NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOMKAMPUS
Fakulteit Ingenieurswese

Benodigdhede vir hierdie vraestel:			
Multikeusekaarte:	☐	Nie-programmeerbare sakrekenaar:	☑
Grafiekpapier:	☐	Draagbare rekenaar:	☐

Oopboek-eksamen: ☐

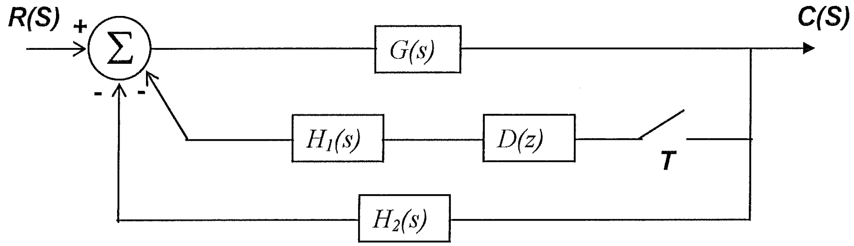
SEMESTERTOETS:	2	GRADE/DIPLOMA:	**B Ing**
VAKKODE:	**EERI 418**	DUUR:	1½ URE
VAK:	BEHEERTEORIE II	MAKS:	40
DOSENT:	PROF. K.R. UREN	DATUM:	**24-04-2015**
		TYD:	**09h00**
MODERATOR:	PROF. G. VAN SCHOOR		

TOTAAL: 40

VRAAG 1/ *QUESTION* 1

Druk *C(z)* in figuur 1 uit as 'n funksie van *R(z)* en die gegewe oordragsfunksies. /

Express C(z) in figure 1 in terms of R(z) and the given transfer functions.



Figuur / *Figure* 1

[10]

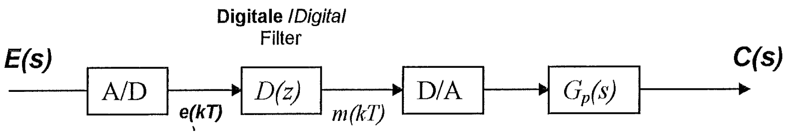
VRAAG 2 / *QUESTION* 2

Die digitale filter in figuur 2 los die volgende verskilvergelyking op: /
The digital filter in figure 2 solves the following difference equation:

$$m(k+2) = e(k) + 1.7m(k+1) - 0.72m(k)$$

Die monsterfrekwensie is 20 Hz. / *The sampling rate is 20 Hz.*
Die aanlegoordragsfunksie word gegee deur: / *The plant transfer function is given by:*

$$G_p(s) = \frac{1}{s+1}$$



Figuur / *Figure* 2

2.1 Bepaal die stelseloordragsfunksie $(\frac{C(z)}{E(z)})$. /

Determine the transfer function of the system $(\frac{C(z)}{E(z)})$. (4)

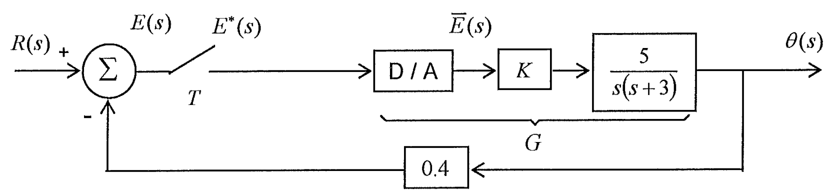
2.2 Bepaal die stelseloordragsfunksie $(\frac{C(z)}{E(z)})$ indien die verwerkingstyd van die digitale filter van 0.051s ook gemodelleer moet word. /

Determine the system transfer function $(\frac{C(z)}{E(z)})$ when a computational delay of 0.051s also needs to be modelled.

