1 Nodal Values To Cell Averages

Definition

$$\bar{q}_j = \mathcal{M} q_j$$

Values

$$\mathcal{M}_A = 2 * sin(dx * k/2)/(dx * k) = \frac{2}{k\Delta x} \sin\left(\frac{k\Delta x}{2}\right)$$

$$\mathcal{M}_1 = 1$$

$$\mathcal{M}_2 = 1$$

$$\mathcal{M}_3 = -cos(dx * k)/12 + 13/12 = \frac{24}{26 - 2\cos(k\Delta x)}$$

2 Reconstruction

2.1 h and G ($\mathcal{R}^+$ and $\mathcal{R}^-$)

Definition:

$$q^+_{j+1/2} = \mathcal{R}^+ q_j$$

and

$$q^-_{j+1/2} = \mathcal{R}^- q_j$$

Values:

$$\mathcal{R}^+_A = \mathcal{R}^-_A = exp(I * dx * k/2) = e^{ik\frac{\Delta x}{2}} = \exp\left(ik\frac{\Delta x}{2}\right)$$

$$\mathcal{R}^+_1 = exp(I * dx * k) = \exp(ik\Delta x)$$
$$\mathcal{R}^-_1 = 1$$

$$\mathcal{R}_2^+ = -exp(2*I*dx*k)/4 + exp(I*dx*k) + 1/4 = \exp(ik\Delta x)\left(1 - \frac{i\sin(k\Delta x)}{2}\right)$$

$$\mathcal{R}_2^- = I*sin(dx*k)/2 + 1 = 1 + \frac{i\sin(k\Delta x)}{2}$$

$$\mathcal{R}_3^+ = (2*exp(2*I*dx*k) - 10*exp(I*dx*k) - 4)/(cos(dx*k) - 13) = \frac{2\exp(2ik\Delta x) - 10\exp(ik\Delta x) - 4}{\cos(k\Delta x) - 13} \quad (1)$$

$$\mathcal{R}_3^- = 2*(-(2*exp(I*dx*k)+5)*exp(I*dx*k)+1)*exp(-I*dx*k)/ (cos(dx*k) - 13) = \frac{2\exp(-ik\Delta x)(1 - \exp(ik\Delta x)(5 + 2\exp(ik\Delta x)))}{\cos(k\Delta x) - 13} = \frac{2\exp(-ik\Delta x) - 4\exp(ik\Delta x) - 10}{\cos(k\Delta x) - 13} \quad (2)$$

2.2 u ($\mathcal{R}^u$)

Definition:

$$q_{j+1/2} = \mathcal{R}^u q_j$$

Values:

$$\mathcal{R}_A^u = exp(I*dx*k/2) = e^{ik\frac{\Delta x}{2}} = \exp\left(ik\frac{\Delta x}{2}\right)$$

$$R_1^u = exp(I*dx*k)/2 + 1/2 = \frac{\exp(ik\Delta x) + 1}{2}$$

$$R_2^u = exp(I*dx*k)/2 + 1/2 = \frac{\exp(ik\Delta x) + 1}{2}$$

$$R_3^u = -exp(2*I*dx*k)/16 + 9*exp(I*dx*k)/16 + 9/16 - exp(-I*dx*k)/16 = \frac{-\exp(-ik\Delta x) + 9\exp(ik\Delta x) - \exp(2ik\Delta x) + 9}{16} \quad (3)$$

3 Elliptic Equation

Definition:

$$G_j = \mathcal{G} u_j$$

values

$$\mathcal{G}_A = H + \frac{H^3}{3} k^2$$

$$\mathcal{G}_{2FD} = -H**3*(2*cos(dx*k)-2)/(3*dx**2)+H = H - \frac{H^3}{3} \frac{2\cos(k\Delta x) - 2}{\Delta x^2}$$

$$\mathcal{G}_3 = -H**3*(32*cos(dx*k)-2*cos(2*dx*k)-30)/(36*dx**2)+H = H - \frac{H^3}{3} \frac{32\cos(k\Delta x) - 2\cos(2k\Delta x) - 30}{12\Delta x^2} \quad (4)$$

$$\mathcal{G}_{2FEM} = (2*H**3*(exp(3*I*dx*k/2)+14*exp(I*dx*k/2)-8*exp(I*dx*k)-8 +exp(-I*dx*k/2))/(3*dx**2)+H*(-exp(3*I*dx*k/2)+8*exp(I*dx*k/2)+2*exp(I*dx*k) +2-exp(-I*dx*k/2))/5)/(-exp(2*I*dx*k)/4+exp(I*dx*k)+I*sin(dx*k)/2+5/4)$$
$$= \left(\frac{2H^3}{3\Delta x^2} \left(\exp\left(ik\frac{3\Delta x}{2}\right) + 14\exp\left(ik\frac{\Delta x}{2}\right) - 8\exp(ik\Delta x) - 8 + \exp\left(-ik\frac{\Delta x}{2}\right)\right) + \frac{H}{5}\left(-\exp\left(ik\frac{3\Delta x}{2}\right) + 8\exp\left(ik\frac{\Delta x}{2}\right) + 2\exp(ik\Delta x) + 2 - \exp\left(-ik\frac{\Delta x}{2}\right)\right)\right) \div \left(-\frac{1}{4}\exp(2i\Delta x k) + \exp(i\Delta x k) + \frac{i}{2}\sin(k\Delta x) + \frac{5}{4}\right) \quad (5)$$

