In[420]:=

```
H In Reals;
dx In Reals;
g In Reals;
k In Reals;
dt In Reals;
dx > 0;
H > 0;
g > 0;
k > 0;
dt > 0;
w = √3 √(g H) k √(1/(3 + H^2 k^2)) + k U;
w1 = (√3 k √(g H (3 + H^2 k^2)) + 3 k U + H^2 k^3 U)/(3 + H^2 k^2);
```

In[439]:=

```
wo1dt = i (√3 k √(g H (3 + H^2 k^2)) + 3 k U + H^2 k^3 U)^2 dt/(2 (3 + H^2 k^2)^2);
wo1dt == i * dt/2 * ((√3 k √(g H (3 + H^2 k^2)) + 3 k U + H^2 k^3 U)/(3 + H^2 k^2))^2;
wo1dt == i * dt/2 * (√3 k √(g H (3 + H^2 k^2))/(3 + H^2 k^2) + k * U)^2;
wo1dt == i * dt/2 * (k * Sqrt[g * H] Sqrt[3/(3 + H^2 k^2)] + k * U)^2;
FullSimplify[wo1dt - i * dt/2 * (k * Sqrt[g * H] Sqrt[3/(3 + H^2 k^2)] + k * U)^2];
wo1dtRed = i * dt/2 * (wp)^2
wo1dx = -1/4 i k^2 (2 √(g H) + √3 U/√(3 + H^2 k^2)) dx
```

Out[444]= $\frac{1}{2}\, i\, dt\, wp^2$

Out[445]= $-\frac{1}{4}\, i\, dx\, k^2 \left(2\sqrt{g\,H} + \frac{\sqrt{3}\,U}{\sqrt{3 + H^2\,k^2}}\right)$

In[730]:=
$$\text{wo2dt} = \frac{1}{6\left(3+H^2k^2\right)^2}k^3\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)dt^2;$$

$$\frac{1}{6\left(3+H^2k^2\right)}dt^2\,k^2\,(wp)\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right);$$

$$\text{wo2dtRed} = \frac{dt^2\ (wp)\text{^}3}{6}$$

$$\text{wo2dx} = \frac{k^3\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)^2U\right)}{24\left(3+H^2k^2\right)^2}dx^2;$$

$$\text{FullSimplify}\left[\left(\frac{k^3\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}\right)}{24\left(3+H^2k^2\right)^2}+k\text{^}3*U/12\right)dx^2-\text{wo2dx}\right];$$

$$\left(\frac{k^3\left(-3\sqrt{3}\ \sqrt{g\,H}\right)}{24\left(3+H^2k^2\right)^{3/2}}+k\text{^}3*U/12\right)dx^2-\text{wo2dx};$$

$$\text{wo2dxRed} = k\text{^}3/12\left(\frac{\left(-3\sqrt{3}\ \sqrt{g\,H}\right)}{2\left(3+H^2k^2\right)^{3/2}}+U\right)$$

Out[732]=
$$\frac{dt^2\,wp^3}{6}$$

Out[736]=
$$\frac{1}{12}k^3\left(-\frac{3\sqrt{3}\ \sqrt{g\,H}}{2\left(3+H^2k^2\right)^{3/2}}+U\right)$$

In[737]:=
$$\text{wo2FEMdt} = \frac{1}{6\left(3+H^2k^2\right)^2}k^3\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+3\,U+H^2k^2\,U\right)$$
$$\left(3\,g\,H+2\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}\ U+3\,U^2+H^2k^2\,U^2\right)dt^2;$$

$$\frac{dt^2\,k^2\,(wp)\left(3\,g\,H+2\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}\ U+3\,U^2+H^2k^2\,U^2\right)}{6\left(3+H^2k^2\right)^1};$$

$$\text{test1} = \text{Expand[w\^{}2]} - \frac{k^2\left(3\,g\,H+2\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}\ U+3\,U^2+H^2k^2\,U^2\right)}{\left(3+H^2k^2\right)^1};$$

