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In[1]:= MA = k * x / ( 2 * Sin[k * x / 2] )
RA = Exp[ I * k * x / 2 ] * k * x / ( 2 * Sin[k * x / 2] )
GA = k * x / ( ( H + H^3 / 3 * k^2 ) * Exp[ -I * k * x / 2 ] * ( 2 * Sin[k * x / 2] ) )
FnnA = 0
FnGA = I * k / ( 1 + H^2 * k^2 / 3 )
FGnA = g * H * I * k
FGGA = 0
FmatA = { { FnnA, FnGA }, { FGnA, FGGA } }
Eigenvalues[FmatA]

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$$\text{Out[1]} = \frac{1}{2} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$$

$$\text{Out[2]} = \frac{1}{2} e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$$

$$\text{Out[3]} = \frac{e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3} \right)}$$

$$\text{Out[4]} = 0$$

$$\text{Out[5]} = \frac{i k}{1 + \frac{H^2 k^2}{3}}$$

$$\text{Out[6]} = i g H k$$

$$\text{Out[7]} = 0$$

$$\text{Out[8]} = \left\{ \left\{ 0, \frac{i k}{1 + \frac{H^2 k^2}{3}} \right\}, \{ i g H k, 0 \} \right\}$$

$$\text{Out[9]} = \left\{ -\frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2} \right\}$$

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In[10]:= M = ( 26 - 2 * Cos[k * x] ) / 24
Series[M - MA, {x, 0, 10}]

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$$\text{Out[10]} = \frac{1}{24} (26 - 2 \operatorname{Cos}[k x])$$

$$\text{Out[11]} = -\frac{3 k^4 x^4}{640} + \frac{3 k^6 x^6}{35840} - \frac{149 k^8 x^8}{51609600} + \frac{29 k^{10} x^{10}}{13624934400} + O[x]^{11}$$

In[12]:= **Rm = (5 - Exp[-I * k * x] + 2 * Exp[I * k * x]) / 6**

Series[Rm - RA, {x, 0, 10}]

Rp = Exp[I * k * x] * (5 + 2 * Exp[-I * k * x] - Exp[I * k * x]) / 6

Series[Rp - RA, {x, 0, 10}]

Ru = (-Exp[-I * k * x] + 9 * Exp[I * k * x] - Exp[2 * I * k * x] + 9) / 16

Series[Ru - Exp[I * k * x / 2], {x, 0, 10}]

Out[12]= $\frac{1}{6} (5 - e^{-i k x} + 2 e^{i k x})$

Out[13]= $-\frac{1}{12} i k^3 x^3 + \frac{k^4 x^4}{120} + \frac{1}{240} i k^5 x^5 - \frac{k^6 x^6}{5040} - \frac{i k^7 x^7}{10080} + \frac{k^8 x^8}{201600} + \frac{i k^9 x^9}{725760} - \frac{k^{10} x^{10}}{39916800} + O[x]^{11}$

Out[14]= $\frac{1}{6} e^{i k x} (5 + 2 e^{-i k x} - e^{i k x})$

Out[15]= $\frac{1}{12} i k^3 x^3 - \frac{3 k^4 x^4}{40} - \frac{3}{80} i k^5 x^5 + \frac{23 k^6 x^6}{1680} + \frac{41 i k^7 x^7}{10080} - \frac{209 k^8 x^8}{201600} - \frac{169 i k^9 x^9}{725760} + \frac{89 k^{10} x^{10}}{1900800} + O[x]^{11}$

Out[16]= $\frac{1}{16} (9 - e^{-i k x} + 9 e^{i k x} - e^{2 i k x})$

Out[17]= $-\frac{3 k^4 x^4}{128} - \frac{3}{256} i k^5 x^5 + \frac{5 k^6 x^6}{1024} + \frac{3 i k^7 x^7}{2048} - \frac{63 k^8 x^8}{163840} - \frac{17 i k^9 x^9}{196608} + \frac{289 k^{10} x^{10}}{16515072} + O[x]^{11}$

In[18]:= **Gold = H - H^3 / 3 * (32 * Cos[k * x] - 2 * Cos[2 * k * x] - 30) / (12 * x^2)**

G = M * Ru / (Gold)

Series[G, {x, 0, 3}]

Series[GA, {x, 0, 3}]

Series[G - GA, {x, 0, 5}]

Out[18]= $H - \frac{H^3 (-30 + 32 \cos[k x] - 2 \cos[2 k x])}{36 x^2}$

Out[19]= $\frac{(9 - e^{-i k x} + 9 e^{i k x} - e^{2 i k x}) (26 - 2 \cos[k x])}{384 \left(H - \frac{H^3 (-30 + 32 \cos[k x] - 2 \cos[2 k x])}{36 x^2} \right)}$

Out[20]= $\frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2 \left(H + \frac{H^3 k^2}{3} \right)} - \frac{k^2 x^2}{12 \left(H + \frac{H^3 k^2}{3} \right)} + O[x]^4$

Out[21]= $\frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2 \left(H + \frac{H^3 k^2}{3} \right)} - \frac{k^2 x^2}{12 \left(H + \frac{H^3 k^2}{3} \right)} + O[x]^4$

Out[22]= $\frac{(-243 k^4 - 49 H^2 k^6) x^4}{960 H (3 + H^2 k^2)^2} - \frac{i (243 k^5 + 49 H^2 k^7) x^5}{1920 H (3 + H^2 k^2)^2} + O[x]^6$

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In[23]:= fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * G;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;
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Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]
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$$\text{Out[27]} = - \frac{(1 - e^{-i k x}) \left(\frac{1}{6} (-5 + e^{-i k x} - 2 e^{i k x}) + \frac{1}{6} e^{i k x} (5 + 2 e^{-i k x} - e^{i k x}) \right) \sqrt{g H}}{2 x}$$

$$\text{Out[28]} = \frac{1}{12} \sqrt{g H} k^4 x^3 - \frac{1}{72} (\sqrt{g H} k^6) x^5 + O[x]^6$$

$$\text{Out[29]} = \frac{(1 - e^{-i k x}) (9 - e^{-i k x} + 9 e^{i k x} - e^{2 i k x}) H (26 - 2 \cos[k x])}{384 x \left(H - \frac{H^3 (-30 + 32 \cos[k x] - 2 \cos[2 k x])}{36 x^2} \right)}$$

$$\text{Out[30]} = - \frac{i (243 k^5 + 49 H^2 k^7) x^4}{960 (3 + H^2 k^2)^2} + O[x]^6$$

$$\text{Out[31]} = - \frac{(1 - e^{-i k x}) \left(\frac{1}{6} (-5 + e^{-i k x} - 2 e^{i k x}) + \frac{1}{6} e^{i k x} (5 + 2 e^{-i k x} - e^{i k x}) \right) \sqrt{g H}}{2 x}$$

$$\text{Out[32]} = \frac{1}{12} \sqrt{g H} k^4 x^3 - \frac{1}{72} (\sqrt{g H} k^6) x^5 + O[x]^6$$

$$\text{Out[33]} = \frac{(1 - e^{-i k x}) \left(\frac{1}{6} e^{i k x} (5 + 2 e^{-i k x} - e^{i k x}) + \frac{1}{6} (5 - e^{-i k x} + 2 e^{i k x}) \right) g H}{2 x}$$

$$\text{Out[34]} = - \frac{1}{30} i g H k^5 x^4 + O[x]^6$$