```
ln[1] = q = q0 * Exp[I * (k * x + w * t)];
     qjn = q0 * Exp[I * (k * xj + w * tn)];
     qjbar = Integrate [q, \{x, xj - dx/2, xj + dx/2\}]/(dx);
     qjnbar = qjbar /. t \rightarrow tn;
     MA = qjn / qjnbar;
     qntbar = Integrate[q, {t, tn, tn + dt}] / (dt);
     qjntbar = qntbar /. x \rightarrow xj;
     MtA = qjntbar / qjn;
     qjphn = q0 * Exp[I * (k * (xj + dx/2) + w * tn)];
     RA = Simplify[MA * qjphn / (qjn)];
     vmultG = H + H^3/3*k^2;
     GnA = -U * RA / vmultG;
     GGA = RA / vmultG;
     GcA = -U * H / vmultG ;
     fn1A = H * vh + U * eh;
     fn1A = fn1A /. vh \rightarrow (GGA*Gca + GnA*eca) /. eh \rightarrow RA*eca;
     fn1Gca0A = fn1A / . Gca \rightarrow 0 ;
     fn1eca0A = fn1A/. eca \rightarrow 0;
     fnnA = Simplify[fn1Gca0A / eca];
     fnGA = fn1eca0A / Gca;
     fncA = H * GcA;
     fG1A = U*Gh + U*H*vh + g*H*eh;
     \texttt{fGlA} = \texttt{fGlA} \ / . \ \texttt{vh} \ \rightarrow \ (\texttt{GGA} \star \texttt{Gca} \ + \ \texttt{GnA} \star \texttt{eca}) \ \ / . \ \ \texttt{eh} \ \rightarrow \ \texttt{RA} \star \texttt{eca} \ / . \ \ \texttt{Gh} \ \rightarrow \ \texttt{RA} \star \texttt{Gca} \ ;
     fG1Gca0A = fG1A / . Gca \rightarrow 0 ;
     fGleca0A = fGlA /. eca \rightarrow 0;
     fGnA = Simplify[fG1Gca0A / eca];
     fGGA = Simplify[fGleca0A / Gca];
     fGcA = U * H * GcA;
     FnnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnnA;
     FnGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fnGA;
     FGnA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGnA;
     FGGA = -MtA * dt / dx * (1 - Exp[-I * k * dx]) * fGGA;
     MatA = { {FnnA, FnGA}, {FGnA, FGGA} } ;
```

```
EA = Simplify[{{1, 0}, {0, 1}} + MatA];
    wAp = U * k + \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};
    wAm = U * k - \frac{\sqrt{3} k \sqrt{g H (3 + H^2 k^2)}}{3 + H^2 k^2};
ln[37]:= M = 1;
    Merr = Series[M - MA, \{dx, 0, 5\}];
    Rm = (1 + I * Sin[k * dx] / 2);
    Rmerr = Series[Rm - RA, \{dx, 0, 4\}];
    Rp = Exp[I*k*dx]*(1 - I*Sin[k*dx]/2);
    Rperr = Series[Rp - RA, {dx, 0, 4}];
     GRHSp1 = -\text{Exp}[-\text{I}*\text{k}*\text{dx}/2] + 2 + 4*\text{Exp}[\text{I}*\text{k}*\text{dx}/2] +
        Exp[I*k*dx]*(4*Exp[-I*k*dx/2]+2-Exp[I*k*dx/2]);
    GRHSp1 = GRHSp1 / Exp[I * k * dx / 2];
    GRHSp1 = Expand[GRHSp1];
    GRHSp1 = ExpToTrig[GRHSp1];
     GRHSp2 = Exp[-I*k*dx/2] - 8 + 7*Exp[I*k*dx/2] +
        \text{Exp}[I*k*dx]*(7*\text{Exp}[-I*k*dx/2] - 8 + \text{Exp}[I*k*dx/2]);
    GRHSp2 = GRHSp2 / Exp[I*k*dx/2];
    GRHSp2 = Expand[GRHSp2];
    GRHSp2 = ExpToTrig[GRHSp2];
    GGLHS = dx / 6 * (Rp + Rm);
    GG2 = GGLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2);
     GG2err = Series[GG2 - GGA, {dx, 0, 5}];
    GnLHS = -U * (dx / 6) * (Rp + Rm);
    Gn2 = GnLHS / (H * dx / 30 * (GRHSp1) + H^3 / (9 * dx) * GRHSp2);
    Gn2err = Series[Gn2 - GnA, {dx, 0, 5}];
     Text[Row[{"M || ", M}]]
     Text[Row[{"M || ", TeXForm[M]}]]
    Text[Row[{"M error || ", TeXForm[Merr]}]]
    Text[Row[{"M error || ", Merr}]]
    Text[" "]
    Text[Row[{"Rm || ", Rm}]]
     Text[Row[{"Rm || ", TeXForm[Rm]}]]
     Text[Row[{"Rm error || ", Rmerr}]]
    Text[Row[{"Rm error || ", TeXForm[Rmerr]}]]
    Text[" "]
    Text[Row[{"Rp || ", Rp}]]
    Text[Row[{"Rp || ", TeXForm[Rp]}]]
```

```
Text[Row[{"Rp error || ", Rperr}]]
                                        Text[Row[{"Rp error || ", TeXForm[Rperr]}]]
                                        Text[" "]
                                          Text[Row[{"GG2 || ", GG2}]]
                                          Text[Row[{"GG2 || ", TeXForm[GG2]}]]
                                        Text[Row[{"GG2 error ||
                                                                                                                                                                                                                                                                 ", GG2err}]]
                                          Text[Row[{"GG2 error || ", TeXForm[GG2err]}]]
                                        Text[" "]
                                        Text[Row[{"Gn2 || ", Gn2}]]
                                          Text[Row[{"Gn2 || ", TeXForm[Gn2]}]]
                                          Text[Row[{"Gn2 error ||
                                                                                                                                                                                                                                                                  ", Gn2err}]]
                                          Text[Row[{"Gn2 error || ", TeXForm[Gn2err]}]]
 Out[57]= M \parallel 1
 Out[58]= M \parallel 1
 \label{eq:outsol} Outsol= Merror \parallel -\frac{k^2}{24}-\frac{4k^2}{5760}+O\left(\frac{k^2}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^2}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^2}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(\frac{k^4}{24}-\frac{4k^4}{5760}\right)+O\left(
 \text{Out[60]=} \ \ M \ error \ \ || \ \ -\frac{k^2 \ dx^2}{24} - \frac{7 \ k^4 \ dx^4}{5760} + O[dx]^6
 Out[61]=
 Out[62]= Rm || 1 + \frac{1}{2} i \sin[dx k]
 \label{eq:outf63} \mbox{Outf63]=} \ Rm \ || \ 1 + \mbox{frac}\{1\}\{2\} \ i \ \mbox{$i$ ($$ text{d}x$} \} \ k)
 Out[64]= Rm error || \frac{k^2 dx^2}{12} - \frac{1}{12} i k^3 dx^3 + \frac{k^4 dx^4}{720} + O[dx]^5
 Out[65]= Rm error |
                                                   \label{eq:continuous} $$ \frac{dx}^2 k^2}{12}-\frac{1}{12} i \text{$k^3 k^3+\frac{dx}^4 k^4}{720}+O\left(\frac{dx}^5\right) i \text{$k^3+\frac{dx}^4 k^4} \right) $$
 Out[67]= Rp || e^{i \operatorname{dx} k} \left(1 - \frac{1}{2} i \operatorname{Sin}[\operatorname{dx} k]\right)
 \label{eq:outloss} \begin{tabular}{ll} Outloss=& Rp & || & e^{i \cdot text\{dx\} k} \cdot left(1-\frac{1}{2} i \cdot sin (\cdot text\{dx\} k) \cdot right) \\ \end{tabular}
 Out[69]= Rp error || \frac{k^2 dx^2}{12} + \frac{1}{6} i k^3 dx^3 - \frac{89 k^4 dx^4}{720} + O[dx]^5
 Out[70]= Rp error ||
                                                   \frac{dx}{dx}^2 k^2}{12}+\frac{1}{6} i \cdot \frac{dx}^3 -\frac{g}{4x}^4 k^4}{720}+O\left(\frac{1}{6x}^5\right) i \cdot \frac{dx}^3 -\frac{g}{4x}^4 k^4}{720}+O\left(\frac{1}{6x}^5\right) i \cdot \frac{1}{6x}^5 -\frac{1}{6x}^5 -\frac{1
 Out[71]=
 \text{Out} \text{[72]=} \quad GG2 \quad \text{[I]} \quad \frac{dx \left(1 + e^{i \, dx \, k} \left(1 - \frac{1}{2} \, i \, \text{Sin}[dx \, k]\right) + \frac{1}{2} \, i \, \text{Sin}[dx \, k]\right)}{6 \left(\frac{1}{30} \, dx \, H\left(8 + 4 \, \text{Cos}\left[\frac{dx \, k}{2}\right] - 2 \, \text{Cos}[dx \, k]\right) + \frac{H^3 \left(14 - 16 \, \text{Cos}\left[\frac{dx \, k}{2}\right] + 2 \, \text{Cos}[dx \, k]\right)}{9 \, dx}\right)} \right)
```

 $\label{eq:condition} Out[73]= GG2 || \frac{\text{k} \left(\frac{x} \left(\frac{x} \right) \left(\frac{1}{2} i \sin \left(\frac{x} k\right)\right)}{6 \left(\frac{x} k\right)} \right)}{6 \left(\frac{x} k\right)} || \frac{1}{2} i \sin \left(\frac{x} k\right)}{6 \left(\frac{x} k\right)} || \frac{x} k\right)}{6 \left(\frac{x} k\right)} || \frac{x} k\right$ 

$$\text{Out} \text{[74]= } \text{GG2 error } \text{[} \text{[} \frac{\left(12\,k^2 + 5\,H^2\,k^4\right)dx^2}{40\,H\left(3 + H^2\,k^2\right)^2} + \frac{i\left(12\,k^3 + 5\,H^2\,k^5\right)dx^3}{80\,H\left(3 + H^2\,k^2\right)^2} + \frac{\left(-6651\,k^4 - 4680\,H^2\,k^6 - 820\,H^4\,k^8\right)dx^4}{4800\,H\left(3 + H^2\,k^2\right)^3} - \frac{i\left(6291\,k^5 + 4410\,H^2\,k^7 + 770\,H^4\,k^9\right)dx^5}{9600\,H\left(3 + H^2\,k^2\right)^3} + O[dx]^6$$

Out[75]= GG2 error |

 $\label{left(4x)^2 \left( \frac{k^2 + 1 \ k^2 \right)}{40 \ H \left( \frac{k^2 \ k^2 + 3 \right)^2} + \frac{i \ k^3 \left( \frac{k^3 \ k^4 + 12 \ k^2 + 3 \right)}{40 \ H \left( \frac{k^2 \ k^2 + 3 \right)^2} + \frac{i \ k^4 \ k^3 \left( \frac{k^3 \ k^4 \ k^6 - 6651 \ k^4 \right)}{40 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} - \frac{i \ k^4 \ k^5 \left( \frac{k^3 \ k^4 \ k^6 - 6651 \ k^4 \right)}{40 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} - \frac{i \ k^4 \ k^5 \left( \frac{k^3 \ k^6 - 6651 \ k^6 + 6291 \ k^5 \right)}{4000 \ H \left( \frac{k^2 \ k^2 + 3 \right)^3} - \frac{i \ k^4 \ k^6 + 6400 \ H^4 \ k^6 + 6400 \ H^6 \ k^6 \ k^6 + 64000 \ H^6 \ k^6 \ k^6 \ h^6 \ k^6 \ k^6 \ h^6 \ k^6 \ h^6 \ h^6 \ h^6 \$ 

Out[76]=

$$\text{Out[77]=} \quad Gn2 \quad || \quad - \frac{\text{dx } U \left( 1 + e^{i \cdot \text{dx } k} \left( 1 - \frac{1}{2} \cdot i \cdot \text{Sin}[\text{dx } k] \right) + \frac{1}{2} \cdot i \cdot \text{Sin}[\text{dx } k] \right) }{6 \left( \frac{1}{30} \cdot \text{dx } H \left( 8 + 4 \cdot \text{Cos} \left[ \frac{\text{dx } k}{2} \right] - 2 \cdot \text{Cos}[\text{dx } k] \right) + \frac{H^3 \left( 14 - 16 \cdot \text{Cos} \left[ \frac{\text{dx } k}{2} \right] + 2 \cdot \text{Cos}[\text{dx } k] \right)}{9 \cdot \text{dx}} \right) } \right) }$$

 $\label{eq:condition} Out[78] Gn2 & -\frac{text{dx} U \left(e^{i \left(x_{dx} k\right) k\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin \left(\text{text{dx} k}\right)}{1 \cdot e^{i}} \left(1-\frac{1}{2} i \sin \left(\text{text{dx} k}\right) + \frac{1}{2} i \sin$ 

$$\begin{array}{ll} \text{Out} \text{[79]=} & Gn2 \; error \; || \\ & - \frac{\left(\left(12 \, k^2 + 5 \, H^2 \, k^4\right) \, U\right) \, dx^2}{40 \left(H \, \left(3 + H^2 \, k^2\right)^2\right)} - \frac{i \left(12 \, k^3 + 5 \, H^2 \, k^5\right) \, U \, dx^3}{80 \, H \, \left(3 + H^2 \, k^2\right)^2} + \frac{\left(6651 \, k^4 + 4680 \, H^2 \, k^6 + 820 \, H^4 \, k^8\right) \, U \, dx^4}{4800 \, H \, \left(3 + H^2 \, k^2\right)^3} + \frac{i \left(6291 \, k^5 + 4410 \, H^2 \, k^7 + 770 \, H^4 \, k^9\right) \, U \, dx^5}{9600 \, H \, \left(3 + H^2 \, k^2\right)^3} + O[dx]^6 \\ \end{array}$$

Out[80]= Gn2 error |

 $-\frac{k^2 \cdot k^4+12 \cdot k^2 \cdot k^4+12 \cdot k^2 \cdot k^4+12 \cdot k^2 \cdot k^4+12 \cdot k^2 \cdot k^4+12 \cdot k^4 \cdot k^4+12 \cdot k^4 \cdot k^4+12 \cdot k$ 

```
In[81]= KurF = (fm * ap - fp * am + am * ap * (qp - qm)) / (ap - am);
KurFWS = KurF /. ap → (U + Sqrt[g * H]) /. am → (U - Sqrt[g * H]);
KurFWSeta =

KurFWS /. fp → (H * v + U * Rpp * n) /. fm → (H * v + U * Rmp * n) /. qp → Rpp * n /.

qm → Rmp * n;
KurFWSeta = KurFWSeta /. v → (GGp * G + Gnp * n);
Kfnnp = FullSimplify[KurFWSeta /. G → 0 /. n → 1];
KfnGp = FullSimplify[KurFWSeta /. n → 0 /. G → 1];
Kfnn = Kfnnp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
KfnG = KfnGp /. Rpp → Rp /. Rmp → Rm /. GGp → GG2 /. Gnp → Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA, {k > 0, U > 0, H > 0, g > 0}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
```

```
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
\texttt{KurFWSG} = \texttt{KurFWS} \ / . \ \texttt{fp} \ \rightarrow \ (\texttt{U} * \texttt{Rpp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rpp} * \texttt{n}) \ / .
       \texttt{fm} \rightarrow (\texttt{U} * \texttt{Rmp} * \texttt{G} + \texttt{U} * \texttt{H} * \texttt{v} + \texttt{g} * \texttt{H} * \texttt{Rmp} * \texttt{n}) \ /. \ \texttt{qp} \rightarrow \texttt{Rpp} * \texttt{G} \ /. \ \texttt{qm} \rightarrow \texttt{Rmp} * \texttt{G};
KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
KfGn = KfGnp /. Rpp \rightarrow Rp /. Rmp \rightarrow Rm /. GGp \rightarrow GG2 /. Gnp \rightarrow Gn2;
KfGG = KfGGp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" -Sqrt(gH) < U < Sqrt(gH)"}]]</pre>
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
Text[" "]
Text[Row[{"FGn || ", KfGnp}]]
Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
Text[Row[{"FGn error || ", FGn2TAr}]]
```

```
Text[" "]
                                              Text[Row[{"FGG || ", KfGGp}]]
                                               Text[Row[{"FGG || ", TeXForm[KfGGp]}]]
                                              Text[Row[{"FGG error ||
                                                                                                                                                                                                                                                                               ", FGG2TAr}]]
                                              Text[Row[{"FGG error ||
                                                                                                                                                                                                                                                                               ", TeXForm[FGG2TAr]}]]
                                              Text[" "]
                                              Text[" "]
                                              Text[Row[{"Omega error ||
                                                                                                                                                                                                                                                                               ", RKstepTayr}]]
                                                                                                                                                                                                                                                                                  ", TeXForm[RKstepTayr]}]]
                                              Text[Row[{"Omega error ||
                                              Text[" "]
                                              Text[Row[{"EA ||
                                                                                                                                                                                                            ", EA}]]
                                               Text[Row[{"EA || ", TeXForm[EA]}]]
                                               Text[Row[{"Eerr || ", Eerr}]]
                                               Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
Out[115]= -Sqrt(gH) < U < Sqrt(gH)
Out[116]=
Out[117]= Fnn \parallel \frac{1}{2} \left( 2 \operatorname{Gnp} H + \operatorname{Rpp} \left( -\sqrt{g H} + U \right) + \operatorname{Rmp} \left( \sqrt{g H} + U \right) \right)
 \label{eq:continuous} Out[118] Fnn \parallel \frac{1}{2} \left( \frac{1}{2} \left( \frac{Rmp} \left( \frac{H}+U\right) + \frac{Rmp}{H} \right) \right) \\
\text{Out} \text{[119]= Fnn error } \text{||} \ \left( -\frac{\left( H^2\,k^3\,U\,w \right)dt^2}{2\left( 3\!+\!H^2\,k^2 \right)} - \frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left( 3\!+\!H^2\,k^2 \right)} + O[dt]^4 \right) + \\
                                                                  \left(-\frac{\imath \left(54 \, k^{3}+45 \, H^{2} \, k^{5}+10 \, H^{4} \, k^{7}\right) \cup dt}{120 \left(3+H^{2} \, k^{2}\right)^{2}}+O[dt]^{4}\right) dx^{2}+\left(-\frac{1}{8} \left(\sqrt{g \, H} \, k^{4}\right) dt+O[dt]^{4}\right) dx^{3}+O[dx]^{4}
 Out[120]= Fnn error ||
                                                         \left(-\frac{t}{t}^2 \left(\frac{t}^2 k^3 U \right)_{2} \left(\frac{t}^2 k^3 U \right)_{2} \left(\frac{t}^2 k^2 + 3\right)_{1} \right) - \left(\frac{t}^3 U \right)_{2} \left(\frac{t}^3 U \right)_{
                                                                                 \left(H^2 k^2+3\right)+O\left(t^4 k^7+45 H^2\right)
                                                                                 \left(-\frac{1}{8}\right) \left(\frac{1}{8}\right) + O\left(\frac{1}{8}\right) + O\left(\frac{1}
Out[121]=
Out[122]= FnG || GGp H
Out[123]= FnG \parallel \text{text}\{GGp\} H
 \text{Out} [124] = \text{ FnG error } || \left( -\frac{3 (k \text{ w}) \text{ d} t^2}{2 \left(3 + \text{H}^2 \text{ k}^2\right)} - \frac{i \text{ k w}^2 \text{ d} t^3}{2 \left(3 + \text{H}^2 \text{ k}^2\right)} + \text{O}[\text{d} t]^4 \right) + \left( -\frac{i \left(12 \text{ k}^3 + 5 \text{ H}^2 \text{ k}^3\right) \text{ d} t}{40 \left(3 + \text{H}^2 \text{ k}^2\right)^2} + \text{O}[\text{d} t]^4 \right) \text{d} x^2 + \text{O}[\text{d} x]^4
```

Text[Row[{"FGn error || ", TeXForm[FGn2TAr]}]]

 $\label{eq:continuous} $$ \inf_{0 \le 1/2} FnG error \| \left(-\frac{3 \text{ kw}}{2 \left(k w\right)}{2 \left(k w\right)}{2 \left(k w\right)} - \frac{i \text{ kc}}{3 k w^2}{2 \left(k w^2} - \frac{i \text{ kc}}{3 k w^2}{2 \left(k w\right)}{2 \left(k w\right)} - \frac{i \text{ kc}}{3 k w^2}{2 \left(k w\right)}{2 \left(k w\right)} - \frac{i \text{ kc}}{3 k w^2}{2 k^2} - \frac{i \text{ kc}}{3 k w^2}{2 k^2} - \frac{i \text{ kc}}{3 k w^2} - \frac{i \text{ kc}}{3 k w^2}{2 k w^2}$ 

Out[126]=

Out[127]= FGn || 
$$\frac{1}{2} \left( g H \left( Rmp + Rpp \right) + \left( 2 Gnp H + \sqrt{g H} \left( Rmp - Rpp \right) \right) U \right)$$

Out[128]= FGn ||

\frac{1}{2} \left(U \left(\sqrt{g H} (\text{Rmp}\-\text{Rpp}))+2 \text{Gnp} H\right)+g H (\text{Rmp}+\text{Rpp})\right)

$$\begin{array}{ll} \text{Out} \text{[129]=} & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w \right) dt^2}{2 \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^2 \, dt^3}{6 \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \\ & \left( -\frac{i \left( 90 \, g \, H \, k^3 + 60 \, g \, H^3 \, k^5 + 10 \, g \, H^5 \, k^7 - 36 \, k^3 \, U^2 - 15 \, H^2 \, k^5 \, U^2 \right) dt}{120 \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{8} \left( \sqrt{g \, H} \, k^4 \, U \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4 \\ & \left( -\frac{1}{8} \left( \sqrt{g \, H} \, k^4 \, U \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) dx^4 + O[dx]^4 + O$$

Out[130]= FGn error |

 $\label{left-proc} $\left(-\frac{\left(text\{dt\}^2 \left(H^2 k^2+3\right)\right)}{2 \left(H^2 k^2+3\right)}-\frac{1}{2 \left(H^2 k^$ 

