In[1]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx / 2, xj + dx / 2}] / (dx);
qjnbar = qjbar /. t → tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x → xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx / 2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / 3 * k^2;
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca;
fn1Gca0A = fn1A /. Gca → 0;
fn1eca0A = fn1A /. eca → 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca /. Gh → RA * Gca;
fG1Gca0A = fG1A /. Gca → 0;
fG1eca0A = fG1A /. eca → 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;


MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
```

```
EA = Simplify[{{1, 0}, {0, 1}} + MatA];
```

$$wAp = U * k + \frac{\sqrt{3}\ k \sqrt{g\,H\left(3 + H^2\,k^2\right)}}{3 + H^2\,k^2};$$

$$wAm = U * k - \frac{\sqrt{3}\ k \sqrt{g\,H\left(3 + H^2\,k^2\right)}}{3 + H^2\,k^2};$$

In[37]:=
```
M = 1;
Merr = Series[M - MA, {dx, 0, 5}];
Rm = (1 + I * Sin[k * dx] / 2);
Rmerr = Series[Rm - RA, {dx, 0, 4}];
Rp = Exp[I * k * dx] * (1 - I * Sin[k * dx] / 2);
Rperr = Series[Rp - RA, {dx, 0, 4}];
GRHSp1 = -Exp[-I * k * dx / 2] + 2 + 4 * Exp[I * k * dx / 2] +
   Exp[I * k * dx] * (4 * Exp[-I * k * dx / 2] + 2 - Exp[I * k * dx / 2]);
GRHSp1 = GRHSp1 / Exp[I * k * dx / 2];
GRHSp1 = Expand[GRHSp1];
GRHSp1 = ExpToTrig[GRHSp1];
GRHSp2 = Exp[-I * k * dx / 2] - 8 + 7 * Exp[I * k * dx / 2] +
   Exp[I * k * dx] * (7 * Exp[-I * k * dx / 2] - 8 + Exp[I * k * dx / 2]);
GRHSp2 = GRHSp2 / Exp[I * k * dx / 2];
GRHSp2 = Expand[GRHSp2];
GRHSp2 = ExpToTrig[GRHSp2];
GGLHS = dx / 6 * (Rp + Rm);
GG2 = GGLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2);
GG2err = Series[GG2 - GGA, {dx, 0, 5}];
GnLHS = -U * (dx / 6) * (Rp + Rm);
Gn2 = GnLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2);
Gn2err = Series[Gn2 - GnA, {dx, 0, 5}];
Text[Row[{"M  ||   ", M}]]
Text[Row[{"M  ||   ", TeXForm[M]}]]
Text[Row[{"M error  ||   ", TeXForm[Merr]}]]
Text[Row[{"M error  ||   ", Merr}]]
Text[" "]
Text[Row[{"Rm  ||   ", Rm}]]
Text[Row[{"Rm  ||   ", TeXForm[Rm]}]]
Text[Row[{"Rm error  ||   ", Rmerr}]]
Text[Row[{"Rm error  ||   ", TeXForm[Rmerr]}]]
Text[" "]
Text[Row[{"Rp  ||   ", Rp}]]
Text[Row[{"Rp  ||   ", TeXForm[Rp]}]]
```

```
Text[Row[{"Rp error  ||   ", Rperr}]]
Text[Row[{"Rp error  ||   ", TeXForm[Rperr]}]]
Text[" "]
Text[Row[{"GG2  ||   ", GG2}]]
Text[Row[{"GG2  ||   ", TeXForm[GG2]}]]
Text[Row[{"GG2 error  ||   ", GG2err}]]
Text[Row[{"GG2 error  ||   ", TeXForm[GG2err]}]]
Text[" "]
Text[Row[{"Gn2  ||   ", Gn2}]]
Text[Row[{"Gn2  ||   ", TeXForm[Gn2]}]]
Text[Row[{"Gn2 error  ||   ", Gn2err}]]
Text[Row[{"Gn2 error  ||   ", TeXForm[Gn2err]}]]
```

Out[57]= M || 1

Out[58]= M || 1

Out[59]= M error || −\frac{\text{dx}^2 k^2}{24}−\frac{7 \text{dx}^4 k^4}{5760}+O\left(\text{dx}^6\right)

Out[60]= M error || $-\frac{k^2\,dx^2}{24}-\frac{7\,k^4\,dx^4}{5760}+O[dx]^6$

Out[61]=

Out[62]= Rm || $1+\frac{1}{2}\,i\,\text{Sin}[dx\,k]$

Out[63]= Rm || 1+\frac{1}{2} i \sin (\text{dx} k)

Out[64]= Rm error || $\frac{k^2\,dx^2}{12}-\frac{1}{12}\,i\,k^3\,dx^3+\frac{k^4\,dx^4}{720}+O[dx]^5$

Out[65]= Rm error ||
\frac{\text{dx}^2 k^2}{12}−\frac{1}{12} i \text{dx}^3 k^3+\frac{\text{dx}^4 k^4}{720}+O\left(\text{dx}^5\right)

Out[66]=

Out[67]= Rp || $e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)$

Out[68]= Rp || e^{i \text{dx} k} \left(1−\frac{1}{2} i \sin (\text{dx} k)\right)

Out[69]= Rp error || $\frac{k^2\,dx^2}{12}+\frac{1}{6}\,i\,k^3\,dx^3-\frac{89\,k^4\,dx^4}{720}+O[dx]^5$

Out[70]= Rp error ||
\frac{\text{dx}^2 k^2}{12}+\frac{1}{6} i \text{dx}^3 k^3−\frac{89 \text{dx}^4 k^4}{720}+O\left(\text{dx}^5\right)

Out[71]=

Out[72]= GG2 || $\frac{dx\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)+\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)}{6\left(\frac{1}{30}\,dx\,H\left(8+4\,\text{Cos}\left[\frac{dx\,k}{2}\right]-2\,\text{Cos}[dx\,k]\right)+\frac{H^3\left(14-16\,\text{Cos}\left[\frac{dx\,k}{2}\right]+2\,\text{Cos}[dx\,k]\right)}{9\,dx}\right)}$

Out[73]= GG2 || \frac{\text{dx} \left(e^{i \text{dx} k} \left(1-\frac{1}{2} i \sin (\text{dx} k)\right)+\frac{1}{2} i \sin (\text{dx} k)+1\right)}{6 \left(\frac{H^3 \left(-16 \cos \left(\frac{\text{dx} k}{2}\right)+2 \cos (\text{dx} k)+14\right)}{9 \text{dx}}+\frac{1}{30} \text{dx} H \left(4 \cos \left(\frac{\text{dx} k}{2}\right)-2 \cos (\text{dx} k)+8\right)\right)}

Out[74]= GG2 error || $\frac{\left(12 k^2+5 H^2 k^4\right) dx^2}{40 H \left(3+H^2 k^2\right)^2}+\frac{i \left(12 k^3+5 H^2 k^5\right) dx^3}{80 H \left(3+H^2 k^2\right)^2}+\frac{\left(-6651 k^4-4680 H^2 k^6-820 H^4 k^8\right) dx^4}{4800 H \left(3+H^2 k^2\right)^3}-\frac{i \left(6291 k^5+4410 H^2 k^7+770 H^4 k^9\right) dx^5}{9600 H \left(3+H^2 k^2\right)^3}+O[dx]^6$

Out[75]= GG2 error ||
\frac{\text{dx}^2 \left(5 H^2 k^4+12 k^2\right)}{40 H \left(H^2 k^2+3\right)^2}+\frac{i \text{dx}^3 \left(5 H^2 k^5+12 k^3\right)}{80 H \left(H^2 k^2+3\right)^2}+\frac{\text{dx}^4 \left(-820 H^4 k^8-4680 H^2 k^6-6651 k^4\right)}{4800 H \left(H^2 k^2+3\right)^3}-\frac{i \text{dx}^5 \left(770 H^4 k^9+4410 H^2 k^7+6291 k^5\right)}{9600 H \left(H^2 k^2+3\right)^3}+O\left(\text{dx}^6\right)

Out[76]=

Out[77]= Gn2 || $-\frac{\text{dx} U \left(1+e^{i \text{dx} k} \left(1-\frac{1}{2} i \text{Sin}[\text{dx} k]\right)+\frac{1}{2} i \text{Sin}[\text{dx} k]\right)}{6 \left(\frac{1}{30} \text{dx} H \left(8+4 \text{Cos}\left[\frac{\text{dx} k}{2}\right]-2 \text{Cos}[\text{dx} k]\right)+\frac{H^3 \left(14-16 \text{Cos}\left[\frac{\text{dx} k}{2}\right]+2 \text{Cos}[\text{dx} k]\right)}{9 \text{dx}}\right)}$

Out[78]= Gn2 || −\frac{\text{dx} U \left(e^{i \text{dx} k} \left(1-\frac{1}{2} i \sin (\text{dx} k)\right)+\frac{1}{2} i \sin (\text{dx} k)+1\right)}{6 \left(\frac{H^3 \left(-16 \cos \left(\frac{\text{dx} k}{2}\right)+2 \cos (\text{dx} k)+14\right)}{9 \text{dx}}+\frac{1}{30} \text{dx} H \left(4 \cos \left(\frac{\text{dx} k}{2}\right)-2 \cos (\text{dx} k)+8\right)\right)}

Out[79]= Gn2 error ||
$-\frac{\left(\left(12 k^2+5 H^2 k^4\right) U\right) dx^2}{40 \left(H \left(3+H^2 k^2\right)^2\right)}-\frac{i \left(12 k^3+5 H^2 k^5\right) U dx^3}{80 H \left(3+H^2 k^2\right)^2}+\frac{\left(6651 k^4+4680 H^2 k^6+820 H^4 k^8\right) U dx^4}{4800 H \left(3+H^2 k^2\right)^3}+\frac{i \left(6291 k^5+4410 H^2 k^7+770 H^4 k^9\right) U dx^5}{9600 H \left(3+H^2 k^2\right)^3}+O[dx]^6$

Out[80]= Gn2 error ||
−\frac{\text{dx}^2 \left(U \left(5 H^2 k^4+12 k^2\right)\right)}{40 \left(H \left(H^2 k^2+3\right)^2\right)}-\frac{i \text{dx}^3 U \left(5 H^2 k^5+12 k^3\right)}{80 H \left(H^2 k^2+3\right)^2}+\frac{\text{dx}^4 U \left(820 H^4 k^8+4680 H^2 k^6+6651 k^4\right)}{4800 H \left(H^2 k^2+3\right)^3}+\frac{i \text{dx}^5 U \left(770 H^4 k^9+4410 H^2 k^7+6291 k^5\right)}{9600 H \left(H^2 k^2+3\right)^3}+O\left(\text{dx}^6\right)

In[81]:=
```
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
```

```
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 ;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ];


Text[Row[{" -Sqrt(gH) < U  < Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn  ||   ", Kfnnp}]]
Text[Row[{"Fnn  ||   ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||   ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||   ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||   ", KfnGp}]]
Text[Row[{"FnG  ||   ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error  ||   ", FnG2TAr}]]
Text[Row[{"FnG error  ||   ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||   ", KfGnp}]]
Text[Row[{"FGn  ||   ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||   ", FGn2TAr}]]
```

```
Text[Row[{"FGn error  ||   ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||   ", KfGGp}]]
Text[Row[{"FGG  ||   ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||   ", FGG2TAr}]]
Text[Row[{"FGG error  ||   ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||   ", RKstepTayr}]]
Text[Row[{"Omega error  ||   ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||   ", EA}]]
Text[Row[{"EA  ||   ", TeXForm[EA]}]]
Text[Row[{"Eerr ||   ", Eerr}]]
Text[Row[{"Eerr ||   ", TeXForm[Eerr]}]]
```

Out[115]= $-\text{Sqrt(gH)} < U < \text{Sqrt(gH)}$

Out[116]=

Out[117]= $\text{Fnn} \;||\; \frac{1}{2}\left(2\,\text{Gnp}\,H + \text{Rpp}\left(-\sqrt{g\,H} + U\right) + \text{Rmp}\left(\sqrt{g\,H} + U\right)\right)$

Out[118]= Fnn || \frac{1}{2} \left(\text{Rmp} \left(\sqrt{g H}+U\right)+\text{Rpp} \left(U-\sqrt{g H}\right)+2 \text{Gnp} H\right)

Out[119]= $\text{Fnn error} \;||\; \left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{i\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)dx^2 + \left(-\frac{1}{8}\left(\sqrt{g\,H}\;k^4\right)dt + O[dt]^4\right)dx^3 + O[dx]^4$

Out[120]= Fnn error ||
\left(-\frac{\text{dt}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 H^2 k^3 U w^2}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(10 H^4 k^7+45 H^2 k^5+54 k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(-\frac{1}{8} \left(\sqrt{g H} k^4\right) \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[121]=

Out[122]= $\text{FnG} \;||\; \text{GGp}\,H$

Out[123]= FnG || \text{GGp} H

Out[124]= $\text{FnG error} \;||\; \left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)} + O[dt]^4\right) + \left(-\frac{i\left(12\,k^3+5\,H^2\,k^5\right)dt}{40\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)dx^2 + O[dx]^4$

Out[125]= FnG error || \left(-\frac{3 \text{dt}^2 (k w)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2}{2 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(5 H^2 k^5+12 k^3\right) \text{dt}}{40 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[126]=

Out[127]= FGn || $\frac{1}{2}\left(g H (Rmp + Rpp) + \left(2 Gnp H + \sqrt{g H}\ (Rmp - Rpp)\right) U\right)$

Out[128]= FGn ||
\frac{1}{2} \left(U \left(\sqrt{g H} (\text{Rmp}-\text{Rpp})+2 \text{Gnp} H\right)+g H (\text{Rmp}+\text{Rpp})\right)

Out[129]= FGn error || $\left(-\frac{\left(k \left(3 g H+g H^3 k^2-3 U^2\right) w\right) dt^2}{2 \left(3+H^2 k^2\right)} - \frac{i k \left(3 g H+g H^3 k^2-3 U^2\right) w^2 dt^3}{6 \left(3+H^2 k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{i \left(90 g H k^3+60 g H^3 k^5+10 g H^5 k^7-36 k^3 U^2-15 H^2 k^5 U^2\right) dt}{120 \left(3+H^2 k^2\right)^2} + O[dt]^4\right) dx^2 + \left(-\frac{1}{8}\left(\sqrt{g H}\ k^4 U\right) dt + O[dt]^4\right) dx^3 + O[dx]^4$

Out[130]= FGn error ||
\left(-\frac{\text{dt}^2 \left(k w \left(g H^3 k^2+3 g H-3 U^2\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2 \left(g H^3 k^2+3 g H-3 U^2\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(10 g H^5 k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(-\frac{1}{8} \left(\sqrt{g H} k^4 U\right) \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[131]=

Out[132]= FGG || $\frac{1}{2}\left(\sqrt{g H}\ Rmp - \sqrt{g H}\ Rpp + (2 GGp H + Rmp + Rpp)\, U\right)$

Out[133]= FGG || \frac{1}{2} \left(\text{Rmp} \sqrt{g H}-\text{Rpp} \sqrt{g H}+U (2 \text{GGp} H+\text{Rmp}+\text{Rpp})\right)

Out[134]= FGG error || $\left(-\frac{\left(k \left(6+H^2 k^2\right) U w\right) dt^2}{2 \left(3+H^2 k^2\right)} - \frac{i k \left(6+H^2 k^2\right) U w^2 dt^3}{6 \left(3+H^2 k^2\right)} + O[dt]^4\right) + \left(-\frac{i \left(126 k^3+75 H^2 k^5+10 H^4 k^7\right) U dt}{120 \left(3+H^2 k^2\right)^2} + O[dt]^4\right) dx^2 +$
$\left(-\frac{1}{8}\left(\sqrt{g H}\ k^4\right) dt + O[dt]^4\right) dx^3 + \left(\frac{i \left(13\,311 k^5 U+11\,430 H^2 k^7 U+3110 H^4 k^9 U+260 H^6 k^{11} U\right) dt}{4800 \left(3+H^2 k^2\right)^3} + O[dt]^4\right) dx^4 + O[dx]^5$

Out[135]= FGG error ||
\left(-\frac{\text{dt}^2 \left(k U w \left(H^2 k^2+6\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k U w^2 \left(H^2 k^2+6\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(10 H^4 k^7+75 H^2 k^5+126 k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(-\frac{1}{8} \left(\sqrt{g H} k^4\right) \text{dt}+O\left(\text{dt}^4\right)\right)+\text{dx}^4 \left(\frac{i \left(260 H^6 U k^{11}+3110 H^4 U k^9+11430 H^2 U k^7+13311 U k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^5\right)

Out[136]=

Out[137]=

Out[138]= Omega error ||

$$\Bigg\{\Bigg(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)dt^2+$$

$$\frac{i\,k^4\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)^2\,dt^3}{8\left(3+H^2\,k^2\right)^2}-\frac{1}{20\left(3+H^2\,k^2\right)^4}\Bigg(k^5\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)^3$$

$$\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)\Bigg)dt^4+O[dt]^5\Bigg)+$$

$$\Bigg(\frac{1}{240\left(3+H^2\,k^2\right)^2}k^3\left(42\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +180\,U+20\,H^4\,k^4\,U+15\,k^2\left(\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +8\,H^2\,U\right)\right)+$$

$$\frac{1}{480\left(3+H^2\,k^2\right)^3}k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(9\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +10\,U\right)+\right.$$

$$5\,k^4\,U^2\left(11\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ +36\,H^4\,U\right)+$$

$$6\,g\,H\left(21\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)+$$

$$\left.3\,k^2\left(15\sqrt{3}\ \sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ +109\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^2+180\,H^2\,U^3\right)\right)dt^2+$$

$$\frac{1}{480\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(102\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +180\,U+20\,H^4\,k^4\,U+\right.\right.$$

$$\left.\left.5\,k^2\left(7\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +24\,H^2\,U\right)\right)\right)dt^3-\frac{1}{960\left(3+H^2\,k^2\right)^4}$$

$$\left(k^7\left(42\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +180\,U+20\,H^4\,k^4\,U+15\,k^2\left(\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +8\,H^2\,U\right)\right)\right.$$

$$\left.\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)^2\right)dt^4+O[dt]^5\Bigg)dx^2+$$

$$\Bigg(-\frac{i\,k^4\left(2\,g\,H\left(3+H^2\,k^2\right)+\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ U\right)}{16\sqrt{g\,H}\ \left(3+H^2\,k^2\right)}-\frac{1}{32\left(3+H^2\,k^2\right)^{3/2}}i\,k^6\left(g\,H\left(6\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\sqrt{3}\ \left(15+4\,H^2\,k^2\right)U\right)+\right.$$

