NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOMKAMPUS

**Benodigdhede vir hierdie vraestel:/Requirements for this paper:**

| | | | |
|---|---|---|---|
| Multikeusekaarte:/ Multi-choice cards: | ☐ | Nie-programmeerbare sakrekenaar:/ Non-programmable calculator: | ☒ |
| Grafiekpapier:/ Graph paper: | ☐ | Skootrekenaar:/ Laptop: | ☐ |

Word ander hulpmiddels toegelaat? / Are other resources allowed? NEE/NO

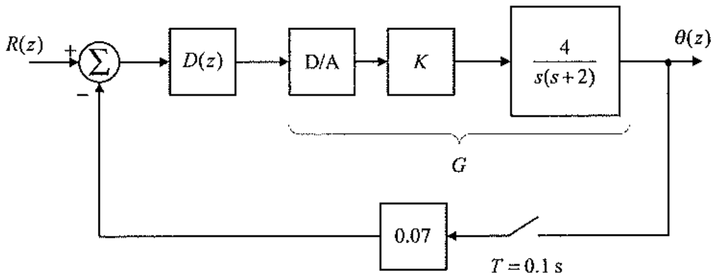
| | | | |
|---|---|---|---|
| SEMESTERTOETS: | 3 | KWALIFIKASIERIGTING: | **B.Ing. / B.Eng.** |
| MODULEKODE: | **EERI 418** | DUUR: | 1.5 URE / HOURS |
| MODULE BESKRYWING: | BEHEERTEORIE II | MAKS / MAX: | 37 |
| EKSAMINATOR: | PROF. K.R. UREN | DATUM: | **19/05/2016** |
| MODERATOR: | PROF. G. VAN SCHOOR | TYD: | **14:00** |

**TOTAAL / TOTAL: 37**

## Vraag 1 / Question 1

Beskou die beheerstelsel in Figuur 1. Die stelsel het 'n monsterperiode van $T = 0.1$ s en $D(z) = 1$. Verder word dit ook gegee dat: / *Consider the control system in Figure 1. This system has a sampling period of* $T = 0.1$ *s and* $D(z) = 1$. *It is also given that:*

$$G(z) = \mathcal{Z}\left[\frac{1-e^{-Ts}}{s}\frac{4K}{s(s+2)}\right] = K\frac{0.01873z + 0.01752}{(z-1)(z-0.8187)}$$




Figuur 1 / *Figure 1*

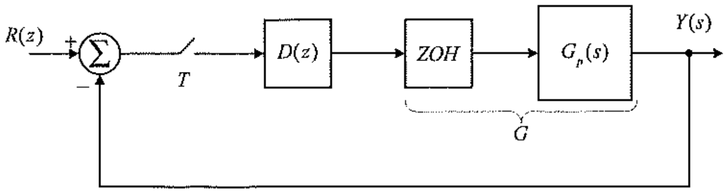
(1.1) Skryf die geslotelus stelsel karakteristieke vergelyking neer. / *Write down the closed-loop system characteristic equation.* (2)

(1.2) Gebruik die Routh-Hurwitz kriterium om te bepaal vir watter waardes van $K$ sal die stelsel stabiel wees. / *Use the Routh-Hurwitz criterion to determine the range of* $K$ *for stability.* (5)

(1.3) Herhaal (1.2) maar gebruik Jury se stabiliteits toets. / *Repeat (1.2) but make use of Jury's stability test.* (5)

**[12]**

## Vraag 2 / Question 2




Figuur 2 / *Figure 2*

Die stelsel in figuur 2 het die volgende oordragsfunksie: / *The system in figure 2 has the following transfer function:*

$$G_p = \frac{10K}{s(s+3)}$$

Vir $K = 1$, is die diskrete oordragsfunksie van die stelsel soos volg: / *For $K = 1$, the discrete transfer function of the system is as follows:*

$$G(z) = \frac{(0.00527z + 0.0051)}{z^2 - 1.906z + 0.9057} \quad T = 0.033 \text{ s}$$

