3\right} U-2 \sqrt{dt}^4\right) U-2 \sqrt{dt}^4\left(\frac{H^2 k^2+3\right)}\right) U-2 \sqrt{dt}^4\left(\frac{H^2 k^2+3\right)}$  $\label{left} $$\left(H^2 k^2+3\right)^3+O\left(\frac{dt}^5\right)\right) \operatorname{left}(H^2 k^3+\left(\frac{dt}^3\right)^3+\left(\frac{dt}^3\right)^3+O\left(\frac$ g H \left(20 H^4 k^4+124 H^2 k^2+177\right)-104 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6  $\$  \\sqrt{g H^5 \\left(H^2 k^2+3\\right)} k^2+9 \\sqrt{g H \\left(H^2 k^2+3\\right)}\\right) \\\right)}{1920}  $\$  \\ \left(H^2 \,\^2+3\\right)^{5/2}}+\\ \frac{k^7 \\left(27 \\right)^2 \\ \right(20 \,\^4 \,\^4+124 \,\^2 \)  $k^2+167$   $h^2+g$  U \left(764 \sqrt{3}  $h^6$  U  $h^6-24$  \left(84 \sqrt{g  $h^9$  \left(H^2  $h^2$   $h^2+3$  \right)}-293 \sqrt{3} H^4 U\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267 \sqrt{3} U-232 \sqrt{g H \left(H^2  $k^2+3\right)$  hight) hight)  $H-16 \left(17 \right) H-16 \left(17 \right) H-13 \left(H^2 k^2+3\right) U^3 k^6+153 \right)$  $H^9 \left( H^2 k^2 + 3 \right) U^3 k^4 + 9 \left( 51 \right) H^5 \left( H^2 k^2 + 3 \right) U^3 + 88 \right)$  $H^7 \left( H^2 k^2 + 3 \right) U \right) U^3 \left( H^2 k^2 + 3 \right) U^3 \left( H^4 k^2$  $\label{eq:linear_condition} $$ \left(\frac{dt}^2}{11520 \right) \right] \left(\frac{4^8 + 4 U^3}{11520 }\right) \left(\frac{4^8 U^4 k^8 + 4 U^3}{11520 }\right) = 0.$ \left(252 H^6 U-83 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+36 U^3 \left(126 H^4  $U-85 \sqrt{g H^2 k^2+3\right} \sqrt{4-9 \left(-1008 H^2 U^4+1039 \sqrt{3}\right)}$ \cart(a UAS \laft(UAD \lah2 + 2\right)) UA2 + 600 \cart(2) \cart(aA2 UA7 \laft(UAD \lah2 \rangle 2) + 2\right)) U\right)

k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U-39 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right)+3 g H U \left(472 H^6 U k^6-4 \left(71 \sqrt{3} \sqrt{g H^9 \left(H^2  $\label{left} $$ k^2+3\right)\right) \end{center} $$ k^2+3\right) \e$  $\left( H^2 k^2+3\right) -\left( H^2 k^2+3\right) -\left( H^2 k^2+3\right) \right) = 17$ \left(H^2 k^2+3\right)\} U-239 \sqrt{3} g H^9\right) k^8-12 U \left(213 \sqrt{3} g^2 H^8+g U \left(979) \sqrt{3} U-349 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7-232 \sqrt{g H^{13} \left(H^2 k^2+3\right)}  $U^3 + k^6 + 9 \left( -2672 \right) - 2 \left( 4384 \right) + k^6 + 9 \left( -2672 \right) - 2 \left( 4384 \right) -$ \sqrt{3} U\right) H^5+1392 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+180 \sqrt{g^5 H^{13} \left(H^2  $k^2+3\right) \$  yield  $k^4+27 \left(-2703 \right) \$  2 U H^4-4070 \sqrt{3} g U^3 H^3+928 \sqrt{9}  $H^5 \left( H^2 k^2 + 3 \right) U^4 + 4515 \left( H^2 k^2 + 3 \right) U^2 + 372 \left( H^5 \right) U^2 + 372 \left($ \left(1527 \sqrt{g H \left(H^2 k^2+3\right)}-1033 \sqrt{3} U\right) U^2-883 \sqrt{3} g^2 H^2 U+157  $\$  \\sqrt{g^5 H^5 \\left(H^2 k^2+3\right)\\right)\\right)\\right)\\ \text{dt}^4}{23040 \\left(\\sqrt{g H} \\left(H^2 k^2+3\right))\\right)\\right}\\ \text{dt}^4}{23040 \\left(\\sqrt{g H} \\left(\\sqrt{g H} \\left(H^2 k^2+3\right))\\right)\\right)\\right\right\\right\\right\\right\right\\right\right\\right\\right\\right\\right\right\right\right\right\right\right\\right\righ 

