In[1]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];

qjp1n = q0 * Exp[I * (k * (xj + dx) + w * tn)];
qjp1F = Simplify[qjp1n / (qjn)];
qjp2n = q0 * Exp[I * (k * (xj + 2 * dx) + w * tn)];
qjp2F = Simplify[qjp2n / (qjn)];
qjm1n = q0 * Exp[I * (k * (xj - dx) + w * tn)];
qjm1F = Simplify[qjm1n / (qjn)];
qjm2n = q0 * Exp[I * (k * (xj - 2 * dx) + w * tn)];
qjm2F = Simplify[qjm2n / (qjn)];
wAp = -U * k - Sqrt[3] k Sqrt[g H (3 + H^2 k^2)] / (3 + H^2 k^2);
wAm = - U * k + Sqrt[3] k Sqrt[g H (3 + H^2 k^2)] / (3 + H^2 k^2);
```

In[13]:=
```
Dx = FullSimplify[(qjp1F - qjm1F) / (2 * dx)];
Dxerr = Series[Dx - (I * k), {dx, 0, 4}];
DxDx = FullSimplify[(qjp1F - 2 + qjm1F) / dx^2];
DxDxerr = Series[DxDx - (-k * k), {dx, 0, 4}];
DxDxDx = FullSimplify[(qjp2F - 2 qjp1F + 2 * qjm1F - qjm2F) / (2 * dx * dx * dx)];
DxDxDxerr = Series[DxDxDx - (-I * k * k * k), {dx, 0, 4}];

Text[Row[{"Dx  ||   ", Dx}]]
Text[Row[{"Dx  ||   ", TeXForm[Dx]}]]
Text[Row[{"Dx error  ||   ", TeXForm[Dxerr]}]]
Text[Row[{"Dx error  ||   ", Dxerr}]]
Text[" "]
Text[Row[{"DxDx  ||   ", DxDx}]]
Text[Row[{"DxDx  ||   ", TeXForm[DxDx]}]]
Text[Row[{"DxDx error  ||   ", TeXForm[DxDxerr]}]]
Text[Row[{"DxDx error  ||   ", DxDxerr}]]
Text[" "]
Text[Row[{"DxDxDx  ||   ", DxDxDx}]]
Text[Row[{"DxDxDx  ||   ", TeXForm[DxDxDx]}]]
Text[Row[{"DxDxDx error  ||   ", TeXForm[DxDxDxerr]}]]
Text[Row[{"DxDxDx error  ||   ", DxDxDxerr}]]
Text[" "]
```

Out[19]= Dx || $\frac{i \, \text{Sin}[\text{dx k}]}{\text{dx}}$

Out[20]= Dx || \frac{i \sin (\text{dx} k)}{\text{dx}}

Out[21]= Dx error || $-\frac{1}{6} i \text{dx}^2 k^3+\frac{1}{120} i \text{dx}^4 k^5+O\left(\text{dx}^5\right)$

Out[22]= Dx error || $-\frac{1}{6} i\, k^3\, dx^2 + \frac{1}{120} i\, k^5\, dx^4 + O[dx]^5$

Out[23]=

Out[24]= DxDx || $\frac{2\,(-1+\text{Cos}[dx\ k])}{dx^2}$

Out[25]= DxDx || $\frac{2 (\cos (\text{dx} k)-1)}{\text{dx}^2}$

Out[26]= DxDx error || $\frac{\text{dx}^2 k^4}{12}-\frac{\text{dx}^4 k^6}{360}+O\left(\text{dx}^5\right)$

Out[27]= DxDx error || $\frac{k^4\, dx^2}{12} - \frac{k^6\, dx^4}{360} + O[dx]^5$

Out[28]=

Out[29]= DxDxDx || $-\frac{4\, i\, \text{Sin}\left[\frac{dx\ k}{2}\right]^2 \text{Sin}[dx\ k]}{dx^3}$

Out[30]= DxDxDx || $-\frac{4 i \sin ^2\left(\frac{\text{dx} k}{2}\right) \sin (\text{dx} k)}{\text{dx}^3}$

Out[31]= DxDxDx error || $\frac{1}{4} i \text{dx}^2 k^5-\frac{1}{40} i \text{dx}^4 k^7+O\left(\text{dx}^5\right)$

Out[32]= DxDxDx error || $\frac{1}{4} i\, k^5\, dx^2 - \frac{1}{40} i\, k^7\, dx^4 + O[dx]^5$

Out[33]=

In[34]:=
```
upsspatderivs = -(g * H * Dx * n + U * H * Dx * v - H^3/3 * U * DxDxDx * v);
upsspatderivsLHS =
  H * v - H^3/3 * DxDx /. v → 1 /. Cos[dx k] - 1 → -2 * Sin[dx k/2]^2;
upsspatderivsu = upsspatderivs /. v → 1 /. n → 0;
upsspatderivsu = Simplify[upsspatderivsu / upsspatderivsLHS];
upsspatderivsn = upsspatderivs /. n → 1 /. v → 0;
upsspatderivsn = Simplify[upsspatderivsn / upsspatderivsLHS];
vph = Simplify[((1 + qjp1F) vnp1 + (1 + qjp1F) v)/4];
vmh = Simplify[((1 + qjm1F) vnp1 + (1 + qjm1F) v)/4];
hph = n * (qjp1F + 1)/2 - dt/(2 * dx) * (H * (qjp1F - 1) * v + U * n * (qjp1F - 1));
hmh = n * (1 + qjm1F)/2 - dt/(2 * dx) * (H * (1 - qjm1F) * v + U * n * (1 - qjm1F));
LWFlux = n - dt / dx * (H * (vph - vmh) + U * (hph - hmh)) /.
