Arbitrarily High Order TVD Reconstruction Polynomials

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1 Goal

We are going to use the following notation for all polynomials

$$P_i^r(x) = a_r^{(r)}(x - x_i)^r + a_r^{(r-1)}(x - x_i)^{r-1} + \cdots + a_r^{(0)}$$

Family of polynomials: B_i^0, B_i^1,B_i^2,B_i^3,... B_i^n

Such that over all the cells to match cell averages with that do not contribute to the cell averages.

Let's assume that we start at cell j and we keep adding points to the left we then would want

$$B_i^1 = a_1^{(1)}(x - x_i)^1 - \frac{1}{x_{j+1/2} - x_{j-1/2}} \int_{x_{j-1/2}}^{x_{j+1/2}} a_1^{(1)}(x - x_i)^1 dx$$

So that

$$\int_{x_{j-1/2}}^{x_{j+1/2}} B_i^1 \; dx = \int_{x_{j-1/2}}^{x_{j+1/2}} a_1^{(1)}(x - x_i)^1 \; dx - \frac{x_{j+1/2} - x_{j-1/2}}{x_{j+1/2} - x_{j-1/2}} \int_{x_{j-1/2}}^{x_{j+1/2}} a_1^{(1)}(x - x_i)^1 dx = 0$$

Then building to the left we would have

$$B_i^2 = a_2^{(2)}(x - x_i)^2 - c_1(x - x_i) - c_0$$

Where

$$\int_{x_{j-1/2}}^{x_{j+1/2}} B_i^2 \; dx = 0$$

and

$$\int_{x_{j-3/2}}^{x_{j-1/2}} B_i^2 \; dx = 0$$

I have the first group of solutions for this problem (admittedly made simpler for having equally sized cells). Would be nice to have these in more general form, anyway

$$B_i^1 = a_1^{(1)}(x - x_i)^1$$

Quadratics

$$B_{i-1,i}^2 = a^{(2)}(x - x_i)^2 + a^{(2)}\Delta x(x - x_i) - \frac{a^{(2)}\Delta x^2}{12}$$

$$B_{i,i+1}^2 = a^{(2)}(x - x_i)^2 - a^{(2)}\Delta x(x - x_i) - \frac{a^{(2)}\Delta x^2}{12}$$

Cubics

$$B_{i-2,i}^3 = a^{(3)}(x - x_i)^3 + 3a^{(3)}\Delta x(x - x_i)^2 + \frac{7a^{(3)}\Delta x^2}{4}(x - x_i) - \frac{a^{(3)}\Delta x^3}{4}$$

$$B_{i,i+2}^3 = a^{(3)}(x - x_i)^3 - 3a^{(3)}\Delta x(x - x_i)^2 + \frac{7a^{(3)}\Delta x^2}{4}(x - x_i) + \frac{a^{(3)}\Delta x^3}{4}$$

So we have the hieracrhy:
Leftwards : B_i^1, $B_{i-1,i}^2$, $B_{i-2,i}^3$
Rightwards : B_i^1, $B_{i,i+1}^2$, $B_{i,i+2}^3$

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