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$$\begin{aligned} & \text{Out} \text{[528]=} \quad \text{EA ||} \quad \left\{ \left\{ 1 \, + \, \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dt w}} \right) \, \text{H}^2 \, \text{k}^3 \, \text{U} \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right]}{6 + 2 \, \text{H}^2 \, \text{k}^2 \right) \, \text{w}}, \, \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dt w}} \right) \, \text{H} \, \text{k} \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right]}{2 \left( \text{H} + \frac{\text{H}^3 \, \text{k}^2}{3} \right) \, \text{w}} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dt w}} \right) \, \text{k} \, \left( \text{g} \, \text{H} \, \left( 3 + \text{H}^2 \, \text{k}^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dt w}} \right) \, \text{k} \, \left( \text{g} \, \text{H} \, \left( 3 + \text{H}^2 \, \text{k}^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dt w}} \right) \, \text{k} \, \left( \text{g} \, \text{H} \, \left( 3 + \text{H}^2 \, \text{k}^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dt w}} \right) \, \text{k} \, \left( \text{g} \, \text{H} \, \left( 3 + \text{H}^2 \, \text{k}^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dt w}} \right) \, \text{k} \, \left( \text{g} \, \text{H} \, \left( 3 + \text{H}^2 \, \text{k}^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dt w}} \right) \, \text{k} \, \left( \text{g} \, \text{H} \, \left( 3 + \text{H}^2 \, \text{k}^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dt w}} \right) \, \text{k} \, \left( \text{g} \, \text{H} \, \left( 3 + \text{H}^2 \, \text{k}^2 \right) - 3 \, \text{U}^2 \right) \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right]}{2} \right\}, \\ & \left\{ \frac{i \, e^{\frac{i \, \text{dx k}}{2}} \left( 1 - e^{-i \, \text{dx k}} \right) \left( -1 + e^{i \, \text{dx k}} \right) \, \text{k} \, \left( \text{g} \, \text{H} \, \left( 1 + e^{i \, \text{dx k}} \right) \, \text{k}^2 \right) \, \text{k}^2 \, \text{Csc} \left[ \frac{\text{dx k}}{2} \right] \, \right\} \right\}$$

Out[529]= EA || \left(

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 $\frac{i e^{\left(\frac{1-e^{-i \cdot text\{dx\} k}\right)} \left(1-e^{-i \cdot text\{dx\} k}\right) \left(1-e^{-i \cdot text\{dx\} k}\right)} \left(1-e^{-i \cdot text\{dx\} k}\right)}$ w\right) H^2 U \csc \left(\frac{\text{dx} k}{2}\right) k^3}{\left(2 H^2 k^2+6\right) w}+1 &  $\frac{i e^{\frac{1}{2}} \left(1-e^{-i \left(k^{2} k\right)}\right)\left(1-e^{-i} \left(k^{2} k\right)\right)}{\left(1-e^{-i} \left(k^{2} k\right)\right)\left(1-e^{-i} \left(k^{2} k\right)\right)}$  $w \rightarrow H \ k \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow W \ k^2 \rightarrow H^3 (3) + H \ w \rightarrow H$ 

