In[3146]:=

```
H In Reals;
dx In Reals;
g In Reals;
k In Reals;
dt In Reals;
dx > 0;
H > 0;
g > 0;
k > 0;
dt > 0;
```

$$w = \sqrt{3}\ \sqrt{g\,H}\ k \sqrt{\frac{1}{3 + H^2 k^2}} + k\,U;$$

$$w1 = \frac{\left(\sqrt{3}\ k \sqrt{g\,H\,\left(3 + H^2 k^2\right)} + 3\,k\,U + H^2 k^3 U\right)}{\left(3 + H^2 k^2\right)};$$

In[3158]:=

```
Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]]
```

$$\text{wo1dt} = \frac{\mathrm{i}\left(\sqrt{3}\,k\sqrt{g\,H\,(3+H^2 k^2)}+3\,k\,U+H^2 k^3 U\right)^2 dt}{2\,(3+H^2 k^2)^2};$$

$$\text{wo1dt} == \mathrm{i}*dt/2*\left(\frac{\left(\sqrt{3}\,k\sqrt{g\,H\,(3+H^2 k^2)}+3\,k\,U+H^2 k^3 U\right)}{(3+H^2 k^2)}\right)\text{^}2;$$

$$\text{wo1dt} == \mathrm{i}*dt/2*\left(\frac{\sqrt{3}\,k\sqrt{g\,H\,(3+H^2 k^2)}}{(3+H^2 k^2)}+k*U\right)\text{^}2;$$

```
wo1dt == i*dt/2*(k*Sqrt[g*H] Sqrt[3/(3+H^2 k^2)] + k*U)^2;
FullSimplify[wo1dt - i*dt/2*(k*Sqrt[g*H] Sqrt[3/(3+H^2 k^2)] + k*U)^2];
wo1dtRed = i*dt/2*(wp)^2
```

$$\text{wo1dx} = -\frac{1}{4}\,\mathrm{i}\,k^2\left(2\sqrt{g\,H}+\frac{\sqrt{3}\,U}{\sqrt{3+H^2 k^2}}\right)dx$$

```
Text[Row[{" U > Sqrt[g*H]  "}]]
```

$$\text{wo1dt1} = \frac{\mathrm{i}\left(\sqrt{3}\,k\sqrt{g\,H\,(3+H^2 k^2)}+3\,k\,U+H^2 k^3 U\right)^2 dt}{2\,(3+H^2 k^2)^2};$$

```
wo1dtRed1 = i*dt/2*(wp)^2
```

$$\text{wo1dx1} = -\frac{1}{4}\,\mathrm{i}\,k^2\left(\sqrt{3}\sqrt{\frac{g\,H}{3+H^2 k^2}}+2\,U\right);$$

$$\text{wo1dxRed1} = -\frac{1}{4}\,\mathrm{i}\,k\,(wp + kU)\,dx$$

```
Text[Row[{" U< -Sqrt[g*H] "}]]
```

$$\text{wo1dt2} = \frac{\mathrm{i}\left(\sqrt{3}\,k\sqrt{g\,H\,(3+H^2 k^2)}+3\,k\,U+H^2 k^3 U\right)^2 dt}{2\,(3+H^2 k^2)^2};$$

```
wo1dtRed2 = i*dt/2*(wp)^2
```

$$\text{wo1dx2} = \frac{1}{4}\,\mathrm{i}\,k^2\left(\sqrt{3}\sqrt{\frac{g\,H}{3+H^2 k^2}}+2\,U\right);$$

$$\text{wo1dxRed2} = \frac{1}{4}\,\mathrm{i}\,k\,(wp + kU)\,dx$$

Out[3158]= −Sqrt[g∗H] < U < Sqrt[g∗H]

Out[3164]= $\frac{1}{2}\,\mathrm{i}\,dt\,wp^2$

Out[3165]= $-\frac{1}{4}\,\mathrm{i}\,dx\,k^2\left(2\sqrt{g\,H}+\frac{\sqrt{3}\,U}{\sqrt{3+H^2 k^2}}\right)$

