In[44]:=
```
MA = k * x / (2 * Sin[k * x / 2])
RA = Exp[I * k * x / 2] * k * x / (2 * Sin[k * x / 2])
GA = k * x / ((H + H^3 / 3 * k^2) * Exp[-I * k * x / 2] * (2 * Sin[k * x / 2]))
FnnA = 0
FnGA = I * k / (1 + H^2 * k^2 / 3)
FGnA = g * H * I * k
FGGA = 0
FmatA = {{FnnA, FnGA}, {FGnA, FGGA}}
wAp = Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2)]
wAm = -Sqrt[g * H] * k * Sqrt[3 / (3 + H^2 * k^2)]
Eigenvalues[FmatA]
```

Out[44]= $\frac{1}{2} k x \text{Csc}\left[\frac{k x}{2}\right]$

Out[45]= $\frac{1}{2} e^{\frac{i k x}{2}} k x \text{Csc}\left[\frac{k x}{2}\right]$

Out[46]= $\frac{e^{\frac{i k x}{2}} k x \text{Csc}\left[\frac{k x}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right)}$

Out[47]= 0

Out[48]= $\frac{i k}{1 + \frac{H^2 k^2}{3}}$

Out[49]= $i g H k$

Out[50]= 0

Out[51]= $\left\{\left\{0, \frac{i k}{1 + \frac{H^2 k^2}{3}}\right\}, \{i g H k, 0\}\right\}$

Out[52]= $\sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$

Out[53]= $-\sqrt{3} \sqrt{g H} k \sqrt{\frac{1}{3 + H^2 k^2}}$

Out[54]= $\left\{-\frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}\right\}$

In[55]:=
```
M2 = 1
Series[M2 - MA, {x, 0, 10}]
```

Out[55]= 1

Out[56]= $-\frac{k^2 x^2}{24} - \frac{7 k^4 x^4}{5760} - \frac{31 k^6 x^6}{967\,680} - \frac{127 k^8 x^8}{154\,828\,800} - \frac{73 k^{10} x^{10}}{3\,503\,554\,560} + O[x]^{11}$

In[57]:=
```
Rm = (1 + I * Sin[k * x] / 2)
Series[Rm - RA, {x, 0, 10}]
Rp = Exp[I * k * x] * (1 - I * Sin[k * x] / 2)
Series[Rp - RA, {x, 0, 10}]
```

Out[57]= $1 + \frac{1}{2}\, i \,\text{Sin}[k\,x]$

Out[58]= $\frac{k^2\,x^2}{12} - \frac{1}{12}\, i\, k^3\, x^3 + \frac{k^4\,x^4}{720} + \frac{1}{240}\, i\, k^5\, x^5 + \frac{k^6\,x^6}{30\,240} - \frac{i\, k^7\, x^7}{10\,080} + \frac{k^8\,x^8}{1\,209\,600} + \frac{i\, k^9\, x^9}{725\,760} + \frac{k^{10}\,x^{10}}{47\,900\,160} + O[x]^{11}$

Out[59]= $e^{i\,k\,x}\left(1 - \frac{1}{2}\, i \,\text{Sin}[k\,x]\right)$

Out[60]= $\frac{k^2\,x^2}{12} + \frac{1}{6}\, i\, k^3\, x^3 - \frac{89\,k^4\,x^4}{720} - \frac{7}{120}\, i\, k^5\, x^5 + \frac{631\,k^6\,x^6}{30\,240} + \frac{31\, i\, k^7\, x^7}{5040} - \frac{1889\,k^8\,x^8}{1\,209\,600} - \frac{127\, i\, k^9\, x^9}{362\,880} + \frac{481\,k^{10}\,x^{10}}{6\,842\,880} + O[x]^{11}$

