Equations:

Generalised Serre - SWWE equations - ϵ introduced which when $\epsilon = 1$ gives Serre and when $\epsilon = 0$ gives SWWE

They are:

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = 0$$

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial x}\left(Gu + \frac{gh^2}{2} - \epsilon\frac{2h^3}{3}\frac{\partial u}{\partial x}\frac{\partial u}{\partial x}\right) = 0$$

where a new conserved quantity, G is given by

$$G = uh - \epsilon\frac{\partial}{\partial x}\left(\frac{h^3}{3}\frac{\partial u}{\partial x}\right).$$

I forced a solution by introducing h^*,u^*,G^* and solving the forced version of these equations:

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = \frac{\partial h^*}{\partial t} + \frac{\partial (u^* h^*)}{\partial x}$$

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial x}\left(Gu + \frac{gh^2}{2} - \epsilon\frac{2h^3}{3}\frac{\partial u}{\partial x}\frac{\partial u^*}{\partial x}\right) = \frac{\partial G^*}{\partial t} + \frac{\partial}{\partial x}\left(G^* u^* + \frac{g(h^*)^2}{2} - \epsilon\frac{2(h^*)^3}{3}\frac{\partial u^*}{\partial x}\frac{\partial u^*}{\partial x}\right)$$

where RHS is calculated analytically and LHS is approximated numerically using FDVM_2 (second-order finite difference volume method)

I used

$$h^* = a_0 + a_1 \exp\left[\frac{(x - a_2 t)^2}{2a_3}\right]$$

$$u^* = a_4 \exp\left[\frac{(x - a_2 t)^2}{2a_3}\right]$$

$$G^* = u^* h^* - \epsilon\frac{\partial}{\partial x}\left(\frac{(h^*)^3}{3}\frac{\partial u^*}{\partial x}\right).$$

With $x \in [-50, 100]$

Number of cells varied like so $n = 100 \times 2^k$ with $k \in [0, 9]$

$$\Delta x = \frac{100 - (-50)}{n}$$

$$\frac{\Delta t}{\Delta x} = \frac{Cr}{a_2 + a_4 + \sqrt{g(a_0 + a_1)}}$$

$Cr = 0.5,\ g = 9.81,\ a0 = 1.0,\ a1 = 1.0,\ a2 = 5.0,\ a3 = 10.0,\ a4 = 1.0$

Results:

I have plotted results for various ϵ values together to demonstrate method can handle both extremes (SWWE and Serre) as well as inbetween values.