 $\label{eq:linear_line$  $\left(\frac{4x}{k}^2 + \frac{3\right}{c} - \frac{4x}{k}^2 + \frac{4$  $w\} \& \frac{i e^{\frac{i \cdot k}{2}} \left(1-e^{-i \cdot k} k}{2}} \left(1-e^{-i \cdot k} k\right) \left(1-e^{i \cdot k} k\right) \left(1-e^{-i \cdot k} k\right) \left(1-e^{-i \cdot k} k\right) }$  $k \left( H^2 k^2 + 6 \right) U \left( \frac{dx}{k}^2 \right) \left( H^2 k^2 + 6 \right) w + 1 \left( H^2 k^2 + 6 \right)$ 

\end{array}

\right)

$$\begin{aligned} & \text{Cut[S30]} & \text{Eerr } \| \ \left\{ \left\{ \left( \frac{(-3 \, \text{g H k}^2 + 3 \, \text{k}^2 \, \text{U}^2 - \text{H}^2 \, \text{k}^4 \, \text{U}^2 - \text{H}^2 \, \text{k}^3 \, \text{U} \, \text{w}^2 \, \text{d}^2}{(3 + \text{H}^2 \, \text{k}^2)} + \frac{1 \, \text{H}^2 \, \text{k}^3 \, \text{U} \, \text{w}^3 \, \text{d}^4}{24 \, (3 + \text{H}^2 \, \text{k}^2)} + \text{O}[\text{d}1]^5 \right) + \\ & \left( -\frac{i \left( 27 \, \text{k}^3 + 9 \, \text{H}^2 \, \text{k}^3 + 17 \, \text{k}^2 \, \text{U}^2 \, \text{d}^2}{16 \, (3 + \text{H}^2 \, \text{k}^2)^2} - \frac{1 \, \text{G}^3 \, \text{H}^2 \, \text{k}^3 \, \text{U}^2 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2 \, \text{d}^2}{24 \, (3 + \text{H}^2 \, \text{k}^2)^2} + \text{O}[\text{d}1]^5 \right) \, \text{d}x^2 + \\ & \left( \frac{1}{8} \, \text{k}^4 \, \text{U} \, \text{d} \, \text{t} \left( -\frac{3 \, i \, \text{g}^2 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2 \, \text{d}^2}{63 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2 \, \text{d}^2} + \frac{1 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2}{63 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2} \right) \, \text{d}^2 + \text{O}[\text{d}1]^5 \right) \, \text{d}x^3 + \\ & \left( \frac{i \left( 405 \, \text{k}^5 \, \text{U} + 35 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2}{84 \, \text{G}^3 \, \text{H}^2 \, \text{k}^2} \right) \, \text{d}^2 + \text{O}[\text{d}1]^5 \right) \, \text{d}x^3 + \\ & \left( \frac{i \left( 405 \, \text{k}^5 \, \text{U} + 35 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2}{84 \, \text{H}^2 \, \text{k}^2} \right) \, \text{d}^2 + \text{O}[\text{d}1]^5 \right) \, \text{d}x^3 + \\ & \left( \frac{i \left( 405 \, \text{k}^5 \, \text{U} + 35 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2 \, \text{d}^2 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2}{84 \, \text{H}^2 \, \text{k}^2} \right) \, \text{d}^2 + \text{O}[\text{d}1]^5 \right) \, \text{d}^2 + \text{O}[\text{d}1]^5 \right) \, \text{d}^2 + \text{O}[\text{d}1]^5 \right) \, \text{d}^2 + \frac{1 \, \left( \frac{14 \, \text{U}^2 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2 \, \text{H}^2 \, \text{k}^2} \, \text{U}^2}{24 \, (3 + \text{H}^2 \, \text{k}^2)^2} \right) \, \text{d}^2 + \frac{1 \, \left( \frac{14 \, \text{U}^2 \, \text{H}^2 \, \text{k}^2 \, \text{U}^2 \, \text{d}^2}{4 \, (3 + \text{H}^2 \, \text{k}^2)^2} \right) \, \text{d}^2 + \text{O}[\text{d}1]^5 \right)} \, \text{d}^2 + \frac{1 \, \left( \frac{14 \, \text{U}^2 \, \text{U}^2 \, \text{H}^2 \, \text{k}^2} \, \text{U}^2 \, \text{U}^2 \, \text{H}^2 \, \text{k}^2} \, \text{U}^2 \, \text{U}^2 \, \text{H}^2 \, \text{k}^2} \right) \, \text{d}^2 + \text{O}[\text{d}1]^5 \right) \, \text{d}^2 + \text{O}[\text{d}1]^5 \right) \, \text{d}^2 + \frac{1 \, \left( \frac{14 \, \text{U}^2 \, \text{H}^2 \, \text{K}^2 \, \text{U}^2 \, \text{U}^2 \, \text{H}^2 \, \text{L}^2 \, \text{U}^2 \, \text{L}^2 \, \text{L}^2$$