In[61]:=
```
GLHS = x / 6 * (Rp + Rm)
GRHSp1 = -Exp[-I * k * x / 2] + 2 + 4 * Exp[I * k * x / 2] +
  Exp[I * k * x] * (4 * Exp[-I * k * x / 2] + 2 - Exp[I * k * x / 2])
GRHSp1 = GRHSp1 / Exp[I * k * x / 2]
GRHSp1 = Expand[GRHSp1]
GRHSp1 = ExpToTrig[GRHSp1]
GRHSp2 = Exp[-I * k * x / 2] - 8 + 7 * Exp[I * k * x / 2] +
  Exp[I * k * x] * (7 * Exp[-I * k * x / 2] - 8 + Exp[I * k * x / 2])
GRHSp2 = GRHSp2 / Exp[I * k * x / 2]
GRHSp2 = Expand[GRHSp2]
GRHSp2 = ExpToTrig[GRHSp2]
G = GLHS / (H * x / 30 * (GRHSp1 ) + H^3 / (9 * x) * GRHSp2)
Series[G, {x, 0, 3} ]
Series[GA, {x, 0, 3} ]
Series[G - GA, {x, 0, 5}]
```

Out[61]= $\frac{1}{6}\, x \left(1 + e^{i\,k\,x}\left(1 - \frac{1}{2}\, i \,\text{Sin}[k\,x]\right) + \frac{1}{2}\, i \,\text{Sin}[k\,x]\right)$

Out[62]= $2 - e^{-\frac{1}{2} i\,k\,x} + 4\, e^{\frac{i\,k\,x}{2}} + e^{i\,k\,x}\left(2 + 4\, e^{-\frac{1}{2} i\,k\,x} - e^{\frac{i\,k\,x}{2}}\right)$

Out[63]= $e^{-\frac{1}{2} i\,k\,x}\left(2 - e^{-\frac{1}{2} i\,k\,x} + 4\, e^{\frac{i\,k\,x}{2}} + e^{i\,k\,x}\left(2 + 4\, e^{-\frac{1}{2} i\,k\,x} - e^{\frac{i\,k\,x}{2}}\right)\right)$

Out[64]= $8 + 2\, e^{-\frac{1}{2} i\,k\,x} + 2\, e^{\frac{i\,k\,x}{2}} - e^{-i\,k\,x} - e^{i\,k\,x}$

Out[65]= $8 + 4\,\text{Cos}\left[\frac{k\,x}{2}\right] - 2\,\text{Cos}[k\,x]$

Out[66]= $-8+e^{-\frac{1}{2}ikx}+7\,e^{\frac{ikx}{2}}+e^{ikx}\left(-8+7\,e^{-\frac{1}{2}ikx}+e^{\frac{ikx}{2}}\right)$

Out[67]= $e^{-\frac{1}{2}ikx}\left(-8+e^{-\frac{1}{2}ikx}+7\,e^{\frac{ikx}{2}}+e^{ikx}\left(-8+7\,e^{-\frac{1}{2}ikx}+e^{\frac{ikx}{2}}\right)\right)$

Out[68]= $14-8\,e^{-\frac{1}{2}ikx}-8\,e^{\frac{ikx}{2}}+e^{-ikx}+e^{ikx}$

Out[69]= $14-16\,\text{Cos}\left[\frac{kx}{2}\right]+2\,\text{Cos}[kx]$

Out[70]= $\left(x\left(1+e^{ikx}\left(1-\frac{1}{2}i\,\text{Sin}[kx]\right)+\frac{1}{2}i\,\text{Sin}[kx]\right)\right)\Big/\left(6\left(\frac{1}{30}Hx\left(8+4\,\text{Cos}\left[\frac{kx}{2}\right]-2\,\text{Cos}[kx]\right)+\frac{H^3\left(14-16\,\text{Cos}\left[\frac{kx}{2}\right]+2\,\text{Cos}[kx]\right)}{9x}\right)\right)$

Out[71]= $\frac{3}{3H+H^3k^2}+\frac{3ikx}{2\left(3H+H^3k^2\right)}+\frac{\left(-18k^2-5H^2k^4\right)x^2}{40H\left(3+H^2k^2\right)^2}+\frac{i\left(12k^3+5H^2k^5\right)x^3}{80H\left(3+H^2k^2\right)^2}+O[x]^4$

Out[72]= $\frac{1}{H+\frac{H^3k^2}{3}}+\frac{ikx}{2\left(H+\frac{H^3k^2}{3}\right)}-\frac{k^2x^2}{12\left(H+\frac{H^3k^2}{3}\right)}+O[x]^4$

