In[1]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];

qjp1n = q0 * Exp[I * (k * (xj + dx) + w * tn)];
qjp1F = Simplify[qjp1n / (qjn)];
qjp2n = q0 * Exp[I * (k * (xj + 2 * dx) + w * tn)];
qjp2F = Simplify[qjp2n / (qjn)];
qjm1n = q0 * Exp[I * (k * (xj - dx) + w * tn)];
qjm1F = Simplify[qjm1n / (qjn)];
qjm2n = q0 * Exp[I * (k * (xj - 2 * dx) + w * tn)];
qjm2F = Simplify[qjm2n / (qjn)];
```

$$wAp = -U * k - \frac{\sqrt{3}\ k \sqrt{g H \left(3 + H^2 k^2\right)}}{3 + H^2 k^2};$$

$$wAm = -U * k + \frac{\sqrt{3}\ k \sqrt{g H \left(3 + H^2 k^2\right)}}{3 + H^2 k^2};$$

In[13]:=
```
Dx = FullSimplify[(qjp1F - qjm1F) / (2 * dx)];
Dxerr = Series[Dx - (I * k), {dx, 0, 4}];
DxDx = FullSimplify[(qjp1F - 2 + qjm1F) / dx^2];
DxDxerr = Series[DxDx - (-k * k), {dx, 0, 4}];
DxDxDx = FullSimplify[(qjp2F - 2 qjp1F + 2 * qjm1F - qjm2F) / (2 * dx * dx * dx)];
DxDxDxerr = Series[DxDxDx - (-I * k * k * k), {dx, 0, 4}];

Text[Row[{"Dx  ||   ", Dx}]]
Text[Row[{"Dx  ||   ", TeXForm[Dx]}]]
Text[Row[{"Dx error  ||   ", TeXForm[Dxerr]}]]
Text[Row[{"Dx error  ||   ", Dxerr}]]
Text[" "]
Text[Row[{"DxDx  ||   ", DxDx}]]
Text[Row[{"DxDx  ||   ", TeXForm[DxDx]}]]
Text[Row[{"DxDx error  ||   ", TeXForm[DxDxerr]}]]
Text[Row[{"DxDx error  ||   ", DxDxerr}]]
Text[" "]
Text[Row[{"DxDxDx  ||   ", DxDxDx}]]
Text[Row[{"DxDxDx  ||   ", TeXForm[DxDxDx]}]]
Text[Row[{"DxDxDx error  ||   ", TeXForm[DxDxDxerr]}]]
Text[Row[{"DxDxDx error  ||   ", DxDxDxerr}]]
Text[" "]
```

Out[19]= Dx || $\frac{i \operatorname{Sin}[dx\, k]}{dx}$

Out[20]= Dx || \frac{i \sin (\text{dx} k)}{\text{dx}}

Out[21]= Dx error || −\frac{1}{6} i \text{dx}^2 k^3+\frac{1}{120} i \text{dx}^4 k^5+O\left(\text{dx}^5\right)

Out[22]= Dx error || $-\frac{1}{6} i\, k^3\, dx^2 + \frac{1}{120} i\, k^5\, dx^4 + O[dx]^5$

Out[23]=

Out[24]= DxDx || $\frac{2\,(-1+\text{Cos}[dx\,k])}{dx^2}$

Out[25]= DxDx || \frac{2 (\cos (\text{dx} k)−1)}{\text{dx}^2}

Out[26]= DxDx error || \frac{\text{dx}^2 k^4}{12}−\frac{\text{dx}^4 k^6}{360}+O\left(\text{dx}^5\right)

Out[27]= DxDx error || $\frac{k^4\, dx^2}{12} - \frac{k^6\, dx^4}{360} + O[dx]^5$

Out[28]=

Out[29]= DxDxDx || $-\frac{4\, i\, \text{Sin}\left[\frac{dx\,k}{2}\right]^2 \text{Sin}[dx\,k]}{dx^3}$

Out[30]= DxDxDx || −\frac{4 i \sin ^2\left(\frac{\text{dx} k}{2}\right) \sin (\text{dx} k)}{\text{dx}^3}

Out[31]= DxDxDx error || \frac{1}{4} i \text{dx}^2 k^5−\frac{1}{40} i \text{dx}^4 k^7+O\left(\text{dx}^5\right)

Out[32]= DxDxDx error || $\frac{1}{4} i\, k^5\, dx^2 - \frac{1}{40} i\, k^7\, dx^4 + O[dx]^5$

Out[33]=

In[34]:=
```
DerivCheck = FullSimplify[DxDx * Dx];
DerivCheck /. Cos[dx k] - 1 → -2 * Sin[dx k / 2]^2;
DxDxDx;
```

In[37]:=
```
etaspatderivs = -(H * Dx * v + U * Dx * n);
etaspatderivsu = etaspatderivs /. v → 1 /. n → 0;
etaspatderivsn = etaspatderivs /. n → 1 /. v → 0;
upsspatderivs = -(g * H * Dx * n + U * H * Dx * v - H^3 / 3 * U * DxDxDx * v);
upsspatderivsLHS =
  H * v - H^3 / 3 * DxDx  /. v → 1  /. Cos[dx k] - 1 → -2 * Sin[dx k / 2]^2;
upsspatderivsu = upsspatderivs /. v → 1 /. n → 0;
upsspatderivsu = Simplify[upsspatderivsu / upsspatderivsLHS ];
upsspatderivsn = upsspatderivs /. n → 1 /. v → 0;
upsspatderivsn = Simplify[upsspatderivsn / upsspatderivsLHS ];
Emat =
  2 * dt * {{etaspatderivsn , etaspatderivsu } , {upsspatderivsn, upsspatderivsu}};
EmatEig  = Eigenvalues[Emat];
EmatEig =
  Series[ wAp - Log[Exp[-I * (wAp) * dt] + EmatEig] / (I * dt), {dx, 0, 4}, {dt, 0, 4}];
```

In[49]:=
```
Text[Row[{"E00  ||   ", etaspatderivsn}]]
Text[Row[{"E00  ||   ", TeXForm[etaspatderivsn]}]]
Text[" "]
Text[Row[{"E01  ||   ", etaspatderivsu}]]
Text[Row[{"E01  ||   ", TeXForm[etaspatderivsu]}]]
Text[" "]
Text[Row[{"E10  ||   ", upsspatderivsn}]]
Text[Row[{"E10  ||   ", TeXForm[upsspatderivsn]}]]
Text[" "]
Text[Row[{"E11  ||   ", upsspatderivsu}]]
