In[1]:=
```
MA = k * x / (2 * Sin[k * x / 2])
RA = Exp[I * k * x / 2] * k * x / (2 * Sin[k * x / 2])
GA = k * x / ((H + H^3 / 3 * k^2) * Exp[-I * k * x / 2] * (2 * Sin[k * x / 2]))
FnnA = 0
FnGA = I * k / (1 + H^2 * k^2 / 3)
FGnA = g * H * I * k
FGGA = 0
FmatA = {{FnnA, FnGA}, {FGnA, FGGA}}
Eigenvalues[FmatA]
```

Out[1]= $\frac{1}{2} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$

Out[2]= $\frac{1}{2} e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$

Out[3]= $\frac{e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right)}$

Out[4]= 0

Out[5]= $\frac{i k}{1 + \frac{H^2 k^2}{3}}$

Out[6]= $i g H k$

Out[7]= 0

Out[8]= $\left\{\left\{0, \frac{i k}{1 + \frac{H^2 k^2}{3}}\right\}, \{i g H k, 0\}\right\}$

Out[9]= $\left\{-\frac{i \sqrt{3}\ k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3}\ k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}\right\}$

In[10]:=
```
M = 1
Series[M - MA, {x, 0, 10}]
```

Out[10]= 1

Out[11]= $-\frac{k^2 x^2}{24} - \frac{7 k^4 x^4}{5760} - \frac{31 k^6 x^6}{967\,680} - \frac{127 k^8 x^8}{154\,828\,800} - \frac{73 k^{10} x^{10}}{3\,503\,554\,560} + O[x]^{11}$

In[12]:=
```
Rm = 1 + I * Sin[k * x] / 2
Series[Rm - RA, {x, 0, 10}]
Rp = Exp[I * k * x] * (1 - I * Sin[k * x] / 2)
Series[Rp - RA, {x, 0, 10}]
Ru = (1 + Exp[I * k * x]) / 2
Series[Ru - Exp[I * k * x / 2], {x, 0, 10}]
```

Out[12]= $1 + \frac{1}{2}\, i\, \text{Sin}[k\, x]$

Out[13]= $\frac{k^2\, x^2}{12} - \frac{1}{12}\, i\, k^3\, x^3 + \frac{k^4\, x^4}{720} + \frac{1}{240}\, i\, k^5\, x^5 + \frac{k^6\, x^6}{30\,240} - \frac{i\, k^7\, x^7}{10\,080} + \frac{k^8\, x^8}{1\,209\,600} + \frac{i\, k^9\, x^9}{725\,760} + \frac{k^{10}\, x^{10}}{47\,900\,160} + O[x]^{11}$

Out[14]= $e^{i\, k\, x} \left(1 - \frac{1}{2}\, i\, \text{Sin}[k\, x]\right)$

Out[15]= $\frac{k^2\, x^2}{12} + \frac{1}{6}\, i\, k^3\, x^3 - \frac{89\, k^4\, x^4}{720} - \frac{7}{120}\, i\, k^5\, x^5 + \frac{631\, k^6\, x^6}{30\,240} + \frac{31\, i\, k^7\, x^7}{5040} - \frac{1889\, k^8\, x^8}{1\,209\,600} - \frac{127\, i\, k^9\, x^9}{362\,880} + \frac{481\, k^{10}\, x^{10}}{6\,842\,880} + O[x]^{11}$

Out[16]= $\frac{1}{2}\left(1 + e^{i\, k\, x}\right)$

Out[17]= $-\frac{k^2\, x^2}{8} - \frac{1}{16}\, i\, k^3\, x^3 + \frac{7\, k^4\, x^4}{384} + \frac{1}{256}\, i\, k^5\, x^5 - \frac{31\, k^6\, x^6}{46\,080} - \frac{i\, k^7\, x^7}{10\,240} + \frac{127\, k^8\, x^8}{10\,321\,920} + \frac{17\, i\, k^9\, x^9}{12\,386\,304} - \frac{73\, k^{10}\, x^{10}}{530\,841\,600} + O[x]^{11}$

