In[1]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx / 2, xj + dx / 2}] / (dx);
qjnbar = qjbar /. t → tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x → xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx / 2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / 3 * k^2;
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca;
fn1Gca0A = fn1A /. Gca → 0;
fn1eca0A = fn1A /. eca → 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca /. Gh → RA * Gca;
fG1Gca0A = fG1A /. Gca → 0;
fG1eca0A = fG1A /. eca → 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;



FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
```

```
EA = Simplify[ {{1, 0}, {0, 1}} + MatA];

Simplify[1 + Eigenvalues[MatA]] /. (k (-Sqrt[3] Sqrt[g H (3 + H^2 k^2)] + (3 + H^2 k^2) U))/(3 + H^2 k^2) -> -w /.

 (k (Sqrt[3] Sqrt[g H (3 + H^2 k^2)] + (3 + H^2 k^2) U))/(3 + H^2 k^2) -> -w;

wAp = U * k + (Sqrt[3] k Sqrt[g H (3 + H^2 k^2)])/(3 + H^2 k^2);

wAm = U * k - (Sqrt[3] k Sqrt[g H (3 + H^2 k^2)])/(3 + H^2 k^2);
```

In[38]:=
```
M = (26 - 2 * Cos[k * dx]) / 24;
Merr = Series[M - MA, {dx, 0, 10}];
Rm = (5 - Exp[-I * k * dx] + 2 * Exp[I * k * dx]) / 6;
Rmerr = Series[Rm - RA, {dx, 0, 10}];
Rp = Exp[I * k * dx] * (5 + 2 * Exp[-I * k * dx] - Exp[I * k * dx]) / 6;
Rperr = Series[Rp - RA, {dx, 0, 10}];
Ru = (-Exp[-I * k * dx] + 9 * Exp[I * k * dx] - Exp[2 * I * k * dx] + 9) / 16;
Ruerr = Series[Ru - Exp[I * k * dx / 2], {dx, 0, 10}];
Gold = H - H^3 / 3 * (32 * Cos[k * dx] - 2 * Cos[2 * k * dx] - 30) / (12 * dx^2);
GG2 = M * Ru / (Gold);
GG2err = Series[GG2 - GGA, {dx, 0, 5}];
Gn2 = -M * Ru * U / (Gold);
Gn2err = Series[Gn2 - GnA, {dx, 0, 5}];

Text[Row[{"M  ||   ", M}]]
Text[Row[{"M  ||   ", TeXForm[M]}]]
Text[Row[{"M error  ||   ", TeXForm[Merr]}]]
Text[Row[{"M error  ||   ", Merr}]]
Text[" "]
Text[Row[{"Rm  ||   ", Rm}]]
Text[Row[{"Rm  ||   ", TeXForm[Rm]}]]
Text[Row[{"Rm error  ||   ", Rmerr}]]
Text[Row[{"Rm error  ||   ", TeXForm[Rmerr]}]]
Text[" "]
Text[Row[{"Rp  ||   ", Rp}]]
Text[Row[{"Rp  ||   ", TeXForm[Rp]}]]
Text[Row[{"Rp error  ||   ", Rperr}]]
Text[Row[{"Rp error  ||   ", TeXForm[Rperr]}]]
Text[" "]
Text[Row[{"GG2  ||   ", GG2}]]
Text[Row[{"GG2  ||   ", TeXForm[GG2]}]]
Text[Row[{"GG2 error  ||   ", GG2err}]]
Text[Row[{"GG2 error  ||   ", TeXForm[GG2err]}]]
Text[" "]
Text[Row[{"Gn2  ||   ", Gn2}]]
Text[Row[{"Gn2  ||   ", TeXForm[Gn2]}]]
Text[Row[{"Gn2 error  ||   ", Gn2err}]]
Text[Row[{"Gn2 error  ||   ", TeXForm[Gn2err]}]]
```

Out[51]= M || $\frac{1}{24}(26 - 2\,\mathrm{Cos}[\mathrm{dx\ k}])$

Out[52]= M || \frac{1}{24} (26−2 \cos (\text{dx} k))

Out[53]= M error ||

$$-\frac{3 \text{dx}^4 k^4}{640}+\frac{3 \text{dx}^6 k^6}{35840}-\frac{149 \text{dx}^8 k^8}{51609600}+\frac{29 \text{dx}^{10} k^{10}}{13624934400}+O\left(\text{dx}^{11}\right)$$

Out[54]= M error || $-\frac{3\,k^4\,dx^4}{640}+\frac{3\,k^6\,dx^6}{35\,840}-\frac{149\,k^8\,dx^8}{51\,609\,600}+\frac{29\,k^{10}\,dx^{10}}{13\,624\,934\,400}+O[dx]^{11}$

Out[55]=

Out[56]= Rm || $\frac{1}{6}\left(5-e^{-i\,dx\,k}+2\,e^{i\,dx\,k}\right)$

Out[57]= Rm || $\frac{1}{6} \left(-e^{-i \text{dx} k}+2 e^{i \text{dx} k}+5\right)$

Out[58]= Rm error || $-\frac{1}{12}\,i\,k^3\,dx^3+\frac{k^4\,dx^4}{120}+\frac{1}{240}\,i\,k^5\,dx^5-\frac{k^6\,dx^6}{5040}-\frac{i\,k^7\,dx^7}{10\,080}+\frac{k^8\,dx^8}{201\,600}+\frac{i\,k^9\,dx^9}{725\,760}-\frac{k^{10}\,dx^{10}}{39\,916\,800}+O[dx]^{11}$

Out[59]= Rm error ||

$$-\frac{1}{12} i \text{dx}^3 k^3+\frac{\text{dx}^4 k^4}{120}+\frac{1}{240} i \text{dx}^5 k^5-\frac{\text{dx}^6 k^6}{5040}-\frac{i \text{dx}^7 k^7}{10080}+\frac{\text{dx}^8 k^8}{201600}+\frac{i \text{dx}^9 k^9}{725760}-\frac{\text{dx}^{10} k^{10}}{39916800}+O\left(\text{dx}^{11}\right)$$

Out[60]=

Out[61]= Rp || $\frac{1}{6}\,e^{i\,dx\,k}\left(5+2\,e^{-i\,dx\,k}-e^{i\,dx\,k}\right)$

Out[62]= Rp || $\frac{1}{6} e^{i \text{dx} k} \left(2 e^{-i \text{dx} k}-e^{i \text{dx} k}+5\right)$

Out[63]= Rp error || $\frac{1}{12}\,i\,k^3\,dx^3-\frac{3\,k^4\,dx^4}{40}-\frac{3}{80}\,i\,k^5\,dx^5+\frac{23\,k^6\,dx^6}{1680}+\frac{41\,i\,k^7\,dx^7}{10\,080}-\frac{209\,k^8\,dx^8}{201\,600}-\frac{169\,i\,k^9\,dx^9}{725\,760}+\frac{89\,k^{10}\,dx^{10}}{1\,900\,800}+O[dx]^{11}$

Out[64]= Rp error ||

$$\frac{1}{12} i \text{dx}^3 k^3-\frac{3 \text{dx}^4 k^4}{40}-\frac{3}{80} i \text{dx}^5 k^5+\frac{23 \text{dx}^6 k^6}{1680}+\frac{41 i \text{dx}^7 k^7}{10080}-\frac{209 \text{dx}^8 k^8}{201600}-\frac{169 i \text{dx}^9 k^9}{725760}+\frac{89 \text{dx}^{10} k^{10}}{1900800}+O\left(\text{dx}^{11}\right)$$

Out[65]=

Out[66]= GG2 || $\frac{\left(9-e^{-i\,dx\,k}+9\,e^{i\,dx\,k}-e^{2\,i\,dx\,k}\right)(26-2\,\text{Cos}[dx\,k])}{384\left(H-\frac{H^3\,(-30+32\,\text{Cos}[dx\,k]-2\,\text{Cos}[2\,dx\,k])}{36\,dx^2}\right)}$

Out[67]= GG2 || $\frac{\left(-e^{-i \text{dx} k}+9 e^{i \text{dx} k}-e^{2 i \text{dx} k}+9\right) (26-2 \cos (\text{dx} k))}{384 \left(H-\frac{H^3 (32 \cos (\text{dx} k)-2 \cos (2 \text{dx} k)-30)}{36 \text{dx}^2}\right)}$

Out[68]= GG2 error || $\frac{\left(-243\,k^4-49\,H^2\,k^6\right)dx^4}{960\,H\left(3+H^2\,k^2\right)^2}-\frac{i\left(243\,k^5+49\,H^2\,k^7\right)dx^5}{1920\,H\left(3+H^2\,k^2\right)^2}+O[dx]^6$

Out[69]= GG2 error ||

$$\frac{\text{dx}^4 \left(-49 H^2 k^6-243 k^4\right)}{960 H \left(H^2 k^2+3\right)^2}-\frac{i \text{dx}^5 \left(49 H^2 k^7+243 k^5\right)}{1920 H \left(H^2 k^2+3\right)^2}+O\left(\text{dx}^6\right)$$

Out[70]=

Out[71]= Gn2 || $\frac{\left(9-e^{-i\,dx\,k}+9\,e^{i\,dx\,k}-e^{2\,i\,dx\,k}\right)U\,(-26+2\,Cos[dx\,k])}{384\left(H-\frac{H^3\,(-30+32\,Cos[dx\,k]-2\,Cos[2\,dx\,k])}{36\,dx^2}\right)}$

Out[72]= Gn2 || \frac{U \left(-e^{-i \text{dx} k}+9 e^{i \text{dx} k}-e^{2 i \text{dx} k}+9\right) (2 \cos (\text{dx} k)-26)}{384 \left(H-\frac{H^3 (32 \cos (\text{dx} k)-2 \cos (2 \text{dx} k)-30)}{36 \text{dx}^2}\right)}

Out[73]= Gn2 error || $\frac{\left(243\,k^4+49\,H^2\,k^6\right)U\,dx^4}{960\,H\left(3+H^2\,k^2\right)^2}+\frac{i\left(243\,k^5+49\,H^2\,k^7\right)U\,dx^5}{1920\,H\left(3+H^2\,k^2\right)^2}+O[dx]^6$

Out[74]= Gn2 error ||
\frac{\text{dx}^4 U \left(49 H^2 k^6+243 k^4\right)}{960 H \left(H^2 k^2+3\right)^2}+\frac{i \text{dx}^5 U \left(49 H^2 k^7+243 k^5\right)}{1920 H \left(H^2 k^2+3\right)^2}+O\left(\text{dx}^6\right)

In[75]:=
```
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 4}];
Fnn2TAr = Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 4}];
FnG2TAr = Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
     fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 4}];
FGn2TAr = Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 4}];
FGG2TAr = Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}];
```

```
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 + Fmat2.Fmat2.Fmat2 / 6;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep =
  Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 + EigvFmat2 * EigvFmat2 * EigvFmat2 / 6 ] /
   (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ];