    vnp1 → vnm1 + 2 * dt * (upsspatderivsu * v + upsspatderivsn * n);
LWFluxun = FullSimplify[LWFlux /. v → 1 /. vnm1 → 0 /. n → 0];
LWFluxunm1 = FullSimplify[LWFlux /. v → 0 /. vnm1 → 1 /. n → 0];
FullSimplify[e^(-i dx k) (-1 + e^(2 i dx k))];
FullSimplify[e^(-i dx k) (-1 + e^(i dx k))^2];
LWFluxn =
  Simplify[LWFlux /. v → 0 /. vnm1 → 0 /. n → 1] /. e^(-i dx k) (-1 + e^(2 i dx k)) →
    2 * I * Sin[k * dx] /. e^(-i dx k) (-1 + e^(i dx k))^2 → 2 (-2 * Sin[dx k/2]^2);
Emat = {{LWFluxn, LWFluxun}, {2 * dt * upsspatderivsn, 2 * dt * upsspatderivsu}};
EmatF = Emat + Exp[-I * (wAp) * dt] {{0, LWFluxunm1}, {0, 1}};
Ematerr =
  Series[EmatF - Exp[I * wAp * dt] * IdentityMatrix[2], {dx, 0, 2}, {dt, 0, 2}];
EmatEig = Eigenvalues[Emat + Exp[-I * (wAp) * dt] {{0, LWFluxunm1}, {0, 1}}];
EmatEig = Series[wAp - Log[EmatEig] / (I * dt), {dx, 0, 2}, {dt, 0, 2}];
```

In[55]:=
```
Text[Row[{"E00  ||   ", LWFluxn}]]
Text[Row[{"E00  ||   ", TeXForm[LWFluxn]}]]
Text[" "]
Text[Row[{"E01  ||   ", LWFluxun}]]
Text[Row[{"E01  ||   ", TeXForm[LWFluxun]}]]
Text[" "]
Text[Row[{"E03  ||   ", LWFluxunm1}]]
Text[Row[{"E03  ||   ", TeXForm[LWFluxunm1]}]]
Text[" "]
Text[Row[{"E10  ||   ", upsspatderivsn}]]
Text[Row[{"E10  ||   ", TeXForm[upsspatderivsn]}]]
Text[" "]
Text[Row[{"E11  ||   ", upsspatderivsu}]]
Text[Row[{"E11  ||   ", TeXForm[upsspatderivsu]}]]
Text[" "]
Text[Row[{"EmatEig  ||   ", EmatEig }]]
Text[Row[{"EmatEig  ||   ", TeXForm[EmatEig ]}]]
Text[" "]
Text[Row[{"Ematerr  ||   ", Ematerr }]]
Text[Row[{"Ematerr ||   ", TeXForm[Ematerr]}]]
```

Out[55]= E00 || $\frac{1}{2}\left(2+\text{dt}^2\left(-\frac{4\,\text{U}^2\,\text{Sin}\left[\frac{\text{dx k}}{2}\right]^2}{\text{dx}^2}+\frac{3\left(-1+e^{2\,i\,\text{dx k}}\right)\text{g H}}{6\,\text{dx}^2+8\,\text{H}^2\,\text{Sin}\left[\frac{\text{dx k}}{2}\right]^2}\right)-\frac{2\,i\,\text{dt U Sin[dx k]}}{\text{dx}}-\frac{3\,i\,\text{dt}^2\,e^{-i\,\text{dx k}}\,\text{g H Sin[dx k]}}{3\,\text{dx}^2+4\,\text{H}^2\,\text{Sin}\left[\frac{\text{dx k}}{2}\right]^2}\right)$

Out[56]= E00 || \frac{1}{2} \left(\text{dt}^2 \left(-\frac{4 U^2 \sin ^2\left(\frac{\text{dx} k}{2}\right)}{\text{dx}^2}+\frac{3 g H \left(-1+e^{2 i \text{dx} k}\right)}{6 \text{dx}^2+8 H^2 \sin ^2\left(\frac{\text{dx} k}{2}\right)}\right)-\frac{3 i \text{dt}^2 g H e^{-i \text{dx} k} \sin (\text{dx} k)}{3 \text{dx}^2+4 H^2 \sin ^2\left(\frac{\text{dx} k}{2}\right)}-\frac{2 i \text{dt} U \sin (\text{dx} k)}{\text{dx}}+2\right)

Out[57]=

Out[58]= E01 || $\frac{\text{dt H (dt U (}-3+2\,\text{Cos[dx k]}+\text{Cos[2 dx k]})-i\,\text{dx Sin[dx k]})}{2\,\text{dx}^2}$

Out[59]= E01 ||
\frac{\text{dt} H (\text{dt} U (2 \cos (\text{dx} k)+\cos (2 \text{dx} k)-3)-i \text{dx} \sin (\text{dx} k))}{2 \text{dx}^2}

Out[60]=

Out[61]= E03 || $-\frac{i\,\text{dt H Sin[dx k]}}{2\,\text{dx}}$

Out[62]= E03 || -\frac{i \text{dt} H \sin (\text{dx} k)}{2 \text{dx}}

Out[63]=

Out[64]= E10 || $-\frac{3\,i\,\text{dx g Sin[dx k]}}{3\,\text{dx}^2+4\,\text{H}^2\,\text{Sin}\left[\frac{\text{dx k}}{2}\right]^2}$

Out[65]= E10 || −\frac{3 i \text{dx} g \sin (\text{dx} k)}{3 \text{dx}^2+4 H^2 \sin ^2\left(\frac{\text{dx} k}{2}\right)}

Out[66]=

Out[67]= E11 || $-\frac{i\, U\, \text{Sin}[dx\, k]}{dx}$

Out[68]= E11 || −\frac{i U \sin (\text{dx} k)}{\text{dx}}

Out[69]=

Out[70]= EmatEig ||

$$\left\{\left(-\frac{3\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}+i\sqrt{3}\ \sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\right)}{2\left(3+H^2\,k^2\right)}+\right.$$

$$\frac{1}{2\left(3+H^2\,k^2\right)^2}i\left(9\,g\,H\,k^2+3\,g\,H^3\,k^4+3\,i\,k\sqrt{g\,H\left(3+H^2\,k^2\right)}\ \sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}-\right.$$