$$\left.U^2\left(12\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +3\sqrt{3}\ U+k^2\left(2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +\sqrt{3}\ H^2\,U\right)\right)\right)dt^2+$$

$$\frac{1}{32\left(3+H^2\,k^2\right)^{5/2}}k^7\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)\left(2\sqrt{3}\ g\,H\left(3+H^2\,k^2\right)+\right.$$

$$\left.U\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +3\sqrt{3}\ U+k^2\left(2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +\sqrt{3}\ H^2\,U\right)\right)\right)dt^3+$$

$$\left(i\,k^8\left(2\,g\,H\left(3+H^2\,k^2\right)+\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ U\right)\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)^2\right.$$

$$\left.dt^4\right)\Big/\left(64\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^3\right)+O[dt]^5\Bigg)dx^3+$$

$$\Bigg(-\Big(\Big(k^5\left(\sqrt{3}\ g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)+2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +\right.\right.\right.$$

$$k^4 \sqrt{g H^9 \left(3 + H^2 k^2\right)}\Big) U\Big)\Big) \Big/ \left(38\,400 \left(\sqrt{g H}\ \left(3 + H^2 k^2\right)^{5/2}\right)\right)\Big) -$$

$$\Big(\Big(k^7 \Big(45 \sqrt{3}\ g^2 H^2 \left(3336 + 2268 H^2 k^2 + 385 H^4 k^4\right) + g H U \Big(447\,588 \sqrt{3}\ H^2 k^2 U +$$

$$16\,705 \sqrt{3}\ H^6 k^6 U + 648 \Big(693 \sqrt{g H \left(3 + H^2 k^2\right)}\ + 688 \sqrt{3}\ U\Big) +$$

$$15 k^4 \Big(3408 \sqrt{g H^9 \left(3 + H^2 k^2\right)}\ + 9985 \sqrt{3}\ H^4 U\Big)\Big) +$$

$$80 \Big(1836 \sqrt{g H \left(3 + H^2 k^2\right)}\ U^3 + 612 k^4 \sqrt{g H^9 \left(3 + H^2 k^2\right)}\ U^3 + 68 k^6 \sqrt{g H^{13} \left(3 + H^2 k^2\right)}$$

$$U^3 + 9 k^2 \Big(421 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)}\ U + 204 \sqrt{g H^5 \left(3 + H^2 k^2\right)}\ U^3\Big)\Big)\Big)\Big) dt^2\Big) \Big/$$

$$\Big(230\,400 \Big(\sqrt{g H}\ \left(3 + H^2 k^2\right)^{7/2}\Big)\Big) - \frac{1}{25\,600 \left(3+H^2 k^2\right)^4} i k^8 \Big(6 g^2 H^2 \left(8046 + 5460 H^2 k^2 + 925 H^4 k^4\right) +$$

$$560 H^8 k^8 U^4 + 432 U^3 \Big(143 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)}\ + 105 U\Big) +$$

$$5 k^4 U^3 \Big(4139 \sqrt{3}\ \sqrt{g H^9 \left(3 + H^2 k^2\right)}\ + 6048 H^4 U\Big) +$$

$$5 k^6 U^3 \Big(461 \sqrt{3}\ \sqrt{g H^{13} \left(3 + H^2 k^2\right)}\ + 1344 H^6 U\Big) +$$

$$12 k^2 \Big(3585 \sqrt{3}\ \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)}\ U + 5161 \sqrt{3}\ \sqrt{g H^5 \left(3 + H^2 k^2\right)}\ U^3 + 5040 H^2 U^4\Big) +$$

$$g H U \Big(63\,720 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)}\ + 282\,852 U + 284\,364 H^2 k^2 U + 10\,640 H^6 k^6 U +$$

$$5 k^4 \Big(1451 \sqrt{3}\ \sqrt{g H^9 \left(3 + H^2 k^2\right)}\ + 19\,056 H^4 U\Big)\Big)\Big) dt^3 + \frac{1}{460\,800 \sqrt{g H}\ \left(3+H^2 k^2\right)^{11/2}}$$

$$k^9 \Big(\sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)}\ + \left(3 + H^2 k^2\right) U\Big)\Big(5 k^8 U^3 \Big(3869 \sqrt{3}\ g H^9 + 928 \sqrt{g H^{17} \left(3 + H^2 k^2\right)}\ U\Big) +$$

$$108 k^2 \Big(2625 \sqrt{g^5 H^9 \left(3 + H^2 k^2\right)}\ + 15\,327 \sqrt{3}\ g^2 H^4 U + 22\,383 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)}\ U^2 +$$

$$19\,261 \sqrt{3}\ g H^3 U^3 + 4640 \sqrt{g H^5 \left(3 + H^2 k^2\right)}\ U^4\Big) +$$

$$1296 \Big(323 \sqrt{g^5 H^5 \left(3 + H^2 k^2\right)}\ + 1268 \sqrt{3}\ g^2 H^2 U + 290 \sqrt{g H \left(3 + H^2 k^2\right)}\ U^4 +$$

$$g H U^2 \Big(1857 \sqrt{g H \left(3 + H^2 k^2\right)}\ + 1202 \sqrt{3}\ U\Big)\Big) + 15 k^6 U \Big(4143 \sqrt{3}\ g^2 H^8 +$$

$$3712 \sqrt{g H^{13} \left(3 + H^2 k^2\right)}\ U^3 + g H^7 U \Big(6019 \sqrt{g H \left(3 + H^2 k^2\right)}\ + 15\,454 \sqrt{3}\ U\Big)\Big) +$$

$$9 k^4 \Big(5325 \sqrt{g^5 H^{13} \left(3 + H^2 k^2\right)}\ + 61\,735 \sqrt{3}\ g^2 H^6 U + 27\,840 \sqrt{g H^9 \left(3 + H^2 k^2\right)}\ U^4 +$$

$$g H^5 U^2 \Big(89\,915 \sqrt{g H \left(3 + H^2 k^2\right)}\ + 115\,737 \sqrt{3}\ U\Big)\Big)\Big) dt^4 + O[dt]^5\Big) dx^4 + O[dx]^5,$$

$$\Bigg(\frac{1}{6 \left(3+H^2 k^2\right)^2} k^3 \Big(-\sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)}\ + \left(3 + H^2 k^2\right) U\Big)\Big(3 g H + U \Big(-2 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)}\ + \left(3 + H^2 k^2\right) U\Big)\Big)$$

$$dt^2 +$$

$$\frac{i k^4 \Big(3 g H + U \Big(-2 \sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}\ + \left(3+H^2 k^2\right) U\Big)\Big)^2 dt^3}{8 \left(3+H^2 k^2\right)^2} - \frac{1}{20 \left(3+H^2 k^2\right)^4}$$

$$\left(k^5\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)^3\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)\right)$$

$$dt^4+O[dt]^5\Bigg)+$$

$$\left(\frac{1}{240\left(3+H^2\,k^2\right)^2}k^3\left(-42\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ -8\,H^2\,U\right)\right)+\right.$$

$$\frac{1}{480\left(3+H^2\,k^2\right)^3}k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(-9\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +10\,U\right)+\right.$$

$$5\,k^4\,U^2\left(-11\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ +36\,H^4\,U\right)+$$

$$6\,g\,H\left(-21\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)-$$

$$\left.3\,k^2\left(15\sqrt{3}\ \sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ +109\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^2-180\,H^2\,U^3\right)\right)dt^2+$$

$$\frac{1}{480\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(-102\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +180\,U+20\,H^4\,k^4\,U-\right.\right.$$

$$\left.\left.5\,k^2\left(7\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ -24\,H^2\,U\right)\right)\right)dt^3-\frac{1}{960\left(3+H^2\,k^2\right)^4}$$

$$\left(k^7\left(-42\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ -8\,H^2\,U\right)\right)\right.$$

$$\left.\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)^2\right)dt^4+O[dt]^5\Bigg)$$

$$dx^2+\left(\frac{1}{16}\,i\,\sqrt{g\,H}\ k^4\left(-2+\frac{\sqrt{3}\ U}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}\right)+\frac{1}{32\left(3+H^2\,k^2\right)^{3/2}}\right.$$

$$i\,k^6\left(g\left(-6\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +15\sqrt{3}\ H\,U+4\sqrt{3}\ H^3\,k^2\,U\right)+\right.$$

$$\left.U^2\left(-12\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +3\sqrt{3}\ U+k^2\left(-2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +\sqrt{3}\ H^2\,U\right)\right)\right)dt^2-$$

$$\frac{1}{32\left(3+H^2\,k^2\right)^{5/2}}\left(k^7\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)\left(2\sqrt{3}\ g\,H\left(3+H^2\,k^2\right)+\right.\right.$$

$$\left.\left.U\left(-9\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +3\sqrt{3}\ U+k^2\left(-2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +\sqrt{3}\ H^2\,U\right)\right)\right)\right)$$

$$dt^3+\left(i\,k^8\left(2\,g\,H\left(3+H^2\,k^2\right)-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ U\right)\right.$$

$$\left.\left.\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)^2dt^4\right)\Big/\left(64\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^3\right)+O[dt]^5\Bigg)$$

$$dx^3+\left(\left(k^5\left(\sqrt{3}\ g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)-\right.\right.\right.$$

$$\left.\left.\left.2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ \right)U\right)\right)\Big/$$

$$\left(38\,400\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{5/2}\right)+\left(k^7\left(45\sqrt{3}\ g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+\right.\right.$$

$$
\begin{aligned}
&g\,H\,U\left(447\,588\sqrt{3}\;H^2k^2U+16\,705\sqrt{3}\;H^6k^6U-648\left(693\sqrt{g\,H\left(3+H^2k^2\right)}-688\sqrt{3}\;U\right)-\right.\\
&\qquad\left.15\,k^4\left(3408\sqrt{g\,H^9\left(3+H^2k^2\right)}-9985\sqrt{3}\;H^4U\right)\right)-\\
&80\left(1836\sqrt{g\,H\left(3+H^2k^2\right)}\;U^3+612\,k^4\sqrt{g\,H^9\left(3+H^2k^2\right)}\;U^3+68\,k^6\sqrt{g\,H^{13}\left(3+H^2k^2\right)}\;U^3+\right.\\
&\qquad\left.\left.9\,k^2\left(421\sqrt{g^3\,H^7\left(3+H^2k^2\right)}\;U+204\sqrt{g\,H^5\left(3+H^2k^2\right)}\;U^3\right)\right)\right)dt^2\Big)\Big/\\
&\left(230\,400\sqrt{g\,H}\;\left(3+H^2k^2\right)^{7/2}\right)-\frac{1}{25\,600\left(3+H^2k^2\right)^4}i\,k^8\left(6\,g^2H^2\left(8046+5460\,H^2k^2+925\,H^4k^4\right)+\right.\\
&560\,H^8k^8U^4+432\,U^3\left(-143\sqrt{3}\;\sqrt{g\,H\left(3+H^2k^2\right)}+105\,U\right)+\\
&5\,k^4U^3\left(-4139\sqrt{3}\;\sqrt{g\,H^9\left(3+H^2k^2\right)}+6048\,H^4U\right)+\\
&5\,k^6U^3\left(-461\sqrt{3}\;\sqrt{g\,H^{13}\left(3+H^2k^2\right)}+1344\,H^6U\right)-\\
&12\,k^2\left(3585\sqrt{3}\;\sqrt{g^3\,H^7\left(3+H^2k^2\right)}\;U+5161\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2k^2\right)}\;U^3-5040\,H^2U^4\right)+\\
&g\,H\,U\left(-63\,720\sqrt{3}\;\sqrt{g\,H\left(3+H^2k^2\right)}+282\,852\,U+284\,364\,H^2k^2U+10\,640\,H^6k^6U+\right.\\
&\qquad\left.\left.5\,k^4\left(-1451\sqrt{3}\;\sqrt{g\,H^9\left(3+H^2k^2\right)}+19\,056\,H^4U\right)\right)\right)dt^3-\frac{1}{460\,800\left(\sqrt{g\,H}\;\left(3+H^2k^2\right)^{11/2}\right)}\\
&\left(k^9\left(\sqrt{3}\;\sqrt{g\,H\left(3+H^2k^2\right)}-\left(3+H^2k^2\right)U\right)\left(5\,k^8U^3\left(-3869\sqrt{3}\;g\,H^9+928\sqrt{g\,H^{17}\left(3+H^2k^2\right)}\;U\right)+\right.\right.\\
&\qquad108\,k^2\left(2625\sqrt{g^5\,H^9\left(3+H^2k^2\right)}-15\,327\sqrt{3}\;g^2H^4U+22\,383\sqrt{g^3\,H^7\left(3+H^2k^2\right)}\;U^2-\right.\\
&\qquad\left.19\,261\sqrt{3}\;g\,H^3U^3+4640\sqrt{g\,H^5\left(3+H^2k^2\right)}\;U^4\right)+\\
&\qquad9\,k^4\left(5325\sqrt{g^5\,H^{13}\left(3+H^2k^2\right)}-61\,735\sqrt{3}\;g^2H^6U+27\,840\sqrt{g\,H^9\left(3+H^2k^2\right)}\;U^4+\right.\\
&\qquad\left.g\,H^5U^2\left(89\,915\sqrt{g\,H\left(3+H^2k^2\right)}-115\,737\sqrt{3}\;U\right)\right)+1296\left(323\sqrt{g^5\,H^5\left(3+H^2k^2\right)}-\right.\\
&\qquad1268\sqrt{3}\;g^2H^2U+290\sqrt{g\,H\left(3+H^2k^2\right)}\;U^4+g\,H\,U^2\left(1857\sqrt{g\,H\left(3+H^2k^2\right)}-\right.\\
&\qquad\left.1202\sqrt{3}\;U\right)\Big)-15\,k^6U\left(4143\sqrt{3}\;g^2H^8-3712\sqrt{g\,H^{13}\left(3+H^2k^2\right)}\;U^3+\right.\\
&\qquad\left.\left.\left.g\,H^7U\left(-6019\sqrt{g\,H\left(3+H^2k^2\right)}+15\,454\sqrt{3}\;U\right)\right)\right)\right)dt^4+O[dt]^5\Big)\,dx^4+O[dx]^5\Big\}
\end{aligned}
$$

Out[139]= Omega error ||

\left\{\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}-\frac{\left(k^5 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{20 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(20 H^4 U k^4+15 \left(8 U H^2+\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+180 U+42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{240 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 U H^4+11 \sqrt{3} \sqrt{g H^9

\left(H^2 k^2+3\right)}\right) k^4+3 \left(180 H^2 U^3+109 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^2+15 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}\right) k^2+54 U^2 \left(10 U+9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U+21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{480 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(9 g H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4+5 \left(24 U H^2+7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+180 U+102 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{480 \left(H^2 k^2+3\right)^3}−\frac{\left(k^7 \left(20 H^4 U k^4+15 \left(8 U H^2+\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+180 U+42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{960 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(−\frac{i k^4 \left(2 g H \left(H^2 k^2+3\right)+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U\right)}{16 \sqrt{g H} \left(H^2 k^2+3\right)}−\frac{i k^6 \left(\left(\left(\sqrt{3} U H^2+2 \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+3 \sqrt{3} U+12 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+g H \left(\sqrt{3} \left(4 H^2 k^2+15\right) U+6 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^{3/2}}+\frac{k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(2 \sqrt{3} g H \left(H^2 k^2+3\right)+U \left(\left(\sqrt{3} U H^2+2 \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+3 \sqrt{3} U+9 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^{5/2}}+\frac{i k^8 \left(2 g H \left(H^2 k^2+3\right)+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \sqrt{g H} \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(−\frac{k^5 \left(\sqrt{3} g H \left(2075 H^4 k^4+12180 H^2 k^2+17856\right)+2080 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{38400 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{5/2}\right)}−\frac{\left(k^7 \left(45 \sqrt{3} g^2 \left(385 H^4 k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6+15 \left(9985 \sqrt{3} U H^4+3408 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+447588 \sqrt{3} H^2 U k^2+648 \left(688 \sqrt{3} U+693 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) H+80 \left(68 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+612 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(204 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+421 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+1836 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text{dt}^2}{230400 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)}−\frac{i k^8 \left(560 H^8 U^4 k^8+5 U^3 \left(1344 U H^6+461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 U H^4+4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+12 \left(5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 U+143 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+g H U \left(10640 H^6 U k^6+5 \left(19056 U H^4+1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U+63720 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{25600 \left(H^2 k^2+3\right)^4}+\frac{k^9 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(5 U^3 \left(3869 \sqrt{3} g H^9+928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U\right) k^8+15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U+6019 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7+3712 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(61735 \sqrt{3} g^2 U H^6+g U^2 \left(115737 \sqrt{3} U+89915 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+108 \left(15327 \sqrt{3} g^2 U H^4+19261 \sqrt{3}

