

```
In[1]:= q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
```

```
qjp1n = q0 * Exp[I * (k * (xj + dx) + w * tn)];
qjp1F = Simplify[qjp1n / (qjn)];
qjp2n = q0 * Exp[I * (k * (xj + 2 * dx) + w * tn)];
qjp2F = Simplify[qjp2n / (qjn)];
qjm1n = q0 * Exp[I * (k * (xj - dx) + w * tn)];
qjm1F = Simplify[qjm1n / (qjn)];
qjm2n = q0 * Exp[I * (k * (xj - 2 * dx) + w * tn)];
qjm2F = Simplify[qjm2n / (qjn)];
```

$$wAp = -U * k - \frac{\sqrt{3} \, k \sqrt{g \, H \, (3 + H^2 \, k^2)}}{3 + H^2 \, k^2};$$

$$wAm = -U * k + \frac{\sqrt{3} \, k \sqrt{g \, H \, (3 + H^2 \, k^2)}}{3 + H^2 \, k^2};$$

```
In[13]:= Dx = FullSimplify[(qjp1F - qjm1F) / (2 * dx)];
Dxerr = Series[Dx - (I * k), {dx, 0, 4}];
DxDx = FullSimplify[(qjp1F - 2 + qjm1F) / dx^2];
DxDxerr = Series[DxDx - (-k * k), {dx, 0, 4}];
DxDxDx = FullSimplify[(qjp2F - 2 qjp1F + 2 * qjm1F - qjm2F) / (2 * dx * dx * dx)];
DxDxDxerr = Series[DxDxDx - (-I * k * k * k), {dx, 0, 4}];
```

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Text[Row[{"Dx || ", Dx}]]
Text[Row[{"Dx || ", TeXForm[Dx]}]]
Text[Row[{"Dx error || ", TeXForm[Dxerr]}]]
Text[Row[{"Dx error || ", Dxerr}]]
Text[" "]
Text[Row[{"DxDx || ", DxDx}]]
Text[Row[{"DxDx || ", TeXForm[DxDx]}]]
Text[Row[{"DxDx error || ", TeXForm[DxDxerr]}]]
Text[Row[{"DxDx error || ", DxDxerr}]]
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Text[Row[{"DxDxDx || ", DxDxDx}]]
Text[Row[{"DxDxDx || ", TeXForm[DxDxDx]}]]
Text[Row[{"DxDxDx error || ", TeXForm[DxDxDxerr]}]]
Text[Row[{"DxDxDx error || ", DxDxDxerr}]]
Text[" "]
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Out[19]= Dx ||  $\frac{i \sin[dx \, k]}{dx}$ 
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$$\text{Out[20]= } D x \parallel \frac{i \sin(\text{dx } k)}{\text{dx}}$$

$$\text{Out[21]= } D x \text{ error } \parallel -\frac{1}{6} i \text{dx}^2 k^3 + \frac{1}{120} i \text{dx}^4 k^5 + O(\text{dx}^5)$$

$$\text{Out[22]= } D x \text{ error } \parallel -\frac{1}{6} i k^3 \text{dx}^2 + \frac{1}{120} i k^5 \text{dx}^4 + O[\text{dx}]^5$$

$$\text{Out[23]=}$$

$$\text{Out[24]= } D x D x \parallel \frac{2(-1 + \cos(\text{dx } k))}{\text{dx}^2}$$

$$\text{Out[25]= } D x D x \parallel \frac{2(\cos(\text{dx } k) - 1)}{\text{dx}^2}$$

$$\text{Out[26]= } D x D x \text{ error } \parallel \frac{\text{dx}^2 k^4}{12} - \frac{\text{dx}^4 k^6}{360} + O(\text{dx}^5)$$

$$\text{Out[27]= } D x D x \text{ error } \parallel \frac{k^4 \text{dx}^2}{12} - \frac{k^6 \text{dx}^4}{360} + O[\text{dx}]^5$$

$$\text{Out[28]=}$$

$$\text{Out[29]= } D x D x D x \parallel -\frac{4 i \sin\left[\frac{\text{dx } k}{2}\right]^2 \sin[\text{dx } k]}{\text{dx}^3}$$

$$\text{Out[30]= } D x D x D x \parallel -\frac{4 i \sin^2\left(\frac{\text{dx } k}{2}\right) \sin(\text{dx } k)}{\text{dx}^3}$$

$$\text{Out[31]= } D x D x D x \text{ error } \parallel \frac{1}{4} i \text{dx}^2 k^5 - \frac{1}{40} i \text{dx}^4 k^7 + O(\text{dx}^5)$$

$$\text{Out[32]= } D x D x D x \text{ error } \parallel \frac{1}{4} i k^5 \text{dx}^2 - \frac{1}{40} i k^7 \text{dx}^4 + O[\text{dx}]^5$$

$$\text{Out[33]=}$$

```
In[34]:= DerivCheck = FullSimplify[DxDx * Dx];
DerivCheck /. Cos[dx k] - 1 -> -2 * Sin[dx k / 2]^2;
DxDxDx;
```

