

```

In[1]:= q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx/2, xj + dx/2}] / (dx);
qjnbar = qjbar /. t -> tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x -> xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / 3 * k^2;
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca;
fn1Gca0A = fn1A /. Gca -> 0;
fn1eca0A = fn1A /. eca -> 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh -> (GGA * Gca + GnA * eca) /. eh -> RA * eca /. Gh -> RA * Gca;
fG1Gca0A = fG1A /. Gca -> 0;
fG1eca0A = fG1A /. eca -> 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
EA = {{1, 0}, {0, 1}} + MatA;

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$$wAp = U * k + \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};$$

$$wAm = U * k - \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};$$

```
In[37]:= M = 1;
Merr = Series[M - MA, {dx, 0, 4}];
Rm = 1;
Rmerr = Series[Rm - RA, {dx, 0, 4}];
Rp = Exp[I * k * dx];
Rperr = Series[Rp - RA, {dx, 0, 4}];
Ru = (1 + Exp[I * k * dx]) / 2;
Ruerr = Series[Ru - Exp[I * k * dx / 2], {dx, 0, 4}];
Gold = H - H^3 / 3 * (2 * Cos[k * dx] - 2) / dx^2;
GG2 = Simplify[Ru / Gold];
GG2err = Series[GG2 - GGA, {dx, 0, 5}];
Gn2 = Simplify[-U * Ru / Gold];
Gn2err = Series[Gn2 - GnA, {dx, 0, 5}];
```

```
In[50]:= Text[Row[{"M" || " ", M}]]
Text[Row[{"M" || " ", TeXForm[M]}]]
Text[Row[{"M error" || " ", TeXForm[Merr]}]]
Text[Row[{"M error" || " ", Merr}]]
Text[" "]
Text[Row[{"Rm" || " ", Rm}]]
Text[Row[{"Rm" || " ", TeXForm[Rm]}]]
Text[Row[{"Rm error" || " ", Rmerr}]]
Text[Row[{"Rm error" || " ", TeXForm[Rmerr]}]]
Text[" "]
Text[Row[{"Rp" || " ", Rp}]]
Text[Row[{"Rp" || " ", TeXForm[Rp]}]]
Text[Row[{"Rp error" || " ", Rperr}]]
Text[Row[{"Rp error" || " ", TeXForm[Rperr]}]]
Text[" "]
Text[Row[{"GG2" || " ", GG2}]]
Text[Row[{"GG2" || " ", TeXForm[GG2]}]]
Text[Row[{"GG2 error" || " ", GG2err}]]
Text[Row[{"GG2 error" || " ", TeXForm[GG2err]}]]
Text[" "]
Text[Row[{"Gn2" || " ", Gn2}]]
Text[Row[{"Gn2" || " ", TeXForm[Gn2]}]]
Text[Row[{"Gn2 error" || " ", Gn2err}]]
Text[Row[{"Gn2 error" || " ", TeXForm[Gn2err]}]]
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Out[50]=  $M \parallel 1$

Out[51]=  $M \parallel 1$

Out[52]=  $M \text{ error} \parallel -\frac{\text{dx}^2 k^2}{24} - \frac{7 \text{dx}^4 k^4}{5760} + O(\text{dx}^5)$

Out[53]=  $M \text{ error} \parallel -\frac{k^2 \text{dx}^2}{24} - \frac{7 k^4 \text{dx}^4}{5760} + O[\text{dx}]^5$

Out[54]=

Out[55]=  $Rm \parallel 1$

Out[56]=  $Rm \parallel 1$

Out[57]=  $Rm \text{ error} \parallel -\frac{1}{2} i k \text{dx} + \frac{k^2 \text{dx}^2}{12} + \frac{k^4 \text{dx}^4}{720} + O[\text{dx}]^5$

Out[58]=  $Rm \text{ error} \parallel -\frac{1}{2} i \text{dx} k + \frac{\text{dx}^2 k^2}{12} + \frac{\text{dx}^4 k^4}{720} + O(\text{dx}^5)$

Out[59]=

Out[60]=  $Rp \parallel e^{i \text{dx} k}$

Out[61]=  $Rp \parallel e^{i \text{dx} k}$

Out[62]=  $Rp \text{ error} \parallel \frac{i k \text{dx}}{2} - \frac{5 k^2 \text{dx}^2}{12} - \frac{1}{6} i k^3 \text{dx}^3 + \frac{31 k^4 \text{dx}^4}{720} + O[\text{dx}]^5$

Out[63]=  $Rp \text{ error} \parallel \frac{i \text{dx} k}{2} - \frac{5 \text{dx}^2 k^2}{12} - \frac{1}{6} i k^3 \text{dx}^3 + \frac{31 \text{dx}^4 k^4}{720} + O(\text{dx}^5)$

Out[64]=

Out[65]=  $GG2 \parallel \frac{1 + e^{i \text{dx} k}}{2 \left( H - \frac{2 H^3 (-1 + \cos[\text{dx} k])}{3 \text{dx}^2} \right)}$

Out[66]=  $GG2 \parallel \frac{1 + e^{i \text{dx} k}}{2 \left( H - \frac{2 H^3 (\cos[\text{dx} k] - 1)}{3 \text{dx}^2} \right)}$

Out[67]=  $GG2 \text{ error} \parallel \frac{(-6 k^2 - H^2 k^4) \text{dx}^2}{4 H (3 + H^2 k^2)^2} - \frac{i (6 k^3 + H^2 k^5) \text{dx}^3}{8 H (3 + H^2 k^2)^2} + \frac{(144 k^4 + 45 H^2 k^6 + 4 H^4 k^8) \text{dx}^4}{240 H (3 + H^2 k^2)^3} - \frac{i (-54 k^5 + H^4 k^9) \text{dx}^5}{480 H (3 + H^2 k^2)^3} + O[\text{dx}]^6$

Out[68]=  $GG2 \text{ error} \parallel \frac{\text{dx}^2 \left( -H^2 k^4 - 6 k^2 \right)}{4 H \left( H^2 k^2 + 3 \right)^2} - \frac{i \text{dx}^3 \left( H^2 k^5 + 6 k^3 \right)}{8 H \left( H^2 k^2 + 3 \right)^2} + \frac{\text{dx}^4 \left( 4 H^4 k^8 + 45 H^2 k^6 + 144 k^4 \right)}{240 H \left( H^2 k^2 + 3 \right)^3} - \frac{i \text{dx}^5 \left( H^4 k^9 - 54 k^5 \right)}{480 H \left( H^2 k^2 + 3 \right)^3} + O(\text{dx}^6)$

Out[69]=

Out[70]=  $Gn2 \parallel -\frac{(1 + e^{i \text{dx} k}) U}{2 \left( H - \frac{2 H^3 (-1 + \cos[\text{dx} k])}{3 \text{dx}^2} \right)}$

Out[71]=  $Gn2 \parallel -\frac{U \left( 1 + e^{i \text{dx} k} \right)}{2 \left( H - \frac{2 H^3 (\cos[\text{dx} k] - 1)}{3 \text{dx}^2} \right)}$

$$\text{Out}[72]= \text{Gn2 error} \parallel \frac{(6k^2+H^2k^4)Udx^2}{4H(3+H^2k^2)^2} + \frac{i(6k^3+H^2k^5)Udx^3}{8H(3+H^2k^2)^2} - \frac{((144k^4+45H^2k^6+4H^4k^8)U)dx^4}{240(H(3+H^2k^2)^3)} + \frac{i(-54k^5+H^4k^9)Udx^5}{480H(3+H^2k^2)^3} + O[dx]^6$$

$$\text{Out}[73]= \text{Gn2 error} \parallel \frac{\text{dx}^2 U \left( H^2 k^4 + 6 k^2 \right)}{4 H \left( H^2 k^2 + 3 \right)^2} + \frac{i \text{dx}^3 U \left( H^2 k^5 + 6 k^3 \right)}{8 H \left( H^2 k^2 + 3 \right)^2} - \frac{\text{dx}^4 U \left( 4 H^4 k^8 + 45 H^2 k^6 + 144 k^4 \right)}{240 H \left( H^2 k^2 + 3 \right)^3} + \frac{i \text{dx}^5 U \left( H^4 k^9 - 54 k^5 \right)}{480 H \left( H^2 k^2 + 3 \right)^3} + O \left( \text{dx}^6 \right)$$

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In[74]:= KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
KurFWS = KurF /. ap -> (U + Sqrt[g*H]) /. am -> (U - Sqrt[g*H]);
KurFWSeta =
  KurFWS /. fp -> (H*v + U*Rpp*n) /. fm -> (H*v + U*Rmp*n) /. qp -> Rpp*n /.
  qm -> Rmp*n;
KurFWSeta = KurFWSeta /. v -> (GGp*G + Gnp*n);
Kfnnp = FullSimplify[KurFWSeta /. G -> 0 /. n -> 1];
KfnGp = FullSimplify[KurFWSeta /. n -> 0 /. G -> 1];
Kfn = Kfnnp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfnG = KfnGp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
Fnn2 = -dt*(1 - Exp[-I*k*dx])/dx*Kfn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TA = Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TA = Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp -> (U*Rpp*G + U*H*v + g*H*Rpp*n) /.
  fm -> (U*Rmp*G + U*H*v + g*H*Rmp*n) /. qp -> Rpp*G /. qm -> Rmp*G;
KurFWSG = KurFWSG /. v -> (GGp*G + Gnp*n);
KfGnp = FullSimplify[KurFWSG /. G -> 0 /. n -> 1];
KfGGp = FullSimplify[KurFWSG /. n -> 0 /. G -> 1];
KfGn = KfGnp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;
KfGG = KfGGp /. Rpp -> Rp /. Rmp -> Rm /. GGp -> GG2 /. Gnp -> Gn2;

FGn2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TA = Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U*H*GG2 + U/2*(Rm + Rp) - (Sqrt[g*H])/2*(Rp - Rm);
FGG2 = -dt*(1 - Exp[-I*k*dx])/dx*KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TA = Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2;
Eerr = Series[Emat2 - Exp[-I*wAp*dt]*IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];

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EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}];

Text[Row[{" -Sqrt(gH) < U < Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
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Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
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Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG || ", KfGGp}]]
Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error || ", FGG2TAr}]]
Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error || ", RKstepTayr}]]
Text[Row[{"Omega error || ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA || ", EA}]]
Text[Row[{"EA || ", TeXForm[EA]}]]
Text[Row[{"Eerr || ", Eerr}]]
Text[Row[{"Eerr || ", TeXForm[Eerr]}]]

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Out[108]=  $-\text{Sqrt}(gH) < U < \text{Sqrt}(gH)$

Out[109]=

$$\text{Out}[110]= \text{Fnn} \parallel \frac{1}{2} \left( 2 \text{Gnp} H + \text{Rpp} \left( -\sqrt{g H} + U \right) + \text{Rmp} \left( \sqrt{g H} + U \right) \right)$$

$$\text{Out}[111]= \text{Fnn} \parallel \frac{1}{2} \left( \text{Rmp} \left( \sqrt{g H} + U \right) + \text{Rpp} \left( U - \sqrt{g H} \right) + 2 \text{Gnp} H \right)$$

$$\text{Out}[112]= \text{Fnn error} \parallel \left( -\frac{H^2 k^3 U w}{2(3+H^2 k^2)} - \frac{i H^2 k^3 U w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left( -\frac{1}{2} \left( \sqrt{g H} k^2 \right) dt + O[dt]^4 \right) dx +$$

$$\left( \frac{i(9 H^2 k^5 + 2 H^4 k^7) U dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left( \frac{1}{24} \sqrt{g H} k^4 dt + O[dt]^4 \right) dx^3 + O[dx]^4$$

$$\text{Out}[113]= \text{Fnn error} \parallel$$

$$\left( -\frac{dt^2}{2} \left( H^2 k^3 U w \right) \right) \frac{1}{2} \left( H^2 k^2 + 3 \right) - \frac{i}{6} \frac{dt^3}{2} H^2 k^3 U w^2 \left( H^2 k^2 + 3 \right) + O(dt^4) + dx \left( -\frac{1}{2} \sqrt{g H} k^2 \right) dt + O(dx^2)$$

$$+ O(dt^4) + dx^2 \left( \frac{i}{12} \left( 2 H^4 k^7 + 9 H^2 k^5 \right) U dt \right) \frac{1}{2} \left( H^2 k^2 + 3 \right)^2 + O(dt^4) + dx^3 \left( \frac{1}{24} \sqrt{g H} k^4 dt \right) + O(dx^4)$$

$$\text{Out}[114]=$$

$$\text{Out}[115]= \text{FnG} \parallel \text{GGp} H$$

$$\text{Out}[116]= \text{FnG} \parallel \text{GGp} H$$

$$\text{Out}[117]= \text{FnG error} \parallel \left( -\frac{3(k w) dt^2}{2(3+H^2 k^2)} - \frac{i k w^2 dt^3}{2(3+H^2 k^2)} + O[dt]^4 \right) + \left( \frac{i(6 k^3 + H^2 k^5) dt}{4(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + O[dx]^4$$

$$\text{Out}[118]= \text{FnG error} \parallel \left( -\frac{3}{2} \frac{dt^2}{2} (k w) \right) \frac{1}{2} \left( H^2 k^2 + 3 \right) - \frac{i}{6} \frac{dt^3}{2} k w^2 \left( H^2 k^2 + 3 \right) + O(dt^4) + dx^2 \left( \frac{i}{4} \left( H^2 k^5 + 6 k^3 \right) \right) dt + O(dx^3)$$

$$+ O(dt^4) + dx^3 \left( \frac{1}{24} \sqrt{g H} k^4 U dt \right) + O(dx^4)$$

$$\text{Out}[119]=$$

$$\text{Out}[120]= \text{FGn} \parallel \frac{1}{2} \left( g H (\text{Rmp} + \text{Rpp}) + \left( 2 \text{Gnp} H + \sqrt{g H} (\text{Rmp} - \text{Rpp}) \right) U \right)$$

$$\text{Out}[121]= \text{FGn} \parallel$$

$$\frac{1}{2} \left( U \left( \sqrt{g H} (\text{Rmp} - \text{Rpp}) + 2 \text{Gnp} H \right) + g H (\text{Rmp} + \text{Rpp}) \right)$$

$$\text{Out}[122]= \text{FGn error} \parallel \left( -\frac{(k(3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2(3+H^2 k^2)} - \frac{i k(3 g H + g H^3 k^2 - 3 U^2) w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left( -\frac{1}{2} \left( \sqrt{g H} k^2 U \right) dt + O[dt]^4 \right) dx +$$

$$\left( \frac{i(18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2) dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left( \frac{1}{24} \sqrt{g H} k^4 U dt + O[dt]^4 \right) dx^3 + O[dx]^4$$

$$\text{Out}[123]= \text{FGn error} \parallel$$

$$\left( -\frac{dt^2}{2} (k w \left( g H^3 k^2 + 3 g H - 3 U^2 \right)) \right) \frac{1}{2} \left( H^2 k^2 + 3 \right) - \frac{i}{6} \frac{dt^3}{2} k w^2 \left( g H^3 k^2 + 3 g H - 3 U^2 \right) + O(dt^4) + dx \left( -\frac{1}{2} \sqrt{g H} k^2 U \right) dt + O(dx^2)$$

$$+ O(dt^4) + dx^2 \left( \frac{i}{12} \left( 2 g H^5 k^7 + 12 g H^3 k^5 - 3 H^2 U^2 k^5 - 18 U^2 k^3 + 18 g H k^3 \right) \right) dt \frac{1}{2} \left( H^2 k^2 + 3 \right)^2 + O(dt^4) + dx^3 \left( \frac{1}{24} \sqrt{g H} k^4 U dt \right) + O(dx^4)$$

Out[124]=

$$\text{Out[125]= FGG} \parallel \frac{1}{2} \left( \sqrt{g H} \text{ Rmp} - \sqrt{g H} \text{ Rpp} + (2 \text{ GGp} H + \text{ Rmp} + \text{ Rpp}) U \right)$$

$$\text{Out[126]= FGG} \parallel \frac{1}{2} \left( \text{Rmp} \sqrt{g H} - \text{Rpp} \sqrt{g H} + U (2 \text{ GGp} H + \text{Rmp} + \text{Rpp}) \right)$$

$$\begin{aligned} \text{Out[127]= FGG error} \parallel & \left( -\frac{k(6+H^2 k^2) U w}{2(3+H^2 k^2)} \frac{dt^2}{dt} - \frac{i k(6+H^2 k^2) U w^2}{6(3+H^2 k^2)} \frac{dt^3}{dt} + O[dt]^4 \right) + \\ & \left( -\frac{1}{2} \left( \sqrt{g H} k^2 \right) dt + O[dt]^4 \right) dx + \left( \frac{i(36 k^3 U + 15 H^2 k^5 U + 2 H^4 k^7 U) dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \\ & \left( \frac{1}{24} \sqrt{g H} k^4 dt + O[dt]^4 \right) dx^3 + \left( -\frac{i(108 k^5 U + 54 H^2 k^7 U + 17 H^4 k^9 U + 2 H^6 k^{11} U) dt}{240(3+H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{aligned}$$

Out[128]= FGG error ||

$$\begin{aligned} & \left( -\frac{1}{2} \left( \text{Rmp} \sqrt{g H} - \text{Rpp} \sqrt{g H} + U (2 \text{ GGp} H + \text{Rmp} + \text{Rpp}) \right) \right) \\ & \left( -\frac{1}{2} \left( \sqrt{g H} k^2 \right) dt + O[dt]^4 \right) dx + \left( \frac{i(36 k^3 U + 15 H^2 k^5 U + 2 H^4 k^7 U) dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \\ & \left( \frac{1}{24} \sqrt{g H} k^4 dt + O[dt]^4 \right) dx^3 + \left( -\frac{i(108 k^5 U + 54 H^2 k^7 U + 17 H^4 k^9 U + 2 H^6 k^{11} U) dt}{240(3+H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{aligned}$$

Out[129]=

Out[130]=

$$\begin{aligned} \text{Out[131]= Omega error} \parallel & \left\{ \frac{i \left( \sqrt{3} k \sqrt{g H (3+H^2 k^2)} + 3 k U + H^2 k^3 U \right)^2 dt}{2(3+H^2 k^2)^2} - \frac{1}{3(3+H^2 k^2)^2} \right. \\ & \left( k^3 \left( \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \left( 3 g H + U \left( 2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \right) dt^2 - \\ & \frac{1}{4(3+H^2 k^2)^3} i k^4 \left( \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \left( 3 g \left( \sqrt{3} H \sqrt{g H (3+H^2 k^2)} + 9 H U + 3 H^3 k^2 U \right) \right. \\ & \left. U^2 \left( H^4 k^4 U + 9 \left( \sqrt{3} \sqrt{g H (3+H^2 k^2)} + U \right) + 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} + 2 H^2 U \right) \right) \right) dt^3 + \\ & \frac{1}{5(3+H^2 k^2)^3} k^5 \left( \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \\ & \left( 9 g^2 H^2 + 6 g H U \left( 2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 3(3+H^2 k^2) U \right) + \right. \\ & \left. U^3 \left( 12 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 9 U + H^4 k^4 U + 2 k^2 \left( 2 \sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} + 3 H^2 U \right) \right) \right) dt^4 + \\ & \left. O[dt]^5 \right\} + \left( -\frac{1}{4} i k^2 \left( 2 \sqrt{g H} + \frac{\sqrt{3} U}{\sqrt{3+H^2 k^2}} \right) + \frac{k^3 \left( 2 \sqrt{g H (3+H^2 k^2)} + \sqrt{3} U \right) \left( \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) dt}{4(3+H^2 k^2)^{3/2}} + \right. \\ & \frac{1}{4(3+H^2 k^2)^{3/2}} i k^4 \left( 2 \sqrt{g H (3+H^2 k^2)} + \sqrt{3} U \right) \left( 3 g H + U \left( 2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) dt^2 - \\ & \left. \frac{1}{4(3+H^2 k^2)^{5/2}} \left( k^5 \left( 2 \sqrt{g H (3+H^2 k^2)} + \sqrt{3} U \right) \left( 3 g \left( \sqrt{3} H \sqrt{g H (3+H^2 k^2)} + 9 H U + 3 H^3 k^2 U \right) + \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
& U^2 \left( H^4 k^4 U + 9 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + U \right) + 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 2 H^2 U \right) \right) \\
& dt^3 - \frac{1}{4(3+H^2 k^2)^{3/2}} i k^6 \left( 2 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} U \right) \\
& \left( 9 g^2 H^2 + 6 g H U \left( 2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^3 \left( 12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + 2 k^2 \left( 2 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 3 H^2 U \right) \right) \right) dt^4 + \\
& O[dt]^5 \left) dx + \left( - \left( k^3 \left( 12 \sqrt{3} g H (4 + H^2 k^2) + U \left( 48 \sqrt{g H (3 + H^2 k^2)} - 9 \sqrt{3} U + \right. \right. \right. \right. \\
& \left. \left. \left. k^2 \left( 16 \sqrt{g H^5 (3 + H^2 k^2)} - 3 \sqrt{3} H^2 U \right) \right) \right) \right) / \left( 96 \left( \sqrt{g H (3 + H^2 k^2)}^{3/2} \right) \right) - \\
& \left( i k^4 \left( 9 U^2 \left( 16 \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} U \right) + k^4 U^2 \left( 16 \sqrt{g H^9 (3 + H^2 k^2)} - 3 \sqrt{3} H^4 U \right) + \right. \right. \\
& 6 k^2 \left( 18 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 - 3 \sqrt{3} H^2 U^3 \right) + \\
& 4 g H \left( 63 \sqrt{g H (3 + H^2 k^2)} + 99 \sqrt{3} U + 63 \sqrt{3} H^2 k^2 U + \right. \\
& \left. k^4 \left( 3 \sqrt{g H^9 (3 + H^2 k^2)} + 10 \sqrt{3} H^4 U \right) \right) \right) dt \Big/ \left( 96 \sqrt{g H (3 + H^2 k^2)}^{5/2} \right) + \\
& \left( k^5 \left( 12 \sqrt{3} g^2 H^2 (30 + 15 H^2 k^2 + 2 H^4 k^4) + 9 U^3 \left( 16 \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} U \right) + \right. \right. \\
& k^4 U^3 \left( 16 \sqrt{g H^9 (3 + H^2 k^2)} - 3 \sqrt{3} H^4 U \right) + \\
& 6 k^2 \left( 56 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^3 - 3 \sqrt{3} H^2 U^4 \right) + \\
& g H U \left( 864 \sqrt{g H (3 + H^2 k^2)} + 675 \sqrt{3} U + 429 \sqrt{3} H^2 k^2 U + \right. \\
& \left. 4 k^4 \left( 6 \sqrt{g H^9 (3 + H^2 k^2)} + 17 \sqrt{3} H^4 U \right) \right) \right) dt^2 \Big/ \left( 96 \sqrt{g H (3 + H^2 k^2)}^{5/2} \right) + \\
& \frac{1}{96 \sqrt{g H (3 + H^2 k^2)}^{7/2}} i k^6 \left( 9 k^2 \left( 84 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 404 \sqrt{3} g^2 H^4 U + 438 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \right. \\
& 314 \sqrt{3} g H^3 U^3 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 9 \sqrt{3} H^2 U^5 \Big) + \\
& 27 \left( 52 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 172 \sqrt{3} g^2 H^2 U + 16 \sqrt{g H (3 + H^2 k^2)} U^4 - \right. \\
& 3 \sqrt{3} U^5 + g H U^2 \left( 210 \sqrt{g H (3 + H^2 k^2)} + 109 \sqrt{3} U \right) \Big) + \\
& 3 k^4 \left( 36 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 304 \sqrt{3} g^2 H^6 U + 48 \sqrt{g H^9 (3 + H^2 k^2)} U^4 - \right. \\
& 9 \sqrt{3} H^4 U^5 + g H^5 U^2 \left( 264 \sqrt{g H (3 + H^2 k^2)} + 301 \sqrt{3} U \right) \Big) + \\
& k^6 U \left( 72 \sqrt{3} g^2 H^8 + 12 g H^7 U \left( 3 \sqrt{g H (3 + H^2 k^2)} + 8 \sqrt{3} U \right) + \right. \\
& \left. U^3 \left( 16 \sqrt{g H^{13} (3 + H^2 k^2)} - 3 \sqrt{3} H^6 U \right) \right) \Big) dt^3 - \frac{1}{96 \left( \sqrt{g H (3 + H^2 k^2)}^{7/2} \right)} \\
& \left( k^7 \left( 36 \sqrt{3} g^3 H^3 (48 + 27 H^2 k^2 + 4 H^4 k^4) + 3 \sqrt{3} g^2 H^2 (4113 + 3075 H^2 k^2 + 712 H^4 k^4 + 48 H^6 k^6) U^2 + \right. \right. \\
& 2 g H U^3 \left( 5076 \sqrt{g H (3 + H^2 k^2)} + 1971 \sqrt{3} U + 1872 \sqrt{3} H^2 k^2 U + \right.
\end{aligned}$$



$$\begin{aligned}
& \left( 3 k^4 \left( 216 \sqrt{g H^9 (3 + H^2 k^2)} + 197 \sqrt{3} H^4 U \right) + 2 k^6 \left( 12 \sqrt{g H^{13} (3 + H^2 k^2)} + 31 \sqrt{3} H^6 U \right) \right) + U \left( 7344 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 432 \sqrt{g H (3 + H^2 k^2)} U^4 - 81 \sqrt{3} U^5 + \right. \\
& k^6 U^4 \left( 16 \sqrt{g H^{13} (3 + H^2 k^2)} - 3 \sqrt{3} H^6 U \right) + 9 k^2 \left( 400 \sqrt{g^5 H^9 (3 + H^2 k^2)} + \right. \\
& 760 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 9 \sqrt{3} H^2 U^5 \left. \right) + \\
& \left. 9 k^4 \left( 48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 16 \sqrt{g H^9 (3 + H^2 k^2)} U^4 - 3 \sqrt{3} H^4 U^5 \right) \right) dt^4 + O[dt]^5 \Big) \\
& dx^2 + \left( \left( i k^4 \left( 3 \sqrt{3} (3 + H^2 k^2) U^3 + 4 g H \left( 12 \sqrt{g H (3 + H^2 k^2)} + 6 \sqrt{3} U + \right. \right. \right. \right. \\
& \left. \left. \left. k^2 \left( 4 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \right) \right) \right) / (384 g H (3 + H^2 k^2)^{3/2}) - \\
& \left( k^5 \left( 8 \sqrt{3} g^2 H^2 (54 + 33 H^2 k^2 + 5 H^4 k^4) + 3 \sqrt{3} (3 + H^2 k^2)^2 U^4 + 2 g H U \right. \right. \\
& \left. \left( 81 \left( 4 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} U \right) + 24 k^2 \left( 7 \sqrt{g H^5 (3 + H^2 k^2)} + 2 \sqrt{3} H^2 U \right) + \right. \right. \\
& \left. \left. k^4 \left( 24 \sqrt{g H^9 (3 + H^2 k^2)} + 7 \sqrt{3} H^4 U \right) \right) \right) dt \Big) / (384 (g H (3 + H^2 k^2)^{5/2})) - \\
& \left( i k^6 \left( 3 k^2 \left( 96 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 380 \sqrt{3} g^2 H^4 U + 224 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \right. \right. \\
& 51 \sqrt{3} g H^3 U^3 + 6 \sqrt{3} H^2 U^5 \Big) + k^4 \left( 16 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 168 \sqrt{3} g^2 H^6 U + \right. \\
& 3 \sqrt{3} H^4 U^5 + 8 g H^5 U^2 \left( 10 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U \right) \Big) + 9 \left( 96 \sqrt{g^5 H^5 (3 + H^2 k^2)} + \right. \\
& 224 \sqrt{3} g^2 H^2 U + 3 \sqrt{3} U^5 + g H U^2 \left( 160 \sqrt{g H (3 + H^2 k^2)} + 27 \sqrt{3} U \right) \Big) dt^2 \Big) / \\
& (384 g H (3 + H^2 k^2)^{5/2}) + \frac{1}{384 g H (3 + H^2 k^2)^{7/2}} k^7 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left( 3 k^2 \left( 184 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 608 \sqrt{3} g^2 H^4 U + 334 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 72 \sqrt{3} g H^3 U^3 - \right. \right. \\
& 3 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 6 \sqrt{3} H^2 U^5 \Big) + k^4 \left( 48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + \right. \\
& 272 \sqrt{3} g^2 H^6 U + 3 \sqrt{3} H^4 U^5 + 2 g H^5 U^2 \left( 56 \sqrt{g H (3 + H^2 k^2)} + 17 \sqrt{3} U \right) \Big) + \\
& 9 \left( 160 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 352 \sqrt{3} g^2 H^2 U - 3 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& 3 \sqrt{3} U^5 + 2 g H U^2 \left( 121 \sqrt{g H (3 + H^2 k^2)} + 19 \sqrt{3} U \right) \Big) dt^3 + \\
& \frac{1}{384 g H (3 + H^2 k^2)^{9/2}} i k^8 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^2 \\
& \left( 3 k^2 \left( 304 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 884 \sqrt{3} g^2 H^4 U + 456 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 95 \sqrt{3} g H^3 U^3 - \right. \right. \\
& 6 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 6 \sqrt{3} H^2 U^5 \Big) + k^4 \left( 96 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + \right. \\
& 400 \sqrt{3} g^2 H^6 U + 3 \sqrt{3} H^4 U^5 + 4 g H^5 U^2 \left( 36 \sqrt{g H (3 + H^2 k^2)} + 11 \sqrt{3} U \right) \Big) + \\
& 27 \left( 80 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 168 \sqrt{3} g^2 H^2 U - 2 \sqrt{g H (3 + H^2 k^2)} U^4 + \sqrt{3} U^5 + \right. \\
& \left. \left. 112 \sqrt{3} g H U^2 \left( 11 \sqrt{g H (3 + H^2 k^2)} + 3 \sqrt{3} U \right) \right) dt^4 + O[dt]^5 \right)
\end{aligned}$$