g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+2625 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+1296 \left(290 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1202 \sqrt{3} U+1857 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+1268 \sqrt{3} g^2 H^2 U+323 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{460800 \sqrt{g H} \left(H^2 k^2+3\right)^{11/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right),\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}-\frac{\left(k^5 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{20 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(20 H^4 U k^4-15 \left(\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-8 H^2 U\right) k^2+180 U-42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{240 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 H^4 U-11 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4-3 \left(-180 H^2 U^3+109 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^2+15 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}\right) k^2+54 U^2 \left(10 U-9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U-21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{480 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(9 g H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4-5 \left(7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-24 H^2 U\right) k^2+180 U-102 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{480 \left(H^2 k^2+3\right)^3}-\frac{\left(k^7 \left(20 H^4 U k^4-15 \left(\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-8 H^2 U\right) k^2+180 U-42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{960 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{16} i \sqrt{g H} k^4 \left(\frac{\sqrt{3} U}{\sqrt{g H \left(H^2 k^2+3\right)}}-2\right)+\frac{i k^6 \left(\left(\left(\sqrt{3} H^2 U-2 \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+3 \sqrt{3} U-12 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+g \left(4 \sqrt{3} k^2 U H^3+15 \sqrt{3} U H-6 \sqrt{g H \left(H^2 k^2+3\right)} H\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^{3/2}}-\frac{\left(k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(2 \sqrt{3} g H \left(H^2 k^2+3\right)+U \left(\left(\sqrt{3} H^2 U-2 \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+3 \sqrt{3} U-9 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^{5/2}}+\frac{i k^8 \left(2 g H \left(H^2 k^2+3\right)-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \sqrt{g H} \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{k^5 \left(\sqrt{3} g H \left(2075 H^4 k^4+12180 H^2 k^2+17856\right)-2080 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{38400 \sqrt{g H} \left(H^2 k^2+3\right)^{5/2}}+\frac{k^7 \left(45 \sqrt{3} g^2 \left(385 H^4 k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6-15 \left(3408 \sqrt{g H^9 \left(H^2 k^2+3\right)}-9985 \sqrt{3} H^4 U\right) k^4+447588 \sqrt{3} H^2 U k^2-648 \left(693 \sqrt{g H \left(H^2 k^2+3\right)}-688 \sqrt{3} U\right)\right) H-80 \left(68 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+612 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(204 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+421 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+1836 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text{dt}^2}{230400 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}+\frac{i k^8 \left(560 H^8 U^4 k^8+5 U^2

\text{dt}^2}{230400 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}-\frac{1 k^8 \left(360 H^8 U^4 k^8+5 U^3 \left(1344 H^6 U-461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 H^4 U-4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4-12 \left(-5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 U-143 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+g H U \left(10640 H^6 U k^6+5 \left(19056 H^4 U-1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U-63720 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{25600 \left(H^2 k^2+3\right)^4}-\frac{\left(k^9 \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}-\left(H^2 k^2+3\right) U\right) \left(5 U^3 \left(928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U-3869 \sqrt{3} g H^9\right) k^8-15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U-6019 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7-3712 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(-61735 \sqrt{3} g^2 U H^6+g U^2 \left(89915 \sqrt{g H \left(H^2 k^2+3\right)}-115737 \sqrt{3} U\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+108 \left(-15327 \sqrt{3} g^2 U H^4-19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+2625 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+1296 \left(290 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1857 \sqrt{g H \left(H^2 k^2+3\right)}-1202 \sqrt{3} U\right) U^2-1268 \sqrt{3} g^2 H^2 U+323 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{460800 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{11/2}\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right\}

Out[140]=

Out[141]= EA || $\left\{\left\{\frac{-H^2 k^2 \left(\left(-1+e^{i\,dt\,w}\right) k\,U-w\right)+3\,w}{\left(3+H^2 k^2\right) w}, -\frac{3\left(-1+e^{i\,dt\,w}\right) k}{\left(3+H^2 k^2\right) w}\right\}, \left\{-\frac{\left(-1+e^{i\,dt\,w}\right) k\left(g\,H\left(3+H^2 k^2\right)-3\,U^2\right)}{\left(3+H^2 k^2\right) w}, 1-\frac{\left(-1+e^{i\,dt\,w}\right) k\left(6+H^2 k^2\right) U}{\left(3+H^2 k^2\right) w}\right\}\right\}$

Out[142]= EA || \left(
\begin{array}{cc}
\frac{3 w-H^2 k^2 \left(\left(-1+e^{i \text{dt} w}\right) k U-w\right)}{\left(H^2 k^2+3\right) w} & -\frac{3 \left(-1+e^{i \text{dt} w}\right) k}{\left(H^2 k^2+3\right) w} \\
-\frac{\left(-1+e^{i \text{dt} w}\right) k \left(g H \left(H^2 k^2+3\right)-3 U^2\right)}{\left(H^2 k^2+3\right) w} & 1-\frac{\left(-1+e^{i \text{dt} w}\right) k \left(H^2 k^2+6\right) U}{\left(H^2 k^2+3\right) w} \\
\end{array}
\right)

Out[143]= Eerr ||

$$\Big\{\Big\{\Big(\frac{\left(-3\,g\,H\,k^2+3\,k^2\,U^2-H^2\,k^4\,U^2-H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{H^2\,k^3\,U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\Big)+$$

$$\Big(-\frac{i\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}-\frac{\left(126\,g\,H\,k^4+45\,g\,H^3\,k^6-72\,k^4\,U^2+30\,H^2\,k^6\,U^2+20\,H^4\,k^8\,U^2\right)dt^2}{240\left(3+H^2\,k^2\right)^2}+O[dt]^5\Big)\,dx^2+$$

$$\Big(-\frac{1}{8}\Big(\sqrt{g\,H}\;k^4\Big)\,dt+\frac{i\sqrt{g\,H}\;\left(3\,k^5+2\,H^2\,k^7\right)U\,dt^2}{16\left(3+H^2\,k^2\right)}+O[dt]^5\Big)\,dx^3+$$

$$\Big(\frac{i\left(729\,k^5\,U+2610\,H^2\,k^7\,U+1570\,H^4\,k^9\,U+260\,H^6\,k^{11}\,U\right)dt}{4800\left(3+H^2\,k^2\right)^3}+\frac{1}{28\,800\left(3+H^2\,k^2\right)^3}\Big(38\,853\,g\,H\,k^6+26\,460\,g\,H^3\,k^8+4500\,g\,H^5\,k^{10}-$$

$$38\,286\,k^6\,U^2-13\,500\,H^2\,k^8\,U^2+4140\,H^4\,k^{10}\,U^2+1460\,H^6\,k^{12}\,U^2\Big)\,dt^2+O[dt]^5\Big)\,dx^4+$$

$$O[dx]^5,\ \Big(\Big(-\frac{3\,k^2\,U}{3+H^2\,k^2}-\frac{3\,k\,w}{2\left(3+H^2\,k^2\right)}\Big)\,dt^2-\frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)}+\frac{k\,w^3\,dt^4}{8\left(3+H^2\,k^2\right)}+O[dt]^5\Big)+$$

$$\Big(-\frac{i\left(12\,k^3+5\,H^2\,k^5\right)dt}{40\left(3+H^2\,k^2\right)^2}-\frac{3\left(\left(14\,k^4+5\,H^2\,k^6\right)U\right)dt^2}{40\left(3+H^2\,k^2\right)^2}+O[dt]^5\Big)\,dx^2+\Big(\frac{3\,i\sqrt{g\,H}\;k^5\,dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\Big)\,dx^3+$$

$$\Big(\frac{i\left(6291\,k^5+4410\,H^2\,k^7+770\,H^4\,k^9\right)dt}{4800\left(3+H^2\,k^2\right)^3}+\frac{\left(12\,951\,k^6\,U+8820\,H^2\,k^8\,U+1500\,H^4\,k^{10}\,U\right)dt^2}{4800\left(3+H^2\,k^2\right)^3}+O[dt]^5\Big)\,dx^4+O[dx]^5\Big\},$$

$$\Big\{\Big(-\frac{\left(k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\left(2\,k\,U+w\right)\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\Big)+$$

$$\Big(-\frac{i\left(90\,g\,H\,k^3+60\,g\,H^3\,k^5+10\,g\,H^5\,k^7-36\,k^3\,U^2-15\,H^2\,k^5\,U^2\right)dt}{120\left(3+H^2\,k^2\right)^2}+\frac{\left(-180\,g\,H\,k^4\,U-120\,g\,H^3\,k^6\,U-20\,g\,H^5\,k^8\,U+126\,k^4\,U^3+45\,H^2\,k^6\,U^3\right)dt^2}{120\left(3+H^2\,k^2\right)^2}+O[dt]^5\Big)$$

$$dx^2+\Big(-\frac{1}{8}\Big(\sqrt{g\,H}\;k^4\,U\Big)\,dt+\frac{i\left(3\,g\,H\sqrt{g\,H}\;k^5+g\,H^3\sqrt{g\,H}\;k^7+H^2\sqrt{g\,H}\;k^7\,U^2\right)dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\Big)\,dx^3+$$

$$\Big(\frac{1}{4800\left(3+H^2\,k^2\right)^3}i\,\Big(7020\,g\,H\,k^5+7020\,g\,H^3\,k^7+2340\,g\,H^5\,k^9+260\,g\,H^7\,k^{11}-6291\,k^5\,U^2-4410\,H^2\,k^7\,U^2-$$

$$770\,H^4\,k^9\,U^2\Big)\,dt+\frac{1}{14\,400\left(3+H^2\,k^2\right)^3}\Big(39\,420\,g\,H\,k^6\,U+39\,420\,g\,H^3\,k^8\,U+13\,140\,g\,H^5\,k^{10}\,U+$$

$$1460\,g\,H^7\,k^{12}\,U-38\,853\,k^6\,U^3-26\,460\,H^2\,k^8\,U^3-4500\,H^4\,k^{10}\,U^3\Big)\,dt^2+O[dt]^5\Big)\,dx^4+O[dx]^5,$$

$$\Big(\frac{\left(-3\,g\,H\,k^2-9\,k^2\,U^2-H^2\,k^4\,U^2-6\,k\,U\,w-H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(6+H^2\,k^2\right)U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{k\left(6+H^2\,k^2\right)U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\Big)+$$

$$\Big(-\frac{i\left(126\,k^3+75\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}-\frac{\left(126\,g\,H\,k^4+45\,g\,H^3\,k^6+432\,k^4\,U^2+210\,H^2\,k^6\,U^2+20\,H^4\,k^8\,U^2\right)dt^2}{240\left(3+H^2\,k^2\right)^2}+O[dt]^5\Big)\,dx^2+$$

$$\Big(-\frac{1}{8}\Big(\sqrt{g\,H}\;k^4\Big)\,dt+\frac{i\sqrt{g\,H}\;k^5\left(15+2\,H^2\,k^2\right)U\,dt^2}{16\left(3+H^2\,k^2\right)}+O[dt]^5\Big)\,dx^3+$$

$$\Big(\frac{i\left(13\,311\,k^5\,U+11\,430\,H^2\,k^7\,U+3110\,H^4\,k^9\,U+260\,H^6\,k^{11}\,U\right)dt}{4800\left(3+H^2\,k^2\right)^3}+\frac{1}{28\,800\left(3+H^2\,k^2\right)^3}\Big(38\,853\,g\,H\,k^6+26\,460\,g\,H^3\,k^8+4500\,g\,H^5\,k^{10}+$$

$$117\,126\,k^6\,U^2+92\,340\,H^2\,k^8\,U^2+22\,140\,H^4\,k^{10}\,U^2+1460\,H^6\,k^{12}\,U^2\Big)\,dt^2+O[dt]^5\Big)\,dx^4+O[dx]^5\Big\}\Big\}$$

Out[144]= Eerr || \left(

\begin{array}{cc}

\left(\frac{\left(-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right) \text{dt}^2}{2 \left(H^2 k^2+3\right)}-\frac{i H^2 k^3 U w^2 \text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(10 H^4 k^7+45 H^2 k^5+54 k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}-\frac{\left(20 H^4 U^2 k^8+45 g H^3 k^6+30 H^2 U^2 k^6-72 U^2 k^4+126 g H k^4\right) \text{dt}^2}{240 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(-\frac{1}{8} \left(\sqrt{g H}\, k^4\right) \text{dt}+\frac{i \sqrt{g H} \left(2 H^2 k^7+3 k^5\right) U \text{dt}^2}{16 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(260 H^6 U k^{11}+1570 H^4 U k^9+2610 H^2 U k^7+729 U k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 H^6 U^2 k^{12}+4500 g H^5 k^{10}+4140 H^4 U^2 k^{10}+26460 g H^3 k^8-13500 H^2 U^2 k^8-38286 U^2 k^6+38853 g H k^6\right) \text{dt}^2}{28800 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) & \left(\left(-\frac{3 U

```
k^2}{H^2 k^2+3}-\frac{3 w k}{2 \left(H^2 k^2+3\right)}\right) \text{dt}^2-\frac{i k w^2 \text{dt}^3}{2 \left(H^2
k^2+3\right)}+\frac{k w^3 \text{dt}^4}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i
\left(5 H^2 k^5+12 k^3\right) \text{dt}}{40 \left(H^2 k^2+3\right)^2}-\frac{3 \left(\left(5 H^2
k^6+14 k^4\right) U\right) \text{dt}^2}{40 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(\frac{3 i \sqrt{g H} k^5 \text{dt}^2}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{i \left(770 H^4 k^9+4410 H^2 k^7+6291 k^5\right) \text{dt}}{4800 \left(H^2
k^2+3\right)^3}+\frac{\left(1500 H^4 U k^{10}+8820 H^2 U k^8+12951 U k^6\right) \text{dt}^2}{4800
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\left(-\frac{\left(k \left(g k^2 H^3+3 g H-3 U^2\right) (2 k U+w)\right) \text{dt}^2}{2 \left(H^2 k^2+3\right)}-\frac{i
k \left(g k^2 H^3+3 g H-3 U^2\right) w^2 \text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{k \left(g k^2 H^3+3
g H-3 U^2\right) w^3 \text{dt}^4}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i
\left(10 g H^5 k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}}{120
\left(H^2 k^2+3\right)^2}+\frac{\left(-20 g H^5 U k^8+45 H^2 U^3 k^6-120 g H^3 U k^6+126 U^3
k^4-180 g H U k^4\right) \text{dt}^2}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(-\frac{1}{8} \left(\sqrt{g H} k^4 U\right) \text{dt}+\frac{i \left(H^2 \sqrt{g H} U^2 k^7+g H^3
\sqrt{g H} k^7+3 g H \sqrt{g H} k^5\right) \text{dt}^2}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{i \left(260 g H^7 k^{11}+2340 g H^5 k^9-770 H^4 U^2 k^9+7020 g H^3 k^7-4410
H^2 U^2 k^7-6291 U^2 k^5+7020 g H k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 g
H^7 U k^{12}-4500 H^4 U^3 k^{10}+13140 g H^5 U k^{10}-26460 H^2 U^3 k^8+39420 g H^3 U k^8-38853
U^3 k^6+39420 g H U k^6\right) \text{dt}^2}{14400 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)
\text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{\left(-H^2 U^2 k^4-H^2 U w k^3-9 U^2 k^2-3 g
H k^2-6 U w k\right) \text{dt}^2}{2 \left(H^2 k^2+3\right)}-\frac{i k \left(H^2 k^2+6\right) U w^2
\text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{k \left(H^2 k^2+6\right) U w^3 \text{dt}^4}{24 \left(H^2
k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(10 H^4 k^7+75 H^2 k^5+126 k^3\right) U
\text{dt}}{120 \left(H^2 k^2+3\right)^2}-\frac{\left(20 H^4 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432
U^2 k^4+126 g H k^4\right) \text{dt}^2}{240 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(-\frac{1}{8} \left(\sqrt{g H} k^4\right) \text{dt}+\frac{i \sqrt{g H} k^5 \left(2 H^2
k^2+15\right) U \text{dt}^2}{16 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i
\left(260 H^6 U k^{11}+3110 H^4 U k^9+11430 H^2 U k^7+13311 U k^5\right) \text{dt}}{4800
\left(H^2 k^2+3\right)^3}+\frac{\left(1460 H^6 U^2 k^{12}+4500 g H^5 k^{10}+22140 H^4 U^2
k^{10}+26460 g H^3 k^8+92340 H^2 U^2 k^8+117126 U^2 k^6+38853 g H k^6\right) \text{dt}^2}{28800
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\end{array}