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In[37]:= etaspatderivs = -(H*Dx*v + U*Dx*n);
etaspatderivsu = etaspatderivs /. v -> 1 /. n -> 0;
etaspatderivsn = etaspatderivs /. n -> 1 /. v -> 0;
upsspatderivs = -(g*H*Dx*n + U*H*Dx*v - H^3/3*U*DxDxDx*v);
upsspatderivsLHS =
  H*v - H^3/3*DxDx /. v -> 1 /. Cos[dx k] - 1 -> -2*Sin[dx k/2]^2;
upsspatderivsu = upsspatderivs /. v -> 1 /. n -> 0;
upsspatderivsu = Simplify[upsspatderivsu / upsspatderivsLHS];
upsspatderivsn = upsspatderivs /. n -> 1 /. v -> 0;
upsspatderivsn = Simplify[upsspatderivsn / upsspatderivsLHS];
Emat =
  2*dt*{{etaspatderivsn, etaspatderivsu}, {upsspatderivsn, upsspatderivsu}};
EmatF = Emat + Exp[-I*(wAp)*dt] {{1, 0}, {0, 1}};
Ematerr =
  Series[EmatF - Exp[I*wAp*dt]*IdentityMatrix[2], {dx, 0, 2}, {dt, 0, 2}];
EmatEig = Eigenvalues[Emat];
EmatEig =
  Series[wAp - Log[Exp[-I*(wAp)*dt] + EmatEig] / (I*dt), {dx, 0, 4}, {dt, 0, 4}];

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In[51]:= Text[Row[{"E00 || ", etaspatderivsn}]]
Text[Row[{"E00 || ", TeXForm[etaspatderivsn]}]]
Text[" "]
Text[Row[{"E01 || ", etaspatderivsu}]]
Text[Row[{"E01 || ", TeXForm[etaspatderivsu]}]]
Text[" "]
Text[Row[{"E10 || ", upsspatderivsn}]]
Text[Row[{"E10 || ", TeXForm[upsspatderivsn]}]]
Text[" "]
Text[Row[{"E11 || ", upsspatderivsu}]]
Text[Row[{"E11 || ", TeXForm[upsspatderivsu]}]]
Text[" "]
Text[Row[{"EmatEig || ", EmatEig}]]
Text[Row[{"EmatEig || ", TeXForm[EmatEig]}]]
Text[" "]
Text[Row[{"Ematerr || ", Ematerr}]]
Text[Row[{"Ematerr || ", TeXForm[Ematerr]}]]

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Out[51]=  $E00 \parallel -\frac{i U \sin[dx k]}{dx}$

Out[52]=  $E00 \parallel -\frac{i U \sin(\text{dx } k)}{\text{dx}}$

Out[53]=

$$\text{Out[54]}= \text{E01} \parallel -\frac{i H \sin[\text{dx} k]}{\text{dx}}$$

$$\text{Out[55]}= \text{E01} \parallel -\frac{i H \sin(\text{dx} k)}{\text{dx}}$$

$$\text{Out[56]}=$$

$$\text{Out[57]}= \text{E10} \parallel -\frac{3 i \text{dx} g \sin[\text{dx} k]}{3 \text{dx}^2 + 4 H^2 \sin\left[\frac{\text{dx} k}{2}\right]^2}$$

$$\text{Out[58]}= \text{E10} \parallel -\frac{3 i \text{dx} g \sin(\text{dx} k)}{3 \text{dx}^2 + 4 H^2 \sin^2\left(\frac{\text{dx} k}{2}\right)}$$

$$\text{Out[59]}=$$

$$\text{Out[60]}= \text{E11} \parallel -\frac{i U \sin[\text{dx} k]}{\text{dx}}$$

$$\text{Out[61]}= \text{E11} \parallel -\frac{i U \sin(\text{dx} k)}{\text{dx}}$$