Out[131]=

Out[132]= FGG || 
$$\frac{1}{2} \left( \sqrt{g H} Rmp - \sqrt{g H} Rpp + (2 GGp H + Rmp + Rpp) U \right)$$

out[133]= FGG || \\frac{1}{2} \\left(\text{Rmp} \\sqrt{g H}\-\text{Rpp} \\sqrt{g H}+U (2 \\text{GGp} H+\\text{Rmp}+\\text{Rmp}+\\text{Rpp})\\right)

$$\begin{array}{ll} \text{Out} \text{[134]=} & FGG \; error \; \parallel \; \left( -\frac{\left( k \left( 6 + H^2 \, k^2 \right) U \, w \right) dt^2}{2 \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 6 + H^2 \, k^2 \right) U \, w^2 \, dt^3}{6 \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( 126 \, k^3 + 75 \, H^2 \, k^5 + 10 \, H^4 \, k^7 \right) U \, dt}{120 \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \\ & \left( -\frac{1}{8} \left( \sqrt{g \, H} \; k^4 \right) dt + O[dt]^4 \right) dx^3 + \left( \frac{i \left( 13311 \, k^5 \, U + 11430 \, H^2 \, k^7 \, U + 3110 \, H^4 \, k^9 \, U + 260 \, H^6 \, k^{11} \, U \right) dt}{4800 \left( 3 + H^2 \, k^2 \right)^3} + O[dt]^4 \right) dx^4 + O[dt]^5 \end{array}$$

Out[135]= FGG error |

 $\label{left-text} $$\left(-\frac{\hat t}^2 \left(U \ \left(U \ \left(H^2 \ k^2+6\right)\right)}{2 \left(H^2 \ k^2+3\right)}-\frac{i \ text{dt}}^3 \ k \ U \ w^2 \left(H^2 \ k^2+6\right)}{6 \left(H^2 \ k^2+3\right)}+O\left(\frac{4}{3}\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}{2 \left(H^2 \ k^2+3\right)}+O\left(\frac{4}{3}\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}{2 \left(H^2 \ k^2+3\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}{2 \left(H^2 \ k^2+3\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}\right)}+O\left(\frac{4}{3}\right)+O\left(\frac{4}{3}$ 

Out[136]=

Out[137]=

Out[138]= Omega error ||

1

$$\begin{cases} \frac{1}{6(2+H^2k^2)^2}k^3\left(\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\!\left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\!\right)dt^2 + \\ \frac{i\,k^2\left[3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right]^2dt^2}{8(3+H^2k^2)} - \frac{1}{20(3+H^2k^2)^2}\left[k^3\left(\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right]^3dt^2 + Q[dt]^3} + \\ \left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right)dt^4 + Q[dt]^3\right] + \\ \left(\frac{1}{240(3+H^2k^2)^2}k^3\left(20\,H^6k^6\,U^3 + 54\,U^2\left(9\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 10\,U\right)\right) + \\ \frac{1}{480(3+H^2k^2)^2}k^3\left(20\,H^6k^6\,U^3 + 54\,U^2\left(9\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 10\,U\right)\right) + \\ 5\,k^4\,U^2\left(11\,\sqrt{3}\ \sqrt{g\,H^3(3+H^2k^2)} + (216+147\,H^2\,k^2 + 25\,H^4\,k^4)\,U\right) + \\ 3\,k^2\left(15\,\sqrt{3}\ \sqrt{g^2\,11^2\,(3+H^2k^2)} + 109\,\sqrt{3}\ \sqrt{g\,H^3\left(3+H^2k^2\right)}\,U^2 + 180\,H^2\,U^3\right)\right)dt^2 + \\ \frac{1}{480(3+H^2k^2)^2}k^3\left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right) \\ \left(9\,g\,H(14+5\,H^2k^2) + U\left(102\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 180\,U + 20\,H^4k^4\,U + 5\,k^2\left(\sqrt{3}\ \sqrt{g\,H^3(3+H^2k^2)} + 8\,H^2\,U\right)\right)\right) \\ \left(k^2\left(42\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 180\,U + 20\,H^4k^4\,U + 15\,k^2\left(\sqrt{3}\ \sqrt{g\,H^3(3+H^2k^2)} + 8\,H^2\,U\right)\right) \\ \left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + 180\,U + 20\,H^4k^4\,U + 15\,k^2\left(\sqrt{3}\ \sqrt{g\,H^3(3+H^2k^2)} + 8\,H^2\,U\right)\right) \\ \left(3\,g\,H + U\left(2\,\sqrt{3}\ \sqrt{g\,H(3+H^2k^2)} + (3+H^2k^2)\,U\right)\right)^2\right)dt^4 + O[dt]^5\right)dx^2 + \\ \left(-\frac{i\,k^2\left(2\,g\,H(3+H^2k^2) + \sqrt{3}\ g\,H(3+H^2k^2)}{10\,\sqrt{g\,H(3+H^2k^2)}} + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + 3\,\sqrt{3}\ U + k^2\left(2\,\sqrt{g\,H^3(3+H^2k^2)} + \sqrt{3}\ H^2\,U\right)\right)dt^2 + \\ \left(i\,k^8\left(2\,g\,H\left(3+H^2k^2\right) + \sqrt{3}\ \sqrt{g\,H\left(3+H^2k^2\right)} + 2075\,H^4k^4\right) + 2080\left(9\,\sqrt{g\,H\left(3+H^2k^2\right)} + 6\,k^2\,\sqrt{g\,H^3(3+H^2k^2)} + 46\,k^2\right)U\right)^2 \right) dt^4 \right) \right\} dt^4 \right) \right\} dt^4 \right) \left(-$$

$$\left(k^{3}\left[-\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right] + \left(3+H^{2}\,k^{2}\right)U\right)^{3}\left(3\,g\,H + U\left(-2\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right) + \left(3+H^{2}\,k^{2}\right)U\right)\right) \right) \\ dt^{4} + O[dt]^{9} + \\ \left(\frac{1}{240(3+H^{2}\,k^{2})^{2}}k^{3}\left(-42\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right) + 180\,U + 20\,H^{4}\,k^{4}\,U - 15\,k^{2}\left(\sqrt{3}\ \sqrt{g\,H^{5}\left(3+H^{2}\,k^{2}\right)}\right) - 8\,H^{2}\,U\right)\right) + \\ \frac{1}{480(3+H^{2}\,k^{2})^{2}}k^{3}\left(20\,H^{6}\,k^{6}\,U^{3} + 54\,U^{2}\left(-9\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right) + 10\,U\right) + \\ 5\,k^{4}\,U^{2}\left(-11\,\sqrt{3}\ \sqrt{g\,H^{9}\left(3+H^{2}\,k^{2}\right)} + 236\,H^{4}\,U\right) + \\ 6\,g\,H\left(-21\,\sqrt{3}\ \sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + 216\,H^{4}\,H^{2}\,k^{2}\right) + 25\,H^{2}\,k^{3}\right)U\right) - \\ 3\,k^{2}\left(15\,\sqrt{3}\ \sqrt{g\,3}\,H^{7}\left(3+H^{2}\,k^{2}\right) + 109\,\sqrt{3}\ \sqrt{g\,H^{5}\left(3+H^{2}\,k^{2}\right)}\,U^{2} - 180\,H^{2}\,U^{3}\right)\right)dt^{2} + \\ \frac{1}{480(3+H^{2}\,k^{2})^{2}}i^{4}k^{6}\left(3\,g\,H + U\left(-2\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + (3+H^{2}\,k^{2})\,U\right)\right) \\ \left(9\,g\,H\left(14+5\,H^{2}\,k^{2}\right) + U\left(-102\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 180\,U + 20\,H^{4}\,k^{4}\,U - \\ 5\,k^{2}\left(7\,\sqrt{3}\ \sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + 180\,U + 20\,H^{4}\,k^{4}\,U - 15\,k^{2}\left(\sqrt{3}\ \sqrt{g\,H^{5}\left(3+H^{2}\,k^{2}\right)} - 8\,H^{2}\,U\right)\right)\right) \\ \left(3\,g\,H + U\left(-2\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 180\,U + 20\,H^{4}\,k^{4}\,U - 15\,k^{2}\left(\sqrt{3}\ \sqrt{g\,H^{5}\left(3+H^{2}\,k^{2}\right)} - 8\,H^{2}\,U\right)\right)\right) \\ dx^{2} + \left(\frac{1}{16}\,i\,\sqrt{g\,H}\,k^{4}\left(-2+\frac{\sqrt{3}\,U}{\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}} + 3\,\sqrt{3}\,U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + \sqrt{3}\,H^{2}\,U\right)\right)\right)dt^{2} - \\ \frac{1}{12(2+H^{2}\,k^{2})^{3/2}}\left(k^{6}\left(g\left(-6\,H\,\sqrt{g\,H}\,(3+H^{2}\,k^{2}\right) + 15\,\sqrt{3}\,U\,H^{2}\,k^{2}\right) + 4\,H^{2}\,k^{2}\right)U\right)\right)^{2}dt^{4} + O[dt]^{5}\right) \\ dx^{3} + \left(\left(k^{6}\left(2\,g\,H\left(3+H^{2}\,k^{2}\right) + 3\,\sqrt{3}\,U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + \sqrt{3}\,H^{2}\,U\right)\right)\right)\right)\right)dt^{2} - \\ \left(3\,g\,H + U\left(-2\,\sqrt{3}\ \sqrt{g\,H\left(3+H^{2}\,k^{2}\right)} + 3\,\sqrt{3}\ U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + \sqrt{3}\,H^{2}\,U\right)\right)\right)\right)dt^{2} - \\ \frac{1}{12(2+H^{2}\,k^{2})^{3/2}}\left(k^{2}\left(3\,g\,H\left(13+H^{2}\,k^{2}\right) + 3\,\sqrt{3}\,U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+H^{2}\,k^{2}\right)} + \sqrt{3}\,H^{2}\,U\right)\right)\right)\right)dt^{2} - \\ \frac{1}{12(2+H^{2}\,k^{2})^{3/2}}\left(k^{2}\left(3\,g\,H^{2}\left(3+H^{2}\,k^{2}\right) + 3\,\sqrt{3}\,U + k^{2}\left(-2\,\sqrt{g\,H^{3}\left(3+$$

$$\begin{split} g\,H\,U\left(447\,588\,\sqrt{3}\,\,H^2\,k^2\,U + 16\,705\,\sqrt{3}\,\,H^6\,k^6\,U - 648\left(693\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} - 688\,\sqrt{3}\,\,U\right) - \\ 15\,k^4\left(3408\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)} - 9985\,\sqrt{3}\,\,H^4\,U\right)\right) - \\ 80\left(1836\,\sqrt{g\,H\left(3 + H^2\,k^2\right)}\,\,U^3 + 612\,k^4\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)}\,\,U^3 + 68\,k^6\,\sqrt{g\,H^{13}\left(3 + H^2\,k^2\right)}\,\,U^3 + \\ 9\,k^2\left(421\,\sqrt{g^3\,H^7\left(3 + H^2\,k^2\right)}\,\,U + 204\,\sqrt{g\,H^5\left(3 + H^2\,k^2\right)}\,\,U^3\right)\right)\right)dt^2\right) / \\ \left(230\,400\,\sqrt{g\,H}\,\left(3 + H^2\,k^2\right)^{7/2}\right) - \frac{1}{25\,600\,(3 + H^2\,k^2)}\,i\,k^8\left(6\,g^2\,H^2\left(8046 + 5460\,H^2\,k^2 + 925\,H^4\,k^4\right) + \right. \\ 560\,H^8\,k^8\,U^4 + 432\,U^3\left(-143\,\sqrt{3}\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 105\,U\right) + \\ 5\,k^4\,U^3\left(-4139\,\sqrt{3}\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)} + 6048\,H^4\,U\right) + \\ 5\,k^6\,U^3\left(-461\,\sqrt{3}\,\sqrt{g\,H^{13}\left(3 + H^2\,k^2\right)} + 1344\,H^6\,U\right) - \\ 12\,k^2\left(3585\,\sqrt{3}\,\sqrt{g^3\,H^7\left(3 + H^2\,k^2\right)}\,U + 5161\,\sqrt{3}\,\sqrt{g\,H^5\left(3 + H^2\,k^2\right)}\,U^3 - 5040\,H^2\,U^4\right) + \\ g\,H\,U\left(-63\,720\,\sqrt{3}\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 282\,852\,U + 284\,364\,H^2\,k^2\,U + 10\,640\,H^6\,k^6\,U + \\ 5\,k^4\left(-1451\,\sqrt{3}\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)} + 19\,056\,H^4\,U\right)\right)\right)dt^3 - \frac{1}{460\,800\left(\sqrt{g\,H}\,\left(3 + H^2\,k^2\right)}\right)U\right) + \\ 108\,k^2\left(2625\,\sqrt{g^5\,H^9\left(3 + H^2\,k^2\right)} - 15\,327\,\sqrt{3}\,g^2\,H^4\,U + 22\,383\,\sqrt{g^3\,H^7\left(3 + H^2\,k^2\right)}\,U^2 - \\ 19\,261\,\sqrt{3}\,g\,H^3\,U^3 + 4640\,\sqrt{g\,H^3\left(3 + H^2\,k^2\right)}\,U^4\right) + \\ g\,H^3\,U^2\left(89\,915\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} - 61\,735\,\sqrt{3}\,g^2\,H^6\,U + 27\,840\,\sqrt{g\,H^9\left(3 + H^2\,k^2\right)}\,U^4 + \\ g\,H^3\,U^2\left(89\,915\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} - 115\,737\,\sqrt{3}\,U\right)\right) + 1296\left(323\,\sqrt{g^5\,H^5\left(3 + H^2\,k^2\right)} - \\ 1268\,\sqrt{3}\,g^2\,H^2\,U + 290\,\sqrt{g\,H\left(3 + H^2\,k^2\right)}\,U^4 + g\,H\,U^2\left(1857\,\sqrt{g\,H\left(3 + H^2\,k^2\right)}\,U^3 + \\ g\,H^7\,U\left(-6019\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 15\,454\,\sqrt{3}\,U\right)\right)\right)\right)dt^4 + O[dt]^5\right)dx^4 + O[dx]^5\right) + \\ g\,H^7\,U\left(-6019\,\sqrt{g\,H\left(3 + H^2\,k^2\right)} + 15\,454\,\sqrt{3}\,U\right)\right)\right)dt^4 + O[dt]^5\right)dx^4 + O[dx]^5\right)$$

Out[139]= Omega error ||

 $\left(\left(\frac{k^3 \left(\frac{k^2+3\right)} U+\right) + \left(\frac{3} \left(\frac{4}{2} k^2+3\right)\right) + \left(\frac{3} k^2+3\right) \right) + \left(\frac{3} k^2+3\right) \left(\frac{4}{2} k^2+3\right) + \left(\frac{4}$ 

 $\left(\frac{H^2 k^2+3\right}{k^2+3\right} k^4+3\left(\frac{180 H^2 U^3+109 \sqrt{3}}{k^2+3}\right)$  $U^2+15 \sqrt{3} \sqrt{10} U^2 + 15 \sqrt{3} \sqrt{10} U^2 \left( U^2 \right)$  $H \left( \frac{h^2 k^2+3 \right)}{tght} + 6 g H \left( \frac{5 h^4 k^4+147 h^2 k^2+216 \right) U+21 \left( \frac{3}{t} \right)$  $\left( H^2 k^2+3\right)\right)\right)$ g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \left(9 g  $H \left( \frac{5 \text{ H}^2 \text{ k}^2 + 14 \text{ right}}{4 \text{ U k}^4 + 5 \text{ left}(24 \text{ U H}^2 + 7 \text{ sqrt}\{3\} \text{ sqrt}\{g \text{ H}^5 \text{ left}(\text{H}^2 + \text{$  $k^2+3\left(H^2 k^2+3\right)\right) + k^2+180 U+102 \left(H^2 k^2+3\right) \left(H^2 k^2+3\right) + k^2+3\left(H^2 k^2+3\left(H^2 k^2+3\right) + k^2+3\left(H^2 k^2+3\right) + k^2+3\left(H^2 k^2+3\right) + k^2+3\left(H^2 k$  $\left(H^2 k^2+3\right)^3-\frac{(k^7 \left(H^2 U k^4+15\right)^3}{h^5}\right)^3$  $\left(H^2 k^2+3\right)\right) \$  $\t x{dt}^4}{960 \left( \frac{h^2 k^2+3\right)^4}{+O\left( \frac{t}{t}^5\right) \right) \left( \frac{k^2+1}{t}^6} \right) }$  $\left(2 g H \left(H^2 k^2+3\right)\right) + \left(H^2 k^2+3\right) \$  $\sqrt{3} U+12 \operatorname{qrt}(g H \left(H^2 k^2+3\right)\right) U^2+g H \left(\frac{4 H^2 k^2+15\right) U+6$  $\label{eq:left} $\operatorname{H}\left(H^2 k^2+3\right)\right)\right) \cdot \left(H^2 k^2+3\right)^{3/2}+\frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2}+\frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2})\right)^{3/2}+\frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2})^{3/2}+\frac{h^2}{2}(32 \left(H^2 k^2+3\right)^{3/2}+\frac{h^2}{2}(32 \left(H^2 k^2+3$  $g H+U \left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3} \sqrt{4} \ln(H^2 k^2+3\right)} \left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3} \sqrt{4} \ln(H^2 k^2+3\right)} \left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3} \ln(H^2 k^2+3)} \left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3} \sqrt{3}} \left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3} \sqrt{3}} \left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3}} \left(\frac{H^$ g H \left( $H^2 k^2+3\right\right)+U \left(\frac{1}{2} K^2+3\right)$  $k^8 \left( 16H(4^2 k^2+3\right) + \left( 16H(4^2 k^2+3\right) \right) + \left( 16H(4^2 k^2+3\right) \left( 16H(4^2 k^2+3\right) \right) + \left( 16H(4^2 k^2+3\right) + \left( 16H(4^2 k^2+3\right) \right) + \left( 16H(4^2 k^2+3\right) + \left( 16H(4^2 k^2 k^2+3\right) + \left( 16H(4^2 k^2 k^2+3\right) + \left( 16H(4^2 k^2+3\right) + \left( 16H(4$  $\label{eq:linear_state} $$ \left( \frac{H^2 k^2+3 \right)^3+O\left( \frac{dt}{5}\right) \operatorname{left}(t) \operatorname{$ g H \left(2075 H^4 k^4+12180 H^2 k^2+17856\right)+2080 \left(\sqrt{g H^9 \left(H^2 k^2+3\right)})  $k^4+6 \operatorname{qt}(H^2 k^2+3\operatorname{right}) k^2+9 \operatorname{qt}(H^2 k^2+3\operatorname{right}) \operatorname{dt}(H^2 k^2+3\operatorname{qt}(H^2 k^2+3\operatorname{$  $\label{left} $$\left( \frac{H^2 k^2+3\right)^{5/2}\right)-\frac{h^4}{2} .$$$ k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6+15 \left(9985 \sqrt{3} U H^4+3408 \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+447588 \sqrt{3} H^2 U k^2+648 \left(688 \sqrt{3} U+693  $k^6 + 612 \sqrt{g H^9 \left( H^2 k^2 + 3\right)} U^3 k^4 + 9 \left( 9 H^5 \right) H^5 \left( H^2 k^2 + 3\right)$  $U^3+421 \cdot g^3 H^7 \cdot (h^2 k^2+3 \cdot g)$  Uright)  $h^2+1836 \cdot g$  H \left( $h^2 k^2+3 \cdot g$ )  $U^3\right) \left( \frac{H^2 k^2+3\right)}{1} \left( \frac{1}{2}(230400 \left( \frac{H^2 k^2+3\right)}{7/2}\right) -\frac{1}{4} \left( \frac{1}{2} \right) \right) \left( \frac{1}{4} \right) \left($ \left(560 H^8 U^4 k^8+5 U^3 \left(1344 U H^6+461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right)  $k^6+5 U^3 \left(048 U H^4+4139 \right) \left(H^9 \left(H^2 k^2+3\right)\right) k^4+12 \left(5040 H^4+13 \right) k^4+12 \left(141 H^2 k^2+3\right) k^4+12 \left(141 H^2 k$  $H^2 U^4+5161 \sqrt{3} \sqrt{4+5161} \sqrt{3} \sqrt{4+5161}$ k^2+3\right) U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105  $U+143 \cdot \{3\} \cdot \{g \in (19056 \cup 1905) + g \in (19056 \cup$ H^4+1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U+63720  $\left(\left(\frac{H^2 k^2+3\right)}{U+\sqrt{3} \left(\frac{H^2 k^2+3\right)}\right)\right)$  $g H^9+928 \sqrt{14} \left( H^2 k^2+3\right) U\right) k^8+15 U \left( 4143 \sqrt{14} \right) g^2 H^8+g U$  $\left(15454 \right) H^7+3712 \right) H^619 YEAR H^6$ k^2+3\right)} U^3\right) k^6+9 \left(61735 \sqrt{3} g^2 U H^6+g U^2 \left(115737 \sqrt{3} U+89915)  $\$  \\sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+27840 \\sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325  $\$  \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)} \left(k^4+108 \left(15327 \right) g^2 U H^4+19261 \right) sqrt{3} g^5 U H^5+19261