\right)
```

In[145]:=
```
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → 0;
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
```

```
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr =  Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2  + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 ;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ];


Text[Row[{" U < -Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn  ||   ", Kfnnp}]]
Text[Row[{"Fnn  ||   ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||   ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||   ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||   ", KfnGp}]]
Text[Row[{"FnG  ||   ", TeXForm[KfnGp]}]]
```

```
Text[Row[{"FnG error  ||   ", FnG2TAr}]]
Text[Row[{"FnG error  ||   ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||   ", KfGnp}]]
Text[Row[{"FGn  ||   ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||   ", FGn2TAr}]]
Text[Row[{"FGn error  ||   ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||   ", KfGGp}]]
Text[Row[{"FGG  ||   ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||   ", FGG2TAr}]]
Text[Row[{"FGG error  ||   ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||   ", RKstepTayr}]]
Text[Row[{"Omega error  ||   ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||   ", EA}]]
Text[Row[{"EA  ||   ", TeXForm[EA]}]]
Text[Row[{"Eerr ||   ", Eerr}]]
Text[Row[{"Eerr ||   ", TeXForm[Eerr]}]]
```

Out[179]= U < −Sqrt(gH)

Out[180]=

Out[181]= Fnn || Gnp H + Rmp U

Out[182]= Fnn || \text{Gnp} H+\text{Rmp} U

Out[183]= Fnn error || $\left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(-\frac{1}{8}\left(k^4\,U\right)dt+O[dt]^4\right)dx^3+O[dx]^4$

Out[184]= Fnn error ||
\left(−\frac{\text{dt}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 H^2 k^3 U w^2}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(−\frac{i \left(10 H^4 k^7+45 H^2 k^5+54 k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(−\frac{1}{8} \left(k^4 U\right) \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[185]=

Out[186]= FnG || GGp H

Out[187]= FnG || \text{GGp} H

Out[188]= FnG error || $\left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(12\,k^3+5\,H^2\,k^5\right)dt}{40\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+O[dx]^4$

Out[189]= FnG error || \left(-\frac{3 \text{dt}^2 (k w)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2}{2 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(5 H^2 k^5+12 k^3\right) \text{dt}}{40 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[190]=

Out[191]= FGn || H (g Rmp + Gnp U)

Out[192]= FGn || H (g \text{Rmp}+\text{Gnp} U)

Out[193]= FGn error || $\left(-\frac{\left(k \left(3 g H+g H^3 k^2-3 U^2\right) w\right) dt^2}{2 \left(3+H^2 k^2\right)}-\frac{i k \left(3 g H+g H^3 k^2-3 U^2\right) w^2 dt^3}{6 \left(3+H^2 k^2\right)}+O[dt]^4\right)+\left(-\frac{i \left(90 g H k^3+60 g H^3 k^5+10 g H^5 k^7-36 k^3 U^2-15 H^2 k^5 U^2\right) dt}{120 \left(3+H^2 k^2\right)^2}+O[dt]^4\right) dx^2+\left(-\frac{1}{8} \left(g H k^4\right) dt+O[dt]^4\right) dx^3+O[dx]^4$

Out[194]= FGn error ||
\left(-\frac{\text{dt}^2 \left(k w \left(g H^3 k^2+3 g H-3 U^2\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2 \left(g H^3 k^2+3 g H-3 U^2\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(10 g H^5 k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(-\frac{1}{8} \left(g H k^4\right) \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[195]=

Out[196]= FGG || (GGp H + Rmp) U

Out[197]= FGG || U (\text{GGp} H+\text{Rmp})

Out[198]= FGG error || $\left(-\frac{\left(k \left(6+H^2 k^2\right) U w\right) dt^2}{2 \left(3+H^2 k^2\right)}-\frac{i k \left(6+H^2 k^2\right) U w^2 dt^3}{6 \left(3+H^2 k^2\right)}+O[dt]^4\right)+\left(-\frac{i \left(126 k^3+75 H^2 k^5+10 H^4 k^7\right) U dt}{120 \left(3+H^2 k^2\right)^2}+O[dt]^4\right) dx^2+\left(-\frac{1}{8} \left(k^4 U\right) dt+O[dt]^4\right) dx^3+\left(\frac{i \left(13\,311 k^5+11\,430 H^2 k^7+3110 H^4 k^9+260 H^6 k^{11}\right) U dt}{4800 \left(3+H^2 k^2\right)^3}+O[dt]^4\right) dx^4+O[dx]^5$

Out[199]= FGG error ||
\left(-\frac{\text{dt}^2 \left(k U w \left(H^2 k^2+6\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k U w^2 \left(H^2 k^2+6\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(10 H^4 k^7+75 H^2 k^5+126 k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(-\frac{1}{8} \left(k^4 U\right) \text{dt}+O\left(\text{dt}^4\right)\right)+\text{dx}^4 \left(\frac{i \left(260 H^6 k^{11}+3110 H^4 k^9+11430 H^2 k^7+13311 k^5\right) U \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^5\right)

Out[200]=

Out[201]=

Out[202]= Omega error ||

$$\left\{\left(\frac{1}{6 \left(3+H^2 k^2\right)^2} k^3 \left(\sqrt{3} \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right) \left(3 g H+U \left(2 \sqrt{3} \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)\right) dt^2+\right.$$

$$\frac{i\,k^4\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\,dt^3}{8\left(3+H^2\,k^2\right)^2}-\frac{1}{20\left(3+H^2\,k^2\right)^4}\left(k^5\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\right.$$

$$\left.\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)dt^4+O[dt]^5\Bigg)+$$

$$\left(\frac{1}{240\left(3+H^2\,k^2\right)^2}k^3\left(42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U+15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+8\,H^2\,U\right)\right)+\right.$$

$$\frac{1}{480\left(3+H^2\,k^2\right)^3}k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(9\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+10\,U\right)+\right.$$

$$5\,k^4\,U^2\left(11\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+36\,H^4\,U\right)+$$

$$6\,g\,H\left(21\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)+$$

$$\left.3\,k^2\left(15\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}+109\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^2+180\,H^2\,U^3\right)\right)dt^2+$$

$$\frac{1}{480\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(102\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U+\right.\right.$$

$$\left.\left.5\,k^2\left(7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+24\,H^2\,U\right)\right)\right)dt^3-\frac{1}{960\left(3+H^2\,k^2\right)^4}$$

$$\left(k^7\left(42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U+15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+8\,H^2\,U\right)\right)\right.$$

$$\left.\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)dt^4+O[dt]^5\Bigg)dx^2+$$

$$\left(-\frac{1}{16}\,i\,k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}}+2\,U\right)-\frac{1}{32\left(3+H^2\,k^2\right)^2}i\,k^6\left(3\,g\,H\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+4\left(3+H^2\,k^2\right)U\right)+\right.\right.$$

$$\left.\left.U^2\left(15\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)dt^2+\right.$$

$$\frac{1}{32\left(3+H^2\,k^2\right)^2}k^7\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(3\,g\,H+U\left(3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\right)dt^3+$$

$$\frac{1}{64\left(3+H^2\,k^2\right)^3}i\,k^8\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)$$

$$\left.\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^4+O[dt]^5\right)dx^3+$$

$$\left(-\left(\left(k^5\left(\sqrt{3}\,g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)+2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\right)\Big/\left(38\,400\left(\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{5/2}\right)\right)\right)-$$

$$\left(\left(k^7\left(45\sqrt{3}\,g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+g\,H\,U\left(447\,588\sqrt{3}\,H^2\,k^2\,U+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.16\,705\sqrt{3}\,H^6\,k^6\,U+648\left(693\sqrt{g\,H\left(3+H^2\,k^2\right)}+688\sqrt{3}\,U\right)+\right.\right.\right.\right.$$

$$15\,k^4\left(3408\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+9985\sqrt{3}\;H^4\,U\right)+$$
$$80\left(1836\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+612\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;U^3+68\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\right.$$
$$\left.\left.U^3+9\,k^2\left(421\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U+204\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^3\right)\right)\right)dt^2\Big)\Big/$$
$$\left(230\,400\left(\sqrt{g\,H}\;\left(3+H^2\,k^2\right)^{7/2}\right)\right)-\frac{1}{25\,600\left(3+H^2\,k^2\right)^4}i\,k^8\left(6\,g^2\,H^2\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)+\right.$$
$$560\,H^8\,k^8\,U^4+432\,U^3\left(143\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+105\,U\right)+$$
$$5\,k^4\,U^3\left(4139\sqrt{3}\;\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+6048\,H^4\,U\right)+$$
$$5\,k^6\,U^3\left(461\sqrt{3}\;\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}+1344\,H^6\,U\right)+$$
$$12\,k^2\left(3585\sqrt{3}\;\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U+5161\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^3+5040\,H^2\,U^4\right)+$$
$$g\,H\,U\left(63\,720\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+282\,852\,U+284\,364\,H^2\,k^2\,U+10\,640\,H^6\,k^6\,U+\right.$$
$$\left.\left.5\,k^4\left(1451\sqrt{3}\;\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+19\,056\,H^4\,U\right)\right)\right)dt^3+\frac{1}{460\,800\sqrt{g\,H}\;\left(3+H^2\,k^2\right)^{11/2}}$$
$$k^9\left(\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(5\,k^8\,U^3\left(3869\sqrt{3}\;g\,H^9+928\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\;U\right)+\right.$$
$$108\,k^2\left(2625\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}+15\,327\sqrt{3}\;g^2\,H^4\,U+22\,383\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U^2+\right.$$
$$\left.19\,261\sqrt{3}\;g\,H^3\,U^3+4640\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^4\right)+$$
$$1296\left(323\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+1268\sqrt{3}\;g^2\,H^2\,U+290\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+\right.$$
$$\left.g\,H\,U^2\left(1857\sqrt{g\,H\left(3+H^2\,k^2\right)}+1202\sqrt{3}\;U\right)\right)+15\,k^6\,U\left(4143\sqrt{3}\;g^2\,H^8+\right.$$
$$\left.3712\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\;U^3+g\,H^7\,U\left(6019\sqrt{g\,H\left(3+H^2\,k^2\right)}+15\,454\sqrt{3}\;U\right)\right)+$$
$$9\,k^4\left(5325\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}+61\,735\sqrt{3}\;g^2\,H^6\,U+27\,840\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;U^4+\right.$$
$$\left.\left.g\,H^5\,U^2\left(89\,915\sqrt{g\,H\left(3+H^2\,k^2\right)}+115\,737\sqrt{3}\;U\right)\right)\right)dt^4+O[dt]^5\Big)dx^4+O[dx]^5,$$

$$\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(3\,g\,H+U\left(-2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$
$$dt^2+$$
$$\frac{i\,k^4\left(3\,g\,H+U\left(-2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^3}{8\left(3+H^2\,k^2\right)^2}-\frac{1}{20\left(3+H^2\,k^2\right)^4}$$
$$\left(k^5\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\left(3\,g\,H+U\left(-2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)$$
$$\left.dt^4+O[dt]^5\right)+$$

$$\left(\frac{1}{240\left(3+H^2\,k^2\right)^2}k^3\left(-42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)+\right.$$
$$\frac{1}{480\left(3+H^2\,k^2\right)^3}k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(-9\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+10\,U\right)+\right.$$
$$5\,k^4\,U^2\left(-11\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+36\,H^4\,U\right)+$$
$$6\,g\,H\left(-21\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)-$$
$$\left.3\,k^2\left(15\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}+109\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^2-180\,H^2\,U^3\right)\right)dt^2+$$
$$\frac{1}{480\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$
$$\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(-102\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-\right.\right.$$
$$\left.\left.5\,k^2\left(7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-24\,H^2\,U\right)\right)\right)dt^3-\frac{1}{960\left(3+H^2\,k^2\right)^4}$$
$$\left(k^7\left(-42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)\right.$$
$$\left.\left.\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)dt^4+O[dt]^5\right)dx^2+$$
$$\left(\frac{1}{16}i\,k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}}-2\,U\right)-\frac{1}{32\left(3+H^2\,k^2\right)^2}i\,k^6\left(-3\,g\,H\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-4\left(3+H^2\,k^2\right)U\right)+\right.\right.$$
$$\left.\left.U^2\left(-15\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(-5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)\right)$$
$$dt^2+\frac{1}{32\left(3+H^2\,k^2\right)^2}k^7\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$
$$\left(3\,g\,H+U\left(-3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\right)dt^3+$$
$$\frac{1}{64\left(3+H^2\,k^2\right)^3}i\,k^8\left(-\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)$$
$$\left.\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^4+O[dt]^5\right)dx^3+$$
$$\left(\left(k^5\left(\sqrt{3}\,g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)-\right.\right.\right.$$
$$\left.\left.\left.2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\right)\Big/$$
$$\left(38\,400\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{5/2}\right)+\left(k^7\left(45\sqrt{3}\,g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+\right.\right.$$
$$g\,H\,U\left(447\,588\sqrt{3}\,H^2\,k^2\,U+16\,705\sqrt{3}\,H^6\,k^6\,U-648\left(693\sqrt{g\,H\left(3+H^2\,k^2\right)}-688\sqrt{3}\,U\right)-\right.$$
$$\left.15\,k^4\left(3408\sqrt{g\,H^9\left(3+H^2\,k^2\right)}-9985\sqrt{3}\,H^4\,U\right)\right)-$$
$$80\left(1836\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+612\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+68\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+\right.$$
$$\left.\left.9\,k^2\left(421\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+204\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\right)dt^2\Big)\Big/$$
