In[1]:=
```
MA = k * x / (2 * Sin[k * x / 2])
RA = Exp[I * k * x / 2] * k * x / (2 * Sin[k * x / 2])
GA = k * x / ((H + H^3 / 3 * k^2) * Exp[-I * k * x / 2] * (2 * Sin[k * x / 2]))
FnnA = 0
FnGA = I * k / (1 + H^2 * k^2 / 3)
FGnA = g * H * I * k
FGGA = 0
FmatA = {{FnnA, FnGA}, {FGnA, FGGA}}
wAp = Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2)]
wAm = -Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2)]
Eigenvalues[FmatA]
```

Out[1]= $\frac{1}{2} k x \text{Csc}\left[\frac{k x}{2}\right]$

Out[2]= $\frac{1}{2} e^{\frac{i k x}{2}} k x \text{Csc}\left[\frac{k x}{2}\right]$

Out[3]= $\frac{e^{\frac{i k x}{2}} k x \text{Csc}\left[\frac{k x}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right)}$

Out[4]= $0$

Out[5]= $\frac{i k}{1 + \frac{H^2 k^2}{3}}$

Out[6]= $i g H k$

Out[7]= $0$

Out[8]= $\left\{\left\{0, \frac{i k}{1 + \frac{H^2 k^2}{3}}\right\}, \{i g H k, 0\}\right\}$

Out[9]= $\sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$

Out[10]= $-\sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$

Out[11]= $\left\{-\frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}\right\}$

In[12]:=
```
M = (26 - 2 * Cos[k * x]) / 24
Series[M - MA, {x, 0, 10}]
```

Out[12]= $\frac{1}{24} (26 - 2 \text{Cos}[k x])$

Out[13]= $-\frac{3 k^4 x^4}{640} + \frac{3 k^6 x^6}{35\,840} - \frac{149 k^8 x^8}{51\,609\,600} + \frac{29 k^{10} x^{10}}{13\,624\,934\,400} + O[x]^{11}$

In[14]:=
```
Rm = (5 - Exp[-I * k * x] + 2 * Exp[I * k * x]) / 6
Series[Rm - RA, {x, 0, 10}]
Rp = Exp[I * k * x] * (5 + 2 * Exp[-I * k * x] - Exp[I * k * x]) / 6
Series[Rp - RA, {x, 0, 10}]
Ru = (-Exp[-I * k * x] + 9 * Exp[I * k * x] - Exp[2 * I * k * x] + 9) / 16
Series[Ru - Exp[I * k * x / 2], {x, 0, 10}]
```

Out[14]= $\frac{1}{6}\left(5 - e^{-i\,k\,x} + 2\,e^{i\,k\,x}\right)$

Out[15]= $-\frac{1}{12}\,i\,k^3\,x^3 + \frac{k^4\,x^4}{120} + \frac{1}{240}\,i\,k^5\,x^5 - \frac{k^6\,x^6}{5040} - \frac{i\,k^7\,x^7}{10\,080} + \frac{k^8\,x^8}{201\,600} + \frac{i\,k^9\,x^9}{725\,760} - \frac{k^{10}\,x^{10}}{39\,916\,800} + O[x]^{11}$

Out[16]= $\frac{1}{6}\,e^{i\,k\,x}\left(5 + 2\,e^{-i\,k\,x} - e^{i\,k\,x}\right)$

Out[17]= $\frac{1}{12}\,i\,k^3\,x^3 - \frac{3\,k^4\,x^4}{40} - \frac{3}{80}\,i\,k^5\,x^5 + \frac{23\,k^6\,x^6}{1680} + \frac{41\,i\,k^7\,x^7}{10\,080} - \frac{209\,k^8\,x^8}{201\,600} - \frac{169\,i\,k^9\,x^9}{725\,760} + \frac{89\,k^{10}\,x^{10}}{1\,900\,800} + O[x]^{11}$

Out[18]= $\frac{1}{16}\left(9 - e^{-i\,k\,x} + 9\,e^{i\,k\,x} - e^{2\,i\,k\,x}\right)$

