CHAPTER 2

2-1. $e(t) = t$; $\quad E(z) = 0 + Tz^{-1} + 2Tz^{-2} + \cdots = \dfrac{Tz}{(z-1)^2}$

2-2. (a) $E(z) = 1 + \epsilon^{-T}z^{-1} + \epsilon^{-2T}z^{-2} + \cdots$

$$= 1 + (\epsilon^{-T}z^{-1})^1 + (\epsilon^{-T}z^{-1})^2 + \cdots = \frac{1}{1-\epsilon^{-T}z^{-1}} = \underline{\frac{z}{z-\epsilon^{-T}}}$$

(b) $E(z) = 1 + (0.9512\,z^{-1})^1 + (0.9512z^{-1})^2 + \cdots$

$$= \frac{z}{z - 0.9512}$$

(c) $\epsilon^{-bT}\Big|_{T=0.2} = \epsilon^{-0.2b} = 0.5$

$$\therefore -0.2b = \ln(0.5) = -0.6931 \implies b = \underline{-3.466}$$

2-3. (a) $e(t) = \epsilon^{-at} \Rightarrow E(z) = 1 + \epsilon^{-aT}z^{-1} + \epsilon^{-2aT}z^{-2} + \cdots = \underline{\dfrac{z}{z-\epsilon^{-aT}}}$

(b) $e(t) = \epsilon^{-(t-T)}u(t-T)$

$$E(z) = z^{-1} + \epsilon^{-T}z^{-2} + \epsilon^{-2T}z^{-3} + \cdots = z^{-1}\left[\frac{z}{z-\epsilon^{-T}}\right] = \underline{\frac{1}{z-\epsilon^{-T}}}$$

(c) $e(t) = \epsilon^{-(t-5T)}u(t-5T)$

$$E(z) = z^{-5} + \epsilon^{-T}z^{-6} + \epsilon^{-2T}z^{-7} + \cdots = z^{-5}\left[\frac{z}{z-\epsilon^{-T}}\right] = \underline{\frac{1}{z^4(z-\epsilon^{-T})}}$$

2-4. $E_1(s) = \dfrac{2}{s(s+2)} = \dfrac{1}{s} + \dfrac{-1}{s+2}$

$$\therefore e_1(t) = (1-\epsilon^{-2t})u(t) \implies e_1(kT) = (1-\epsilon^{-2kT})u(kT)$$

$$\therefore E_1(z) = (1 + z^{-1} + z^{-2} + \cdots) - (1 - \epsilon^{-2T}z^{-1} + \epsilon^{-4T}z^{-2} + \cdots)$$

$$= \frac{1}{1-z^{-1}} - \frac{1}{1-\epsilon^{-2T}z^{-1}} = \frac{z}{z-1} - \frac{z}{z-\epsilon^{-2T}} = \underline{\frac{(1-\epsilon^{-2})z}{(z-1)(z-\epsilon^{-2})}},\ T=1$$

$$E(z) = E_1(z) - z^{-5}E_1(z) = \frac{(1-\epsilon^{-2})(z^5-1)}{z^4(z-1)(z-\epsilon^{-2})} = \underline{\frac{0.8647(z^5-1)}{z^4(z-1)(z-0.1353)}}$$

2-5. (a) $e(k) = \displaystyle\sum_{\text{residues}} \frac{0.1z^{k-1}}{z(z-0.9)} = \sum_{\text{residues}} \frac{0.1z^{k-2}}{z-0.9}$

2-5. (a) $k=0$: $\text{fcn} = \dfrac{0.1}{z^2(z-0.9)}$, $\therefore$ residue$\Big|_{z=0.9} = \dfrac{0.1}{(0.9)^2} = \underline{0.1235}$

residue$\Big|_{z=0} = \dfrac{d}{dz}\left[\dfrac{0.1}{z-0.9}\right]_{z=0} = \dfrac{-0.1(1)}{(z-0.9)^2}\Big|_{z=0} = \dfrac{-0.1}{(0.9)^2} = \underline{-0.1235}$

$\therefore\ e(0) = \underline{0}$

$k=1$: $e(1) = \dfrac{0.1}{z-0.9}\Big|_{z=0} + \dfrac{0.1}{z}\Big|_{z=0.9} = \underline{0}$

$k=10$: $e(10) = \underline{0.1(0.9)^8}$

(b) $e(0) = \lim\limits_{z\to\infty} E(z) = \lim\limits_{z\to\infty} \dfrac{0.1}{z(z-0.9)} = \underline{0}$