$$\begin{aligned}
& 9888 \sqrt{3} \, g^2 H^{10} U^3 - 75 \sqrt{3} \, H^8 U^7 + 16 g H^9 U^4 \left( 96 \sqrt{g H (3 + H^2 k^2)} - 5 \sqrt{3} \, U \right) \Big) dt^3 + \\
& \frac{1}{92160 (g H)^{3/2} (3 + H^2 k^2)^{9/2}} k^9 \left( 144 \sqrt{3} \, g^4 H^4 (23832 + 22104 H^2 k^2 + 7395 H^4 k^4 + 1000 H^6 k^6 + 40 H^8 k^8) + \right. \\
& 24 \sqrt{3} \, g^3 H^3 (893700 + 953451 H^2 k^2 + 376029 H^4 k^4 + 64744 H^6 k^6 + 4080 H^8 k^8) U^2 + \\
& 3 \sqrt{3} \, g^2 H^2 (3 + H^2 k^2)^2 (197625 + 114352 H^2 k^2 + 16944 H^4 k^4) U^4 + \\
& 2 g H U^5 \left( 238464 \sqrt{g H (3 + H^2 k^2)} - 8505 \sqrt{3} \, U - 13365 \sqrt{3} \, H^2 k^2 U + \right. \\
& 27 k^4 \left( 5888 \sqrt{g H^9 (3 + H^2 k^2)} - 285 \sqrt{3} \, H^4 U \right) + 3 k^6 \left( 11776 \sqrt{g H^{13} (3 + H^2 k^2)} - 645 \right. \\
& \left. \left. \sqrt{3} \, H^6 U \right) + 4 k^8 \left( 736 \sqrt{g H^{17} (3 + H^2 k^2)} - 45 \sqrt{3} \, H^8 U \right) \Big) + \right. \\
& 3 U \left( 135 \left( 33920 \sqrt{g^7 H^7 (3 + H^2 k^2)} + 39424 \sqrt{g^5 H^5 (3 + H^2 k^2)} U^2 - 45 \sqrt{3} \, U^7 \right) + \right. \\
& 36 k^2 \left( 105472 \sqrt{g^7 H^{11} (3 + H^2 k^2)} + 144512 \sqrt{g^5 H^9 (3 + H^2 k^2)} \right. \\
& \left. U^2 + 5888 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^4 - 225 \sqrt{3} \, H^2 U^7 \right) + \\
& 18 k^4 \left( 60608 \sqrt{g^7 H^{15} (3 + H^2 k^2)} + 100096 \sqrt{g^5 H^{13} (3 + H^2 k^2)} U^2 - 225 \sqrt{3} \, H^4 U^7 \right) + \\
& 12 k^6 \left( 9600 \sqrt{g^7 H^{19} (3 + H^2 k^2)} + 20864 \sqrt{g^5 H^{17} (3 + H^2 k^2)} U^2 - 75 \sqrt{3} \, H^6 U^7 \right) + \\
& \left. \left. 5 k^8 \left( 384 \sqrt{g^7 H^{23} (3 + H^2 k^2)} + 2048 \sqrt{g^5 H^{21} (3 + H^2 k^2)} U^2 - 15 \sqrt{3} \, H^8 U^7 \right) \right) \right) dt^4 + \\
& O[dt]^5 \Big) dx^4 + O[dx]^5, \left( \frac{i \left( -\sqrt{3} \, k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^3 U \right)^2 dt}{2 (3 + H^2 k^2)^2} - \frac{1}{3 (3 + H^2 k^2)^2} \right. \\
& \left( k^3 \left( -\sqrt{3} \, \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \left. \left( 3 g H + U \left( -2 \sqrt{3} \, \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right) dt^2 - \\
& \frac{1}{4 (3 + H^2 k^2)^3} i k^4 \left( -\sqrt{3} \, \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left( 3 g H \left( -\sqrt{3} \, \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& U^2 \left( -9 \sqrt{3} \, \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left( \sqrt{3} \, \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \Big) dt^3 + \\
& \frac{1}{5 (3 + H^2 k^2)^3} k^5 \left( -\sqrt{3} \, \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left( 9 g^2 H^2 + 6 g H U \left( -2 \sqrt{3} \, \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& U^3 \left( -12 \sqrt{3} \, \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left( -4 \sqrt{3} \, \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \Big) dt^4 + O[ \\
& dt]^5 \Big) + \\
& \left( \frac{1}{2 (3 + H^2 k^2)^3} k^3 \left( 2 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} \, U \right) \left( -\sqrt{3} \, \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) dt \right.
\end{aligned}$$

$$\begin{aligned}
& \left( -\frac{1}{4} i k^2 \left( 2 \sqrt{g H} - \frac{\sqrt{3} U}{\sqrt{3+H^2 k^2}} \right) + \frac{\left( \sqrt{g H} - \frac{\sqrt{3} U}{\sqrt{3+H^2 k^2}} \right) \left( \sqrt{g H} - \frac{\sqrt{3} U}{\sqrt{3+H^2 k^2}} \right)}{4 (3+H^2 k^2)^{3/2}} + \right. \\
& \quad \frac{1}{4 (3+H^2 k^2)^{3/2}} \\
& \quad i k^4 \left( 2 \sqrt{g H (3+H^2 k^2)} - \sqrt{3} U \right) \\
& \quad \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \right) \\
& \quad dt^2 - \frac{1}{4 (3+H^2 k^2)^{3/2}} \\
& \quad \left( k^2 \left( 2 \sqrt{g H (3+H^2 k^2)} - \sqrt{3} U \right) \left( 3 g H \left( -\sqrt{3} \sqrt{g H (3+H^2 k^2)} + 3 (3+H^2 k^2) U \right) + \right. \right. \\
& \quad \left. \left. U^2 \left( -9 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} - 2 H^2 U \right) \right) \right) \right) \\
& \quad dt^3 - \frac{1}{4 (3+H^2 k^2)^{3/2}} i k^6 \left( 2 \sqrt{g H (3+H^2 k^2)} - \sqrt{3} U \right) \\
& \quad \left( 9 g^2 H^2 + 6 g H U \left( -2 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 3 (3+H^2 k^2) U \right) + \right. \\
& \quad \left. U^3 \left( -12 \sqrt{3} \sqrt{g H (3+H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left( -4 \sqrt{3} \sqrt{g H^5 (3+H^2 k^2)} + 6 H^2 U \right) \right) \right) dt^4 + \\
& \quad O[dt]^5 \Bigg) dx + \left( \left( k^3 \left( 12 \sqrt{3} g H (4+H^2 k^2) - U \left( 48 \sqrt{g H (3+H^2 k^2)} + 9 \sqrt{3} U + \right. \right. \right. \right. \\
& \quad \left. \left. \left. k^2 \left( 16 \sqrt{g H^5 (3+H^2 k^2)} + 3 \sqrt{3} H^2 U \right) \right) \right) \right) / \left( 96 \sqrt{g H} (3+H^2 k^2)^{3/2} \right) - \\
& \quad \left( i k^4 \left( 9 U^2 \left( 16 \sqrt{g H (3+H^2 k^2)} + 3 \sqrt{3} U \right) + k^4 U^2 \left( 16 \sqrt{g H^9 (3+H^2 k^2)} + 3 \sqrt{3} H^4 U \right) + \right. \right. \\
& \quad 6 k^2 \left( 18 \sqrt{g^3 H^7 (3+H^2 k^2)} + 16 \sqrt{g H^5 (3+H^2 k^2)} U^2 + 3 \sqrt{3} H^2 U^3 \right) + \\
& \quad 4 g H \left( 63 \sqrt{g H (3+H^2 k^2)} - 99 \sqrt{3} U - 63 \sqrt{3} H^2 k^2 U + \right. \\
& \quad \left. k^4 \left( 3 \sqrt{g H^9 (3+H^2 k^2)} - 10 \sqrt{3} H^4 U \right) \right) \Bigg) dt \Bigg) / \left( 96 \sqrt{g H} (3+H^2 k^2)^{5/2} \right) + \\
& \quad \left( k^5 \left( -12 \sqrt{3} g^2 H^2 (30 + 15 H^2 k^2 + 2 H^4 k^4) + 9 U^3 \left( 16 \sqrt{g H (3+H^2 k^2)} + 3 \sqrt{3} U \right) + \right. \right. \\
& \quad k^4 U^3 \left( 16 \sqrt{g H^9 (3+H^2 k^2)} + 3 \sqrt{3} H^4 U \right) + \\
& \quad 6 k^2 \left( 56 \sqrt{g^3 H^7 (3+H^2 k^2)} U + 16 \sqrt{g H^5 (3+H^2 k^2)} U^3 + 3 \sqrt{3} H^2 U^4 \right) + \\
& \quad g H U \left( 864 \sqrt{g H (3+H^2 k^2)} - 675 \sqrt{3} U - 429 \sqrt{3} H^2 k^2 U + \right. \\
& \quad \left. 4 k^4 \left( 6 \sqrt{g H^9 (3+H^2 k^2)} - 17 \sqrt{3} H^4 U \right) \right) \Bigg) dt^2 \Bigg) / \left( 96 \sqrt{g H} (3+H^2 k^2)^{5/2} \right) + \\
& \quad \frac{1}{96 \sqrt{g H} (3+H^2 k^2)^{7/2}} i k^6 \left( 9 k^2 \left( 84 \sqrt{g^5 H^9 (3+H^2 k^2)} - 404 \sqrt{3} g^2 H^4 U + 438 \sqrt{g^3 H^7 (3+H^2 k^2)} U^2 - \right. \right. \\
& \quad 314 \sqrt{3} g H^3 U^3 + 48 \sqrt{g H^5 (3+H^2 k^2)} U^4 + 9 \sqrt{3} H^2 U^5 \Bigg) + \\
& \quad 3 k^4 \left( 36 \sqrt{g^5 H^{13} (3+H^2 k^2)} - 304 \sqrt{3} g^2 H^6 U + 48 \sqrt{g H^9 (3+H^2 k^2)} U^4 + \right. \\
& \quad \left. 9 \sqrt{3} H^4 U^5 + 9 H^5 U^2 \left( 264 \sqrt{g H (3+H^2 k^2)} - 301 \sqrt{3} U \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& \left( -\sqrt{g^5 H^5 (3 + H^2 k^2)} - 172 \sqrt{3} g^2 H^2 U + 16 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. 3 \sqrt{3} U^5 + g H U^2 \left( 210 \sqrt{g H (3 + H^2 k^2)} - 109 \sqrt{3} U \right) \right) - \\
& \quad k^6 U \left( 72 \sqrt{3} g^2 H^8 + 12 g H^7 U \left( -3 \sqrt{g H (3 + H^2 k^2)} + 8 \sqrt{3} U \right) - \right. \\
& \quad \left. U^3 \left( 16 \sqrt{g H^{13} (3 + H^2 k^2)} + 3 \sqrt{3} H^6 U \right) \right) \Bigg) dt^3 + \frac{1}{96 \sqrt{g H (3 + H^2 k^2)^{7/2}}} \\
& \quad k^7 \left( 36 \sqrt{3} g^3 H^3 (48 + 27 H^2 k^2 + 4 H^4 k^4) + 3 \sqrt{3} g^2 H^2 (4113 + 3075 H^2 k^2 + 712 H^4 k^4 + 48 H^6 k^6) U^2 + \right. \\
& \quad \left. 2 g H U^3 \left( -5076 \sqrt{g H (3 + H^2 k^2)} + 1971 \sqrt{3} U + 1872 \sqrt{3} H^2 k^2 U + \right. \right. \\
& \quad \left. \left. 3 k^4 \left( -216 \sqrt{g H^9 (3 + H^2 k^2)} + 197 \sqrt{3} H^4 U \right) + \right. \right. \\
& \quad \left. \left. 2 k^6 \left( -12 \sqrt{g H^{13} (3 + H^2 k^2)} + 31 \sqrt{3} H^6 U \right) \right) \right) - \\
& \quad U \left( 7344 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 432 \sqrt{g H (3 + H^2 k^2)} U^4 + 81 \sqrt{3} U^5 + \right. \\
& \quad \left. k^6 U^4 \left( 16 \sqrt{g H^{13} (3 + H^2 k^2)} + 3 \sqrt{3} H^6 U \right) + 9 k^2 \left( 400 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 760 \right. \right. \\
& \quad \left. \left. \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 48 \sqrt{g H^5 (3 + H^2 k^2)} U^4 + 9 \sqrt{3} H^2 U^5 \right) + \right. \\
& \quad \left. \left. 9 k^4 \left( 48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 16 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + 3 \sqrt{3} H^4 U^5 \right) \right) \right) dt^4 + O[dt]^5 \Bigg) dx^2 + \\
& \quad \left( - \left( i k^4 \left( 3 \sqrt{3} (3 + H^2 k^2) U^3 + 4 g H \left( -12 \sqrt{g H (3 + H^2 k^2)} + 6 \sqrt{3} U + \right. \right. \right. \right. \\
& \quad \left. \left. \left. k^2 \left( -4 \sqrt{g H^5 (3 + H^2 k^2)} + \sqrt{3} H^2 U \right) \right) \right) \right) / (384 g H (3 + H^2 k^2)^{3/2}) \right) + \\
& \quad \left( k^5 \left( 8 \sqrt{3} g^2 H^2 (54 + 33 H^2 k^2 + 5 H^4 k^4) + 3 \sqrt{3} (3 + H^2 k^2)^2 U^4 - 2 g H U \right. \right. \\
& \quad \left. \left. \left( 324 \sqrt{g H (3 + H^2 k^2)} - 81 \sqrt{3} U + 24 k^2 \left( 7 \sqrt{g H^5 (3 + H^2 k^2)} - 2 \sqrt{3} H^2 U \right) + \right. \right. \right. \\
& \quad \left. \left. \left. k^4 \left( 24 \sqrt{g H^9 (3 + H^2 k^2)} - 7 \sqrt{3} H^4 U \right) \right) \right) \right) / (384 g H (3 + H^2 k^2)^{5/2}) - \\
& \quad \left( i k^6 \left( 3 k^2 \left( 96 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 380 \sqrt{3} g^2 H^4 U + 224 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \right. \\
& \quad \left. \left. \left. 51 \sqrt{3} g H^3 U^3 - 6 \sqrt{3} H^2 U^5 \right) + 9 \left( 96 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 224 \sqrt{3} g^2 H^2 U - \right. \right. \right. \\
& \quad \left. \left. \left. 3 \sqrt{3} U^5 + g H U^2 \left( 160 \sqrt{g H (3 + H^2 k^2)} - 27 \sqrt{3} U \right) \right) + k^4 \left( 16 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - \right. \right. \right. \\
& \quad \left. \left. \left. 168 \sqrt{3} g^2 H^6 U - 3 \sqrt{3} H^4 U^5 + 8 g H^5 U^2 \left( 10 \sqrt{g H (3 + H^2 k^2)} - 3 \sqrt{3} U \right) \right) \right) \right) dt^2 \Bigg) / \\
& \quad \left( 384 g H (3 + H^2 k^2)^{5/2} \right) - \frac{1}{384 (g H (3 + H^2 k^2)^{7/2})} \left( k^7 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \quad \left. \left( -3 k^2 \left( 184 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 608 \sqrt{3} g^2 H^4 U + 334 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 - \right. \right. \right. \\
& \quad \left. \left. \left. 72 \sqrt{3} g H^3 U^3 - 3 \sqrt{g H^5 (3 + H^2 k^2)} U^4 - 6 \sqrt{3} H^2 U^5 \right) - \right. \right. \\
& \quad \left. \left. 9 \left( 160 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 352 \sqrt{3} g^2 H^2 U - 3 \sqrt{g H (3 + H^2 k^2)} U^4 - 3 \sqrt{3} U^5 + \right. \right. \right. \\
& \quad \left. \left. \left. 2 g H U^2 \left( 121 \sqrt{g H (3 + H^2 k^2)} - 19 \sqrt{3} U \right) \right) - k^4 \left( 48 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 272 \sqrt{3} \, g^2 H^6 U - 3 \sqrt{3} \, H^4 U^5 + 2 g H^5 U^2 \left( 56 \sqrt{g H (3 + H^2 k^2)} - 17 \sqrt{3} \, U \right) \Big) \Big) \\
& dt^3 - \frac{1}{384 g H (3 + H^2 k^2)^{9/2}} i k^8 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \\
& \left( -3 k^2 \left( 304 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 884 \sqrt{3} \, g^2 H^4 U + 456 \sqrt{g^3 H^7 (3 + H^2 k^2)} \, U^2 - \right. \right. \\
& \quad \left. 95 \sqrt{3} \, g H^3 U^3 - 6 \sqrt{g H^5 (3 + H^2 k^2)} \, U^4 - 6 \sqrt{3} \, H^2 U^5 \right) - \\
& \quad 27 \left( 80 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 168 \sqrt{3} \, g^2 H^2 U - 2 \sqrt{g H (3 + H^2 k^2)} \, U^4 - \right. \\
& \quad \left. \sqrt{3} \, U^5 + g H U^2 \left( 112 \sqrt{g H (3 + H^2 k^2)} - 17 \sqrt{3} \, U \right) \right) - \\
& \quad k^4 \left( 96 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 400 \sqrt{3} \, g^2 H^6 U - 3 \sqrt{3} \, H^4 U^5 + \right. \\
& \quad \left. 4 g H^5 U^2 \left( 36 \sqrt{g H (3 + H^2 k^2)} - 11 \sqrt{3} \, U \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^3 + \\
& \left( \left( k^5 \left( -16 \sqrt{3} \, g^2 H^2 (144 + 48 H^2 k^2 + 5 H^4 k^4) + 75 \sqrt{3} \, (3 + H^2 k^2)^2 U^4 + 8 g H (3 + H^2 k^2) U \right. \right. \right. \\
& \quad \left. \left. \left( 96 \sqrt{g H (3 + H^2 k^2)} + k^2 \left( 32 \sqrt{g H^5 (3 + H^2 k^2)} - 5 \sqrt{3} \, H^2 U \right) \right) \right) \right) / \\
& \quad \left( 30720 (g H)^{3/2} (3 + H^2 k^2)^{5/2} \right) + \frac{1}{92160 (g H)^{3/2} (3 + H^2 k^2)^{7/2}} \\
& \quad i k^6 \left( 9 k^2 \left( 8448 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 16160 \sqrt{3} \, g^2 H^4 U + 6144 \sqrt{g^3 H^7 (3 + H^2 k^2)} \, U^2 + \right. \right. \\
& \quad \left. 240 \sqrt{3} \, g H^3 U^3 + 675 \sqrt{3} \, H^2 U^5 \right) + k^6 \left( 1920 \sqrt{g^5 H^{17} (3 + H^2 k^2)} - \right. \\
& \quad \left. 4368 \sqrt{3} \, g^2 H^8 U + 2048 g H^7 \sqrt{g H (3 + H^2 k^2)} \, U^2 + 225 \sqrt{3} \, H^6 U^5 \right) + \\
& \quad 3 k^4 \left( 6720 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 14512 \sqrt{3} \, g^2 H^6 U + 675 \sqrt{3} \, H^4 U^5 + \right. \\
& \quad \left. 24 g H^5 U^2 \left( 256 \sqrt{g H (3 + H^2 k^2)} + 5 \sqrt{3} \, U \right) \right) + 27 \left( 3968 \sqrt{g^5 H^5 (3 + H^2 k^2)} - \right. \\
& \quad \left. 6016 \sqrt{3} \, g^2 H^2 U + 225 \sqrt{3} \, U^5 + 8 g H U^2 \left( 256 \sqrt{g H (3 + H^2 k^2)} + 15 \sqrt{3} \, U \right) \right) \Big) dt - \\
& \frac{1}{92160 (g H)^{3/2} (3 + H^2 k^2)^{7/2}} \left( k^7 \left( -48 \sqrt{3} \, g^3 H^3 (6192 + 5004 H^2 k^2 + 1425 H^4 k^4 + 140 H^6 k^6) - \right. \right. \\
& \quad 8 \sqrt{3} \, g^2 H^2 (60426 + 55557 H^2 k^2 + 17121 H^4 k^4 + 1772 H^6 k^6) U^2 + \\
& \quad g H U^3 \left( 89856 \sqrt{g H (3 + H^2 k^2)} + 5265 \sqrt{3} \, U + 4590 \sqrt{3} \, H^2 k^2 U + \right. \\
& \quad \left. 9 k^4 \left( 3328 \sqrt{g H^9 (3 + H^2 k^2)} + 145 \sqrt{3} \, H^4 U \right) + 8 k^6 \left( 416 \sqrt{g H^{13} (3 + H^2 k^2)} + \right. \\
& \quad \left. 15 \sqrt{3} \, H^6 U \right) \Big) + 3 U \left( 81 \left( 2816 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 25 \sqrt{3} \, U^5 \right) + \right. \\
& \quad \left. 3 k^2 \left( 52352 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 9984 \sqrt{g^3 H^7 (3 + H^2 k^2)} \, U^2 + 675 \sqrt{3} \, H^2 U^5 \right) + \right. \\
& \quad \left. 3 k^4 \left( 11776 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 225 \sqrt{3} \, H^4 U^5 \right) + \right. \\
& \quad \left. \left. 5 k^6 \left( 512 \sqrt{g^5 H^{17} (3 + H^2 k^2)} + 15 \sqrt{3} \, H^6 U^5 \right) \right) \right) \Big) dt^2 - \frac{1}{30720 (g H)^{3/2} (3 + H^2 k^2)^{9/2}} \\
& i k^8 \left( 27 k^2 \left( 20416 \sqrt{g^7 H^{11} (3 + H^2 k^2)} - 80544 \sqrt{3} \, g^3 H^5 U + 82240 \sqrt{g^5 H^9 (3 + H^2 k^2)} \, U^2 - \right. \right.
\end{aligned}$$

$$\begin{aligned} & 46\,960\sqrt{3}\,g^2H^4U^3 + 6144\sqrt{g^3H^7(3+H^2k^2)}\,U^4 + 305\sqrt{3}\,gH^3U^5 + 300\sqrt{3}\,H^2U^7 \Big) + \\ & k^8 \Big( 480\sqrt{g^7H^{23}(3+H^2k^2)} - 11\,520\sqrt{3}\,g^3H^{11}U + 5760\sqrt{g^5H^{21}(3+H^2k^2)}\,U^2 - \\ & 9888\sqrt{3}\,g^2H^{10}U^3 + 75\sqrt{3}\,H^8U^7 + 16gH^9U^4 \Big( 96\sqrt{gH(3+H^2k^2)} + 5\sqrt{3}\,U \Big) \Big) + \\ & 243 \Big( 2592\sqrt{g^7H^7(3+H^2k^2)} - 8128\sqrt{3}\,g^3H^3U + 9152\sqrt{g^5H^5(3+H^2k^2)}\,U^2 - \\ & 4168\sqrt{3}\,g^2H^2U^3 + 25\sqrt{3}\,U^7 + gH^4U^4 \Big( 512\sqrt{gH(3+H^2k^2)} + 25\sqrt{3}\,U \Big) \Big) + \\ & 3k^6 \Big( 6720\sqrt{g^7H^{19}(3+H^2k^2)} - 55\,216\sqrt{3}\,g^3H^9U + 39\,488\sqrt{g^5H^{17}(3+H^2k^2)}\,U^2 - \\ & 41\,728\sqrt{3}\,g^2H^8U^3 + 300\sqrt{3}\,H^6U^7 + 3gH^7U^4 \Big( 2048\sqrt{gH(3+H^2k^2)} + 105\sqrt{3}\,U \Big) \Big) + \\ & 9k^4 \Big( 19\,040\sqrt{g^7H^{15}(3+H^2k^2)} - 99\,856\sqrt{3}\,g^3H^7U + 88\,512\sqrt{g^5H^{13}(3+H^2k^2)}\,U^2 - 66\,296 \\ & \sqrt{3}\,g^2H^6U^3 + 450\sqrt{3}\,H^4U^7 + 3gH^5U^4 \Big( 3072\sqrt{gH(3+H^2k^2)} + 155\sqrt{3}\,U \Big) \Big) \Big) dt^3 + \\ & \frac{1}{92\,160(gH)^{3/2}(3+H^2k^2)^{9/2}} k^9 \Big( -144\sqrt{3}\,g^4H^4(23\,832 + 22\,104H^2k^2 + 7395H^4k^4 + 1000H^6k^6 + 40H^8k^8) - \\ & 24\sqrt{3}\,g^3H^3(893\,700 + 953\,451H^2k^2 + 376\,029H^4k^4 + 64\,744H^6k^6 + 4080H^8k^8)U^2 - \\ & 3\sqrt{3}\,g^2H^2(3+H^2k^2)^2(197\,625 + 114\,352H^2k^2 + 16\,944H^4k^4)U^4 + \\ & 2gH^5U^5 \Big( 238\,464\sqrt{gH(3+H^2k^2)} + 8505\sqrt{3}\,U + 13\,365\sqrt{3}\,H^2k^2U + \\ & 27k^4 \Big( 5888\sqrt{gH^9(3+H^2k^2)} + 285\sqrt{3}\,H^4U \Big) + 3k^6 \Big( 11\,776\sqrt{gH^{13}(3+H^2k^2)} + 645 \\ & \sqrt{3}\,H^6U \Big) + 4k^8 \Big( 736\sqrt{gH^{17}(3+H^2k^2)} + 45\sqrt{3}\,H^8U \Big) \Big) \Big) + \\ & 3U \Big( 135 \Big( 33\,920\sqrt{g^7H^7(3+H^2k^2)} + 39\,424\sqrt{g^5H^5(3+H^2k^2)}\,U^2 + 45\sqrt{3}\,U^7 \Big) + \\ & 36k^2 \Big( 105\,472\sqrt{g^7H^{11}(3+H^2k^2)} + 144\,512\sqrt{g^5H^9(3+H^2k^2)} \\ & U^2 + 5888\sqrt{g^3H^7(3+H^2k^2)}\,U^4 + 225\sqrt{3}\,H^2U^7 \Big) + \\ & 18k^4 \Big( 60\,608\sqrt{g^7H^{15}(3+H^2k^2)} + 100\,096\sqrt{g^5H^{13}(3+H^2k^2)}\,U^2 + 225\sqrt{3}\,H^4U^7 \Big) + \\ & 12k^6 \Big( 9600\sqrt{g^7H^{19}(3+H^2k^2)} + 20\,864\sqrt{g^5H^{17}(3+H^2k^2)}\,U^2 + 75\sqrt{3}\,H^6U^7 \Big) + \\ & 5k^8 \Big( 384\sqrt{g^7H^{23}(3+H^2k^2)} + 2048\sqrt{g^5H^{21}(3+H^2k^2)} \\ & U^2 + 15\sqrt{3}\,H^8U^7 \Big) \Big) \Big) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5 \Big\} \end{aligned}$$

