In[1255]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx / 2, xj + dx / 2}] / (dx);
qjnbar = qjbar /. t → tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x → xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx / 2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / 3 * k^2;
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca;
fn1Gca0A = fn1A /. Gca → 0;
fn1eca0A = fn1A /. eca → 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca /. Gh → RA * Gca;
fG1Gca0A = fG1A /. Gca → 0;
fG1eca0A = fG1A /. eca → 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
WfrommatA =
  Simplify[1 - MtA * dt * Eigenvalues[MatA], {k > 0, H > 0, g > 0, dx > 0, dt > 0}];
```

$$wAp = U * k + \frac{\sqrt{3}\ k \sqrt{g\,H\left(3 + H^2 k^2\right)}}{3 + H^2 k^2};$$

$$wAm = U * k - \frac{\sqrt{3}\ k \sqrt{g\,H\left(3 + H^2 k^2\right)}}{3 + H^2 k^2};$$

In[1291]:=
```
M2 = 1
Series[M2 - MA, {dx, 0, 10}]
```

Out[1291]= 1

Out[1292]= $-\frac{k^2\,dx^2}{24} - \frac{7\,k^4\,dx^4}{5760} - \frac{31\,k^6\,dx^6}{967\,680} - \frac{127\,k^8\,dx^8}{154\,828\,800} - \frac{73\,k^{10}\,dx^{10}}{3\,503\,554\,560} + O[dx]^{11}$

In[1293]:=
```
Rm = (1 + I * Sin[k * dx] / 2)
Series[Rm - RA, {dx, 0, 4}]
Rp = Exp[I * k * dx] * (1 - I * Sin[k * dx] / 2)
Series[Rp - RA, {dx, 0, 4}]
```

Out[1293]= $1 + \frac{1}{2}\,i\,\text{Sin}[dx\,k]$

Out[1294]= $\frac{k^2\,dx^2}{12} - \frac{1}{12}\,i\,k^3\,dx^3 + \frac{k^4\,dx^4}{720} + O[dx]^5$

Out[1295]= $e^{i\,dx\,k}\left(1 - \frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)$

Out[1296]= $\frac{k^2\,dx^2}{12} + \frac{1}{6}\,i\,k^3\,dx^3 - \frac{89\,k^4\,dx^4}{720} + O[dx]^5$

In[1297]:=
```
GRHSp1 = -Exp[-I * k * dx / 2] + 2 + 4 * Exp[I * k * dx / 2] +
  Exp[I * k * dx] * (4 * Exp[-I * k * dx / 2] + 2 - Exp[I * k * dx / 2])
GRHSp1 = GRHSp1 / Exp[I * k * dx / 2];
GRHSp1 = Expand[GRHSp1];
GRHSp1 = ExpToTrig[GRHSp1]
GRHSp2 = Exp[-I * k * dx / 2] - 8 + 7 * Exp[I * k * dx / 2] +
  Exp[I * k * dx] * (7 * Exp[-I * k * dx / 2] - 8 + Exp[I * k * dx / 2])
GRHSp2 = GRHSp2 / Exp[I * k * dx / 2];
GRHSp2 = Expand[GRHSp2];
GRHSp2 = ExpToTrig[GRHSp2]
```

Out[1297]= $2 - e^{-\frac{1}{2} i\, dx\, k} + 4\, e^{\frac{i\, dx\, k}{2}} + e^{i\, dx\, k} \left(2 + 4\, e^{-\frac{1}{2} i\, dx\, k} - e^{\frac{i\, dx\, k}{2}}\right)$

Out[1300]= $8 + 4 \operatorname{Cos}\left[\frac{dx\, k}{2}\right] - 2 \operatorname{Cos}[dx\, k]$

Out[1301]= $-8 + e^{-\frac{1}{2} i\, dx\, k} + 7\, e^{\frac{i\, dx\, k}{2}} + e^{i\, dx\, k} \left(-8 + 7\, e^{-\frac{1}{2} i\, dx\, k} + e^{\frac{i\, dx\, k}{2}}\right)$

Out[1304]= $14 - 16 \operatorname{Cos}\left[\frac{dx\, k}{2}\right] + 2 \operatorname{Cos}[dx\, k]$

In[1305]:=
```
GGLHS = dx / 6 * (Rp + Rm)
GG2 = GGLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2)
Series[GG2, {dx, 0, 3}];
Series[GGA, {dx, 0, 3}];
Series[GG2 - GGA, {dx, 0, 5}]
```

Out[1305]= $\frac{1}{6} dx \left(1 + e^{i\, dx\, k} \left(1 - \frac{1}{2} i \operatorname{Sin}[dx\, k]\right) + \frac{1}{2} i \operatorname{Sin}[dx\, k]\right)$

Out[1306]= $\left(dx \left(1 + e^{i\, dx\, k} \left(1 - \frac{1}{2} i \operatorname{Sin}[dx\, k]\right) + \frac{1}{2} i \operatorname{Sin}[dx\, k]\right)\right) \Big/ \left(6 \left(\frac{1}{30} dx\, H \left(8 + 4 \operatorname{Cos}\left[\frac{dx\, k}{2}\right] - 2 \operatorname{Cos}[dx\, k]\right) + \frac{H^3 \left(14 - 16 \operatorname{Cos}\left[\frac{dx\, k}{2}\right] + 2 \operatorname{Cos}[dx\, k]\right)}{9\, dx}\right)\right)$

Out[1309]= $\frac{\left(12\, k^2 + 5\, H^2\, k^4\right) dx^2}{40\, H \left(3 + H^2\, k^2\right)^2} + \frac{i \left(12\, k^3 + 5\, H^2\, k^5\right) dx^3}{80\, H \left(3 + H^2\, k^2\right)^2} + \frac{\left(-6651\, k^4 - 4680\, H^2\, k^6 - 820\, H^4\, k^8\right) dx^4}{4800\, H \left(3 + H^2\, k^2\right)^3} - \frac{i \left(6291\, k^5 + 4410\, H^2\, k^7 + 770\, H^4\, k^9\right) dx^5}{9600\, H \left(3 + H^2\, k^2\right)^3} + O[dx]^6$

In[1310]:=
```
GnLHS = -U * (dx / 6) * (Rp + Rm)
Gn2 = GnLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2)
Series[Gn2, {dx, 0, 3}];
Series[GnA, {dx, 0, 3}];
Series[Gn2 - GnA, {dx, 0, 5}]
```

Out[1310]= $$-\frac{1}{6}\,dx\,U\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,Sin[dx\,k]\right)+\frac{1}{2}\,i\,Sin[dx\,k]\right)$$

Out[1311]= $$-\left(\left(dx\,U\left(1+e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,Sin[dx\,k]\right)+\frac{1}{2}\,i\,Sin[dx\,k]\right)\right)\Big/\left(6\left(\frac{1}{30}\,dx\,H\left(8+4\,Cos\left[\frac{dx\,k}{2}\right]-2\,Cos[dx\,k]\right)+\frac{H^3\left(14-16\,Cos\left[\frac{dx\,k}{2}\right]+2\,Cos[dx\,k]\right)}{9\,dx}\right)\right)\right)$$

Out[1314]= $$-\frac{\left(\left(12\,k^2+5\,H^2\,k^4\right)U\right)dx^2}{40\left(H\left(3+H^2\,k^2\right)^2\right)}-\frac{i\left(12\,k^3+5\,H^2\,k^5\right)U\,dx^3}{80\,H\left(3+H^2\,k^2\right)^2}+\frac{\left(6651\,k^4+4680\,H^2\,k^6+820\,H^4\,k^8\right)U\,dx^4}{4800\,H\left(3+H^2\,k^2\right)^3}+\frac{i\left(6291\,k^5+4410\,H^2\,k^7+770\,H^4\,k^9\right)U\,dx^5}{9600\,H\left(3+H^2\,k^2\right)^3}+O[dx]^6$$

In[1315]:=
```
GcLHS = -U * H * dx / 3
Gc2 = GcLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2)
Series[Gc2, {dx, 0, 3}]
Series[GcA, {dx, 0, 3}]
Series[Gc2 - GcA, {dx, 0, 5}]
```

Out[1315]= $$-\frac{1}{3}\,dx\,H\,U$$

Out[1316]= $$-\left((dx\,H\,U)\Big/\left(3\left(\frac{1}{30}\,dx\,H\left(8+4\,Cos\left[\frac{dx\,k}{2}\right]-2\,Cos[dx\,k]\right)+\frac{H^3\left(14-16\,Cos\left[\frac{dx\,k}{2}\right]+2\,Cos[dx\,k]\right)}{9\,dx}\right)\right)\right)$$

Out[1317]= $$-\frac{3\,U}{3+H^2\,k^2}+\frac{\left(18\,k^2+5\,H^2\,k^4\right)U\,dx^2}{40\left(3+H^2\,k^2\right)^2}+O[dx]^4$$

Out[1318]= $$-\frac{H\,U}{H+\frac{H^3\,k^2}{3}}$$

Out[1319]= $$\frac{\left(18\,k^2+5\,H^2\,k^4\right)U\,dx^2}{40\left(3+H^2\,k^2\right)^2}-\frac{\left(\left(423\,k^4+200\,H^2\,k^6+20\,H^4\,k^8\right)U\right)dx^4}{1600\left(3+H^2\,k^2\right)^3}+O[dx]^6$$

In[1320]:=
```
Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]]
Text[Row[{"Fnn and FnG "}]]
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnn = FullSimplify[KurFWSeta /. G → 0 /. n → 1]
KfnG = FullSimplify[KurFWSeta /. n → 0 /. G → 1]
Kfnn = Kfnn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]]
KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGn = FullSimplify[KurFWSG /. G → 0 /. n → 1]
KfGG = FullSimplify[KurFWSG /. n → 0 /. G → 1]
KfGn = KfGn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2  + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2^2 / 2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ]
```

