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In[1]:= MA = k * x / (2 * Sin[k * x / 2])
RA = Exp[I * k * x / 2] * k * x / (2 * Sin[k * x / 2])
GA = k * x / ((H + H^3 / 3 * k^2) * Exp[-I * k * x / 2] * (2 * Sin[k * x / 2]))
FnnA = 0
FnGA = I * k / (1 + H^2 * k^2 / 3)
FGnA = g * H * I * k
FGGA = 0
FmatA = {{FnnA, FnGA}, {FGnA, FGGA}}
Eigenvalues[FmatA]

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$$\text{Out[1]} = \frac{1}{2} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$$

$$\text{Out[2]} = \frac{1}{2} e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]$$

$$\text{Out[3]} = \frac{e^{\frac{i k x}{2}} k x \operatorname{Csc}\left[\frac{k x}{2}\right]}{2 \left(H + \frac{H^3 k^2}{3}\right)}$$

$$\text{Out[4]} = 0$$

$$\text{Out[5]} = \frac{i k}{1 + \frac{H^2 k^2}{3}}$$

$$\text{Out[6]} = i g H k$$

$$\text{Out[7]} = 0$$

$$\text{Out[8]} = \left\{ \left\{ 0, \frac{i k}{1 + \frac{H^2 k^2}{3}} \right\}, \{i g H k, 0\} \right\}$$

$$\text{Out[9]} = \left\{ -\frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2}, \frac{i \sqrt{3} k \sqrt{3 g H + g H^3 k^2}}{3 + H^2 k^2} \right\}$$

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In[10]:= M2 = 1
Series[M2 - MA, {x, 0, 10}]

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$$\text{Out[10]} = 1$$

$$\text{Out[11]} = -\frac{k^2 x^2}{24} - \frac{7 k^4 x^4}{5760} - \frac{31 k^6 x^6}{967680} - \frac{127 k^8 x^8}{154828800} - \frac{73 k^{10} x^{10}}{3503554560} + O[x]^{11}$$

In[12]:= **Rm = (1 + I * Sin[k * x] / 2)**

Series[Rm - RA, {x, 0, 10}]

Rp = Exp[I * k * x] * (1 - I * Sin[k * x] / 2)

Series[Rp - RA, {x, 0, 10}]

Out[12]= $1 + \frac{1}{2} i \sin[k x]$

Out[13]= $\frac{k^2 x^2}{12} - \frac{1}{12} i k^3 x^3 + \frac{k^4 x^4}{720} + \frac{1}{240} i k^5 x^5 + \frac{k^6 x^6}{30240} - \frac{i k^7 x^7}{10080} + \frac{k^8 x^8}{1209600} + \frac{i k^9 x^9}{725760} + \frac{k^{10} x^{10}}{47900160} + O[x]^{11}$

Out[14]= $e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right)$

Out[15]= $\frac{k^2 x^2}{12} + \frac{1}{6} i k^3 x^3 - \frac{89 k^4 x^4}{720} - \frac{7}{120} i k^5 x^5 + \frac{631 k^6 x^6}{30240} + \frac{31 i k^7 x^7}{5040} - \frac{1889 k^8 x^8}{1209600} - \frac{127 i k^9 x^9}{362880} + \frac{481 k^{10} x^{10}}{6842880} + O[x]^{11}$

In[16]:= **GLHS = x / 6 * (Rp + Rm)**

GRHSp1 = -Exp[-I * k * x / 2] + 2 + 4 * Exp[I * k * x / 2] +

Exp[I * k * x] * (4 * Exp[-I * k * x / 2] + 2 - Exp[I * k * x / 2])

GRHSp1 = GRHSp1 / Exp[I * k * x / 2]

GRHSp1 = Expand[GRHSp1]

GRHSp1 = ExpToTrig[GRHSp1]

Out[16]= $\frac{1}{6} x \left(1 + e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right) + \frac{1}{2} i \sin[k x] \right)$

Out[17]= $2 - e^{-\frac{1}{2} i k x} + 4 e^{\frac{i k x}{2}} + e^{i k x} \left(2 + 4 e^{-\frac{1}{2} i k x} - e^{\frac{i k x}{2}} \right)$

Out[18]= $e^{-\frac{1}{2} i k x} \left(2 - e^{-\frac{1}{2} i k x} + 4 e^{\frac{i k x}{2}} + e^{i k x} \left(2 + 4 e^{-\frac{1}{2} i k x} - e^{\frac{i k x}{2}} \right) \right)$

Out[19]= $8 + 2 e^{-\frac{1}{2} i k x} + 2 e^{\frac{i k x}{2}} - e^{-i k x} - e^{i k x}$

Out[20]= $8 + 4 \cos\left[\frac{k x}{2}\right] - 2 \cos[k x]$

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In[21]:= GRHSp2 = Exp[-I * k * x / 2] - 8 + 7 * Exp[I * k * x / 2] +
          Exp[I * k * x] * (7 * Exp[-I * k * x / 2] - 8 + Exp[I * k * x / 2])
GRHSp2 = GRHSp2 / Exp[I * k * x / 2]
GRHSp2 = Expand[GRHSp2]
GRHSp2 = ExpToTrig[GRHSp2]
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$$\text{Out[21]} = -8 + e^{-\frac{1}{2} i k x} + 7 e^{\frac{i k x}{2}} + e^{i k x} \left(-8 + 7 e^{-\frac{1}{2} i k x} + e^{\frac{i k x}{2}} \right)$$

