Generalised Serre-Green-Naghdi Model

June 4, 2020

0.1 Numerical Experiments

0.1.1 Smooth Dambreak

$$h(x,0) = h_0 + \frac{h_1 - h_0}{2}\left(1 + \tanh\left(\frac{x}{\alpha}\right)\right) \tag{1}$$

$$u(x,0) = 0 \tag{2}$$

$$G(x,0) = 0 \tag{3}$$

$\alpha = 2$
$\alpha = 0.1$

References

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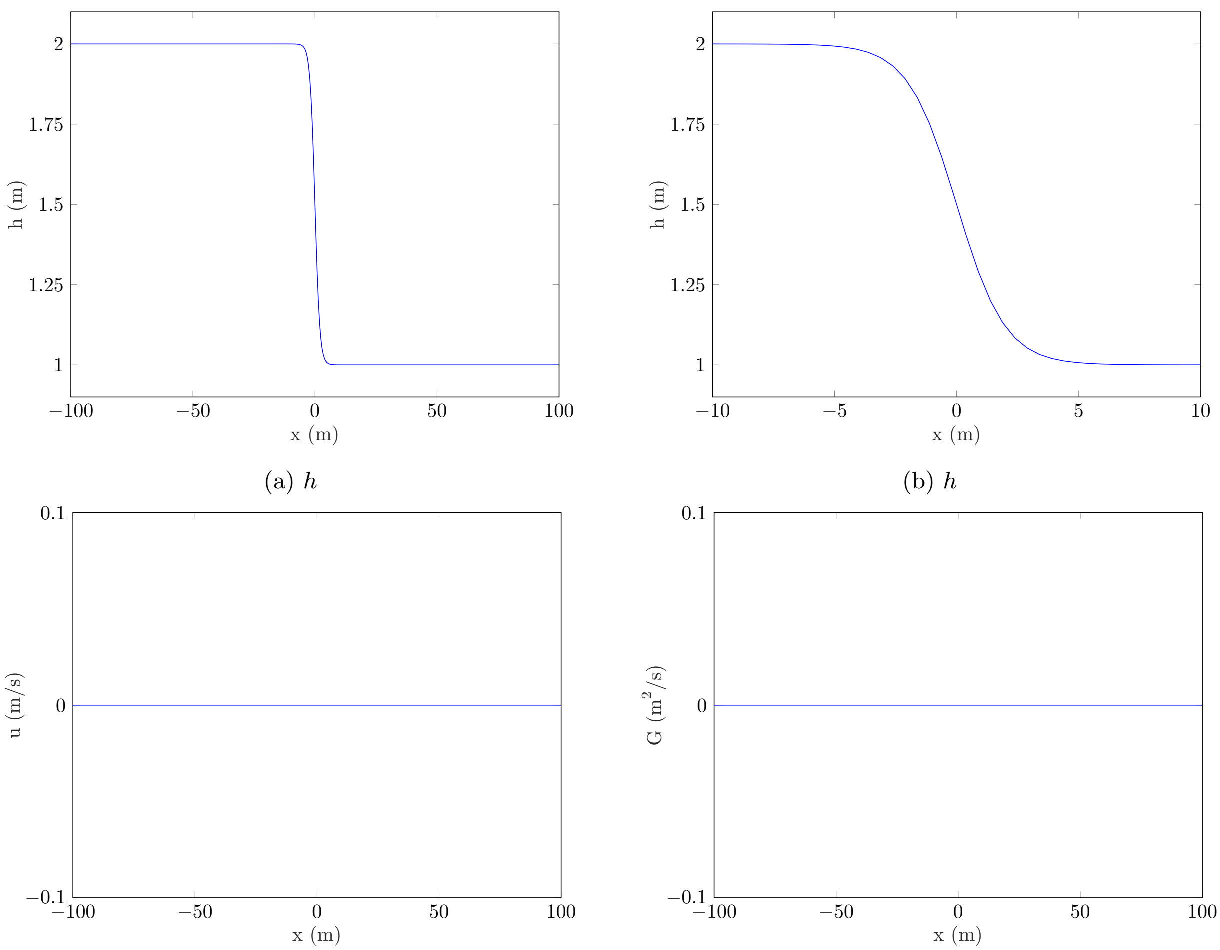