Example Solutions

Convergence with second order slope shown

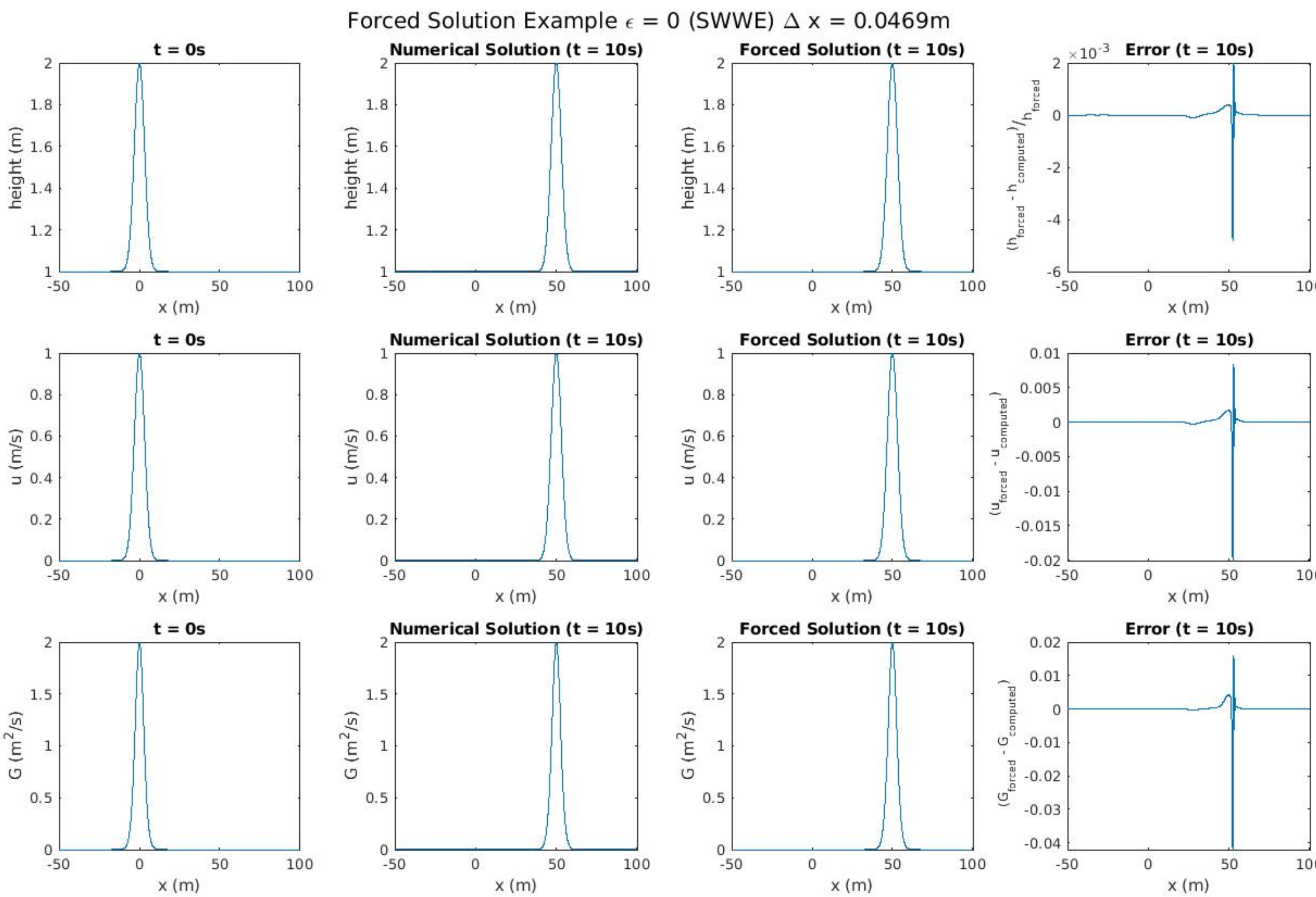

Figure 1: $\epsilon = 0$ (SWWE)