Text[Row[{" -Sqrt(gH) < U  < Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn  ||   ", Kfnnp}]]
Text[Row[{"Fnn  ||   ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||   ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||   ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||   ", KfnGp}]]
Text[Row[{"FnG  ||   ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error  ||   ", FnG2TAr}]]
Text[Row[{"FnG error  ||   ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||   ", KfGnp}]]
Text[Row[{"FGn  ||   ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||   ", FGn2TAr}]]
Text[Row[{"FGn error  ||   ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||   ", KfGGp}]]
Text[Row[{"FGG  ||   ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||   ", FGG2TAr}]]
Text[Row[{"FGG error  ||   ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||   ", RKstepTayr}]]
Text[Row[{"Omega error  ||   ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||   ", EA}]]
Text[Row[{"EA  ||   ", TeXForm[EA]}]]
Text[Row[{"Eerr ||   ", Eerr}]]
Text[Row[{"Eerr ||   ", TeXForm[Eerr]}]]
```

Out[109]= $-\text{Sqrt(gH)} < U < \text{Sqrt(gH)}$

Out[110]=

Out[111]= $\text{Fnn} \;||\; \frac{1}{2}\left(2\,\text{Gnp}\,H + \text{Rpp}\left(-\sqrt{g\,H} + U\right) + \text{Rmp}\left(\sqrt{g\,H} + U\right)\right)$

Out[112]= Fnn || \frac{1}{2} \left(\text{Rmp} \left(\sqrt{g H}+U\right)+\text{Rpp} \left(U-\sqrt{g H}\right)+2 \text{Gnp} H\right)

Out[113]= $\text{Fnn error} \;||\; \left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + \frac{H^2\,k^3\,U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)} + O[dt]^5\right) +$
$\left(-\frac{1}{12}\left(\sqrt{g\,H}\;k^4\right)dt + O[dt]^5\right)dx^3 + \left(\frac{i\left(45\,k^5\,U+143\,H^2\,k^7\,U+32\,H^4\,k^9\,U\right)dt}{960\left(3+H^2\,k^2\right)^2} + O[dt]^5\right)dx^4 + O[dx]^5$

Out[114]= Fnn error ||
\left(-\frac{\text{dt}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 H^2 k^3 U w^2}{6 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 H^2 k^3 U w^3}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^3 \left(-\frac{1}{12} \left(\sqrt{g H} k^4\right) \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+O\left(\text{dx}^5\right)

Out[115]=

Out[116]= $\text{FnG} \;||\; \text{GGp}\,H$

Out[117]= FnG || \text{GGp} H

Out[118]= $\text{FnG error} \;||\; \left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)} + \frac{k\,w^3\,dt^4}{8\left(3+H^2\,k^2\right)} + O[dt]^5\right) + \left(\frac{i\left(243\,k^5+49\,H^2\,k^7\right)dt}{960\left(3+H^2\,k^2\right)^2} + O[dt]^5\right)dx^4 + O[dx]^5$

Out[119]= FnG error || \left(-\frac{3 \text{dt}^2 (k w)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2}{2 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k w^3}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(49 H^2 k^7+243 k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+O\left(\text{dx}^5\right)

Out[120]=

Out[121]= $\text{FGn} \;||\; \frac{1}{2}\left(g\,H\,(\text{Rmp} + \text{Rpp}) + \left(2\,\text{Gnp}\,H + \sqrt{g\,H}\;(\text{Rmp} - \text{Rpp})\right)U\right)$

Out[122]= FGn ||
\frac{1}{2} \left(U \left(\sqrt{g H} (\text{Rmp}-\text{Rpp})+2 \text{Gnp} H\right)+g H (\text{Rmp}+\text{Rpp})\right)

Out[123]= $\text{FGn error} \;||\; \left(-\frac{\left(k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w\right)dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + \frac{k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^3\,dt^4}{24\left(3+H^2\,k^2\right)} + O[dt]^5\right) +$
$\left(-\frac{1}{12}\left(\sqrt{g\,H}\;k^4\,U\right)dt + O[dt]^5\right)dx^3 + \left(\frac{i\left(288\,g\,H\,k^5+192\,g\,H^3\,k^7+32\,g\,H^5\,k^9-243\,k^5\,U^2-49\,H^2\,k^7\,U^2\right)dt}{960\left(3+H^2\,k^2\right)^2} + O[dt]^5\right)dx^4 + O[dx]^5$

Out[124]= FGn error ||

\left(-\frac{\text{dt}^2 \left(k w \left(g H^3 k^2+3 g H-3 U^2\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2 \left(g H^3 k^2+3 g H-3 U^2\right)}{6 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k w^3 \left(g H^3 k^2+3 g H-3 U^2\right)}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^3 \left(-\frac{1}{12} \left(\sqrt{g H} k^4 U\right) \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+O\left(\text{dx}^5\right)

Out[125]=

Out[126]= FGG || $\frac{1}{2}\left(\sqrt{g H}\ \text{Rmp}-\sqrt{g H}\ \text{Rpp}+(2\,\text{GGp}\,H+\text{Rmp}+\text{Rpp})\,U\right)$

Out[127]= FGG || \frac{1}{2} \left(\text{Rmp} \sqrt{g H}-\text{Rpp} \sqrt{g H}+U (2 \text{GGp} H+\text{Rmp}+\text{Rpp})\right)

Out[128]= FGG error || $\left(-\frac{k\left(6+H^2 k^2\right) U w\, dt^2}{2\left(3+H^2 k^2\right)}-\frac{i k\left(6+H^2 k^2\right) U w^2\, dt^3}{6\left(3+H^2 k^2\right)}+\frac{k\left(6+H^2 k^2\right) U w^3\, dt^4}{24\left(3+H^2 k^2\right)}+O[dt]^5\right)+$
$\left(-\frac{1}{12}\left(\sqrt{g H}\ k^4\right) dt+O[dt]^5\right) dx^3+\left(\frac{i\left(531 k^5 U+241 H^2 k^7 U+32 H^4 k^9 U\right) dt}{960\left(3+H^2 k^2\right)^2}+O[dt]^5\right) dx^4+O[dx]^5$

Out[129]= FGG error ||

\left(-\frac{\text{dt}^2 \left(k U w \left(H^2 k^2+6\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k U w^2 \left(H^2 k^2+6\right)}{6 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k U w^3 \left(H^2 k^2+6\right)}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^3 \left(-\frac{1}{12} \left(\sqrt{g H} k^4\right) \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(32 H^4 U k^9+241 H^2 U k^7+531 U k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+O\left(\text{dx}^5\right)

Out[130]=

Out[131]=

Out[132]= Omega error ||

$\left\{\left(-\frac{1}{24\left(3+H^2 k^2\right)^3} i k^4\left(\sqrt{3}\ \sqrt{g H\left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)\left(3 g\left(\sqrt{3}\ H \sqrt{g H\left(3+H^2 k^2\right)}+9 H U+3 H^3 k^2 U\right)+\right.\right.$
$\left.\left. U^2\left(H^4 k^4 U+9\left(\sqrt{3}\ \sqrt{g H\left(3+H^2 k^2\right)}+U\right)+3 k^2\left(\sqrt{3}\ \sqrt{g H^5\left(3+H^2 k^2\right)}+2 H^2 U\right)\right)\right) dt^3+$
$\frac{1}{30\left(3+H^2 k^2\right)^3} k^5\left(\sqrt{3}\ \sqrt{g H\left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)$
$\left(9 g^2 H^2+6 g H U\left(2\sqrt{3}\ \sqrt{g H\left(3+H^2 k^2\right)}+3\left(3+H^2 k^2\right) U\right)+\right.$
$\left. U^3\left(12\sqrt{3}\ \sqrt{g H\left(3+H^2 k^2\right)}+9 U+H^4 k^4 U+2 k^2\left(2\sqrt{3}\ \sqrt{g H^5\left(3+H^2 k^2\right)}+3 H^2 U\right)\right)\right) dt^4+$
$O[dt]^5\Bigg)+\Bigg(-\frac{i k^4\left(2 g H\left(3+H^2 k^2\right)+\sqrt{3}\ \sqrt{g H\left(3+H^2 k^2\right)}\ U\right)}{24\sqrt{g H}\ \left(3+H^2 k^2\right)}-\frac{1}{144\left(3+H^2 k^2\right)^{5/2}}$

$\left(k^7\left(6\sqrt{3}\ g^2 H^2\left(3+H^2 k^2\right)+9 U^3\left(5\sqrt{g H\left(3+H^2 k^2\right)}+\sqrt{3}\ U\right)+k^4 U^3\left(2\sqrt{g H^9\left(3+H^2 k^2\right)}+\right.\right.\right.$
$\left.\left.\left.\sqrt{3}\ H^4 U\right)+3 g H U\left(21\sqrt{g H\left(3+H^2 k^2\right)}+\sqrt{3}\ \left(27+15 H^2 k^2+2 H^4 k^4\right) U\right)+$
$3 k^2\left(6\sqrt{3 H^7\left(3+H^2 k^2\right)}\ U+7\sqrt{H^5\left(3+H^2 k^2\right)}\ U^3+2\sqrt{3}\ H^2 U^4\right)\right)\right) k^3$

$$3\,k^2\left(6\sqrt{g^2\,H^7\left(3+H^2\,k^2\right)}\;U+7\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^2+2\sqrt{3}\;H^2\,U^3\right)\Big)\Big)\,dt^2-$$

$$\frac{1}{144\left(3+H^2\,k^2\right)^{5/2}}i\,k^8\left(3\sqrt{3}\;g^2\,H^2\left(27+8\,H^2\,k^2\right)U+k^4\,U^4\left(2\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;+\sqrt{3}\;H^4\,U\right)+\right.$$

$$2\,g\,H\,U^2\left(72\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+\sqrt{3}\;\left(63+33\,H^2\,k^2+4\,H^4\,k^4\right)U\right)+$$

$$9\left(2\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}\;+6\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+\sqrt{3}\;U^5\right)+$$

$$\left.\left.6\,k^2\left(6\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U^2+4\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^4+\sqrt{3}\;H^2\,U^5\right)\right)dt^4+O[dt]^5\right)dx^3+$$

$$\left(-\frac{1}{5760\left(3+H^2\,k^2\right)^2}k^5\left(531\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+1728\,U+192\,H^4\,k^4\,U+\right.\right.$$

$$\left.k^2\left(145\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;+1152\,H^2\,U\right)\right)+$$

$$\left(i\,k^8\left(k^6\,U^3\left(721\sqrt{3}\;g\,H^7+192\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\;U\right)+9\,k^2\left(145\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}\;+1350\sqrt{3}\;g^2\right.\right.\right.$$

$$H^4\,U+2118\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U^2+2227\sqrt{3}\;g\,H^3\,U^3+576\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^4\Big)+$$

$$81\left(59\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}\;+241\sqrt{3}\;g^2\,H^2\,U+64\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+\right.$$

$$\left.g\,H\,U^2\left(369\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+251\sqrt{3}\;U\right)\right)+3\,k^4\,U\left(627\sqrt{3}\;g^2\,H^6+\right.$$

$$\left.\left.\left.576\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;U^3+g\,H^5\,U\left(1011\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+2195\sqrt{3}\;U\right)\right)\right)$$

$$dt^3\Big)\Big/\left(34\,560\sqrt{g\,H}\;\left(3+H^2\,k^2\right)^{7/2}\right)-\frac{1}{34\,560\left(\sqrt{g\,H}\;\left(3+H^2\,k^2\right)^{7/2}\right)}$$

$$\left(k^9\left(9\sqrt{3}\;g^3\,H^3\left(531+145\,H^2\,k^2\right)+54\sqrt{3}\;g^2\,H^2\left(915+578\,H^2\,k^2+91\,H^4\,k^4\right)U^2+\right.\right.$$

$$g\,H\,U^3\left(25\,227\sqrt{3}\;H^2\,k^2\,U+913\sqrt{3}\;H^6\,k^6\,U+405\left(124\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+63\sqrt{3}\;U\right)+\right.$$

$$\left.3\,k^4\left(1732\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;+2771\sqrt{3}\;H^4\,U\right)\right)+$$

$$12\,U\left(2025\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}\;+432\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+144\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;U^4+\right.$$

$$16\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\;U^4+3\,k^2\left(193\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}\;+898\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\right.$$

$$\left.\left.\left.U^2+144\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^4\right)\right)\right)\right)dt^4+O[dt]^5\Big)dx^4+O[dx]^5,$$

$$\left(-\frac{1}{24\left(3+H^2\,k^2\right)^3}i\,k^4\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+\left(3+H^2\,k^2\right)U\right)\left(3\,g\,H\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+3\left(3+H^2\,k^2\right)U\right)+\right.\right.$$

$$\left.U^2\left(-9\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+9\,U+H^4\,k^4\,U-3\,k^2\left(\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;-2\,H^2\,U\right)\right)\right)dt^3+$$

$$\frac{1}{30\left(3+H^2\,k^2\right)^3}k^5\left(-\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+\left(3+H^2\,k^2\right)U\right)$$

$$\left(9\,g^2\,H^2+6\,g\,H\,U\left(-2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+3\left(3+H^2\,k^2\right)U\right)+\right.$$

$$\left.U^3\left(-12\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}\;+9\,U+H^4\,k^4\,U+k^2\left(-4\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;+6\,H^2\,U\right)\right)\right)dt^4+$$

$$O[dt]^5\Big) + \Bigg(-\frac{i\,k^4\left(2\,g\,H\left(3+H^2\,k^2\right)-\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)}{24\sqrt{g\,H}\,\left(3+H^2\,k^2\right)} + \frac{1}{144\left(3+H^2\,k^2\right)^{5/2}}$$

$$k^7\left(6\sqrt{3}\,g^2\,H^2\left(3+H^2\,k^2\right)+9\,U^3\left(-5\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\,U\right)+\right.$$
$$k^4\,U^3\left(-2\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^4\,U\right)+$$
$$3\,g\,H\,U\left(-21\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\,\left(27+15\,H^2\,k^2+2\,H^4\,k^4\right)U\right)-$$
$$\left.3\,k^2\left(6\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U+7\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3-2\sqrt{3}\,H^2\,U^4\right)\right)dt^3+$$
$$\frac{1}{144\left(3+H^2\,k^2\right)^{5/2}}i\,k^8\left(3\sqrt{3}\,g^2\,H^2\left(27+8\,H^2\,k^2\right)U+k^4\,U^4\left(-2\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+\sqrt{3}\,H^4\,U\right)+\right.$$
$$2\,g\,H\,U^2\left(-72\sqrt{g\,H\left(3+H^2\,k^2\right)}+\sqrt{3}\,\left(63+33\,H^2\,k^2+4\,H^4\,k^4\right)U\right)+$$
$$9\left(-2\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}-6\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+\sqrt{3}\,U^5\right)-$$
$$\left.6\,k^2\left(6\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2+4\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4-\sqrt{3}\,H^2\,U^5\right)\right)dt^4+O[dt]^5\Bigg)dx^3+$$

$$\left(\frac{1}{5760\left(3+H^2\,k^2\right)^2}k^5\left(531\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}-1728\,U-192\,H^4\,k^4\,U+\right.\right.$$
$$\left.k^2\left(145\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}-1152\,H^2\,U\right)\right)+\frac{1}{34\,560\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}}$$
$$i\,k^8\left(k^6\,U^3\left(-721\sqrt{3}\,g\,H^7+192\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U\right)+9\,k^2\left(145\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}-1350\sqrt{3}\right.\right.$$
$$g^2\,H^4\,U+2118\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2-2227\sqrt{3}\,g\,H^3\,U^3+576\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\Big)+$$
$$81\left(59\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}-241\sqrt{3}\,g^2\,H^2\,U+64\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+\right.$$
$$g\,H\,U^2\left(369\sqrt{g\,H\left(3+H^2\,k^2\right)}-251\sqrt{3}\,U\right)\Big)-3\,k^4\,U\Big(627\sqrt{3}\,g^2\,H^6-$$
$$\left.\left.576\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3+g\,H^5\,U\left(-1011\sqrt{g\,H\left(3+H^2\,k^2\right)}+2195\sqrt{3}\,U\right)\right)\right)dt^3+$$
$$\frac{1}{34\,560\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}}k^9\left(9\sqrt{3}\,g^3\,H^3\left(531+145\,H^2\,k^2\right)+54\sqrt{3}\,g^2\,H^2\left(915+578\,H^2\,k^2+91\,H^4\,k^4\right)U^2+\right.$$
$$g\,H\,U^3\left(25\,227\sqrt{3}\,H^2\,k^2\,U+913\sqrt{3}\,H^6\,k^6\,U+405\left(-124\sqrt{g\,H\left(3+H^2\,k^2\right)}+63\sqrt{3}\,U\right)+\right.$$
$$3\,k^4\left(-1732\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+2771\sqrt{3}\,H^4\,U\right)\Big)-$$
$$12\,U\left(2025\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}+432\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^4+144\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^4+\right.$$
$$16\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\,U^4+3\,k^2\left(193\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}+898\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\right.$$
$$\left.\left.\left.U^2+144\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^4\right)\right)\right)dt^4+O[dt]^5\Bigg)dx^4+O[dx]^5\Bigg\}$$

Out[133]= Omega error ||