(a) h (b) h

(c) u (d) G

Figure 1: Initial Conditions $\alpha = 2$

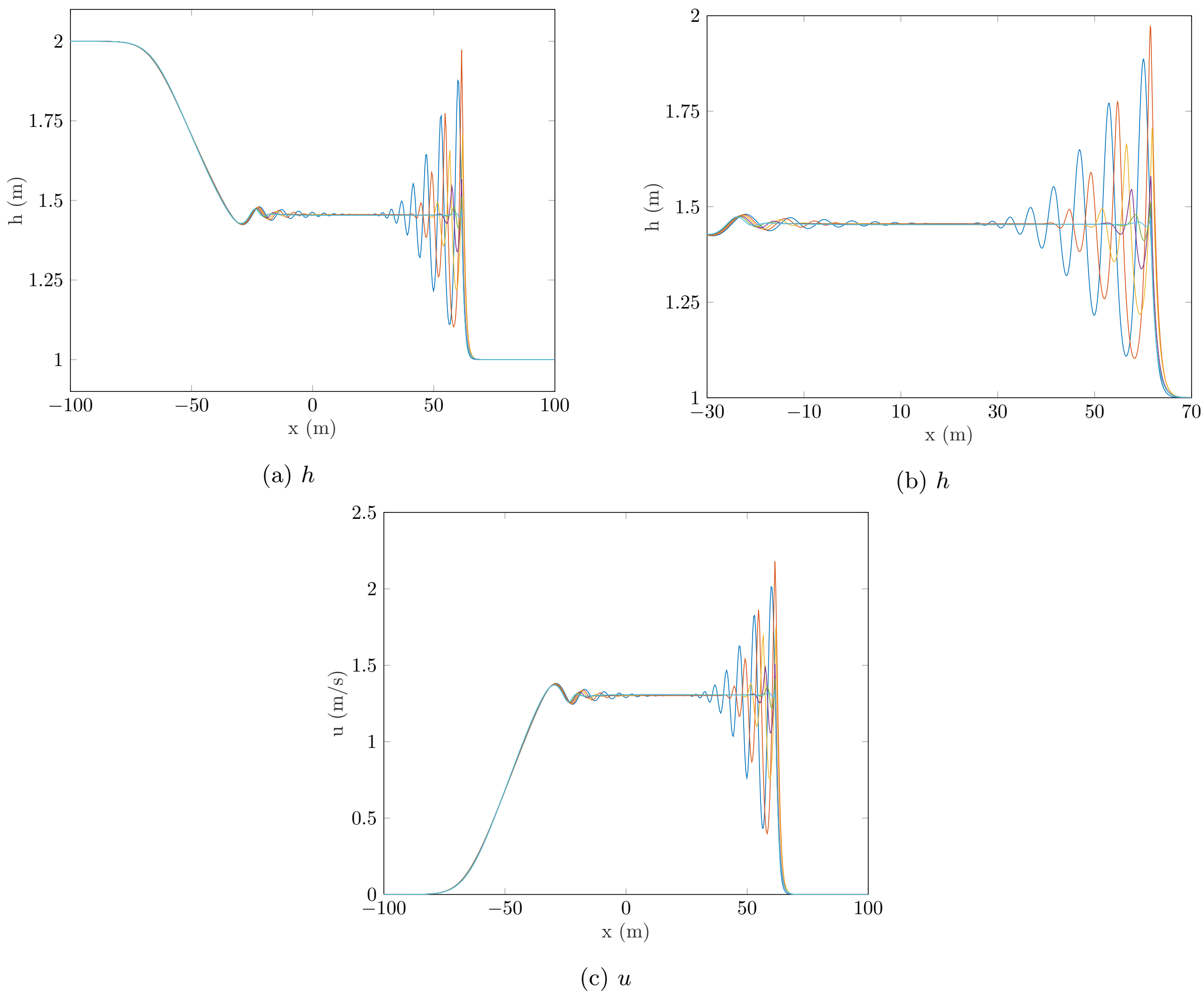


Figure 2: Improve Dispersion Serre Family $\beta_1 = \beta_2$ for smooth dam-break $\alpha = 2$ at $t = 15s$. $\beta_1 = \beta_2 = 0.1$ to $\beta_1 = \beta_2 = 1$.

