In[1668]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx / 2, xj + dx / 2}] / (dx);
qjnbar = qjbar /. t → tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x → xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx / 2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / 3 * k^2;
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca;
fn1Gca0A = fn1A /. Gca → 0;
fn1eca0A = fn1A /. eca → 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca /. Gh → RA * Gca;
fG1Gca0A = fG1A /. Gca → 0;
fG1eca0A = fG1A /. eca → 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
```

$$wAp = U * k + \frac{\sqrt{3}\ k \sqrt{g H \left(3 + H^2 k^2\right)}}{3 + H^2 k^2};$$

$$wAm = U * k - \frac{\sqrt{3}\ k \sqrt{g H \left(3 + H^2 k^2\right)}}{3 + H^2 k^2};$$

In[1703]:=
```
M = (26 - 2 * Cos[k * dx]) / 24
Series[M - MA, {dx, 0, 10}]
```

Out[1703]= $\frac{1}{24}\ (26 - 2\ \text{Cos}[\text{dx k}])$

Out[1704]= $-\frac{3\ k^4\ dx^4}{640} + \frac{3\ k^6\ dx^6}{35\,840} - \frac{149\ k^8\ dx^8}{51\,609\,600} + \frac{29\ k^{10}\ dx^{10}}{13\,624\,934\,400} + O[dx]^{11}$

In[1705]:=
```
Rm = (5 - Exp[-I * k * dx] + 2 * Exp[I * k * dx]) / 6
Series[Rm - RA, {dx, 0, 10}]
Rp = Exp[I * k * dx] * (5 + 2 * Exp[-I * k * dx] - Exp[I * k * dx]) / 6
Series[Rp - RA, {dx, 0, 10}]
Ru = (-Exp[-I * k * dx] + 9 * Exp[I * k * dx] - Exp[2 * I * k * dx] + 9) / 16
Series[Ru - Exp[I * k * dx / 2], {dx, 0, 10}]
```

Out[1705]= $\frac{1}{6}\ \left(5 - e^{-i\ \text{dx k}} + 2\ e^{i\ \text{dx k}}\right)$

Out[1706]= $-\frac{1}{12}\ i\ k^3\ dx^3 + \frac{k^4\ dx^4}{120} + \frac{1}{240}\ i\ k^5\ dx^5 - \frac{k^6\ dx^6}{5040} - \frac{i\ k^7\ dx^7}{10\,080} + \frac{k^8\ dx^8}{201\,600} + \frac{i\ k^9\ dx^9}{725\,760} - \frac{k^{10}\ dx^{10}}{39\,916\,800} + O[dx]^{11}$

Out[1707]= $\frac{1}{6}\ e^{i\ \text{dx k}}\ \left(5 + 2\ e^{-i\ \text{dx k}} - e^{i\ \text{dx k}}\right)$

Out[1708]= $\frac{1}{12}\ i\ k^3\ dx^3 - \frac{3\ k^4\ dx^4}{40} - \frac{3}{80}\ i\ k^5\ dx^5 + \frac{23\ k^6\ dx^6}{1680} + \frac{41\ i\ k^7\ dx^7}{10\,080} - \frac{209\ k^8\ dx^8}{201\,600} - \frac{169\ i\ k^9\ dx^9}{725\,760} + \frac{89\ k^{10}\ dx^{10}}{1\,900\,800} + O[dx]^{11}$

Out[1709]= $\frac{1}{16}\ \left(9 - e^{-i\ \text{dx k}} + 9\ e^{i\ \text{dx k}} - e^{2\ i\ \text{dx k}}\right)$

Out[1710]= $-\frac{3\ k^4\ dx^4}{128} - \frac{3}{256}\ i\ k^5\ dx^5 + \frac{5\ k^6\ dx^6}{1024} + \frac{3\ i\ k^7\ dx^7}{2048} - \frac{63\ k^8\ dx^8}{163\,840} - \frac{17\ i\ k^9\ dx^9}{196\,608} + \frac{289\ k^{10}\ dx^{10}}{16\,515\,072} + O[dx]^{11}$

In[1711]:=
```
Gold = H - H^3 / 3 * (32 * Cos[k * dx] - 2 * Cos[2 * k * dx] - 30) / (12 * dx^2);
GG = M * Ru / (Gold)
Series[GG - GGA, {dx, 0, 5}]
Gn = -M * Ru * U / (Gold)
Series[Gn - GnA, {dx, 0, 5}]
```

Out[1712]=
$$\frac{\left(9 - e^{-i\,dx\,k} + 9\,e^{i\,dx\,k} - e^{2\,i\,dx\,k}\right)\,(26 - 2\,\text{Cos}[dx\,k])}{384\left(H - \frac{H^3\,(-30+32\,\text{Cos}[dx\,k]-2\,\text{Cos}[2\,dx\,k])}{36\,dx^2}\right)}$$

Out[1713]=
$$\frac{\left(-243\,k^4 - 49\,H^2\,k^6\right)\,dx^4}{960\,H\left(3 + H^2\,k^2\right)^2} - \frac{i\,\left(243\,k^5 + 49\,H^2\,k^7\right)\,dx^5}{1920\,H\left(3 + H^2\,k^2\right)^2} + O[dx]^6$$

Out[1714]=
$$\frac{\left(9 - e^{-i\,dx\,k} + 9\,e^{i\,dx\,k} - e^{2\,i\,dx\,k}\right)\,U\,(-26 + 2\,\text{Cos}[dx\,k])}{384\left(H - \frac{H^3\,(-30+32\,\text{Cos}[dx\,k]-2\,\text{Cos}[2\,dx\,k])}{36\,dx^2}\right)}$$

Out[1715]=
$$\frac{\left(243\,k^4 + 49\,H^2\,k^6\right)\,U\,dx^4}{960\,H\left(3 + H^2\,k^2\right)^2} + \frac{i\,\left(243\,k^5 + 49\,H^2\,k^7\right)\,U\,dx^5}{1920\,H\left(3 + H^2\,k^2\right)^2} + O[dx]^6$$

In[1716]:=
```
Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]]
Text[Row[{"Fnn and FnG "}]]
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnn = FullSimplify[KurFWSeta /. G → 0 /. n → 1]
KfnG = FullSimplify[KurFWSeta /. n → 0 /. G → 1]
Kfnn = Kfnn /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;
KfnG = KfnG /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]]
KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGn = FullSimplify[KurFWSG /. G → 0 /. n → 1]
KfGG = FullSimplify[KurFWSG /. n → 0 /. G → 1]
KfGn = KfGn /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;
KfGG = KfGG /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 ^2 / 2 + EigvFmat2^3 / 6 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ]
```

Out[1716]= $-\text{Sqrt}[g*H] < U < \text{Sqrt}[g*H]$

Out[1717]= Fnn and FnG

Out[1722]= $\frac{1}{2}\left(2\,\text{Gnp}\,H + \text{Rpp}\left(-\sqrt{g\,H} + U\right) + \text{Rmp}\left(\sqrt{g\,H} + U\right)\right)$

Out[1723]= GGp H

Out[1728]= $\left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) + \left(-\frac{1}{12}\left(\sqrt{g\,H}\,k^4\right)\,dt + O[dt]^4\right)\,dx^3 +$
$\left(\frac{i\left(45\,k^5\,U + 143\,H^2\,k^7\,U + 32\,H^4\,k^9\,U\right)\,dt}{960\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)\,dx^4 + O[dx]^5$

Out[1731]= $\left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)} + O[dt]^4\right) + \left(\frac{i\left(243\,k^5 + 49\,H^2\,k^7\right)\,dt}{960\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)\,dx^4 + O[dx]^5$

Out[1732]= FGn and FGG

Out[1735]= $\frac{1}{2}\left(g\,H\,(\text{Rmp}+\text{Rpp}) + \left(2\,\text{Gnp}\,H + \sqrt{g\,H}\,(\text{Rmp}-\text{Rpp})\right)\,U\right)$

Out[1736]= $\frac{1}{2}\left(\sqrt{g\,H}\,\text{Rmp} - \sqrt{g\,H}\,\text{Rpp} + (2\,\text{GGp}\,H + \text{Rmp} + \text{Rpp})\,U\right)$

Out[1741]= $\left(-\frac{\left(k\left(3\,g\,H + g\,H^3\,k^2 - 3\,U^2\right)\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\left(3\,g\,H + g\,H^3\,k^2 - 3\,U^2\right)\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{1}{12}\left(\sqrt{g\,H}\,k^4\,U\right)\,dt + O[dt]^4\right)\,dx^3 +$
$\left(\left(i\left(288\,g\,H\,k^5 + 192\,g\,H^3\,k^7 + 32\,g\,H^5\,k^9 - 243\,k^5\,U^2 - 49\,H^2\,k^7\,U^2\right)\,dt\right) \Big/ \left(960\left(3+H^2\,k^2\right)^2\right) +$
$O[dt]^4\right)\,dx^4 + O[dx]^5$

Out[1745]= $\left(-\frac{\left(k\left(6+H^2\,k^2\right)\,U\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\left(6+H^2\,k^2\right)\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) + \left(-\frac{1}{12}\left(\sqrt{g\,H}\,k^4\right)\,dt + O[dt]^4\right)\,dx^3 +$
$\left(\frac{i\left(531\,k^5\,U + 241\,H^2\,k^7\,U + 32\,H^4\,k^9\,U\right)\,dt}{960\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)\,dx^4 + O[dx]^5$

Out[1746]= W : omega

Out[1751]= $\left\{\left(-\left(\left(i\,k^4\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)\,U\right)\left(3\,g\left(\sqrt{3}\,H\sqrt{g\,H\left(3+H^2\,k^2\right)} + 9\,H\,U + 3\,H^3\,k^2\,U\right) + U^2\left(H^4\,k^4\,U + 9\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + U\right) + 3\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + 2\,H^2\,U\right)\right)\right)\,dt^3\right) \Big/ \left(24\left(3+H^2\,k^2\right)^3\right)\right) + \left(k^5\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)\,U\right)\left(9\,g^2\,H^2 + 6\,g\,H\,U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\left(3+H^2\,k^2\right)\,U\right) + U^3\left(12\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 9\,U + H^4\,k^4\,U + 2\,k^2\left(2\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + 3\,H^2\,U\right)\right)\right)\,dt^4\right) \Big/ \left(30\left(3+H^2\,k^2\right)^3\right) + O[dt]^5\right) +$

$$\left(-\frac{i\,k^4\left(2\,g\,H\left(3+H^2\,k^2\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)}{24\,\sqrt{g\,H}\,\left(3+H^2\,k^2\right)}-\right.$$

$$\left(\left(k^7\left(6\sqrt{3}\,g^2\,H^2\left(3+H^2\,k^2\right)+9\,U^3\left(5\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\,U\right)+\right.\right.\right.$$

$$k^4\,U^3\left(2\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^4\,U\right)+$$

$$3\,g\,H\,U\left(21\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\,\left(27+15\,H^2\,k^2+2\,H^4\,k^4\right)U\right)+$$