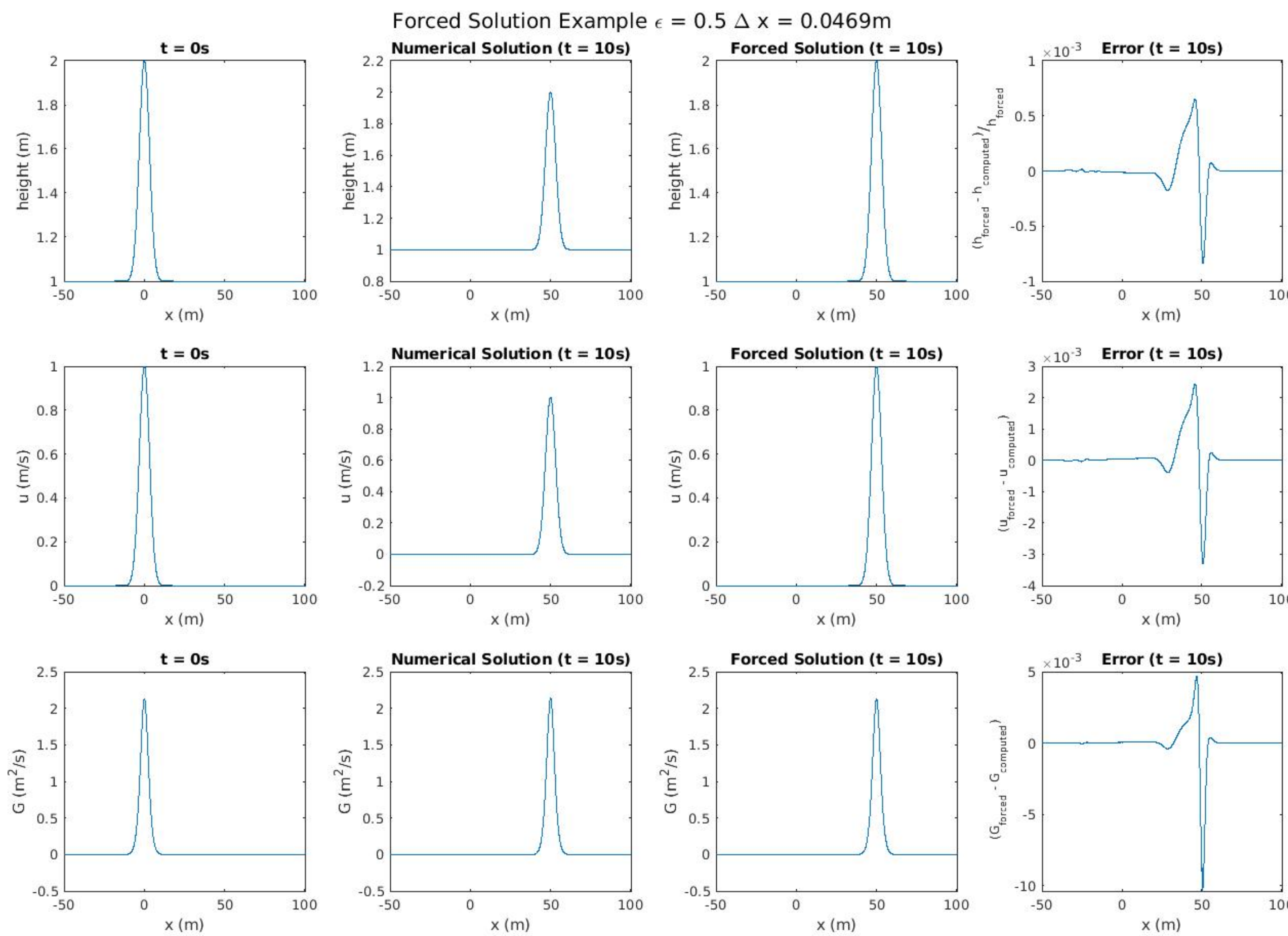

Figure 2: $\epsilon = 0.5$

