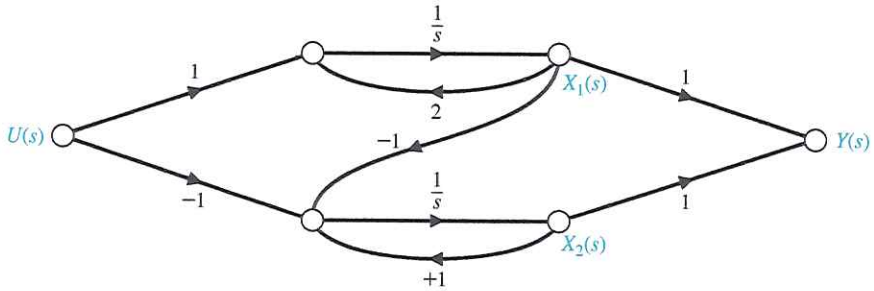
Exercise 2

Consider the following system given by

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u \text{ and } y = \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{x}$$

Check for controllability and observability using the $\mathbf{P}_c$ and $\mathbf{P}_o$ matrices.

Exercise 2



(a)

Check for controllability and observability using $\mathbb{P}_c$ and $\mathbb{P}_o$

$\mathbb{P}_c$:

$$A\mathbb{B} = \underset{2\times2}{\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

$$= \begin{bmatrix} 2+0 \\ -1+-1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\mathbb{P}_c = [\mathbb{B} \quad A\mathbb{B}] = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$\det(\mathbb{P}_c) = (1\times-2) - (2\times-1)$$

$$= -2+2 = 0 \quad \therefore \text{ not controllable.}$$

$\mathbb{P}_o$:

$$\mathbb{C} = [1 \quad 1]$$

$$\mathbb{C}A = \underset{1\times2}{[1 \quad 1]} \underset{2\times2}{\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}} = [(1\times2)+(1\times-1) \quad (1\times0)+(1\times1)]$$

$$= [1 \quad 1]$$

$$\mathbb{P}_o = \begin{bmatrix} \mathbb{C} \\ \mathbb{C}A \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\det(\mathbb{P}_o) = 1\times1 - (1\times1)$$

$$= 0 \quad \therefore \text{ not observable}$$

Note. $y = x_1 + x_2$ → depends on $x_1(0)$ $x_2(0)$ → does not allow us to determine $x_1(0)$ and $x_2(0)$ independantly.

$$\boxed{\begin{aligned} \dot{x}_1 &= 2x_1 + u(t) \\ \dot{x}_2 &= -x_1 + x_2 - u(t) \end{aligned}}$$

$$\dot{x}_1 + \dot{x}_2 = x_1 + x_2$$

⇒ system state variables do not depend on $u(t)$ ⇒ not controllable