Out[3166]= U > Sqrt[g*H]

Out[3168]= $\frac{1}{2}\, i\, dt\, wp^2$

Out[3170]= $-\frac{1}{4}\, i\, dx\, k\, (kU + wp)$

Out[3171]= U< −Sqrt[g*H]

Out[3173]= $\frac{1}{2}\, i\, dt\, wp^2$

Out[3175]= $\frac{1}{4}\, i\, dx\, k\, (kU + wp)$

In[3176]:=

```
Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]]
```

$$wo2dt = \frac{1}{6\left(3 + H^2 k^2\right)^2} k^3 \left(\sqrt{3}\, \sqrt{g\, H\left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U\right) \left(3\, g\, H + U\left(2\sqrt{3}\, \sqrt{g\, H\left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U\right)\right) dt^2;$$

$$\frac{1}{6\left(3 + H^2 k^2\right)} dt^2\, k^2\, (wp)\, \left(3\, g\, H + U\left(2\sqrt{3}\, \sqrt{g\, H\left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U\right)\right);$$

$$wo2dtRed = \frac{dt^2\; (wp)\verb|^|3}{6}$$

$$wo2dx = \frac{k^3\left(-3\sqrt{3}\, \sqrt{g\, H\left(3 + H^2 k^2\right)} + 2\left(3 + H^2 k^2\right)^2 U\right)}{24\left(3 + H^2 k^2\right)^2} dx^2;$$

$$FullSimplify\left[\left(\frac{k^3\left(-3\sqrt{3}\, \sqrt{g\, H\left(3 + H^2 k^2\right)}\right)}{24\left(3 + H^2 k^2\right)^2} + k\verb|^|3 * U/12\right) dx^2 - wo2dx\right];$$

$$\left(\frac{k^3\left(-3\sqrt{3}\, \sqrt{g\, H}\right)}{24\left(3 + H^2 k^2\right)^{3/2}} + k\verb|^|3 * U/12\right) dx^2 - wo2dx;$$

$$wo2dxRed = k\verb|^|3/12\left(\frac{\left(-3\sqrt{3}\, \sqrt{g\, H}\right)}{2\left(3 + H^2 k^2\right)^{3/2}} + U\right) dx^2$$

```
Text[Row[{" U > Sqrt[g*H]  "}]]
```

$$wo2dt1 = \frac{1}{6\left(3 + H^2 k^2\right)^2} k^3 \left(\sqrt{3}\, \sqrt{g\, H\left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U\right) \left(3\, g\, H + U\left(2\sqrt{3}\, \sqrt{g\, H\left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U\right)\right) dt^2;$$

$$\text{wo2dtRed1} = \frac{dt^2\ (wp)\hat{}3}{6}$$

$$\text{wo2dx1} = \frac{k^3 \left(-3\sqrt{3}\ \sqrt{g\,H\,\left(3+H^2\,k^2\right)} + 2\ \left(3+H^2\,k^2\right)^2\,U\right)}{24\ \left(3+H^2\,k^2\right)^2}\ dx^2;$$

$$\text{wo2dxRed1} = k\hat{}3\big/12 \left(\frac{\left(-3\sqrt{3}\ \sqrt{g\,H}\ \right)}{2\ \left(3+H^2\,k^2\right)^{3/2}} + U\right) dx^2$$

```
Text[Row[{" U< -Sqrt[g*H] "}]]
```

$$\text{wo2dt2} = \frac{1}{6\ \left(3+H^2\,k^2\right)^2}\ k^3\ \left(\sqrt{3}\ \sqrt{g\,H\,\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)\ U\right)$$
$$\left(3\,g\,H + U\left(2\sqrt{3}\ \sqrt{g\,H\,\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)\ U\right)\right)\ dt^2;$$

$$\text{wo2dtRed2} = \frac{dt^2\ (wp)\hat{}3}{6}$$

$$\text{wo2dx2} = \frac{k^3 \left(-3\sqrt{3}\ \sqrt{g\,H\,\left(3+H^2\,k^2\right)} + 2\ \left(3+H^2\,k^2\right)^2\,U\right)}{24\ \left(3+H^2\,k^2\right)^2}\ dx^2;$$

$$\text{wo2dxRed2} = k\hat{}3\big/12 \left(\frac{\left(-3\sqrt{3}\ \sqrt{g\,H}\ \right)}{2\ \left(3+H^2\,k^2\right)^{3/2}} + U\right) dx^2$$