Text[Row[{"E11  ||   ", TeXForm[upsspatderivsu]}]]
Text[" "]
Text[Row[{"EmatEig  ||   ", EmatEig }]]
Text[Row[{"EmatEig  ||   ", TeXForm[EmatEig ]}]]
```

Out[49]= E00 || $-\frac{i\, U\, \text{Sin}[dx\, k]}{dx}$

Out[50]= E00 || -\frac{i U \sin (\text{dx} k)}{\text{dx}}

Out[51]=

Out[52]= E01 || $-\frac{i\, H\, \text{Sin}[dx\, k]}{dx}$

Out[53]= E01 || -\frac{i H \sin (\text{dx} k)}{\text{dx}}

Out[54]=

Out[55]= E10 || $-\frac{3\, i\, dx\, g\, \text{Sin}[dx\, k]}{3\, dx^2+4\, H^2\, \text{Sin}\left[\frac{dx\, k}{2}\right]^2}$

Out[56]= E10 || -\frac{3 i \text{dx} g \sin (\text{dx} k)}{3 \text{dx}^2+4 H^2 \sin ^2\left(\frac{\text{dx} k}{2}\right)}

Out[57]=

Out[58]= E11 || $-\frac{i\, U\, \text{Sin}[dx\, k]}{dx}$

Out[59]= E11 || -\frac{i U \sin (\text{dx} k)}{\text{dx}}

Out[60]=

Out[61]= EmatEig ||

$$\left\{\left(\frac{1}{3\left(3+H^2\, k^2\right)^2}\left(3\sqrt{3}\ g\, H\, k^3\sqrt{g\, H\left(3+H^2\, k^2\right)}+27\, g\, H\, k^3\, U+9\, g\, H^3\, k^5\, U+9\sqrt{3}\ k^3\sqrt{g\, H\left(3+H^2\, k^2\right)}\ U^2+\right.\right.\right.$$
$$\left.\left.\left.3\sqrt{3}\ H^2\, k^5\sqrt{g\, H\left(3+H^2\, k^2\right)}\ U^2+9\, k^3\, U^3+6\, H^2\, k^5\, U^3+H^4\, k^7\, U^3\right)dt^2+\frac{1}{3\left(3+H^2\, k^2\right)^2}\right.\right.$$
$$i\left(9\, g^2\, H^2\, k^4+12\sqrt{3}\ g\, H\, k^4\sqrt{g\, H\left(3+H^2\, k^2\right)}\ U+54\, g\, H\, k^4\, U^2+18\, g\, H^3\, k^6\, U^2+12\sqrt{3}\ k^4\right.$$
$$\left.\sqrt{g\, H\left(3+H^2\, k^2\right)}\ U^3+4\sqrt{3}\ H^2\, k^6\sqrt{g\, H\left(3+H^2\, k^2\right)}\ U^3+9\, k^4\, U^4+6\, H^2\, k^6\, U^4+H^4\, k^8\, U^4\right)dt^3-$$

$$\frac{1}{60\left(3+H^2\,k^2\right)^3}11\Big(9\sqrt{3}\;g^2\,H^2\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}+135\,g^2\,H^2\,k^5\,U+45\,g^2\,H^4\,k^7\,U+90\sqrt{3}\;g\,H$$
$$k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+30\sqrt{3}\;g\,H^3\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+270\,g\,H\,k^5\,U^3+180\,g\,H^3\,k^7\,U^3+$$
$$30\,g\,H^5\,k^9\,U^3+45\sqrt{3}\;k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+30\sqrt{3}\;H^2\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+$$
$$5\sqrt{3}\;H^4\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+27\,k^5\,U^5+27\,H^2\,k^7\,U^5+9\,H^4\,k^9\,U^5+H^6\,k^{11}\,U^5\Big)\,dt^4+O[dt]^5\Big)+$$
$$\Big(-\frac{1}{12\left(3+H^2\,k^2\right)^2}\Big(12\sqrt{3}\;k^3\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\sqrt{3}\;H^2\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}+36\,k^3\,U+$$
$$24\,H^2\,k^5\,U+4\,H^4\,k^7\,U\Big)-\frac{1}{12\left(3+H^2\,k^2\right)^3}i\,k^4\Big(\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,U+H^2\,k^2\,U\Big)$$
$$\Big(12\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}+36\,U+24\,H^2\,k^2\,U+4\,H^4\,k^4\,U\Big)\,dt+$$
$$\frac{1}{24\left(3+H^2\,k^2\right)^3}k^5\Big(12\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}+36\,U+24\,H^2\,k^2\,U+4\,H^4\,k^4\,U\Big)$$
$$\Big(3\,g\,H+2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+3\,U^2+H^2\,k^2\,U^2\Big)\,dt^2-\frac{1}{72\left(3+H^2\,k^2\right)^4}$$
$$i\,k^6\Big(12\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}+36\,U+24\,H^2\,k^2\,U+4\,H^4\,k^4\,U\Big)$$
$$\Big(3\sqrt{3}\;g\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}+27\,g\,H\,U+9\,g\,H^3\,k^2\,U+9\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+$$
$$3\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+9\,U^3+6\,H^2\,k^2\,U^3+H^4\,k^4\,U^3\Big)\,dt^3+\frac{1}{96\left(3+H^2\,k^2\right)^4}$$
$$5\,k^7\Big(12\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}+36\,U+24\,H^2\,k^2\,U+4\,H^4\,k^4\,U\Big)$$
$$\Big(9\,g^2\,H^2+12\sqrt{3}\;g\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+54\,g\,H\,U^2+18\,g\,H^3\,k^2\,U^2+12\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+$$