g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}  $U^2 + 2625 \sqrt{g^5 H^9 \left( H^2 k^2 + 3\right)} k^2 + 1296 \left( 290 \sqrt{g^2 H^2 k^2 + 3\right) } k^2 + 1296 \left( 290 \sqrt{g^2 H^2 k^2 + 3} k^2 + 3\right) k^2 + 1296 \left( 290 \sqrt{g^2 H^2 k^2 + 3} k^2 +$  $U^4+g H \left(1202 \right) U+1857 \left(14 H^2 k^2+3\right) U+1857 \left(1$  $H^2 U+323 \left( \frac{g^5 H^5 \left( \frac{40800 \left( + \right)} \right)} \right)} {40800 \left( \frac{40800 \left( + \right)} \right)} {40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( + \right)} {40800 \left( \frac{40800 \left( + \right)} {40800 \left( \frac{40800 \left( + \right)} {40800 \left( +$  $\left(\left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \sqrt{3} \right) + \left(\frac{H^2 k^2+3\right)}\right) \left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \left(\frac{H^2 k^2+3\right)}}\right)$  $k^2+3\right) U-2 \sqrt{3} \sqrt{4} \ln(H^2 k^2+3\right) \int U-2 \sqrt{3} \left( H^2 k^2+3\right) \left($  $\label{eq:k-2+3-right} $$ k^2+3\right)^2+\frac{k^4\left(\frac{3}{4} +U\left(\frac{k^2 k^2+3\right) U-2 \right)U-2 \left(\frac{3}{4} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3\right)U-2 }{2} +U\left(\frac{k^2 k^2+3}{4} +U\left(\frac{k^2 k^2+3}{4$  $\label{left(H^2 k^2+3\right)} $$ \left( H^2 k^2 + 3\right)^2 \left( H^2 k^2 + 3\right)^2 - \frac{h^2}{h^2} \right) $$ \left( H^2 k^2 + 3\right)^2 - \frac{h^2}{h^2} \left( H^2 k^2 + 3\right)^2 \right) $$ \left( H^2 k^2 + 3\right)^2 + \frac{h^2}{h^2} \left( H^2 k^2 + 3\right)^2$  $k^2+3\right\in U-\sqrt{3} \left( U-\sqrt{3} \right) -3 \left( U-\sqrt{3} \right)$  $k^2+3\left( U-2 \right) U-2 \left( H^2 \left( H^2 \right) \right) \left( H^2 \right) \left( H$  $k^2+3\left(\frac{4t}^5\right)+\left(\frac{4t}^5\right)$  $\left(H^2 k^2+3\right)^2+\frac{h^2 u^2}{h^2 u^2} + \frac{h^2 u^2 \left(H^2 u^2 + \frac{h^2 u^2}{h^2 u^2}\right)}{h^2 u^2 u^2} + \frac{h^2 u^2 u^2}{h^2 u^2} + \frac{h^2 u^2}{h^2 u^2} + \frac{h^2$  $\label{left(H^2 k^2+3\wedge left(H^2 k^2+3\wedge left($  $U^2+15 \sqrt{3} \sqrt{6^3 H^7 \left(H^2 k^2+3\right)} \right) + 2 + 4 U^2 \left(10 U-9 \right)$ H^2 k^2+14\right)+U \left(20 H^4 U k^4-5 \left(7 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)}-24 H^2  $U = U + 180 U - 102 \left( \frac{3} \left( \frac{480 e^2 + 3 e^2 + 3 e^2}{180 e^2} \right) \right)$  $k^2 + 3 \cdot (k^7 \cdot (k^7 \cdot k^7 \cdot k^7$  $H^2 U \cdot k^2 + 180 U - 42 \cdot k^2$  $k^2+3\right) U-2 \sqrt{3} \sqrt{g} H \left(\frac{h^2 k^2+3\right)}\right) \left(\frac{h^2 k^2+3\right)}{16h} \left(\frac{h^2 k^2+3\right)}{16h}\right) \left(\frac{h^2 k^2+3\right)}{16h}$  $\left(\frac{4}{2} - \frac{4}{1} \right)^4 + O\left(\frac{4t}{5}\right) \cdot \left(\frac{4t}{2} - \frac{4t}{2}\right)$  $\left(\frac{3}{U}\right) = \frac{1}{2\pi} \left(\frac{3}{U}\right)$  $U-2 \left( H^2 k^2+3\right) + 12 \left( H^2 k^2+3\right) + 12$  $U^2+g \left( \frac{1}{sqrt} \right) H^2 U H^3+15 \left( \frac{1}{3} U H^6 \right) H^6 \left( \frac{1}{2} K^2+3\right) H^7 U H^6 \left( \frac{1}{3} U H^6 \right) H^7 U H^6 \left( \frac{1}{3} U H^6 \right) H^7 U H^6 U H^6$  $\label{eq:left} $$ \operatorname{dt}^2_{32}\left(H^2 k^2+3\right)^{3/2}-\frac{(k^7 \left(h^7 \left(h^2 k^2+3\right)^3)^{3/2})^{-h^2}}{h^3}\right)^{3/2} + H^2 \left(h^2 k^2+3\right)^{3/2} + H^2 \left(h^2$  $U-2 \sqrt{3} \left( H \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} \right) \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)} \left( \frac{h^2 k^2+3\right)}{\sinh(h^2 k^2+3)} \left( \frac{h^2 k^2+3\right)}{h^2 k^2+3} \left( \frac{h^2 k^2+3}{h^2 k^2+3} \right) \left( \frac{h^2 k^2+3}{h^2 k^2+3} \left( \frac{h^2 k^2+3}{h^2 k^2+3} \right) \left( \frac{h^2 k^2+3}{h^2 k^2+3} \left( \frac{h^2 k^2+3}{h^2 k^2+3} \right) \left( \frac{h^2 k$  $\left(\left(\frac{3}{4}\right) + \frac{3}{4}\right) = \frac{1}{4}$ \left(H^2 k^2+3\right)\right)\right)\right)\right)\right)\\text{dt}^3}{32 \left(H^2 k^2+3\right)^{5/2}}+\frac{i k^8 \left(2)}{2}  $\label{eq:linear_line$  $k^2+3\right)^3+O\left(\frac{dt}^5\right)\right)$  $H^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 \right] + W^4 k^4 + 12180 H^2 k^2 + 17856 H^2 k^2 + 1786 H$  $H^5 \left( H^2 k^2 + 3\right) \left( H^2 k^2 + 3\right)$  $\left(H^2 k^2+3\right)^{5/2}+\frac{k^7 \left(45 \right)^2}{9^2 \left(45 \right)^2}$  $H^2+g U \left(16705 \right) -15 \left(14705 \right)$  $H^4 U \cdot k^4 + 447588 \cdot k^4 + 447588 \cdot k^4 + 447588 \cdot k^4 - 447588 \cdot k^4 + 447688 \cdot k^4 + 44768$  $\sqrt{3} U\right) + M-80 \left(68 \right) + M-90 \left(6$  $\left(H^2 k^2+3\right) U^3 k^4+9\left(204 \right) 16$  $H^7 \left(H^2 k^2+3\right) U\right) U\right) H^7 \left(H^2 k^2+3\right) U^3\right) U^3\right)$ 

\left(1344 H^6 U-461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 H^4  $U-4139 \sqrt{3} \sqrt{4-12 \left( -5040 \text{ H}^2 \text{ U}^4+5161 \right)} \right)$  $\sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \right]$  $k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 U-143 \right)$  $H \left( \frac{h^2 k^2+3\right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 \left( \frac{19056 H^4 U-1451 \right)}{h} \right)}{h} \ U \left( \frac{h^6 U k^6+5 U k^$ H^9 \left(H^2 k^2+3\right)\right) k^4+284364 H^2 U k^2+282852 U-63720 \sqrt{3} \sqrt{g H \left(H^2  $\label{left} $$ k^2+3\right)\right) \right) \left( k^9 \left( k^2 + 3\right) \right) \left( k^9 \left( k^9 \right) \right) \left( k^9 \left( k^9 \right) \right) \right) \left( k^9 \left( k^9 \left( k^9 \right) \right) \left( k^9 \left( k^9 \right) \right) \left( k^9 \left( k^9 \left( k^9 \right) \right) \left( k^9 \left( k^9 \right$  $H \left( H^2 k^2 + 3 \right) - \left( H^2 k^2 + 3 \right) - \left( H^2 k^2 + 3 \right) \left( H^2 k^2 + 3 \right) - \left( H$ k^2+3\right)} U-3869 \sqrt{3} g H^9\right) k^8-15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3})  $U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-3 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3\right} U-6019 \sqrt{H^2 k^2+3} U-6019 U$ k^6+9 \left(-61735 \sqrt{3} g^2 U H^6+g U^2 \left(89915 \sqrt{g H \left(H^2 k^2+3\right)}-115737 \sqrt{3} U\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)\right) k^4+108 \left(-15327 \sqrt{3} g^2 U H^4-19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g}  $H^5 \left(H^2 k^2+3\right) U^4+22383 \left(H^2 k^2+3\right) U^4+22383 \right) U^4+22383 \left(H^2 k^2+3\right) U^4+24383 \left(H^4 k^2+3\right) U^4+24383 \left(H^4 k^2+3$  $H^9 \left( H^2 k^2 + 3 \right) \right) + H^9 \left( H^2 k^2 + 3 \right) + H^9 \left( H^8 k^2 + 3 \right) + H$ \left(1857 \sqrt{g H \left(H^2 k^2+3\right)}-1202 \sqrt{3} U\right) U^2-1268 \sqrt{3} g^2 H^2 U+323  $\$  \sqrt{g^5 H^5 \left(H^2 k^2+3\right)}\right)\right)\right) \text{dt}^4}{460800 \left(\sqrt{g H} \left(H^2 L^2+3)\right)}  $k^2+3\right)^{1/2}\right)+O\left(\frac{dt}^5\right)\right)$ 

 $\text{Out} \text{[141]=} \quad EA \text{ ||} \quad \left\{ \left\{ \frac{-H^2 \ k^2 \left( \left( -1 + e^{i \ d \ w} \right) k \ U - w \right) + 3 \ w}{\left( 3 + H^2 \ k^2 \right) w}, \ -\frac{3 \left( -1 + e^{i \ d \ w} \right) k}{\left( 3 + H^2 \ k^2 \right) w} \right\}, \ \left\{ -\frac{\left( -1 + e^{i \ d \ w} \right) k \left( g \ H \left( 3 + H^2 \ k^2 \right) - 3 \ U^2 \right)}{\left( 3 + H^2 \ k^2 \right) w}, \ 1 \ -\frac{\left( -1 + e^{i \ d \ w} \right) k \left( 6 + H^2 \ k^2 \right) U}{\left( 3 + H^2 \ k^2 \right) w} \right\} \right\}$ \begin{array}{cc}  $\frac{3 \text{ w-H}^2 \text{ k}^2 \left(\frac{1+e^{i \text{ text}}}{u}\right)}{\left(\frac{1+e^{i \text{ text}}}{u}\right)} k U-w\right)}{\left(\frac{1+e^{i \text{ text}}}{u}\right)}$  $k^2+3\right) w$  &  $-\frac{3 \left(1+e^{i \left(t\right) w}\right) k}{\left(t\right) w} \$  $-\frac{1+e^{i \cdot text\{dt\} w}\right| k \left(H^2 k^2+3\right)}{\|h\|^2 k^2+3\right)} \|h\|^2 \|h\|^2$ 

\right)

\end{array}

Out[140]=

$$\begin{aligned} &\operatorname{Coll}_{(443)^8} \;\; \operatorname{Eerr} \, \| \;\; \left\{ \left\{ \left( \frac{(-3\,g\,H\,k^2+3\,k^2\,U^2-H^2\,k^2\,U\,u^3)\,d^2}{2\,(3+H^2\,k^2)} - \frac{i\,H^2\,k^2\,U\,u^3\,d^2}{6\,(3+H^2\,k^2)} + \operatorname{O}[\,d\,I]^5 \right) + \\ &\left( -\frac{i\,(5\,4\,k^2+43\,H^2\,k^2+3\,H^2\,k^2\,U\,u^4}{12\,(3+H^2\,k^2)} \right) dt^2}{2\,40\,(3+H^2\,k^2)^2} + \operatorname{O}[\,d\,I]^5 \right) dx^2 + \\ &\left( -\frac{1}{8}\left( \sqrt{g\,H} \;\;k^4 \right) dt + \frac{i\,\sqrt{g\,H}}{16\,(3+H^2\,k^2)} \right) dt^2}{16\,(3+H^2\,k^2)} + \operatorname{O}[\,d\,I]^5 \right) dx^3 + \\ &\left( -\frac{1}{8}\left( \sqrt{g\,H} \;\;k^4 \right) dt + \frac{i\,\sqrt{g\,H}}{16\,(3+H^2\,k^2)} \right) dt^2}{4800\,(3+H^2\,k^2)} + \operatorname{O}[\,d\,I]^5 \right) dx^3 + \\ &\left( \frac{i\,(729\,k^2\,U+26\,10\,H^2\,k^2\,U+570\,H^2\,k^2\,U+26\,H^2\,k^2\,U\,U\,d^2}{4800\,(3+H^2\,k^2)} + \frac{1}{2800\,(3+H^2\,k^2)} \right) dt^2 + \frac{1}{2800\,(3+H^2\,k^2)} \left( 38\,853\,g\,H\,k^6 + 26\,460\,g\,H^3\,k^8 + 4500\,g\,H^5\,k^{10} - 38\,286\,k^6\,U^2 - 13\,500\,H^2\,k^8\,U + 24\,410\,H^2\,k^{10}\,U^2 + 1460\,H^6\,k^{12}\,U^2 \right) dt^2 + \operatorname{O}[\,d\,I]^5 \right) dx^4 + \\ &\left( O[\,d\,X]^5, \left( \left( -\frac{3\,k^2\,U}{3+H^2\,k^2} - \frac{3\,k^4\,w}{2\,(3+H^2\,k^2)} \right) dt^2 - \frac{i\,k^4\,u^2\,d^3}{2\,(3+H^2\,k^2)} + \frac{k^4\,u^3\,d^4}{8\,(3+H^2\,k^2)} + \operatorname{O}[\,d\,I]^5 \right) dx^3 + \\ &\left( -\frac{i\,(22\,k^2+5\,H^2\,k^2)\,d}{3\,(3+H^2\,k^2)} + \frac{3\,(2\,H^2\,k^2\,d^3\,u^2\,k^2\,U+2)}{4\,(3+H^2\,k^2)} + \operatorname{O}[\,d\,I]^5 \right) dx^3 + \\ &\left( \frac{i\,(22\,k^2+5\,H^2\,k^2)\,d^2}{4\,(3+H^2\,k^2)} + \frac{i\,(295\,k^2\,U+830\,H^2\,k^2\,U+350\,H^2\,k^2\,U+150\,H^2\,u^2\,U^2}{8\,(3+H^2\,k^2)} + \operatorname{O}[\,d\,I]^5 \right) dx^3 + \\ &\left( \frac{i\,(22\,k^2+5\,H^2\,k^2)\,d^2}{4\,(3+H^2\,k^2)} + \frac{i\,(295\,k^2\,U+830\,H^2\,k^2\,U+150\,H^2\,u^2\,U^2}{8\,(3+H^2\,k^2)} + \operatorname{O}[\,d\,I]^5 \right) dx^3 + \\ &\left( \frac{i\,(29\,H^2\,k^2+40\,H^2\,k^2\,U+3)\,d^2}{4\,(3+H^2\,k^2)} + \frac{i\,(295\,H^2\,k^2\,U+350\,H^2\,k^2\,U+150\,H^2\,u^2\,U^2}{8\,(3+H^2\,k^2)} + \frac{i\,(295\,H^2\,k^2\,U+35\,H^2\,k^2\,U^2}{8\,(3+H^2\,k^2)} + \operatorname{O}[\,d\,I]^5 \right) dx^3 + \\ &\left( -\frac{i\,(20\,k^2\,H^2\,k^2)\,U+3}{4\,(30\,(3+H^2\,k^2)^2} \right) dt^2 + \frac{i\,(3\,g\,H^2\,u^2\,U+340\,H^2\,k^2\,U^2\,U+340\,H^2\,k^2\,U^2}{8\,(3+H^2\,k^2)} + \frac{i\,(24\,H^2\,k^2\,U^2\,U+340\,H^2\,u^2\,U^2}{12\,(3+H^2\,k^2)} + \operatorname{O}[\,d\,I]^5 \right) dx^3 + \\ &\left( -\frac{i\,(30\,g\,H^2\,k^2\,U+36\,H^2\,k^2\,U-340\,H^2\,k^2\,U-15\,H^2\,k^2\,U-15\,H^2\,k^2\,U^2}{12\,(3+H^2\,k^2)^2} + \frac{i\,(4\,H^2\,k^2\,U-140\,H^2\,k^2\,U-140\,H^2\,k^2\,U^2}{12\,(3+H^2\,k^2)^2} + \operatorname{O}[\,d\,I]^5 \right) dx^3 + \\ &\left( -\frac{i\,$$