Out[1320]= $-\text{Sqrt}[g*H] < U < \text{Sqrt}[g*H]$

Out[1321]= Fnn and FnG

Out[1326]= $\frac{1}{2}\left(2\,\text{Gnp}\,H + \text{Rpp}\left(-\sqrt{g\,H} + U\right) + \text{Rmp}\left(\sqrt{g\,H} + U\right)\right)$

Out[1327]= GGp H

Out[1332]= $\left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{i\left(54\,k^3 + 45\,H^2\,k^5 + 10\,H^4\,k^7\right)\,U\,dt}{120\left(3+H^2\,k^2\right)^2} + O[dt]^4\right) dx^2 + \left(-\frac{1}{8}\left(\sqrt{g\,H}\,k^4\right)\,dt + O[dt]^4\right) dx^3 +$
$\left(\frac{i\left(729\,k^5\,U + 2610\,H^2\,k^7\,U + 1570\,H^4\,k^9\,U + 260\,H^6\,k^{11}\,U\right)\,dt}{4800\left(3+H^2\,k^2\right)^3} + O[dt]^4\right) dx^4 + O[dx]^5$

Out[1335]= $\left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)} + O[dt]^4\right) + \left(-\frac{i\left(12\,k^3 + 5\,H^2\,k^5\right)\,dt}{40\left(3+H^2\,k^2\right)^2} + O[dt]^4\right) dx^2 +$
$\left(\frac{i\left(6291\,k^5 + 4410\,H^2\,k^7 + 770\,H^4\,k^9\right)\,dt}{4800\left(3+H^2\,k^2\right)^3} + O[dt]^4\right) dx^4 + O[dx]^5$

Out[1336]= FGn and FGG

Out[1339]= $\frac{1}{2}\left(g\,H\,(\text{Rmp}+\text{Rpp}) + \left(2\,\text{Gnp}\,H + \sqrt{g\,H}\,(\text{Rmp}-\text{Rpp})\right)\,U\right)$

Out[1340]= $\frac{1}{2}\left(\sqrt{g\,H}\,\text{Rmp} - \sqrt{g\,H}\,\text{Rpp} + (2\,\text{GGp}\,H + \text{Rmp} + \text{Rpp})\,U\right)$

Out[1345]= $\left(-\frac{\left(k\left(3\,g\,H + g\,H^3\,k^2 - 3\,U^2\right)\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\left(3\,g\,H + g\,H^3\,k^2 - 3\,U^2\right)\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\left(\left(i\left(90\,g\,H\,k^3 + 60\,g\,H^3\,k^5 + 10\,g\,H^5\,k^7 - 36\,k^3\,U^2 - 15\,H^2\,k^5\,U^2\right)\,dt\right) \Big/ \left(120\left(3+H^2\,k^2\right)^2\right)\right) +\right.$
$\left. O[dt]^4\right) dx^2 + \left(-\frac{1}{8}\left(\sqrt{g\,H}\,k^4\,U\right)\,dt + O[dt]^4\right) dx^3 +$
$\left(\left(i\left(7020\,g\,H\,k^5 + 7020\,g\,H^3\,k^7 + 2340\,g\,H^5\,k^9 + 260\,g\,H^7\,k^{11} - 6291\,k^5\,U^2 - 4410\,H^2\,k^7\,U^2 - \right.\right.\right.$
$\left.\left.\left. 770\,H^4\,k^9\,U^2\right)\,dt\right) \Big/ \left(4800\left(3+H^2\,k^2\right)^3\right) + O[dt]^4\right) dx^4 + O[dx]^5$

Out[1349]= $\left(-\frac{\left(k\left(6+H^2\,k^2\right)\,U\,w\right)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\left(6+H^2\,k^2\right)\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{i\left(126\,k^3 + 75\,H^2\,k^5 + 10\,H^4\,k^7\right)\,U\,dt}{120\left(3+H^2\,k^2\right)^2} + O[dt]^4\right) dx^2 + \left(-\frac{1}{8}\left(\sqrt{g\,H}\,k^4\right)\,dt + O[dt]^4\right) dx^3 +$
$\left(\left(i\left(13\,311\,k^5\,U + 11\,430\,H^2\,k^7\,U + 3110\,H^4\,k^9\,U + 260\,H^6\,k^{11}\,U\right)\,dt\right) \Big/ \left(4800\left(3+H^2\,k^2\right)^3\right) + O[dt]^4\right)$
$dx^4 + O[dx]^5$

Out[1350]= W : omega

Out[1355]= $\left\{\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)\,U\right)\right.$

$$\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)dt^2+$$
$$\frac{1}{8\left(3+H^2\,k^2\right)^2}i\,k^4\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^3-$$
$$\left(\left(k^5\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\left(3\,g\,H+\right.\right.\right.$$
$$\left.\left.\left.U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)dt^4\right)\Big/\left(20\left(3+H^2\,k^2\right)^4\right)+O[dt]^5\Bigg)+$$
$$\left(\left(k^3\left(42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U+15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+8\,H^2\,U\right)\right)\right)\Big/\right.$$
$$\left(240\left(3+H^2\,k^2\right)^2\right)+$$
$$\left(k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(9\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+10\,U\right)+5\,k^4\,U^2\left(11\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+\right.\right.\right.$$
$$\left.\left.\left.36\,H^4\,U\right)+6\,g\,H\left(21\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)+\right.\right.$$
$$\left.\left.3\,k^2\left(15\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}+109\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^2+180\,H^2\,U^3\right)\right)dt^2\right)\Big/$$
$$\left(480\left(3+H^2\,k^2\right)^3\right)+\left(i\,k^6\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$
$$\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(102\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U+\right.\right.$$
$$\left.\left.5\,k^2\left(7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+24\,H^2\,U\right)\right)\right)dt^3\Big)\Big/\left(480\left(3+H^2\,k^2\right)^3\right)-$$
$$\left(\left(k^7\left(42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U+15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+8\,H^2\,U\right)\right)\right.\right.$$
$$\left.\left.\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)dt^4\Big)\Big/\left(960\left(3+H^2\,k^2\right)^4\right)+$$
$$O[dt]^5\Big)\,dx^2+\Bigg(-\frac{i\,k^4\left(2\,g\,H\left(3+H^2\,k^2\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)}{16\sqrt{g\,H}\,\left(3+H^2\,k^2\right)}-$$
$$\left(i\,k^6\left(g\,H\left(6\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\,\left(15+4\,H^2\,k^2\right)U\right)+U^2\left(12\sqrt{g\,H\left(3+H^2\,k^2\right)}+\right.\right.\right.$$
$$\left.\left.\left.3\sqrt{3}\,U+k^2\left(2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^2\,U\right)\right)\right)dt^2\right)\Big/\left(32\left(3+H^2\,k^2\right)^{3/2}\right)+$$
$$\left(k^7\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\left(2\sqrt{3}\,g\,H\left(3+H^2\,k^2\right)+\right.\right.$$
$$\left.\left.U\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\sqrt{3}\,U+k^2\left(2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^2\,U\right)\right)\right)dt^3\right)\Big/$$
$$\left(32\left(3+H^2\,k^2\right)^{5/2}\right)+\left(i\,k^8\left(2\,g\,H\left(3+H^2\,k^2\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)\right.$$
$$\left.\left(3\,g\,H+U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^4\right)\Big/$$
$$\left.\left(64\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^3\right)+O[dt]^5\right)dx^3+$$
$$\Bigg(-\left(\left(k^5\left(\sqrt{3}\,g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)+\right.\right.\right.$$

$$2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\Big)\Big)\Big/$$
$$\left(38\,400\left(\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{5/2}\right)\right)\Big)-\frac{1}{230\,400\left(\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}\right)}$$
$$\Big(k^7\Big(45\sqrt{3}\,g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+g\,H\,U$$