(c) $\dfrac{E(z)}{z} = \dfrac{0.1}{z^2(z-0.9)} = \dfrac{k_1}{z^2} + \dfrac{k_2}{z} + \dfrac{k_3}{z-0.9}$

$k_1 = \dfrac{-0.1}{0.9} = \underline{-\dfrac{1}{9}}$; $k_3 = \dfrac{0.1}{(0.9)^2} = \underline{\dfrac{1}{8.1}}$

$k_2 = \dfrac{d}{dz}\left[\dfrac{0.1}{z-0.9}\right]_{z=0} = \underline{\dfrac{-1}{8.1}}$, from (a)

$\therefore\ e(k) = \dfrac{-1}{8.1}\delta(k) - \dfrac{1}{9}\delta(k-1) + \dfrac{1}{8.1}(0.9)^k$

$x(0) = -\dfrac{1}{8.1} + 0 + \dfrac{1}{8.1} = \underline{0}$; $x(1) = -0 - \dfrac{1}{9} + \dfrac{0.9}{8.1} = \underline{0}$

$x(10) = -0 - 0 + \dfrac{0.1}{(0.9)^2}(0.9)^{10} = \underline{0.1(0.9)^8}$

(d) $E(z) = \dfrac{1.98z}{z^5 + \cdots} = 1.98z^{-4} + (\cdot)z^{-5} + (\cdot)z^{-6} + \cdots$

$\therefore\ e(0) = e(1) = e(2) = e(3) = \underline{0}$; $e(4) = \underline{1.98}$

(e) $E(z) = \dfrac{2z}{z-0.8} = \dfrac{2z}{z-\epsilon^{-aT}}$ $\therefore\ \epsilon^{-aT} = 0.8 \Rightarrow aT = 0.2231$

$\therefore\ a = \dfrac{0.2231}{0.1} = 2.231$, $\therefore\ \underline{e(t) = 2\epsilon^{-2.231t}u(t)}$

(f) $E(z) = \dfrac{2z}{z-(-0.8)}$; $\therefore\ \epsilon^{-aT}\epsilon^{j\pi} = -0.8 \Rightarrow aT = 2.231$

$\therefore\ \underline{e(t) = 2e^{-2.231t}\cos 10\pi t}$ where $\dfrac{\omega_s}{2} = 10\pi$

(g) (e) $e(k) = (0.8)^k$; (f) $e(k) = (-0.8)^k$

$\therefore$ sign alternates on $e(k)$.

2-6. (a) $\mathcal{Z}[e(t-2T)u(t-2T)] = \dfrac{(z^3-2z)z^{-2}}{z^4-0.9z^2+0.8}$

(b) $e(0)=0$, $e(1)=1$

$\therefore \mathcal{Z}[e(t+T)u(t)] = z[E(z) - e(0) - e(1)z^{-1}]$

$= z\left[\dfrac{z^3-2z}{z^4-0.9z^2+0.8} - \dfrac{1}{z}\right] = \dfrac{-1.1z^2+0.8}{z^4-0.9z^2+0.8}$

(c) $\mathcal{Z}[e(t-T)u(t-2T)] = e(T)z^{-2} + e(2T)z^{-3} + \cdots$

$= z^{-1}[E(z) - e(0)] = z^{-1}E(z)$, since $e(0)=0$

$= \dfrac{z^2-z}{z^4-0.9z^2+0.8}$

2-7. (a) $e(\infty) = \lim_{z\to 1}(z-1)E(z) = \dfrac{z(z-1)}{(z+1)^2}\Big|_{z=1} = \underline{0}$

(b) $e(k) = \mathcal{Z}^{-1}\left[\dfrac{z}{(z-1)^2}\right] = k(-1)^k$, $\therefore e(\infty)$ unbounded

(c) (a) $e(\infty) = \lim_{z\to 1}(z-1)\dfrac{z}{(z-1)^2}$, $\therefore$ unbounded

(b) $e(k) = k$, $\therefore$ unbounded

(d) (a) $e(\infty) = \lim_{z\to 1}(z-1)\dfrac{z}{(z-0.9)^2} = \underline{0}$

(b) $e(k) = k(0.9)^k$; $\therefore e(\infty)\to \underline{0}$

(e) (a) $e(\infty) = \lim_{z\to 1}(z-1)\dfrac{z}{(z-1.1)^2} = \underline{0}$

(b) $e(k) = k(1.1)^k$; $\therefore e(\infty)$ is unbounded.