Out[144]= Eerr || \left(

\begin{array}{cc}

 $\left( \frac{-H^2 U^2 k^4 - H^2 U w k^3 + 3 U^2 k^2 - 3 g H k^2 + 3 U w^3 \frac{12 u w^3 u w^3 \frac{12 u w^3 \frac{12 u w^3 u w^3 \frac{12 u w^3 u w^3$ 

```
k^2_{H^2} k^2+3-\frac{3 w k}{2 \left(H^2 k^2+3\right)\right)\right) \times (h^2 k^2+3)} 
                                                                                                                                                       \label{left} $$\left(5\ H^2\ k^5+12\ k^3\right) \left(\frac{40\ \left(H^2\ k^2+3\right)}{2}-\frac{3\ \left(\frac{40\ H^2\ k^2+3\right)}{2}-\frac{3\ h^2\ k^2+3}{2}-\frac{3\ h^2
                                                                                                                                                       k^6+14 k^4\right) U\right) U\right) \text{dt}^2{40 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^5\right)\right)
                                                                                                                                                       \text{text}\{dx\}^3 + \text{left}(\frac{770 \text{ H}^4 \text{ k}^9 + 4410 \text{ H}^2 \text{ k}^7 + 6291 \text{ k}^5} \right) \text{ text}\{dt\} \{4800 \text{ left}(\text{H}^2 \text{ k}^7 + 6291 \text
                                                                                                                                                       k^2+3\right)^3}+\frac{\left(1500 H^4 U k^{10}+8820 H^2 U k^8+12951 U k^6\right) \text{dt}^2}{4800}
                                                                                                                                                       \label{left(H^2 k^2+3\right)^3} + O\left(\frac{dt}^5\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^5\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^5\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^4\right) \cdot \left(\frac{dx}^4\right) \cdot \left(\frac{dx}^4 + O\left(\frac{dx}^4\right) \cdot \left(\frac{dx}^4\right) \cdot \left(
                                                                                      \left(-\frac{k^2 H^3+3 g H^3 U^2\right)}{(2 k U+w)\right) + \left(\frac{k^2 H^3+3 g H^3 U^2\right)}{(2 k U+w)\right) + \left(\frac{k^2 H^3}{(2 k^2 H^3+3 g H^3 U^2)}\right) + \left(\frac{k^2 H^3}{(2 k^2 H^3 H^3 U^2)}
                                                                                                                                                       k \left( \frac{k^2 H^3+3 g H-3 U^2\right) w^2 \left( \frac{k^2 H^3+3}{6 \left( \frac{k^2 H^3+
                                                                                                                                                       g H-3 U^2\right) + O(\frac{1}{2} + \frac{1}{24} \left(\frac{H^2 k^2+3\right)}+O(\frac{1}{2} \left(\frac{H^2 k^2
                                                                                                                                                       \left(10 g H^5 k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}\{120
                                                                                                                                                       \left(H^2 k^2+3\right)^2}+\frac{\left(-20 g H^5 U k^8+45 H^2 U^3 k^6-120 g H^3 U k^6+126 U^3
                                                                                                                                                       k^4-180 \text{ g H U } k^4 \text{ (ight) } \text{ (i
                                                                                                                                                       \label{eq:left-def} $$ \left( \frac{1}{8} \left( \frac{1}{8} \right) \left( \frac{dt}{+ \frac{dt}{+ \frac{dt}{4}}} \right) \right) $$
                                                                                                                                                       \operatorname{H} k^7+3 g \operatorname{H} \operatorname{Sqrt}(g H) k^5 \right) \times (t k^2+3 \right) + O\left(t k^7+3 g \operatorname{H} k^5 \right) \cdot (t k^2+3 \right) + O\left(t k^7+3 g \operatorname{H} k^7+3 g
                                                                                                                                                       \text{text}_{dx}^3 + \text{left}_{frac}_{i} \left[ \frac{60 \text{ g H}^7 \text{ k}^1}{11} + 2340 \text{ g H}^5 \text{ k}^9 - 770 \text{ H}^4 \text{ U}^2 \text{ k}^9 + 7020 \text{ g H}^3 \text{ k}^7 - 4410 \right]
                                                                                                                                                       H^2 U^2 k^7-6291 U^2 k^5+7020 g H k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\\left(1460 g)}
                                                                                                                                                       H^7 U k^{12}-4500 H^4 U^3 k^{10}+13140 g H^5 U k^{10}-26460 H^2 U^3 k^8+39420 g H^3 U k^8-38853
                                                                                                                                                       U^3 k^6 + 39420 g H U k^6 \left( \frac{dt}^2}{14400 \left( \frac{k^2 + 3 \right)}{14400 \left( \frac{k^2 + 3 \right)}{14400
                                                                                                                                                       \t (dx)^4 + O(eft(\text{kx}_{dx})^5) \cdot (eft(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hrac}(\text{hr
                                                                                                                                                       H k^2 - 6 U w k \left( \frac{h^2 k^2 + 3 \right) - \left( k \left( \frac{h^2 k^2 + 3 \right) - \left( \frac{h^2 k^2 + 6 \right) U w^2}{h^2 k^2 + 6 \right)} 
                                                                                                                                                       \label{left} $$ \left(\frac{dt}^3}{6\left(\frac{4t}^2 k^2+3\right)}+\frac{k \left(\frac{4t}^2 k^2+6\right) U w^3 \left(\frac{4t}^4\right)^2}{4\left(\frac{4t}^2 k^2+6\right) U w^3 \left(\frac{4t}^4\right)^2} \right) $$
                                                                                                                                                       k^2+3\right)+O\left(\frac{10 H^4 k^7+75 H^2 k^5+126 k^3\right)}{U}
                                                                                                                                                       \text{text}(dt){120 \left(H^2 k^2+3\right)^2}-\frac{\left(20 H^4 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^2 U^2 k^6+432 H^3 k^6+210 H^2 U^2 k^6+432 H^3 L^4 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 k^6+210 H^2 U^2 k^6+432 H^3 U^2 k^8+45 g H^3 U^
                                                                                                                                                       U^2 k^4+126 g H k^4 \right) \left( \frac{4t}^2}{240 \left( \frac{4^2}{2} + \frac{4^2+3\right)}{2} + O\left( \frac{4t}^5 \right) \right) \right)
                                                                                                                                                       \t \{dx\}^2 + \left(-\frac{1}{8} \left(-\frac{1}{8} \right) + \frac{4 \cdot 1}{1} \right) + \frac{1}{8} \left(-\frac{1}{8} \left(-\frac{1}{8} \right) + \frac{1}{8} \right) + \frac{1}{8} \left(-\frac{1}{8} \left(-\frac{1}{8} \left(-\frac{1}{8} \right) + \frac{1}{8} \left(-\frac{1}{8} \left(-\frac{1}{8} \left(-\frac{1}{8} \right) + \frac{1}{8} \left(-\frac{1}{8} \left(-\frac{1
                                                                                                                                                       k^2+15\right) U \text{ } U \text{ } t \text{ } t^2} 16 \text{ } \left( H^2 k^2+3\right) + O\left( t \text{ } t^3\right) + O\left( t \text{ } t^3\right) \right) \text{ } t \text{ } t^3+\left( t^3\right) + O\left( t \text{ } t^3\right) + O\left( t \text{ } t^3\right) \right) 
                                                                                                                                                       \left(260 H^6 U k^{11}+3110 H^4 U k^9+11430 H^2 U k^7+13311 U k^5\right) \text{dt}}{4800}
                                                                                                                                                       \left(H^2 k^2+3\right)^3}+\frac{\left(1460 H^6 U^2 k^{12}+4500 g H^5 k^{10}+22140 H^4 U^2
                                                                                                                                                       k^{10}+26460 g H^3 k^8+92340 H^2 U^2 k^8+117126 U^2 k^6+38853 g H k^6\right) \text{dt}^2\{28800
                                                                                                                                                     \left(\frac{4+0\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac{4x}{4}\right)^{3}+O\left(\frac
                                                                                \end{array}
                                                                                \right)
ln[145] = KurF = (fm*ap - fp*am + am*ap*(qp - qm)) / (ap - am);
                                                                                KurFWS = KurF /. ap \rightarrow (U + Sqrt[g * H]) /. am \rightarrow 0;
                                                                                KurFWSeta =
                                                                                                                     \texttt{KurFWS} \ /. \ \texttt{fp} \ \rightarrow \ (\texttt{H} * \texttt{v} \ + \ \texttt{U} * \texttt{Rpp} * \texttt{n}) \ /. \ \texttt{fm} \ \rightarrow \ (\texttt{H} * \texttt{v} \ + \ \texttt{U} * \texttt{Rmp} * \texttt{n}) \ /. \ \texttt{qp} \ \rightarrow \ \texttt{Rpp} * \texttt{n} \ /.
                                                                                                                                         qm \rightarrow Rmp * n;
                                                                                KurFWSeta = KurFWSeta / . v \rightarrow (GGp * G + Gnp * n);
                                                                                  Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1];  
                                                                                KfnGp = FullSimplify[KurFWSeta /. n \rightarrow 0 /. G \rightarrow 1];
```

```
Kfnn = Kfnnp / Rpp \rightarrow Rp / Rmp \rightarrow Rm / GGp \rightarrow GG2 / Gnp \rightarrow Gn2;
KfnG = KfnGp / Rpp \rightarrow Rp / Rmp \rightarrow Rm / GGp \rightarrow GG2 / Gnp \rightarrow Gn2;
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA , \{k > 0, U > 0, H > 0, g > 0\}];
KurFWSG = KurFWS / . fp \rightarrow (U * Rpp * G + U * H * v + g * H * Rpp * n) / .
       fm \rightarrow (U*Rmp*G+U*H*v+g*H*Rmp*n)/.qp \rightarrow Rpp*G/.qm \rightarrow Rmp*G;
KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
KfGnp = FullSimplify[KurFWSG /. G \rightarrow 0 /. n \rightarrow 1];
\texttt{KfGn} \; = \; \texttt{KfGnp} \; / \; . \; \; \texttt{Rpp} \; \rightarrow \; \texttt{Rp} \; \; / \; . \; \; \; \texttt{Rmp} \; \rightarrow \; \texttt{Rm} \; \; / \; . \; \; \; \; \texttt{GGp} \; \rightarrow \; \texttt{GG2} \; \; / \; . \; \; \; \; \texttt{Gnp} \; \rightarrow \; \texttt{Gn2} \; ;
KfGG = KfGGp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g*H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
RKStep = Log[1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" U < -Sqrt(gH)"}]]</pre>
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
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Text[Row[{"FnG error || ", FnG2TAr}]]

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Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
                                                    Text[" "]
                                                     Text[Row[{"FGn ||
                                                                                                                                                                                                                                                        ", KfGnp}]]
                                                     Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
                                                     Text[Row[{"FGn error ||
                                                                                                                                                                                                                                                                                                                   ", FGn2TAr}]]
                                                     Text[Row[{"FGn error ||
                                                                                                                                                                                                                                                                                                                 ", TeXForm[FGn2TAr]}]]
                                                     Text[" "]
                                                    Text[Row[{"FGG || ", KfGGp}]]
                                                     Text[Row[{"FGG ||
                                                                                                                                                                                                                                                        ", TeXForm[KfGGp]}]]
                                                     Text[Row[{"FGG error ||
                                                                                                                                                                                                                                                                                                                   ", FGG2TAr}]]
                                                    Text[Row[{"FGG error ||
                                                                                                                                                                                                                                                                                                                 ", TeXForm[FGG2TAr]}]]
                                                     Text[" "]
                                                     Text[" "]
                                                    Text[Row[{"Omega error ||
                                                                                                                                                                                                                                                                                                                   ", RKstepTayr}]]
                                                     Text[Row[{"Omega error ||
                                                                                                                                                                                                                                                                                                                     ", TeXForm[RKstepTayr]}]]
                                                     Text[" "]
                                                    Text[Row[{"EA ||
                                                                                                                                                                                                                                      ", EA}]]
                                                     Text[Row[{"EA || ", TeXForm[EA]}]]
                                                     Text[Row[{"Eerr || ", Eerr}]]
                                                    Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
 Out[179]= U < -Sqrt(gH)
 Out[180]=
 Out[181]= Fnn || Gnp H + Rmp U
 Out[182]= Fnn || \text{text}\{Gnp\}\ H+\text{text}\{Rmp\}\ U
 \begin{array}{ll} \text{Out} \text{[183]=} & Fnn \; error \; || \; \left( -\frac{\left( H^2 \, k^3 \, U \, w \right) \, dt^2}{2 \, \left( 3 + H^2 \, k^2 \right)} - \frac{i \, H^2 \, k^3 \, U \, w^2 \, dt^3}{6 \, \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \\ & \left( -\frac{i \, \left( 54 \, k^3 + 45 \, H^2 \, k^5 + 10 \, H^4 \, k^7 \right) U \, dt}{120 \, \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{8} \left( k^4 \, U \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) \\ \end{array} 
 Out[184]= Fnn error |
                                                                 \label{left(-frac{text{dt}^2 \left( h^2 k^3 U w\right)}{2 \left( h^2 k^2 + 3 \right) - \frac{1}{2} \left( h^2 k^3 U w^2 \right)} - \frac{1}{2} \left( h^2 k^3 U w^2 \right)} = \frac{1}{2} \left( h^2 k^3 U w^2 \right) + \frac{1}{2} \left( h^2 k^3 U w^
                                                                                           \label{left} $$\left(H^2 k^2+3\right)+O\left(t(t)^4\right)+C\left(H^2 k^2+3\right)+O\left(H^2 k
                                                                                           k^5 + 54 k^3 \cdot U \cdot t + (dt)^{120} \cdot (H^2 k^2 + 3 \cdot t)^2 + O \cdot (t \cdot t \cdot (dt)^4 \cdot t) + (t \cdot (dt)^4 \cdot t) + (t \cdot t \cdot
                                                                                           \left(-\frac{1}{8}\right)\left(\frac{4 \operatorname{Uright}}{\operatorname{tht}}+O\left(\frac{4}{4}\right)\right)\right)
 Out[185]=
Out[186]= FnG || GGp H
 Out[187]= FnG \parallel \text{text}\{GGp\} H
 \text{Out} [\text{188}] = FnG \ error \ || \ \left( -\frac{3 \ (k \ w) \ dt^2}{2 \left(3 + H^2 \ k^2\right)} - \frac{i \ k \ w^2 \ dt^3}{2 \left(3 + H^2 \ k^2\right)} + O[dt]^4 \right) + \left( -\frac{i \left(12 \ k^3 + 5 \ H^2 \ k^5\right) dt}{40 \left(3 + H^2 \ k^2\right)^2} + O[dt]^4 \right) dx^2 + O[dx]^4
```

 $\label{left-frac} $$ \int_{\infty} FnG error \| \left( -\frac{3 \left( w \right)}{2 \left( w \right)} \right) -\frac{1}{2} \left( w \right) -\frac{1}{2} \left( w \right) \right) -\frac{1}{2} \left( w \right) + \frac{1}{2} \left( w \right) -\frac{1}{2} \left( w \right) \right) -\frac{1}{2} \left( w \right) -\frac{1}{2} \left( w \right) -\frac{1}{2} \left( w \right) + \frac{1}{2} \left($  $k^2+3\right)+O\left(\frac{dt}^4\right)\right)+\left(\frac{dt}^4\right)$ 

Out[190]=

Out[191]=  $FGn \parallel H(gRmp + Gnp U)$ 

Out[192]=  $FGn \parallel H(g \text{text}\{Rmp\}+\text{text}\{Gnp\} U)$ 

$$\begin{array}{ll} \text{Out} \text{[193]=} & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w \right) \, dt^2}{2 \left( 3 + H^2 \, k^2 \right)} - \frac{i \, k \left( 3 \, g \, H + g \, H^3 \, k^2 - 3 \, U^2 \right) w^2 \, dt^3}{6 \left( 3 + H^2 \, k^2 \right)} + O[dt]^4 \right) + \\ & \left( -\frac{i \left( 90 \, g \, H \, k^3 + 60 \, g \, H^3 \, k^5 + 10 \, g \, H^5 \, k^7 - 36 \, k^3 \, U^2 - 15 \, H^2 \, k^5 \, U^2 \right) dt}{120 \left( 3 + H^2 \, k^2 \right)^2} + O[dt]^4 \right) dx^2 + \left( -\frac{1}{8} \left( g \, H \, k^4 \right) dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) dx^4 + O[dt]^4 \right) dx^4 + O[dt]^4 +$$

Out[194]= FGn error ||

 $\left(-\frac{t}{4t}^2 \left( \frac{t}^2 \right)^2 \right)^{2} \left( \frac{t}^2 \left( \frac{t}^2 \right)^2 \left($  $\text{text}\{dt\}^3 \text{ k w}^2 \left\{ H^3 \text{ k}^2 + 3 \text{ g H} - 3 \text{ U}^2\right\}$  $k^2+3\right)+O\left(\frac{dt}^4\right)+\int \left(\frac{dt}^4\right)+\int \left(\frac{dt}^4\right)+\int \left(\frac{dt}^2\right)+\int \left(\frac{dt}^4\right)+\int \left(\frac{d$ k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}}{120

 $\label{left(H^2 k^2+3\left|right()^2\right|+O\left(text{dt}^4\right)-text{dx}^3 \left(-\frac{1}{8}\right)} $$ \left(-\frac{1}{8}\right) $$ (A) $$ (A)$  $\left( H k^4\right) \left( H k^4\right) \left( \frac{dt}{dt} + O\left( \frac{dt}{dt} \right) \right) + O\left( \frac{dt}{dt} \right) + O\left( \frac{dt}$ 

Out[195]=

Out[196]=  $FGG \parallel (GGp H + Rmp) U$ 

 $\label{eq:outsign} \mbox{Outsign} \mbox{ outsign} = FGG \ || \ U \ (\text{text}\{GGp\} \ H + \text{text}\{Rmp\})$ 

$$\begin{array}{ll} \text{Out[198]=} & FGG \; error \; \parallel \; \left( -\frac{\left( k \left( 6+H^2 \; k^2 \right) U \; w \right) \; dt^2}{2 \left( 3+H^2 \; k^2 \right)} - \frac{i \; k \left( 6+H^2 \; k^2 \right) U \; w^2 \; dt^3}{6 \left( 3+H^2 \; k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( 126 \; k^3+75 \; H^2 \; k^5+10 \; H^4 \; k^7 \right) U \; dt}{120 \left( 3+H^2 \; k^2 \right)^2} + O[dt]^4 \right) dx^2 + \\ & \left( -\frac{1}{8} \left( k^4 \; U \right) dt + O[dt]^4 \right) dx^3 + \left( \frac{i \left( 13311 \; k^5+11430 \; H^2 \; k^7+3110 \; H^4 \; k^9+260 \; H^6 \; k^{11} \right) U \; dt}{4800 \left( 3+H^2 \; k^2 \right)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{array}$$

Out[199]= FGG error ||

 $\label{left} $\left(-\frac{t_4}{k^2 + \theta\right)^2 \left(H^2 k^2 + \theta\right)}{2 \left(H^2 k^2 + \theta\right)^2 \left(H^2 k^2$  $U w^2 \left(\frac{H^2 k^2+6\right)}{6\left(\frac{H^2 k^2+3\right)}+O\left(\frac{H^2 k^2+3\right)}{4\left(\frac{H^2 k^2+6\right)}}+\frac{1}{4\left(\frac{H^2 k^2+6\right)}}$ \left(-\frac{i \left(10 H^4 k^7+75 H^2 k^5+126 k^3\right) U \text{dt}}{120 \left(H^2  $k^2+3\left(\frac{dt}^4\right)+\left(\frac{dt}^4\right)$  $U \rightarrow \frac{dt}{-4 \cdot dt} - \frac{dt}{-4 \cdot dt} -$ H^6 k^{11}+3110 H^4 k^9+11430 H^2 k^7+13311 k^5\right) U \text{dt}}{4800  $\left(H^2 k^2+3\right)^3+O\left(\left(text\left(dt\right)^4\right)\right)+O\left(text\left(dx\right)^5\right)$ 

Out[200]=

Out[201]=

Out[202]= Omega error ||

$$\left\{ \left[ \frac{1}{6\left(3+H^{2}\,k^{2}\right)^{2}}k^{3}\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right. + \left(3+H^{2}\,k^{2}\right)U\right) \left(3\,g\,H + U\left(2\,\sqrt{3}\,\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right. + \left(3+H^{2}\,k^{2}\right)U\right)\right)dt^{2} + \left(3+H^{2}\,k^{2}\right)U\right] + \left(3+H^{2}\,k^{2}\right)U +$$

$$\frac{ik! \left[3 g \Pi : U \left[2 \sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} \cdot (3 + \Pi^2 k^2) U\right]\right]^{\frac{3}{2}} e^{t}}{8 (3 + \Pi^2 k^2)} - \frac{1}{20 (3 + \Pi^2 k^2)!} \left(k^5 \left(\sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} + (3 + \Pi^2 k^2) U\right)\right)^3} \\ \left(3 g \Pi + U \left(2 \sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} + (3 + \Pi^2 k^2) U\right)\right) dt^4 + O[dt]^5\right) + \\ \left(\frac{1}{200 (3 + \Pi^2 k^2)!} k^3 \left(42 \sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} + 180 U + 20 \Pi^4 k^4 U + 15 k^2 \left(\sqrt{3} \sqrt{g \Pi^5 (3 + \Pi^2 k^2)} + 8 \Pi^2 U\right)\right) + \\ \frac{1}{480 (3 + \Pi^2 k^2)!} k^5 \left(20 \Pi^6 k^6 U^3 + 54 U^2 \left(9 \sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} + 10 U\right) + \\ 6 g \Pi \left(21 \sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} + (216 + 147 \Pi^2 k^2 + 25 \Pi^4 k^4) U\right) + \\ 3 k^2 \left(15 \sqrt{3} \sqrt{g^3 \Pi^7 (3 + \Pi^2 k^2)} + 109 \sqrt{3} \sqrt{g \Pi^5 (3 + \Pi^2 k^2)} U^2 + 180 \Pi^2 U^3\right)\right) dt^2 + \\ \frac{1}{480 (3 + \Pi^2 k^2)!} i^4 k^6 \left(3 g \Pi + U \left(2 \sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} + (3 + \Pi^2 k^2) U\right)\right) \\ \left(9 g \Pi \left(14 + 5 \Pi^2 k^2\right) + U \left(102 \sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} + 180 U + 20 \Pi^4 k^4 U + \\ 5 k^2 \left(7 \sqrt{3} \sqrt{g \Pi^3 (3 + \Pi^2 k^2)} + 180 U + 20 \Pi^4 k^4 U + 15 k^2 \left(\sqrt{3} \sqrt{g \Pi^5 (3 + \Pi^2 k^2)} + 8 \Pi^2 U\right)\right) \\ \left(3 g \Pi + U \left(2 \sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} + 180 U + 20 \Pi^4 k^4 U + 15 k^2 \left(\sqrt{3} \sqrt{g \Pi^5 (3 + \Pi^2 k^2)} + 8 \Pi^2 U\right)\right) \\ \left(3 g \Pi + U \left(3 \sqrt{3} \sqrt{g \Pi (3 + \Pi^2 k^2)} + 18 U + 2 \Pi^4 k^4 U + k^2 \left(\sqrt{3} \sqrt{g \Pi^3 (3 + \Pi^2 k^2)} + 12 \Pi^2 U\right)\right)\right) dt^2 + \\ \frac{1}{22 (3 + \Pi^2 k^2)^2} k^2 \left(3 g \Pi (3 + \Pi^2 k^2) + 18 U + 2 \Pi^4 k^4 U + k^2 \left(\sqrt{3} \sqrt{g \Pi^3 (3 + \Pi^2 k^2)} + 12 \Pi^2 U\right)\right)\right) dt^2 + \\ \frac{1}{22 (3 + \Pi^2 k^2)^2} k^2 \left(3 g \Pi (3 + \Pi^2 k^2) + 18 U + 2 \Pi^4 k^4 U + k^2 \left(\sqrt{3} \sqrt{g \Pi^3 (3 + \Pi^2 k^2)} + 12 \Pi^2 U\right)\right)\right) dt^2 + \\ \frac{1}{24 (3 + \Pi^2 k^2)^2} k^2 \left(3 g \Pi (3 + \Pi^2 k^2) + 18 U + 2 \Pi^4 k^4 U + k^2 \left(\sqrt{3} \sqrt{g \Pi^3 (3 + \Pi^2 k^2)} + 12 \Pi^2 U\right)\right)\right) dt^2 + \\ \frac{1}{44 (3 + \Pi^2 k^2)^2} k^2 \left(3 g \Pi (3 + \Pi^2 k^2) + 18 U + 2 \Pi^4 k^4 U + k^2 \left(\sqrt{3} \sqrt{g \Pi^3 (3 + \Pi^2 k^2)} + 12 \Pi^2 U\right)\right)\right) dt^2 + \\ \frac{1}{44 (3 + \Pi^2 k^2)^2} k^2 \left(3 g \Pi (3 + \Pi^2 k^2) + 18 U + 2 \Pi^4 k^4 U + k^2 \left(\sqrt{3} \sqrt{g \Pi^3 (3 + \Pi^2 k^2)} + 12 \Pi^2 U\right)\right)\right) dt^2 + \\ \frac{1}{44 (3 + \Pi^2 k^2)^2} k^2 \left(3 g \Pi (3 + \Pi^2 k^2) + 18 \Pi^2 U + 18$$

ι

$$\begin{split} & 15\,k^4\left(3408\,\sqrt{g}\,H^9\left(3+H^2\,k^2\right) + 9985\,\sqrt{3}\,H^3\,U\right) + \\ & 80\left(1836\,\sqrt{g}\,H^3(3+H^2\,k^2)\,U^3 + 612\,k^4\,\sqrt{g}\,H^9\left(3+H^2\,k^2\right)\,U^3 + 68\,k^6\,\sqrt{g}\,H^{13}\left(3+H^2\,k^2\right)}\right. \\ & U^3 + 9\,k^2\left(421\,\sqrt{g^3\,H^2\left(3+H^2\,k^2\right)}\,U + 204\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right)\,U^3\right) \right) \right) d^2\right) \Big/ \\ & \left(230400\left(\sqrt{g}\,H\right)\left(3+H^2\,k^2\right)^{7/2}\right) - \frac{1}{25600(3+H^2\,k^2)^2}\,i\,k^8\left(6\,g^2\,H^2\left(8046 + 5460\,H^2\,k^2 + 925\,H^4\,k^4\right) + 560\,H^8\,k^8\,U^4 + 4322\,U^3\left(143\,\sqrt{3}\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) + 105\,U\right) + 5\,k^4\,U^3\left(4139\,\sqrt{3}\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) + 6048\,H^4\,U\right) + \\ & 5\,k^4\,U^3\left(461\,\sqrt{3}\,\sqrt{g}\,H^{13}\left(3+H^2\,k^2\right) + 282\,885\,U + 284\,364\,H^2\,k^2\,U^3 + 5040\,H^2\,U^4\right) + \\ & g\,H\,U\left(63\,720\,\sqrt{3}\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) + 282\,885\,U + 284\,364\,H^2\,k^2\,U + 10\,640\,H^6\,k^6\,U + \\ & 5\,k^4\left(1451\,\sqrt{3}\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) + 19\,056\,H^4\,U\right)\right) d^3 + \frac{1}{460\,800\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right)}\,U\right) + \\ & 108\,k^2\left(2625\,\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)} + 15\,327\,\sqrt{3}\,g^2\,H^4\,U + 22\,383\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2 + \\ & 19\,261\,\sqrt{3}\,g\,H^3\,U^2 + 4640\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) - 19\,05\,H^4\,U\right) + 15\,k^6\,U\left(4143\,\sqrt{3}\,g^2\,H^8 + 3712\,\sqrt{g}\,H^{13}\,(3+H^2\,k^2) + 1268\,\sqrt{3}\,g^2\,H^2\,U + 290\,\sqrt{g}\,H\left(3+H^2\,k^2\right)\,U^4 + \\ & g\,H\,U^2\left(1857\,\sqrt{g}\,H\left(3+H^2\,k^2\right) + 1268\,\sqrt{3}\,g^2\,H^2\,U + 290\,\sqrt{g}\,H\left(3+H^2\,k^2\right)\,U^4 + \\ & g\,H^3\left(33\,H^2\,k^2\right) + 29\,H^3\left(3+H^2\,k^2\right) + 115\,737\,\sqrt{3}\,U\right)\right) + 15\,k^6\,U\left(4143\,\sqrt{3}\,g^2\,H^8 + 3712\,\sqrt{g}\,H^{13}\,(3+H^2\,k^2\right) + 61\,735\,\sqrt{3}\,g^2\,H^6\,U + 27\,840\,\sqrt{g}\,H^9\left(3+H^2\,k^2\right)\,U^4 + \\ & g\,H^5\,U^2\left(89\,915\,\sqrt{g}\,H\left(3+H^2\,k^2\right) + 115\,737\,\sqrt{3}\,U\right)\right)\right) d^4 + O(dH^3)\,dx^4 + O(dx)^5, \\ d^2 + \\ \frac{i^4}{6(3+H^2\,k^2)^2}\,g^3\,H^{13}\,(3+H^2\,k^2) + (3+H^2\,k^2)\,U\right)^3\left(3\,g\,H + U\left(-2\,\sqrt{3}\,\sqrt{g}\,H\left(3+H^2\,k^2\right) + (3+H^2\,k^2)\,U\right)\right)\right) \\ d^4 + O(dt)^3 + H^3\,k^2 + H^3\,k$$