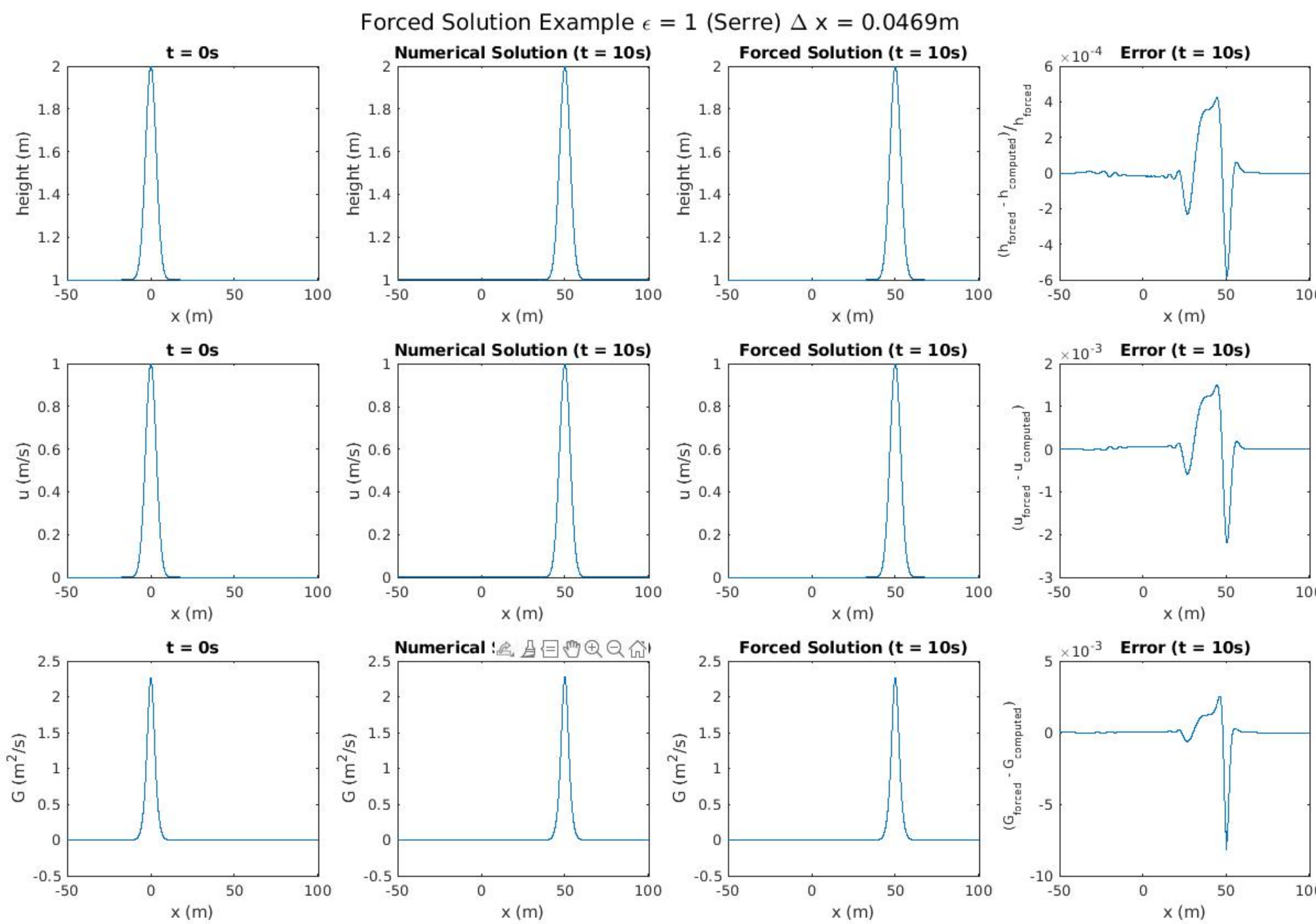

Figure 3: $\epsilon = 1$ (Serre)