(6)

[10]

VRAAG 3 / *QUESTION 3*



Figuur / *Figure 3*

Beskou die stelsel in figuur 3. / *Consider the system in figure 3.*

3.1 Bepaal die stelseloordragsfunksie $(\frac{\theta(z)}{R(z)})$ in terme van *G(z)*. /

Determine the system transfer function $(\frac{\theta(z)}{R(z)})$ *in terms of G(z).* (1)

3.2 Bepaal die oordragsfunksie vir $K = 10$ en $T = 0.2$ s. Wat is die tipe van die stelsel? /
Determine the transfer function for K = 10 and T = 0.2 s. Find the system type. (5)

3.3 Bepaal die bestendige toestand fout van die diskrete stelsel vir 'n eenheidshellingsinset. /
Determine the steady state error of the discrete system for a unit ramp input. (4)

3.4 Bepaal die demping asook die natuurlike frekwensie van die diskrete stelsel. /
Determine the damping as well as the natural frequency of the discrete system. (5)

3.5 Spreek jou uit oor die sinvollheid van die keuse van die monstertempo. Wat is die effek van die keuse van monstertempo op die respons van die gediskretiseerde stelsel. Maak 'n aanbeveling oor die monstertempo wat die diskretiseringsfout sal minimeer, maar nie die modelleringstyd onnodig sal verleng nie. /

Discuss the meaningfulness of the choice of the sampling rate. What is the effect of this sampling rate on the response of the system Make a recommendation on the sampling rate that would minimise the discretisation error without unnecessarily increasing the modelling time. (5)

Addisionele inligting / *additional information:*

$$\zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}}$$

$$\omega_n = \frac{1}{T}\sqrt{\ln^2 r + \theta^2}$$ [20]

$$\tau = \frac{1}{\zeta\omega_n}$$

Table 1. Properties of the z transform

Sequence	Transform
$e(k)$	$E(z) = \sum_{k=0}^{\infty} e(k) z^{-k}$
$a_1 e_1(k) + a_2 e_2(k)$	$a_1 E_1(z) + a_2 E_2(z)$
$e(k-n)u(k-n); \quad n \geq 0$	$z^{-n} E(z)$
$e(k+n)u(k); \quad n \geq 1$	$z^n \left[E(z) - \sum_{k=0}^{n-1} e(k) z^{-k} \right]$
$\epsilon^{ak} e(k)$	$E(z\epsilon^{-a})$
$ke(k)$	$-z \frac{dE(z)}{dz}$
$e_1(k) * e_2(k)$	$E_1(z) E_2(z)$
$e_1(k) = \sum_{n=0}^{k} e(n)$	$E_1(z) = \frac{z}{z-1} E(z)$

Initial value: $e(0) = \lim_{z \to \infty} E(z)$

Final value: $e(\infty) = \lim_{z \to 1} (z-1) E(z)$, if $e(\infty)$ exists

Table 2. z-transforms

Sequence	z-Transform
$\delta(k-n)$	z^{-n}
1	$\frac{z}{z-1}$
k	$\frac{z}{(z-1)^2}$
k^2	$\frac{z(z+1)}{(z-1)^3}$
a^k	$\frac{z}{z-a}$
ka^k	$\frac{az}{(z-a)^2}$
$\sin ak$	$\frac{z \sin a}{z^2 - 2z \cos a + 1}$
$\cos ak$	$\frac{z(z - \cos a)}{z^2 - 2z \cos a + 1}$
$a^k \sin bk$	$\frac{az \sin b}{z^2 - 2az \cos b + a^2}$
$a^k \cos bk$	$\frac{z^2 - az \cos b}{z^2 - 2az \cos b + a^2}$