$$\left(230\,400\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}\right)-\frac{1}{25\,600\left(3+H^2\,k^2\right)^4}i\,k^8\left(6\,g^2\,H^2\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)+\right.$$
$$560\,H^8\,k^8\,U^4+432\,U^3\left(-143\sqrt{3}\,\sqrt{H\left(3+H^2\,k^2\right)}+105\,U\right)+$$

$$
\begin{aligned}
&560\,H^6\,k^6\,U^3 + 432\,U^3\left(-145\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)} + 105\,U\right) + \\
&5\,k^4\,U^3\left(-4139\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)} + 6048\,H^4\,U\right) + \\
&5\,k^6\,U^3\left(-461\sqrt{3}\ \sqrt{g\,H^{13}\left(3+H^2\,k^2\right)} + 1344\,H^6\,U\right) - \\
&12\,k^2\left(3585\sqrt{3}\ \sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U + 5161\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3 - 5040\,H^2\,U^4\right) + \\
&g\,H\,U\left(-63\,720\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)} + 282\,852\,U + 284\,364\,H^2\,k^2\,U + 10\,640\,H^6\,k^6\,U + \right. \\
&\qquad \left. 5\,k^4\left(-1451\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)} + 19\,056\,H^4\,U\right)\right)\Big)\,dt^3 - \frac{1}{460\,800\left(\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{11/2}\right)} \\
&\left(k^9\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)} - \left(3+H^2\,k^2\right)U\right)\left(5\,k^8\,U^3\left(-3869\sqrt{3}\ g\,H^9 + 928\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\ U\right) + \right.\right. \\
&\qquad 108\,k^2\left(2625\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)} - 15\,327\sqrt{3}\ g^2\,H^4\,U + 22\,383\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U^2 - \right. \\
&\qquad\qquad \left. 19\,261\sqrt{3}\ g\,H^3\,U^3 + 4640\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^4\right) + \\
&\qquad 9\,k^4\left(5325\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)} - 61\,735\sqrt{3}\ g^2\,H^6\,U + 27\,840\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^4 + \right. \\
&\qquad\qquad g\,H^5\,U^2\left(89\,915\sqrt{g\,H\left(3+H^2\,k^2\right)} - 115\,737\sqrt{3}\ U\right)\Big) + 1296\left(323\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)} - \\
&\qquad\qquad 1268\sqrt{3}\ g^2\,H^2\,U + 290\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^4 + g\,H\,U^2\left(1857\sqrt{g\,H\left(3+H^2\,k^2\right)} - \right. \\
&\qquad\qquad\qquad \left. 1202\sqrt{3}\ U\right)\Big) - 15\,k^6\,U\left(4143\sqrt{3}\ g^2\,H^8 - 3712\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ U^3 + \right. \\
&\qquad\qquad g\,H^7\,U\left(-6019\sqrt{g\,H\left(3+H^2\,k^2\right)} + 15\,454\sqrt{3}\ U\right)\Big)\Big)\Big)\,dt^4 + O[dt]^5\Big)\,dx^4 + O[dx]^5\Big\}
$$

Out[203]= Omega error ||

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\left\{\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U
    \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6
    \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H
    \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}-\frac{\left(k^5 \left(\left(H^2
    k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2
    k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{20 \left(H^2
    k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(20 H^4 U k^4+15 \left(8 U H^2+\sqrt{3}
    \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+180 U+42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{240
    \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 U H^4+11 \sqrt{3} \sqrt{g H^9
    \left(H^2 k^2+3\right)}\right) k^4+3 \left(180 H^2 U^3+109 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}
    U^2+15 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}\right) k^2+54 U^2 \left(10 U+9 \sqrt{3} \sqrt{g
    H \left(H^2 k^2+3\right)}\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U+21 \sqrt{3}
    \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{480 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3
    g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(9 g
    H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4+5 \left(24 U H^2+7 \sqrt{3} \sqrt{g H^5 \left(H^2
    k^2+3\right)}\right) k^2+180 U+102 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{480
    \left(H^2 k^2+3\right)^3}-\frac{\left(k^7 \left(20 H^4 U k^4+15 \left(8 U H^2+\sqrt{3} \sqrt{g H^5 \left(H^2
    k^2+3\right)}\right) k^2+180 U+42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2
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k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{960 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(−\frac{1}{16} i k^4 \left(2 U+\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}\right)−\frac{i k^6 \left(\left(2 H^4 U k^4+\left(12 U H^2+5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g H \left(4 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^2}+\frac{k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 g H+U \left(2 \left(H^2 k^2+3\right) U+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(2 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(−\frac{k^5 \left(\sqrt{3} g H \left(2075 H^4 k^4+12180 H^2 k^2+17856\right)+2080 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{38400 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{5/2}\right)}−\frac{\left(k^7 \left(45 \sqrt{3} g^2 \left(385 H^4 k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6+15 \left(9985 \sqrt{3} U H^4+3408 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+447588 \sqrt{3} H^2 U k^2+648 \left(688 \sqrt{3} U+693 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) H+80 \left(68 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+612 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(204 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+421 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+1836 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right)\right) \text{dt}^2}{230400 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)}−\frac{i k^8 \left(560 H^8 U^4 k^8+5 U^3 \left(1344 U H^6+461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 U H^4+4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+12 \left(5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 U+143 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+g H U \left(10640 H^6 U k^6+5 \left(19056 U H^4+1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U+63720 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{25600 \left(H^2 k^2+3\right)^4}+\frac{k^9 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(5 U^3 \left(3869 \sqrt{3} g H^9+928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U\right) k^8+15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U+6019 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7+3712 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(61735 \sqrt{3} g^2 U H^6+g U^2 \left(115737 \sqrt{3} U+89915 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+108 \left(15327 \sqrt{3} g^2 U H^4+19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+2625 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+1296 \left(290 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1202 \sqrt{3} U+1857 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+1268 \sqrt{3} g^2 H^2 U+323 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{460800 \sqrt{g H} \left(H^2 k^2+3\right)^{11/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right),\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U−\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}−\frac{\left(k^5 \left(\left(H^2 k^2+3\right) U−\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2

k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{20 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(20 H^4 U k^4−15 \left(\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−8 H^2 U\right) k^2+180 U−42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{240 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 H^4 U−11 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4−3 \left(−180 H^2 U^3+109 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^2+15 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}\right) k^2+54 U^2 \left(10 U−9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U−21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{480 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(9 g H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4−5 \left(7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−24 H^2 U\right) k^2+180 U−102 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{480 \left(H^2 k^2+3\right)^3}−\frac{\left(k^7 \left(20 H^4 U k^4−15 \left(\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−8 H^2 U\right) k^2+180 U−42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{960 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{16} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}−2 U\right)−\frac{i k^6 \left(U^2 \left(2 H^4 U k^4+\left(12 H^2 U−5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U−15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)−3 g H \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}−4 \left(H^2 k^2+3\right) U\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^2}+\frac{k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 g H+U \left(2 \left(H^2 k^2+3\right) U−3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(2 \left(H^2 k^2+3\right) U−\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{k^5 \left(\sqrt{3} g H \left(2075 H^4 k^4+12180 H^2 k^2+17856\right)−2080 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{38400 \sqrt{g H} \left(H^2 k^2+3\right)^{5/2}}+\frac{k^7 \left(45 \sqrt{3} g^2 \left(385 H^4 k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6−15 \left(3408 \sqrt{g H^9 \left(H^2 k^2+3\right)}−9985 \sqrt{3} H^4 U\right) k^4+447588 \sqrt{3} H^2 U k^2−648 \left(693 \sqrt{g H \left(H^2 k^2+3\right)}−688 \sqrt{3} U\right)\right) H−80 \left(68 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+612 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(204 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+421 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+1836 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text{dt}^2}{230400 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}−\frac{i k^8 \left(560 H^8 U^4 k^8+5 U^3 \left(1344 H^6 U−461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 H^4 U−4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4−12 \left(−5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 U−143 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+g H U \left(10640 H^6 U k^6+5 \left(19056 H^4 U−1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U−63720 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{25600 \left(H^2 k^2+3\right)^4}−\frac{\left(k^9 \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}−\left(H^2 k^2+3\right) U\right) \left(5 U^3 \left(928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U−3869 \sqrt{3} g H^9\right) k^8−15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U−6019 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7−3712 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right)

\sqrt{g H \left(H^2 k^2+3\right)}\right) H^7−3712 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(−61735 \sqrt{3} g^2 U H^6+g U^2 \left(89915 \sqrt{g H \left(H^2 k^2+3\right)}−115737 \sqrt{3} U\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+108 \left(−15327 \sqrt{3} g^2 U H^4−19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+2625 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+1296 \left(290 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1857 \sqrt{g H \left(H^2 k^2+3\right)}−1202 \sqrt{3} U\right) U^2−1268 \sqrt{3} g^2 H^2 U+323 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{460800 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{11/2}\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right\}

Out[204]=

Out[205]= EA || $\left\{\left\{\frac{-H^2\,k^2\left(\left(-1+e^{i\,dt\,w}\right)k\,U-w\right)+3\,w}{\left(3+H^2\,k^2\right)w},\ -\frac{3\left(-1+e^{i\,dt\,w}\right)k}{\left(3+H^2\,k^2\right)w}\right\},\ \left\{-\frac{\left(-1+e^{i\,dt\,w}\right)k\left(g\,H\left(3+H^2\,k^2\right)-3\,U^2\right)}{\left(3+H^2\,k^2\right)w},\ 1-\frac{\left(-1+e^{i\,dt\,w}\right)k\left(6+H^2\,k^2\right)U}{\left(3+H^2\,k^2\right)w}\right\}\right\}$

Out[206]= EA || \left(
\begin{array}{cc}
\frac{3 w−H^2 k^2 \left(\left(−1+e^{i \text{dt} w}\right) k U−w\right)}{\left(H^2 k^2+3\right) w} & −\frac{3 \left(−1+e^{i \text{dt} w}\right) k}{\left(H^2 k^2+3\right) w} \\
−\frac{\left(−1+e^{i \text{dt} w}\right) k \left(g H \left(H^2 k^2+3\right)−3 U^2\right)}{\left(H^2 k^2+3\right) w} & 1−\frac{\left(−1+e^{i \text{dt} w}\right) k \left(H^2 k^2+6\right) U}{\left(H^2 k^2+3\right) w} \\
\end{array}
\right)

Out[207]= Eerr ||
$$\left\{\left\{\left(\frac{\left(-3\,g\,H\,k^2+3\,k^2\,U^2-H^2\,k^4\,U^2-H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{H^2\,k^3\,U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\right)+\right.$$
$$\left(-\frac{i\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}-\frac{\left(126\,g\,H\,k^4+45\,g\,H^3\,k^6-72\,k^4\,U^2+30\,H^2\,k^6\,U^2+20\,H^4\,k^8\,U^2\right)dt^2}{240\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$
$$\left(-\frac{1}{8}\left(k^4\,U\right)dt+\left(\frac{3\,i\,g\,H\,k^5}{16\left(3+H^2\,k^2\right)}+\frac{i\,H^2\,k^7\,U^2}{8\left(3+H^2\,k^2\right)}\right)dt^2+O[dt]^5\right)dx^3+$$
$$\left(\frac{i\left(729\,k^5\,U+2610\,H^2\,k^7\,U+1570\,H^4\,k^9\,U+260\,H^6\,k^{11}\,U\right)dt}{4800\left(3+H^2\,k^2\right)^3}+\frac{1}{28\,800\left(3+H^2\,k^2\right)^3}\left(38\,853\,g\,H\,k^6+26\,460\,g\,H^3\,k^8+4500\,g\,H^5\,k^{10}-\right.\right.$$
$$\left.\left.38\,286\,k^6\,U^2-13\,500\,H^2\,k^8\,U^2+4140\,H^4\,k^{10}\,U^2+1460\,H^6\,k^{12}\,U^2\right)dt^2+O[dt]^5\right)dx^4+$$
$$O[dx]^5,\left(\left(-\frac{3\,k^2\,U}{3+H^2\,k^2}-\frac{3\,k\,w}{2\left(3+H^2\,k^2\right)}\right)dt^2-\frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)}+\frac{k\,w^3\,dt^4}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)+$$