$$6\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U-2\sqrt{3}\ H^2\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U-$$

$$\left.6\,i\sqrt{3}\ k\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U-2\,i\sqrt{3}\ H^2\,k^3\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U\right)dt+$$

$$\frac{1}{36\,g\,H\left(3+H^2\,k^2\right)^2}\left(54\sqrt{3}\ g^2\,H^2\,k^3\sqrt{g\,H\left(3+H^2\,k^2\right)}+48\,i\sqrt{3}\ g^2\,H^2\,k^2\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}-\right.$$

$$324\,g^2\,H^2\,k^3\,U-108\,g^2\,H^4\,k^5\,U-144\,i\,g\,H\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ \sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U+$$

$$108\sqrt{3}\ g\,H\,k^3\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^2+36\sqrt{3}\ g\,H^3\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^2+$$

$$54\,i\sqrt{3}\ g\,H\,k^2\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U^2+18\,i\sqrt{3}\ g\,H^3\,k^4\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U^2+$$

$$81\,g\,H\,k^3\,U^3+54\,g\,H^3\,k^5\,U^3+9\,g\,H^5\,k^7\,U^3+3\,i\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ \sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U^3+$$

$$\left.\left.i\,H^2\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\ \sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U^3\right)dt^2+O[dt]^3\right)+$$

$$\left(\left(i\left(24\sqrt{3}\ g\,H\,k^4+6\sqrt{3}\ g\,H^3\,k^6-3\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U-H^2\,k^6\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U-\right.\right.\right.$$

$$\left.\left.\left.27\,i\,k^3\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U-9\,i\,H^2\,k^5\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U\right)\right)\Big/$$

$$\left(9\left(-12-4\,H^2\,k^2\right)\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\right)+\frac{1}{432\left(3+H^2\,k^2\right)^2\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}}$$

$$\left(-360\,g\,H\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}-90\,g\,H^3\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}-648\,i\,g\,H\,k^4\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}-\right.$$

$$162\,i\,g\,H^3\,k^6\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}+1548\sqrt{3}\ g\,H\,k^5\,U+1008\sqrt{3}\ g\,H^3\,k^7\,U+$$

$$164\sqrt{3}\ g\,H^5\,k^9\,U+216\,i\sqrt{3}\ k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\ \sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U+$$

$$72\,i\sqrt{3}\ H^2\,k^6\sqrt{g\,H\left(3+H^2\,k^2\right)}\ \sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U-279\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^2-$$

$$186\,H^2\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^2-31\,H^4\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^2-891\,i\,k^4\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}$$

$$\left.U^2-594\,i\,H^2\,k^6\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U^2-99\,i\,H^4\,k^8\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\ U^2\right)dt+$$

$$\frac{1}{11\,664\left(3+H^2\,k^2\right)^3\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}}\Big(57\,024\,i\,\sqrt{3}\;g^2\,H^2\,k^6+33\,264\,i\,\sqrt{3}\;g^2\,H^4\,k^8+$$
$$4752\,i\,\sqrt{3}\;g^2\,H^6\,k^{10}-17\,496\,\sqrt{3}\;g\,H\,k^5\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}-$$
$$4374\,\sqrt{3}\;g\,H^3\,k^7\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}-168\,480\,i\,g\,H\,k^6\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U-$$
