In[1]:=
```
q = q0 * Exp[I * (k * x + w * t)];
qjn = q0 * Exp[I * (k * xj + w * tn)];
qjbar = Integrate[q, {x, xj - dx / 2, xj + dx / 2}] / (dx);
qjnbar = qjbar /. t → tn;
MA = qjn / qjnbar;

qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
qjntbar = qntbar /. x → xj;
MtA = qjntbar / qjn;

qjphn = q0 * Exp[I * (k * (xj + dx / 2) + w * tn)];
RA = Simplify[MA * qjphn / (qjn)];

vmultG = H + H^3 / 3 * k^2;
GnA = -U * RA / vmultG;
GGA = RA / vmultG;
GcA = -U * H / vmultG;

fn1A = H * vh + U * eh;
fn1A = fn1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca;
fn1Gca0A = fn1A /. Gca → 0;
fn1eca0A = fn1A /. eca → 0;
fnnA = Simplify[fn1Gca0A / eca];
fnGA = fn1eca0A / Gca;
fncA = H * GcA;

fG1A = U * Gh + U * H * vh + g * H * eh;
fG1A = fG1A /. vh → (GGA * Gca + GnA * eca) /. eh → RA * eca /. Gh → RA * Gca;
fG1Gca0A = fG1A /. Gca → 0;
fG1eca0A = fG1A /. eca → 0;
fGnA = Simplify[fG1Gca0A / eca];
fGGA = Simplify[fG1eca0A / Gca];
fGcA = U * H * GcA;

FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;

MatA = {{FnnA, FnGA}, {FGnA, FGGA}};
EA = {{1, 0}, {0, 1}} + MatA;
```

$$\text{wAp} = \text{U} * \text{k} + \frac{\sqrt{3}\ \text{k} \sqrt{\text{g H} \left(3 + \text{H}^2\ \text{k}^2\right)}}{3 + \text{H}^2\ \text{k}^2};$$

$$\text{wAm} = \text{U} * \text{k} - \frac{\sqrt{3}\ \text{k} \sqrt{\text{g H} \left(3 + \text{H}^2\ \text{k}^2\right)}}{3 + \text{H}^2\ \text{k}^2};$$

In[37]:=
```
M = 1;
Merr = Series[M - MA, {dx, 0, 5}];
Rm = (1 + I * Sin[k * dx] / 2);
Rmerr = Series[Rm - RA, {dx, 0, 5}];
Rp = Exp[I * k * dx] * (1 - I * Sin[k * dx] / 2);
Rperr = Series[Rp - RA, {dx, 0, 5}];
Ru = (1 + Exp[I * k * dx]) / 2;
Ruerr = Series[Ru - Exp[I * k * dx / 2], {dx, 0, 5}];
Gold = H - H^3 / 3 * (2 * Cos[k * dx] - 2) / dx^2;
GG2 = Ru / Gold;
GG2err = Series[GG2 - GGA, {dx, 0, 5}];
Gn2 = -U * Ru / Gold;
Gn2err = Series[Gn2 - GnA, {dx, 0, 5}];

Text[Row[{"M  ||   ", M}]]
Text[Row[{"M  ||   ", TeXForm[M]}]]
Text[Row[{"M error  ||   ", TeXForm[Merr]}]]
Text[Row[{"M error  ||   ", Merr}]]
Text[" "]
Text[Row[{"Rm  ||   ", Rm}]]
Text[Row[{"Rm  ||   ", TeXForm[Rm]}]]
Text[Row[{"Rm error  ||   ", Rmerr}]]
Text[Row[{"Rm error  ||   ", TeXForm[Rmerr]}]]
Text[" "]
Text[Row[{"Rp  ||   ", Rp}]]
Text[Row[{"Rp  ||   ", TeXForm[Rp]}]]
Text[Row[{"Rp error  ||   ", Rperr}]]
Text[Row[{"Rp error  ||   ", TeXForm[Rperr]}]]
Text[" "]
Text[Row[{"GG2  ||   ", GG2}]]
Text[Row[{"GG2  ||   ", TeXForm[GG2]}]]
Text[Row[{"GG2 error  ||   ", GG2err}]]
Text[Row[{"GG2 error  ||   ", TeXForm[GG2err]}]]
Text[" "]
Text[Row[{"Gn2  ||   ", Gn2}]]
Text[Row[{"Gn2  ||   ", TeXForm[Gn2]}]]
Text[Row[{"Gn2 error  ||   ", Gn2err}]]
Text[Row[{"Gn2 error  ||   ", TeXForm[Gn2err]}]]
```

Out[50]= M || 1

Out[51]= M || 1

Out[52]= M error || $-\frac{\text{dx}^2 k^2}{24}-\frac{7 \text{dx}^4 k^4}{5760}+O\left(\text{dx}^6\right)$

Out[53]= M error || $-\frac{k^2\,dx^2}{24}-\frac{7\,k^4\,dx^4}{5760}+O[dx]^6$

Out[54]=

Out[55]= Rm || $1+\frac{1}{2}\,i\,\text{Sin}[dx\,k]$

Out[56]= Rm || 1+\frac{1}{2} i \sin (\text{dx} k)

Out[57]= Rm error || $\frac{k^2\,dx^2}{12}-\frac{1}{12}\,i\,k^3\,dx^3+\frac{k^4\,dx^4}{720}+\frac{1}{240}\,i\,k^5\,dx^5+O[dx]^6$

Out[58]= Rm error || \frac{\text{dx}^2 k^2}{12}-\frac{1}{12} i \text{dx}^3 k^3+\frac{\text{dx}^4 k^4}{720}+\frac{1}{240} i \text{dx}^5 k^5+O\left(\text{dx}^6\right)

Out[59]=

Out[60]= Rp || $e^{i\,dx\,k}\left(1-\frac{1}{2}\,i\,\text{Sin}[dx\,k]\right)$

Out[61]= Rp || e^{i \text{dx} k} \left(1-\frac{1}{2} i \sin (\text{dx} k)\right)

Out[62]= Rp error || $\frac{k^2\,dx^2}{12}+\frac{1}{6}\,i\,k^3\,dx^3-\frac{89\,k^4\,dx^4}{720}-\frac{7}{120}\,i\,k^5\,dx^5+O[dx]^6$

Out[63]= Rp error || \frac{\text{dx}^2 k^2}{12}+\frac{1}{6} i \text{dx}^3 k^3-\frac{89 \text{dx}^4 k^4}{720}-\frac{7}{120} i \text{dx}^5 k^5+O\left(\text{dx}^6\right)

Out[64]=

Out[65]= GG2 || $\frac{1+e^{i\,dx\,k}}{2\left(H-\frac{H^3\,(-2+2\,\text{Cos}[dx\,k])}{3\,dx^2}\right)}$

Out[66]= GG2 || \frac{1+e^{i \text{dx} k}}{2 \left(H-\frac{H^3 (2 \cos (\text{dx} k)-2)}{3 \text{dx}^2}\right)}

Out[67]= GG2 error || $\frac{\left(-6\,k^2-H^2\,k^4\right)dx^2}{4\,H\left(3+H^2\,k^2\right)^2}-\frac{i\left(6\,k^3+H^2\,k^5\right)dx^3}{8\,H\left(3+H^2\,k^2\right)^2}+\frac{\left(144\,k^4+45\,H^2\,k^6+4\,H^4\,k^8\right)dx^4}{240\,H\left(3+H^2\,k^2\right)^3}-\frac{i\left(-54\,k^5+H^4\,k^9\right)dx^5}{480\,H\left(3+H^2\,k^2\right)^3}+O[dx]^6$

Out[68]= GG2 error || \frac{\text{dx}^2 \left(-H^2 k^4-6 k^2\right)}{4 H \left(H^2 k^2+3\right)^2}-\frac{i \text{dx}^3 \left(H^2 k^5+6 k^3\right)}{8 H \left(H^2 k^2+3\right)^2}+\frac{\text{dx}^4 \left(4 H^4 k^8+45 H^2 k^6+144 k^4\right)}{240 H \left(H^2 k^2+3\right)^3}-\frac{i \text{dx}^5 \left(H^4 k^9-54 k^5\right)}{480 H \left(H^2 k^2+3\right)^3}+O\left(\text{dx}^6\right)

Out[69]=

Out[70]= Gn2 || $-\frac{\left(1+e^{i\,dx\,k}\right)U}{2\left(H-\frac{H^3\,(-2+2\,\text{Cos}[dx\,k])}{3\,dx^2}\right)}$

Out[71]= Gn2 || -\frac{U \left(1+e^{i \text{dx} k}\right)}{2 \left(H-\frac{H^3 (2 \cos (\text{dx} k)-2)}{3 \text{dx}^2}\right)}

Out[72]= Gn2 error || $\frac{\left(6\,k^2+H^2\,k^4\right)U\,dx^2}{4\,H\left(3+H^2\,k^2\right)^2}+\frac{i\left(6\,k^3+H^2\,k^5\right)U\,dx^3}{8\,H\left(3+H^2\,k^2\right)^2}-\frac{\left(\left(144\,k^4+45\,H^2\,k^6+4\,H^4\,k^8\right)U\right)dx^4}{240\left(H\left(3+H^2\,k^2\right)^3\right)}+\frac{i\left(-54\,k^5+H^4\,k^9\right)U\,dx^5}{480\,H\left(3+H^2\,k^2\right)^3}+O[dx]^6$

Out[73]= Gn2 error ||

\frac{\text{dx}^2 U \left(H^2 k^4+6 k^2\right)}{4 H \left(H^2 k^2+3\right)^2}+\frac{i \text{dx}^3 U \left(H^2 k^5+6 k^3\right)}{8 H \left(H^2 k^2+3\right)^2}−\frac{\text{dx}^4 \left(U \left(4 H^4 k^8+45 H^2 k^6+144 k^4\right)\right)}{240 \left(H \left(H^2 k^2+3\right)^3\right)}+\frac{i \text{dx}^5 U \left(H^4 k^9−54 k^5\right)}{480 H \left(H^2 k^2+3\right)^3}+O\left(\text{dx}^6\right)

In[74]:=
```
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (U - Sqrt[g * H]);
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 ;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
```

```
RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ];


Text[Row[{" -Sqrt(gH) < U  < Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn  ||   ", Kfnnp}]]
Text[Row[{"Fnn  ||   ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||   ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||   ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||   ", KfnGp}]]
Text[Row[{"FnG  ||   ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error  ||   ", FnG2TAr}]]
Text[Row[{"FnG error  ||   ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||   ", KfGnp}]]
Text[Row[{"FGn  ||   ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||   ", FGn2TAr}]]
Text[Row[{"FGn error  ||   ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||   ", KfGGp}]]
Text[Row[{"FGG  ||   ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||   ", FGG2TAr}]]
Text[Row[{"FGG error  ||   ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||   ", RKstepTayr}]]
Text[Row[{"Omega error  ||   ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||   ", EA}]]
Text[Row[{"EA  ||   ", TeXForm[EA]}]]
Text[Row[{"Eerr ||   ", Eerr}]]
Text[Row[{"Eerr ||   ", TeXForm[Eerr]}]]
```

Out[108]= $-\text{Sqrt(gH)} < \text{U} \ < \text{Sqrt(gH)}$

Out[109]=

Out[110]= Fnn || $\frac{1}{2}\left(2\,\text{Gnp}\,H + \text{Rpp}\left(-\sqrt{g\,H} + U\right) + \text{Rmp}\left(\sqrt{g\,H} + U\right)\right)$

Out[111]= Fnn || \frac{1}{2} \left(\text{Rmp} \left(\sqrt{g H}+U\right)+\text{Rpp} \left(U−\sqrt{g H}\right)+2 \text{Gnp} H\right)

Out[112]= Fnn error || $\left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{i\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)U\,dt}{12\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)dx^2 + \left(-\frac{1}{8}\left(\sqrt{g\,H}\;k^4\right)dt + O[dt]^4\right)dx^3 + O[dx]^4$

Out[113]= Fnn error ||
\left(−\frac{\text{dt}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 H^2 k^3 U w^2}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(−\frac{i \left(H^4 k^7+9 H^2 k^5+27 k^3\right) U \text{dt}}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(−\frac{1}{8} \left(\sqrt{g H} k^4\right) \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[114]=

Out[115]= FnG || GGp H

Out[116]= FnG || \text{GGp} H

Out[117]= FnG error || $\left(-\frac{3\,(k\,w)\,dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\,w^2\,dt^3}{2\left(3+H^2\,k^2\right)} + O[dt]^4\right) + \left(\frac{i\left(6\,k^3+H^2\,k^5\right)dt}{4\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)dx^2 + O[dx]^4$

Out[118]= FnG error || \left(−\frac{3 \text{dt}^2 (k w)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 k w^2}{2 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(\frac{i \left(H^2 k^5+6 k^3\right) \text{dt}}{4 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[119]=

Out[120]= FGn || $\frac{1}{2}\left(g\,H\,(\text{Rmp} + \text{Rpp}) + \left(2\,\text{Gnp}\,H + \sqrt{g\,H}\;(\text{Rmp} - \text{Rpp})\right)U\right)$

Out[121]= FGn ||
\frac{1}{2} \left(U \left(\sqrt{g H} (\text{Rmp}−\text{Rpp})+2 \text{Gnp} H\right)+g H (\text{Rmp}+\text{Rpp})\right)

Out[122]= FGn error || $\left(-\frac{\left(k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w\right)dt^2}{2\left(3+H^2\,k^2\right)} - \frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)w^2\,dt^3}{6\left(3+H^2\,k^2\right)} + O[dt]^4\right) +$
$\left(-\frac{i\left(9\,g\,H\,k^3+6\,g\,H^3\,k^5+g\,H^5\,k^7+18\,k^3\,U^2+3\,H^2\,k^5\,U^2\right)dt}{12\left(3+H^2\,k^2\right)^2} + O[dt]^4\right)dx^2 + \left(-\frac{1}{8}\left(\sqrt{g\,H}\;k^4\,U\right)dt + O[dt]^4\right)dx^3 + O[dx]^4$

Out[123]= FGn error ||
\left(−\frac{\text{dt}^2 \left(k w \left(g H^3 k^2+3 g H−3 U^2\right)\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 k w^2 \left(g H^3 k^2+3 g H−3 U^2\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(−\frac{i \left(g H^5 k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(−\frac{1}{8} \left(\sqrt{g H} k^4 U\right) \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[124]=

Out[125]= FGG || $\frac{1}{2}\left(\sqrt{g\,H}\ \text{Rmp}-\sqrt{g\,H}\ \text{Rpp}+(2\,\text{GGp}\,H+\text{Rmp}+\text{Rpp})\,U\right)$

Out[126]= FGG || \frac{1}{2} \left(\text{Rmp} \sqrt{g H}-\text{Rpp} \sqrt{g H}+U (2 \text{GGp} H+\text{Rmp}+\text{Rpp})\right)

Out[127]= FGG error || $\left(-\frac{\left(k\left(6+H^2\,k^2\right)U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,k\left(6+H^2\,k^2\right)U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(-9\,k^3+3\,H^2\,k^5+H^4\,k^7\right)U\,dt}{12\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+$
$\left(-\frac{1}{8}\left(\sqrt{g\,H}\ k^4\right)dt+O[dt]^4\right)dx^3+\left(\frac{i\left(297\,k^5\,U+351\,H^2\,k^7\,U+118\,H^4\,k^9\,U+13\,H^6\,k^{11}\,U\right)dt}{240\left(3+H^2\,k^2\right)^3}+O[dt]^4\right)dx^4+O[dx]^5$

Out[128]= FGG error ||
\left(-\frac{\text{dt}^2 \left(k U w \left(H^2 k^2+6\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k U w^2 \left(H^2 k^2+6\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(H^4 k^7+3 H^2 k^5-9 k^3\right) U \text{dt}}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(-\frac{1}{8} \left(\sqrt{g H} k^4\right) \text{dt}+O\left(\text{dt}^4\right)\right)+\text{dx}^4 \left(\frac{i \left(13 H^6 U k^{11}+118 H^4 U k^9+351 H^2 U k^7+297 U k^5\right) \text{dt}}{240 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^5\right)