Table 3. z-transforms

Laplace transform $E(s)$	Time function $e(t)$	z-Transform $E(z)$	Modified z-transform $E(z,m)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1}+\frac{T}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2}\left[\frac{m^2}{z-1}+\frac{2m+1}{(z-1)^2}+\frac{2}{(z-1)^3}\right]$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim\limits_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$\lim\limits_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{1}{s+a}$	ϵ^{-at}	$\frac{z}{z-\epsilon^{-aT}}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$	$\frac{Tz\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}$	$\frac{T\epsilon^{-amT}[\epsilon^{-aT}+m(z-\epsilon^{-aT})]}{(z-\epsilon^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k\epsilon^{-at}$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-\epsilon^{-at}$	$\frac{z(1-\epsilon^{-aT})}{(z-1)(z-\epsilon^{-aT})}$	$\frac{1}{z-1}-\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1-\epsilon^{-at}}{a}$	$\frac{z[(aT-1+\epsilon^{-aT})z+(1-\epsilon^{-aT}-aT\epsilon^{-aT})]}{a(z-1)^2(z-\epsilon^{-aT})}$	$\frac{T}{(z-1)^2}+\frac{amT-1}{a(z-1)}+\frac{\epsilon^{-amT}}{a(z-\epsilon^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\epsilon^{-at}$	$\frac{z}{z-1}-\frac{z}{z-\epsilon^{-aT}}-\frac{aT\epsilon^{-aT}z}{(z-\epsilon^{-aT})^2}$	$\frac{1}{z-1}-\left[\frac{1+amT}{z-\epsilon^{-aT}}+\frac{aT\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}\right]\epsilon^{-amT}$
$\frac{b-a}{(s+a)(s+b)}$	$\epsilon^{-at}-\epsilon^{-bt}$	$\frac{(\epsilon^{-aT}-\epsilon^{-bT})z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}-\frac{\epsilon^{-bmT}}{z-\epsilon^{-bT}}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z\sin(aT)}{z^2-2z\cos(aT)+1}$	$\frac{z\sin(amT)+\sin(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{s}{s^2+a^2}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2-2z\cos aT+1}$	$\frac{z\cos(amT)-\cos(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\epsilon^{-at}\sin bt$	$\frac{1}{b}\left[\frac{z\epsilon^{-aT}\sin bT}{z^2-2z\epsilon^{-aT}\cos(bT)+\epsilon^{-2aT}}\right]$	$\frac{1}{b}\left[\frac{\epsilon^{-amT}[z\sin bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$\epsilon^{-at}\cos bt$	$\frac{z^2-z\epsilon^{-aT}\cos bT}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$	$\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-\epsilon^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$	$\frac{z(Az+B)}{(z-1)(z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT})}$ $A=1-\epsilon^{-aT}\left(\cos bT+\frac{a}{b}\sin bT\right)$ $B=\epsilon^{-2aT}+\epsilon^{-aT}\left(\frac{a}{b}\sin bT-\cos bT\right)$	$\frac{1}{z-1}$ $-\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$ $+\frac{\frac{a}{b}\{\epsilon^{-amT}[z\sin bmT-\epsilon^{-aT}\sin(1-m)bT]\}}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}+\frac{\epsilon^{-at}}{a(a-b)}$ $+\frac{\epsilon^{-bt}}{b(b-a)}$	$\frac{(Az+B)z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})(z-1)}$	$A=\frac{b(1-\epsilon^{-aT})-a(1-\epsilon^{-bT})}{ab(b-a)}$ $B=\frac{a\epsilon^{-aT}(1-\epsilon^{-bT})-b\epsilon^{-bT}(1-\epsilon^{-aT})}{ab(b-a)}$

Table 4. Laplace transform properties

Name	Theorem
Derivative	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0^+)$
nth-order derivative	$\mathscr{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^+) - \cdots f^{(n-1)}(0^+)$
Integral	$\mathscr{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{F(s)}{s}$
Shifting	$\mathscr{L}[f(t - t_0)u(t - t_0)] = e^{-t_0 s} F(s)$
Initial value	$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$
Final value	$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$
Frequency shift	$\mathscr{L}[e^{-at} f(t)] = F(s + a)$
Convolution integral	$\mathscr{L}^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t - \tau) f_2(\tau)\,d\tau = \int_0^t f_1(\tau) f_2(t - \tau)\,d\tau$

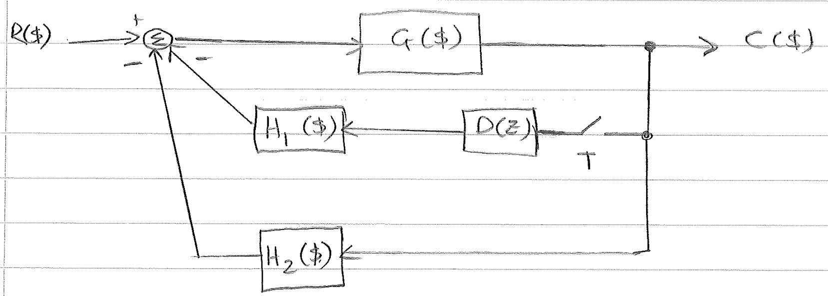
Table 5 Optimum coefficients of T(s) Based on the ITAE Criterion for a Step Input

$$s + \omega_n$$
$$s^2 + 1.4\omega_n s + \omega_n^2$$
$$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$$
$$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$$
$$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$$
$$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.60\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$$