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Out[132]= Omega error ||
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$$\frac{\left(\frac{i}{\sqrt{H^2 U^3 + 3 U + \sqrt{3}}} \sqrt{g H \left(H^2 k^2 + 3\right)} k\right)^2 \text{ (dt)}^2 \left(H^2 k^2 + 3\right)^2 - \frac{\left(k^3 \left(\left(H^2 k^2 + 3\right) U + \sqrt{3}\right) \sqrt{g H \left(H^2 k^2 + 3\right)}\right) \left(\left(3 g H + U \left(\left(H^2 k^2 + 3\right) U + 2 \sqrt{3}\right) \sqrt{g H \left(H^2 k^2 + 3\right)}\right)\right) \text{ (dt)}^2 \right)^3 \left(H^2 k^2 + 3\right)^2 - \frac{i k^4 \left(\left(H^2 k^2 + 3\right) U + \sqrt{3}\right) \sqrt{g H \left(H^2 k^2 + 3\right)}}{\left(H^4 U k^4 + 3 \left(2 U H^2 + \sqrt{3}\right) \sqrt{g H^5 \left(H^2 k^2 + 3\right)}\right) k^2 + 9 \left(U + \sqrt{3}\right) \sqrt{g H \left(H^2 k^2 + 3\right)}} U^2 + 3 g \left(3 k^2 U H^3 + 9 U H + \sqrt{3}\right) \sqrt{g H \left(H^2 k^2 + 3\right)} H\right) \text{ (dt)}^3 \left(4 \left(H^2 k^2 + 3\right)^3 + \frac{k^5 \left(\left(H^2 k^2 + 3\right) U + \sqrt{3}\right) \sqrt{g H \left(H^2 k^2 + 3\right)}}{\left(H^4 U k^4 + 3 \left(2 U H^2 + \sqrt{3}\right) \sqrt{g H^5 \left(H^2 k^2 + 3\right)}\right) k^2 + 9 \left(U + \sqrt{3}\right) \sqrt{g H \left(H^2 k^2 + 3\right)}}\right)$$

[illegible]



$$\begin{aligned} & H^4 U^5 + 16 \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} U^4 + 48 \sqrt{g^5 H^{13} \left( H^2 k^2 + 3 \right)} \right) \\ & k^4 + 9 \left( -9 \sqrt{3} H^2 U^5 + 48 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} U^4 + 760 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} U^2 + 400 \sqrt{g^5 H^9 \left( H^2 k^2 + 3 \right)} \right) \\ & k^2 - 81 \sqrt{3} U^5 + 432 \sqrt{g H \left( H^2 k^2 + 3 \right)} U^4 + 7344 \sqrt{g^5 H^5 \left( H^2 k^2 + 3 \right)} \right) \right) \text{t}^4 \{ 96 \\ & \left( \sqrt{g H} \left( H^2 k^2 + 3 \right)^{7/2} \right) + O \left( \text{t}^5 \right) \right) \text{d}^2 + \left( \frac{i}{k^4 \left( 3 \sqrt{3} \left( H^2 k^2 + 3 \right) U^3 + 4 g H \left( \left( \sqrt{3} U H^2 + 4 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) \right) \right) \right.} \right. \\ & k^2 + 6 \sqrt{3} U + 12 \sqrt{g H \left( H^2 k^2 + 3 \right)} \left. \right) \} 384 g H \left( H^2 k^2 + 3 \right)^{3/2} - \frac{\left( k^5 \left( 3 \sqrt{3} \left( H^2 k^2 + 3 \right) U^4 + 2 g H \left( \left( 7 \sqrt{3} U H^4 + 24 \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} \right) \right) \right) \right.} \\ & k^4 + 24 \left( 2 \sqrt{3} U H^2 + 7 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) \left. \right) k^2 + 81 \left( \sqrt{3} U + 4 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \\ & U + 8 \sqrt{3} g^2 H^2 \left( 5 H^4 k^4 + 33 H^2 k^2 + 54 \right) \right) \text{t} \{ 384 \left( g H \left( H^2 k^2 + 3 \right)^{5/2} \right) - \frac{i}{k^6 \left( \left( 168 \sqrt{3} g^2 U H^6 + 8 g U^2 \left( 3 \sqrt{3} U + 10 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \right.} \right. \\ & H^5 + 3 \sqrt{3} U^5 H^4 + 16 \sqrt{g^5 H^{13} \left( H^2 k^2 + 3 \right)} \left. \right) k^4 + 3 \left( 6 \sqrt{3} H^2 U^5 + 51 \sqrt{3} g H^3 U^3 + 224 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} U^2 + 380 \sqrt{3} g^2 H^4 U + 96 \sqrt{g^5 H^9 \left( H^2 k^2 + 3 \right)} \right) \\ & k^2 + 9 \left( 3 \sqrt{3} U^5 + g H \left( 27 \sqrt{3} U + 160 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \\ & U^2 + 224 \sqrt{3} g^2 H^2 U + 96 \sqrt{g^5 H^5 \left( H^2 k^2 + 3 \right)} \left. \right) \text{t}^2 \{ 384 g H \left( H^2 k^2 + 3 \right)^{5/2} \} + \frac{k^7 \left( \left( H^2 k^2 + 3 \right) U + \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \left( \left( 272 \sqrt{3} g^2 U H^6 + 2 g U^2 \left( 17 \sqrt{3} U + 56 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \right.} \\ & H^5 + 3 \sqrt{3} U^5 H^4 + 48 \sqrt{g^5 H^{13} \left( H^2 k^2 + 3 \right)} \left. \right) k^4 + 3 \left( 6 \sqrt{3} H^2 U^5 - 3 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} U^4 + 72 \sqrt{3} g H^3 U^3 + 334 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} U^2 + 608 \sqrt{3} g^2 H^4 U + 184 \sqrt{g^5 H^9 \left( H^2 k^2 + 3 \right)} \right) \\ & k^2 + 9 \left( 3 \sqrt{3} U^5 - 3 \sqrt{g H \left( H^2 k^2 + 3 \right)} U^4 + 2 g H \left( 19 \sqrt{3} U + 121 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \\ & U^2 + 352 \sqrt{3} g^2 H^2 U + 160 \sqrt{g^5 H^5 \left( H^2 k^2 + 3 \right)} \left. \right) \text{t}^3 \{ 384 g H \left( H^2 k^2 + 3 \right)^{7/2} \} + \frac{i}{k^8 \left( \left( H^2 k^2 + 3 \right) U + \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \left( \left( 400 \sqrt{3} g^2 U H^6 + 4 g U^2 \left( 11 \sqrt{3} U + 36 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \right.} \\ & H^5 + 3 \sqrt{3} U^5 H^4 + 96 \sqrt{g^5 H^{13} \left( H^2 k^2 + 3 \right)} \left. \right) k^4 + 3 \left( 6 \sqrt{3} H^2 U^5 - 6 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} U^4 + 95 \sqrt{3} g H^3 U^3 + 456 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} U^2 + 884 \sqrt{3} g^2 H^4 U + 304 \sqrt{g^5 H^9 \left( H^2 k^2 + 3 \right)} \right) \\ & k^2 + 27 \left( \sqrt{3} U^5 - 2 \sqrt{g H \left( H^2 k^2 + 3 \right)} U^4 + g H \left( 17 \sqrt{3} U + 112 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \\ & U^2 + 168 \sqrt{3} g^2 H^2 U + 80 \sqrt{g^5 H^5 \left( H^2 k^2 + 3 \right)} \left. \right) \text{t}^4 \{ 384 g H \left( H^2 k^2 + 3 \right)^{9/2} \} + O \left( \text{t}^5 \right) \right) \text{d}^3 + \left( \frac{k^5 \left( \left( H^2 k^2 + 3 \right) U^4 + 8 g H \left( H^2 k^2 + 3 \right) \left( \left( 5 \sqrt{3} U H^2 + 32 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) \right) \right.} \right. \\ & k^2 + 96 \sqrt{g H \left( H^2 k^2 + 3 \right)} \left. \right) U + 16 \sqrt{3} g^2 H^2 \left( 5 H^4 k^4 + 48 H^2 k^2 + 144 \right) \right) \} 30720 \left( g H \right)^{3/2} \left( H^2 k^2 + 3 \right)^{5/2} + \frac{i}{k^6 \left( \left( 4368 \sqrt{3} g^2 U H^8 + 2048 g \sqrt{g H \left( H^2 k^2 + 3 \right)} U^2 H^7 - 225 \sqrt{3} U^5 H^6 + 1920 \sqrt{g^5 H^{17} \left( H^2 k^2 + 3 \right)} \right) \right) \\ & k^6 + 3 \left( 14512 \sqrt{3} g^2 U H^6 + 24 g U^2 \left( 256 \sqrt{g H \left( H^2 k^2 + 3 \right)} - 5 \sqrt{3} U \right) H^5 - 675 \sqrt{3} U^5 H^4 + 6720 \sqrt{g^5 H^{13} \left( H^2 k^2 + 3 \right)} \right) \\ & k^4 + 9 \left( -675 \sqrt{3} H^2 U^5 - 240 \sqrt{3} g H^3 U^3 + 6144 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} U^2 + 16160 \sqrt{3} g^2 H^4 U + 8448 \sqrt{g^5 H^9 \left( H^2 k^2 + 3 \right)} \right) \\ & k^2 + 27 \left( -225 \sqrt{3} U^5 + 8 g H \left( 256 \sqrt{g H \left( H^2 k^2 + 3 \right)} - 15 \sqrt{3} U \right) U^2 + 6016 \sqrt{3} g^2 H^2 U + 3968 \sqrt{g^5 H^5 \left( H^2 k^2 + 3 \right)} \right) \end{aligned}$$

$$\begin{aligned}
& k^2+3\right)\right)\right) \text{ \texttt{dt}}\}\{92160 (g H)^{3/2} \left(H^2 k^2+3\right)^{7/2}\}-\frac{\left(k^7\right. \\
& \left.\left(48 \sqrt{3} g^3 \left(140 H^6 k^6+1425 H^4 k^4+5004 H^2 k^2+6192\right) H^3+8 \sqrt{3} g^2\right. \right. \\
& \left.\left(1772 H^6 k^6+17121 H^4 k^4+55557 H^2 k^2+60426\right) U^2 H^2+g U^3 \left(8 \left(416\right. \right. \right. \\
& \left.\left.\sqrt{3} g H^{13} \left(H^2 k^2+3\right)\right)\right)-15 \sqrt{3} H^6 U\right) k^6+9 \left(3328 \sqrt{3} g H^9 \left(H^2\right. \right. \\
& \left.\left.k^2+3\right)\right)\right)-145 \sqrt{3} H^4 U\right) k^4-4590 \sqrt{3} H^2 U k^2-5265 \sqrt{3} U+89856 \sqrt{3} g \\
& H \left(H^2 k^2+3\right)\right)\right) H+3 U \left(5 \left(512 \sqrt{3} g^5 H^{17} \left(H^2 k^2+3\right)\right)\right)-15 \\
& \sqrt{3} H^6 U^5\right) k^6+3 \left(11776 \sqrt{3} g^5 H^{13} \left(H^2 k^2+3\right)\right)\right)-225 \sqrt{3} \\
& H^4 U^5\right) k^4+3 \left(-675 \sqrt{3} H^2 U^5+9984 \sqrt{3} g^3 H^7 \left(H^2 k^2+3\right)\right)\right) \\
& U^2+52352 \sqrt{3} g^5 H^9 \left(H^2 k^2+3\right)\right)\right) k^2+81 \left(2816 \sqrt{3} g^5 H^5 \left(H^2\right. \right. \\
& \left.\left.k^2+3\right)\right)\right)-25 \sqrt{3} U^5\right)\right)\right) \text{ \texttt{dt}}^2\}\{92160 \left((g H)^{3/2} \left(H^2\right. \right. \\
& \left.\left.k^2+3\right)\right)^{7/2}\right)\right)-\frac{i k^8 \left(\left(11520 \sqrt{3} g^3 U H^{11}+9888 \sqrt{3} g^2 U^3\right. \right. \\
& \left.\left.H^{10}+16 g U^4 \left(96 \sqrt{3} g H \left(H^2 k^2+3\right)\right)\right)-5 \sqrt{3} U\right) H^9-75 \sqrt{3} \\
& U^7 H^8+5760 \sqrt{3} g^5 H^{21} \left(H^2 k^2+3\right)\right) U^2+480 \sqrt{3} g^7 H^{23} \left(H^2\right. \\
& \left.\left.k^2+3\right)\right)\right) k^8+3 \left(55216 \sqrt{3} g^3 U H^9+41728 \sqrt{3} g^2 U^3 H^8+3 g U^4\right. \\
& \left.\left(2048 \sqrt{3} g H \left(H^2 k^2+3\right)\right)\right)-105 \sqrt{3} U\right) H^7-300 \sqrt{3} U^7 H^6+39488 \\
& \sqrt{3} g^5 H^{17} \left(H^2 k^2+3\right)\right) U^2+6720 \sqrt{3} g^7 H^{19} \left(H^2 k^2+3\right)\right)\right) k^6+9 \\
& \left(99856 \sqrt{3} g^3 U H^7+66296 \sqrt{3} g^2 U^3 H^6+3 g U^4 \left(3072 \sqrt{3} g H \left(H^2\right. \right. \right. \\
& \left.\left.\left.k^2+3\right)\right)\right)-155 \sqrt{3} U\right) H^5-450 \sqrt{3} U^7 H^4+88512 \sqrt{3} g^5 H^{13} \left(H^2\right. \\
& \left.\left.k^2+3\right)\right)\right) U^2+19040 \sqrt{3} g^7 H^{15} \left(H^2 k^2+3\right)\right)\right) k^4+27 \left(-300 \sqrt{3} \\
& H^2 U^7-305 \sqrt{3} g H^3 U^5+6144 \sqrt{3} g^3 H^7 \left(H^2 k^2+3\right)\right) U^4+46960 \sqrt{3} g^2 \\
& H^4 U^3+82240 \sqrt{3} g^5 H^9 \left(H^2 k^2+3\right)\right) U^2+80544 \sqrt{3} g^3 H^5 U+20416 \sqrt{3} g^7 \\
& H^{11} \left(H^2 k^2+3\right)\right)\right) k^2+243 \left(-25 \sqrt{3} U^7+g H \left(512 \sqrt{3} g H \left(H^2\right. \right. \right. \\
& \left.\left.\left.k^2+3\right)\right)\right)-25 \sqrt{3} U\right) U^4+4168 \sqrt{3} g^2 H^2 U^3+9152 \sqrt{3} g^5 H^5 \left(H^2\right. \\
& \left.\left.k^2+3\right)\right)\right) U^2+8128 \sqrt{3} g^3 H^3 U+2592 \sqrt{3} g^7 H^7 \left(H^2 k^2+3\right)\right)\right)\right) \\
& \text{ \texttt{dt}}^3\}\{30720 (g H)^{3/2} \left(H^2 k^2+3\right)^{9/2}\}+\frac{k^9 \left(2 g H \left(4 \left(736 \sqrt{3} g\right. \right. \right. \\
& \left.\left.\left.H^{17} \left(H^2 k^2+3\right)\right)\right)-45 \sqrt{3} H^8 U\right) k^8+3 \left(11776 \sqrt{3} g H^{13} \left(H^2\right. \right. \right. \\
& \left.\left.\left.k^2+3\right)\right)\right)-645 \sqrt{3} H^6 U\right) k^6+27 \left(5888 \sqrt{3} g H^9 \left(H^2 k^2+3\right)\right)\right)-285 \\
& \sqrt{3} H^4 U\right) k^4-13365 \sqrt{3} H^2 U k^2-8505 \sqrt{3} U+238464 \sqrt{3} g H \left(H^2\right. \\
& \left.\left.k^2+3\right)\right)\right) U^5+3 \sqrt{3} g^2 H^2 \left(H^2 k^2+3\right)^2 \left(16944 H^4 k^4+114352\right. \\
& \left.\left.H^2 k^2+197625\right) U^4+24 \sqrt{3} g^3 H^3 \left(4080 H^8 k^8+64744 H^6 k^6+376029 H^4\right. \right. \\
& \left.\left.k^4+953451 H^2 k^2+893700\right) U^2+3 \left(5 \left(-15 \sqrt{3} U^7 H^8+2048 \sqrt{3} g^5 H^{21}\right. \right. \right. \\
& \left.\left.\left(H^2 k^2+3\right)\right)\right) U^2+384 \sqrt{3} g^7 H^{23} \left(H^2 k^2+3\right)\right)\right) k^8+12 \left(-75\right. \\
& \left.\sqrt{3} H^6 U^7+20864 \sqrt{3} g^5 H^{17} \left(H^2 k^2+3\right)\right) U^2+9600 \sqrt{3} g^7 H^{19} \left(H^2\right. \\
& \left.\left.k^2+3\right)\right)\right) k^6+18 \left(-225 \sqrt{3} H^4 U^7+100096 \sqrt{3} g^5 H^{13} \left(H^2 k^2+3\right)\right)\right) \\
& U^2+60608 \sqrt{3} g^7 H^{15} \left(H^2 k^2+3\right)\right)\right) k^4+36 \left(-225 \sqrt{3} H^2 U^7+5888\right. \\
& \left.\sqrt{3} g^3 H^7 \left(H^2 k^2+3\right)\right) U^4+144512 \sqrt{3} g^5 H^9 \left(H^2 k^2+3\right)\right) U^2+105472 \\
& \sqrt{3} g^7 H^{11} \left(H^2 k^2+3\right)\right)\right) k^2+135 \left(-45 \sqrt{3} U^7+39424 \sqrt{3} g^5\right. \\
& \left.\left.H^5 \left(H^2 k^2+3\right)\right) U^2+33920 \sqrt{3} g^7 H^7 \left(H^2 k^2+3\right)\right)\right)\right) U+144 \\
& \sqrt{3} g^4 H^4 \left(40 H^8 k^8+1000 H^6 k^6+7395 H^4 k^4+22104 H^2 k^2+23832\right)\right)\right) \\
& \text{ \texttt{dt}}^4\}\{92160 (g H)^{3/2} \left(H^2 k^2+3\right)^{9/2}\}+O\left(\text{ \texttt{dt}}^5\right)\right) \\
& \text{ \texttt{dx}}^4+O\left(\text{ \texttt{dx}}^5\right), \left(\frac{i \left(H^2 U k^3+3 U k-\sqrt{3}\right) \sqrt{3} g H \left(H^2\right. \right. \\
& \left.\left.k^2+3\right)\right) k\right)^2 \text{ \texttt{dt}}\}\{2 \left(H^2 k^2+3\right)^2-\frac{\left(k^3 \left(H^2 k^2+3\right)\right. \right. \\
& \left.\left.U-\sqrt{3}\right) \sqrt{3} g H \left(H^2 k^2+3\right)\right) \left(3 g H+U \left(H^2 k^2+3\right) U-2 \sqrt{3}\right)\}
\end{aligned}$$

$$\begin{aligned}
& \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \right) \text{dt}^2 \{ 3 \left( H^2 k^2 + 3 \right)^2 - \frac{i}{k^4} \left( \left( H^2 k^2 + 3 \right) U - \sqrt{3} \right) \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \left( \left( H^4 U \right. \right. \\
& k^4 - 3 \left. \left( \sqrt{3} \right) \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} - 2 H^2 U \right) k^2 + 9 U - 9 \sqrt{3} \right) \sqrt{g H} \\
& \left. \left( H^2 k^2 + 3 \right) \right) \right) U^2 + 3 g H \left( 3 \left( H^2 k^2 + 3 \right) U - \sqrt{3} \right) \sqrt{g H \left( H^2 \right.} \\
& k^2 + 3 \left. \right) \right) \right) \text{dt}^3 \{ 4 \left( H^2 k^2 + 3 \right)^3 + \frac{k^5}{\left( H^2 k^2 + 3 \right)} U - \sqrt{3} \right) \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \left( \left( H^4 U \right. \right. \\
& k^4 + \left. \left( 6 H^2 U - 4 \sqrt{3} \right) \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) k^2 + 9 U - 12 \sqrt{3} \right) \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \\
& U^3 + 6 g H \left( 3 \left( H^2 k^2 + 3 \right) U - 2 \sqrt{3} \right) \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U + 9 g^2 \\
& H^2 \right) \text{dt}^4 \{ 5 \left( H^2 k^2 + 3 \right)^3 + O \left( \text{dt}^5 \right) \right) + \left( - \frac{1}{4} \right) \{ 4 \\
& i k^2 \left( 2 \sqrt{g H} - \frac{\sqrt{3}}{\sqrt{3}} U \right) \sqrt{H^2 k^2 + 3} \right) + \frac{k^3}{\left( 2 \sqrt{g H} \right.} \\
& \left. \left( H^2 k^2 + 3 \right) - \sqrt{3} U \right) \left( \left( H^2 k^2 + 3 \right) U - \sqrt{3} \right) \sqrt{g H \left( H^2 \right.} \\
& k^2 + 3 \left. \right) \right) \text{dt} \{ 4 \left( H^2 k^2 + 3 \right)^{3/2} + \frac{i k^4}{\left( 2 \sqrt{g H \left( H^2 \right.} \right.} \\
& k^2 + 3 \left. \right) - \sqrt{3} U \right) \left( 3 g H + U \left( \left( H^2 k^2 + 3 \right) U - 2 \sqrt{3} \right) \sqrt{g H} \right. \\
& \left. \left( H^2 k^2 + 3 \right) \right) \right) \text{dt}^2 \{ 4 \left( H^2 k^2 + 3 \right)^{3/2} - \frac{\left( k^5 \right.} \\
& \left. \left( 2 \sqrt{g H \left( H^2 k^2 + 3 \right) - \sqrt{3} U \right) \left( \left( H^4 U \right. \right. \right.} \\
& k^4 - 3 \left. \left( \sqrt{3} \right) \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} - 2 H^2 U \right) k^2 + 9 U - 9 \sqrt{3} \right) \sqrt{g H \left( H^2 \right.} \\
& k^2 + 3 \left. \right) \right) U^2 + 3 g H \left( 3 \left( H^2 k^2 + 3 \right) U - \sqrt{3} \right) \sqrt{g H \left( H^2 \right.} \\
& k^2 + 3 \left. \right) \right) \right) \text{dt}^3 \{ 4 \left( H^2 k^2 + 3 \right)^{5/2} - \frac{i k^6}{\left( 2 \sqrt{g} \right.} \\
& H \left( H^2 k^2 + 3 \right) - \sqrt{3} U \right) \left( \left( H^4 U \right. \right. \left. \left( 6 H^2 U - 4 \sqrt{3} \right) \sqrt{g H^5} \right. \\
& \left. \left( H^2 k^2 + 3 \right) \right) \right) k^2 + 9 U - 12 \sqrt{3} \right) \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U^3 + 6 g H \\
& \left( 3 \left( H^2 k^2 + 3 \right) U - 2 \sqrt{3} \right) \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U + 9 g^2 H^2 \right) \\
& \text{dt}^4 \{ 4 \left( H^2 k^2 + 3 \right)^{5/2} + O \left( \text{dt}^5 \right) \right) \text{dx} + \left( \frac{k^3}{\left( 12 \sqrt{3} \right.} \right. \\
& g H \left( H^2 k^2 + 4 \right) - U \left( \left( 3 \sqrt{3} \right) U H^2 + 16 \sqrt{g H^5 \left( H^2 \right.} \right. \\
& k^2 + 3 \left. \right) \right) \right) k^2 + 9 \sqrt{3} U + 48 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \{ 96 \sqrt{g H} \\
& \left( H^2 k^2 + 3 \right)^{3/2} - \frac{i k^4}{\left( U^2 \left( 3 \sqrt{3} \right) U H^4 + 16 \sqrt{g H^9 \left( H^2 \right.} \right.} \\
& k^2 + 3 \left. \right) \right) \right) k^4 + 6 \left( 3 \sqrt{3} \right) H^2 U^3 + 16 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) U^2 + 18 \\
& \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} \right) k^2 + 9 U^2 \left( 3 \sqrt{3} \right) U + 16 \sqrt{g H \left( H^2 \right.} \\
& k^2 + 3 \left. \right) \right) + 4 g H \left( \left( 3 \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} - 10 \sqrt{3} \right) H^4 U \right) \\
& k^4 - 63 \sqrt{3} H^2 U k^2 - 99 \sqrt{3} U + 63 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \text{dt} \{ 96 \\
& \sqrt{g H} \left( H^2 k^2 + 3 \right)^{5/2} + \frac{k^5}{\left( U^3 \left( 3 \sqrt{3} \right) U H^4 + 16 \sqrt{g H^9} \right.} \\
& \left. \left( H^2 k^2 + 3 \right) \right) \right) k^4 + 6 \left( 3 \sqrt{3} \right) H^2 U^4 + 16 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) \\
& U^3 + 56 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} \right) U \right) k^2 - 12 \sqrt{3} g^2 H^2 \left( 2 H^4 k^4 + 15 \right. \\
& H^2 k^2 + 30 \right) + 9 U^3 \left( 3 \sqrt{3} \right) U + 16 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) + g H U \\
& \left( 4 \left( 6 \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} - 17 \sqrt{3} \right) H^4 U \right) k^4 - 429 \sqrt{3} H^2 \\
& U k^2 - 675 \sqrt{3} U + 864 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \text{dt}^2 \{ 96 \sqrt{g H} \\
& \left( H^2 k^2 + 3 \right)^{5/2} + \frac{i k^6}{\left( - U \left( 72 \sqrt{3} \right) g^2 H^8 + 12 g U \left( 8 \sqrt{3} \right) U - 3 \right.} \\
& \left. \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) H^7 - U^3 \left( 3 \sqrt{3} \right) U H^6 + 16 \sqrt{g H^{13} \left( H^2 \right.} \\
& k^2 + 3 \left. \right) \right) \right) k^6 + 3 \left( - 304 \sqrt{3} \right) g^2 U H^6 + g U^2 \left( 264 \sqrt{g H \left( H^2 \right.} \right. \\
& k^2 + 3 \left. \right) - 301 \sqrt{3} U \right) H^5 + 9 \sqrt{3} U^5 H^4 + 48 \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} \\
& U^4 + 36 \sqrt{g^5 H^{13} \left( H^2 k^2 + 3 \right)} \right) k^4 + 9 \left( 9 \sqrt{3} \right) H^2 U^5 + 48 \sqrt{g} \\
& H^5 \left( H^2 k^2 + 3 \right) U^4 - 314 \sqrt{3} g H^3 U^3 + 438 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} \\
& U^2 - 404 \sqrt{3} g^2 H^4 U + 84 \sqrt{g^5 H^9 \left( H^2 k^2 + 3 \right)} \right) k^2 + 27 \left( 3 \sqrt{3} \right) \\
& U^5 + 16 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U^4 + g H \left( 210 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) - 109
\end{aligned}$$

