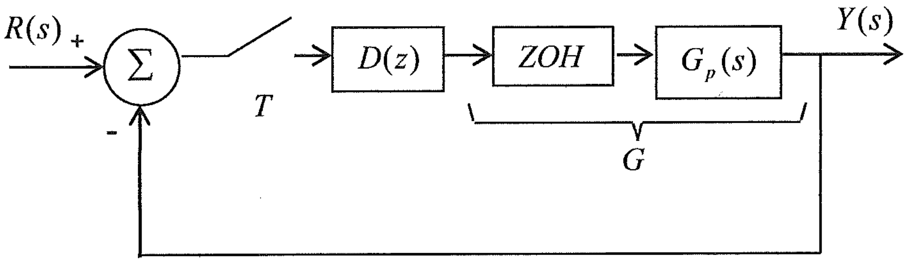
Example of phase lag and phase lead design using Bode diagrams



Figuur / *Figure* 1

Die stelsel in figuur 1 het die volgende oordragsfunksie: /
The system in figure 1 has the following transfer function:

$$G_p(s) = \frac{20K}{s(s+1)(s+4)}$$

Vir K = 1, is die diskrete oordragsfunksie van die stelsel soos volg: /
For K = 1, the discrete transfer function of the system is as follows:

$$G(z) = \frac{3.292\cdot 10^{-6} z^2 + 1.3\cdot 10^{-5} z + 3.211\cdot 10^{-6})}{(z-1)(z-0.99)(z-0.9608)}, \qquad T = 0.01s$$

Figuur 2 toon die bodediagram van G(jω) vir K = 1. /
Figure 2 shows the bode diagram of G(jω) for K = 1.

4.1 Hou K = 1 en ontwerp 'n fasenaloopnetwerk *D(z)* wat 'n fasegrens van 45° tot gevolg sal hê, maar nie die bestendige gedrag van die stelsel sal verander nie. /
Keep K = 1 and design a phase lag compensator D(z) that will give a phase margin of 45° for the system without changing the system's steady state performance.

4.2 Hou K = 1 en ontwerp 'n fasevoorloopnetwerk *D(z)* wat 'n fasegrens van 30° tot gevolg sal hê, maar nie die bestendige gedrag van die stelsel sal verander nie. /
Keep K = 1 and design a phase lead compensator D(z) that will give a phase margin of 30° for the system without changing the system's steady state performance.

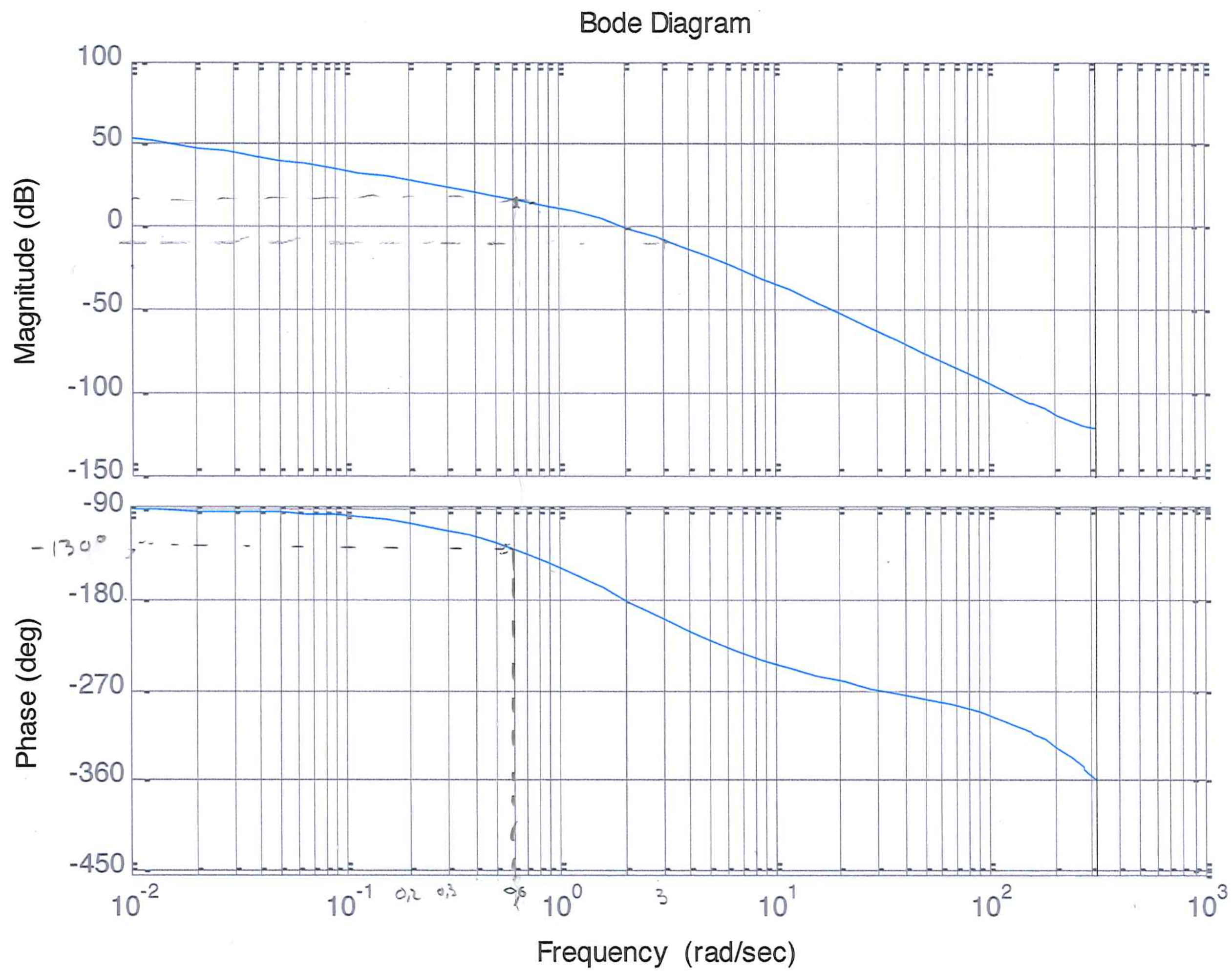
Addisionele inligting / *Additional information:*