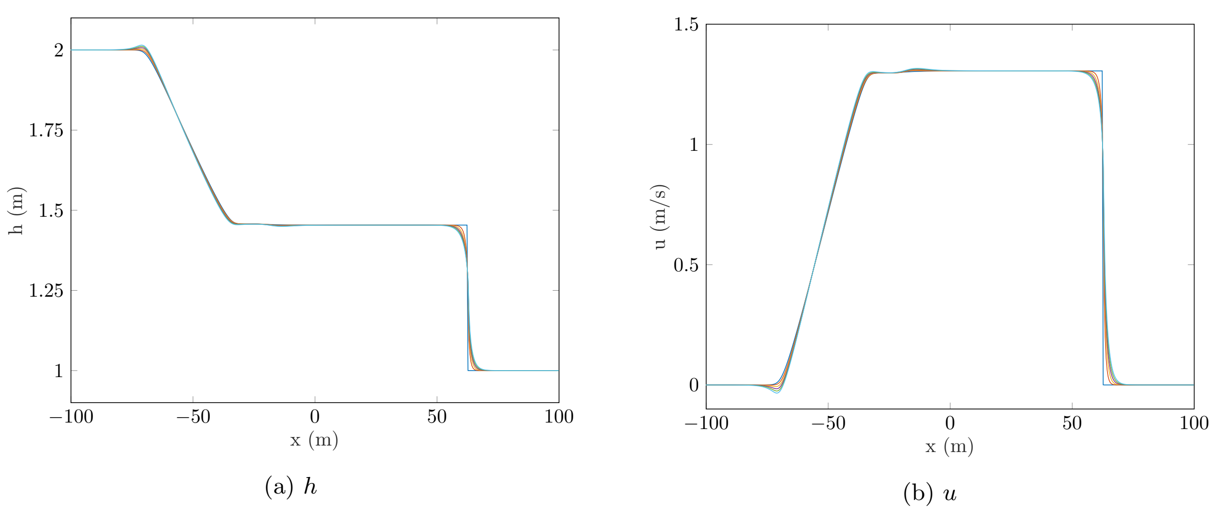
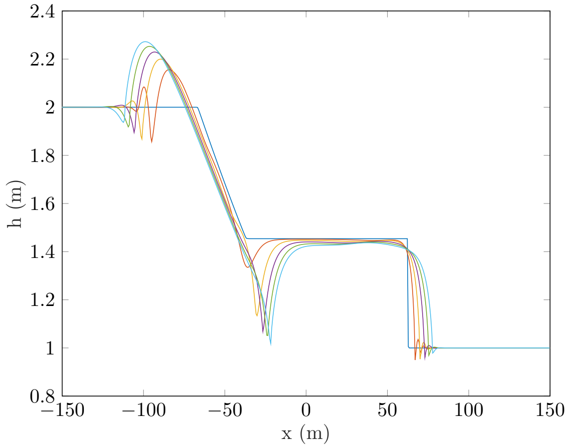
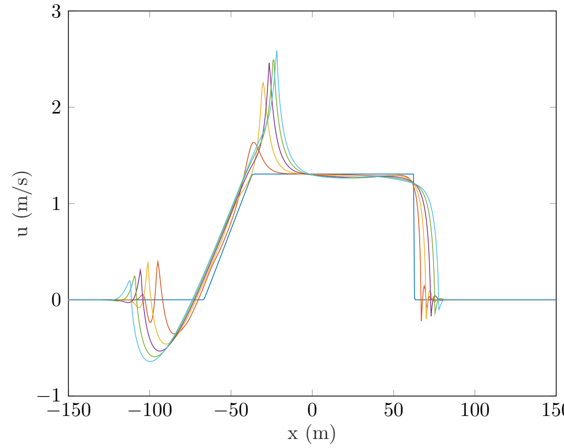


Figure 3: Regularised SWWE Family $\beta_1 = \beta_2 - \frac{2}{3}$ for smooth dam-break $\alpha = 2$ at $t = 15s$. $\beta_1 = \beta_2 - \frac{2}{3}$ with $\beta_2 = 0$ to $\beta_2 = 5$.