$$\left.\left.\left.3\,k^2\left(6\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+7\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3+2\sqrt{3}\,H^2\,U^4\right)\right)\right)\,dt^3\Big)\Big/$$

$$\left(144\left(3+H^2\,k^2\right)^{5/2}\right)-\frac{1}{144\left(3+H^2\,k^2\right)^{5/2}}i\,k^8\left(3\sqrt{3}\,g^2\,H^2\left(27+8\,H^2\,k^2\right)U+\right.$$

$$k^4\,U^4\left(2\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^4\,U\right)+$$

$$2\,g\,H\,U^2\left(72\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\,\left(63+33\,H^2\,k^2+4\,H^4\,k^4\right)U\right)+$$

$$9\left(2\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+6\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+\sqrt{3}\,U^5\right)+$$

$$\left.\left.6\,k^2\left(6\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2+4\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4+\sqrt{3}\,H^2\,U^5\right)\right)\,dt^4+O[dt]^5\right)\,dx^3+$$

$$\left(-\left(\left(k^5\left(531\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+1728\,U+192\,H^4\,k^4\,U+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.k^2\left(145\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+1152\,H^2\,U\right)\right)\right)\Big/\left(5760\left(3+H^2\,k^2\right)^2\right)\right)+$$

$$\frac{1}{34\,560\,\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}}i\,k^8\left(k^6\,U^3\left(721\sqrt{3}\,g\,H^7+192\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U\right)+\right.$$

$$9\,k^2\left(145\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}+1350\sqrt{3}\,g^2\,H^4\,U+\right.$$

$$2118\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2+2227\sqrt{3}\,g\,H^3\,U^3+576\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\Big)+$$

$$81\left(59\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+241\sqrt{3}\,g^2\,H^2\,U+64\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+\right.$$

$$\left.g\,H\,U^2\left(369\sqrt{g\,H\left(3+H^2\,k^2\right)}+251\sqrt{3}\,U\right)\right)+$$

$$3\,k^4\,U\left(627\sqrt{3}\,g^2\,H^6+576\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+\right.$$

$$\left.\left.\left.g\,H^5\,U\left(1011\sqrt{g\,H\left(3+H^2\,k^2\right)}+2195\sqrt{3}\,U\right)\right)\right)\,dt^3-\frac{1}{34\,560\left(\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}\right)}$$

$$\left(k^9\left(9\sqrt{3}\,g^3\,H^3\left(531+145\,H^2\,k^2\right)+54\sqrt{3}\,g^2\,H^2\left(915+578\,H^2\,k^2+91\,H^4\,k^4\right)U^2+\right.\right.$$

$$g\,H\,U^3\left(25\,227\sqrt{3}\,H^2\,k^2\,U+913\sqrt{3}\,H^6\,k^6\,U+405\left(124\sqrt{g\,H\left(3+H^2\,k^2\right)}+63\sqrt{3}\,U\right)+\right.$$

$$\left.\left.3\,k^4\left(1732\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+2771\sqrt{3}\,H^4\,U\right)\right)+$$

$$12\,U\left(2025\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+432\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+144\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right.$$

$$U^4 + 16\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\;U^4 + 3\,k^2\left(193\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}\;+ 898\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U^2 + 144\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^4\right)\Big)\Big)\Big)\,dt^4 + O[dt]^5\Big)$$

$$dx^4 + O[dx]^5,\ \Big(-\Big(\Big(i\,k^4\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\right)\left(3\,g\,H\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\left(3+H^2\,k^2\right)U\right) + U^2\left(-9\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)} + 9\,U + H^4\,k^4\,U - 3\,k^2\left(\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)} - 2\,H^2\,U\right)\right)\right)dt^3\Big)\Big/\left(24\left(3+H^2\,k^2\right)^3\right)\Big) + \Big(k^5\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\right)\left(9\,g^2\,H^2 + 6\,g\,H\,U\left(-2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\left(3+H^2\,k^2\right)U\right) + U^3\left(-12\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)} + 9\,U + H^4\,k^4\,U + k^2\left(-4\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + 6\,H^2\,U\right)\right)\right)dt^4\Big)\Big/\left(30\left(3+H^2\,k^2\right)^3\right) + O[dt]^5\Big) +$$

