

The regularised Serre equations are

$$h_t + (uh)_x = 0 \quad (1a)$$

$$(uh)_t + \left( u^2 h + \frac{gh^2}{2} + \epsilon h^2 \left[ h \left[ (u_x)^2 - uu_{xx} - u_{xt} \right] - g \left( h \frac{\partial^2 h}{\partial x^2} + \frac{1}{2} \frac{\partial h}{\partial x} \frac{\partial h}{\partial x} \right) \right] \right)_x = 0 \quad (1b)$$

Lets group time derivatives

$$h_t + u_x h + uh_x = 0 \quad (2a)$$

$$(uh)_t + 3\epsilon h^2 h_x u_{xt} + \epsilon h^3 u_{xtx} + \left( u^2 h + \frac{gh^2}{2} + \epsilon h^2 \left[ h \left[ (u_x)^2 - uu_{xx} - u_{xt} \right] - g \left( h \frac{\partial^2 h}{\partial x^2} + \frac{1}{2} \frac{\partial h}{\partial x} \frac{\partial h}{\partial x} \right) \right] \right)_x = 0 \quad (2b)$$

equivalently

$$h_t + u_x h + uh_x = 0 \quad (3a)$$

$$u_t + uu_x + gh_x + \frac{\epsilon}{h} \left( h^3 \left[ (u_x)^2 - uu_{xx} - u_{xt} \right] - gh^2 \left( h \frac{\partial^2 h}{\partial x^2} + \frac{1}{2} \frac{\partial h}{\partial x} \frac{\partial h}{\partial x} \right) \right)_x = 0 \quad (3b)$$

Obvious way:

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = 0 \quad (4a)$$

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial x} \left[ uG + \frac{gh^2}{2} - \epsilon h^2 \left( 2h \frac{\partial u}{\partial x} \frac{\partial u}{\partial x} + gh \frac{\partial^2 h}{\partial x^2} + \frac{g}{2} \frac{\partial h}{\partial x} \frac{\partial h}{\partial x} \right) \right] = 0 \quad (4b)$$

where

$$G = uh - \epsilon \frac{\partial}{\partial x} \left( h^3 \frac{\partial u}{\partial x} \right) \quad (5)$$

Lets see

$$\frac{\partial G}{\partial x} = \frac{\partial(uh)}{\partial x} - \epsilon \frac{\partial^2}{\partial x^2} \left( h^3 \frac{\partial u}{\partial x} \right) \quad (6)$$

Then

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \left[ G + \epsilon \frac{\partial}{\partial x} \left( h^3 \frac{\partial u}{\partial x} \right) \right] = 0 \quad (7)$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} \left[ G + \epsilon \left[ 3h^2 h_x u_x + h^3 u_{xx} \right] \right] = 0 \quad (8)$$

$$\frac{\partial h}{\partial t} + G_x + \epsilon \left[ \frac{\partial}{\partial x} \left[ G + \epsilon \left[ 3h^2 h_x u_x + h^3 u_{xx} \right] \right] \right] = 0 \quad (9)$$