$$\left\{\left(-\frac{i k^4 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(\left(H^4 U k^4+3 \left(2 U H^2+\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+9 \left(U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) U^2+3 g \left(3 k^2 U H^3+9 U H+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} H\right)\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^3}+\frac{k^5 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(\left(H^4 U k^4+2 \left(3 U H^2+2 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+9 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^3+6 g H \left(3 \left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right) \text{dt}^4}{30 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i k^4 \left(2 g H \left(H^2 k^2+3\right)+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U\right)}{24 \sqrt{g H} \left(H^2 k^2+3\right)}-\frac{\left(k^7 \left(U^3 \left(\sqrt{3} U H^4+2 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+3 \left(2 \sqrt{3} H^2 U^4+7 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+6 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 \sqrt{3} g^2 H^2 \left(H^2 k^2+3\right)+9 U^3 \left(\sqrt{3} U+5 \sqrt{g H \left(H^2 k^2+3\right)}\right)+3 g H U \left(\sqrt{3} \left(2 H^4 k^4+15 H^2 k^2+27\right) U+21 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{144 \left(H^2 k^2+3\right)^{5/2}}-\frac{i k^8 \left(k^4 \left(\sqrt{3} U H^4+2 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) U^4+2 g H \left(\sqrt{3} \left(4 H^4 k^4+33 H^2 k^2+63\right) U+72 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 \sqrt{3} g^2 H^2 \left(8 H^2 k^2+27\right) U+9 \left(\sqrt{3} U^5+6 \sqrt{g H \left(H^2 k^2+3\right)} U^4+2 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)+6 k^2 \left(\sqrt{3} H^2 U^5+4 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+6 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2\right)\right) \text{dt}^4}{144 \left(H^2 k^2+3\right)^{5/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(-\frac{k^5 \left(192 H^4 U k^4+\left(1152 U H^2+145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+1728 U+531 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{5760 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(U^3 \left(721 \sqrt{3} g H^7+192 \sqrt{g H^{13}} \left(H^2 k^2+3\right)\right) U\right) k^6+3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3} U+1011 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+576 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3\right) k^4+9 \left(1350 \sqrt{3} g^2 U H^4+2227 \sqrt{3} g U^3 H^3+576 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+2118 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+145 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(64 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(251 \sqrt{3} U+369 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{34560 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}-\frac{\left(k^9 \left(9 \sqrt{3} g^3 \left(145 H^2 k^2+531\right) H^3+54 \sqrt{3} g^2 \left(91 H^4 k^4+578 H^2 k^2+915\right) U^2 H^2+g U^3 \left(913 \sqrt{3} H^6 U k^6+3 \left(2771 \sqrt{3} U H^4+1732 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+25227 \sqrt{3} H^2 U k^2+405 \left(63 \sqrt{3} U+124 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) H+12 U \left(16 \sqrt{g H^{13}} \left(H^2 k^2+3\right)\right) U^4 k^6+144 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4 k^4+3 \left(144 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+898 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+193 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+432 \sqrt{g H \left(H^2 k^2+3\right)} U^4+2025 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{34560 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)}+O\left(\text{dt}^5\right)\right) \text{dt}^4+O\left(\text{dx}^4\right)^5\right),\left(-\frac{i k^4 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(\left(H^4 U k^4-3 \left(\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-2 H^2 U\right) k^2+9 U-9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g H \left(3 \left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^3}+\frac{k^5 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(\left(H^4 U k^4+\left(6 H^2 U-4 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+9 U-12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^3+6 g H \left(3 \left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right)$$

H \left(3 \left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right) \text{dt}^4}{30 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)+\left(−\frac{i k^4 \left(2 g H \left(H^2 k^2+3\right)−\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U\right)}{24 \sqrt{g H} \left(H^2 k^2+3\right)}+\frac{k^7 \left(U^3 \left(\sqrt{3} H^4 U−2 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4−3 \left(−2 \sqrt{3} H^2 U^4+7 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+6 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 \sqrt{3} g^2 H^2 \left(H^2 k^2+3\right)+9 U^3 \left(\sqrt{3} U−5 \sqrt{g H \left(H^2 k^2+3\right)}\right)+3 g H U \left(\sqrt{3} \left(2 H^4 k^4+15 H^2 k^2+27\right) U−21 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{144 \left(H^2 k^2+3\right)^{5/2}}+\frac{i k^8 \left(k^4 \left(\sqrt{3} H^4 U−2 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) U^4+2 g H \left(\sqrt{3} \left(4 H^4 k^4+33 H^2 k^2+63\right) U−72 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 \sqrt{3} g^2 H^2 \left(8 H^2 k^2+27\right) U+9 \left(\sqrt{3} U^5−6 \sqrt{g H \left(H^2 k^2+3\right)} U^4−2 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)−6 k^2 \left(−\sqrt{3} H^2 U^5+4 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+6 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2\right)\right) \text{dt}^4}{144 \left(H^2 k^2+3\right)^{5/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{k^5 \left(−192 H^4 U k^4+\left(145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−1152 H^2 U\right) k^2−1728 U+531 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{5760 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(U^3 \left(192 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U−721 \sqrt{3} g H^7\right) k^6−3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3} U−1011 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5−576 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3\right) k^4+9 \left(−1350 \sqrt{3} g^2 U H^4−2227 \sqrt{3} g U^3 H^3+576 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+2118 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+145 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(64 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(369 \sqrt{g H \left(H^2 k^2+3\right)}−251 \sqrt{3} U\right) U^2−241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{34560 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}+\frac{k^9 \left(9 \sqrt{3} g^3 \left(145 H^2 k^2+531\right) H^3+54 \sqrt{3} g^2 \left(91 H^4 k^4+578 H^2 k^2+915\right) U^2 H^2+g U^3 \left(913 \sqrt{3} H^6 U k^6+3 \left(2771 \sqrt{3} H^4 U−1732 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+25227 \sqrt{3} H^2 U k^2+405 \left(63 \sqrt{3} U−124 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) H−12 U \left(16 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^4 k^6+144 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4 k^4+3 \left(144 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+898 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+193 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+432 \sqrt{g H \left(H^2 k^2+3\right)} U^4+2025 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{34560 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right\}

Out[134]=

Out[135]= $$\text{EA} \parallel \left\{\left\{\frac{-H^2 k^2 \left(\left(-1+e^{i\,dt\,w}\right) k\,U-w\right)+3\,w}{\left(3+H^2 k^2\right) w},\ -\frac{3\left(-1+e^{i\,dt\,w}\right) k}{\left(3+H^2 k^2\right) w}\right\},\ \left\{-\frac{\left(-1+e^{i\,dt\,w}\right) k \left(g\,H\left(3+H^2 k^2\right)-3\,U^2\right)}{\left(3+H^2 k^2\right) w},\ 1-\frac{\left(-1+e^{i\,dt\,w}\right) k \left(6+H^2 k^2\right) U}{\left(3+H^2 k^2\right) w}\right\}\right\}$$

Out[136]=
$$\text{EA} \parallel \left(\begin{array}{cc} \frac{3\,w-H^2 k^2 \left(\left(-1+e^{i\,\text{dt}\,w}\right) k\,U-w\right)}{\left(H^2 k^2+3\right) w} & -\frac{3\left(-1+e^{i\,\text{dt}\,w}\right) k}{\left(H^2 k^2+3\right) w} \\ -\frac{\left(-1+e^{i\,\text{dt}\,w}\right) k \left(g\,H\left(H^2 k^2+3\right)-3\,U^2\right)}{\left(H^2 k^2+3\right) w} & 1-\frac{\left(-1+e^{i\,\text{dt}\,w}\right) k \left(H^2 k^2+6\right) U}{\left(H^2 k^2+3\right) w} \end{array}\right)$$

Out[137]= Eerr || $$\left\{\left\{\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U+3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}+\left(-\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3+\frac{i\left(18\,g\,H\,k^3\,U+9\,g\,H^3\,k^5\,U-18\,k^3\,U^3-3\,H^2\,k^5\,U^3+H^4\,k^7\,U^3\right)}{6\left(3+H^2\,k^2\right)^2}\right)dt^3-\frac{1}{24}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4dt^4+O[dt]^5\right)+\left(-\frac{1}{12}\left(\sqrt{g\,H}\ k^4\right)dt+\frac{i\sqrt{g\,H}\ \left(3\,k^5+2\,H^2\,k^7\right)U\,dt^2}{24\left(3+H^2\,k^2\right)}+\frac{\sqrt{g\,H}\ \left(9\,g\,H\,k^6+3\,g\,H^3\,k^8-3\,k^6\,U^2+3\,H^2\,k^8\,U^2+H^4\,k^{10}\,U^2\right)dt^3}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^3+\left(\frac{i\left(45\,k^5\,U+143\,H^2\,k^7\,U+32\,H^4\,k^9\,U\right)dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(531\,g\,H\,k^6+145\,g\,H^3\,k^8-486\,k^6\,U^2+94\,H^2\,k^8\,U^2+64\,H^4\,k^{10}\,U^2\right)dt^2}{1920\left(3+H^2\,k^2\right)^2}-\frac{1}{1920\left(3+H^2\,k^2\right)^3}i\,k^7\left(1683\,g\,H\,U+1348\,g\,H^3\,k^2\,U+241\,g\,H^5\,k^4\,U-1593\,U^3-678\,H^2\,k^2\,U^3+47\,H^4\,k^4\,U^3+32\,H^6\,k^6\,U^3\right)dt^3+O[dt]^5\right)dx^4+O[dx]^5,\left(-\frac{3\,i\,k\,dt}{3+H^2\,k^2}-\frac{3\left(k^2\,U\right)dt^2}{3+H^2\,k^2}+\frac{3\,i\left(g\,H\,k^3+3\,k^3\,U^2+H^2\,k^5\,U^2\right)dt^3}{2\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)+\left(\frac{i\sqrt{g\,H}\ k^5\,dt^2}{4\left(3+H^2\,k^2\right)}+\left(\frac{7\sqrt{g\,H}\ k^6\,U}{8\left(3+H^2\,k^2\right)^2}+\frac{H^2\sqrt{g\,H}\ k^8\,U}{4\left(3+H^2\,k^2\right)^2}\right)dt^3+O[dt]^5\right)dx^3+\left(\frac{i\left(243\,k^5+49\,H^2\,k^7\right)dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(531\,k^6+145\,H^2\,k^8\right)U\,dt^2}{960\left(3+H^2\,k^2\right)^2}-\frac{i\left(774\,g\,H\,k^7+194\,g\,H^3\,k^9+2457\,k^7\,U^2+1542\,H^2\,k^9\,U^2+241\,H^4\,k^{11}\,U^2\right)dt^3}{1920\left(3+H^2\,k^2\right)^3}+O[dt]^5\right)dx^4+O[dx]^5\right\},\left\{\left(\left(-i\,g\,H\,k+\frac{3\,i\,k\,U^2}{3+H^2\,k^2}\right)dt-\frac{k^2\,U\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)dt^2}{3+H^2\,k^2}+\frac{1}{2\left(3+H^2\,k^2\right)^2}i\left(3\,g^2\,H^2\,k^3+g^2\,H^4\,k^5+6\,g\,H\,k^3\,U^2+6\,g\,H^3\,k^5\,U^2+g\,H^5\,k^7\,U^2-9\,k^3\,U^4-3\,H^2\,k^5\,U^4\right)dt^3+O[dt]^5\right)+\left(-\frac{1}{12}\left(\sqrt{g\,H}\ k^4\,U\right)dt+\frac{i\left(3\,g\,H\sqrt{g\,H}\ k^5+g\,H^3\sqrt{g\,H}\ k^7+H^2\sqrt{g\,H}\ k^7\,U^2\right)dt^2}{12\left(3+H^2\,k^2\right)}+\frac{1}{24\left(3+H^2\,k^2\right)^2}\left(24\,g\,H\sqrt{g\,H}\ k^6\,U+14\,g\,H^3\sqrt{g\,H}\ k^8\,U+2\,g\,H^5\sqrt{g\,H}\ k^{10}\,U-12\sqrt{g\,H}\ k^6\,U^3+H^4\sqrt{g\,H}\ k^{10}\,U^3\right)dt^3+O[dt]^5\right)dx^3+\left(\frac{i\left(288\,g\,H\,k^5+192\,g\,H^3\,k^7+32\,g\,H^5\,k^9-243\,k^5\,U^2-49\,H^2\,k^7\,U^2\right)dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(576\,g\,H\,k^6\,U+384\,g\,H^3\,k^8\,U+64\,g\,H^5\,k^{10}\,U-531\,k^6\,U^3-145\,H^2\,k^8\,U^3\right)dt^2}{960\left(3+H^2\,k^2\right)^2}-\frac{1}{5760\left(3+H^2\,k^2\right)^3}i\left(2457\,g^2\,H^2\,k^7+1542\,g^2\,H^4\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H\,k^7\,U^2+7194\,g\,H^3\,k^9\,U^2+2592\,g\,H^5\,k^{11}\,U^2+288\,g\,H^7\,k^{13}\,U^2-7371\,k^7\,U^4-4626\,H^2\,k^9\,U^4-723\,H^4\,k^{11}\,U^4\right)dt^3+O[dt]^5\right)dx^4+O[dx]^5,\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}-3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U-3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}+\left(-\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3+\frac{i\left(36\,g\,H\,k^3\,U+9\,g\,H^3\,k^5\,U+36\,k^3\,U^3+15\,H^2\,k^5\,U^3+H^4\,k^7\,U^3\right)}{6\left(3+H^2\,k^2\right)^2}\right)dt^3-1\left(\ i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}\ \ i\,k\,U\right)^4dt^4+O[dt]^5\right)+$$

$$\frac{1}{24}\left(-\frac{\dots}{3+H^2 k^2} - i k U\right) dt^4 + O[dt]^5\Bigg) +$$

$$\left(-\frac{1}{12}\left(\sqrt{g H}\ k^4\right) dt + \frac{i\sqrt{g H}\ \left(15 k^5+2 H^2 k^7\right) U\, dt^2}{24\left(3+H^2 k^2\right)} + \frac{1}{24\left(3+H^2 k^2\right)^2}\Big(9 g H\sqrt{g H}\ k^6 + 3 g H^3\sqrt{g H}\ k^8 + 39\sqrt{g H}\ k^6 U^2 + 15 H^2\sqrt{g H}\ k^8 U^2 + H^4\sqrt{g H}\ k^{10} U^2\Big) dt^3 + O[dt]^5\Big) dx^3 + \left(\frac{i\left(531 k^5 U+241 H^2 k^7 U+32 H^4 k^9 U\right) dt}{960\left(3+H^2 k^2\right)^2} + \frac{\left(531 g H k^6+145 g H^3 k^8+1638 k^6 U^2+674 H^2 k^8 U^2+64 H^4 k^{10} U^2\right) dt^2}{1920\left(3+H^2 k^2\right)^2} - \frac{1}{1920\left(3+H^2 k^2\right)^3}\right.$$
$$\left. i\left(3231 g H k^7 U + 1736 g H^3 k^9 U + 241 g H^5 k^{11} U + 3321 k^7 U^3 + 2406 H^2 k^9 U^3 + 529 H^4 k^{11} U^3 + 32 H^6 k^{13} U^3\right) dt^3 + O[dt]^5\Big) dx^4 + O[dx]^5\Big)\Big\}$$

Out[138]= Eerr || \left(
\begin{array}{cc}
\left(\frac{i \left(3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \text{dt}}{H^2 k^2+3}+\frac{\left(3 U^2 k^2+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2\right) \text{dt}^2}{H^2 k^2+3}+\left(\frac{i \left(H^4 U^3 k^7-3 H^2 U^3 k^5+9 g H^3 U k^5-18 U^3 k^3+18 g H U k^3\right)}{6 \left(H^2 k^2+3\right)^2}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^3\right) \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{1}{12} \left(\sqrt{g H} k^4\right) \text{dt}+\frac{i \sqrt{g H} \left(2 H^2 k^7+3 k^5\right) U \text{dt}^2}{24 \left(H^2 k^2+3\right)}+\frac{\sqrt{g H} \left(H^4 U^2 k^{10}+3 g H^3 k^8+3 H^2 U^2 k^8-3 U^2 k^6+9 g H k^6\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 H^4 U^2 k^{10}+145 g H^3 k^8+94 H^2 U^2 k^8-486 U^2 k^6+531 g H k^6\right) \text{dt}^2}{1920 \left(H^2 k^2+3\right)^2}-\frac{i k^7 \left(32 H^6 U^3 k^6+47 H^4 U^3 k^4+241 g H^5 U k^4-678 H^2 U^3 k^2+1348 g H^3 U k^2-1593 U^3+1683 g H U\right) \text{dt}^3}{1920 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) & \left(-\frac{3 i k \text{dt}}{H^2 k^2+3}-\frac{3 \left(k^2 U\right) \text{dt}^2}{H^2 k^2+3}+\frac{3 i \left(H^2 U^2 k^5+3 U^2 k^3+g H k^3\right) \text{dt}^3}{2 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+\left(\frac{i \sqrt{g H} k^5 \text{dt}^2}{4 \left(H^2 k^2+3\right)}+\left(\frac{H^2 \sqrt{g H} U k^8}{4 \left(H^2 k^2+3\right)^2}+\frac{7 \sqrt{g H} U k^6}{8 \left(H^2 k^2+3\right)^2}\right) \text{dt}^3+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(49 H^2 k^7+243 k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(145 H^2 k^8+531 k^6\right) U \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(241 H^4 U^2 k^{11}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7\right) \text{dt}^3}{1920 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\left(\left(\frac{3 i k U^2}{H^2 k^2+3}-i g H k\right) \text{dt}-\frac{k^2 U \left(g k^2 H^3+3 g H-3 U^2\right) \text{dt}^2}{H^2 k^2+3}+\frac{i \left(g H^5 U^2 k^7+g^2 H^4 k^5-3 H^2 U^4 k^5+6 g H^3 U^2 k^5-9 U^4 k^3+3 g^2 H^2 k^3+6 g H U^2 k^3\right) \text{dt}^3}{2 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+\left(-\frac{1}{12} \left(\sqrt{g H} k^4 U\right) \text{dt}+\frac{i \left(H^2 \sqrt{g H} U^2 k^7+g H^3 \sqrt{g H} k^7+3 g H \sqrt{g H} k^5\right) \text{dt}^2}{12 \left(H^2 k^2+3\right)}+\frac{\left(H^4 \sqrt{g H} U^3 k^{10}+2 g H^5 \sqrt{g H} U k^{10}+14 g H^3 \sqrt{g H} U k^8-12 \sqrt{g H} U^3 k^6+24 g H \sqrt{g H} U k^6\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) \text{dt}}{960 \left(H^2