$$\left(447\,588\sqrt{3}\,H^2\,k^2\,U+16\,705\sqrt{3}\,H^6\,k^6\,U+648\left(693\sqrt{g\,H\left(3+H^2\,k^2\right)}+688\sqrt{3}\,U\right)+\right.$$
$$\left.15\,k^4\left(3408\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+9985\sqrt{3}\,H^4\,U\right)\right)+$$
$$80\left(1836\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+612\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+68\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\right.$$
$$\left.\left.\left.U^3+9\,k^2\left(421\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+204\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\right)\Big)\,dt^2-$$
$$\frac{1}{25\,600\left(3+H^2\,k^2\right)^4}i\,k^8\left(6\,g^2\,H^2\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)+560\,H^8\,k^8\,U^4+432\,U^3\right.$$
$$\left(143\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+105\,U\right)+5\,k^4\,U^3\left(4139\sqrt{3}\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+6048\,H^4\,U\right)+$$
$$5\,k^6\,U^3\left(461\sqrt{3}\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}+1344\,H^6\,U\right)+$$
$$12\,k^2\left(3585\sqrt{3}\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+5161\sqrt{3}\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3+5040\,H^2\,U^4\right)+$$
$$g\,H\,U\left(63\,720\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+282\,852\,U+284\,364\,H^2\,k^2\,U+\right.$$
$$\left.\left.10\,640\,H^6\,k^6\,U+5\,k^4\left(1451\sqrt{3}\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+19\,056\,H^4\,U\right)\right)\right)\,dt^3+$$
$$\frac{1}{460\,800\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{11/2}}k^9\left(\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)$$
$$\left(5\,k^8\,U^3\left(3869\sqrt{3}\,g\,H^9+928\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\,U\right)+\right.$$
$$108\,k^2\left(2625\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}+15\,327\sqrt{3}\,g^2\,H^4\,U+\right.$$
$$\left.22\,383\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2+19\,261\sqrt{3}\,g\,H^3\,U^3+4640\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\right)+$$
$$1296\left(323\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+1268\sqrt{3}\,g^2\,H^2\,U+290\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+\right.$$
$$\left.g\,H\,U^2\left(1857\sqrt{g\,H\left(3+H^2\,k^2\right)}+1202\sqrt{3}\,U\right)\right)+15\,k^6\,U\left(4143\sqrt{3}\,g^2\,H^8+\right.$$
$$\left.3712\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+g\,H^7\,U\left(6019\sqrt{g\,H\left(3+H^2\,k^2\right)}+15\,454\sqrt{3}\,U\right)\right)+$$
$$9\,k^4\left(5325\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}+61\,735\sqrt{3}\,g^2\,H^6\,U+27\,840\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^4+\right.$$
$$\left.\left.\left.g\,H^5\,U^2\left(89\,915\sqrt{g\,H\left(3+H^2\,k^2\right)}+115\,737\sqrt{3}\,U\right)\right)\right)\,dt^4+O[dt]^5\right)\,dx^4+$$
$$O[dx]^5,\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(-\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right.$$
$$\left.\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$
$$\left.dt^2+\frac{1}{8\left(3+H^2\,k^2\right)^2}\right.$$

$$i\,k^4\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\,dt^3-$$
$$\left(\left(k^5\left(-\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\left(3\,g\,H+\right.\right.\right.$$
$$\left.\left.\left.U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)\,dt^4\right)\Big/\left(20\left(3+H^2\,k^2\right)^4\right)+O[dt]^5\Bigg)+$$
$$\left(\left(k^3\left(-42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)\right)\Big/\right.$$
$$\left(240\left(3+H^2\,k^2\right)^2\right)+$$
$$\left(k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(-9\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+10\,U\right)+\right.\right.$$
$$5\,k^4\,U^2\left(-11\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+36\,H^4\,U\right)+$$
$$6\,g\,H\left(-21\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)-$$
$$\left.\left.3\,k^2\left(15\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}+109\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^2-180\,H^2\,U^3\right)\right)\,dt^2\right)\Big/$$
$$\left(480\left(3+H^2\,k^2\right)^3\right)+\left(i\,k^6\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$
$$\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(-102\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-\right.\right.$$
$$\left.\left.5\,k^2\left(7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-24\,H^2\,U\right)\right)\right)\,dt^3\Big)\Big/\left(480\left(3+H^2\,k^2\right)^3\right)-$$
$$\left(\left(k^7\left(-42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)\right.\right.$$
$$\left.\left.\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)\,dt^4\Big)\Big/$$
$$\left(960\left(3+H^2\,k^2\right)^4\right)+O[dt]^5\Bigg)\,dx^2+\left(\frac{1}{16}\,i\,\sqrt{g\,H}\,k^4\left(-2+\frac{\sqrt{3}\,U}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}\right)+\right.$$
$$\left(i\,k^6\left(g\left(-6\,H\sqrt{g\,H\left(3+H^2\,k^2\right)}+15\sqrt{3}\,H\,U+4\sqrt{3}\,H^3\,k^2\,U\right)+U^2\left(-12\sqrt{g\,H\left(3+H^2\,k^2\right)}+\right.\right.\right.$$
$$\left.\left.\left.3\sqrt{3}\,U+k^2\left(-2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^2\,U\right)\right)\right)\,dt^2\Big)\Big/\left(32\left(3+H^2\,k^2\right)^{3/2}\right)-$$
$$\left(\left(k^7\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\left(2\sqrt{3}\,g\,H\left(3+H^2\,k^2\right)+\right.\right.\right.$$
$$\left.\left.\left.U\left(-9\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\sqrt{3}\,U+k^2\left(-2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^2\,U\right)\right)\right)\right)\,dt^3\Big)\Big/$$
$$\left(32\left(3+H^2\,k^2\right)^{5/2}\right)+\left(i\,k^8\left(2\,g\,H\left(3+H^2\,k^2\right)-\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)\right.$$
$$\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\,dt^4\Big)\Big/$$
$$\left.\left(64\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^3\right)+O[dt]^5\right)\,dx^3+$$
$$\left(\left(k^5\left(\sqrt{3}\,g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)-\right.\right.\right.$$
$$\left.\left.\left.2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\right)\Big/$$

$$\left(38\,400\,\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{5/2}\right)+\frac{1}{230\,400\,\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}}$$

$$k^7\left(45\,\sqrt{3}\,g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+\right.$$

