```
ln[1] = q = q0 * Exp[I * (k * x + w * t)];
    qjn = q0 * Exp[I * (k * xj + w * tn)];
    qjbar = Integrate [q, \{x, xj - dx/2, xj + dx/2\}]/(dx);
    qjnbar = qjbar /. t \rightarrow tn;
    MA = qjn / qjnbar;
    qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
    qjntbar = qntbar /. x \rightarrow xj;
    MtA = qjntbar / qjn;
    qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
    RA = Simplify[MA * qjphn / (qjn)];
    vmultG = H + H^3/3*k^2;
    GnA = -U * RA / vmultG;
    GGA = RA / vmultG;
    GcA = -U * H / vmultG ;
     fn1A = H * vh + U * eh;
     fn1A = fn1A /. vh \rightarrow (GGA * Gca + GnA * eca) /. eh \rightarrow RA * eca;
    fn1Gca0A = fn1A / . Gca \rightarrow 0 ;
     fn1eca0A = fn1A/. eca \rightarrow 0;
     fnnA = Simplify[fn1Gca0A / eca];
     fnGA = fn1eca0A / Gca;
     fncA = H * GcA;
    fG1A = U*Gh + U*H*vh + g*H*eh;
     \texttt{fG1A} = \texttt{fG1A} \ / \ . \ \texttt{vh} \ \rightarrow \ (\texttt{GGA} * \texttt{Gca} + \ \texttt{GnA} * \texttt{eca}) \ / \ . \ \texttt{eh} \ \rightarrow \ \texttt{RA} * \texttt{eca} \ / \ . \ \texttt{Gh} \ \rightarrow \ \texttt{RA} * \texttt{Gca} \ ;
     fG1Gca0A = fG1A / . Gca \rightarrow 0 ;
     fGleca0A = fGlA /. eca \rightarrow 0;
     fGnA = Simplify[fG1Gca0A / eca];
     fGGA = Simplify[fGleca0A / Gca];
     fGcA = U * H * GcA;
    FnnA = -MtA*dt/dx*(1 - Exp[-I*k*dx])*fnnA;
    FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
    FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
    FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;
    MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
```

$$\frac{\mathbf{k}\left(\sqrt{3}\sqrt{\mathbf{g}\,\mathbf{H}\,\left(3+\mathbf{H}^2\,\mathbf{k}^2\right)}\,+\left(3+\mathbf{H}^2\,\mathbf{k}^2\right)\,\mathbf{U}\right)}{\left(3+\mathbf{H}^2\,\mathbf{k}^2\right)}\,\,\rightarrow\,\,-\mathbf{w};$$

wAp = 
$$U * k + \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};$$
  
wAm =  $U * k - \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};$ 

```
ln[38] = M = (26 - 2 * Cos[k * dx]) / 24;
     Merr = Series[M - MA, \{dx, 0, 10\}];
     Rm = (5 - Exp[-I*k*dx] + 2*Exp[I*k*dx]) / 6;
     Rmerr = Series[Rm - RA, \{dx, 0, 10\}];
     Rp = Exp[I*k*dx]*(5 + 2*Exp[-I*k*dx] - Exp[I*k*dx])/6;
     Rperr = Series[Rp - RA, \{dx, 0, 10\}];
     Ru = \left(-Exp[-I*k*dx] + 9*Exp[I*k*dx] - Exp[2*I*k*dx] + 9\right) / 16;
     Ruerr = Series \left[ Ru - Exp \left[ I * k * dx / 2 \right], \left\{ dx, 0, 10 \right\} \right];
     Gold = H - H^3/3 * (32 * Cos[k * dx] - 2 * Cos[2 * k * dx] - 30) / (12 * dx^2);
     GG2 = M * Ru / (Gold);
     GG2err = Series[GG2 - GGA, {dx, 0, 5}];
     Gn2 = -M * Ru * U / (Gold);
     Gn2err = Series[Gn2 - GnA, {dx, 0, 5}];
     Text[Row[{"M ||
                         ", M}]]
     Text[Row[{"M || ", TeXForm[M]}]]
     Text[Row[{"M error || ", TeXForm[Merr]}]]
     Text[Row[{"M error || ", Merr}]]
     Text[" "]
     Text[Row[{"Rm || ", Rm}]]
     Text[Row[{"Rm || ", TeXForm[Rm]}]]
     Text[Row[{"Rm error || ", Rmerr}]]
     Text[Row[{"Rm error || ", TeXForm[Rmerr]}]]
     Text[" "]
     Text[Row[{"Rp ||
                          ", Rp}]]
     Text[Row[{"Rp || ", TeXForm[Rp]}]]
     Text[Row[{"Rp error || ", Rperr}]]
     Text[Row[{"Rp error || ", TeXForm[Rperr]}]]
     Text[" "]
     Text[Row[{"GG2 || ", GG2}]]
     Text[Row[{"GG2 || ", TeXForm[GG2]}]]
     Text[Row[{"GG2 error || ", GG2err}]]
     Text[Row[{"GG2 error || ", TeXForm[GG2err]}]]
     Text[" "]
     Text[Row[{"Gn2 || ", Gn2}]]
     Text[Row[{"Gn2 || ", TeXForm[Gn2]}]]
     Text[Row[{"Gn2 error || ", Gn2err}]]
     Text[Row[{"Gn2 error || ", TeXForm[Gn2err]}]]
Out[51]= M || \frac{1}{24} (26 - 2 \cos[dx \, k])
Out[52]= M || \frac{1}{24} (26-2 \cos (\text{d}x) k)
```

Out[53]= M error |

 $-\frac{3 \text{3 } \text{4 k}^4}{640} + \frac{3 \text{4 k}^6 k^6}{35840} - \frac{149 \text{4 k}^8 k^8}{51609600} + \frac{29 \text{4 k}^10}{10} \frac{10}{13624934400} + O\left(\frac{4x}^{11}\right) + O\left(\frac{4x}{11}\right) + O\left(\frac{4x}{11}$ 

$$\text{Out} \text{[54]=} \quad M \; error \; \mid \mid \; -\frac{3 \; k^4 \; dx^4}{640} + \frac{3 \; k^6 \; dx^6}{35 \; 840} - \frac{149 \; k^8 \; dx^8}{51 \; 609 \; 600} + \frac{29 \; k^{10} \; dx^{10}}{13 \; 624 \; 934 \; 400} + O[dx]^{11}$$

Out[55]=

Out[56]= Rm || 
$$\frac{1}{6} (5 - e^{-i \, dx \, k} + 2 \, e^{i \, dx \, k})$$

Out[57]= Rm ||  $\frac{1}{6} \left| -e^{-i \cdot text \cdot dx} k \right| + 2 e^{i \cdot text \cdot dx} k + 5 \right|$ 

$$\text{Out} \text{[58]=} \quad Rm \; error \; \mid \mid \; \; -\frac{1}{12} \, i \, k^3 \; dx^3 \, + \, \frac{k^4 \, dx^4}{120} \, + \, \frac{1}{240} \, i \, k^5 \; dx^5 \, - \, \frac{k^6 \, dx^6}{5040} \, - \, \frac{i \, k^7 \, dx^7}{10080} \, + \, \frac{k^8 \, dx^8}{201600} \, + \, \frac{i \, k^9 \, dx^9}{725760} \, - \, \frac{k^{10} \, dx^{10}}{39916800} \, + \, O[dx]^{11}$$

Out[59]= Rm error ||

 $-\frac{1}{12} i \cdot \frac{dx}^3 k^3 + \frac{dx}^4 k^4}{120} + \frac{1}{240} i \cdot \frac{dx}^5 k^5 - \frac{dx}^6 k^6}{5040} - \frac{i \cdot \frac{dx}^7 k^7}{10080} + \frac{dx}^8 k^8}{201600} + \frac{i \cdot \frac{dx}^7 k^7}{10080} + \frac{dx}^9 k^9}{725760} - \frac{\cot dx}^10} k^10}{39916800} + O\cdot \frac{dx}^11}{ \cdot \frac{dx}^9 k^9}{725760} - \frac{\cot dx}^10} k^10} k^10} + \frac{dx}^9 k^9}{725760} - \frac{\cot dx}^10} k^10} k^10} k^10} + \frac{dx}^9 k^9}{725760} - \frac{\cot dx}^9 k^10}{39916800} + O\cdot \frac{dx}^9 k^9}{11} + \frac{dx}^9$ 

Out[60]=

Out[61]= Rp || 
$$\frac{1}{6} e^{i \, dx \, k} \left( 5 + 2 e^{-i \, dx \, k} - e^{i \, dx \, k} \right)$$

 $\label{eq:continuity} $$\operatorname{Out}(0)= Rp \mid \int \frac{1}{6} e^{i \cdot k} \left( 2 e^{i \cdot k} \right) \left( 2$ 

$$\text{Out} \text{[63]=} \quad Rp \; error \; \mid \mid \quad \frac{1}{12} \; \dot{\textit{i}} \; k^3 \; dx^3 \; - \; \frac{3 \, k^4 \, dx^4}{40} \; - \; \frac{3}{80} \; \dot{\textit{i}} \; k^5 \; dx^5 \; + \; \frac{23 \, k^6 \, dx^6}{1680} \; + \; \frac{41 \, \dot{\textit{i}} \; k^7 \, dx^7}{10080} \; - \; \frac{209 \, k^8 \, dx^8}{201600} \; - \; \frac{169 \, \dot{\textit{i}} \; k^9 \, dx^9}{725760} \; + \; \frac{89 \, k^{10} \, dx^{10}}{1900800} \; + \; O[dx]^{11} \; + \; O[dx]^{11} \; dx^4 \; dx^4 \; dx^4 \; + \; O[dx]^{11} \; dx^4 \; dx^$$

Out[64]= Rp error ||

 $\label{eq:condition} $$ \frac{1}{12} i \text{d}^3 k^3-\frac{3 \ker(dx)^4 k^4}{40}-\frac{3}{80} i \text{d}^5 k^5+\frac{23 \ker(dx)^6 k^6}{1680}+\frac{41 i \det(dx)^7 k^7}{10080}-\frac{209 \det(dx)^8 k^8}{201600}-\frac{169 i \det(dx)^9 k^9}{725760}+\frac{89 \det(dx)^{10} k^{10}}{1900800}+O\left(\frac{dx}^7 1\right)\right) $$$ 

Out[65]=

$$\text{Out} \text{[G6]=} \quad GG2 \quad \text{[I]} \quad \frac{\left(9 - e^{-i\,dx\,k} + 9\,\,e^{i\,dx\,k} + e^{2\,i\,dx\,k}\right) \left(26 - 2\,\text{Cos}[dx\,k]\right)}{384 \left(H - \frac{H^3\,(-30 + 32\,\text{Cos}[dx\,k] - 2\,\text{Cos}[2\,dx\,k])}{36\,dx^2}\right)}$$

Out[67]= GG2 ||  $\frac{e^{-i \text{dx} k}+9 e^{i \text{dx} k}-e^{2 i \text{dx} k}+9 \text{right}}{26-2 \cos (\text{dx} k)}}{384 \left(H-\frac{H^3 (32 \cos (\text{dx} k)-2 \cos (2 \text{dx} k)-30)}{36 \text{dx} k}\right)}$ 

$$\text{Out[68]=} \quad GG2 \; error \; \mid \mid \quad \frac{\left(-243 \, k^4 - 49 \, H^2 \, k^6\right) dx^4}{960 \, H \, (3 + H^2 \, k^2)^2} - \frac{i \, \left(243 \, k^5 + 49 \, H^2 \, k^7\right) dx^5}{1920 \, H \, \left(3 + H^2 \, k^2\right)^2} + O[dx]^6$$

Out[69]= GG2 error ||

 $\label{left(49 H^2 k^6-243 k^4+right)} $$ H^2 k^7+243 k^5\right] + H^2 k^7+243 k^7+244 k^7+244 k^7+244 k^7+244 k^7+24$ 

Out[70]=

```
\text{Out} [71] = \quad Gn2 \quad || \quad \frac{\left(9 - e^{-i\,dx\,k} + 9\,\,e^{i\,dx\,k} - e^{2\,i\,dx\,k}\right)\,U\,(-26 + 2\,Cos[\,dx\,k])}{384\left(H - \frac{H^3\,(-30 + 32\,Cos[\,dx\,k] - 2\,Cos[\,2\,dx\,k])}{384}\right)}
Outf72] = Gn2 \parallel \frac{U \left(-e^{-i \cdot text\{dx\} k} + 9 e^{i \cdot text\{dx\} k} + 9 e^{-i 
                                        k)-26}{384 \left(H-\frac{H^3 (32 \cos (\text{dx} k)-2 \cos (2 \text{dx} k)-30)}{36 \text{dx}^2}\right)}
 \text{Out} \text{[73]= } \text{Gn2 error } \text{[]} \text{ } \frac{ \left( 243\,k^4 + 49\,H^2\,k^6 \right)\,U\,dx^4 }{ 960\,H\,(3 + H^2\,k^2)^2 } + \frac{i\left( 243\,k^5 + 49\,H^2\,k^7 \right)\,U\,dx^5 }{ 1920\,H\,(3 + H^2\,k^2)^2 } + O[dx]^6 
Out[74]= Gn2 error |
                           \frac{d^4 U \left(49 + 2 k^6 + 243 k^4 \right)}{960 H \left(49 + 2 k^2 + 3 \right)}{960 H \left(49 + 2 k^2 + 3 \right)} + \frac{1}{100}
                                       \left(49 \text{ H}^2 \text{ k}^7 + 243 \text{ k}^5\right) \left(1920 \text{ H} \left(4^2 \text{ k}^2 + 3\right)^2\right) + O\left(\frac{40 \text{ k}^2 + 3\right)^2}{1920 \text{ H}^2} + O\left(\frac{40 \text{ k}^2 + 3\right)^2}{1920 \text{ k}^2} + 
  ln[75]:= KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
                     KurFWS = KurF /. ap \rightarrow (U + Sqrt[g * H]) /. am \rightarrow (U - Sqrt[g * H]);
                     KurFWSeta =
                              KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
                                    qm \rightarrow Rmp * n;
                     KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
                     Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1];
                     KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1];
                     Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
                     KfnG = KfnGp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
                     Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
                     Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 4}];
                     Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                     FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
                     FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 4}];
                     FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                     KurFWSG = KurFWS /. fp \rightarrow (U*Rpp*G + U*H*v + g*H*Rpp*n) /.
                                              fm \rightarrow (U * Rmp * G + U * H * V + g * H * Rmp * n) /. qp \rightarrow Rpp * G /. qm \rightarrow Rmp * G;
                     KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
                     KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
                     KfGn = KfGnp /. Rpp \rightarrow Rp /. Rmp \rightarrow Rm /. GGp \rightarrow GG2 /. Gnp \rightarrow Gn2;
                     KfGG = KfGGp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
                     FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
                     FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 4}];
                     FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
                      fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
                     FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
                     FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 4}];
```

 $FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];$ 

```
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2/2 + Fmat2.Fmat2.Fmat2/6;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
RKStep =
  Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 + EigvFmat2 * EigvFmat2 * EigvFmat2 / 6]/
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" -Sqrt(gH) < U < Sqrt(gH)"}]]</pre>
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn || ", KfGnp}]]
Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error || ", FGn2TAr}]]
Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG || ", KfGGp}]]
Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error || ", FGG2TAr}]]
Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error || ", RKstepTayr}]]
Text[Row[{"Omega error || ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||
                   ", EA}]]
Text[Row[{"EA || ", TeXForm[EA]}]]
Text[Row[{"Eerr || ", Eerr}]]
Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
```

Out[109]= -Sqrt(gH) < U < Sqrt(gH)

Out[110]=

Out[111]= Fnn || 
$$\frac{1}{2} \left( 2 \operatorname{Gnp} H + \operatorname{Rpp} \left( -\sqrt{g H} + U \right) + \operatorname{Rmp} \left( \sqrt{g H} + U \right) \right)$$

 $\label{eq:output} \begin{tabular}{ll} $$\operatorname{Fnn} & \frac{1}{2} \left( \operatorname{Rmp} \left( \operatorname{Rmp} \right) + \operatorname{H-V-ight} \right) + \operatorname{H-V-ight} \right) $$ is the second of the se$ 

$$\begin{array}{ll} \text{Out} \text{[113]=} & Fnn \; error \; \mid \mid \; \left( -\frac{\left( H^2 \; k^3 \; U \; w \right) dt^2}{2 \left( 3 + H^2 \; k^2 \right)} - \frac{i \; H^2 \; k^3 \; U \; w^2 \; dt^3}{6 \left( 3 + H^2 \; k^2 \right)} + \frac{H^2 \; k^3 \; U \; w^3 \; dt^4}{24 \left( 3 + H^2 \; k^2 \right)} + O[dt]^5 \right) + \\ & \left( -\frac{1}{12} \left( \sqrt{g \; H} \; \; k^4 \right) dt + O[dt]^5 \right) dx^3 + \left( \frac{i \left( 45 \; k^5 \; U + 143 \; H^2 \; k^7 \; U + 32 \; H^4 \; k^9 \; U \right) dt}{960 \left( 3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \\ \end{array}$$

Out[114]= Fnn error |

 $\left(-\frac{t}{2 \cdot t}\right)^2 \left(-\frac{t}{2 \cdot$  $\left(H^2 k^2+3\right)+\frac{dt}^4 H^2 k^3 U w^3}{24 \left(H^2 k^2 + 3\right)}$ 

 $k^2+3\left(\frac{1}{1}12\right)\left(\frac{1}12\right)\left(\frac$ \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i}{12 H^4 U k^9+143 H^2 U k^7+45 U  $k^5 \right) \left( \frac{dt}{960 \left( \frac{A^2 k^2+3\right)^2}+O\left( \frac{dt}{5}\right) \right)}{1} + O\left( \frac{dt}{5}\right) \right) \left( \frac{dt}{5}\right) \left($ 

Out[115]=

Out[116]=  $FnG \parallel GGp H$ 

Out[117]=  $FnG \parallel \text{text}\{GGp\} H$ 

$$\text{Out} \\ \text{[118]= } FnG \ error \ || \ \left( -\frac{3 \, (k \, w) \, dt^2}{2 \, (3 + H^2 \, k^2)} - \frac{i \, k \, w^2 \, dt^3}{2 \, (3 + H^2 \, k^2)} + \frac{k \, w^3 \, dt^4}{8 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) \\ + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^5 + 49 \, H^2 \, k^7) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dx]^5 \right) dx^4 \\ + O[dx]^5 +$$