Out[531]= Eerr || \left(

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\left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{H^2 k^2 + 3 right}{-H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2\right) \left(\frac{-H^2 U w k^3 + 3 U w k^3 + 3 U w k^3 + 3 U w k^3\right) \left(\frac{-H^2 U w k^3 + 3 U w k^3 + 3 U w k^3\right) \left(\frac{-H^2 U w k^3 + 3 U w k^3 + 3 U w k^3\right) \left(\frac{-H^2 U w k^3 + 3 U w k^3 + 3 U w k^3\right) \left(\frac{-H^2 U w k^3 + 3 U w k^3\right) \left(\frac{-H^2 U w k^3 + 3 U w k^3\right) \left(\frac{-H^2 U w k^3 + 3 U w k^3\right) \left(\frac{-H^2 U w k^3 + 3 U w k^3\right) \left(\frac{-H^2 U w k^3}{2 U w k^3}\right) \left(\frac{-H^2 U w k^3 + 3 U w k^3\right) \left(\frac{-H^2 U w k^3}{2 U w k^3}\right) \left(\frac{-H^2 U w 
                                             H^2 k^3 U w^2 \text{dt}^3{6 \left(H^2 k^2+3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2 + 3\right)} + \frac{H^2 k^3 U w^3 U w^
                                             k^2+3\right)+O\left(\frac{t}{t}\right)+O\left(\frac{t}{t}\right)+O\left(\frac{t}{t}\right)+O\left(\frac{t}{t}\right)
                                             U \text{dt}}{12 \left(H^2 k^2+3\right)^2}-\frac{\left(2 H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2)}
                                             k^4-9 g H k^4right) \text{dt}^2}{24 \left(H^2 k^2+3\right)^2}+O\left(\frac{dt}^5\right)^1
                                             \t (-\frac{1}{8} k^4 U \text{ } -\frac{1}{8} k^4 U \text{ } 
                                             i g H k^5 \{ 16 \left( H^2 k^2 + 3 \right) \right) \left( t + t dt \right)^2 + O\left( t + t dt \right)^5 \right) 
                                             \text{dx}^3+\left(\frac{i \left(13 H^6 U k^{11}+116 H^4 U k^9+351 H^2 U k^7+405 U k^5\right)}
                                             U^2 k^{10}+837 g H^3 k^8+297 H^2 U^2 k^8-351 U^2 k^6+1161 g H k^6\right) \text{dt}^2\{1440
                                             & \left(\left(\frac{3 U k^2}{H^2 k^2+3}-\frac{3 w k}{2 \left(H^2 k^2+3\right)}\right)\right)\right)
                                             \t \{dt\}^2-\frac{i \ k \ w^2 \ t(dt)^3}{2 \ (dt)^4} 
                                             \left(\frac{4t}^2 k^2+3\right)+O\left(\frac{4t}^5\right)+O\left(\frac{4t}^5\right)
                                             \label{eq:continuity} $$ k^2+\frac{3 i k^3}{2 \left(H^2 k^2+3\right)^2}\right) \text{ } \text{ } \text{ } k^4 U \text{ } t^2}{4t}^2}$
                                             \label{left} $$\left(H^2 k^2+3\right)^2+O\left(text\{dt\}^5\right)\right) \text\{dx\}^2+\left(-\frac{3 i k^5 U text\{dt\}^2}{8}\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text\{dt\}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{text}^2\right)^2+O\left(text{tex
