```
In[1]:= q = q0 * Exp[I * (k * x + w * t)];

qjn = q0 * Exp[I * (k * xj + w * tn)];

qjbar = Integrate[q, {x, xj - dx/2, xj + dx/2}] / (dx);

qjnbar = qjbar /. t → tn;

MA = qjn / qjnbar;


qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);

qjntbar = qntbar /. x → xj;

MtA = qjntbar / qjn;


qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];

RA = Simplify[MA * qjphn / (qjn)];


vmultG = H + H^3/3 * k^2;

GnA = -U * RA / vmultG ;

GGA = RA / vmultG ;

GcA = -U * H / vmultG ;


fn1A = H * vh + U * eh;

fn1A = fn1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca;

fn1Gca0A = fn1A /. Gca → 0 ;

fn1eca0A = fn1A /. eca → 0 ;

fnnA = Simplify[fn1Gca0A / eca];

fnGA = fn1eca0A / Gca;

fncA = H * GcA;


fG1A = U * Gh + U * H * vh + g * H * eh;

fG1A = fG1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca /. Gh → RA * Gca ;

fG1Gca0A = fG1A /. Gca → 0 ;

fG1eca0A = fG1A /. eca → 0 ;

fGnA = Simplify[fG1Gca0A / eca];

fGGA = Simplify[fG1eca0A / Gca];

fGcA = U * H * GcA;


FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;

FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;

FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;

FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;


MatA = {{FnnA, FnGA}, {FGnA, FGGA}} ;
```

$$\text{wAp} \; = \; \text{U} * \text{k} \; + \; \frac{\sqrt{3} \; \text{k} \; \sqrt{g \, H \, \left(3 + H^2 \, k^2\right)}}{3 + H^2 \, k^2} \; ;$$

$$\text{wAm} \; = \; \text{U} * \text{k} \; - \; \frac{\sqrt{3} \; \text{k} \; \sqrt{g \, H \, \left(3 + H^2 \, k^2\right)}}{3 + H^2 \, k^2} \; ;$$

In[36]:= **M = $\left(26 - 2 * \text{Cos}[\text{k} * \text{dx}]\right) \big/ 24$**

Series[M - MA, {dx, 0, 10}]

Out[36]= $\dfrac{1}{24} \, \left(26 - 2 \, \text{Cos}[\text{dx} \; k]\right)$

Out[37]= $-\dfrac{3 \, k^4 \, dx^4}{640} + \dfrac{3 \, k^6 \, dx^6}{35\,840} - \dfrac{149 \, k^8 \, dx^8}{51\,609\,600} + \dfrac{29 \, k^{10} \, dx^{10}}{13\,624\,934\,400} + O[dx]^{11}$

In[38]:= **Rm = $\left(5 - \text{Exp}[-\text{I} * \text{k} * \text{dx}] + 2 * \text{Exp}[\text{I} * \text{k} * \text{dx}]\right) \big/ 6$**

Series[Rm - RA, {dx, 0, 10}]

Rp = $\text{Exp}[\text{I} * \text{k} * \text{dx}] * \left(5 + 2 * \text{Exp}[-\text{I} * \text{k} * \text{dx}] - \text{Exp}[\text{I} * \text{k} * \text{dx}]\right) \big/ 6$

Series[Rp - RA, {dx, 0, 10}]

Ru = $\left(-\text{Exp}[-\text{I} * \text{k} * \text{dx}] + 9 * \text{Exp}[\text{I} * \text{k} * \text{dx}] - \text{Exp}[2 * \text{I} * \text{k} * \text{dx}] + 9\right) \big/ 16$

Series$\left[\text{Ru} - \text{Exp}\left[\text{I} * \text{k} * \text{dx} \big/ 2\right], \{\text{dx}, 0, 10\}\right]$

Out[38]= $\dfrac{1}{6} \, \left(5 - e^{-i \, dx \, k} + 2 \, e^{i \, dx \, k}\right)$

Out[39]= $-\dfrac{1}{12} \, i \, k^3 \, dx^3 + \dfrac{k^4 \, dx^4}{120} + \dfrac{1}{240} \, i \, k^5 \, dx^5 - \dfrac{k^6 \, dx^6}{5040} -$
$\dfrac{i \, k^7 \, dx^7}{10\,080} + \dfrac{k^8 \, dx^8}{201\,600} + \dfrac{i \, k^9 \, dx^9}{725\,760} - \dfrac{k^{10} \, dx^{10}}{39\,916\,800} + O[dx]^{11}$