Out[19]= $-\frac{3\,k^4\,x^4}{128} - \frac{3}{256}\,i\,k^5\,x^5 + \frac{5\,k^6\,x^6}{1024} + \frac{3\,i\,k^7\,x^7}{2048} - \frac{63\,k^8\,x^8}{163\,840} - \frac{17\,i\,k^9\,x^9}{196\,608} + \frac{289\,k^{10}\,x^{10}}{16\,515\,072} + O[x]^{11}$

In[20]:=
```
Gold = H - H^3 / 3 * (32 * Cos[k * x] - 2 * Cos[2 * k * x] - 30) / (12 * x^2)
G = M * Ru / (Gold)
Series[G, {x, 0, 3} ]
Series[GA, {x, 0, 3} ]
Series[G - GA, {x, 0, 5}]
```

Out[20]= $H - \frac{H^3\,(-30 + 32\,\text{Cos}[k\,x] - 2\,\text{Cos}[2\,k\,x])}{36\,x^2}$

Out[21]= $\frac{\left(9 - e^{-i\,k\,x} + 9\,e^{i\,k\,x} - e^{2\,i\,k\,x}\right)\,(26 - 2\,\text{Cos}[k\,x])}{384\left(H - \frac{H^3\,(-30+32\,\text{Cos}[k\,x]-2\,\text{Cos}[2\,k\,x])}{36\,x^2}\right)}$

Out[22]= $\frac{1}{H + \frac{H^3\,k^2}{3}} + \frac{i\,k\,x}{2\left(H + \frac{H^3\,k^2}{3}\right)} - \frac{k^2\,x^2}{12\left(H + \frac{H^3\,k^2}{3}\right)} + O[x]^4$

Out[23]= $\frac{1}{H + \frac{H^3\,k^2}{3}} + \frac{i\,k\,x}{2\left(H + \frac{H^3\,k^2}{3}\right)} - \frac{k^2\,x^2}{12\left(H + \frac{H^3\,k^2}{3}\right)} + O[x]^4$

Out[24]= $\frac{\left(-243\,k^4 - 49\,H^2\,k^6\right)\,x^4}{960\,H\left(3 + H^2\,k^2\right)^2} - \frac{i\left(243\,k^5 + 49\,H^2\,k^7\right)\,x^5}{1920\,H\left(3 + H^2\,k^2\right)^2} + O[x]^6$

In[25]:=
```
fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * G;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;

Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]

Fmat = {{Fnn , Fng}, {Fgn , Fgg}}
EigvFmat = Eigenvalues[Fmat];
 Simplify[Series[EigvFmat , {x, 0, 5}]]
RKStep = Log[ 1 - t * EigvFmat + (t * EigvFmat)^2 / 2 - (t * EigvFmat)^3 / 6] / (I * t);
RKstepTay = Series[RKStep, {x, 0, 4}, {t, 0, 4}]
Simplify[RKstepTay, k * H > 0 ]
Simplify[RKstepTay - {wAp, wAm}, k * H > 0]
```

Out[29]=
$$-\frac{\left(1-e^{-i\,k\,x}\right)\left(\frac{1}{6}\left(-5+e^{-i\,k\,x}-2\,e^{i\,k\,x}\right)+\frac{1}{6}\,e^{i\,k\,x}\left(5+2\,e^{-i\,k\,x}-e^{i\,k\,x}\right)\right)\sqrt{g\,H}}{2\,x}$$

Out[30]=
$$\frac{1}{12}\sqrt{g\,H}\;k^4\,x^3-\frac{1}{72}\left(\sqrt{g\,H}\;k^6\right)x^5+O[x]^6$$

Out[31]=
$$\frac{\left(1-e^{-i\,k\,x}\right)\left(9-e^{-i\,k\,x}+9\,e^{i\,k\,x}-e^{2\,i\,k\,x}\right)H\,(26-2\,\text{Cos}[k\,x])}{384\,x\left(H-\frac{H^3\,(-30+32\,\text{Cos}[k\,x]-2\,\text{Cos}[2\,k\,x])}{36\,x^2}\right)}$$