Out[3176]= −Sqrt[g∗H] < U < Sqrt[g∗H]

Out[3179]= $\frac{dt^2\ wp^3}{6}$

Out[3183]= $\frac{1}{12}\ dx^2\ k^3\ \left(-\frac{3\sqrt{3}\ \sqrt{g\,H}}{2\ \left(3+H^2\,k^2\right)^{3/2}} + U\right)$

Out[3184]= U > Sqrt[g∗H]

Out[3186]= $\frac{dt^2\ wp^3}{6}$

Out[3188]= $\frac{1}{12}\ dx^2\ k^3\ \left(-\frac{3\sqrt{3}\ \sqrt{g\,H}}{2\ \left(3+H^2\,k^2\right)^{3/2}} + U\right)$

Out[3189]= U< −Sqrt[g∗H]

Out[3191]= $\frac{dt^2\ wp^3}{6}$

Out[3193]= $\frac{1}{12}\ dx^2\ k^3\ \left(-\frac{3\sqrt{3}\ \sqrt{g\,H}}{2\ \left(3+H^2\,k^2\right)^{3/2}} + U\right)$

In[3194]:=

```
Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H]  "}]]
```

$$\text{wo2FEMdt} = \frac{1}{6\left(3+H^2 k^2\right)^2} k^3 \left(\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right) \left(3 g H + U\left(2\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right)\right) dt^2;$$

$$\frac{1}{6\left(3+H^2 k^2\right)} dt^2 k^2 (wp) \left(3 g H + U\left(2\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right)\right);$$

$$\text{wo2FEMdtRed} = \frac{dt^2\ (wp)\hat{}3}{6}$$

$$\text{wo2FEMdx} = \frac{1}{240\left(3+H^2 k^2\right)^2} \left(42\sqrt{3} k^3 \sqrt{g H\left(3+H^2 k^2\right)} + 15\sqrt{3} H^2 k^5 \sqrt{g H\left(3+H^2 k^2\right)} + 180 k^3 U + 120 H^2 k^5 U + 20 H^4 k^7 U\right) dx^2;$$

FullSimplify[wo2FEMdx];

$$\left(dx^2 k^3 \left(\sqrt{3}\sqrt{g H}\sqrt{\left(3+H^2 k^2\right)}\left(42 + 15 H^2 k^2\right) + 20\left(3+H^2 k^2\right)^2 U\right)\right) \Big/ \left(240\left(3+H^2 k^2\right)^2\right);$$

$$\text{wo2FEMdxRed} = \frac{dx^2 k^3 \left(\sqrt{3}\sqrt{g H}\left(42 + 15 H^2 k^2\right) / \left(3+H^2 k^2\right)^{3/2} + 20 U\right)}{240}$$

Text[Row[{" U > Sqrt[g*H] "}]]

$$\text{wo2FEMdt1} = \frac{1}{6\left(3+H^2 k^2\right)^2} k^3 \left(\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + 3 U + H^2 k^2 U\right) \left(3 g H + 2\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} U + 3 U^2 + H^2 k^2 U^2\right) dt^2;$$

$$\text{wo2FEMdtRed1} = \frac{dt^2\ (wp)\hat{}3}{6}$$

$$\text{wo2FEMdx1} = \frac{1}{240\left(3+H^2 k^2\right)^2} \left(42\sqrt{3} k^3 \sqrt{g H\left(3+H^2 k^2\right)} + 15\sqrt{3} H^2 k^5 \sqrt{g H\left(3+H^2 k^2\right)} + 180 k^3 U + 120 H^2 k^5 U + 20 H^4 k^7 U\right) dx^2;$$

$$\text{wo2FEMdxRed1} = \frac{dx^2 k^3 \left(\sqrt{3}\sqrt{g H}\left(42 + 15 H^2 k^2\right) / \left(3+H^2 k^2\right)^{3/2} + 20 U\right)}{240}$$