$$4\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+9\,U^4+6\,H^2\,k^2\,U^4+H^4\,k^4\,U^4\Big)\,dt^4+O[dt]^5\Big)\,dx^2+$$
$$\Big(\frac{1}{960\left(3+H^2\,k^2\right)^3}\Big(144\sqrt{3}\;k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}+48\sqrt{3}\;H^2\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}+$$
$$5\sqrt{3}\;H^4\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}+432\,k^5\,U+432\,H^2\,k^7\,U+144\,H^4\,k^9\,U+16\,H^6\,k^{11}\,U\Big)+$$
$$\frac{1}{2880\left(3+H^2\,k^2\right)^3}i\Big(5616\,g\,H\,k^6+2592\,g\,H^3\,k^8+315\,g\,H^5\,k^{10}+3744\sqrt{3}\;k^6\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+$$
$$2112\sqrt{3}\;H^2\,k^8\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+303\sqrt{3}\;H^4\,k^{10}\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+$$
$$5616\,k^6\,U^2+5616\,H^2\,k^8\,U^2+1872\,H^4\,k^{10}\,U^2+208\,H^6\,k^{12}\,U^2\Big)\,dt-$$
$$\frac{1}{5760\left(3+H^2\,k^2\right)^4}\Big(k^7\Big(18\,576\sqrt{3}\;g\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}+9072\sqrt{3}\;g\,H^3\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}+$$
$$1125\sqrt{3}\;g\,H^5\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}+167\,184\,g\,H\,U+147\,312\,g\,H^3\,k^2\,U+43\,110\,g\,H^5\,k^4\,U+$$
$$4194\,g\,H^7\,k^6\,U+55\,728\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+52\,416\sqrt{3}\;H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+$$
$$16\,413\sqrt{3}\;H^4\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+1711\sqrt{3}\;H^6\,k^6\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^2+55\,728\,U^3+$$
$$74\,304\,H^2\,k^2\,U^3+37\,152\,H^4\,k^4\,U^3+8256\,H^6\,k^6\,U^3+688\,H^8\,k^8\,U^3\Big)\Big)\,dt^2-\frac{1}{1920\left(3+H^2\,k^2\right)^4}$$
$$i\,k^8\Big(16\,848\,g^2\,H^2+8496\,g^2\,H^4\,k^2+1065\,g^2\,H^6\,k^4+22\,464\sqrt{3}\;g\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+$$
$$12\,240\sqrt{3}\;g\,H^3\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+1649\sqrt{3}\;g\,H^5\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U+$$
$$101\,088\,g\,H\,U^2+92\,880\,g\,H^3\,k^2\,U^2+28\,299\,g\,H^5\,k^4\,U^2+2857\,g\,H^7\,k^6\,U^2+$$

$$
\begin{aligned}
&101\,088\,g\,H\,U^3 + 92\,880\,g\,H^3\,k^2\,U^3 + 28\,299\,g\,H^5\,k^4\,U^3 + 2857\,g\,H^7\,k^6\,U^3 + \\
&22\,464\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + 21\,552\sqrt{3}\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + \\
&6875\sqrt{3}\,H^4\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + 729\sqrt{3}\,H^6\,k^6\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + \\
&16\,848\,U^4 + 22\,464\,H^2\,k^2\,U^4 + 11\,232\,H^4\,k^4\,U^4 + 2496\,H^6\,k^6\,U^4 + 208\,H^8\,k^8\,U^4\Big)\,dt^3 + \\
&\frac{1}{4}\,i\Bigg(-\frac{1}{108\left(3+H^2\,k^2\right)^7}i\,k^9\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\,U + H^2\,k^2\,U\right)^3 \\
&\left(12\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\sqrt{3}\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)} + 36\,U + 24\,H^2\,k^2\,U + 4\,H^4\,k^4\,U\right)^2 + \\
&\frac{1}{384\left(3+H^2\,k^2\right)^5}i\,k^9\left(144\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 48\sqrt{3}\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)} + \right. \\
&\left. 5\sqrt{3}\,H^4\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)} + 432\,U + 432\,H^2\,k^2\,U + 144\,H^4\,k^4\,U + 16\,H^6\,k^6\,U\right)\Big(9\,g^2\,H^2 + \\
&12\sqrt{3}\,g\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U + 54\,g\,H\,U^2 + 18\,g\,H^3\,k^2\,U^2 + 12\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + \\
&4\sqrt{3}\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + 9\,U^4 + 6\,H^2\,k^2\,U^4 + H^4\,k^4\,U^4\Big)\Big)\,dt^4 + O[dt]^5\Bigg)\,dx^4 + \\
&O[dx]^5,\Bigg(-\frac{4\left(\sqrt{3}\,k\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)}{3+H^2\,k^2} - \frac{4\,i\left(-3\,g\,H\,k^2+\sqrt{3}\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)dt}{3+H^2\,k^2} + \frac{1}{3\left(3+H^2\,k^2\right)^2} \\