$$\begin{split} & \left(\frac{1}{246[3+H^2k^2]^2}k^2\left(-42\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + 180\,U + 20\,H^4\,k^4\,U - 15\,k^2\left(\sqrt{3}\sqrt{g\,H^3(3+H^2\,k^2)} - 8\,H^2\,U\right)\right) + \\ & \frac{1}{480[3+H^2k^2]^2}k^2\left(20\,H^6\,k^6\,U^3 + 54\,U^2\left(-9\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + 10\,U\right) + \\ & 5\,k^4\,U^2\left(-11\sqrt{3}\sqrt{g\,H^3(3+H^2\,k^2)} + 26\,H^4\,U\right) + \\ & 6\,g\,H\left(-21\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + 216+147\,H^2\,k^2 + 25\,H^4\,k^4\right)U\right) - \\ & 3\,k^2\left(15\sqrt{3}\sqrt{g^2\,H^3(3+H^2\,k^2)} + 109\sqrt{3}\sqrt{g\,H^3(3+H^2\,k^2)} + 120\,H^2\,k^3\right)\right)dt^2 + \\ & \frac{1}{480[3+H^2k^2]^4}k^6\left(3\,g\,H + U\left(-2\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\right)\right) \\ & \left(9\,g\,H\left(14+5\,H^2\,k^2\right) + U\left(-102\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + 180\,U + 20\,H^4\,k^4\,U - 5\,k^2\left(7\sqrt{3}\sqrt{g\,H^3(3+H^2\,k^2)} - 8\,H^2\,U\right)\right)\right)dt^2 - \\ & \left(k^2\left(-42\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + 180\,U + 20\,H^4\,k^4\,U - 15\,k^2\left(\sqrt{3}\sqrt{g\,H^3(3+H^2\,k^2)} - 8\,H^2\,U\right)\right) \\ & \left(3\,g\,H + U\left(-2\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + (3+H^2\,k^2)\,U\right)\right)^2\right)dt^4 + O[dt]^5\right)dx^2 + \\ & \left(\frac{1}{16}\,k^4\left(\sqrt{3}\sqrt{\frac{g\,H^3(3+H^2\,k^2)}{3+H^2\,k^2}} - 2\,U\right) - \frac{1}{32(3+H^2\,k^2)}k^6\left(-3\,g\,H\left(\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} - 4\left(3+H^2\,k^2\right)\,U\right)\right) + \\ & U^2\left(-15\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + 18\,U + 2\,H^4\,k^4\,U + k^2\left(-5\sqrt{3}\sqrt{g\,H^3(3+H^2\,k^2)} + 12\,H^2\,U\right)\right)\right) \\ & dt^2 + \frac{1}{32(3+H^2\,k^2)}k^2\left(3\,g\,H + U\left(-2\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + 2\left(3+H^2\,k^2\right)\,U\right)\right)dt^3 + \\ & \frac{1}{64(3+H^2\,k^2)}k^2\left(-\sqrt{3}\sqrt{g\,H(3+H^2\,k^2)} + 2\left(3+H^2\,k^2\right)U\right)\right)dt^3 + \\ & \frac{1}{64(3+H^2\,k^2)}k^2\left(-\sqrt{3}\sqrt{g\,H(3+H^$$

## Out[203]= Omega error ||

 $\left(\frac{K^3 \left(\frac{K^2 + 3\right)}{H^2 + 1}}{U+\sqrt{K^2 + 1}}\right) U + \sqrt{K^2 + 1}}\right)$  $\left(\left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3}}\right) + \left(\frac{H^2 k^2+3\right)}{\left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3}}\right)$  $\label{left(H^2 k^2+3\left(H^2 k^2+3\right) U+2 sqrt{3} sqrt{g H U \left(H^2 k^2+3\right) U+2 sqrt{3} sqrt{g H U (H^2 k^2+3\right)}}$  $\label{left} $\left(H^2 \times^2 + 3\right)\right]/r(ght)\right]/r(ght)^2 \left(dt\right)^3 \left(H^2 \times^2 + 3\right)/r(ght)^2 - \frac{1}{2} \left(H^2 \times^2 + 3\right)/r(ght)^2 \right). $\left(H^2 \times^2 + 3\right)/r(ght)^2 - \frac{1}{2} \left(H^2 \times^2 + 3\right)/r(ght)^2 \right). $\left(H^2 \times^2 + 3\right)/r(ght)^2 - \frac{1}{2} \left(H^2 \times^2 + 3\right)/r(ght)^2 \right). $\left(H^2 \times^2 + 3\right)/r(ght)^2 - \frac{1}{2} \left(H^2 \times^2 + 3\right)/r(ght)^2 \right). $\left(H^2 \times^2 + 3\right)/r(ght)^2 - \frac{1}{2} \left(H^2 \times^2 + 3\right)/r(ght)^2 + \frac{1}{2} \left(H^2 \times^2$  $k^2+3\right) U+\sqrt{3} \sqrt{2+3}\right) U+\sqrt{3} \ln(4^2 k^2+3\right)$  $k^2+3\left( U+2 \right) +2 \left( U+2 \right)$  $k^2+3\right/h^4+O\left(\frac{dt}^5\right)h^4+O$  $\left(H^2 k^2+3\right)^2+\frac{h^2 u^2}{h^2 u^2} + \frac{h^2 u^2 \left(H^2 u^2 + \frac{h^2 u^2}{h^2 u^2}\right)}{h^2 u^2 u^2} + \frac{h^2 u^2 u^2}{h^2 u^2} + \frac{h^2 u^2 u^2}{h^2 u^2} + \frac{h^2 u^2}{h^2 u^2} + \frac{$  $\left(\frac{H^2 k^2+3\right}{k^2+3\right} k^4+3\left(\frac{180 H^2 U^3+109 \sqrt{3}}{k^2+3}\right)$  $U^2+15 \sqrt{3} \sqrt{10} U^2 + 15 \sqrt{3} \sqrt{10} U^2 \left( U^2 \left( U^2 \right) \right)$ H \left(H^2 k^2+3\right)\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U+21 \sqrt{3}  $\left(\frac{H^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)}\right)$  $g \ H+U \left( \left( H^2 \ k^2+3\right) \right) \ U+2 \ \left( H^2 \ k^2+3\right) \right) \ \left( H^2 \ k^2+3\right) \ \left( H^2$ H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4+5 \left(24 U H^2+7 \sqrt{3} \sqrt{g H^5 \left(H^2  $k^2+3\right) \ k^2+180 \ U+102 \ \sqrt{1} \ k^2+3\right) \ k^2+3\right) \ k^2+3\right) \ k^2+3\ U+102 \ \sqrt{1} \ k^2+3\right) \ k^2+3\left(\frac{1}{2} \ k^2+3\right) \ k^2$  $\left( H^2 k^2 + 3\right) - \left( H^2 k^2$  $k^2+3\left(k^2+3\right)\right) + k^2+180 + 42 \left(180 + 42 \right) + k^2+3\left(180 + 42 \right) +$   $H^5 \left(H^2 k^2+3\right) \ k^2+18 \ U+15 \ yqrt{g} \ H\left(H^2 k^2+3\right) \ U+2+3 \ U+15 \ yqrt{g} \ H\left(H^2 k^2+3\right) \ U+2+3 \ U+15 \ U+2+3 \ U+15 \ U+2+3 \ U+15 \ U+15$  $\label{eq:heft} $$\left(H^2 k^2+3\right)^2+\frac{k^7 \left(g H+U \left(H^2 k^2+3\right) + gH}{2}\right)^2} + \frac{k^7 \left(H^2 k^2 H+U \left(H^2 k^2+3\right) + gH}{2}\right)^2} + \frac{k^7 \left(H^2 k^2 H+U \left(H^2 k^2$  $\left(H^2 k^2+3\right)\right) \left(g H+U \left(3 g H+U \right) \left(H^2 k^2+3\right) U+3 \right)$  $H \left(H^2 k^2+3\right)\right) \left(H^2 k^2+3\right) \left(H^2 k^2+3\right)^2 + \left(H^2 k^2+3\right)^$  $\label{left(H^2 k^2+3\wedge light) U+sqrt{3} sqrt{g H \left(H^2 k^2+3\wedge light)\right) \left(g H+U \left(H^2 k^2+3\right) \right)} left(3 g H+U \left(H^2 k^2+3\right) left(4 g H+U \left(H^2 k^2+3\right) lef$  $k^2+3\right) U+2 \sqrt{3} \sqrt{4}{64 \left(H^2 k^2+3\right)\right)} \left(H^2 k^2+3\right) \left(H^2 k^2+3\right)$  $k^2+3\right)^3+O\left(\frac{t}{5\right)^3}+O\left(\frac{t}{5\right)^3}+O\left(\frac{t}{5}\right)^3+O\left(\frac{t}{5$  $H^4 k^4 + 12180 H^2 k^2 + 17856 \right) + 2080 \left( \frac{H^9 \left( \frac{K^2 + 3\right)}{k^4 + 6} \right) + 2080 \right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^4 + 3\right)$  $H^5 \left(H^2 k^2+3\right) k^2+9 \left(H^2 k^2+3\right) U\right) H^5 \left(H^2 k^2+3\right) U\right) H^5 \left(H^2 k^2+3\right) U\right)$ H} \left( $H^2 k^2+3 \right)^{5/2}\right] - \frac{k^4 k^4+2268}{k^7 \left(k^7 \left(k^7$ H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6+15 \left(9985 \sqrt{3} U H^4+3408 \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+447588 \sqrt{3} H^2 U k^2+648 \left(688 \sqrt{3} U+693  $k^6+612 \sqrt{g H^9 \left(\frac{h^2 k^2+3\right)} U^3 k^4+9 \left(\frac{4 \sqrt{g H^5 \left(\frac{h^2 k^2+3\right)}} U^3 k^4+\frac{h^2 k^2+3 v^2}{h^2 k^2+3 v^2}\right)}$  $U^3+421 \cdot g^3 H^7 \cdot (h^2 k^2+3 \cdot g)$  Uright)  $h^2+1836 \cdot g$  H \left( $h^2 k^2+3 \cdot g$ )  $\label{left} $$U^3\right)\right) \operatorname{left}\left(\frac{H^2 k^2+3\right)}{(30400 \left(\frac{1}{4}\right)^2}{(30400 \left(\frac{1}{4}\right)^2 k^2+3\right)}-\frac{1}{4}\left(\frac{1}{4}\right)^2}{(30400 \left(\frac{1}{4}\right)^2 k^2+3\right)^2}$ \left(560 H^8 U^4 k^8+5 U^3 \left(1344 U H^6+461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 U H^4+4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+12 \left(5040  $H^2 U^4+5161 \sqrt{3} \sqrt{4+5161} \sqrt{3} \sqrt{4+5161}$ k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 H^2 k^2+8046\right)+432 U^3 \right(105 H^2 k^2+80468\right)+432 U^3 \right(105 H^2 k^2+804688\right(105 H^2 k^2+804688\right(105 H^2 k^2+804688\right(105 H^2 k^2+804688\right(105 U+143 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+g H U \left(10640 H^6 U k^6+5 \left(19056 U H^4+1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U+63720  $\label{left(H^2 k^2+3\wedge U+\sqrt{3} \operatorname{H \left(16^2 k^2+3\right)} \left(5 U^3 \left(5 U^3 \left(5 U^3 \left(5 U^3 \right) U+\right) \right) \right) } \\$ g H^9+928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U\right) k^8+15 U \left(4143 \sqrt{3} g^2 H^8+g U  $\left(15454 \right) H^7+3712 \right) H^619 YEAR H^6$  $k^2+3 \right) U^3 \right) U^3 \right) k^6+9 \left( 61735 \right) \left( 3 \right) g^2 U H^6+g U^2 \left( 115737 \right) \left( 115737 \right) H^6+g U^2 \right) U^4 \left( 115737 \right) U$ \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+108 \left(15327 \sqrt{3} g^2 U H^4+19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}  $U^2 + 2625 \sqrt{g^5 H^9 \left( H^2 k^2 + 3\right)} k^2 + 1296 \left( 290 \sqrt{g^2 H^2 k^2 + 3\right) } k^2 + 1296 \left( 290 \sqrt{g^2 H^2 k^2 + 3} k^2 + 3\right) k^2 + 1296 \left( 290 \sqrt{g^2 H^2 k^2 + 3} k^2 +$  $U^4+g H \left(1202 \right) U+1857 \left(14 H^2 k^2+3\right) U+1857 \left(1$  $H^2 U+323 \left( \frac{g^5 H^5 \left( \frac{40800 \left( + \right)} \right)} \right)} {40800 \left( \frac{40800 \left( + \right)} \right)} {40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( + \right)} {40800 \left( \frac{40800 \left( + \right)} {40800 \left( \frac{40800 \left( + \right)} {40800 \left( +$  $\left(\left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \sqrt{3} \right) + \left(\frac{H^2 k^2+3\right)}\right) \left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \left(\frac{H^2 k^2+3\right)}}\right)$  $k^2+3\right) U-2 \sqrt{3} \left( H^2 k^2+3\right) \left$  $k^2+3\right)^2+\frac{1}{2}+\frac{1}{2} +\frac{1}{2} +$  $k^2+3\left(\frac{h^2}{k^2+3\right)}\right) \left(\frac{h^2}{k^2+3\right)} \left(\frac{h^2}{k^2+3}\right) \left($ 

 $k^2+3\left(\frac{4t}^5\right)+\left(\frac{4t}^5\right)+\left(\frac{4t}^5\right)+\frac{4t}{4}$  $k^2+3\right)$ {240 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 H^4 U^5 H^6 U^5  $H^5 \left(H^2 k^2+3\right) U^2+15 \right] V^2+15 \right] V^2+15 \right]$  $\left(10 \text{ U} - 9 \right) \left(10 \text{ U}$  $k^2 + 216 \right) U-21 \left(3 \right) \left(H^2 k^2 + 3\right) \right) \left(H^2 k^2 + 3\right) \left(H^2 k^2 + 3\right)$  $k^2+3\right)^3+\frac{i}{k^6}\left(\frac{1}{2}H+U\right)^3+\frac{1}{2}h^2\left(\frac{1}{2}H+U\right)^2+\frac{1}{2}h^2\left(\frac{1}{2}H+U\right)^2+\frac{1}{2}h^2\left(\frac{1}{2}H+U\right)^2+\frac{1}{2}h^$ k^2+3\right)\right)\right)\left(9 g H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4-5 \left(7 \sqrt{3})  $\label{left} $$ \kappa^2+3\right\right)\right) \operatorname{left}(M^7 \end{0.00} 1.3 + 3\operatorname{left}(M^7 \end{0.00} 1.3 + 3\operatorname{left}$  $k^4-15 \left( \frac{4^2 + 1}{160} \right) -8 H^2 U\right) k^2+180 U-42 \left( \frac{4^2 + 1}{160} \right) -8 H^2 U\right) k^2+180 U-42 \left( \frac{4^2 + 1}{160} \right) + 2 H^2 U\right) k^4-15 H^2 U$  $\left(H^2 k^2+3\right)\right)$  $k^2+3\right)\right)\$  $\t x_d^2+\left(\frac{1}{16} i k^4 \left(\frac{3} \right)^2 + \left(\frac{1}{16} i k^4 \right)^2 k^2 + \frac{1}{16} i k^6$  $\left(U^2 \left(H^4 \cup A^4 + \left(12 H^2 \cup -5 \right)\right)\right) \right)$  $k^2+18 U-15 \sqrt{3} \sqrt{4} H \left(H^2 k^2+3\right)/right} = g H \left(H^2 k^2+3\right)/right} = g H \left(H^2 k^2+3\right)/right}$  $\label{left} $$k^2+3\right)^4\left(\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\right)^2}+\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3}+\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\right)^2}{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac{h^2 k^2+3\left(\frac$  $\left(3 g H+U \left(\frac{A^2 k^2+3\right) U-2 \right) + \left(\frac{B H}{B k^2 + 3 \right)}\right)$  $\label{left} $$\left(3 + U \left(2 \left(H^2 k^2 + 3\right)\right) U - 3 \right(H^2 k^2 + 3\right) \right) + \left(1 + U \left(H^2 k^2 + 3\right)\right) \right) + \left(1 + U \left(H^2 k^2 + 3\right)\right) + \left(1 + U \left(H^2 k^2 k^2 + 3\right)\right)$  $\t (dt)^3 {32 \left( H^2 k^2 + 3\right) U - \left( h^2 k^2 + 3$  $\left(H^2 k^2+3\right)\right)$  $\label{eq:continuity} $$ k^2+3\right)\right)^2 \left( k^2+3\right)^3+O\left( k^$  $\text{text}_{dx}^3 + \text{left}_{fac}_{k^5} \left[ \frac{3}{9} H \left( \frac{2075 \text{ H}^4 \text{ k}^4 + 12180 \text{ H}^2 \text{ k}^2 + 17856} \right) - 2080 \right]$  $\left( \sqrt{g H^2 k^2 + 3 + 6 \cdot grt\{g H^5 \cdot left(H^2 k^2 + 3 \cdot grt\{g H^5 + 3 \cdot grt\{g$  $H \left( H^2 k^2 + 3\right) \left$ \left(45 \sqrt{3} g^2 \left(385 H^4 k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U  $k^6 - 15 \left(3408 \right) k^4 + 447588 \left(4^2 + 3\right) - 9985 \left(4^3 + 447588 \right) k^6 - 15 \left(4^3 + 447588 \right) k^6$ H^2 U k^2-648 \left(693 \sqrt{g H \left(H^2 k^2+3\right)}-688 \sqrt{3} U\right)\right) H-80 \left(68  $\$  \\sqrt{g H^{13} \\left(H^2 k^2+3\right)} U^3 k^6+612 \\sqrt{g H^9 \\left(H^2 k^2+3\right)} U^3 k^4+9 \\  $\left(204 \right) H^5 \left(H^2 k^2+3\right) U^3+421 \right) H^7 \left(H^2 k^2+3\right) U\right) U$ k^2+3\right)^{7/2}}-\frac{i k^8 \left(560 H^8 U^4 k^8+5 U^3 \left(1344 H^6 U-461 \sqrt{3} \sqrt{g} k^2+3\right)\right) k^4-12 \left(-5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 U \left(10640 H^6 U k^6+5 \left(19056 H^4 U-1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right)  $k^4 + 284364 \ H^2 \ U \ k^2 + 282852 \ U - 63720 \ sqrt{3} \ sqrt{g H \left(H^2 \ k^2 + 3\right)} \right) \right) \ right) \ r$  $\t \{dt\}^3\}\{25600 \left(H^2 k^2+3\right)^4-\frac{(k^9 \left(\sqrt{3} \sqrt{3} \right)^3)}{25600} \left(H^2 k^2+3\right)^4-\frac{(k^9 \left(\sqrt{3} \right)^4)}{25600} \right)$  $k^2+3\right)-\left(H^2 k^2+3\right) - \left(H^2 k^2+3\right) U\right) + \left(U^3 \left(928 \right) - \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) - \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) + \left($ U-3869 \sqrt{3} g H^9\right) k^8-15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U-6019 \aart(a II\laft(II\) !\A2 + 2\right\\\right\\ II\A7 - 2712\\aart(a II\(12)\\laft(II\A2 !\A2 + 2\right\\) II\A2\right\

\squt{g π \iett(π 2 κ 2+5\light)} υ 2 κ 1/2 - 2 / 12 \light (π 1/2) \iett(π 2 κ 2+5\light) υ 5\light (π 1/2) k^6+9 \left(-61735 \sqrt{3} g^2 U H^6+g U^2 \left(89915 \sqrt{g H \left(H^2 k^2+3\right)}-115737 \sqrt{3} U\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)\right) k^4+108 \left(-15327 \sqrt{3} g^2 U H^4-19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g}  $H^5 \left(H^2 k^2+3\right) U^4+22383 \left(H^7 \left(H^2 k^2+3\right) U^2+2625 \right) U^5 + 2810 U^4 + 2810 U^4$  $H^9 \left( H^2 k^2 + 3 \right) \right) + H^9 \left( H^2 k^2 + 3 \right) + H^9 \left( H^8 k^2 + 3 \right) + H$ \left(1857 \sqrt{g H \left(H^2 k^2+3\right)}-1202 \sqrt{3} U\right) U^2-1268 \sqrt{3} g^2 H^2 U+323 

Out[204]=

$$\text{Out} [\text{205}] = \ EA \ || \ \left. \left\{ \left\{ \frac{-H^2 \ k^2 \left( \left( -1 + e^{i \ dt \ w} \right) k \ U - w \right) + 3 \ w}{\left( 3 + H^2 \ k^2 \right) w}, \ - \frac{3 \left( -1 + e^{i \ dt \ w} \right) k}{\left( 3 + H^2 \ k^2 \right) w} \right\}, \ \left\{ - \frac{\left( -1 + e^{i \ dt \ w} \right) k \left( g \ H \left( 3 + H^2 \ k^2 \right) - 3 \ U^2 \right)}{\left( 3 + H^2 \ k^2 \right) w}, \ 1 - \frac{\left( -1 + e^{i \ dt \ w} \right) k \left( 6 + H^2 \ k^2 \right) U}{\left( 3 + H^2 \ k^2 \right) w} \right\} \right\}$$

Out[206]= EA || \left(

\begin{array}{cc}

 $\frac{3 \text{ w-H}^2 \text{ k}^2 \left(\frac{1+e^{i \text{ text}}}{u}\right)}{\left(\frac{1+e^{i \text{ text}}}{u}\right)} k U-w\right)}{\left(\frac{1+e^{i \text{ text}}}{u}\right)}$ 

 $-\frac{1+e^{i \cdot text\{dt\} w}\right| k \left(H^2 k^2+3\right)}{\|h\|^2 k^2+3\right|}$ 

 $w\} \& 1-\frac{dt}{m^2 k^2+6\right} \| w\|^2 k^2+6\right) \| w\|^2 \|$ 

\end{array}

\right)