$$\left(-\frac{i\left(12\,k^3+5\,H^2\,k^5\right)dt}{40\left(3+H^2\,k^2\right)^2}-\frac{3\left(\left(14\,k^4+5\,H^2\,k^6\right)U\right)dt^2}{40\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+\left(\frac{3\,i\,k^5\,U\,dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$
$$\left(\frac{i\left(6291\,k^5+4410\,H^2\,k^7+770\,H^4\,k^9\right)dt}{4800\left(3+H^2\,k^2\right)^3}+\frac{\left(12\,951\,k^6\,U+8820\,H^2\,k^8\,U+1500\,H^4\,k^{10}\,U\right)dt^2}{4800\left(3+H^2\,k^2\right)^3}+O[dt]^5\right)dx^4+O[dx]^5\Big\},$$
$$\left\{\left(-\frac{\left(k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\left(2\,k\,U+w\right)\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\right)+\right.$$
$$\left(-\frac{i\left(90\,g\,H\,k^3+60\,g\,H^3\,k^5+10\,g\,H^5\,k^7-36\,k^3\,U^2-15\,H^2\,k^5\,U^2\right)dt}{120\left(3+H^2\,k^2\right)^2}+\frac{\left(-180\,g\,H\,k^4\,U-120\,g\,H^3\,k^6\,U-20\,g\,H^5\,k^8\,U+126\,k^4\,U^3+45\,H^2\,k^6\,U^3\right)dt^2}{120\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)$$
$$dx^2+\left(-\frac{1}{8}\left(g\,H\,k^4\right)dt+\frac{i\left(6\,g\,H\,k^5\,U+2\,g\,H^3\,k^7\,U-3\,k^5\,U^3\right)dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$
$$\left(\frac{1}{4800\left(3+H^2\,k^2\right)^3}i\left(7020\,g\,H\,k^5+7020\,g\,H^3\,k^7+2340\,g\,H^5\,k^9+260\,g\,H^7\,k^{11}-6291\,k^5\,U^2-4410\,H^2\,k^7\,U^2-\right.\right.$$
$$\left.\left.770\,H^4\,k^9\,U^2\right)dt+\frac{1}{14\,400\left(3+H^2\,k^2\right)^3}\left(39\,420\,g\,H\,k^6\,U+39\,420\,g\,H^3\,k^8\,U+13\,140\,g\,H^5\,k^{10}\,U+\right.\right.$$
$$\left.\left.1460\,g\,H^7\,k^{12}\,U-38\,853\,k^6\,U^3-26\,460\,H^2\,k^8\,U^3-4500\,H^4\,k^{10}\,U^3\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5,$$
$$\left(\frac{\left(-3\,g\,H\,k^2-9\,k^2\,U^2-H^2\,k^4\,U^2-6\,k\,U\,w-H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(6+H^2\,k^2\right)U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{k\left(6+H^2\,k^2\right)U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\right)+$$
$$\left(-\frac{i\left(126\,k^3+75\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}-\frac{\left(126\,g\,H\,k^4+45\,g\,H^3\,k^6+432\,k^4\,U^2+210\,H^2\,k^6\,U^2+20\,H^4\,k^8\,U^2\right)dt^2}{240\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$
$$\left(-\frac{1}{8}\left(k^4\,U\right)dt+\frac{i\,k^5\left(3\,g\,H+12\,U^2+2\,H^2\,k^2\,U^2\right)dt^2}{16\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$
$$\left(\frac{i\left(13\,311\,k^5+11\,430\,H^2\,k^7+3110\,H^4\,k^9+260\,H^6\,k^{11}\right)U\,dt}{4800\left(3+H^2\,k^2\right)^3}+\frac{1}{28\,800\left(3+H^2\,k^2\right)^3}\left(38\,853\,g\,H\,k^6+26\,460\,g\,H^3\,k^8+4500\,g\,H^5\,k^{10}+\right.\right.$$
$$\left.\left.117\,126\,k^6\,U^2+92\,340\,H^2\,k^8\,U^2+22\,140\,H^4\,k^{10}\,U^2+1460\,H^6\,k^{12}\,U^2\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5\Big\}\Big\}$$

Out[208]= Eerr || \left(

\begin{array}{cc}

\left(\frac{\left(-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right) \text{dt}^2}{2 \left(H^2 k^2+3\right)}-\frac{i H^2 k^3 U w^2 \text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+ & \left(\left(-\frac{3 U k^2}{H^2 k^2+3}-\frac{3 w k}{2 \left(H^2 k^2+3\right)}\right) \text{dt}^2-\frac{i k w^2 \text{dt}^3}{2 \left(H^2 k^2+3\right)}+\frac{k w^3 \text{dt}^4}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+ \\

\left(-\frac{i \left(54 k^3+45 H^2 k^5+10 H^4 k^7\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}-\frac{\left(126 g H k^4+45 g H^3 k^6-72 k^4 U^2+30 H^2 k^6 U^2+20 H^4 k^8 U^2\right) \text{dt}^2}{240 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+ & \left(-\frac{i \left(10 H^4 k^7+45 H^2 k^5+54 k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}-\frac{\left(20 H^4 U^2 k^8+45 g H^3 k^6+30 H^2 U^2 k^6-72 U^2 k^4+126 g H k^4\right) \text{dt}^2}{240 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+ \\

\left(-\frac{1}{8} \left(k^4 U\right) \text{dt}+\left(\frac{3 i g H k^5}{16 \left(H^2 k^2+3\right)}+\frac{i H^2 k^7 U^2}{8 \left(H^2 k^2+3\right)}\right) \text{dt}^2+O\left(\text{dt}^5\right)\right) \text{dx}^3+ & \left(-\frac{1}{8} \left(k^4 U\right) \text{dt}+\frac{i k^5 \left(3 g H+12 U^2+2 H^2 k^2 U^2\right) \text{dt}^2}{16 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+ \\

\left(\frac{i \left(729 k^5 U+2610 H^2 k^7 U+1570 H^4 k^9 U+260 H^6 k^{11} U\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{1}{28800 \left(H^2 k^2+3\right)^3} \left(38853 g H k^6+26460 g H^3 k^8+4500 g H^5 k^{10}-38286 k^6 U^2-13500 H^2 k^8 U^2+4140 H^4 k^{10} U^2+1460 H^6 k^{12} U^2\right) \text{dt}^2+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{i \left(13311 k^5+11430 H^2 k^7+3110 H^4 k^9+260 H^6 k^{11}\right) U \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{1}{28800 \left(H^2 k^2+3\right)^3} \left(38853 g H k^6+26460 g H^3 k^8+4500 g H^5 k^{10}+117126 k^6 U^2+92340 H^2 k^8 U^2+22140 H^4 k^{10} U^2+1460 H^6 k^{12} U^2\right) \text{dt}^2+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)

\end{array}\right)

```
w^3 \text{dt}^4}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(5
H^2 k^5+12 k^3\right) \text{dt}}{40 \left(H^2 k^2+3\right)^2}-\frac{3 \left(\left(5 H^2 k^6+14
k^4\right) U\right) \text{dt}^2}{40 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(\frac{3 i k^5 U \text{dt}^2}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{i \left(770 H^4 k^9+4410 H^2 k^7+6291 k^5\right) \text{dt}}{4800 \left(H^2
k^2+3\right)^3}+\frac{\left(1500 H^4 U k^{10}+8820 H^2 U k^8+12951 U k^6\right) \text{dt}^2}{4800
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\left(-\frac{\left(k \left(g k^2 H^3+3 g H-3 U^2\right) (2 k U+w)\right) \text{dt}^2}{2 \left(H^2 k^2+3\right)}-\frac{i
k \left(g k^2 H^3+3 g H-3 U^2\right) w^2 \text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{k \left(g k^2 H^3+3
g H-3 U^2\right) w^3 \text{dt}^4}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i
\left(10 g H^5 k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}}{120
\left(H^2 k^2+3\right)^2}+\frac{\left(-20 g H^5 U k^8+45 H^2 U^3 k^6-120 g H^3 U k^6+126 U^3
k^4-180 g H U k^4\right) \text{dt}^2}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(-\frac{1}{8} \left(g H k^4\right) \text{dt}+\frac{i \left(2 g H^3 U k^7-3 U^3 k^5+6 g H
U k^5\right) \text{dt}^2}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i
\left(260 g H^7 k^{11}+2340 g H^5 k^9-770 H^4 U^2 k^9+7020 g H^3 k^7-4410 H^2 U^2 k^7-6291
U^2 k^5+7020 g H k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 g H^7 U
k^{12}-4500 H^4 U^3 k^{10}+13140 g H^5 U k^{10}-26460 H^2 U^3 k^8+39420 g H^3 U k^8-38853
U^3 k^6+39420 g H U k^6\right) \text{dt}^2}{14400 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)
\text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{\left(-H^2 U^2 k^4-H^2 U w k^3-9 U^2 k^2-3 g
H k^2-6 U w k\right) \text{dt}^2}{2 \left(H^2 k^2+3\right)}-\frac{i k \left(H^2 k^2+6\right) U w^2
\text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{k \left(H^2 k^2+6\right) U w^3 \text{dt}^4}{24 \left(H^2
k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(10 H^4 k^7+75 H^2 k^5+126 k^3\right) U
\text{dt}}{120 \left(H^2 k^2+3\right)^2}-\frac{\left(20 H^4 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432
U^2 k^4+126 g H k^4\right) \text{dt}^2}{240 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(-\frac{1}{8} \left(k^4 U\right) \text{dt}+\frac{i k^5 \left(2 H^2 k^2 U^2+12 U^2+3 g
H\right) \text{dt}^2}{16 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i
\left(260 H^6 k^{11}+3110 H^4 k^9+11430 H^2 k^7+13311 k^5\right) U \text{dt}}{4800 \left(H^2
k^2+3\right)^3}+\frac{\left(1460 H^6 U^2 k^{12}+4500 g H^5 k^{10}+22140 H^4 U^2 k^{10}+26460
g H^3 k^8+92340 H^2 U^2 k^8+117126 U^2 k^6+38853 g H k^6\right) \text{dt}^2}{28800
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\end{array}
\right)
```

In[209]:=
```
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → 0 /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
```

```
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 ;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ];


Text[Row[{" U < -Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn  ||   ", Kfnnp}]]
Text[Row[{"Fnn  ||   ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||   ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||   ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||   ", KfnGp}]]
Text[Row[{"FnG  ||   ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error  ||   ", FnG2TAr}]]
Text[Row[{"FnG error  ||   ", TeXForm[FnG2TAr]}]]
```

```
Text[" "]
Text[Row[{"FGn  ||   ", KfGnp}]]
Text[Row[{"FGn  ||   ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||   ", FGn2TAr}]]
Text[Row[{"FGn error  ||   ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||   ", KfGGp}]]
Text[Row[{"FGG  ||   ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||   ", FGG2TAr}]]
Text[Row[{"FGG error  ||   ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||   ", RKstepTayr}]]
Text[Row[{"Omega error  ||   ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||   ", EA}]]
Text[Row[{"EA  ||   ", TeXForm[EA]}]]
Text[Row[{"Eerr ||   ", Eerr}]]
Text[Row[{"Eerr ||   ", TeXForm[Eerr]}]]
```

Out[243]= U < −Sqrt(gH)

Out[244]=

Out[245]= Fnn || Gnp H + Rpp U

Out[246]= Fnn || \text{Gnp} H+\text{Rpp} U

Out[247]= Fnn error ||
$$\left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(\frac{1}{8}\,k^4\,U\,dt+O[dt]^4\right)dx^3+O[dx]^4$$

Out[248]= Fnn error ||
\left(−\frac{\text{dt}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 H^2 k^3 U w^2}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(−\frac{i \left(10 H^4 k^7+45 H^2 k^5+54 k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(\frac{1}{8} k^4 U \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[249]=

Out[250]= FnG || GGp H

Out[251]= FnG || \text{GGp} H

Out[252]= FnG error ||
$$\left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(12\,k^3+5\,H^2\,k^5\right)dt}{40\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+O[dx]^4$$

Out[253]= FnG error || \left(−\frac{3 \text{dt}^2 (k w)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 k w^2}{2 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(−\frac{i \left(5 H^2 k^5+12 k^3\right) \text{dt}}{40 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[254]=

Out[255]= FGn || H (g Rpp + Gnp U)

Out[256]= FGn || H (g \text{Rpp}+\text{Gnp} U)

Out[257]= FGn error || $\left(-\frac{\left(k \left(3\, g\, H+g\, H^3\, k^2-3\, U^2\right) w\right) dt^2}{2 \left(3+H^2\, k^2\right)}-\frac{i\, k \left(3\, g\, H+g\, H^3\, k^2-3\, U^2\right) w^2\, dt^3}{6 \left(3+H^2\, k^2\right)}+O[dt]^4\right)+\left(-\frac{i \left(90\, g\, H\, k^3+60\, g\, H^3\, k^5+10\, g\, H^5\, k^7-36\, k^3\, U^2-15\, H^2\, k^5\, U^2\right) dt}{120 \left(3+H^2\, k^2\right)^2}+O[dt]^4\right) dx^2+\left(\frac{1}{8} g\, H\, k^4\, dt+O[dt]^4\right) dx^3+O[dx]^4$

Out[258]= FGn error || \left(−\frac{\text{dt}^2 \left(k w \left(g H^3 k^2+3 g H−3 U^2\right)\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 k w^2 \left(g H^3 k^2+3 g H−3 U^2\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(−\frac{i \left(10 g H^5 k^7+60 g H^3 k^5−15 H^2 U^2 k^5−36 U^2 k^3+90 g H k^3\right) \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(\frac{1}{8} g H k^4 \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[259]=

Out[260]= FGG || (GGp H + Rpp) U

Out[261]= FGG || U (\text{GGp} H+\text{Rpp})

Out[262]= FGG error || $\left(-\frac{\left(k \left(6+H^2\, k^2\right) U\, w\right) dt^2}{2 \left(3+H^2\, k^2\right)}-\frac{i\, k \left(6+H^2\, k^2\right) U\, w^2\, dt^3}{6 \left(3+H^2\, k^2\right)}+O[dt]^4\right)+\left(-\frac{i \left(126\, k^3+75\, H^2\, k^5+10\, H^4\, k^7\right) U\, dt}{120 \left(3+H^2\, k^2\right)^2}+O[dt]^4\right) dx^2+\left(\frac{1}{8} k^4\, U\, dt+O[dt]^4\right) dx^3+\left(\frac{i \left(13\,311\, k^5+11\,430\, H^2\, k^7+3110\, H^4\, k^9+260\, H^6\, k^{11}\right) U\, dt}{4800 \left(3+H^2\, k^2\right)^3}+O[dt]^4\right) dx^4+O[dx]^5$

Out[263]= FGG error || \left(−\frac{\text{dt}^2 \left(k U w \left(H^2 k^2+6\right)\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 k U w^2 \left(H^2 k^2+6\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(−\frac{i \left(10 H^4 k^7+75 H^2 k^5+126 k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(\frac{1}{8} k^4 U \text{dt}+O\left(\text{dt}^4\right)\right)+\text{dx}^4 \left(\frac{i \left(260 H^6 k^{11}+3110 H^4 k^9+11430 H^2 k^7+13311 k^5\right) U \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^5\right)

Out[264]=

Out[265]=

Out[266]= Omega error ||

$\left\{\left(\frac{1}{6 \left(3+H^2\, k^2\right)^2} k^3 \left(\sqrt{3}\ \sqrt{g\, H \left(3+H^2\, k^2\right)}+\left(3+H^2\, k^2\right) U\right) \left(3\, g\, H+U \left(2 \sqrt{3}\ \sqrt{g\, H \left(3+H^2\, k^2\right)}+\left(3+H^2\, k^2\right) U\right)\right) dt^2+\right.$

$$\frac{i\,k^4\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\,dt^3}{8\left(3+H^2\,k^2\right)^2}-\frac{1}{20\left(3+H^2\,k^2\right)^4}\left(k^5\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\right.$$

$$\left.\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)dt^4+O[dt]^5\Bigg)+$$

$$\left(\frac{1}{240\left(3+H^2\,k^2\right)^2}k^3\left(42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U+15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+8\,H^2\,U\right)\right)+\right.$$

$$\frac{1}{480\left(3+H^2\,k^2\right)^3}k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(9\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+10\,U\right)+\right.$$

$$5\,k^4\,U^2\left(11\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+36\,H^4\,U\right)+$$

$$6\,g\,H\left(21\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)+$$

$$\left.3\,k^2\left(15\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}+109\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^2+180\,H^2\,U^3\right)\right)dt^2+$$

$$\frac{1}{480\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(102\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U+\right.\right.$$

$$\left.\left.5\,k^2\left(7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+24\,H^2\,U\right)\right)\right)dt^3-\frac{1}{960\left(3+H^2\,k^2\right)^4}$$