```
k^2+3\right)^2}+\frac{\left(64 g H^5 U k^{10}-145 H^2 U^3 k^8+384 g H^3 U k^8-531 U^3 k^6+576
g H U k^6\right) \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(288 g H^7 U^2 k^{13}+241 g^2
H^6 k^{11}-723 H^4 U^4 k^{11}+2592 g H^5 U^2 k^{11}+1542 g^2 H^4 k^9-4626 H^2 U^4 k^9+7194
g H^3 U^2 k^9-7371 U^4 k^7+2457 g^2 H^2 k^7+5454 g H U^2 k^7\right) \text{dt}^3}{5760 \left(H^2
k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{i \left(\sqrt{3}
k \sqrt{g H \left(H^2 k^2+3\right)}-3 k U\right) \text{dt}}{H^2 k^2+3}+\frac{\left(\sqrt{3} k^2 \sqrt{g H
\left(H^2 k^2+3\right)} U-3 k^2 U^2\right) \text{dt}^2}{H^2 k^2+3}+\left(\frac{i \left(H^4 U^3 k^7+15
H^2 U^3 k^5+9 g H^3 U k^5+36 U^3 k^3+36 g H U k^3\right)}{6 \left(H^2 k^2+3\right)^2}-\frac{1}{6}
\left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^3\right)
\text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^4
\text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{1}{12} \left(\sqrt{g H} k^4\right) \text{dt}+\frac{i
\sqrt{g H} \left(2 H^2 k^7+15 k^5\right) U \text{dt}^2}{24 \left(H^2 k^2+3\right)}+\frac{\left(H^4 \sqrt{g H}
U^2 k^{10}+15 H^2 \sqrt{g H} U^2 k^8+3 g H^3 \sqrt{g H} k^8+39 \sqrt{g H} U^2 k^6+9 g H \sqrt{g H}
k^6\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i
\left(32 H^4 U k^9+241 H^2 U k^7+531 U k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64
H^4 U^2 k^{10}+145 g H^3 k^8+674 H^2 U^2 k^8+1638 U^2 k^6+531 g H k^6\right) \text{dt}^2}{1920
\left(H^2 k^2+3\right)^2}-\frac{i \left(32 H^6 U^3 k^{13}+529 H^4 U^3 k^{11}+241 g H^5 U
k^{11}+2406 H^2 U^3 k^9+1736 g H^3 U k^9+3321 U^3 k^7+3231 g H U k^7\right) \text{dt}^3}{1920
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\end{array}
\right)
```

In[139]:=
```
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → 0;
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 4}];
Fnn2TAr = Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 4}];
FnG2TAr = Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
```