Out[73]= $\frac{\left(12k^2+5H^2k^4\right)x^2}{40H\left(3+H^2k^2\right)^2}+\frac{i\left(12k^3+5H^2k^5\right)x^3}{80H\left(3+H^2k^2\right)^2}+\frac{\left(-6651k^4-4680H^2k^6-820H^4k^8\right)x^4}{4800H\left(3+H^2k^2\right)^3}-\frac{i\left(6291k^5+4410H^2k^7+770H^4k^9\right)x^5}{9600H\left(3+H^2k^2\right)^3}+O[x]^6$

In[74]:=
```
fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * G;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;

Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]

Fmat = {{Fnn , Fng}, {Fgn , Fgg}}
EigvFmat = Eigenvalues[Fmat];
 Simplify[Series[EigvFmat , {x, 0, 5}]]

RKStep = Log[ 1 - t * EigvFmat + (t * EigvFmat)^2 / 2] / (I * t);
RKstepTay = Series[RKStep, {x, 0, 4}, {t, 0, 4}]
Simplify[RKstepTay, k * H > 0 ]
Simplify[RKstepTay - {wAp, wAm}, k * H > 0]
```

Out[78]=
$$-\frac{1}{2\,x}\left(1-e^{-i\,k\,x}\right)\sqrt{g\,H}\,\left(-1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)$$

Out[79]=
$$\frac{1}{8}\sqrt{g\,H}\;k^4\,x^3-\frac{1}{48}\left(\sqrt{g\,H}\;k^6\right)x^5+O[x]^6$$

Out[80]=
$$\left(\left(1-e^{-i\,k\,x}\right)H\left(1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)+\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)\right)\Big/\left(\left(6\left(\frac{1}{30}\,H\,x\left(8+4\,\text{Cos}\left[\frac{k\,x}{2}\right]-2\,\text{Cos}[k\,x]\right)+\frac{H^3\left(14-16\,\text{Cos}\left[\frac{k\,x}{2}\right]+2\,\text{Cos}[k\,x]\right)}{9\,x}\right)\right)\right)$$

Out[81]=
$$\frac{i\left(12\,k^3+5\,H^2\,k^5\right)x^2}{40\left(3+H^2\,k^2\right)^2}-\frac{i\left(6291\,k^5+4410\,H^2\,k^7+770\,H^4\,k^9\right)x^4}{4800\left(3+H^2\,k^2\right)^3}+O[x]^6$$

Out[82]=
$$-\frac{1}{2\,x}\left(1-e^{-i\,k\,x}\right)\sqrt{g\,H}\,\left(-1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)$$

Out[83]=
$$\frac{1}{8}\sqrt{g\,H}\;k^4\,x^3-\frac{1}{48}\left(\sqrt{g\,H}\;k^6\right)x^5+O[x]^6$$

Out[84]=
$$\frac{\left(1-e^{-i\,k\,x}\right)g\,H\left(1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)+\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)}{2\,x}$$

Out[85]= $\frac{1}{12}\,\mathrm{i}\,g\,H\,k^3\,x^2-\frac{13}{240}\,\mathrm{i}\,g\,H\,k^5\,x^4+O[x]^6$