$$106\,650\,i\,g\,H^3\,k^8\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U-16\,830\,i\,g\,H^5\,k^{10}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+$$
$$170\,586\,g\,H\,k^5\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U+109\,350\,g\,H^3\,k^7\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U+$$
$$17\,496\,g\,H^5\,k^9\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U+159\,894\,i\,\sqrt{3}\;g\,H\,k^6\,U^2+161\,676\,i\,\sqrt{3}\;g\,H^3\,k^8\,U^2+$$
$$54\,486\,i\,\sqrt{3}\;g\,H^5\,k^{10}\,U^2+6120\,i\,\sqrt{3}\;g\,H^7\,k^{12}\,U^2-52\,488\,\sqrt{3}\;k^5\,\sqrt{g\,H\left(3+H^2\,k^2\right)}$$
$$\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^2-34\,992\,\sqrt{3}\;H^2\,k^7\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^2-$$
$$5832\,\sqrt{3}\;H^4\,k^9\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^2-29\,241\,i\,k^6\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3-$$
$$29\,133\,i\,H^2\,k^8\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3-9675\,i\,H^4\,k^{10}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3-$$
$$1071\,i\,H^6\,k^{12}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+32\,805\,k^5\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3+$$
$$32\,805\,H^2\,k^7\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3+10\,935\,H^4\,k^9\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3+$$
$$1215\,H^6\,k^{11}\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3-1296\,i\,\sqrt{3}\;k^6\,U^4-1728\,i\,\sqrt{3}\;H^2\,k^8\,U^4-$$
$$864\,i\,\sqrt{3}\;H^4\,k^{10}\,U^4-192\,i\,\sqrt{3}\;H^6\,k^{12}\,U^4-16\,i\,\sqrt{3}\;H^8\,k^{14}\,U^4\Big)\,dt^2+O[dt]^3\Bigg)\,dx^2+$$

$$O[dx]^3,\Bigg(-\frac{3\left(\sqrt{3}\;k\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-i\,\sqrt{3}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\right)}{2\left(3+H^2\,k^2\right)}+\frac{1}{2\left(3+H^2\,k^2\right)^2}$$

$$i\Big(9\,g\,H\,k^2+3\,g\,H^3\,k^4-3\,i\,k\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}-$$
$$6\,\sqrt{3}\;k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U-2\,\sqrt{3}\;H^2\,k^4\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+$$
$$6\,i\,\sqrt{3}\;k\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U+2\,i\,\sqrt{3}\;H^2\,k^3\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U\Big)\,dt+$$
$$\frac{1}{36\,g\,H\left(3+H^2\,k^2\right)^2}\Big(54\,\sqrt{3}\;g^2\,H^2\,k^3\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-48\,i\,\sqrt{3}\;g^2\,H^2\,k^2\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}-$$
$$324\,g^2\,H^2\,k^3\,U-108\,g^2\,H^4\,k^5\,U+144\,i\,g\,H\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U+$$
$$108\,\sqrt{3}\;g\,H\,k^3\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+36\,\sqrt{3}\;g\,H^3\,k^5\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2-$$
$$54\,i\,\sqrt{3}\;g\,H\,k^2\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^2-18\,i\,\sqrt{3}\;g\,H^3\,k^4\,\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^2+$$
$$81\,g\,H\,k^3\,U^3+54\,g\,H^3\,k^5\,U^3+9\,g\,H^5\,k^7\,U^3-3\,i\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3-$$
$$i\,H^2\,k^4\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;\sqrt{-g\,H\,k^2\left(3+H^2\,k^2\right)}\;U^3\Big)\,dt^2+O[dt]^3\Bigg)+$$

$$\Bigg(-\Big(\Big(i\Big(24\,\sqrt{3}\;g\,H\,k^4+6\,\sqrt{3}\;g\,H^3\,k^6-3\,k^4\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U-H^2\,k^6\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+$$

$$27\,i\,k^3\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U+9\,i\,H^2k^5\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U\Big)\Big)\Big/$$

$$\left(9\left(-12-4H^2k^2\right)\sqrt{-g H k^2\left(3+H^2k^2\right)}\right)\Big)+\frac{1}{432\left(3+H^2k^2\right)^2\sqrt{-g H k^2\left(3+H^2k^2\right)}}$$