Out[129]=

Out[130]=

Out[131]= Omega error ||

$$\left\{\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)dt^2+\right.$$
$$\frac{1}{8\left(3+H^2\,k^2\right)^3}i\,k^4\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(3\,g\left(\sqrt{3}\ H\sqrt{g\,H\left(3+H^2\,k^2\right)}+9\,H\,U+3\,H^3\,k^2\,U\right)+\right.$$
$$\left.U^2\left(H^4\,k^4\,U+9\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+U\right)+3\,k^2\left(\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}+2\,H^2\,U\right)\right)\right)dt^3-$$
$$\frac{1}{20\left(3+H^2\,k^2\right)^3}\left(k^5\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right.$$
$$\left(9\,g^2\,H^2+6\,g\,H\,U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+3\left(3+H^2\,k^2\right)U\right)+U^3\left(12\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\right.\right.$$
$$\left.\left.\left.9\,U+H^4\,k^4\,U+2\,k^2\left(2\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}+3\,H^2\,U\right)\right)\right)\right)dt^4+O[dt]^5\Bigg)+$$
$$\left(\frac{k^3\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2U\right)}{24\left(3+H^2\,k^2\right)^2}+\frac{1}{48\left(3+H^2\,k^2\right)^3}k^5\left(g\left(-9\sqrt{3}\ H\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,H^3\,k^2\,U+6\,H^5\,k^4\,U\right)+\right.\right.$$
$$U^2\left(27\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+54\,U+2\,H^6\,k^6\,U+3\,k^2\left(7\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}+18\,H^2\,U\right)+\right.$$
$$\left.\left.\left.2\,k^4\left(2\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}+9\,H^4\,U\right)\right)\right)dt^2+$$
$$\frac{1}{48\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$
$$\left(-9\,g\,H+U\left(3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+2\,k^2\left(\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}+6\,H^2\,U\right)\right)\right)$$
$$dt^3-\frac{1}{96\left(3+H^2\,k^2\right)^4}\left(k^7\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2U\right)\right.$$
$$\left(9\,g^2\,H^2+6\,g\,H\,U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+3\left(3+H^2\,k^2\right)U\right)+U^3\left(12\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\right.\right.$$

$$9\,U + H^4\,k^4\,U + 2\,k^2\left(2\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + 3\,H^2\,U\right)\Big)\Big)\Big)\,dt^4 + O[dt]^5\Bigg)\,dx^2 +$$

$$\Bigg(-\frac{i\,k^4\left(2\,g\,H\left(3+H^2\,k^2\right)+\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)}{16\sqrt{g\,H}\,\left(3+H^2\,k^2\right)} - \frac{1}{32\left(3+H^2\,k^2\right)^{3/2}}i\,k^6\left(g\,H\left(6\sqrt{g\,H\left(3+H^2\,k^2\right)} + \sqrt{3}\,\left(15+4\,H^2\,k^2\right)U\right) +$$

$$U^2\left(12\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\sqrt{3}\,U + k^2\left(2\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + \sqrt{3}\,H^2\,U\right)\right)\Big)\,dt^2 +$$

$$\frac{1}{32\left(3+H^2\,k^2\right)^{5/2}}k^7\left(3\,g\,H + U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + \left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(2\sqrt{3}\,g\,H\left(3+H^2\,k^2\right) + U\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\sqrt{3}\,U + k^2\left(2\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + \sqrt{3}\,H^2\,U\right)\right)\right)$$

$$dt^3 + \Big(i\,k^8\left(2\,g\,H\left(3+H^2\,k^2\right) + \sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U\right)$$

$$\left(9\,g^2\,H^2 + 6\,g\,H\,U\left(2\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 3\left(3+H^2\,k^2\right)U\right) +\right.$$

$$\left.U^3\left(12\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 9\,U + H^4\,k^4\,U + 2\,k^2\left(2\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)} + 3\,H^2\,U\right)\right)\right)$$

$$dt^4\Big)\Big/\left(64\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^3\right) + O[dt]^5\Bigg)\,dx^3 +$$

$$\Bigg(-\Big(\Big(k^5\left(3\sqrt{3}\,g\,H\left(177+124\,H^2\,k^2+20\,H^4\,k^4\right) + 104\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)} + 6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)} +\right.\right.$$

$$\left.\left.k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\Big)\Big/\left(1920\left(\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{5/2}\right)\right)\Big) -$$

$$\Big(\Big(k^7\left(27\sqrt{3}\,g^2\,H^2\left(167+124\,H^2\,k^2+20\,H^4\,k^4\right) + g\,H\,U\left(21\,429\sqrt{3}\,H^2\,k^2\,U +\right.\right.$$

$$764\sqrt{3}\,H^6\,k^6\,U + 81\left(232\sqrt{g\,H\left(3+H^2\,k^2\right)} + 267\sqrt{3}\,U\right) +$$

$$\left.24\,k^4\left(84\sqrt{g\,H^9\left(3+H^2\,k^2\right)} + 293\sqrt{3}\,H^4\,U\right)\right) +$$

$$16\left(459\sqrt{g\,H\left(3+H^2\,k^2\right)}\,U^3 + 153\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\,U^3 + 17\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\right.$$

$$\left.\left.U^3 + 9\,k^2\left(88\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U + 51\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3\right)\right)\Big)\Big)\Big)\,dt^2\Big)\Big/$$

$$\left(11\,520\left(\sqrt{g\,H}\,\left(3+H^2\,k^2\right)^{7/2}\right)\right) - \frac{1}{3840\left(3+H^2\,k^2\right)^4}i\,k^8\left(54\,g^2\,H^2\left(81+62\,H^2\,k^2+10\,H^4\,k^4\right) +\right.$$

$$84\,H^8\,k^8\,U^4 + 243\,U^3\left(39\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 28\,U\right) + 36\,k^4\,U^3$$

$$\left(85\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)} + 126\,H^4\,U\right) + 4\,k^6\,U^3\left(83\sqrt{3}\,\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)} + 252\,H^6\,U\right) +$$

$$9\,k^2\left(600\sqrt{3}\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U + 1039\sqrt{3}\,\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\,U^3 + 1008\,H^2\,U^4\right) +$$

$$3\,g\,H\,U\left(13\,500\,H^2\,k^2\,U + 472\,H^6\,k^6\,U + 27\left(97\sqrt{3}\,\sqrt{g\,H\left(3+H^2\,k^2\right)} + 504\,U\right) +\right.$$

$$\left.\left.4\,k^4\left(71\sqrt{3}\,\sqrt{g\,H^9\left(3+H^2\,k^2\right)} + 1101\,H^4\,U\right)\right)\right)\,dt^3 +$$

$$\frac{1}{23040\sqrt{g H}\,(3+H^2k^2)^{9/2}}k^9\Big(81\sqrt{3}\,g^3H^3\left(157+124H^2k^2+20H^4k^4\right)+$$
$$6\sqrt{3}\,g^2H^2\left(32535+32481H^2k^2+10584H^4k^4+1124H^6k^6\right)U^2+$$
$$3gHU^3\Big(69120\sqrt{gH\left(3+H^2k^2\right)}+34155\sqrt{3}\,U+44982\sqrt{3}\,H^2k^2U+$$
$$396\sqrt{3}\,H^8k^8U+27k^4\Big(816\sqrt{gH^9\left(3+H^2k^2\right)}+821\sqrt{3}\,H^4U\Big)+$$
$$28k^6\Big(84\sqrt{gH^{13}\left(3+H^2k^2\right)}+173\sqrt{3}\,H^6U\Big)\Big)+8U\Big(348k^6\sqrt{gH^{13}\left(3+H^2k^2\right)}\,U^4+$$
$$29k^8\sqrt{gH^{17}\left(3+H^2k^2\right)}\,U^4+81\Big(130\sqrt{g^5H^5\left(3+H^2k^2\right)}+29\sqrt{gH\left(3+H^2k^2\right)}\,U^4\Big)+$$
$$54k^2\Big(137\sqrt{g^5H^9\left(3+H^2k^2\right)}+472\sqrt{g^3H^7\left(3+H^2k^2\right)}\,U^2+58\sqrt{gH^5\left(3+H^2k^2\right)}\,U^4\Big)+$$
$$27k^4\Big(43\sqrt{g^5H^{13}\left(3+H^2k^2\right)}+58\sqrt{gH^9\left(3+H^2k^2\right)}\,U^4\Big)\Big)\Big)dt^4+O[dt]^5\Big)dx^4+O[dx]^5,$$

$$\Bigg(\frac{1}{6\left(3+H^2k^2\right)^2}k^3\Big(-\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\Big)\Big(3gH+U\Big(-2\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\Big)\Big)$$
$$dt^2+$$
$$\frac{ik^4\left(3gH+U\left(-2\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2dt^3}{8\left(3+H^2k^2\right)^2}-$$
$$\frac{1}{20\left(3+H^2k^2\right)^4}$$
$$\Big(k^5\Big(-\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\Big)^3\Big(3gH+U\Big(-2\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\Big)\Big)\Big)$$
$$dt^4+O[dt]^5\Bigg)+$$

$$\Bigg(\frac{k^3\left(3\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)^2U\right)}{24\left(3+H^2k^2\right)^2}+\frac{1}{48\left(3+H^2k^2\right)^3}k^5\Big(3g\Big(3\sqrt{3}\,H\sqrt{gH\left(3+H^2k^2\right)}+6H^3k^2U+2H^5k^4U\Big)+$$
$$U^2\Big(-27\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+54U+2H^6k^6U-3k^2\Big(7\sqrt{3}\,\sqrt{gH^5\left(3+H^2k^2\right)}-18H^2U\Big)-$$
$$2k^4\Big(2\sqrt{3}\,\sqrt{gH^9\left(3+H^2k^2\right)}-9H^4U\Big)\Big)\Big)dt^2+$$
$$\frac{1}{48\left(3+H^2k^2\right)^3}ik^6\Big(3gH+U\Big(-2\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\Big)\Big)$$
$$\Big(-9gH+U\Big(-3\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+18U+2H^4k^4U-2k^2\Big(\sqrt{3}\,\sqrt{gH^5\left(3+H^2k^2\right)}-6H^2U\Big)\Big)\Big)$$
$$dt^3-\frac{1}{96\left(3+H^2k^2\right)^4}$$
$$\Big(k^7\Big(3\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+2\left(3+H^2k^2\right)^2U\Big)\Big(3gH+U\Big(-2\sqrt{3}\,\sqrt{gH\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\Big)\Big)^2\Big)$$

$$\left. dt^4 + O[dt]^5 \right| dx^2 +$$

$$\left(\frac{1}{16} i \sqrt{g H}\ k^4 \left(-2 + \frac{\sqrt{3}\ U}{\sqrt{g H \left(3+H^2 k^2\right)}} \right) + \frac{1}{32 \left(3+H^2 k^2\right)^{3/2}} \right.$$

$$i k^6 \left(g \left(-6 H \sqrt{g H \left(3 + H^2 k^2\right)} + 15 \sqrt{3}\ H U + 4 \sqrt{3}\ H^3 k^2 U \right) + \right.$$

$$\left. \left. U^2 \left(-12 \sqrt{g H \left(3 + H^2 k^2\right)} + 3 \sqrt{3}\ U + k^2 \left(-2 \sqrt{g H^5 \left(3 + H^2 k^2\right)} + \sqrt{3}\ H^2 U \right) \right) \right) \right) dt^2 -$$

$$\frac{1}{32 \left(3+H^2 k^2\right)^{5/2}} \left(k^7 \left(3 g H + U \left(-2 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U \right) \right) \left(2 \sqrt{3}\ g H \left(3 + H^2 k^2\right) + \right. \right.$$

$$\left. \left. \left. U \left(-9 \sqrt{g H \left(3 + H^2 k^2\right)} + 3 \sqrt{3}\ U + k^2 \left(-2 \sqrt{g H^5 \left(3 + H^2 k^2\right)} + \sqrt{3}\ H^2 U \right) \right) \right) \right)$$

$$dt^3 + \left(i k^8 \left(2 g H \left(3 + H^2 k^2\right) - \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)}\ U \right) \right.$$

$$\left. \left(3 g H + U \left(-2 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} + \left(3 + H^2 k^2\right) U \right) \right)^2 dt^4 \right) \Big/ \left(64 \sqrt{g H}\ \left(3 + H^2 k^2\right)^3 \right) + O[dt]^5 \Bigg)$$

$$dx^3 + \left(\left(k^5 \left(3 \sqrt{3}\ g H \left(177 + 124 H^2 k^2 + 20 H^4 k^4\right) - 104 \left(9 \sqrt{g H \left(3 + H^2 k^2\right)} + \right. \right. \right.$$

$$\left. \left. \left. 6 k^2 \sqrt{g H^5 \left(3 + H^2 k^2\right)} + k^4 \sqrt{g H^9 \left(3 + H^2 k^2\right)} \right) U \right) \right) \Big/$$

$$\left(1920 \sqrt{g H}\ \left(3 + H^2 k^2\right)^{5/2} \right) + \left(k^7 \left(27 \sqrt{3}\ g^2 H^2 \left(167 + 124 H^2 k^2 + 20 H^4 k^4\right) + \right. \right.$$

$$g H U \left(21\,429 \sqrt{3}\ H^2 k^2 U + 764 \sqrt{3}\ H^6 k^6 U + 81 \left(-232 \sqrt{g H \left(3 + H^2 k^2\right)} + 267 \sqrt{3}\ U \right) - \right.$$

$$\left. 24 k^4 \left(84 \sqrt{g H^9 \left(3 + H^2 k^2\right)} - 293 \sqrt{3}\ H^4 U \right) \right) -$$

$$16 \left(459 \sqrt{g H \left(3 + H^2 k^2\right)}\ U^3 + 153 k^4 \sqrt{g H^9 \left(3 + H^2 k^2\right)}\ U^3 + 17 k^6 \sqrt{g H^{13} \left(3 + H^2 k^2\right)}\ U^3 + \right.$$

$$\left. \left. 9 k^2 \left(88 \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)}\ U + 51 \sqrt{g H^5 \left(3 + H^2 k^2\right)}\ U^3 \right) \right) \right) dt^2 \Big) \Big/$$

$$\left(11\,520 \sqrt{g H}\ \left(3 + H^2 k^2\right)^{7/2} \right) - \frac{1}{3840 \left(3+H^2 k^2\right)^4} i k^8 \left(54 g^2 H^2 \left(81 + 62 H^2 k^2 + 10 H^4 k^4\right) + 84 H^8 k^8 U^4 + \right.$$

$$243 U^3 \left(-39 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} + 28 U \right) + 36 k^4 U^3 \left(-85 \sqrt{3}\ \sqrt{g H^9 \left(3 + H^2 k^2\right)} + 126 H^4 U \right) +$$

$$4 k^6 U^3 \left(-83 \sqrt{3}\ \sqrt{g H^{13} \left(3 + H^2 k^2\right)} + 252 H^6 U \right) -$$

$$9 k^2 \left(600 \sqrt{3}\ \sqrt{g^3 H^7 \left(3 + H^2 k^2\right)}\ U + 1039 \sqrt{3}\ \sqrt{g H^5 \left(3 + H^2 k^2\right)}\ U^3 - 1008 H^2 U^4 \right) +$$

$$3 g H U \left(13\,500 H^2 k^2 U + 472 H^6 k^6 U + 27 \left(-97 \sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} + 504 U \right) - \right.$$

$$\left. \left. 4 k^4 \left(71 \sqrt{3}\ \sqrt{g H^9 \left(3 + H^2 k^2\right)} - 1101 H^4 U \right) \right) \right) dt^3 - \frac{1}{23\,040 \left(\sqrt{g H}\ \left(3+H^2 k^2\right)^{11/2} \right)}$$

$$\left(k^9 \left(\sqrt{3}\ \sqrt{g H \left(3 + H^2 k^2\right)} - \left(3 + H^2 k^2\right) U \right) \left(4 k^8 U^3 \left(-239 \sqrt{3}\ g H^9 + 58 \sqrt{g H^{17} \left(3 + H^2 k^2\right)}\ U \right) + \right.$$

$$\left. 27 k^2 \left(272 \sqrt{5 H^9 \left(3 + H^2 k^2\right)} - 2703 \sqrt{3} \right) \right.$$

$$27\,k^2\left(372\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}-2703\sqrt{3}\;g^2\,H^4\,U+4515\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U^2-4070\sqrt{3}\;g\,H^3\,U^3+928\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^4\right)+9\,k^4\left(180\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}-2672\sqrt{3}\;g^2\,H^6\,U+1392\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;U^4+g\,H^5\,U^2\left(4384\sqrt{g\,H\left(3+H^2\,k^2\right)}-5997\sqrt{3}\;U\right)\right)+81\left(157\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}-883\sqrt{3}\;g^2\,H^2\,U+232\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+g\,H\,U^2\left(1527\sqrt{g\,H\left(3+H^2\,k^2\right)}-1033\sqrt{3}\;U\right)\right)-12\,k^6\,U\left(213\sqrt{3}\;g^2\,H^8-232\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\;U^3+g\,H^7\,U\left(-349\sqrt{g\,H\left(3+H^2\,k^2\right)}+979\sqrt{3}\;U\right)\right)\Big)\Big)\,dt^4+O[dt]^5\Big)\,dx^4+O[dx]^5\Big\}$$