```
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 4}];
FGn2TAr = Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 4}];
FGG2TAr = Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 + Fmat2.Fmat2.Fmat2 / 6;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep =
  Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 + EigvFmat2 * EigvFmat2 * EigvFmat2 / 6 ] /
   (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ];


Text[Row[{" U > -Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn  ||   ", Kfnnp}]]
Text[Row[{"Fnn  ||   ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||   ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||   ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||   ", KfnGp}]]
Text[Row[{"FnG  ||   ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error  ||   ", FnG2TAr}]]
Text[Row[{"FnG error  ||   ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||   ", KfGnp}]]
Text[Row[{"FGn  ||   ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||   ", FGn2TAr}]]
Text[Row[{"FGn error  ||   ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||   ", KfGGp}]]
Text[Row[{"FGG  ||   ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||   ", FGG2TAr}]]
```

```
Text[Row[{"FGG error  ||   ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||   ", RKstepTayr}]]
Text[Row[{"Omega error  ||   ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||   ", EA}]]
Text[Row[{"EA  ||   ", TeXForm[EA]}]]
Text[Row[{"Eerr ||   ", Eerr}]]
Text[Row[{"Eerr ||   ", TeXForm[Eerr]}]]
```

Out[173]= U > −Sqrt(gH)

Out[174]=

Out[175]= Fnn || Gnp H + Rmp U

Out[176]= Fnn || \text{Gnp} H+\text{Rmp} U

Out[177]= Fnn error || $\left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{H^2\,k^3\,U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\right)+$
$\left(-\frac{1}{12}\left(k^4\,U\right)dt+O[dt]^5\right)dx^3+\left(\frac{i\left(45\,k^5\,U+143\,H^2\,k^7\,U+32\,H^4\,k^9\,U\right)dt}{960\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^4+O[dx]^5$

Out[178]= Fnn error ||
\left(−\frac{\text{dt}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 H^2 k^3 U w^2}{6 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 H^2 k^3 U w^3}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^3 \left(−\frac{1}{12} \left(k^4 U\right) \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+O\left(\text{dx}^5\right)

Out[179]=

Out[180]= FnG || GGp H

Out[181]= FnG || \text{GGp} H

Out[182]= FnG error || $\left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)}+\frac{k\,w^3\,dt^4}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)+\left(\frac{i\left(243\,k^5+49\,H^2\,k^7\right)dt}{960\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^4+O[dx]^5$

Out[183]= FnG error || \left(−\frac{3 \text{dt}^2 (k w)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 k w^2}{2 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k w^3}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\left(\frac{i \left(49 H^2 k^7+243 k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)\text{dx}^4+O\left(\text{dx}^5\right)

Out[184]=

Out[185]= FGn || H (g Rmp + Gnp U)

Out[186]= FGn || H (g \text{Rmp}+\text{Gnp} U)

Out[187]= FGn error || $\left(-\frac{\left(k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\right)+$
$\left(-\frac{1}{12}\left(g\,H\,k^4\right)dt+O[dt]^5\right)dx^3+\left(\frac{i\left(288\,g\,H\,k^5+192\,g\,H^3\,k^7+32\,g\,H^5\,k^9-243\,k^5\,U^2-49\,H^2\,k^7\,U^2\right)dt}{960\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^4+O[dx]^5$

Out[188]= FGn error ||
\left(-\frac{\text{dt}^2 \left(k w \left(g H^3 k^2+3 g H-3 U^2\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2 \left(g H^3 k^2+3 g H-3 U^2\right)}{6 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k w^3 \left(g H^3 k^2+3 g H-3 U^2\right)}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^3 \left(-\frac{1}{12} \left(g H k^4\right) \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+O\left(\text{dx}^5\right)

Out[189]=

Out[190]= FGG || (GGp H + Rmp) U

Out[191]= FGG || U (\text{GGp} H+\text{Rmp})

Out[192]= FGG error || $\left(-\frac{\left(k\left(6+H^2\,k^2\right)U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(6+H^2\,k^2\right)U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{k\left(6+H^2\,k^2\right)U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\right)+$
$\left(-\frac{1}{12}\left(k^4\,U\right)dt+O[dt]^5\right)dx^3+\left(\frac{i\left(531\,k^5+241\,H^2\,k^7+32\,H^4\,k^9\right)U\,dt}{960\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^4+O[dx]^5$

Out[193]= FGG error ||
\left(-\frac{\text{dt}^2 \left(k U w \left(H^2 k^2+6\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k U w^2 \left(H^2 k^2+6\right)}{6 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k U w^3 \left(H^2 k^2+6\right)}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^3 \left(-\frac{1}{12} \left(k^4 U\right) \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(32 H^4 k^9+241 H^2 k^7+531 k^5\right) U \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+O\left(\text{dx}^5\right)

Out[194]=

Out[195]=

Out[196]= Omega error || $\left\{\left(-\frac{i\left(\sqrt{3}\;k\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,k\,U+H^2\,k^3\,U\right)^4dt^3}{24\left(3+H^2\,k^2\right)^4}+\frac{\left(\sqrt{3}\;k\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,k\,U+H^2\,k^3\,U\right)^5dt^4}{30\left(3+H^2\,k^2\right)^5}+O[dt]^5\right)+\right.$

$\left(-\frac{1}{24}\,i\,k^4\left(\sqrt{3}\;\sqrt{\frac{g\,H}{3+H^2\,k^2}}+2\,U\right)-\frac{1}{144\left(3+H^2\,k^2\right)^2}\right.$

$\left(k^7\left(9\,g^2\,H^2+3\,g\,H\,U\left(5\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+9\left(3+H^2\,k^2\right)U\right)+\right.\right.$

$\left.\left.\left.U^3\left(21\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(7\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)\right)$

$dt^3-\frac{1}{144\left(3+H^2\,k^2\right)^3}i\,k^8\left(3\,g\,H+U\left(2\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$

$\left(3\,g\,H\left(\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+4\left(3+H^2\,k^2\right)U\right)+\right.$

$$U^2\left(15\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +18\,U+2\,H^4\,k^4\,U+k^2\left(5\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +12\,H^2\,U\right)\right)\right)dt^4+$$

$$O[dt]^5\Bigg)dx^3+\Bigg(-\frac{1}{5760\left(3+H^2\,k^2\right)^2}k^5\left(531\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +1728\,U+192\,H^4\,k^4\,U+\right.$$

$$\left.k^2\left(145\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +1152\,H^2\,U\right)\right)+$$

$$\left(i\,k^8\left(k^6\,U^3\left(721\sqrt{3}\ g\,H^7+192\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ U\right)+9\,k^2\left(145\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}\ +1350\sqrt{3}\ g^2\right.\right.\right.$$

$$H^4\,U+2118\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U^2+2227\sqrt{3}\ g\,H^3\,U^3+576\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^4\Big)+$$

$$81\left(59\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}\ +241\sqrt{3}\ g^2\,H^2\,U+64\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^4+\right.$$

$$g\,H\,U^2\left(369\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +251\sqrt{3}\ U\right)\Big)+3\,k^4\,U\Big(627\sqrt{3}\ g^2\,H^6+$$

$$\left.\left.576\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^3+g\,H^5\,U\left(1011\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +2195\sqrt{3}\ U\right)\right)\right)dt^3\Bigg)\Big/$$

$$\left(34\,560\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{7/2}\right)-\Big(\Big(k^9\Big(3\,g\,H+U\Big(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\Big)\Big)$$

$$\left(3\sqrt{3}\ g^2\,H^2\left(531+145\,H^2\,k^2\right)+g\,H\,U\Big(4914\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\sqrt{3}\ \left(5049+3270\,H^2\,k^2+\right.\right.\right.$$

$$\left.\left.\left.529\,H^4\,k^4\right)U\Big)+6\Big(288\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^3+32\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^3+\right.\right.$$

$$\left.\left.k^2\left(241\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U+192\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3\right)\right)\right)\Big)$$

$$dt^4\Big)\Big/\left(34\,560\left(\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{7/2}\right)+O[dt]^5\right)dx^4+O[dx]^5,$$

$$\Bigg(-\frac{i\left(-\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +3\,k\,U+H^2\,k^3\,U\right)^4dt^3}{24\left(3+H^2\,k^2\right)^4}+\frac{\left(-\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +3\,k\,U+H^2\,k^3\,U\right)^5dt^4}{30\left(3+H^2\,k^2\right)^5}+$$

$$O[dt]^5\Bigg)+$$

$$\left(\frac{1}{24}\,i\,k^4\left(\sqrt{3}\ \sqrt{\frac{g\,H}{3+H^2\,k^2}}\ -2\,U\right)-\frac{1}{144\left(3+H^2\,k^2\right)^2}\right.$$

$$\Big(k^7\left(9\,g^2\,H^2+3\,g\,H\,U\left(-5\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +9\left(3+H^2\,k^2\right)U\right)+U^3\left(-21\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\right.\right.$$

$$\left.\left.18\,U+2\,H^4\,k^4\,U+k^2\left(-7\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +12\,H^2\,U\right)\right)\right)dt^3-$$

$$\frac{1}{144\left(3+H^2\,k^2\right)^3}i\,k^8\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)$$

$$\Big(-3\,g\,H\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ -4\left(3+H^2\,k^2\right)U\right)+U^2\left(-15\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +$$

$$\left.\left.18\,U+2\,H^4\,k^4\,U+k^2\left(-5\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +12\,H^2\,U\right)\right)\right)dt^4+O[dt]^5\Bigg)dx^3+$$

$$\Bigg(\frac{1}{5760\left(3+H^2\,k^2\right)^2}k^5\left(531\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ -1728\,U-192\,H^4\,k^4\,U+\right.$$

$$k^2\left(145\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ -1152\,H^2\,U\right)\Big)+\frac{1}{34\,560\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{7/2}}$$

$$i\,k^8\Big(k^6\,U^3\left(-721\sqrt{3}\ g\,H^7+192\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ U\right)+9\,k^2\Big(145\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}\ -1350\sqrt{3}$$

$$g^2\,H^4\,U+2118\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U^2-2227\sqrt{3}\ g\,H^3\,U^3+576\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^4\Big)+$$

$$81\Big(59\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}\ -241\sqrt{3}\ g^2\,H^2\,U+64\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^4+$$

$$g\,H\,U^2\left(369\sqrt{g\,H\left(3+H^2\,k^2\right)}\ -251\sqrt{3}\ U\right)\Big)-3\,k^4\,U$$

$$\left(627\sqrt{3}\ g^2\,H^6-576\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^3+g\,H^5\,U\left(-1011\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +2195\sqrt{3}\ U\right)\right)\Big)$$

$$dt^3+\Big(k^9\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)\Big(3\sqrt{3}\ g^2\,H^2\left(531+145\,H^2\,k^2\right)+$$

$$g\,H\,U\left(-4914\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\sqrt{3}\ \left(5049+3270\,H^2\,k^2+529\,H^4\,k^4\right)U\right)-$$

$$6\Big(288\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^3+32\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^3+$$

$$k^2\left(241\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U+192\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3\right)\Big)\Big)$$

$$dt^4\Big)\Big/\Big(34\,560\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{7/2}\Big)+O[dt]^5\Big)\,dx^4+O[dx]^5\Big\}$$

Out[197]= Omega error ||

\left\{\left(-\frac{i \left(H^2 U k^3+3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^4 \text{dt}^3}{24 \left(H^2 k^2+3\right)^4}+\frac{\left(H^2 U k^3+3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^5 \text{dt}^4}{30 \left(H^2 k^2+3\right)^5}+O\left(\text{dt}^5\right)\right)+\left(-\frac{1}{24} i k^4 \left(2 U+\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}\right)-\frac{\left(k^7 \left(\left(2 H^4 U k^4+\left(12 U H^2+7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^3+3 g H \left(9 \left(H^2 k^2+3\right) U+5 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right)\right) \text{dt}^3}{144 \left(H^2 k^2+3\right)^2}-\frac{i k^8 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(\left(2 H^4 U k^4+\left(12 U H^2+5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g H \left(4 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{144 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(-\frac{k^5 \left(192 H^4 U k^4+\left(1152 U H^2+145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+1728 U+531 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{5760 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(U^3 \left(721 \sqrt{3} g H^7+192 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U\right) k^6+3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3} U+1011 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+576 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3\right) k^4+9 \left(1350 \sqrt{3} g^2 U H^4+2227 \sqrt{3} g U^3 H^3+576 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+2118 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+145 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(64 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(251 \sqrt{3} U+369 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{34560 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}-\frac{\left(k^9 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 \sqrt{3} g^2 \left(145 H^2 k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right) U+4914 \sqrt{g H \left(H^2 k^2+3\right)}\right) H+6 \left(32 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+\left(192

\sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+241 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+288 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right)\right) \text{dt}^4}{34560 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right),\left(-\frac{i \left(H^2 U k^3+3 U k-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^4 \text{dt}^3}{24 \left(H^2 k^2+3\right)^4}+\frac{\left(H^2 U k^3+3 U k-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^5 \text{dt}^4}{30 \left(H^2 k^2+3\right)^5}+O\left(\text{dt}^5\right)\right)+\left(\frac{1}{24} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}-2 U\right)-\frac{\left(k^7 \left(\left(2 H^4 U k^4+\left(12 H^2 U-7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U-21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^3+3 g H \left(9 \left(H^2 k^2+3\right) U-5 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right)\right) \text{dt}^3}{144 \left(H^2 k^2+3\right)^2}-\frac{i k^8 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(U^2 \left(2 H^4 U k^4+\left(12 H^2 U-5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U-15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)-3 g H \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}-4 \left(H^2 k^2+3\right) U\right)\right) \text{dt}^4}{144 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{k^5 \left(-192 H^4 U k^4+\left(145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-1152 H^2 U\right) k^2-1728 U+531 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{5760 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(U^3 \left(192 \sqrt{g H^{13}} \left(H^2 k^2+3\right)\right) U-721 \sqrt{3} g H^7\right) k^6-3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3} U-1011 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5-576 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3\right) k^4+9 \left(-1350 \sqrt{3} g^2 U H^4-2227 \sqrt{3} g U^3 H^3+576 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+2118 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+145 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(64 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(369 \sqrt{g H \left(H^2 k^2+3\right)}-251 \sqrt{3} U\right) U^2-241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{34560 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}+\frac{k^9 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 \sqrt{3} g^2 \left(145 H^2 k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right) U-4914 \sqrt{g H \left(H^2 k^2+3\right)}\right) H-6 \left(32 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+\left(192 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+241 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+288 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text{dt}^4}{34560 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right)\}

Out[198]=

Out[199]= EA || $\left\{\left\{\frac{-H^2 k^2 \left(\left(-1+e^{i\,dt\,w}\right) k\,U-w\right)+3\,w}{\left(3+H^2 k^2\right) w}, -\frac{3\left(-1+e^{i\,dt\,w}\right) k}{\left(3+H^2 k^2\right) w}\right\}, \left\{-\frac{\left(-1+e^{i\,dt\,w}\right) k \left(g\,H\left(3+H^2 k^2\right)-3\,U^2\right)}{\left(3+H^2 k^2\right) w}, 1-\frac{\left(-1+e^{i\,dt\,w}\right) k \left(6+H^2 k^2\right) U}{\left(3+H^2 k^2\right) w}\right\}\right\}$

Out[200]= EA || \left(
\begin{array}{cc}
 \frac{3 w-H^2 k^2 \left(\left(-1+e^{i \text{dt} w}\right) k U-w\right)}{\left(H^2 k^2+3\right) w} & -\frac{3 \left(-1+e^{i \text{dt} w}\right) k}{\left(H^2 k^2+3\right) w} \\
 -\frac{\left(-1+e^{i \text{dt} w}\right) k \left(g H \left(H^2 k^2+3\right)-3 U^2\right)}{\left(H^2 k^2+3\right) w} & 1-\frac{\left(-1+e^{i \text{dt} w}\right) k \left(H^2 k^2+6\right) U}{\left(H^2 k^2+3\right) w} \\
\end{array}
\right)

Eqm || $\left\{\left\{\left(i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2 k^2\right)}+3\,k\,U\right)\right)\text{dt}\right.\right.$ $\left.\left.\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2 k^2\right)}\ U+3\,k^2\,U^2\right)\text{dt}^2\right.$

Out[201]= Eeff || {{{

$$\frac{}{3+H^2\,k^2}+\frac{}{3+H^2\,k^2}+$$

$$\left(-\frac{1}{6}\left(-\frac{i\sqrt{3}\;k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3+\frac{i\left(18\,g\,H\,k^3\,U+9\,g\,H^3\,k^5\,U-18\,k^3\,U^3-3\,H^2\,k^5\,U^3+H^4\,k^7\,U^3\right)}{6\left(3+H^2\,k^2\right)^2}\right)dt^3-$$

$$\frac{1}{24}\left(-\frac{i\sqrt{3}\;k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4dt^4+O[dt]^5\Bigg)+$$

$$\left(-\frac{1}{12}\left(k^4\,U\right)dt+\frac{i\left(3\,g\,H\,k^5+2\,H^2\,k^7\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)}+\frac{U\left(15\,g\,H\,k^6+6\,g\,H^3\,k^8-9\,k^6\,U^2+H^4\,k^{10}\,U^2\right)dt^3}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(45\,k^5\,U+143\,H^2\,k^7\,U+32\,H^4\,k^9\,U\right)dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(531\,g\,H\,k^6+145\,g\,H^3\,k^8-486\,k^6\,U^2+94\,H^2\,k^8\,U^2+64\,H^4\,k^{10}\,U^2\right)dt^2}{1920\left(3+H^2\,k^2\right)^2}-\frac{1}{1920\left(3+H^2\,k^2\right)^3}\right.$$

$$\left.i\,k^7\left(1683\,g\,H\,U+1348\,g\,H^3\,k^2\,U+241\,g\,H^5\,k^4\,U-1593\,U^3-678\,H^2\,k^2\,U^3+47\,H^4\,k^4\,U^3+32\,H^6\,k^6\,U^3\right)dt^3+O[dt]^5\right)dx^4+O[dx]^5,$$

$$\left(-\frac{3\,i\,k\,dt}{3+H^2\,k^2}-\frac{3\left(k^2\,U\right)dt^2}{3+H^2\,k^2}+\frac{3\,i\left(g\,H\,k^3+3\,k^3\,U^2+H^2\,k^5\,U^2\right)dt^3}{2\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)+\left(\frac{i\,k^5\,U\,dt^2}{4\left(3+H^2\,k^2\right)}+\frac{\left(g\,H\,k^6+6\,k^6\,U^2+2\,H^2\,k^8\,U^2\right)dt^3}{8\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(243\,k^5+49\,H^2\,k^7\right)dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(531\,k^6+145\,H^2\,k^8\right)U\,dt^2}{960\left(3+H^2\,k^2\right)^2}-\frac{i\left(774\,g\,H\,k^7+194\,g\,H^3\,k^9+2457\,k^7\,U^2+1542\,H^2\,k^9\,U^2+241\,H^4\,k^{11}\,U^2\right)dt^3}{1920\left(3+H^2\,k^2\right)^3}+O[dt]^5\right)dx^4+$$

$$O[dx]^5\Bigg\},$$

$$\Bigg\{\Bigg\{\left(\left(-i\,g\,H\,k+\frac{i\,H\,k\,U^2}{H+\frac{H^3\,k^2}{3}}\right)dt-\frac{k^2\,U\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)dt^2}{3+H^2\,k^2}+\frac{1}{2\left(3+H^2\,k^2\right)^2}\right.$$

$$\left.i\left(3\,g^2\,H^2\,k^3+g^2\,H^4\,k^5+6\,g\,H\,k^3\,U^2+6\,g\,H^3\,k^5\,U^2+g\,H^5\,k^7\,U^2-9\,k^3\,U^4-3\,H^2\,k^5\,U^4\right)dt^3+O[dt]^5\right)+$$

$$\left(-\frac{1}{12}\left(g\,H\,k^4\right)dt+\frac{i\left(6\,g\,H\,k^5\,U+2\,g\,H^3\,k^7\,U-3\,k^5\,U^3\right)dt^2}{12\left(3+H^2\,k^2\right)}+\frac{1}{24\left(3+H^2\,k^2\right)^2}\left(6\,g^2\,H^2\,k^6+2\,g^2\,H^4\,k^8+24\,g\,H\,k^6\,U^2+\right.\right.$$

$$\left.\left.18\,g\,H^3\,k^8\,U^2+3\,g\,H^5\,k^{10}\,U^2-18\,k^6\,U^4-6\,H^2\,k^8\,U^4\right)dt^3+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(288\,g\,H\,k^5+192\,g\,H^3\,k^7+32\,g\,H^5\,k^9-243\,k^5\,U^2-49\,H^2\,k^7\,U^2\right)dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(576\,g\,H\,k^6\,U+384\,g\,H^3\,k^8\,U+64\,g\,H^5\,k^{10}\,U-531\,k^6\,U^3-145\,H^2\,k^8\,U^3\right)dt^2}{960\left(3+H^2\,k^2\right)^2}-\right.$$

$$\left.\frac{1}{5760\left(3+H^2\,k^2\right)^3}i\left(2457\,g^2\,H^2\,k^7+1542\,g^2\,H^4\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H\,k^7\,U^2+7194\,g\,H^3\,k^9\,U^2+\right.\right.$$

$$\left.\left.2592\,g\,H^5\,k^{11}\,U^2+288\,g\,H^7\,k^{13}\,U^2-7371\,k^7\,U^4-4626\,H^2\,k^9\,U^4-723\,H^4\,k^{11}\,U^4\right)dt^3+O[dt]^5\right)$$

$$dx^4+O[dx]^5,\left(\frac{i\left(\sqrt{3}\;k\sqrt{g\,H\left(3+H^2\,k^2\right)}\;-3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\;k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U-3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}+\right.$$

$$\left(-\frac{1}{6}\left(-\frac{i\sqrt{3}\;k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3+\frac{i\left(36\,g\,H\,k^3\,U+9\,g\,H^3\,k^5\,U+36\,k^3\,U^3+15\,H^2\,k^5\,U^3+H^4\,k^7\,U^3\right)}{6\left(3+H^2\,k^2\right)^2}\right)dt^3-$$

$$\frac{1}{24}\left(-\frac{i\sqrt{3}\;k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4dt^4+O[dt]^5\Bigg)+$$

$$\left(-\frac{1}{12}\left(k^4\,U\right)dt+\frac{i\,k^5\left(3\,g\,H+12\,U^2+2\,H^2\,k^2\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)}+\frac{\left(21\,g\,H\,k^6\,U+6\,g\,H^3\,k^8\,U+27\,k^6\,U^3+12\,H^2\,k^8\,U^3+H^4\,k^{10}\,U^3\right)dt^3}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^3+$$

$$\left(i\left(531\,k^5+241\,H^2\,k^7+32\,H^4\,k^9\right)U\,dt-\left(531\,g\,H\,k^6+145\,g\,H^3\,k^8+1638\,k^6\,U^2+674\,H^2\,k^8\,U^2+64\,H^4\,k^{10}\,U^2\right)dt^2\right.$$

$$\left(\frac{i\left(531\,k^2+241\,H^2\,k^4+32\,H^4\,k^6\right)U\,dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(531\,g\,H\,k^6+145\,g\,H^3\,k^8+1638\,k^6\,U^2+674\,H^2\,k^8\,U^2+64\,H^4\,k^{10}\,U^2\right)dt^2}{1920\left(3+H^2\,k^2\right)^2}-\frac{1}{1920\left(3+H^2\,k^2\right)^3}i\left(3231\,g\,H\,k^7\,U+1736\,g\,H^3\,k^9\,U+241\,g\,H^5\,k^{11}\,U+3321\,k^7\,U^3+2406\,H^2\,k^9\,U^3+529\,H^4\,k^{11}\,U^3+32\,H^6\,k^{13}\,U^3\right)dt^3+O[dt]^5\right)dx^4+O[dx]^5\right\}\right\}$$

Out[202]= Eerr || \left(

\begin{array}{cc}

\left(\frac{i \left(3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \text{dt}}{H^2 k^2+3}+\frac{\left(3 U^2 k^2+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2\right) \text{dt}^2}{H^2 k^2+3}+\left(\frac{i \left(H^4 U^3 k^7-3 H^2 U^3 k^5+9 g H^3 U k^5-18 U^3 k^3+18 g H U k^3\right)}{6 \left(H^2 k^2+3\right)^2}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^3\right) \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{1}{12} \left(k^4 U\right) \text{dt}+\frac{i \left(2 H^2 U^2 k^7+3 g H k^5\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)}+\frac{U \left(H^4 U^2 k^{10}+6 g H^3 k^8-9 U^2 k^6+15 g H k^6\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 H^4 U^2 k^{10}+145 g H^3 k^8+94 H^2 U^2 k^8-486 U^2 k^6+531 g H k^6\right) \text{dt}^2}{1920 \left(H^2 k^2+3\right)^2}-\frac{i k^7 \left(32 H^6 U^3 k^6+47 H^4 U^3 k^4+241 g H^5 U k^4-678 H^2 U^3 k^2+1348 g H^3 U k^2-1593 U^3+1683 g H U\right) \text{dt}^3}{1920 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) & \left(-\frac{3 i k \text{dt}}{H^2 k^2+3}-\frac{3 \left(k^2 U\right) \text{dt}^2}{H^2 k^2+3}+\frac{3 i \left(H^2 U^2 k^5+3 U^2 k^3+g H k^3\right) \text{dt}^3}{2 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+\left(\frac{i k^5 U \text{dt}^2}{4 \left(H^2 k^2+3\right)}+\frac{\left(2 H^2 U^2 k^8+6 U^2 k^6+g H k^6\right) \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(49 H^2 k^7+243 k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(145 H^2 k^8+531 k^6\right) U \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(241 H^4 U^2 k^{11}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7\right) \text{dt}^3}{1920 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\

\left(\left(\frac{i H k U^2}{\frac{k^2 H^3}{3}+H}-i g H k\right) \text{dt}-\frac{k^2 U \left(g k^2 H^3+3 g H-3 U^2\right) \text{dt}^2}{H^2 k^2+3}+\frac{i \left(g H^5 U^2 k^7+g^2 H^4 k^5-3 H^2 U^4 k^5+6 g H^3 U^2 k^5-9 U^4 k^3+3 g^2 H^2 k^3+6 g H U^2 k^3\right) \text{dt}^3}{2 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+\left(-\frac{1}{12} \left(g H k^4\right) \text{dt}+\frac{i \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \text{dt}^2}{12 \left(H^2 k^2+3\right)}+\frac{\left(3 g H^5 U^2 k^{10}+2 g^2 H^4 k^8-6 H^2 U^4 k^8+18 g H^3 U^2 k^8-18 U^4 k^6+6 g^2 H^2 k^6+24 g H U^2 k^6\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 g H^5 U k^{10}-145 H^2 U^3 k^8+384 g H^3 U k^8-531 U^3 k^6+576 g H U k^6\right) \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(288 g H^7 U^2 k^{13}+241 g^2 H^6 k^{11}-723 H^4 U^4 k^{11}+2592 g H^5 U^2 k^{11}+1542 g^2 H^4 k^9-4626 H^2 U^4 k^9+7194 g H^3 U^2 k^9-7371 U^4 k^7+2457 g^2 H^2 k^7+5454 g H U^2 k^7\right) \text{dt}^3}{5760 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{i \left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}-3 k U\right) \text{dt}}{H^2 k^2+3}+\frac{\left(\sqrt{3} k^2 \sqrt{g H \left(H^2 k^2+3\right)} U-3 k^2 U^2\right)

\text{dt}^2}{H^2 k^2+3}+\left(\frac{i \left(H^4 U^3 k^7+15 H^2 U^3 k^5+9 g H^3 U k^5+36 U^3 k^3+36 g H U k^3\right)}{6 \left(H^2 k^2+3\right)^2}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^3\right) \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{1}{12} \left(k^4 U\right) \text{dt}+\frac{i k^5 \left(2 H^2 k^2 U^2+12 U^2+3 g H\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)}+\frac{\left(H^4 U^3 k^{10}+12 H^2 U^3 k^8+6 g H^3 U k^8+27 U^3 k^6+21 g H U k^6\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(32 H^4 k^9+241 H^2 k^7+531 k^5\right) U \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 H^4 U^2 k^{10}+145 g H^3 k^8+674 H^2 U^2 k^8+1638 U^2 k^6+531 g H k^6\right) \text{dt}^2}{1920 \left(H^2 k^2+3\right)^2}-\frac{i \left(32 H^6 U^3 k^{13}+529 H^4 U^3 k^{11}+241 g H^5 U k^{11}+2406 H^2 U^3 k^9+1736 g H^3 U k^9+3321 U^3 k^7+3231 g H U k^7\right) \text{dt}^3}{1920 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\end{array}
\right)

In[203]:=
```
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → 0 /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 4}, {dt, 0, 4}];
Fnn2TAr = Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 4}, {dt, 0, 4}];
FnG2TAr = Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 4}, {dt, 0, 4}];
FGn2TAr = Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}];
```