$$g\,H\,U\left(447\,588\,\sqrt{3}\,H^2\,k^2\,U+16\,705\,\sqrt{3}\,H^6\,k^6\,U-648\left(693\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-688\,\sqrt{3}\,U\right)-\right.$$

$$\left.15\,k^4\left(3408\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}-9985\,\sqrt{3}\,H^4\,U\right)\right)-$$

$$80\left(1836\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+612\,k^4\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+68\,k^6\,\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+\right.$$

$$\left.\left.9\,k^2\left(421\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+204\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\right)\,dt^2-$$

$$\frac{1}{25\,600\left(3+H^2\,k^2\right)^4}i\,k^8\left(6\,g^2\,H^2\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)+560\,H^8\,k^8\,U^4+\right.$$

$$432\,U^3\left(-143\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+105\,U\right)+$$

$$5\,k^4\,U^3\left(-4139\,\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+6048\,H^4\,U\right)+$$

$$5\,k^6\,U^3\left(-461\,\sqrt{3}\,\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}+1344\,H^6\,U\right)-$$

$$12\,k^2\left(3585\,\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+5161\,\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3-5040\,H^2\,U^4\right)+$$

$$g\,H\,U\left(-63\,720\,\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+282\,852\,U+284\,364\,H^2\,k^2\,U+\right.$$

$$\left.\left.10\,640\,H^6\,k^6\,U+5\,k^4\left(-1451\,\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+19\,056\,H^4\,U\right)\right)\right)\,dt^3-$$

$$\frac{1}{460\,800\left(\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{11/2}\right)}\left(k^9\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-\left(3+H^2\,k^2\right)U\right)\right.$$

$$\left(5\,k^8\,U^3\left(-3869\,\sqrt{3}\,g\,H^9+928\,\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\,U\right)+108\,k^2\left(2625\,\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}-\right.\right.$$

$$15\,327\,\sqrt{3}\,g^2\,H^4\,U+22\,383\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2-19\,261\,\sqrt{3}\,g\,H^3\,U^3+$$

$$4640\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\Big)+9\,k^4\left(5325\,\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}-61\,735\,\sqrt{3}\,g^2\,H^6\,U+\right.$$

$$\left.27\,840\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^4+g\,H^5\,U^2\left(89\,915\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-115\,737\,\sqrt{3}\,U\right)\right)+$$

$$1296\left(323\,\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}-1268\,\sqrt{3}\,g^2\,H^2\,U+290\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+\right.$$

$$\left.g\,H\,U^2\left(1857\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-1202\,\sqrt{3}\,U\right)\right)-$$

$$15\,k^6\,U\left(4143\,\sqrt{3}\,g^2\,H^8-3712\,\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+\right.$$

$$\left.\left.\left.\left.g\,H^7\,U\left(-6019\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+15\,454\,\sqrt{3}\,U\right)\right)\right)\right)\,dt^4+O[dt]^5\right)\,dx^4+O[dx]^5\Big\}$$

In[1356]:=
```
Text[Row[{" U > Sqrt[g*H]  "}]]
Text[Row[{"Fnn and FnG "}]]
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (0);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnn = FullSimplify[KurFWSeta /. G → 0 /. n → 1]
KfnG = FullSimplify[KurFWSeta /. n → 0 /. G → 1]
Kfnn = Kfnn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]]
KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGn = FullSimplify[KurFWSG /. G → 0 /. n → 1]
KfGG = FullSimplify[KurFWSG /. n → 0 /. G → 1]
KfGn = KfGn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2  + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2^2 / 2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ]
```

Out[1356]= U > Sqrt[g*H]

Out[1357]= Fnn and FnG

Out[1362]= Gnp H + Rmp U

Out[1363]= GGp H

Out[1368]= $$\left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)\,U\,dt}{120\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(-\frac{1}{8}\,\left(k^4\,U\right)\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(729\,k^5\,U+2610\,H^2\,k^7\,U+1570\,H^4\,k^9\,U+260\,H^6\,k^{11}\,U\right)\,dt}{4800\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1371]= $$\left(-\frac{3\,(k\,w)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(12\,k^3+5\,H^2\,k^5\right)\,dt}{40\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{i\,\left(6291\,k^5+4410\,H^2\,k^7+770\,H^4\,k^9\right)\,dt}{4800\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1372]= FGn and FGG

Out[1375]= H (g Rmp + Gnp U)

Out[1376]= (GGp H + Rmp) U

Out[1381]= $$\left(-\frac{\left(k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\left(\left(i\,\left(90\,g\,H\,k^3+60\,g\,H^3\,k^5+10\,g\,H^5\,k^7-36\,k^3\,U^2-15\,H^2\,k^5\,U^2\right)\,dt\right)\Big/\left(120\,\left(3+H^2\,k^2\right)^2\right)\right)+O[dt]^4\right)\,dx^2+\left(-\frac{1}{8}\,\left(g\,H\,k^4\right)\,dt+O[dt]^4\right)\,dx^3+\left(\left(i\,\left(7020\,g\,H\,k^5+7020\,g\,H^3\,k^7+2340\,g\,H^5\,k^9+260\,g\,H^7\,k^{11}-6291\,k^5\,U^2-4410\,H^2\,k^7\,U^2-770\,H^4\,k^9\,U^2\right)\,dt\right)\Big/\left(4800\,\left(3+H^2\,k^2\right)^3\right)+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1385]= $$\left(-\frac{\left(k\,\left(6+H^2\,k^2\right)\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(6+H^2\,k^2\right)\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(126\,k^3+75\,H^2\,k^5+10\,H^4\,k^7\right)\,U\,dt}{120\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(-\frac{1}{8}\,\left(k^4\,U\right)\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(13\,311\,k^5+11\,430\,H^2\,k^7+3110\,H^4\,k^9+260\,H^6\,k^{11}\right)\,U\,dt}{4800\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1386]= W : omega

Out[1391]= $$\left\{\left(\frac{1}{6\,\left(3+H^2\,k^2\right)^2}k^3\,\left(\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)\,U\right)\left(3\,g\,H+U\,\left(2\,\sqrt{3}\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)\,U\right)\right)\,dt^2+\right.$$

$$\frac{1}{8\left(3+H^2k^2\right)^2} i k^4 \left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2 dt^3 -$$

$$\left(\left(k^5\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)^3\left(3gH+\right.\right.\right.$$

$$\left.\left.\left.U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right)dt^4\right)\Big/\left(20\left(3+H^2k^2\right)^4\right)+O[dt]^5\Bigg)+$$

$$\left(\left(k^3\left(42\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+180U+20H^4k^4U+15k^2\left(\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+8H^2U\right)\right)\right)\Big/\right.$$