$$\begin{aligned}
& \sqrt{3} \, U \right) U^{2-1/2} \sqrt{3} \, g^2 H^2 U + 52 \sqrt{3} g^5 H^5 \left( H^2 k^2 + 3 \right) \right) \right) \right) \\
& \text{dt}^3 \{ 96 \sqrt{3} g H \left( H^2 k^2 + 3 \right)^{7/2} \} + \frac{k^7 \left( 36 \sqrt{3} g^3 \left( 4 H^4 k^4 + 27 \right. \right. \\
& H^2 k^2 + 48 \right) H^3 + 3 \sqrt{3} g^2 \left( 48 H^6 k^6 + 712 H^4 k^4 + 3075 H^2 k^2 + 4113 \right) \\
& U^2 H^2 + 2 g U^3 \left( 2 \left( 31 \sqrt{3} H^6 U - 12 \sqrt{3} g H^3 \left( H^2 k^2 + 3 \right) \right) \right) \\
& k^6 + 3 \left( 197 \sqrt{3} H^4 U - 216 \sqrt{3} g H^9 \left( H^2 k^2 + 3 \right) \right) \right) k^4 + 1872 \sqrt{3} \\
& H^2 U k^2 + 1971 \sqrt{3} U - 5076 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) \right) H - U \left( U^4 \left( 3 \right. \right. \\
& \sqrt{3} U H^6 + 16 \sqrt{3} g H^3 \left( H^2 k^2 + 3 \right) \right) \right) k^6 + 9 \left( 3 \sqrt{3} H^4 U^5 + 16 \right. \\
& \sqrt{3} g H^9 \left( H^2 k^2 + 3 \right) \right) U^4 + 48 \sqrt{3} g^5 H^3 \left( H^2 k^2 + 3 \right) \right) k^4 + 9 \\
& \left( 9 \sqrt{3} H^2 U^5 + 48 \sqrt{3} g H^5 \left( H^2 k^2 + 3 \right) \right) U^4 + 760 \sqrt{3} g^3 H^7 \left( H^2 \right. \\
& k^2 + 3 \right) \right) U^2 + 400 \sqrt{3} g^5 H^9 \left( H^2 k^2 + 3 \right) \right) k^2 + 81 \sqrt{3} U^5 + 432 \sqrt{3} g \\
& H \left( H^2 k^2 + 3 \right) \right) U^4 + 7344 \sqrt{3} g^5 H^5 \left( H^2 k^2 + 3 \right) \right) \right) \text{dt}^4 \{ 96 \\
& \sqrt{3} g H \left( H^2 k^2 + 3 \right)^{7/2} \} + O \left( \text{dt}^5 \right) \right) \text{dx}^2 + \left( -\frac{i k^4}{\left( 3 \sqrt{3} \right. \right. \\
& \left. \left. \left( H^2 k^2 + 3 \right) U^3 + 4 g H \left( \left( \sqrt{3} H^2 U - 4 \sqrt{3} g H^5 \left( H^2 \right. \right. \right. \right. \right. \right. \right. \\
& k^2 + 3 \right) \right) \right) k^2 + 6 \sqrt{3} U - 12 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) \right) \} \{ 384 g H \left( H^2 \right. \\
& k^2 + 3 \right)^{3/2} \} + \frac{k^5 \left( 3 \sqrt{3} \left( H^2 k^2 + 3 \right)^2 U^4 - 2 g H \left( 24 \sqrt{3} g \right. \right. \\
& H^9 \left( H^2 k^2 + 3 \right) \right) - 7 \sqrt{3} H^4 U \right) k^4 + 24 \left( 7 \sqrt{3} g H^5 \left( H^2 k^2 + 3 \right) \right) - 2 \\
& \sqrt{3} H^2 U \right) k^2 - 81 \sqrt{3} U + 324 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) U + 8 \sqrt{3} g^2 \\
& H^2 \left( 5 H^4 k^4 + 33 H^2 k^2 + 54 \right) \right) \text{dt} \} \{ 384 g H \left( H^2 k^2 + 3 \right)^{5/2} \} - \frac{i}{\left( k^6 \left( \left( -168 \sqrt{3} \right. \right. \right. \\
& g^2 U H^6 + 8 g U^2 \left( 10 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) - 3 \sqrt{3} U \right) H^5 - 3 \sqrt{3} U^5 H^4 + 16 \sqrt{3} g^5 H^3 \left( H^2 k^2 + 3 \right) \right) \right) \\
& k^4 + 3 \left( -6 \sqrt{3} H^2 U^5 - 51 \sqrt{3} g H^3 U^3 + 224 \sqrt{3} g^3 H^7 \left( H^2 k^2 + 3 \right) \right) U^2 - 380 \sqrt{3} \\
& g^2 H^4 U + 96 \sqrt{3} g^5 H^9 \left( H^2 k^2 + 3 \right) \right) k^2 + 9 \left( -3 \sqrt{3} U^5 + g H \left( 160 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) \right. \\
& - 27 \sqrt{3} U \right) U^2 - 224 \sqrt{3} g^2 H^2 U + 96 \sqrt{3} g^5 H^5 \left( H^2 k^2 + 3 \right) \right) \right) \text{dt}^2 \} \{ 384 g H \left( H^2 k^2 + 3 \right)^{5/2} \} - \frac{i}{\left( \left( k^7 \left( \left( H^2 k^2 + 3 \right) \right. \right. \right. \\
& U - \sqrt{3} \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) \right) \left( -\left( -272 \sqrt{3} \right. \right. \\
& g^2 U H^6 + 2 g U^2 \left( 56 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) - 17 \sqrt{3} U \right) H^5 - 3 \sqrt{3} U^5 \\
& H^4 + 48 \sqrt{3} g^5 H^3 \left( H^2 k^2 + 3 \right) \right) \right) k^4 - 3 \left( -6 \sqrt{3} H^2 U^5 - 3 \sqrt{3} g H^5 \left( H^2 k^2 + 3 \right) \right) \\
& U^4 - 72 \sqrt{3} g H^3 U^3 + 334 \sqrt{3} g^3 H^7 \left( H^2 k^2 + 3 \right) \right) U^2 - 608 \sqrt{3} \\
& g^2 H^4 U + 184 \sqrt{3} g^5 H^9 \left( H^2 k^2 + 3 \right) \right) k^2 - 9 \left( -3 \sqrt{3} U^5 - 3 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) \\
& U^4 + 2 g H \left( 121 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) - 19 \sqrt{3} U \right) U^2 - 352 \sqrt{3} g^2 H^2 U + 160 \sqrt{3} g^5 H^5 \left( H^2 k^2 + 3 \right) \right) \right) \right) \\
& \text{dt}^3 \} \{ 384 \left( g H \left( H^2 k^2 + 3 \right)^{7/2} \right) - \frac{i k^8 \left( \sqrt{3} \sqrt{3} g H \left( H^2 \right. \right. \\
& k^2 + 3 \right) \right) - \left( H^2 k^2 + 3 \right) U \right)^2 \left( -\left( -400 \sqrt{3} g^2 U H^6 + 4 g U^2 \left( 36 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) \right. \right. \\
& - 11 \sqrt{3} U \right) H^5 - 3 \sqrt{3} U^5 H^4 + 96 \sqrt{3} g^5 H^3 \left( H^2 k^2 + 3 \right) \right) \\
& \left. \left. \left( H^2 k^2 + 3 \right) \right) \right) k^4 - 3 \left( -6 \sqrt{3} H^2 U^5 - 6 \sqrt{3} g H^5 \left( H^2 k^2 + 3 \right) \right) \right. \\
& U^4 - 95 \sqrt{3} g H^3 U^3 + 456 \sqrt{3} g^3 H^7 \left( H^2 k^2 + 3 \right) \right) U^2 - 884 \sqrt{3} g^2 H^4 \\
& U + 304 \sqrt{3} g^5 H^9 \left( H^2 k^2 + 3 \right) \right) k^2 - 27 \left( -\sqrt{3} U^5 - 2 \sqrt{3} g H \left( H^2 \right. \right. \\
& k^2 + 3 \right) \right) U^4 + g H \left( 112 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) - 17 \sqrt{3} U \right) U^2 - 168 \\
& \sqrt{3} g^2 H^2 U + 80 \sqrt{3} g^5 H^5 \left( H^2 k^2 + 3 \right) \right) \right) \text{dt}^4 \} \{ 384 g H \left( H^2 \right. \\
& k^2 + 3 \right)^{9/2} \} + O \left( \text{dt}^5 \right) \right) \text{dx}^3 + \left( \frac{k^5 \left( 75 \sqrt{3} \left( H^2 \right. \right. \right. \\
& k^2 + 3 \right)^2 U^4 + 8 g H \left( H^2 k^2 + 3 \right) \left( \left( 32 \sqrt{3} g H^5 \left( H^2 k^2 + 3 \right) \right) - 5 \sqrt{3} H^2 U \right) \\
& k^2 + 96 \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) U - 16 \sqrt{3} g^2 H^2 \left( 5 H^4 k^4 + 48 H^2 k^2 + 144 \right) \right) \right) \} \{ 30720 \left( g H \right)^{3/2} \left( H^2 k^2 + 3 \right)^{5/2} \} + \frac{i}{\left( k^6 \left( \left( -4368 \sqrt{3} \right. \right. \right. \\
& g^2 U H^8 + 2048 g \sqrt{3} g H \left( H^2 k^2 + 3 \right) \right) U^2 H^7 + 225 \sqrt{3} \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \left( \left( -4508 \sqrt{3} \right) g^2 U H^6 + 2048 g^2 \sqrt{3} H^6 \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 H^7 + 225 \sqrt{3} U^5 H^6 + 1920 \sqrt{3} g^5 H^{17} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^6 + 3 \left( -14512 \sqrt{3} g^2 U H^6 + 24 g U^2 \left( 5 \sqrt{3} U + 256 \sqrt{3} g H \left( \left( H^2 k^2 + 3 \right) \right) \right) H^5 + 675 \sqrt{3} U^5 H^4 + 6720 \sqrt{3} g^5 H^{13} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^4 + 9 \left( 675 \sqrt{3} H^2 U^5 + 240 \sqrt{3} g H^3 U^3 + 6144 \sqrt{3} g^3 H^7 \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 - 16160 \sqrt{3} g^2 H^4 U + 8448 \sqrt{3} g^5 H^9 \left( \left( H^2 k^2 + 3 \right) \right) k^2 + 27 \left( 225 \sqrt{3} U^5 + 8 g H \left( 15 \sqrt{3} U + 256 \sqrt{3} g H \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 - 6016 \sqrt{3} g^2 H^2 U + 3968 \sqrt{3} g^5 H^5 \left( \left( H^2 k^2 + 3 \right) \right) \right) \right) \text{dt} \} \{ 92160 (g H)^{3/2} \left( \left( H^2 k^2 + 3 \right) \right)^{7/2} \} - \frac{\left( k^7 \left( -48 \sqrt{3} g^3 \left( 140 H^6 k^6 + 1425 H^4 k^4 + 5004 H^2 k^2 + 6192 \right) H^3 - 8 \sqrt{3} g^2 \left( 1772 H^6 k^6 + 17121 H^4 k^4 + 55557 H^2 k^2 + 60426 \right) U^2 H^2 + g U^3 \left( 8 \left( 15 \sqrt{3} U + H^6 + 416 \sqrt{3} g H^{13} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^6 + 9 \left( 145 \sqrt{3} U H^4 + 3328 \sqrt{3} g H^9 \left( \left( H^2 k^2 + 3 \right) \right) \right) k^4 + 4590 \sqrt{3} H^2 U k^2 + 5265 \sqrt{3} U + 89856 \sqrt{3} g H \left( \left( H^2 k^2 + 3 \right) \right) \right) H + 3 U \left( 5 \left( 15 \sqrt{3} U^5 H^6 + 512 \sqrt{3} g^5 H^{17} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^6 + 3 \left( 225 \sqrt{3} H^4 U^5 + 11776 \sqrt{3} g^5 H^{13} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^4 + 3 \left( 675 \sqrt{3} H^2 U^5 + 9984 \sqrt{3} g^3 H^7 \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 + 52352 \sqrt{3} g^5 H^9 \left( \left( H^2 k^2 + 3 \right) \right) \right) k^2 + 81 \left( 25 \sqrt{3} U^5 + 2816 \sqrt{3} g^5 H^5 \left( \left( H^2 k^2 + 3 \right) \right) \right) \right) \right) \text{dt}^2 \} \{ 92160 \left( (g H)^{3/2} \left( \left( H^2 k^2 + 3 \right) \right)^{7/2} \right) \} - \frac{i k^8 \left( \left( -11520 \sqrt{3} g^3 U H^{11} - 9888 \sqrt{3} g^2 U^3 H^{10} + 16 g U^4 \left( 5 \sqrt{3} U + 96 \sqrt{3} g H \left( \left( H^2 k^2 + 3 \right) \right) \right) H^9 + 75 \sqrt{3} U^7 H^8 + 5760 \sqrt{3} g^5 H^{21} \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 + 480 \sqrt{3} g^7 H^{23} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^8 + 3 \left( -55216 \sqrt{3} g^3 U H^9 - 41728 \sqrt{3} g^2 U^3 H^8 + 3 g U^4 \left( 105 \sqrt{3} U + 2048 \sqrt{3} g H \left( \left( H^2 k^2 + 3 \right) \right) \right) H^7 + 300 \sqrt{3} U^7 H^6 + 39488 \sqrt{3} g^5 H^{17} \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 + 6720 \sqrt{3} g^7 H^{19} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^6 + 9 \left( -99856 \sqrt{3} g^3 U H^7 - 66296 \sqrt{3} g^2 U^3 H^6 + 3 g U^4 \left( 155 \sqrt{3} U + 3072 \sqrt{3} g H \left( \left( H^2 k^2 + 3 \right) \right) \right) H^5 + 450 \sqrt{3} U^7 H^4 + 88512 \sqrt{3} g^5 H^{13} \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 + 19040 \sqrt{3} g^7 H^{15} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^4 + 27 \left( 300 \sqrt{3} H^2 U^7 + 305 \sqrt{3} g H^3 U^5 + 6144 \sqrt{3} g^3 H^7 \left( \left( H^2 k^2 + 3 \right) \right) \right) U^4 - 46960 \sqrt{3} g^2 H^4 U^3 + 82240 \sqrt{3} g^5 H^9 \left( \left( H^2 k^2 + 3 \right) \right) U^2 - 80544 \sqrt{3} g^3 H^5 U + 20416 \sqrt{3} g^7 H^{11} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^2 + 243 \left( 25 \sqrt{3} U^7 + g H \left( 25 \sqrt{3} U + 512 \sqrt{3} g H \left( \left( H^2 k^2 + 3 \right) \right) \right) U^4 - 4168 \sqrt{3} g^2 H^2 U^3 + 9152 \sqrt{3} g^5 H^5 \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 - 8128 \sqrt{3} g^3 H^3 U + 2592 \sqrt{3} g^7 H^7 \left( \left( H^2 k^2 + 3 \right) \right) \right) \text{dt}^3 \} \{ 30720 (g H)^{3/2} \left( \left( H^2 k^2 + 3 \right) \right)^{9/2} \} + \frac{k^9 \left( 2 g H \left( 4 \left( 45 \sqrt{3} U H^8 + 736 \sqrt{3} g H^{17} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^8 + 3 \left( 645 \sqrt{3} U H^6 + 11776 \sqrt{3} g H^{13} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^6 + 27 \left( 285 \sqrt{3} U H^4 + 5888 \sqrt{3} g H^9 \left( \left( H^2 k^2 + 3 \right) \right) \right) k^4 + 13365 \sqrt{3} H^2 U k^2 + 8505 \sqrt{3} U + 238464 \sqrt{3} g H \left( \left( H^2 k^2 + 3 \right) \right) \right) U^5 - 3 \sqrt{3} g^2 H^2 \left( \left( H^2 k^2 + 3 \right) \right)^2 \left( 16944 H^4 k^4 + 114352 H^2 k^2 + 197625 \right) U^4 - 24 \sqrt{3} g^3 H^3 \left( 4080 H^8 k^8 + 64744 H^6 k^6 + 376029 H^4 k^4 + 953451 H^2 k^2 + 893700 \right) U^2 + 3 \left( 5 \left( 15 \sqrt{3} U^7 H^8 + 2048 \sqrt{3} g^5 H^{21} \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 + 384 \sqrt{3} g^7 H^{23} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^8 + 12 \left( 75 \sqrt{3} H^6 U^7 + 20864 \sqrt{3} g^5 H^{17} \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 + 9600 \sqrt{3} g^7 H^{19} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^6 + 18 \left( 225 \sqrt{3} H^4 U^7 + 100096 \sqrt{3} g^5 H^{13} \left( \left( H^2 k^2 + 3 \right) \right) \right) U^2 + 60608 \sqrt{3} g^7 H^{15} \left( \left( H^2 k^2 + 3 \right) \right) \right) k^4 + 36 \left( 225 \sqrt{3} H^2 U^7 + 5888 \sqrt{3} g^3 H^7 \left( \left( H^2 k^2 + 3 \right) \right) \right) U^4 + 144512 \sqrt{3} g^5 H^9 \left( \left( H^2 k^2 + 3 \right) \right) U^2 + 105472 \sqrt{3} g^7 H^{11} \left( \left( H^2 k^2 + 3 \right) \right) \right) \right)
\end{aligned}$$

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k^2+135 \left(45 \sqrt{3} U^7+39424 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)} U^2+33920 \sqrt{g^7
H^7 \left(H^2 k^2+3\right)}\right)\right) U-144 \sqrt{3} g^4 H^4 \left(40 H^8 k^8+1000 H^6
k^6+7395 H^4 k^4+22104 H^2 k^2+23832\right)\right) \text{dt}^4\{92160 (g H)^{3/2} \left(H^2
k^2+3\right)^{9/2}\}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right)

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Out[133]=

$$\text{Out[134]= } EA \parallel \left\{ \left\{ 1 + \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) H^2 k^3 U \text{Csc}\left[\frac{dx k}{2}\right]}{(6 + 2 H^2 k^2) w}, \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) H k \text{Csc}\left[\frac{dx k}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right) w} \right\}, \right. \\ \left. \left\{ \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) k (g H (3 + H^2 k^2) - 3 U^2) \text{Csc}\left[\frac{dx k}{2}\right]}{(6 + 2 H^2 k^2) w}, 1 + \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) k (6 + H^2 k^2) U \text{Csc}\left[\frac{dx k}{2}\right]}{(6 + 2 H^2 k^2) w} \right\} \right\}$$

Out[135]=

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EA \parallel \left(
\begin{array}{cc}
\frac{i e^{\frac{i}{\text{dx}} k}{2}}{\text{dx}} \left(1 - e^{-i \text{dx} k}\right) \left(1 - e^{i \text{dt} w}\right) H^2 U \csc\left(\frac{\text{dx} k}{2}\right) k^3 \left(\left(2 H^2 k^2 + 6\right) w + 1\right) & \\
& \frac{i e^{\frac{i}{\text{dx}} k}{2}}{\text{dx}} \left(1 - e^{-i \text{dx} k}\right) \left(1 - e^{i \text{dt} w}\right) H k \csc\left(\frac{\text{dx} k}{2}\right) \left\{\left(k^2 H^3\right) + H\right\} w \\
\frac{i e^{\frac{i}{\text{dx}} k}{2}}{\text{dx}} \left(1 - e^{-i \text{dx} k}\right) \left(1 - e^{i \text{dt} w}\right) k \left(g H \left(3 + H^2 k^2\right) - 3 U^2\right) \csc\left(\frac{\text{dx} k}{2}\right) & \\
& \frac{i e^{\frac{i}{\text{dx}} k}{2}}{\text{dx}} \left(1 - e^{-i \text{dx} k}\right) \left(1 - e^{i \text{dt} w}\right) k \left(6 + H^2 k^2\right) U \csc\left(\frac{\text{dx} k}{2}\right)
\end{array}
\right)
\end{pre}

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$$\begin{aligned}
\text{Out[136]= Err} \parallel & \left\{ \left( \frac{i \left( \sqrt{3} k \sqrt{g H (3+H^2 k^2)} + 3 k U \right) dt}{3+H^2 k^2} - \frac{1}{2} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^2 dt^2 - \right. \right. \\
& \left. \frac{1}{6} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^3 dt^3 - \frac{1}{24} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^4 dt^4 + O[dt]^5 \right) + \\
& \left( -\frac{1}{2} \left( \sqrt{g H} k^2 \right) dt + O[dt]^5 \right) dx + \left( \frac{i (9 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( \frac{1}{24} \sqrt{g H} k^4 dt + O[dt]^5 \right) dx^3 + \\
& \left( -\frac{i (54 H^2 k^7 + 19 H^4 k^9 + 2 H^6 k^{11}) U dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5, \\
& \left( -\frac{3 i k dt}{3+H^2 k^2} + O[dt]^5 \right) + \left( \frac{i (6 k^3 + H^2 k^5) dt}{4 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( \frac{i (-54 k^5 + H^4 k^9) dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5 \}, \\
& \left\{ \left( (-i g H k + \frac{3 i k U^2}{3+H^2 k^2}) dt + O[dt]^5 \right) + \left( -\frac{1}{2} \left( \sqrt{g H} k^2 U \right) dt + O[dt]^5 \right) dx + \right. \\
& \left( \frac{i (18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2) dt}{12 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( \frac{1}{24} \sqrt{g H} k^4 U dt + O[dt]^5 \right) dx^3 + \\
& \left( -\frac{i (54 g H k^5 + 54 g H^3 k^7 + 18 g H^5 k^9 + 2 g H^7 k^{11} - 54 k^5 U^2 + H^4 k^9 U^2) dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5, \\
& \left( \frac{i \left( \sqrt{3} k \sqrt{g H (3+H^2 k^2)} - 3 k U \right) dt}{3+H^2 k^2} - \frac{1}{2} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^2 dt^2 - \right. \\
& \left. \frac{1}{6} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^3 dt^3 - \frac{1}{24} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^4 dt^4 + O[dt]^5 \right) + \\
& \left( -\frac{1}{2} \left( \sqrt{g H} k^2 \right) dt + O[dt]^5 \right) dx + \left( \frac{i (36 k^3 U + 15 H^2 k^5 U + 2 H^4 k^7 U) dt}{12 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \\
& \left( \frac{1}{24} \sqrt{g H} k^4 dt + O[dt]^5 \right) dx^3 + \left( -\frac{i (108 k^5 U + 54 H^2 k^7 U + 17 H^4 k^9 U + 2 H^6 k^{11} U) dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5 \} \}
\end{aligned}$$

```

Out[137]= Eerr || \left(
\begin{array}{cc}
\left(\frac{i}{\left(3 U k+\sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right)}} k\right) \text {dt}\right)\left\{H^2\right. \\
k^2+3\left.\right\}-\frac{1}{2} \left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)}} k\right)\left\{H^2\right. \\
k^2+3\left.\right\} \text {dt}\right)^2-\frac{1}{6} \left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)}} k\right)\left\{H^2\right. \\
k^2+3\left.\right\} \text {dt}\right)^3-\frac{1}{24} \left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2\right.}\right. \\
k^2+3\left.\left.\right\right) k\right)\left\{H^2 k^2+3\right\} \text {dt}\right)^4+O\left(\left(\text {dt}\right)^5\right) \text {right}\right)+\left(-\frac{1}{2}\right) \\
\left(\sqrt{g H} k^2\right) \text {dt}\right)+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)+\left(\frac{i}{\left(2 H^4\right.}\right. \\
k^7+9 H^2 k^5\left.\left.\right\right) U \text {dt}\right)\left\{12 \left(H^2 k^2+3\right)^2+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \\
\text {dx}\right)^2+\left(\frac{1}{24} \sqrt{g H} k^4 \text {dt}\right)+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \\
\text {dx}\right)^3+\left(-\frac{1}{\left(2 H^6 k^{11}+19 H^4 k^9+54 H^2 k^7\right) U \text {dt}\right)\left\{240 \left(H^2\right.}\right. \\
k^2+3\left.\left.\right\}^3+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)^4+O\left(\left(\text {dx}\right)^5\right) \& \left(-\frac{3 i k}{\right.} \\
\left.\text {dt}\right)\left\{H^2 k^2+3\right\}+O\left(\left(\text {dt}\right)^5\right) \text {right}\right)+\left(\frac{i}{\left(H^2 k^5+6 k^3\right)} \text {dt}\right)\left\{4\right. \\
\left.\left(H^2 k^2+3\right)^2+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)^2+\left(\frac{i}{\left(H^4 k^9-54 k^5\right)}\right. \\
\left.\text {dt}\right)\left\{240 \left(H^2 k^2+3\right)^3+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)^4+O\left(\left(\text {dx}\right)^5\right) \backslash \\
\left(\left(\frac{3 i k U^2}{\left(H^2 k^2+3\right)}-i g H k\right) \text {dt}\right)+O\left(\left(\text {dt}\right)^5\right) \text {right}\right)+\left(-\frac{1}{2}\right) \\
\left(\sqrt{g H} k^2 U\right) \text {dt}\right)+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)+\left(\frac{i}{\left(2\right.}\right. \\
g H^5 k^7+12 g H^3 k^5-3 H^2 U^2 k^5-18 U^2 k^3+18 g H k^3\left.\left.\right\right) \text {dt}\right)\left\{12 \left(H^2\right.}\right. \\
k^2+3\left.\left.\right\}^2+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)^2+\left(\frac{1}{24} \sqrt{g H} k^4 U\right. \\
\left.\text {dt}\right)+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)^3+\left(-\frac{i}{\left(2 g H^7 k^{11}+18 g\right.}\right. \\
H^5 k^9+H^4 U^2 k^9+54 g H^3 k^7-54 U^2 k^5+54 g H k^5\left.\left.\right\right) \text {dt}\right)\left\{240 \left(H^2\right.}\right. \\
k^2+3\left.\left.\right\}^3+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)^4+O\left(\left(\text {dx}\right)^5\right) \& \left(\frac{i}{\left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}-3 k U\right) \text {dt}\right)\left\{H^2 k^2+3\right\}-\frac{1}{2}\right) \\
\left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)}} k\right)\left\{H^2 k^2+3\right\} \text {dt}\right)^2 \\
\text {dt}\right)^2-\frac{1}{6} \left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)}} k\right)\left\{H^2\right. \\
k^2+3\left.\right\} \text {dt}\right)^3-\frac{1}{24} \left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)}} k\right)\left\{H^2\right. \\
k^2+3\left.\right\} \text {dt}\right)^4+O\left(\left(\text {dt}\right)^5\right) \text {right}\right)+\left(-\frac{1}{2}\right) \left(\sqrt{g}\right. \\
H) k^2\right) \text {dt}\right)+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)+\left(\frac{i}{\left(2 H^4 U k^7+15\right.}\right. \\
H^2 U k^5+36 U k^3\left.\left.\right\right) \text {dt}\right)\left\{12 \left(H^2 k^2+3\right)^2+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \\
\text {dx}\right)^2+\left(\frac{1}{24} \sqrt{g H} k^4 \text {dt}\right)+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)^3+\left(-\frac{i}{\left(2 H^6 U k^{11}+17 H^4 U k^9+54 H^2 U k^7+108 U k^5\right)}\right. \\
\left.\text {dt}\right)\left\{240 \left(H^2\right.}\right. \\
k^2+3\left.\left.\right\}^3+O\left(\left(\text {dt}\right)^5\right) \text {right}\right) \text {dx}\right)^4+O\left(\left(\text {dx}\right)^5\right) \backslash \\
\end{array}
\right)

```

```

In[138]:= KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap -> (U + Sqrt[g * H]) /. am -> 0;
KurFWSeta =
  KurFWS /. fp -> (H * v + U * Rpp * n) /. fm -> (H * v + U * Rmp * n) /. qp -> Rpp * n /.
  qm -> Rmp * n;
KurFWSeta = KurFWSeta /. v -> (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G -> 0 /. n -> 1];
KfnGp = FullSimplify[KurFWSeta /. n -> 0 /. G -> 1];

```



```

Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TA = Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TA = Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
  fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TA = Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TA = Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}];

Text[Row[{" U > Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TA}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TA]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]

```

```

Text[Row[{"FnG error  ||  ", FnG2TAr}]]
Text[Row[{"FnG error  ||  ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||  ", KfGnp}]]
Text[Row[{"FGn  ||  ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||  ", FGn2TAr}]]
Text[Row[{"FGn error  ||  ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||  ", KfGGp}]]
Text[Row[{"FGG  ||  ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||  ", FGG2TAr}]]
Text[Row[{"FGG error  ||  ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||  ", RKstepTayr}]]
Text[Row[{"Omega error  ||  ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA  ||  ", EA}]]
Text[Row[{"EA  ||  ", TeXForm[EA]}]]
Text[Row[{"Eerr  ||  ", Eerr}]]
Text[Row[{"Eerr  ||  ", TeXForm[Eerr]}]]