$$\left(k^7\left(42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U+15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+8\,H^2\,U\right)\right)\right.$$

$$\left.\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)dt^4+O[dt]^5\Bigg)dx^2+$$

$$\left(\frac{1}{16}\,i\,k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}}+2\,U\right)+\frac{1}{32\left(3+H^2\,k^2\right)^2}i\,k^6\left(3\,g\,H\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+4\left(3+H^2\,k^2\right)U\right)+\right.\right.$$

$$\left.\left.U^2\left(15\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)dt^2-\right.$$

$$\frac{1}{32\left(3+H^2\,k^2\right)^2}\left(k^7\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$

$$\left.\left(3\,g\,H+U\left(3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\right)\right)dt^3-$$

$$\frac{1}{64\left(3+H^2\,k^2\right)^3}i\,k^8\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)$$

$$\left.\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^4+O[dt]^5\right)dx^3+$$

$$\left(-\left(\left(k^5\left(\sqrt{3}\,g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)+2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\right)\Big/\left(38\,400\left(\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{5/2}\right)\right)\right)-$$

$$\left(\left(k^7\left(45\sqrt{3}\,g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+g\,H\,U\left(447\,588\sqrt{3}\,H^2\,k^2\,U+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.16\,705\sqrt{3}\,H^6\,k^6\,U+648\left(693\sqrt{g\,H\left(3+H^2\,k^2\right)}+688\sqrt{3}\,U\right)+\right.\right.\right.\right.$$

$$15\,k^4\left(3408\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+9985\sqrt{3}\;H^4\,U\right)+$$
$$80\left(1836\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+612\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;U^3+68\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\right.$$
$$\left.\left.U^3+9\,k^2\left(421\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U+204\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^3\right)\right)\right)dt^2\Big)\Big/$$
$$\left(230\,400\left(\sqrt{g\,H}\;\left(3+H^2\,k^2\right)^{7/2}\right)\right)-\frac{1}{25\,600\left(3+H^2\,k^2\right)^4}i\,k^8\left(6\,g^2\,H^2\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)+\right.$$
$$560\,H^8\,k^8\,U^4+432\,U^3\left(143\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+105\,U\right)+$$
$$5\,k^4\,U^3\left(4139\sqrt{3}\;\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+6048\,H^4\,U\right)+$$
$$5\,k^6\,U^3\left(461\sqrt{3}\;\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}+1344\,H^6\,U\right)+$$
$$12\,k^2\left(3585\sqrt{3}\;\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U+5161\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^3+5040\,H^2\,U^4\right)+$$
$$g\,H\,U\left(63\,720\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+282\,852\,U+284\,364\,H^2\,k^2\,U+10\,640\,H^6\,k^6\,U+\right.$$
$$\left.\left.\left.5\,k^4\left(1451\sqrt{3}\;\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+19\,056\,H^4\,U\right)\right)\right)dt^3+\frac{1}{460\,800\sqrt{g\,H}\;\left(3+H^2\,k^2\right)^{11/2}}$$
$$k^9\left(\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(5\,k^8\,U^3\left(3869\sqrt{3}\;g\,H^9+928\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\;U\right)+\right.$$
$$108\,k^2\left(2625\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}+15\,327\sqrt{3}\;g^2\,H^4\,U+22\,383\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U^2+\right.$$
$$\left.19\,261\sqrt{3}\;g\,H^3\,U^3+4640\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^4\right)+$$
$$1296\left(323\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+1268\sqrt{3}\;g^2\,H^2\,U+290\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+\right.$$
$$\left.g\,H\,U^2\left(1857\sqrt{g\,H\left(3+H^2\,k^2\right)}+1202\sqrt{3}\;U\right)\right)+15\,k^6\,U\left(4143\sqrt{3}\;g^2\,H^8+\right.$$
$$\left.3712\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\;U^3+g\,H^7\,U\left(6019\sqrt{g\,H\left(3+H^2\,k^2\right)}+15\,454\sqrt{3}\;U\right)\right)+$$
$$9\,k^4\left(5325\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}+61\,735\sqrt{3}\;g^2\,H^6\,U+27\,840\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;U^4+\right.$$
$$\left.\left.g\,H^5\,U^2\left(89\,915\sqrt{g\,H\left(3+H^2\,k^2\right)}+115\,737\sqrt{3}\;U\right)\right)\right)dt^4+O[dt]^5\Big)dx^4+O[dx]^5,$$

$$\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(3\,g\,H+U\left(-2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$
$$dt^2+$$
$$\frac{i\,k^4\left(3\,g\,H+U\left(-2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^3}{8\left(3+H^2\,k^2\right)^2}-\frac{1}{20\left(3+H^2\,k^2\right)^4}$$
$$\left(k^5\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\left(3\,g\,H+U\left(-2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$
$$\left.\left.dt^4+O[dt]^5\right)+\right.$$

$$\left(\frac{1}{240\left(3+H^2\,k^2\right)^2}k^3\left(-42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)+\right.$$
$$\frac{1}{480\left(3+H^2\,k^2\right)^3}k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(-9\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+10\,U\right)+\right.$$
$$5\,k^4\,U^2\left(-11\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+36\,H^4\,U\right)+$$
$$6\,g\,H\left(-21\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)-$$
$$\left.3\,k^2\left(15\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}+109\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^2-180\,H^2\,U^3\right)\right)dt^2+$$
$$\frac{1}{480\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$
$$\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(-102\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-\right.\right.$$
$$\left.\left.5\,k^2\left(7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-24\,H^2\,U\right)\right)\right)dt^3-\frac{1}{960\left(3+H^2\,k^2\right)^4}$$
$$\left(k^7\left(-42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)\right.$$
$$\left.\left.\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)dt^4+O[dt]^5\right)dx^2+$$
$$\left(-\frac{1}{16}\,i\,k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}}-2\,U\right)+\frac{1}{32\left(3+H^2\,k^2\right)^2}i\,k^6\left(-3\,g\,H\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-4\left(3+H^2\,k^2\right)U\right)+\right.\right.$$
$$\left.\left.U^2\left(-15\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(-5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)\right)$$
$$dt^2-\frac{1}{32\left(3+H^2\,k^2\right)^2}\left(k^7\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$
$$\left.\left(3\,g\,H+U\left(-3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\right)\right)dt^3-$$
$$\frac{1}{64\left(3+H^2\,k^2\right)^3}i\,k^8\left(-\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)$$
$$\left.\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^4+O[dt]^5\right)dx^3+$$
$$\left(\left(k^5\left(\sqrt{3}\,g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)-\right.\right.\right.$$
$$\left.\left.\left.2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\right)\Big/$$
$$\left(38\,400\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{5/2}\right)+\left(k^7\left(45\sqrt{3}\,g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+\right.\right.$$
$$g\,H\,U\left(447\,588\sqrt{3}\,H^2\,k^2\,U+16\,705\sqrt{3}\,H^6\,k^6\,U-648\left(693\sqrt{g\,H\left(3+H^2\,k^2\right)}-688\sqrt{3}\,U\right)-\right.$$
$$\left.15\,k^4\left(3408\sqrt{g\,H^9\left(3+H^2\,k^2\right)}-9985\sqrt{3}\,H^4\,U\right)\right)-$$
$$80\left(1836\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+612\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+68\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+\right.$$
$$\left.\left.9\,k^2\left(421\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+204\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\right)dt^2\Big)\Big/$$
$$\left(230\,400\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}\right)-\frac{1}{25\,600\left(3+H^2\,k^2\right)^4}i\,k^8\left(6\,g^2\,H^2\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)+\right.$$
$$560\,H^8\,k^8\,U^4+432\,U^3\left(-143\sqrt{3}\,\sqrt{H\left(3+H^2\,k^2\right)}+105\,U\right)+$$

$$
\begin{aligned}
&560\,H^6\,k^6\,U^3 + 432\,U^3\left(-145\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ + 105\,U\right) + \\
&5\,k^4\,U^3\left(-4139\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ + 6048\,H^4\,U\right) + \\
&5\,k^6\,U^3\left(-461\sqrt{3}\ \sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ + 1344\,H^6\,U\right) - \\
&12\,k^2\left(3585\sqrt{3}\ \sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U + 5161\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3 - 5040\,H^2\,U^4\right) + \\
&g\,H\,U\left(-63\,720\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ + 282\,852\,U + 284\,364\,H^2\,k^2\,U + 10\,640\,H^6\,k^6\,U + \right. \\
&\qquad \left. 5\,k^4\left(-1451\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ + 19\,056\,H^4\,U\right)\right)\Big)\,dt^3 - \frac{1}{460\,800\left(\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{11/2}\right)} \\
&\left(k^9\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ - \left(3+H^2\,k^2\right)U\right)\left(5\,k^8\,U^3\left(-3869\sqrt{3}\ g\,H^9 + 928\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\ U\right) + \right.\right. \\
&\qquad 108\,k^2\left(2625\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}\ - 15\,327\sqrt{3}\ g^2\,H^4\,U + 22\,383\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U^2 - \right. \\
&\qquad\quad \left. 19\,261\sqrt{3}\ g\,H^3\,U^3 + 4640\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^4\right) + \\
&\qquad 9\,k^4\left(5325\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}\ - 61\,735\sqrt{3}\ g^2\,H^6\,U + 27\,840\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^4 + \right. \\
&\qquad\quad g\,H^5\,U^2\left(89\,915\sqrt{g\,H\left(3+H^2\,k^2\right)}\ - 115\,737\sqrt{3}\ U\right)\Big) + 1296\left(323\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}\ - \right. \\
&\qquad\quad 1268\sqrt{3}\ g^2\,H^2\,U + 290\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^4 + g\,H\,U^2\left(1857\sqrt{g\,H\left(3+H^2\,k^2\right)}\ - \right. \\
&\qquad\qquad \left. 1202\sqrt{3}\ U\right)\Big) - 15\,k^6\,U\left(4143\sqrt{3}\ g^2\,H^8 - 3712\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ U^3 + \right. \\
&\qquad\quad \left.\left. g\,H^7\,U\left(-6019\sqrt{g\,H\left(3+H^2\,k^2\right)}\ + 15\,454\sqrt{3}\ U\right)\right)\Big)\Big)\,dt^4 + O[dt]^5\Big)\,dx^4 + O[dx]^5\Big\}
\end{aligned}
$$

Out[267]= Omega error ||

\left\{\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}-\frac{\left(k^5 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{20 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(20 H^4 U k^4+15 \left(8 U H^2+\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+180 U+42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{240 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 U H^4+11 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+3 \left(180 H^2 U^3+109 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^2+15 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}\right) k^2+54 U^2 \left(10 U+9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U+21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{480 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(9 g H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4+5 \left(24 U H^2+7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+180 U+102 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{480 \left(H^2 k^2+3\right)^3}-\frac{\left(k^7 \left(20 H^4 U k^4+15 \left(8 U H^2+\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+180 U+42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2

k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{960 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{16} i k^4 \left(2 U+\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}\right)+\frac{i k^6 \left(\left(2 H^4 U k^4+\left(12 U H^2+5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g H \left(4 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^2}-\frac{\left(k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 g H+U \left(2 \left(H^2 k^2+3\right) U+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^2}-\frac{i k^8 \left(2 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(-\frac{k^5 \left(\sqrt{3} g H \left(2075 H^4 k^4+12180 H^2 k^2+17856\right)+2080 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{38400 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{5/2}\right)}-\frac{\left(k^7 \left(45 \sqrt{3} g^2 \left(385 H^4 k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6+15 \left(9985 \sqrt{3} U H^4+3408 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+447588 \sqrt{3} H^2 U k^2+648 \left(688 \sqrt{3} U+693 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) H+80 \left(68 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+612 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(204 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+421 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+1836 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right)\right) \text{dt}^2}{230400 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)}-\frac{i k^8 \left(560 H^8 U^4 k^8+5 U^3 \left(1344 U H^6+461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 U H^4+4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+12 \left(5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 U+143 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+g H U \left(10640 H^6 U k^6+5 \left(19056 U H^4+1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U+63720 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{25600 \left(H^2 k^2+3\right)^4}+\frac{k^9 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(5 U^3 \left(3869 \sqrt{3} g H^9+928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U\right) k^8+15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U+6019 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7+3712 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(61735 \sqrt{3} g^2 U H^6+g U^2 \left(115737 \sqrt{3} U+89915 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+108 \left(15327 \sqrt{3} g^2 U H^4+19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+2625 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+1296 \left(290 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1202 \sqrt{3} U+1857 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+1268 \sqrt{3} g^2 H^2 U+323 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{460800 \sqrt{g H} \left(H^2 k^2+3\right)^{11/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right),\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}-\frac{\left(k^5 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2