Out[208]= Eerr || \left(

\begin{array}{cc}

```
W J MEATHULF 4560 METH 1 LA C+JAIGHT) TO METH MEATHULF JAIGHT MIGHT METH METH METH JAIGHT 
                                                                                                                                                           H^2 k^5+12 k^3\right) \left( \frac{4}{40} \left( \frac{h^2 k^2+3\right)}{2} - \frac{3 \left( \frac{h^2 k^6+14}{40} \right) \left( \frac{h^2 k^6+14}{40} \right) \right) \right)
                                                                                                                                                           \label{eq:left} $$ \left(\frac{dt}^2 + \left(\frac{dt}^5 U \left(\frac{dt}^2}{8}\right)\right) + O\left(\frac{dt}^5 \right)\right) + O\left(\frac{dt}^5 \right) \right) $$
                                                                                                                                                           \text{dx}^3+\left(\frac{i \left(770 H^4 k^9+4410 H^2 k^7+6291 k^5\right) \text{dt}}{4800 \left(H^2
                                                                                                                                                           k^2+3\right)^3+\frac{1500 H^4 U k^{10}+8820 H^2 U k^8+12951 U k^6\right) \text{ } text{dt}^2}{4800 L^2 U k^8+12951 U k^6\right) \text{ } text{dt}^2}{4800 L^2 U k^8+12951 U k^6\right) \text{ } text{dt}^2}
                                                                                                                                                           \label{left} $\left(-\frac{k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3+3 g H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3 U^2\right)(2 k U+w)\right) + \left(t^2 k^2 H^3 U^2\right)(2 k U+w)
                                                                                                                                                           k \left( \frac{k^2 H^3+3 g H-3 U^2\right) w^2 \left( \frac{k^2 H^3+3 g H-3 U^2\right) + \frac{k^2 H^3+3 g H-3 U^2\right) w^2 \left( \frac{k^2 H^3+3 U^2}{2 U^2}\right) w^2 \left( \frac{k^
                                                                                                                                                           g H-3 U^2\right) + 0 \left( \frac{4}{24 \left( \frac{4}{24} \left( \frac{4}{24} \right)^2 + \frac{4}{24} \right) + O\left( \frac{4}{3} \right)^2 + O\left( \frac{4}{3} \right)^
                                                                                                                                                           \left(10 \text{ g H}^5 \text{ k}^7 + 60 \text{ g H}^3 \text{ k}^5 - 15 \text{ H}^2 \text{ U}^2 \text{ k}^5 - 36 \text{ U}^2 \text{ k}^3 + 90 \text{ g H k}^3\right) \left(10 \text{ g H}^3 \text{ k}^5 - 15 \text{ H}^2 \text{ U}^2 \text{ k}^5 - 36 \text{ U}^2 \text{ k}^3 + 90 \text{ g H k}^3\right) \left(10 \text{ g H}^3 \text{ k}^5 - 15 \text{ H}^2 \text{ U}^2 \text{ k}^5 - 36 \text{ U}^2 \text{ k}^3 + 90 \text{ g H k}^3\right) \left(10 \text{ g H}^3 \text{ k}^5 - 15 \text{ H}^2 \text{ U}^2 \text{ k}^5 - 36 \text{ U}^2 \text{ k}^3 + 90 \text{ g H k}^3\right) \left(10 \text{ g H}^3 \text{ k}^3 + 90 \text{ g H}^3 \text{ k}^3\right) \left(10 \text{ g H}^3 \text{ k}^3 + 90 \text{ g H}^3 \text{ k}^3\right) \left(10 \text{ g H}^3 \text{ k}^3 + 90 \text{ g H}^3 \text{ k}^3\right) \left(10 \text{ g H}^3 \text{ k}^3 + 90 \text{ g H}^3 \text{ k}^3\right) \left(10 \text{ g H}^3 \text{ k}^3 + 90 \text{ g H}^3 \text{ k}^3\right) \left(10 \text{ g H}^3 \text{ k}^3 + 90 \text{ g H}^3 \text{ k}^3\right) \left(10 \text{ g H}^3 \text{ k}^3 + 90 \text{ g H}^3 \text{ k}^3\right) \left(10 \text{ g H}^3 \text{
                                                                                                                                                           \left(H^2 k^2+3\right)^2}+\frac{\left(-20 g H^5 U k^8+45 H^2 U^3 k^6-120 g H^3 U k^6+126 U^3 H^2 U^3 Left(H^2 k^2+3) Left(H^2 
                                                                                                                                                           k^4-180 \text{ g H U } k^4 \text{ hight} \text{ $k^4-180 \text{ g H U } k^4 \text{ hight} \ \text{ hight} \ \text{ hight}} = k^2+3 \text{ hight}^2 + O\left(\frac{dt}{3}\right) \text{ hight} 
                                                                                                                                                           \t text{dx}^2 + \left( -\frac{1}{8} \right) text{dt} + \frac{1}{8} \left( H k^4 \right) text{dt} + \frac{1}{8} \left( H k^7 - 3 U^3 k^5 + 6 g H H H^2 \right) text{dt} + \frac{1}{8} \left( H h^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U^3 k^5 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3 U k^7 + 6 g H H^3 U k^7 - 3
                                                                                                                                                           \left(260 g H^7 k^{11}+2340 g H^5 k^9-770 H^4 U^2 k^9+7020 g H^3 k^7-4410 H^2 U^2 k^7-6291
                                                                                                                                                           U^2 k^5+7020 g H k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1460 g H^7 U h
                                                                                                                                                           k^{12}-4500 H^4 U^3 k^{10}+13140 g H^5 U k^{10}-26460 H^2 U^3 k^8+39420 g H^3 U k^8-38853
                                                                                                                                                           U^3 \ k^6 + 39420 \ g \ H \ U \ k^6 \ | \ k^2 \ k^2 + 3 \ | \ k^2 + 3 
                                                                                                                                                           H k^2 - 6 U w k \right) \left( \frac{dt}^2}{2 \left( \frac{dt}^2} \right) - \frac{k^2 - 6 U w k \right) \left( \frac{dt}^2}{2 \left( \frac{dt}^2} \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \left( \frac{dt}{2 \left( \frac{dt}^2} \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) \right) - \frac{dt}{2 \left( \frac{dt}^2} \right) - \frac
                                                                                                                                                           \label{left(H^2 k^2+3\right)} $$\left(\frac{h^2 k^2+3\right)}+\frac{k^2 k^2+6\right) U w^3 \text{ } text{dt}^4}{24 \left(\frac{h^2 k^2+6\right)}{24 ent}} $$
                                                                                                                                                           k^2+3\left(\frac{1}{4}\right)+O\left(\frac{1}{4}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+\frac{1}{2}k^5+
                                                                                                                                                           \label{left} $$ \operatorname{dt}_{120 \left(H^2 k^2 + 3\right)^2}-\operatorname{frac}\left(H^2 u^2 k^8 + 45 g H^3 k^6 + 210 H^2 u^2 k^6 + 432 H^2 u^2 k^6 
                                                                                                                                                           U^2 k^4 + 126 g H k^4 + 126 
                                                                                                                                                           \t (dx)^2 + \left( -\frac{1}{8} \right) \left( -\frac{1
                                                                                                                                                           H\right(\frac{dt}^2}{16 \left(\frac{h^2 k^2+3\right)}+O\left(\frac{dt}^5\right)} + O\left(\frac{dt}^5\right) \cdot \frac{dt}^3+\left(\frac{h^2 k^2+3\right)}{16 \left(\frac{h^2 k^2+3
                                                                                                                                                           \left(4800 + 6 k^{11} + 3110 + 4 k^9 + 11430 + 2 k^7 + 13311 k^5 \right) U \text{dt}{4800 \left(H^2 k^7 + 13311 k^5 k^7 + 13311 k^5 k^7 + 13311 k^7 
                                                                                                                                                           k^2+3\right)^3+\frac{10}{2}k^{10}+22140 H^4 U^2 k^{10}+26460
                                                                                                                                                           g H^3 k^8+92340 H^2 U^2 k^8+117126 U^2 k^6+38853 g H k^6\right) \text{dt}^2\{28800
                                                                                                                                                           \label{left} $$\left(H^2 k^2+3\right)^3$+O\left(\frac{dt}^5\right)\right) \text{dx}^4+O\left(\frac{dx}^5\right) \ (\ text{dx}^5\right) \ (\ text{dx}^5\right) \ (\ text{dx}^6) \ (\ text{dx}^6
                                                                                    \end{array}
                                                                                  \right)
\ln[209] = \text{KurF} = (\text{fm} * \text{ap} - \text{fp} * \text{am} + \text{am} * \text{ap} * (\text{qp} - \text{qm})) / (\text{ap} - \text{am});
                                                                                  KurFWS = KurF /. ap \rightarrow 0 /. am \rightarrow (U - Sqrt[g * H]);
                                                                                  KurFWSeta =
                                                                                                                         KurFWS /. fp \rightarrow (H * v + U * Rpp * n) /. fm \rightarrow (H * v + U * Rmp * n) /. qp \rightarrow Rpp * n /.
                                                                                                                                             qm \rightarrow Rmp * n;
                                                                                  \texttt{KurFWSeta} = \texttt{KurFWSeta} \ / \ . \ \ v \ \rightarrow \ (\texttt{GGp} * \texttt{G} \ + \ \texttt{Gnp} * \texttt{n}) \ ;
                                                                                  Kfnnp = FullSimplify[KurFWSeta /. G \rightarrow 0 /. n \rightarrow 1];
                                                                                  Kfnn = Kfnnp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
                                                                                  KfnG = KfnGp / . Rpp → Rp / . Rmp → Rm / . GGp → GG2 / . Gnp → Gn2;
```

```
Fnn2 = -dt * (1 - Exp[-I * k * dx]) / dx * Kfnn;
Fnn2TA = Series[Fnn2 - FnnA, {dx, 0, 3}, {dt, 0, 3}];
Fnn2TAr = Refine[Fnn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
FnG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfnG;
FnG2TA = Series[FnG2 - FnGA, {dx, 0, 3}, {dt, 0, 3}];
FnG2TAr = Refine[FnG2TA , \{k > 0, U > 0, H > 0, g > 0\}];
\texttt{KurFWSG} = \texttt{KurFWS} \ / . \ \texttt{fp} \ \rightarrow \ (\texttt{U} * \texttt{Rpp} * \texttt{G} \ + \ \texttt{U} * \texttt{H} * \texttt{v} \ + \ \texttt{g} * \texttt{H} * \texttt{Rpp} * \texttt{n}) \ / .
       \texttt{fm} \rightarrow (\texttt{U} * \texttt{Rmp} * \texttt{G} + \texttt{U} * \texttt{H} * \texttt{v} + \texttt{g} * \texttt{H} * \texttt{Rmp} * \texttt{n}) \ /. \ \texttt{qp} \rightarrow \texttt{Rpp} * \texttt{G} \ /. \ \texttt{qm} \rightarrow \texttt{Rmp} * \texttt{G};
KurFWSG = KurFWSG / . v \rightarrow (GGp * G + Gnp * n);
KfGn = KfGnp /. Rpp \rightarrow Rp /. Rmp \rightarrow Rm /. GGp \rightarrow GG2 /. Gnp \rightarrow Gn2;
KfGG = KfGGp / . Rpp \rightarrow Rp / . Rmp \rightarrow Rm / . GGp \rightarrow GG2 / . Gnp \rightarrow Gn2;
FGn2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGn;
FGn2TA = Series[FGn2 - FGnA, {dx, 0, 3}, {dt, 0, 3}];
FGn2TAr = Refine[FGn2TA, \{k > 0, U > 0, H > 0, g > 0\}];
fGG2 = U * H * GG2 + U / 2 * (Rm + Rp) - (Sqrt[g * H]) / (2) * (Rp - Rm);
FGG2 = -dt * (1 - Exp[-I * k * dx]) / dx * KfGG;
FGG2TA = Series[FGG2 - FGGA, {dx, 0, 4}, {dt, 0, 3}];
FGG2TAr = Refine[FGG2TA, \{k > 0, U > 0, H > 0, g > 0\}];
Fmat2 = {{Fnn2, FnG2}, {FGn2, FGG2}};
Emat2 = IdentityMatrix[2] + Fmat2 + Fmat2.Fmat2 / 2;
Eerr = Series[Emat2 - EA, {dx, 0, 4}, {dt, 0, 4}];
EigvFmat2 = Eigenvalues[Fmat2];
RKStep = Log[ 1 + EigvFmat2 + EigvFmat2 * EigvFmat2 / 2 ] / (I * dt);
RKstepTay = Series[RKStep, {dx, 0, 4}, {dt, 0, 4}];
RKstepTayr = Simplify[-RKstepTay - \{wAp, wAm\}, \{k > 0, H > 0, g > 0, U > 0\}];
Text[Row[{" U < -Sqrt(gH)"}]]</pre>
Text[" "]
Text[Row[{"Fnn || ", Kfnnp}]]
Text[Row[{"Fnn || ", TeXForm[Kfnnp]}]]
Text[Row[{"Fnn error || ", Fnn2TAr}]]
Text[Row[{"Fnn error || ", TeXForm[Fnn2TAr]}]]
Text[" "]
Text[Row[{"FnG || ", KfnGp}]]
Text[Row[{"FnG || ", TeXForm[KfnGp]}]]
Text[Row[{"FnG error || ", FnG2TAr}]]
Text[Row[{"FnG error || ", TeXForm[FnG2TAr]}]]
```

```
Text[" "]
                                                                Text[Row[{"FGn || ", KfGnp}]]
                                                                Text[Row[{"FGn || ", TeXForm[KfGnp]}]]
                                                                 Text[Row[{"FGn error ||
                                                                                                                                                                                                                                                                                                                                                                                      ", FGn2TAr}]]
                                                                 Text[Row[{"FGn error ||
                                                                                                                                                                                                                                                                                                                                                                                     ", TeXForm[FGn2TAr]}]]
                                                                Text[" "]
                                                                 Text[Row[{"FGG ||
                                                                                                                                                                                                                                                                                                                ", KfGGp}]]
                                                                Text[Row[{"FGG ||
                                                                                                                                                                                                                                                                                                              ", TeXForm[KfGGp]}]]
                                                                Text[Row[{"FGG error ||
                                                                                                                                                                                                                                                                                                                                                                                     ", FGG2TAr}]]
                                                                 Text[Row[{"FGG error ||
                                                                                                                                                                                                                                                                                                                                                                                     ", TeXForm[FGG2TAr]}]]
                                                                Text[" "]
                                                                Text[" "]
                                                                 Text[Row[{"Omega error ||
                                                                                                                                                                                                                                                                                                                                                                                         ", RKstepTayr}]]
                                                                 Text[Row[{"Omega error ||
                                                                                                                                                                                                                                                                                                                                                                                      ", TeXForm[RKstepTayr]}]]
                                                                Text[" "]
                                                                Text[Row[{"EA ||
                                                                                                                                                                                                                                                                                         ", EA}]]
                                                                 Text[Row[{"EA || ", TeXForm[EA]}]]