(2.1) Verstel $K$ na 10 om die bestendige toestand foutvereiste te bevredig./ *Adjust $K$ to 10 to satisfy the steady state error requirement.*
Ontwerp 'n fasevoorloopnetwerk $D(z)$ in die bodediagramvlak wat 'n fasegrens van 30° tot gevolg sal hê. / *Design a phase lead compensator D(z) in the bode diagram plane that will give a phase margin of* 30°. Figuur 3 toon die bodediagram van $G(j)$ vir $K = 1$. / *Figure 3 shows the bode diagram of $G(j\omega)$ for $K = 1$.* (15)

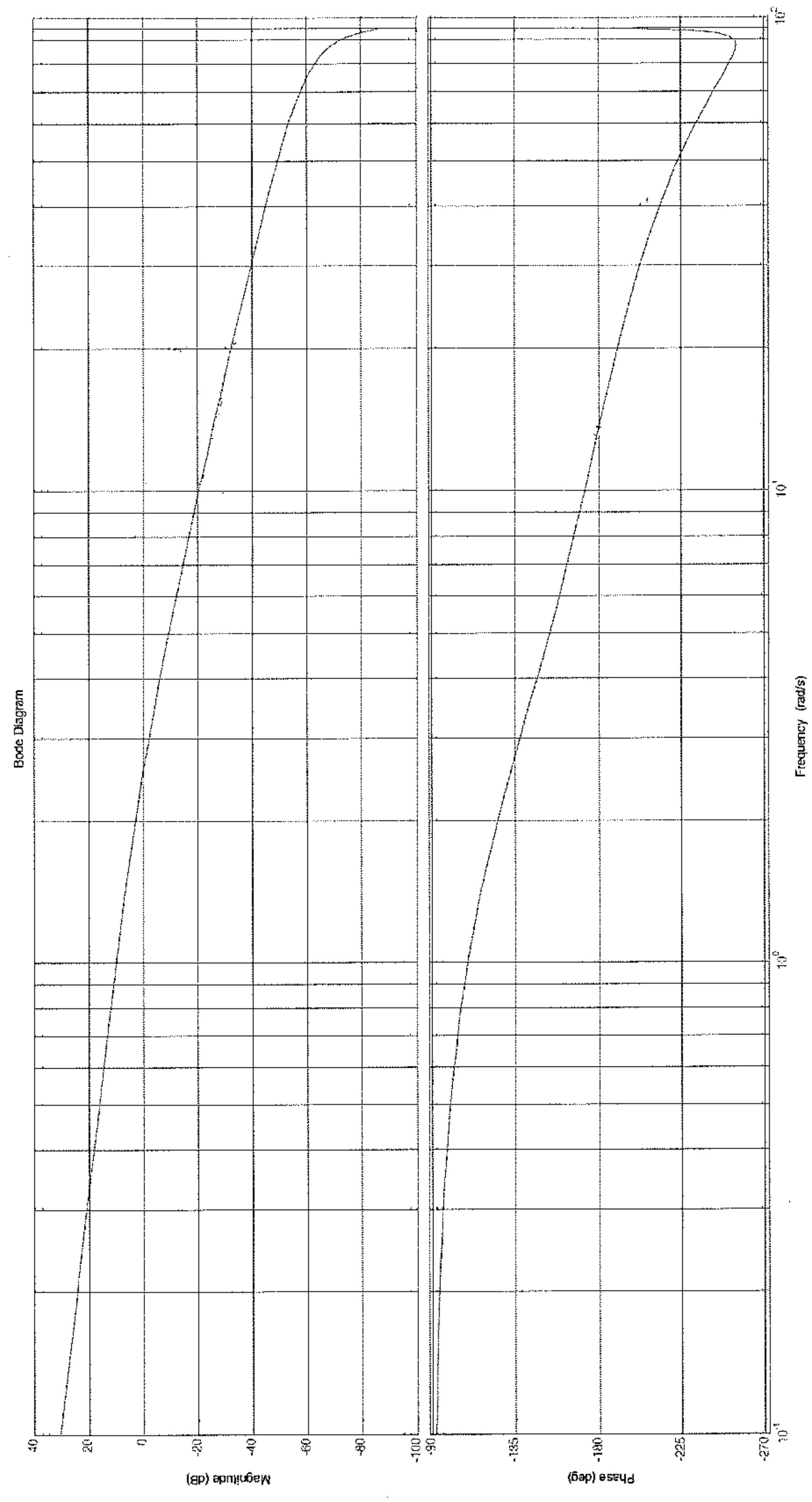
(2.2) Die stelsel moet bedryf word by die punt van kritiese demping. Teken die benaderde wortellokus van die stelsel en bepaal die wins van die stelsel vir krities gedempte pole. /
*The system must be operated at the point of critical damping. Draw the approximate root locus of the system and determine the gain of the system for crytically damped poles.*
Ontwerp 'n fase-naloopkompensator wat die krities gedempte poolposisies behou, maar die bestendige toestand fout van die stelsel met 'n faktor 4 verminder./
*Design a phase lag compensator that will retain the crytically damped pole positions, but reduce the steady state error of the system by a factor 4.* (10)

