Lemma : For $a \geq 0$, $b \geq 0$ and $x \geq 0$ we have that (all quantities are reals)

$$f(x) = \frac{ax^2 + 2}{bx^2 + 2} \tag{1}$$

$$f(x) = 1 + \frac{(b-a)\,x^2}{bx^2 + 2} \tag{2}$$

$$f(x) = 1 + \frac{(b-a)}{b + \dfrac{2}{x^2}} \tag{3}$$

is a montone function. We know that $\frac{2}{x^2}$ is monotone decreasing when $x \geq 0$.
Proof:

$$x_0 \leq x_1 \implies x_0^2 \leq x_1^2 \implies \frac{2}{x_1^2} \leq \frac{2}{x_0^2}$$

It then follows that $b + \frac{2}{x^2}$ is monotone decreasing when $x \geq 0$. Then $\dfrac{1}{b + \frac{2}{x^2}}$ is monotone increasing. Since $x_0 \leq x_1 \implies \dfrac{1}{b + \frac{2}{x_0^2}} \geq \dfrac{1}{b + \frac{2}{x_1^2}}$, due to inequality flipping for division.

Then if $a \leq b$ it follows that $f(x)$ is monotone increasing whilst if $a \geq b$ then $f(x)$ is monotone decreasing. With the special case of $a = b$ giving the constant function $f(x) = 1$.