2-8. (a) i)

$$\begin{array}{r l}
 & 0.5z^{-1} + 0.8z^{-2} + 0.98z^{-3} + \cdots \\
z^2 - 1.6z + 0.6 \,\big) & 0.5z \\
 & \underline{0.5z - 0.8 + 0.3z^{-1}} \\
 & 0.8 - 0.3z^{-1} \\
 & \underline{0.8 - 1.28z^{-1} + \cdots} \\
 & 0.98z^{-1} + \cdots
\end{array}$$

ii) $\dfrac{E(z)}{z} = \dfrac{0.5}{(z-1)(z-0.6)} = \dfrac{1.25}{z-1} + \dfrac{-1.25}{z-0.6}$; $\therefore E(z) = \dfrac{1.25z}{z-1} - \dfrac{1.25z}{z-0.6}$

$\therefore e(k) = \underline{1.25(1 - 0.6^k)u(k)}$

(iii) $z^{k-1}E(z) = \dfrac{0.5z^k}{(z-1)(z-0.6)}$

$$e(k) = \frac{0.5(1)^k}{1-0.6} + \frac{0.5(0.6)^k}{0.6-1} = \underline{1.25(1-0.6^k)u(k)}$$

(iv) $E_1(z) = \dfrac{0.5z}{z-0.6} \Rightarrow e_1(k) = 0.5(0.6)^k$

$E_2(z) = \dfrac{1}{z-1} \Rightarrow e_2(0)=0\ ;\ e_2(k)=1,\ k \geq 1$

$e(0) = e_1(0)e_2(0) = (0.5)(0) = \underline{0}$

$e(1) = e_1(0)e_2(1) + e_1(1)e_2(0) = (0.5)(1) + (0.3)(0) = \underline{0.5}$

$e(2) = e_1(0)e_2(2) + e_1(1)e_2(1) + e_1(2)e_2(0)$

$= 0.5\times 1 + 0.3\times 1 + 0.18\times 0 = \underline{0.8}$

$e(3) = 0.5\times 1 + 0.3\times 1 + 0.18\times 1 + 0.108\times 0 = \underline{0.98}$

(b) $e(0)=0$

$e(k) = 1.25 - 2.083(0.6)^k,\ k \geq 1$

$E(z) = \underline{0.5z^{-2} + 0.8z^{-3} + 0.98z^{-4} + 1.088z^{-5}} + \cdots$

(c) $e(0)=0\ ;\ e(k) = 2.5 - 3.33(0.6)^k,\ k \geq 1$

$E(z) = \underline{0.5z^{-1} + 1.30z^{-2} + 1.78z^{-3} + 2.068z^{-4} + 2.2408z^{-5} + \cdots}$

(d) $e(k) = 0.75 + 0.25(0.6)^k$

$\underline{E(z) = 1 + 0.9z^{-1} + 0.84z^{-2} + 0.804z^{-3} + \cdots}$

(e)

```
num=[0 0 0.5];
den=[1 -1.6 0.6];
[r,p,k]=residue(num,den)
```

2-9. (a) poles: $z = \dfrac{2\cos a \pm \sqrt{4\cos^2 a - 4}}{2} = \cos(a) \pm j\sin(a)$

$\therefore$ pole $= \cos a$, provided $\sin a = 0 \Rightarrow a = 0, \pm\pi, \pm 2\pi, \ldots, \pm n\pi$

Then $\cos a = (-1)^n$ $\therefore$ poles $= \underline{\cos a}$

(b) $E(z) = \dfrac{z(z-\cos a)}{(z-\cos a)(z-\cos a)} = \underline{\dfrac{z}{z-\cos a}}$, $a = \pm n\pi,\ n = 0, 1, \ldots$

(c) $E(z) = \dfrac{z}{z-\cos a} = \underline{\dfrac{z}{z-1}}$, $\therefore \underline{\cos a = 1}$, $a = 0, \pm 2\pi, \pm 4\pi, \ldots$

2-10. $x(k) - 3x(k-1) + 2x(k-2) = e(k)$, $e(k) = \begin{cases} 1, & k = 0, 1 \\ 0, & k \geq 2 \end{cases}$