$$\Bigg(-\frac{i\,k^4\left(2\,g\,H\left(3+H^2\,k^2\right) - \sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U\right)}{24\sqrt{g\,H}\;\left(3+H^2\,k^2\right)} +$$

$$\Big(k^7\left(6\sqrt{3}\;g^2\,H^2\left(3+H^2\,k^2\right) + 9\,U^3\left(-5\sqrt{g\,H\left(3+H^2\,k^2\right)} + \sqrt{3}\;U\right) + k^4\,U^3\left(-2\sqrt{g\,H^9\left(3+H^2\,k^2\right)} + \sqrt{3}\;H^4\,U\right) + 3\,g\,H\,U\left(-21\sqrt{g\,H\left(3+H^2\,k^2\right)} + \sqrt{3}\;\left(27+15\,H^2\,k^2+2\,H^4\,k^4\right)U\right) - 3\,k^2\left(6\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U + 7\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^3 - 2\sqrt{3}\;H^2\,U^4\right)\right)dt^3\Big)\Big/\left(144\left(3+H^2\,k^2\right)^{5/2}\right) + \frac{1}{144\left(3+H^2\,k^2\right)^{5/2}}$$

$$i\,k^8\left(3\sqrt{3}\;g^2\,H^2\left(27+8\,H^2\,k^2\right)U + k^4\,U^4\left(-2\sqrt{g\,H^9\left(3+H^2\,k^2\right)} + \sqrt{3}\;H^4\,U\right) + 2\,g\,H\,U^2\left(-72\sqrt{g\,H\left(3+H^2\,k^2\right)} + \sqrt{3}\;\left(63+33\,H^2\,k^2+4\,H^4\,k^4\right)U\right) + 9\left(-2\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)} - 6\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4 + \sqrt{3}\;U^5\right) - 6\,k^2\left(6\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U^2 + 4\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^4 - \sqrt{3}\;H^2\,U^5\right)\Big)\,dt^4 + O[dt]^5\Bigg)\,dx^3 +$$

$$\Bigg(\Big(k^5\left(531\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)} - 1728\,U - 192\,H^4\,k^4\,U + k^2\left(145\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)} - 1152\,H^2\,U\right)\right)\Big)\Big/\left(5760\left(3+H^2\,k^2\right)^2\right) + \frac{1}{34\,560\sqrt{g\,H}\;\left(3+H^2\,k^2\right)^{7/2}}i\,k^8\Big(k^6\,U^3\left(-721\sqrt{3}\;g\,H^7 + 192\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\;U\right) + 9\,k^2\left(145\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)} - 1350\sqrt{3}\;g^2\,H^4\,U +$$

$$2118\sqrt{g^3 H^7 \left(3+H^2 k^2\right)}\,U^2 - 2227\sqrt{3}\,g H^3 U^3 + 576\sqrt{g H^5 \left(3+H^2 k^2\right)}\,U^4\Big) +$$
$$81\left(59\sqrt{g^5 H^5 \left(3+H^2 k^2\right)} - 241\sqrt{3}\,g^2 H^2 U + 64\sqrt{g H \left(3+H^2 k^2\right)}\,U^4 +\right.$$
$$g H U^2\left(369\sqrt{g H \left(3+H^2 k^2\right)} - 251\sqrt{3}\,U\right)\Big) - 3 k^4 U\left(627\sqrt{3}\,g^2 H^6 -\right.$$
$$\left.\left.576\sqrt{g H^9 \left(3+H^2 k^2\right)}\,U^3 + g H^5 U\left(-1011\sqrt{g H \left(3+H^2 k^2\right)} + 2195\sqrt{3}\,U\right)\right)\right) dt^3 +$$
$$\frac{1}{34\,560\sqrt{g H}\left(3+H^2 k^2\right)^{7/2}} k^9\left(9\sqrt{3}\,g^3 H^3\left(531+145 H^2 k^2\right) +\right.$$
$$54\sqrt{3}\,g^2 H^2\left(915+578 H^2 k^2+91 H^4 k^4\right)U^2 +$$
$$g H U^3\left(25\,227\sqrt{3}\,H^2 k^2 U + 913\sqrt{3}\,H^6 k^6 U + 405\left(-124\sqrt{g H \left(3+H^2 k^2\right)} + 63\sqrt{3}\,U\right) +\right.$$
$$\left.3 k^4\left(-1732\sqrt{g H^9 \left(3+H^2 k^2\right)} + 2771\sqrt{3}\,H^4 U\right)\right) -$$
$$12 U\left(2025\sqrt{g^5 H^5 \left(3+H^2 k^2\right)} + 432\sqrt{g H \left(3+H^2 k^2\right)}\,U^4 + 144 k^4\sqrt{g H^9 \left(3+H^2 k^2\right)}\,U^4 +\right.$$
$$16 k^6\sqrt{g H^{13}\left(3+H^2 k^2\right)}\,U^4 + 3 k^2\left(193\sqrt{g^5 H^9\left(3+H^2 k^2\right)} + 898\sqrt{g^3 H^7\left(3+H^2 k^2\right)}\right.$$
$$\left.\left.\left.U^2 + 144\sqrt{g H^5\left(3+H^2 k^2\right)}\,U^4\right)\right)\right) dt^4 + O[dt]^5\Big)\,dx^4 + O[dx]^5\Big\}$$

In[1752]:=
```
Text[Row[{" U > Sqrt[g*H]  "}]]
Text[Row[{"Fnn and FnG "}]]
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (0);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnn = FullSimplify[KurFWSeta /. G → 0 /. n → 1]
KfnG = FullSimplify[KurFWSeta /. n → 0 /. G → 1]
Kfnn = Kfnn /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;
KfnG = KfnG /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]]
KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGn = FullSimplify[KurFWSG /. G → 0 /. n → 1]
KfGG = FullSimplify[KurFWSG /. n → 0 /. G → 1]
KfGn = KfGn /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;
KfGG = KfGG /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2 + EigvFmat2 ^2 / 2 + EigvFmat2^3 / 6] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}]
```

Out[1752]= U > Sqrt[g*H]

Out[1753]= Fnn and FnG

Out[1758]= Gnp H + Rmp U

Out[1759]= GGp H

Out[1764]= $$\left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{1}{12}\,\left(k^4\,U\right)\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(45\,k^5\,U+143\,H^2\,k^7\,U+32\,H^4\,k^9\,U\right)\,dt}{960\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1767]= $$\left(-\frac{3\,(k\,w)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(\frac{i\,\left(243\,k^5+49\,H^2\,k^7\right)\,dt}{960\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1768]= FGn and FGG

Out[1771]= H (g Rmp + Gnp U)

Out[1772]= (GGp H + Rmp) U

Out[1777]= $$\left(-\frac{\left(k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{1}{12}\,\left(g\,H\,k^4\right)\,dt+O[dt]^4\right)\,dx^3+\left(\left(i\,\left(288\,g\,H\,k^5+192\,g\,H^3\,k^7+32\,g\,H^5\,k^9-243\,k^5\,U^2-49\,H^2\,k^7\,U^2\right)\,dt\right)\Big/\left(960\,\left(3+H^2\,k^2\right)^2\right)+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1781]= $$\left(-\frac{\left(k\,\left(6+H^2\,k^2\right)\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(6+H^2\,k^2\right)\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{1}{12}\,\left(k^4\,U\right)\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(531\,k^5+241\,H^2\,k^7+32\,H^4\,k^9\right)\,U\,dt}{960\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1782]= W : omega

Out[1787]= $$\left\{\left(-\frac{i\,\left(\sqrt{3}\,k\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+3\,k\,U+H^2\,k^3\,U\right)^4\,dt^3}{24\,\left(3+H^2\,k^2\right)^4}+\frac{\left(\sqrt{3}\,k\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+3\,k\,U+H^2\,k^3\,U\right)^5\,dt^4}{30\,\left(3+H^2\,k^2\right)^5}+O[dt]^5\right)+\left(-\frac{1}{24}\,i\,k^4\,\left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}}+2\,U\right)-\left(\left(k^7\,\left(9\,g^2\,H^2+3\,g\,H\,U\,\left(5\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+9\,\left(3+H^2\,k^2\right)\,U\right)+U^3\,\left(21\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\,\left(7\,\sqrt{3}\,\sqrt{g\,H^5\,\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)\right)\right)$$

$$dt^3\Big)\Big/\Big(144\left(3+H^2k^2\right)^2\Big)-\Big(i\,k^8\Big(3\,g\,H+U\Big(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\Big)\Big)$$
$$\Big(3\,g\,H\Big(\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+4\left(3+H^2k^2\right)U\Big)+U^2\Big(15\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+18\,U+$$
$$2\,H^4k^4\,U+k^2\Big(5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2k^2\right)}+12\,H^2U\Big)\Big)\Big)\,dt^4\Big)\Big/\Big(144\left(3+H^2k^2\right)^3\Big)+$$
$$O[dt]^5\Big)\,dx^3+\Bigg(-\Big(\Big(k^5\Big(531\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+1728\,U+192\,H^4k^4\,U+$$
$$k^2\Big(145\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2k^2\right)}+1152\,H^2U\Big)\Big)\Big)\Big/\Big(5760\left(3+H^2k^2\right)^2\Big)\Big)+$$
$$\frac{1}{34\,560\sqrt{g\,H}\,\left(3+H^2k^2\right)^{7/2}}i\,k^8\Big(k^6U^3\Big(721\sqrt{3}\,g\,H^7+192\sqrt{g\,H^{13}\left(3+H^2k^2\right)}\,U\Big)+$$
$$9\,k^2\Big(145\sqrt{g^5H^9\left(3+H^2k^2\right)}+1350\sqrt{3}\,g^2H^4U+$$
$$2118\sqrt{g^3H^7\left(3+H^2k^2\right)}\,U^2+2227\sqrt{3}\,g\,H^3U^3+576\sqrt{g\,H^5\left(3+H^2k^2\right)}\,U^4\Big)+$$
$$81\Big(59\sqrt{g^5H^5\left(3+H^2k^2\right)}+241\sqrt{3}\,g^2H^2U+64\sqrt{g\,H\left(3+H^2k^2\right)}\,U^4+$$
$$g\,H\,U^2\Big(369\sqrt{g\,H\left(3+H^2k^2\right)}+251\sqrt{3}\,U\Big)\Big)+3\,k^4U\Big(627\sqrt{3}\,g^2H^6+$$