$$\left.\left(240\left(3+H^2k^2\right)^2\right)+\right.$$

$$\left(k^5\left(20H^6k^6U^3+54U^2\left(9\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+10U\right)+5k^4U^2\left(11\sqrt{3}\sqrt{gH^9\left(3+H^2k^2\right)}+\right.\right.\right.$$

$$\left.\left.\left.36H^4U\right)+6gH\left(21\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(216+147H^2k^2+25H^4k^4\right)U\right)+\right.\right.$$

$$\left.\left.3k^2\left(15\sqrt{3}\sqrt{g^3H^7\left(3+H^2k^2\right)}+109\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}U^2+180H^2U^3\right)\right)dt^2\right)\Big/$$

$$\left(480\left(3+H^2k^2\right)^3\right)+\left(ik^6\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right.$$

$$\left(9gH\left(14+5H^2k^2\right)+U\left(102\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+180U+20H^4k^4U+\right.\right.$$

$$\left.\left.5k^2\left(7\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+24H^2U\right)\right)\right)dt^3\Big)\Big/\left(480\left(3+H^2k^2\right)^3\right)-$$

$$\left(\left(k^7\left(42\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+180U+20H^4k^4U+15k^2\left(\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+8H^2U\right)\right)\right.\right.$$

$$\left.\left.\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2\right)dt^4\right)\Big/$$

$$\left(960\left(3+H^2k^2\right)^4\right)+O[dt]^5\Bigg)dx^2+\left(-\frac{1}{16}ik^4\left(\sqrt{3}\sqrt{\frac{gH}{3+H^2k^2}}+2U\right)-\right.$$

$$\left(ik^6\left(3gH\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+4\left(3+H^2k^2\right)U\right)+U^2\left(15\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\right.\right.\right.$$

$$\left.\left.\left.18U+2H^4k^4U+k^2\left(5\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+12H^2U\right)\right)\right)dt^2\right)\Big/$$

$$\left(32\left(3+H^2k^2\right)^2\right)+\left(k^7\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right.$$

$$\left.\left(3gH+U\left(3\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)U\right)\right)dt^3\right)\Big/$$

$$\left(32\left(3+H^2k^2\right)^2\right)+\left(ik^8\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)U\right)\right.$$

$$\left.\left.\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2dt^4\right)\Big/\left(64\left(3+H^2k^2\right)^3\right)+O[dt]^5\right)$$

$$dx^3+\left(-\left(\left(k^5\left(\sqrt{3}\,gH\left(17\,856+12\,180H^2k^2+2075H^4k^4\right)+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.2080\left(9\sqrt{gH\left(3+H^2k^2\right)}+6k^2\sqrt{gH^5\left(3+H^2k^2\right)}+k^4\sqrt{gH^9\left(3+H^2k^2\right)}\right)U\right)\right)\Big/\right.$$

$$\left.\left(38\,400\left(\sqrt{gH}\left(3+H^2k^2\right)^{5/2}\right)\right)\right)-\frac{1}{230\,400\left(\sqrt{gH}\left(3+H^2k^2\right)^{7/2}\right)}$$

$$\left(k^7\left(45\sqrt{3}\,g^2H^2\left(3336+2268H^2k^2+385H^4k^4\right)+gHU\right.\right.$$

$$\left(447\,588\sqrt{3}\ H^2 k^2 U + 16\,705\sqrt{3}\ H^6 k^6 U + 648\left(693\sqrt{g H\left(3+H^2 k^2\right)} + 688\sqrt{3}\ U\right) + 15 k^4\left(3408\sqrt{g H^9\left(3+H^2 k^2\right)} + 9985\sqrt{3}\ H^4 U\right)\right) + 80\left(1836\sqrt{g H\left(3+H^2 k^2\right)}\ U^3 + 612 k^4\sqrt{g H^9\left(3+H^2 k^2\right)}\ U^3 + 68 k^6\sqrt{g H^{13}\left(3+H^2 k^2\right)}\ U^3 + 9 k^2\left(421\sqrt{g^3 H^7\left(3+H^2 k^2\right)}\ U + 204\sqrt{g H^5\left(3+H^2 k^2\right)}\ U^3\right)\right)\right)\right) dt^2 -$$

$$\frac{1}{25\,600\left(3+H^2 k^2\right)^4} i k^8\left(6 g^2 H^2\left(8046 + 5460 H^2 k^2 + 925 H^4 k^4\right) + 560 H^8 k^8 U^4 + 432 U^3\left(143\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + 105 U\right) + 5 k^4 U^3\left(4139\sqrt{3}\sqrt{g H^9\left(3+H^2 k^2\right)} + 6048 H^4 U\right) + 5 k^6 U^3\left(461\sqrt{3}\sqrt{g H^{13}\left(3+H^2 k^2\right)} + 1344 H^6 U\right) + 12 k^2\left(3585\sqrt{3}\sqrt{g^3 H^7\left(3+H^2 k^2\right)}\ U + 5161\sqrt{3}\sqrt{g H^5\left(3+H^2 k^2\right)}\ U^3 + 5040 H^2 U^4\right) + g H U\left(63\,720\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + 282\,852 U + 284\,364 H^2 k^2 U + 10\,640 H^6 k^6 U + 5 k^4\left(1451\sqrt{3}\sqrt{g H^9\left(3+H^2 k^2\right)} + 19\,056 H^4 U\right)\right)\right) dt^3 +$$

$$\frac{1}{460\,800\sqrt{g H}\left(3+H^2 k^2\right)^{11/2}} k^9\left(\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right)\left(5 k^8 U^3\left(3869\sqrt{3}\ g H^9 + 928\sqrt{g H^{17}\left(3+H^2 k^2\right)}\ U\right) + 108 k^2\left(2625\sqrt{g^5 H^9\left(3+H^2 k^2\right)} + 15\,327\sqrt{3}\ g^2 H^4 U + 22\,383\sqrt{g^3 H^7\left(3+H^2 k^2\right)}\ U^2 + 19\,261\sqrt{3}\ g H^3 U^3 + 4640\sqrt{g H^5\left(3+H^2 k^2\right)}\ U^4\right) + 1296\left(323\sqrt{g^5 H^5\left(3+H^2 k^2\right)} + 1268\sqrt{3}\ g^2 H^2 U + 290\sqrt{g H\left(3+H^2 k^2\right)}\ U^4 + g H U^2\left(1857\sqrt{g H\left(3+H^2 k^2\right)} + 1202\sqrt{3}\ U\right)\right) + 15 k^6 U\left(4143\sqrt{3}\ g^2 H^8 + 3712\sqrt{g H^{13}\left(3+H^2 k^2\right)}\ U^3 + g H^7 U\left(6019\sqrt{g H\left(3+H^2 k^2\right)} + 15\,454\sqrt{3}\ U\right)\right) + 9 k^4\left(5325\sqrt{g^5 H^{13}\left(3+H^2 k^2\right)} + 61\,735\sqrt{3}\ g^2 H^6 U + 27\,840\sqrt{g H^9\left(3+H^2 k^2\right)}\ U^4 + g H^5 U^2\left(89\,915\sqrt{g H\left(3+H^2 k^2\right)} + 115\,737\sqrt{3}\ U\right)\right)\right) dt^4 + O[dt]^5\Bigg) dx^4 +$$

$$O[dx]^5, \left(\frac{1}{6\left(3+H^2 k^2\right)^2} k^3\left(-\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right)\left(3 g H + U\left(-2\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right)\right)\right) dt^2 + \frac{1}{8\left(3+H^2 k^2\right)^2} i k^4\left(3 g H + U\left(-2\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right)\right)^2 dt^3 -$$

$$\left(\left(k^5\left(-\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right)^3\left(3 g H + U\left(-2\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right)\right)\right) dt^4\right) \Big/ \left(20\left(3+H^2 k^2\right)^4\right) + O[dt]^5\Bigg) +$$