 $\text{text}\{dt\}^3 \text{ k w}^2\}\{2 \left(H^2 \text{ k}^2 + 3\right)\} + \frac{dt}^4 \text{ k w}^3\}\{8 \left(H^2 \text{ k}^2 + 3\right)\}$  $k^2+3\right)+O\left(\frac{d4}{h^2}\right)+\left(\frac{d4}{h^2}\right)$  $\text{dt}{960 \left(H^2 k^2+3\right)^2}+O\left(\left(text{dt}^5\right)\right)+O\left(text{dt}^5\right)+O\left(text{dx}^5\right)+O\left(text{d$ 

Out[120]=

$$\text{Out} [\text{121}] = \text{ } FGn \text{ } || \text{ } \frac{1}{2} \left( g \text{ } H \text{ } (Rmp + Rpp) + \left( 2 \text{ } Gnp \text{ } H + \sqrt{g \text{ } H} \text{ } (Rmp - Rpp) \right) U \right)$$

Out[122]= FGn ||

$$\begin{array}{ll} \text{Out} \text{[123]=} & FGn \; error \; || \; \left( -\frac{\left( k\left( 3\,g\,H+g\,H^3\,k^2-3\,U^2\right) w\right) dt^2}{2\left( 3+H^2\,k^2\right)} - \frac{i\,k\left( 3\,g\,H+g\,H^3\,k^2-3\,U^2\right) w^2\,dt^3}{6\left( 3+H^2\,k^2\right)} + \frac{k\left( 3\,g\,H+g\,H^3\,k^2-3\,U^2\right) w^3\,dt^4}{24\left( 3+H^2\,k^2\right)} + O[dt]^5 \right) + \\ & \left( -\frac{1}{12} \left( \sqrt{g\,H} \; k^4\,U \right) dt + O[dt]^5 \right) dx^3 + \left( \frac{i\left( 288\,g\,H\,k^5+192\,g\,H^3\,k^7+32\,g\,H^5\,k^9-243\,k^5\,U^2-49\,H^2\,k^7\,U^2\right) dt}{960\left( 3+H^2\,k^2\right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \end{array}$$

Out[124]= FGn error |

 $\label{left-frac} $\left(-\frac{\left(\frac{4t}^2 \left( H^2 k^2+3\right)-\frac{4t}{2} \right)}{2 \left( H^2 k^2+3\right)}-\frac{4t}{3} k^2+3 g H-3 U^2\right)} \\ \left(-\frac{4t}^3 k^2-\frac{4t}^3 k^2+3 g H-3 U^2\right)}{6 \left( H^2 k^2+3\right)}+\frac{4t}^4 k w^3 \left( H^2 k^2+3 g H-3 U^2\right)}{6 \left( H^2 k^2+3\right)}+O\left(\frac{4t}^5\right)+\frac{4t}^4 k w^3 \left( H^2 k^2+3 g H-3 U^2\right)}{24 \left( H^2 k^2+3\right)}+O\left(\frac{4t}^5\right)+\frac{4t}^4 h^3 k^2+3 g H-3 U^2\right)}+\frac{4t}^4 h^3 k^2+3 g H-3 U^2\right)}+\frac{4t}^4 h^3 k^2+3 g H^3 k^4 U\right)+\frac{4t}^4 h^4 U\right)+\frac{4t}^4 h^4 U^2 k^4 h^4 U^2 k^4 U^2 k^4 U^2 k^4 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 h^4 U^2 k^4 U^2 k^4 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^4 U^2 k^5 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^4 U^2 k^5 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^4 U^2 k^5 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^4 U^2 k^5 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^4 U^2 k^5 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^4 U^2 k^5 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^4 U^2 k^5 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^4 U^2 k^5 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^4 U^2 k^5 U^2 k^5+288 g H k^5\right)+\frac{4t}^4 U^2 k^5 U^2 k^5$ 

Out[125]=

Out[126]= FGG || 
$$\frac{1}{2} \left( \sqrt{g H} Rmp - \sqrt{g H} Rpp + (2 GGp H + Rmp + Rpp) U \right)$$

$$\begin{array}{ll} \text{Out} [\text{128}] = & FGG \; error \; \mid \mid \; \left( -\frac{\left( k \left( 6 + H^2 \; k^2 \right) U \; w \right) \; dt^2}{2 \left( 3 + H^2 \; k^2 \right)} - \frac{i \; k \left( 6 + H^2 \; k^2 \right) U \; w^2 \; dt^3}{6 \left( 3 + H^2 \; k^2 \right)} + \frac{k \left( 6 + H^2 \; k^2 \right) U \; w^3 \; dt^4}{24 \left( 3 + H^2 \; k^2 \right)} + O[dt]^5 \right) + \\ & \left( -\frac{1}{12} \left( \sqrt{g \; H} \; \; k^4 \right) dt + O[dt]^5 \right) dx^3 + \left( \frac{i \left( 531 \; k^5 \; U + 241 \; H^2 \; k^7 \; U + 32 \; H^4 \; k^9 \; U \right) dt}{960 \left( 3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \\ \end{array}$$

Out[129]= FGG error ||

 $\label{left-deft} $$\left(-\frac{\hat U w \left(\frac{4t}^2 k^2+6\right)}{2 \left(\frac{4t}^2 k^2+3\right)}-\frac{i \left(\frac{4t}^3 k U w^2 \left(\frac{4t}^2 k^2+6\right)}{4 k U w^3 \left(\frac{4t}^2 k^2+6\right)}\right)} \right) $$\left(\frac{4t}^2 k^2+6\right)}{4 k U w^3 \left(\frac{4t}^2 k^2+6\right)} \left(\frac{4t}^2 k^2+6\right)} \left(\frac{4t}^2 k^2+6\right)}{24 \left(\frac{4t}^2 k^2+3\right)} +O\left(\frac{4t}^2 k^2+3\right)} +O\left(\frac{4t}^2 k^2+3\right)} \right) $$\left(\frac{4t}^4 k U w^3 \left(\frac{4t}^2 k^2+6\right)}{24 \left(\frac{4t}^4 k^2 k^2+3\right)} \right) +O\left(\frac{4t}^4 k^4 k^2 k^2+6\right)} \left(\frac{4t}^4 k U w^3 \left(\frac{4t}^4 k U w^3 \left(\frac{4t}^4 k^2 k^2+6\right)}{24 \left(\frac{4t}^4 k^2 k^2+3\right)} \right) +O\left(\frac{4t}^4 k U k^3 k^4 U k^4 U k^3 k^4 U k^3 k^4 U k^3 k^4 U k^4 U k^3 k^4 U k^3 k^4 U k^4 U k^3 k^4 U k^4 U k^3 k^4 U k$ 

Out[130]=

Out[131]=

Out[132]= Omega error ||

$$\left\{ \left( -\frac{1}{24(3+H^2k^2)^3} i \, k^4 \left( \sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + \left( 3 + H^2\,k^2 \right) \, U \right) \left( 3 \, g \left( \sqrt{3} \, H \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + 9 \, H \, U + 3 \, H^3 \, k^2 \, U \right) + U^2 \left( H^4 \, k^4 \, U + 9 \left( \sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + U \right) + 3 \, k^2 \left( \sqrt{3} \, \sqrt{g \, H^5 \, (3+H^2\,k^2)} \right. + 2 \, H^2 \, U \right) \right) \right) dt^3 + U^2 \left( H^4 \, k^4 \, U + 9 \left( \sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + \left. (3+H^2\,k^2) \, U \right) \right) + U^3 \left( 3 \, H^2 \, k^2 \right) + 2 \, H^2 \, U \right) \right) dt^3 + U^3 \left( 12 \, \sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + \left. (3+H^2\,k^2) \, U \right) + U^3 \left( 12 \, \sqrt{3} \, \sqrt{g \, H \, (3+H^2\,k^2)} \right. + 9 \, U + H^4 \, k^4 \, U + 2 \, k^2 \left( 2 \, \sqrt{3} \, \sqrt{g \, H^5 \, (3+H^2\,k^2)} \right. + 3 \, H^2 \, U \right) \right) \right) dt^4 + U^3 \left( 12 \, \sqrt{g \, H \, (3+H^2\,k^2)} \right) dt^3 + U^3 \, U^$$

$$\begin{array}{c} {}^{3} K^{*}\left(6 \vee g \cdot H^{*}\left(3 + H^{*}\left(K^{*}\right)\right) + V + V \cdot g \cdot H^{*}\left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + \sqrt{3} \cdot H^{9} \cdot U\right) + V \cdot g \cdot H^{9} \cdot H^{9} \cdot \left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + \sqrt{3} \cdot H^{9} \cdot U\right) + V \cdot g \cdot H^{9} \cdot \left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + \sqrt{3} \cdot H^{9} \cdot U\right) + V \cdot g \cdot H^{9} \cdot \left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + \sqrt{3} \cdot H^{9} \cdot U\right) + V \cdot g \cdot H^{9} \cdot \left(2 \vee g \cdot H^{9}\left(3 + H^{2} \cdot K^{2}\right) + V \cdot \left(63 + 33 \cdot H^{2} \cdot K^{2}\right) \cdot U^{4} + \sqrt{3} \cdot U^{5}\right) + V \cdot g \cdot H^{9} \cdot \left(6 \vee g^{3} \cdot H^{3}\left(3 + H^{2} \cdot K^{2}\right) + 6 \vee g \cdot H^{3}\left(3 + H^{2} \cdot K^{2}\right) \cdot U^{4} + \sqrt{3} \cdot U^{5}\right) + V \cdot g \cdot H^{9} \cdot \left(6 \vee g^{3} \cdot H^{3}\left(3 + H^{2} \cdot K^{2}\right) + 1728 \cdot U + 192 \cdot H^{4} \cdot K^{2} \cdot U^{4} + V \cdot 3 \cdot H^{2} \cdot U^{3}\right)\right) dt^{4} + O(dt)^{5} \cdot dx^{3} + V \cdot g \cdot H^{9} \cdot \left(145 \vee \sqrt{3} \cdot \sqrt{3} \cdot \frac{147}{3} \cdot H^{2} \cdot K^{2}\right) + 1152 \cdot H^{2} \cdot U\right) + V \cdot g \cdot H^{9} \cdot \left(145 \vee \sqrt{3} \cdot \frac{147}{3} \cdot H^{2} \cdot K^{2}\right) \cdot U^{3} + V \cdot g \cdot H^{2} \cdot \left(145 \vee \sqrt{3} \cdot \frac{147}{3} \cdot H^{2} \cdot K^{2}\right) \cdot U^{3} + V \cdot g \cdot H^{2} \cdot \left(145 \vee \sqrt{3} \cdot \frac{147}{3} \cdot H^{2} \cdot K^{2}\right) \cdot U^{3} + V \cdot g \cdot H^{2} \cdot \left(145 \vee \sqrt{3} \cdot \frac{147}{3} \cdot H^{2} \cdot K^{2}\right) \cdot U^{3} + V \cdot g \cdot H^{2} \cdot \left(145 \vee \sqrt{3} \cdot \frac{147}{3} \cdot H^{2} \cdot K^{2}\right) \cdot U^{3} + V \cdot g \cdot H^{2} \cdot \left(145 \vee \sqrt{3} \cdot \frac{147}{3} \cdot H^{2} \cdot K^{2}\right) \cdot U^{3} + V \cdot g \cdot H^{2} \cdot H^{2$$

$$O[dil]^{5} + \left[ -\frac{ik^{4} \left[ 2 \pm H(2+H^{2}k^{2}) - \sqrt{3} \sqrt{g \pm H(2+H^{2}k^{2})} \right]}{24 \sqrt{g + H^{2}(3+H^{2}k^{2})}} + \frac{1}{144(3+H^{2}k^{2})} + \sqrt{3} \right] + \frac{1}{144(3+H^{2}k^{2})^{3/2}}$$

$$k^{7} \left( 6\sqrt{3} \right) \frac{2^{2}}{g^{2}} H^{2} \left( 3 + H^{2}k^{2} \right) + 9 U^{3} \left( -5\sqrt{g H} \left( 3 + H^{2}k^{2} \right) + \sqrt{3} \right) U \right) + \frac{1}{144(3+H^{2}k^{2})} \left( -2\sqrt{g H^{9}} \left( 3 + H^{2}k^{2} \right) + \sqrt{3} \right) H^{2} U \right) + \frac{1}{3} \frac{1}{3} H^{2} U \left( -21\sqrt{g H} \left( 3 + H^{2}k^{2} \right) + \sqrt{3} \right) \left( 27 + 15 H^{2}k^{2} + 2 H^{4}k^{4} \right) U \right) - \frac{1}{3} k^{2} \left( 6\sqrt{g^{3}} H^{7} \left( 3 + H^{2}k^{2} \right) U + 7\sqrt{g H^{3}} \left( 3 + H^{2}k^{2} \right) U^{3} - 2\sqrt{3} H^{2}U^{4} \right) \right) dt^{3} + \frac{1}{144(3+H^{2}k^{2})^{2}} k^{2} \left( 3\sqrt{3} \right) \frac{g^{2}}{g^{2}} H^{2} \left( 27 + 8 H^{2}k^{2} \right) U + k^{2}U^{4} \left( -2\sqrt{g H^{9}} \left( 3 + H^{2}k^{2} \right) U + \sqrt{3} H^{4}U \right) + \frac{2}{g H^{12}} \left( -2\sqrt{g^{2}} H^{3} \left( 3 + H^{2}k^{2} \right) U + \sqrt{3} H^{2}U^{3} \right) \right) dt^{4} + O[dt]^{3} \right] dt^{3} + \frac{2}{g H^{12}} \left( 3 + H^{2}k^{2} \right) U^{2} - 6\sqrt{g H} \left( 3 + H^{2}k^{2} \right) U^{4} + \sqrt{3} U^{5} \right) - \frac{1}{34500\sqrt{g H}} \left( 3 + H^{2}k^{2} \right) U^{2} + 4\sqrt{g H^{5}} \left( 3 + H^{2}k^{2} \right) U^{4} + \sqrt{3} U^{5} \right) dt^{4} + O[dt]^{3} \right] dt^{3} + \frac{1}{g^{2}} \left( 1 + \frac{1}{g^{2}} H^{2} \left( 3 + H^{2}k^{2} \right) U^{2} + 4\sqrt{g H^{5}} \left( 3 + H^{2}k^{2} \right) U^{4} + \sqrt{3} H^{2}U^{5} \right) dt^{4} + O[dt]^{3} \right) dt^{4} + O[dt]^{3} \right) dt^{4} + O[dt]^{3} dt^{3} + \frac{1}{g^{2}} H^{2} U^{2} U^{2} + \frac{1}{g^{2}} H^{2} U^{2} U^{2}$$