Out[40]= $\dfrac{1}{6} \, e^{i \, dx \, k} \, \left(5 + 2 \, e^{-i \, dx \, k} - e^{i \, dx \, k}\right)$

Out[41]= $\dfrac{1}{12} \, i \, k^3 \, dx^3 - \dfrac{3 \, k^4 \, dx^4}{40} - \dfrac{3}{80} \, i \, k^5 \, dx^5 + \dfrac{23 \, k^6 \, dx^6}{1680} +$
$\dfrac{41 \, i \, k^7 \, dx^7}{10\,080} - \dfrac{209 \, k^8 \, dx^8}{201\,600} - \dfrac{169 \, i \, k^9 \, dx^9}{725\,760} + \dfrac{89 \, k^{10} \, dx^{10}}{1\,900\,800} + O[dx]^{11}$

Out[42]= $\dfrac{1}{16} \, \left(9 - e^{-i \, dx \, k} + 9 \, e^{i \, dx \, k} - e^{2 \, i \, dx \, k}\right)$

Out[43]= $-\dfrac{3 \, k^4 \, dx^4}{128} - \dfrac{3}{256} \, i \, k^5 \, dx^5 + \dfrac{5 \, k^6 \, dx^6}{1024} + \dfrac{3 \, i \, k^7 \, dx^7}{2048} - \dfrac{63 \, k^8 \, dx^8}{163\,840} - \dfrac{17 \, i \, k^9 \, dx^9}{196\,608} + \dfrac{289 \, k^{10} \, dx^{10}}{16\,515\,072} + O[dx]^{11}$

In[44]:= **Gold = H - H^3 $\big/$ 3 * $\big(32$ * Cos[k * dx] - 2 * Cos[2 * k * dx] - 30$\big)$ $\big/$ $\big(12$ * dx^2$\big)$;**

GG = M * Ru / (Gold)

Series[GG - GGA, {dx, 0, 5}]

Gn = -M * Ru * U / (Gold)

Series[Gn - GnA, {dx, 0, 5}]

Out[45]= $\dfrac{\left(9 - e^{-i\, dx\, k} + 9\, e^{i\, dx\, k} - e^{2\, i\, dx\, k}\right)\, (26 - 2\, \text{Cos}[dx\, k])}{384\, \left(H - \dfrac{H^3\, (-30 + 32\, \text{Cos}[dx\, k] - 2\, \text{Cos}[2\, dx\, k])}{36\, dx^2}\right)}$

Out[46]= $\dfrac{\left(-243\, k^4 - 49\, H^2\, k^6\right)\, dx^4}{960\, H\, \left(3 + H^2\, k^2\right)^2} - \dfrac{i\, \left(243\, k^5 + 49\, H^2\, k^7\right)\, dx^5}{1920\, H\, \left(3 + H^2\, k^2\right)^2} + O[dx]^6$

Out[47]= $\dfrac{\left(9 - e^{-i\, dx\, k} + 9\, e^{i\, dx\, k} - e^{2\, i\, dx\, k}\right)\, U\, (-26 + 2\, \text{Cos}[dx\, k])}{384\, \left(H - \dfrac{H^3\, (-30 + 32\, \text{Cos}[dx\, k] - 2\, \text{Cos}[2\, dx\, k])}{36\, dx^2}\right)}$

Out[48]= $\dfrac{\left(243\, k^4 + 49\, H^2\, k^6\right)\, U\, dx^4}{960\, H\, \left(3 + H^2\, k^2\right)^2} + \dfrac{i\, \left(243\, k^5 + 49\, H^2\, k^7\right)\, U\, dx^5}{1920\, H\, \left(3 + H^2\, k^2\right)^2} + O[dx]^6$

```
In[49]:= fnn = H * Gn + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
        Fnn = -dt * (1 - Exp[-I * k * dx]) / dx * fnn
        FnnTA = Series[Fnn - FnnA, {dx, 0, 4}, {dt, 0, 3}];
        Refine[FnnTA , {k > 0, U > 0, H > 0, g > 0}]
        fnG = H * GG;
        FnG = -dt * (1 - Exp[-I * k * dx]) / dx * fnG
        FnGTA = Series[FnG - FnGA, {dx, 0, 4}, {dt, 0, 3}];
        Refine[FnGTA , {k > 0, U > 0, H > 0, g > 0}]
```