                                                                Text[Row[{"Eerr || ", Eerr}]]
                                                                 Text[Row[{"Eerr || ", TeXForm[Eerr]}]]
 Out[243]= U < -Sqrt(gH)
Out[244]=
 Out[245]= Fnn \parallel Gnp H + Rpp U
Out[246]= Fnn || \text{text}\{Gnp\}\ H+\text{text}\{Rpp\}\ U
 Out[247]= Fnn error |
                                                                               \left(-\frac{\left(H^2\,k^3\,U\,w\right)dt^2}{2\left(3+H^2\,k^2\right)}-\frac{i\,H^2\,k^3\,U\,w^2\,dt^3}{6\left(3+H^2\,k^2\right)}+O[dt]^4\right)+\left(-\frac{i\left(54\,k^3+45\,H^2\,k^5+10\,H^4\,k^7\right)U\,dt}{120\left(3+H^2\,k^2\right)^2}+O[dt]^4\right)dx^2+\left(\frac{1}{8}\,k^4\,U\,dt+O[dt]^4\right)dx^3+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O[dx]^4+O
 Out[248]= Fnn error |
                                                                               \label{left(-frac{text{dt}^2 \left( h^2 k^3 U w\right)}{2 \left( h^2 k^2 + 3 \right) - \frac{1}{2} \left( h^2 k^3 U w^2 \right)} - \frac{1}{2} \left( h^2 k^3 U w^2 \right)} = \frac{1}{2} \left( h^2 k^3 U w^2 \right) + \frac{1}{2} \left( h^2 k^3 U w^
                                                                                                               \label{left} $$\left(H^2 k^2+3\right)+O\left(t(t)^4\right)+C\left(H^2 k^2+3\right)+O\left(H^2 k
                                                                                                               k^5+54 k^3 + U \text{ (dt)}{120 \left( \frac{k^2 k^2+3\right)^2} + O\left( \frac{t}{4}\right)^4 \right) + text{dx}^3
                                                                                                               \left(\frac{1}{8} k^4 U \left(\frac{dt}{-0}\right)\right) + O\left(\frac{1}{8} k^4 U \left(\frac{dt}{-0}\right)\right) + O\left(\frac{dt}{-0}\right) + O\left(
 Out[249]=
Out[250]= FnG \parallel GGpH
 Out[251]= FnG \parallel \text{text}\{GGp\} H
 \text{Out} \text{[252]= } FnG \ error \ || \ \left( -\frac{3 \ (k \ w) \ dt^2}{2 \left( 3 + H^2 \ k^2 \right)} - \frac{i \ k \ w^2 \ dt^3}{2 \left( 3 + H^2 \ k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( 12 \ k^3 + 5 \ H^2 \ k^3 \right) \ dt}{40 \left( 3 + H^2 \ k^2 \right)^2} + O[dt]^4 \right) dx^2 + O[dx]^4
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Out[254]=

Out[255]=  $FGn \parallel H(gRpp + Gnp U)$ 

Out[256]=  $FGn \parallel H(g \text{text}{Rpp}+\text{text}{Gnp} U)$ 

$$\begin{array}{ll} \text{Out} \text{[257]=} & FGn \; error \; \mid \mid \; \left( -\frac{\left( k \left( 3 \; g \; H + g \; H^3 \; k^2 - 3 \; U^2 \right) w \right) \; dt^2}{2 \left( 3 + H^2 \; k^2 \right)} - \frac{i \; k \left( 3 \; g \; H + g \; H^3 \; k^2 - 3 \; U^2 \right) w^2 \; dt^3}{6 \left( 3 + H^2 \; k^2 \right)} + O[dt]^4 \right) + \\ & \left( -\frac{i \left( 90 \; g \; H \; k^3 + 60 \; g \; H^3 \; k^5 + 10 \; g \; H^5 \; k^7 - 36 \; k^3 \; U^2 - 15 \; H^2 \; k^5 \; U^2 \right) dt}{120 \left( 3 + H^2 \; k^2 \right)^2} + O[dt]^4 \right) dx^2 + \left( \frac{1}{8} \; g \; H \; k^4 \; dt + O[dt]^4 \right) dx^3 + O[dx]^4 \right) dx^4 + O[dt]^4 + O$$

Out[258]= FGn error |

 $\label{left-def} $$\left(-\frac{t}^2 \left( x^2 - 3 \right)^2\right)^{2} \left( x^2 - 3 \right)^{-2} g H-3 U^2\right)^{2} \left( x^2 - 3 \right)^{-2} \left( x^2 - 3 \right)^{-2} g H-3 U^2\right)^{6} \left( x^2 - 3 \right)^{-2} k^2 - 3 \right)^{-2} \left( x^2 - 3 \right)^{-2} \left( x^2$ 

g H^5 k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}}{120 \left(H^2 k^2+3\right)^2}+O\left(\text{dt}^4\right)\right)\right)+\text{dx}^3

 $\left(\frac{1}{8} g H k^4 \left(\frac{dt}{-4\right)\right) + O\left(\frac{1}{8} g H k^4 \left(\frac{dt}{-4\right)\right) + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right) + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right) + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right) + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right) + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right) + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt}{-4\right) + O\left(\frac{dt}{-4\right)} + O\left(\frac{dt$ 

Out[259]=

Out[260]=  $FGG \parallel (GGp H + Rpp) U$ 

 $\label{eq:output} \mbox{Out} \mbox{[261]= } FGG \ || \ U \ (\text{GGp} \ H+\text{Rpp})$ 

$$\begin{array}{ll} \text{Out} [\text{262}] = & FGG \; error \; \mid \mid \; \left( -\frac{\left( k \left( 6 + H^2 \; k^2 \right) U \; w \right) \; dt^2}{2 \left( 3 + H^2 \; k^2 \right)} - \frac{i \; k \left( 6 + H^2 \; k^2 \right) U \; w^2 \; dt^3}{6 \left( 3 + H^2 \; k^2 \right)} + O[dt]^4 \right) + \left( -\frac{i \left( 126 \; k^3 + 75 \; H^2 \; k^5 + 10 \; H^4 \; k^7 \right) U \; dt}{120 \left( 3 + H^2 \; k^2 \right)^2} + O[dt]^4 \right) dx^2 + \\ & \left( \frac{1}{8} \; k^4 \; U \; dt + O[dt]^4 \right) dx^3 + \left( \frac{i \left( 13311 \; k^5 + 11430 \; H^2 \; k^7 + 3110 \; H^4 \; k^9 + 260 \; H^6 \; k^{11} \right) U \; dt}{4800 \left( 3 + H^2 \; k^2 \right)^3} + O[dt]^4 \right) dx^4 + O[dx]^5 \end{array}$$

Out[263]= FGG error ||

 $\label{left} $$\left(-\frac{dt}^2 \left( u \cdot k^2 + 6\right)\right)_{1} = \frac{dt}^2 \left( u \cdot k^2 + 6\right)^2 \left( u \cdot k^$ 

 $U w^2 \left( H^2 k^2 + 6\right) + O\left( H^2 k^2 + 3\right) + O\left( H^2 k^2 + 3\right)$ 

\left(-\frac{i \left(10 H^4 k^7+75 H^2 k^5+126 k^3\right) U \text{dt}}{120 \left(H^2

 $k^2+3\left(\frac{dt}^4\right)+\left(\frac{dt}^4\right)$ 

 $k^4 \ U \ \text{$t$} + O\left(\frac{dt}^4\right) + \left(\frac{dt}^4\right) + \left(\frac{dt}^$ 

H^6 k^{11}+3110 H^4 k^9+11430 H^2 k^7+13311 k^5\right) U \text{dt}}{4800

 $\left(H^2 k^2+3\right)^3+O\left(\left(text\left(dt\right)^4\right)\right)+O\left(text\left(dx\right)^5\right)$ 

Out[264]=

Out[265]=

Out[266]= Omega error ||

$$\left\{ \frac{1}{6\left(3+H^{2}\,k^{2}\right)^{2}}k^{3}\left(\sqrt{3}\,\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right. \\ \left. + \left(3+H^{2}\,k^{2}\right)U\right)\!\left(3\,g\,H+U\left(2\,\sqrt{3}\,\sqrt{g\,H\left(3+H^{2}\,k^{2}\right)}\right. \\ \left. + \left(3+H^{2}\,k^{2}\right)U\right)\!\right)dt^{2} + \left(3+H^{2}\,k^{2}\right)U\right) + \left(3+H^{2}\,k^{2}\right)U\right) + \left(3+H^{2}\,k^{2}\right)U\right) + \left(3+H^{2}\,k^{2}\right)U\right) + \left(3+H^{2}\,k^{2}\right)U\right) + \left(3+H^{2}\,k^{2}\right)U$$

$$\frac{ik! \left[3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U\right)\right]^2 e^4}{8 (3 + H^2 k^2)} - \frac{1}{20 (3 + H^2 k^2)} \left(k^5 \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U\right)\right)^3}{\left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 180 U + 20 H^4 k^4 U + 15 k^2 \left(\sqrt{3} \sqrt{g H^5 (3 + H^2 k^2)} + 8 H^2 U\right)\right) + \frac{1}{200 (3 + H^2 k^2)} k^5 \left(20 H^6 k^6 U^3 + 54 U^2 \left(9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 10 U\right) + \frac{1}{200 (3 + H^2 k^2)} k^5 \left(20 H^6 k^6 U^3 + 54 U^2 \left(9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 10 U\right) + \frac{1}{200 (3 + H^2 k^2)} k^5 \left(20 H^6 k^6 U^3 + 54 U^2 \left(9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 10 U\right) + \frac{1}{200 (3 + H^2 k^2)} k^5 \left(20 H^6 k^6 U^3 + 54 U^2 \left(9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 10 U\right) + \frac{1}{200 (3 + H^2 k^2)} k^6 \left(3 g H (3 + H^2 k^2) + 109 \sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} + 120 H^2 U^3\right)\right) dt^2 + \frac{1}{200 (3 + H^2 k^2)} k^6 \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 140 U + 20 H^4 k^4 U + \frac{1}{200 (3 + H^2 k^2)} k^6\right) \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 24 H^2 U\right)\right)\right) dt^2 - \frac{1}{200 (3 + H^2 k^2)} k^6 \left(3 g H (3 + H^2 k^2) + 180 U + 20 H^4 k^4 U + \frac{1}{200 (3 + H^2 k^2)} k^6\right) \left(3 g H + U \left(2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 120 U + 20 H^4 k^4 U + 15 k^2 \left(\sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} + 8 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 4 \left(3 + H^2 k^2\right) U\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)\right)\right) dt^2 - \frac{1}{20 (3 + H^2 k^2)} k^2 \left(3 g H \left(3 + H^2 k^2\right) + 12 H^2 U\right)$$

ι

$$\begin{split} & 15\,k^4\left(3408\,\sqrt{g}\,H^9\left(3+H^2\,k^2\right) + 9985\,\sqrt{3}\,H^3\,U\right) + \\ & 80\left(1836\,\sqrt{g}\,H^3(3+H^2\,k^2)\,U^3 + 612\,k^4\,\sqrt{g}\,H^9\left(3+H^2\,k^2\right)\,U^3 + 68\,k^6\,\sqrt{g}\,H^{13}\left(3+H^2\,k^2\right)}\right. \\ & U^3 + 9\,k^2\left(421\,\sqrt{g^3\,H^2\left(3+H^2\,k^2\right)}\,U + 204\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right)\,U^3\right) \right) \right) d^2\right) \Big/ \\ & \left(230400\left(\sqrt{g}\,H\right)\left(3+H^2\,k^2\right)^{7/2}\right) - \frac{1}{25600(3+H^2\,k^2)^2}\,i\,k^8\left(6\,g^2\,H^2\left(8046 + 5460\,H^2\,k^2 + 925\,H^4\,k^4\right) + 560\,H^8\,k^8\,U^4 + 4322\,U^3\left(143\,\sqrt{3}\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) + 105\,U\right) + 5\,k^4\,U^3\left(4139\,\sqrt{3}\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) + 6048\,H^4\,U\right) + \\ & 5\,k^4\,U^3\left(461\,\sqrt{3}\,\sqrt{g}\,H^{13}\left(3+H^2\,k^2\right) + 282\,885\,U + 284\,364\,H^2\,k^2\,U^3 + 5040\,H^2\,U^4\right) + \\ & g\,H\,U\left(63\,720\,\sqrt{3}\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) + 282\,885\,U + 284\,364\,H^2\,k^2\,U + 10\,640\,H^6\,k^6\,U + \\ & 5\,k^4\left(1451\,\sqrt{3}\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) + 19\,056\,H^4\,U\right)\right) d^3 + \frac{1}{460\,800\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right)}\,U\right) + \\ & 108\,k^2\left(2625\,\sqrt{g^5\,H^9\left(3+H^2\,k^2\right)} + 15\,327\,\sqrt{3}\,g^2\,H^4\,U + 22\,383\,\sqrt{g^3\,H^7\left(3+H^2\,k^2\right)}\,U^2 + \\ & 19\,261\,\sqrt{3}\,g\,H^3\,U^2 + 4640\,\sqrt{g}\,H^3\left(3+H^2\,k^2\right) - 19\,05\,H^4\,U\right) + 15\,k^6\,U\left(4143\,\sqrt{3}\,g^2\,H^8 + 3712\,\sqrt{g}\,H^{13}\,(3+H^2\,k^2) + 1268\,\sqrt{3}\,g^2\,H^2\,U + 290\,\sqrt{g}\,H\left(3+H^2\,k^2\right)\,U^4 + \\ & g\,H\,U^2\left(1857\,\sqrt{g}\,H\left(3+H^2\,k^2\right) + 1268\,\sqrt{3}\,g^2\,H^2\,U + 290\,\sqrt{g}\,H\left(3+H^2\,k^2\right)\,U^4 + \\ & g\,H^3\left(33\,H^2\,k^2\right) + 29\,H^3\left(3+H^2\,k^2\right) + 115\,737\,\sqrt{3}\,U\right)\right) + 15\,k^6\,U\left(4143\,\sqrt{3}\,g^2\,H^8 + 3712\,\sqrt{g}\,H^{13}\,(3+H^2\,k^2\right) + 61\,735\,\sqrt{3}\,g^2\,H^6\,U + 27\,840\,\sqrt{g}\,H^9\left(3+H^2\,k^2\right)\,U^4 + \\ & g\,H^5\,U^2\left(89\,915\,\sqrt{g}\,H\left(3+H^2\,k^2\right) + 115\,737\,\sqrt{3}\,U\right)\right)\right) d^4 + O(dH^3)\,dx^4 + O(dx)^5, \\ d^2 + \\ \frac{i^4}{6(3+H^2\,k^2)^2}\,g^3\,H^{13}\,(3+H^2\,k^2) + (3+H^2\,k^2)\,U\right)^3\left(3\,g\,H + U\left(-2\,\sqrt{3}\,\sqrt{g}\,H\left(3+H^2\,k^2\right) + (3+H^2\,k^2)\,U\right)\right)\right) \\ d^4 + O(dt)^3 + H^3\,k^2 + H^3\,k$$

$$\begin{split} & \frac{1}{480[5:H^2 W)^2} k^2 \left( -42 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 180 U + 20 H^4 k^4 U - 15 k^2 \left( \sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} - 8 H^2 U \right) \right) + \\ & \frac{1}{480[5:H^2 W)^2} k^5 \left( 20 H^6 k^6 U^3 + 54 U^2 \left( -9 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 10 U \right) + \\ & 5 k^4 U^2 \left( -11 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 26 H^4 U \right) + \\ & 6 g H \left( -21 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2(16 + 147 H^2 k^2 + 25 H^4 k^4) U \right) - \\ & 3 k^2 \left( 15 \sqrt{3} \sqrt{g^3 H^3 (3 + H^2 k^2)} + 109 \sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} U^2 - 180 H^2 U^3 \right) \right) dt^2 + \\ & \frac{1}{480(1 + H^2 W)^2} k^4 k^6 \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) - 180 H^2 U^3 \right) \right) dt^2 + \\ & \frac{1}{480(1 + H^2 W)^2} k^4 k^6 \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 24 H^2 U \right) \right) \right) dt^3 - \frac{1}{800(1 + H^2 W)^2} \\ & \left( k^2 \left( -42 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 180 U + 20 H^4 k^4 U - 15 k^2 \left( \sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} - 8 H^2 U \right) \right) \right) \\ & \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + (3 + H^2 k^2) U \right) \right)^2 \right) dt^4 + O(dt)^3 \right) dx^2 + \\ & \left( -\frac{1}{16} i k^4 \left( \sqrt{3} \sqrt{\frac{g H^3}{3 H^2 W^2}} - 2 U \right) + \frac{1}{32[5 H^2 W^2]^2} i k^6 \left( -3 g H \left( \sqrt{3} \sqrt{g H (3 + H^2 k^2)} - 4 \left( 3 + H^2 k^2 \right) U \right) + \right) \\ & U^2 \left( -15 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 18 U + 2 H^4 k^4 U + k^2 \left( -5 \sqrt{3} \sqrt{g H^3 (3 + H^2 k^2)} + 12 H^2 U \right) \right) \right) \\ & dt^2 - \frac{1}{32[5 H^2 W^2]^2} \left( k^2 \left( 3 g H + U \left( -2 \sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 \left( 3 + H^2 k^2 \right) U \right) \right) \right) dt^3 - \\ & \frac{1}{36[5 H^2 W^2]^2} k^2 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 \left( 3 + H^2 k^2 \right) U \right) \right) \right) dt^3 - \\ & \frac{1}{36[5 H^2 W^2]^2} k^2 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 \left( 3 + H^2 k^2 \right) U \right) \right) dt^3 - \\ & \frac{1}{36[5 H^2 W^2]^2} k^2 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 \left( 3 + H^2 k^2 \right) U \right) \right) \right) dt^3 - \\ & \frac{1}{36[5 H^2 W^2]^2} k^2 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 \left( 3 + H^2 k^2 \right) U \right) \right) dt^3 - \\ & \frac{1}{36[5 H^2 W^2]^2} k^2 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 \left( 3 + H^2 k^2 \right) U \right) \right) dt^3 - \\ & \frac{1}{36[5 H^2 W^2]^2} k^2 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^2)} + 2 \left( 3 + H^2 k^2 \right) U \right) \right) dt^3 + \\ & \frac{1}{36[5 H^2 W^2]^2} k^2 \left( -\sqrt{3} \sqrt{g H (3 + H^2 k^$$

$$\begin{array}{c} 560 \ H^{\circ} \ K^{\circ} \ U^{\circ} + 432 \ U^{\circ} \left(-143 \ \sqrt{3} \ \sqrt{g} \ H^{\circ} (3 + H^{2} \ k^{2}) \right. \\ + 6048 \ H^{4} \ U \right) + \\ 5 k^{6} \ U^{3} \left(-461 \ \sqrt{3} \ \sqrt{g} \ H^{13} \ (3 + H^{2} \ k^{2}) \right. \\ + 1344 \ H^{6} \ U \right) - \\ 12 k^{2} \left(3585 \ \sqrt{3} \ \sqrt{g} \ 3 \ H^{7} \ (3 + H^{2} \ k^{2}) \right. \\ + 12 k^{2} \left(3585 \ \sqrt{3} \ \sqrt{g} \ H^{3} \ (3 + H^{2} \ k^{2}) \right. \\ + 282 852 \ U + 284 364 \ H^{2} \ k^{2} \ U + 10 640 \ H^{6} \ k^{6} \ U + \\ 5 k^{4} \left(-1451 \ \sqrt{3} \ \sqrt{g} \ H^{9} \ (3 + H^{2} \ k^{2}) \right. \\ + 19 056 \ H^{4} \ U \right) \right) dt^{3} - \frac{1}{460 800 \left(\sqrt{g} \ H} \ (3 + H^{2} \ k^{2}) \right. \\ \left(k^{9} \left(\sqrt{3} \ \sqrt{g} \ H \ (3 + H^{2} \ k^{2}) \right. \\ - \left(3 + H^{2} \ k^{2}\right) U\right) \left(5 k^{8} \ U^{3} \left(-3869 \ \sqrt{3} \ g \ H^{9} + 928 \ \sqrt{g} \ H^{17} \ (3 + H^{2} \ k^{2}) \right. U\right) + \\ 108 k^{2} \left(2625 \ \sqrt{g^{5}} \ H^{9} \ (3 + H^{2} \ k^{2}) \right. \\ - 15 327 \ \sqrt{3} \ g^{2} \ H^{4} \ U + 22 383 \ \sqrt{g^{3}} \ H^{7} \ (3 + H^{2} \ k^{2}) \ U^{2} - \\ 19 261 \ \sqrt{3} \ g \ H^{3} \ 3 + 4640 \ \sqrt{g} \ H^{5} \ (3 + H^{2} \ k^{2}) \ U^{4} \right) + \\ g \ H^{5} \ U^{2} \left(89915 \ \sqrt{g} \ H \ (3 + H^{2} \ k^{2}) \right. \\ - 115 737 \ \sqrt{3} \ U\right) + 1296 \left(323 \ \sqrt{g^{5}} \ H^{5} \ (3 + H^{2} \ k^{2}) - \\ 1268 \ \sqrt{3} \ g^{2} \ H^{2} \ U + 290 \ \sqrt{g} \ H \ (3 + H^{2} \ k^{2}) \ U^{4} + g \ H \ U^{2} \left(1857 \ \sqrt{g} \ H \ (3 + H^{2} \ k^{2}) - \\ 1202 \ \sqrt{3} \ U\right) - 15 k^{6} \ U \left(4143 \ \sqrt{3} \ g^{2} \ H^{8} - 3712 \ \sqrt{g} \ H^{13} \ (3 + H^{2} \ k^{2}) \ U^{3} + \\ g \ H^{7} \ U \left(-6019 \ \sqrt{g} \ H \ (3 + H^{2} \ k^{2}) + 15 454 \ \sqrt{3} \ U\right) \right) \right) dt^{4} + O[dt]^{5} dx^{4} + O[dt]^{5} \right)$$

