

In[2649]:= **Text[Row[{" dt error for all Fnn"}]]**

$$\mathbf{FGndt} = -\frac{dt^2 k}{2 w} \left(gH - \frac{3 U^2}{(3 + H^2 k^2)} \right)$$

Out[2649]= dt error for all Fnn

$$\text{Out[2650]} = -\frac{1}{2} dt^2 k \left(gH - \frac{3 U^2}{3 + H^2 k^2} \right) w$$

In[2651]:=

Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H] " }]]

$$\mathbf{FGn1FDdxdt} = -\frac{1}{2} \left(\sqrt{g H} k^2 U \right) dt * dx$$

Text[Row[{" U > Sqrt[g*H] " }]]

$$\mathbf{FGn1FDdxdt1} = -\frac{1}{2} (g H k^2) dt dx$$

Text[Row[{" U< -Sqrt[g*H] " }]]

$$\mathbf{FGn1FDdxdt2} = \frac{1}{2} g H k^2 dt dx$$

Out[2651]= -Sqrt[g*H] < U < Sqrt[g*H]

$$\text{Out[2652]} = -\frac{1}{2} dt dx \sqrt{g H} k^2 U$$

Out[2653]= U > Sqrt[g*H]

$$\text{Out[2654]} = -\frac{1}{2} dt dx g H k^2$$

Out[2655]= U< -Sqrt[g*H]

$$\text{Out[2656]} = \frac{1}{2} dt dx g H k^2$$

In[2657]:=

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Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]]
FGn2FDdxdt = - 
$$\frac{i \left( 9 g H k^3 + 6 g H^3 k^5 + g H^5 k^7 + 18 k^3 U^2 + 3 H^2 k^5 U^2 \right) dt}{12 \left( 3 + H^2 k^2 \right)^2} * dx^2;$$

Expand[ $\left( 3 + H^2 k^2 \right)^2$ ];
FGn2FDdxdtRed = -  $\left( i * dt * k^3 * dx^2 / 12 \right) * \left( gH + \frac{U^2 \left( 18 + 3 H^2 k^2 \right)}{\left( 3 + H^2 k^2 \right)^2} \right)$ 
Text[Row[{" U > Sqrt[g*H]  "}]]
FGn2FDdxdt1 = - 
$$\frac{i \left( 9 g H k^3 + 6 g H^3 k^5 + g H^5 k^7 + 18 k^3 U^2 + 3 H^2 k^5 U^2 \right) dt}{12 \left( 3 + H^2 k^2 \right)^2} * dx^2;$$

FGn2FDdxdtRed1 = -  $\left( i * dt * k^3 * dx^2 / 12 \right) * \left( gH + \frac{U^2 \left( 18 + 3 H^2 k^2 \right)}{\left( 3 + H^2 k^2 \right)^2} \right)$ 
Text[Row[{" U< -Sqrt[g*H]  "}]]
FGn2FDdxdt2 = - 
$$\frac{i \left( 9 g H k^3 + 6 g H^3 k^5 + g H^5 k^7 + 18 k^3 U^2 + 3 H^2 k^5 U^2 \right) dt}{12 \left( 3 + H^2 k^2 \right)^2} * dx^2;$$

FGn2FDdxdtRed2 = -  $\left( i * dt * k^3 * dx^2 / 12 \right) * \left( gH + \frac{U^2 \left( 18 + 3 H^2 k^2 \right)}{\left( 3 + H^2 k^2 \right)^2} \right)$ 

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Out[2657]= $-Sqrt[g*H] < U < Sqrt[g*H]$ Out[2660]=
$$-\frac{1}{12} dt dx^2 i k^3 \left(gH + \frac{\left(18 + 3 H^2 k^2 \right) U^2}{\left(3 + H^2 k^2 \right)^2} \right)$$
Out[2661]= $U > Sqrt[g*H]$ Out[2663]=
$$-\frac{1}{12} dt dx^2 i k^3 \left(gH + \frac{\left(18 + 3 H^2 k^2 \right) U^2}{\left(3 + H^2 k^2 \right)^2} \right)$$
Out[2664]= $U < -Sqrt[g*H]$ Out[2666]=
$$-\frac{1}{12} dt dx^2 i k^3 \left(gH + \frac{\left(18 + 3 H^2 k^2 \right) U^2}{\left(3 + H^2 k^2 \right)^2} \right)$$