```

Out[172]=  $U > \text{Sqrt}(gH)$

Out[173]=

Out[174]=  $F_{nn} \parallel G_{np} H + R_{mp} U$

Out[175]=  $F_{nn} \parallel \text{\texttt{Gnp}} H + \text{\texttt{Rmp}} U$

Out[176]= 
$$F_{nn} \text{ error } \parallel \left( -\frac{(H^2 k^3 U w) dt^2}{2(3+H^2 k^2)} - \frac{i H^2 k^3 U w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) +$$

$$\left( -\frac{1}{2} (k^2 U) dt + O[dt]^4 \right) dx + \left( \frac{i(9 H^2 k^5 + 2 H^4 k^7) U dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left( \frac{1}{24} k^4 U dt + O[dt]^4 \right) dx^3 + O[dx]^4$$

Out[177]= 
$$F_{nn} \text{ error } \parallel$$

$$\begin{aligned} & \left( -\frac{1}{2} \left( \text{\texttt{dt}}^2 \left( H^2 k^3 U w \right) \right) \right) \{ 2 \left( H^2 k^2 + 3 \right) \} - \frac{i}{6} \left( \text{\texttt{dt}}^3 H^2 k^3 U w^2 \right) \{ 6 \\ & \left( H^2 k^2 + 3 \right) \} + O \left( \text{\texttt{dt}}^4 \right) \right) + \text{\texttt{dx}} \left( -\frac{1}{2} \left( k^2 U \right) \text{\texttt{dt}} + O \left( \text{\texttt{dt}}^4 \right) \right) + \text{\texttt{dx}}^2 \left( \frac{i}{12} \left( 2 H^4 k^7 + 9 H^2 k^5 \right) U \text{\texttt{dt}} \right) \{ 12 \left( H^2 k^2 + 3 \right)^2 \} + O \left( \text{\texttt{dt}}^4 \right) + \text{\texttt{dx}}^3 \\ & \left( \frac{1}{24} k^4 U \text{\texttt{dt}} + O \left( \text{\texttt{dt}}^4 \right) \right) + O \left( \text{\texttt{dx}}^4 \right) \end{aligned}$$

Out[178]=

Out[179]=  $F_{nG} \parallel G_{Gp} H$

Out[180]=  $F_{nG} \parallel \text{\texttt{GGp}} H$

$$\text{Out[181]= FnG error} \parallel \left( -\frac{3(k w) dt^2}{2(3+H^2 k^2)} - \frac{i k w^2 dt^3}{2(3+H^2 k^2)} + O[dt]^4 \right) + \left( \frac{i(6 k^3 + H^2 k^5) dt}{4(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + O[dx]^4$$

$$\text{Out[182]= FnG error} \parallel \left( -\frac{3}{2} \frac{\text{dt}^2 (k w)}{\text{left}(H^2 k^2 + 3 \text{right})} - \frac{i}{2} \frac{\text{dt}^3 k w^2}{\text{left}(H^2 k^2 + 3 \text{right})} + O[\text{left}(\text{dt}^4 \text{right})] + \text{dx}^2 \left( \frac{i}{2} \frac{\text{left}(H^2 k^5 + 6 k^3 \text{right})}{\text{left}(\text{dt})^4 \text{left}(H^2 k^2 + 3 \text{right})^2} + O[\text{left}(\text{dt}^4 \text{right})] + O[\text{left}(\text{dx}^4 \text{right})] \right)$$

Out[183]=

$$\text{Out[184]= FGn} \parallel H(g \text{Rmp} + \text{Gnp} U)$$

$$\text{Out[185]= FGn} \parallel H(g \text{Rmp} + \text{Gnp} U)$$

$$\text{Out[186]= FGn error} \parallel \left( -\frac{(k(3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2(3+H^2 k^2)} - \frac{i k(3 g H + g H^3 k^2 - 3 U^2) w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left( -\frac{1}{2} (g H k^2) dt + O[dt]^4 \right) dx + \left( \frac{i(18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2) dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left( \frac{1}{24} g H k^4 dt + O[dt]^4 \right) dx^3 + O[dx]^4$$

$$\text{Out[187]= FGn error} \parallel \left( -\frac{\text{dt}^2 \text{left}(k w \text{left}(g H^3 k^2 + 3 g H - 3 U^2 \text{right}) \text{right})}{2 \text{left}(H^2 k^2 + 3 \text{right})} - \frac{i}{2} \frac{\text{dt}^3 k w^2 \text{left}(g H^3 k^2 + 3 g H - 3 U^2 \text{right})}{6 \text{left}(H^2 k^2 + 3 \text{right})} + O[\text{left}(\text{dt}^4 \text{right})] + \text{dx} \left( \frac{1}{2} \text{left}(g H k^2 \text{right}) \text{dt} + O[\text{left}(\text{dt}^4 \text{right})] \right) + \text{dx}^2 \left( \frac{i}{2} \frac{\text{left}(2 g H^5 k^7 + 12 g H^3 k^5 - 3 H^2 U^2 k^5 - 18 U^2 k^3 + 18 g H k^3 \text{right})}{\text{left}(\text{dt})^4 \text{left}(H^2 k^2 + 3 \text{right})^2} + O[\text{left}(\text{dt}^4 \text{right})] + \text{dx}^3 \left( \frac{1}{24} g H k^4 \text{dt} + O[\text{left}(\text{dt}^4 \text{right})] + O[\text{left}(\text{dx}^4 \text{right})] \right) \right)$$

Out[188]=

$$\text{Out[189]= FGG} \parallel (G G p H + \text{Rmp}) U$$

$$\text{Out[190]= FGG} \parallel U (\text{GGp} H + \text{Rmp})$$

$$\text{Out[191]= FGG error} \parallel \left( -\frac{(k(6+H^2 k^2) U w) dt^2}{2(3+H^2 k^2)} - \frac{i k(6+H^2 k^2) U w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left( -\frac{1}{2} (k^2 U) dt + O[dt]^4 \right) dx + \left( \frac{i(36 k^3 + 15 H^2 k^5 + 2 H^4 k^7) U dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left( \frac{1}{24} k^4 U dt + O[dt]^4 \right) dx^3 + \left( -\frac{i(108 k^5 + 54 H^2 k^7 + 17 H^4 k^9 + 2 H^6 k^{11}) U dt}{240(3+H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$$

$$\text{Out[192]= FGG error} \parallel \left( -\frac{\text{dt}^2 \text{left}(k U w \text{left}(H^2 k^2 + 6 \text{right}) \text{right})}{2 \text{left}(H^2 k^2 + 3 \text{right})} - \frac{i}{2} \frac{\text{dt}^3 k U w^2 \text{left}(H^2 k^2 + 6 \text{right})}{6 \text{left}(H^2 k^2 + 3 \text{right})} + O[\text{left}(\text{dt}^4 \text{right})] + \text{dx} \left( \frac{1}{2} \text{left}(k^2 U \text{right}) \text{dt} + O[\text{left}(\text{dt}^4 \text{right})] \right) + \text{dx}^2 \left( \frac{i}{2} \frac{\text{left}(2 H^4 k^7 + 15 H^2 k^5 + 36 k^3 \text{right}) U \text{left}(\text{dt})}{12 \text{left}(H^2 k^2 + 3 \text{right})^2} + O[\text{left}(\text{dt}^4 \text{right})] + \text{dx}^3 \left( \frac{1}{24} k^4 U \text{dt} + O[\text{left}(\text{dt}^4 \text{right})] + \text{dx}^4 \left( -\frac{i}{240} \frac{\text{left}(2 H^6 k^{11} + 17 H^4 k^9 + 54 H^2 k^7 + 108 k^5 \text{right}) U \text{left}(\text{dt})}{\text{left}(H^2 k^2 + 3 \text{right})^3} + O[\text{left}(\text{dt}^4 \text{right})] + O[\text{left}(\text{dx}^5 \text{right})] \right) \right) \right)$$

Out[193]=

Out[194]=

$$\begin{aligned}
\text{Out[195]= } \Omega \text{ error } & \parallel \left\{ \frac{i \left( \sqrt{3} \, k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^3 U \right)}{2 (3 + H^2 k^2)^2} - \frac{1}{3 (3 + H^2 k^2)^2} \right. \\
& \left( k^3 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( 3 g H + U \left( 2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right) dt^2 - \\
& \frac{1}{4 (3 + H^2 k^2)^3} i k^4 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( 3 g \left( \sqrt{3} H \sqrt{g H (3 + H^2 k^2)} + 9 H U + 3 H^3 k^2 U \right) + \right. \\
& \left. U^2 \left( H^4 k^4 U + 9 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + U \right) + 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 2 H^2 U \right) \right) \right) dt^3 + \\
& \frac{1}{5 (3 + H^2 k^2)^3} k^5 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left( 9 g^2 H^2 + 6 g H U \left( 2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^3 \left( 12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + 2 k^2 \left( 2 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 3 H^2 U \right) \right) \right) dt^4 + \\
& \left. O[dt]^5 \right) + \left( -\frac{1}{4} i k^2 \left( \sqrt{3} \sqrt{\frac{g H}{3 + H^2 k^2}} + 2 U \right) + \frac{k^3 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( \sqrt{3} g H + 2 \sqrt{g H (3 + H^2 k^2)} U \right) dt}{4 \sqrt{g H} (3 + H^2 k^2)^{3/2}} + \right. \\
& \left( i k^4 \left( \sqrt{3} g H + 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left( 3 g H + U \left( 2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) dt^2 \right) / \\
& \left( 4 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) - \\
& \left( k^5 \left( \sqrt{3} g H + 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left( 3 g \left( \sqrt{3} H \sqrt{g H (3 + H^2 k^2)} + 9 H U + 3 H^3 k^2 U \right) + \right. \right. \\
& \left. \left. U^2 \left( H^4 k^4 U + 9 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + U \right) + 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 2 H^2 U \right) \right) \right) \right) \\
& dt^3 \Big/ \left( 4 \left( \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) \right) - \left( i k^6 \left( \sqrt{3} g H + 2 \sqrt{g H (3 + H^2 k^2)} U \right) \right. \\
& \left( 9 g^2 H^2 + 6 g H U \left( 2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^3 \left( 12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + 2 k^2 \left( 2 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 3 H^2 U \right) \right) \right) \\
& dt^4 \Big/ \left( 4 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + O[dt]^5 \Big) dx + \\
& \left( -\left( k^3 \left( 3 \sqrt{3} g H (13 + 3 H^2 k^2) + 16 \left( 3 \sqrt{g H (3 + H^2 k^2)} + k^2 \sqrt{g H^5 (3 + H^2 k^2)} \right) U \right) \right) / \right. \\
& \left( 96 \left( \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) \right) - \\
& \left( i k^4 \left( g H \left( 144 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (369 + 234 H^2 k^2 + 37 H^4 k^4) U \right) + 4 \left( 63 \sqrt{g H (3 + H^2 k^2)} U^2 + \right. \right. \\
& \left. \left. 7 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 3 k^2 \left( 3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 14 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) \\
& dt \Big/ \left( 96 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \left( k^5 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \left( g H \left( 171 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (477 + 306 H^2 k^2 + 49 H^4 k^4) U \right) + 5 \left( 72 \sqrt{g H (3 + H^2 k^2)} U^2 + \right. \right. \\
& \left. \left. 7 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 3 k^2 \left( 3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 14 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) \\
& dt \Big/ \left( 96 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + O[dt]^5 \Big) dx +
\end{aligned}$$

[illegible]

$$\begin{aligned}
& \left( \left( k^7 \left( 3 \sqrt{3} \sqrt{g H (543 + 146 H^2 k^2 + 15 H^4 k^2)} + 256 \left( 9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^2 (3 + H^2 k^2)} + \right. \right. \right. \right. \\
& \quad \left. \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \left( 30720 \sqrt{g H (3 + H^2 k^2)^{5/2}} \right) + \\
& \left( i k^6 \left( 3 g H \left( 18432 \sqrt{g H (3 + H^2 k^2)} + 51039 \sqrt{3} U + 45735 \sqrt{3} H^2 k^2 U + \right. \right. \right. \\
& \quad \left. \left. 1381 \sqrt{3} H^6 k^6 U + k^4 \left( 960 \sqrt{g H^9 (3 + H^2 k^2)} + 13717 \sqrt{3} H^4 U \right) \right) \right) + \\
& 128 \left( 837 \sqrt{g H (3 + H^2 k^2)} U^2 + 279 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 31 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. 27 k^2 \left( 7 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 31 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt \Big/ \\
& \left( 92160 \sqrt{g H (3 + H^2 k^2)^{7/2}} \right) - \left( \left( k^7 \left( 9 \sqrt{3} g^2 H^2 (12999 + 6258 H^2 k^2 + 775 H^4 k^4) + \right. \right. \right. \\
& \quad g H U \left( 617661 \sqrt{3} H^2 k^2 U + 20551 \sqrt{3} H^6 k^6 U + 81 \left( 5888 \sqrt{g H (3 + H^2 k^2)} + \right. \right. \\
& \quad \left. \left. 8053 \sqrt{3} U \right) + 3 k^4 \left( 12288 \sqrt{g H^9 (3 + H^2 k^2)} + 65021 \sqrt{3} H^4 U \right) \right) \right) + \\
& 128 \left( 2322 \sqrt{g H (3 + H^2 k^2)} U^3 + 774 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 86 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} \right. \\
& \quad \left. U^3 + 9 k^2 \left( 229 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 258 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \Big) \\
& dt^2 \Big/ \left( 92160 \left( \sqrt{g H (3 + H^2 k^2)^{7/2}} \right) \right) - \frac{1}{30720 \sqrt{g H (3 + H^2 k^2)^{9/2}}} \\
& i k^8 \left( k^8 U^3 \left( 21253 \sqrt{3} g H^9 + 7776 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& 243 \left( 832 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 4423 \sqrt{3} g^2 H^2 U + 2592 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad g H U^2 \left( 8832 \sqrt{g H (3 + H^2 k^2)} + 7823 \sqrt{3} U \right) \Big) + 3 k^6 U \left( 9181 \sqrt{3} g^2 H^8 + \right. \\
& \quad \left. 31104 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + 4 g H^7 U \left( 5312 \sqrt{g H (3 + H^2 k^2)} + 21787 \sqrt{3} U \right) \right) \Big) + \\
& 27 k^2 \left( 35319 \sqrt{3} g^2 H^4 U + 91580 \sqrt{3} g H^3 U^3 + \right. \\
& \quad \left. 64 \left( 59 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 1155 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 486 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) \right) \Big) + \\
& 9 k^4 \left( 31231 \sqrt{3} g^2 H^6 U + 2 g H^5 U^2 \left( 34336 \sqrt{g H (3 + H^2 k^2)} + 67003 \sqrt{3} U \right) + \right. \\
& \quad \left. 288 \left( 5 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 162 \sqrt{g H^9 (3 + H^2 k^2)} U^4 \right) \right) \Big) dt^3 + \\
& \frac{1}{92160 \sqrt{g H (3 + H^2 k^2)^{11/2}}} k^9 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left( k^8 U^3 \left( 111559 \sqrt{3} g H^9 + 42368 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& 27 k^2 \left( 18078 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 172047 \sqrt{3} g^2 H^4 U + 372075 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. 477940 \sqrt{3} g H^3 U^3 + 169472 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) \Big) + \\
& 81 \left( 11603 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 63917 \sqrt{3} g^2 H^2 U + 42368 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad g H U^2 \left( 132513 \sqrt{g H (3 + H^2 k^2)} + 122207 \sqrt{3} U \right) \Big) + 3 k^6 U \left( 45573 \sqrt{3} g^2 H^8 + \right. \\
& \quad \left. 169472 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + g H^7 U \left( 108089 \sqrt{g H (3 + H^2 k^2)} + 456644 \sqrt{3} U \right) \right) \Big) +
\end{aligned}$$

$$\begin{aligned}
& \left( 9 k^4 \left( 7035 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 153703 \sqrt{3} g^2 H^6 U + 254208 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + \right. \right. \\
& \quad \left. \left. g H^5 U^2 \left( 347651 \sqrt{g H (3 + H^2 k^2)} + 700818 \sqrt{3} U \right) \right) \right) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5, \\
& \left( \frac{i \left( -\sqrt{3} k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^3 U \right)^2}{2 (3 + H^2 k^2)^2} dt - \frac{1}{3 (3 + H^2 k^2)^2} \left( k^3 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \right. \\
& \quad \left. \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right) dt^2 - \\
& \quad \frac{1}{4 (3 + H^2 k^2)^3} i k^4 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \quad \left( 3 g H \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \quad \left. U^2 \left( -9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) dt^3 + \\
& \quad \frac{1}{5 (3 + H^2 k^2)^5} k^5 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \quad \left( 9 g^2 H^2 + 6 g H U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \quad \left. U^3 \left( -12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left( -4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) dt^4 + O[ \\
& \quad dt]^5 \Big) + \\
& \left( \frac{1}{4} i k^2 \left( \sqrt{3} \sqrt{\frac{g H}{3 + H^2 k^2}} - 2 U \right) - \frac{\left( k^3 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( \sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \right) dt}{4 \left( \sqrt{g H} (3 + H^2 k^2)^{3/2} \right)} - \right. \\
& \quad \left( i k^4 \left( \sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) dt^2 \right) / \\
& \quad \left( 4 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) + \\
& \quad \left( k^5 \left( \sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left( 3 g H \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + U^2 \right. \right. \\
& \quad \left. \left. \left( -9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) dt^3 \right) / \\
& \quad \left( 4 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \left( i k^6 \left( \sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \right. \\
& \quad \left. \left( 9 g^2 H^2 + 6 g H U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \right. \\
& \quad \left. \left. U^3 \left( -12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left( -4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) \right) \\
& \quad \left. dt^4 \right) / \left( 4 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + O[dt]^5 \Big) dx +
\end{aligned}$$

$$\left( \frac{k^3 \left( 3 \sqrt{3} \sqrt{g H (13 + 3 H^2 k^2)} - 16 \left( 3 \sqrt{g H (3 + H^2 k^2)} U + k^2 \sqrt{g H^5 (3 + H^2 k^2)} U \right) \right)}{96 \sqrt{g H} (3 + H^2 k^2)^{3/2}} + \right. \\ \left( i k^4 \left( g H \left( -144 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (369 + 234 H^2 k^2 + 37 H^4 k^4) U \right) - \right. \\ \left. 4 \left( 63 \sqrt{g H (3 + H^2 k^2)} U^2 + 7 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \right. \\ \left. \left. 3 k^2 \left( 3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 14 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt \Big/ \left( 96 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) - \\ \left( \left( k^5 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right) \left( g H \left( 171 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (477 + 306 H^2 k^2 + \right. \right. \right. \\ \left. \left. 49 H^4 k^4) U \right) + 5 \left( 72 \sqrt{g H (3 + H^2 k^2)} U^2 + 8 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \right. \\ \left. \left. 3 k^2 \left( 3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt^2 \Big/ \\ \left( 96 \left( \sqrt{g H} (3 + H^2 k^2)^{7/2} \right) \right) + \left( i k^6 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \right. \\ \left( 468 \sqrt{g H (3 + H^2 k^2)} U^2 + 52 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\ \left. g H \left( 198 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (585 + 378 H^2 k^2 + 61 H^4 k^4) U \right) + \right. \\ \left. 6 k^2 \left( 9 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 52 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt^3 \Big/ \left( 96 \sqrt{g H} (3 + H^2 k^2)^{9/2} \right) + \\ \left( k^7 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^3 \left( 576 \sqrt{g H (3 + H^2 k^2)} U^2 + 64 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \right. \\ \left. \left. g H \left( 225 \sqrt{g H (3 + H^2 k^2)} - \sqrt{3} (693 + 450 H^2 k^2 + 73 H^4 k^4) U \right) + \right. \right. \\ \left. \left. 3 k^2 \left( 21 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 128 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt^4 \right) \Big/ \left( 96 \sqrt{g H} (3 + H^2 k^2)^{11/2} \right) + \\ \left. O[dt]^5 \right) dx^2 + \left( \frac{i k^4 \left( -\sqrt{3} \sqrt{g H (33 + 7 H^2 k^2)} + 16 \left( 3 \sqrt{g H (3 + H^2 k^2)} + k^2 \sqrt{g H^5 (3 + H^2 k^2)} U \right) \right)}{384 \sqrt{g H} (3 + H^2 k^2)^{3/2}} + \right. \\ \left( k^5 \left( g H \left( -72 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (207 + 126 H^2 k^2 + 19 H^4 k^4) U \right) - \right. \right. \\ \left. 16 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^2 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \right. \\ \left. \left. k^2 \left( \sqrt{g^3 H^7 (3 + H^2 k^2)} + 6 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \\ \left( i k^6 \left( 9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \left( -96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) - \right. \right. \\ \left. 32 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^3 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left( 2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \right. \\ \left. \left. \left. 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) dt^2 \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) - \\ \left( \left( k^7 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( 24 \sqrt{3} g^2 H^2 (21 + 5 H^2 k^2) + \right. \right. \right. \\ \left. \left. 3 g H U \left( -717 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1011 + 646 H^2 k^2 + 103 H^4 k^4) U \right) - \right. \right. \right. \\ \left. \left. \left. 3 k^2 \left( 3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt^2 \right) \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) - \\ \left. \left( k^8 \left( 9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \left( -96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) - \right. \right. \right. \right. \\ \left. \left. \left. 32 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^3 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left( 2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \right) dt^2 \right) \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) - \right. \\ \left. \left( k^9 \left( 9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \left( -96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) - \right. \right. \right. \right. \\ \left. \left. \left. 32 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^3 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left( 2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \right) dt^2 \right) \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) - \right. \\ \left. \left( k^{10} \left( 9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \left( -96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) - \right. \right. \right. \right. \\ \left. \left. \left. 32 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^3 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left( 2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \right) dt^2 \right) \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) - \right. \\ \left. \left( k^{11} \left( 9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \left( -96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) - \right. \right. \right. \right. \\ \left. \left. \left. 32 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^3 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left( 2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \right) dt^2 \right) \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) - \right. \\ \left. \left( k^{12} \left( 9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \left( -96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) - \right. \right. \right. \right. \\ \left. \left. \left. 32 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^3 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left( 2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \right) dt^2 \right) \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) - \right. \\ \left. \left( k^{13} \left( 9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \left( -96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) - \right. \right. \right. \right. \\ \left. \left. \left. 32 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^3 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left( 2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \right) dt^2 \right) \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) - \right. \\ \left. \left( k^{14} \left( 9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \left( -96 \sqrt{$$



$$\begin{aligned}
& 5 \left( 288 \sqrt{g H (3 + H^2 k^2)} U^3 + 32 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. 3 k^2 \left( 39 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 64 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) dt^3 \Big/ \\
& \left( 384 \left( \sqrt{g H (3 + H^2 k^2)}^{7/2} \right) - \left( i k^8 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \left( 5 \sqrt{3} g^2 H^2 \right. \right. \right. \\
& \quad \left. \left. (45 + 11 H^2 k^2) + g H U \left( -990 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1449 + 930 H^2 k^2 + 149 H^4 k^4) U \right) - \right. \right. \\
& \quad \left. \left. 2 \left( 360 \sqrt{g H (3 + H^2 k^2)} U^3 + 40 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + k^2 \left( 137 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \right. \right. \\
& \quad \left. \left. \left. 240 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) dt^4 \Big/ \left( 128 \sqrt{g H (3 + H^2 k^2)}^{9/2} \right) + O[dt]^5 \Big) dx^3 + \\
& \left( k^5 \left( -3 \sqrt{3} g H (543 + 146 H^2 k^2 + 15 H^4 k^4) + 256 \left( 9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + \right. \right. \right. \\
& \quad \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} U \right) \right) \Big/ \left( 30720 \sqrt{g H (3 + H^2 k^2)}^{5/2} \right) + \\
& \left( i k^6 \left( -3 g H \left( -18432 \sqrt{g H (3 + H^2 k^2)} + 51039 \sqrt{3} U + 45735 \sqrt{3} H^2 k^2 U + \right. \right. \right. \\
& \quad \left. \left. 1381 \sqrt{3} H^6 k^6 U + k^4 \left( -960 \sqrt{g H^9 (3 + H^2 k^2)} + 13717 \sqrt{3} H^4 U \right) \right) \right) + \\
& \quad 128 \left( 837 \sqrt{g H (3 + H^2 k^2)} U^2 + 279 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 31 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. 27 k^2 \left( 7 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 31 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt \Big/ \\
& \left( 92160 \sqrt{g H (3 + H^2 k^2)}^{7/2} \right) + \left( k^7 \left( 9 \sqrt{3} g^2 H^2 (12999 + 6258 H^2 k^2 + 775 H^4 k^4) + \right. \right. \\
& \quad g H U \left( 617661 \sqrt{3} H^2 k^2 U + 20551 \sqrt{3} H^6 k^6 U + 81 \left( -5888 \sqrt{g H (3 + H^2 k^2)} + \right. \right. \\
& \quad \left. \left. 8053 \sqrt{3} U \right) - 3 k^4 \left( 12288 \sqrt{g H^9 (3 + H^2 k^2)} - 65021 \sqrt{3} H^4 U \right) \right) - \\
& \quad 128 \left( 2322 \sqrt{g H (3 + H^2 k^2)} U^3 + 774 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 86 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. 9 k^2 \left( 229 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 258 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) dt^2 \Big/ \\
& \left( 92160 \sqrt{g H (3 + H^2 k^2)}^{7/2} \right) - \frac{1}{30720 \sqrt{g H (3 + H^2 k^2)}^{9/2}} i k^8 \\
& \left( k^8 U^3 \left( -21253 \sqrt{3} g H^9 + 7776 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& \quad 243 \left( 832 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 4423 \sqrt{3} g^2 H^2 U + 2592 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad g H U^2 \left( 8832 \sqrt{g H (3 + H^2 k^2)} - 7823 \sqrt{3} U \right) - 3 k^6 U \left( 9181 \sqrt{3} g^2 H^8 - \right. \\
& \quad \left. 31104 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + 4 g H^7 U \left( -5312 \sqrt{g H (3 + H^2 k^2)} + 21787 \sqrt{3} U \right) \right) + \\
& \quad \left. 27 k^2 \left( -35319 \sqrt{3} g^2 H^4 U - 91580 \sqrt{3} g H^3 U^3 + \right. \right. \\
& \quad \left. \left. 64 \left( 59 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 1155 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 486 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) \right) \right) + \\
& \quad 9 k^4 \left( -31231 \sqrt{3} g^2 H^6 U + 2 g H^5 U^2 \left( 34336 \sqrt{g H (3 + H^2 k^2)} - 67003 \sqrt{3} U \right) + \right.
\end{aligned}$$

$$\begin{aligned}
& \left( 288 \left( 5 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 162 \sqrt{g H^9 (3 + H^2 k^2)} U^4 \right) \right) dt^3 - \frac{1}{92160 \left( \sqrt{g H} (3 + H^2 k^2)^{11/2} \right)} \\
& \left( k^9 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right) \left( k^8 U^3 \left( -111559 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} U \right) \right) \right. \\
& 27 k^2 \left( 18078 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 172047 \sqrt{3} \sqrt{g^2 H^4 U + 372075 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2} - \right. \\
& 477940 \sqrt{3} \sqrt{g H^3 U^3 + 169472 \sqrt{g H^5 (3 + H^2 k^2)} U^4} \left. \right) + \\
& 9 k^4 \left( 7035 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 153703 \sqrt{3} \sqrt{g^2 H^6 U + 254208 \sqrt{g H^9 (3 + H^2 k^2)} U^4} + \right. \\
& g H^5 U^2 \left( 347651 \sqrt{g H (3 + H^2 k^2)} - 700818 \sqrt{3} U \right) \left. \right) + \\
& 81 \left( 11603 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 63917 \sqrt{3} \sqrt{g^2 H^2 U + 42368 \sqrt{g H (3 + H^2 k^2)} U^4} + \right. \\
& g H U^2 \left( 132513 \sqrt{g H (3 + H^2 k^2)} - 122207 \sqrt{3} U \right) \left. \right) - \\
& 3 k^6 U \left( 45573 \sqrt{3} \sqrt{g^2 H^8 - 169472 \sqrt{g H^{13} (3 + H^2 k^2)} U^3} + \right. \\
& \left. \left. g H^7 U \left( -108089 \sqrt{g H (3 + H^2 k^2)} + 456644 \sqrt{3} U \right) \right) \right) dt^4 + O[dt]^5 \Big\} dx^4 + O[dx]^5 \Big\}
\end{aligned}$$