k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{20 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(20 H^4 U k^4−15 \left(\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−8 H^2 U\right) k^2+180 U−42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{240 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 H^4 U−11 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4−3 \left(−180 H^2 U^3+109 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^2+15 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}\right) k^2+54 U^2 \left(10 U−9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U−21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{480 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(9 g H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4−5 \left(7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−24 H^2 U\right) k^2+180 U−102 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{480 \left(H^2 k^2+3\right)^3}−\frac{\left(k^7 \left(20 H^4 U k^4−15 \left(\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−8 H^2 U\right) k^2+180 U−42 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{960 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(−\frac{1}{16} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}−2 U\right)+\frac{i k^6 \left(U^2 \left(2 H^4 U k^4+\left(12 H^2 U−5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U−15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)−3 g H \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}−4 \left(H^2 k^2+3\right) U\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^2}−\frac{\left(k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 g H+U \left(2 \left(H^2 k^2+3\right) U−3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^2}−\frac{i k^8 \left(2 \left(H^2 k^2+3\right) U−\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{k^5 \left(\sqrt{3} g H \left(2075 H^4 k^4+12180 H^2 k^2+17856\right)−2080 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{38400 \sqrt{g H} \left(H^2 k^2+3\right)^{5/2}}+\frac{k^7 \left(45 \sqrt{3} g^2 \left(385 H^4 k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6−15 \left(3408 \sqrt{g H^9 \left(H^2 k^2+3\right)}−9985 \sqrt{3} H^4 U\right) k^4+447588 \sqrt{3} H^2 U k^2−648 \left(693 \sqrt{g H \left(H^2 k^2+3\right)}−688 \sqrt{3} U\right)\right) H−80 \left(68 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+612 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(204 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+421 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+1836 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text{dt}^2}{230400 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}−\frac{i k^8 \left(560 H^8 U^4 k^8+5 U^3 \left(1344 H^6 U−461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 H^4 U−4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4−12 \left(−5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 U−143 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+g H U \left(10640 H^6 U k^6+5 \left(19056 H^4 U−1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U−63720 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{25600 \left(H^2 k^2+3\right)^4}−\frac{\left(k^9 \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}−\left(H^2 k^2+3\right) U\right) \left(5 U^3 \left(928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U−3869 \sqrt{3} g H^9\right) k^8−15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U−6019 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7−3712 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right)

\sqrt{g H \left(H^2 k^2+3\right)}\right) H^7−3712 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(−61735 \sqrt{3} g^2 U H^6+g U^2 \left(89915 \sqrt{g H \left(H^2 k^2+3\right)}−115737 \sqrt{3} U\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+108 \left(−15327 \sqrt{3} g^2 U H^4−19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+2625 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+1296 \left(290 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1857 \sqrt{g H \left(H^2 k^2+3\right)}−1202 \sqrt{3} U\right) U^2−1268 \sqrt{3} g^2 H^2 U+323 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{460800 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{11/2}\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right\}

Out[268]=

Out[269]= EA || $\left\{\left\{\frac{-H^2 k^2 \left(\left(-1+e^{i\,\text{dt}\,w}\right) k\,U-w\right)+3\,w}{\left(3+H^2 k^2\right) w},\ -\frac{3\left(-1+e^{i\,\text{dt}\,w}\right) k}{\left(3+H^2 k^2\right) w}\right\},\ \left\{-\frac{\left(-1+e^{i\,\text{dt}\,w}\right) k\left(g\,H\left(3+H^2 k^2\right)-3\,U^2\right)}{\left(3+H^2 k^2\right) w},\ 1-\frac{\left(-1+e^{i\,\text{dt}\,w}\right) k\left(6+H^2 k^2\right) U}{\left(3+H^2 k^2\right) w}\right\}\right\}$

Out[270]= EA || \left(
\begin{array}{cc}
\frac{3 w−H^2 k^2 \left(\left(−1+e^{i \text{dt} w}\right) k U−w\right)}{\left(H^2 k^2+3\right) w} & −\frac{3 \left(−1+e^{i \text{dt} w}\right) k}{\left(H^2 k^2+3\right) w} \\
−\frac{\left(−1+e^{i \text{dt} w}\right) k \left(g H \left(H^2 k^2+3\right)−3 U^2\right)}{\left(H^2 k^2+3\right) w} & 1−\frac{\left(−1+e^{i \text{dt} w}\right) k \left(H^2 k^2+6\right) U}{\left(H^2 k^2+3\right) w} \\
\end{array}
\right)

Out[271]= Eerr ||

$$\Big\{\Big\{\Big(\frac{\left(-3\,g\,H\,k^2+3\,k^2\,U^2-H^2\,k^4\,U^2-H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{H^2\,k^3\,U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\Big)+$$
$$\Big(-\frac{i\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}-\frac{\left(126\,g\,H\,k^4+45\,g\,H^3\,k^6-72\,k^4\,U^2+30\,H^2\,k^6\,U^2+20\,H^4\,k^8\,U^2\right)dt^2}{240\left(3+H^2\,k^2\right)^2}+O[dt]^5\Big)\,dx^2+$$
$$\Big(\frac{1}{8}\,k^4\,U\,dt+\Big(-\frac{3\,i\,g\,H\,k^5}{16\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^7\,U^2}{8\left(3+H^2\,k^2\right)}\Big)dt^2+O[dt]^5\Big)\,dx^3+$$
$$\Big(\frac{i\left(729\,k^5\,U+2610\,H^2\,k^7\,U+1570\,H^4\,k^9\,U+260\,H^6\,k^{11}\,U\right)dt}{4800\left(3+H^2\,k^2\right)^3}+\frac{1}{28\,800\left(3+H^2\,k^2\right)^3}\Big(38\,853\,g\,H\,k^6+26\,460\,g\,H^3\,k^8+4500\,g\,H^5\,k^{10}-$$
$$38\,286\,k^6\,U^2-13\,500\,H^2\,k^8\,U^2+4140\,H^4\,k^{10}\,U^2+1460\,H^6\,k^{12}\,U^2\Big)dt^2+O[dt]^5\Big)\,dx^4+$$
$$O[dx]^5,\ \Big(\Big(-\frac{3\,k^2\,U}{3+H^2\,k^2}-\frac{3\,k\,w}{2\left(3+H^2\,k^2\right)}\Big)dt^2-\frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)}+\frac{k\,w^3\,dt^4}{8\left(3+H^2\,k^2\right)}+O[dt]^5\Big)+$$
$$\Big(-\frac{i\left(12\,k^3+5\,H^2\,k^5\right)dt}{40\left(3+H^2\,k^2\right)^2}-\frac{3\left(\left(14\,k^4+5\,H^2\,k^6\right)U\right)dt^2}{40\left(3+H^2\,k^2\right)^2}+O[dt]^5\Big)\,dx^2+\Big(-\frac{3\,i\,k^5\,U\,dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\Big)\,dx^3+$$
$$\Big(\frac{i\left(6291\,k^5+4410\,H^2\,k^7+770\,H^4\,k^9\right)dt}{4800\left(3+H^2\,k^2\right)^3}+\frac{\left(12\,951\,k^6\,U+8820\,H^2\,k^8\,U+1500\,H^4\,k^{10}\,U\right)dt^2}{4800\left(3+H^2\,k^2\right)^3}+O[dt]^5\Big)\,dx^4+O[dx]^5\Big\},$$
$$\Big\{\Big(-\frac{\left(k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\left(2\,k\,U+w\right)\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\Big)+$$
$$\Big(-\frac{i\left(90\,g\,H\,k^3+60\,g\,H^3\,k^5+10\,g\,H^5\,k^7-36\,k^3\,U^2-15\,H^2\,k^5\,U^2\right)dt}{120\left(3+H^2\,k^2\right)^2}+\frac{\left(-180\,g\,H\,k^4\,U-120\,g\,H^3\,k^6\,U-20\,g\,H^5\,k^8\,U+126\,k^4\,U^3+45\,H^2\,k^6\,U^3\right)dt^2}{120\left(3+H^2\,k^2\right)^2}+O[dt]^5\Big)$$
$$dx^2+\Big(\frac{1}{8}\,g\,H\,k^4\,dt-\frac{i\left(6\,g\,H\,k^5\,U+2\,g\,H^3\,k^7\,U-3\,k^5\,U^3\right)dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\Big)\,dx^3+$$
$$\Big(\frac{1}{4800\left(3+H^2\,k^2\right)^3}i\,\Big(7020\,g\,H\,k^5+7020\,g\,H^3\,k^7+2340\,g\,H^5\,k^9+260\,g\,H^7\,k^{11}-6291\,k^5\,U^2-4410\,H^2\,k^7\,U^2-$$
$$770\,H^4\,k^9\,U^2\Big)\,dt+\frac{1}{14\,400\left(3+H^2\,k^2\right)^3}\Big(39\,420\,g\,H\,k^6\,U+39\,420\,g\,H^3\,k^8\,U+13\,140\,g\,H^5\,k^{10}\,U+$$
$$1460\,g\,H^7\,k^{12}\,U-38\,853\,k^6\,U^3-26\,460\,H^2\,k^8\,U^3-4500\,H^4\,k^{10}\,U^3\Big)dt^2+O[dt]^5\Big)\,dx^4+O[dx]^5,$$
$$\Big(\frac{\left(-3\,g\,H\,k^2-9\,k^2\,U^2-H^2\,k^4\,U^2-6\,k\,U\,w-H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(6+H^2\,k^2\right)U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{k\left(6+H^2\,k^2\right)U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\Big)+$$
$$\Big(-\frac{i\left(126\,k^3+75\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}-\frac{\left(126\,g\,H\,k^4+45\,g\,H^3\,k^6+432\,k^4\,U^2+210\,H^2\,k^6\,U^2+20\,H^4\,k^8\,U^2\right)dt^2}{240\left(3+H^2\,k^2\right)^2}+O[dt]^5\Big)\,dx^2+$$
$$\Big(\frac{1}{8}\,k^4\,U\,dt-\frac{i\,k^5\left(3\,g\,H+12\,U^2+2\,H^2\,k^2\,U^2\right)dt^2}{16\left(3+H^2\,k^2\right)}+O[dt]^5\Big)\,dx^3+$$
$$\Big(\frac{i\left(13\,311\,k^5+11\,430\,H^2\,k^7+3110\,H^4\,k^9+260\,H^6\,k^{11}\right)U\,dt}{4800\left(3+H^2\,k^2\right)^3}+\frac{1}{28\,800\left(3+H^2\,k^2\right)^3}\Big(38\,853\,g\,H\,k^6+26\,460\,g\,H^3\,k^8+4500\,g\,H^5\,k^{10}+$$
$$117\,126\,k^6\,U^2+92\,340\,H^2\,k^8\,U^2+22\,140\,H^4\,k^{10}\,U^2+1460\,H^6\,k^{12}\,U^2\Big)dt^2+O[dt]^5\Big)\,dx^4+O[dx]^5\Big\}\Big\}$$

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\left(\frac{\left(-H^2 U^2 k^4-H^2 U w k^3+3 U^2 k^2-3 g H k^2\right) \text{dt}^2}{2 \left(H^2 k^2+3\right)}-\frac{i H^2 k^3 U w^2 \text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{H^2 k^3 U w^3 \text{dt}^4}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(10 H^4 k^7+45 H^2 k^5+54 k^3\right) U \text{dt}}{120 \left(H^2 k^2+3\right)^2}-\frac{\left(20 H^4 U^2 k^8+45 g H^3 k^6+30 H^2 U^2 k^6-72 U^2 k^4+126 g H k^4\right) \text{dt}^2}{240 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{8} k^4 U \text{dt}+\left(-\frac{i H^2 U^2 k^7}{8 \left(H^2 k^2+3\right)}-\frac{3 i g H k^5}{16 \left(H^2 k^2+3\right)}\right) \text{dt}^2+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(260 H^6 U k^{11}+1570 H^4 U k^9+2610 H^2 U k^7+729 U k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 H^6 U^2 k^{12}+4500 g H^5 k^{10}+4140 H^4 U^2 k^{10}+26460 g H^3 k^8-13500 H^2 U^2 k^8-38286 U^2 k^6+38853 g H k^6\right) \text{dt}^2}{28800 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) & \left(\left(-\frac{3 U k^2}{H^2 k^2+3}-\frac{3 w k}{2 \left(H^2 k^2+3\right)}\right) \text{dt}^2-\frac{i k w^2 \text{dt}^3}{2 \left(H^2 k^2+3\right)}+\frac{k w^3 \text{dt}^4}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(5 H^2 k^5+12 k^3\right) \text{dt}}{40 \left(H^2 k^2+3\right)^2}-\frac{3 \left(5 H^2

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\left(3 H^2 k^3+12 k^3\right) \text{dt}}{40 \left(H^2 k^2+3\right)^2}-\frac{3 \left(\left(3 H^2
k^6+14 k^4\right) U\right) \text{dt}^2}{40 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(-\frac{3 i k^5 U \text{dt}^2}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{i \left(770 H^4 k^9+4410 H^2 k^7+6291 k^5\right) \text{dt}}{4800 \left(H^2
k^2+3\right)^3}+\frac{\left(1500 H^4 U k^{10}+8820 H^2 U k^8+12951 U k^6\right) \text{dt}^2}{4800
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\left(-\frac{\left(k \left(g k^2 H^3+3 g H-3 U^2\right) (2 k U+w)\right) \text{dt}^2}{2 \left(H^2 k^2+3\right)}-\frac{i
k \left(g k^2 H^3+3 g H-3 U^2\right) w^2 \text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{k \left(g k^2 H^3+3
g H-3 U^2\right) w^3 \text{dt}^4}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i
\left(10 g H^5 k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}}{120
\left(H^2 k^2+3\right)^2}+\frac{\left(-20 g H^5 U k^8+45 H^2 U^3 k^6-120 g H^3 U k^6+126 U^3
k^4-180 g H U k^4\right) \text{dt}^2}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(\frac{1}{8} g H k^4 \text{dt}-\frac{i \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right)
\text{dt}^2}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(260
g H^7 k^{11}+2340 g H^5 k^9-770 H^4 U^2 k^9+7020 g H^3 k^7-4410 H^2 U^2 k^7-6291 U^2
k^5+7020 g H k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 g H^7 U k^{12}-4500
H^4 U^3 k^{10}+13140 g H^5 U k^{10}-26460 H^2 U^3 k^8+39420 g H^3 U k^8-38853 U^3
k^6+39420 g H U k^6\right) \text{dt}^2}{14400 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)
\text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{\left(-H^2 U^2 k^4-H^2 U w k^3-9 U^2 k^2-3 g
H k^2-6 U w k\right) \text{dt}^2}{2 \left(H^2 k^2+3\right)}-\frac{i k \left(H^2 k^2+6\right) U w^2
\text{dt}^3}{6 \left(H^2 k^2+3\right)}+\frac{k \left(H^2 k^2+6\right) U w^3 \text{dt}^4}{24 \left(H^2
k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(10 H^4 k^7+75 H^2 k^5+126 k^3\right) U
\text{dt}}{120 \left(H^2 k^2+3\right)^2}-\frac{\left(20 H^4 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432
U^2 k^4+126 g H k^4\right) \text{dt}^2}{240 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(\frac{1}{8} k^4 U \text{dt}-\frac{i k^5 \left(2 H^2 k^2 U^2+12 U^2+3 g H\right)
\text{dt}^2}{16 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i
\left(260 H^6 k^{11}+3110 H^4 k^9+11430 H^2 k^7+13311 k^5\right) U \text{dt}}{4800 \left(H^2
k^2+3\right)^3}+\frac{\left(1460 H^6 U^2 k^{12}+4500 g H^5 k^{10}+22140 H^4 U^2 k^{10}+26460
g H^3 k^8+92340 H^2 U^2 k^8+117126 U^2 k^6+38853 g H k^6\right) \text{dt}^2}{28800
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
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