**[25]**

Addisionele inligting / *Additional information:*

$$D(w) = a_0\left[\frac{1 + w/(a_0/a_1)}{1 + w/(1/b_1)}\right], \quad a_1 = \frac{1 - a_0|G(j\omega_{w1})|\cos\theta}{\omega_{w1}|G(j\omega_{w1})|\sin\theta}, \quad b_1 = \frac{\cos\theta - a_0|G(j\omega_{w1})|}{\omega_{w1}\sin\theta}$$

$$K_d = a_0\left[\frac{\omega_{wp}(\omega_{w0} + 2/T)}{\omega_{w0}(\omega_{wp} + 2/T)}\right], \quad z_0 = \left[\frac{2/T - \omega_{w0}}{2/T + \omega_{w0}}\right], \quad z_p = \left[\frac{2/T - \omega_{wp}}{2/T + \omega_{wp}}\right]$$




Figuur 3/*Figure 3*

Vraag 2. [25]

2.1 $\angle G(j\omega_{w1}) < -180° + 30° = -150°$

$|G(j\omega_{w1})| < 1$

Kies dus $\omega_{w1} > 4{,}4$ rad/s en $> 9{,}71$ rad/s

Dus $\omega_{w1} = 15$ rad/s.

en $\cos\theta > |G(j\omega_{w1})| = 0{,}43$

$$\theta = 180° + 30° + 183° = 33°$$

$\therefore$ $\cos 33° = 0{,}8387 > 0{,}43$

$$\therefore a_1 = \frac{1 - a_0 |G(j\omega_{w1})| \cos\theta}{\omega_{w1} |G(j\omega_{w1})| \sin\theta} = 0{,}182$$

$$b_1 = \frac{\cos\theta - a_0 |G(j\omega_{w1})|}{\omega_{w1} \cdot \sin\theta} = 0{,}05$$

$\therefore$ $\omega_{wo} = \frac{1}{a_1} = 5{,}4943$ rad/s

$\omega_{wp} = \frac{1}{b_1} = 19{,}99$ rad/s

$$K_d = \frac{19{,}99 \left(5{,}4943 + \frac{2}{0{,}033}\right)}{5{,}4943 \left(19{,}99 + \frac{2}{0{,}033}\right)} = 2{,}984$$

$$z_o = \frac{\frac{2}{0{,}033} - 5{,}4943}{\frac{2}{0{,}033} + 5{,}4943} = 0{,}8338$$

$$z_p = \frac{\frac{2}{0{,}033} - 19{,}99}{\frac{2}{0{,}033} + 19{,}99} = 0{,}5039$$

$$\therefore D(z) = 2{,}984 \frac{(z - 0{,}8338)}{(z - 0{,}5039)}$$

(15)

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2.2 $$G(z) = \frac{k \cdot 0{,}00527\,(z + 0{,}9677)}{(z-1)(z-0{,}9029)}$$

By die punt van kritiese demping

$$z = \frac{1+0{,}9029}{2} = 0{,}952$$

en $$K_u = \frac{|1-0{,}952|\,|0{,}952-0{,}9029|}{0{,}00527\,|0{,}952+0{,}9677|}$$

$$= 0{,}233.$$

$$K_c = 4 \cdot K_u = 0{,}932.$$

$$\therefore\ K_d = \frac{K_u}{K_c} = \frac{1}{4} = 0{,}25$$

Laat $z_p = 0{,}999$

$$\therefore\ z_o = 1 - \frac{1-0{,}999}{0{,}25} = 0{,}996$$

$$\therefore\ D(z) = 0{,}25\,\frac{(z-0{,}996)}{(z-0{,}999)}$$

(10)