 $\left(\frac{h^4 \left(\frac{h^2 k^2+3\right)} U+\sqrt{1} u}{1}\right) U+\sqrt{1} u}$  $k^4+3 \left( U H^2+\sqrt{3} \right) \left( U H^2+\sqrt{3} \right)$  $H \left( H^2 k^2 + 3 \right) \right) \left( H^2 k^2 + 3 \right) \left( H^$  $U+\sqrt{3} \sqrt{4+2 \left(H^2 k^2+3\right)}\right) \left(H^4 U k^4+2 H^2 U H^2+2 \right)$  $\$  \\ \sqrt{g H^5 \\ left(H^2 k^2+3\right)\\ right) \\  $U^3+6 g H \left( \frac{h^2 k^2+3 \right) U+2 \left( \frac{4}{3} \left( \frac{h^2 k^2+3 \right) U+2 \left( \frac{4}{3} \right) U+2$  $H^2\left(\frac{dt}^4}{30 \left(\frac{k^2 + 3 \right)}{3} + O\left(\frac{t}{t}^5 \right) + \left(\frac{k^4 \left(\frac{k^4 \left(\frac{k^4 - k^2 + 3 \right)}{3} + O\left(\frac{k^4 \left(\frac{k^4 - k^4 \right)}{3} + O\left(\frac{k^4 - k^4 \right)}{3} + O\left(\frac{k^4 - k^4 \right)}{3} + O\left(\frac{k^4 - k^4 - k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 - k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 - k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + A + O\left(\frac{k^4 + O\left(k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + O\left(k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + O\left(\frac{k^4 + O\left(k$  $g H \left( H^2 k^2 + 3 \right) + \left( H^2 k^2 + 3 \right) U \left( H^2 k^2 + 3 \right) U$  $\label{eq:k-2+3-right} $$ k^2+3\right-\frac{h^2 \left(k^7 \left(k^7 \left(k^7 \left(k^7 \left(k^4 + 2 \left(k^4 + k^2 + k^4 + k$  $k^4+3 \left( \frac{3}{4^2} \right) U^4+7 \left( \frac{4^2 k^2+3 \right) U^3+6 \left( \frac{3}{4^2} \right) U^3+6 \left( \frac{3}{4^2$  $k^2+3 \right) U \right) U \right) + (k^2+6 \right) + (k^2+3 + k^2) + (k$ H \left(H^2 k^2+3\right)\right)+3 g H U \left(\sqrt{3} \left(2 H^4 k^4+15 H^2 k^2+27\right) U+21  $\left(\frac{H^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)}\right)$ \left(4 H^4 k^4+33 H^2 k^2+63\right) U+72 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 \sqrt{3} g^2 H^2 \left(8 H^2 k^2+27\right) U+9 \left(\sqrt{3} U^5+6 \sqrt{g H \left(H^2 k^2+3\right)} U^4+2 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)\right)+6 k^2 \left(\sqrt{3} H^2 U^5+4 \sqrt{g H^5 \left(H^2  $k^2+3\right) U^4+6 \left( 4^2 k^2+3\right) U^2\right) U^4+6 \left( 4^2 k^2+3\right) U^2\right) U^2\right) U^4+6 \left( 4^4\right) U^4+6 \left( 4^2 k^2+3\right) U^4+6 U^$  $k^2+3\left(-\frac{k^5}{19}+O\left(\frac{19}{2}\right)^{5/2}\right)+O\left(\frac{19}{2}\right)^{5/2}+O\left($  $U H^2 + 145 \sqrt{3} \sqrt{g} H^5 \left(H^2 k^2 + 3\right) \right) + h^2 + 1728 U + 531 \sqrt{3} \sqrt{g} H \left(H^2 k^2 + 3\right) \right) + h^2 + 1728 U + 531 \sqrt{3} \right)$  $k^2+3\right)/100$ \sqrt{g H^{13} \left(H^2 k^2+3\right)} U\right) k^6+3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3})  $U+1011 \sqrt{H^2 k^2+3\right} + H^5+576 \sqrt{H^2 k^2+3\right} + U+1011 \sqrt{H^2 k^2+3} + U+1011$  $k^4+9 \left(1350 \right) q^2 U H^4+2227 \right] U^3 H^3+576 \left(H^2 k^2+3\right)$  $U^4+2118 \sqrt{4^2 + 145 \sqrt{g^5 H^9 \left(\frac{h^2 k^2+3\right)}} U^2+145 \sqrt{g^5 H^9 \left(\frac{h^2 k^2+3\right)}\right)}$  $k^2+81 \left(64 \right) U^4+g H\left(65 \right) U^4+g H\left(65 \right) U^4+g H\left(65 \right) U^4+g H\left(65 \right) U^4+g U$  $k^2+3\right) \left( U^2+241 \right) U^2+241 \right) U^2+241 \right)$  $\text{dt}^3}{34560 \operatorname{gH} \left( \frac{^2 k^2+3\right)^{7/2}}-\frac{(k^9 \left( \frac{9 \operatorname{gH}}{2} \right)^{3/2}}{2} \right)}$ H^2 k^2+531\right) H^3+54 \sqrt{3} g^2 \left(91 H^4 k^4+578 H^2 k^2+915\right) U^2 H^2+g U^3  $\left(913 \right) + 16 U k^6 + 3 \left(2771 \right) + 1732 \left(4^2 + 1732 \right) + 1841$ k^4+25227 \sqrt{3} H^2 U k^2+405 \left(63 \sqrt{3} U+124 \sqrt{g H \left(H^2 k^2+3\right)\right)\right)\right)  $H+12 U \left( 16 \right) H+12 U \left( 16 \right) H-13 \left( 16$ U^4 k^4+3 \left(144 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+898 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}  $U^2+193 \cdot qrt\{g^5 + h^9 \cdot (h^2 k^2+3 \cdot ght)\} \cdot h^2+432 \cdot qrt\{g + \left(h^2 k^2+3 \cdot ght\right)\} \cdot h^2+2025 \cdot h^2 \cdot h$  $\left(\frac{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \$  $\label{left(H^2 k^2+3\left(H^4 U-\sqrt{3} \right) \left(H^4 U$  $k^4-3 \left( \frac{3} \right) \left( \frac{4^2 + 1}{100} + \frac{4^2 + 1}{100} \right) = H^2 U \right) k^2+9 U-9 \left( \frac{3}{100} \right)$  $\left(H^2 k^2+3\right) U-3 g H \left(H^2 k^2+3\right) U-3 g$  $k^2+3\left(\frac{t}^3}{24 \left(\frac{t^2}{k^2+3\right)}^3}+\frac{k^5\left(\frac{t^2}{k^5}\right)^3}{k^5\left(\frac{t^2}{k^5}\right)^3} + \frac{k^5\left(\frac{t^2}{k^5}\right)^3}{k^5\left(\frac{t^2}{k^5}\right)^3} + \frac{k^5\left(\frac{t^2}{k^5}\right)^3}{k^5\left(\frac{t^2}{$  $U-\sqrt{3} \sqrt{4 + \left(\frac{H^2 U^4 - 1}{16H(H^2 U^4 - 1)}\right)} \left(\frac{H^4 U K^4 + \left(\frac{H^2 U - 4 \right)}{16H(H^2 U K^2 - 1)}}{16H(H^2 U K^2 + 1)} \right)$  $H^5 \left(H^2 k^2+3\right) \right) + U-12 \left(H^2 k^2+3\right) \right) + U-12 \left(H^2 k^2+3\right) \right) + U-12 \left(H^2 k^2+3\right) +$ TT \1\_0/2 \1\_0/2TT02 1-02 - 2\...'-1.0 TT - 2 \-....(2) \-....(-TT \1\_0/TT02 1-02 - 2\...'-1.0)\...'-1.0 TT - 0 - 02 TT02\...'-1.0

 $H \left( H \right) \left$  $\text{dt}^4{30 \left(H^2 k^2+3\right)}+O\left(\left(text{dt}^5\right)\right)+\left(-\left(text{dt}^6\right)\right)+O\left(text{dt}^6\right)}+O\left(text{dt}^6\right)+O\left(te$  $H \left(H^2 k^2+3\right) - \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) + \left($  $k^2+3\right)+\frac{k^7 \left(U^3 \left(V^3 \left(V^4 U-2 \right) + H^4 U-2\right)}{H^4 U-2 \left(H^2 k^2+3\right)\right)}\right)}$  $k^4-3 \left( -2 \right) H^2 U^4+7 \left( H^2 k^2+3 \right) U^3+6 U^3+6$  $H \left( H^2 k^2 + 3\right) \right) \ g \ H \ U \left( \frac{3}{4} \left( H^4 k^4 + 15 H^2 k^2 + 27\right) \right) \ U-21$  $\$  \\left(H^2 k^2+3\right)\\right)\\right)\\right)\\right)\\ \text{dt}^3\{144 \\left(H^2 k^2+3\right)^{5/2}}+\\\frac{i}{}  $k^8 \left( \frac{4 \left( \frac{4}{1} \right) H^4 U - 2 \left( \frac{4}{1} \right) \left( \frac{4}{1} \right) U^4 + 2 g H \left( \frac{4}{1} \right) U^4$  $\left(4 + ^4 + ^4 + ^3 + ^2 + ^2 + ^3\right) U - 72 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^3 + ^2\right) U - 24 \left(4 + ^4 + ^4 + ^4\right) U - 24 \left(4 + ^4 + ^4\right) U - 24 \left(4 + ^4 + ^4\right) U - 24 \left(4 + ^4\right) U$ g^2 H^2 \left(8 H^2 k^2+27\right) U+9 \left(\sqrt{3} U^5-6 \sqrt{g H \left(H^2 k^2+3\right)} U^4-2  $k^2+3\right) U^4+6 \left( 4^2 k^2+3\right) U^2\right) U^4+6 \left( 4^2 k^2+3\right) U^2\right) U^2\right) U^4+6 \left( 4^4\right) U^4+6 \left( 4^2 k^2+3\right) U^4+6 U^$  $k^2+3\right)^{5/2}+O\left(text{dt}^5\right)\right)$  $k^4+\left(145 \right)^{15} \left(145 \right)^{$  $\left( H^2 k^2+3\right) \left( H$  $\sqrt{9} \left( \frac{4^2 h^6+g}{13} \right) \left( \frac{4^2 h^6+g}$ U \left(2195 \sqrt{3} U-1011 \sqrt{g H \left(H^2 k^2+3\right)\right) \hat\text{hright)}\right) \hat\text{hright} H^5-576 \sqrt{g H^9 \left(H^2 k^2+3\right)}  $k^2+3\right) U^3\right) V^3+9 \left(-1350 \right) 13 q^2 U H^4-2227 \right] U^3 H^3+576 \right]$  $H^5 \left(H^2 k^2+3\right) U^4+2118 \left(H^2 k^2+3\right) U^2+145 \left(H^2 k^2+3\right) U$  $\left(\frac{4 \operatorname{ft}(H^2 k^2+3\right)} \right) h^2+81 \left(\frac{64 \operatorname{ft}(H^2 k^2+3\right)} U^4+g H \right)$ \sqrt{3} g^3 \left(145 H^2 k^2+531\right) H^3+54 \sqrt{3} g^2 \left(91 H^4 k^4+578 H^2 k^2+915\right) U^2 H^2+g U^3 \left(913 \sqrt{3} H^6 U k^6+3 \left(2771 \sqrt{3} H^4 U-1732 \sqrt{g H^9 \left(H^2  $k^2+3\right) \ h^4+25227 \ h^2 U \ h^2+405 \ h^3 \ U-124 \ h^2 U \ h^2$ k^2+3\right)\right)\right) H-12 U \left(16 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^4 k^6+144 \sqrt{g H^9} \left(H^2 k^2+3\right)} U^4 k^4+3 \left(144 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+898 \sqrt{g^3 H^7}  $\left(\frac{H^2 k^2+3\right)}{U^2+193 \left(\frac{4^2 k^2+3\right)} L^2+193 \left(\frac{4^2 k^2+3\right)}{L^2+193 \left(\frac{4^2 k^2+3\right)} L^2+193 \left(\frac{4^2 k^2+3\right)}{L^2+193 \left(\frac{4^2 k^2+3\right)}{L^$  $k^2+3\right) U^4+2025 \sqrt{6^5 H^5 \left(H^2 k^2+3\right)}\right) \left(14+2025 \right) 1$  $\left(\frac{4x}{2} + \frac{4x}{4}\right)^{7/2}+O\left(\frac{4x}{5}\right)\right)$ 

 $\text{Out} [\text{135}] = \ EA \ || \ \left. \left\{ \left\{ \frac{-H^2 \, k^2 \, \left( \left( -1 + e^{i \, dt \, w} \right) k \, U - w \right) + 3 \, w}{\left( 3 + H^2 \, k^2 \right) w}, \ - \frac{3 \, \left( -1 + e^{i \, dt \, w} \right) k}{\left( 3 + H^2 \, k^2 \right) w} \right\}, \ \left\{ - \frac{\left( -1 + e^{i \, dt \, w} \right) k \left( g \, H \, \left( 3 + H^2 \, k^2 \right) - 3 \, U^2 \right)}{\left( 3 + H^2 \, k^2 \right) w}, \ 1 \, - \frac{\left( -1 + e^{i \, dt \, w} \right) k \left( 6 + H^2 \, k^2 \right) U}{\left( 3 + H^2 \, k^2 \right) w} \right\} \right\}$ Out[136]= EA || \left( \begin{array}{cc}  $\frac{3 w-H^2 k^2 \left(\left(-1+e^{i \cdot text \cdot dt} w\right)\right)}{\left(-1+e^{i \cdot text \cdot dt} w\right)} k U-w\right)}{\left(-1+e^{i \cdot text \cdot dt} w\right)}$ 

 $k^2+3\right) w$  &  $-\frac{3 \left(1+e^{i \left(t\right) w}\right) k}{\left(t^2 k^2+3\right) w} \$  $-\frac{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)} \left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}{\left(\frac{1+e^{i \cdot k}}{h^2 \cdot k^2 + 3 \cdot k}\right)}$ \end{array}

\right)

Out[134]=

$$\begin{aligned} & \log(37) - \text{ Eerr } \| \ \left\{ \left[ \frac{4 \left[ \sqrt{3} - k \sqrt{g \, H(13 \, HF \, E)} - 4 \, k \, U \right]^2 + \frac{4 \left[ (8 \, g \, HF \, U \, U \, S \, g \, H(13 \, HF \, E)}{3 \, HF \, E} + \frac{1}{2} \right] e^2}{3 \, HF \, E^2} \right. \\ & + \left[ -\frac{1}{6} \left[ -\frac{4 \sqrt{3} - k \sqrt{g \, H(13 \, HF \, E)}}{3 \, HF \, E^2} - 4 \, k \, U \right]^3 + \frac{4 \left[ (8 \, g \, HF \, U \, U \, S \, g \, HF \, U \, U \, U \, S \, HF \, U \, U \, U \, S \, HF \, E^2 \, U^2}{6 \, (3 \, HF \, E)^2} \right] dt^3 - \\ & + \left[ -\frac{1}{12} \left( \sqrt{g \, H} - k^4 \right) dt + \frac{4 \sqrt{g \, H(13 \, HF \, E)}}{24 \, HF \, E^2} - 4 \, k \, U \right]^3 + \frac{4 \left[ (8 \, g \, HF \, U \, U \, S \, g \, HF \, U \, U \, S \, HF \, U \, U \, S \, HF \, E^2 \, U^2 \, HF \, E^2 \, U^2 \right]}{24 \, HF \, E^2} + \frac{4 \sqrt{g \, H}}{24 \, HF \, E^2} + \frac{4 \sqrt{g \, H}}{24 \, HF \, E^2} \frac{1 (9 \, g \, HF \, E^2 \, HF \, E^2 \, HF \, E^2 \, U^2 \, HF \, E^2 \, U^2 + 4 \, C_1 \, HF^2 \, E^2 \, U^2 + 4 \, C_2 \, HF^2 \, E^2 \, U^2 \, U^2$$

$$\frac{1}{24} \left[ -\frac{1}{3+H^2 k^2} - \iota \, k \, U \right] \, dt + O[ut] \right]^{+}$$

$$\left( -\frac{1}{12} \left( \sqrt{g \, H} \, k^4 \right) dt + \frac{i \sqrt{g \, H} \, \left( 15 \, k^5 + 2 \, H^2 \, k^7 \right) U \, dt^2}{24 \, \left( 3 + H^2 \, k^2 \right)} + \frac{1}{24 \, \left( 3 + H^2 \, k^2 \right)^2} \left( 9 \, g \, H \, \sqrt{g \, H} \, k^6 + 3 \, g \, H^3 \, \sqrt{g \, H} \, k^8 + 3 \, g \, H^3 \, \sqrt{g \, H} \, k^8 + 3 \, g \, H^3 \, \sqrt{g \, H} \, k^8 + 3 \, g \, H^3 \, \sqrt{g \, H} \, k^8 + 3 \, g \, H^3 \, \sqrt{g \, H} \, k^8 + 3 \, g \, H^3 \, \sqrt{g \, H} \, k^8 + 3 \, g \, H^3 \, k^8 + 1 \, g \, H^3 \, k^8 + 1$$

Out[138]= Eerr || \left(

\begin{array}{cc}

 $\label{eq:left} $\left(\frac{u + \sqrt{u^2 + \sqrt{u$  $\label{left} $$k^2+\sqrt{3} \right] \leq H^2 k^2+3\right) U k^2\right) \cdot (h^2 k^2+3) + \left( h^2 k^2+3 \right) \cdot (h^2 k^2+3) \cdot$ \left(H^4 U^3 k^7-3 H^2 U^3 k^5+9 g H^3 U k^5-18 U^3 k^3+18 g H U k^3\right)\}{6 \left(H^2

 $k^2+3\left(H^2 + \left(H^2 + \frac{1}{6}\right) + \left(H^2 + \frac{1$  $k^2+3\right\right)^3\right) + t_1^2\left(\frac{1}{24}\left(-iU_k-\frac{1}{24}\right)^3\right)$ k{ $H^2 k^2+3$ }right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{1}{12} \left(\sqrt{g H} k^4\right)

 $\t x_{dt}+\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) + \frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right) + \frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right) + \frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right) + \frac{1}{2} \left( \frac{1}{2} + \frac{1}{2}$ H} \left(H^4 U^2 k^{10}+3 g H^3 k^8+3 H^2 U^2 k^8-3 U^2 k^6+9 g H k^6\right) \text{dt}^3}{24 \left(H^2 U^2 k^8-3 U^2 k^6+9 g H k^6\right)}

 $k^2+3\right)^2+O\left(\frac{t}{2} + C\left(\frac{t}^5\right)\right) + C\left(\frac{t}{2} + C\right) + C\left(\frac{t}{2} + C\left(\frac{t}{2} + C\right) + C\left(\frac{t}{2} + C\right) + C\left(\frac{t}{2} + C\right) + C\left(\frac{t}{2} + C\right) + C\left(\frac{t}{2} + C\left(\frac{t}{2} + C\right) + C(\frac{t}{2} + C\right) + C\left(\frac{t}{2} + C\right) + C(\frac{t}{2} + C) + C(\frac{t}$ k^7+45 U k^5\right)\text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 H^4 U^2 k^{10}+145 g H^3)}  $k^8+94 \; H^2 \; U^2 \; k^8-486 \; U^2 \; k^6+531 \; g \; H \; k^6 \\ \text{right} \; \text{text} \\ \text{dt}^2 \\ \text{1920 } \\ \text{left} (H^2 \; k^2+3 \\ \text{right})^2 \\ \text{-} \\ \text{frac} \\ \text{in } \\ \text{text} \\ \text{text} \\ \text{dt}^2 \\ \text{dt}^2$ 

k^7 \left(32 H^6 U^3 k^6+47 H^4 U^3 k^4+241 g H^5 U k^4-678 H^2 U^3 k^2+1348 g H^3 U k^2-1593  $\label{left} $$ \operatorname{dx}^4+O\operatorname{left}(\text{dx}^5\right) & \operatorname{left}(-\frac{3 i k \text{dt}}{H^2 k^2+3}-\frac{3 \operatorname{left}(k^2 U\operatorname{right})}{H^2 k^2+3} - \frac{3 \operatorname{left}(k^2 U\operatorname{right})}{H^2 k^2+3$ 

 $\text{dt}^2 H^2 k^2+3+\frac{1}{2} \left( H^2 U^2 k^5+3 U^2 k^3+g H k^3\right) \left( \frac{1}{2} \left( H^2 U^2 k^5+3 U^2 k^3+g H k^3\right) \right)$  $k^2+3\right)^2+O\left(\frac{dt}^5\right)+\left(\frac{dt}^5\right)+\left(\frac{dt}^2}{4\left(\frac{dt}^2}\right)^2}+O\left(\frac{dt}^2\right)^2+O\left(\frac{dt}^2\right)^2}$ 

 $\left(\frac{H^2 k^2+3\right)}{2}\right) \text{ (thrule } 4^2+3\right) \left(\frac{H^2 k^2+3\right)}{2}\right) \text{ (thrule } 4^2+3\right) \left(\frac{H^2 k^2+3\right)}{2}\right) \text{ (thrule } 4^2+3\right) \left(\frac{H^2 k^2+3\right)}{2}\right) \text{ (thrule } 4^2+3\right)$ 

 $\left(49 \text{ H}^2 \text{ k}^7 + 243 \text{ k}^5\right) \left(49 \text{ H}^2 \text{ k}^2 + 3\right)^2 + \frac{145 \text{ H}^2}{2}$ 

k^8+531 k^6\right) U \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(241 H^4 U^2)}{2} k^{11}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7\right) \text{dt}^3}{1920

 $\left(H^2 k^2+3\right)^3+O\left(\frac{dt}^5\right) \left(\frac{dt}^3+O\left(\frac{dt}^5\right)\right)$ 

\left(\left(\frac{3 i k U^2}{H^2 k^2+3}-i g H k\right) \text{dt}-\frac{k^2 U \left(g k^2 H^3+3 g H-3)}  $U^2\right) \det dt^2 H^2 k^2+3+\frac{1}{4} U^2 k^7+g^2 H^4 k^5-3 H^2 U^4$ k^5+6 g H^3 U^2 k^5-9 U^4 k^3+3 g^2 H^2 k^3+6 g H U^2 k^3\right) \text{dt}^3}{2 \left| \left( H^2 \right)^2 \right|^2}