&\Big(69\sqrt{3}\,g\,H\,k^3\sqrt{g\,H\left(3+H^2\,k^2\right)} - 297\,g\,H\,k^3\,U - 99\,g\,H^3\,k^5\,U + 27\sqrt{3}\,k^3\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2 + \\
&9\sqrt{3}\,H^2\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2 + 9\,k^3\,U^3 + 6\,H^2\,k^5\,U^3 + H^4\,k^7\,U^3\Big)\,dt^2 + \frac{1}{3\left(3+H^2\,k^2\right)^2} \\
&i\Big(-441\,g^2\,H^2\,k^4 + 282\sqrt{3}\,g\,H\,k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U - 432\,g\,H\,k^4\,U^2 - 144\,g\,H^3\,k^6\,U^2 + 6\sqrt{3} \\
&k^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + 2\sqrt{3}\,H^2\,k^6\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + 9\,k^4\,U^4 + 6\,H^2\,k^6\,U^4 + H^4\,k^8\,U^4\Big)\,dt^3 + \\
&\frac{1}{60\left(3+H^2\,k^2\right)^3}\Big(-20\,061\sqrt{3}\,g^2\,H^2\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)} + 144\,315\,g^2\,H^2\,k^5\,U + 48\,105\,g^2\,H^4\,k^7\,U - \\
&36\,090\sqrt{3}\,g\,H\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2 - 12\,030\sqrt{3}\,g\,H^3\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2 + \\
&21\,870\,g\,H\,k^5\,U^3 + 14\,580\,g\,H^3\,k^7\,U^3 + 2430\,g\,H^5\,k^9\,U^3 + 855\sqrt{3}\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4 + \\
&570\sqrt{3}\,H^2\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4 + 95\sqrt{3}\,H^4\,k^9\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4 - \\
&297\,k^5\,U^5 - 297\,H^2\,k^7\,U^5 - 99\,H^4\,k^9\,U^5 - 11\,H^6\,k^{11}\,U^5\Big)\,dt^4 + O[dt]^5\Bigg) + \\
&\Bigg(-\frac{1}{12\left(3+H^2\,k^2\right)^2}\Big(-12\sqrt{3}\,k^3\sqrt{g\,H\left(3+H^2\,k^2\right)} - 3\sqrt{3}\,H^2\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)} + \\
&36\,k^3\,U + 24\,H^2\,k^5\,U + 4\,H^4\,k^7\,U\Big) - \frac{1}{12\left(3+H^2\,k^2\right)^3} \\
&i\,k^4\left(-3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\,U + H^2\,k^2\,U\right)\Big(-12\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} - \\
&3\sqrt{3}\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)} + 36\,U + 24\,H^2\,k^2\,U + 4\,H^4\,k^4\,U\Big)\,dt + \frac{1}{24\left(3+H^2\,k^2\right)^3} \\
&k^5\left(-12\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} - 3\sqrt{3}\,H^2\,k^2\sqrt{g\,H\left(3+H^2\,k^2\right)} + 36\,U + 24\,H^2\,k^2\,U + 4\,H^4\,k^4\,U\right)
\end{aligned}
$$

$$\left(51\,g\,H-14\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+3\,U^2+H^2\,k^2\,U^2\right)dt^2-\frac{1}{72\left(3+H^2\,k^2\right)^4}$$
$$i\,k^6\left(-12\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-3\,\sqrt{3}\,H^2\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+36\,U+24\,H^2\,k^2\,U+4\,H^4\,k^4\,U\right)$$
$$\left(435\,\sqrt{3}\,g\,H\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-1701\,g\,H\,U-567\,g\,H^3\,k^2\,U+153\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+\right.$$
$$\left.51\,\sqrt{3}\,H^2\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+9\,U^3+6\,H^2\,k^2\,U^3+H^4\,k^4\,U^3\right)dt^3+\frac{1}{288\left(3+H^2\,k^2\right)^4}$$
$$k^7\left(-12\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-3\,\sqrt{3}\,H^2\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+36\,U+24\,H^2\,k^2\,U+4\,H^4\,k^4\,U\right)$$
$$\left(-14\,841\,g^2\,H^2+8820\,\sqrt{3}\,g\,H\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U-13\,014\,g\,H\,U^2-4338\,g\,H^3\,k^2\,U^2+\right.$$
$$372\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+124\,\sqrt{3}\,H^2\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+$$
$$\left.135\,U^4+90\,H^2\,k^2\,U^4+15\,H^4\,k^4\,U^4\right)dt^4+O[dt]^5\Big)\,dx^2+$$
$$\left(\frac{1}{960\left(3+H^2\,k^2\right)^3}\left(-144\,\sqrt{3}\,k^5\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-48\,\sqrt{3}\,H^2\,k^7\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-\right.\right.$$