Out[86]= $\left\{\left\{-\frac{1}{2\,x}\left(1-e^{-\mathrm{i}\,k\,x}\right)\sqrt{g\,H}\,\left(-1+e^{\mathrm{i}\,k\,x}\left(1-\frac{1}{2}\,\mathrm{i}\,\mathrm{Sin}[k\,x]\right)-\frac{1}{2}\,\mathrm{i}\,\mathrm{Sin}[k\,x]\right),\right.\right.$
$\left.\left(\left(1-e^{-\mathrm{i}\,k\,x}\right)H\left(1+e^{\mathrm{i}\,k\,x}\left(1-\frac{1}{2}\,\mathrm{i}\,\mathrm{Sin}[k\,x]\right)+\frac{1}{2}\,\mathrm{i}\,\mathrm{Sin}[k\,x]\right)\right)\Big/$
$\left.\left(6\left(\frac{1}{30}\,H\,x\left(8+4\,\mathrm{Cos}\left[\frac{k\,x}{2}\right]-2\,\mathrm{Cos}[k\,x]\right)+\frac{H^3\left(14-16\,\mathrm{Cos}\left[\frac{k\,x}{2}\right]+2\,\mathrm{Cos}[k\,x]\right)}{9\,x}\right)\right)\right\},$
$\left\{\frac{\left(1-e^{-\mathrm{i}\,k\,x}\right)g\,H\left(1+e^{\mathrm{i}\,k\,x}\left(1-\frac{1}{2}\,\mathrm{i}\,\mathrm{Sin}[k\,x]\right)+\frac{1}{2}\,\mathrm{i}\,\mathrm{Sin}[k\,x]\right)}{2\,x},\right.$
$\left.\left.-\frac{1}{2\,x}\left(1-e^{-\mathrm{i}\,k\,x}\right)\sqrt{g\,H}\,\left(-1+e^{\mathrm{i}\,k\,x}\left(1-\frac{1}{2}\,\mathrm{i}\,\mathrm{Sin}[k\,x]\right)-\frac{1}{2}\,\mathrm{i}\,\mathrm{Sin}[k\,x]\right)\right\}\right\}$

Out[88]= $\left\{-\frac{\mathrm{i}\,\sqrt{3}\,g\,H\,k}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}-\frac{\mathrm{i}\,\sqrt{3}\,g^2\,H^2\,k^3\left(14+5\,H^2\,k^2\right)x^2}{80\left(g\,H\left(3+H^2\,k^2\right)\right)^{3/2}}+\frac{1}{8}\sqrt{g\,H}\,k^4\,x^3+\right.$
$\frac{\mathrm{i}\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)x^4}{12\,800\,\sqrt{3}\left(3+H^2\,k^2\right)^3}-\frac{1}{48}\left(\sqrt{g\,H}\,k^6\right)x^5+O[x]^6,$
$\frac{\mathrm{i}\,\sqrt{3}\,g\,H\,k}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}+\frac{\mathrm{i}\,\sqrt{3}\,g^2\,H^2\,k^3\left(14+5\,H^2\,k^2\right)x^2}{80\left(g\,H\left(3+H^2\,k^2\right)\right)^{3/2}}+\frac{1}{8}\sqrt{g\,H}\,k^4\,x^3-$
$\left.\frac{\mathrm{i}\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)x^4}{12\,800\,\sqrt{3}\left(3+H^2\,k^2\right)^3}-\frac{1}{48}\left(\sqrt{g\,H}\,k^6\right)x^5+O[x]^6\right\}$

Out[90]= $\left\{\left(\frac{\sqrt{3}\,k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}+\frac{\sqrt{3}\,g\,H\,k^3\sqrt{g\,H\left(3+H^2\,k^2\right)}\,t^2}{2\left(3+H^2\,k^2\right)^2}-\right.\right.$
$\left.\left.\frac{9\,\mathrm{i}\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}-\frac{9\left(\sqrt{3}\,g^2\,H^2\,k^5\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)t^4}{20\left(3+H^2\,k^2\right)^3}+O[t]^5\right)+\right.$
$\left(\frac{\sqrt{3}\left(14\,g\,H\,k^3+5\,g\,H^3\,k^5\right)}{80\left(3+H^2\,k^2\right)\sqrt{g\,H\left(3+H^2\,k^2\right)}}+\frac{3\sqrt{3}\,g^2\,k^5\left(14\,H^2+5\,H^4\,k^2\right)t^2}{160\left(3+H^2\,k^2\right)^2\sqrt{g\,H\left(3+H^2\,k^2\right)}}-\right.$
$\left.\left.\frac{9\,\mathrm{i}\,g^2\,H^2\,k^6\left(14+5\,H^2\,k^2\right)t^3}{160\left(3+H^2\,k^2\right)^3}-\frac{9\left(\sqrt{3}\,g^3\,H^3\,k^7\left(14+5\,H^2\,k^2\right)\right)t^4}{320\left(\left(3+H^2\,k^2\right)^3\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)}+O[t]^5\right)x^2+\right.$
$\left(\frac{1}{8}\,\mathrm{i}\,\sqrt{g\,H}\,k^4+\frac{3\,\mathrm{i}\,g\,H\sqrt{g\,H}\,k^6\,t^2}{16\left(3+H^2\,k^2\right)}+\frac{3\sqrt{3}\,g\,H\sqrt{g\,H}\,k^7\sqrt{g\,H\left(3+H^2\,k^2\right)}\,t^3}{16\left(3+H^2\,k^2\right)^2}-\right.$