Text[Row[{" U< -Sqrt[g*H] "}]]

$$\text{wo2FEMdt2} = \frac{1}{6\left(3+H^2 k^2\right)^2} k^3 \left(\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right) \left(3 g H + U\left(2\sqrt{3}\sqrt{g H\left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right)\right) dt^2;$$

$$\text{wo2FEMdtRed2} = \frac{dt^2\ (wp)\hat{}3}{6}$$

$$\text{wo2FEMdx2} = \frac{1}{240\left(3+H^2 k^2\right)^2} \left(42\sqrt{3} k^3 \sqrt{g H\left(3+H^2 k^2\right)} +\right.$$

$$15\sqrt{3}\ H^2 k^5 \sqrt{g H \left(3+H^2 k^2\right)} + 180 k^3 U + 120 H^2 k^5 U + 20 H^4 k^7 U\Big) dx^2;$$

$$\text{wo2FEMdxRed2} = \frac{dx^2 k^3 \left(\sqrt{3}\ \sqrt{g H}\ \left(42\ + 15\ H^2 k^2\right) / \left(3+H^2 k^2\right)^{3/2} + 20\ U\right)}{240}$$

Out[3194]= $-\text{Sqrt}[g*H] < U < \text{Sqrt}[g*H]$

Out[3197]= $\frac{dt^2\, wp^3}{6}$

Out[3201]= $\frac{1}{240} dx^2 k^3 \left(\frac{\sqrt{3}\ \sqrt{g H}\ \left(42+15 H^2 k^2\right)}{\left(3+H^2 k^2\right)^{3/2}} + 20 U\right)$

Out[3202]= $U > \text{Sqrt}[g*H]$

Out[3204]= $\frac{dt^2\, wp^3}{6}$

Out[3206]= $\frac{1}{240} dx^2 k^3 \left(\frac{\sqrt{3}\ \sqrt{g H}\ \left(42+15 H^2 k^2\right)}{\left(3+H^2 k^2\right)^{3/2}} + 20 U\right)$

Out[3207]= $U < -\text{Sqrt}[g*H]$

Out[3209]= $\frac{dt^2\, wp^3}{6}$

Out[3211]= $\frac{1}{240} dx^2 k^3 \left(\frac{\sqrt{3}\ \sqrt{g H}\ \left(42+15 H^2 k^2\right)}{\left(3+H^2 k^2\right)^{3/2}} + 20 U\right)$

In[3212]:= Text[Row[{" -Sqrt[g*H] < U < Sqrt[g*H] "}]]

$$\text{wo3dt} = -\frac{1}{24\left(3+H^2 k^2\right)^3} i\, k^4 \left(\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)} + \left(3+H^2 k^2\right) U\right) \left(3 g \left(\sqrt{3}\ H \sqrt{g H \left(3+H^2 k^2\right)} + 9 H U + 3 H^3 k^2 U\right) + U^2 \left(H^4 k^4 U + 9 \left(\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)} + U\right) + 3 k^2 \left(\sqrt{3}\ \sqrt{g H^5 \left(3+H^2 k^2\right)} + 2 H^2 U\right)\right)\right) dt^3;$$

$$-\frac{1}{24\left(3+H^2 k^2\right)^2} i\, k^3\ (wp) \left(3 g \left(\sqrt{3}\ H \sqrt{g H \left(3+H^2 k^2\right)} + 9 H U + 3 H^3 k^2 U\right) + U^2 \left(H^4 k^4 U + 9 \left(\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)} + U\right) + 3 k^2 \left(\sqrt{3}\ \sqrt{g H^5 \left(3+H^2 k^2\right)} + 2 H^2 U\right)\right)\right) dt^3;$$

$$\text{Expand}\left[\frac{1}{\left(3+H^2 k^2\right)^2} k^3 \left(3 g \left(\sqrt{3}\ H \sqrt{g H \left(3+H^2 k^2\right)} + 9 H U + 3 H^3 k^2 U\right) + U^2 \left(H^4 k^4 U + 9 \left(\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)} + U\right) + 3 k^2 \left(\sqrt{3}\ \sqrt{g H^5 \left(3+H^2 k^2\right)} + 2 H^2 U\right)\right)\right)\right];$$