Out[196]= Omega error ||

$$\begin{aligned}
& \left( \frac{1}{\sqrt{3}} \left( \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right) \left( k^8 U^3 \left( -111559 \sqrt{3} \sqrt{g H^9 (3 + H^2 k^2)} U \right) \right) \right. \\
& 27 k^2 \left( 18078 \sqrt{g^5 H^9 (3 + H^2 k^2)} - 172047 \sqrt{3} \sqrt{g^2 H^4 U + 372075 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2} - \right. \\
& 477940 \sqrt{3} \sqrt{g H^3 U^3 + 169472 \sqrt{g H^5 (3 + H^2 k^2)} U^4} \left. \right) + \\
& 9 k^4 \left( 7035 \sqrt{g^5 H^{13} (3 + H^2 k^2)} - 153703 \sqrt{3} \sqrt{g^2 H^6 U + 254208 \sqrt{g H^9 (3 + H^2 k^2)} U^4} + \right. \\
& g H^5 U^2 \left( 347651 \sqrt{g H (3 + H^2 k^2)} - 700818 \sqrt{3} U \right) \left. \right) + \\
& 81 \left( 11603 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 63917 \sqrt{3} \sqrt{g^2 H^2 U + 42368 \sqrt{g H (3 + H^2 k^2)} U^4} + \right. \\
& g H U^2 \left( 132513 \sqrt{g H (3 + H^2 k^2)} - 122207 \sqrt{3} U \right) \left. \right) - \\
& 3 k^6 U \left( 45573 \sqrt{3} \sqrt{g^2 H^8 - 169472 \sqrt{g H^{13} (3 + H^2 k^2)} U^3} + \right. \\
& \left. \left. g H^7 U \left( -108089 \sqrt{g H (3 + H^2 k^2)} + 456644 \sqrt{3} U \right) \right) \right) dt^4 + O[dt]^5 \Big\} dx^4 + O[dx]^5 \Big\}
\end{aligned}$$







$$\begin{aligned}
& H^2 k^2 + 129 \text{right}) U - 96 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} H - 32 \left( \sqrt{g H^9 \left( H^2 k^2 + 3 \text{right}) \right)} U^3 k^4 + 2 \left( 3 \sqrt{g H^5 \left( H^2 k^2 + 3 \text{right}) \right)} U^3 + 2 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \text{right}) \right)} U \text{right}) k^2 + 9 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} U^3 \text{right}) \text{right}) \text{text{dt}}^2 \{ 128 \sqrt{g H} \left( H^2 k^2 + 3 \text{right}) \right)^{5/2} \} - \frac{\left( k^7 \left( \left( H^2 k^2 + 3 \text{right}) \right) U - \sqrt{3} \right) \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} \left( 24 \sqrt{3} g^2 \left( 5 H^2 k^2 + 21 \text{right}) \right) H^2 + 3 g U \left( \sqrt{3} \left( 103 H^4 k^4 + 646 H^2 k^2 + 1011 \text{right}) \right) U - 717 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} \right) H - 5 \left( 32 \sqrt{g H^9 \left( H^2 k^2 + 3 \text{right}) \right)} U^3 k^4 + 3 \left( 64 \sqrt{g H^5 \left( H^2 k^2 + 3 \text{right}) \right)} U^3 + 39 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \text{right}) \right)} U \text{right}) k^2 + 288 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} U^3 \text{right}) \text{right}) \text{right}) \text{text{dt}}^3 \{ 384 \left( \sqrt{g H} \left( H^2 k^2 + 3 \text{right}) \right)^{7/2} \right) \} - \frac{i k^8 \left( \sqrt{3} \right) \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} - \left( H^2 k^2 + 3 \text{right}) U \text{right})^2 \left( 5 \sqrt{3} g^2 \left( 11 H^2 k^2 + 45 \text{right}) \right) H^2 + g U \left( \sqrt{3} \left( 149 H^4 k^4 + 930 H^2 k^2 + 1449 \text{right}) \right) U - 990 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} \right) H - 2 \left( 40 \sqrt{g H^9 \left( H^2 k^2 + 3 \text{right}) \right)} U^3 k^4 + \left( 240 \sqrt{g H^5 \left( H^2 k^2 + 3 \text{right}) \right)} U^3 + 137 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \text{right}) \right)} U \text{right}) k^2 + 360 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} U^3 \text{right}) \text{right}) \text{text{dt}}^4 \{ 128 \sqrt{g H} \left( H^2 k^2 + 3 \text{right}) \right)^{9/2} \} + O \left( \text{text{dt}}^5 \text{right}) \text{right}) \text{text{dx}}^3 + \left( \frac{k^5 \left( 256 \left( \sqrt{g H^9 \left( H^2 k^2 + 3 \text{right}) \right)} k^4 + 6 \sqrt{g H^5 \left( H^2 k^2 + 3 \text{right}) \right)} k^2 + 9 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} \right) U - 3 \sqrt{3} g H \left( 15 H^4 k^4 + 146 H^2 k^2 + 543 \text{right}) \right) \{ 30720 \sqrt{g H} \left( H^2 k^2 + 3 \text{right}) \right)^{5/2} \} + \frac{i k^6 \left( 128 \left( 31 \sqrt{g H^{13}} \left( H^2 k^2 + 3 \text{right}) \right) U^2 k^6 + 279 \sqrt{g H^9 \left( H^2 k^2 + 3 \text{right}) \right)} U^2 k^4 + 27 \left( 31 \sqrt{g H^5 \left( H^2 k^2 + 3 \text{right}) \right)} U^2 + 7 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \text{right}) \right)} \right) \text{right}) k^2 + 837 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} U^2 \text{right}) - 3 g H \left( 1381 \sqrt{3} H^6 U k^6 + \left( 13717 \sqrt{3} H^4 U - 960 \sqrt{g H^9 \left( H^2 k^2 + 3 \text{right}) \right)} \right) k^4 + 45735 \sqrt{3} H^2 U k^2 + 51039 \sqrt{3} U - 18432 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} \right) \text{right}) \text{right}) \text{text{dt}} \{ 92160 \sqrt{g H} \left( H^2 k^2 + 3 \text{right}) \right)^{7/2} \} + \frac{k^7 \left( 9 \sqrt{3} g^2 \left( 775 H^4 k^4 + 6258 H^2 k^2 + 12999 \text{right}) \right) H^2 + g U \left( 20551 \sqrt{3} H^6 U k^6 - 3 \left( 12288 \sqrt{g H^9 \left( H^2 k^2 + 3 \text{right}) \right)} - 65021 \sqrt{3} H^4 U \text{right}) k^4 + 617661 \sqrt{3} H^2 U k^2 + 81 \left( 8053 \sqrt{3} U - 5888 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} \right) \text{right}) \right) H - 128 \left( 86 \sqrt{g H^{13}} \left( H^2 k^2 + 3 \text{right}) \right) U^3 k^6 + 774 \sqrt{g H^9 \left( H^2 k^2 + 3 \text{right}) \right)} U^3 k^4 + 9 \left( 258 \sqrt{g H^5 \left( H^2 k^2 + 3 \text{right}) \right)} U^3 + 229 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \text{right}) \right)} U \text{right}) k^2 + 2322 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} U^3 \text{right}) \text{right}) \text{text{dt}}^2 \{ 92160 \sqrt{g H} \left( H^2 k^2 + 3 \text{right}) \right)^{7/2} \} - \frac{i k^8 \left( U^3 \left( 7776 \sqrt{g H^{17}} \left( H^2 k^2 + 3 \text{right}) \right) U - 21253 \sqrt{3} g H^9 \text{right}) k^8 - 3 U \left( 9181 \sqrt{3} g^2 H^8 + 4 g U \left( 21787 \sqrt{3} U - 5312 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} \right) \right) H^7 - 31104 \sqrt{g H^{13}} \left( H^2 k^2 + 3 \text{right}) \right) U^3 \text{right}) k^6 + 9 \left( -31231 \sqrt{3} g^2 U H^6 + 2 g U^2 \left( 34336 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} - 67003 \sqrt{3} U \text{right}) H^5 + 288 \left( 162 \sqrt{g H^9 \left( H^2 k^2 + 3 \text{right}) \right)} U^4 + 5 \sqrt{g^5 H^{13}} \left( H^2 k^2 + 3 \text{right}) \right) \right) \text{right}) \right) k^4 + 27 \left( -35319 \sqrt{3} g^2 U H^4 - 91580 \sqrt{3} g U^3 H^3 + 64 \left( 486 \sqrt{g H^5 \left( H^2 k^2 + 3 \text{right}) \right)} U^4 + 1155 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \text{right}) \right)} U^2 + 59 \sqrt{g^5 H^9 \left( H^2 k^2 + 3 \text{right}) \right)} \right) \text{right}) \right) k^2 + 243 \left( 2592 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} U^4 + g H \left( 8832 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} - 7823 \sqrt{3} U \text{right}) U^2 - 4423 \sqrt{3} g^2 H^2 U + 832 \sqrt{g^5 H^5 \left( H^2 k^2 + 3 \text{right}) \right)} \right) \text{right}) \text{right}) \text{text{dt}}^3 \{ 30720 \sqrt{g H} \left( H^2 k^2 + 3 \text{right}) \right)^{9/2} \} - \frac{\left( k^9 \left( \sqrt{3} \right) \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} - \left( H^2 k^2 + 3 \text{right}) U \text{right}) \left( U^3 \left( 42368 \sqrt{g H^{17}} \left( H^2 k^2 + 3 \text{right}) \right) U - 111559 \sqrt{3} g H^9 \text{right}) k^8 - 3 U \left( 45573 \sqrt{3} g^2 H^8 + g U \left( 456644 \sqrt{3} U - 108089 \sqrt{g H \left( H^2 k^2 + 3 \text{right}) \right)} \right) H^7 - 169472 \sqrt{g H^{13}} \left( H^2 k^2 + 3 \text{right}) \right) U^3 \text{right}) k^6 + 9 \left( -153703 \right. \right.
\end{aligned}$$

$$\begin{aligned} & \sqrt[3]{g^2 U H^6 + g U^2 \left( 347651 \sqrt{g H \left( H^2 k^2 + 3 \right)} - 700818 \sqrt[3]{U} \right) H^5 + 254208 \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} U^4 + 7035 \sqrt{g^5 H^{13} \left( H^2 k^2 + 3 \right)} \right) \\ & k^4 + 27 \left( -172047 \sqrt[3]{g^2 U H^4} - 477940 \sqrt[3]{g U^3 H^3} + 169472 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) U^4 + 372075 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} U^2 + 18078 \sqrt{g^5 H^9 \left( H^2 k^2 + 3 \right)} \right) k^2 + 81 \left( 42368 \sqrt{g H \left( H^2 k^2 + 3 \right)} U^4 + g H \left( 132513 \sqrt{g H \left( H^2 k^2 + 3 \right)} - 122207 \sqrt[3]{U} \right) U^2 - 63917 \sqrt[3]{g^2 H^2 U} + 11603 \sqrt{g^5 H^5 \left( H^2 k^2 + 3 \right)} \right) \right) \text{dt}^4 \left( \sqrt{g H} \left( H^2 k^2 + 3 \right) \right)^{11/2} \right) + O \left( \text{dt}^5 \right) \text{dx}^4 + O \left( \text{dx}^5 \right) \end{aligned}$$

Out[197]=

$$\begin{aligned} \text{Out[198]} = \text{EA} \parallel & \left\{ \left\{ 1 + \frac{i e^{\frac{i \text{dx} k}{2}} (1 - e^{-i \text{dx} k}) (-1 + e^{i \text{dt} w}) H^2 k^3 U \text{Csc} \left[ \frac{\text{dx} k}{2} \right]}{(6 + 2 H^2 k^2) w}, \frac{i e^{\frac{i \text{dx} k}{2}} (1 - e^{-i \text{dx} k}) (-1 + e^{i \text{dt} w}) H k \text{Csc} \left[ \frac{\text{dx} k}{2} \right]}{2 \left( H + \frac{H^3 k^2}{3} \right) w} \right\}, \right. \\ & \left. \left\{ \frac{i e^{\frac{i \text{dx} k}{2}} (1 - e^{-i \text{dx} k}) (-1 + e^{i \text{dt} w}) k (g H (3 + H^2 k^2) - 3 U^2) \text{Csc} \left[ \frac{\text{dx} k}{2} \right]}{(6 + 2 H^2 k^2) w}, 1 + \frac{i e^{\frac{i \text{dx} k}{2}} (1 - e^{-i \text{dx} k}) (-1 + e^{i \text{dt} w}) k (6 + H^2 k^2) U \text{Csc} \left[ \frac{\text{dx} k}{2} \right]}{(6 + 2 H^2 k^2) w} \right\} \right\} \end{aligned}$$

Out[199]=

$$\begin{aligned} \text{EA} \parallel & \left( \begin{array}{c} \frac{i e^{\frac{i}{\text{dx} k}}}{\frac{i}{\text{dx} k}} \left( 1 - e^{-i \frac{\text{dx} k}{2}} \right) \left( -1 + e^{i \text{dt} w} \right) H^2 U \csc \left( \frac{\text{dx} k}{2} \right) k^3 \left( \left( 2 H^2 k^2 + 6 \right) w \right) + 1 \\ & \& \frac{i e^{\frac{i}{\text{dx} k}}}{\frac{i}{\text{dx} k}} \left( 1 - e^{-i \frac{\text{dx} k}{2}} \right) \left( -1 + e^{i \text{dt} w} \right) H k \csc \left( \frac{\text{dx} k}{2} \right) \left( \left( 2 H^2 k^2 + 6 \right) w \right) \\ & \frac{i e^{\frac{i}{\text{dx} k}}}{\frac{i}{\text{dx} k}} \left( 1 - e^{-i \frac{\text{dx} k}{2}} \right) \left( -1 + e^{i \text{dt} w} \right) k \left( g H (3 + H^2 k^2) - 3 U^2 \right) \csc \left( \frac{\text{dx} k}{2} \right) \\ & \& \frac{i e^{\frac{i}{\text{dx} k}}}{\frac{i}{\text{dx} k}} \left( 1 - e^{-i \frac{\text{dx} k}{2}} \right) \left( -1 + e^{i \text{dt} w} \right) k (6 + H^2 k^2) U \csc \left( \frac{\text{dx} k}{2} \right) \end{array} \right) \\ & \end{array} \end{aligned}$$

$$\begin{aligned}
\text{Out[200]= Err} \parallel & \left\{ \left\{ \frac{i \left( \sqrt{3} k \sqrt{g H (3+H^2 k^2)} + 3 k U \right) dt}{3+H^2 k^2} - \frac{1}{2} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^2 dt^2 - \right. \right. \\
& \left. \frac{1}{6} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^3 dt^3 - \frac{1}{24} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^4 dt^4 + O[dt]^5 \right\} + \\
& \left( -\frac{1}{2} (k^2 U) dt + O[dt]^5 \right) dx + \left( \frac{i (9 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( \frac{1}{24} k^4 U dt + O[dt]^5 \right) dx^3 + \\
& \left( -\frac{i (54 H^2 k^7 + 19 H^4 k^9 + 2 H^6 k^{11}) U dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5, \\
& \left( -\frac{3 i k dt}{3+H^2 k^2} + O[dt]^5 \right) + \left( \frac{i (6 k^3 + H^2 k^5) dt}{4 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( \frac{i (-54 k^5 + H^4 k^9) dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5 \Big\}, \\
& \left\{ \left( \left( -i g H k + \frac{i H k U^2}{H + \frac{H^3 k^2}{3}} \right) dt + O[dt]^5 \right) + \left( -\frac{1}{2} (g H k^2) dt + O[dt]^5 \right) dx + \right. \\
& \left( \frac{i (18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2) dt}{12 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( \frac{1}{24} g H k^4 dt + O[dt]^5 \right) dx^3 + \\
& \left( -\frac{i (54 g H k^5 + 54 g H^3 k^7 + 18 g H^5 k^9 + 2 g H^7 k^{11} - 54 k^5 U^2 + H^4 k^9 U^2) dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5, \\
& \left( \frac{i \left( \sqrt{3} k \sqrt{g H (3+H^2 k^2)} - 3 k U \right) dt}{3+H^2 k^2} - \frac{1}{2} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^2 dt^2 - \right. \\
& \left. \frac{1}{6} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^3 dt^3 - \frac{1}{24} \left( -\frac{i \sqrt{3} k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^4 dt^4 + O[dt]^5 \right\} + \\
& \left( -\frac{1}{2} (k^2 U) dt + O[dt]^5 \right) dx + \left( \frac{i (36 k^3 + 15 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( \frac{1}{24} k^4 U dt + O[dt]^5 \right) dx^3 + \\
& \left( -\frac{i (108 k^5 + 54 H^2 k^7 + 17 H^4 k^9 + 2 H^6 k^{11}) U dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5 \Big\}
\end{aligned}$$



```

Out[201]= Eerr || \left(
\begin{array}{cc}
\left(\frac{i}{\left(3 U k+\sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right)}} k\right) \text {dt}\left\{H^2 k^2+3\right\}-\frac{1}{2}\right) \left(-i U
k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \left(H^2 k^2+3\right)^2 \text {dt}^2-\frac{1}{6}\right)
\left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \left(H^2 k^2+3\right)^3
\text {dt}^3-\frac{1}{24}\right) \left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \left(H^2
k^2+3\right)^4 \text {dt}^4+O\left(\text {dt}^5\right) \right) \left(-\frac{1}{2}\right) \left(k^2 U\right)
\text {dt}+O\left(\text {dt}^5\right) \right) \text {dx}+\left(\frac{i}{\left(2 H^4 k^7+9 H^2 k^5\right) U}
\text {dt}\right)\left\{12\left(H^2 k^2+3\right)^2+O\left(\text {dt}^5\right) \right) \text {dx}^2+\left(\frac{1}{24}\right)
k^4 U \text {dt}+O\left(\text {dt}^5\right) \right) \text {dx}^3+\left(-\frac{i}{\left(2 H^6 k^{11}+19 H^4
k^9+54 H^2 k^7\right) U} \text {dt}\right)\left\{240\left(H^2 k^2+3\right)^3+O\left(\text {dt}^5\right) \right)
\text {dx}^4+O\left(\text {dx}^5\right) \& \left(-\frac{3 i k}{\text {dt}}\right)\left\{H^2
k^2+3\right\}+O\left(\text {dt}^5\right) \right) \left(\frac{i}{\left(H^2 k^5+6 k^3\right) \text {dt}}\right)\left\{4\left(H^2
k^2+3\right)^2+O\left(\text {dt}^5\right) \right) \text {dx}^2+\left(\frac{i}{\left(H^4 k^9-54 k^5\right)
\text {dt}}\right)\left\{240\left(H^2 k^2+3\right)^3+O\left(\text {dt}^5\right) \right) \text {dx}^4+O\left(\text {dx}^5\right) \right) \backslash
\left(\frac{i H k U^2}{k^2 H^3}\right)\left\{3+H\right\}-i g H k\right)
\text {dt}+O\left(\text {dt}^5\right) \right) \left(-\frac{1}{2}\right) \left(g H k^2\right)
\text {dt}+O\left(\text {dt}^5\right) \right) \text {dx}+\left(\frac{i}{\left(2 g H^5 k^7+12 g H^3 k^5-3 H^2 U^2
k^5-18 U^2 k^3+18 g H k^3\right) \text {dt}}\right)\left\{12\left(H^2 k^2+3\right)^2+O\left(\text {dt}^5\right) \right)
\text {dx}^2+\left(\frac{1}{24}\right) g H k^4 \text {dt}+O\left(\text {dt}^5\right) \right) \text {dx}^3+\left(-\frac{i}{\left(2 g H^7 k^{11}+18 g H^5 k^9+H^4 U^2 k^9+54 g H^3 k^7-54 U^2 k^5+54 g H k^5\right)
\text {dt}}\right)\left\{240\left(H^2 k^2+3\right)^3+O\left(\text {dt}^5\right) \right) \text {dx}^4+O\left(\text {dx}^5\right) \right)
\& \left(\frac{i}{\left(\sqrt{3}\right) k \sqrt{g H \left(H^2 k^2+3\right)}}-3 k U\right) \text {dt}\left\{H^2
k^2+3\right\}-\frac{1}{2}\right) \left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \left(H^2
k^2+3\right)^2 \text {dt}^2-\frac{1}{6}\right) \left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \left(H^2 k^2+3\right)^3 \text {dt}^3-\frac{1}{24}\right) \left(-i U k-\frac{i}{\sqrt{3}} \sqrt{g H \left(H^2
k^2+3\right)} k\right) \left(H^2 k^2+3\right)^4 \text {dt}^4+O\left(\text {dt}^5\right) \right) \left(-\frac{1}{2}\right)
\left(k^2 U\right) \text {dt}+O\left(\text {dt}^5\right) \right) \text {dx}+\left(\frac{i}{\left(2 H^4 k^7+15
H^2 k^5+36 k^3\right) U} \text {dt}\right)\left\{12\left(H^2 k^2+3\right)^2+O\left(\text {dt}^5\right) \right)
\text {dx}^2+\left(\frac{1}{24}\right) k^4 U \text {dt}+O\left(\text {dt}^5\right) \right) \text {dx}^3+\left(-\frac{i}{\left(2 H^6 k^{11}+17 H^4 k^9+54 H^2 k^7+108 k^5\right) U} \text {dt}\right)\left\{240\left(H^2
k^2+3\right)^3+O\left(\text {dt}^5\right) \right) \text {dx}^4+O\left(\text {dx}^5\right) \right) \backslash
\end{array}
\right)

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```

In[202]:= KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
KurFWS = KurF /. ap -> 0 /. am -> (U - Sqrt[g*H]);
KurFWSeta =
  KurFWS /. fp -> (H*v + U*Rpp*n) /. fm -> (H*v + U*Rmp*n) /. qp -> Rpp*n /.
  qm -> Rmp*n;
KurFWSeta = KurFWSeta /. v -> (GGp*G + Gnp*n);
Kfnnp = FullSimplify[KurFWSeta /. G -> 0 /. n -> 1];
KfnGp = FullSimplify[KurFWSeta /. n -> 0 /. G -> 1];

```

```

Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA, {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
  fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA, {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA, {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}];

Text[Row[{" U < -Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn  ||  ", Kfnnp}]]
Text[Row[{"Fnn  ||  ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||  ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||  ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||  ", KfnGp}]]
Text[Row[{"FnG  ||  ", TeXForm[KfnGp]}]]