Out[132]= Omega error ||

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\left\{\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U
    \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2
    k^2+3\right)^2}+\frac{i k^4 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)
    \left(\left(H^4 U k^4+3 \left(2 U H^2+\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+9 \left(U+\sqrt{3}
    \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) U^2+3 g \left(3 k^2 U H^3+9 U H+\sqrt{3} \sqrt{g H \left(H^2
    k^2+3\right)} H\right)\right) \text{dt}^3}{8 \left(H^2 k^2+3\right)^3}-\frac{\left(k^5 \left(\left(H^2 k^2+3\right)
    U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(\left(H^4 U k^4+2 \left(3 U H^2+2 \sqrt{3} \sqrt{g
    H^5 \left(H^2 k^2+3\right)}\right) k^2+9 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^3+6 g H
    \left(3 \left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right)\right)
    \text{dt}^4}{20 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(2 \left(H^2
    k^2+3\right)^2 U-3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{24 \left(H^2 k^2+3\right)^2}+\frac{k^5
    \left(\left(2 H^6 U k^6+2 \left(9 U H^4+2 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+3 \left(18
    U H^2+7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+54 U+27 \sqrt{3} \sqrt{g H \left(H^2
    k^2+3\right)}\right) U^2+g \left(6 k^4 U H^5+18 k^2 U H^3-9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}
    H\right)\right) \text{dt}^2}{48 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right)
    U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(U \left(2 H^4 U k^4+2 \left(6 U H^2+\sqrt{3}
    \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)-9
    g H\right) \text{dt}^3}{48 \left(H^2 k^2+3\right)^3}-\frac{\left(k^7 \left(2 \left(H^2 k^2+3\right)^2
    U-3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(\left(H^4 U k^4+2 \left(3 U H^2+2 \sqrt{3}
    \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+9 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)
    U^3+6 g H \left(3 \left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)
    U+9 g^2 H^2\right)\right) \text{dt}^4}{96 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)
    \text{dx}^2+\left(-\frac{i k^4 \left(2 g H \left(H^2 k^2+3\right)+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}
    U\right)}{16 \sqrt{g H} \left(H^2 k^2+3\right)}-\frac{i k^6 \left(\left(\left(\sqrt{3} U H^2+2 \sqrt{g H^5
    \left(H^2 k^2+3\right)}\right) k^2+3 \sqrt{3} U+12 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+g H
    \left(\sqrt{3} \left(4 H^2 k^2+15\right) U+6 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{32
    \left(H^2 k^2+3\right)^{3/2}}+\frac{k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H
    \left(H^2 k^2+3\right)}\right)\right) \left(2 \sqrt{3} g H \left(H^2 k^2+3\right)+U \left(\left(\sqrt{3} U H^2+2
    \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+3 \sqrt{3} U+9 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)
    \text{dt}^3}{32 \left(H^2 k^2+3\right)^{5/2}}+\frac{i k^8 \left(2 g H \left(H^2 k^2+3\right)+\sqrt{3} \sqrt{g
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H \left(H^2 k^2+3\right)} U\right) \left(\left(H^4 U k^4+2 \left(3 U H^2+2 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+9 U+12 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^3+6 g H \left(3 \left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U+9 g^2 H^2\right) \text{dt}^4}{64 \sqrt{g H} \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(-\frac{k^5 \left(3 \sqrt{3} g H \left(20 H^4 k^4+124 H^2 k^2+177\right)+104 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{1920 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{5/2}\right)}-\frac{\left(k^7 \left(27 \sqrt{3} g^2 \left(20 H^4 k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6+24 \left(293 \sqrt{3} U H^4+84 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267 \sqrt{3} U+232 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) H+16 \left(17 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+153 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(51 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+88 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+459 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right)\right) \text{dt}^2}{11520 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)}-\frac{i k^8 \left(84 H^8 U^4 k^8+4 U^3 \left(252 U H^6+83 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+36 U^3 \left(126 U H^4+85 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+9 \left(1008 H^2 U^4+1039 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+600 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U+39 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+3 g H U \left(472 H^6 U k^6+4 \left(1101 U H^4+71 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+13500 H^2 U k^2+27 \left(504 U+97 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{3840 \left(H^2 k^2+3\right)^4}+\frac{k^9 \left(81 \sqrt{3} g^3 \left(20 H^4 k^4+124 H^2 k^2+157\right) H^3+6 \sqrt{3} g^2 \left(1124 H^6 k^6+10584 H^4 k^4+32481 H^2 k^2+32535\right) U^2 H^2+3 g U^3 \left(396 \sqrt{3} H^8 U k^8+28 \left(173 \sqrt{3} U H^6+84 \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+27 \left(821 \sqrt{3} U H^4+816 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+44982 \sqrt{3} H^2 U k^2+34155 \sqrt{3} U+69120 \sqrt{g H \left(H^2 k^2+3\right)}\right) H+8 U \left(29 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U^4 k^8+348 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^4 k^6+27 \left(58 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+43 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+54 \left(58 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+472 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+137 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(29 \sqrt{g H \left(H^2 k^2+3\right)} U^4+130 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{23040 \sqrt{g H} \left(H^2 k^2+3\right)^{9/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right),\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}-\frac{\left(k^5 \left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{20 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(2 U \left(H^2 k^2+3\right)^2+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{24 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(\left(2 H^6 U k^6-2 \left(2 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}-9 H^4 U\right) k^4-3 \left(7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-18 H^2 U\right) k^2+54 U-27 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g \left(2 k^4 U H^5+6 k^2 U H^3+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} H\right)\right) \text{dt}^2}{48 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(U \left(2 H^4 U k^4-2

\left(\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−6 H^2 U\right) k^2+18 U−3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)−9 g H\right) \text{dt}^3}{48 \left(H^2 k^2+3\right)^3}−\frac{\left(k^7 \left(2 U \left(H^2 k^2+3\right)^2+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{96 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{16} i \sqrt{g H} k^4 \left(\frac{\sqrt{3} U}{\sqrt{g H \left(H^2 k^2+3\right)}}−2\right)+\frac{i k^6 \left(\left(\left(\sqrt{3} H^2 U−2 \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+3 \sqrt{3} U−12 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+g \left(4 \sqrt{3} k^2 U H^3+15 \sqrt{3} U H−6 \sqrt{g H \left(H^2 k^2+3\right)} H\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^{3/2}}−\frac{\left(k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(2 \sqrt{3} g H \left(H^2 k^2+3\right)+U \left(\left(\sqrt{3} H^2 U−2 \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+3 \sqrt{3} U−9 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^{5/2}}+\frac{i k^8 \left(2 g H \left(H^2 k^2+3\right)−\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \sqrt{g H} \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{k^5 \left(3 \sqrt{3} g H \left(20 H^4 k^4+124 H^2 k^2+177\right)−104 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{1920 \sqrt{g H} \left(H^2 k^2+3\right)^{5/2}}+\frac{k^7 \left(27 \sqrt{3} g^2 \left(20 H^4 k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6−24 \left(84 \sqrt{g H^9 \left(H^2 k^2+3\right)}−293 \sqrt{3} H^4 U\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267 \sqrt{3} U−232 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) H−16 \left(17 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+153 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(51 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+88 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+459 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text{dt}^2}{11520 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}−\frac{i k^8 \left(84 H^8 U^4 k^8+4 U^3 \left(252 H^6 U−83 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+36 U^3 \left(126 H^4 U−85 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4−9 \left(−1008 H^2 U^4+1039 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+600 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U−39 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+3 g H U \left(472 H^6 U k^6−4 \left(71 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}−1101 H^4 U\right) k^4+13500 H^2 U k^2+27 \left(504 U−97 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{3840 \left(H^2 k^2+3\right)^4}−\frac{\left(k^9 \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}−\left(H^2 k^2+3\right) U\right) \left(4 U^3 \left(58 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U−239 \sqrt{3} g H^9\right) k^8−12 U \left(213 \sqrt{3} g^2 H^8+g U \left(979 \sqrt{3} U−349 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7−232 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(−2672 \sqrt{3} g^2 U H^6+g U^2 \left(4384 \sqrt{g H \left(H^2 k^2+3\right)}−5997 \sqrt{3} U\right) H^5+1392 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+180 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+27 \left(−2703 \sqrt{3} g^2 U H^4−4070 \sqrt{3} g U^3 H^3+928 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+4515 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+372 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(232 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1527 \sqrt{g H \left(H^2 k^2+3\right)}−1033 \sqrt{3} U\right) U^2−883 \sqrt{3} g^2 H^2 U+157 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{23040 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{11/2}\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right\}

Out[133]=

Out[134]= EA || $$\left\{\left\{1+\frac{i\, e^{\frac{i\,\text{dx}\,k}{2}}\left(1-e^{-i\,\text{dx}\,k}\right)\left(-1+e^{i\,\text{dt}\,w}\right)H^2\,k^3\,U\,\text{Csc}\left[\frac{\text{dx}\,k}{2}\right]}{\left(6+2\,H^2\,k^2\right)w},\ \frac{i\, e^{\frac{i\,\text{dx}\,k}{2}}\left(1-e^{-i\,\text{dx}\,k}\right)\left(-1+e^{i\,\text{dt}\,w}\right)H\,k\,\text{Csc}\left[\frac{\text{dx}\,k}{2}\right]}{2\left(H+\frac{H^3\,k^2}{3}\right)w}\right\},\right.$$
$$\left.\left\{\frac{i\, e^{\frac{i\,\text{dx}\,k}{2}}\left(1-e^{-i\,\text{dx}\,k}\right)\left(-1+e^{i\,\text{dt}\,w}\right)k\left(g\,H\left(3+H^2\,k^2\right)-3\,U^2\right)\text{Csc}\left[\frac{\text{dx}\,k}{2}\right]}{\left(6+2\,H^2\,k^2\right)w},\ 1+\frac{i\, e^{\frac{i\,\text{dx}\,k}{2}}\left(1-e^{-i\,\text{dx}\,k}\right)\left(-1+e^{i\,\text{dt}\,w}\right)k\left(6+H^2\,k^2\right)U\,\text{Csc}\left[\frac{\text{dx}\,k}{2}\right]}{\left(6+2\,H^2\,k^2\right)w}\right\}\right\}$$

Out[135]= EA || \left(

\begin{array}{cc}

\frac{i e^{\frac{i \text{dx} k}{2}} \left(1-e^{-i \text{dx} k}\right) \left(-1+e^{i \text{dt} w}\right) H^2 U \csc \left(\frac{\text{dx} k}{2}\right) k^3}{\left(2 H^2 k^2+6\right) w}+1 & \frac{i e^{\frac{i \text{dx} k}{2}} \left(1-e^{-i \text{dx} k}\right) \left(-1+e^{i \text{dt} w}\right) H k \csc \left(\frac{\text{dx} k}{2}\right)}{2 \left(\frac{k^2 H^3}{3}+H\right) w} \\

\frac{i e^{\frac{i \text{dx} k}{2}} \left(1-e^{-i \text{dx} k}\right) \left(-1+e^{i \text{dt} w}\right) k \left(g H \left(H^2 k^2+3\right)-3 U^2\right) \csc \left(\frac{\text{dx} k}{2}\right)}{\left(2 H^2 k^2+6\right) w} & \frac{i e^{\frac{i \text{dx} k}{2}} \left(1-e^{-i \text{dx} k}\right) \left(-1+e^{i \text{dt} w}\right) k \left(H^2 k^2+6\right) U \csc \left(\frac{\text{dx} k}{2}\right)}{\left(2 H^2 k^2+6\right) w}+1 \\

\end{array}

\right)

Out[136]= Eerr ||

$$\left\{\left\{\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U+3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}-\right.\right.$$

$$\left.\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3dt^3-\frac{1}{24}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4dt^4+O[dt]^5\right)+$$

$$\left(-\frac{i\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)U\,dt}{12\left(3+H^2\,k^2\right)^2}-\frac{\left(-9\,g\,H\,k^4+36\,k^4\,U^2+12\,H^2\,k^6\,U^2+2\,H^4\,k^8\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(-\frac{1}{8}\left(\sqrt{g\,H}\ k^4\right)dt+\frac{i\sqrt{g\,H}\ \left(3\,k^5+2\,H^2\,k^7\right)U\,dt^2}{16\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(405\,k^5\,U+351\,H^2\,k^7\,U+116\,H^4\,k^9\,U+13\,H^6\,k^{11}\,U\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{1}{1440\left(3+H^2\,k^2\right)^3}\left(1161\,g\,H\,k^6+837\,g\,H^3\,k^8+135\,g\,H^5\,k^{10}-\right.\right.$$

$$\left.\left.351\,k^6\,U^2+297\,H^2\,k^8\,U^2+387\,H^4\,k^{10}\,U^2+73\,H^6\,k^{12}\,U^2\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5,$$

$$\left(-\frac{3\,i\,k\,dt}{3+H^2\,k^2}-\frac{3\left(k^2\,U\right)dt^2}{3+H^2\,k^2}+O[dt]^5\right)+\left(\left(\frac{3\,i\,k^3}{2\left(3+H^2\,k^2\right)^2}+\frac{i\,H^2\,k^5}{4\left(3+H^2\,k^2\right)^2}\right)dt+\frac{3\,k^4\,U\,dt^2}{4\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(\frac{3\,i\sqrt{g\,H}\ k^5\,dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+\left(\frac{i\left(-54\,k^5+H^4\,k^9\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{\left(387\,k^6\,U+279\,H^2\,k^8\,U+45\,H^4\,k^{10}\,U\right)dt^2}{240\left(3+H^2\,k^2\right)^3}+O[dt]^5\right)dx^4+O[dx]^5\Big\},$$

$$\left\{\left(-\frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)dt}{3+H^2\,k^2}-\frac{k^2\,U\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)dt^2}{3+H^2\,k^2}+O[dt]^5\right)+\right.$$

$$\left(-\frac{i\left(9\,g\,H\,k^3+6\,g\,H^3\,k^5+g\,H^5\,k^7+18\,k^3\,U^2+3\,H^2\,k^5\,U^2\right)dt}{12\left(3+H^2\,k^2\right)^2}+\frac{\left(-18\,g\,H\,k^4\,U-12\,g\,H^3\,k^6\,U-2\,g\,H^5\,k^8\,U-9\,k^4\,U^3\right)dt^2}{12\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(-\frac{1}{8}\left(\sqrt{g\,H}\ k^4\,U\right)dt+\frac{i\left(3\,g\,H\sqrt{g\,H}\ k^5+g\,H^3\sqrt{g\,H}\ k^7+H^2\sqrt{g\,H}\ k^7\,U^2\right)dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(351\,g\,H\,k^5+351\,g\,H^3\,k^7+117\,g\,H^5\,k^9+13\,g\,H^7\,k^{11}+54\,k^5\,U^2-H^4\,k^9\,U^2\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{1}{720\left(3+H^2\,k^2\right)^3}\left(1971\,g\,H\,k^6\,U+1971\,g\,H^3\,k^8\,U+657\,g\right.\right.$$

$$\left.\left.H^5\,k^{10}\,U+73\,g\,H^7\,k^{12}\,U-1161\,k^6\,U^3-837\,H^2\,k^8\,U^3-135\,H^4\,k^{10}\,U^3\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5,$$

$$\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}-3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U-3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}-\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3dt^3-\right.$$

$$\left.\frac{1}{24}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4dt^4+O[dt]^5\right)+$$

$$\left(-\frac{i\left(-9\,k^3\,U+3\,H^2\,k^5\,U+H^4\,k^7\,U\right)dt}{12\left(3+H^2\,k^2\right)^2}+\frac{\left(9\,g\,H\,k^4-12\,H^2\,k^6\,U^2-2\,H^4\,k^8\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(-\frac{1}{8}\left(\sqrt{g\,H}\ k^4\right)dt+\frac{i\sqrt{g\,H}\ k^5\left(15+2\,H^2\,k^2\right)U\,dt^2}{16\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(297\,k^5\,U+351\,H^2\,k^7\,U+118\,H^4\,k^9\,U+13\,H^6\,k^{11}\,U\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{1}{1440\left(3+H^2\,k^2\right)^3}\left(1161\,g\,H\,k^6+837\,g\,H^3\,k^8+135\,g\,H^5\,k^{10}+\right.\right.$$

$$\left.\left.4293\,k^6\,U^2+3645\,H^2\,k^8\,U^2+927\,H^4\,k^{10}\,U^2+73\,H^6\,k^{12}\,U^2\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5\Big\}\Big\}$$