$$\Big(360\,g H k^5\sqrt{g H\left(3+H^2k^2\right)}+90\,g H^3k^7\sqrt{g H\left(3+H^2k^2\right)}-648\,i\,g H k^4\sqrt{-g H k^2\left(3+H^2k^2\right)}-$$
$$162\,i\,g H^3k^6\sqrt{-g H k^2\left(3+H^2k^2\right)}-1548\sqrt{3}\;g H k^5U-1008\sqrt{3}\;g H^3k^7U-$$
$$164\sqrt{3}\;g H^5k^9U+216\,i\sqrt{3}\;k^4\sqrt{g H\left(3+H^2k^2\right)}\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U+$$
$$72\,i\sqrt{3}\;H^2k^6\sqrt{g H\left(3+H^2k^2\right)}\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U+279\,k^5\sqrt{g H\left(3+H^2k^2\right)}\;U^2+$$
$$186\,H^2k^7\sqrt{g H\left(3+H^2k^2\right)}\;U^2+31\,H^4k^9\sqrt{g H\left(3+H^2k^2\right)}\;U^2-891\,i\,k^4\sqrt{-g H k^2\left(3+H^2k^2\right)}$$
$$U^2-594\,i\,H^2k^6\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U^2-99\,i\,H^4k^8\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U^2\Big)\,dt+$$

$$\frac{1}{11\,664\left(3+H^2k^2\right)^3\sqrt{-g H k^2\left(3+H^2k^2\right)}}\Big(-57\,024\,i\sqrt{3}\;g^2H^2k^6-33\,264\,i\sqrt{3}\;g^2H^4k^8-$$
$$4752\,i\sqrt{3}\;g^2H^6k^{10}-17\,496\sqrt{3}\;g H k^5\sqrt{g H\left(3+H^2k^2\right)}\sqrt{-g H k^2\left(3+H^2k^2\right)}-$$
$$4374\sqrt{3}\;g H^3k^7\sqrt{g H\left(3+H^2k^2\right)}\sqrt{-g H k^2\left(3+H^2k^2\right)}+168\,480\,i\,g H k^6\sqrt{g H\left(3+H^2k^2\right)}\;U+$$
$$106\,650\,i\,g H^3k^8\sqrt{g H\left(3+H^2k^2\right)}\;U+16\,830\,i\,g H^5k^{10}\sqrt{g H\left(3+H^2k^2\right)}\;U+$$
$$170\,586\,g H k^5\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U+109\,350\,g H^3k^7\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U+$$
$$17\,496\,g H^5k^9\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U-159\,894\,i\sqrt{3}\;g H k^6U^2-161\,676\,i\sqrt{3}\;g H^3k^8U^2-$$
$$54\,486\,i\sqrt{3}\;g H^5k^{10}U^2-6120\,i\sqrt{3}\;g H^7k^{12}U^2-52\,488\sqrt{3}\;k^5\sqrt{g H\left(3+H^2k^2\right)}$$
$$\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U^2-34\,992\sqrt{3}\;H^2k^7\sqrt{g H\left(3+H^2k^2\right)}\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U^2-$$
$$5832\sqrt{3}\;H^4k^9\sqrt{g H\left(3+H^2k^2\right)}\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U^2+29\,241\,i\,k^6\sqrt{g H\left(3+H^2k^2\right)}\;U^3+$$
$$29\,133\,i\,H^2k^8\sqrt{g H\left(3+H^2k^2\right)}\;U^3+9675\,i\,H^4k^{10}\sqrt{g H\left(3+H^2k^2\right)}\;U^3+$$
$$1071\,i\,H^6k^{12}\sqrt{g H\left(3+H^2k^2\right)}\;U^3+32\,805\,k^5\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U^3+$$
$$32\,805\,H^2k^7\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U^3+10\,935\,H^4k^9\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U^3+$$
$$1215\,H^6k^{11}\sqrt{-g H k^2\left(3+H^2k^2\right)}\;U^3+1296\,i\sqrt{3}\;k^6U^4+1728\,i\sqrt{3}\;H^2k^8U^4+$$
$$864\,i\sqrt{3}\;H^4k^{10}U^4+192\,i\sqrt{3}\;H^6k^{12}U^4+16\,i\sqrt{3}\;H^8k^{14}U^4\Big)\,dt^2+O[dt]^3\Bigg)\,dx^2+O[dx]^3\Big\}$$

Out[71]= EmatEig || \left\{\left(−\frac{3 \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k+i \sqrt{3} \sqrt{−g H k^2 \left(H^2 k^2+3\right)}\right)}{2 \left(H^2 k^2+3\right)}+\frac{i \left(3 g H^3 k^4−2 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^4−2 i \sqrt{3} H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^3+9 g H k^2−6 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2−6 i \sqrt{3} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k+3 i \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k\right) \text{dt}}{2 \left(H^2 k^2+3\right)^2}+\frac{\left(9 g H^5 U^3 k^7+54 g H^3 U^3 k^5+36 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5−108 g^2 H^4 U k^5+i H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g