In[18]:=
```
Gold = H - H^3 / 3 * (2 * Cos[k * x] - 2) / x^2
G = Ru / Gold
Series[G, {x, 0, 3} ]
Series[GA, {x, 0, 3} ]
Series[G - GA, {x, 0, 5}]
```

Out[18]= $$H - \frac{H^3 \, (-2 + 2 \, \text{Cos}[k \, x])}{3 \, x^2}$$

Out[19]= $$\frac{1 + e^{i \, k \, x}}{2 \left(H - \frac{H^3 \, (-2+2 \, \text{Cos}[k \, x])}{3 \, x^2}\right)}$$

Out[20]= $$\frac{1}{H + \frac{H^3 \, k^2}{3}} + \frac{i \, k \, x}{2 \left(H + \frac{H^3 \, k^2}{3}\right)} + \frac{\left(-9 \, k^2 - 2 \, H^2 \, k^4\right) x^2}{4 \, H \left(3 + H^2 \, k^2\right)^2} - \frac{i \left(6 \, k^3 + H^2 \, k^5\right) x^3}{8 \, H \left(3 + H^2 \, k^2\right)^2} + O[x]^4$$

Out[21]= $$\frac{1}{H + \frac{H^3 \, k^2}{3}} + \frac{i \, k \, x}{2 \left(H + \frac{H^3 \, k^2}{3}\right)} - \frac{k^2 \, x^2}{12 \left(H + \frac{H^3 \, k^2}{3}\right)} + O[x]^4$$

Out[22]= $$\frac{\left(-6 \, k^2 - H^2 \, k^4\right) x^2}{4 \, H \left(3 + H^2 \, k^2\right)^2} - \frac{i \left(6 \, k^3 + H^2 \, k^5\right) x^3}{8 \, H \left(3 + H^2 \, k^2\right)^2} + \frac{\left(144 \, k^4 + 45 \, H^2 \, k^6 + 4 \, H^4 \, k^8\right) x^4}{240 \, H \left(3 + H^2 \, k^2\right)^3} - \frac{i \left(-54 \, k^5 + H^4 \, k^9\right) x^5}{480 \, H \left(3 + H^2 \, k^2\right)^3} + O[x]^6$$

In[23]:=
```
fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * G;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;

Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]
```

Out[27]= $$-\frac{1}{2\,x}\left(1-e^{-i\,k\,x}\right)\sqrt{g\,H}\,\left(-1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)$$

Out[28]= $$\frac{1}{8}\sqrt{g\,H}\;k^4\,x^3-\frac{1}{48}\left(\sqrt{g\,H}\;k^6\right)x^5+O[x]^6$$

Out[29]= $$\frac{\left(1-e^{-i\,k\,x}\right)\left(1+e^{i\,k\,x}\right)H}{2\,x\left(H-\frac{H^3\,(-2+2\,\text{Cos}[k\,x])}{3\,x^2}\right)}$$

Out[30]= $$-\frac{i\left(6\,k^3+H^2\,k^5\right)x^2}{4\left(3+H^2\,k^2\right)^2}-\frac{i\left(-54\,k^5+H^4\,k^9\right)x^4}{240\left(3+H^2\,k^2\right)^3}+O[x]^6$$

Out[31]= $$-\frac{1}{2\,x}\left(1-e^{-i\,k\,x}\right)\sqrt{g\,H}\,\left(-1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)$$

Out[32]= $$\frac{1}{8}\sqrt{g\,H}\;k^4\,x^3-\frac{1}{48}\left(\sqrt{g\,H}\;k^6\right)x^5+O[x]^6$$

Out[33]= $$\frac{\left(1-e^{-i\,k\,x}\right)g\,H\left(1+e^{i\,k\,x}\left(1-\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)+\frac{1}{2}\,i\,\text{Sin}[k\,x]\right)}{2\,x}$$

Out[34]= $$\frac{1}{12}\,i\,g\,H\,k^3\,x^2-\frac{13}{240}\,i\,g\,H\,k^5\,x^4+O[x]^6$$