$$\left(\left(k^3\left(-42\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)\right)\Big/\left(240\left(3+H^2\,k^2\right)^2\right)+\left(k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(-9\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+10\,U\right)+5\,k^4\,U^2\left(-11\sqrt{3}\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+36\,H^4\,U\right)+6\,g\,H\left(-21\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)-3\,k^2\left(15\sqrt{3}\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}+109\sqrt{3}\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^2-180\,H^2\,U^3\right)\right)\,dt^2\right)\Big/\left(480\left(3+H^2\,k^2\right)^3\right)+\left(i\,k^6\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(-102\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-5\,k^2\left(7\sqrt{3}\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-24\,H^2\,U\right)\right)\right)\,dt^3\right)\Big/\left(480\left(3+H^2\,k^2\right)^3\right)-\left(\left(k^7\left(-42\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)\,dt^4\right)\Big/\left(960\left(3+H^2\,k^2\right)^4\right)+O[dt]^5\right)\,dx^2+\left(\frac{1}{16}\,i\,k^4\left(\sqrt{3}\sqrt{\frac{g\,H}{3+H^2\,k^2}}-2\,U\right)-\left(i\,k^6\left(-3\,g\,H\left(\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}-4\left(3+H^2\,k^2\right)U\right)+U^2\left(-15\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(-5\sqrt{3}\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)\,dt^2\right)\Big/\left(32\left(3+H^2\,k^2\right)^2\right)+\left(k^7\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\left(3\,g\,H+U\left(-3\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\right)\,dt^3\right)\Big/\left(32\left(3+H^2\,k^2\right)^2\right)+\left(i\,k^8\left(-\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\left(3\,g\,H+U\left(-2\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\,dt^4\right)\Big/\left(64\left(3+H^2\,k^2\right)^3\right)+O[dt]^5\right)$$

$$dx^3+\left(\left(k^5\left(\sqrt{3}\,g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)-2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\right)\Big/\left(38\,400\sqrt{g\,H}\left(3+H^2\,k^2\right)^{5/2}\right)+\frac{1}{230\,400\sqrt{g\,H}\left(3+H^2\,k^2\right)^{7/2}}k^7\left(45\sqrt{3}\,g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+g\,H\,U\left(447\,588\sqrt{3}\,H^2\,k^2\,U+16\,705\sqrt{3}\,H^6\,k^6\,U-648\left(693\sqrt{g\,H\left(3+H^2\,k^2\right)}-688\sqrt{3}\,U\right)-15\,k^4\left(3408\sqrt{g\,H^9\left(3+H^2\,k^2\right)}-9985\sqrt{3}\,H^4\,U\right)\right)-80\left(1836\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3+612\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+68\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^3+\right.$$

$$9\,k^2\left(421\sqrt{g^3 H^7\left(3+H^2 k^2\right)}\,U+204\sqrt{g H^5\left(3+H^2 k^2\right)}\,U^3\right)\Big)\Big)\,dt^2-$$

$$\frac{1}{25\,600\left(3+H^2 k^2\right)^4}i\,k^8\left(6\,g^2 H^2\left(8046+5460\,H^2 k^2+925\,H^4 k^4\right)+560\,H^8 k^8 U^4+\right.$$

$$432\,U^3\left(-143\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)}+105\,U\right)+$$

$$5\,k^4 U^3\left(-4139\sqrt{3}\sqrt{g H^9\left(3+H^2 k^2\right)}+6048\,H^4 U\right)+$$

$$5\,k^6 U^3\left(-461\sqrt{3}\sqrt{g H^{13}\left(3+H^2 k^2\right)}+1344\,H^6 U\right)-$$

$$12\,k^2\left(3585\sqrt{3}\sqrt{g^3 H^7\left(3+H^2 k^2\right)}\,U+5161\sqrt{3}\sqrt{g H^5\left(3+H^2 k^2\right)}\,U^3-5040\,H^2 U^4\right)+$$

$$g H U\left(-63\,720\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)}+282\,852\,U+284\,364\,H^2 k^2 U+\right.$$

$$10\,640\,H^6 k^6 U+5\,k^4\left(-1451\sqrt{3}\sqrt{g H^9\left(3+H^2 k^2\right)}+19\,056\,H^4 U\right)\Big)\Big)\,dt^3-$$

$$\frac{1}{460\,800\left(\sqrt{g H}\left(3+H^2 k^2\right)^{11/2}\right)}\left(k^9\left(\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)}-\left(3+H^2 k^2\right)U\right)\right.$$

$$\left(5\,k^8 U^3\left(-3869\sqrt{3}\,g H^9+928\sqrt{g H^{17}\left(3+H^2 k^2\right)}\,U\right)+108\,k^2\left(2625\sqrt{g^5 H^9\left(3+H^2 k^2\right)}-\right.\right.$$

$$15\,327\sqrt{3}\,g^2 H^4 U+22\,383\sqrt{g^3 H^7\left(3+H^2 k^2\right)}\,U^2-19\,261\sqrt{3}\,g H^3 U^3+$$

$$4640\sqrt{g H^5\left(3+H^2 k^2\right)}\,U^4\Big)+9\,k^4\left(5325\sqrt{g^5 H^{13}\left(3+H^2 k^2\right)}-61\,735\sqrt{3}\,g^2 H^6 U+\right.$$

$$27\,840\sqrt{g H^9\left(3+H^2 k^2\right)}\,U^4+g H^5 U^2\left(89\,915\sqrt{g H\left(3+H^2 k^2\right)}-115\,737\sqrt{3}\,U\right)\Big)+$$

$$1296\left(323\sqrt{g^5 H^5\left(3+H^2 k^2\right)}-1268\sqrt{3}\,g^2 H^2 U+290\sqrt{g H\left(3+H^2 k^2\right)}\,U^4+\right.$$

$$g H U^2\left(1857\sqrt{g H\left(3+H^2 k^2\right)}-1202\sqrt{3}\,U\right)\Big)-$$

$$15\,k^6 U\left(4143\sqrt{3}\,g^2 H^8-3712\sqrt{g H^{13}\left(3+H^2 k^2\right)}\,U^3+\right.$$

$$\left.\left.\left.\left.g H^7 U\left(-6019\sqrt{g H\left(3+H^2 k^2\right)}+15\,454\sqrt{3}\,U\right)\right)\right)\right)\,dt^4+O[dt]^5\Big)\,dx^4+O[dx]^5\Big\}$$

In[1392]:=
```
Text[Row[{" U < -Sqrt[g*H]"}]]
Text[Row[{"Fnn and FnG "}]]
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (0) /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnn = FullSimplify[KurFWSeta /. G → 0 /. n → 1]
KfnG = FullSimplify[KurFWSeta /. n → 0 /. G → 1]
Kfnn = Kfnn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}]
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"FGn and FGG "}]]
KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGn = FullSimplify[KurFWSG /. G → 0 /. n → 1]
KfGG = FullSimplify[KurFWSG /. n → 0 /. G → 1]
KfGn = KfGn /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGG /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}]
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}]
Text[Row[{"W : omega"}]]
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[1 + EigvFmat2 + EigvFmat2^2 / 2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0}]
```

Out[1392]= $U < -\text{Sqrt}[g*H]$

Out[1393]= Fnn and FnG

Out[1398]= Gnp H + Rpp U

Out[1399]= GGp H

Out[1404]= $$\left(-\frac{\left(H^2\,k^3\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)\,U\,dt}{120\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{1}{8}\,k^4\,U\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(729\,k^5\,U+2610\,H^2\,k^7\,U+1570\,H^4\,k^9\,U+260\,H^6\,k^{11}\,U\right)\,dt}{4800\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1407]= $$\left(-\frac{3\,(k\,w)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(12\,k^3+5\,H^2\,k^5\right)\,dt}{40\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{i\,\left(6291\,k^5+4410\,H^2\,k^7+770\,H^4\,k^9\right)\,dt}{4800\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1408]= FGn and FGG

Out[1411]= H (g Rpp + Gnp U)

Out[1412]= (GGp H + Rpp) U