Out[137]= Eerr || \left(

\begin{array}{cc}

\left(\frac{i \left(3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \text{dt}}{H^2 k^2+3}+\frac{\left(3 U^2

```
k^2+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2\right) \text{dt}^2}{H^2 k^2+3}-\frac{1}{6} \left(-i
U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^3 \text{dt}^3-\frac{1}{24}
\left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^4
\text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(H^4 k^7+9 H^2 k^5+27 k^3\right) U \text{dt}}{12
\left(H^2 k^2+3\right)^2}-\frac{\left(2 H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H k^4\right)
\text{dt}^2}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(-\frac{1}{8}
\left(\sqrt{g H} k^4\right) \text{dt}+\frac{i \sqrt{g H} \left(2 H^2 k^7+3 k^5\right) U \text{dt}^2}{16
\left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(13 H^6 U k^{11}+116
H^4 U k^9+351 H^2 U k^7+405 U k^5\right) \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73
H^6 U^2 k^{12}+135 g H^5 k^{10}+387 H^4 U^2 k^{10}+837 g H^3 k^8+297 H^2 U^2 k^8-351 U^2
k^6+1161 g H k^6\right) \text{dt}^2}{1440 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)
\text{dx}^4+O\left(\text{dx}^5\right) & \left(-\frac{3 i k \text{dt}}{H^2 k^2+3}-\frac{3 \left(k^2
U\right) \text{dt}^2}{H^2 k^2+3}+O\left(\text{dt}^5\right)\right)+\left(\left(\frac{i H^2 k^5}{4 \left(H^2
k^2+3\right)^2}+\frac{3 i k^3}{2 \left(H^2 k^2+3\right)^2}\right) \text{dt}+\frac{3 k^4 U \text{dt}^2}{4
\left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{3 i \sqrt{g H} k^5 \text{dt}^2}{8
\left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(H^4 k^9-54 k^5\right)
\text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(45 H^4 U k^{10}+279 H^2 U k^8+387 U k^6\right)
\text{dt}^2}{240 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\left(-\frac{i k \left(g k^2 H^3+3 g H-3 U^2\right) \text{dt}}{H^2 k^2+3}-\frac{k^2 U \left(g k^2 H^3+3
g H-3 U^2\right) \text{dt}^2}{H^2 k^2+3}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(g
H^5 k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12 \left(H^2
k^2+3\right)^2}+\frac{\left(-2 g H^5 U k^8-12 g H^3 U k^6-9 U^3 k^4-18 g H U k^4\right)
\text{dt}^2}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(-\frac{1}{8}
\left(\sqrt{g H} k^4 U\right) \text{dt}+\frac{i \left(H^2 \sqrt{g H} U^2 k^7+g H^3 \sqrt{g H} k^7+3
g H \sqrt{g H} k^5\right) \text{dt}^2}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{i \left(13 g H^7 k^{11}+117 g H^5 k^9-H^4 U^2 k^9+351 g H^3 k^7+54
U^2 k^5+351 g H k^5\right) \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73 g H^7 U
k^{12}-135 H^4 U^3 k^{10}+657 g H^5 U k^{10}-837 H^2 U^3 k^8+1971 g H^3 U k^8-1161 U^3
k^6+1971 g H U k^6\right) \text{dt}^2}{720 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)
\text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{i \left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}-3 k
U\right) \text{dt}}{H^2 k^2+3}+\frac{\left(\sqrt{3} k^2 \sqrt{g H \left(H^2 k^2+3\right)} U-3 k^2 U^2\right)
\text{dt}^2}{H^2 k^2+3}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2
k^2+3}\right)^3 \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}
k}{H^2 k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(H^4 U k^7+3
H^2 U k^5-9 U k^3\right) \text{dt}}{12 \left(H^2 k^2+3\right)^2}+\frac{\left(-2 H^4 U^2 k^8-12 H^2
U^2 k^6+9 g H k^4\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(-\frac{1}{8} \left(\sqrt{g H} k^4\right) \text{dt}+\frac{i \sqrt{g H} k^5 \left(2
H^2 k^2+15\right) U \text{dt}^2}{16 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{i \left(13 H^6 U k^{11}+118 H^4 U k^9+351 H^2 U k^7+297 U k^5\right)
\text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73 H^6 U^2 k^{12}+135 g H^5 k^{10}+927 H^4
U^2 k^{10}+837 g H^3 k^8+3645 H^2 U^2 k^8+4293 U^2 k^6+1161 g H k^6\right) \text{dt}^2}{1440
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\end{array}
```

\right)

In[138]:=
```
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → 0;
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
      fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[ g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 ;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ];
```

```
Text[Row[{" U < -Sqrt(gH) < U "}]]
Text[" "]
Text[Row[{"Fnn  ||   ", Kfnnp}]]
Text[Row[{"Fnn  ||   ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||   ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||   ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||   ", KfnGp}]]
Text[Row[{"FnG  ||   ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error  ||   ", FnG2TAr}]]
Text[Row[{"FnG error  ||   ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||   ", KfGnp}]]
Text[Row[{"FGn  ||   ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||   ", FGn2TAr}]]
Text[Row[{"FGn error  ||   ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||   ", KfGGp}]]
Text[Row[{"FGG  ||   ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||   ", FGG2TAr}]]
Text[Row[{"FGG error  ||   ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||   ", RKstepTayr}]]
Text[Row[{"Omega error  ||   ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||   ", EA}]]
Text[Row[{"EA  ||   ", TeXForm[EA]}]]
Text[Row[{"Eerr ||   ", Eerr}]]
Text[Row[{"Eerr ||   ", TeXForm[Eerr]}]]
```

Out[172]= U < −Sqrt(gH) < U

Out[173]=

Out[174]= Fnn || Gnp H + Rmp U

Out[175]= Fnn || \text{Gnp} H+\text{Rmp} U

Out[176]= Fnn error ||

$$\left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)U\,dt}{12\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(-\frac{1}{8}\left(k^4\,U\right)dt+O[dt]^4\right)dx^3+O[dx]^4$$

Out[177]= Fnn error ||

\left(−\frac{\text{dt}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 H^2 k^3 U w^2}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(−\frac{i \left(H^4 k^7+9 H^2 k^5+27 k^3\right) U \text{dt}}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(−\frac{1}{8} \left(k^4 U\right) \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[178]=

Out[179]= FnG || GGp H

Out[180]= FnG || \text{GGp} H

Out[181]= FnG error || $\left(-\frac{3\,(k\,w)\,dt^2}{2\,(3+H^2\,k^2)}-\frac{i\,k\,w^2\,dt^3}{2\,(3+H^2\,k^2)}+O[dt]^4\right)+\left(\frac{i\,(6\,k^3+H^2\,k^5)\,dt}{4\,(3+H^2\,k^2)^2}+O[dt]^4\right)dx^2+O[dx]^4$

Out[182]= FnG error || \left(−\frac{3 \text{dt}^2 (k w)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 k w^2}{2 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(\frac{i \left(H^2 k^5+6 k^3\right) \text{dt}}{4 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[183]=

Out[184]= FGn || H (g Rmp + Gnp U)

Out[185]= FGn || H (g \text{Rmp}+\text{Gnp} U)

Out[186]= FGn error || $\left(-\frac{(k\,(3\,g\,H+g\,H^3\,k^2-3\,U^2)\,w)\,dt^2}{2\,(3+H^2\,k^2)}-\frac{i\,k\,(3\,g\,H+g\,H^3\,k^2-3\,U^2)\,w^2\,dt^3}{6\,(3+H^2\,k^2)}+O[dt]^4\right)+$
$\left(-\frac{i\,(9\,g\,H\,k^3+6\,g\,H^3\,k^5+g\,H^5\,k^7+18\,k^3\,U^2+3\,H^2\,k^5\,U^2)\,dt}{12\,(3+H^2\,k^2)^2}+O[dt]^4\right)dx^2+\left(-\frac{1}{8}\,(g\,H\,k^4)\,dt+O[dt]^4\right)dx^3+O[dx]^4$

Out[187]= FGn error ||

\left(−\frac{\text{dt}^2 \left(k w \left(g H^3 k^2+3 g H−3 U^2\right)\right)}{2 \left(H^2 k^2+3\right)}−\frac{i \text{dt}^3 k w^2 \left(g H^3 k^2+3 g H−3 U^2\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(−\frac{i \left(g H^5 k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(−\frac{1}{8} \left(g H k^4\right) \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[188]=

Out[189]= FGG || (GGp H + Rmp) U

Out[190]= FGG || U (\text{GGp} H+\text{Rmp})

Out[191]= FGG error || $\left(-\frac{(k\,(6+H^2\,k^2)\,U\,w)\,dt^2}{2\,(3+H^2\,k^2)}-\frac{i\,k\,(6+H^2\,k^2)\,U\,w^2\,dt^3}{6\,(3+H^2\,k^2)}+O[dt]^4\right)+\left(-\frac{i\,(-9\,k^3+3\,H^2\,k^5+H^4\,k^7)\,U\,dt}{12\,(3+H^2\,k^2)^2}+O[dt]^4\right)dx^2+$
$\left(-\frac{1}{8}\,(k^4\,U)\,dt+O[dt]^4\right)dx^3+\left(\frac{i\,(297\,k^5+351\,H^2\,k^7+118\,H^4\,k^9+13\,H^6\,k^{11})\,U\,dt}{240\,(3+H^2\,k^2)^3}+O[dt]^4\right)dx^4+O[dx]^5$

Out[192]= FGG error ||

\left(-\frac{\text{dt}^2 \left(k U w \left(H^2 k^2+6\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k U w^2 \left(H^2 k^2+6\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(H^4 k^7+3 H^2 k^5-9 k^3\right) U \text{dt}}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(-\frac{1}{8} \left(k^4 U\right) \text{dt}+O\left(\text{dt}^4\right)\right)+\text{dx}^4 \left(\frac{i \left(13 H^6 k^{11}+118 H^4 k^9+351 H^2 k^7+297 k^5\right) U \text{dt}}{240 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^5\right)

Out[193]=

Out[194]=

Out[195]= Omega error ||

$$\left\{\left(\frac{1}{6\left(3+H^2 k^2\right)^2} k^3 \left(\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)\left(3 g H+U\left(2\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)\right) dt^2+\right.$$

$$\frac{i k^4 \left(3 g H+U\left(2\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)\right)^2 dt^3}{8\left(3+H^2 k^2\right)^2}-\frac{1}{20\left(3+H^2 k^2\right)^4}\left(k^5\left(\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)^3\right.$$

$$\left.\left.\left(3 g H+U\left(2\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)\right)\right) dt^4+O[dt]^5\right)+$$

$$\left(\frac{k^3\left(-3\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+2\left(3+H^2 k^2\right)^2 U\right)}{24\left(3+H^2 k^2\right)^2}+\frac{1}{48\left(3+H^2 k^2\right)^3} k^5\left(g\left(-9\sqrt{3}\ H\sqrt{g H \left(3+H^2 k^2\right)}+18 H^3 k^2 U+6 H^5 k^4 U\right)+\right.\right.$$

$$U^2\left(27\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+54 U+2 H^6 k^6 U+3 k^2\left(7\sqrt{3}\ \sqrt{g H^5 \left(3+H^2 k^2\right)}+18 H^2 U\right)+$$

$$\left.\left.2 k^4\left(2\sqrt{3}\ \sqrt{g H^9 \left(3+H^2 k^2\right)}+9 H^4 U\right)\right)\right) dt^2+$$

$$\frac{1}{48\left(3+H^2 k^2\right)^3} i k^6\left(3 g H+U\left(2\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)\right)$$

$$\left(-9 g H+U\left(3\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+18 U+2 H^4 k^4 U+2 k^2\left(\sqrt{3}\ \sqrt{g H^5 \left(3+H^2 k^2\right)}+6 H^2 U\right)\right)\right)$$

$$dt^3-\frac{1}{96\left(3+H^2 k^2\right)^4}\left(k^7\left(-3\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+2\left(3+H^2 k^2\right)^2 U\right)\right.$$

$$\left.\left.\left(3 g H+U\left(2\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+\left(3+H^2 k^2\right) U\right)\right)^2\right) dt^4+O[dt]^5\right) dx^2+$$

$$\left(-\frac{1}{16} i k^4\left(\sqrt{3}\ \sqrt{\frac{g H}{3+H^2 k^2}}+2 U\right)-\frac{1}{32\left(3+H^2 k^2\right)^2} i k^6\left(3 g H\left(\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+4\left(3+H^2 k^2\right) U\right)+\right.\right.$$

$$\left.\left.U^2\left(15\sqrt{3}\ \sqrt{g H \left(3+H^2 k^2\right)}+18 U+2 H^4 k^4 U+k^2\left(5\sqrt{3}\ \sqrt{g H^5 \left(3+H^2 k^2\right)}+12 H^2 U\right)\right)\right) k^2$$

$$U^2\left(15\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +18\,U+2\,H^2\,k^2\,U+k^2\left(5\sqrt{3}\ \sqrt{g\,H^3\left(3+H^2\,k^2\right)}\ +12\,H^2\,U\right)\right)\Big)\Big)\,dt^2+$$

$$\frac{1}{32\left(3+H^2\,k^2\right)^2}k^7\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(3\,g\,H+U\left(3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +2\left(3+H^2\,k^2\right)U\right)\right)dt^3+$$

$$\frac{1}{64\left(3+H^2\,k^2\right)^3}i\,k^8\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +2\left(3+H^2\,k^2\right)U\right)$$

$$\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\right)^2dt^4+O[dt]^5\Bigg)dx^3+$$

$$\Bigg(-\Bigg(\Bigg(k^5\left(3\sqrt{3}\ g\,H\left(177+124\,H^2\,k^2+20\,H^4\,k^4\right)+104\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ +\right.\right.$$

$$\left.\left.k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ \right)U\right)\Bigg)\Big/\left(1920\left(\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{5/2}\right)\right)\Bigg)-$$

$$\Bigg(\Bigg(k^7\left(27\sqrt{3}\ g^2\,H^2\left(167+124\,H^2\,k^2+20\,H^4\,k^4\right)+g\,H\,U\left(21\,429\sqrt{3}\ H^2\,k^2\,U+\right.\right.$$

$$764\sqrt{3}\ H^6\,k^6\,U+81\left(232\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +267\sqrt{3}\ U\right)+$$

$$24\,k^4\left(84\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ +293\sqrt{3}\ H^4\,U\right)\Big)+$$

$$16\left(459\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^3+153\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^3+17\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\right.$$

$$\left.\left.\left.U^3+9\,k^2\left(88\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U+51\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3\right)\right)\right)\Bigg)dt^2\Bigg)\Big/$$

$$\left(11\,520\left(\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{7/2}\right)\right)-\frac{1}{3840\left(3+H^2\,k^2\right)^4}i\,k^8\left(54\,g^2\,H^2\left(81+62\,H^2\,k^2+10\,H^4\,k^4\right)+\right.$$

$$84\,H^8\,k^8\,U^4+243\,U^3\left(39\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +28\,U\right)+36\,k^4\,U^3$$

$$\left(85\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ +126\,H^4\,U\right)+4\,k^6\,U^3\left(83\sqrt{3}\ \sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ +252\,H^6\,U\right)+$$

$$9\,k^2\left(600\sqrt{3}\ \sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U+1039\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^3+1008\,H^2\,U^4\right)+$$

$$3\,g\,H\,U\left(13\,500\,H^2\,k^2\,U+472\,H^6\,k^6\,U+27\left(97\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +504\,U\right)+\right.$$

$$\left.4\,k^4\left(71\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ +1101\,H^4\,U\right)\right)\Big)dt^3+\frac{1}{23\,040\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{11/2}}$$

$$k^9\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}\ +\left(3+H^2\,k^2\right)U\right)\Big(4\,k^8\,U^3\left(239\sqrt{3}\ g\,H^9+58\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\ U\right)+$$

$$27\,k^2\left(372\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}\ +2703\sqrt{3}\ g^2\,H^4\,U+4515\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\ U^2+\right.$$

$$\left.4070\sqrt{3}\ g\,H^3\,U^3+928\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\ U^4\right)+12\,k^6\,U$$

$$\left(213\sqrt{3}\ g^2\,H^8+232\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\ U^3+g\,H^7\,U\left(349\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +979\sqrt{3}\ U\right)\right)+$$

$$81\left(157\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}\ +883\sqrt{3}\ g^2\,H^2\,U+232\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U^4+\right.$$

$$\left.g\,H\,U^2\left(1527\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +1033\sqrt{3}\ U\right)\right)+$$

$$9\,k^4\left(180\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}\ +2672\sqrt{3}\ g^2\,H^6\,U+1392\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\ U^4+\right.$$

$$\left.\left.g\,H^5\,U^2\left(4384\sqrt{g\,H\left(3+H^2\,k^2\right)}\ +5997\sqrt{3}\ U\right)\right)\right)dt^4+O[dt]^5\Bigg)dx^4+O[dx]^5,$$

$$\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$

$$dt^2+$$

$$\frac{i\,k^4\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\,dt^3}{8\left(3+H^2\,k^2\right)^2}-\frac{1}{20\left(3+H^2\,k^2\right)^4}$$

$$\left(k^5\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)$$

$$\left.dt^4+O[dt]^5\right)+$$

$$\left(\frac{k^3\left(3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2U\right)}{24\left(3+H^2\,k^2\right)^2}+\frac{1}{48\left(3+H^2\,k^2\right)^3}k^5\left(3\,g\left(3\sqrt{3}\ H\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,H^3\,k^2\,U+2\,H^5\,k^4\,U\right)+\right.\right.$$

$$U^2\left(-27\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+54\,U+2\,H^6\,k^6\,U-3\,k^2\left(7\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}-18\,H^2\,U\right)-\right.$$

$$\left.\left.2\,k^4\left(2\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}-9\,H^4\,U\right)\right)\right)dt^2+$$

$$\frac{1}{48\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(-9\,g\,H+U\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U-2\,k^2\left(\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}-6\,H^2\,U\right)\right)\right)$$

$$dt^3-\frac{1}{96\left(3+H^2\,k^2\right)^4}$$

$$\left(k^7\left(3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2U\right)\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)$$

$$\left.dt^4+O[dt]^5\right)dx^2+$$

$$\left(\frac{1}{16}i\,k^4\left(\sqrt{3}\ \sqrt{\frac{g\,H}{3+H^2\,k^2}}-2\,U\right)-\frac{1}{32\left(3+H^2\,k^2\right)^2}i\,k^6\left(-3\,g\,H\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}-4\left(3+H^2\,k^2\right)U\right)+\right.\right.$$

$$\left.\left.U^2\left(-15\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(-5\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)$$

$$dt^2+\frac{1}{32\left(3+H^2\,k^2\right)^2}k^7\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(3\,g\,H+U\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\right)dt^3+$$