H k^2 \left(H^2 k^2+3\right)} U^3 k^4+18 i \sqrt{3} g H^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^4+81 g H U^3 k^3+108 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^3−324 g^2 H^2 U k^3+54 \sqrt{3} g^2 H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^3+3 i \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^2+54 i \sqrt{3} g H \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^2−144 i g H \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^2+48 i \sqrt{3} g^2 H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^2\right) \text{dt}^2}{36 g H \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^3\right)\right)+\left(\frac{i \left(6 \sqrt{3} g H^3 k^6−H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^6−9 i H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^5+24 \sqrt{3} g H k^4−3 \sqrt{g H \left(H^2 k^2+3\right)} U k^4−27 i \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^3\right)}{9 \left(−4 H^2 k^2−12\right) \sqrt{−g H k^2 \left(H^2 k^2+3\right)}}+\frac{\left(−31 H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^9+164 \sqrt{3} g H^5 U k^9−99 i H^4 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^8−186 H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^7+1008 \sqrt{3} g H^3 U k^7−90 g H^3 \sqrt{g H \left(H^2 k^2+3\right)} k^7−594 i H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^6+72 i \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^6−162 i g H^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^6−279 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5+1548 \sqrt{3} g H U k^5−360 g H \sqrt{g H \left(H^2 k^2+3\right)} k^5−891 i \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^4+216 i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^4−648 i g H \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^4\right) \text{dt}}{432 \left(H^2 k^2+3\right)^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)}}+\frac{\left(−16 i \sqrt{3} H^8 U^4 k^{14}−192 i \sqrt{3} H^6 U^4 k^{12}−1071 i H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^{12}+6120 i \sqrt{3} g H^7 U^2 k^{12}+1215 H^6 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^{11}+4752 i \sqrt{3} g^2 H^6 k^{10}−864 i \sqrt{3} H^4 U^4 k^{10}−9675 i H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^{10}+54486 i \sqrt{3} g H^5 U^2 k^{10}−16830 i g H^5 \sqrt{g H \left(H^2 k^2+3\right)} U k^{10}+10935 H^4 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^9−5832 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^9+17496 g H^5 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^9+33264 i \sqrt{3} g^2 H^4 k^8−1728 i \sqrt{3} H^2 U^4 k^8−29133 i H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^8+161676 i \sqrt{3} g H^3 U^2 k^8−106650 i g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U k^8+32805 H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^7−34992 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^7+109350 g H^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^7−4374 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^7−1296 i \sqrt{3} U^4 k^6−29241 i \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6+57024 i \sqrt{3} g^2 H^2 k^6+159894 i \sqrt{3} g H U^2 k^6−168480 i g H \sqrt{g H \left(H^2 k^2+3\right)} U k^6+32805 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^5−52488 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^5+170586 g H \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^5−17496 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^5\right) \text{dt}^2}{11664 \left(H^2 k^2+3\right)^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)}}+O\left(\text{dt}^3\right)\right) \text{dx}^2+O\left(\text{dx}^3\right),\left(−\frac{3 \left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}−i \sqrt{3} \sqrt{−g H k^2 \left(H^2 k^2+3\right)}\right)}{2 \left(H^2 k^2+3\right)}+\frac{i \left(3 g H^3 k^4−2 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^4+2 i \sqrt{3} H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^3+9 g H k^2−6 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2+6 i \sqrt{3} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k−3 i \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k\right) \text{dt}}{2 \left(H^2 k^2+3\right)^2}+\frac{\left(9 g H^5 U^3 k^7+54 g H^3 U^3 k^5+36 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5−108 g^2 H^4 U k^5−i H^2 \sqrt{g H

\left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^4−18 i \sqrt{3} g H^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^4+81 g H U^3 k^3+108 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^3−324 g^2 H^2 U k^3+54 \sqrt{3} g^2 H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^3−3 i \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^2−54 i \sqrt{3} g H \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^2+144 i g H \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^2−48 i \sqrt{3} g^2 H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^2\right) \text{dt}^2}{36 g H \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^3\right)\right)+\left(−\frac{i \left(6 \sqrt{3} g H^3 k^6−H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^6+9 i H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^5+24 \sqrt{3} g H k^4−3 \sqrt{g H \left(H^2 k^2+3\right)} U k^4+27 i \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^3\right)}{9 \left(−4 H^2 k^2−12\right) \sqrt{−g H k^2 \left(H^2 k^2+3\right)}}+\frac{\left(31 H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^9−164 \sqrt{3} g H^5 U k^9−99 i H^4 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^8+186 H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^7−1008 \sqrt{3} g H^3 U k^7+90 g H^3 \sqrt{g H \left(H^2 k^2+3\right)} k^7−594 i H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^6+72 i \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^6−162 i g H^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^6+279 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5−1548 \sqrt{3} g H U k^5+360 g H \sqrt{g H \left(H^2 k^2+3\right)} k^5−891 i \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^4+216 i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^4−648 i g H \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^4\right) \text{dt}}{432 \left(H^2 k^2+3\right)^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)}}+\frac{\left(16 i \sqrt{3} H^8 U^4 k^{14}+192 i \sqrt{3} H^6 U^4 k^{12}+1071 i H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^{12}−6120 i \sqrt{3} g H^7 U^2 k^{12}+1215 H^6 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^{11}−4752 i \sqrt{3} g^2 H^6 k^{10}+864 i \sqrt{3} H^4 U^4 k^{10}+9675 i H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^{10}−54486 i \sqrt{3} g H^5 U^2 k^{10}+16830 i g H^5 \sqrt{g H \left(H^2 k^2+3\right)} U k^{10}+10935 H^4 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^9−5832 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^9+17496 g H^5 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^9−33264 i \sqrt{3} g^2 H^4 k^8+1728 i \sqrt{3} H^2 U^4 k^8+29133 i H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^8−161676 i \sqrt{3} g H^3 U^2 k^8+106650 i g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U k^8+32805 H^2 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^7−34992 