$$\left.5\,\sqrt{3}\,H^4\,k^9\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+432\,k^5\,U+432\,H^2\,k^7\,U+144\,H^4\,k^9\,U+16\,H^6\,k^{11}\,U\right)+$$
$$\frac{1}{2880\left(3+H^2\,k^2\right)^3}i\left(8208\,g\,H\,k^6+3456\,g\,H^3\,k^8+405\,g\,H^5\,k^{10}-4608\,\sqrt{3}\,k^6\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U-\right.$$
$$2688\,\sqrt{3}\,H^2\,k^8\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U-399\,\sqrt{3}\,H^4\,k^{10}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+$$
$$\left.5616\,k^6\,U^2+5616\,H^2\,k^8\,U^2+1872\,H^4\,k^{10}\,U^2+208\,H^6\,k^{12}\,U^2\right)dt-\frac{1}{5760\left(3+H^2\,k^2\right)^4}$$
$$\left(k^7\left(-73\,872\,\sqrt{3}\,g\,H\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-33\,264\,\sqrt{3}\,g\,H^3\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-4005\,\sqrt{3}\,g\right.\right.$$
$$H^5\,k^4\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+483\,408\,g\,H\,U+430\,704\,g\,H^3\,k^2\,U+128\,250\,g\,H^5\,k^4\,U+12\,798$$
$$g\,H^7\,k^6\,U-105\,840\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2-102\,528\,\sqrt{3}\,H^2\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2-$$
$$33\,117\,\sqrt{3}\,H^4\,k^4\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2-3567\,\sqrt{3}\,H^6\,k^6\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+55\,728\,U^3+$$
$$\left.\left.74\,304\,H^2\,k^2\,U^3+37\,152\,H^4\,k^4\,U^3+8256\,H^6\,k^6\,U^3+688\,H^8\,k^8\,U^3\right)\right)dt^2-\frac{1}{5760\left(3+H^2\,k^2\right)^4}$$
$$i\,k^8\left(861\,840\,g^2\,H^2+399\,600\,g^2\,H^4\,k^2+48\,645\,g^2\,H^6\,k^4-766\,368\,\sqrt{3}\,g\,H\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U-\right.$$
$$417\,456\,\sqrt{3}\,g\,H^3\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U-58\,035\,\sqrt{3}\,g\,H^5\,k^4\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U+$$
$$2\,073\,600\,g\,H\,U^2+1\,956\,528\,g\,H^3\,k^2\,U^2+615\,807\,g\,H^5\,k^4\,U^2+64\,677\,g\,H^7\,k^6\,U^2-$$
$$228\,960\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3-226\,224\,\sqrt{3}\,H^2\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3-$$
$$74\,481\,\sqrt{3}\,H^4\,k^4\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3-8171\,\sqrt{3}\,H^6\,k^6\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+50\,544\,U^4+$$
$$\left.67\,392\,H^2\,k^2\,U^4+33\,696\,H^4\,k^4\,U^4+7488\,H^6\,k^6\,U^4+624\,H^8\,k^8\,U^4\right)dt^3+\frac{1}{4}\,i\left(-\frac{1}{108\left(3+H^2\,k^2\right)^6}i\,k^9\right.$$
$$\left(-12\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-3\,\sqrt{3}\,H^2\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+36\,U+24\,H^2\,k^2\,U+4\,H^4\,k^4\,U\right)^2$$
$$\left(-447\,\sqrt{3}\,g\,H\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+1647\,g\,H\,U+549\,g\,H^3\,k^2\,U-153\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\right.$$
$$\left.U^2-51\,\sqrt{3}\,H^2\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^2+9\,U^3+6\,H^2\,k^2\,U^3+H^4\,k^4\,U^3\right)+$$
$$\frac{1}{5760\left(3+H^2\,k^2\right)^5}i\,k^9\left(-144\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-48\,\sqrt{3}\,H^2\,k^2\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-\right.$$

$$5\sqrt{3}\ H^4 k^4 \sqrt{g H\left(3+H^2 k^2\right)}\ +432 U+432 H^2 k^2 U+144 H^4 k^4 U+16 H^6 k^6 U\Big)$$
$$\Big(-14\,841\, g^2 H^2+8820\sqrt{3}\ g H\sqrt{g H\left(3+H^2 k^2\right)}\ U-13\,014\, g H U^2-4338\, g H^3 k^2 U^2+$$
$$372\sqrt{3}\ \sqrt{g H\left(3+H^2 k^2\right)}\ U^3+124\sqrt{3}\ H^2 k^2\sqrt{g H\left(3+H^2 k^2\right)}\ U^3+$$
$$135 U^4+90 H^2 k^2 U^4+15 H^4 k^4 U^4\Big)\Big)\text{dt}^4+O[\text{dt}]^5\Big)\text{dx}^4+O[\text{dx}]^5\Big\}$$