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 $\left(H^2 k^2+3\right) + O\left(\frac{dt}{3}\right) + O\left(\frac{dt}{3}\right$ \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(45 H^4 U k^{10}+279 H^2 U k^8+387 U k^6\right)}  $\label{left} $\left(-\frac{k^2 H^3+3 g H-3 U^2\right)(2 k U+w)\right) \operatorname{text}(dt)^2}{2 \left(-\frac{k^2 H^3+3 g H-3 U^2\right)(2 k U+w)\right)}-\frac{1}{2}\left(-\frac{k^2 H^3+3 g H-3 U^2\right)(2 k U+w)\right)^2}$  $k \left( \frac{k^2 H^3+3 g H-3 U^2\right) w^2 \left( \frac{k^2 H^3+3}{6 \left( \frac{k^2 H^3+$  $g H-3 U^2\left(t\right) w^3 \left(t\right)^4\left($ \left(g H^5 k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12 \left(H^2 k^2+3\right)^2}+\frac{\left(-2 g H^5 U k^8-12 g H^3 U k^6-9 U^3 k^4-18 g H U k^4\right)}  $\label{eq:local_$ g H k^4 \text{dt}-\frac{i \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \text{dt}^2}{8 \left(H^2  $k^2+3\right)+O\left(\frac{dt}{5\right)}+O\left(\frac{dt}{5\right)}+O\left(\frac{dt}{5\right)}$ H^5 k^9-H^4 U^2 k^9+351 g H^3 k^7+54 U^2 k^5+351 g H k^5\right) \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73 g H^7 U k^{12}-135 H^4 U^3 k^{10}+657 g H^5 U k^{10}-837  $H^2 U^3 k^8 + 1971 g H^3 U k^8 - 1161 U^3 k^6 + 1971 g H U k^6 \right) \text{ $$\operatorname{t}(H^2 = 161 U^3 k^6 + 1971 g H U k^6 ) $$ $$\operatorname{t}(H^2 = 161 U^3 k^6 + 1971 g H U k^6 ) $$$  $\label{left} $$k^2+3\right)^3$+O\left(\frac{dt}^5\right)^3\right)+O\left(\frac{dt}^5\right)^3$+O\left(\frac{dt}^5\right)^3$+O\left(\frac{dt}^6\right)^3$+O$ U^2 k^4-H^2 U w k^3-9 U^2 k^2-3 g H k^2-6 U w k\right) \text{dt}^2\{2 \left(H^2 k^2+3\right)}-\frac{i}  $k \left( \frac{h^2 k^2 + 6 \right) U w^2 \left( \frac{h^2 k^2 + 3 \right)}{h^2 k^2 + 6 \right) + \frac{h^2 k^2 + 6 \right) } + \frac{h^2 k^2 + 6 \right)$ k^7+3 H^2 U k^5-9 U k^3\right) \text{dt}}{12 \left(H^2 k^2+3\right)^2}+\frac{\left(-2 H^4 U^2 k^8-12)}{12 \left(H^2 k^2+3\right)^2}+\frac{\left(H^2 k^2+3\right)^2}{12 \left(H^2 k^2+3\right)^2}+\frac{\left(H^2 k^2+3\r  $H^2 U^2 k^6 + 9 g H k^4 + g H k^4$ \text{dx}^2+\left(\frac{1}{8} k^4 U \text{dt}-\frac{i k^5 \left(2 H^2 k^2 U^2+12 U^2+3 g H\right)} \left(13 H^6 k^{11}+118 H^4 k^9+351 H^2 k^7+297 k^5\right) U \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73 H^6 U^2 k^{12}+135 g H^5 k^{10}+927 H^4 U^2 k^{10}+837 H^4 U^2 k^2 H^4 U^2 h^4 U^2 k^2 H^4 U^2 k^2 H^4 U^2 g H^3 k^8+3645 H^2 U^2 k^8+4293 U^2 k^6+1161 g H k^6\right) \text{dt}^2}{1440 \left(H^2 

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