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q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx/2, xj + dx/2}] / (dx);
qjnbar = qjbar /. t -> tn;
MA = qjn / qjnbar;
Text[Row[{"M (qj = M qbar): ", TraditionalForm[MA]}]]

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x -> xj;
MtA = qjntbar / qjn;
Text[Row[{"Mta (1/dt * int(qj,tm,tn+1) = MtA q): ", TraditionalForm[MtA]}]]

qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];
Text[Row[{"R (qjph = RA qbar): ", TraditionalForm[RA]}]]

vmultG = H + H^3 / (3 * k^2);
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;
Text[Row[{"G (vjph = Gna nbarj + GGA Gbarj + GcA ): ",
TraditionalForm[GnA], TraditionalForm[GGA], TraditionalForm[GcA]}]]

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca;
fn1Gca0A = fn1A /. Gca -> 0;
fn1eca0A = fn1A /. eca -> 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca /. Gh -> RA * Gca;
fG1Gca0A = fG1A /. Gca -> 0;
fG1eca0A = fG1A /. eca -> 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA / dx * (1 - Exp[-I * k * dx]) * fGnA;

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$$\mathbf{FGGA} = -\mathbf{MtA} / \mathbf{dx} * \left(1 - \mathbf{Exp}[-\mathbf{I} * \mathbf{k} * \mathbf{dx}]\right) * \mathbf{fGGA};$$

$$\mathbf{MatA} = \{\{\mathbf{FnnA}, \mathbf{FnGA}\}, \{\mathbf{FGnA}, \mathbf{FGGA}\}\};$$

$$\mathbf{Text}[\mathbf{Row}[\{\mathbf{"Fn (Fn = Fnn nbarj + FGG Gbarj) : "}, \mathbf{TraditionalForm}[\mathbf{FnnA}], \mathbf{TraditionalForm}[\mathbf{FnGA}]\}]]$$

$$\text{Out[133]= } \mathbf{M}(\mathbf{qj} = \mathbf{M} \mathbf{qbar}): \frac{1}{2} \mathbf{dx} \mathbf{k} \csc\left(\frac{\mathbf{dx} \mathbf{k}}{2}\right)$$

$$\text{Out[137]= } \mathbf{Mta} (1/\mathbf{dt} * \mathbf{int}(\mathbf{qj}, \mathbf{tm}, \mathbf{tn}+1) = \mathbf{MtA} \mathbf{q}): -\frac{i(-1+e^{i \mathbf{dt} w})}{\mathbf{dt} w}$$

$$\text{Out[140]= } \mathbf{R}(\mathbf{qjph} = \mathbf{RA} \mathbf{qbar}): \frac{1}{2} \mathbf{dx} \mathbf{k} e^{\frac{i \mathbf{dx} \mathbf{k}}{2}} \csc\left(\frac{\mathbf{dx} \mathbf{k}}{2}\right)$$

$$\text{Out[145]= } \mathbf{G}(\mathbf{vjph} = \mathbf{Gna} \mathbf{nbarj} + \mathbf{GGa} \mathbf{Gbarj} + \mathbf{GcA}): -\frac{\mathbf{dx} \mathbf{k} \mathbf{U} e^{\frac{i \mathbf{dx} \mathbf{k}}{2}} \csc\left(\frac{\mathbf{dx} \mathbf{k}}{2}\right) \mathbf{dx} \mathbf{k} e^{\frac{i \mathbf{dx} \mathbf{k}}{2}} \csc\left(\frac{\mathbf{dx} \mathbf{k}}{2}\right)}{2\left(\frac{\mathbf{H}^3 \mathbf{k}^2}{3} + \mathbf{H}\right)} - \frac{\mathbf{H} \mathbf{U}}{\frac{\mathbf{H}^3 \mathbf{k}^2}{3} + \mathbf{H}}$$

$$\text{Out[165]= } \mathbf{Fn}(\mathbf{Fn} = \mathbf{Fnn} \mathbf{nbarj} + \mathbf{FGG} \mathbf{Gbarj}): \frac{i \mathbf{H}^2 \mathbf{k}^3 \mathbf{U} (-1+e^{i \mathbf{dt} w}) e^{\frac{i \mathbf{dx} \mathbf{k}}{2}} (1-e^{-i \mathbf{dx} \mathbf{k}}) \csc\left(\frac{\mathbf{dx} \mathbf{k}}{2}\right) i \mathbf{H} \mathbf{k} (-1+e^{i \mathbf{dt} w}) e^{\frac{i \mathbf{dx} \mathbf{k}}{2}} (1-e^{-i \mathbf{dx} \mathbf{k}}) \csc\left(\frac{\mathbf{dx} \mathbf{k}}{2}\right)}{\mathbf{dt} w (2 \mathbf{H}^2 \mathbf{k}^2 + 6)} \frac{1}{2 \mathbf{dt} w \left(\frac{\mathbf{H}^3 \mathbf{k}^2}{3} + \mathbf{H}\right)}$$