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\left\{\left(\frac{\left(H^4 U^3 k^7+6 H^2 U^3 k^5+3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5+9 g H^3 U k^5+9 U^3 k^3+9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^3+27 g H U k^3+3 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} k^3\right) \text{dt}^2}{3 \left(H^2 k^2+3\right)^2}+\frac{i \left(H^4 U^4 k^8+6 H^2 U^4 k^6+4 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6+18 g H^3 U^2 k^6+9 U^4 k^4+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^4+9 g^2 H^2 k^4+54 g H U^2 k^4+12 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U k^4\right) \text{dt}^3}{3 \left(H^2 k^2+3\right)^2}-\frac{11 \left(H^6 U^5 k^{11}+9 H^4 U^5 k^9+5 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^4 k^9+30 g H^5 U^3 k^9+27 H^2 U^5 k^7+30 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^4 k^7+180 g H^3 U^3 k^7+30 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^7+45 g^2 H^4 U k^7+27 U^5 k^5+45 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^4 k^5+270 g H U^3 k^5+90 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5+135 g^2 H^2 U k^5+9 \sqrt{3} g^2 H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^5\right) \text{dt}^4}{60 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)+\left(-\frac{4 H^4 U k^7+24 H^2 U k^5+3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^5+36 U k^3+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k^3}{12 \left(H^2 k^2+3\right)^2}-\frac{i k^4 \left(H^2 U k^2+3 U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(4 H^4 U k^4+24 H^2 U k^2+3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \text{dt}}{12 \left(H^2 k^2+3\right)^3}+\frac{k^5 \left(4 H^4 U k^4+24 H^2 U k^2+3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(H^2 k^2 U^2+3 U^2+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U+3 g H\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)^3}-\frac{i k^6 \left(4 H^4 U k^4+24 H^2 U k^2+3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(H^4 U^3 k^4+6 H^2 U^3 k^2+3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2+9 g H^3 U k^2+9 U^3+9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2+27 g H U+3 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)}\right) \text{dt}^3}{72 \left(H^2 k^2+3\right)^4}+\frac{5 k^7 \left(4 H^4 U k^4+24 H^2 U k^2+3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(H^4 k^4 U^4+6 H^2 k^2 U^4+9 U^4+4 \sqrt{3} H^2 k^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3+18 g H^3 k^2 U^2+54 g H U^2+12 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U+9 g^2 H^2\right) \text{dt}^4}{96 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{16 H^6 U k^{11}+144 H^4 U k^9+5 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} k^9+432 H^2 U k^7+48 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^7+432 U k^5+144 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k^5}{960 \left(H^2 k^2+3\right)^3}+\frac{i \left(208 H^6 U^2 k^{12}+315 g H^5 k^{10}+1872 H^4 U^2 k^{10}+303 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U k^{10}+2592 g H^3 k^8+5616 H^2 U^2 k^8+2112 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^8+5616 U^2 k^6+5616 g H k^6+3744 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^6\right) \text{dt}}{2880 \left(H^2 k^2+3\right)^3}-\frac{\left(k^7 \left(688 H^8 U^3 k^8+8256 H^6 U^3 k^6+1711 \sqrt{3} H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^6+4194 g H^7 U k^6+37152 H^4 U^3 k^4+16413 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^2