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^7+109350 g H^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^7−4374 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^7+1296 i \sqrt{3} U^4 k^6+29241 i \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6−57024 i \sqrt{3} g^2 H^2 k^6−159894 i \sqrt{3} g H U^2 k^6+168480 i g H \sqrt{g H \left(H^2 k^2+3\right)} U k^6+32805 \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^3 k^5−52488 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U^2 k^5+170586 g H \sqrt{−g H k^2 \left(H^2 k^2+3\right)} U k^5−17496 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} \sqrt{−g H k^2 \left(H^2 k^2+3\right)} k^5\right) \text{dt}^2}{11664 \left(H^2 k^2+3\right)^3 \sqrt{−g H k^2 \left(H^2 k^2+3\right)}}+O\left(\text{dt}^3\right)\right) \text{dx}^2+O\left(\text{dx}^3\right)\right\}

Out[72]=

Out[73]= Ematerr ||

$$\left\{\left\{\left(\frac{i\sqrt{3}\,k\sqrt{g\,H\left(3+H^2k^2\right)}\,dt}{3+H^2k^2}+\left(-\frac{3\,g\,H\,k^2}{2\left(3+H^2k^2\right)}+\frac{\sqrt{3}\,k^2\sqrt{g\,H\left(3+H^2k^2\right)}\,U}{3+H^2k^2}\right)dt^2+O[dt]^3\right)+\right.$$

$$\left(\frac{1}{6}i\,k^3\,U\,dt+\left(\frac{3\,g\,H\,k^4}{\left(3+H^2k^2\right)^2}+\frac{3\,g\,H^3\,k^6}{4\left(3+H^2k^2\right)^2}+\frac{k^4U^2}{24}\right)dt^2+O[dt]^3\right)dx^2+O[dx]^3,$$

$$\left(-i\,H\,k\,dt+\left(\frac{\sqrt{3}\,H\,k^2\sqrt{g\,H\left(3+H^2k^2\right)}}{2\left(3+H^2k^2\right)}-H\,k^2\,U\right)dt^2+O[dt]^3\right)+$$

$$\left.\left(\frac{1}{6}i\,H\,k^3\,dt+\left(-\frac{H\,k^4\sqrt{g\,H\left(3+H^2k^2\right)}}{4\sqrt{3}\,\left(3+H^2k^2\right)}+\frac{7}{24}H\,k^4\,U\right)dt^2+O[dt]^3\right)dx^2+O[dx]^3\right\},$$

$$\left\{\left(-\frac{6\,i\,g\,k\,dt}{3+H^2k^2}+O[dt]^3\right)+\left(\frac{i\,g\left(6\,k^3+H^2k^5\right)dt}{2\left(3+H^2k^2\right)^2}+O[dt]^3\right)dx^2+O[dx]^3,\right.$$

$$\left.\left(\frac{2\,i\sqrt{3}\,k\sqrt{g\,H\left(3+H^2k^2\right)}\,dt}{3+H^2k^2}+O[dt]^3\right)+\left(\frac{1}{3}i\,k^3\,U\,dt+O[dt]^3\right)dx^2+O[dx]^3\right\}\right\}$$

Out[74]=

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Ematerr ||  \left(
\begin{array}{cc}
 \left(\frac{i \sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)} \text{dt}}{H^2 k^2+3}+\left(\frac{\sqrt{3} k^2 \sqrt{g
      H \left(H^2 k^2+3\right)} U}{H^2 k^2+3}-\frac{3 g H k^2}{2 \left(H^2 k^2+3\right)}\right)
      \text{dt}^2+O\left(\text{dt}^3\right)\right)+\left(\frac{1}{6} i k^3 U \text{dt}+\left(\frac{3
      g H^3 k^6}{4 \left(H^2 k^2+3\right)^2}+\frac{U^2 k^4}{24}+\frac{3 g H k^4}{\left(H^2
      k^2+3\right)^2}\right) \text{dt}^2+O\left(\text{dt}^3\right)\right) \text{dx}^2+O\left(\text{dx}^3\right)
      & \left(-i H k \text{dt}+\left(\frac{\sqrt{3} H k^2 \sqrt{g H \left(H^2 k^2+3\right)}}{2 \left(H^2
      k^2+3\right)}-H k^2 U\right) \text{dt}^2+O\left(\text{dt}^3\right)\right)+\left(\frac{1}{6} i H k^3
      \text{dt}+\left(\frac{7}{24} H k^4 U-\frac{H k^4 \sqrt{g H \left(H^2 k^2+3\right)}}{4 \sqrt{3} \left(H^2
      k^2+3\right)}\right) \text{dt}^2+O\left(\text{dt}^3\right)\right) \text{dx}^2+O\left(\text{dx}^3\right) \\
 \left(-\frac{6 i g k \text{dt}}{H^2 k^2+3}+O\left(\text{dt}^3\right)\right)+\left(\frac{i g \left(H^2
      k^5+6 k^3\right) \text{dt}}{2 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^3\right)\right)
      \text{dx}^2+O\left(\text{dx}^3\right) & \left(\frac{2 i \sqrt{3} k \sqrt{g H \left(H^2
      k^2+3\right)} \text{dt}}{H^2 k^2+3}+O\left(\text{dt}^3\right)\right)+\left(\frac{1}{3} i
      k^3 U \text{dt}+O\left(\text{dt}^3\right)\right) \text{dx}^2+O\left(\text{dx}^3\right) \\
\end{array}
\right)
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