$$\text{In[166]:= } \mathbf{MatA}$$

$$\text{Out[166]= } \left\{ \left\{ \frac{i e^{\frac{i \mathbf{dx} \mathbf{k}}{2}} (1 - e^{-i \mathbf{dx} \mathbf{k}}) (-1 + e^{i \mathbf{dt} w}) \mathbf{H}^2 \mathbf{k}^3 \mathbf{U} \csc\left[\frac{\mathbf{dx} \mathbf{k}}{2}\right]}{\mathbf{dt} (6 + 2 \mathbf{H}^2 \mathbf{k}^2) w}, \right. \right. \\ \left. \frac{i e^{\frac{i \mathbf{dx} \mathbf{k}}{2}} (1 - e^{-i \mathbf{dx} \mathbf{k}}) (-1 + e^{i \mathbf{dt} w}) \mathbf{H} \mathbf{k} \csc\left[\frac{\mathbf{dx} \mathbf{k}}{2}\right]}{2 \mathbf{dt} \left(\mathbf{H} + \frac{\mathbf{H}^3 \mathbf{k}^2}{3}\right) w} \right\}, \\ \left\{ \frac{i e^{\frac{i \mathbf{dx} \mathbf{k}}{2}} (1 - e^{-i \mathbf{dx} \mathbf{k}}) (-1 + e^{i \mathbf{dt} w}) \mathbf{k} (\mathbf{g} \mathbf{H} (3 + \mathbf{H}^2 \mathbf{k}^2) - 3 \mathbf{U}^2) \csc\left[\frac{\mathbf{dx} \mathbf{k}}{2}\right]}{\mathbf{dt} (6 + 2 \mathbf{H}^2 \mathbf{k}^2) w}, \right. \\ \left. \frac{i e^{\frac{i \mathbf{dx} \mathbf{k}}{2}} (1 - e^{-i \mathbf{dx} \mathbf{k}}) (-1 + e^{i \mathbf{dt} w}) \mathbf{k} (6 + \mathbf{H}^2 \mathbf{k}^2) \mathbf{U} \csc\left[\frac{\mathbf{dx} \mathbf{k}}{2}\right]}{\mathbf{dt} (6 + 2 \mathbf{H}^2 \mathbf{k}^2) w} \right\} \right\}$$

$$\text{In[167]:= } \mathbf{Simplify}[\mathbf{MatA}]$$

$$\text{Out[167]= } \left\{ \left\{ -\frac{(-1 + e^{i \mathbf{dt} w}) \mathbf{H}^2 \mathbf{k}^3 \mathbf{U}}{\mathbf{dt} (3 + \mathbf{H}^2 \mathbf{k}^2) w}, \frac{3 \mathbf{k} - 3 e^{i \mathbf{dt} w} \mathbf{k}}{3 \mathbf{dt} w + \mathbf{dt} \mathbf{H}^2 \mathbf{k}^2 w} \right\}, \right. \\ \left. \left\{ -\frac{(-1 + e^{i \mathbf{dt} w}) \mathbf{k} (\mathbf{g} \mathbf{H} (3 + \mathbf{H}^2 \mathbf{k}^2) - 3 \mathbf{U}^2)}{\mathbf{dt} (3 + \mathbf{H}^2 \mathbf{k}^2) w}, -\frac{(-1 + e^{i \mathbf{dt} w}) \mathbf{k} (6 + \mathbf{H}^2 \mathbf{k}^2) \mathbf{U}}{\mathbf{dt} (3 + \mathbf{H}^2 \mathbf{k}^2) w} \right\} \right\}$$

$$\text{In[168]:= } \mathbf{Simplify}[\mathbf{Eigenvalues}[\mathbf{MatA}]]$$

$$\text{Out[168]= } \left\{ -\frac{(-1 + e^{i \mathbf{dt} w}) \mathbf{k} \left(-\sqrt{3} \sqrt{\mathbf{g} \mathbf{H} (3 + \mathbf{H}^2 \mathbf{k}^2)} + (3 + \mathbf{H}^2 \mathbf{k}^2) \mathbf{U} \right)}{\mathbf{dt} (3 + \mathbf{H}^2 \mathbf{k}^2) w}, \right. \\ \left. -\frac{(-1 + e^{i \mathbf{dt} w}) \mathbf{k} \left(\sqrt{3} \sqrt{\mathbf{g} \mathbf{H} (3 + \mathbf{H}^2 \mathbf{k}^2)} + (3 + \mathbf{H}^2 \mathbf{k}^2) \mathbf{U} \right)}{\mathbf{dt} (3 + \mathbf{H}^2 \mathbf{k}^2) w} \right\}$$

In[174]:= **EA = 1 + dt * Simplify[Eigenvalues[MatA]] /.**

$$\frac{k \left(-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)}{(3 + H^2 k^2)} \rightarrow -w /.$$

$$k * \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) / (3 + H^2 k^2) \rightarrow -w$$

Eigenvectors[MatA]

Out[174]= $\{e^{i dt w}, e^{i dt w}\}$

$$\text{Out[175]= } \left\{ \left\{ -\frac{\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 U}{3 g H + g H^3 k^2 - 3 U^2}, 1 \right\}, \left\{ -\frac{-\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 U}{3 g H + g H^3 k^2 - 3 U^2}, 1 \right\} \right\}$$