$$\text{Out[22]} = e^{-\frac{1}{2} i k x} \left(-8 + e^{-\frac{1}{2} i k x} + 7 e^{\frac{i k x}{2}} + e^{i k x} \left(-8 + 7 e^{-\frac{1}{2} i k x} + e^{\frac{i k x}{2}} \right) \right)$$

$$\text{Out[23]} = 14 - 8 e^{-\frac{1}{2} i k x} - 8 e^{\frac{i k x}{2}} + e^{-i k x} + e^{i k x}$$

$$\text{Out[24]} = 14 - 16 \cos\left[\frac{k x}{2}\right] + 2 \cos[k x]$$

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In[25]:= GFEM2 = GLHS / (H * x / 30 * (GRHSp1) + H^3 / (9 * x) * GRHSp2)
Series[GFEM2, {x, 0, 3}]
Series[GA, {x, 0, 3}]
Series[GFEM2 - GA, {x, 0, 5}]
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$$\text{Out[25]} = \left(x \left(1 + e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right) \right) + \frac{1}{2} i \sin[k x] \right) \Bigg/ \left(6 \left(\frac{1}{30} H x \left(8 + 4 \cos\left[\frac{k x}{2}\right] - 2 \cos[k x] \right) + \frac{H^3 \left(14 - 16 \cos\left[\frac{k x}{2}\right] + 2 \cos[k x] \right)}{9 x} \right) \right)$$

$$\text{Out[26]} = \frac{3}{3 H + H^3 k^2} + \frac{3 i k x}{2 (3 H + H^3 k^2)} + \frac{(-18 k^2 - 5 H^2 k^4) x^2}{40 H (3 + H^2 k^2)^2} + \frac{i (12 k^3 + 5 H^2 k^5) x^3}{80 H (3 + H^2 k^2)^2} + O[x]^4$$

$$\text{Out[27]} = \frac{1}{H + \frac{H^3 k^2}{3}} + \frac{i k x}{2 \left(H + \frac{H^3 k^2}{3} \right)} - \frac{k^2 x^2}{12 \left(H + \frac{H^3 k^2}{3} \right)} + O[x]^4$$

$$\text{Out[28]} = \frac{(12 k^2 + 5 H^2 k^4) x^2}{40 H (3 + H^2 k^2)^2} + \frac{i (12 k^3 + 5 H^2 k^5) x^3}{80 H (3 + H^2 k^2)^2} + \frac{(-6651 k^4 - 4680 H^2 k^6 - 820 H^4 k^8) x^4}{4800 H (3 + H^2 k^2)^3} - \frac{i (6291 k^5 + 4410 H^2 k^7 + 770 H^4 k^9) x^5}{9600 H (3 + H^2 k^2)^3} + O[x]^6$$

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In[29]:= fnn = - Sqrt[g * H] / 2 * (Rp - Rm);
fng = H * GFEM2;
fgg = - Sqrt[g * H] / 2 * (Rp - Rm);
fgn = g * H * (Rp + Rm) / 2;
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Fnn = (1 - Exp[-I * k * x]) / x * fnn
Series[Fnn - FnnA, {x, 0, 5}]
Fng = (1 - Exp[-I * k * x]) / x * fng
Series[Fng - FnGA, {x, 0, 5}]
Fgg = (1 - Exp[-I * k * x]) / x * fgg
Series[Fgg - FGGA, {x, 0, 5}]
Fgn = (1 - Exp[-I * k * x]) / x * fgn
Series[Fgn - FGnA, {x, 0, 5}]
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$$\text{Out[33]} = - \frac{(1 - e^{-i k x}) \sqrt{g H} \left(-1 + e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right) - \frac{1}{2} i \sin[k x] \right)}{2 x}$$

$$\text{Out[34]} = \frac{1}{8} \sqrt{g H} k^4 x^3 - \frac{1}{48} \left(\sqrt{g H} k^6 \right) x^5 + O[x]^6$$

$$\text{Out[35]} = \frac{(1 - e^{-i k x}) H \left(1 + e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right) + \frac{1}{2} i \sin[k x] \right)}{6 \left(\frac{1}{30} H x \left(8 + 4 \cos\left[\frac{k x}{2}\right] - 2 \cos[k x] \right) + \frac{H^3 \left(14 - 16 \cos\left[\frac{k x}{2}\right] + 2 \cos[k x] \right)}{9 x} \right)}$$

$$\text{Out[36]} = \frac{i \left(12 k^3 + 5 H^2 k^5 \right) x^2}{40 \left(3 + H^2 k^2 \right)^2} - \frac{i \left(6291 k^5 + 4410 H^2 k^7 + 770 H^4 k^9 \right) x^4}{4800 \left(3 + H^2 k^2 \right)^3} + O[x]^6$$

$$\text{Out[37]} = - \frac{(1 - e^{-i k x}) \sqrt{g H} \left(-1 + e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right) - \frac{1}{2} i \sin[k x] \right)}{2 x}$$

$$\text{Out[38]} = \frac{1}{8} \sqrt{g H} k^4 x^3 - \frac{1}{48} \left(\sqrt{g H} k^6 \right) x^5 + O[x]^6$$

$$\text{Out[39]} = \frac{(1 - e^{-i k x}) g H \left(1 + e^{i k x} \left(1 - \frac{1}{2} i \sin[k x] \right) + \frac{1}{2} i \sin[k x] \right)}{2 x}$$

$$\text{Out[40]} = \frac{1}{12} i g H k^3 x^2 - \frac{13}{240} i g H k^5 x^4 + O[x]^6$$