Out[50]= $-\dfrac{1}{dx} dt \left(1 - e^{-i\,dx\,k}\right) \left(-\dfrac{1}{2} \left(\dfrac{1}{6} \left(-5 + e^{-i\,dx\,k} - 2\,e^{i\,dx\,k}\right) + \dfrac{1}{6}\,e^{i\,dx\,k} \left(5 + 2\,e^{-i\,dx\,k} - e^{i\,dx\,k}\right) \right) \sqrt{g\,H} + \right.$

$\qquad \dfrac{1}{2} \left(\dfrac{1}{6}\,e^{i\,dx\,k} \left(5 + 2\,e^{-i\,dx\,k} - e^{i\,dx\,k}\right) + \dfrac{1}{6} \left(5 - e^{-i\,dx\,k} + 2\,e^{i\,dx\,k}\right) \right) U +$

$\qquad \left. \dfrac{\left(9 - e^{-i\,dx\,k} + 9\,e^{i\,dx\,k} - e^{2\,i\,dx\,k}\right) H\,U\,(-26 + 2\,\text{Cos}[dx\,k])}{384 \left(H - \dfrac{H^3\,(-30 + 32\,\text{Cos}[dx\,k] - 2\,\text{Cos}[2\,dx\,k])}{36\,dx^2}\right)} \right)$

Out[52]= $\left(-\dfrac{\left(H^2\,k^3\,U\,w\right) dt^2}{2\left(3 + H^2\,k^2\right)} - \dfrac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3 + H^2\,k^2\right)} + O[dt]^4 \right) + \left(-\dfrac{1}{12} \left(\sqrt{g\,H}\,k^4\right) dt + O[dt]^4 \right) dx^3 +$

$\qquad \left(\dfrac{i\left(45\,k^5\,U + 143\,H^2\,k^7\,U + 32\,H^4\,k^9\,U\right) dt}{960\left(3 + H^2\,k^2\right)^2} + O[dt]^4 \right) dx^4 + O[dx]^5$

Out[54]= $-\left(\left(dt\left(1 - e^{-i\,dx\,k}\right) \left(9 - e^{-i\,dx\,k} + 9\,e^{i\,dx\,k} - e^{2\,i\,dx\,k}\right) H\,(26 - 2\,\text{Cos}[dx\,k])\right) \middle/ \right.$

$\qquad \left. \left(384\,dx \left(H - \dfrac{H^3\,(-30 + 32\,\text{Cos}[dx\,k] - 2\,\text{Cos}[2\,dx\,k])}{36\,dx^2}\right)\right) \right)$

Out[56]= $\left(-\dfrac{3\,(k\,w)\,dt^2}{2\left(3 + H^2\,k^2\right)} - \dfrac{i\,k\,w^2\,dt^3}{2\left(3 + H^2\,k^2\right)} + O[dt]^4 \right) + \left(\dfrac{i\left(243\,k^5 + 49\,H^2\,k^7\right) dt}{960\left(3 + H^2\,k^2\right)^2} + O[dt]^4 \right) dx^4 + O[dx]^5$

In[57]:= `fGn = U * H * Gn + g * H * (Rm + Rp) / 2 + (U * Sqrt[ g * H]) / (2) * (Rm - Rp);`