Out[1417]= $$\left(-\frac{\left(k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(90\,g\,H\,k^3+60\,g\,H^3\,k^5+10\,g\,H^5\,k^7-36\,k^3\,U^2-15\,H^2\,k^5\,U^2\right)\,dt}{120\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{1}{8}\,g\,H\,k^4\,dt+O[dt]^4\right)\,dx^3+\left(\frac{1}{4800\,\left(3+H^2\,k^2\right)^3}i\,\left(7020\,g\,H\,k^5+7020\,g\,H^3\,k^7+2340\,g\,H^5\,k^9+260\,g\,H^7\,k^{11}-6291\,k^5\,U^2-4410\,H^2\,k^7\,U^2-770\,H^4\,k^9\,U^2\right)\,dt+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1421]= $$\left(-\frac{\left(k\,\left(6+H^2\,k^2\right)\,U\,w\right)\,dt^2}{2\,\left(3+H^2\,k^2\right)}-\frac{i\,k\,\left(6+H^2\,k^2\right)\,U\,w^2\,dt^3}{6\,\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\,\left(126\,k^3+75\,H^2\,k^5+10\,H^4\,k^7\right)\,U\,dt}{120\,\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)\,dx^2+\left(\frac{1}{8}\,k^4\,U\,dt+O[dt]^4\right)\,dx^3+\left(\frac{i\,\left(13\,311\,k^5+11\,430\,H^2\,k^7+3110\,H^4\,k^9+260\,H^6\,k^{11}\right)\,U\,dt}{4800\,\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)\,dx^4+O[dx]^5$$

Out[1422]= W : omega

Out[1427]=

$$\left\{\left(\frac{1}{6\left(3+H^2k^2\right)^2}k^3\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right.\right.$$
$$\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)dt^2+$$
$$\frac{ik^4\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2dt^3}{8\left(3+H^2k^2\right)^2}-$$
$$\frac{1}{20\left(3+H^2k^2\right)^4}\left(k^5\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)^3\right.$$
$$\left.\left.\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right)dt^4+O[dt]^5\right)+$$
$$\left(\left(k^3\left(42\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+180U+20H^4k^4U+15k^2\left(\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+8H^2U\right)\right)\right)\Big/\right.$$
$$\left(240\left(3+H^2k^2\right)^2\right)+$$
$$\left(k^5\left(20H^6k^6U^3+54U^2\left(9\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+10U\right)+5k^4U^2\left(11\sqrt{3}\sqrt{gH^9\left(3+H^2k^2\right)}+\right.\right.\right.$$
$$\left.\left.36H^4U\right)+6gH\left(21\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(216+147H^2k^2+25H^4k^4\right)U\right)+$$
$$\left.\left.3k^2\left(15\sqrt{3}\sqrt{g^3H^7\left(3+H^2k^2\right)}+109\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}U^2+180H^2U^3\right)\right)dt^2\right)\Big/$$
$$\left(480\left(3+H^2k^2\right)^3\right)+\left(ik^6\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right.$$
$$\left(9gH\left(14+5H^2k^2\right)+U\left(102\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+180U+20H^4k^4U+\right.\right.$$
$$\left.\left.5k^2\left(7\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+24H^2U\right)\right)\right)dt^3\Big)\Big/\left(480\left(3+H^2k^2\right)^3\right)-$$
$$\left(\left(k^7\left(42\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+180U+20H^4k^4U+15k^2\left(\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+8H^2U\right)\right)\right.\right.$$
$$\left.\left.\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2\right)dt^4\Big)\Big/\left(960\left(3+H^2k^2\right)^4\right)+$$
$$O[dt]^5\Big)dx^2+\left(\frac{1}{16}ik^4\left(\sqrt{3}\sqrt{\frac{gH}{3+H^2k^2}}+2U\right)+\frac{1}{32\left(3+H^2k^2\right)^2}\right.$$
$$ik^6\left(3gH\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+4\left(3+H^2k^2\right)U\right)+\right.$$
$$\left.\left.U^2\left(15\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+18U+2H^4k^4U+k^2\left(5\sqrt{3}\sqrt{gH^5\left(3+H^2k^2\right)}+12H^2U\right)\right)\right)$$
$$dt^2-\frac{1}{32\left(3+H^2k^2\right)^2}\left(k^7\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right.$$
$$\left.\left(3gH+U\left(3\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)U\right)\right)\right)dt^3-$$
$$\frac{1}{64\left(3+H^2k^2\right)^3}ik^8\left(\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)U\right)$$
$$\left.\left(3gH+U\left(2\sqrt{3}\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2dt^4+O[dt]^5\right)dx^3+$$

$$\left(-\left(\left(k^5\left(\sqrt{3}\ g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\right)\Big/$$
$$\left.\left.\left(38\,400\left(\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{5/2}\right)\right)\right)-\frac{1}{230\,400\left(\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{7/2}\right)}$$
$$\left(k^7\left(45\sqrt{3}\ g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+g\,H\,U\right.\right.$$
$$\left(447\,588\sqrt{3}\ H^2\,k^2\,U+16\,705\sqrt{3}\ H^6\,k^6\,U+648\left(693\sqrt{g\,H\left(3+H^2\,k^2\right)}+688\sqrt{3}\ U\right)+\right.$$
$$\left.15\,k^4\left(3408\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+9985\sqrt{3}\ H^4\,U\right)\right)+$$
$$80\left(1836\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^3+612\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^3+68\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\right.$$
$$\left.\left.\left.\left.U^3+9\,k^2\left(421\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U+204\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3\right)\right)\right)\right)\,dt^2-$$
$$\frac{1}{25\,600\left(3+H^2\,k^2\right)^4}i\,k^8\left(6\,g^2\,H^2\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)+560\,H^8\,k^8\,U^4+432\,U^3\right.$$
$$\left(143\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+105\,U\right)+5\,k^4\,U^3\left(4139\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}+6048\,H^4\,U\right)+$$
$$5\,k^6\,U^3\left(461\sqrt{3}\ \sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}+1344\,H^6\,U\right)+$$
$$12\,k^2\left(3585\sqrt{3}\ \sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U+5161\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3+5040\,H^2\,U^4\right)+$$
$$g\,H\,U\left(63\,720\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+282\,852\,U+284\,364\,H^2\,k^2\,U+\right.$$
$$\left.\left.10\,640\,H^6\,k^6\,U+5\,k^4\left(1451\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}+19\,056\,H^4\,U\right)\right)\right)\,dt^3+$$
$$\frac{1}{460\,800\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{11/2}}k^9\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)$$
$$\left(5\,k^8\,U^3\left(3869\sqrt{3}\ g\,H^9+928\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\ U\right)+\right.$$
$$108\,k^2\left(2625\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}+15\,327\sqrt{3}\ g^2\,H^4\,U+\right.$$
$$\left.22\,383\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U^2+19\,261\sqrt{3}\ g\,H^3\,U^3+4640\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^4\right)+$$
$$1296\left(323\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+1268\sqrt{3}\ g^2\,H^2\,U+290\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^4+\right.$$
$$\left.g\,H\,U^2\left(1857\sqrt{g\,H\left(3+H^2\,k^2\right)}+1202\sqrt{3}\ U\right)\right)+15\,k^6\,U\left(4143\sqrt{3}\ g^2\,H^8+\right.$$
$$3712\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ U^3+g\,H^7\,U\left(6019\sqrt{g\,H\left(3+H^2\,k^2\right)}+15\,454\sqrt{3}\ U\right)\Big)+$$