$$\text{Out[62]}=$$

$$\text{Out[63]}= \text{EmatEig} \parallel$$

$$\begin{aligned} & \left\{ \left( \frac{1}{3(3+H^2 k^2)^2} \left( 3 \sqrt{3} g H k^3 \sqrt{g H (3+H^2 k^2)} + 27 g H k^3 U + 9 g H^3 k^5 U + 9 \sqrt{3} k^3 \sqrt{g H (3+H^2 k^2)} U^2 + \right. \right. \right. \\ & \quad \left. \left. 3 \sqrt{3} H^2 k^5 \sqrt{g H (3+H^2 k^2)} U^2 + 9 k^3 U^3 + 6 H^2 k^5 U^3 + H^4 k^7 U^3 \right) dt^2 + \frac{1}{3(3+H^2 k^2)^2} \right. \\ & \quad \left. i \left( 9 g^2 H^2 k^4 + 12 \sqrt{3} g H k^4 \sqrt{g H (3+H^2 k^2)} U + 54 g H k^4 U^2 + 18 g H^3 k^6 U^2 + 12 \sqrt{3} k^4 \right. \right. \\ & \quad \left. \left. \sqrt{g H (3+H^2 k^2)} U^3 + 4 \sqrt{3} H^2 k^6 \sqrt{g H (3+H^2 k^2)} U^3 + 9 k^4 U^4 + 6 H^2 k^6 U^4 + H^4 k^8 U^4 \right) dt^3 - \right. \\ & \quad \left. \frac{1}{60(3+H^2 k^2)^3} 11 \left( 9 \sqrt{3} g^2 H^2 k^5 \sqrt{g H (3+H^2 k^2)} + 135 g^2 H^2 k^5 U + 45 g^2 H^4 k^7 U + 90 \sqrt{3} g H \right. \right. \\ & \quad \left. \left. k^5 \sqrt{g H (3+H^2 k^2)} U^2 + 30 \sqrt{3} g H^3 k^7 \sqrt{g H (3+H^2 k^2)} U^2 + 270 g H k^5 U^3 + 180 g H^3 k^7 U^3 + \right. \right. \\ & \quad \left. \left. 30 g H^5 k^9 U^3 + 45 \sqrt{3} k^5 \sqrt{g H (3+H^2 k^2)} U^4 + 30 \sqrt{3} H^2 k^7 \sqrt{g H (3+H^2 k^2)} U^4 + \right. \right. \\ & \quad \left. \left. 5 \sqrt{3} H^4 k^9 \sqrt{g H (3+H^2 k^2)} U^4 + 27 k^5 U^5 + 27 H^2 k^7 U^5 + 9 H^4 k^9 U^5 + H^6 k^{11} U^5 \right) dt^4 + O[dt]^5 \right) + \\ & \quad \left( -\frac{1}{12(3+H^2 k^2)^2} \left( 12 \sqrt{3} k^3 \sqrt{g H (3+H^2 k^2)} + 3 \sqrt{3} H^2 k^5 \sqrt{g H (3+H^2 k^2)} + 36 k^3 U + \right. \right. \\ & \quad \left. \left. 24 H^2 k^5 U + 4 H^4 k^7 U \right) - \frac{1}{12(3+H^2 k^2)^3} i k^4 \left( \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 3 U + H^2 k^2 U \right) \right. \\ & \quad \left. \left( 12 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 3 \sqrt{3} H^2 k^2 \sqrt{g H (3+H^2 k^2)} + 36 U + 24 H^2 k^2 U + 4 H^4 k^4 U \right) dt + \right. \\ & \quad \left. \frac{1}{24(3+H^2 k^2)^3} k^5 \left( 12 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 3 \sqrt{3} H^2 k^2 \sqrt{g H (3+H^2 k^2)} + 36 U + 24 H^2 k^2 U + 4 H^4 k^4 U \right) \right. \\ & \quad \left. \left( 3 g H + 2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} U + 3 U^2 + H^2 k^2 U^2 \right) dt^2 - \frac{1}{72(3+H^2 k^2)^4} \right. \\ & \quad \left. i k^6 \left( 12 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 3 \sqrt{3} H^2 k^2 \sqrt{g H (3+H^2 k^2)} + 36 U + 24 H^2 k^2 U + 4 H^4 k^4 U \right) \right. \\ & \quad \left. \left( 3 \sqrt{3} g H \sqrt{g H (3+H^2 k^2)} + 27 g H U + 9 g H^3 k^2 U + 9 \sqrt{3} \sqrt{g H (3+H^2 k^2)} U^2 + \right. \right. \\ & \quad \left. \left. 3 \sqrt{3} H^2 k^2 \sqrt{g H (3+H^2 k^2)} U^2 + 9 U^3 + 6 H^2 k^2 U^3 + H^4 k^4 U^3 \right) dt^3 + \frac{1}{96(3+H^2 k^2)^4} \right. \\ & \quad \left. 5 k^7 \left( 12 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 3 \sqrt{3} H^2 k^2 \sqrt{g H (3+H^2 k^2)} + 36 U + 24 H^2 k^2 U + 4 H^4 k^4 U \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left( 9 g^2 H^2 + 12 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} U + 54 g H U^2 + 18 g H^3 k^2 U^2 + 12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. 4 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^3 + 9 U^4 + 6 H^2 k^2 U^4 + H^4 k^4 U^4 \right) dt^4 + O[dt]^5 \Big) dx^2 + \\
& \left( \frac{1}{960 (3 + H^2 k^2)^3} \left( 144 \sqrt{3} k^5 \sqrt{g H (3 + H^2 k^2)} + 48 \sqrt{3} H^2 k^7 \sqrt{g H (3 + H^2 k^2)} + \right. \right. \\
& \quad \left. 5 \sqrt{3} H^4 k^9 \sqrt{g H (3 + H^2 k^2)} + 432 k^5 U + 432 H^2 k^7 U + 144 H^4 k^9 U + 16 H^6 k^{11} U \right) + \\
& \quad \frac{1}{2880 (3 + H^2 k^2)^3} i \left( 5616 g H k^6 + 2592 g H^3 k^8 + 315 g H^5 k^{10} + 3744 \sqrt{3} k^6 \sqrt{g H (3 + H^2 k^2)} U + \right. \\
& \quad 2112 \sqrt{3} H^2 k^8 \sqrt{g H (3 + H^2 k^2)} U + 303 \sqrt{3} H^4 k^{10} \sqrt{g H (3 + H^2 k^2)} U + \\
& \quad \left. 5616 k^6 U^2 + 5616 H^2 k^8 U^2 + 1872 H^4 k^{10} U^2 + 208 H^6 k^{12} U^2 \right) dt - \\
& \quad \frac{1}{5760 (3 + H^2 k^2)^4} \left( k^7 \left( 18576 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} + 9072 \sqrt{3} g H^3 k^2 \sqrt{g H (3 + H^2 k^2)} + \right. \right. \\