`FGn = -dt * (1 - Exp[-I * k * dx]) / dx * fGn`

`FGnTA = Series[FGn - FGnA, {dx, 0, 4}, {dt, 0, 3}];`

`Refine[FGnTA , {k > 0, U > 0, H > 0, g > 0}]`

`fGG = U * H * GG + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);`

`FGG = -dt * (1 - Exp[-I * k * dx]) / dx * fGG`

`FGGTA = Series[FGG - FGGA, {dx, 0, 4}, {dt, 0, 3}];`

`Refine[FGGTA , {k > 0, U > 0, H > 0, g > 0}]`

Out[58]=
$$-\frac{1}{dx}dt \left(1 - e^{-i\,dx\,k}\right) \left(\frac{1}{2} \left(\frac{1}{6} e^{i\,dx\,k} \left(5 + 2 e^{-i\,dx\,k} - e^{i\,dx\,k}\right) + \frac{1}{6} \left(5 - e^{-i\,dx\,k} + 2 e^{i\,dx\,k}\right) \right) g\,H + \right.$$
$$\frac{1}{2} \left(-\frac{1}{6} e^{i\,dx\,k} \left(5 + 2 e^{-i\,dx\,k} - e^{i\,dx\,k}\right) + \frac{1}{6} \left(5 - e^{-i\,dx\,k} + 2 e^{i\,dx\,k}\right) \right) \sqrt{g\,H}\ U +$$
$$\left. \frac{\left(9 - e^{-i\,dx\,k} + 9 e^{i\,dx\,k} - e^{2\,i\,dx\,k}\right) H\,U^2 \left(-26 + 2\,Cos[dx\,k]\right)}{384 \left(H - \frac{H^3\,(-30+32\,Cos[dx\,k]-2\,Cos[2\,dx\,k])}{36\,dx^2}\right)} \right)$$

Out[60]=
$$\left(-\frac{\left(k \left(3\,g\,H + g\,H^3\,k^2 - 3\,U^2\right) w\right) dt^2}{2 \left(3 + H^2\,k^2\right)} - \frac{i\,k \left(3\,g\,H + g\,H^3\,k^2 - 3\,U^2\right) w^2\,dt^3}{6 \left(3 + H^2\,k^2\right)} + O[dt]^4 \right) +$$
$$\left(-\frac{1}{12} \left(\sqrt{g\,H}\ k^4\,U\right) dt + O[dt]^4 \right) dx^3 +$$
$$\left(\left(i \left(288\,g\,H\,k^5 + 192\,g\,H^3\,k^7 + 32\,g\,H^5\,k^9 - 243\,k^5\,U^2 - 49\,H^2\,k^7\,U^2\right) dt \right) \Big/ \left(960 \left(3 + H^2\,k^2\right)^2\right) + \right.$$
$$\left. O[dt]^4 \right) dx^4 + O[dx]^5$$

Out[62]=
$$-\frac{1}{dx}dt \left(1 - e^{-i\,dx\,k}\right) \left(-\frac{1}{2} \left(\frac{1}{6} \left(-5 + e^{-i\,dx\,k} - 2 e^{i\,dx\,k}\right) + \frac{1}{6} e^{i\,dx\,k} \left(5 + 2 e^{-i\,dx\,k} - e^{i\,dx\,k}\right) \right) \sqrt{g\,H} + \right.$$
$$\frac{1}{2} \left(\frac{1}{6} e^{i\,dx\,k} \left(5 + 2 e^{-i\,dx\,k} - e^{i\,dx\,k}\right) + \frac{1}{6} \left(5 - e^{-i\,dx\,k} + 2 e^{i\,dx\,k}\right) \right) U +$$
$$\left. \frac{\left(9 - e^{-i\,dx\,k} + 9 e^{i\,dx\,k} - e^{2\,i\,dx\,k}\right) H\,U \left(26 - 2\,Cos[dx\,k]\right)}{384 \left(H - \frac{H^3\,(-30+32\,Cos[dx\,k]-2\,Cos[2\,dx\,k])}{36\,dx^2}\right)} \right)$$

Out[64]=
$$\left(-\frac{\left(k \left(6 + H^2\,k^2\right) U\,w\right) dt^2}{2 \left(3 + H^2\,k^2\right)} - \frac{i\,k \left(6 + H^2\,k^2\right) U\,w^2\,dt^3}{6 \left(3 + H^2\,k^2\right)} + O[dt]^4 \right) + \left(-\frac{1}{12} \left(\sqrt{g\,H}\ k^4\right) dt + O[dt]^4 \right) dx^3 +$$
$$\left(\frac{i \left(531\,k^5\,U + 241\,H^2\,k^7\,U + 32\,H^4\,k^9\,U\right) dt}{960 \left(3 + H^2\,k^2\right)^2} + O[dt]^4 \right) dx^4 + O[dx]^5$$