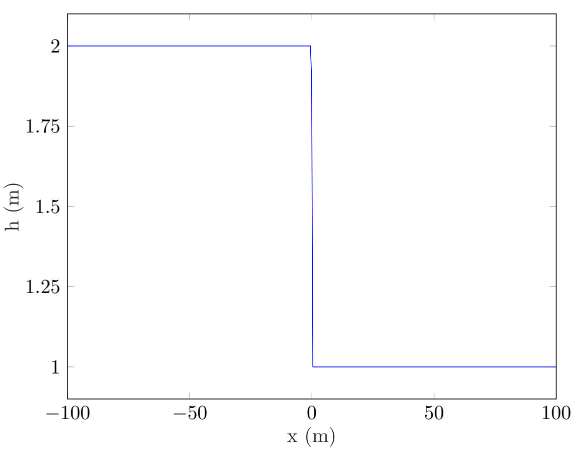


(a) h

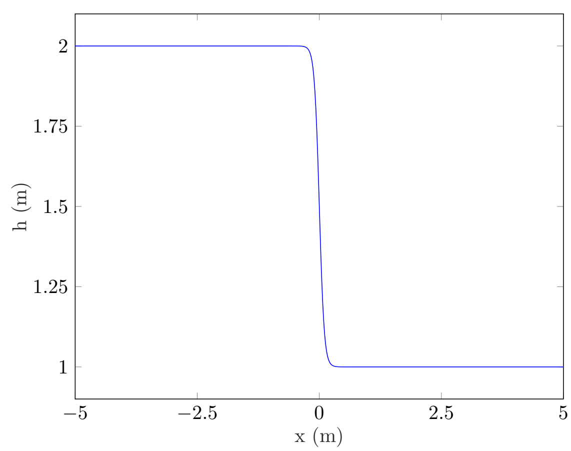


(b) u

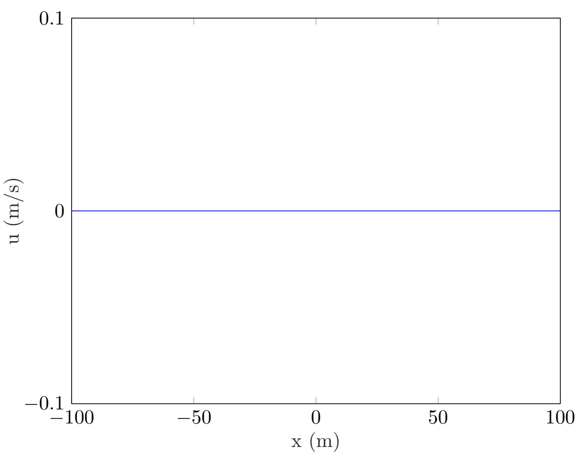
Figure 4: Region 2 Family $\beta_1 = \beta_2 - \frac{2}{3} - k$ and $\beta_2 = 2k$ (should find something better so we get expanding wave train), so $\alpha = 2$ for smooth dam-break $\alpha = 2$ at $t = 15s$.