$$\frac{9\,i\,g^2\,H^2\,\sqrt{g\,H}\;k^8\,t^4}{32\,\left(3+H^2\,k^2\right)^2}+O[t]^5\Bigg)\,x^3+$$

$$\Bigg(-\frac{17\,856\,\sqrt{3}\;g\,H\,k^5+12\,180\,\sqrt{3}\;g\,H^3\,k^7+2075\,\sqrt{3}\;g\,H^5\,k^9}{38\,400\,\left(\left(3+H^2\,k^2\right)^2\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\right)}-$$

$$\frac{\left(k^7\,\left(3336\,\sqrt{3}\;g^2\,H^2+2268\,\sqrt{3}\;g^2\,H^4\,k^2+385\,\sqrt{3}\;g^2\,H^6\,k^4\right)\right)\,t^2}{5120\,\left(\left(3+H^2\,k^2\right)^3\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\right)}+$$

$$\frac{3\,i\,g^2\,\left(8046\,H^2\,k^8+5460\,H^4\,k^{10}+925\,H^6\,k^{12}\right)\,t^3}{12\,800\,\left(3+H^2\,k^2\right)^4}+$$

$$\frac{3\,\left(15\,504\,\sqrt{3}\;g^3\,H^3\,k^9+10\,500\,\sqrt{3}\;g^3\,H^5\,k^{11}+1775\,\sqrt{3}\;g^3\,H^7\,k^{13}\right)\,t^4}{51\,200\,\left(3+H^2\,k^2\right)^4\sqrt{g\,H\,\left(3+H^2\,k^2\right)}}+O[t]^5\Bigg)\,x^4+O[x]^5,$$

$$\Bigg(-\frac{\sqrt{3}\;k\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-\frac{\left(\sqrt{3}\;g\,H\,k^3\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\right)\,t^2}{2\,\left(3+H^2\,k^2\right)^2}-\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{8\,\left(3+H^2\,k^2\right)^2}+$$

$$\frac{9\,\sqrt{3}\;g^2\,H^2\,k^5\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\;t^4}{20\,\left(3+H^2\,k^2\right)^3}+O[t]^5\Bigg)+$$

$$\Bigg(-\frac{\sqrt{3}\;\left(14\,g\,H\,k^3+5\,g\,H^3\,k^5\right)}{80\,\left(\left(3+H^2\,k^2\right)\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\right)}-\frac{3\,\left(\sqrt{3}\;g^2\,k^5\,\left(14\,H^2+5\,H^4\,k^2\right)\right)\,t^2}{160\,\left(\left(3+H^2\,k^2\right)^2\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\right)}-$$

$$\frac{9\,i\,g^2\,H^2\,k^6\,\left(14+5\,H^2\,k^2\right)\,t^3}{160\,\left(3+H^2\,k^2\right)^3}+\frac{9\,\sqrt{3}\;g^3\,H^3\,k^7\,\left(14+5\,H^2\,k^2\right)\,t^4}{320\,\left(3+H^2\,k^2\right)^3\sqrt{g\,H\,\left(3+H^2\,k^2\right)}}+O[t]^5\Bigg)\,x^2+$$

$$\Bigg(\frac{1}{8}\,i\,\sqrt{g\,H}\;k^4+\frac{3\,i\,g\,H\,\sqrt{g\,H}\;k^6\,t^2}{16\,\left(3+H^2\,k^2\right)}-\frac{3\,\left(\sqrt{3}\;g\,H\,\sqrt{g\,H}\;k^7\,\sqrt{g\,H\,\left(3+H^2\,k^2\right)}\right)\,t^3}{16\,\left(3+H^2\,k^2\right)^2}-$$

