1 Elliptic Equation

The linearised elliptic equation is

$$G = Hv - \frac{H^3}{3}\left(\frac{\partial^2 v}{\partial x^2}\right)$$

Taking the weak version of this we get that

$$\int_\Omega Gv \ dx = H \int_\Omega vv \ dx - \frac{H^3}{3} \int_\Omega \frac{\partial^2 v}{\partial x^2} v \ dx$$

$$\int_\Omega Gv \ dx = H \int_\Omega vv \ dx + \frac{H^3}{3} \int_\Omega \frac{\partial v}{\partial x}\frac{\partial v}{\partial x} \ dx$$

In particular for the basis function ϕ_j we must have

$$\int_\Omega G\phi_j \ dx = H \int_\Omega v\phi_j \ dx + \frac{H^3}{3} \int_\Omega \frac{\partial v}{\partial x}\frac{\partial (\phi_j)}{\partial x} \ dx$$

We use the FEM discretisation from []

$$G = \sum_j G^+_{j-1/2}\psi^+_{j-1/2} + G^-_{j+1/2}\psi^-_{j+1/2}$$

and

$$v = \sum_j v_{j-1/2}\phi_{j-1/2} + v_j\phi_j + v_{j+1/2}\phi_{j+1/2} \tag{1}$$