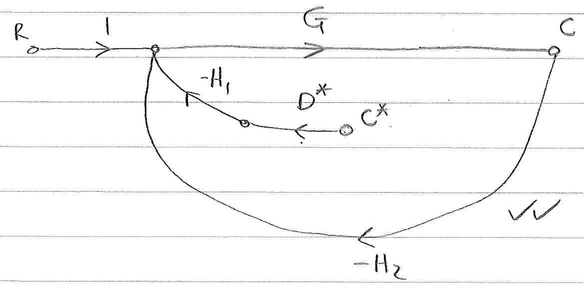
(1)

SEMESTER TEST 2 EERI418 - memo 24-04-2015

Question 1



Signal-flow diagram:



Inputs: R, C*

Outputs: C ✓

✓

$$\therefore \quad C = G\left(R - H_1 D^* C^* - H_2 C\right)$$

$$C + G H_2 C = G R - G H_1 D^* C^* \quad ✓$$

$$C = \frac{GR - GH_1 D^* C^*}{(1 + G H_2)} \quad ✓$$

$$C^* = \left(\frac{GR}{1 + GH_2}\right) - \left(\frac{G_1 H_1}{1 + G H_2}\right) D^* C^*$$

$$C^*\left[1+\left(\frac{GH_1}{1+GH_2}\right)D^*\right] = \left[\frac{\overline{GR}}{1+GH_2}\right]^*$$

$$C^* = \frac{\left(\frac{\overline{GR}}{1+GH_2}\right)^*}{1+\left(\frac{GH_1}{1+GH_2}\right)^* D^*}$$

$$C(z) = \frac{\frac{GR}{1+GH_2}(z)}{1+\frac{GH_1}{1+GH_2}(z)\,D(z)}$$

[10]

Question 2

2.1 ① Determine $D(z)$

$$m(k+2) = e(k) + 1{,}7\,m(k+1) - 0{,}72\,m(k)$$

$$z^2M(z) = E(z) + 1{,}7\,M(z) - 0{,}72\,M(z)$$

$$M(z)\left[z^2 - 1{,}7z + 0{,}72\right] = E(z)$$

$$D(z) = \frac{M(z)}{E(z)} = \frac{1}{z^2 - 1{,}7z + 0{,}72}$$

② Determine the system transfer function $\frac{C(z)}{E(z)}$

$$\frac{C(z)}{E(z)} = D(z)\cdot G(z)$$

$$G(s) = \frac{1-e^{-sT}}{s}\cdot\frac{1}{s+1}$$

$$= (1-z^{-1})\cdot \mathcal{Z}\left[\frac{1}{s(s+1)}\right]$$

$$= \frac{z-1}{z}\;\frac{z(1-e^{-T})}{z-0{,}9512}$$

$T = \frac{1}{20} = 0{,}05\,s$

$$\frac{C(z)}{E(z)} = D(z)\,G(z) = \frac{0{,}0488}{(z^2-1{,}7z+0{,}72)(z-0{,}9512)}$$

[4]

2.2 $\frac{C(z)}{E(z)} = z^{-k} G(z,m) D(z)$ ✓

$t_o = 0{,}051\ s = T + 0{,}02T \quad \therefore \; k = 1 \quad \Delta = 0{,}02$

$m = 0{,}98$

$mT = 0{,}049$ ✓

$$G(z,m) = \mathcal{Z}_m \left[\frac{1 - e^{-sT}}{s(s+1)} \right]$$

$$= \left[1 - z^{-1}\right] \mathcal{Z}_m \left[\frac{1}{s(s+1)} \right]$$ ✓

$$= \frac{z-1}{z} \left[\frac{1}{z-1} - \frac{e^{-mT}}{z - e^{-T}} \right]$$

$$= \frac{z-1}{z} \left[\frac{(z - 0{,}9512) - (z-1)0{,}9522}{(z-1)(z-0{,}9512)} \right]$$

$$= \frac{(0{,}0478 z + 0{,}001)}{z(z - 0{,}9512)}$$ ✓

$$\frac{C(z)}{E(z)} = \frac{(0{,}0478 z + 0{,}001)}{z^2 (z - 0{,}9512)} \cdot \frac{1}{z^2 - 1{,}7z + 0{,}72}$$ ✓ ✓