 $k^2+3\left(\frac{1}{12}\right)^2+O\left(\frac{dt}^5\right)\right)+\left(\frac{1}{12}\right)^2+O\left(\frac{dt}^5\right)^2$  $\text{text}(dt) + \frac{1}{\ln(H^2 \cdot H^2 \cdot H$ \text{dt}^2}{12 \left(H^2 k^2+3\right)}+\frac{\left(H^4 \sqrt{g H} U^3 k^{10}+2 g H^5 \sqrt{g H} U

k^{10}+14 g H^3 \sqrt{g H} U k^8-12 \sqrt{g H} U^3 k^6+24 g H \sqrt{g H} U k^6\right) \text{dt}^3\{24  $\label{left} $$\left(H^2 k^2+3\right)^2$+O\left(\frac{dt}^5\right)\right) \det(H^2 k^2+3\right)^2$+O\left(\frac{dt}^5\right)$ 

k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) \text{dt}}{960 \left(H^2

k^2+3\right)^2}+\frac{\\left(64 g H^5 U k^{10}-145 H^2 U^3 k^8+384 g H^3 U k^8-531 U^3 k^6+576 g H U  $k^6\right$ ight) \text{dt}^2}{960 \left(H^2  $k^2$ +3\right)^2}-\frac{i \left(288 g H^7 U^2  $k^1$ 3+241 g^2  $^{4}$  H^6 k^{11}-723 H^4 U^4 k^{11}+2592 g H^5 U^2 k^{11}+1542 g^2 H^4 k^9-4626 H^2 U^4 k^9+7194 H^6 k^{11}-723 H^4 U^4 k^9+7194 H^6 L^7 U^4 L g H^3 U^2 k^9-7371 U^4 k^7+2457 g^2 H^2 k^7+5454 g H U^2 k^7\right) \text{dt}^3}{5760 \left(H^2  $k^2+3\left(\frac{dt}^5\right)+O\left(\frac{dt}^5\right)$  $k \cdot \left( \frac{dt}{dt} \right) + \frac{h^2 k^2 + 3 \cdot (k^2 + 3) + \frac{dt}{dt}}{h^2 k^2 + 3 \cdot (k^2 + 3) + \frac{dt}{dt}}$  $\label{left} $$\left(H^2 k^2+3\right) U-3 k^2 U^2\right) \text{dt}^2 H^2 k^2+3+\left(\frac{1}{4} U^3 k^7+15 U^2\right) $$$ H^2 U^3 k^5+9 g H^3 U k^5+36 U^3 k^3+36 g H U k^3\right)\\{6 \left(H^2 k^2+3\right)^2\}-\\frac{1}\\{6\}  $\left(-i U k - \frac{1}{k} \right) k}{H^2 k^2+3\right) k}{H^2 k^2+3\right)$  $\label{eq:left} $\operatorname{dt}^3-\frac{1}{24} \left(i \ k-\frac{3} \right) + \left(i \ k^2 + 3\right) k}{H^2 \ k^2 + 3\right)^4}$ U^2 k^{10}+15 H^2 \sqrt{g H} U^2 k^8+3 g H^3 \sqrt{g H} k^8+39 \sqrt{g H} U^2 k^6+9 g H \sqrt{g H}  $k^6 + k^6 + k^3 + k^4 + k^3 + k^4 + k^4$  $\left(32 \text{ H}^4 \text{ U k}^9 + 241 \text{ H}^2 \text{ U k}^7 + 531 \text{ U k}^5\right) \left(44\right) \left(47\right) \left(47\right$ H<sup>4</sup> U<sup>2</sup> k<sup>10</sup>+145 g H<sup>3</sup> k<sup>8</sup>+674 H<sup>2</sup> U<sup>2</sup> k<sup>8</sup>+1638 U<sup>2</sup> k<sup>6</sup>+531 g H k<sup>6</sup>(right) \text{dt}<sup>2</sup>{1920 \left(H^2 k^2+3\right)^2}-\frac{i \left(32 H^6 U^3 k^{13}+529 H^4 U^3 k^{11}+241 g H^5 U  $k^{11}+2406 H^2 U^3 k^9+1736 g H^3 U k^9+3321 U^3 k^7+3231 g H U k^7 \right)$   $\frac{1}{2} U^3 k^9+3321 U^3 k^7+3231 g H U k^7 \right)$ \end{array}

```
\right)
ln[139] = KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
      KurFWS = KurF /. ap \rightarrow (U + Sqrt[g * H]) /. am \rightarrow 0;
      KurFWSeta =
         KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
           qm \rightarrow Rmp * n;
      KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
       KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1]; 
      Kfnn = Kfnnp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
      KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
      Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
      Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 4}];
      Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
      FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
      FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 4}];
      FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
      \texttt{KurFWSG} = \texttt{KurFWS} \ / . \ \texttt{fp} \ \rightarrow \ (\texttt{U} * \texttt{Rpp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rpp} * \texttt{n}) \ / .
              fm \rightarrow (U*Rmp*G + U*H*v + g*H*Rmp*n) /. qp \rightarrow Rpp*G /. qm \rightarrow Rmp*G;
      KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
      KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
```

```
KfGn = KfGnp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 4}];
FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g*H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 4}];
FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2/2 + Fmat2.Fmat2.Fmat2/6;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
RKStep =
  Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 + EigvFmat2 * EigvFmat2 * EigvFmat2 / 6]/
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" U > -Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn || ", KfGnp}]]
Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error || ", FGn2TAr}]]
Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG || ", KfGGp}]]
Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error || ", FGG2TAr}]]
```

```
Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
                                                  Text[" "]
                                                  Text[" "]
                                                  Text[Row[{"Omega error ||
                                                                                                                                                                                                                                                                                                             ", RKstepTayr}]]
                                                  Text[Row[{"Omega error ||
                                                                                                                                                                                                                                                                                                              ", TeXForm[RKstepTayr]}]]
                                                  Text[" "]
                                                  Text[Row[{"EA || ", EA}]]
                                                  Text[Row[{"EA || ", TeXForm[EA]}]]
                                                  Text[Row[{"Eerr || ", Eerr}]]
                                                   Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
  Out[173]= U > -Sqrt(gH)
 Out[174]=
  Out[175]= Fnn || Gnp H + Rmp U
  Out[176] = Fnn \parallel \text{text}\{Gnp\} H + \text{text}\{Rmp\} U
 \begin{array}{ll} \text{Out} \text{[177]=} & Fnn \; error \; \mid \mid \; \left( -\frac{\left( H^2 \, k^3 \, U \, w \right) \, dt^2}{2 \, \left( 3 + H^2 \, k^2 \right)} \, - \, \frac{i \, H^2 \, k^3 \, U \, w^2 \, dt^3}{6 \, \left( 3 + H^2 \, k^2 \right)} \, + \, \frac{H^2 \, k^3 \, U \, w^3 \, dt^4}{24 \, \left( 3 + H^2 \, k^2 \right)} \, + \, O[dt]^5 \right) + \\ & \left( -\frac{1}{12} \left( k^4 \, U \right) dt \, + \, O[dt]^5 \right) dx^3 \, + \left( \frac{i \, \left( 45 \, k^3 \, U + 143 \, H^2 \, k^7 \, U + 32 \, H^4 \, k^9 \, U \right) dt}{960 \, \left( 3 + H^2 \, k^2 \right)^2} \, + \, O[dt]^5 \right) dx^4 \, + \, O[dx]^5 \\ \end{array} 
  Out[178]= Fnn error |
                                                             \left(-\frac{t_4^2 \ker\{dt\}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2 + 3\right)} - \frac{t_4^3 U w^2}{6} \right)} - \frac{t_4^3 U w^2}{6} 
                                                                                    \left(H^2 k^2+3\right)+\left(H^2 k^3 U w^3\right)^2+\left(H^2 k^3 U w^3\right)^2
                                                                                      k^2+3\right)+O\left(\frac{dt}^5\right)+O\left(\frac{dt}^5\right)+O\left(\frac{dt}^3\right)+O\left(\frac{dt}^3\right)
                                                                                      \text{dt}+O\left(\text{dt}}^5\right)\right)+\text{dx}^4 \left(\frac{i} \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^
                                                                                      k^5 \right) \left( \frac{dt}{960 \left( \frac{A^2 k^2+3\right)^2}+O\left( \frac{dt}{5}\right) \right)}{1} + O\left( \frac{dt}{5}\right) \right) \left( \frac{dt}{5}\right) \left(
  Out[179]=
  Out[180]= FnG || GGp H
  Out[181]= FnG \parallel \text{text}\{GGp\} H
  \text{Out} \\ \text{[182]=} \quad FnG \; error \; || \; \left( -\frac{3 \, (k \, w) \, dt^2}{2 \, (3 + H^2 \, k^2)} - \frac{i \, k \, w^2 \, dt^3}{2 \, (3 + H^2 \, k^2)} + \frac{k \, w^3 \, dt^4}{8 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) \\ + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)^2} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)} + O[dt]^5 \right) dx^4 \\ + O[dx]^5 + \left( \frac{i \, (243 \, k^3 + 49 \, H^2 \, k^2) \, dt}{960 \, (3 + H^2 \, k^2)} + O[dx]^5 + O[dx]^5
  \text{text}\{dt\}^3 \text{ k w}^2\}\{2 \left(H^2 \text{ k}^2 + 3\right)\} + \frac{dt}^4 \text{ k w}^3\}\{8 \left(H^2 \text{ k}^2 + 3\right)\}
                                                                                      k^2+3\right)+O\left(\frac{dt}^5\right)+\frac{dt}^5\right)
                                                                                      \label{left} $$ \operatorname{dt}_{960}\left(\frac{A^2 k^2+3\right)^2}+O\left(\frac{t}{t}^5\right)\right)^2+O\left(\frac{t}{t}^5\right)^2\right)^2$
  Out[184]=
  Out[185]= FGn \parallel H(gRmp + Gnp U)
```

Out[186]=  $FGn \parallel H(g \text{text}\{Rmp\}+\text{text}\{Gnp\} U)$ 

$$\begin{array}{ll} \text{Out} \text{[187]=} & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w \right) dt^2}{2 \, \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^2 \, dt^3}{6 \, \left( 3 + H^2 \, k^2 \right)} + \frac{k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^3 \, dt^4}{24 \, \left( 3 + H^2 \, k^2 \right)} + O[dt]^5 \right) + \\ & \left( -\frac{1}{12} \left( g \, H \, k^4 \right) dt + O[dt]^5 \right) dx^3 + \left( \frac{i \left( 288 \, g \, H \, k^5 + 192 \, g \, H^3 \, k^7 + 32 \, g \, H^5 \, k^9 - 243 \, k^5 \, U^2 - 49 \, H^2 \, k^7 \, U^2 \right) dt}{960 \, \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \end{array}$$

Out[188]= FGn error |

 $\left(-\frac{t}{4t}^2 \left( \frac{t}^2 \right)^2 \right)^{2} \left( \frac{t}^2 \left( \frac{t}^2 \right)^2 \left($  $\text{text}_dt^3 k w^2 \left( \frac{H^3 k^2+3 g H-3 U^2\right)}_6 \left( \frac{h^2 k^2+3\right)}{h^2 k^2+3 (text_dt)^4 k w^3} \right)$  $\left(\frac{H^2 k^2+3 g H-3 U^2\left(\frac{H^2 k^2+3\right)}+O\left(\frac{t}{t}\right)}{1}\right)}{24\left(\frac{H^2 k^2+3\right)}+O\left(\frac{t}{t}\right)}+O\left(\frac{t}{t}\right)}$  $\left(-\frac{1}{12}\left(g H k^4\right)\right) \times \left(dt\right) - \left(text\left(dt\right)^5\right)\right) + text\left(dx\right)^4$ \left(\frac{i} \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right)  $\text{dt}{960 \left(H^2 k^2+3\right)^2}+O\left(\left(text{dt}^5\right)\right)+O\left(text{dt}^5\right)+O\left(text{dx}^5\right)+O\left(text{d$ 

Out[189]=

Out[190]= 
$$FGG \parallel (GGp H + Rmp) U$$

Out[191]= 
$$FGG \parallel U (\text{text}\{GGp\} H + \text{text}\{Rmp\})$$

$$\begin{array}{ll} \text{Out} \text{[192]=} & FGG \; error \; \mid \mid \; \left( -\frac{\left( k \left( 6 + H^2 \; k^2 \right) U \; w \right) \; dt^2}{2 \left( 3 + H^2 \; k^2 \right)} - \frac{i \; k \left( 6 + H^2 \; k^2 \right) U \; w^2 \; dt^3}{6 \left( 3 + H^2 \; k^2 \right)} + \frac{k \left( 6 + H^2 \; k^2 \right) U \; w^3 \; dt^4}{24 \left( 3 + H^2 \; k^2 \right)} + O[dt]^5 \right) + \\ & \left( -\frac{1}{12} \left( k^4 \; U \right) dt + O[dt]^5 \right) dx^3 + \left( \frac{i \left( 531 \; k^5 + 241 \; H^2 \; k^7 + 32 \; H^4 \; k^9 \right) U \; dt}{960 \left( 3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \\ \end{array}$$

Out[193]= FGG error ||

 $\label{left} $\left(-\frac{t_4^2 \ker\{dt\}^2 \left(U \le \left(H^2 k^2 + 6\right)\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_4^2 k^2 + 6\left(H^2 k^2 + 6\right)}{2 \left(H^2 k^2 + 6\right)} - \frac{t_$  $\left(H^2 k^2+6\right) \left(H^2 k^2+6\right) + \left(H^2 k^2+3\right) + \left(H^2 k^2+6\right) + \left(H^2 k^2+6\right)$  $\label{left} $$ \operatorname{left}(\operatorname{dt}^5\right)+\operatorname{dt}^4 \operatorname{left}(\operatorname{dt}^4)^4 \operatorname{left}$  $k^5$  | U\text{dt}{6960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}\^5\right)\right)+O\left(\text{dt}\^5\right)

Out[194]=

Out[195]=

Out[196]= Omega error 
$$\| \left\{ \left( -\frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H} (3+\text{H}^2 \text{ k}^2)} + 3 \text{ k U} + \text{H}^2 \text{ k}^3 \text{ U}} \right)^4 \text{dt}^3}{24 \left( 3+\text{H}^2 \text{ k}^2 \right)^4} + \frac{\left( \sqrt{3} \text{ k} \sqrt{\text{g H} (3+\text{H}^2 \text{ k}^2)} + 3 \text{ k U} + \text{H}^2 \text{ k}^3 \text{ U}} \right)^5 \text{dt}^4}{30 \left( 3+\text{H}^2 \text{ k}^2 \right)^5} + \text{O}[\text{dt}]^5 \right\} + \left( \left( -\frac{1}{24} i \text{ k}^4 \left( \sqrt{3} \sqrt{\frac{\text{g H}}{3+\text{H}^2 \text{ k}^2}} + 2 \text{ U} \right) - \frac{1}{144 \left( 3+\text{H}^2 \text{ k}^2 \right)^2} \right) + \left( \frac{1}{144 \left( 3+\text{H}^2 \text{ k}^2 \right)^2} + 2 \text{ g H U} \left( 5 \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 9 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{ U} \right) + \left( \frac{3}{2} \left( 21 \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 18 \text{ U} + 2 \text{ H}^4 \text{ k}^4 \text{ U} + \text{k}^2 \left( 7 \sqrt{3} \sqrt{\text{g H}^5 \left( 3+\text{H}^2 \text{ k}^2 \right)} + 12 \text{ H}^2 \text{ U} \right) \right) \right) \right) \\ dt^3 - \frac{1}{144 \left( 3+\text{H}^2 \text{ k}^2 \right)^3} i \text{ k}^8 \left( 3 \text{ g H} + \text{U} \left( 2 \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + \left( 3+\text{H}^2 \text{ k}^2 \right) \text{ U} \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \text{U} \right) \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \right) \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+\text{H}^2 \text{ k}^2 \right) \right) \right) + \left( 3 \text{ g H} \left( \sqrt{3} \sqrt{\text{g H} \left( 3+\text{H}^2 \text{ k}^2 \right)} + 4 \left( 3+$$

$$\begin{split} U^2\left(15\,\sqrt{3}\,\sqrt{g\,H(3+H^2\,k^2)}\right. + 18\,U + 2\,H^4\,k^4\,U + k^2\left(5\,\sqrt{3}\,\sqrt{g\,H^3(3+H^2\,k^2)}\right. + 12\,H^2\,U\right)\right)\right)dt^4 + \\ O[dI]^3 dx^3 + \left(-\frac{1}{5780(3+H^2\,k^2)}k^2\left(531\,\sqrt{3}\,\sqrt{g\,H(3+H^2\,k^2)}\right) + 1728\,U + 192\,H^4\,k^4\,U + k^2\left(145\,\sqrt{3}\,\sqrt{g\,H^3(3+H^2\,k^2)}\right) + 1152\,H^2\,U\right)\right) + \\ \left(i\,k^8\left(k^6\,U^3\left(721\,\sqrt{3}\,g\,H^7 + 192\,\sqrt{g\,H^{13}(3+H^2\,k^2)}\right)U\right) + 9\,k^2\left(145\,\sqrt{g^5\,H^3(3+H^2\,k^2)}\right) + 1350\,\sqrt{3}\,g^2 + 11^4\,U + 2118\,\sqrt{g^3\,H^7(3+H^2\,k^2)}\right)U^2 + 2227\,\sqrt{3}\,g\,H^3\,U^3 + 576\,\sqrt{g\,H^3(3+H^2\,k^2)}\right)U^3\right) + \\ 81\left(59\,\sqrt{g^5\,H^3(3+H^2\,k^2)} + 241\,\sqrt{3}\,g^2\,H^2\,U + 64\,\sqrt{g\,H(3+H^2\,k^2)}\right)U^4 + g\,H\,U^2\left(369\,\sqrt{g\,H(3+H^2\,k^2)}\right)U^3 + g\,H^3\,U\left(1011\,\sqrt{g\,H(3+H^2\,k^2)}\right) + 2195\,\sqrt{3}\,U\right)\right)\right)dt^3\right) / \\ \left(34\,560\,\sqrt{g\,H^3(3+H^2\,k^2)}\right) - \left(\left(k^9\left(3\,g\,H + U\,U\left(2\,\sqrt{3}\,\sqrt{g\,H(3+H^2\,k^2)}\right) + 1218\,U^2\right)U\right)\right) \\ \left(3\,\sqrt{3}\,g^2\,H^2\left(531+145\,H^2\,k^2\right) + g\,H\,U\left(49\,H\,\sqrt{g\,H(3+H^2\,k^2)}\right) + 312\,H^2\,k^2\right)U\right)\right) \\ \left(3\,\sqrt{3}\,g^2\,H^2\left(531+145\,H^2\,k^2\right) + g\,H\,U\left(49\,H\,\sqrt{g\,H(3+H^2\,k^2)}\right) + 32\,k^4\,\sqrt{g\,H^9(3+H^2\,k^2)}\right)U^3 + \\ k^2\left(241\,\sqrt{g^2\,H^3(3+H^2\,k^2)}\right) + 0[dI]^3\right)dx^4 + O[dx]^5, \\ \left(\frac{4}{2^{34}}k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H(3+H^2\,k^2)}{3^{32}H^3}} + 2U\right) - \frac{1}{344(5+H^2\,k^2)}\right)^{\frac{3}{2}}d^4 + \\ O[dI]^3\right) + \\ \left(\frac{1}{2^{34}}k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H(3+H^2\,k^2)}{3^{32}H^3}} + 2U\right) - \frac{1}{344(5+H^2\,k^2)}\right) + 2U^3\left(3+H^2\,k^2\right) + 2U\right) + U^3\left(-21\,\sqrt{3}\,\sqrt{g\,H(3+H^2\,k^2)}\right) + \\ \left(\frac{1}{2^{34}}k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H(3+H^2\,k^2)}{3^{32}H^3}} + 2U\right) - \frac{1}{344(5+H^2\,k^2)}\right)^{\frac{3}{2}}d^4} + \\ \left(\frac{1}{2^{36}}k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H(3+H^2\,k^2)}{3^3}} + 2U\right) - \frac{1}{344(5+H^2\,k^2)}\right) + 2U^3\left(3+H^2\,k^2\right) + 2U\right) + U^3\left(-21\,\sqrt{3}\,\sqrt{g\,H(3+H^2\,k^2)}\right) + \\ \left(\frac{1}{2^{36}}k^4\left(\sqrt{3}\,\sqrt{\frac{g\,H(3+H^2\,k^2)}{3^3}} + 2U\right) - \frac{1}{344(5+H^2\,k^2)}\right) + 2U^3\left(-21\,\sqrt{3}\,\sqrt{g\,H(3+H^2\,k^2)}\right) + 2U^3\left(-21\,\sqrt{3}\,\sqrt{g$$