```
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 4}];
FGG2TAr = Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 + Fmat2.Fmat2.Fmat2 / 6;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep =
  Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 + EigvFmat2 * EigvFmat2 * EigvFmat2 / 6 ] /
   (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ];


Text[Row[{"U <  -Sqrt(gH) < U"}]]
Text[" "]
Text[Row[{"Fnn  ||   ", Kfnnp}]]
Text[Row[{"Fnn  ||   ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||   ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||   ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||   ", KfnGp}]]
Text[Row[{"FnG  ||   ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error  ||   ", FnG2TAr}]]
Text[Row[{"FnG error  ||   ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||   ", KfGnp}]]
Text[Row[{"FGn  ||   ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||   ", FGn2TAr}]]
Text[Row[{"FGn error  ||   ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||   ", KfGGp}]]
Text[Row[{"FGG  ||   ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||   ", FGG2TAr}]]
Text[Row[{"FGG error  ||   ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||   ", RKstepTayr}]]
Text[Row[{"Omega error  ||   ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||   ", EA}]]
```

```
Text[Row[{"EA  ||   ", TeXForm[EA]}]]
Text[Row[{"Eerr ||   ", Eerr}]]
Text[Row[{"Eerr ||   ", TeXForm[Eerr]}]]
```

Out[237]= $U < -\text{Sqrt}(gH) < U$

Out[238]=

Out[239]= Fnn || Gnp H + Rpp U

Out[240]= Fnn || \text{Gnp} H+\text{Rpp} U

Out[241]= Fnn error || $\left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{H^2\,k^3\,U\,w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\right)+$
$\left(\frac{1}{12}\,k^4\,U\,dt+O[dt]^5\right)dx^3+\left(\frac{i\left(45\,k^5\,U+143\,H^2\,k^7\,U+32\,H^4\,k^9\,U\right)dt}{960\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^4+O[dx]^5$

Out[242]= Fnn error ||
\left(−\frac{\text{dt}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 H^2 k^3 U w^2}{6 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 H^2 k^3 U w^3}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^3 \left(\frac{1}{12} k^4 U \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)+O\left(\text{dx}^5\right)

Out[243]=

Out[244]= FnG || GGp H

Out[245]= FnG || \text{GGp} H

Out[246]= FnG error || $\left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)}+\frac{k\,w^3\,dt^4}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)+\left(\frac{i\left(243\,k^5+49\,H^2\,k^7\right)dt}{960\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^4+O[dx]^5$

Out[247]= FnG error || \left(−\frac{3 \text{dt}^2 (k w)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 k w^2}{2 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k w^3}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(49 H^2 k^7+243 k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)+O\left(\text{dx}^5\right)

Out[248]=

Out[249]= FGn || H (g Rpp + Gnp U)

Out[250]= FGn || H (g \text{Rpp}+\text{Gnp} U)

Out[251]= FGn error || $\left(-\frac{\left(k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+\frac{k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^3\,dt^4}{24\left(3+H^2\,k^2\right)}+O[dt]^5\right)+$
$\left(\frac{1}{12}\,g\,H\,k^4\,dt+O[dt]^5\right)dx^3+\left(\frac{i\left(288\,g\,H\,k^5+192\,g\,H^3\,k^7+32\,g\,H^5\,k^9-243\,k^5\,U^2-49\,H^2\,k^7\,U^2\right)dt}{960\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^4+O[dx]^5$

Out[252]= FGn error ||

\left(-\frac{\text{dt}^2 \left(k w \left(g H^3 k^2+3 g H-3 U^2\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2 \left(g H^3 k^2+3 g H-3 U^2\right)}{6 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k w^3 \left(g H^3 k^2+3 g H-3 U^2\right)}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^3 \left(\frac{1}{12} g H k^4 \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+O\left(\text{dx}^5\right)

Out[253]=

Out[254]= FGG || (GGp H + Rpp) U

Out[255]= FGG || U (\text{GGp} H+\text{Rpp})

Out[256]= FGG error || $\left(-\frac{\left(k \left(6+H^2 k^2\right) U w\right) dt^2}{2 \left(3+H^2 k^2\right)}-\frac{i k \left(6+H^2 k^2\right) U w^2 dt^3}{6 \left(3+H^2 k^2\right)}+\frac{k \left(6+H^2 k^2\right) U w^3 dt^4}{24 \left(3+H^2 k^2\right)}+O[dt]^5\right)+\left(\frac{1}{12} k^4 U dt+O[dt]^5\right) dx^3+\left(\frac{i \left(531 k^5+241 H^2 k^7+32 H^4 k^9\right) U dt}{960 \left(3+H^2 k^2\right)^2}+O[dt]^5\right) dx^4+O[dx]^5$

Out[257]= FGG error ||

\left(-\frac{\text{dt}^2 \left(k U w \left(H^2 k^2+6\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k U w^2 \left(H^2 k^2+6\right)}{6 \left(H^2 k^2+3\right)}+\frac{\text{dt}^4 k U w^3 \left(H^2 k^2+6\right)}{24 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)+\text{dx}^3 \left(\frac{1}{12} k^4 U \text{dt}+O\left(\text{dt}^5\right)\right)+\text{dx}^4 \left(\frac{i \left(32 H^4 k^9+241 H^2 k^7+531 k^5\right) U \text{dt}}{960 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+O\left(\text{dx}^5\right)

Out[258]=

Out[259]=

Out[260]= Omega error || $\left\{-\frac{i \left(\sqrt{3}\ k \sqrt{g H \left(3+H^2 k^2\right)}+3 k U+H^2 k^3 U\right)^4 dt^3}{24 \left(3+H^2 k^2\right)^4}+\frac{\left(\sqrt{3}\ k \sqrt{g H \left(3+H^2 k^2\right)}+3 k U+H^2 k^3 U\right)^5 dt^4}{30 \left(3+H^2 k^2\right)^5}+O[dt]^5\right\}+$

$\left(\frac{1}{24} i k^4 \left(\sqrt{3}\ \sqrt{\frac{g H}{3+H^2 k^2}}+2 U\right)+\frac{1}{144 \left(3+H^2 k^2\right)^2} k^7 \left(9 g^2 H^2+3 g H U \left(5 \sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+9 \left(3+H^2 k^2\right) U\right)+U^3 \left(21 \sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+18 U+2 H^4 k^4 U+k^2 \left(7 \sqrt{3}\ \sqrt{g H^5 \left(3+H^2 k^2\right)}+12 H^2 U\right)\right)\right) dt^3+\frac{1}{144 \left(3+H^2 k^2\right)^3} i k^8 \left(3 g H+U \left(2 \sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)\right) \left(3 g H \left(\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+4 \left(3+H^2 k^2\right) U\right)+U^2 \left(15 \sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+18 U+2 H^4 k^4 U+k^2 \left(5 \sqrt{3}\ \sqrt{g H^5 \left(3+H^2 k^2\right)}+12 H^2 U\right)\right)\right) dt^4+O[dt]^5\right) dx^3+\left(-\frac{1}{5760 \left(3+H^2 k^2\right)^2} k^5 \left(531 \sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+1728 U+192 H^4 k^4 U+k^2 \left(145 \sqrt{3}\ \sqrt{g H^5 \left(3+H^2 k^2\right)}+1152 H^2 U\right)\right)+\left(i k^8 \left(k^6 U^3 \left(721 \sqrt{3}\ g H^7+192 \sqrt{g H^{13} \left(3+H^2 k^2\right)}\right) U\right)+9 k^2 \left(145 \sqrt{g^5 H^9 \left(3+H^2 k^2\right)}+1350 \sqrt{3}\ g^2

$$\left(i k \left(k^6 U^3 \left(-721 \sqrt{3}\ g H^7 + 192 \sqrt{g H^{13} \left(3 + H^2 k^2\right)}\ U\right) + 9 k^2 \left(145 \sqrt{g^5 H^9 \left(3 + H^2 k^2\right)} - 1350 \sqrt{3}\ g^2 H^4 U + 2118 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)}\ U^2 + 2227 \sqrt{3}\ g H^3 U^3 + 576 \sqrt{g H^5 \left(3 + H^2 k^2\right)}\ U^4\right) + 81 \left(59 \sqrt{g^5 H^5 \left(3 + H^2 k^2\right)} + 241 \sqrt{3}\ g^2 H^2 U + 64 \sqrt{g H \left(3 + H^2 k^2\right)}\ U^4 + g H U^2 \left(369 \sqrt{g H \left(3 + H^2 k^2\right)} + 251 \sqrt{3}\ U\right)\right) + 3 k^4 U \left(627 \sqrt{3}\ g^2 H^6 + 576 \sqrt{g H^9 \left(3 + H^2 k^2\right)}\ U^3 + g H^5 U \left(1011 \sqrt{g H \left(3 + H^2 k^2\right)} + 2195 \sqrt{3}\ U\right)\right)\right) dt^3\right) \Big/ \left(34\,560 \sqrt{g H}\ \left(3 + H^2 k^2\right)^{7/2}\right) - \left(\left(k^9 \left(3 g H + U \left(2 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U\right)\right) \left(3 \sqrt{3}\ g^2 H^2 \left(531 + 145 H^2 k^2\right) + g H U \left(4914 \sqrt{g H \left(3 + H^2 k^2\right)} + \sqrt{3}\ \left(5049 + 3270 H^2 k^2 + 529 H^4 k^4\right) U\right) + 6 \left(288 \sqrt{g H \left(3 + H^2 k^2\right)}\ U^3 + 32 k^4 \sqrt{g H^9 \left(3 + H^2 k^2\right)}\ U^3 + k^2 \left(241 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)}\ U + 192 \sqrt{g H^5 \left(3 + H^2 k^2\right)}\ U^3\right)\right)\right)\right) dt^4\right) \Big/ \left(34\,560 \left(\sqrt{g H}\ \left(3 + H^2 k^2\right)^{7/2}\right)\right) + O[dt]^5\right) dx^4 + O[dx]^5,$$

$$\left(-\frac{i \left(-\sqrt{3}\ k \sqrt{g H \left(3+H^2 k^2\right)} + 3 k U + H^2 k^3 U\right)^4 dt^3}{24 \left(3+H^2 k^2\right)^4} + \frac{\left(-\sqrt{3}\ k \sqrt{g H \left(3+H^2 k^2\right)} + 3 k U + H^2 k^3 U\right)^5 dt^4}{30 \left(3+H^2 k^2\right)^5} + O[dt]^5\right) +$$

$$\left(-\frac{1}{24} i k^4 \left(\sqrt{3}\ \sqrt{\frac{g H}{3+H^2 k^2}} - 2 U\right) + \frac{1}{144 \left(3+H^2 k^2\right)^2} k^7 \left(9 g^2 H^2 + 3 g H U \left(-5 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} + 9 \left(3 + H^2 k^2\right) U\right) + U^3 \left(-21 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} + 18 U + 2 H^4 k^4 U + k^2 \left(-7 \sqrt{3}\ \sqrt{g H^5 \left(3 + H^2 k^2\right)} + 12 H^2 U\right)\right)\right) dt^3 + \frac{1}{144 \left(3+H^2 k^2\right)^3} i k^8 \left(3 g H + U \left(-2 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U\right)\right) \left(-3 g H \left(\sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} - 4 \left(3 + H^2 k^2\right) U\right) + U^2 \left(-15 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} + 18 U + 2 H^4 k^4 U + k^2 \left(-5 \sqrt{3}\ \sqrt{g H^5 \left(3 + H^2 k^2\right)} + 12 H^2 U\right)\right)\right) dt^4 + O[dt]^5\right) dx^3 +$$

$$\left(\frac{1}{5760 \left(3+H^2 k^2\right)^2} k^5 \left(531 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} - 1728 U - 192 H^4 k^4 U + k^2 \left(145 \sqrt{3}\ \sqrt{g H^5 \left(3 + H^2 k^2\right)} - 1152 H^2 U\right)\right) + \frac{1}{34\,560 \sqrt{g H}\ \left(3+H^2 k^2\right)^{7/2}} i k^8 \left(k^6 U^3 \left(-721 \sqrt{3}\ g H^7 + 192 \sqrt{g H^{13} \left(3 + H^2 k^2\right)}\ U\right) + 9 k^2 \left(145 \sqrt{g^5 H^9 \left(3 + H^2 k^2\right)} - 1350 \sqrt{3}\ g^2 H^4 U + 2118 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)}\ U^2 - 2227 \sqrt{3}\ g H^3 U^3 + 576 \sqrt{g H^5 \left(3 + H^2 k^2\right)}\ U^4\right) + 81 \left(59 \sqrt{g^5 H^5 \left(3 + H^2 k^2\right)} - 241 \sqrt{3}\ g^2 H^2 U + 64 \sqrt{g H \left(3 + H^2 k^2\right)}\ U^4 +\right.$$

$$
\begin{aligned}
&\left(\mathrm{g\,H\,U^2}\left(369\sqrt{\mathrm{g\,H}\left(3+\mathrm{H^2\,k^2}\right)}-251\sqrt{3}\ \mathrm{U}\right)\right)-3\,\mathrm{k^4\,U}\\
&\left(627\sqrt{3}\ \mathrm{g^2\,H^6}-576\sqrt{\mathrm{g\,H^9}\left(3+\mathrm{H^2\,k^2}\right)}\ \mathrm{U^3}+\mathrm{g\,H^5\,U}\left(-1011\sqrt{\mathrm{g\,H}\left(3+\mathrm{H^2\,k^2}\right)}+2195\sqrt{3}\ \mathrm{U}\right)\right)\Big)\\
&\mathrm{dt^3}+\left(\mathrm{k^9}\left(3\,\mathrm{g\,H}+\mathrm{U}\left(-2\sqrt{3}\ \sqrt{\mathrm{g\,H}\left(3+\mathrm{H^2\,k^2}\right)}+\left(3+\mathrm{H^2\,k^2}\right)\mathrm{U}\right)\right)\left(3\sqrt{3}\ \mathrm{g^2\,H^2}\left(531+145\,\mathrm{H^2\,k^2}\right)+\right.\right.\\
&\mathrm{g\,H\,U}\left(-4914\sqrt{\mathrm{g\,H}\left(3+\mathrm{H^2\,k^2}\right)}+\sqrt{3}\ \left(5049+3270\,\mathrm{H^2\,k^2}+529\,\mathrm{H^4\,k^4}\right)\mathrm{U}\right)-\\
&6\left(288\sqrt{\mathrm{g\,H}\left(3+\mathrm{H^2\,k^2}\right)}\ \mathrm{U^3}+32\,\mathrm{k^4}\sqrt{\mathrm{g\,H^9}\left(3+\mathrm{H^2\,k^2}\right)}\ \mathrm{U^3}+\right.\\
&\left.\left.\mathrm{k^2}\left(241\sqrt{\mathrm{g^3\,H^7}\left(3+\mathrm{H^2\,k^2}\right)}\ \mathrm{U}+192\sqrt{\mathrm{g\,H^5}\left(3+\mathrm{H^2\,k^2}\right)}\ \mathrm{U^3}\right)\right)\right)\\
&\left.\mathrm{dt^4}\right)\Big/\left(34\,560\sqrt{\mathrm{g\,H}}\ \left(3+\mathrm{H^2\,k^2}\right)^{7/2}\right)+\mathrm{O[dt]^5}\Big)\,\mathrm{dx^4}+\mathrm{O[dx]^5}\Big\}
\end{aligned}
$$

Out[261]= Omega error ||

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\left\{\left(-\frac{i \left(H^2 U k^3+3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^4 \text{dt}^3}{24 \left(H^2
    k^2+3\right)^4}+\frac{\left(H^2 U k^3+3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^5
    \text{dt}^4}{30 \left(H^2 k^2+3\right)^5}+O\left(\text{dt}^5\right)\right)+\left(\frac{1}{24} i k^4 \left(2
    U+\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}\right)+\frac{k^7 \left(\left(2 H^4 U k^4+\left(12 U H^2+7 \sqrt{3}
    \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^3+3
    g H \left(9 \left(H^2 k^2+3\right) U+5 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right)
    \text{dt}^3}{144 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3}
    \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(\left(2 H^4 U k^4+\left(12 U H^2+5 \sqrt{3} \sqrt{g H^5
    \left(H^2 k^2+3\right)}\right) k^2+18 U+15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g H \left(4
    \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{144 \left(H^2
    k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(-\frac{k^5 \left(192 H^4 U k^4+\left(1152
    U H^2+145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+1728 U+531 \sqrt{3} \sqrt{g H \left(H^2
    k^2+3\right)}\right)}{5760 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(U^3 \left(721 \sqrt{3} g H^7+192
    \sqrt{g H^{13} \left(H^2 k^2+3\right)} U\right) k^6+3 U \left(627 \sqrt{3} g^2 H^6+g U \left(2195 \sqrt{3}
    U+1011 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+576 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3\right)
    k^4+9 \left(1350 \sqrt{3} g^2 U H^4+2227 \sqrt{3} g U^3 H^3+576 \sqrt{g H^5 \left(H^2 k^2+3\right)}
    U^4+2118 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+145 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right)
    k^2+81 \left(64 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(251 \sqrt{3} U+369 \sqrt{g H \left(H^2
    k^2+3\right)}\right) U^2+241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)