[6]

Question 3

3.1 $\quad \dfrac{\Theta(z)}{R(z)} = \dfrac{G(z)}{1+0{,}4\,G(z)}$ ✓ (1)

3.2 $\quad G(z) = \mathcal{Z}\left[\dfrac{1-e^{-sT}}{s}\cdot\dfrac{5K}{s(s+3)}\right]$ ✓

$$= \frac{z-1}{z}\cdot 50\cdot\mathcal{Z}\left[\frac{1}{s^2(s+3)}\right]$$ ✓

$$= \frac{z-1}{z}\cdot\frac{50}{3}\cdot\mathcal{Z}\left[\frac{3}{s^3(s+3)}\right]_{T=0{,}2}$$ ✓

$$= \frac{z-1}{z}\cdot\frac{50}{3}\cdot\frac{z\,[0{,}1492+0{,}1219]}{3\,(z-1)^2(z-0{,}549)}$$

$$= \frac{50\times 0{,}149\,(z+0{,}818)}{(z-1)(z-0{,}549)}$$ ✓✓ (5)

3.3 $\quad E(z) = R(z) - 0{,}4\,\Theta(z)$

$$= R(z) - 0{,}4\,G(z)\,E(z)$$

$$E(z) = \frac{R(z)}{1+0{,}4\,G(z)}$$ ✓

$$e(kT)\big|_{k\to\infty} = \lim_{z\to 1}(z-1)\,E(z)$$ ✓

For $\quad R(z) = \dfrac{Tz}{(z-1)^2}$

$$\therefore\ e(kT)\big|_{k\to\infty} = \lim_{z\to 1}(z-1)\frac{Tz}{(z-1)^2}\cdot\frac{1}{1+\dfrac{0{,}4\cdot 0{,}828\,(z+0{,}818)}{(z-1)(z-0{,}549)}}$$ ✓

$$= \frac{T(1-0{,}549)}{0{,}4\cdot 0{,}828\cdot 1{,}818} = 0{,}15$$ ✓ (4)

3.4 Pole van die stelsel is by

$$z^2 - 1{,}2178z + 0{,}82 = 0$$

$$z = \frac{1{,}2178 \pm \sqrt{(1{,}2178)^2 - 4 \cdot 0{,}82}}{2}$$

$$= 0{,}609 \pm j\,0{,}67$$

$$= 0{,}905 \angle \pm 0{,}833$$

$$= r \angle \pm\theta$$

$$\zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}} = \frac{-\ln 0{,}905}{\sqrt{\ln^2 0{,}905 + 0{,}833^2}} = 0{,}119$$

$$\omega_n = \frac{1}{T}\sqrt{\ln^2 r + \theta^2}$$

$$= \frac{1}{0{,}2}\sqrt{\ln^2 0{,}905 + 0{,}833^2}$$

$$= 4{,}19 \text{ rad/s}$$

(5)

3.5 Without discretization, the system transferfunction is given by.

$$T(s) = \frac{KG_p(s)}{1 + 0{,}4 \cdot K\,G_p(s)}$$

$$= \frac{\frac{10 \cdot 5}{s(s+3)}}{1 + 0{,}4\,\frac{10 \cdot 5}{s(s+3)}}$$

$$= \frac{50}{s(s+3) + 20} = \frac{50}{s^2 + 3s + 20}$$

$$\therefore 2\zeta\omega_n = 3 \qquad \therefore \frac{1}{\zeta\omega_n} = \frac{2}{3} = 0{,}67\text{ s} = \tau$$

'n better choice will be $T = \frac{\tau}{10} = 0{,}067$

6.

The sampling rate is too low. The overshoot will be larger than expected.

(5)