4 Conservation Equation

4.1 mass flux

Definition

$$F^{\eta}_{j+1/2} = \mathcal{F}^{\eta,\eta}\eta_j + \mathcal{F}^{\eta,\upsilon}\upsilon_j$$

4.1.1 $\mathcal{F}^{\eta,\eta}$

$$\mathcal{F}_A^{\eta,\eta} = 0$$

$$\mathcal{F}_g^{\eta,\eta} = sqrt(H*g)*(Rms - Rps)/2 = -\sqrt{gH}\frac{\mathcal{R}^+ - \mathcal{R}^-}{2}$$

$$\mathcal{F}_3^{\eta,\eta} = sqrt(H*g)*(exp(I*dx*k)-1)*(cos(dx*k)-13)**2*((2*exp(I*dx*k)+5)\\ exp(I*dx*k) - (-exp(2*I*dx*k) + 5*exp(I*dx*k) + 2)\\ exp(I*dx*k) - 1)*exp(-2*I*dx*k)/(1728*dx)$$

$$\mathcal{F}_3^{\eta,\eta} = \sqrt{gH}\left(\exp\left(ik\Delta x\right) - 1\right)\left(\cos\left(k\Delta x\right) - 13\right)^2\Bigg((2\exp\left(ik\Delta x\right) + 5)\exp\left(ik\Delta x\right)\\ - \left(-\exp\left(2ik\Delta x\right) + 5\exp\left(ik\Delta x\right) + 2\right)\exp\left(ik\Delta x\right) - 1\Bigg)\frac{\exp\left(-2ik\Delta x\right)}{1728\Delta x}$$

4.1.2 $\mathcal{F}^{\eta,\upsilon}$

$$\begin{aligned}\mathcal{F}_A^{\eta,\upsilon} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} H\upsilon_{j+1/2} &= \frac{1}{i\omega\Delta t}\left(\exp\left(i\omega\Delta t\right) - 1\right)H\upsilon_{j+1/2}\\ &= \frac{1}{i\omega\Delta t}\left(\exp\left(i\omega\Delta t\right) - 1\right)\exp\left(ik\frac{\Delta x}{2}\right)H\upsilon_j \quad (6)\end{aligned}$$

$$\mathcal{F}_g^{\eta,\upsilon} = H*Rus = H\mathcal{R}^u$$

4.2 momentum flux

Definition

$$F^G_{j+1/2} = \mathcal{F}^{G,\eta}\eta_j + \mathcal{F}^{G,\upsilon}\upsilon_j$$

4.2.1 $\mathcal{F}^{G,\eta}$

$$\mathcal{F}_A^{G,\eta} = \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} gH\eta_{j+1/2} = \frac{1}{i\omega\Delta t}\left(\exp\left(i\omega\Delta t\right)-1\right)gH\eta_{j+1/2}$$
$$= \frac{1}{i\omega\Delta t}\left(\exp\left(i\omega\Delta t\right)-1\right)\exp\left(ik\frac{\Delta x}{2}\right)gH\eta_j \quad (7)$$

$$\mathcal{F}_g^{G,\eta} = H*g*(Rms+Rps)/2 = gH\frac{\mathcal{R}^+ + \mathcal{R}^-}{2}$$

$$\begin{aligned}\mathcal{F}_3^{G,\eta} = {} & H*g*(1-exp(-I*dx*k))*(-((2*exp(I*dx*k)+5)*exp(I*dx*k)-1)*(cos(dx*k)\\ & -13)*exp(-I*dx*k)/72+(cos(dx*k)-13)\\ & (exp(2*I*dx*k)-5*exp(I*dx*k)-2)/72)\\ & (-cos(dx*k)/12+13/12)\\ & /(2*dx*(-H**3*(32*cos(dx*k)-2*cos(2*dx*k)-30)/(36*dx**2)+H))\end{aligned} \quad (8)$$

$$\begin{aligned}\mathcal{F}_3^{G,\eta} = {} & gH\left(1-\exp(-ik\Delta x)\right)\Bigg[-((2\exp(ik\Delta x)+5)\exp(ik\Delta x)-1)\left(\cos\left(k\Delta x\right)-13\right)\\ & \frac{\exp(-ik\Delta x)}{72}+\frac{1}{72}\left(\cos\left(k\Delta x\right)-13\right)\left(\exp(2ik\Delta x)-5\exp(ik\Delta x)-2\right)\Bigg]\left(\frac{-\cos\left(k\Delta x\right)}{12}+\frac{13}{12}\right)\\ & \div\left(2\Delta x\left(-H^3\frac{32\cos\left(k\Delta x\right)-2\cos\left(2k\Delta x\right)-30}{36\Delta x^2}+H\right)\right)\end{aligned} \quad (9)$$

4.2.2 $\mathcal{F}^{G,v}$

$$\mathcal{F}_A^{G,v} = 0$$

$$\mathcal{F}_g^{G,\eta} = Gs*sqrt(H*g)*(Rms-Rps)/2 = -\sqrt{gH}\frac{\mathcal{R}^+ - \mathcal{R}^-}{2}$$