k^4+43110 g H^5 U k^4+1125 \sqrt{3} g H^5 \sqrt{g H \left(H^2 k^2+3\right)} k^4+74304 H^2 U^3 k^2+52416 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2+147312 g H^3 U k^2+9072 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} k^2+55728 U^3+55728 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2+167184 g H U+18576 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{5760 \left(H^2 k^2+3\right)^4}-\frac{i k^8 \left(208 H^8 U^4 k^8+2496 H^6 U^4 k^6+729 \sqrt{3} H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6+2857 g H^7 U^2 k^6+1065 g^2 H^6 k^4+11232 H^4 U^4 k^4+6875 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^4+28299 g H^5 U^2 k^4+1649 \sqrt{3} g H^5 \sqrt{g H \left(H^2 k^2+3\right)} U k^4+8496 g^2 H^4 k^2+22464 H^2 U^4 k^2+21552 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^2+92880 g H^3 U^2 k^2+12240 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U k^2+16848 U^4+22464 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3+16848 g^2 H^2+101088 g H U^2+22464 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U\right) \text{dt}^3}{1920 \left(H^2 k^2+3\right)^4}+\frac{1}{4} i \left(\frac{i k^9 \left(16 H^6 U k^6+144 H^4 U k^4+5 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} k^4+432 H^2 U k^2+48 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+432 U+144 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(H^4 k^4 U^4+6 H^2 k^2 U^4+9 U^4+4 \sqrt{3} H^2 k^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3+18 g H^3 k^2 U^2+54 g H U^2+12 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U+9 g^2 H^2\right)}{384 \left(H^2 k^2+3\right)^5}-\frac{i k^9 \left(H^2 U k^2+3 U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(4 H^4 U k^4+24 H^2 U k^2+3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^2}{108 \left(H^2 k^2+3\right)^7}\right) \text{dt}^4+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right),\left(-\frac{4 \left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}\right)}{H^2 k^2+3}-\frac{4 i \left(\sqrt{3} k^2 \sqrt{g H \left(H^2 k^2+3\right)} U-3 g H k^2\right) \text{dt}}{H^2 k^2+3}+\frac{\left(H^4 U^3 k^7+6 H^2 U^3 k^5+9 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5-99 g H^3 U k^5+9 U^3 k^3+27 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^3-297 g H U k^3+69 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} k^3\right) \text{dt}^2}{3 \left(H^2 k^2+3\right)^2}+\frac{i \left(H^4 U^4 k^8+6 H^2 U^4 k^6+2 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6-144 g H^3 U^2 k^6+9 U^4 k^4+6 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^4-441 g^2 H^2 k^4-432 g H U^2 k^4+282 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U k^4\right) \text{dt}^3}{3 \left(H^2 k^2+3\right)^2}+\frac{\left(-11 H^6 U^5 k^{11}-99 H^4 U^5 k^9+95 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^4 k^9+2430 g H^5 U^3 k^9-297 H^2 U^5 k^7+570 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^4 k^7+14580 g H^3 U^3 k^7-12030 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^7+48105 g^2 H^4 U k^7-297 U^5 k^5+855 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^4 k^5+21870 g H U^3 k^5-36090 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^5+144315 g^2 H^2 U k^5-20061 \sqrt{3} g^2 H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^5\right) \text{dt}^4}{60 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)+\left(-\frac{4 H^4 U k^7+24 H^2 U k^5-3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^5+36 U k^3-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k^3}{12 \left(H^2 k^2+3\right)^2}-\frac{i k^4 \left(H^2 U k^2+3 U-3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(4 H^4 U k^4+24 H^2 U k^2-3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \text{dt}}{12 \left(H^2 k^2+3\right)^3}+\frac{k^5 \left(4 H^4 U k^4+24 H^2 U k^2-3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(H^2 k^2 U^2+3 U^2-14 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U+51 g H\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)^3}-\frac{i k^6 \left(4 H^4 U k^4+24 H^2 U k^2-3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(H^4 U^3 k^4+6 H^2 U^3 k^2+51

\sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2−567 g H^3 U k^2+9 U^3+153 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2−1701 g H U+435 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)}\right) \text{dt}^3}{72 \left(H^2 k^2+3\right)^4}+\frac{k^7 \left(4 H^4 U k^4+24 H^2 U k^2−3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U−12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(15 H^4 k^4 U^4+90 H^2 k^2 U^4+135 U^4+124 \sqrt{3} H^2 k^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3+372 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3−4338 g H^3 k^2 U^2−13014 g H U^2+8820 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U−14841 g^2 H^2\right) \text{dt}^4}{288 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{16 H^6 U k^{11}+144 H^4 U k^9−5 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} k^9+432 H^2 U k^7−48 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^7+432 U k^5−144 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k^5}{960 \left(H^2 k^2+3\right)^3}+\frac{i \left(208 H^6 U^2 k^{12}+405 g H^5 k^{10}+1872 H^4 U^2 k^{10}−399 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U k^{10}+3456 g H^3 k^8+5616 H^2 U^2 k^8−2688 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U k^8+5616 U^2 k^6+8208 g H k^6−4608 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^6\right) \text{dt}}{2880 \left(H^2 k^2+3\right)^3}−\frac{\left(k^7 \left(688 H^8 U^3 k^8+8256 H^6 U^3 k^6−3567 \sqrt{3} H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^6+12798 g H^7 U k^6+37152 H^4 U^3 k^4−33117 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^4+128250 g H^5 U k^4−4005 \sqrt{3} g H^5 \sqrt{g H \left(H^2 k^2+3\right)} k^4+74304 H^2 U^3 k^2−102528 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2+430704 g H^3 U k^2−33264 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} k^2+55728 U^3−105840 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2+483408 g H U−73872 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{5760 \left(H^2 k^2+3\right)^4}−\frac{i k^8 \left(624 H^8 U^4 k^8+7488 H^6 U^4 k^6−8171 \sqrt{3} H^6 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^6+64677 g H^7 U^2 k^6+48645 g^2 H^6 k^4+33696 H^4 U^4 k^4−74481 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^4+615807 g H^5 U^2 k^4−58035 \sqrt{3} g H^5 \sqrt{g H \left(H^2 k^2+3\right)} U k^4+399600 g^2 H^4 k^2+67392 H^2 U^4 k^2−226224 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3 k^2+1956528 g H^3 U^2 k^2−417456 \sqrt{3} g H^3 \sqrt{g H \left(H^2 k^2+3\right)} U k^2+50544 U^4−228960 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3+861840 g^2 H^2+2073600 g H U^2−766368 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U\right) \text{dt}^3}{5760 \left(H^2 k^2+3\right)^4}+\frac{1}{4} i \left(\frac{i k^9 \left(16 H^6 U k^6+144 H^4 U k^4−5 \sqrt{3} H^4 \sqrt{g H \left(H^2 k^2+3\right)} k^4+432 H^2 U k^2−48 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+432 U−144 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(15 H^4 k^4 U^4+90 H^2 k^2 U^4+135 U^4+124 \sqrt{3} H^2 k^2 \sqrt{g H \left(H^2 k^2+3\right)} U^3+372 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^3−4338 g H^3 k^2 U^2−13014 g H U^2+8820 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)} U−14841 g^2 H^2\right)}{5760 \left(H^2 k^2+3\right)^5}−\frac{i k^9 \left(4 H^4 U k^4+24 H^2 U k^2−3 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} k^2+36 U−12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^2 \left(H^4 U^3 k^4+6 H^2 U^3 k^2−51 \sqrt{3} H^2 \sqrt{g H \left(H^2 k^2+3\right)} U^2 k^2+549 g H^3 U k^2+9 U^3−153 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2+1647 g H U−447 \sqrt{3} g H \sqrt{g H \left(H^2 k^2+3\right)}\right)}{108 \left(H^2 k^2+3\right)^6}\right) \text{dt}^4+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right\}