$$\frac{1}{64\left(3+H^2\,k^2\right)^3}i\,k^8\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)$$

$$\left.\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^4+O[dt]^5\right)$$

$$dx^3 + \Bigg(\Big(k^5\Big(3\sqrt{3}\ g\,H\left(177 + 124\,H^2\,k^2 + 20\,H^4\,k^4\right) - 104\left(9\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 6\,k^2\sqrt{g\,H^5\left(3 + H^2\,k^2\right)} + k^4\sqrt{g\,H^9\left(3 + H^2\,k^2\right)}\right)U\Big)\Big)\Big/\left(1920\sqrt{g\,H}\ \left(3 + H^2\,k^2\right)^{5/2}\right) + \Big(k^7\Big(27\sqrt{3}\ g^2\,H^2\left(167 + 124\,H^2\,k^2 + 20\,H^4\,k^4\right) + g\,H\,U\Big(21\,429\sqrt{3}\ H^2\,k^2\,U + 764\sqrt{3}\ H^6\,k^6\,U + 81\Big(-232\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 267\sqrt{3}\ U\Big) - 24\,k^4\Big(84\sqrt{g\,H^9\left(3 + H^2\,k^2\right)} - 293\sqrt{3}\ H^4\,U\Big)\Big) - 16\Big(459\sqrt{g\,H\left(3 + H^2\,k^2\right)}\ U^3 + 153\,k^4\sqrt{g\,H^9\left(3 + H^2\,k^2\right)}\ U^3 + 17\,k^6\sqrt{g\,H^{13}\left(3 + H^2\,k^2\right)}\ U^3 + 9\,k^2\Big(88\sqrt{g^3\,H^7\left(3 + H^2\,k^2\right)}\ U + 51\sqrt{g\,H^5\left(3 + H^2\,k^2\right)}\ U^3\Big)\Big)\Big)\,dt^2\Big)\Big/\left(11\,520\sqrt{g\,H}\ \left(3 + H^2\,k^2\right)^{7/2}\right) - \frac{1}{3840\left(3+H^2\,k^2\right)^4}i\,k^8\Big(54\,g^2\,H^2\left(81 + 62\,H^2\,k^2 + 10\,H^4\,k^4\right) + 84\,H^8\,k^8\,U^4 + 243\,U^3\Big(-39\sqrt{3}\ \sqrt{g\,H\left(3 + H^2\,k^2\right)} + 28\,U\Big) + 36\,k^4\,U^3\Big(-85\sqrt{3}\ \sqrt{g\,H^9\left(3 + H^2\,k^2\right)} + 126\,H^4\,U\Big) + 4\,k^6\,U^3\Big(-83\sqrt{3}\ \sqrt{g\,H^{13}\left(3 + H^2\,k^2\right)} + 252\,H^6\,U\Big) - 9\,k^2\Big(600\sqrt{3}\ \sqrt{g^3\,H^7\left(3 + H^2\,k^2\right)}\ U + 1039\sqrt{3}\ \sqrt{g\,H^5\left(3 + H^2\,k^2\right)}\ U^3 - 1008\,H^2\,U^4\Big) + 3\,g\,H\,U\Big(13\,500\,H^2\,k^2\,U + 472\,H^6\,k^6\,U + 27\Big(-97\sqrt{3}\ \sqrt{g\,H\left(3 + H^2\,k^2\right)} + 504\,U\Big) - 4\,k^4\Big(71\sqrt{3}\ \sqrt{g\,H^9\left(3 + H^2\,k^2\right)} - 1101\,H^4\,U\Big)\Big)\Big)\,dt^3 - \frac{1}{23\,040\left(\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{11/2}\right)}\Big(k^9\Big(\sqrt{3}\ \sqrt{g\,H\left(3 + H^2\,k^2\right)} - \left(3 + H^2\,k^2\right)U\Big)\Big(4\,k^8\,U^3\Big(-239\sqrt{3}\ g\,H^9 + 58\sqrt{g\,H^{17}\left(3 + H^2\,k^2\right)}\ U\Big) + 27\,k^2\Big(372\sqrt{g^5\,H^9\left(3 + H^2\,k^2\right)} - 2703\sqrt{3}\ g^2\,H^4\,U + 4515\sqrt{g^3\,H^7\left(3 + H^2\,k^2\right)}\ U^2 - 4070\sqrt{3}\ g\,H^3\,U^3 + 928\sqrt{g\,H^5\left(3 + H^2\,k^2\right)}\ U^4\Big) + 9\,k^4\Big(180\sqrt{g^5\,H^{13}\left(3 + H^2\,k^2\right)} - 2672\sqrt{3}\ g^2\,H^6\,U + 1392\sqrt{g\,H^9\left(3 + H^2\,k^2\right)}\ U^4 + g\,H^5\,U^2\Big(4384\sqrt{g\,H\left(3 + H^2\,k^2\right)} - 5997\sqrt{3}\ U\Big)\Big) + 81\Big(157\sqrt{g^5\,H^5\left(3 + H^2\,k^2\right)} - 883\sqrt{3}\ g^2\,H^2\,U + 232\sqrt{g\,H\left(3 + H^2\,k^2\right)}\ U^4 + g\,H\,U^2\Big(1527\sqrt{g\,H\left(3 + H^2\,k^2\right)} - 1033\sqrt{3}\ U\Big)\Big) - 12\,k^6\,U\Big(213\sqrt{3}\ g^2\,H^8 - 232\sqrt{g\,H^{13}\left(3 + H^2\,k^2\right)}\ U^3 + g\,H^7\,U\Big(-349\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 979\sqrt{3}\ U\Big)\Big)\Big)\,dt^4 + O[dt]^5\Bigg)\,dx^4 + O[dx]^5\Bigg\}$$

Out[196]= Omega error ||

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\left\{\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U
    \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6
    \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H
    \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}-\frac{\left(k^5 \left(\left(H^2
    k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2
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k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{20 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(2 \left(H^2 k^2+3\right)^2 U−3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{24 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(\left(2 H^6 U k^6+2 \left(9 U H^4+2 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+3 \left(18 U H^2+7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+54 U+27 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+g \left(6 k^4 U H^5+18 k^2 U H^3−9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} H\right)\right) \text{dt}^2}{48 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(U \left(2 H^4 U k^4+2 \left(6 U H^2+\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)−9 g H\right) \text{dt}^3}{48 \left(H^2 k^2+3\right)^3}−\frac{\left(k^7 \left(2 \left(H^2 k^2+3\right)^2 U−3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{96 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(−\frac{1}{16} i k^4 \left(2 U+\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}\right)−\frac{i k^6 \left(\left(2 H^4 U k^4+\left(12 U H^2+5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g H \left(4 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^2}+\frac{k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 g H+U \left(2 \left(H^2 k^2+3\right) U+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(2 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(−\frac{k^5 \left(3 \sqrt{3} g H \left(20 H^4 k^4+124 H^2 k^2+177\right)+104 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{1920 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{5/2}\right)}−\frac{\left(k^7 \left(27 \sqrt{3} g^2 \left(20 H^4 k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6+24 \left(293 \sqrt{3} U H^4+84 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267 \sqrt{3} U+232 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) H+16 \left(17 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+153 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(51 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+88 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+459 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text{dt}^2}{11520 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)}−\frac{i k^8 \left(84 H^8 U^4 k^8+4 U^3 \left(252 U H^6+83 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+36 U^3 \left(126 U H^4+85 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+9 \left(1008 H^2 U^4+1039 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+600 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U+39 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+3 g H U \left(472 H^6 U k^6+4 \left(1101 U H^4+71 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+13500 H^2 U k^2+27 \left(504 U+97 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{3840 \left(H^2 k^2+3\right)^4}+\frac{k^9 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(4 U^3 \left(239 \sqrt{3} g H^9+58 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U\right) k^8+12 U \left(213 \sqrt{3} g^2 H^8+g U \left(979 \sqrt{3} U+349 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7+232 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(2672 \sqrt{3} g^2 U H^6+g U^2 \left(5997 \sqrt{3} U+4384 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+1392 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+180 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+27 \left(2703 \sqrt{3} g^2 U H^4+4070 \sqrt{3} g

U^3 H^3+928 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+4515 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}
U^2+372 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(232 \sqrt{g H \left(H^2 k^2+3\right)}
U^4+g H \left(1033 \sqrt{3} U+1527 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+883 \sqrt{3} g^2
H^2 U+157 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{23040 \sqrt{g H} \left(H^2
k^2+3\right)^{11/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right),\left(\frac{k^3
\left(\left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2
k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2
k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2
k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}-\frac{\left(k^5 \left(\left(H^2
k^2+3\right) U-\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2
k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{20 \left(H^2
k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(2 U \left(H^2 k^2+3\right)^2+3
\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{24 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(\left(2 H^6 U
k^6-2 \left(2 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}-9 H^4 U\right) k^4-3 \left(7 \sqrt{3} \sqrt{g H^5
\left(H^2 k^2+3\right)}-18 H^2 U\right) k^2+54 U-27 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)
U^2+3 g \left(2 k^4 U H^5+6 k^2 U H^3+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} H\right)\right)
\text{dt}^2}{48 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3}
\sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(U \left(2 H^4 U k^4-2 \left(\sqrt{3} \sqrt{g H^5 \left(H^2
k^2+3\right)}-6 H^2 U\right) k^2+18 U-3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)-9 g H\right)
\text{dt}^3}{48 \left(H^2 k^2+3\right)^3}-\frac{\left(k^7 \left(2 U \left(H^2 k^2+3\right)^2+3 \sqrt{3} \sqrt{g
H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2
k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{96 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(\frac{1}{16} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}-2 U\right)-\frac{i k^6
\left(U^2 \left(2 H^4 U k^4+\left(12 H^2 U-5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right)
k^2+18 U-15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)-3 g H \left(\sqrt{3} \sqrt{g H \left(H^2
k^2+3\right)}-4 \left(H^2 k^2+3\right) U\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^2}+\frac{k^7
\left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)
\left(3 g H+U \left(2 \left(H^2 k^2+3\right) U-3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)
\text{dt}^3}{32 \left(H^2 k^2+3\right)^2}+\frac{i k^8 \left(2 \left(H^2 k^2+3\right) U-\sqrt{3} \sqrt{g H
\left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U-2 \sqrt{3} \sqrt{g H \left(H^2
k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{k^5 \left(3 \sqrt{3} g H \left(20 H^4 k^4+124 H^2 k^2+177\right)-104 \left(\sqrt{g
H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2
k^2+3\right)}\right) U\right)}{1920 \sqrt{g H} \left(H^2 k^2+3\right)^{5/2}}+\frac{k^7 \left(27 \sqrt{3}
g^2 \left(20 H^4 k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6-24 \left(84 \sqrt{g
H^9 \left(H^2 k^2+3\right)}-293 \sqrt{3} H^4 U\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267
\sqrt{3} U-232 \sqrt{g H \left(H^2 k^2+3\right)}\right) H-16 \left(17 \sqrt{g H^{13}} \left(H^2
k^2+3\right)} U^3 k^6+153 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(51 \sqrt{g H^5 \left(H^2
k^2+3\right)} U^3+88 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+459 \sqrt{g H \left(H^2
k^2+3\right)} U^3\right)\right) \text{dt}^2}{11520 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}-\frac{i k^8
\left(84 H^8 U^4 k^8+4 U^3 \left(252 H^6 U-83 \sqrt{3} \sqrt{g H^{13}} \left(H^2 k^2+3\right)\right)
k^6+36 U^3 \left(126 H^4 U-85 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4-9 \left(-1008
H^2 U^4+1039 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+600 \sqrt{3} \sqrt{g^3 H^7 \left(H^2
k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U-39

k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U−39 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+3 g H U \left(472 H^6 U k^6−4 \left(71 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}−1101 H^4 U\right) k^4+13500 H^2 U k^2+27 \left(504 U−97 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{3840 \left(H^2 k^2+3\right)^4}−\frac{\left(k^9 \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}−\left(H^2 k^2+3\right) U\right) \left(4 U^3 \left(58 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U−239 \sqrt{3} g H^9\right) k^8−12 U \left(213 \sqrt{3} g^2 H^8+g U \left(979 \sqrt{3} U−349 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7−232 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(−2672 \sqrt{3} g^2 U H^6+g U^2 \left(4384 \sqrt{g H \left(H^2 k^2+3\right)}−5997 \sqrt{3} U\right) H^5+1392 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+180 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+27 \left(−2703 \sqrt{3} g^2 U H^4−4070 \sqrt{3} g U^3 H^3+928 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+4515 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+372 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(232 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1527 \sqrt{g H \left(H^2 k^2+3\right)}−1033 \sqrt{3} U\right) U^2−883 \sqrt{3} g^2 H^2 U+157 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{23040 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{11/2}\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right\}

Out[197]=

Out[198]= $$\mathrm{EA} \parallel \left\{\left\{1+\frac{i\, e^{\frac{i\,\mathrm{dx}\,k}{2}}\left(1-e^{-i\,\mathrm{dx}\,k}\right)\left(-1+e^{i\,\mathrm{dt}\,w}\right) H^2\, k^3\, U \operatorname{Csc}\left[\frac{\mathrm{dx}\,k}{2}\right]}{\left(6+2\,H^2\,k^2\right) w},\ \frac{i\, e^{\frac{i\,\mathrm{dx}\,k}{2}}\left(1-e^{-i\,\mathrm{dx}\,k}\right)\left(-1+e^{i\,\mathrm{dt}\,w}\right) H\, k \operatorname{Csc}\left[\frac{\mathrm{dx}\,k}{2}\right]}{2\left(H+\frac{H^3\,k^2}{3}\right) w}\right\},\ \left\{\frac{i\, e^{\frac{i\,\mathrm{dx}\,k}{2}}\left(1-e^{-i\,\mathrm{dx}\,k}\right)\left(-1+e^{i\,\mathrm{dt}\,w}\right) k\left(g\,H\left(3+H^2\,k^2\right)-3\,U^2\right) \operatorname{Csc}\left[\frac{\mathrm{dx}\,k}{2}\right]}{\left(6+2\,H^2\,k^2\right) w},\ 1+\frac{i\, e^{\frac{i\,\mathrm{dx}\,k}{2}}\left(1-e^{-i\,\mathrm{dx}\,k}\right)\left(-1+e^{i\,\mathrm{dt}\,w}\right) k\left(6+H^2\,k^2\right) U \operatorname{Csc}\left[\frac{\mathrm{dx}\,k}{2}\right]}{\left(6+2\,H^2\,k^2\right) w}\right\}\right\}$$

Out[199]= EA || \left(
\begin{array}{cc}
\frac{i e^{\frac{i \text{dx} k}{2}} \left(1−e^{−i \text{dx} k}\right) \left(−1+e^{i \text{dt} w}\right) H^2 U \csc \left(\frac{\text{dx} k}{2}\right) k^3}{\left(2 H^2 k^2+6\right) w}+1 & \frac{i e^{\frac{i \text{dx} k}{2}} \left(1−e^{−i \text{dx} k}\right) \left(−1+e^{i \text{dt} w}\right) H k \csc \left(\frac{\text{dx} k}{2}\right)}{2 \left(\frac{k^2 H^3}{3}+H\right) w} \\
\frac{i e^{\frac{i \text{dx} k}{2}} \left(1−e^{−i \text{dx} k}\right) \left(−1+e^{i \text{dt} w}\right) k \left(g H \left(H^2 k^2+3\right)−3 U^2\right) \csc \left(\frac{\text{dx} k}{2}\right)}{\left(2 H^2 k^2+6\right) w} & \frac{i e^{\frac{i \text{dx} k}{2}} \left(1−e^{−i \text{dx} k}\right) \left(−1+e^{i \text{dt} w}\right) k \left(H^2 k^2+6\right) U \csc \left(\frac{\text{dx} k}{2}\right)}{\left(2 H^2 k^2+6\right) w}+1 \\
\end{array}
\right)