In[2667]:=

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Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]]
FGn2FEMdxdt =
  - ( (i (90 g H k^3 + 60 g H^3 k^5 + 10 g H^5 k^7 - 36 k^3 U^2 - 15 H^2 k^5 U^2) dt) / (120 (3 + H^2 k^2)^2) ) *
    dx^2;
Expand[ (3 + H^2 k^2)^2 ];
FGn2FEMdxdtRed = - (i * dt * k^3 * dx^2 / 12) * (gH - (U^2 (36 + 15 H^2 k^2) / (10 (3 + H^2 k^2)^2)))

Text[Row[{" U > Sqrt[g*H]  "}]]
FGn2FEMdxdt1 =
  - ( (i (90 g H k^3 + 60 g H^3 k^5 + 10 g H^5 k^7 - 36 k^3 U^2 - 15 H^2 k^5 U^2) dt) / (120 (3 + H^2 k^2)^2) ) *
    dx^2;
FGn2FEMdxdtRed1 = - (i * dt * k^3 * dx^2 / 12) * (gH - (U^2 (36 + 15 H^2 k^2) / (10 (3 + H^2 k^2)^2)))

Text[Row[{" U< -Sqrt[g*H]  "}]]
FGn2FEMdxdt2 =
  - ( (i (90 g H k^3 + 60 g H^3 k^5 + 10 g H^5 k^7 - 36 k^3 U^2 - 15 H^2 k^5 U^2) dt) / (120 (3 + H^2 k^2)^2) ) *
    dx^2;
FGn2FEMdxdtRed2 = - (i * dt * k^3 * dx^2 / 12) * (gH - (U^2 (36 + 15 H^2 k^2) / (10 (3 + H^2 k^2)^2)))

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Out[2667]= $-\text{Sqrt}[g*H] < U < \text{Sqrt}[g*H]$

$$\text{Out[2670]} = -\frac{1}{12} dt dx^2 i k^3 \left(gH - \frac{(36 + 15 H^2 k^2) U^2}{10 (3 + H^2 k^2)^2} \right)$$

Out[2671]= $U > \text{Sqrt}[g*H]$

$$\text{Out[2673]} = -\frac{1}{12} dt dx^2 i k^3 \left(gH - \frac{(36 + 15 H^2 k^2) U^2}{10 (3 + H^2 k^2)^2} \right)$$

Out[2674]= $U < -\text{Sqrt}[g*H]$

$$\text{Out[2676]} = -\frac{1}{12} dt dx^2 i k^3 \left(gH - \frac{(36 + 15 H^2 k^2) U^2}{10 (3 + H^2 k^2)^2} \right)$$

In[2677]:=

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Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]]
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$$\text{FGn3FDdxdt} = -\frac{1}{12} \left(\sqrt{g H} k^4 U \right) dt * dx^3$$

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Text[Row[{" U > Sqrt[g*H]  "}]]
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$$\text{FGn3FDdxdt1} = -\frac{1}{12} \left(k^4 U \right) dt * dx^3$$

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Text[Row[{" U< -Sqrt[g*H]  "}]]
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$$\text{FGn3FDdxdt2} = \frac{1}{12} k^4 U dt * dx^3$$

Out[2677]= $-\text{Sqrt}[g*H] < U < \text{Sqrt}[g*H]$ Out[2678]= $-\frac{1}{12} dt dx^3 \sqrt{g H} k^4 U$ Out[2679]= $U > \text{Sqrt}[g*H]$ Out[2680]= $-\frac{1}{12} dt dx^3 k^4 U$ Out[2681]= $U < -\text{Sqrt}[g*H]$ Out[2682]= $\frac{1}{12} dt dx^3 k^4 U$