In[65]:= `Fmat = {{Fnn, FnG}, {FGn, FGG}};`

`EigvFmat = Eigenvalues[Fmat];`

`RKStep = Log[ 1 + EigvFmat + EigvFmat^2 / 2 + EigvFmat^3 / 6] / (I * dt);`

`RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];`

`Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}]`

Out[69]= $\Bigg\{ \Bigg(-\dfrac{1}{24\left(3+H^2\,k^2\right)^3}$

$\quad \dot{\imath}\,k^4\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(3\,g\left(\sqrt{3}\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}+9\,H\,U+3\,H^3\,k^2\,U\right)+\right.$

$\quad\quad \left.U^2\left(H^4\,k^4\,U+9\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+U\right)+3\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+2\,H^2\,U\right)\right)\right)\right)dt^3+$

$\quad \dfrac{1}{30\left(3+H^2\,k^2\right)^3}k^5\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)$

$\quad \left(9\,g^2\,H^2+6\,g\,H\,U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\left(3+H^2\,k^2\right)U\right)+\right.$

$\quad\quad \left.U^3\left(12\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+9\,U+H^4\,k^4\,U+2\,k^2\left(2\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+3\,H^2\,U\right)\right)\right)dt^4+$

$\quad O[dt]^5\Bigg)+\Bigg(-\dfrac{\dot{\imath}\,k^4\left(2\,g\,H\left(3+H^2\,k^2\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)}{24\sqrt{g\,H}\left(3+H^2\,k^2\right)}-$

$\quad \left(\left(k^7\left(6\sqrt{3}\,g^2\,H^2\left(3+H^2\,k^2\right)+9\,U^3\left(5\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\,U\right)+\right.\right.\right.$

$\quad\quad k^4\,U^3\left(2\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^4\,U\right)+$

$\quad\quad 3\,g\,H\,U\left(21\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\left(27+15\,H^2\,k^2+2\,H^4\,k^4\right)U\right)+$

$\quad\quad \left.\left.\left.3\,k^2\left(6\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+7\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3+2\sqrt{3}\,H^2\,U^4\right)\right)\right)dt^3\right)\Bigg/$

$\quad \left(144\left(3+H^2\,k^2\right)^{5/2}\right)-\dfrac{1}{144\left(3+H^2\,k^2\right)^{5/2}}\dot{\imath}\,k^8\left(3\sqrt{3}\,g^2\,H^2\left(27+8\,H^2\,k^2\right)U+\right.$

$\quad k^4\,U^4\left(2\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^4\,U\right)+$

$\quad 2\,g\,H\,U^2\left(72\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\left(63+33\,H^2\,k^2+4\,H^4\,k^4\right)U\right)+$

$\quad 9\left(2\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+6\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+\sqrt{3}\,U^5\right)+$

$\quad \left.6\,k^2\left(6\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2+4\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4+\sqrt{3}\,H^2\,U^5\right)\right)dt^4+O[dt]^5\Bigg)dx^3+$

$\quad \Bigg(-\left(\left(k^5\left(531\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+1728\,U+192\,H^4\,k^4\,U+\right.\right.\right.$

$\quad\quad \left.\left.\left.k^2\left(145\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+1152\,H^2\,U\right)\right)\right)\right)\Bigg/\left(5760\left(3+H^2\,k^2\right)^2\right)\Bigg)+$

$\quad \dfrac{1}{34560\sqrt{g\,H}\left(3+H^2\,k^2\right)^{7/2}}\dot{\imath}\,k^8\left(k^6\,U^3\left(721\sqrt{3}\,g\,H^7+192\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U\right)+\right.$

$\quad 9\,k^2\left(145\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}+1350\sqrt{3}\,g^2\,H^4\,U+2118\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2+\right.$

$\quad\quad \left.2227\sqrt{3}\,g\,H^3\,U^3+576\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\right)+81\left(59\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+241\sqrt{3}\right.$

$\quad\quad \left.g^2\,H^2\,U+64\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+g\,H\,U^2\left(369\sqrt{g\,H\left(3+H^2\,k^2\right)}+251\sqrt{3}\,U\right)\right)+$