Out[200]= Eerr ||

$$\left\{\left\{\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U+3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}-\right.\right.$$

$$\left.\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3 dt^3-\frac{1}{24}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4 dt^4+O[dt]^5\right)+$$

$$\left(-\frac{i\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)U\,dt}{12\left(3+H^2\,k^2\right)^2}-\frac{\left(-9\,g\,H\,k^4+36\,k^4\,U^2+12\,H^2\,k^6\,U^2+2\,H^4\,k^8\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(-\frac{1}{8}\left(k^4\,U\right)dt+\left(\frac{3\,i\,g\,H\,k^5}{16\left(3+H^2\,k^2\right)}+\frac{i\,H^2\,k^7\,U^2}{8\left(3+H^2\,k^2\right)}\right)dt^2+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(405\,k^5\,U+351\,H^2\,k^7\,U+116\,H^4\,k^9\,U+13\,H^6\,k^{11}\,U\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{1}{1440\left(3+H^2\,k^2\right)^3}\left(1161\,g\,H\,k^6+837\,g\,H^3\,k^8+135\,g\,H^5\,k^{10}-\right.\right.$$

$$\left.\left.351\,k^6\,U^2+297\,H^2\,k^8\,U^2+387\,H^4\,k^{10}\,U^2+73\,H^6\,k^{12}\,U^2\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5,$$

$$\left(-\frac{3\,i\,k\,dt}{3+H^2\,k^2}-\frac{3\left(k^2\,U\right)dt^2}{3+H^2\,k^2}+O[dt]^5\right)+\left(\left(\frac{3\,i\,k^3}{2\left(3+H^2\,k^2\right)^2}+\frac{i\,H^2\,k^5}{4\left(3+H^2\,k^2\right)^2}\right)dt+\frac{3\,k^4\,U\,dt^2}{4\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(\frac{3\,i\,k^5\,U\,dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+\left(\frac{i\left(-54\,k^5+H^4\,k^9\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{\left(387\,k^6\,U+279\,H^2\,k^8\,U+45\,H^4\,k^{10}\,U\right)dt^2}{240\left(3+H^2\,k^2\right)^3}+O[dt]^5\right)dx^4+O[dx]^5\Big\},$$

$$\left\{\left(-\frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)dt}{3+H^2\,k^2}-\frac{k^2\,U\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)dt^2}{3+H^2\,k^2}+O[dt]^5\right)+\right.$$

$$\left(-\frac{i\left(9\,g\,H\,k^3+6\,g\,H^3\,k^5+g\,H^5\,k^7+18\,k^3\,U^2+3\,H^2\,k^5\,U^2\right)dt}{12\left(3+H^2\,k^2\right)^2}+\frac{\left(-18\,g\,H\,k^4\,U-12\,g\,H^3\,k^6\,U-2\,g\,H^5\,k^8\,U-9\,k^4\,U^3\right)dt^2}{12\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(-\frac{1}{8}\left(g\,H\,k^4\right)dt+\frac{i\left(6\,g\,H\,k^5\,U+2\,g\,H^3\,k^7\,U-3\,k^5\,U^3\right)dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(351\,g\,H\,k^5+351\,g\,H^3\,k^7+117\,g\,H^5\,k^9+13\,g\,H^7\,k^{11}+54\,k^5\,U^2-H^4\,k^9\,U^2\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{1}{720\left(3+H^2\,k^2\right)^3}\left(1971\,g\,H\,k^6\,U+1971\,g\,H^3\,k^8\,U+657\,g\right.\right.$$

$$\left.\left.H^5\,k^{10}\,U+73\,g\,H^7\,k^{12}\,U-1161\,k^6\,U^3-837\,H^2\,k^8\,U^3-135\,H^4\,k^{10}\,U^3\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5,$$

$$\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}-3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U-3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}-\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3 dt^3-\right.$$

$$\left.\frac{1}{24}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4 dt^4+O[dt]^5\right)+$$

$$\left(-\frac{i\left(-9\,k^3\,U+3\,H^2\,k^5\,U+H^4\,k^7\,U\right)dt}{12\left(3+H^2\,k^2\right)^2}+\frac{\left(9\,g\,H\,k^4-12\,H^2\,k^6\,U^2-2\,H^4\,k^8\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(-\frac{1}{8}\left(k^4\,U\right)dt+\frac{i\,k^5\left(3\,g\,H+12\,U^2+2\,H^2\,k^2\,U^2\right)dt^2}{16\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(297\,k^5+351\,H^2\,k^7+118\,H^4\,k^9+13\,H^6\,k^{11}\right)U\,dt}{240\left(3+H^2\,k^2\right)^3}+\frac{1}{1440\left(3+H^2\,k^2\right)^3}\left(1161\,g\,H\,k^6+837\,g\,H^3\,k^8+135\,g\,H^5\,k^{10}+\right.\right.$$

$$\left.\left.4293\,k^6\,U^2+3645\,H^2\,k^8\,U^2+927\,H^4\,k^{10}\,U^2+73\,H^6\,k^{12}\,U^2\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5\Big\}\Big\}$$

Out[201]= Eerr || \left(

\begin{array}{cc}

\left(\frac{i \left(3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \text{dt}}{H^2 k^2+3}+\frac{\left(3 U^2 k^2+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2\right) \text{dt}^2}{H^2 k^2+3}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2 k^2+3}\right)^3 \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2

```
k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(H^4 k^7+9 H^2 k^5+27
k^3\right) U \text{dt}}{12 \left(H^2 k^2+3\right)^2}-\frac{\left(2 H^4 U^2 k^8+12 H^2 U^2 k^6+36
U^2 k^4-9 g H k^4\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(-\frac{1}{8} \left(k^4 U\right) \text{dt}+\left(\frac{i H^2 U^2 k^7}{8 \left(H^2
k^2+3\right)}+\frac{3 i g H k^5}{16 \left(H^2 k^2+3\right)}\right) \text{dt}^2+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{i \left(13 H^6 U k^{11}+116 H^4 U k^9+351 H^2 U k^7+405 U k^5\right)
\text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73 H^6 U^2 k^{12}+135 g H^5 k^{10}+387 H^4
U^2 k^{10}+837 g H^3 k^8+297 H^2 U^2 k^8-351 U^2 k^6+1161 g H k^6\right) \text{dt}^2}{1440
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)
& \left(-\frac{3 i k \text{dt}}{H^2 k^2+3}-\frac{3 \left(k^2 U\right) \text{dt}^2}{H^2
k^2+3}+O\left(\text{dt}^5\right)\right)+\left(\left(\frac{i H^2 k^5}{4 \left(H^2 k^2+3\right)^2}+\frac{3
i k^3}{2 \left(H^2 k^2+3\right)^2}\right) \text{dt}+\frac{3 k^4 U \text{dt}^2}{4 \left(H^2
k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{3 i k^5 U \text{dt}^2}{8 \left(H^2
k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(H^4 k^9-54 k^5\right) \text{dt}}{240
\left(H^2 k^2+3\right)^3}+\frac{\left(45 H^4 U k^{10}+279 H^2 U k^8+387 U k^6\right) \text{dt}^2}{240
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\left(-\frac{i k \left(g k^2 H^3+3 g H-3 U^2\right) \text{dt}}{H^2 k^2+3}-\frac{k^2 U \left(g k^2 H^3+3
g H-3 U^2\right) \text{dt}^2}{H^2 k^2+3}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(g
H^5 k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12 \left(H^2
k^2+3\right)^2}+\frac{\left(-2 g H^5 U k^8-12 g H^3 U k^6-9 U^3 k^4-18 g H U k^4\right)
\text{dt}^2}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(-\frac{1}{8} \left(g
H k^4\right) \text{dt}+\frac{i \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \text{dt}^2}{8 \left(H^2
k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(13 g H^7 k^{11}+117 g H^5 k^9-H^4
U^2 k^9+351 g H^3 k^7+54 U^2 k^5+351 g H k^5\right) \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73
g H^7 U k^{12}-135 H^4 U^3 k^{10}+657 g H^5 U k^{10}-837 H^2 U^3 k^8+1971 g H^3 U k^8-1161
U^3 k^6+1971 g H U k^6\right) \text{dt}^2}{720 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)
\text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{i \left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}-3 k
U\right) \text{dt}}{H^2 k^2+3}+\frac{\left(\sqrt{3} k^2 \sqrt{g H \left(H^2 k^2+3\right)} U-3 k^2 U^2\right)
\text{dt}^2}{H^2 k^2+3}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2
k^2+3}\right)^3 \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}
k}{H^2 k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(H^4 U k^7+3
H^2 U k^5-9 U k^3\right) \text{dt}}{12 \left(H^2 k^2+3\right)^2}+\frac{\left(-2 H^4 U^2 k^8-12 H^2
U^2 k^6+9 g H k^4\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(-\frac{1}{8} \left(k^4 U\right) \text{dt}+\frac{i k^5 \left(2 H^2 k^2 U^2+12
U^2+3 g H\right) \text{dt}^2}{16 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{i \left(13 H^6 k^{11}+118 H^4 k^9+351 H^2 k^7+297 k^5\right) U
\text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73 H^6 U^2 k^{12}+135 g H^5 k^{10}+927 H^4
U^2 k^{10}+837 g H^3 k^8+3645 H^2 U^2 k^8+4293 U^2 k^6+1161 g H k^6\right) \text{dt}^2}{1440
\left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\end{array}
\right)
```

In[202]:=
```
KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → 0 /. am → (U - Sqrt[g * H]);
```

```
KurFWSeta =
  KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.
   qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA , {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA , {k > 0, U > 0, H > 0, g > 0}];

KurFWSG = KurFWS /. fp → (U * Rpp * G + U * H * v + g * H * Rpp * n) /.
     fm → (U * Rmp * G + U * H * v + g * H * Rmp * n) /. qp → Rpp * G /. qm → Rmp * G;
KurFWSG = KurFWSG /. v → (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G → 0 /. n → 1];
KfGGp = FullSimplify[KurFWSG /. n → 0 /. G → 1];
KfGn = KfGnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfGG = KfGGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;

FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA , {k > 0, U > 0, H > 0, g > 0}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA , {k > 0, U > 0, H > 0, g > 0}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2 ;
Eerr = Series[Emat2 - Exp[-I * wAp * dt] * IdentityMatrix[2], {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];

RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - {wAp, wAm}, {k > 0, H > 0, g > 0, U > 0} ];


Text[Row[{"  U  > Sqrt(gH)"}]]
Text[" "]
Text[Row[{"Fnn  ||   ", Kfnnp}]]
```

```
Text[Row[{"Fnn  ||   ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error  ||   ", Fnn2TAr}]]
Text[Row[{"Fnn error  ||   ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG  ||   ", KfnGp}]]
Text[Row[{"FnG  ||   ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error  ||   ", FnG2TAr}]]
Text[Row[{"FnG error  ||   ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn  ||   ", KfGnp}]]
Text[Row[{"FGn  ||   ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error  ||   ", FGn2TAr}]]
Text[Row[{"FGn error  ||   ", TeXForm[FGn2TAr]}]]
Text[" "]
Text[Row[{"FGG  ||   ", KfGGp}]]
Text[Row[{"FGG  ||   ", TeXForm[KfGGp]}]]
Text[Row[{"FGG error  ||   ", FGG2TAr}]]
Text[Row[{"FGG error  ||   ", TeXForm[FGG2TAr]}]]
Text[" "]
Text[" "]
Text[Row[{"Omega error  ||   ", RKstepTayr}]]
Text[Row[{"Omega error  ||   ", TeXForm[RKstepTayr]}]]
Text[" "]
Text[Row[{"EA ||   ", EA}]]
Text[Row[{"EA  ||   ", TeXForm[EA]}]]
Text[Row[{"Eerr ||   ", Eerr}]]
Text[Row[{"Eerr ||   ", TeXForm[Eerr]}]]
```

Out[236]= U > Sqrt(gH)

Out[237]=

Out[238]= Fnn || Gnp H + Rpp U

Out[239]= Fnn || \text{Gnp} H+\text{Rpp} U

Out[240]= Fnn error ||

$$\left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)U\,dt}{12\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(\frac{1}{8}\,k^4\,U\,dt+O[dt]^4\right)dx^3+O[dx]^4$$

Out[241]= Fnn error ||

```
\left(-\frac{\text{dt}^2 \left(H^2 k^3 U w\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 H^2 k^3 U w^2}{6
    \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(H^4 k^7+9 H^2
    k^5+27 k^3\right) U \text{dt}}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3
    \left(\frac{1}{8} k^4 U \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)
```

Out[242]=

Out[243]= FnG || GGp H

Out[244]= FnG || \text{GGp} H

Out[245]= FnG error || $\left(-\frac{3\,(k\,w)\,dt^2}{2\,(3+H^2\,k^2)}-\frac{i\,k\,w^2\,dt^3}{2\,(3+H^2\,k^2)}+O[dt]^4\right)+\left(\frac{i\,(6\,k^3+H^2\,k^5)\,dt}{4\,(3+H^2\,k^2)^2}+O[dt]^4\right)dx^2+O[dx]^4$

Out[246]= FnG error || \left(-\frac{3 \text{dt}^2 (k w)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2}{2 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(\frac{i \left(H^2 k^5+6 k^3\right) \text{dt}}{4 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[247]=

Out[248]= FGn || H (g Rpp + Gnp U)

Out[249]= FGn || H (g \text{Rpp}+\text{Gnp} U)

Out[250]= FGn error || $\left(-\frac{(k\,(3\,g\,H+g\,H^3\,k^2-3\,U^2)\,w)\,dt^2}{2\,(3+H^2\,k^2)}-\frac{i\,k\,(3\,g\,H+g\,H^3\,k^2-3\,U^2)\,w^2\,dt^3}{6\,(3+H^2\,k^2)}+O[dt]^4\right)+$
$\left(-\frac{i\,(9\,g\,H\,k^3+6\,g\,H^3\,k^5+g\,H^5\,k^7+18\,k^3\,U^2+3\,H^2\,k^5\,U^2)\,dt}{12\,(3+H^2\,k^2)^2}+O[dt]^4\right)dx^2+\left(\frac{1}{8}\,g\,H\,k^4\,dt+O[dt]^4\right)dx^3+O[dx]^4$

Out[251]= FGn error ||
\left(-\frac{\text{dt}^2 \left(k w \left(g H^3 k^2+3 g H-3 U^2\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k w^2 \left(g H^3 k^2+3 g H-3 U^2\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(g H^5 k^7+6 g H^3 k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(\frac{1}{8} g H k^4 \text{dt}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^4\right)

Out[252]=

Out[253]= FGG || (GGp H + Rpp) U

Out[254]= FGG || U (\text{GGp} H+\text{Rpp})

Out[255]= FGG error || $\left(-\frac{(k\,(6+H^2\,k^2)\,U\,w)\,dt^2}{2\,(3+H^2\,k^2)}-\frac{i\,k\,(6+H^2\,k^2)\,U\,w^2\,dt^3}{6\,(3+H^2\,k^2)}+O[dt]^4\right)+\left(-\frac{i\,(-9\,k^3+3\,H^2\,k^5+H^4\,k^7)\,U\,dt}{12\,(3+H^2\,k^2)^2}+O[dt]^4\right)dx^2+$
$\left(\frac{1}{8}\,k^4\,U\,dt+O[dt]^4\right)dx^3+\left(\frac{i\,(297\,k^5+351\,H^2\,k^7+118\,H^4\,k^9+13\,H^6\,k^{11})\,U\,dt}{240\,(3+H^2\,k^2)^3}+O[dt]^4\right)dx^4+O[dx]^5$

Out[256]= FGG error ||
\left(-\frac{\text{dt}^2 \left(k U w \left(H^2 k^2+6\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{i \text{dt}^3 k U w^2 \left(H^2 k^2+6\right)}{6 \left(H^2 k^2+3\right)}+O\left(\text{dt}^4\right)\right)+\text{dx}^2 \left(-\frac{i \left(H^4 k^7+3 H^2 k^5-9 k^3\right) U \text{dt}}{12 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)+\text{dx}^3 \left(\frac{1}{8} k^4 U \text{dt}+O\left(\text{dt}^4\right)\right)+\text{dx}^4 \left(\frac{i \left(13 H^6 k^{11}+118 H^4 k^9+351 H^2 k^7+297 k^5\right) U \text{dt}}{240 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^4\right)\right)+O\left(\text{dx}^5\right)