$$\frac{9\,i\,g^2\,H^2\,\sqrt{g\,H}\;k^8\,t^4}{32\,\left(3+H^2\,k^2\right)^2}+O[t]^5\Bigg)\,x^3+$$

$$\Bigg(\frac{17\,856\,\sqrt{3}\;g\,H\,k^5+12\,180\,\sqrt{3}\;g\,H^3\,k^7+2075\,\sqrt{3}\;g\,H^5\,k^9}{38\,400\,\left(3+H^2\,k^2\right)^2\sqrt{g\,H\,\left(3+H^2\,k^2\right)}}+$$

$$\frac{k^7\,\left(3336\,\sqrt{3}\;g^2\,H^2+2268\,\sqrt{3}\;g^2\,H^4\,k^2+385\,\sqrt{3}\;g^2\,H^6\,k^4\right)\,t^2}{5120\,\left(3+H^2\,k^2\right)^3\sqrt{g\,H\,\left(3+H^2\,k^2\right)}}+$$

$$\frac{3\,i\,g^2\left(8046\,H^2\,k^8+5460\,H^4\,k^{10}+925\,H^6\,k^{12}\right)t^3}{12\,800\left(3+H^2\,k^2\right)^4}-$$

$$\frac{3\left(15\,504\sqrt{3}\;g^3\,H^3\,k^9+10\,500\sqrt{3}\;g^3\,H^5\,k^{11}+1775\sqrt{3}\;g^3\,H^7\,k^{13}\right)t^4}{51\,200\left(\left(3+H^2\,k^2\right)^4\sqrt{g\,H\left(3+H^2\,k^2\right)}\right)}+O[t]^5\Bigg)x^4+O[x]^5\Bigg\}$$

Out[91]=

$$\Bigg\{\Bigg(\frac{\sqrt{3}\;g\,H\,k}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}+\frac{1}{2}\sqrt{3}\;k^3\left(\frac{g\,H}{3+H^2\,k^2}\right)^{3/2}t^2-$$

$$\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}-\frac{9}{20}\left(\sqrt{3}\;k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}\right)t^4+O[t]^5\Bigg)+$$

$$\Bigg(\frac{\sqrt{3}\;\sqrt{g\,H}\;k^3\left(14+5\,H^2\,k^2\right)}{80\left(3+H^2\,k^2\right)^{3/2}}+\frac{3\sqrt{3}\;(g\,H)^{3/2}\,k^5\left(14+5\,H^2\,k^2\right)t^2}{160\left(3+H^2\,k^2\right)^{5/2}}-\frac{9\,i\,g^2\,H^2\,k^6\left(14+5\,H^2\,k^2\right)t^3}{160\left(3+H^2\,k^2\right)^3}-$$

$$\frac{9\left(\sqrt{3}\;(g\,H)^{5/2}\,k^7\left(14+5\,H^2\,k^2\right)\right)t^4}{320\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\Bigg)x^2+$$

$$\Bigg(\frac{1}{8}\,i\,\sqrt{g\,H}\;k^4+\frac{3\,i\,(g\,H)^{3/2}\,k^6\,t^2}{16\left(3+H^2\,k^2\right)}+\frac{3\sqrt{3}\;g^2\,H^2\,k^7\,t^3}{16\left(3+H^2\,k^2\right)^{3/2}}-\frac{9\,i\,(g\,H)^{5/2}\,k^8\,t^4}{32\left(3+H^2\,k^2\right)^2}+O[t]^5\Bigg)x^3+$$

$$\Bigg(-\frac{\sqrt{g\,H}\;k^5\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)}{12\,800\left(\sqrt{3}\;\left(3+H^2\,k^2\right)^{5/2}\right)}-$$

$$\frac{\left(\sqrt{3}\;(g\,H)^{3/2}\,k^7\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)\right)t^2}{5120\left(3+H^2\,k^2\right)^{7/2}}+$$

$$\frac{3\,i\,g^2\,H^2\,k^8\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)t^3}{12\,800\left(3+H^2\,k^2\right)^4}+$$