 $k^{2} \left(145 \sqrt{3} \sqrt{g H^{5} (3 + H^{2} k^{2})} - 1152 H^{2} U\right) + \frac{1}{34560 \sqrt{g H} (3 + H^{2} k^{2})^{7/2}}$  $i\,k^{8}\left(k^{6}\,U^{3}\left(-721\,\sqrt{3}\,g\,H^{7}+192\,\sqrt{g\,H^{13}\left(3+H^{2}\,k^{2}\right)}\,U\right)+9\,k^{2}\left(145\,\sqrt{g^{5}\,H^{9}\left(3+H^{2}\,k^{2}\right)}\,-1350\,\sqrt{3}\,H^{2}\,k^{2}\right)\right)$  $g^2 \ H^4 \ U \ + \ 2118 \ \sqrt{g^3 \ H^7 \left(3 + H^2 \ k^2\right)} \ U^2 \ - \ 2227 \ \sqrt{3} \ g \ H^3 \ U^3 \ + \ 576 \ \sqrt{g \ H^5 \left(3 + H^2 \ k^2\right)} \ U^4 \right) \ + \ \left(3 + H^2 \ k^2\right) \ U^4 \ U^4 \ U^4 \ U^4 \ U^4 \ U^4 \ U^4$  $81 \left(59 \, \sqrt{\,g^5 \, H^5 \left(3 + H^2 \, k^2\right)} \, - 241 \, \sqrt{\,3\,} \, g^2 \, H^2 \, U + 64 \, \sqrt{\,g \, H \left(3 + H^2 \, k^2\right)} \, \, U^4 + \right.$  $g H U^{2} \left(369 \sqrt{g H (3 + H^{2} k^{2})} - 251 \sqrt{3} U\right) - 3 k^{4} U$  $\left(627\sqrt{3} \text{ g}^2 \text{ H}^6 - 576\sqrt{\text{g} \text{ H}^9 \left(3 + \text{H}^2 \text{ k}^2\right)} \text{ U}^3 + \text{g} \text{ H}^5 \text{ U} \left(-1011\sqrt{\text{g} \text{ H} \left(3 + \text{H}^2 \text{ k}^2\right)} + 2195\sqrt{3} \text{ U}\right)\right)\right)$  $dt^{3} + \left(k^{9} \left(3 \text{ g H} + U \left(-2 \sqrt{3} \sqrt{\text{ g H} \left(3 + H^{2} k^{2}\right)}\right.\right. + \left(3 + H^{2} k^{2}\right) U\right)\right)\left(3 \sqrt{3} \text{ g}^{2} H^{2} \left(531 + 145 H^{2} k^{2}\right) + \left(3 + H^{2} k^{2}\right) U\right)\right)$  $g H U \left(-4914 \sqrt{g H (3 + H^2 k^2)} + \sqrt{3} (5049 + 3270 H^2 k^2 + 529 H^4 k^4) U\right) 6 \left(288 \, \sqrt{g \, H \left(3 + H^2 \, k^2\right)} \, \, U^3 + 32 \, k^4 \, \sqrt{g \, H^9 \left(3 + H^2 \, k^2\right)} \, \, U^3 + \right.$  $k^{2}$   $\left(241\sqrt{g^{3}H^{7}(3+H^{2}k^{2})}U+192\sqrt{gH^{5}(3+H^{2}k^{2})}U^{3}\right)\right)$  $dt^4$  /  $(34560 \sqrt{g H} (3 + H^2 k^2)^{7/2}) + O[dt]^5) dx^4 + O[dx]^5)$ 

## Out[197]= Omega error ||

 $\left( -\frac{1}{\left( -\frac{1}{2} U k^3 + 3 U k + \sqrt{1} k^2 H \left( -\frac{1}{2} k^2 + 3 \right) k \right) k \right) k \left( -\frac{1}{2} U k^3 + 3 U k + \sqrt{1} k \right) k \left( -\frac{1}{2} U k^3 + 3 U k \right) k \left( -\frac{1}{2} U k^3 + 3 U k \right) k \left( -\frac{1}{2} U k^3 + 3 U k \right) k \left( -\frac{1}{2} U k^3 + 3 U k \right) k \left( -\frac{1}{2} U k \right) k \left( -\frac{1}{2}$  $k^2+3\right)^4+\frac{h^2 U k^3+3 U k+\sqrt{3} \sqrt{4} +\frac{h^2 U k^2+3 \pi(H^2 U k^3+3 U k+\sqrt{3} \sqrt{4} +\frac{h^2 U k^2+3 \pi(H^2 U k^3+3 U k+\sqrt{3} +\frac{h^2 U k^3+3 U k+\sqrt{3} \sqrt{4} +\frac{h^2 U k^3+3 U k+\sqrt{3} U k+\sqrt{3} +\frac{h^2 U k^3+3 U k+\sqrt{3} U$  $\label{eq:left} $$ \operatorname{dt}^4{30 \operatorname{left}(H^2 k^2+3\operatorname{right})^5}+O\operatorname{left}(\operatorname{text}_{dt}^5\operatorname{right})\operatorname{right})+\operatorname{left}(-\operatorname{frac}_{1}^{24} i k^4 \operatorname{left}(2 k^2+3\operatorname{right})^5)+O\operatorname{left}(k^2 k^2+3\operatorname{ri$  $U+\sqrt{3} \sqrt{1} L^2 k^2+3}\rightright)-\frac{1}{4} L^2 k^2+3}$  $\sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^3+3$ g H \left( $9 \cdot (H^2 k^2+3\right) \cdot U+5 \cdot (H^2 k^2+3\right) + (H^2 k^2+3\right)$  $\label{eq:linear_continuous_series} $$ \left(\frac{dt}^3}{144 \left(\frac{H^2 k^2+3\right)}{h^2 k^2}-\frac{k^8 \left(\frac{H^2 k^8 + U \left(\frac{H^2 k^2+3\right)}{h^2 k^2}+\frac{H^2 k^2+3}{h^2 k^2}+\frac{$  $\sqrt{g H \left(\frac{h^2 k^2+3\right)}\right)}\right)$ \left(H^2 k^2+3\right)\right) k^2+18 U+15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)\right) U^2+3 g H \left(4  $\label{left(H^2 k^2+3\left| h(H^2 k^2+3\right| h(H^2 k^2+3\right| h(H^2 k^2+3\right)} \left( \frac{H^2 k^2+3\left| h(H^2 k^2+3\right| h(H^2 k^2+3\right| h(H^2 k^2+3\right)}{\left( \frac{H^2 k^2+3\right| h(H^2 k^2+3\right| h(H^2 k^2+3\right)}{\left( \frac{H^2 k^2+3\right| h(H^2 k^2+3\right| h(H^2 k^2+3\right)}{\left( \frac{H^2 k^2+3\right)}$  $\label{eq:linear_continuous_con$  $\label{eq:continuity} $$U H^2+145 \left(3\right) \right] + 1728 U+531 \left(3\right) \left(H^2 k^2+3\right)\right) + 1728 U+531 \left(3\right) \left(H^2 k^2+3\right) \right) + 1728 U+531 \left(13\right) \left(13\right)$  $k^2+3\right)/10$ \sqrt{g H^{13} \left(H^2 k^2+3\right)} U\right) k^6+3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3})  $U+1011 \sqrt{H^2 k^2+3 \cdot h} H^5+576 \sqrt{g H^9 \left(H^2 k^2+3 \cdot h\right)} U^3 \right)$ k^4+9 \left(1350 \sqrt{3} g^2 U H^4+2227 \sqrt{3} g U^3 H^3+576 \sqrt{g H^5 \left(H^2 k^2+3\right)}  $k^2+81 \left(64 \right) + H\left(64 \right) + H\left$ k^2+3\right)\right) U^2+241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)\right)\right)\right  $\label{eq:continuity} $$ k^2+3\right) U+2 \operatorname{qt}{3} \operatorname{H}\left(H^2 k^2+3\right)\right) \left(145 H^2 k^2+3\right) \left(145 H^2 k^2+3\right$  $k^2+531\right) H^2+g U \left( \sqrt{3} \right) H^4 k^4+3270 H^2 k^2+5049\right) U+4914 \sqrt{3} H^2 k^2+5049\right)$  $H \left( \frac{H^2 k^2+3\right)}{h^2 k^2+3\right)} H+6 \left( \frac{H^2 k^2+3\right)}{h^2 k^2+3\right) U^3 k^4+\left( \frac{H^2 k^2+3\right)}{h^2 k^2+3\right) U^3 k^4+\left( \frac{H^2 k^4+3\right) U^3 k^4+\left( \frac{H$ 

 $\$  \\sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+241 \\sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\\right) k^2+288  $\left(H^2 \cup k^3 + 3 \cup k - \sqrt{4}\right) k - \left(H^2 \cup k^3 + 3 \cup k^3 + 3$  $k^2+3\right)^4+\frac{h^2 U k^3+3 U k-\sqrt{3}}{gH \left(\frac{h^2 U k^3+3 U k-\sqrt{3}}{gH \left(\frac{h^2 U k^2+3\right)}{gH \left(\frac{h^2 U k^3+3 U k-\sqrt{3}}{gH \left(\frac{h^2 U k-\sqrt{3$  $\label{left} $$ \operatorname{dt}^4{30 \left(\frac{1}{24} i k^2 + 3\right)} + \operatorname{left}(\text{text}dt)^5\right) + \left(\frac{1}{24} i k^4 \left(\frac{3}{24} i k^4 \right)\right)^2 + \operatorname{left}(\frac{3}{24} i k^4 \left(\frac{3}{24} i k^4 \right)^2 + \operatorname{left}(\frac{3}{24} i k^4 \right)^2 + \operatorname{left}(\frac{3}{24} i k^4 \left(\frac{3}{24} i k^4 \right)^2 + \operatorname{left}(\frac{3}{24} i k^4 \right)^2 + \operatorname{left}(\frac{3}{24} i k^4 \left(\frac{3}{24} i k^4 \right)^2 + \operatorname{left}(\frac{3}{24} i k^4 \right)^2 + \operatorname{left}(\frac{3}{24} i k^4 + k^4 \left(\frac{3}{24} i k^4 \right)^2 + \operatorname{left}(\frac{3}{24} i k^4 + k^4 \left(\frac{3}{24} i k^4 \right)^2 + \operatorname{left}(\frac{3}{24} i k^4 + k^4 \right)^2 + \operatorname{left}(\frac{3}{24} i k^4 + k^4 \left(\frac{3}{24} i k^4$  $\label{eq:left} $\operatorname{H}^4 W^2 +^2} = U\right-\int_{\mathbb{R}^2} U - \frac{1}{4} \operatorname{Heft}(e^7 \left(e^7 \left(e$  $H^5 \left(H^2 k^2+3\right)\right) + U-21 \left(H^2 k^2+3\right) + U-21 \left(H^2 k^2+3\right)\right) + U-3+3gH$  $\left(9 \left( 4^2 k^2 + 3\right) U - 5 \left( 4^2 k^2 + 3\right) U - 5 \left( 4^2 k^2 + 3\right) U - 5 \left( 4^2 k^2 + 3\right) U - 6 \left( 4^2 k^2 + 3\right)$  $\text{dt}^3$ {144 \left(H^2 k^2+3\right)^2}-\frac{i k^8 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2)}-\frac{i k^8 \left(3 g H+U \left(\left(H^2 k^2 k^2+3\right) U-2)}-\frac{i k^8 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2)}-\frac{i k^8 \left(3 g H+U \left(3 g H+  $\sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U-15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) k^2+18 U-15 \sqrt{3}$  $g H \left( \sqrt{3} \right) \left( \frac{4 \left( \frac{4^2 k^2 + 3\right)}{4^2 k^2 + 3\right)} - 4 \left( \frac{4^2 k^2 + 3\right)}{4^2 k^2 + 3\right)} \right) \left( \frac{4^2 k^2 + 3\right)}{4^2 k^2 + 3\right)}$  $\left(H^2 k^2+3\right)^3+O\left(text\left(dt\right)^5\right)\right) \$ k^4+\left(145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-1152 H^2 U\right) k^2-1728 U+531 \sqrt{3}  $\left( H^2 k^2+3\right) \left( H$  $\sqrt{g H^{13} \left( H^2 k^2 + 3\right)} U - 721 \right) k^6 - 3 U \left( h^2 k^2 + 3\right) U - 721 \right)$ U \left(2195 \sqrt{3} U-1011 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5-576 \sqrt{g H^9 \left(H^2 k^2+3\right)}  $k^2+3\right) U^3\right) V^3+9 \left(-1350 \right) 137$  $H^5 \left(H^2 k^2+3\right) U^4+2118 \left(H^2 k^2+3\right) U^2+145 \left(H^2 k^2+3\right) U$  $\left(H^2 k^2+3\right) V^4+g H\left(H^2 k^2+3\right) V^4+g H\left(H^2 k^2+3\right) U^4+g U^$  $\label{eq:left} $$ k^2+3\right\right)\right) \operatorname{H}(H^2 k^2+3\right)^{7/2}+\frac{h^2}{2}.$  $H+U \left( \left( \frac{h^2 k^2+3\right)}{U-2 \left( \frac{3}{3} \right) } \right) \\$ g^2 \left(145 H^2 k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right)  $U-4914 \sqrt{H^2 k^2+3\right} H-6 \left(32 \sqrt{H^9 \left(H^2 k^2+3\right)}\right) U-3$  $k^4+\left(192 \right) + 16t(192 \right) U^3+241 \right) U^3+241$  $U\left( h^2 + 288 \right) + U\left( h^2 + 24\right) + U^3\right) + U^3) + U^3\right) + U^3) + U^3\right) + U^3) + U^3\right) + U^3) + U^3) + U^3$  + UV^3) + U^3) + U^3) + U^3) + U^3 + UV^3) + UV^3) + U^3 + UV^3) + UV^3) + U^3 + UV^3) + UV^3 + UV^3) + UV^3 + UV^3) + UV^3  $\left(H^2 k^2+3\right)^{7/2}+O\left(text\left(dt\right)^5\right)\right) \$ 

Out[198]=

$$\text{Out} \text{[199]=} \quad EA \text{ ||} \quad \left\{ \left\{ \frac{-H^2 \ k^2 \left( \left( -1 + e^{i \ dt \ w} \right) k \ U - w \right) + 3 \ w}{\left( 3 + H^2 \ k^2 \right) w}, \right. \\ \left. -\frac{3 \left( -1 + e^{i \ dt \ w} \right) k}{\left( 3 + H^2 \ k^2 \right) w} \right\}, \\ \left. \left\{ -\frac{\left( -1 + e^{i \ dt \ w} \right) k \left( g \ H \left( 3 + H^2 \ k^2 \right) - 3 \ U^2 \right)}{\left( 3 + H^2 \ k^2 \right) w}, \right. \\ \left. 1 - \frac{\left( -1 + e^{i \ dt \ w} \right) k \left( 6 + H^2 \ k^2 \right) U}{\left( 3 + H^2 \ k^2 \right) w} \right\} \right\}$$

Out[200]= EA || \left(

\begin{array}{cc}

 $\label{left} $$ \operatorname{W-H^2 k^2 \left(\left(-1+e^{i \cdot t_{dt} w\right) right) k U-w\right)}(\end{tikzpicture} $$ \operatorname{W-H^2 k^2 \left(\left(-1+e^{i \cdot t_{dt} w\right) right) k U-w\right)}(\end{tikzpicture} $$ \operatorname{W-H^2 k^2 \left(\left(-1+e^{i \cdot t_{dt} w\right) right) k U-w\right)}(\end{tikzpicture} $$ \operatorname{W-H^2 k^2 \left(\left(-1+e^{i \cdot t_{dt} w\right) right) k U-w\right)}(\end{tikzpicture} $$ \operatorname{W-H^2 k^2 k^2 k end}(\end{tikzpicture} $$ \operatorname{W-W-inject}(\end{tikzpicture} $$) $$ \end{tikzpicture} $$ \operatorname{W-W-inject}(\end{tikzpicture} $$) $$ \end{tikzpicture} $$ \end{tikzpictu$ 