(a) $x(0) = e(0) = 1$

$x(1) = e(1) + 3x(0) = 4$

2-10. (a) $x(2) = e(2) + 3x(1) - 2x(0) = 10$

$x(3) = 0 + 3(10) - 2(4) = 22$

$x(4) = 0 + 3(22) - 2(10) = 46$

(b) $[1 - 3z^{-1} + 2z^{-2}]X(z) = E(z) = 1 + z^{-1} = \frac{z+1}{z}$

$X(z) = \frac{z^2}{(z-1)(z-2)} \times \frac{z+1}{z} = \frac{z(z+1)}{(z-1)(z-2)} = z\left[\frac{-2}{z-1} + \frac{3}{z-2}\right]$

$\therefore x(k) = \underline{-2 + 3(2)^k}$

(c) No, since the final value does not exist.

2-11. (a) $E(z) = \mathcal{Z}[u(k-1)] = z^{-1}\left[\frac{z}{z-1}\right] = \frac{1}{z-1}$

$[z^2 - \frac{3}{4}z + \frac{1}{8}]Y(z) = E(z)$

$\frac{Y(z)}{z} = \frac{1}{z(z-\frac{1}{2})(z-\frac{1}{4})} \cdot \frac{1}{z-1} = \frac{-8}{z} + \frac{8/3}{z-1} + \frac{-16}{z-1/2} + \frac{64/3}{z-1/4}$

$\therefore y(k) = \underline{-8\delta(0) + 8/3 - 16(\frac{1}{2})^k + \frac{64}{3}(\frac{1}{4})^k}$

$\therefore y(0) = \underline{0}$; $y(1) = \underline{0}$; $y(2) = \underline{0}$; $y(3) = \underline{1}$; $y(4) = \underline{\frac{7}{4}}$

(b) $y(k+2) = e(k) + \frac{3}{4}y(k+1) - \frac{1}{8}y(k)$

$y(2) = 0 + \frac{3}{4}(0) - \frac{1}{8}(0) = \underline{0}$

$y(3) = 1 + \frac{3}{4}(0) - \frac{1}{8}(0) = \underline{1}$

$y(4) = 1 + \frac{3}{4}(1) - \frac{1}{8}(0) = \underline{7/4}$

(c) (a) $y(k+2) - \frac{3}{4}y(k+1) + \frac{1}{8}y(k) = 0$

$\therefore z^2[Y(z) - y(0) - y(1)z^{-1}] - \frac{3}{4}z[Y(z) - y(0)] + \frac{1}{8}Y(z) = 0$

$\therefore [z^2 - \frac{3}{4}z + \frac{1}{8}]Y(z) = z^2 - 2z - \frac{3}{4}z$

$\therefore Y(z) = z\left[\frac{z - 1/4}{(z-1/2)(z-1/4)}\right] = z\left[\frac{-9}{z-1/2} + \frac{10}{z-1/4}\right] \Rightarrow y(k) = \underline{-9(\frac{1}{2})^k + 10(\frac{1}{4})^k}$

$y(0) = \underline{1}$, $y(1) = \underline{-2}$, $y(2) = \underline{-13/8}$, $y(3) = \underline{-31/32}$ $y(4) = \underline{-67/128}$

(b) $y(k+2) = \frac{3}{4}y(k+1) - \frac{1}{8}y(k)$

$y(2) = \frac{3}{4}(-2) - \frac{1}{8}(1) = \underline{-13/8}$ $\qquad y(4) = \frac{3}{4}(-\frac{31}{32}) - \frac{1}{8}(-\frac{13}{8}) = -\frac{67}{128}$

$y(3) = \frac{3}{4}(-\frac{13}{8}) - \frac{1}{8}(-2) = \underline{-31/32}$

2-12.(a) $[1-z^{-1}+z^{-2}]X(z) = E(z) = \frac{z}{z-1}$

$$X(z) = \frac{z^3}{(z-1)(z^2-z+1)}, \qquad \text{poles: } z = \frac{1}{2} \pm j\frac{\sqrt{3}}{2} = 1\angle{\pm 60°}$$

$$\frac{X(z)}{z} = \frac{1}{z-1} + \frac{k_1}{z-p_1} + \frac{k_1^*}{z-p_1^*} \qquad \text{with } p = 1\angle 60°$$

$$k_1 = \frac{z^2}{(z-1)(z-1\angle{-60°})}\bigg|_{z=1\angle 60°} = \frac{1\angle 120°}{(.5+j.866-1)(.5+j.866-.5+j.866)}$$

