

enr48pkjun14 Q1, 2 doodled.

2014 Q1

Critically damped: $\zeta = 1$

$$\zeta = 0.5 = \frac{\gamma}{\omega_n} \Rightarrow \omega_n = 8.$$

$$P_c = [B \quad AB \quad A^2B]$$

$$\begin{aligned} \psi(s) &= s^2 + 2\omega_n s + \omega_n^2 \\ &= s^2 + 16s + 64 \end{aligned}$$

$$= \begin{bmatrix} 0 & 0 & 2 \\ 0 & 1 & -6 \\ 1 & -4 & 16 \end{bmatrix}$$

$$\begin{aligned} \underline{k} &= [0 \ 0 \ 1] P_c^{-1} \psi(A) \\ &= [0.5 \ 0 \ 0] \cdot A^2 + 16A + 64I \end{aligned}$$

$$P_c^{-1} = \begin{bmatrix} 4 & 4 & 1 \\ 3 & 1 & 6 \\ 0.5 & 0 & 0 \end{bmatrix}$$

$$\underline{k} = [33 \quad -4 \quad 32].$$

Q2

$$2.1. \quad x(k) + x(k-1) - x(k-2) = e(k-1) + ze(k)$$

$$X(z) + X(z)z^{-1} - X(z)z^{-2} = E(z)z^{-1} + zE(z)$$

$$X(z)[1 + z^{-1} - z^{-2}] = E(z)[z^{-1} + z]$$

$$\frac{X(z)}{E(z)} = \frac{z^{-1} + z}{1 + z^{-1} - z^{-2}}$$

$$= \frac{z + z^2}{z^2 + z - 1}$$

2.2.