$$576\sqrt{g\,H^9\left(3+H^2k^2\right)}\,U^3+g\,H^5U\Big(1011\sqrt{g\,H\left(3+H^2k^2\right)}+2195\sqrt{3}\,U\Big)\Big)\Big)\,dt^3-$$
$$\frac{1}{34\,560\left(\sqrt{g\,H}\,\left(3+H^2k^2\right)^{7/2}\right)}\Big(k^9\Big(3\,g\,H+U\Big(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\Big)\Big)$$
$$\Big(3\sqrt{3}\,g^2H^2\left(531+145\,H^2k^2\right)+$$
$$g\,H\,U\Big(4914\sqrt{g\,H\left(3+H^2k^2\right)}+\sqrt{3}\,\left(5049+3270\,H^2k^2+529\,H^4k^4\right)U\Big)+$$
$$6\Big(288\sqrt{g\,H\left(3+H^2k^2\right)}\,U^3+32\,k^4\sqrt{g\,H^9\left(3+H^2k^2\right)}\,U^3+$$
$$k^2\Big(241\sqrt{g^3H^7\left(3+H^2k^2\right)}\,U+192\sqrt{g\,H^5\left(3+H^2k^2\right)}\,U^3\Big)\Big)\Big)\,dt^4+O[dt]^5\Big)\,dx^4+$$
$$O[dx]^5,\Bigg(-\frac{i\left(-\sqrt{3}\,k\sqrt{g\,H\left(3+H^2k^2\right)}+3\,k\,U+H^2k^3U\right)^4dt^3}{24\left(3+H^2k^2\right)^4}+$$
$$\frac{\left(-\sqrt{3}\,k\sqrt{g\,H\left(3+H^2k^2\right)}+3\,k\,U+H^2k^3U\right)^5dt^4}{30\left(3+H^2k^2\right)^5}+$$
$$O[dt]^5\Bigg)+$$
$$\Bigg(\frac{1}{24}i\,k^4\Big(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2k^2}}-2\,U\Big)-$$
$$\Big(\Big(k^7\Big(9\,g^2H^2+3\,g\,H\,U\Big(-5\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+9\left(3+H^2k^2\right)U\Big)+$$

$$U^3 \left(-21\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(-7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\Big)\Big)\,dt^3\Big)\Big/\left(144\left(3+H^2\,k^2\right)^2\right)-\left(i\,k^8\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\left(-3\,g\,H\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-4\left(3+H^2\,k^2\right)U\right)+U^2\left(-15\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(-5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)\,dt^4\Big)\Big/\left(144\left(3+H^2\,k^2\right)^3\right)+O[dt]^5\Big)\,dx^3+\Bigg(\left(k^5\left(531\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-1728\,U-192\,H^4\,k^4\,U+k^2\left(145\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-1152\,H^2\,U\right)\right)\right)\Big/\left(5760\left(3+H^2\,k^2\right)^2\right)+\frac{1}{34\,560\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}}i\,k^8\left(k^6\,U^3\left(-721\sqrt{3}\,g\,H^7+192\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U\right)+9\,k^2\left(145\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}-1350\sqrt{3}\,g^2\,H^4\,U+2118\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2-2227\sqrt{3}\,g\,H^3\,U^3+576\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\right)+81\left(59\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}-241\sqrt{3}\,g^2\,H^2\,U+64\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+g\,H\,U^2\left(369\sqrt{g\,H\left(3+H^2\,k^2\right)}-251\sqrt{3}\,U\right)\right)-3\,k^4\,U\left(627\sqrt{3}\,g^2\,H^6-576\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+g\,H^5\,U\left(-1011\sqrt{g\,H\left(3+H^2\,k^2\right)}+2195\sqrt{3}\,U\right)\right)\right)\,dt^3+\frac{1}{34\,560\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}}k^9\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\left(3\sqrt{3}\,g^2\,H^2\left(531+145\,H^2\,k^2\right)+g\,H\,U\left(-4914\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\,\left(5049+3270\,H^2\,k^2+529\,H^4\,k^4\right)U\right)-6\left(288\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+32\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+k^2\left(241\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+192\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\right)\,dt^4+O[dt]^5\Bigg)\,dx^4+O[dx]^5\Big\}$$

In[1788]:=
```
Text[Row[{" U < -Sqrt[g*H] "}]]
Text[Row[{"Fnn and FnG "}]]
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (0) /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnn = FullSimplify[KurFWSeta /. G → 0 /. n → 1]
KfnG = FullSimplify[KurFWSeta /. n → 0 /. G → 1]
Kfnn = Kfnn /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;
KfnG = KfnG /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]]
KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
     fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGn = FullSimplify[KurFWSG /. G → 0 /. n → 1]
KfGG = FullSimplify[KurFWSG /. n → 0 /. G → 1]
KfGn = KfGn /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;
KfGG = KfGG /. Rpp → Rp /. Rmp → Rm /. GGp → GG /. Gnp → Gn;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 ^2 / 2 + EigvFmat2^3 / 6 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ]
```

Out[1788]= $U < -\text{Sqrt}[g*H]$

Out[1789]= Fnn and FnG

Out[1794]= Gnp H + Rpp U

Out[1795]= GGp H

Out[1800]= $$\left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(\frac{1}{12}\,k^4\,U\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(45\,k^5\,U+143\,H^2\,k^7\,U+32\,H^4\,k^9\,U\right)\,dt}{960\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1803]= $$\left(-\frac{3\,(k\,w)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(\frac{i\,\left(243\,k^5+49\,H^2\,k^7\right)\,dt}{960\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1804]= FGn and FGG

Out[1807]= H (g Rpp + Gnp U)

Out[1808]= (GGp H + Rpp) U

Out[1813]= $$\left(-\frac{\left(k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(\frac{1}{12}\,g\,H\,k^4\,dt+O[dt]^4\right)\,dx^3+\left(\left(i\,\left(288\,g\,H\,k^5+192\,g\,H^3\,k^7+32\,g\,H^5\,k^9-243\,k^5\,U^2-49\,H^2\,k^7\,U^2\right)\,dt\right)\Big/\left(960\,\left(3+H^2\,k^2\right)^2\right)+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1817]= $$\left(-\frac{\left(k\,\left(6+H^2\,k^2\right)\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(6+H^2\,k^2\right)\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(\frac{1}{12}\,k^4\,U\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(531\,k^5+241\,H^2\,k^7+32\,H^4\,k^9\right)\,U\,dt}{960\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1818]= W : omega

Out[1823]= $$\left\{\left(-\frac{i\,\left(\sqrt{3}\,k\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+3\,k\,U+H^2\,k^3\,U\right)^4\,dt^3}{24\,\left(3+H^2\,k^2\right)^4}+\frac{\left(\sqrt{3}\,k\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+3\,k\,U+H^2\,k^3\,U\right)^5\,dt^4}{30\,\left(3+H^2\,k^2\right)^5}+O[dt]^5\right)+\left(\frac{1}{24}\,i\,k^4\,\left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}}+2\,U\right)+\left(k^7\,\left(9\,g^2\,H^2+3\,g\,H\,U\,\left(5\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+9\,\left(3+H^2\,k^2\right)\,U\right)+U^3\,\left(21\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\,\left(7\,\sqrt{3}\,\sqrt{g\,H^5\,\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)\,dt^3\right)\Big/\left(144\,\left(3+H^2\,k^2\right)^2\right)+\left(i\,k^8\,\left(3\,g\,H+U\,\left(2\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)\,U\right)\right)\left(3\,g\,H\,\left(\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+4\,\left(3+H^2\,k^2\right)\,U\right)+U^2\,\left(15\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+18\,U+\right.\right.$$

$$2\,H^4\,k^4\,U + k^2\left(5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + 12\,H^2\,U\right)\Big)\Big)\,dt^4\Big)\Big/\left(144\left(3+H^2\,k^2\right)^3\right) + O[dt]^5\Big)\,dx^3 + \Bigg(-\Bigg(\Bigg(k^5\left(531\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 1728\,U + 192\,H^4\,k^4\,U + k^2\left(145\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + 1152\,H^2\,U\right)\Big)\Big)\Big/\left(5760\left(3+H^2\,k^2\right)^2\right)\Bigg) + \frac{1}{34\,560\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}}\,i\,k^8\Big(k^6\,U^3\left(721\sqrt{3}\,g\,H^7 + 192\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U\right) + 9\,k^2\left(145\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)} + 1350\sqrt{3}\,g^2\,H^4\,U + 2118\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2 + 2227\sqrt{3}\,g\,H^3\,U^3 + 576\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\right) + 81\left(59\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)} + 241\sqrt{3}\,g^2\,H^2\,U + 64\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4 + g\,H\,U^2\left(369\sqrt{g\,H\left(3+H^2\,k^2\right)} + 251\sqrt{3}\,U\right)\right) + 3\,k^4\,U\left(627\sqrt{3}\,g^2\,H^6 + 576\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3 + g\,H^5\,U\left(1011\sqrt{g\,H\left(3+H^2\,k^2\right)} + 2195\sqrt{3}\,U\right)\right)\Big)\,dt^3 - \frac{1}{34\,560\left(\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}\right)}\Big(k^9\Big(3\,g\,H + U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\right)\Big)\Big(3\sqrt{3}\,g^2\,H^2\left(531+145\,H^2\,k^2\right) + g\,H\,U\left(4914\sqrt{g\,H\left(3+H^2\,k^2\right)} + \sqrt{3}\,\left(5049+3270\,H^2\,k^2+529\,H^4\,k^4\right)U\right) + 6\left(288\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + 32\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3 + k^2\left(241\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U + 192\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\Big)\Big)\,dt^4 + O[dt]^5\Bigg)\,dx^4 + O[dx]^5,\Bigg(-\frac{i\left(-\sqrt{3}\,k\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\,k\,U + H^2\,k^3\,U\right)^4\,dt^3}{24\left(3+H^2\,k^2\right)^4} + \frac{\left(-\sqrt{3}\,k\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\,k\,U + H^2\,k^3\,U\right)^5\,dt^4}{30\left(3+H^2\,k^2\right)^5} + O[dt]^5\Bigg) + \Bigg(-\frac{1}{24}\,i\,k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}} - 2\,U\right) + \Big(k^7\Big(9\,g^2\,H^2 + 3\,g\,H\,U\left(-5\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 9\left(3+H^2\,k^2\right)U\right) + U^3\left(-21\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 18\,U + 2\,H^4\,k^4\,U + k^2\left(-7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + 12\,H^2\,U\right)\right)\Big)\,dt^3\Big)\Big/$$

$$\left(144\left(3+H^2k^2\right)^2\right)+\left(i\,k^8\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right.$$
$$\left(-3\,g\,H\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}-4\left(3+H^2k^2\right)U\right)+U^2\left(-15\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+18\,U+\right.\right.$$
$$\left.\left.2\,H^4k^4\,U+k^2\left(-5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2k^2\right)}+12\,H^2\,U\right)\right)\right)dt^4\Big)\Big/\left(144\left(3+H^2k^2\right)^3\right)+$$
$$O[dt]^5\Big)\,dx^3+\left(\left(k^5\left(531\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}-1728\,U-192\,H^4k^4\,U+\right.\right.\right.$$
$$\left.\left.\left.k^2\left(145\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2k^2\right)}-1152\,H^2\,U\right)\right)\right)\Big/\left(5760\left(3+H^2k^2\right)^2\right)+$$
$$\frac{1}{34\,560\sqrt{g\,H}\,\left(3+H^2k^2\right)^{7/2}}i\,k^8\left(k^6\,U^3\left(-721\sqrt{3}\,g\,H^7+192\sqrt{g\,H^{13}\left(3+H^2k^2\right)}\,U\right)+\right.$$
$$9\,k^2\left(145\sqrt{g^5\,H^9\left(3+H^2k^2\right)}-1350\sqrt{3}\,g^2\,H^4\,U+\right.$$
$$\left.2118\sqrt{g^3\,H^7\left(3+H^2k^2\right)}\,U^2-2227\sqrt{3}\,g\,H^3\,U^3+576\sqrt{g\,H^5\left(3+H^2k^2\right)}\,U^4\right)+$$
$$81\left(59\sqrt{g^5\,H^5\left(3+H^2k^2\right)}-241\sqrt{3}\,g^2\,H^2\,U+64\sqrt{g\,H\left(3+H^2k^2\right)}\,U^4+\right.$$
$$\left.g\,H\,U^2\left(369\sqrt{g\,H\left(3+H^2k^2\right)}-251\sqrt{3}\,U\right)\right)-3\,k^4\,U\left(627\sqrt{3}\,g^2\,H^6-\right.$$
$$\left.\left.576\sqrt{g\,H^9\left(3+H^2k^2\right)}\,U^3+g\,H^5\,U\left(-1011\sqrt{g\,H\left(3+H^2k^2\right)}+2195\sqrt{3}\,U\right)\right)\right)dt^3+$$
$$\frac{1}{34\,560\sqrt{g\,H}\,\left(3+H^2k^2\right)^{7/2}}k^9\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)$$
$$\left(3\sqrt{3}\,g^2\,H^2\left(531+145\,H^2k^2\right)+\right.$$
$$g\,H\,U\left(-4914\sqrt{g\,H\left(3+H^2k^2\right)}+\sqrt{3}\,\left(5049+3270\,H^2k^2+529\,H^4k^4\right)U\right)-$$
$$6\left(288\sqrt{g\,H\left(3+H^2k^2\right)}\,U^3+32\,k^4\sqrt{g\,H^9\left(3+H^2k^2\right)}\,U^3+k^2\left(241\sqrt{g^3\,H^7\left(3+H^2k^2\right)}\,U+\right.\right.$$
$$\left.\left.\left.192\sqrt{g\,H^5\left(3+H^2k^2\right)}\,U^3\right)\right)\right)dt^4+O[dt]^5\Big)\,dx^4+O[dx]^5\Big\}$$