$$D(w) = a_0\left[\frac{1 + w/(a_0/a_1)}{1 + w/(1/b_1)}\right]$$

$$a_1 = \frac{1 - a_0|G(j\omega_{w1})|\cos\theta}{\omega_{w1}|G(j\omega_{w1})|\sin\theta}, \quad b_1 = \frac{\cos\theta - a_0|G(j\omega_{w1})|}{\omega_{w1}\sin\theta}$$

[20]

$$K_d = a_0\left[\frac{\omega_{wp}(\omega_{w0} + 2/T)}{\omega_{w0}(\omega_{wp} + 2/T)}\right], \quad z_0 = \left[\frac{2/T - \omega_{w0}}{2/T + \omega_{w0}}\right], \quad z_p = \left[\frac{2/T - \omega_{wp}}{2/T + \omega_{wp}}\right]$$



Figuur / *Figure* 2

Memo

4.1 Determine ω_{w1} at which the phase angle is ;

$$\phi \approx (-180^\circ + \phi_m + 5^\circ)$$
$$\approx -130^\circ$$

$\therefore$ $\omega_{w1} \approx 0{,}63$ rad/sec.

Choose $\omega_{wo} = 0{,}1 \cdot \omega_{w1} = 0{,}063$ rad/sec

Then $$\omega_{wp} = \frac{0{,}1\,\omega_{w1}}{a_0 \left| G(j\omega_{w1}) \right|}$$

To keep steady steady state response requirement

$a_0 = 1$

Then $20 \log \left| G(j\omega_{w1}) \right| = 18$ dB

$$\left| G(j\omega_{w1}) \right| = 10^{18/20} = 7{,}94 \approx 8$$

$$\therefore \omega_{wp} = \frac{0{,}1 \cdot 0{,}63}{8} = 0{,}008 \text{ rad/sec}$$

$$k_d = a_0 \left[\frac{\omega_{wp}(\omega_{wo} + 2/T)}{\omega_{wo}(\omega_{wp} + 2/T)} \right] = 1 \left[\frac{0{,}008\left(0{,}063 + \frac{2}{0{,}01}\right)}{0{,}063\left(0{,}008 + \frac{2}{0{,}01}\right)} \right]$$

$= 0{,}13$

$$z_0 = \frac{2/T - \omega_{wo}}{2/T + \omega_{wo}} = \frac{\frac{2}{0{,}01} - 0{,}063}{2/0{,}01 + 0{,}063} = 0{,}9994$$

$$z_p = \frac{2/T - \omega_{wp}}{2/T + \omega_{wp}} = \frac{2/0{,}01 - 0{,}008}{2/0{,}01 + 0{,}008} = 0{,}9999$$

$$\therefore \quad D(z) = \frac{0{,}13\,(z - 0{,}9994)}{(z - 0{,}9999)} \qquad \square$$

4.2 We start with the requirements for a phase lead:

$$\angle G(j\omega_{w1}) < -180^\circ + 30^\circ$$
$$< -150^\circ$$

and $|G(j\omega_{w1})| < 1$

So choose $\omega_{w1} > 1$ rad/sec

$\therefore$ choose $\omega_{w1} = 3$ rad/sec

and then check.

$$\cos\theta > |G(j\omega_{w1})| = 0{,}464$$

$$\theta = 180^\circ + 30^\circ + 198^\circ = 48^\circ$$

$$\cos 48^\circ = 0{,}67 > 0{,}464$$

$$a_1 = \frac{1 - a_0 |G(j\omega_{w1})| \cos\theta}{\omega_{w1} |G(j\omega_{w1})| \sin\theta}$$

$$= \frac{1 - 1\cdot(0{,}464)\cdot 0{,}67}{3 \cdot 0{,}464 \cdot \sin 48^\circ} = 0{,}666$$

$$b_1 = \frac{0{,}67 - 0{,}464}{3 \sin 48^\circ} = 0{,}0924$$

$$\omega_{wo} = \frac{1}{a_1} = 1{,}5 \text{ rad/sec}$$

$$\omega_{wp} = 1/b_1 = 10{,}8 \text{ rad/sec}$$

$$K_d = \frac{10{,}8\,(1{,}5 + 200)}{1{,}5\,(10{,}8 + 200)} = 6{,}88$$

$$z_o = \frac{200 - 1{,}5}{200 + 1{,}5} = 0{,}985$$

$$z_p = \frac{200 - 10{,}8}{200 + 10{,}8} = 0{,}898$$

$$D(z) = \frac{6{,}88\,(z - 0{,}985)}{(z - 0{,}898)}$$ ∎