 $k^2+3\right) w$  &  $-\frac{3 \left(1+e^{i \left(t\right) w}\right) k}{\left(t^2 k^2+3\right) w} \$ 

 $-\frac{1+e^{i \cdot text}}{dt}$  w\right) k \left(g H \left(H^2 k^2+3\right)-3 U^2\right)\{\left(H^2 k^2+3\right)-3 U^2\right)}

 $w\} \& 1-\frac{\left(-1+e^{i \cdot k^2+3\right)} k \left(H^2 k^2+6\right)}{U}\left(H^2 k^2+3\right) w} \wedge W \\$ 

\end{array}

\right)

$$\begin{aligned} & \left( -\frac{1}{6} \left( -\frac{i\sqrt{3}}{3 + 16 + 16} \frac{\sqrt{g} H(s + 16 + 1)}{3 + 16 + 16} - i k U \right)^{\frac{3}{4}} + \frac{i(18 g H k^{\frac{3}{4}} U + 9 g H^{\frac{3}{4}} U - 18 k^{\frac{3}{4}} U^{\frac{3}{4}} H^{\frac{3}{4}} U^{\frac{3}{4}})}{6 (3 + 16 + 1)^{\frac{3}{4}}} + O[d1]^{\frac{3}{4}} dt^{\frac{3}{4}} - O[d1]^{\frac{3}{4}} \right) + \\ & \left( -\frac{1}{12} \left[ k^{\frac{3}{4}} U \right] dt + \frac{i(18 g H k^{\frac{3}{4}} U + 16 g H k^{\frac{3}{4}} U - 18 k^{\frac{3}{4}} U -$$

```
\frac{_{+241\,H^{-}\,K'^{+}+32\,H^{+}\,K'^{2})\,\cup\,\alpha\Gamma}}{960\,(3+H^{2}\,k^{2})^{2}}\,+\,\frac{_{(231\,g\,H\,K''+143\,g\,H'\,K''+1638\,K''\,U^{-}+6/4\,H^{-}\,K''\,U^{-}+64\,H^{+}\,K'''\,U^{-})\,\alpha\Gamma}}{1920\,(3+H^{2}\,k^{2})^{2}}
\frac{1}{^{1920} \, (^{3} + \mathrm{H}^{2} \, \mathrm{k}^{2})^{3}} \dot{\imath} \, \big(3231 \, \mathrm{g} \, \mathrm{H} \, \mathrm{k}^{7} \, \mathrm{U} + 1736 \, \mathrm{g} \, \mathrm{H}^{3} \, \mathrm{k}^{9} \, \mathrm{U} + 241 \, \mathrm{g} \, \mathrm{H}^{5} \, \mathrm{k}^{11} \, \mathrm{U} + 3321 \, \mathrm{k}^{7} \, \mathrm{U}^{3} + \\
                  2406\,H^{2}\,k^{9}\,U^{3} + 529\,H^{4}\,k^{11}\,U^{3} + 32\,H^{6}\,k^{13}\,U^{3}\big)\,dt^{3} + O[dt]^{5}\big)\,dx^{4} + O[dx]^{5}\big\}\big\}
```

Out[202]= Eerr || \left(

\begin{array}{cc}

 $\label{eq:left} $\left(\frac{u + \sqrt{u^2 + \sqrt{u$  $k^2 + \sqrt{3} \operatorname{H} \left( \frac{4t}^2 k^2 + 3\right) \setminus \frac{4t}^2 \right)$ \left(H^4 U^3 k^7-3 H^2 U^3 k^5+9 g H^3 U k^5-18 U^3 k^3+18 g H U k^3\right)\{6 \left(H^2  $k^2+3\right)^2-\frac{1}{6}\left(-iUk-\frac{1}{3} \right)^2-\frac{1}{6}\left(-iUk-\frac{1}{3}\right)^2-\frac{1}$  $\label{left} $$k^2+3\right\right) \operatorname{left}(-i\ U\ k-\frac{i\ \sqrt{3}\ \sqrt{4}\ H\ (H^2)}{24} \operatorname{left}(-i\ U\ k-\frac{i\ \sqrt{3}\ V\ (H^2)}{24} \operatorname{left}(-i$  $k^2+3\left(t\right)^{k} \left(t\right)^{k} \left(t\right)^{k$  $\left(k^4 U\right) \left(k^4 U\right) \left(k^$  $k^2+3\right)+\frac{10}{6} H^4 U^2 k^{10}+6 H^3 k^8-9 U^2 k^6+15 H^6 H^6 H^6 L^2 k^6+15 H^6 H^6 L^2 H^6 L^4 H^$  $\label{left} $$\left(H^2 k^2+3\right)^2+O\left(\frac{dt}^5\right)\right) \operatorname{text}_{dx}^3+\left(\frac{dt}^3\right)^2+O\left(\frac{dt}^3\right)^2+O\left(\frac{dt}^3\right)^3+O\left(\frac$  $k^9+143 H^2 U k^7+45 U k^5 \right) \left( \frac{4H^4 U k^5+16H^2 k^2+3\right)^2}{h^2 U k^7+45 U k^5 \right) \left( \frac{4H^4 U k^5+16H^2 U k^7+45 U k^5+16H^2 U k^7+45 U k^5 \right) \left( \frac{4H^4 U k^5+16H^2 U k^7+45 U k^5+16H^2 U k^7+45 U k^5+16H^2 U k^7+45 U k^5 \right) \left( \frac{4H^4 U k^5+16H^2 U k^5+$ U^2 k^{10}+145 g H^3 k^8+94 H^2 U^2 k^8-486 U^2 k^6+531 g H k^6\right) \text{dt}^2\{1920 \left(H^2 k^2+3\right)^2}-\frac{i k^7 \left(32 H^6 U^3 k^6+47 H^4 U^3 k^4+241 g H^5 U k^4-678 H^2 U^3 k^2+1348 g H^3 U k^2-1593 U^3+1683 g H U\right) \text{dt}^3}{1920 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}\^5\right)\right)\right)\\text{dx}\^4+O\left(\text{dx}\^5\right) & \left(-\frac{3} i k  $\text{text}_{dt}^{H^2} k^2+3$ -\frac{3 \left(k^2 U\right) \text{dt}^2}{H^2 k^2+3}+\frac{3 i \left(H^2 U^2 k^5+3)}{H^2 k^2+3}+ (H^2 k^2+3)+(H^2 k^2+3)+(H^  $U^2 k^3 + g H k^3 \right) \left( \frac{dt}^3}{2 \left( \frac{k^2 + 3 \right)^2}{+ O\left( \frac{dt}^5 - \frac{dt}^5 \right)^2} \right) \left( \frac{dt}^5 - \frac{dt$ k^5 U \text{dt}^2}{4 \left(H^2 k^2+3\right)}+\frac{\left(2 H^2 U^2 k^8+6 U^2 k^6+g H k^6\right)}  $\t (t_3)_{8 \left(t_4^2 + \frac{dt}^3}_{8 \left(t_4^2 + \frac{dt}^3}\right) \right) \\$  $\left(49 \text{ H}^2 \text{ k}^7 + 243 \text{ k}^5\right) \left(49 \text{ H}^2 \text{ k}^2 + 3\right)^2 + \frac{145 \text{ H}^2}{2} + \frac{145 \text{ H}^2}{2$ k^8+531 k^6\right) U \text{dt}^2\{960 \left(H^2 k^2+3\right)^2\}-\frac\{i \left(241 H^4 U^2 \)  $k^{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^9+1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^9+1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^9+1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9+1542 H^2 U^2 k^9+1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H^3 k^9 +1744 g H k^7 \right) \left( \frac{1}{1}+194 g H k$  $\left(H^2 k^2+3\right)^3+O\left(\frac{dt}^5\right)\right) \left(\frac{dt}^3+O\left(\frac{dt}^5\right)\right)$  $\left(\frac{k^2 H^3}{3} + H\right) - ig H \left(\frac{k^2 H^3}{3$ k^5+6 g H^3 U^2 k^5-9 U^4 k^3+3 g^2 H^2 k^3+6 g H U^2 k^3\right) \text{dt}^3}{2 \left(H^2  $\label{eq:linear_line$  $\left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^2 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \left(2 g H U k^7-3 U^3 k^5+6 g H U k^7-3 U^3 k^5\right) \left(2 g H U k^7-3 U^3 k^7-3$ g H^5 U^2 k^{10}+2 g^2 H^4 k^8-6 H^2 U^4 k^8+18 g H^3 U^2 k^8-18 U^4 k^6+6 g^2 H^2  $k^6+24 g H U^2 k^6 \right) \left( \frac{dt}^3}{24 \left( \frac{H^2 k^2+3\right)}{2} + O\left( \frac{dt}^5 \right) \right)} \right)$ \text{dx}^3+\left(\frac{i} \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H  $k^5 \right) \left( \frac{dt}{960 \left( \frac{4^2 + 3\right)}} \right) \left( \frac{4^2 + 3\right)}{10^2 + \frac{4^2 + 3\right)}} \left( \frac{4^2 + 3\right)}{10^2 + \frac{4^2 + 3\right)}{10^2 + \frac{4^2 + 3\right)}} \left( \frac{4^2 + 3\right)}{10^2 + 3} \left( \frac{4^2 + 3\right)}{10^2 + 3}$ g H^3 U k^8-531 U^3 k^6+576 g H U k^6\right) \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i} \left(288 g H^7 U^2 k^{13}+241 g^2 H^6 k^{11}-723 H^4 U^4 k^{11}+2592 g H^5 U^2 k^{11}+1542 g^2 H^4 k^9-4626 H^2 U^4 k^9+7194 g H^3 U^2 k^9-7371 U^4 k^7+2457 g^2 H^2 k^7+5454 g H U<sup>2</sup> k<sup>7</sup>\right) \text{dt}<sup>3</sup>{5760 \left(H<sup>2</sup> k<sup>2</sup>+ $^3$ \right)<sup>3</sup>}+O\left(\text{dt}<sup>5</sup>\right)\right)  $\text{dx}^4+O\left(\frac{dx}^5\right) & \left(\frac{dx}^4+O\left(\frac{dx}^5\right) & \left(\frac{dx}^4+O\left(\frac{dx}^4\right) & \left(\frac{dx}^4\right) & \left(\frac{dx}^4\right)$ 

\text{dt}^2\{H^2 k^2+3}+\\left(\frac{i \left(H^4 U^3 k^7+15 H^2 U^3 k^5+9 g H^3 U k^5+36 U^3 k^3+36  $g \ H \ U \ k^3\right] \ \{6 \left( h^2 \ k^2 + 3\right)^2 - \frac{1}{6} \left( i \ U \ k - \frac{3}{3} \right) \ sqrt{g \ H \ left(H^2 \ k^2 + 3\right)^2} - \frac{1}{6} \left( i \ U \ k - \frac{3}{3} \right) \ sqrt{g \ H \ left(H^2 \ k^2 + 3\right)} \ sqrt{g \ H \ left(H^2 \ k^2 + 3)} \ sqrt{g \ H \ l$  $\label{eq:k-2+3-inj} k_{H^2 k^2+3}\right) \left( \frac{1}{24} \left( i \ U \ k-\frac{i \ \sqrt{1} \ \sqrt{1}}{24} \right) \right) \right) \\$  $\label{left(H^2 k^2+3\rightarrow k} $$\left(H^2 k^2+3\right)^4 \left(\frac{dt}^4+O\left(\frac{dt}^5\right)\right)^4$$  $\left( \frac{4 \text{ U}}{f^2} \right) \left( \frac{4t}{-2} \right) \left( \frac{4t}-2 \right) \left( \frac{4t}{-2} \right) \left( \frac{4t}{-2} \right) \left( \frac{4t}{-2} \right) \left( \frac{4t}$ k^2+3\right)}+\frac{\left(H^4 U^3 k^{10}+12 H^2 U^3 k^8+6 g H^3 U k^8+27 U^3 k^6+21 g H U  $\left(32 \text{ H}^4 \text{ k}^9 + 241 \text{ H}^2 \text{ k}^7 + 531 \text{ k}^5\right) \text{ U} \left(43\right) \left(960 \left(4^2 \text{ k}^2 + 3\right)\right)^2 + \frac{660}{100}$ H<sup>4</sup> U<sup>2</sup> k<sup>10</sup>+145 g H<sup>3</sup> k<sup>8</sup>+674 H<sup>2</sup> U<sup>2</sup> k<sup>8</sup>+1638 U<sup>2</sup> k<sup>6</sup>+531 g H k<sup>6</sup>right) \text{dt}<sup>2</sup>{1920 \left(H^2 k^2+3\right)^2}-\frac{i \left(32 H^6 U^3 k^{13}+529 H^4 U^3 k^{11}+241 g H^5 U k^{11}+2406 H^2 U^3 k^9+1736 g H^3 U k^9+3321 U^3 k^7+3231 g H U k^7\right) \text{dt}^3\{1920  $\left(\frac{dt}^2 k^2+3\right)^3+O\left(\frac{dt}^5\right)\right) \left(\frac{dt}^4+O\left(\frac{dt}^5\right)\right)$ 

\end{array}

```
\right)
\ln[203] = \text{KurF} = (\text{fm} * \text{ap} - \text{fp} * \text{am} + \text{am} * \text{ap} * (\text{qp} - \text{qm})) / (\text{ap} - \text{am});
       KurFWS = KurF /. ap \rightarrow 0 /. am \rightarrow (U - Sqrt[g * H]);
       KurFWSeta =
          KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
            qm \rightarrow Rmp * n;
       KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
         Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1]; 
        KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1]; 
       Kfnn = Kfnnp /. Rpp \rightarrow Rp /. Rmp \rightarrow Rm /. GGp \rightarrow GG2 /. Gnp \rightarrow Gn2;
       KfnG = KfnGp / Rpp \rightarrow Rp / Rmp \rightarrow Rm / GGp \rightarrow GG2 / Gnp \rightarrow Gn2;
       Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
       Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 4}];
       Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
       FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
       FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 4}];
       FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
       \texttt{KurFWSG} = \texttt{KurFWS} \ / . \ \texttt{fp} \ \rightarrow \ (\texttt{U} * \texttt{Rpp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rpp} * \texttt{n}) \ / .
               fm \rightarrow (U*Rmp*G + U*H*v + g*H*Rmp*n) / . qp \rightarrow Rpp*G / . qm \rightarrow Rmp*G;
       KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
       KfGGp = FullSimplify[KurFWSG /. n \rightarrow 0 /. G \rightarrow 1];
       KfGn = KfGnp /. Rpp \rightarrow Rp /. Rmp \rightarrow Rm /. GGp \rightarrow GG2 /. Gnp \rightarrow Gn2;
       KfGG = KfGGp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
       FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
       FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 4}];
       FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
```

```
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 4}];
FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2/2 + Fmat2.Fmat2.Fmat2/6;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
RKStep =
  Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 + EigvFmat2 * EigvFmat2 * EigvFmat2 / 6]/
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{"U < -Sqrt(gH) < U"}]]</pre>
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn || ", KfGnp}]]
Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error || ", FGn2TAr}]]
Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG || ", KfGGp}]]
Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error || ", FGG2TAr}]]
Text[Row[{"FGG error || ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error || ", RKstepTayr}]]
Text[Row[{"Omega error || ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA || ", EA}]]
```

```
Text[Row[{"EA ||
                  ", TeXForm[EA]}]]
Text[Row[{"Eerr || ", Eerr}]]
Text[Row[{"Eerr ||
                    ", TeXForm[Eerr] } ] ]
```

Out[237]= U < -Sqrt(gH) < U

Out[238]=

Out[239]= Fnn || Gnp H + Rpp U

 $Out[240] = Fnn \parallel \text{text}\{Gnp\} H + \text{text}\{Rpp\} U$ 

$$\begin{array}{ll} \text{Out} [241] = & Fnn \; error \; \mid \mid \; \left( -\frac{\left( H^2 \; k^3 \; U \; w \right) \; dt^2}{2 \left( 3 + H^2 \; k^2 \right)} \; - \; \frac{i \; H^2 \; k^3 \; U \; w^2 \; dt^3}{6 \left( 3 + H^2 \; k^2 \right)} \; + \; \frac{H^2 \; k^3 \; U \; w^3 \; dt^4}{24 \left( 3 + H^2 \; k^2 \right)} \; + \; O[dt]^5 \right) + \\ & \left( \frac{1}{12} \; k^4 \; U \; dt \; + \; O[dt]^5 \right) dx^3 \; + \left( \frac{i \left( 45 \; k^3 \; U + 143 \; H^2 \; k^7 \; U + 32 \; H^4 \; k^9 \; U \right) dt}{960 \left( 3 + H^2 \; k^2 \right)^2} \; + \; O[dt]^5 \right) dx^4 \; + \; O[dx]^5 \\ \end{array}$$

Out[242]= Fnn error |

 $\left(-\frac{t}{2 \cdot t}\right)^2 \left(-\frac{t}{2 \cdot$  $\left(H^2 k^2+3\right)+\frac{dt}^4 H^2 k^3 U w^3}{24 \left(H^2 k^2 + 3\right)}$ 

 $k^2+3\right)+O\left(\frac{dt}^5\right)+\frac{dt}^3\left(\frac{dt}^5\right)+\frac{dt}^3\left(\frac{dt}^3\right)+\frac{dt}^3\left(\frac{dt}^3\right)}{2}$ 

\text{dt}+O\left(\text{dt}}^5\right)\right)+\text{dx}^4 \left(\frac{i} \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^  $k^5 \right) \left( \frac{dt}{960 \left( \frac{A^2 k^2+3\right)^2}+O\left( \frac{dt}{5}\right) \right)}{1} + O\left( \frac{dt}{5}\right) \right) \left( \frac{dt}{5}\right) \left($ 

Out[243]=

Out[244]= FnG || GGp H

Out[245]=  $FnG \parallel \text{text}\{GGp\} H$ 

$$\text{Out} [246] = \text{FnG error } || \left( -\frac{3 \left( k \, \text{w} \right) \, dt^2}{2 \left( 3 + \text{H}^2 \, \text{k}^2 \right)} - \frac{i \, k \, \text{w}^2 \, dt^3}{2 \left( 3 + \text{H}^2 \, \text{k}^2 \right)} + \frac{k \, \text{w}^3 \, dt^4}{8 \left( 3 + \text{H}^2 \, \text{k}^2 \right)} + \text{O}[dt]^5 \right) + \left( \frac{i \left( 243 \, k^5 + 49 \, \text{H}^2 \, k^7 \right) \, dt}{960 \left( 3 + \text{H}^2 \, k^2 \right)^2} + \text{O}[dt]^5 \right) dx^4 + \text{O}[dx]^5$$