$$= \frac{1\angle 120°}{1\angle 120°\,[j2(0.866)]} = 0.5774\angle{-90°}$$

$$\therefore aT = \ln(|p_1|) = \underline{0}\ ; \quad bT = \arg P_1 = \underline{\frac{\pi}{3}}$$

$$A = 2|k_1| = \underline{1.155}\ ; \quad \theta = \arg k_1 = \underline{-90}°$$

$$\therefore x(k) = 1 + 1.155\cos\left(\frac{\pi}{3}k - 90°\right) = 1 + 1.155\sin\left(\frac{\pi}{3}k\right)$$

$$\therefore x(0) = \underline{1}, \quad x(1) = \underline{2}, \quad x(2) = \underline{2}$$

(b)

$$\begin{array}{r} 1 + 2z^{-1} + 2z^{-2} + \cdots \\ z^3 - 2z^2 + 2z - 1\ \overline{)\ z^3 \qquad\qquad} \\ \underline{z^3 - 2z^2 + 2z - 1} \\ 2z^2 - 2z + 1 \\ \underline{2z^2 - 4z + 4 - 2z^{-1}} \\ 2z + \cdots \end{array}$$

$\therefore x(0) = \underline{1}$

$x(1) = \underline{2}$

$x(2) = \underline{2}$

(c) $x(k) = 1 + x(k-1) - x(k-2)$

$x(0) = 1 + 0 - 0 = \underline{1}$

$x(1) = 1 + 1 - 0 = \underline{2}$

$x(2) = 1 + 2 - 1 = \underline{2}$

(d) No, 3 poles for $X(z)$ on the unit circle.

2-13.(a) $z^2[X(z) - x(0) - x(1)z^{-1}] + 3z[X(z) - x(0)] + 2X(z) = E(z) = 1$

$$\therefore X(z) = \frac{1 + z^2 - z + 3z}{z^2 - 3z + 2} = \frac{z^2 + 2z + 1}{z^2 + 3z + 2} = \underline{\frac{z+1}{z+2}}$$

$$\therefore X(z) = z\left[\frac{z+1}{z(z+2)}\right] = z\left[\frac{1/2}{z} + \frac{1/2}{z+2}\right]$$

$$\therefore x(k) = \frac{1}{2}\delta(k) + \frac{1}{2}(-2)^k$$

2-13.(b) $x(0) = \underline{1}$, $x(1) = \underline{-1}$, $x(2) = \underline{2}$, $x(3) = \underline{-4}$

(c)

$$
\begin{array}{r l}
 & 1 - z^{-1} + 2z^{-2} - 4z^{-3} + \cdots \\
z+2 \,\big) & z + 1 \\
 & \underline{z + 2} \\
 & -1 \\
 & \underline{-1 - 2z^{-1}} \\
 & 2z^{-1} \\
 & \underline{2z^{-1} + 4z^{-2}} \\
 & -4z^{-2} \\
 & \cdots
\end{array}
$$

(d) $x(k+2) = e(k) - 3x(k+1) - 2x(k)$

$x(2) = 1 - 3(-1) - 2(1) = \underline{2}$

$x(3) = 0 - 3(2) - 2(-1) = \underline{-4}$

2-14.(a)

```
x0 = 0;
x1 = 0;
x2 = 0;
for k = 0:5;
   x3 = 2.2*x2 - 1.57*x1 + 0.36*x0 + 1
   x0 = x1;
   x1 = x2;
   x2 = x3;
end
```

(b) $x(k+3) = e(k) + 2.2x(k+2) - 1.57x(k+1) + 0.36x(k)$

$x(3) = 1 + 0 - 0 + 0 = \underline{1}$

$x(4) = 1 + 2.2(1) - 0 + 0 = \underline{3.2}$

$x(5) = 1 + 2.2(3.2) - 1.57(1) = \underline{6.47}$

(c) $[z^3 - 2.2z^2 + 1.57z - 0.36]X(z) = E(z) = \dfrac{z}{z-1}$

$$X(z) = \frac{z}{(z-1)(z^3 - 2.2z^2 + 1.57z - 0.36)}$$

$$
\begin{array}{r l}
 & z^{-3} + 3.2z^{-4} + 6.47z^{-5} + \cdots \\
z^4 - 3.2z^3 + 3.77z^2 - 1.93z + 0.36 \,\big) & z \\
 & \underline{z - 3.2 + 3.77z^{-1} \cdots} \\
 & 3.2 - 3.77z^{-1} \\
 & \underline{3.2 - 10.24z^{-1} + \cdots} \\
 & 6.47z^{-1} + \cdots \\
 & \cdots
\end{array}
$$