Out[32]=
$$-\frac{i\left(243\,k^5+49\,H^2\,k^7\right)x^4}{960\left(3+H^2\,k^2\right)^2}+O[x]^6$$

Out[33]=
$$-\frac{\left(1-e^{-i\,k\,x}\right)\left(\frac{1}{6}\left(-5+e^{-i\,k\,x}-2\,e^{i\,k\,x}\right)+\frac{1}{6}\,e^{i\,k\,x}\left(5+2\,e^{-i\,k\,x}-e^{i\,k\,x}\right)\right)\sqrt{g\,H}}{2\,x}$$

Out[34]=
$$\frac{1}{12}\sqrt{g\,H}\;k^4\,x^3-\frac{1}{72}\left(\sqrt{g\,H}\;k^6\right)x^5+O[x]^6$$

Out[35]=
$$\frac{\left(1-e^{-i\,k\,x}\right)\left(\frac{1}{6}\,e^{i\,k\,x}\left(5+2\,e^{-i\,k\,x}-e^{i\,k\,x}\right)+\frac{1}{6}\left(5-e^{-i\,k\,x}+2\,e^{i\,k\,x}\right)\right)g\,H}{2\,x}$$

Out[36]=
$$-\frac{1}{30}\,i\,g\,H\,k^5\,x^4+O[x]^6$$

Out[37]=
$$\left\{\left\{-\frac{\left(1-e^{-i k x}\right)\left(\frac{1}{6}\left(-5+e^{-i k x}-2 e^{i k x}\right)+\frac{1}{6} e^{i k x}\left(5+2 e^{-i k x}-e^{i k x}\right)\right)\sqrt{g H}}{2 x},\right.\right.$$
$$\left.\frac{\left(1-e^{-i k x}\right)\left(9-e^{-i k x}+9 e^{i k x}-e^{2 i k x}\right) H\,(26-2\,\text{Cos}[k x])}{384 x\left(H-\frac{H^3\,(-30+32\,\text{Cos}[k x]-2\,\text{Cos}[2 k x])}{36 x^2}\right)}\right\},$$
$$\left\{\frac{\left(1-e^{-i k x}\right)\left(\frac{1}{6} e^{i k x}\left(5+2 e^{-i k x}-e^{i k x}\right)+\frac{1}{6}\left(5-e^{-i k x}+2 e^{i k x}\right)\right) g H}{2 x},\right.$$
$$\left.\left.-\frac{\left(1-e^{-i k x}\right)\left(\frac{1}{6}\left(-5+e^{-i k x}-2 e^{i k x}\right)+\frac{1}{6} e^{i k x}\left(5+2 e^{-i k x}-e^{i k x}\right)\right)\sqrt{g H}}{2 x}\right\}\right\}$$

Out[39]=
$$\left\{-\frac{i\sqrt{3}\,g H k}{\sqrt{g H\left(3+H^2 k^2\right)}}+\frac{1}{12}\sqrt{g H}\,k^4 x^3+\frac{i g^2 H^2 k^5\left(531+145 H^2 k^2\right) x^4}{1920\sqrt{3}\,\left(g H\left(3+H^2 k^2\right)\right)^{3/2}}-\frac{1}{72}\left(\sqrt{g H}\,k^6\right) x^5+O[x]^6,\right.$$
$$\left.\frac{i\sqrt{3}\,g H k}{\sqrt{g H\left(3+H^2 k^2\right)}}+\frac{1}{12}\sqrt{g H}\,k^4 x^3-\frac{i g^2 H^2 k^5\left(531+145 H^2 k^2\right) x^4}{1920\sqrt{3}\,\left(g H\left(3+H^2 k^2\right)\right)^{3/2}}-\frac{1}{72}\left(\sqrt{g H}\,k^6\right) x^5+O[x]^6\right\}$$