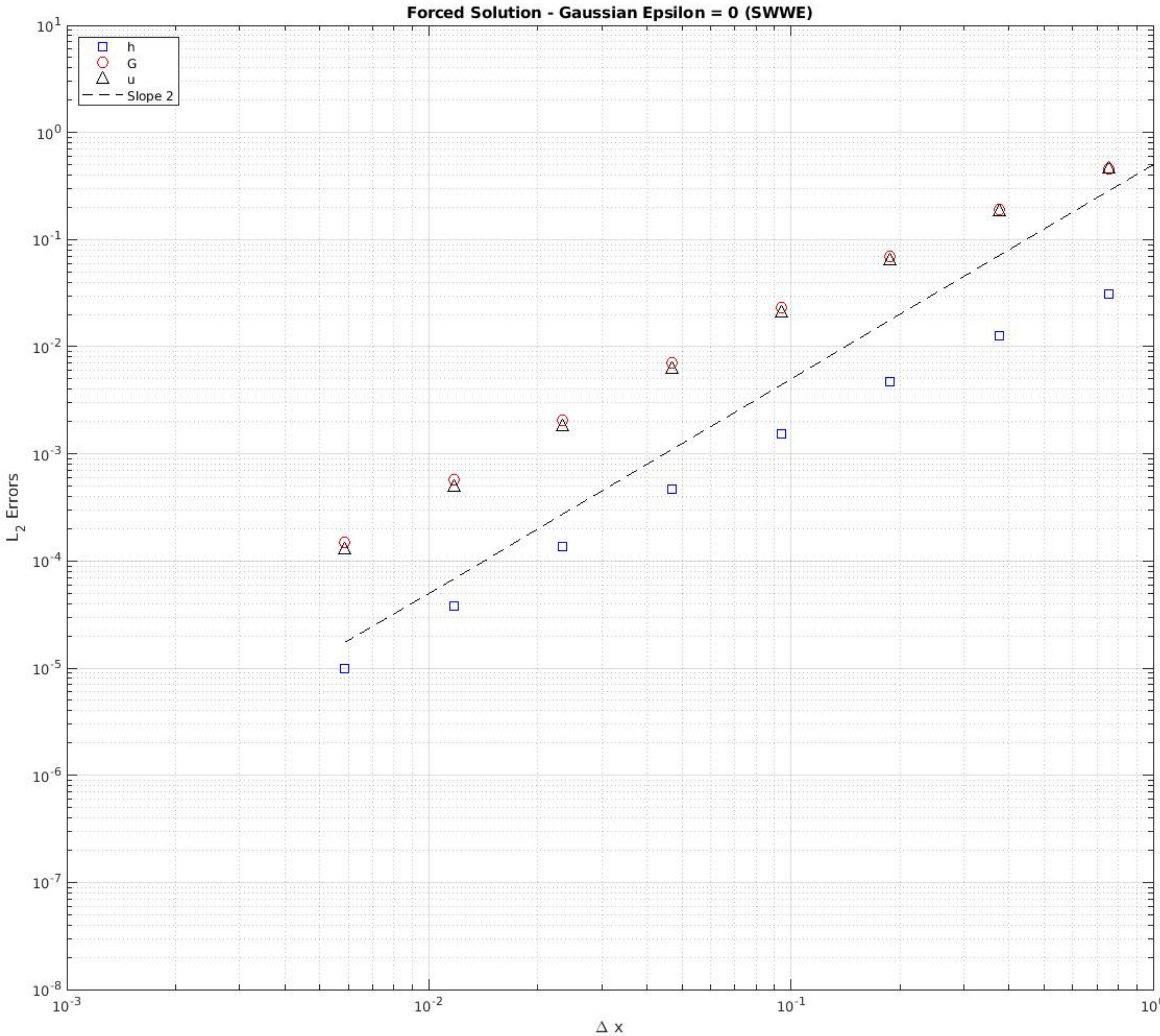

Figure 4: $\epsilon = 0$ (SWWE)