```

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Text[Row[{"FnG error  ||  ", FnG2TAr}]]
Text[Row[{"FnG error  ||  ", TeXForm[FnG2TAr]}]]]
Text[" "]
Text[Row[{"FGn  ||  ", KfGnp}]]]
Text[Row[{"FGn  ||  ", TeXForm[KfGnp]}]]]
Text[Row[{"FGn error  ||  ", FGn2TAr}]]]
Text[Row[{"FGn error  ||  ", TeXForm[FGn2TAr]}]]]
Text[" "]
Text[Row[{"FGG  ||  ", KfGGp}]]]
Text[Row[{"FGG  ||  ", TeXForm[KfGGp]}]]]
Text[Row[{"FGG error  ||  ", FGG2TAr}]]]
Text[Row[{"FGG error  ||  ", TeXForm[FGG2TAr]}]]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||  ", RKstepTayr}]]]
Text[Row[{"Omega error  ||  ", TeXForm[RKstepTayr]}]]]
Text[" "]
Text[Row[{"EA  ||  ", EA}]]]
Text[Row[{"EA  ||  ", TeXForm[EA]}]]]
Text[Row[{"Eerr  ||  ", Eerr}]]]
Text[Row[{"Eerr  ||  ", TeXForm[Eerr]}]]]

```

Out[236]=  $U < -\text{Sqrt}(gH)$

Out[237]=

Out[238]=  $\text{Fnn} \parallel \text{Gnp} H + \text{Rpp} U$

Out[239]=  $\text{Fnn} \parallel \text{\texttt{Gnp}} H + \text{\texttt{Rpp}} U$

Out[240]= 
$$\text{Fnn error} \parallel \left( -\frac{H^2 k^3 U w}{2(3+H^2 k^2)} \frac{dt^2}{dt} - \frac{i H^2 k^3 U w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left( \frac{1}{2} k^2 U dt + O[dt]^4 \right) dx +$$

$$\left( \frac{i(9 H^2 k^5 + 2 H^4 k^7) U dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{24} (k^4 U) dt + O[dt]^4 \right) dx^3 + O[dx]^4$$

Out[241]=  $\text{Fnn error} \parallel$

$$\begin{aligned} & \left( -\frac{\text{\texttt{dt}}^2}{2} \left( H^2 k^3 U w \right) \right) \{ 2 \left( H^2 k^2 + 3 \right) \} - \frac{i \text{\texttt{dt}}^3 H^2 k^3 U w^2}{6} \{ 6 \\ & \left( H^2 k^2 + 3 \right) \} + O \left( \text{\texttt{dt}}^4 \right) \} + \text{\texttt{dx}} \left\{ \frac{1}{2} k^2 U \right. \\ & \left. \text{\texttt{dt}} + O \left( \text{\texttt{dt}}^4 \right) \right\} + \text{\texttt{dx}}^2 \left\{ \frac{i \left( 2 H^4 k^7 + 9 H^2 k^5 \right) U \text{\texttt{dt}}}{12 \left( H^2 k^2 + 3 \right)^2} + O \left( \text{\texttt{dt}}^4 \right) \right\} + \text{\texttt{dx}}^3 \\ & \left\{ -\frac{1}{24} \left( k^4 U \right) \text{\texttt{dt}} + O \left( \text{\texttt{dt}}^4 \right) \right\} + O \left( \text{\texttt{dx}}^4 \right) \} \end{aligned}$$

Out[242]=

Out[243]=  $\text{FnG} \parallel \text{GGp} H$

Out[244]=  $\text{FnG} \parallel \text{\texttt{GGp}} H$

$$\text{Out[245]= FnG error} \parallel \left( -\frac{3(k w) dt^2}{2(3+H^2 k^2)} - \frac{i k w^2 dt^3}{2(3+H^2 k^2)} + O[dt]^4 \right) + \left( \frac{i(6 k^3 + H^2 k^5) dt}{4(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + O[dx]^4$$

$$\text{Out[246]= FnG error} \parallel \left( -\frac{3}{2} \text{dt}^2 (k w)^2 \left( H^2 k^2 + 3 \right) \right) - \frac{i}{2} \text{dt}^3 k w^2 \left( H^2 k^2 + 3 \right) + O(\text{dt}^4) + \text{dx}^2 \left( \frac{i}{2} \left( H^2 k^5 + 6 k^3 \right) \text{dt} \right) \text{dt}^4 \left( H^2 k^2 + 3 \right)^2 + O(\text{dt}^4) + O(\text{dx}^4)$$

Out[247]=

$$\text{Out[248]= FGn} \parallel H(g R_{pp} + G_{np} U)$$

$$\text{Out[249]= FGn} \parallel H(g \text{Rpp} + \text{Gnp} U)$$

$$\text{Out[250]= FGn error} \parallel \left( -\frac{(k(3 g H + g H^3 k^2 - 3 U^2) w) dt^2}{2(3+H^2 k^2)} - \frac{i k(3 g H + g H^3 k^2 - 3 U^2) w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left( \frac{1}{2} g H k^2 dt + O[dt]^4 \right) dx + \left( \frac{i(18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2) dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{24} (g H k^4) dt + O[dt]^4 \right) dx^3 + O[dx]^4$$

$$\text{Out[251]= FGn error} \parallel \left( -\frac{3}{2} \text{dt}^2 (k w \left( g H^3 k^2 + 3 g H - 3 U^2 \right))^2 \left( H^2 k^2 + 3 \right) \right) - \frac{i}{2} \text{dt}^3 k w^2 \left( g H^3 k^2 + 3 g H - 3 U^2 \right) \left( H^2 k^2 + 3 \right) + O(\text{dt}^4) + \text{dx} \left( \frac{1}{2} g H k^2 \text{dt} + O(\text{dt}^4) \right) \text{dt}^4 \left( H^2 k^2 + 3 \right)^2 + O(\text{dt}^4) + \text{dx}^2 \left( \frac{i}{2} \left( 2 g H^5 k^7 + 12 g H^3 k^5 - 3 H^2 U^2 k^5 - 18 U^2 k^3 + 18 g H k^3 \right) \text{dt} \right) \text{dt}^4 \left( H^2 k^2 + 3 \right)^2 + O(\text{dt}^4) + \text{dx}^3 \left( -\frac{1}{24} (g H k^4) \text{dt} + O(\text{dt}^4) \right) \text{dt}^4 \left( H^2 k^2 + 3 \right)^2 + O(\text{dt}^4) + O(\text{dx}^4)$$

Out[252]=

$$\text{Out[253]= FGG} \parallel (G_{Gp} H + R_{pp}) U$$

$$\text{Out[254]= FGG} \parallel U(\text{GGp} H + \text{Rpp})$$

$$\text{Out[255]= FGG error} \parallel \left( -\frac{(k(6+H^2 k^2) U w) dt^2}{2(3+H^2 k^2)} - \frac{i k(6+H^2 k^2) U w^2 dt^3}{6(3+H^2 k^2)} + O[dt]^4 \right) + \left( \frac{1}{2} k^2 U dt + O[dt]^4 \right) dx + \left( \frac{i(36 k^3 + 15 H^2 k^5 + 2 H^4 k^7) U dt}{12(3+H^2 k^2)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{24} (k^4 U) dt + O[dt]^4 \right) dx^3 + \left( -\frac{i(108 k^5 + 54 H^2 k^7 + 17 H^4 k^9 + 2 H^6 k^{11}) U dt}{240(3+H^2 k^2)^3} + O[dt]^4 \right) dx^4 + O[dx]^5$$

$$\text{Out[256]= FGG error} \parallel \left( -\frac{3}{2} \text{dt}^2 (k U w \left( H^2 k^2 + 6 \right))^2 \left( H^2 k^2 + 3 \right) \right) - \frac{i}{2} \text{dt}^3 k U w^2 \left( H^2 k^2 + 6 \right) \left( H^2 k^2 + 3 \right) + O(\text{dt}^4) + \text{dx} \left( \frac{1}{2} k^2 U \text{dt} + O(\text{dt}^4) \right) \text{dt}^4 \left( H^2 k^2 + 3 \right)^2 + O(\text{dt}^4) + \text{dx}^2 \left( \frac{i}{2} \left( 2 H^4 k^7 + 15 H^2 k^5 + 36 k^3 \right) U \text{dt} \right) \text{dt}^4 \left( H^2 k^2 + 3 \right)^2 + O(\text{dt}^4) + \text{dx}^3 \left( -\frac{1}{24} (k^4 U) \text{dt} + O(\text{dt}^4) \right) \text{dt}^4 \left( H^2 k^2 + 3 \right)^2 + O(\text{dt}^4) + \text{dx}^4 \left( -\frac{i}{240} \left( 2 H^6 k^{11} + 17 H^4 k^9 + 54 H^2 k^7 + 108 k^5 \right) U \text{dt} \right) \text{dt}^4 \left( H^2 k^2 + 3 \right)^2 + O(\text{dt}^4) + O(\text{dx}^5)$$

Out[257]=

Out[258]=

$$\begin{aligned}
\text{Out[259]= } \Omega \text{ error } & \parallel \left\{ \frac{i \left( \sqrt{3} \, k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^3 U \right)}{2 (3 + H^2 k^2)^2} - \frac{1}{3 (3 + H^2 k^2)^2} \right. \\
& \left( k^3 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( 3 g H + U \left( 2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) \right) dt^2 - \\
& \frac{1}{4 (3 + H^2 k^2)^3} i k^4 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( 3 g \left( \sqrt{3} H \sqrt{g H (3 + H^2 k^2)} + 9 H U + 3 H^3 k^2 U \right) + \right. \\
& \left. U^2 \left( H^4 k^4 U + 9 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + U \right) + 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 2 H^2 U \right) \right) \right) dt^3 + \\
& \frac{1}{5 (3 + H^2 k^2)^3} k^5 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left( 9 g^2 H^2 + 6 g H U \left( 2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^3 \left( 12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + 2 k^2 \left( 2 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 3 H^2 U \right) \right) \right) dt^4 + \\
& \left. O[dt]^5 \right) + \left( \frac{1}{4} i k^2 \left( \sqrt{3} \sqrt{\frac{g H}{3 + H^2 k^2}} + 2 U \right) - \frac{k^3 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( \sqrt{3} g H + 2 \sqrt{g H (3 + H^2 k^2)} U \right)}{4 \left( \sqrt{g H} (3 + H^2 k^2)^{3/2} \right)} - \right. \\
& \left( i k^4 \left( \sqrt{3} g H + 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left( 3 g H + U \left( 2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) dt^2 \right) / \\
& \left( 4 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) + \\
& \left( k^5 \left( \sqrt{3} g H + 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left( 3 g \left( \sqrt{3} H \sqrt{g H (3 + H^2 k^2)} + 9 H U + 3 H^3 k^2 U \right) + \right. \\
& \left. U^2 \left( H^4 k^4 U + 9 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + U \right) + 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 2 H^2 U \right) \right) \right) dt^3 \right) / \\
& \left( 4 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \left( i k^6 \left( \sqrt{3} g H + 2 \sqrt{g H (3 + H^2 k^2)} U \right) \right. \\
& \left( 9 g^2 H^2 + 6 g H U \left( 2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^3 \left( 12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + 2 k^2 \left( 2 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 3 H^2 U \right) \right) \right) \\
& \left. dt^4 \right) / \left( 4 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + O[dt]^5 \Bigg) dx + \\
& \left( - \left( \left( k^3 \left( 3 \sqrt{3} g H (13 + 3 H^2 k^2) + 16 \left( 3 \sqrt{g H (3 + H^2 k^2)} + k^2 \sqrt{g H^5 (3 + H^2 k^2)} \right) U \right) \right) / \right. \right. \\
& \left. \left( 96 \left( \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) \right) \right) - \\
& \left( i k^4 \left( g H \left( 144 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (369 + 234 H^2 k^2 + 37 H^4 k^4) U \right) + 4 \left( 63 \sqrt{g H (3 + H^2 k^2)} U^2 + \right. \right. \\
& \left. \left. 7 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 3 k^2 \left( 3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 14 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) \\
& dt \right) / \left( 96 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \left( k^5 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \left( g H \left( 171 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (477 + 306 H^2 k^2 + 49 H^4 k^4) U \right) + 5 \left( 72 \sqrt{g H (3 + H^2 k^2)} U^2 + \right. \right. \\
& \left. \left. 7 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 3 k^2 \left( 3 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 14 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) \\
& dt \Bigg) / \left( 96 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + O[dt]^5 \Bigg) dx +
\end{aligned}$$

$$\begin{aligned}
& \left( 8 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 3 k^2 \left( 3 \sqrt{g^3 H' (3 + H^2 k^2)} + 16 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \\
& dt^2 \Big/ \left( 96 \sqrt{g H (3 + H^2 k^2)^{7/2}} \right) + \left( i k^6 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^2 \right. \\
& \left( 468 \sqrt{g H (3 + H^2 k^2)} U^2 + 52 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \\
& \left. g H \left( 198 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (585 + 378 H^2 k^2 + 61 H^4 k^4) U \right) + \right. \\
& \left. 6 k^2 \left( 9 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 52 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt^3 \Big/ \left( 96 \sqrt{g H (3 + H^2 k^2)^{9/2}} \right) - \\
& \left( \left( k^7 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^3 \left( 576 \sqrt{g H (3 + H^2 k^2)} U^2 + 64 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \right. \right. \\
& \left. g H \left( 225 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (693 + 450 H^2 k^2 + 73 H^4 k^4) U \right) + \right. \\
& \left. \left. 3 k^2 \left( 21 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 128 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) \\
& dt^4 \Big/ \left( 96 \left( \sqrt{g H (3 + H^2 k^2)^{11/2}} \right) \right) + O[dt]^5 dx^2 + \\
& \left( - \left( \left( i k^4 \left( \sqrt{3} g H (33 + 7 H^2 k^2) + 16 \left( 3 \sqrt{g H (3 + H^2 k^2)} + k^2 \sqrt{g H^5 (3 + H^2 k^2)} \right) U \right) \right) \right) \right. \\
& \left. \left( 384 \sqrt{g H (3 + H^2 k^2)^{3/2}} \right) \right) + \\
& \left( k^5 \left( g H \left( 72 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (207 + 126 H^2 k^2 + 19 H^4 k^4) U \right) + \right. \right. \\
& \left. 16 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^2 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + \right. \right. \\
& \left. \left. k^2 \left( \sqrt{g^3 H^7 (3 + H^2 k^2)} + 6 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt \Big/ \left( 128 \sqrt{g H (3 + H^2 k^2)^{5/2}} \right) + \\
& \left( i k^6 \left( 9 \sqrt{3} g^2 H^2 (13 + 3 H^2 k^2) + 5 g H U \left( 96 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) + \right. \right. \\
& \left. 32 \left( 9 \sqrt{g H (3 + H^2 k^2)} U^3 + k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 2 k^2 \left( 2 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \right. \\
& \left. \left. \left. 3 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) dt^2 \Big/ \left( 128 \sqrt{g H (3 + H^2 k^2)^{5/2}} \right) - \\
& \left( \left( k^7 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( 24 \sqrt{3} g^2 H^2 (21 + 5 H^2 k^2) + \right. \right. \right. \\
& \left. 3 g H U \left( 717 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1011 + 646 H^2 k^2 + 103 H^4 k^4) U \right) + \right. \\
& \left. 5 \left( 288 \sqrt{g H (3 + H^2 k^2)} U^3 + 32 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \right. \\
& \left. \left. \left. 3 k^2 \left( 39 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 64 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \right) dt^3 \Big/ \\
& \left( 384 \left( \sqrt{g H (3 + H^2 k^2)^{7/2}} \right) \right) - \left( i k^8 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right)^2 \right. \\
& \left( 5 \sqrt{3} g^2 H^2 (45 + 11 H^2 k^2) + 720 \sqrt{g H (3 + H^2 k^2)} U^3 + 80 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + \right. \\
& g H U \left( 990 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1449 + 930 H^2 k^2 + 149 H^4 k^4) U \right) + \\
& \left. 2 k^2 \left( 137 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 240 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \\
& dt^4 \Big/ \left( 128 \sqrt{g H (3 + H^2 k^2)^{9/2}} \right) + O[dt]^5 dx^3 +
\end{aligned}$$

$$\begin{aligned}
& \left( \left( k^5 \left( 3 \sqrt{3} \, g H (543 + 146 H^2 k^2 + 15 H^4 k^4) + 256 \left( 9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + \right. \right. \right. \right. \\
& \quad \left. \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \right) / \left( 30720 \sqrt{g H (3 + H^2 k^2)^{5/2}} \right) + \\
& \left( i k^6 \left( 3 g H \left( 18432 \sqrt{g H (3 + H^2 k^2)} + 51039 \sqrt{3} U + 45735 \sqrt{3} H^2 k^2 U + \right. \right. \right. \\
& \quad \left. \left. \left. 1381 \sqrt{3} H^6 k^6 U + k^4 \left( 960 \sqrt{g H^9 (3 + H^2 k^2)} + 13717 \sqrt{3} H^4 U \right) \right) \right) + \right. \\
& \quad 128 \left( 837 \sqrt{g H (3 + H^2 k^2)} U^2 + 279 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 31 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. \left. 27 k^2 \left( 7 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 31 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) \right) dt \Big/ \\
& \left( 92160 \sqrt{g H (3 + H^2 k^2)^{7/2}} \right) - \left( \left( k^7 \left( 9 \sqrt{3} \, g^2 H^2 (12999 + 6258 H^2 k^2 + 775 H^4 k^4) + \right. \right. \right. \\
& \quad g H U \left( 617661 \sqrt{3} H^2 k^2 U + 20551 \sqrt{3} H^6 k^6 U + 81 \left( 5888 \sqrt{g H (3 + H^2 k^2)} + \right. \right. \\
& \quad \left. \left. 8053 \sqrt{3} U \right) + 3 k^4 \left( 12288 \sqrt{g H^9 (3 + H^2 k^2)} + 65021 \sqrt{3} H^4 U \right) \right) + \\
& \quad \left. 128 \left( 2322 \sqrt{g H (3 + H^2 k^2)} U^3 + 774 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 86 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} \right. \right. \\
& \quad \left. \left. U^3 + 9 k^2 \left( 229 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 258 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) \right) \Big/ \\
& \left( 92160 \left( \sqrt{g H (3 + H^2 k^2)^{7/2}} \right) \right) - \frac{1}{30720 \sqrt{g H (3 + H^2 k^2)^{9/2}}} \\
& i k^8 \left( k^8 U^3 \left( 21253 \sqrt{3} \, g H^9 + 7776 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& \quad 243 \left( 832 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 4423 \sqrt{3} \, g^2 H^2 U + 2592 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. g H U^2 \left( 8832 \sqrt{g H (3 + H^2 k^2)} + 7823 \sqrt{3} U \right) \right) + 3 k^6 U \left( 9181 \sqrt{3} \, g^2 H^8 + \right. \\
& \quad \left. 31104 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + 4 g H^7 U \left( 5312 \sqrt{g H (3 + H^2 k^2)} + 21787 \sqrt{3} U \right) \right) + \\
& \quad 27 k^2 \left( 35319 \sqrt{3} \, g^2 H^4 U + 91580 \sqrt{3} \, g H^3 U^3 + \right. \\
& \quad \left. 64 \left( 59 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 1155 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 486 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) \right) + \\
& \quad 9 k^4 \left( 31231 \sqrt{3} \, g^2 H^6 U + 2 g H^5 U^2 \left( 34336 \sqrt{g H (3 + H^2 k^2)} + 67003 \sqrt{3} U \right) + \right. \\
& \quad \left. \left. 288 \left( 5 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 162 \sqrt{g H^9 (3 + H^2 k^2)} U^4 \right) \right) \right) dt^3 + \\
& \frac{1}{92160 \sqrt{g H (3 + H^2 k^2)^{11/2}}} k^9 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \\
& \left( k^8 U^3 \left( 111559 \sqrt{3} \, g H^9 + 42368 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& \quad 27 k^2 \left( 18078 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 172047 \sqrt{3} \, g^2 H^4 U + 372075 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. 477940 \sqrt{3} \, g H^3 U^3 + 169472 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) + \\
& \quad 81 \left( 11603 \sqrt{g^5 H^5 (3 + H^2 k^2)} + 63917 \sqrt{3} \, g^2 H^2 U + 42368 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad \left. g H U^2 \left( 132513 \sqrt{g H (3 + H^2 k^2)} + 122207 \sqrt{3} U \right) \right) + 3 k^6 U \left( 45573 \sqrt{3} \, g^2 H^8 + \right.
\end{aligned}$$

$$\begin{aligned}
& 169472 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + g H^7 U \left( 108089 \sqrt{g H (3 + H^2 k^2)} + 456644 \sqrt{3} U \right) + \\
& 9 k^4 \left( 7035 \sqrt{g^5 H^{13} (3 + H^2 k^2)} + 153703 \sqrt{3} g^2 H^6 U + 254208 \sqrt{g H^9 (3 + H^2 k^2)} U^4 + \right. \\
& \left. g H^5 U^2 \left( 347651 \sqrt{g H (3 + H^2 k^2)} + 700818 \sqrt{3} U \right) \right) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5, \\
& \left( \frac{i \left( -\sqrt{3} k \sqrt{g H (3 + H^2 k^2)} + 3 k U + H^2 k^3 U \right)^2}{2 (3 + H^2 k^2)^2} dt - \frac{1}{3 (3 + H^2 k^2)^2} \left( k^3 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \right. \\
& \left. \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) dt^2 - \right. \\
& \left. \frac{1}{4 (3 + H^2 k^2)^3} i k^4 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \left( 3 g H \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^2 \left( -9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) dt^3 + \\
& \left. \frac{1}{5 (3 + H^2 k^2)^3} k^5 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right. \\
& \left( 9 g^2 H^2 + 6 g H U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^3 \left( -12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left( -4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) dt^4 + O[ \\
& dt]^5 \Big) + \\
& \left( -\frac{1}{4} i k^2 \left( \sqrt{3} \sqrt{\frac{g H}{3 + H^2 k^2}} - 2 U \right) + \frac{k^3 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \left( \sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) dt}{4 \sqrt{g H} (3 + H^2 k^2)^{3/2}} + \right. \\
& \left( i k^4 \left( \sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right) dt^2 \right) / \\
& \left( 4 \sqrt{g H} (3 + H^2 k^2)^{3/2} \right) - \\
& \left( \left( k^5 \left( \sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \left( 3 g H \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \right. \right. \\
& \left. U^2 \left( -9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U - 3 k^2 \left( \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} - 2 H^2 U \right) \right) \right) \right) \\
& dt^3 \Big) / \left( 4 \left( \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) \right) - \left( i k^6 \left( \sqrt{3} g H - 2 \sqrt{g H (3 + H^2 k^2)} U \right) \right. \\
& \left( 9 g^2 H^2 + 6 g H U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 3 (3 + H^2 k^2) U \right) + \right. \\
& \left. U^3 \left( -12 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 9 U + H^4 k^4 U + k^2 \left( -4 \sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 6 H^2 U \right) \right) \right) \\
& dt^4 \Big) / \left( 4 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + O[dt]^5 \Big) dx +
\end{aligned}$$



$$\begin{aligned}
& \left( \frac{k^3 \left( 3\sqrt{3} \, g H (13+3 H^2 k^2) - 16 \left( 3\sqrt{g H (3+H^2 k^2)} U + k^2 \sqrt{g H^5 (3+H^2 k^2)} U \right) \right)}{96 \sqrt{g H} (3+H^2 k^2)^{3/2}} + \right. \\
& \left( i k^4 \left( g H \left( -144 \sqrt{g H (3+H^2 k^2)} + \sqrt{3} (369 + 234 H^2 k^2 + 37 H^4 k^4) U \right) - \right. \\
& \quad 4 \left( 63 \sqrt{g H (3+H^2 k^2)} U^2 + 7 k^4 \sqrt{g H^9 (3+H^2 k^2)} U^2 + \right. \\
& \quad \left. \left. 3 k^2 \left( 3 \sqrt{g^3 H^7 (3+H^2 k^2)} + 14 \sqrt{g H^5 (3+H^2 k^2)} U^2 \right) \right) \right) dt \Big/ \left( 96 \sqrt{g H} (3+H^2 k^2)^{5/2} \right) - \\
& \left( \left( k^5 \left( \sqrt{3} \sqrt{g H (3+H^2 k^2)} - (3+H^2 k^2) U \right) \left( g H \left( 171 \sqrt{g H (3+H^2 k^2)} - \sqrt{3} (477 + 306 H^2 k^2 + \right. \right. \right. \\
& \quad \left. \left. 49 H^4 k^4) U \right) + 5 \left( 72 \sqrt{g H (3+H^2 k^2)} U^2 + 8 k^4 \sqrt{g H^9 (3+H^2 k^2)} U^2 + \right. \right. \\
& \quad \left. \left. 3 k^2 \left( 3 \sqrt{g^3 H^7 (3+H^2 k^2)} + 16 \sqrt{g H^5 (3+H^2 k^2)} U^2 \right) \right) \right) dt^2 \Big/ \\
& \left( 96 \left( \sqrt{g H} (3+H^2 k^2)^{7/2} \right) \right) + \left( i k^6 \left( \sqrt{3} \sqrt{g H (3+H^2 k^2)} - (3+H^2 k^2) U \right)^2 \right. \\
& \quad \left( 468 \sqrt{g H (3+H^2 k^2)} U^2 + 52 k^4 \sqrt{g H^9 (3+H^2 k^2)} U^2 + \right. \\
& \quad g H \left( 198 \sqrt{g H (3+H^2 k^2)} - \sqrt{3} (585 + 378 H^2 k^2 + 61 H^4 k^4) U \right) + \\
& \quad \left. \left. 6 k^2 \left( 9 \sqrt{g^3 H^7 (3+H^2 k^2)} + 52 \sqrt{g H^5 (3+H^2 k^2)} U^2 \right) \right) \right) dt^3 \Big/ \left( 96 \sqrt{g H} (3+H^2 k^2)^{9/2} \right) + \\
& \left( k^7 \left( \sqrt{3} \sqrt{g H (3+H^2 k^2)} - (3+H^2 k^2) U \right)^3 \left( 576 \sqrt{g H (3+H^2 k^2)} U^2 + 64 k^4 \sqrt{g H^9 (3+H^2 k^2)} U^2 + \right. \right. \\
& \quad g H \left( 225 \sqrt{g H (3+H^2 k^2)} - \sqrt{3} (693 + 450 H^2 k^2 + 73 H^4 k^4) U \right) + \\
& \quad \left. \left. 3 k^2 \left( 21 \sqrt{g^3 H^7 (3+H^2 k^2)} + 128 \sqrt{g H^5 (3+H^2 k^2)} U^2 \right) \right) \right) dt^4 \Big/ \left( 96 \sqrt{g H} (3+H^2 k^2)^{11/2} \right) + \\
& \left. O[dt]^5 \right) dx^2 + \left( \frac{i k^4 \left( \sqrt{3} \, g H (33+7 H^2 k^2) - 16 \left( 3\sqrt{g H (3+H^2 k^2)} U + k^2 \sqrt{g H^5 (3+H^2 k^2)} U \right) \right)}{384 \sqrt{g H} (3+H^2 k^2)^{3/2}} + \right. \\
& \left( k^5 \left( g H \left( 72 \sqrt{g H (3+H^2 k^2)} - \sqrt{3} (207 + 126 H^2 k^2 + 19 H^4 k^4) U \right) + 16 \left( 9 \sqrt{g H (3+H^2 k^2)} U^2 + \right. \right. \\
& \quad k^4 \sqrt{g H^9 (3+H^2 k^2)} U^2 + k^2 \left( \sqrt{g^3 H^7 (3+H^2 k^2)} + 6 \sqrt{g H^5 (3+H^2 k^2)} U^2 \right) \right) dt \Big/ \\
& \left( 128 \sqrt{g H} (3+H^2 k^2)^{5/2} \right) + \left( i k^6 \left( -9 \sqrt{3} \, g^2 H^2 (13 + 3 H^2 k^2) - \right. \right. \\
& \quad 5 g H U \left( -96 \sqrt{g H (3+H^2 k^2)} + \sqrt{3} (129 + 82 H^2 k^2 + 13 H^4 k^4) U \right) + \\
& \quad 32 \left( 9 \sqrt{g H (3+H^2 k^2)} U^3 + k^4 \sqrt{g H^9 (3+H^2 k^2)} U^3 + 2 k^2 \left( 2 \sqrt{g^3 H^7 (3+H^2 k^2)} U + \right. \right. \\
& \quad \left. \left. 3 \sqrt{g H^5 (3+H^2 k^2)} U^3 \right) \right) dt^2 \Big/ \left( 128 \sqrt{g H} (3+H^2 k^2)^{5/2} \right) + \\
& \left( k^7 \left( -\sqrt{3} \sqrt{g H (3+H^2 k^2)} + (3+H^2 k^2) U \right) \left( 24 \sqrt{3} \, g^2 H^2 (21 + 5 H^2 k^2) + \right. \right. \\
& \quad \left. \left. 3 g H U \left( -717 \sqrt{g H (3+H^2 k^2)} + \sqrt{3} (1011 + 646 H^2 k^2 + 103 H^4 k^4) U \right) - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 5 \left( 288 \sqrt{g H (3 + H^2 k^2)} U^3 + 32 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 3 k^2 \left( 39 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \\
& \quad \left. \left. 64 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) dt^3 \Big/ \left( 384 \sqrt{g H} (3 + H^2 k^2)^{7/2} \right) + \\
& \left( i k^8 \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - (3 + H^2 k^2) U \right)^2 \left( 5 \sqrt{3} g^2 H^2 (45 + 11 H^2 k^2) + \right. \right. \\
& \quad g H U \left( -990 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (1449 + 930 H^2 k^2 + 149 H^4 k^4) U \right) - \\
