

Numerical Scheme for the Generalised Serre-Green-Naghdi Model

Contents

1	Abstract	1
2	Introduction	1
3	Generalised Serre-Green-Naghdi Equations	3
3.1	Dispersion Relation of Linearised gSGN	4
3.1.1	Wave Speed Bounds	4
3.2	Alternative Conservative Form of the gSGN	5
4	Numerical Method	6
4.1	Overview	6
4.2	Example Implementation	6
4.2.1	Reconstruction of h and G	7
4.2.2	Reconstruction of u	7
4.2.3	Flux Approximation	7
4.2.4	Evolution Step	7
4.2.5	Runge-Kutta Time Stepping	7
5	Validation	7
5.1	Analytic Solutions	7
5.1.1	Serre Equations ($\beta_1 = \beta_2 = 0$) - Solitary Travelling Wave Solution	7
5.1.2	SWWE ($\beta_1 = -\frac{2}{3}$ and $\beta_2 = 0$) - Dam-break Solution	9
5.2	Forced Solutions	12
6	Conclusion	14

Papers Primary focus - to extend the approach outlined in Zoppou,Hank to the gSGN equations. This achieves the following goals:

- Extension of numerical scheme (elliptic + conservation solvers) of previous papers for SGN to gSGN
- Straightforward 2nd order implementation - demonstrating the use of this method
- Validation of the example implementation using analytic solutions for SWWE and SGN (dam-break and soliton) and forced solutions for more general members (only showing a select few)
- Method is promising : can handle discontinuities, maintain order of accuracy. However, there are additional challenges (weak discontinuities) and limiting of derivative which will be investigated in another paper.

1. Abstract

2. Introduction

In previous work we have developed and validated numerical methods for a conservative reformulation of the Serre-Green-Naghdi (SGN) equations [1, 2, 3, 4]. These numerical methods have all been based on solving an elliptic equation containing only spatial derivatives, to obtain all the primitive variables in the conservation equation, and then updating the conservation equation using a finite volume method. The central benefit of