$\quad 3\,k^4\,U\left(627\sqrt{3}\,g^2\,H^6+576\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+g\,H^5\,U\right.$

$$\left(1011 \sqrt{g H \left(3 + H^2 k^2\right)} + 2195 \sqrt{3} U\right)\right)\right) dt^3 - \frac{1}{34560 \left(\sqrt{g H} \left(3 + H^2 k^2\right)^{7/2}\right)}$$

$$\left(k^9 \left(9 \sqrt{3} g^3 H^3 \left(531 + 145 H^2 k^2\right) + 54 \sqrt{3} g^2 H^2 \left(915 + 578 H^2 k^2 + 91 H^4 k^4\right) U^2 + \right.\right.$$

$$g H U^3 \left(25227 \sqrt{3} H^2 k^2 U + 913 \sqrt{3} H^6 k^6 U + 405 \left(124 \sqrt{g H \left(3 + H^2 k^2\right)} + 63 \sqrt{3} U\right) + \right.$$

$$\left.\left. 3 k^4 \left(1732 \sqrt{g H^9 \left(3 + H^2 k^2\right)} + 2771 \sqrt{3} H^4 U\right)\right)\right) + $$

$$12 U \left(2025 \sqrt{g^5 H^5 \left(3 + H^2 k^2\right)} + 432 \sqrt{g H \left(3 + H^2 k^2\right)} U^4 + 144 k^4 \sqrt{g H^9 \left(3 + H^2 k^2\right)}\right.$$

$$U^4 + 16 k^6 \sqrt{g H^{13} \left(3 + H^2 k^2\right)} U^4 + 3 k^2 \left(193 \sqrt{g^5 H^9 \left(3 + H^2 k^2\right)} + \right.$$

$$\left.\left.\left.\left.\left. 898 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)} U^2 + 144 \sqrt{g H^5 \left(3 + H^2 k^2\right)} U^4\right)\right)\right)\right) dt^4 + O[dt]^5\right) dx^4 + $$

$$O[dx]^5, \left(-\frac{1}{24 \left(3 + H^2 k^2\right)^3} i k^4 \left(-\sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U\right)\right.$$

$$\left(3 g H \left(-\sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 3 \left(3 + H^2 k^2\right) U\right) + \right.$$

$$\left.\left. U^2 \left(-9 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 9 U + H^4 k^4 U - 3 k^2 \left(\sqrt{3} \sqrt{g H^5 \left(3 + H^2 k^2\right)} - 2 H^2 U\right)\right)\right) dt^3 + $$

$$\frac{1}{30 \left(3 + H^2 k^2\right)^3} k^5 \left(-\sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U\right)$$

$$\left(9 g^2 H^2 + 6 g H U \left(-2 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 3 \left(3 + H^2 k^2\right) U\right) + \right.$$

$$\left.\left. U^3 \left(-12 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} + 9 U + H^4 k^4 U + k^2 \left(-4 \sqrt{3} \sqrt{g H^5 \left(3 + H^2 k^2\right)} + 6 H^2 U\right)\right)\right)\right)$$

$$dt^4 + O[dt]^5 + \left(-\frac{i k^4 \left(2 g H \left(3 + H^2 k^2\right) - \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} U\right)}{24 \sqrt{g H} \left(3 + H^2 k^2\right)} + \right.$$

$$\left(k^7 \left(6 \sqrt{3} g^2 H^2 \left(3 + H^2 k^2\right) + 9 U^3 \left(-5 \sqrt{g H \left(3 + H^2 k^2\right)} + \sqrt{3} U\right) + \right.\right.$$

$$k^4 U^3 \left(-2 \sqrt{g H^9 \left(3 + H^2 k^2\right)} + \sqrt{3} H^4 U\right) + $$

$$3 g H U \left(-21 \sqrt{g H \left(3 + H^2 k^2\right)} + \sqrt{3} \left(27 + 15 H^2 k^2 + 2 H^4 k^4\right) U\right) - $$

$$\left.\left. 3 k^2 \left(6 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)} U + 7 \sqrt{g H^5 \left(3 + H^2 k^2\right)} U^3 - 2 \sqrt{3} H^2 U^4\right)\right)$$

$$dt^3\right) \Big/ \left(144 \left(3 + H^2 k^2\right)^{5/2}\right) + \frac{1}{144 \left(3 + H^2 k^2\right)^{5/2}}$$

$$i k^8 \left(3 \sqrt{3} g^2 H^2 \left(27 + 8 H^2 k^2\right) U + k^4 U^4 \left(-2 \sqrt{g H^9 \left(3 + H^2 k^2\right)} + \sqrt{3} H^4 U\right) + \right.$$