Expand[w^3];

$\text{Expand}\left[\left(3+H^2 k^2\right)^2\right];$

$$\text{wo3dtRed} = -\frac{i\ (wp)\text{^}4\, dt^3}{24}$$

$$\text{wo3dx} = -\frac{i\,k^4\left(2\,g\,H\left(3+H^2k^2\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}\,U\right)}{24\sqrt{g\,H}\left(3+H^2k^2\right)}\,dx^3;$$

FullSimplify[wo3dx];

$$-\frac{i\,dx^3\,k^4\left(2\,g\,H\left(3+H^2k^2\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}\,U\right)}{24\sqrt{g\,H}\left(3+H^2k^2\right)};$$

$$-\frac{1}{24}i\,dx^3\,k^4\left(2\,g\,H\left(3+H^2k^2\right)\Big/\left(\sqrt{g\,H}\left(3+H^2k^2\right)\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2k^2\right)}\,U\Big/\left(\sqrt{g\,H}\left(3+H^2k^2\right)\right)\right);$$

$$\text{wo3dxRed} = -\frac{1}{24}\,i\,dx^3\,k^4\left(2\sqrt{g\,H}+\frac{\sqrt{3}\,U}{\sqrt{\left(3+H^2k^2\right)}}\right)$$

Text[Row[{" U > Sqrt[g*H] "}]]

$$\text{wo3dt1} = -\frac{i\left(\sqrt{3}\,k\sqrt{g\,H\left(3+H^2k^2\right)}+3\,k\,U+H^2k^3U\right)^4dt^3}{24\left(3+H^2k^2\right)^4};$$

$$\text{wo3dtRed1} = -\frac{i\ (wp)\,\hat{}\,4\,dt^3}{24}$$

$$\text{wo3dx1} = -\frac{1}{24}\,i\,k^4\left(\sqrt{3}\sqrt{\frac{g\,H}{3+H^2k^2}}+2\,U\right)dx^3;$$

$$\text{wo3dxRed1} = -\frac{1}{24}\,i\,k^3\,(wp+k\,U)$$

Text[Row[{" U< -Sqrt[g*H] "}]]

$$\text{wo3dt2} = -\frac{i\left(\sqrt{3}\,k\sqrt{g\,H\left(3+H^2k^2\right)}+3\,k\,U+H^2k^3U\right)^4dt^3}{24\left(3+H^2k^2\right)^4};$$

$$\text{wo3dtRed2} = -\frac{i\ (wp)\,\hat{}\,4\,dt^3}{24}$$

$$\text{wo3dx2} = \frac{1}{24}\,i\,k^4\left(\sqrt{3}\sqrt{\frac{g\,H}{3+H^2k^2}}+2\,U\right)dx^3;$$

$$\text{wo3dxRed2} = \frac{1}{24}\,i\,k^3\,(wp+k\,U)$$

Out[3212]= $-\text{Sqrt}[g*H] < U < \text{Sqrt}[g*H]$

Out[3218]= $-\frac{1}{24}\,i\,dt^3\,wp^4$

Out[3223]= $-\frac{1}{24}\,i\,dx^3\,k^4\left(2\sqrt{g\,H}+\frac{\sqrt{3}\,U}{\sqrt{3+H^2k^2}}\right)$

Out[3224]= U > Sqrt[g*H]

Out[3226]= $-\frac{1}{24}\, i\, dt^3\, wp^4$

Out[3228]= $-\frac{1}{24}\, i\, k^3\, (k\, U + wp)$

Out[3229]= U< −Sqrt[g*H]

Out[3231]= $-\frac{1}{24}\, i\, dt^3\, wp^4$

Out[3233]= $\frac{1}{24}\, i\, k^3\, (k\, U + wp)$