& \quad 2 \left( 360 \sqrt{g H (3 + H^2 k^2)} U^3 + 40 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + k^2 \left( 137 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + \right. \right. \\
& \quad \left. \left. 240 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) dt^4 \Big/ \left( 128 \sqrt{g H} (3 + H^2 k^2)^{9/2} \right) + O[dt]^5 \Big) dx^3 + \\
& \left( \left( k^5 \left( -3 \sqrt{3} g H (543 + 146 H^2 k^2 + 15 H^4 k^4) + 256 \left( 9 \sqrt{g H (3 + H^2 k^2)} + 6 k^2 \sqrt{g H^5 (3 + H^2 k^2)} + \right. \right. \right. \right. \\
& \quad \left. \left. k^4 \sqrt{g H^9 (3 + H^2 k^2)} \right) U \right) \Big/ \left( 30720 \sqrt{g H} (3 + H^2 k^2)^{5/2} \right) + \\
& \quad \left( i k^6 \left( -3 g H \left( -18432 \sqrt{g H (3 + H^2 k^2)} + 51039 \sqrt{3} U + 45735 \sqrt{3} H^2 k^2 U + \right. \right. \right. \\
& \quad \left. \left. 1381 \sqrt{3} H^6 k^6 U + k^4 \left( -960 \sqrt{g H^9 (3 + H^2 k^2)} + 13717 \sqrt{3} H^4 U \right) \right) \right) + \\
& \quad 128 \left( 837 \sqrt{g H (3 + H^2 k^2)} U^2 + 279 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^2 + 31 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^2 + \right. \\
& \quad \left. 27 k^2 \left( 7 \sqrt{g^3 H^7 (3 + H^2 k^2)} + 31 \sqrt{g H^5 (3 + H^2 k^2)} U^2 \right) \right) dt \Big/ \\
& \quad \left( 92160 \sqrt{g H} (3 + H^2 k^2)^{7/2} \right) + \left( k^7 \left( 9 \sqrt{3} g^2 H^2 (12999 + 6258 H^2 k^2 + 775 H^4 k^4) + \right. \right. \\
& \quad g H U \left( 617661 \sqrt{3} H^2 k^2 U + 20551 \sqrt{3} H^6 k^6 U + 81 \left( -5888 \sqrt{g H (3 + H^2 k^2)} + \right. \right. \\
& \quad \left. \left. 8053 \sqrt{3} U \right) - 3 k^4 \left( 12288 \sqrt{g H^9 (3 + H^2 k^2)} - 65021 \sqrt{3} H^4 U \right) \right) - \\
& \quad 128 \left( 2322 \sqrt{g H (3 + H^2 k^2)} U^3 + 774 k^4 \sqrt{g H^9 (3 + H^2 k^2)} U^3 + 86 k^6 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + \right. \\
& \quad \left. 9 k^2 \left( 229 \sqrt{g^3 H^7 (3 + H^2 k^2)} U + 258 \sqrt{g H^5 (3 + H^2 k^2)} U^3 \right) \right) dt^2 \Big/ \\
& \quad \left( 92160 \sqrt{g H} (3 + H^2 k^2)^{7/2} \right) - \frac{1}{30720 \sqrt{g H} (3 + H^2 k^2)^{9/2}} i k^8 \\
& \quad \left( k^8 U^3 \left( -21253 \sqrt{3} g H^9 + 7776 \sqrt{g H^{17} (3 + H^2 k^2)} U \right) + \right. \\
& \quad 243 \left( 832 \sqrt{g^5 H^5 (3 + H^2 k^2)} - 4423 \sqrt{3} g^2 H^2 U + 2592 \sqrt{g H (3 + H^2 k^2)} U^4 + \right. \\
& \quad g H U^2 \left( 8832 \sqrt{g H (3 + H^2 k^2)} - 7823 \sqrt{3} U \right) - 3 k^6 U \left( 9181 \sqrt{3} g^2 H^8 - \right. \\
& \quad \left. 31104 \sqrt{g H^{13} (3 + H^2 k^2)} U^3 + 4 g H^7 U \left( -5312 \sqrt{g H (3 + H^2 k^2)} + 21787 \sqrt{3} U \right) \right) + \\
& \quad \left. 27 k^2 \left( -35319 \sqrt{3} g^2 H^4 U - 91580 \sqrt{3} g H^3 U^3 + \right. \right. \\
& \quad \left. \left. 64 \left( 59 \sqrt{g^5 H^9 (3 + H^2 k^2)} + 1155 \sqrt{g^3 H^7 (3 + H^2 k^2)} U^2 + 486 \sqrt{g H^5 (3 + H^2 k^2)} U^4 \right) \right) \right) +
\end{aligned}$$

$$\begin{aligned} & 9k^4 \left( -31231 \sqrt{3} g^2 H^6 U + 2gH^5 U^2 \left( 34336 \sqrt{gH(3+H^2 k^2)} - 67003 \sqrt{3} U \right) + \right. \\ & \quad \left. 288 \left( 5 \sqrt{g^5 H^{13} (3+H^2 k^2)} + 162 \sqrt{gH^9 (3+H^2 k^2)} U^4 \right) \right) dt^3 - \frac{1}{92160 \left( \sqrt{gH(3+H^2 k^2)}^{11/2} \right)} \\ & \left( k^9 \left( \sqrt{3} \sqrt{gH(3+H^2 k^2)} - (3+H^2 k^2) U \right) \left( k^8 U^3 \left( -111559 \sqrt{3} gH^9 + 42368 \sqrt{gH^{17} (3+H^2 k^2)} U \right) + \right. \right. \\ & \quad \left. 27k^2 \left( 18078 \sqrt{g^5 H^9 (3+H^2 k^2)} - 172047 \sqrt{3} g^2 H^4 U + 372075 \sqrt{g^3 H^7 (3+H^2 k^2)} U^2 - \right. \right. \\ & \quad \left. 477940 \sqrt{3} gH^3 U^3 + 169472 \sqrt{gH^5 (3+H^2 k^2)} U^4 \right) + \\ & \quad 9k^4 \left( 7035 \sqrt{g^5 H^{13} (3+H^2 k^2)} - 153703 \sqrt{3} g^2 H^6 U + 254208 \sqrt{gH^9 (3+H^2 k^2)} U^4 + \right. \\ & \quad \left. gH^5 U^2 \left( 347651 \sqrt{gH(3+H^2 k^2)} - 700818 \sqrt{3} U \right) \right) + \\ & \quad 81 \left( 11603 \sqrt{g^5 H^5 (3+H^2 k^2)} - 63917 \sqrt{3} g^2 H^2 U + 42368 \sqrt{gH(3+H^2 k^2)} U^4 + \right. \\ & \quad \left. gHU^2 \left( 132513 \sqrt{gH(3+H^2 k^2)} - 122207 \sqrt{3} U \right) \right) - \\ & \quad 3k^6 U \left( 45573 \sqrt{3} g^2 H^8 - 169472 \sqrt{gH^{13} (3+H^2 k^2)} U^3 + \right. \\ & \quad \left. gH^7 U \left( -108089 \sqrt{gH(3+H^2 k^2)} + 456644 \sqrt{3} U \right) \right) \Big) dt^4 + O[dt]^5 \Big) dx^4 + O[dx]^5 \Big\} \end{aligned}$$

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Out[260]= Omega error ||
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[illegible]

[illegible]

$$\begin{aligned}
& k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)} U \right)\{30720 \\
& \sqrt{g H} \left(H^2 k^2+3\right)^{5/2}\}+\frac{i k^6 \left(3 g H \left(1381 \sqrt{3} H^6 U k^6+\left(13717 \sqrt{3} U H^4+960 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+45735 \sqrt{3} H^2 U k^2+51039 \sqrt{3} U+18432 \sqrt{g H \left(H^2 k^2+3\right)}\right) \right)+128 \left(31 \sqrt{g H^{13}} \left(H^2 k^2+3\right)\right) U^2 k^6+279 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^2 k^4+27 \left(31 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^2+7 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}\right) k^2+837 \sqrt{g H \left(H^2 k^2+3\right)} U^2 \right) \right) \text{dt}}{92160 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}-\frac{\left(k^7 \left(9 \sqrt{3} g^2 \left(775 H^4 k^4+6258 H^2 k^2+12999\right) H^2+g U \left(20551 \sqrt{3} H^6 U k^6+3 \left(65021 \sqrt{3} U H^4+12288 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+617661 \sqrt{3} H^2 U k^2+81 \left(8053 \sqrt{3} U+5888 \sqrt{g H \left(H^2 k^2+3\right)}\right) \right) H+128 \left(86 \sqrt{g H^{13}} \left(H^2 k^2+3\right)\right) U^3 k^6+774 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(258 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+229 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+2322 \sqrt{g H \left(H^2 k^2+3\right)} U^3 \right) \right) \text{dt}}{92160 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)}-\frac{i k^8 \left(U^3 \left(21253 \sqrt{3} g H^9+7776 \sqrt{g H^{17}} \left(H^2 k^2+3\right)\right) U \right) k^8+3 U \left(9181 \sqrt{3} g^2 H^8+4 g U \left(21787 \sqrt{3} U+5312 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7+31104 \sqrt{g H^{13}} \left(H^2 k^2+3\right)\right) U^3 \right) k^6+9 \left(31231 \sqrt{3} g^2 U H^6+2 g U^2 \left(67003 \sqrt{3} U+34336 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+288 \left(162 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5 \sqrt{g^5 H^{13}} \left(H^2 k^2+3\right)\right) \right) k^4+27 \left(35319 \sqrt{3} g^2 U H^4+91580 \sqrt{3} g U^3 H^3+64 \left(486 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+1155 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+59 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+243 \left(2592 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(7823 \sqrt{3} U+8832 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+4423 \sqrt{3} g^2 H^2 U+832 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right) \right) \text{dt}}{30720 \sqrt{g H} \left(H^2 k^2+3\right)^{9/2}}+\frac{k^9 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(U^3 \left(111559 \sqrt{3} g H^9+42368 \sqrt{g H^{17}} \left(H^2 k^2+3\right)\right) U \right) k^8+3 U \left(45573 \sqrt{3} g^2 H^8+g U \left(456644 \sqrt{3} U+108089 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7+169472 \sqrt{g H^{13}} \left(H^2 k^2+3\right)\right) U^3 \right) k^6+9 \left(153703 \sqrt{3} g^2 U H^6+g U^2 \left(700818 \sqrt{3} U+347651 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+254208 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+7035 \sqrt{g^5 H^{13}} \left(H^2 k^2+3\right)\right) k^4+27 \left(172047 \sqrt{3} g^2 U H^4+477940 \sqrt{3} g U^3 H^3+169472 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+372075 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+18078 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(42368 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(122207 \sqrt{3} U+132513 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+63917 \sqrt{3} g^2 H^2 U+11603 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right) \right) \text{dt}}{92160 \sqrt{g H} \left(H^2 k^2+3\right)^{11/2}}+O\left(\text{dt}^5\right) \text{dt}^4+O\left(\text{dx}^5\right) \text{dt}, \left(\frac{i \left(H^2 U k^3+3 U k-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \text{dt}}{2 \left(H^2 k^2+3\right)^2}-\frac{\left(k^3 \left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \text{dt}}{\left(3 g H+U \left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \text{dt}}\right) \text{dt}^3 \left(H^2 k^2+3\right)^2-\frac{i k^4 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \text{dt}}{\left(H^2 U \right) k^2+9 U-9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U^2+3 g H \left(3 \left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \text{dt}}\right) \text{dt}^3+4 \left(H^2 k^2+3\right)^3+\frac{k^5 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \text{dt}}{\left(H^2 k^2+3\right)^3}\right)
\end{aligned}$$

$$\begin{aligned}
& \left( \left( H^4 U k^4 + \left( 6 H^2 U - 4 \sqrt{3} \right) \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) \right) k^2 + 9 \\
& U - 12 \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U^3 + 6 g H \left( 3 \left( H^2 k^2 + 3 \right) \right. \\
& U - 2 \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U + 9 g^2 H^2 \right) \text{dt}^4 \{ 5 \left( H^2 \right. \\
& k^2 + 3 \right) ^3 + O \left( \text{dt}^5 \right) \right) + \left( -\frac{1}{4} i k^2 \left( \sqrt{3} \sqrt{\frac{g}{H}} \right. \right. \\
& H \{ H^2 k^2 + 3 \} - 2 U \right) + \frac{k^3 \left( \left( H^2 k^2 + 3 \right) U - \sqrt{3} \sqrt{g H \left( H^2 \right.} \right. \\
& k^2 + 3 \right) \right) \left( \sqrt{3} g H - 2 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U \right) \text{dt} \{ 4 \sqrt{g H} \\
& \left( H^2 k^2 + 3 \right) ^{3/2} \} + \frac{i k^4 \left( \sqrt{3} g H - 2 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U \right) \\
& \left( 3 g H + U \left( \left( H^2 k^2 + 3 \right) U - 2 \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \right) \\
& \text{dt}^2 \{ 4 \sqrt{g H} \left( H^2 k^2 + 3 \right) ^{3/2} \} - \frac{\left( k^5 \left( \sqrt{3} g H - 2 \sqrt{g H} \right. \right. \\
& \left. \left( H^2 k^2 + 3 \right) U \right) \left( \left( H^4 U k^4 - 3 \left( \sqrt{3} \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) - 2 \right. \right. \right. \\
& H^2 U \right) k^2 + 9 U - 9 \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U^2 + 3 g H \left( 3 \left( H^2 \right. \right. \\
& k^2 + 3 \right) U - \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \right) \right) \text{dt}^3 \{ 4 \left( \sqrt{g} \right. \\
& H \} \left( H^2 k^2 + 3 \right) ^{5/2} \right) - \frac{i k^6 \left( \sqrt{3} g H - 2 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) \\
& U \right) \left( \left( H^4 U k^4 + \left( 6 H^2 U - 4 \sqrt{3} \right) \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) \right) \\
& k^2 + 9 U - 12 \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U^3 + 6 g H \left( 3 \left( H^2 k^2 + 3 \right) \right. \\
& U - 2 \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) U + 9 g^2 H^2 \right) \text{dt}^4 \{ 4 \sqrt{g H} \\
& \left( H^2 k^2 + 3 \right) ^{5/2} \} + O \left( \text{dt}^5 \right) \right) \text{dx} + \left( \frac{k^3 \left( 3 \sqrt{3} g \right. \right. \\
& H \left( 3 H^2 k^2 + 13 \right) - 16 \left( \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \right) U k^2 + 3 \sqrt{g H \left( H^2 \right.} \right. \\
& k^2 + 3 \right) \} U \right) \right) \{ 96 \sqrt{g H} \left( H^2 k^2 + 3 \right) ^{3/2} \} + \frac{i k^4 \left( g H \left( \sqrt{3} \right. \right. \\
& \left. \left( 37 H^4 k^4 + 234 H^2 k^2 + 369 \right) U - 144 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) - 4 \left( 7 \right. \\
& \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} \} U^2 k^4 + 3 \left( 14 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \} U^2 + 3 \right. \\
& \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} \right) k^2 + 63 \sqrt{g H \left( H^2 k^2 + 3 \right)} \} U^2 \right) \right) \\
& \text{dt} \{ 96 \sqrt{g H} \left( H^2 k^2 + 3 \right) ^{5/2} \} - \frac{\left( k^5 \left( \sqrt{3} \sqrt{g H \left( H^2 \right.} \right. \right. \\
& k^2 + 3 \right) - \left( H^2 k^2 + 3 \right) U \right) \left( g H \left( 171 \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) - \sqrt{3} \right) \\
& \left( 49 H^4 k^4 + 306 H^2 k^2 + 477 \right) U \right) + 5 \left( 8 \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} \} U^2 \right. \\
& k^4 + 3 \left( 16 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \} U^2 + 3 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} \right) \right) \\
& k^2 + 72 \sqrt{g H \left( H^2 k^2 + 3 \right)} \} U^2 \right) \right) \text{dt}^2 \{ 96 \left( \sqrt{g H} \right. \\
& \left. \left( H^2 k^2 + 3 \right) ^{7/2} \right) + \frac{i k^6 \left( \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} \right) - \left( H^2 \right. \\
& k^2 + 3 \right) U \right) ^2 \left( 52 \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} \} U^2 k^4 + 6 \left( 52 \sqrt{g H^5} \right. \\
& \left. \left( H^2 k^2 + 3 \right) \} U^2 + 9 \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} \right) k^2 + 468 \sqrt{g H \left( H^2 \right.} \\
& k^2 + 3 \right) \} U^2 + g H \left( 198 \sqrt{g H \left( H^2 k^2 + 3 \right)} - \sqrt{3} \right) \left( 61 H^4 k^4 + 378 \right. \\
& H^2 k^2 + 585 \right) U \right) \right) \text{dt}^3 \{ 96 \sqrt{g H} \left( H^2 k^2 + 3 \right) ^{9/2} \} + \frac{k^7 \\
& \left( \sqrt{3} \sqrt{g H \left( H^2 k^2 + 3 \right)} - \left( H^2 k^2 + 3 \right) U \right) ^3 \left( 64 \sqrt{g} \right. \\
& H^9 \left( H^2 k^2 + 3 \right) \} U^2 k^4 + 3 \left( 128 \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \} U^2 + 21 \right. \\
& \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} \right) k^2 + 576 \sqrt{g H \left( H^2 k^2 + 3 \right)} \} U^2 + g \\
& H \left( 225 \sqrt{g H \left( H^2 k^2 + 3 \right)} - \sqrt{3} \right) \left( 73 H^4 k^4 + 450 H^2 k^2 + 693 \right) \\
& U \right) \right) \text{dt}^4 \{ 96 \sqrt{g H} \left( H^2 k^2 + 3 \right) ^{11/2} \} + O \left( \text{dt}^5 \right) \right) \\
& \text{dx}^2 + \left( \frac{i k^4 \left( \sqrt{3} g H \left( 7 H^2 k^2 + 33 \right) - 16 \left( \sqrt{g H^5 \left( H^2 \right.} \right. \right. \right. \\
& k^2 + 3 \right) \} U k^2 + 3 \sqrt{g H \left( H^2 k^2 + 3 \right)} \} U \right) \right) \{ 384 \sqrt{g H} \left( H^2 \right. \\
& k^2 + 3 \right) ^{3/2} \} + \frac{k^5 \left( g H \left( 72 \sqrt{g H \left( H^2 k^2 + 3 \right)} - \sqrt{3} \right) \left( 19 H^4 \right. \\
& k^4 + 126 H^2 k^2 + 207 \right) U \right) + 16 \left( \sqrt{g H^9 \left( H^2 k^2 + 3 \right)} \} U^2 k^4 + \left( 6 \right. \\
& \sqrt{g H^5 \left( H^2 k^2 + 3 \right)} \} U^2 + \sqrt{g^3 H^7 \left( H^2 k^2 + 3 \right)} \right) k^2 + 9 \sqrt{g H} \\
& \left. \left( H^2 k^2 + 3 \right) \} U^2 \right) \right) \text{dt} \{ 128 \sqrt{g H} \left( H^2 k^2 + 3 \right) ^{5/2} \} + \frac{i}{4}
\end{aligned}$$



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k^2+3\right))\right) H^/-1694/2 \sqrt{g H^{13}} \left(H^2 k^2+3\right) U^3\right) k^6+9 \left(-153/03
\sqrt{3} g^2 U H^6+g U^2 \left(347651 \sqrt{g H \left(H^2 k^2+3\right)}-700818 \sqrt{3} U\right)
H^5+254208 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+7035 \sqrt{g^5 H^{13}} \left(H^2 k^2+3\right)\right)
k^4+27 \left(-172047 \sqrt{3} g^2 U H^4-477940 \sqrt{3} g U^3 H^3+169472 \sqrt{g H^5 \left(H^2
k^2+3\right)} U^4+372075 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+18078 \sqrt{g^5 H^9 \left(H^2
k^2+3\right)}\right) k^2+81 \left(42368 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(132513
\sqrt{g H \left(H^2 k^2+3\right)}-122207 \sqrt{3} U\right) U^2-63917 \sqrt{3} g^2 H^2 U+11603
\sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4\{92160 \left(\sqrt{g H} \left(H^2
k^2+3\right)\right)^{11/2}\right)\}+O\left(\text{dt}^5\right) \text{dx}^4+O\left(\text{dx}^5\right)\right)

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Out[261]=

$$\text{Out[262]} = \text{EA} \parallel \left\{ \left\{ 1 + \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) H^2 k^3 U \text{Csc}\left[\frac{dx k}{2}\right]}{(6 + 2 H^2 k^2) w}, \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) H k \text{Csc}\left[\frac{dx k}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right) w} \right\}, \right. \\ \left. \left\{ \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) k (g H (3 + H^2 k^2) - 3 U^2) \text{Csc}\left[\frac{dx k}{2}\right]}{(6 + 2 H^2 k^2) w}, 1 + \frac{i e^{\frac{i dx k}{2}} (1 - e^{-i dx k}) (-1 + e^{i dt w}) k (6 + H^2 k^2) U \text{Csc}\left[\frac{dx k}{2}\right]}{(6 + 2 H^2 k^2) w} \right\} \right\}$$

Out[263]=

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EA \parallel \left(
\begin{array}{cc}
\frac{i e^{\frac{i}{\text{dx}} k}{2}} \left(1 - e^{-i \text{dx} k}\right) \left(1 + e^{i \text{dt} w}\right) H^2 U \text{csc}\left(\frac{\text{dx} k}{2}\right) k^3}{\left(2 H^2 k^2 + 6\right) w} + 1 \\
& \frac{i e^{\frac{i}{\text{dx}} k}{2}} \left(1 - e^{-i \text{dx} k}\right) \left(1 + e^{i \text{dt} w}\right) H k \text{csc}\left(\frac{\text{dx} k}{2}\right)}{2 \left(H + \frac{H^3 k^2}{3}\right) w} \\
\frac{i e^{\frac{i}{\text{dx}} k}{2}} \left(1 - e^{-i \text{dx} k}\right) \left(1 + e^{i \text{dt} w}\right) k \left(g H \left(H^2 k^2 + 3\right) - 3 U^2\right) \text{csc}\left(\frac{\text{dx} k}{2}\right)}{\left(2 H^2 k^2 + 6\right) w} & \frac{i e^{\frac{i}{\text{dx}} k}{2}} \left(1 - e^{-i \text{dx} k}\right) \left(1 + e^{i \text{dt} w}\right) k \left(6 + H^2 k^2\right) U \text{csc}\left(\frac{\text{dx} k}{2}\right)}{\left(2 H^2 k^2 + 6\right) w}
\end{array}
\right)

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$$\begin{aligned}
\text{Out[264]= Err} \parallel & \left\{ \left\{ \frac{i \left( \sqrt{3} \, k \sqrt{g H (3+H^2 k^2)} + 3 k U \right) dt}{3+H^2 k^2} - \frac{1}{2} \left( -\frac{i \sqrt{3} \, k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^2 dt^2 - \right. \right. \\
& \left. \frac{1}{6} \left( -\frac{i \sqrt{3} \, k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^3 dt^3 - \frac{1}{24} \left( -\frac{i \sqrt{3} \, k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^4 dt^4 + O[dt]^5 \right\} + \\
& \left( \frac{1}{2} k^2 U dt + O[dt]^5 \right) dx + \left( \frac{i (9 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( -\frac{1}{24} (k^4 U) dt + O[dt]^5 \right) dx^3 + \\
& \left( -\frac{i (54 H^2 k^7 + 19 H^4 k^9 + 2 H^6 k^{11}) U dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5, \\
& \left( -\frac{3 i k dt}{3+H^2 k^2} + O[dt]^5 \right) + \left( \frac{i (6 k^3 + H^2 k^5) dt}{4 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( \frac{i (-54 k^5 + H^4 k^9) dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5 \Big\}, \\
& \left\{ \left( \left( -i g H k + \frac{i H k U^2}{H + \frac{H^3 k^2}{3}} \right) dt + O[dt]^5 \right) + \left( \frac{1}{2} g H k^2 dt + O[dt]^5 \right) dx + \right. \\
& \left( \frac{i (18 g H k^3 + 12 g H^3 k^5 + 2 g H^5 k^7 - 18 k^3 U^2 - 3 H^2 k^5 U^2) dt}{12 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( -\frac{1}{24} (g H k^4) dt + O[dt]^5 \right) dx^3 + \\
& \left( -\frac{i (54 g H k^5 + 54 g H^3 k^7 + 18 g H^5 k^9 + 2 g H^7 k^{11} - 54 k^5 U^2 + H^4 k^9 U^2) dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5, \\
& \left( \frac{i \left( \sqrt{3} \, k \sqrt{g H (3+H^2 k^2)} - 3 k U \right) dt}{3+H^2 k^2} - \frac{1}{2} \left( -\frac{i \sqrt{3} \, k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^2 dt^2 - \right. \\
& \left. \frac{1}{6} \left( -\frac{i \sqrt{3} \, k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^3 dt^3 - \frac{1}{24} \left( -\frac{i \sqrt{3} \, k \sqrt{g H (3+H^2 k^2)}}{3+H^2 k^2} - i k U \right)^4 dt^4 + O[dt]^5 \right\} + \\
& \left( \frac{1}{2} k^2 U dt + O[dt]^5 \right) dx + \left( \frac{i (36 k^3 + 15 H^2 k^5 + 2 H^4 k^7) U dt}{12 (3+H^2 k^2)^2} + O[dt]^5 \right) dx^2 + \left( -\frac{1}{24} (k^4 U) dt + O[dt]^5 \right) dx^3 + \\
& \left( -\frac{i (108 k^5 + 54 H^2 k^7 + 17 H^4 k^9 + 2 H^6 k^{11}) U dt}{240 (3+H^2 k^2)^3} + O[dt]^5 \right) dx^4 + O[dx]^5 \Big\}
\end{aligned}$$

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Out[265]= Eerr || \left(
\begin{array}{cc}
\left(\frac{i}{\left(3 U k+\sqrt{3}\right) \sqrt{g H \left(H^2 k^2+3\right)}} k\right) \text {dt}\right)\left\{H^2\right. \\
k^2+3\}-\frac{1}{2}\left(-i U k-\frac{i}{\sqrt{3}}\right) \sqrt{g H \left(H^2 k^2+3\right)} k\right\} H^2 \\
k^2+3\right)^2 \text {dt}^2-\frac{1}{6}\left(-i U k-\frac{i}{\sqrt{3}}\right) \sqrt{g H \left(H^2 k^2+3\right)} \\
k\right\} H^2 k^2+3\right)^3 \text {dt}^3-\frac{1}{24}\left(-i U k-\frac{i}{\sqrt{3}}\right) \sqrt{g H \left(H^2\right.} \\
k^2+3\right) k\right\} H^2 k^2+3\right)^4 \text {dt}^4+O\left(\text {dt}^5\right) \text {right}+\left(\frac{1}{2}\right) \\
k^2 U \text {dt}+O\left(\text {dt}^5\right) \text {right} \text {dx}+\left(\frac{i}{\left(2 H^4 k^7+9\right.}\right. \\
H^2 k^5\right) U \text {dt}\right)\left\{12 \left(H^2 k^2+3\right)^2+O\left(\text {dt}^5\right) \text {right}\right\} \\
\text {dx}^2+\left(-\frac{1}{24}\right) \left(k^4 U\right) \text {dt}+O\left(\text {dt}^5\right) \text {right} \\
\text {dx}^3+\left(-\frac{i}{\left(2 H^6 k^{11}+19 H^4 k^9+54 H^2 k^7\right)} U \text {dt}\right)\left\{240 \left(H^2\right.\right. \\
k^2+3\right)^3+O\left(\text {dt}^5\right) \text {right}\} \text {dx}^4+O\left(\text {dx}^5\right) \& \left(-\frac{3}{i k}\right. \\
\left.\text {dt}\right)\left\{H^2 k^2+3\right\}+O\left(\text {dt}^5\right) \text {right}+\left(\frac{i}{\left(H^2 k^5+6 k^3\right)} \text {dt}\right)\left\{4\right. \\
\left.\left(H^2 k^2+3\right)^2+O\left(\text {dt}^5\right) \text {right}\} \text {dx}^2+\left(\frac{i}{\left(H^4 k^9-54 k^5\right)}\right. \\
\left.\text {dt}\right)\left\{240 \left(H^2 k^2+3\right)^3+O\left(\text {dt}^5\right) \text {right}\} \text {dx}^4+O\left(\text {dx}^5\right) \backslash\right. \\
\left.\left(\frac{i}{\left(H k U^2\right)\left(k^2 H^3\right)\{3\}+H}\right)-i g H k\right) \text {dt}+O\left(\text {dt}^5\right) \text {right}+\left(\frac{1}{2}\right)\left\{2\right. \\
g H k^2 \text {dt}+O\left(\text {dt}^5\right) \text {right}\} \text {dx}+\left(\frac{i}{\left(2 g H^5 k^7+12 g H^3 k^5-3 H^2\right.}\right. \\
U^2 k^5-18 U^2 k^3+18 g H k^3\right) \text {dt}\right)\left\{12 \left(H^2 k^2+3\right)^2+O\left(\text {dt}^5\right) \text {right}\right\} \\
\text {dx}^2+\left(-\frac{1}{24}\right) \left(g H k^4\right) \text {dt}+O\left(\text {dt}^5\right) \text {right} \\
\text {dx}^3+\left(-\frac{i}{\left(2 g H^7 k^{11}+18 g H^5 k^9+H^4 U^2 k^9+54 g H^3 k^7-54\right.}\right. \\
U^2 k^5+54 g H k^5\right) \text {dt}\right)\left\{240 \left(H^2 k^2+3\right)^3+O\left(\text {dt}^5\right) \text {right}\right\} \\
\text {dx}^4+O\left(\text {dx}^5\right) \& \left(\frac{i}{\left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}\right)-3 k}\right. \\
U\right) \text {dt}\right)\left\{H^2 k^2+3\right\}-\frac{1}{2}\left(-i U k-\frac{i}{\sqrt{3}}\right) \sqrt{g H \left(H^2 k^2+3\right)} \\
k\right\} H^2 k^2+3\right)^2 \text {dt}^2-\frac{1}{6}\left(-i U k-\frac{i}{\sqrt{3}}\right) \sqrt{g H \left(H^2\right.} \\
k^2+3\right) k\right\} H^2 k^2+3\right)^3 \text {dt}^3-\frac{1}{24}\left(-i U k-\frac{i}{\sqrt{3}}\right) \sqrt{g H} \\
\left(H^2 k^2+3\right) k\right\} H^2 k^2+3\right)^4 \text {dt}^4+O\left(\text {dt}^5\right) \text {right}+\left(\frac{1}{2}\right) \\
k^2 U \text {dt}+O\left(\text {dt}^5\right) \text {right} \text {dx}+\left(\frac{i}{\left(2 H^4 k^7+15 H^2\right.}\right. \\
k^5+36 k^3\right) U \text {dt}\right)\left\{12 \left(H^2 k^2+3\right)^2+O\left(\text {dt}^5\right) \text {right}\right\} \\
\text {dx}^2+\left(-\frac{1}{24}\right) \left(k^4 U\right) \text {dt}+O\left(\text {dt}^5\right) \text {right} \\
\text {dx}^3+\left(-\frac{i}{\left(2 H^6 k^{11}+17 H^4 k^9+54 H^2 k^7+108 k^5\right)} U \text {dt}\right)\left\{240\right. \\
\left.\left(H^2 k^2+3\right)^3+O\left(\text {dt}^5\right) \text {right}\} \text {dx}^4+O\left(\text {dx}^5\right) \backslash\right. \\
\end{array}
\right)

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In[266]:=