$$\text{wo2FEMdtred} = \frac{dt^2\,(wp\text{\^{}}3)}{6}$$

$$\text{wo2FEMdx} = \frac{1}{240\left(3+H^2k^2\right)^2}\left(42\sqrt{3}\,k^3\sqrt{g\,H\left(3+H^2k^2\right)}+\right.$$
$$\left.15\sqrt{3}\,H^2k^5\sqrt{g\,H\left(3+H^2k^2\right)}+180\,k^3\,U+120\,H^2k^5\,U+20\,H^4k^7\,U\right)dx^2;$$

$$\text{FullSimplify[wo2FEMdx]};$$

$$\frac{dx^2\,k^3\left(\sqrt{3}\ \sqrt{g\,H}\ \sqrt{\left(3+H^2k^2\right)}\ \left(42\ +15\ H^2k^2\right)+20\left(3+H^2k^2\right)^2U\right)}{240\left(3+H^2k^2\right)^2};$$

$$\text{wo2FEMdxRed} = \frac{dx^2\,k^3\left(\sqrt{3}\ \sqrt{g\,H}\ \left(42\ +15\ H^2k^2\right)/\left(3+H^2k^2\right)^{3/2}+20\ U\right)}{240}$$

Out[740]=
$$\frac{dt^2\,wp^3}{6}$$

Out[744]=
$$\frac{1}{240}\,dx^2\,k^3\left(\frac{\sqrt{3}\ \sqrt{g\,H}\ \left(42+15\,H^2\,k^2\right)}{\left(3+H^2\,k^2\right)^{3/2}}+20\,U\right)$$

In[766]:=
$$\text{wo3dt} = -\frac{1}{24\left(3+H^2k^2\right)^3}\,i\,k^4\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\left(3\,g\left(\sqrt{3}\,H\sqrt{g\,H\left(3+H^2k^2\right)}+9\,H\,U+3\,H^3k^2U\right)+U^2\left(H^4k^4U+9\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+U\right)+3\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2k^2\right)}+2\,H^2U\right)\right)\right)dt^3;$$

$$-\frac{1}{24\left(3+H^2k^2\right)^2}i\,k^3\,(wp)\left(3\,g\left(\sqrt{3}\,H\sqrt{g\,H\left(3+H^2k^2\right)}+9\,H\,U+3\,H^3k^2U\right)+U^2\left(H^4k^4U+9\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+U\right)+3\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2k^2\right)}+2\,H^2U\right)\right)\right)dt^3;$$

$$\text{Expand}\left[\frac{1}{\left(3+H^2k^2\right)^2}k^3\left(3\,g\left(\sqrt{3}\,H\sqrt{g\,H\left(3+H^2k^2\right)}+9\,H\,U+3\,H^3k^2U\right)+U^2\left(H^4k^4U+9\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}+U\right)+3\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2k^2\right)}+2\,H^2U\right)\right)\right)\right];$$

$$\text{Expand}[w\text{^}3];$$

$$\text{Expand}\left[\left(3+H^2k^2\right)^2\right];$$

$$\text{wo3dtRed} = -\frac{i\,(wp)\text{^}4\,dt^3}{24}$$

$$\text{wo3dx} = -\frac{i\,k^4\left(2\,g\,H\left(3+H^2k^2\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}\,U\right)}{24\sqrt{g\,H}\,\left(3+H^2k^2\right)}dx^3;$$

$$\text{FullSimplify}[\text{wo3dx}];$$

$$-\frac{i\,dx^3\,k^4\left(2\,g\,H\left(3+H^2k^2\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}\,U\right)}{24\sqrt{g\,H}\,\left(3+H^2k^2\right)};$$

$$-\frac{1}{24}i\,dx^3\,k^4\left(2\,g\,H\left(3+H^2k^2\right)\Big/\left(\sqrt{g\,H}\,\left(3+H^2k^2\right)\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}\,U\Big/\left(\sqrt{g\,H}\,\left(3+H^2k^2\right)\right)\right);$$

$$\text{wo3dxRed} = -\frac{1}{24}i\,dx^3\,k^4\left(2\sqrt{g\,H}+\frac{\sqrt{3}\,U}{\sqrt{\left(3+H^2k^2\right)}}\right)$$

Out[771]=
$$-\frac{1}{24}i\,dt^3\,wp^4$$

Out[776]=
$$-\frac{1}{24}i\,dx^3\,k^4\left(2\sqrt{g\,H}+\frac{\sqrt{3}\,U}{\sqrt{3+H^2k^2}}\right)$$