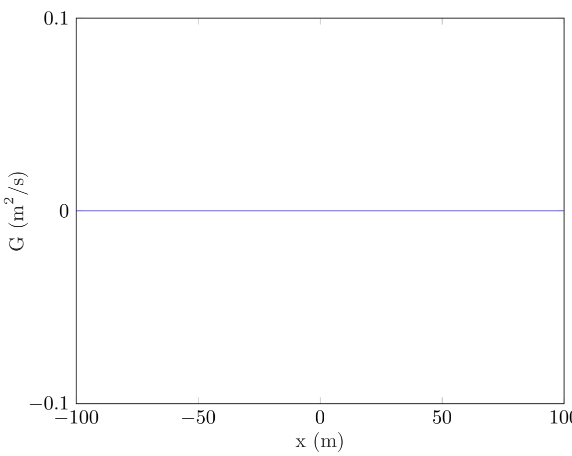
(a) h



(b) h

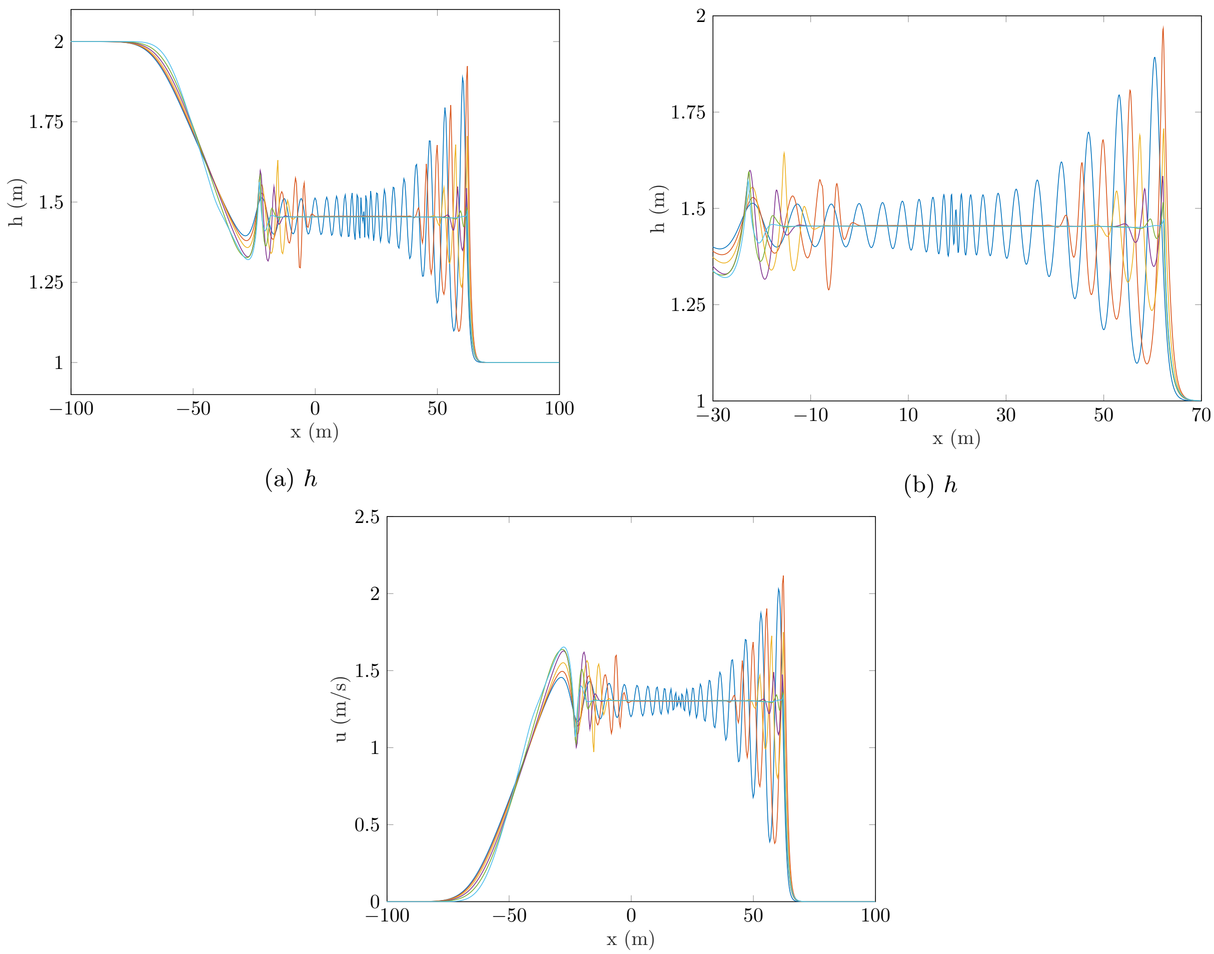


(c) u



(d) G

Figure 5: Initial Conditions $\alpha = 0.1$

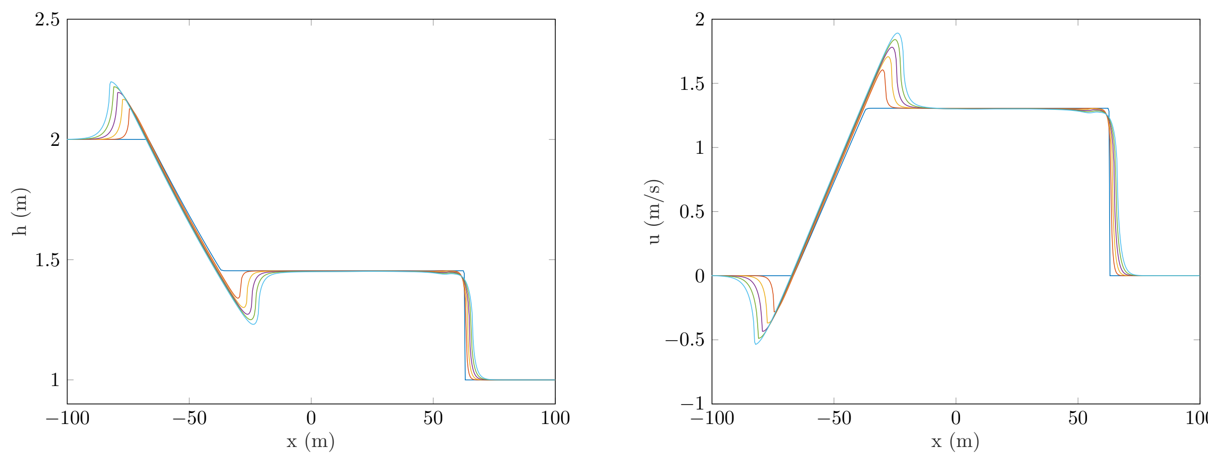


(a) h

(b) h

(c) u

Figure 6: Improve Dispersion Serre Family $\beta_1 = \beta_2$ for smooth dam-break $\alpha = 0.1$ at $t = 15s$. $\beta_1 = \beta_2 = 0.1$ to $\beta_1 = \beta_2 = 1$.



(a) h

(b) u

Figure 7: Improve Dispersion Serre Family $\beta_1 = \beta_2$ for smooth dam-break $\alpha = 2$ at $t = 15s$. $\beta_1 = \beta_2 - \frac{2}{3}$ with $\beta_2 = 0$ to $\beta_2 = 5$.