 $\text{text}\{dt\}^3 \text{ k w}^2\}\{2 \left(H^2 \text{ k}^2 + 3\right)\} + \frac{dt}^4 \text{ k w}^3\}\{8 \left(H^2 \text{ k}^2 + 3\right)\}$ 

 $k^2+3\right)+O\left(\frac{dt}^5\right)+\frac{dt}^5\right)$ 

 $\text{dt}{960 \left(H^2 k^2+3\right)^2}+O\left(\left(text{dt}^5\right)\right)+O\left(text{dt}^5\right)+O\left(text{dx}^5\right)+O\left(text{d$ 

Out[248]=

Out[249]=  $FGn \parallel H(gRpp + Gnp U)$ 

Out[250]=  $FGn \parallel H(g \text{text}{Rpp}+\text{text}{Gnp} U)$ 

$$\begin{array}{ll} \text{Out} \text{[251]=} & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w \right) dt^2}{2 \, \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^2 \, dt^3}{6 \, \left( 3 + H^2 \, k^2 \right)} + \frac{k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^3 \, dt^4}{24 \, \left( 3 + H^2 \, k^2 \right)} + O[dt]^5 \right) + \\ & \left( \frac{1}{12} \, g \, H \, k^4 \, dt + O[dt]^5 \right) dx^3 + \left( \frac{i \, \left( 288 \, g \, H \, k^5 + 192 \, g \, H^3 \, k^7 + 32 \, g \, H^5 \, k^9 - 243 \, k^5 \, U^2 - 49 \, H^2 \, k^7 \, U^2 \right) dt}{960 \, \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \end{array}$$

Out[252]= FGn error |

 $\text{text}(dt)^3 k \text{ w}^2 \left( H^3 k^2 + 3 g H - 3 U^2 \right) (6 \left( H^2 k^2 + 3 \right) + \frac{1}{4} k w^3$  $\left(\frac{H^2 k^2+3 g H-3 U^2\right)}{24 \left(\frac{H^2 k^2+3\right)}+O\left(\frac{dk}{3}\right)}+O\left(\frac{dk}{3}\right)}$  $\left(\frac{1}{12} g H k^4 \left(\frac{dt}{+0}\right)\right) + \left(\frac{dt}{4}\right) + \left(\frac{dt}{5}\right) \right)$ \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right)  $\text{text}_{dt}^{960} \left( \frac{H^2 k^2+3\right)^2}+O\left( \frac{dt}^5\right)\right) + O\left( \frac{H^2 k^2+3\right)^2}+O\left( \frac{dt}^5\right)\right)$ 

Out[253]=

Out[254]= 
$$FGG \parallel (GGp H + Rpp) U$$

Out[255]= 
$$FGG \parallel U (\text{text}\{GGp\} H+\text{text}\{Rpp\})$$

$$\begin{array}{ll} \text{Out} \text{[256]=} & FGG \; error \; \mid \mid \; \left( -\frac{\left( k \left( 6 + H^2 \; k^2 \right) \text{U} \; w \right) \, dt^2}{2 \left( 3 + H^2 \; k^2 \right)} - \frac{i \; k \left( 6 + H^2 \; k^2 \right) \text{U} \; w^2 \; dt^3}{6 \left( 3 + H^2 \; k^2 \right)} + \frac{k \left( 6 + H^2 \; k^2 \right) \text{U} \; w^3 \; dt^4}{24 \left( 3 + H^2 \; k^2 \right)} + O[dt]^5 \right) + \\ & \left( \frac{1}{12} \; k^4 \; \text{U} \; dt + O[dt]^5 \right) dx^3 + \left( \frac{i \left( 531 \; k^5 + 241 \; H^2 \; k^7 + 32 \; H^4 \; k^9 \right) \text{U} \; dt}{960 \left( 3 + H^2 \; k^2 \right)^2} + O[dt]^5 \right) dx^4 + O[dx]^5 \\ \end{array}$$

Out[257]= FGG error ||

 $\label{left} $\left(-\frac{d^2 \ker d^2 \cdot (H^2 k^2+6\right)}{2 \left(H^2 k^2+6\right)}^2 \left(H^2 k^2+3\right)^{-\frac{1}{2}} \right) -\frac{1}{2} \left(H^2 k^2+3\right)^2 \left(H^2 k$  $k U w^2 \left( \frac{h^2 k^2 + 6 \right)}{6 \left( \frac{h^2 k^2 + 3 \right)} + \frac{dt}{4} k U w^3 \left( \frac{h^2 k^2 + 3 \right)}{6 \left( \frac{h^2 k^2 + 3 \right$  $k^2 + 6 \cdot (h^2 k^2 + 3 \cdot (t^2 k^2$ 

Out[258]=

Out[259]=

Out[260]= Omega error 
$$\| \left\{ \left( -\frac{i \left[ \sqrt{3} \text{ k} \sqrt{g \text{ H} (3 + \text{H}^2 \text{ k}^2)} + 3 \text{ k} \text{ U} + \text{H}^2 \text{ k}^3 \text{ U}} \right]^{\frac{1}{4}} \text{ dt}^{\frac{3}{4}}}{24 (3 + \text{H}^2 \text{ k}^2)^4} + \frac{\left[ \sqrt{3} \text{ k} \sqrt{g \text{ H} (3 + \text{H}^2 \text{ k}^2)} + 3 \text{ k} \text{ U} + \text{H}^2 \text{ k}^3 \text{ U}} \right]^{\frac{5}{4}} \text{ dt}^{\frac{4}{4}}}{30 (3 + \text{H}^2 \text{ k}^2)^5} + O[\text{dt}]^5 \right\} + \left( \frac{1}{24} i \text{ k}^4 \left[ \sqrt{3} \sqrt{g \text{ H} (3 + \text{H}^2 \text{ k}^2)} + 2 \text{ U} \right] + \frac{1}{144 (3 + \text{H}^2 \text{ k}^2)^2} \text{ k}^7 \left( 9 \text{ g}^2 \text{ H}^2 + 3 \text{ g} \text{ H} \text{ U} \left( 5 \sqrt{3} \sqrt{g \text{ H} (3 + \text{H}^2 \text{ k}^2)} + 9 \left( 3 + \text{H}^2 \text{ k}^2 \right) \text{ U} \right) + U^3 \left( 21 \sqrt{3} \sqrt{g \text{ H} (3 + \text{H}^2 \text{ k}^2)} + 18 \text{ U} + 2 \text{ H}^4 \text{ k}^4 \text{ U} + \text{k}^2 \left( 7 \sqrt{3} \sqrt{g \text{ H}^5 (3 + \text{H}^2 \text{ k}^2)} + 12 \text{ H}^2 \text{ U} \right) \right) \right) \text{ dt}^3 + \frac{1}{144 (3 + \text{H}^2 \text{ k}^2)^3} i \text{ k}^8 \left( 3 \text{ g} \text{ H} + \text{U} \left( 2 \sqrt{3} \sqrt{g \text{ H} (3 + \text{H}^2 \text{ k}^2)} + (3 + \text{H}^2 \text{ k}^2) \text{ U} \right) \right)$$

$$\left( 3 \text{ g} \text{ H} \left( \sqrt{3} \sqrt{g \text{ H} (3 + \text{H}^2 \text{ k}^2)} + 4 \left( 3 + \text{H}^2 \text{ k}^2 \right) \text{ U} \right) + \frac{1}{2} \left( 15 \sqrt{3} \sqrt{g \text{ H} (3 + \text{H}^2 \text{ k}^2)} + 18 \text{ U} + 2 \text{ H}^4 \text{ k}^4 \text{ U} + \text{k}^2 \left( 5 \sqrt{3} \sqrt{g \text{ H}^5 (3 + \text{H}^2 \text{ k}^2)} + 12 \text{ H}^2 \text{ U} \right) \right) \right) \right) \text{ dt}^4 + \frac{1}{2} \left( 145 \sqrt{3} \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 12 \text{ H}^2 \text{ U} \right) \right) + \frac{1}{2} \left( 145 \sqrt{3} \sqrt{g \text{ H}^3 (3 + \text{H}^2 \text{ k}^2)} + 1350 \sqrt{3} \text{ g}^2 \right) + 1350 \sqrt{3} \text{ g}^2 \right)$$

$$\begin{aligned} &\Pi^4 U + 2118 \sqrt{g^3 \Pi^2 (3 + \Pi^2 k^2)} & U^2 + 2227 \sqrt{3} g\Pi^3 U^3 + 576 \sqrt{g}\Pi^5 (3 + \Pi^2 k^2)} U^4 + \\ & 81 \left(59 \sqrt{g^3 H^3 (3 + H^2 k^2)} + 241 \sqrt{3} g^2 H^2 U + 64 \sqrt{g}H (3 + H^2 k^2)} U^4 + \\ & gH U^2 \left(369 \sqrt{g}H (3 + H^2 k^2) + 251 \sqrt{3} U\right) + 3 k^4 U \left(627 \sqrt{3} g^2 H^6 + 576 \sqrt{g}\Pi^9 (3 + H^2 k^2)} U^3 + gH^5 U \left(1011 \sqrt{g}H (3 + H^2 k^2) + 2195 \sqrt{3} U\right)\right) dt^3 \right) / \\ & \left(34560 \sqrt{g}H (3 + H^2 k^2)^{7/2}\right) - \left(\left(k^9 \left(3 gH + U \left(2 \sqrt{3} \sqrt{g}H (3 + H^2 k^2) + 43 H^2 k^2\right) + 0\right)\right) \right) \\ & \left(3\sqrt{3} g^2 H^2 \left(531 + 145 H^2 k^2\right) + gH U \left(4914 \sqrt{g}H (3 + H^2 k^2) + 43 H^2 k^2\right) U\right) \right) \\ & \left(3\sqrt{3} g^2 H^2 \left(531 + 145 H^2 k^2\right) + gH U \left(4914 \sqrt{g}H (3 + H^2 k^2) + 43 H^2 k^2\right) U\right) + k^2 \left(241 \sqrt{g^3 \Pi^2 (3 + H^2 k^2)} U + 192 \sqrt{g} H^3 \left(3 + H^2 k^2\right) U^3 + 32 k^4 \sqrt{g} H^9 \left(3 + H^2 k^2\right) U^3 + k^2 \left(241 \sqrt{g^3 \Pi^2 (3 + H^2 k^2)} U + 192 \sqrt{g} H^3 \left(3 + H^2 k^2\right) U^3\right)\right) \right) \\ & dt^4 \right) / \left(34560 \left(\sqrt{g}H (3 + H^2 k^2)^{7/2}\right) + O[dd]^5 \right) dx^4 + O[dx]^5, \\ & \left(-\frac{4}{24} \frac{1}{4} \sqrt{3} \sqrt{\frac{g}{3} H^3 + 2} + 2 U\right) + \frac{1}{144 \left(3 + H^2 k^2\right)} \frac{1}{2} dt^4 + \frac{1}{2} \left(3 \frac{1}{2} \frac{1}{4} \frac{1}{2} \frac{1}{4} \frac{1}{2} \frac{1}{4} \frac{1}{2} \frac{1$$

$$g H U^{2} \left(369 \sqrt{g H (3 + H^{2} k^{2})} - 251 \sqrt{3} U\right) \right) - 3 k^{4} U$$

$$\left(627 \sqrt{3} g^{2} H^{6} - 576 \sqrt{g H^{9} (3 + H^{2} k^{2})} U^{3} + g H^{5} U \left(-1011 \sqrt{g H (3 + H^{2} k^{2})} + 2195 \sqrt{3} U\right)\right) \right)$$

$$dt^{3} + \left(k^{9} \left(3 g H + U \left(-2 \sqrt{3} \sqrt{g H (3 + H^{2} k^{2})} + (3 + H^{2} k^{2}) U\right)\right) \left(3 \sqrt{3} g^{2} H^{2} (531 + 145 H^{2} k^{2}) + g H U \left(-4914 \sqrt{g H (3 + H^{2} k^{2})} + \sqrt{3} (5049 + 3270 H^{2} k^{2} + 529 H^{4} k^{4}) U\right) - 6 \left(288 \sqrt{g H (3 + H^{2} k^{2})} U^{3} + 32 k^{4} \sqrt{g H^{9} (3 + H^{2} k^{2})} U^{3} + k^{2} \left(241 \sqrt{g^{3} H^{7} (3 + H^{2} k^{2})} U + 192 \sqrt{g H^{5} (3 + H^{2} k^{2})} U^{3}\right) \right)$$

$$dt^{4} \right) / \left(34560 \sqrt{g H} \left(3 + H^{2} k^{2}\right)^{7/2}\right) + O[dt]^{5} dx^{4} + O[dx]^{5} \right)$$

## Out[261]= Omega error ||

 $\left( -\frac{1}{\left( -\frac{1}{2} U k^3 + 3 U k + \sqrt{1} k^2 H \left( -\frac{1}{2} k^2 + 3 \right) k \right) k \right) k \left( -\frac{1}{2} U k^3 + 3 U k + \sqrt{1} k \right) k \left( -\frac{1}{2} U k^3 + 3 U k \right) k \left( -\frac{1}{2} U k^3 + 3 U k \right) k \left( -\frac{1}{2} U k^3 + 3 U k \right) k \left( -\frac{1}{2} U k^3 + 3 U k \right) k \left( -\frac{1}{2} U k \right) k \left( -\frac{1}{2}$  $k^2+3\right)^4+\frac{h^2 U k^3+3 U k+\sqrt{3} \sqrt{4} +\frac{h^2 U k^2+3 \pi(H^2 U k^3+3 U k+\sqrt{3} \sqrt{4} +\frac{h^2 U k^2+3 \pi(H^2 U k^3+3 U k+\sqrt{3} +\frac{h^2 U k^3+3 U k+\sqrt{3} \sqrt{4} +\frac{h^2 U k^3+3 U k+\sqrt{3} U k+\sqrt{3} +\frac{h^2 U k^3+3 U k+\sqrt{3} U$  $\label{left} $$ \operatorname{dt}^4{30 \left(\frac{h^2 k^2+3\right)^5}+O\left(\frac{dt}^5\right)^5\right)^5}$$  $U+\sqrt{3} \sqrt{1} U +\sqrt{3} \sqrt{1} U +\sqrt{3} \sqrt{1} U +\sqrt{3} \sqrt{1} U +\sqrt{3} U$  $\sqrt{g H^5 \left(H^2 k^2+3\right)}\right) \ k^2+18 U+21 \ \sqrt{3} \ \sqrt{g H \left(H^2 k^2+3\right)}\right) \ U^3+3$  $g\ H \left( \frac{4^2 k^2+3\right)}{U+5 \left( \frac{4^2 k^2+3\right$  $\label{eq:linear_continuous_state} $$ \left(\frac{dt}^3}{144 \left(\frac{H^2 k^2+3\right)^2}+\frac{k^8 \left(\frac{dt}{3} H+U \left(\frac{H^2 k^2+3\right)}{U+2 \left(\frac{dt}{3}\right)}\right)}{U+2 \left(\frac{dt}{3}\right)} \right) $$$  $\left(H^2 k^2+3\right)\right)\right)$  $\left(\frac{H^2 k^2+3\right}{k^2+3\right} k^2+18 U+15 \sqrt{3} \right) + \left(\frac{H^2 k^2+3\right}{k^2+3\right} U^2+3 gH \left(\frac{H^2 k^2+3\right}{k^2+3\right} gH \left(\frac{H^2 k^2+3\right}{k^2+3}\right) gH \left(\frac{H^2 k^2+3\right}{k^2+3} gH \left(\frac{H^2 k^2+3\right)}{k^2+3} gH \left(\frac{H^2 k^2+3\right)}{k^2+$  $\label{left(H^2 k^2+3\wedge ight) U+\sqrt{1}} \left( H^2 k^2 + 3\wedge ight \right) \\ \label{left(H^2 k^2+3\wedge ight) \ell} \\ \label{le$  $\label{left} $$k^2+3\right)^3+O\left(\frac{dt}^5\right)^3+O\left(\frac{dt}^$  $\label{eq:continuity} $$U H^2+145 \left(3\right) \right] + 1728 U+531 \left(3\right) \left(H^2 k^2+3\right)\right) + 1728 U+531 \left(3\right) \left(H^2 k^2+3\right) \right) + 1728 U+531 \left(13\right) \left(13\right)$  $k^2+3\right)/100$ \sqrt{g H^{13} \left(H^2 k^2+3\right)} U\right) k^6+3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3})  $U+1011 \operatorname{l} H \left(H^2 k^2+3\right) \right) + H^5+576 \operatorname{l} H^9 \left(H^2 k^2+3\right) U^3 \right)$ k^4+9 \left(1350 \sqrt{3} g^2 U H^4+2227 \sqrt{3} g U^3 H^3+576 \sqrt{g H^5 \left(H^2 k^2+3\right)}  $k^2+81 \left(64 \right) + H\left(64 \right) + H\left$  $k^2+3\right)$  \\right) \  $\t text{dt}^3}{34560 \operatorname{left}(H^2 k^2+3\right)} - \t (h^2 k^2+3) + \t (h^2 k^2$  $k^2+3\left( U+2 \right) + \left( 145 H^2 k^2+3\right) \left( 145 H^2 k$  $k^2+531\right) H^2+g U \left( \sqrt{3} \right) H^4 k^4+3270 H^2 k^2+5049\right) U+4914 \sqrt{3} H^2 k^2+5049\right)$  $H \left( \frac{H^2 k^2+3\right)}{h^2 k^2+3\right) \ H+6 \left( \frac{H^2 k^2+3\right) \ U^3 k^4+\left( \frac{H^2$  $\$  \\sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+241 \\sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\\right) k^2+288  $\sqrt{g H \left(\frac{h^2 k^2+3\right)} U^3\right)} U^3\right) \$ \left(H^2 U k^3+3 U k\_\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^4 \text{dt}^3}{24 \left(H^2  $\label{left} $$k^2+3\right)^4}+\frac{(H^2Uk^3+3Uk-\sqrt{3}) \left(H^2Uk^2+3\right)^5}$  $\label{eq:local_continuous_cont$ \sart{\frac{o H}{H^2 k^2+3}}=2 II\right)+\frac{k^7 \left(\left(2 H^4 II k^4+\left(12 H^2 II-7 \sart{3} \sart{o