& \quad 1125 \sqrt{3} g H^5 k^4 \sqrt{g H (3 + H^2 k^2)} + 167184 g H U + 147312 g H^3 k^2 U + 43110 g H^5 k^4 U + \\
& \quad 4194 g H^7 k^6 U + 55728 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^2 + 52416 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^2 + \\
& \quad 16413 \sqrt{3} H^4 k^4 \sqrt{g H (3 + H^2 k^2)} U^2 + 1711 \sqrt{3} H^6 k^6 \sqrt{g H (3 + H^2 k^2)} U^2 + 55728 U^3 + \\
& \quad \left. 74304 H^2 k^2 U^3 + 37152 H^4 k^4 U^3 + 8256 H^6 k^6 U^3 + 688 H^8 k^8 U^3 \right) \Big) dt^2 - \frac{1}{1920 (3 + H^2 k^2)^4} \\
& \quad i k^8 \left( 16848 g^2 H^2 + 8496 g^2 H^4 k^2 + 1065 g^2 H^6 k^4 + 22464 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} U + \right. \\
& \quad 12240 \sqrt{3} g H^3 k^2 \sqrt{g H (3 + H^2 k^2)} U + 1649 \sqrt{3} g H^5 k^4 \sqrt{g H (3 + H^2 k^2)} U + \\
& \quad 101088 g H U^2 + 92880 g H^3 k^2 U^2 + 28299 g H^5 k^4 U^2 + 2857 g H^7 k^6 U^2 + \\
& \quad 22464 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^3 + 21552 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^3 + \\
& \quad 6875 \sqrt{3} H^4 k^4 \sqrt{g H (3 + H^2 k^2)} U^3 + 729 \sqrt{3} H^6 k^6 \sqrt{g H (3 + H^2 k^2)} U^3 + \\
& \quad \left. 16848 U^4 + 22464 H^2 k^2 U^4 + 11232 H^4 k^4 U^4 + 2496 H^6 k^6 U^4 + 208 H^8 k^8 U^4 \right) dt^3 + \\
& \quad \frac{1}{4} i \left( -\frac{1}{108 (3 + H^2 k^2)^7} i k^9 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 U + H^2 k^2 U \right)^3 \right. \\
& \quad \left( 12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} + 36 U + 24 H^2 k^2 U + 4 H^4 k^4 U \right)^2 + \\
& \quad \frac{1}{384 (3 + H^2 k^2)^5} i k^9 \left( 144 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 48 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} + \right. \\
& \quad 5 \sqrt{3} H^4 k^4 \sqrt{g H (3 + H^2 k^2)} + 432 U + 432 H^2 k^2 U + 144 H^4 k^4 U + 16 H^6 k^6 U \Big) \Big) \left( 9 g^2 H^2 + \right. \\
& \quad 12 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} U + 54 g H U^2 + 18 g H^3 k^2 U^2 + 12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^3 + \\
& \quad \left. 4 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^3 + 9 U^4 + 6 H^2 k^2 U^4 + H^4 k^4 U^4 \right) \Big) dt^4 + O[dt]^5 \Big) dx^4 + \\
& \quad O[dx]^5, \left( -\frac{4 \left( \sqrt{3} k \sqrt{g H (3 + H^2 k^2)} \right)}{3 + H^2 k^2} - \frac{4 i \left( -3 g H k^2 + \sqrt{3} k^2 \sqrt{g H (3 + H^2 k^2)} U \right) dt}{3 + H^2 k^2} + \frac{1}{3 (3 + H^2 k^2)^2} \right. \\
& \quad \left( 69 \sqrt{3} g H k^3 \sqrt{g H (3 + H^2 k^2)} - 297 g H k^3 U - 99 g H^3 k^5 U + 27 \sqrt{3} k^3 \sqrt{g H (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. 9 \sqrt{3} H^2 k^5 \sqrt{g H (3 + H^2 k^2)} U^2 + 9 k^3 U^3 + 6 H^2 k^5 U^3 + H^4 k^7 U^3 \right) dt^2 + \frac{1}{\dots}
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{1}{60(3+H^2k^2)^3} \left( -20061\sqrt{3}g^2H^2k^5\sqrt{gH(3+H^2k^2)} + 144315g^2H^2k^5U + 48105g^2H^4k^7U - \right. \right. \\
& \quad 36090\sqrt{3}gHk^5\sqrt{gH(3+H^2k^2)}U^2 - 12030\sqrt{3}gH^3k^7\sqrt{gH(3+H^2k^2)}U^2 + \\
& \quad 21870gHk^5U^3 + 14580gH^3k^7U^3 + 2430gH^5k^9U^3 + 855\sqrt{3}k^5\sqrt{gH(3+H^2k^2)}U^4 + \\
& \quad 570\sqrt{3}H^2k^7\sqrt{gH(3+H^2k^2)}U^4 + 95\sqrt{3}H^4k^9\sqrt{gH(3+H^2k^2)}U^4 - \\
& \quad \left. \left. 297k^5U^5 - 297H^2k^7U^5 - 99H^4k^9U^5 - 11H^6k^{11}U^5 \right) dt^4 + O[dt]^5 \right) + \\
& \left( -\frac{1}{12(3+H^2k^2)^2} \left( -12\sqrt{3}k^3\sqrt{gH(3+H^2k^2)} - 3\sqrt{3}H^2k^5\sqrt{gH(3+H^2k^2)} + \right. \right. \\
& \quad \left. \left. 36k^3U + 24H^2k^5U + 4H^4k^7U \right) - \frac{1}{12(3+H^2k^2)^3} \right. \\
& \quad \left. i k^4 \left( -3\sqrt{3}\sqrt{gH(3+H^2k^2)} + 3U + H^2k^2U \right) \left( -12\sqrt{3}\sqrt{gH(3+H^2k^2)} - \right. \right. \\
& \quad \left. \left. 3\sqrt{3}H^2k^2\sqrt{gH(3+H^2k^2)} + 36U + 24H^2k^2U + 4H^4k^4U \right) dt + \frac{1}{24(3+H^2k^2)^3} \right. \\
& \quad \left. k^5 \left( -12\sqrt{3}\sqrt{gH(3+H^2k^2)} - 3\sqrt{3}H^2k^2\sqrt{gH(3+H^2k^2)} + 36U + 24H^2k^2U + 4H^4k^4U \right) \right. \\
& \quad \left. \left( 51gH - 14\sqrt{3}\sqrt{gH(3+H^2k^2)}U + 3U^2 + H^2k^2U^2 \right) dt^2 - \frac{1}{72(3+H^2k^2)^4} \right. \\
& \quad \left. i k^6 \left( -12\sqrt{3}\sqrt{gH(3+H^2k^2)} - 3\sqrt{3}H^2k^2\sqrt{gH(3+H^2k^2)} + 36U + 24H^2k^2U + 4H^4k^4U \right) \right. \\