$$9\,k^4\left(5325\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}+61\,735\sqrt{3}\ g^2\,H^6\,U+27\,840\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^4+\right.$$
$$\left.\left.g\,H^5\,U^2\left(89\,915\sqrt{g\,H\left(3+H^2\,k^2\right)}+115\,737\sqrt{3}\ U\right)\right)\right)\,dt^4+O[dt]^5\Bigg)\,dx^4+$$

$$O[dx]^5,\ \left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right.$$

$$\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)dt^2+$$

$$\frac{i\,k^4\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^3}{8\left(3+H^2\,k^2\right)^2}-$$

$$\frac{1}{20\left(3+H^2\,k^2\right)^4}$$

$$\left(k^5\left(-\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\right.$$

$$\left.\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)dt^4+O[dt]^5\right)+$$

$$\left(\left(k^3\left(-42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)\right)\Big/\right.$$

$$\left(240\left(3+H^2\,k^2\right)^2\right)+$$

$$\left(k^5\left(20\,H^6\,k^6\,U^3+54\,U^2\left(-9\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+10\,U\right)+\right.\right.$$

$$5\,k^4\,U^2\left(-11\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+36\,H^4\,U\right)+$$

$$6\,g\,H\left(-21\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(216+147\,H^2\,k^2+25\,H^4\,k^4\right)U\right)-$$

$$\left.\left.3\,k^2\left(15\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}+109\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^2-180\,H^2\,U^3\right)\right)dt^2\right)\Big/$$

$$\left(480\left(3+H^2\,k^2\right)^3\right)+\left(i\,k^6\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$

$$\left(9\,g\,H\left(14+5\,H^2\,k^2\right)+U\left(-102\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-\right.\right.$$

$$\left.\left.5\,k^2\left(7\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-24\,H^2\,U\right)\right)\right)dt^3\Big)\Big/\left(480\left(3+H^2\,k^2\right)^3\right)-$$

$$\left(\left(k^7\left(-42\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+180\,U+20\,H^4\,k^4\,U-15\,k^2\left(\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-8\,H^2\,U\right)\right)\right.\right.$$

$$\left.\left.\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)dt^4\Big)\Big/\left(960\left(3+H^2\,k^2\right)^4\right)+$$

$$O[dt]^5\Big)\,dx^2+\left(-\frac{1}{16}\,i\,k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H}{3+H^2\,k^2}}-2\,U\right)+\frac{1}{32\left(3+H^2\,k^2\right)^2}\right.$$

$$i\,k^6\left(-3\,g\,H\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-4\left(3+H^2\,k^2\right)U\right)+U^2\right.$$

$$\left.\left.\left(-15\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(-5\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)$$

$$dt^2-\frac{1}{32\left(3+H^2\,k^2\right)^2}\left(k^7\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$

$$\left.\left(3\,g\,H+U\left(-3\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\right)\right)dt^3-$$

$$\frac{1}{64\left(3+H^2\,k^2\right)^3}i\,k^8\left(-\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)$$

$$\left.\left(3\,g\,H+U\left(-2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^4+O[dt]^5\right)dx^3+$$

$$\Bigg(\Big(k^5\Big(\sqrt{3}\ g\,H\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)-$$

$$2080\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\Big)\Big)\Big/$$

$$\left(38\,400\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{5/2}\right)+\frac{1}{230\,400\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{7/2}}$$

$$k^7\Big(45\sqrt{3}\ g^2\,H^2\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)+$$

$$g\,H\,U\Big(447\,588\sqrt{3}\ H^2\,k^2\,U+16\,705\sqrt{3}\ H^6\,k^6\,U-648\left(693\sqrt{g\,H\left(3+H^2\,k^2\right)}-688\sqrt{3}\ U\right)-$$

$$15\,k^4\left(3408\sqrt{g\,H^9\left(3+H^2\,k^2\right)}-9985\sqrt{3}\ H^4\,U\right)\Big)-$$

$$80\Big(1836\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^3+612\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^3+68\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ U^3+$$

$$9\,k^2\left(421\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U+204\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3\right)\Big)\Big)\,dt^2-$$

$$\frac{1}{25\,600\left(3+H^2\,k^2\right)^4}i\,k^8\Big(6\,g^2\,H^2\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)+560\,H^8\,k^8\,U^4+$$

$$432\,U^3\left(-143\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+105\,U\right)+$$

$$5\,k^4\,U^3\left(-4139\sqrt{3}\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+6048\,H^4\,U\right)+$$

$$5\,k^6\,U^3\left(-461\sqrt{3}\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}+1344\,H^6\,U\right)-$$

$$12\,k^2\left(3585\sqrt{3}\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U+5161\sqrt{3}\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3-5040\,H^2\,U^4\right)+$$

$$g\,H\,U\Big(-63\,720\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}+282\,852\,U+284\,364\,H^2\,k^2\,U+$$

$$10\,640\,H^6\,k^6\,U+5\,k^4\left(-1451\sqrt{3}\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+19\,056\,H^4\,U\right)\Big)\Big)\,dt^3-$$

$$\frac{1}{460\,800\left(\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{11/2}\right)}\Big(k^9\left(\sqrt{3}\sqrt{g\,H\left(3+H^2\,k^2\right)}-\left(3+H^2\,k^2\right)U\right)$$

$$\Big(5\,k^8\,U^3\left(-3869\sqrt{3}\ g\,H^9+928\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\ U\right)+108\,k^2\left(2625\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}-$$

$$15\,327\sqrt{3}\ g^2\,H^4\,U+22\,383\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U^2-19\,261\sqrt{3}\ g\,H^3\,U^3+$$

$$4640\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^4\Big)+9\,k^4\left(5325\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}-61\,735\sqrt{3}\ g^2\,H^6\,U+\right.$$

$$\left.27\,840\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^4+g\,H^5\,U^2\left(89\,915\sqrt{g\,H\left(3+H^2\,k^2\right)}-115\,737\sqrt{3}\ U\right)\right)+$$

$$1296\left(323\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}-1268\sqrt{3}\ g^2\,H^2\,U+290\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^4+\right.$$

$$\left.g\,H\,U^2\left(1857\sqrt{g\,H\left(3+H^2\,k^2\right)}-1202\sqrt{3}\ U\right)\right)-$$

$$15\,k^6\,U\left(4143\sqrt{3}\ g^2\,H^8-3712\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ U^3+\right.$$

$$\left.g\,H^7\,U\left(-6019\sqrt{g\,H\left(3+H^2\,k^2\right)}+15\,454\sqrt{3}\ U\right)\right)\Big)\Big)\,dt^4+O[dt]^5\Bigg)\,dx^4+O[dx]^5\Big\}$$