Out[41]=
$$\left\{\left(\frac{\sqrt{3}\,k\sqrt{g H\left(3+H^2 k^2\right)}}{3+H^2 k^2}+\frac{3 i g^2 H^2 k^4 t^3}{8\left(3+H^2 k^2\right)^2}+\frac{3\sqrt{3}\,g^2 H^2 k^5\sqrt{g H\left(3+H^2 k^2\right)}\,t^4}{10\left(3+H^2 k^2\right)^3}+O[t]^5\right)+\right.$$
$$\left(\frac{1}{12} i\sqrt{g H}\,k^4-\frac{\left(g H\sqrt{g H}\,k^7\sqrt{g H\left(3+H^2 k^2\right)}\right)t^3}{8\left(\sqrt{3}\,\left(3+H^2 k^2\right)^2\right)}+\frac{i g^2 H^2\sqrt{g H}\,k^8 t^4}{8\left(3+H^2 k^2\right)^2}+O[t]^5\right) x^3+$$
$$\left(-\frac{\sqrt{g H\left(3+H^2 k^2\right)}\,\left(531 k^5+145 H^2 k^7\right)}{1920\left(\sqrt{3}\,\left(3+H^2 k^2\right)^2\right)}-\frac{i g^2 H^2 k^8\left(531+145 H^2 k^2\right) t^3}{3840\left(3+H^2 k^2\right)^3}-\right.$$
$$\left.\left.\frac{\left(g^2 H^2 k^9\sqrt{g H\left(3+H^2 k^2\right)}\,\left(531+145 H^2 k^2\right)\right)t^4}{1280\left(\sqrt{3}\,\left(3+H^2 k^2\right)^4\right)}+O[t]^5\right) x^4+O[x]^5,$$
$$\left(-\frac{\sqrt{3}\,k\sqrt{g H\left(3+H^2 k^2\right)}}{3+H^2 k^2}+\frac{3 i g^2 H^2 k^4 t^3}{8\left(3+H^2 k^2\right)^2}-\frac{3\left(\sqrt{3}\,g^2 H^2 k^5\sqrt{g H\left(3+H^2 k^2\right)}\right)t^4}{10\left(3+H^2 k^2\right)^3}+O[t]^5\right)+$$
$$\left(\frac{1}{12} i\sqrt{g H}\,k^4+\frac{g H\sqrt{g H}\,k^7\sqrt{g H\left(3+H^2 k^2\right)}\,t^3}{8\sqrt{3}\,\left(3+H^2 k^2\right)^2}+\frac{i g^2 H^2\sqrt{g H}\,k^8 t^4}{8\left(3+H^2 k^2\right)^2}+O[t]^5\right) x^3+$$
$$\left(\frac{\sqrt{g H\left(3+H^2 k^2\right)}\,\left(531 k^5+145 H^2 k^7\right)}{1920\sqrt{3}\,\left(3+H^2 k^2\right)^2}-\frac{i g^2 H^2 k^8\left(531+145 H^2 k^2\right) t^3}{3840\left(3+H^2 k^2\right)^3}+\right.$$
$$\left.\left.\frac{g^2 H^2 k^9\sqrt{g H\left(3+H^2 k^2\right)}\,\left(531+145 H^2 k^2\right) t^4}{1280\sqrt{3}\,\left(3+H^2 k^2\right)^4}+O[t]^5\right) x^4+O[x]^5\right\}$$

Out[42]=

$$\left\{\left(\frac{\sqrt{3}\ g\,H\,k}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}+\frac{3\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}+\frac{3}{10}\sqrt{3}\ k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}t^4+O[t]^5\right)+\right.$$

$$\left(\frac{1}{12}\,i\,\sqrt{g\,H}\ k^4-\frac{\left(g^2\,H^2\,k^7\right)t^3}{8\left(\sqrt{3}\ \left(3+H^2\,k^2\right)^{3/2}\right)}+\frac{i\,(g\,H)^{5/2}\,k^8\,t^4}{8\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+$$