& \quad \left. \left( 435\sqrt{3}gH\sqrt{gH(3+H^2k^2)} - 1701gHU - 567gH^3k^2U + 153\sqrt{3}\sqrt{gH(3+H^2k^2)}U^2 + \right. \right. \\
& \quad \left. \left. 51\sqrt{3}H^2k^2\sqrt{gH(3+H^2k^2)}U^2 + 9U^3 + 6H^2k^2U^3 + H^4k^4U^3 \right) dt^3 + \frac{1}{288(3+H^2k^2)^4} \right. \\
& \quad \left. k^7 \left( -12\sqrt{3}\sqrt{gH(3+H^2k^2)} - 3\sqrt{3}H^2k^2\sqrt{gH(3+H^2k^2)} + 36U + 24H^2k^2U + 4H^4k^4U \right) \right. \\
& \quad \left( -14841g^2H^2 + 8820\sqrt{3}gH\sqrt{gH(3+H^2k^2)}U - 13014gHU^2 - 4338gH^3k^2U^2 + \right. \\
& \quad \left. 372\sqrt{3}\sqrt{gH(3+H^2k^2)}U^3 + 124\sqrt{3}H^2k^2\sqrt{gH(3+H^2k^2)}U^3 + \right. \\
& \quad \left. \left. 135U^4 + 90H^2k^2U^4 + 15H^4k^4U^4 \right) dt^4 + O[dt]^5 \right) dx^2 + \\
& \left( \frac{1}{960(3+H^2k^2)^3} \left( -144\sqrt{3}k^5\sqrt{gH(3+H^2k^2)} - 48\sqrt{3}H^2k^7\sqrt{gH(3+H^2k^2)} - \right. \right. \\
& \quad \left. \left. 5\sqrt{3}H^4k^9\sqrt{gH(3+H^2k^2)} + 432k^5U + 432H^2k^7U + 144H^4k^9U + 16H^6k^{11}U \right) + \right. \\
& \quad \left. \frac{1}{2880(3+H^2k^2)^3} i \left( 8208gHk^6 + 3456gH^3k^8 + 405gH^5k^{10} - 4608\sqrt{3}k^6\sqrt{gH(3+H^2k^2)}U - \right. \right. \\
& \quad \left. \left. 2688\sqrt{3}H^2k^8\sqrt{gH(3+H^2k^2)}U - 399\sqrt{3}H^4k^{10}\sqrt{gH(3+H^2k^2)}U + \right. \right. \\
& \quad \left. \left. 5616k^6U^2 + 5616H^2k^8U^2 + 1872H^4k^{10}U^2 + 208H^6k^{12}U^2 \right) dt - \frac{1}{5760(3+H^2k^2)^4} \right. \\
& \quad \left. \left( 117 \left( -22872\sqrt{3} - 11\sqrt{3}H(2+H^2k^2) - 22264\sqrt{3} - 11312\sqrt{3}H(2+H^2k^2) - 4005\sqrt{3} - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{5} \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - \frac{1}{204} \sqrt{3} \sqrt{g H^2 k^2} \sqrt{g H (3 + H^2 k^2)} - 4005 \sqrt{3} g \right. \\
& \quad H^5 k^4 \sqrt{g H (3 + H^2 k^2)} + 483408 g H U + 430704 g H^3 k^2 U + 128250 g H^5 k^4 U + 12798 \\
& \quad g H^7 k^6 U - 105840 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^2 - 102528 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^2 - \\
& \quad 33117 \sqrt{3} H^4 k^4 \sqrt{g H (3 + H^2 k^2)} U^2 - 3567 \sqrt{3} H^6 k^6 \sqrt{g H (3 + H^2 k^2)} U^2 + 55728 U^3 + \\
& \quad \left. 74304 H^2 k^2 U^3 + 37152 H^4 k^4 U^3 + 8256 H^6 k^6 U^3 + 688 H^8 k^8 U^3 \right) dt^2 - \frac{1}{5760 (3 + H^2 k^2)^4} \\
& \quad i k^8 \left( 861840 g^2 H^2 + 399600 g^2 H^4 k^2 + 48645 g^2 H^6 k^4 - 766368 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} U - \right. \\
& \quad 417456 \sqrt{3} g H^3 k^2 \sqrt{g H (3 + H^2 k^2)} U - 58035 \sqrt{3} g H^5 k^4 \sqrt{g H (3 + H^2 k^2)} U + \\
& \quad 2073600 g H U^2 + 1956528 g H^3 k^2 U^2 + 615807 g H^5 k^4 U^2 + 64677 g H^7 k^6 U^2 - \\
& \quad 228960 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^3 - 226224 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^3 - \\
& \quad 74481 \sqrt{3} H^4 k^4 \sqrt{g H (3 + H^2 k^2)} U^3 - 8171 \sqrt{3} H^6 k^6 \sqrt{g H (3 + H^2 k^2)} U^3 + 50544 U^4 + \\
& \quad \left. 67392 H^2 k^2 U^4 + 33696 H^4 k^4 U^4 + 7488 H^6 k^6 U^4 + 624 H^8 k^8 U^4 \right) dt^3 + \frac{1}{4} i \left( -\frac{1}{108 (3 + H^2 k^2)^6} i k^9 \right. \\
& \quad \left( -12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} + 36 U + 24 H^2 k^2 U + 4 H^4 k^4 U \right)^2 \\
& \quad \left( -447 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} + 1647 g H U + 549 g H^3 k^2 U - 153 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} \right. \\
& \quad \left. U^2 - 51 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^2 + 9 U^3 + 6 H^2 k^2 U^3 + H^4 k^4 U^3 \right) + \\
& \quad \frac{1}{5760 (3 + H^2 k^2)^5} i k^9 \left( -144 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - 48 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} - \right. \\
& \quad \left. 5 \sqrt{3} H^4 k^4 \sqrt{g H (3 + H^2 k^2)} + 432 U + 432 H^2 k^2 U + 144 H^4 k^4 U + 16 H^6 k^6 U \right) \\
& \quad \left( -14841 g^2 H^2 + 8820 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} U - 13014 g H U^2 - 4338 g H^3 k^2 U^2 + \right. \\
& \quad 372 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^3 + 124 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^3 + \\
& \quad \left. 135 U^4 + 90 H^2 k^2 U^4 + 15 H^4 k^4 U^4 \right) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5 \Big\}
\end{aligned}$$

