Numerical Study of Steep Gradient Problems for Generalised Serre-Green-Naghdi Model

Contents

Papers Primary focus - Numerical study of dam-break problem for important families of gSGN equations (improved dispersion, serre to swwe, regularised shallow water).

- Discussion of different implementations and effect of reconstruction (particularly of derivatives) on solutions
- First robust numerical solutions for the family of equations for steep gradient problem
- Robustness of method in presence of weak discontinuity (spike in G)
- Number of different implementations of methods demonstrating same behaviour (justifying our observations)

1. Abstract

2. Introduction

3. Generalised Serre-Green-Naghdi Equations

3.1. Properties

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3.1.2. Dispersion

3.1.3. Challenges

4. Extensions To gSGN Scheme

4.1. FEM + Weno Reconstruction

4.2. FDM + Weno Reconstruction

5. Dam-break Study

5.1. Regularised Shallow Water Wave Equations Family

5.2. Improved Dispersion SGN Family

5.3. SGN to SWWE Family

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