$$\frac{3\sqrt{3}\;(g\,H)^{5/2}\,k^9\left(15\,504+10\,500\,H^2\,k^2+1775\,H^4\,k^4\right)t^4}{51\,200\left(3+H^2\,k^2\right)^{9/2}}+O[t]^5\Bigg)x^4+O[x]^5,$$

$$\Bigg(-\frac{\sqrt{3}\;g\,H\,k}{\sqrt{g\,H\left(3+H^2\,k^2\right)}}-\frac{1}{2}\left(\sqrt{3}\;k^3\left(\frac{g\,H}{3+H^2\,k^2}\right)^{3/2}\right)t^2-\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}+$$

$$\frac{9}{20}\sqrt{3}\;k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}t^4+O[t]^5\Bigg)+$$

$$\Bigg(-\frac{\sqrt{3}\;\sqrt{g\,H}\;k^3\left(14+5\,H^2\,k^2\right)}{80\left(3+H^2\,k^2\right)^{3/2}}-\frac{3\left(\sqrt{3}\;(g\,H)^{3/2}\,k^5\left(14+5\,H^2\,k^2\right)\right)t^2}{160\left(3+H^2\,k^2\right)^{5/2}}-$$

$$\frac{9\,i\,g^2\,H^2\,k^6\left(14+5\,H^2\,k^2\right)t^3}{160\left(3+H^2\,k^2\right)^3}+\frac{9\sqrt{3}\;(g\,H)^{5/2}\,k^7\left(14+5\,H^2\,k^2\right)t^4}{320\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\Bigg)x^2+$$

$$\left(\frac{1}{8}\,i\,\sqrt{g\,H}\;k^4+\frac{3\,i\,(g\,H)^{3/2}\,k^6\,t^2}{16\left(3+H^2\,k^2\right)}-\frac{3\left(\sqrt{3}\;g^2\,H^2\,k^7\right)t^3}{16\left(3+H^2\,k^2\right)^{3/2}}-\frac{9\,i\,(g\,H)^{5/2}\,k^8\,t^4}{32\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+$$

$$\left(\frac{\sqrt{g\,H}\;k^5\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)}{12\,800\,\sqrt{3}\;\left(3+H^2\,k^2\right)^{5/2}}+\right.$$

$$\frac{\sqrt{3}\;(g\,H)^{3/2}\,k^7\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)t^2}{5120\left(3+H^2\,k^2\right)^{7/2}}+$$

$$\frac{3\,i\,g^2\,H^2\,k^8\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)t^3}{12\,800\left(3+H^2\,k^2\right)^4}-$$

$$\left.\frac{3\left(\sqrt{3}\;(g\,H)^{5/2}\,k^9\left(15\,504+10\,500\,H^2\,k^2+1775\,H^4\,k^4\right)\right)t^4}{51\,200\left(3+H^2\,k^2\right)^{9/2}}+O[t]^5\right)x^4+O[x]^5\Big\}$$

Out[92]=

$$\left\{\left(\frac{1}{2}\sqrt{3}\,k^3\left(\frac{g\,H}{3+H^2\,k^2}\right)^{3/2}t^2-\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}-\frac{9}{20}\left(\sqrt{3}\,k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}\right)t^4+O[t]^5\right)+\right.$$

$$\left(\frac{\sqrt{3}\,\sqrt{g\,H}\,k^3\left(14+5\,H^2\,k^2\right)}{80\left(3+H^2\,k^2\right)^{3/2}}+\frac{3\sqrt{3}\,(g\,H)^{3/2}\,k^5\left(14+5\,H^2\,k^2\right)t^2}{160\left(3+H^2\,k^2\right)^{5/2}}-\right.$$

$$\left.\frac{9\,i\,g^2\,H^2\,k^6\left(14+5\,H^2\,k^2\right)t^3}{160\left(3+H^2\,k^2\right)^3}-\frac{9\left(\sqrt{3}\,(g\,H)^{5/2}\,k^7\left(14+5\,H^2\,k^2\right)\right)t^4}{320\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\right)x^2+$$