Out[64]= EmatEig ||

$$\begin{aligned}
& \left( \frac{1}{5} \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - \frac{1}{204} \sqrt{3} \sqrt{g H^2 k^2} \sqrt{g H (3 + H^2 k^2)} - 4005 \sqrt{3} g \right. \\
& \quad H^5 k^4 \sqrt{g H (3 + H^2 k^2)} + 483408 g H U + 430704 g H^3 k^2 U + 128250 g H^5 k^4 U + 12798 \\
& \quad g H^7 k^6 U - 105840 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^2 - 102528 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^2 - \\
& \quad 33117 \sqrt{3} H^4 k^4 \sqrt{g H (3 + H^2 k^2)} U^2 - 3567 \sqrt{3} H^6 k^6 \sqrt{g H (3 + H^2 k^2)} U^2 + 55728 U^3 + \\
& \quad \left. 74304 H^2 k^2 U^3 + 37152 H^4 k^4 U^3 + 8256 H^6 k^6 U^3 + 688 H^8 k^8 U^3 \right) dt^2 - \frac{1}{5760 (3 + H^2 k^2)^4} \\
& \quad i k^8 \left( 861840 g^2 H^2 + 399600 g^2 H^4 k^2 + 48645 g^2 H^6 k^4 - 766368 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} U - \right. \\
& \quad 417456 \sqrt{3} g H^3 k^2 \sqrt{g H (3 + H^2 k^2)} U - 58035 \sqrt{3} g H^5 k^4 \sqrt{g H (3 + H^2 k^2)} U + \\
& \quad 2073600 g H U^2 + 1956528 g H^3 k^2 U^2 + 615807 g H^5 k^4 U^2 + 64677 g H^7 k^6 U^2 - \\
& \quad 228960 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^3 - 226224 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^3 - \\
& \quad 74481 \sqrt{3} H^4 k^4 \sqrt{g H (3 + H^2 k^2)} U^3 - 8171 \sqrt{3} H^6 k^6 \sqrt{g H (3 + H^2 k^2)} U^3 + 50544 U^4 + \\
& \quad \left. 67392 H^2 k^2 U^4 + 33696 H^4 k^4 U^4 + 7488 H^6 k^6 U^4 + 624 H^8 k^8 U^4 \right) dt^3 + \frac{1}{4} i \left( -\frac{1}{108 (3 + H^2 k^2)^6} i k^9 \right. \\
& \quad \left( -12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} + 36 U + 24 H^2 k^2 U + 4 H^4 k^4 U \right)^2 \\
& \quad \left( -447 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} + 1647 g H U + 549 g H^3 k^2 U - 153 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} \right. \\
& \quad \left. U^2 - 51 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^2 + 9 U^3 + 6 H^2 k^2 U^3 + H^4 k^4 U^3 \right) + \\
& \quad \frac{1}{5760 (3 + H^2 k^2)^5} i k^9 \left( -144 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - 48 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} - \right. \\
& \quad \left. 5 \sqrt{3} H^4 k^4 \sqrt{g H (3 + H^2 k^2)} + 432 U + 432 H^2 k^2 U + 144 H^4 k^4 U + 16 H^6 k^6 U \right) \\
& \quad \left( -14841 g^2 H^2 + 8820 \sqrt{3} g H \sqrt{g H (3 + H^2 k^2)} U - 13014 g H U^2 - 4338 g H^3 k^2 U^2 + \right. \\
& \quad 372 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} U^3 + 124 \sqrt{3} H^2 k^2 \sqrt{g H (3 + H^2 k^2)} U^3 + \\
& \quad \left. 135 U^4 + 90 H^2 k^2 U^4 + 15 H^4 k^4 U^4 \right) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5 \Big\}
\end{aligned}$$