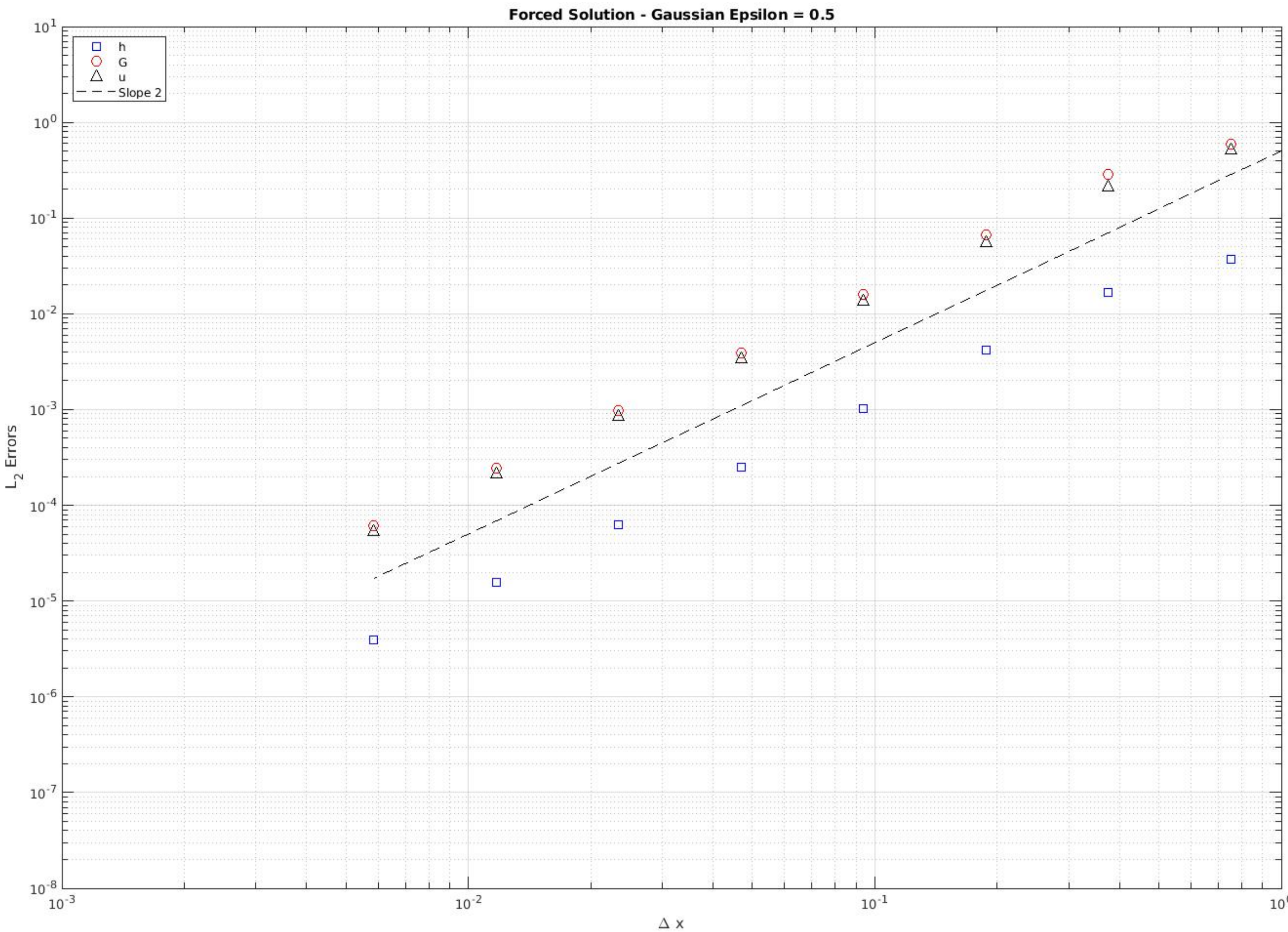

Figure 5: $\epsilon = 0.5$

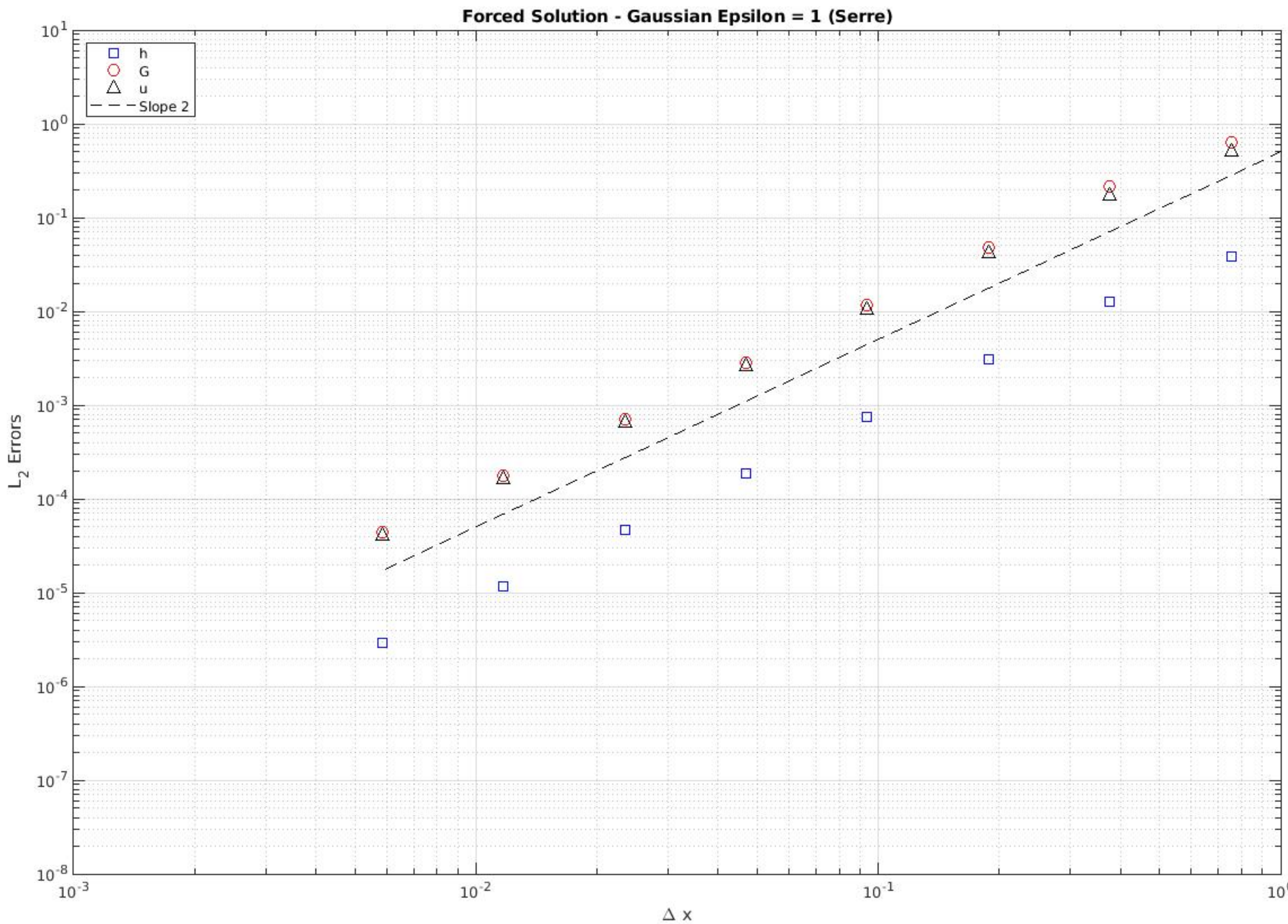

Figure 6: $\epsilon = 1$ (Serre)