$$2 g H U^2 \left(-72 \sqrt{g H \left(3 + H^2 k^2\right)} + \sqrt{3} \left(63 + 33 H^2 k^2 + 4 H^4 k^4\right) U\right) + $$

$$9 \left(-2 \sqrt{g^5 H^5 \left(3 + H^2 k^2\right)} - 6 \sqrt{g H \left(3 + H^2 k^2\right)} U^4 + \sqrt{3} U^5\right) - $$

$$\left.\left. 6 k^2 \left(6 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)} U^2 + 4 \sqrt{g H^5 \left(3 + H^2 k^2\right)} U^4 - \sqrt{3} H^2 U^5\right)\right) dt^4 + O[dt]^5\right) dx^3 + $$

$$\left(\left(k^5 \left(531 \sqrt{3} \sqrt{g H \left(3 + H^2 k^2\right)} - 1728 U - 192 H^4 k^4 U + \right.\right.\right.$$

$$\left.\left.k^2 \left(145 \sqrt{3} \sqrt{g H^5 \left(3 + H^2 k^2\right)} - 1152 H^2 U\right)\right)\right) \Big/ \left(5760 \left(3 + H^2 k^2\right)^2\right) +$$

$$\frac{1}{34560 \sqrt{g H} \left(3 + H^2 k^2\right)^{7/2}} \, i \, k^8 \left(k^6 U^3 \left(-721 \sqrt{3} g H^7 + 192 \sqrt{g H^{13} \left(3 + H^2 k^2\right)} \, U\right) + \right.$$

$$9 k^2 \left(145 \sqrt{g^5 H^9 \left(3 + H^2 k^2\right)} - 1350 \sqrt{3} g^2 H^4 U + \right.$$

$$\left. 2118 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)} \, U^2 - 2227 \sqrt{3} g H^3 U^3 + 576 \sqrt{g H^5 \left(3 + H^2 k^2\right)} \, U^4\right) +$$

$$81 \left(59 \sqrt{g^5 H^5 \left(3 + H^2 k^2\right)} - 241 \sqrt{3} g^2 H^2 U + 64 \sqrt{g H \left(3 + H^2 k^2\right)} \, U^4 + \right.$$

$$\left. g H U^2 \left(369 \sqrt{g H \left(3 + H^2 k^2\right)} - 251 \sqrt{3} U\right)\right) - 3 k^4 U \left(627 \sqrt{3} g^2 H^6 - \right.$$

$$\left. 576 \sqrt{g H^9 \left(3 + H^2 k^2\right)} \, U^3 + g H^5 U \left(-1011 \sqrt{g H \left(3 + H^2 k^2\right)} + 2195 \sqrt{3} U\right)\right)\right) dt^3 +$$

$$\frac{1}{34560 \sqrt{g H} \left(3 + H^2 k^2\right)^{7/2}} k^9 \left(9 \sqrt{3} g^3 H^3 \left(531 + 145 H^2 k^2\right) + \right.$$

$$54 \sqrt{3} g^2 H^2 \left(915 + 578 H^2 k^2 + 91 H^4 k^4\right) U^2 +$$

$$g H U^3 \left(25227 \sqrt{3} H^2 k^2 U + 913 \sqrt{3} H^6 k^6 U + 405 \left(-124 \sqrt{g H \left(3 + H^2 k^2\right)} + 63 \sqrt{3} U\right) + \right.$$

$$\left. 3 k^4 \left(-1732 \sqrt{g H^9 \left(3 + H^2 k^2\right)} + 2771 \sqrt{3} H^4 U\right)\right) -$$

$$12 U \left(2025 \sqrt{g^5 H^5 \left(3 + H^2 k^2\right)} + 432 \sqrt{g H \left(3 + H^2 k^2\right)} \, U^4 + 144 k^4 \sqrt{g H^9 \left(3 + H^2 k^2\right)} \, U^4 + \right.$$

$$16 k^6 \sqrt{g H^{13} \left(3 + H^2 k^2\right)} \, U^4 + 3 k^2 \left(193 \sqrt{g^5 H^9 \left(3 + H^2 k^2\right)} + 898 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)} \right.$$

$$\left.\left.\left.\left. U^2 + 144 \sqrt{g H^5 \left(3 + H^2 k^2\right)} \, U^4\right)\right)\right) dt^4 + O[dt]^5\right) dx^4 + O[dx]^5\right\}$$