Out[257]=

Out[258]=

Out[259]= Omega error ||

$$\left\{\left(\frac{1}{6\left(3+H^2\,k^2\right)^2}k^3\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)dt^2+\right.\right.$$

$$\frac{i\,k^4\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^3}{8\left(3+H^2\,k^2\right)^2}-\frac{1}{20\left(3+H^2\,k^2\right)^4}\left(k^5\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)^3\right.$$

$$\left.\left.\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)dt^4+O[dt]^5\right)+$$

$$\left(\frac{k^3\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2U\right)}{24\left(3+H^2\,k^2\right)^2}+\frac{1}{48\left(3+H^2\,k^2\right)^3}k^5\left(g\left(-9\sqrt{3}\ H\sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,H^3\,k^2\,U+6\,H^5\,k^4\,U\right)+\right.\right.$$

$$U^2\left(27\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+54\,U+2\,H^6\,k^6\,U+3\,k^2\left(7\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}+18\,H^2\,U\right)+$$

$$\left.\left.2\,k^4\left(2\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}+9\,H^4\,U\right)\right)\right)dt^2+$$

$$\frac{1}{48\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(-9\,g\,H+U\left(3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+2\,k^2\left(\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}+6\,H^2\,U\right)\right)\right)$$

$$dt^3-\frac{1}{96\left(3+H^2\,k^2\right)^4}\left(k^7\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2U\right)\right.$$

$$\left.\left.\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)dt^4+O[dt]^5\right)dx^2+$$

$$\left(\frac{1}{16}i\,k^4\left(\sqrt{3}\ \sqrt{\frac{g\,H}{3+H^2\,k^2}}+2\,U\right)+\frac{1}{32\left(3+H^2\,k^2\right)^2}i\,k^6\left(3\,g\,H\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+4\left(3+H^2\,k^2\right)U\right)+\right.\right.$$

$$\left.\left.U^2\left(15\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(5\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\right)dt^2-$$

$$\frac{1}{32\left(3+H^2\,k^2\right)^2}\left(k^7\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right.$$

$$\left.\left.\left(3\,g\,H+U\left(3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\right)\right)dt^3-$$

$$\frac{1}{64\left(3+H^2\,k^2\right)^3}i\,k^8\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)$$

$$\left.\left(3\,g\,H+U\left(2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^4+O[dt]^5\right)dx^3+$$

$$\left(-\left(\left(k^5\left(3\sqrt{3}\ g\,H\left(177+124\,H^2k^2+20\,H^4k^4\right)+104\left(9\sqrt{g\,H\left(3+H^2k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2k^2\right)}\right)U\right)\right)\Big/\left(1920\left(\sqrt{g\,H}\ \left(3+H^2k^2\right)^{5/2}\right)\right)\right)-\left(\left(k^7\left(27\sqrt{3}\ g^2H^2\left(167+124\,H^2k^2+20\,H^4k^4\right)+g\,H\,U\left(21\,429\sqrt{3}\ H^2k^2U+764\sqrt{3}\ H^6k^6U+81\left(232\sqrt{g\,H\left(3+H^2k^2\right)}+267\sqrt{3}\ U\right)+24\,k^4\left(84\sqrt{g\,H^9\left(3+H^2k^2\right)}+293\sqrt{3}\ H^4U\right)\right)+16\left(459\sqrt{g\,H\left(3+H^2k^2\right)}\ U^3+153\,k^4\sqrt{g\,H^9\left(3+H^2k^2\right)}\ U^3+17\,k^6\sqrt{g\,H^{13}\left(3+H^2k^2\right)}\ U^3+9\,k^2\left(88\sqrt{g^3\,H^7\left(3+H^2k^2\right)}\ U+51\sqrt{g\,H^5\left(3+H^2k^2\right)}\ U^3\right)\right)\right)\right)dt^2\right)\Big/\left(11\,520\left(\sqrt{g\,H}\ \left(3+H^2k^2\right)^{7/2}\right)\right)-\frac{1}{3840\left(3+H^2k^2\right)^4}i\,k^8\left(54\,g^2H^2\left(81+62\,H^2k^2+10\,H^4k^4\right)+84\,H^8k^8U^4+243\,U^3\left(39\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+28\,U\right)+36\,k^4U^3\left(85\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2k^2\right)}+126\,H^4U\right)+4\,k^6U^3\left(83\sqrt{3}\ \sqrt{g\,H^{13}\left(3+H^2k^2\right)}+252\,H^6U\right)+9\,k^2\left(600\sqrt{3}\ \sqrt{g^3\,H^7\left(3+H^2k^2\right)}\ U+1039\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2k^2\right)}\ U^3+1008\,H^2U^4\right)+3\,g\,H\,U\left(13\,500\,H^2k^2U+472\,H^6k^6U+27\left(97\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+504\,U\right)+4\,k^4\left(71\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2k^2\right)}+1101\,H^4U\right)\right)\right)dt^3+\frac{1}{23\,040\sqrt{g\,H}\ \left(3+H^2k^2\right)^{11/2}}k^9\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\left(4\,k^8U^3\left(239\sqrt{3}\ g\,H^9+58\sqrt{g\,H^{17}\left(3+H^2k^2\right)}\ U\right)+27\,k^2\left(372\sqrt{g^5\,H^9\left(3+H^2k^2\right)}+2703\sqrt{3}\ g^2H^4U+4515\sqrt{g^3\,H^7\left(3+H^2k^2\right)}\ U^2+4070\sqrt{3}\ g\,H^3U^3+928\sqrt{g\,H^5\left(3+H^2k^2\right)}\ U^4\right)+12\,k^6U\left(213\sqrt{3}\ g^2H^8+232\sqrt{g\,H^{13}\left(3+H^2k^2\right)}\ U^3+g\,H^7U\left(349\sqrt{g\,H\left(3+H^2k^2\right)}+979\sqrt{3}\ U\right)\right)+81\left(157\sqrt{g^5\,H^5\left(3+H^2k^2\right)}+883\sqrt{3}\ g^2H^2U+232\sqrt{g\,H\left(3+H^2k^2\right)}\ U^4+g\,H\,U^2\left(1527\sqrt{g\,H\left(3+H^2k^2\right)}+1033\sqrt{3}\ U\right)\right)+9\,k^4\left(180\sqrt{g^5\,H^{13}\left(3+H^2k^2\right)}+2672\sqrt{3}\ g^2H^6U+1392\sqrt{g\,H^9\left(3+H^2k^2\right)}\ U^4+g\,H^5U^2\left(4384\sqrt{g\,H\left(3+H^2k^2\right)}+5997\sqrt{3}\ U\right)\right)\right)dt^4+O[dt]^5\Big)dx^4+O[dx]^5,$$

$$\left(\frac{1}{6\left(3+H^2k^2\right)^2}k^3\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)\right.$$

$$\left.dt^2+\frac{i\,k^4\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)}+\left(3+H^2k^2\right)U\right)\right)^2dt^3}{8\left(3+H^2k^2\right)^2}-\frac{1}{20\left(3+H^2k^2\right)^4}\right)$$

$$\left(k^3\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)\right)$$

$$dt^4+O[dt]^5\Bigg)+$$

$$\Bigg(\frac{k^3\left(3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2U\right)}{24\left(3+H^2\,k^2\right)^2}+\frac{1}{48\left(3+H^2\,k^2\right)^3}k^5\left(3\,g\left(3\sqrt{3}\ H\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,H^3\,k^2\,U+2\,H^5\,k^4\,U\right)+$$

$$U^2\left(-27\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+54\,U+2\,H^6\,k^6\,U-3\,k^2\left(7\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}-18\,H^2\,U\right)-$$

$$2\,k^4\left(2\sqrt{3}\ \sqrt{g\,H^9\left(3+H^2\,k^2\right)}-9\,H^4\,U\right)\right)\Big)\,dt^2+$$

$$\frac{1}{48\left(3+H^2\,k^2\right)^3}i\,k^6\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(-9\,g\,H+U\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U-2\,k^2\left(\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}-6\,H^2\,U\right)\right)\right)$$

$$dt^3-\frac{1}{96\left(3+H^2\,k^2\right)^4}$$

$$\left(k^7\left(3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)^2U\right)\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2\right)$$

$$dt^4+O[dt]^5\Bigg)\,dx^2+$$

$$\Bigg(-\frac{1}{16}\,i\,k^4\left(\sqrt{3}\ \sqrt{\frac{g\,H}{3+H^2\,k^2}}-2\,U\right)+\frac{1}{32\left(3+H^2\,k^2\right)^2}i\,k^6\left(-3\,g\,H\left(\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}-4\left(3+H^2\,k^2\right)U\right)+$$

$$U^2\left(-15\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+18\,U+2\,H^4\,k^4\,U+k^2\left(-5\sqrt{3}\ \sqrt{g\,H^5\left(3+H^2\,k^2\right)}+12\,H^2\,U\right)\right)\Big)$$

$$dt^2-\frac{1}{32\left(3+H^2\,k^2\right)^2}\Big(k^7\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)$$

$$\left(3\,g\,H+U\left(-3\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)\right)\Big)\,dt^3-$$

$$\frac{1}{64\left(3+H^2\,k^2\right)^3}i\,k^8\left(-\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+2\left(3+H^2\,k^2\right)U\right)$$

$$\left(3\,g\,H+U\left(-2\sqrt{3}\ \sqrt{g\,H\left(3+H^2\,k^2\right)}+\left(3+H^2\,k^2\right)U\right)\right)^2dt^4+O[dt]^5\Bigg)$$

$$dx^3+\Bigg(\Big(k^5\left(3\sqrt{3}\ g\,H\left(177+124\,H^2\,k^2+20\,H^4\,k^4\right)-\right.$$

$$\left.104\left(9\sqrt{g\,H\left(3+H^2\,k^2\right)}+6\,k^2\sqrt{g\,H^5\left(3+H^2\,k^2\right)}+k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\right)U\right)\Big)\Big/$$

$$\left(1920\sqrt{g\,H}\ \left(3+H^2\,k^2\right)^{5/2}\right)+\Big(k^7\Big(27\sqrt{3}\ g^2\,H^2\left(167+124\,H^2\,k^2+20\,H^4\,k^4\right)+$$

$$g\,H\,U\left(21\,429\sqrt{3}\ H^2\,k^2\,U+764\sqrt{3}\ H^6\,k^6\,U+81\left(-232\sqrt{g\,H\left(3+H^2\,k^2\right)}+267\sqrt{3}\ U\right)-\right.$$

$$\left.24\,k^4\left(84\sqrt{g\,H^9\left(3+H^2\,k^2\right)}-293\sqrt{3}\ H^4\,U\right)\right)-$$

$$16\left(459\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^3+153\,k^4\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;U^3+17\,k^6\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\;U^3+9\,k^2\left(88\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U+51\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^3\right)\right)dt^2\Bigg)\Bigg/$$
$$\left(11\,520\sqrt{g\,H}\;\left(3+H^2\,k^2\right)^{7/2}\right)-\frac{1}{3840\left(3+H^2\,k^2\right)^4}i\,k^8\left(54\,g^2\,H^2\left(81+62\,H^2\,k^2+10\,H^4\,k^4\right)+84\,H^8\,k^8\,U^4+243\,U^3\left(-39\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+28\,U\right)+36\,k^4\,U^3\left(-85\sqrt{3}\;\sqrt{g\,H^9\left(3+H^2\,k^2\right)}+126\,H^4\,U\right)+4\,k^6\,U^3\left(-83\sqrt{3}\;\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}+252\,H^6\,U\right)-9\,k^2\left(600\sqrt{3}\;\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U+1039\sqrt{3}\;\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^3-1008\,H^2\,U^4\right)+3\,g\,H\,U\left(13\,500\,H^2\,k^2\,U+472\,H^6\,k^6\,U+27\left(-97\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}+504\,U\right)-4\,k^4\left(71\sqrt{3}\;\sqrt{g\,H^9\left(3+H^2\,k^2\right)}-1101\,H^4\,U\right)\right)\right)dt^3-\frac{1}{23\,040\left(\sqrt{g\,H}\;\left(3+H^2\,k^2\right)^{11/2}\right)}$$
$$\left(k^9\left(\sqrt{3}\;\sqrt{g\,H\left(3+H^2\,k^2\right)}-\left(3+H^2\,k^2\right)U\right)\left(4\,k^8\,U^3\left(-239\sqrt{3}\;g\,H^9+58\sqrt{g\,H^{17}\left(3+H^2\,k^2\right)}\;U\right)+27\,k^2\left(372\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)}-2703\sqrt{3}\;g^2\,H^4\,U+4515\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\;U^2-4070\sqrt{3}\;g\,H^3\,U^3+928\sqrt{g\,H^5\left(3+H^2\,k^2\right)}\;U^4\right)+9\,k^4\left(180\sqrt{g^5\,H^{13}\left(3+H^2\,k^2\right)}-2672\sqrt{3}\;g^2\,H^6\,U+1392\sqrt{g\,H^9\left(3+H^2\,k^2\right)}\;U^4+g\,H^5\,U^2\left(4384\sqrt{g\,H\left(3+H^2\,k^2\right)}-5997\sqrt{3}\;U\right)\right)+81\left(157\sqrt{g^5\,H^5\left(3+H^2\,k^2\right)}-883\sqrt{3}\;g^2\,H^2\,U+232\sqrt{g\,H\left(3+H^2\,k^2\right)}\;U^4+g\,H\,U^2\left(1527\sqrt{g\,H\left(3+H^2\,k^2\right)}-1033\sqrt{3}\;U\right)\right)-12\,k^6\,U\left(213\sqrt{3}\;g^2\,H^8-232\sqrt{g\,H^{13}\left(3+H^2\,k^2\right)}\;U^3+g\,H^7\,U\left(-349\sqrt{g\,H\left(3+H^2\,k^2\right)}+979\sqrt{3}\;U\right)\right)\right)dt^4+O[dt]^5\Bigg)dx^4+O[dx]^5\Bigg\}$$

Out[260]= Omega error ||

\left\{\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}-\frac{\left(k^5 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{20 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(2 \left(H^2 k^2+3\right)^2 U-3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{24 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(\left(2 H^6 U k^6+2 \left(9 U H^4+2 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+3 \left(18 U H^2+7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+54 U+27 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+g \left(6 k^4 U H^5+18 k^2 U H^3-9 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} H\right)\right) \text{dt}^2}{48 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(U \left(2 H^4 U k^4+2 \left(6 U H^2+\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)-9 g H\right)

\text{dt}^3}{48 \left(H^2 k^2+3\right)^3}−\frac{\left(k^7 \left(2 \left(H^2 k^2+3\right)^2 U−3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{96 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{16} i k^4 \left(2 U+\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}\right)+\frac{i k^6 \left(\left(2 H^4 U k^4+\left(12 U H^2+5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U+15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g H \left(4 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^2}−\frac{\left(k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 g H+U \left(2 \left(H^2 k^2+3\right) U+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^2}−\frac{i k^8 \left(2 \left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(−\frac{k^5 \left(3 \sqrt{3} g H \left(20 H^4 k^4+124 H^2 k^2+177\right)+104 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{1920 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{5/2}\right)}−\frac{\left(k^7 \left(27 \sqrt{3} g^2 \left(20 H^4 k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6+24 \left(293 \sqrt{3} U H^4+84 \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267 \sqrt{3} U+232 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) H+16 \left(17 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3 k^6+153 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(51 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+88 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+459 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right)\right) \text{dt}^2}{11520 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{7/2}\right)}−\frac{i k^8 \left(84 H^8 U^4 k^8+4 U^3 \left(252 U H^6+83 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+36 U^3 \left(126 U H^4+85 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+9 \left(1008 H^2 U^4+1039 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+600 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U+39 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+3 g H U \left(472 H^6 U k^6+4 \left(1101 U H^4+71 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+13500 H^2 U k^2+27 \left(504 U+97 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{3840 \left(H^2 k^2+3\right)^4}+\frac{k^9 \left(\left(H^2 k^2+3\right) U+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(4 U^3 \left(239 \sqrt{3} g H^9+58 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U\right) k^8+12 U \left(213 \sqrt{3} g^2 H^8+g U \left(979 \sqrt{3} U+349 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7+232 \sqrt{g H^{13} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(2672 \sqrt{3} g^2 U H^6+g U^2 \left(5997 \sqrt{3} U+4384 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+1392 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+180 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+27 \left(2703 \sqrt{3} g^2 U H^4+4070 \sqrt{3} g U^3 H^3+928 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+4515 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+372 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(232 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1033 \sqrt{3} U+1527 \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+883 \sqrt{3} g^2 H^2 U+157 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{23040 \sqrt{g H} \left(H^2 k^2+3\right)^{11/2}}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right),\left(\frac{k^3 \left(\left(H^2 k^2+3\right) U−\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^2}{6 \left(H^2 k^2+3\right)^2}+\frac{i k^4 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^3}{8 \left(H^2 k^2+3\right)^2}−\frac{\left(k^5 \left(\left(H^2

k^2+3\right) U−\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)^3 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \text{dt}^4}{20 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right)+\left(\frac{k^3 \left(2 U \left(H^2 k^2+3\right)^2+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)}{24 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(\left(2 H^6 U k^6−2 \left(2 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}−9 H^4 U\right) k^4−3 \left(7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−18 H^2 U\right) k^2+54 U−27 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) U^2+3 g \left(2 k^4 U H^5+6 k^2 U H^3+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} H\right)\right) \text{dt}^2}{48 \left(H^2 k^2+3\right)^3}+\frac{i k^6 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(U \left(2 H^4 U k^4−2 \left(\sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}−6 H^2 U\right) k^2+18 U−3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)−9 g H\right) \text{dt}^3}{48 \left(H^2 k^2+3\right)^3}−\frac{\left(k^7 \left(2 U \left(H^2 k^2+3\right)^2+3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2\right) \text{dt}^4}{96 \left(H^2 k^2+3\right)^4}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(−\frac{1}{16} i k^4 \left(\sqrt{3} \sqrt{\frac{g H}{H^2 k^2+3}}−2 U\right)+\frac{i k^6 \left(U^2 \left(2 H^4 U k^4+\left(12 H^2 U−5 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}\right) k^2+18 U−15 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)−3 g H \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}−4 \left(H^2 k^2+3\right) U\right)\right) \text{dt}^2}{32 \left(H^2 k^2+3\right)^2}−\frac{\left(k^7 \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) \left(3 g H+U \left(2 \left(H^2 k^2+3\right) U−3 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{32 \left(H^2 k^2+3\right)^2}−\frac{i k^8 \left(2 \left(H^2 k^2+3\right) U−\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right) \left(3 g H+U \left(\left(H^2 k^2+3\right) U−2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)^2 \text{dt}^4}{64 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{k^5 \left(3 \sqrt{3} g H \left(20 H^4 k^4+124 H^2 k^2+177\right)−104 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)} k^4+6 \sqrt{g H^5 \left(H^2 k^2+3\right)} k^2+9 \sqrt{g H \left(H^2 k^2+3\right)}\right) U\right)}{1920 \sqrt{g H} \left(H^2 k^2+3\right)^{5/2}}+\frac{k^7 \left(27 \sqrt{3} g^2 \left(20 H^4 k^4+124 H^2 k^2+167\right) H^2+g U \left(764 \sqrt{3} H^6 U k^6−24 \left(84 \sqrt{g H^9 \left(H^2 k^2+3\right)}−293 \sqrt{3} H^4 U\right) k^4+21429 \sqrt{3} H^2 U k^2+81 \left(267 \sqrt{3} U−232 \sqrt{g H \left(H^2 k^2+3\right)}\right)\right) H−16 \left(17 \sqrt{g H^{13}} \left(H^2 k^2+3\right)} U^3 k^6+153 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^3 k^4+9 \left(51 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+88 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+459 \sqrt{g H \left(H^2 k^2+3\right)} U^3\right)\right) \text{dt}^2}{11520 \sqrt{g H} \left(H^2 k^2+3\right)^{7/2}}−\frac{i k^8 \left(84 H^8 U^4 k^8+4 U^3 \left(252 H^6 U−83 \sqrt{3} \sqrt{g H^{13}} \left(H^2 k^2+3\right)}\right) k^6+36 U^3 \left(126 H^4 U−85 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4−9 \left(−1008 H^2 U^4+1039 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+600 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+54 g^2 H^2 \left(10 H^4 k^4+62 H^2 k^2+81\right)+243 U^3 \left(28 U−39 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+3 g H U \left(472 H^6 U k^6−4 \left(71 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}−1101 H^4 U\right) k^4+13500 H^2 U k^2+27 \left(504 U−97 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^3}{3840 \left(H^2 k^2+3\right)^4}−\frac{\left(k^9 \left(\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}−\left(H^2 k^2+3\right) U\right) \left(4 U^3 \left(58 \sqrt{g H^{17}} \left(H^2 k^2+3\right)} U−239 \sqrt{3} g H^9\right) k^8−12 U \left(213 \sqrt{3} g^2 H^8+g U \left(979 \sqrt{3} U−349 \sqrt{g H \left(H^2 k^2+3\right)}\right) H^7−232 \sqrt{g H^{13}} \left(H^2 k^2+3\right)} U^3\right) k^6+9 \left(−2672 \sqrt{3} g^2 U H^6+g U^2 \left(4384 \sqrt{g H \left(H^2 k^2+3\right)}−5997