## Out[267]= Omega error ||

 $\left(\frac{K^3 \left(\frac{K^2 + 3\right)}{H^2 + 1}}{U+\sqrt{K^2 + 1}}\right) U + \sqrt{K^2 + 1}}\right)$  $\left(\left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3}}\right) + \left(\frac{H^2 k^2+3\right)}{\left(\frac{H^2 k^2+3\right)}{U+2 \sqrt{3}}\right)$  $\label{left(H^2 k^2+3\left(H^2 k^2+3\right) U+2 sqrt{3} sqrt{g H U \left(H^2 k^2+3\right) U+2 sqrt{3} sqrt{g H U (H^2 k^2+3\right)}}$  $\label{left} $$\left(H^2 k^2+3\right)\right]/r(h^2) \left(H^2 k^2+3\right)^2 \left(H^2 k^2+3\right)^2-\frac{h^2(h^2 k^2+3)^2}{h^2(h^2 k^2+3)^2}\right)^2 + \frac{h^2(h^2 k^2+3)^2}{h^2(h^2 k^2+3)^2} \left(H^2 k^2+3\right)^2 + \frac{h^2(h^2 k^2+3)^2}{h^2(h^$  $k^2+3\right) U+\sqrt{3} \sqrt{2+3}\right) U+\sqrt{3} \ln(4^2 k^2+3\right)$  $k^2+3\left( U+2 \right) +2 \left( U+2 \right)$  $k^2+3\right/h^4+O\left(\frac{dt}^5\right)h^4+O$  $\left(H^2 k^2+3\right)^2+\frac{h^2 u^2}{h^2 u^2} + \frac{h^2 u^2 \left(H^2 u^2 + \frac{h^2 u^2}{h^2 u^2}\right)}{h^2 u^2 u^2} + \frac{h^2 u^2 u^2}{h^2 u^2} + \frac{h^2 u^2}{h^2 u^2} + \frac{h^2$  $\left(\frac{H^2 k^2+3\right}{k^2+3\right} k^4+3\left(\frac{180 H^2 U^3+109 \sqrt{3}}{k^2+3}\right)$  $U^2+15 \sqrt{3} \sqrt{10} U^2 + 15 \sqrt{3} \sqrt{10} U^2 \left( U^2 \left( U^2 \right) \right)$ H \left(H^2 k^2+3\right)\right)+6 g H \left(\left(25 H^4 k^4+147 H^2 k^2+216\right) U+21 \sqrt{3}  $\left(\frac{H^2 k^2+3\right)}{\sinh(h^2 k^2+3\right)}\right)$ g H+U \left(\left(H^2 k^2+3\right) U+2 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)\right)\right) \left(9 g  $H \left(5 \text{ H}^2 \text{ k}^2 + 14\right) + U \left(20 \text{ H}^4 \text{ U k}^4 + 5 \right) + U \left(24 \text{ U H}^2 + 7 \right) + U \left(24 \text{ U H}^2 + 14\right) + U \left$  $k^2+3\right) \ k^2+180 \ U+102 \ \sqrt{1} \ k^2+3\right) \ k^2+3\right) \ k^2+3\right) \ k^2+3\ U+102 \ \sqrt{1} \ k^2+3\right) \ k^2+3\left(\frac{1}{2} \ k^2+3\right) \ k^2$  $\left( H^2 k^2 + 3\right) - \left( H^2 k^2$  $k^2+3\left(k^2+3\right)\right) + k^2+180 + 42 \left(180 + 42 \right) + k^2+3\left(180 + 42 \right) +$   $k^2+3\left( U+2 \right) U+2 \left( U+2 \right) \left( U+2 \right) U+2 \left( U+2 \right) U+$  $H^5 \left(H^2 k^2+3\right) \ k^2+18 \ U+15 \ yqrt{g} \ H\left(H^2 k^2+3\right) \ U+2+3 \ U+15 \ yqrt{g} \ H\left(H^2 k^2+3\right) \ U+2+3 \ U+15 \ U+2+3 \ U+15 \ U+2+3 \ U+15 \ U+15$  $\label{left} $$\left(H^2 k^2+3\right)^2-\frac{k^2+3\left(k^7 \left(k^7 \left(3 gH+U \left(h^2 k^2+3\right) U+2 \right)U+2 \right)}{H^2 k^2+3\left(k^2+3\right)}\right)$$$  $H \left(H^2 k^2+3\right)\right) \left(H^2 k^2+3\right) + \left(H^2 k^2+$  $H \left(\frac{4^2 k^2+3\right)}{\sinh(2k^2+3\right)} \left(\frac{4k^2+3}{\sinh(2k^2+3)}\right) \left(\frac{k^2+3}{\sinh(2k^2+3)}\right) \left(\frac{k^2+3}{h^2}\right) \left(\frac{k^2+$  $\label{eq:left} $\left(H^2 k^2 + 3\right) \cdot U + \sqrt{3} \right(H^2 k^2 + 3\right) \cdot \left(H^2 k^2 + 3\right) \cdot \left($  $k^2+3\right) U+2 \sqrt{3} \sqrt{4}{64 \left(H^2 k^2+3\right)\right)} \left(H^2 k^2+3\right) \left(H^2 k^2+3\right)$  $k^2+3\right)^3+O\left(\frac{t}{5\right)^3}+O\left(\frac{t}{5\right)^3}+O\left(\frac{t}{5}\right)^3+O\left(\frac{t}{5$  $H^4 k^4 + 12180 H^2 k^2 + 17856 \right) + 2080 \left( \frac{H^9 \left( \frac{K^2 + 3\right)}{k^4 + 6} \right) + 2080 \right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^4 + 12180 H^2 k^2 + 3\right) + 2080 \left( \frac{H^2 k^4 + 3\right)$  $H^5 \left(H^2 k^2+3\right) k^2+9 \left(H^2 k^2+3\right) U\right) H^5 \left(H^2 k^2+3\right) U\right) H^5 \left(H^2 k^2+3\right) U\right)$ H} \left( $H^2 k^2+3 \right)^{5/2}\right] - \frac{k^4 k^4+2268}{k^7 \left(k^7 \left(k^7 \right)^2 k^2 k^2+3 \right)^6}$ H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U k^6+15 \left(9985 \sqrt{3} U H^4+3408 \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+447588 \sqrt{3} H^2 U k^2+648 \left(688 \sqrt{3} U+693  $k^6+612 \cdot H^5 \cdot H^2 \cdot H^2 \cdot H^2 \cdot H^2 \cdot H^2 \cdot H^3 \cdot$  $U^3+421 \cdot g^3 H^7 \cdot (h^2 k^2+3 \cdot g)$  Uright)  $h^2+1836 \cdot g$  H \left( $h^2 k^2+3 \cdot g$ )  $\label{left} $$U^3\right)\right) \operatorname{left}\left(\frac{H^2 k^2+3\right)}{(30400 \left(\frac{1}{4}\right)^2}{(30400 \left(\frac{1}{4}\right)^2 k^2+3\right)}-\frac{1}{4}\left(\frac{1}{4}\right)^2}{(30400 \left(\frac{1}{4}\right)^2 k^2+3\right)^2}$ \left(560 H^8 U^4 k^8+5 U^3 \left(1344 U H^6+461 \sqrt{3} \sqrt{g H^{13} \left(H^2 k^2+3\right)}\right) k^6+5 U^3 \left(6048 U H^4+4139 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)\right) k^4+12 \left(5040  $H^2 U^4+5161 \sqrt{3} \sqrt{4+5161} \sqrt{3} \sqrt{4+5161}$ k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 H^2 k^2+8046\right)+432 U^3 \left(105 H^2 k^2+8046\right)+432 U^3 \right(105 H^2 k^2+80468\right)+432 U^3 \right(105 H^2 k^2+804688\right(105 H^2 k^2+804688\right(105 H^2 k^2+804688\right(105 H^2 k^2+804688\right(105 U+143 \sqrt{3} \sqrt{g H \left(H^2 k^2+3\right)}\right)+g H U \left(10640 H^6 U k^6+5 \left(19056 U H^4+1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right) k^4+284364 H^2 U k^2+282852 U+63720  $\label{left(H^2 k^2+3\wedge U+\sqrt{3} \operatorname{H \left(16^2 k^2+3\right)} \left(5 U^3 \left(5 U^3 \left(5 U^3 \left(5 U^3 \right) U+\right) \right) \right) } \\$ g H^9+928 \sqrt{g H^{17} \left(H^2 k^2+3\right)} U\right) k^8+15 U \left(4143 \sqrt{3} g^2 H^8+g U  $\left(15454 \right) H^7+3712 \right) H^619 YEAR H^6$  $k^2+3 \right) U^3 \right) U^3 \right) k^6+9 \left( 61735 \right) \left( 3 \right) g^2 U H^6+g U^2 \left( 115737 \right) \left( 115737 \right) H^6+g U^2 \right) U^4 \left( 115737 \right) U$ \sqrt{g H \left(H^2 k^2+3\right)}\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)}\right) k^4+108 \left(15327 \sqrt{3} g^2 U H^4+19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g H^5 \left(H^2 k^2+3\right)} U^4+22383 \sqrt{g^3 H^7 \left(H^2 k^2+3\right)}  $U^2 + 2625 \sqrt{g^5 H^9 \left( H^2 k^2 + 3\right)} k^2 + 1296 \left( 290 \sqrt{g^2 H^2 k^2 + 3\right) } k^2 + 1296 \left( 290 \sqrt{g^2 H^2 k^2 + 3} k^2 + 3\right) k^2 + 1296 \left( 290 \sqrt{g^2 H^2 k^2 + 3} k^2 +$  $U^4+g H \left(1202 \right) U+1857 \left(14 H^2 k^2+3\right) U+1857 \left(1$  $H^2 U+323 \left( \frac{g^5 H^5 \left( \frac{40800 \left( + \right)} \right)} \right)} {40800 \left( \frac{40800 \left( + \right)} \right)} {40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( \frac{40800 \left( + \right)} {40800 \left( \frac{40800 \left( + \right)} {40800 \left( \frac{40800 \left( + \right)} {40800 \left( +$  $\left(\left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \sqrt{3} \right) + \left(\frac{H^2 k^2+3\right)}\right) \left(\frac{H^2 k^2+3\right)}{U-\sqrt{3} \left(\frac{H^2 k^2+3\right)}}\right)$  $k^2+3\right) U-2 \sqrt{3} \left( H^2 k^2+3\right) \left$  $k^2+3\right)^2+\frac{1}{2}+\frac{1}{2} +\frac{1}{2} +$  $\label{left} $$ \frac{k^2+3\right\right)\right} \cdot (k^2+3\right)^2 \left( \frac{dt}^3}{8 \left( \frac{k^2+3\right)^2}{-\frac{t^2}^2} \right)^2} -\frac{t^2}{k^2+3\right)^2} -\frac{t^2}{k^2+3\left( \frac{k^2+3\right)^2}{-\frac{t^2}^2} \right)^2} -\frac{t^2}{k^2+3\left( \frac{k^2+3\right)^2}{-\frac{t^2}^2} -\frac{t^2}{k^2+3\left( \frac{k^2+3\right)^2} -\frac{t^2}{k^2+3\left$ 

 $k^2+3\left(\frac{4t}^5\right)+\left(\frac{4t}^5\right)+\left(\frac{4t}^5\right)+\frac{4t}{4}$  $k^2+3\right)$ {240 \left(H^2 k^2+3\right)^2}+\frac{k^5 \left(20 H^6 U^3 k^6+5 U^2 \left(36 H^4 U^5 H^6 U^5  $H^5 \left(H^2 k^2+3\right) U^2+15 \right] V^2+15 \right] V^2+15 \right]$  $\left(10 \text{ U} - 9 \right) \left(10 \text{ U}$  $k^2 + 216 \right) U-21 \left(3 \right) \left(H^2 k^2 + 3\right) \right) \left(H^2 k^2 + 3\right) \left(H^2 k^2 + 3\right)$  $k^2+3\right)^3+\frac{i}{k^6}\left(\frac{1}{2}H+U\right)^3+\frac{1}{2}h^2} L^2+3\right)^3+\frac{1}{2}h^2} L^2+3h^2} L^2+3$ k^2+3\right)\right)\right)\left(9 g H \left(5 H^2 k^2+14\right)+U \left(20 H^4 U k^4-5 \left(7 \sqrt{3})  $\label{left} $$ \kappa^2+3\right\right)\right) \operatorname{left}(M^7 \end{0.05} + 3\operatorname{left}(M^7 \en$  $k^4-15 \left( \frac{4^2 + 1}{160} \right) -8 H^2 U\right) k^2+180 U-42 \left( \frac{4^2 + 1}{160} \right) -8 H^2 U\right) k^2+180 U-42 \left( \frac{4^2 + 1}{160} \right) + 2 H^2 U\right) k^4-15 H^2 U$  $k^2+3\right)\right)\$  $\text{dx}^2+\left(-\frac{1}{16} i k^4 \left(\frac{3} \right) \right) + \frac{4 k^2 k^2+3}{-2 k^2+3}$  $\left(U^2 \left(H^4 U ^4+\left(H^2 U^5 \right)\right) \right)$  $U-15 \sqrt{3} \sqrt{3} \sqrt{g H \left(\frac{h^2 k^2+3\right)}} g H \left(\frac{3} \sqrt{3} \right) + (h^2 k^2+3\right) + (h^2 k^2+3) + (h^2 k^2+3)$  $\left(\frac{H^2 k^2+3\right)}{U\right)} \left(\frac{H^2 k^2+3\right)}{U\right)} \left(\frac{H^2 k^2+3\right)}{U\right)}$  $g H+U \left( \frac{H^2 k^2+3\right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) \right) - U-2 \sqrt{g} H} \left( \frac{h^2 k^2+3\right) \right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H} \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H} \right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H} \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H} \right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H} \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H} \right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H} \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H} \right) - U-2 \sqrt{g} H \left( \frac{h^2 k^2+3\right) - U-2 \sqrt{g} H} \left( \frac{h^2 k^2+3\right) - U$ g H+U \left(2 \left(H^2  $k^2+3 \cdot U-3 \cdot y + U \t (dt)^3 {32 \left( H^2 k^2 + 3\right) U - \left( h^2 k^2 + 3$  $\left(H^2 k^2+3\right)\right)$  $\label{eq:continuity} $$ k^2+3\right)\right)^2 \left( k^2+3\right)^3+O\left( k^$  $\text{text}_{dx}^3 + \text{left}_{fac}_{k^5} \left[ \frac{3}{9} H \left( \frac{2075 \text{ H}^4 \text{ k}^4 + 12180 \text{ H}^2 \text{ k}^2 + 17856} \right) - 2080 \right]$  $\left( \sqrt{g H^2 k^2 + 3 + 6 \cdot grt\{g H^5 \cdot left(H^2 k^2 + 3 \cdot grt\{g H^5 + 3 \cdot grt\{g$  $H \left( H^2 k^2 + 3\right) \left$ \left(45 \sqrt{3} g^2 \left(385 H^4 k^4+2268 H^2 k^2+3336\right) H^2+g U \left(16705 \sqrt{3} H^6 U  $k^6 - 15 \left(3408 \right) k^4 + 447588 \left(4^2 + 3\right) - 9985 \left(4^3 + 447588 \right) k^6 - 15 \left(4^3 + 447588 \right) k^6$ H^2 U k^2-648 \left(693 \sqrt{g H \left(H^2 k^2+3\right)}-688 \sqrt{3} U\right)\right) H-80 \left(68  $\$  \\sqrt{g H^{13} \\left(H^2 k^2+3\right)} U^3 k^6+612 \\sqrt{g H^9 \\left(H^2 k^2+3\right)} U^3 k^4+9 \\  $\left(204 \right) H^5 \left(H^2 k^2+3\right) U^3+421 \right) H^7 \left(H^2 k^2+3\right) U\right) U$ k^2+3\right)^{7/2}}-\frac{i k^8 \left(560 H^8 U^4 k^8+5 U^3 \left(1344 H^6 U-461 \sqrt{3} \sqrt{g}  $H^{13} \left(H^2 k^2+3\right)\right) \$ k^2+3\right)\right) k^4-12 \left(-5040 H^2 U^4+5161 \sqrt{3} \sqrt{g H^5 \left(H^2 k^2+3\right)} U^3+3585 \sqrt{3} \sqrt{g^3 H^7 \left(H^2 k^2+3\right)} U\right) k^2+6 g^2 H^2 \left(925 H^4 k^4+5460 U \left(10640 H^6 U k^6+5 \left(19056 H^4 U-1451 \sqrt{3} \sqrt{g H^9 \left(H^2 k^2+3\right)}\right)  $k^4 + 284364 \ H^2 \ U \ k^2 + 282852 \ U - 63720 \ sqrt{3} \ sqrt{g H \left(H^2 \ k^2 + 3\right)} \right) \right) \ right) \ r$  $\t \{dt\}^3\}\{25600 \left(H^2 k^2+3\right)^4-\frac{(k^9 \left(\sqrt{3} \sqrt{3}\right) + H^2}{2}\right)$  $k^2+3\right)-\left(H^2 k^2+3\right) - \left(H^2 k^2+3\right) U\right) + \left(U^3 \left(928 \right) - \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) - \left(H^2 k^2+3\right) U\right) + \left(H^2 k^2+3\right) + \left($ U-3869 \sqrt{3} g H^9\right) k^8-15 U \left(4143 \sqrt{3} g^2 H^8+g U \left(15454 \sqrt{3} U-6019 \aart(a II\laft(II\) !\A2 + 2\right\\\right\\ II\A7 - 2712\\aart(a II\(12)\\laft(II\A2 !\A2 + 2\right\\) II\A2\right\

\squt{g π \iett(π 2 κ 2+5\light)} υ 2 κ 1/2 - 2 / 12 \light (π 1/2) \iett(π 2 κ 2+5\light) υ 5\light (π 1/2) k^6+9 \left(-61735 \sqrt{3} g^2 U H^6+g U^2 \left(89915 \sqrt{g H \left(H^2 k^2+3\right)}-115737 \sqrt{3} U\right) H^5+27840 \sqrt{g H^9 \left(H^2 k^2+3\right)} U^4+5325 \sqrt{g^5 H^{13} \left(H^2 k^2+3\right)\right) k^4+108 \left(-15327 \sqrt{3} g^2 U H^4-19261 \sqrt{3} g U^3 H^3+4640 \sqrt{g}  $H^5 \left(H^2 k^2+3\right) U^4+22383 \left(H^7 \left(H^2 k^2+3\right) U^2+2625 \right) U^5 + 2810 U^4 + 2810 U^4$  $H^9 \left( H^2 k^2 + 3 \right) \right) + H^9 \left( H^2 k^2 + 3 \right) + H^9 \left( H^8 k^2 + 3 \right) + H$ \left(1857 \sqrt{g H \left(H^2 k^2+3\right)}-1202 \sqrt{3} U\right) U^2-1268 \sqrt{3} g^2 H^2 U+323 

Out[268]=

$$\text{Out} [269] = \quad EA \mid \mid \quad \left\{ \left\{ \frac{-H^2 \; k^2 \left( \left( -1 + e^{i \, d \, t \, w} \right) k \; U - w \right) + 3 \; w}{\left( 3 + H^2 \; k^2 \right) w}, \; - \frac{3 \left( -1 + e^{i \, d \, t \, w} \right) k}{\left( 3 + H^2 \; k^2 \right) w} \right\}, \; \left\{ - \frac{\left( -1 + e^{i \, d \, t \, w} \right) k \left( g \; H \left( 3 + H^2 \; k^2 \right) - 3 \; U^2 \right)}{\left( 3 + H^2 \; k^2 \right) w}, \; 1 \; - \frac{\left( -1 + e^{i \, d \, t \, w} \right) k \left( 6 + H^2 \; k^2 \right) U}{\left( 3 + H^2 \; k^2 \right) w} \right\} \right\}$$

Out[270]= EA || \left(

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 $\frac{3 \text{ w-H}^2 \text{ k}^2 \left(\frac{1+e^{i \text{ text}}}{u}\right)}{\left(\frac{1+e^{i \text{ text}}}{u}\right)} k U-w\right)}{\left(\frac{1+e^{i \text{ text}}}{u}\right)}$ 

 $-\frac{1+e^{i \cdot text\{dt\} w}\right| k \left(H^2 k^2+3\right)}{\|h\|^2 k^2+3\right|}$ 

 $w\} \& 1-\frac{dt}{m^2 k^2+6\right} \| w\|^2 k^2+6\right) \| w\|^2 \|$ 

\end{array}

\right)

$$\begin{aligned} \text{Out271} &= \text{Eerr} \, \| \, \, \left\{ \left\{ \left( \frac{(-3 \, \text{g H k}^2 + 3)^2 \, U^2 - H^2 \, k^4 \, U^3 \, \text{H}^2 \, k^3 \, U \, \text{d}^2}{6(3 \, \text{H k}^2 \, k)} - \frac{i \, H^2 \, k^3 \, U \, w^3 \, dh^4}{24(3 \, \text{H k}^2 \, k)^2} + O[\, dt]^5 \right) \, + \\ &- \left( \frac{i \, (54 \, k)^4 \, 45 \, H^2 \, k + 10 \, H^2 \, k^3 \, U^2}{120 \, (3 \, \text{H}^2 \, k^2)^2} \right) \, - \frac{i \, (126 \, g \, H^2 \, k^4 + 45 \, g \, H^3 \, k + 22 \, U^4 \, 30 \, H^3 \, k^2 \, U^2 + O[\, dt]^5 \right) \, dx^2 \, + \\ &- \left( \frac{i \, (29 \, k)^3 \, U + 2610 \, H^2 \, k^2 \, U^2}{16(3 \, \text{H}^2 \, k)^2} \right) \, - \frac{i \, (126 \, g \, H^2 \, k^4 \, U)}{8(3 \, \text{H}^2 \, k)^2} \right) \, dt^2 \, + O[\, dt]^5 \right) \, dx^3 \, + \\ &- \left( \frac{i \, (29 \, k)^3 \, U + 2610 \, H^2 \, k^4 \, U \, dt + \left( -\frac{3 \, k^2 \, U}{8(3 \, \text{H}^2 \, k)^2} \right) \, dt^2 \, + O[\, dt]^5 \right) \, dx^3 \, + \\ &- \left( \frac{i \, (29 \, k)^3 \, U + 2610 \, H^2 \, k^4 \, U \, dt + 2610 \, H^2 \, k^4 \, U \, dt + 4100 \, H^4 \, k^{10} \, U^2 \, + 1460 \, H^6 \, k^{12} \, U^2 \right) \, dt^2 \, + O[\, dt]^5 \right) \, dx^4 \, + \\ &- \left( \frac{3 \, k^2 \, U}{3 \, \text{H}^2 \, k^2} \right) \, dt^2 \, - \frac{i \, k^2 \, w^2 \, dt^3}{2(3 \, \text{H}^2 \, k)^2} \, + \frac{k^2 \, w^3 \, dt^4}{8(3 \, \text{H}^2 \, k)^2} \right) \, + O[\, dt]^5 \right) \, dx^3 \, + \\ &- \left( \frac{i \, (291 \, k^2 + 4 \, H^3 \, k^2 \, H^2 \, k^2 \, U^2 \, U)}{40(3 \, \text{H}^2 \, k^2)^2} \, + \frac{k^2 \, w^3 \, dt^4}{2(3 \, \text{H}^2 \, k)^2} \right) \, + O[\, dt]^5 \right) \, dx^3 \, + \\ &- \left( \frac{i \, (291 \, k^2 + 4 \, H^3 \, H^2 \, k^2 \, U^2 \, U^2 \, W^2 \, H^2 \, k^2 \, U^2 \, W^2 \, H^2 \, W^2}{40(3 \, \text{H}^2 \, k^2)^2} \, + \frac{k^2 \, w^3 \, dt^4}{830 \, (3 \, \text{H}^2 \, k^2)} \, + O[\, dt]^5 \right) \, dx^3 \, + \\ &- \left( \frac{i \, (291 \, k^2 \, 4 \, H^3 \, H^2 \, k^2 \, U^2 \, U^2 \, W^2 \, H^2 \, W^2}{40(3 \, \text{H}^2 \, k^2)^2} \, + \frac{k^2 \, w^3 \, dt^4}{830 \, (3 \, \text{H}^2 \, k^2)} \, + O[\, dt]^5 \right) \, dx^3 \, + \\ &- \left( \frac{i \, (291 \, k^2 \, 4 \, H^3 \, W^2 \, H^2 \, W^2 \, H^2 \, W^2}{40(3 \, \text{H}^2 \, k^2)^2} \, + \frac{k^2 \, w^3 \, dt^4}{830 \, (3 \, \text{H}^2 \, k^2)} \, + O[\, dt]^5 \right) \, dx^4 \, + O[\, dt]^5 \right) \, dx^4 \, + O[\, dt]^5 \right) \, dx^2 \, + \\ &- \left( \frac{i \, (291 \, k^2 \, k^2 \, W^2 \, W^2 \, W^2 \, W^2}{40 \, (3 \, \text{H}^2 \, k^2)} \, + \frac{k^2 \, k^2 \, U \, dt^2}{4(3 \, \text{H}^2 \, k^2)} \, + O[\, dt]^5 \right) \, dx^4 \, + O[\, dt]^5 \right) \, dx^2 \, + \\$$

Out[272]= Eerr || \left(

\begin{array}{cc}

 $k^6+14 k^4\right) U\right) \left(t k^2+3\right) \left(t k^4\right) \left(t k$  $\label{left} $$ \operatorname{dx}^2+\left(-\frac{3 i k^5 U \operatorname{dt}^2}{8 \left(H^2 k^2+3\right)}+O\left(\frac{dt}^5\right)\right)$ is the proof of the proof$ \text{dx}^3+\left(\frac{i \left(770 H^4 k^9+4410 H^2 k^7+6291 k^5\right) \text{dt}}{4800 \left(H^2 k^2+3\right)^3}+\frac{\left(1500 H^4 U k^{10}+8820 H^2 U k^8+12951 U k^6\right) \text{dt}^2}{4800}  $\left(\frac{dx}^4+O\left(\frac{dx}^5\right)^3\right)+O\left(\frac{dx}^5\right) \right)$ 

 $\left(-\frac{k^2 H^3 + 3 H^3 + 3 H^3 U^2\right)}{2 kU+w}\right) \left(2 kU+w\right)\right) \left(2 kU+w\right)\right) \left(2 kU+w\right)\right)$  $k \left( \frac{k^2 H^3+3 g H-3 U^2\right) w^2 \left( \frac{k^2 H^3+3}{6 \left( \frac{k^2 H^3+$  $g H-3 U^2\right) + 0 \left( \frac{4}{24} \left( \frac{4}{24} \left( \frac{4}{24} \right) + 0 \right) + 0 \left( \frac{4}{3} \right) \right) + 0 \left( \frac{4}{3} \right)$ \left(10 g H^5 k^7+60 g H^3 k^5-15 H^2 U^2 k^5-36 U^2 k^3+90 g H k^3\right) \text{dt}\{120 \left(H^2 k^2+3\right)^2}+\frac{\left(-20 g H^5 U k^8+45 H^2 U^3 k^6-120 g H^3 U k^6+126 U^3  $k^4-180 g H U k^4 + k^2 + 2 {120 \left( \frac{h^2 k^2+3 \right)^2} + O\left( \frac{dt}{5}\right)^5 \right)$ \text{dx}^2+\left(\frac{1}{8} g H k^4 \text{dt}-\frac{i \left(2 g H^3 U k^7-3 U^3 k^5+6 g H U k^5\right) g H^7 k^{11}+2340 g H^5 k^9-770 H^4 U^2 k^9+7020 g H^3 k^7-4410 H^2 U^2 k^7-6291 U^2  $k^5 + 7020 \text{ g H k}^5 \text{ hight) } \text{ left} (H^2 k^2 + 3 \text{ hight)}^3 + \text{ frac} (left (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3) + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} + 3 \text{ left})^3 + \text{ left} (1460 \text{ g H}^7 \text{ U k}^{2} +$ H<sup>4</sup> U<sup>3</sup> k<sup>1</sup> 10}+13140 g H<sup>5</sup> U k<sup>1</sup> 10}-26460 H<sup>2</sup> U<sup>3</sup> k<sup>8</sup>+39420 g H<sup>3</sup> U k<sup>8</sup>-38853 U<sup>3</sup>  $k^6+39420 \text{ g H U } k^6 \text{ hight} \text{ text} dt^2 14400 \left( \frac{k^2 + 3 \right)^3} + O\left( \frac{dt}{\hbar} \right)$  $\label{eq:hamiltonian} H \ k^2-6 \ U \ w \ k\rightarrow (h^2 \ k^2+6\rightarrow U \ w \ k\rightarrow (h^2 \ k))$  $\t t_{dt}^3 = \frac{h^2 k^2+3 \right}{h^2 k^2+3 \right} + \frac{h^2 k^2+6 \right}{u^2 k^2+6 \right} \t U w^3 \left( \frac{h^2 k^2+3 \right)}{h^2 k^2+6 \right}$  $k^2+3\right)+O\left(\frac{10 H^4 k^7+75 H^2 k^5+126 k^3\right)}{U}$  $\label{eq:local_local_local_local_local_local} $$ \left( \frac{120 \left( \frac{4}{2} k^2 + 3\right)^2}{120 \left( \frac{4}{2} k^2 + 3\right)^2} \right) - \frac{4}{2} k^2 + \frac{4}{2} k^3 + \frac{4}{2} k^$  $U^2 k^4 + 126 g H k^4 \right) \text{ } text{dt}^2{240 \left(H^2 k^2 + 3\right)} + O\left(\frac{4t}^5\right) \right)$  $\t (x^2-\left(\frac{1}{8}\right) k^4 U \left(\frac{1}{6}\right) - \frac{1}{8} k^4 U \left(\frac{1}{6}\right) - \frac{1}{8} k^5 \left(\frac{1}{6}\right) k^5 \left(\frac{1}{6}\right)$  $\t (text{dt}^2){16 \left(H^2 k^2+3\right)}+O\left(text{dt}^5\right)\right) \t (text{dt}^5\right) \t (text{dx}^3+\left(text{dx}^3+text{dx}^3+text{dx}^3+text{dx}^3\right)$  $\left(4800 + 6 k^{11} + 3110 + 4 k^9 + 11430 + 2 k^7 + 13311 k^5 \right)$  \\ \text{dt}{4800 \\ left(H^2 \)  $k^2+3\left(10\right)+2140\ H^4\ U^2\ k^{12}+4500\ g\ H^5\ k^{10}+22140\ H^4\ U^2\ k^{10}+26460$ g H^3 k^8+92340 H^2 U^2 k^8+117126 U^2 k^6+38853 g H k^6\right) \text{dt}^2\{28800 

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