    \text{dt}^3}{34560 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}-\frac{\left(k^9 \left(3 g H+U \left(\left(H^2
    k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 \sqrt{3} g^2 \left(145 H^2
    k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right) U+4914 \sqrt{g
    H \left(H^2 k^2+3\right)}\right) H+6 \left(32 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+\left(192
    \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+241 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+288
    \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right)\right) \text{dt}^4}{34560 \left(\sqrt{g H} \left(H^2
    k^2+3\right)^{7/2}\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right),\left(-\frac{i
    \left(H^2 U k^3+3 U k-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^4 \text{dt}^3}{24 \left(H^2
    k^2+3\right)^4}+\frac{\left(H^2 U k^3+3 U k-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right)^5
    \text{dt}^4}{30 \left(H^2 k^2+3\right)^5}+O\left(\text{dt}^5\right)\right)+\left(-\frac{1}{24} i k^4 \left(\sqrt{3}
    \sqrt{\frac{g H}{H^2 k^2+3}}-2 U\right)+\frac{k^7 \left(\left(2 H^4 U k^4+\left(12 H^2 U-7 \sqrt{3} \sqrt{g
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\sqrt{\frac{g H}{H^2 k^2+3}}-2 U\right)+\frac{k^7 \left(\left(2 H^4 U k^4+\left(12 H^2 U-5 \sqrt{3} \sqrt{g
H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U-21 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^3+3 g
H \left(9 \left(H^2 k^2+3\right) U-5 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right)
\text{dt}^3}{144 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2
\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(U^2 \left(2 H^4 U k^4+\left(12 H^2 U-5 \sqrt{3}
\sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U-15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)-3
g H \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}-4 \left(H^2 k^2+3\right) U\right)\right) \text{dt}^4}{144
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{k^5 \left(-192 H^4 U
k^4+\left(145 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-1152 H^2 U\right) k^2-1728 U+531 \sqrt{3}
\sqrt{g H \left(H^2 k^2+3\right)}\right)}{5760 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(U^3 \left(192
\sqrt{g H^{13} \left(H^2 k^2+3\right)} U-721 \sqrt{3} g H^7\right) k^6-3 U \left(627 \sqrt{3} g^2 H^6+g
U \left(2195 \sqrt{3} U-1011 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5-576 \sqrt{g H^9 \left(H^2
k^2+3\right)} U^3\right) k^4+9 \left(-1350 \sqrt{3} g^2 U H^4-2227 \sqrt{3} g U^3 H^3+576 \sqrt{g
H^5 \left(H^2 k^2+3\right)} U^4+2118 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+145 \sqrt{g^5 H^9
\left(H^2 k^2+3\right)}\right) k^2+81 \left(64 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(369 \sqrt{g
H \left(H^2 k^2+3\right)}-251 \sqrt{3} U\right) U^2-241 \sqrt{3} g^2 H^2 U+59 \sqrt{g^5 H^5 \left(H^2
k^2+3\right)}\right)\right) \text{dt}^3}{34560 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}+\frac{k^9 \left(3 g
H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 \sqrt{3}
g^2 \left(145 H^2 k^2+531\right) H^2+g U \left(\sqrt{3} \left(529 H^4 k^4+3270 H^2 k^2+5049\right)
U-4914 \sqrt{g H \left(H^2 k^2+3\right)}\right) H-6 \left(32 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3
k^4+\left(192 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+241 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}
U\right) k^2+288 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text{dt}^4}{34560 \sqrt{g H}
\left(H^2 k^2+3\right)^{7/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right\}

Out[262]=

Out[263]= EA || $\left\{\left\{\frac{-H^2 k^2 \left(\left(-1+e^{i\,\text{dt}\,w}\right) k\,U-w\right)+3\,w}{\left(3+H^2 k^2\right) w},\ -\frac{3\left(-1+e^{i\,\text{dt}\,w}\right) k}{\left(3+H^2 k^2\right) w}\right\},\left\{-\frac{\left(-1+e^{i\,\text{dt}\,w}\right) k\left(g\,H\left(3+H^2 k^2\right)-3\,U^2\right)}{\left(3+H^2 k^2\right) w},\ 1-\frac{\left(-1+e^{i\,\text{dt}\,w}\right) k\left(6+H^2 k^2\right) U}{\left(3+H^2 k^2\right) w}\right\}\right\}$

Out[264]= EA || \left(
\begin{array}{cc}
\frac{3 w-H^2 k^2 \left(\left(-1+e^{i \text{dt} w}\right) k U-w\right)}{\left(H^2
k^2+3\right) w} & -\frac{3 \left(-1+e^{i \text{dt} w}\right) k}{\left(H^2 k^2+3\right) w} \\
-\frac{\left(-1+e^{i \text{dt} w}\right) k \left(g H \left(H^2 k^2+3\right)-3 U^2\right)}{\left(H^2 k^2+3\right)
w} & 1-\frac{\left(-1+e^{i \text{dt} w}\right) k \left(H^2 k^2+6\right) U}{\left(H^2 k^2+3\right) w} \\
\end{array}
\right)

Out[265]= Eerr || $\left\{\left\{\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2 k^2\right)}+3\,k\,U\right)\text{dt}}{3+H^2 k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2 k^2\right)}\ U+3\,k^2\,U^2\right)\text{dt}^2}{3+H^2 k^2}+\right.\right.$

$\left.\left(-\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2 k^2\right)}}{3+H^2 k^2}-i\,k\,U\right)^3+\frac{i\left(18\,g\,H\,k^3\,U+9\,g\,H^3 k^5\,U-18\,k^3\,U^3-3\,H^2 k^5\,U^3+H^4 k^7\,U^3\right)}{6\left(3+H^2 k^2\right)^2}\right)\text{dt}^3-\right.$

$\left.\left(-\sqrt{\qquad}\right)^4\right)$

$$\frac{1}{24}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)\ dt^4+O[dt]^5\Bigg)+$$

$$\left(\frac{1}{12}\,k^4\,U\,dt-\frac{i\left(3\,g\,H\,k^5+2\,H^2\,k^7\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)}-\frac{\left(U\left(15\,g\,H\,k^6+6\,g\,H^3\,k^8-9\,k^6\,U^2+H^4\,k^{10}\,U^2\right)\right)dt^3}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(45\,k^5\,U+143\,H^2\,k^7\,U+32\,H^4\,k^9\,U\right)dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(531\,g\,H\,k^6+145\,g\,H^3\,k^8-486\,k^6\,U^2+94\,H^2\,k^8\,U^2+64\,H^4\,k^{10}\,U^2\right)dt^2}{1920\left(3+H^2\,k^2\right)^2}-\frac{1}{1920\left(3+H^2\,k^2\right)^3}\right.$$

$$i\,k^7\left(1683\,g\,H\,U+1348\,g\,H^3\,k^2\,U+241\,g\,H^5\,k^4\,U-1593\,U^3-678\,H^2\,k^2\,U^3+47\,H^4\,k^4\,U^3+32\,H^6\,k^6\,U^3\right)$$

$$\left.dt^3+O[dt]^5\right)dx^4+O[dx]^5,$$

$$\left(-\frac{3\,i\,k\,dt}{3+H^2\,k^2}-\frac{3\left(k^2\,U\right)dt^2}{3+H^2\,k^2}+\frac{3\,i\left(g\,H\,k^3+3\,k^3\,U^2+H^2\,k^5\,U^2\right)dt^3}{2\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)+\left(-\frac{i\,k^5\,U\,dt^2}{4\left(3+H^2\,k^2\right)}+\frac{\left(-g\,H\,k^6-6\,k^6\,U^2-2\,H^2\,k^8\,U^2\right)dt^3}{8\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(243\,k^5+49\,H^2\,k^7\right)dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(531\,k^6+145\,H^2\,k^8\right)U\,dt^2}{960\left(3+H^2\,k^2\right)^2}-\frac{i\left(774\,g\,H\,k^7+194\,g\,H^3\,k^9+2457\,k^7\,U^2+1542\,H^2\,k^9\,U^2+241\,H^4\,k^{11}\,U^2\right)dt^3}{1920\left(3+H^2\,k^2\right)^3}+O[dt]^5\right)dx^4+$$

$$O[dx]^5\Bigg\},$$

$$\left\{\left(\left(-i\,g\,H\,k+\frac{i\,H\,k\,U^2}{H+\frac{H^3\,k^2}{3}}\right)dt-\frac{k^2\,U\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)dt^2}{3+H^2\,k^2}+\frac{1}{2\left(3+H^2\,k^2\right)^2}\right.\right.$$

$$\left.i\left(3\,g^2\,H^2\,k^3+g^2\,H^4\,k^5+6\,g\,H\,k^3\,U^2+6\,g\,H^3\,k^5\,U^2+g\,H^5\,k^7\,U^2-9\,k^3\,U^4-3\,H^2\,k^5\,U^4\right)dt^3+O[dt]^5\right)+$$

$$\left(\frac{1}{12}\,g\,H\,k^4\,dt-\frac{i\left(6\,g\,H\,k^5\,U+2\,g\,H^3\,k^7\,U-3\,k^5\,U^3\right)dt^2}{12\left(3+H^2\,k^2\right)}-\frac{1}{24\left(3+H^2\,k^2\right)^2}\left(6\,g^2\,H^2\,k^6+2\,g^2\,H^4\,k^8+24\,g\,H\,k^6\,U^2+\right.\right.$$

$$\left.18\,g\,H^3\,k^8\,U^2+3\,g\,H^5\,k^{10}\,U^2-18\,k^6\,U^4-6\,H^2\,k^8\,U^4\right)dt^3+O[dt]^5\Bigg)dx^3+$$

$$\left(\frac{i\left(288\,g\,H\,k^5+192\,g\,H^3\,k^7+32\,g\,H^5\,k^9-243\,k^5\,U^2-49\,H^2\,k^7\,U^2\right)dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(576\,g\,H\,k^6\,U+384\,g\,H^3\,k^8\,U+64\,g\,H^5\,k^{10}\,U-531\,k^6\,U^3-145\,H^2\,k^8\,U^3\right)dt^2}{960\left(3+H^2\,k^2\right)^2}-\right.$$

$$\frac{1}{5760\left(3+H^2\,k^2\right)^3}i\left(2457\,g^2\,H^2\,k^7+1542\,g^2\,H^4\,k^9+241\,g^2\,H^6\,k^{11}+5454\,g\,H\,k^7\,U^2+7194\,g\,H^3\,k^9\,U^2+\right.$$

$$\left.\left.2592\,g\,H^5\,k^{11}\,U^2+288\,g\,H^7\,k^{13}\,U^2-7371\,k^7\,U^4-4626\,H^2\,k^9\,U^4-723\,H^4\,k^{11}\,U^4\right)dt^3+O[dt]^5\Bigg)$$

$$dx^4+O[dx]^5,\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}\ -3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U-3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}+\right.$$

$$\left(-\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3+\frac{i\left(36\,g\,H\,k^3\,U+9\,g\,H^3\,k^5\,U+36\,k^3\,U^3+15\,H^2\,k^5\,U^3+H^4\,k^7\,U^3\right)}{6\left(3+H^2\,k^2\right)^2}\right)dt^3-$$

$$\frac{1}{24}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4\ dt^4+O[dt]^5\Bigg)+$$

$$\left(\frac{1}{12}\,k^4\,U\,dt-\frac{i\,k^5\left(3\,g\,H+12\,U^2+2\,H^2\,k^2\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)}-\frac{\left(21\,g\,H\,k^6\,U+6\,g\,H^3\,k^8\,U+27\,k^6\,U^3+12\,H^2\,k^8\,U^3+H^4\,k^{10}\,U^3\right)dt^3}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(531\,k^5+241\,H^2\,k^7+32\,H^4\,k^9\right)U\,dt}{960\left(3+H^2\,k^2\right)^2}+\frac{\left(531\,g\,H\,k^6+145\,g\,H^3\,k^8+1638\,k^6\,U^2+674\,H^2\,k^8\,U^2+64\,H^4\,k^{10}\,U^2\right)dt^2}{1920\left(3+H^2\,k^2\right)^2}-\right.$$

$$\frac{1}{1920\left(3+H^2\,k^2\right)^3}i\left(3231\,g\,H\,k^7\,U+1736\,g\,H^3\,k^9\,U+241\,g\,H^5\,k^{11}\,U+3321\,k^7\,U^3+\right.$$

$$\left.\left.\left.2406\,H^2\,k^9\,U^3+529\,H^4\,k^{11}\,U^3+32\,H^6\,k^{13}\,U^3\right)dt^3+O[dt]^5\right)dx^4+O[dx]^5\right)\Bigg\}$$

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$$\left(\frac{i \left(3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \text{dt}}{H^2 k^2+3}+\frac{\left(3 U^2 k^2+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2\right) \text{dt}^2}{H^2 k^2+3}+\left(\frac{i \left(H^4 U^3 k^7-3 H^2 U^3 k^5+9 g H^3 U k^5-18 U^3 k^3+18 g H U k^3\right)}{6 \left(H^2 k^2+3\right)^2}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^3\right) \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(\frac{1}{12} k^4 U \text{dt}-\frac{i \left(2 H^2 U^2 k^7+3 g H k^5\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)}-\frac{\left(U \left(H^4 U^2 k^{10}+6 g H^3 k^8-9 U^2 k^6+15 g H k^6\right)\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(32 H^4 U k^9+143 H^2 U k^7+45 U k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 H^4 U^2 k^{10}+145 g H^3 k^8+94 H^2 U^2 k^8-486 U^2 k^6+531 g H k^6\right) \text{dt}^2}{1920 \left(H^2 k^2+3\right)^2}-\frac{i k^7 \left(32 H^6 U^3 k^6+47 H^4 U^3 k^4+241 g H^5 U k^4-678 H^2 U^3 k^2+1348 g H^3 U k^2-1593 U^3+1683 g H U\right) \text{dt}^3}{1920 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) & \left(-\frac{3 i k \text{dt}}{H^2 k^2+3}-\frac{3 \left(k^2 U\right) \text{dt}^2}{H^2 k^2+3}+\frac{3 i \left(H^2 U^2 k^5+3 U^2 k^3+g H k^3\right) \text{dt}^3}{2 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i k^5 U \text{dt}^2}{4 \left(H^2 k^2+3\right)}+\frac{\left(-2 H^2 U^2 k^8-6 U^2 k^6-g H k^6\right) \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(49 H^2 k^7+243 k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(145 H^2 k^8+531 k^6\right) U \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(241 H^4 U^2 k^{11}+194 g H^3 k^9+1542 H^2 U^2 k^9+2457 U^2 k^7+774 g H k^7\right) \text{dt}^3}{1920 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\$$

$$\left(\left(\frac{i H k U^2}{\frac{k^2 H^3}{3}+H}-i g H k\right) \text{dt}-\frac{k^2 U \left(g k^2 H^3+3 g H-3 U^2\right) \text{dt}^2}{H^2 k^2+3}+\frac{i \left(g H^5 U^2 k^7+g^2 H^4 k^5-3 H^2 U^4 k^5+6 g H^3 U^2 k^5-9 U^4 k^3+3 g^2 H^2 k^3+6 g H U^2 k^3\right) \text{dt}^3}{2 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)+\left(\frac{1}{12} g H k^4 \text{dt}-\frac{i \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \text{dt}^2}{12 \left(H^2 k^2+3\right)}-\frac{\left(3 g H^5 U^2 k^{10}+2 g^2 H^4 k^8-6 H^2 U^4 k^8+18 g H^3 U^2 k^8-18 U^4 k^6+6 g^2 H^2 k^6+24 g H U^2 k^6\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(32 g H^5 k^9+192 g H^3 k^7-49 H^2 U^2 k^7-243 U^2 k^5+288 g H k^5\right) \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64 g H^5 U k^{10}-145 H^2 U^3 k^8+384 g H^3 U k^8-531 U^3 k^6+576 g H U k^6\right) \text{dt}^2}{960 \left(H^2 k^2+3\right)^2}-\frac{i \left(288 g H^7 U^2 k^{13}+241 g^2 H^6 k^{11}-723 H^4 U^4 k^{11}+2592 g H^5 U^2 k^{11}+1542 g^2 H^4 k^9-4626 H^2 U^4 k^9+7194 g H^3 U^2 k^9-7371 U^4 k^7+2457 g^2 H^2 k^7+5454 g H U^2 k^7\right) \text{dt}^3}{5760 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{i \left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}-3 k U\right) \text{dt}}{H^2 k^2+3}+\frac{\left(\sqrt{3} k^2 \sqrt{g H \left(H^2 k^2+3\right)} U-3 k^2 U^2\right) \text{dt}^2}{H^2 k^2+3}+\left(\frac{i \left(H^4 U^3 k^7+15 H^2 U^3 k^5+9 g H^3 U k^5+36 U^3 k^3+36 g H U k^3\right)}{6 \left(H^2 k^2+3\right)^2}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^3\right) \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(\frac{1}{12} k^4 U \text{dt}-\frac{i k^5 \left(2 H^2 k^2 U^2+12 U^2+3 g H\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)}-\frac{\left(H^4 U^3 k^{10}+12 H^2 U^3 k^8+6 g H^3 U k^8+27 U^3 k^6+21 g H U k^6\right) \text{dt}^3}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i$$

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\left(32 H^4 k^9+241 H^2 k^7+531 k^5\right) U \text{dt}}{960 \left(H^2 k^2+3\right)^2}+\frac{\left(64
H^4 U^2 k^{10}+145 g H^3 k^8+674 H^2 U^2 k^8+1638 U^2 k^6+531 g H k^6\right) \text{dt}^2}{1920
\left(H^2 k^2+3\right)^2}−\frac{i \left(32 H^6 U^3 k^{13}+529 H^4 U^3 k^{11}+241 g H^5 U
k^{11}+2406 H^2 U^3 k^9+1736 g H^3 U k^9+3321 U^3 k^7+3231 g H U k^7\right) \text{dt}^3}{1920
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
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