\sqrt{3} U\right) H^5+1392 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+180 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+27 \left(−2703 \sqrt{3} g^2 U H^4−4070 \sqrt{3} g U^3 H^3+928 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+4515 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U^2+372 \sqrt{g^5 H^9 \left(H^2 k^2+3\right)}\right) k^2+81 \left(232 \sqrt{g H \left(H^2 k^2+3\right)} U^4+g H \left(1527 \sqrt{g H \left(H^2 k^2+3\right)}−1033 \sqrt{3} U\right) U^2−883 \sqrt{3} g^2 H^2 U+157 \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{23040 \left(\sqrt{g H} \left(H^2 k^2+3\right)^{11/2}\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right)\right\}

Out[261]=

Out[262]= EA || $\left\{\left\{1+\frac{i e^{\frac{i \text{dx} k}{2}} \left(1-e^{-i \text{dx} k}\right) \left(-1+e^{i \text{dt} w}\right) H^2 k^3 U \csc\left[\frac{\text{dx} k}{2}\right]}{\left(6+2 H^2 k^2\right) w},\ \frac{i e^{\frac{i \text{dx} k}{2}} \left(1-e^{-i \text{dx} k}\right) \left(-1+e^{i \text{dt} w}\right) H k \csc\left[\frac{\text{dx} k}{2}\right]}{2\left(H+\frac{H^3 k^2}{3}\right) w}\right\},\right.$

$\left.\left\{\frac{i e^{\frac{i \text{dx} k}{2}} \left(1-e^{-i \text{dx} k}\right) \left(-1+e^{i \text{dt} w}\right) k \left(g H \left(3+H^2 k^2\right)-3 U^2\right) \csc\left[\frac{\text{dx} k}{2}\right]}{\left(6+2 H^2 k^2\right) w},\ 1+\frac{i e^{\frac{i \text{dx} k}{2}} \left(1-e^{-i \text{dx} k}\right) \left(-1+e^{i \text{dt} w}\right) k \left(6+H^2 k^2\right) U \csc\left[\frac{\text{dx} k}{2}\right]}{\left(6+2 H^2 k^2\right) w}\right\}\right\}$

Out[263]= EA || \left(

\begin{array}{cc}

\frac{i e^{\frac{i \text{dx} k}{2}} \left(1−e^{−i \text{dx} k}\right) \left(−1+e^{i \text{dt} w}\right) H^2 U \csc \left(\frac{\text{dx} k}{2}\right) k^3}{\left(2 H^2 k^2+6\right) w}+1 & \frac{i e^{\frac{i \text{dx} k}{2}} \left(1−e^{−i \text{dx} k}\right) \left(−1+e^{i \text{dt} w}\right) H k \csc \left(\frac{\text{dx} k}{2}\right)}{2 \left(\frac{k^2 H^3}{3}+H\right) w} \\

\frac{i e^{\frac{i \text{dx} k}{2}} \left(1−e^{−i \text{dx} k}\right) \left(−1+e^{i \text{dt} w}\right) k \left(g H \left(H^2 k^2+3\right)−3 U^2\right) \csc \left(\frac{\text{dx} k}{2}\right)}{\left(2 H^2 k^2+6\right) w} & \frac{i e^{\frac{i \text{dx} k}{2}} \left(1−e^{−i \text{dx} k}\right) \left(−1+e^{i \text{dt} w}\right) k \left(H^2 k^2+6\right) U \csc \left(\frac{\text{dx} k}{2}\right)}{\left(2 H^2 k^2+6\right) w}+1 \\

\end{array}

\right)

Out[264]= Eerr ||

$$\left\{\left\{\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}+3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U+3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}-\right.\right.$$

$$\left.\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3dt^3-\frac{1}{24}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4dt^4+O[dt]^5\right)+$$

$$\left(-\frac{i\left(27\,k^3+9\,H^2\,k^5+H^4\,k^7\right)U\,dt}{12\left(3+H^2\,k^2\right)^2}-\frac{\left(-9\,g\,H\,k^4+36\,k^4\,U^2+12\,H^2\,k^6\,U^2+2\,H^4\,k^8\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(\frac{1}{8}\,k^4\,U\,dt+\left(-\frac{3\,i\,g\,H\,k^5}{16\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^7\,U^2}{8\left(3+H^2\,k^2\right)}\right)dt^2+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(405\,k^5\,U+351\,H^2\,k^7\,U+116\,H^4\,k^9\,U+13\,H^6\,k^{11}\,U\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{1}{1440\left(3+H^2\,k^2\right)^3}\left(1161\,g\,H\,k^6+837\,g\,H^3\,k^8+135\,g\,H^5\,k^{10}-\right.\right.$$

$$\left.\left.351\,k^6\,U^2+297\,H^2\,k^8\,U^2+387\,H^4\,k^{10}\,U^2+73\,H^6\,k^{12}\,U^2\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5,$$

$$\left(-\frac{3\,i\,k\,dt}{3+H^2\,k^2}-\frac{3\left(k^2\,U\right)dt^2}{3+H^2\,k^2}+O[dt]^5\right)+\left(\left(\frac{3\,i\,k^3}{2\left(3+H^2\,k^2\right)^2}+\frac{i\,H^2\,k^5}{4\left(3+H^2\,k^2\right)^2}\right)dt+\frac{3\,k^4\,U\,dt^2}{4\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(-\frac{3\,i\,k^5\,U\,dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+\left(\frac{i\left(-54\,k^5+H^4\,k^9\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{\left(387\,k^6\,U+279\,H^2\,k^8\,U+45\,H^4\,k^{10}\,U\right)dt^2}{240\left(3+H^2\,k^2\right)^3}+O[dt]^5\right)dx^4+O[dx]^5\Big\},$$

$$\left\{\left(-\frac{i\,k\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)dt}{3+H^2\,k^2}-\frac{k^2\,U\left(3\,g\,H+g\,H^3\,k^2-3\,U^2\right)dt^2}{3+H^2\,k^2}+O[dt]^5\right)+\right.$$

$$\left(-\frac{i\left(9\,g\,H\,k^3+6\,g\,H^3\,k^5+g\,H^5\,k^7+18\,k^3\,U^2+3\,H^2\,k^5\,U^2\right)dt}{12\left(3+H^2\,k^2\right)^2}+\frac{\left(-18\,g\,H\,k^4\,U-12\,g\,H^3\,k^6\,U-2\,g\,H^5\,k^8\,U-9\,k^4\,U^3\right)dt^2}{12\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(\frac{1}{8}\,g\,H\,k^4\,dt-\frac{i\left(6\,g\,H\,k^5\,U+2\,g\,H^3\,k^7\,U-3\,k^5\,U^3\right)dt^2}{8\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(351\,g\,H\,k^5+351\,g\,H^3\,k^7+117\,g\,H^5\,k^9+13\,g\,H^7\,k^{11}+54\,k^5\,U^2-H^4\,k^9\,U^2\right)dt}{240\left(3+H^2\,k^2\right)^3}+\frac{1}{720\left(3+H^2\,k^2\right)^3}\left(1971\,g\,H\,k^6\,U+1971\,g\,H^3\,k^8\,U+657\,g\right.\right.$$

$$\left.\left.H^5\,k^{10}\,U+73\,g\,H^7\,k^{12}\,U-1161\,k^6\,U^3-837\,H^2\,k^8\,U^3-135\,H^4\,k^{10}\,U^3\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5,$$

$$\left(\frac{i\left(\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}-3\,k\,U\right)dt}{3+H^2\,k^2}+\frac{\left(\sqrt{3}\ k^2\sqrt{g\,H\left(3+H^2\,k^2\right)}\ U-3\,k^2\,U^2\right)dt^2}{3+H^2\,k^2}-\frac{1}{6}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^3dt^3-\right.$$

$$\left.\frac{1}{24}\left(-\frac{i\sqrt{3}\ k\sqrt{g\,H\left(3+H^2\,k^2\right)}}{3+H^2\,k^2}-i\,k\,U\right)^4dt^4+O[dt]^5\right)+$$

$$\left(-\frac{i\left(-9\,k^3\,U+3\,H^2\,k^5\,U+H^4\,k^7\,U\right)dt}{12\left(3+H^2\,k^2\right)^2}+\frac{\left(9\,g\,H\,k^4-12\,H^2\,k^6\,U^2-2\,H^4\,k^8\,U^2\right)dt^2}{24\left(3+H^2\,k^2\right)^2}+O[dt]^5\right)dx^2+$$

$$\left(\frac{1}{8}\,k^4\,U\,dt-\frac{i\,k^5\left(3\,g\,H+12\,U^2+2\,H^2\,k^2\,U^2\right)dt^2}{16\left(3+H^2\,k^2\right)}+O[dt]^5\right)dx^3+$$

$$\left(\frac{i\left(297\,k^5+351\,H^2\,k^7+118\,H^4\,k^9+13\,H^6\,k^{11}\right)U\,dt}{240\left(3+H^2\,k^2\right)^3}+\frac{1}{1440\left(3+H^2\,k^2\right)^3}\left(1161\,g\,H\,k^6+837\,g\,H^3\,k^8+135\,g\,H^5\,k^{10}+\right.\right.$$

$$\left.\left.4293\,k^6\,U^2+3645\,H^2\,k^8\,U^2+927\,H^4\,k^{10}\,U^2+73\,H^6\,k^{12}\,U^2\right)dt^2+O[dt]^5\right)dx^4+O[dx]^5\Big\}\Big\}$$

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\left(\frac{i \left(3 U k+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k\right) \text{dt}}{H^2 k^2+3}+\frac{\left(3 U^2 k^2+\sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} U k^2\right) \text{dt}^2}{H^2 k^2+3}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}}{H^2 k^2+3}\right)^3 \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2

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k^2+3\right)} k}{H^2 k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{i
\left(H^4 k^7+9 H^2 k^5+27 k^3\right) U \text{dt}}{12 \left(H^2 k^2+3\right)^2}-\frac{\left(2
H^4 U^2 k^8+12 H^2 U^2 k^6+36 U^2 k^4-9 g H k^4\right) \text{dt}^2}{24 \left(H^2
k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{8} k^4 U \text{dt}+\left(-\frac{i
H^2 U^2 k^7}{8 \left(H^2 k^2+3\right)}-\frac{3 i g H k^5}{16 \left(H^2 k^2+3\right)}\right)
\text{dt}^2+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(13 H^6 U k^{11}+116 H^4 U
k^9+351 H^2 U k^7+405 U k^5\right) \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73 H^6
U^2 k^{12}+135 g H^5 k^{10}+387 H^4 U^2 k^{10}+837 g H^3 k^8+297 H^2 U^2 k^8-351 U^2
k^6+1161 g H k^6\right) \text{dt}^2}{1440 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)
\text{dx}^4+O\left(\text{dx}^5\right) & \left(-\frac{3 i k \text{dt}}{H^2 k^2+3}-\frac{3 \left(k^2
U\right) \text{dt}^2}{H^2 k^2+3}+O\left(\text{dt}^5\right)\right)+\left(\left(\frac{i H^2 k^5}{4 \left(H^2
k^2+3\right)^2}+\frac{3 i k^3}{2 \left(H^2 k^2+3\right)^2}\right) \text{dt}+\frac{3 k^4 U \text{dt}^2}{4
\left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(-\frac{3 i k^5 U \text{dt}^2}{8
\left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i \left(H^4 k^9-54 k^5\right)
\text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(45 H^4 U k^{10}+279 H^2 U k^8+387 U k^6\right)
\text{dt}^2}{240 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
\left(-\frac{i k \left(g k^2 H^3+3 g H-3 U^2\right) \text{dt}}{H^2 k^2+3}-\frac{k^2 U \left(g k^2 H^3+3 g H-3
U^2\right) \text{dt}^2}{H^2 k^2+3}+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(g H^5 k^7+6 g H^3
k^5+3 H^2 U^2 k^5+18 U^2 k^3+9 g H k^3\right) \text{dt}}{12 \left(H^2 k^2+3\right)^2}+\frac{\left(-2
g H^5 U k^8-12 g H^3 U k^6-9 U^3 k^4-18 g H U k^4\right) \text{dt}^2}{12 \left(H^2
k^2+3\right)^2}+O\left(\text{dt}^5\right)\right) \text{dx}^2+\left(\frac{1}{8} g H k^4 \text{dt}-\frac{i \left(2 g
H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) \text{dt}^2}{8 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right)
\text{dx}^3+\left(\frac{i \left(13 g H^7 k^{11}+117 g H^5 k^9-H^4 U^2 k^9+351 g H^3 k^7+54
U^2 k^5+351 g H k^5\right) \text{dt}}{240 \left(H^2 k^2+3\right)^3}+\frac{\left(73 g H^7 U
k^{12}-135 H^4 U^3 k^{10}+657 g H^5 U k^{10}-837 H^2 U^3 k^8+1971 g H^3 U k^8-1161 U^3
k^6+1971 g H U k^6\right) \text{dt}^2}{720 \left(H^2 k^2+3\right)^3}+O\left(\text{dt}^5\right)\right)
\text{dx}^4+O\left(\text{dx}^5\right) & \left(\frac{i \left(\sqrt{3} k \sqrt{g H \left(H^2 k^2+3\right)}-3 k
U\right) \text{dt}}{H^2 k^2+3}+\frac{\left(\sqrt{3} k^2 \sqrt{g H \left(H^2 k^2+3\right)} U-3 k^2 U^2\right)
\text{dt}^2}{H^2 k^2+3}-\frac{1}{6} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)} k}{H^2
k^2+3}\right)^3 \text{dt}^3-\frac{1}{24} \left(-i U k-\frac{i \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}
k}{H^2 k^2+3}\right)^4 \text{dt}^4+O\left(\text{dt}^5\right)\right)+\left(-\frac{i \left(H^4 U k^7+3
H^2 U k^5-9 U k^3\right) \text{dt}}{12 \left(H^2 k^2+3\right)^2}+\frac{\left(-2 H^4 U^2 k^8-12 H^2
U^2 k^6+9 g H k^4\right) \text{dt}^2}{24 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
\text{dx}^2+\left(\frac{1}{8} k^4 U \text{dt}-\frac{i k^5 \left(2 H^2 k^2 U^2+12 U^2+3 g H\right)
\text{dt}^2}{16 \left(H^2 k^2+3\right)}+O\left(\text{dt}^5\right)\right) \text{dx}^3+\left(\frac{i
\left(13 H^6 k^{11}+118 H^4 k^9+351 H^2 k^7+297 k^5\right) U \text{dt}}{240 \left(H^2
k^2+3\right)^3}+\frac{\left(73 H^6 U^2 k^{12}+135 g H^5 k^{10}+927 H^4 U^2 k^{10}+837
g H^3 k^8+3645 H^2 U^2 k^8+4293 U^2 k^6+1161 g H k^6\right) \text{dt}^2}{1440 \left(H^2
k^2+3\right)^3}+O\left(\text{dt}^5\right)\right) \text{dx}^4+O\left(\text{dx}^5\right) \\
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