$$\begin{aligned} & k^2+3\text{right)} U-3 \text{ g H } k^2\text{right)} \text{text{dt}}\}\{H^2 k^2+3\}+\frac{1}{\sqrt{3}}\sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5-99 \text{ g H }^3 U k^5+9 U^3 k^3+27 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^3-297 \text{ g H } U k^3+69 \sqrt{3} \text{ g H } \sqrt{g H \left(H^2 k^2+3\right)} \\ & k^3\text{right)} \text{text{dt}}^2)\}\{3 \left(H^2 k^2+3\right)^2\}+\frac{i}{\sqrt{3}}\sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6-144 \text{ g H }^3 U^2 k^6+9 U^4 k^4+6 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^4-441 \text{ g }^2 H^2 k^4-432 \text{ g H } U^2 k^4+282 \sqrt{3} \text{ g H } \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^4 \\ & k^2+3\text{right)} U k^4\text{right)} \text{text{dt}}^3)\}\{3 \left(H^2 k^2+3\right)^2\}+\frac{\left(-11 H^6 U^5 k^{11}\right)-99 H^4 U^5 k^9+95 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^4 k^9+2430 \text{ g H }^5 U^3 k^9-297 H^2 U^5 k^7+570 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^4 k^7+14580 \text{ g H }^3 U^3 k^7-12030 \\ & \sqrt{3} \text{ g H }^3 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^7+48105 \text{ g }^2 H^4 U k^7-297 U^5 k^5+855 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^4 k^5+21870 \text{ g H } U^3 k^5-36090 \sqrt{3} \text{ g H } \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5+144315 \text{ g }^2 H^2 U k^5-20061 \sqrt{3} \text{ g }^2 H^2 \sqrt{g H \left(H^2 k^2+3\right)} \\ & k^5\text{right)} \text{text{dt}}^4)\}\{60 \left(H^2 k^2+3\right)^3\}+O\left(\text{text{dt}}^5\text{right)}\right)+\left(-\frac{4}{\sqrt{3}} H^4 U k^7+24 H^2 U k^5-3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^5+36 U k^3-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k^3\right)\{12 \left(H^2 k^2+3\right)^2\}-\frac{i}{\sqrt{3}} k^4 \left(H^2 U k^2+3 U-3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(4 H^4 U k^4+24 H^2 U k^2-3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \right) \text{text{dt}}\}\{12 \left(H^2 k^2+3\right)^3\}+\frac{1}{\sqrt{3}}\left\{k^5 \left(4 H^4 U k^4+24 H^2 U k^2-3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(4 H^4 U k^4+24 H^2 U k^2-3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \right) \left(H^2 U^2+3 U^2-14 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U+51 \text{ g H}\right) \text{text{dt}}^2)\}\{24 \left(H^2 k^2+3\right)^3\}-\frac{i}{\sqrt{3}} k^6 \left(4 H^4 U k^4+24 H^2 U k^2-3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \right) \left(H^4 U^3 k^4+6 H^2 U^3 k^2+51 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2-567 \text{ g H }^3 U k^2+9 U^3+153 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2-1701 \text{ g H } U+435 \sqrt{3} \text{ g H } \sqrt{g H \left(H^2 k^2+3\right)}\right) \right) \text{text{dt}}^3)\}\{72 \left(H^2 k^2+3\right)^4\}+\frac{1}{\sqrt{3}}\left\{k^7 \left(4 H^4 U k^4+24 H^2 U k^2-3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \right) \left(15 H^4 k^4 U^4+90 H^2 k^2 U^4+135 U^4+124 \sqrt{3} H^2 k^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3+372 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3-4338 \text{ g H }^3 k^2 U^2-13014 \text{ g H } U^2+8820 \sqrt{3} \text{ g H } \sqrt{g H \left(H^2 k^2+3\right)} U-14841 \text{ g }^2 H^2\right) \text{text{dt}}^4)\}\{288 \left(H^2 k^2+3\right)^4\}+O\left(\text{text{dt}}^5\text{right)}\right) \text{text{dx}}^2+\left(\frac{16}{\sqrt{3}} H^6 U k^{11}\right)+144 H^4 U k^9-5 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} k^9+432 H^2 U k^7-48 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^7+432 U k^5-144 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k^5\}\{960 \left(H^2 k^2+3\right)^3\}+\frac{i}{\sqrt{3}}\left\{208 H^6 U^2 k^{12}\right\}+405 \text{ g H }^5 k^{10}\}+1872 H^4 U^2 k^{10}-399 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U k^{10}\}+3456 \text{ g H }^3 k^8+5616 H^2 U^2 k^8-2688 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^8+5616 U^2 k^6+8208 \text{ g H } k^6-4608 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^6\text{right)} \text{text{dt}}\}\{2880 \left(H^2 k^2+3\right)^3\}-\frac{1}{\sqrt{3}}\left\{k^7 \left(688 H^8 U^3 k^8+8256 H^6 U^3 k^6-3567 \sqrt{3} H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^6+12798 \text{ g H }^7 U k^6+37152 H^4 U^3 k^4-33117 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^4+128250 \text{ g H }^5 U k^4-4005 \sqrt{3} \text{ g H }^5 \sqrt{g H \left(H^2 k^2+3\right)} k^4+74304 H^2 U^3 k^2-102528 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2+430704 \text{ g H }^3 U k^2-33264 \sqrt{3} \text{ g H }^3 \sqrt{g H \left(H^2 k^2+3\right)} k^2+55728 U^3-105840 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2+483408 \text{ g H } U-73872 \sqrt{3} \text{ g H } \sqrt{g H \left(H^2 k^2+3\right)}\right) \right) \text{text{dt}}^2)\}\{5760 \left(H^2 k^2+3\right)^4\}-\frac{i}{\sqrt{3}} k^8 \left(624 H^8 U^4 k^8+7488 H^6 U^4 k^6-8171 \sqrt{3} H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6+12798 \text{ g H }^7 U k^6+37152 H^4 U^3 k^4-33117 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^4+128250 \text{ g H }^5 U k^4-4005 \sqrt{3} \text{ g H }^5 \sqrt{g H \left(H^2 k^2+3\right)} k^4+74304 H^2 U^3 k^2-102528 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2+430704 \text{ g H }^3 U k^2-33264 \sqrt{3} \text{ g H }^3 \sqrt{g H \left(H^2 k^2+3\right)} k^2+55728 U^3-105840 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2+483408 \text{ g H } U-73872 \sqrt{3} \text{ g H } \sqrt{g H \left(H^2 k^2+3\right)}\right) \right) \text{text{dt}}^2)\}\{5760 \left(H^2 k^2+3\right)^4\}-\frac{i}{\sqrt{3}} k^8 \left(624 H^8 U^4 k^8+7488 H^6 U^4 k^6-8171 \sqrt{3} H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6+12798 \text{ g H }^7 U k^6+37152 H^4 U^3 k^4-33117 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^4+128250 \text{ g H }^5 U k^4-4005 \sqrt{3} \text{ g H }^5 \sqrt{g H \left(H^2 k^2+3\right)} k^4+74304 H^2 U^3 k^2-102528 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2+430704 \text{ g H }^3 U k^2-33264 \sqrt{3} \text{ g H }^3 \sqrt{g H \left(H^2 k^2+3\right)} k^2+55728 U^3-105840 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2+483408 \text{ g H } U-73872 \sqrt{3} \text{ g H } \sqrt{g H \left(H^2 k^2+3\right)}\right) \right) \text{text{dt}}^2)\}\{5760 \left(H^2 k^2+3\right)^4\}-\frac{i}{\sqrt{3}} k^8 \left(624 H^8 U^4 k^8+7488 H^6 U^4 k^6-8171 \sqrt{3} H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6+12798 \text{ g H }^7 U k^6+37152 H^4 U^3 k^4-33117 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^4+128250 \text{ g H }^5 U k^4-4005 \sqrt{3} \text{ g H }^5 \sqrt{g H \left(H^2 k^2+3\right)} k^4+74304 H^2 U^3 k^2-10$$