$$\left(\frac{1}{8}\,i\,\sqrt{g\,H}\,k^4+\frac{3\,i\,(g\,H)^{3/2}\,k^6\,t^2}{16\left(3+H^2\,k^2\right)}+\frac{3\sqrt{3}\,g^2\,H^2\,k^7\,t^3}{16\left(3+H^2\,k^2\right)^{3/2}}-\frac{9\,i\,(g\,H)^{5/2}\,k^8\,t^4}{32\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+$$

$$\left(-\frac{\sqrt{g\,H}\,k^5\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)}{12\,800\left(\sqrt{3}\,\left(3+H^2\,k^2\right)^{5/2}\right)}-\right.$$

$$\frac{\left(\sqrt{3}\,(g\,H)^{3/2}\,k^7\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)\right)t^2}{5120\left(3+H^2\,k^2\right)^{7/2}}+$$

$$\frac{3\,i\,g^2\,H^2\,k^8\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)t^3}{12\,800\left(3+H^2\,k^2\right)^4}+$$

$$\left.\frac{3\sqrt{3}\,(g\,H)^{5/2}\,k^9\left(15\,504+10\,500\,H^2\,k^2+1775\,H^4\,k^4\right)t^4}{51\,200\left(3+H^2\,k^2\right)^{9/2}}+O[t]^5\right)x^4+O[x]^5,$$

$$\left(-\frac{1}{2}\left(\sqrt{3}\,k^3\left(\frac{g\,H}{3+H^2\,k^2}\right)^{3/2}\right)t^2-\frac{9\,i\,g^2\,H^2\,k^4\,t^3}{8\left(3+H^2\,k^2\right)^2}+\frac{9}{20}\sqrt{3}\,k^5\left(\frac{g\,H}{3+H^2\,k^2}\right)^{5/2}t^4+O[t]^5\right)+$$

$$\left(-\frac{\sqrt{3}\,\sqrt{g\,H}\,k^3\left(14+5\,H^2\,k^2\right)}{80\left(3+H^2\,k^2\right)^{3/2}}-\frac{3\left(\sqrt{3}\,(g\,H)^{3/2}\,k^5\left(14+5\,H^2\,k^2\right)\right)t^2}{160\left(3+H^2\,k^2\right)^{5/2}}-\right.$$

$$\left.\frac{9\,i\,g^2\,H^2\,k^6\left(14+5\,H^2\,k^2\right)t^3}{160\left(3+H^2\,k^2\right)^3}+\frac{9\sqrt{3}\,(g\,H)^{5/2}\,k^7\left(14+5\,H^2\,k^2\right)t^4}{320\left(3+H^2\,k^2\right)^{7/2}}+O[t]^5\right)x^2+$$

$$\left(\frac{1}{8}\,i\,\sqrt{g\,H}\,k^4+\frac{3\,i\,(g\,H)^{3/2}\,k^6\,t^2}{16\left(3+H^2\,k^2\right)}-\frac{3\left(\sqrt{3}\,g^2\,H^2\,k^7\right)t^3}{16\left(3+H^2\,k^2\right)^{3/2}}-\frac{9\,i\,(g\,H)^{5/2}\,k^8\,t^4}{32\left(3+H^2\,k^2\right)^2}+O[t]^5\right)x^3+$$

$$\left(\frac{\sqrt{g\,H}\,k^5\left(17\,856+12\,180\,H^2\,k^2+2075\,H^4\,k^4\right)}{12\,800\sqrt{3}\,\left(3+H^2\,k^2\right)^{5/2}}+\right.$$

$$\frac{\sqrt{3}\,(g\,H)^{3/2}\,k^7\left(3336+2268\,H^2\,k^2+385\,H^4\,k^4\right)t^2}{5120\left(3+H^2\,k^2\right)^{7/2}}+$$

$$\frac{3\,i\,g^2\,H^2\,k^8\left(8046+5460\,H^2\,k^2+925\,H^4\,k^4\right)t^3}{12\,800\left(3+H^2\,k^2\right)^4}-$$

$$\left.\left.\frac{3\left(\sqrt{3}\,(g\,H)^{5/2}\,k^9\left(15\,504+10\,500\,H^2\,k^2+1775\,H^4\,k^4\right)\right)t^4}{51\,200\left(3+H^2\,k^2\right)^{9/2}}+O[t]^5\right)x^4+O[x]^5\right\}$$