 $H^5 \left( H^2 k^2 + 3 \right) \right) + 2 + 18 U - 21 \left( H^2 k^2 + 3 \right) + 2 + 18 U - 21 \left( H^2 k^2 + 3 \right) \right) + 2 + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 \left( H^2 k^2 + 3 \right) + 18 U - 21 U H \left( \frac{h^2 k^2+3\right) U-5 \left( H^2 k^2+3\right) U-5 \left( H^2 k^2+3\right) U-9 e^2 H^2\right) U-5 \left( H^2 k^2+3\right) U-5 \left( H^2 k^2+3$  $\text{dt}^3$ {144 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2)}+\frac{i k^8 \left(3 g H+U \left(H^2 k^2+3\right) U-2)}+\frac{i k^8 \left(3 g H+U \left(H^2 k^2+3\right) U-2)}+\frac{i k^8 \left(3 g H+U  $\sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+18 U-15 \sqrt{3} \right)$  $g \ H \left( \frac{3} \right) + \left( \frac{4}{144} \right) - 4 \left( \frac{4}{2} \right) - 4 \left( \frac{4}{144} \right) - 4 \left( \frac{4}$  $k^4+\left(145 \right)^{15} \left(145 \right)^{$  $\left( H^2 k^2+3\right) \left( H$  $\sqrt{g H^{13} \left(H^2 k^2+3\right)} U-721 \right] H^7\right]$ U \left(2195 \sqrt{3} U-1011 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5-576 \sqrt{g H^9 \left(H^2 k^2+3\right)}  $k^2+3\right) U^3\right) V^3+9 \left(-1350 \right) 13 q^2 U H^4-2227 \right] U^3 H^3+576 \right]$  $H^5 \left(H^2 k^2+3\right) U^4+2118 \left(H^2 k^2+3\right) U^2+145 \left(H^2 k^2+3\right) U$ H \left(H^2 k^2+3\right)\-251 \sqrt{3} U\right) U^2-241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2  $k^2+3\right) \right) \left( \frac{dt}^3}{34560 \cdot \frac{H}\left( \frac{h^2 k^2+3\right)^{7/2}}{h^2 t^2}} \right) \left( \frac{h^2 k^2+3\right)^{7/2}}{h^2 t^2} + \frac{h^2 k^2+3\left( \frac{h^2 k^2+3\right)^{7/2}}{h^2 t^2} \right) \left( \frac{h^2 k^2+3\right)^{7/2}}{h^2 t^2} + \frac{h^2 k^2+3\left( \frac{h^2 k^2+3}{h^2} \right) \left( \frac{h^2 k^2+3}{h^2} \right) \left( \frac{h^2 k^2+3\left( \frac{h^2 k^2+3}{h^2} \right) \left( \frac{h^2 k^2+3}{h^2} \right) \left( \frac{h^2 k^2+3}{h^2} \right) \left( \frac{h^2 k^2+3}{h$  $H+U \left( \frac{h^2 k^2+3\right)}{U-2 \left( \frac{h^2 k^2+3\right)$ g^2 \left(145 H^2 k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right)  $U-4914 \sqrt{H^2 k^2+3}\right) H-6 \left(32 \sqrt{H^9 \left(H^2 k^2+3\right)}\right) U-3$  $k^4+\left(192 \right) + 16 \left(192 \right) +$  $U \rightarrow k^2 + 288 \qquad k^2 + 3 \rightarrow k^2 + k^2$ 

 $\left(H^2 k^2+3\right)^{7/2}+O\left(text\left(dt\right)^5\right)\right) \$ 

Out[262]=

$$\text{Out} [263] = \ EA \ || \ \left. \left\{ \left\{ \frac{-H^2 \ k^2 \left( \left( -1 + e^{i \ d \ w} \right) k \ U - w \right) + 3 \ w}{\left( 3 + H^2 \ k^2 \right) w}, \ - \frac{3 \left( -1 + e^{i \ d \ w} \right) k}{\left( 3 + H^2 \ k^2 \right) w} \right\}, \ \left\{ - \frac{\left( -1 + e^{i \ d \ w} \right) k \left( g \ H \left( 3 + H^2 \ k^2 \right) - 3 \ U^2 \right)}{\left( 3 + H^2 \ k^2 \right) w}, \ 1 \ - \frac{\left( -1 + e^{i \ d \ w} \right) k \left( 6 + H^2 \ k^2 \right) U}{\left( 3 + H^2 \ k^2 \right) w} \right\} \right\}$$

Out[264]= EA || \left(

\begin{array}{cc}

 $\label{left} $$ \operatorname{W-H^2 k^2 \left(\left(\frac{1+e^{i \det\{dt\} w}\right)} k U-w\right)}_{\label{left(H^2)}} $$$ 

 $k^2+3\right) w \& -\frac{3 \left(1+e^{i \operatorname{dt} w}\right) k}{\left(1+e^2 k^2+3\right) w} \le k^2+3\right) w & -\frac{3 \left(1+e^{i \operatorname{dt} w}\right) k}{\left(1+e^2 k^2+3\right) w}$ 

 $-\frac{1+e^{i \cdot k}}{k} \frac{1}{k} \frac$ 

\end{array}

\right)

Out[265]= Eerr || 
$$\left\{ \left\{ \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + 3 \text{ k U} \right) dt}{3 + \text{H}^2 \text{ k}^2} + \frac{\left( \sqrt{3} \text{ k}^2 \sqrt{\text{g H (3+H^2 k^2)}} \text{ U+3 k}^2 \text{ U}^2 \right) dt^2}{3 + \text{H}^2 \text{ k}^2} + \frac{\left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + 1 \right) dt^2}{3 + \text{H}^2 \text{ k}^2} + \frac{i \left( 18 \text{ g H k}^3 \text{ U+9 g H}^3 \text{ k}^5 \text{ U-18 k}^3 \text{ U}^3 - 3 \text{ H}^2 \text{ k}^5 \text{ U}^3 + \text{H}^4 \text{ k}^7 \text{ U}^3 \right)}{6 \left( 3 + \text{H}^2 \text{ k}^2 \right)^2} \right\} dt^3 - \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( 18 \text{ g H k}^3 \text{ U+9 g H}^3 \text{ k}^5 \text{ U-18 k}^3 \text{ U}^3 - 3 \text{ H}^2 \text{ k}^5 \text{ U}^3 + \text{H}^4 k}^7 \text{ U}^3 \right)}{6 \left( 3 + \text{H}^2 \text{ k}^2 \right)^2} \right) dt^3 - \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( 18 \text{ g H k}^3 \text{ U+9 g H}^3 \text{ k}^5 \text{ U-18 k}^3 \text{ U}^3 - 3 \text{ H}^2 \text{ k}^5 \text{ U}^3 + \text{H}^4 k}^7 \text{ U}^3 \right)}{6 \left( 3 + \text{H}^2 \text{ k}^2 \right)^2} \right) dt^3 - \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( 18 \text{ g H k}^3 \text{ U+9 g H}^3 \text{ k}^5 \text{ U-18 k}^3 \text{ U}^3 - 3 \text{ H}^2 \text{ k}^5 \text{ U}^3 + \text{H}^4 k}^7 \text{ U}^3 \right)}{6 \left( 3 + \text{H}^2 \text{ k}^2 \right)^2} \right] dt^3 - \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + 1 \right) \right)}{3 + \text{H}^2 k^2} \right] dt^3 - \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + 1 \right) \right)}{3 + \text{H}^2 k^2} \right) dt^3 - \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + 1 \right) \right)}{3 + \text{H}^2 k^2} \right) dt^3 - \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + 1 \right) \right)}{3 + \text{H}^2 k^2} \right) dt^3 - \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + 1 \right) \right)}{3 + \text{H}^2 k^2} \right) dt^3 - \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H^2 k^2)}} + \frac{i \left( \sqrt{3} \text{ k} \sqrt{\text{g H (3+H$$

$$\frac{1}{24} \left[ -\frac{i\sqrt{3}}{34H^2k^2} - ikU \right] dt^4 + O[dt]^5 \right] + \\ \frac{1}{(12}k^4 U dt - \frac{i(1gHk^4 + 2H^2k^2)^2)d^2}{34(34H^2k^2)} - ikU \right] dt^4 + O[dt]^5 \right] + \\ \frac{1}{(243K^2U 143H^2k^2U 32H^2k^2)d^2}{990(34H^2k^2)} + \frac{(21gHk^4 + 48gH^2k^2)gH^2k^2 - 04kU^2 3H^2kU^3 6H^2k^2)d^2}{1920(34H^2k^2)^2} - \frac{1}{1920(34H^2k^2)^2} \\ \frac{ik^2 \left( 1633g H U + 1348g H^3 k^2 U + 241g H^3 k^4 U - 1593 U^3 - 678 H^2 k^2 U^3 + 47 H^4 k^4 U^3 + 32 H^6 k^6 U^3 \right)}{dt^3 + O[dt]^3} dt^4 + O[dt]^5 \right) d$$

Out[266]= Eerr || \left( \begin{array}{cc}  $\label{left} $\left(\frac{U + \sqrt{2} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{2} + 3 + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4} + \sqrt{4}}{h^2 k^2 + 3 + \sqrt{4}}\right) \left(\frac{U + \sqrt{4}}{h^2 k^2 + 3 +$  $k^2 + \sqrt{3} \sqrt{3} + \frac{1}{4} \left( H^2 k^2 + 3\right) + \left($ k^7-3 H^2 U^3 k^5+9 g H^3 U k^5-18 U^3 k^3+18 g H U k^3\right)\{6 \left(H^2 k^2+3\right)^2\}-\frac{1}{6}  $\left(-i U k - \frac{1}{sqrt}3\right) + \left(\frac{H^2 k^2 + 3\right) k}{H^2 k^2 + 3\right)$  $\label{left} $\operatorname{dt}^3-\frac{1}{24} \left(-i U k-\frac{3} \right) + \left(\frac{4h^2 k^2+3\right)^4 k}{4h^2 k^2+3\right)^4}.$  $\label{eq:linear_continuous_con$ g H k^5\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)} - \frac{\\left(U \\left(H^4 U^2 k^{10})+6 g H^3 k^8-9)}{1} + (10)  $U^2 k^6 + 15 g H k^6 \left( \frac{1}{2} + \frac{1}{2} +$ \text{dx}^3+\left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 H^4 U^2 k^{10}+145 g H^3 k^8+94 H^2 U^2 k^8-486 U^2 k^6+531 g H k^6\right) \text{dt}^2}{1920 \left(H^2 k^2+3\right)^2}-\frac{i k^7 \left(32 H^6 U^3 k^6+47 H^4 U^3 k^4+241 H^4 U^5 k^4+241 H^4 U^5 k^4+241 H^4 U^5 k^4+241 H^4 U^5 k^5+41 H^5 U^5 k^5+41 H g H^5 U k^4-678 H^2 U^3 k^2+1348 g H^3 U k^2-1593 U^3+1683 g H U\right) \text{dt}^3}{1920 \left(H^2  $k^2+3\right)^3+O\left(text\left(dt\right)^5\right)\right) \text\left(dx\right)^4+O\left(text\left(dx\right)^5\right) \ \& \left(-\frac{3 i k}{2}\right)^2$  $k^3+g H k^3\right) \left( \frac{dt}^3}{2 \left( \frac{k^2+3\right)^2}{+O\left( \frac{dt}^5\right) \right)} \left( \frac{k^2+3\right)^2}{+O\left( \frac{dt}^5\right) \left( \frac{dt}^5\right)$  $k^5 U \text{dt}^2{4 \left(\frac{H^2 k^2+3\right)}}+\frac{1}{U^2 k^8-6 U^2 k^8-6 U^2 k^6-g H k^6\right)}$  $\label{left(h^2 k^2+3\right)} $$\left(\frac{dt}^5\right)\right) \cdot \left(\frac{dt}^3}{8 \left(\frac{dx}^3+\left(\frac{dx}^3\right)}{1-\frac{dx}^3}\right)} $$$  $\left(49 \text{ H}^2 \text{ k}^7 + 243 \text{ k}^5\right) \left(44\right) \left(44\right) \left(44\right) \left(45\right) \left(45\right$ k^8+531 k^6\right) U \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(241 H^4 U^2)}{2} k^{11}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7\right) \text{dt}^3}{1920  $\left(H^2 k^2+3\right)^3+O\left(\frac{dt}^5\right)\right) \left(\frac{dt}^3+O\left(\frac{dt}^5\right)\right)$  $\left(\left(\frac{k^2 H^3}{3} + H\right) - g H \right) \left(\frac{k^2 H^3}{3} + H\right) - i g H \right) \left(\frac{k^2 H^3}{3} + H\right) - i g H \left(\frac{k^2 H^3}{3} + H\right) - i g$ k^5+6 g H^3 U^2 k^5-9 U^4 k^3+3 g^2 H^2 k^3+6 g H U^2 k^3\right) \text{dt}^3}{2 \left(H^2  $k^2+3\right)^2+O\left(\frac{dt}^5\right)^1+\left(\frac{1}{12}gHk^4\left(\frac{dt}-\frac{dt}{12}gHk^3\right)^2\right)^2+O\left(\frac{dt}{12}gHk^3\right)^2$  $k^7-3 U^3 k^5+6 g H U k^5 \right) \left( \frac{k^2}{12} \left( \frac{k^2 + 3 \right)}{10} \right) - \frac{k^5 U^2 k^{10}+2}{10}$ g^2 H^4 k^8-6 H^2 U^4 k^8+18 g H^3 U^2 k^8-18 U^4 k^6+6 g^2 H^2 k^6+24 g H U^2 k^6\right)  $\label{left} $$ \left(\frac{dt}^3}{24 \left(\frac{h^2 k^2+3\right)^2}+O\left(\frac{dt}^5\right)\right) \operatorname{left}(\frac{dt}^5\right) \left(\frac{dt}^3+\left(\frac{dt}^3+\left(\frac{dt}^3\right)\right)\right) }{\left(\frac{dt}^3}\right) \left(\frac{dt}^3+\frac{dt}^3\right)}$ g H<sup>5</sup> k<sup>9</sup>+192 g H<sup>3</sup> k<sup>7</sup>-49 H<sup>2</sup> U<sup>2</sup> k<sup>7</sup>-243 U<sup>2</sup> k<sup>5</sup>+288 g H k<sup>5</sup>\right) \text{dt}}{960 \left(H<sup>2</sup> k^2+3\right)^2}+\frac{\left(64 g H^5 U k^{10}-145 H^2 U^3 k^8+384 g H^3 U k^8-531 U^3 k^6+576 g H U k^6\right) \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(288 g H^7 U^2 k^{13}+241 g^2)}  $\text{H}^6 \text{ k}^{11} - 723 \text{ H}^4 \text{ U}^4 \text{ k}^{11} + 2592 \text{ g} \text{ H}^5 \text{ U}^2 \text{ k}^{11} + 1542 \text{ g}^2 \text{ H}^4 \text{ k}^9 - 4626 \text{ H}^2 \text{ U}^4 \text{ k}^9 + 7194 \text{ H}^6 \text{ k}^{11} + 1542 \text{ g}^2 \text{ H}^4 \text{ k}^9 - 4626 \text{ H}^2 \text{ U}^4 \text{ k}^9 + 7194 \text{ H}^6 \text{ k}^9 + 7194 \text{$ g H^3 U^2 k^9-7371 U^4 k^7+2457 g^2 H^2 k^7+5454 g H U^2 k^7\right) \text{dt}^3}{5760 \left(H^2  $\label{left} $$k^2+3\right)^3}+O\left(\frac{dt}^5\right)^3+O\left(\frac{dt}$  $\left( \frac{3}{k \cdot \frac{4}{k}} \right) + \left( \frac{3}{k \cdot \frac{4}{k}} \right) - 3 k \left( \frac{4}{k} \right) + \left( \frac{3}{k \cdot \frac{4}{k}} \right)$ \left(H^4 U^3 k^7+15 H^2 U^3 k^5+9 g H^3 U k^5+36 U^3 k^3+36 g H U k^3\right)\{6 \left(H^2  $k^2+3\right)^2-\frac{1}{6}\left(i U k-\frac{1}{3}\right)^2-\frac{1}{6}\left(i U k-\frac{1}{3}\right)^2$  $\label{left} $$k^2+3\right\right) \operatorname{left}(-i\ U\ k-\frac{i\ \sqrt{3}\ \sqrt{4}\ H\ (H^2)}{24} \operatorname{left}(-i\ U\ k-\frac{i\ \sqrt{3}\ V\ (H^2)}{24} \operatorname{left}(-i$  $k^2+3\right) k_{H^2 k^2+3}\right) + \label{eq:k^2+3}\right) + \label{eq:k^2+3}$ k^4 U \text{dt}-\frac{i k^5 \left(2 H^2 k^2 U^2+12 U^2+3 g H\right) \text{dt}\^2}{24 \left(H^2 k^2+3\right)}-\frac{\left(H^4 U^3 k^{10}+12 H^2 U^3 k^8+6 g H^3 U k^8+27 U^3 k^6+21 g H U  $k^6 + k^6 + k^3 + 24 \left(\frac{dt}^3}{24 \left(\frac{dt}^3}{24 \left(\frac{dt}^5\right)} \right) \left(\frac{dt}^3 + \frac{dt}^3}{24 \left(\frac{dt}^5\right)} \right) \left(\frac{dt}^5 + \frac{dt}^5}{24 \left(\frac{dt}^5\right)} \right) \left(\frac{dt}^5 + \frac{dt}^5$ 

 $\label{left} $$ \left(32\ H^4\ k^9+241\ H^2\ k^7+531\ k^5\right)$ U \left(44\right)^{960}\left(H^2\ k^2+3\right)^2+\frac{1}{64} H^2\ k^7+531\ k^5\right)$ U \left(44\right)^{960} \left(H^2\ k^2+3\right)^2+\frac{1}{64} H^2\ k^7+531\ k^7+531\$  $\label{left} $$\left(H^2 k^2 + 3\right)^2 - \frac{i \left(32 H^6 U^3 k^{13} + 529 H^4 U^3 k^{11} + 241 g H^5 U\right)^2}{h^6 U^3 k^6 U^3 k$  $\label{left} $$\left(H^2 k^2+3\right)^3+O\left(\frac{dt}^5\right)\right) \leq k^2+3\left(\frac{dx}^4+O\left(\frac{dx}^5\right)\right) \\$ 

\end{array} \right)

In[267]:=