Non-linear dispersive water wave models

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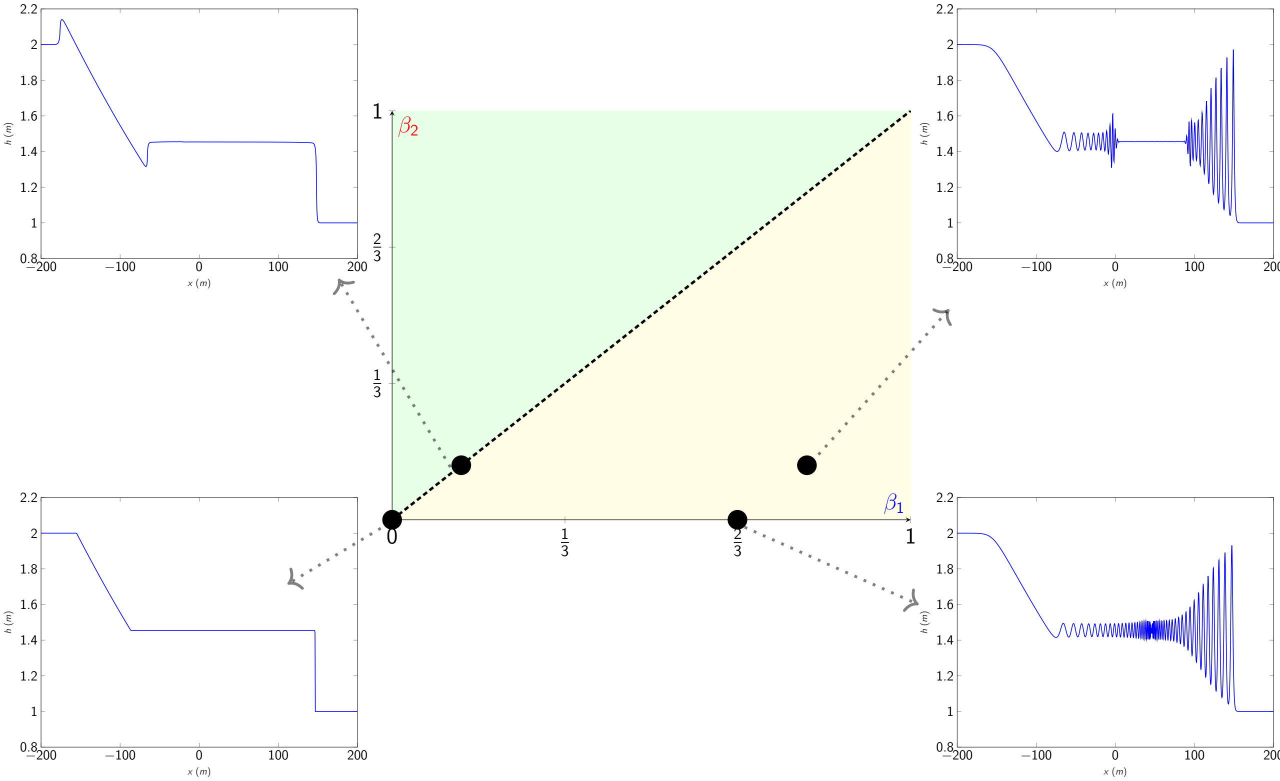

Figure 1: Example numerical solutions for water depth (h) to the dam break problem for the generalised Serre-Green-Naghdi equations

In this talk we discuss the family of equations described by the generalised Serre-Green-Naghdi equations which model water waves using the water depth (h), the depth-averaged horizontal velocity (u) and the acceleration due to gravity (g). There are two free parameters β_1 and β_2 which allow the dispersion properties of the equations to be changed. The generalised Serre-Green-Naghdi equations for one spatial dimension (x) are given by

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0$$
$$\frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x}\left(hu^2 + \frac{1}{2}gh^2 + \beta_1 \Phi - \beta_2 \Psi\right) = 0$$

where

$$\Phi = \frac{h^3}{2}\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial^2 u}{\partial x \partial t} - u\frac{\partial^2 u}{\partial x^2}\right)$$
$$\Psi = \frac{gh^2}{2}\left(h\frac{\partial^2 h}{\partial x^2} + \frac{1}{2}\frac{\partial h}{\partial x}\frac{\partial h}{\partial x}\right)$$

The focus of the talk will be introducing these equations and comparing the numerical solutions to the dam-break problem (stationary Riemann problem) displayed in Figure 1 to the linear theory.