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\sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6+64677 g H^7 U^2 k^6+48645 g^2 H^6 k^4+33696 H^4 U^4
k^4-74481 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^4+615807 g H^5 U^2 k^4-58035 \sqrt{3}
g H^5 \sqrt{g H \left(H^2 k^2+3\right)} U k^4+399600 g^2 H^4 k^2+67392 H^2 U^4 k^2-226224
\sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^2+1956528 g H^3 U^2 k^2-417456 \sqrt{3} g H^3
\sqrt{g H \left(H^2 k^2+3\right)} U k^2+50544 U^4-228960 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}
U^3+861840 g^2 H^2+2073600 g H U^2-766368 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U\right)
\text{dt}^3\}\{5760 \left(H^2 k^2+3\right)^4+\frac{1}{4} i \left(\frac{k^9 \left(16 H^6 U k^6+144 H^4
U k^4-5 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} k^4+432 H^2 U k^2-48 \sqrt{3} H^2 \sqrt{g H
\left(H^2 k^2+3\right)} k^2+432 U-144 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(15 H^4
k^4 U^4+90 H^2 k^2 U^4+135 U^4+124 \sqrt{3} H^2 k^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3+372
\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3-4338 g H^3 k^2 U^2-13014 g H U^2+8820 \sqrt{3} g
H \sqrt{g H \left(H^2 k^2+3\right)} U-14841 g^2 H^2\right)\}\{5760 \left(H^2 k^2+3\right)^5)-\frac{i
k^9 \left(4 H^4 U k^4+24 H^2 U k^2-3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U-12
\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^2 \left(H^4 U^3 k^4+6 H^2 U^3 k^2-51 \sqrt{3} H^2
\sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2+549 g H^3 U k^2+9 U^3-153 \sqrt{3} \sqrt{g H \left(H^2
k^2+3\right)} U^2+1647 g H U-447 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)}\right)\}\{108 \left(H^2
k^2+3\right)^6\right) \text{dt}^4+O\left(\text{dt}^5\right) \text{right)} \text{dx}^4+O\left(\text{dx}^5\right) \text{right)}

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Out[65]=

$$\begin{aligned}
\text{Out[66]} = \text{Ematerr} \parallel & \left\{ \left\{ \frac{2i\sqrt{3}k\sqrt{gH(3+H^2k^2)}dt}{3+H^2k^2} + O[dt]^3 \right\} + \left( \frac{1}{3}ik^3Udt + O[dt]^3 \right) dx^2 + O[dx]^3, \right. \\
& \left. \left( -2iHkdt + O[dt]^3 \right) + \left( \frac{1}{3}iHk^3dt + O[dt]^3 \right) dx^2 + O[dx]^3 \right\}, \\
& \left\{ \left( -\frac{6igkdt}{3+H^2k^2} + O[dt]^3 \right) + \left( \frac{ig(6k^3+H^2k^5)dt}{2(3+H^2k^2)^2} + O[dt]^3 \right) dx^2 + O[dx]^3, \right. \\
& \left. \left( \frac{2i\sqrt{3}k\sqrt{gH(3+H^2k^2)}dt}{3+H^2k^2} + O[dt]^3 \right) + \left( \frac{1}{3}ik^3Udt + O[dt]^3 \right) dx^2 + O[dx]^3 \right\}
\end{aligned}$$

Out[67]=

```

Ematerr \parallel \left(
\begin{array}{cc}
\left(\frac{2i\sqrt{3}k\sqrt{gH\left(H^2k^2+3\right)}}{3+H^2k^2}+O\left(\text{dt}^3\right)\right)+\left(\frac{1}{3}ik^3U\text{dt}+O\left(\text{dt}^3\right)\right)
\text{dx}^2+O\left(\text{dx}^3\right) \& \left(-2iHk\text{dt}+O\left(\text{dt}^3\right)\right)+\left(\frac{1}{3}iHk^3\text{dt}+O\left(\text{dt}^3\right)\right)
\text{dx}^2+O\left(\text{dx}^3\right) \\
\left(-\frac{6igk\text{dt}}{3+H^2k^2}+O\left(\text{dt}^3\right)\right)+\left(\frac{ig\left(H^2k^2+3\right)dt}{2\left(3+H^2k^2\right)^2}+O\left(\text{dt}^3\right)\right)
\text{dx}^2+O\left(\text{dx}^3\right) \& \left(\frac{2i\sqrt{3}k\sqrt{gH\left(H^2k^2+3\right)}}{3+H^2k^2}+O\left(\text{dt}^3\right)\right)+\left(\frac{1}{3}ik^3U\text{dt}+O\left(\text{dt}^3\right)\right)
\text{dx}^2+O\left(\text{dx}^3\right)
\end{array}
\right)

```