$$\left(-\frac{\sqrt{g\,H}\ k^5\left(531+145\,H^2\,k^2\right)}{1920\left(\sqrt{3}\ \left(3+H^2\,k^2\right)^{3/2}\right)}-\frac{i\,g^2\,H^2\,k^8\left(531+145\,H^2\,k^2\right)t^3}{3840\left(3+H^2\,k^2\right)^3}-\right.$$

$$\left.\left.\frac{\left((g\,H)^{5/2}\,k^9\left(531+145\,H^2\,k^2\right)\right)t^4}{1280\left(\sqrt{3}\ \left(3+H^2\,k^2\right)^{7/2}\right)}+O[t]^5\right)x^4+O[x]^5,\right.$$

$$\left(-\frac{\sqrt{3}\ g\,H\,k}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}+\frac{3\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}-\frac{3}{10}\left(\sqrt{3}\ k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}\right)t^4+O[t]^5\right)+$$

$$\left(\frac{1}{12}\,i\,\sqrt{g\,H}\ k^4+\frac{g^2\,H^2\,k^7\,t^3}{8\sqrt{3}\ \left(3+H^2\,k^2\right)^{3/2}}+\frac{i\,(g\,H)^{5/2}\,k^8\,t^4}{8\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+$$

$$\left(\frac{\sqrt{g\,H}\ k^5\left(531+145\,H^2\,k^2\right)}{1920\sqrt{3}\ \left(3+H^2\,k^2\right)^{3/2}}-\frac{i\,g^2\,H^2\,k^8\left(531+145\,H^2\,k^2\right)t^3}{3840\left(3+H^2\,k^2\right)^3}+\right.$$

$$\left.\left.\frac{(g\,H)^{5/2}\,k^9\left(531+145\,H^2\,k^2\right)t^4}{1280\sqrt{3}\ \left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\right)x^4+O[x]^5\right\}$$

Out[43]=

$$\left\{\left(\frac{3\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}+\frac{3}{10}\sqrt{3}\;k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}t^4+O[t]^5\right)+\right.$$

$$\left(\frac{1}{12}\,i\,\sqrt{g\,H}\;k^4-\frac{\left(g^2\,H^2\,k^7\right)t^3}{8\left(\sqrt{3}\;\left(3+H^2\,k^2\right)^{3/2}\right)}+\frac{i\,(g\,H)^{5/2}\,k^8\,t^4}{8\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+$$

$$\left(-\frac{\sqrt{g\,H}\;k^5\left(531+145\,H^2\,k^2\right)}{1920\left(\sqrt{3}\;\left(3+H^2\,k^2\right)^{3/2}\right)}-\frac{i\,g^2\,H^2\,k^8\left(531+145\,H^2\,k^2\right)t^3}{3840\left(3+H^2\,k^2\right)^3}-\right.$$

$$\left.\left.\frac{\left((g\,H)^{5/2}\,k^9\left(531+145\,H^2\,k^2\right)\right)t^4}{1280\left(\sqrt{3}\;\left(3+H^2\,k^2\right)^{7/2}\right)}+O[t]^5\right)x^4+O[x]^5,\right.$$

$$\left(\frac{3\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}-\frac{3}{10}\left(\sqrt{3}\;k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}\right)t^4+O[t]^5\right)+$$

$$\left(\frac{1}{12}\,i\,\sqrt{g\,H}\;k^4+\frac{g^2\,H^2\,k^7\,t^3}{8\sqrt{3}\;\left(3+H^2\,k^2\right)^{3/2}}+\frac{i\,(g\,H)^{5/2}\,k^8\,t^4}{8\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+$$

$$\left(\frac{\sqrt{g\,H}\;k^5\left(531+145\,H^2\,k^2\right)}{1920\sqrt{3}\;\left(3+H^2\,k^2\right)^{3/2}}-\frac{i\,g^2\,H^2\,k^8\left(531+145\,H^2\,k^2\right)t^3}{3840\left(3+H^2\,k^2\right)^3}+\right.$$

$$\left.\left.\frac{(g\,H)^{5/2}\,k^9\left(531+145\,H^2\,k^2\right)t^4}{1280\sqrt{3}\;\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\right)x^4+O[x]^5\right\}$$