$\therefore$ $x(3) = \underline{1}$

$x(4) = \underline{3.2}$

$x(5) = \underline{6.47}$

2-15.(a) $y(k+1) = y(k) + Tx(k)$

(b) $zY(z) = Y(z) + TX(z) \Rightarrow \dfrac{Y(z)}{X(z)} = \dfrac{T}{z-1}$

2-15. (c) $y(k+1) = y(k) + T x(k+1)$

(d) $zY(z) = Y(z) + TzX(z) \Rightarrow \dfrac{Y(z)}{X(z)} = \dfrac{Tz}{z-1}$

(e) $y(1) = y(0) + Tx(0)$

$y(2) = y(1) + Tx(1) = y(0) + T(x(0) + x(1))$

$y(3) = y(2) + Tx(2) = y(0) + T[x(0) + x(1) + x(2)]$

$\therefore \quad y(k) = y(0) + T\sum_{n=0}^{k-1} x(n)$

(f) $y(1) = y(0) + Tx(1)$

$y(2) = y(1) + Tx(2) = y(0) + T[x(1) + x(2)]$

$\therefore \quad y(k) = y(0) + T\sum_{n=1}^{k} x(n)$

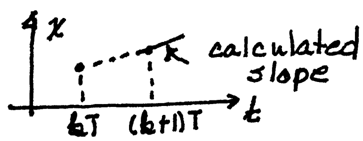
2-16. (a) $y(k+1) = y(k) + T\dfrac{x(k) + x(k+1)}{2}$

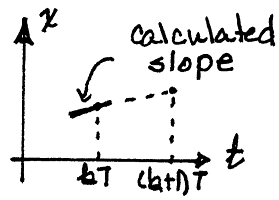
(b) $zY(z) = Y(z) + \frac{T}{2}[X(z) + zX(z)] \Rightarrow \underline{Y(z) = \frac{T}{2}\,\frac{z+1}{z-1}X(z)}$

2-17. (a) $TzW(z) = zX(z) - X(z)$

$w(k+1) = \frac{1}{T}[x(k+1) - x(k)]$

(b)





(c) $TW(z) = zX(z) - X(z)$

$w(k) = \frac{1}{T}[x(k+1) - x(k)]$

2.18. (a) $y(k) = \beta_2 e(k) + \beta_1 e(k-1) + \beta_0 e(k-2) - \alpha_1 y(k-1) - \alpha_0 y(k-2)$

(b) $[1 + \alpha_1 z^{-1} + \alpha_0 z^{-2}]Y(z) = [\beta_2 + \beta_1 z^{-1} + \beta_0 z^{-2}]E(z)$

$$\frac{Y(z)}{E(z)} = \underline{\frac{\beta_2 z^2 + \beta_1 z + \beta_0}{z^2 + \alpha_1 z + \alpha_0}}$$

(c) $f(k) = e(k) - a_1 f(k-1) - a_0 f(k-2)$

$y(k) = b_2 f(k) + b_1 f(k-1) + b_0 f(k-2)$

(d) $F(z) = E(z) - (a_1 z^{-1} + a_0 z^{-2})F(z) \Rightarrow F(z) = \dfrac{E(z)}{1 + a_1 z^{-1} + a_0 z^{-2}}$

2.18.(d) $Y(z) = (b_2 + b_1 z^{-1} + b_0 z^{-2})F(z) = \underline{\dfrac{b_2 z^2 + b_1 z + b_0}{z^2 + a_1 z + a_0} E(z)}$

(e) $\underline{\alpha_i = a_i}$ and $\underline{\beta_i = b_i}$, $i = 1, 2$

(f)

```
ykminus2 = 0;
ykminus1 = 0;
ekminus2 = 0;
ekminus1 = 0;
ek = 1;
for k = 0:5
   yk=b2*ek+b1*ekminus1+b0*ekminus2-a1*ykminus1-a0*ykminus2;
   [k, ek, yk]
   ekminus2 = ekminus1;
   ekminus1 = ek;
   ykminus2 = ykminus1;
   ykminus1 = yk;
end
```

(g)

```
fkminus2 = 0;
fkminus1 = 0;
ek = 1;
for k = 0:5
   fk=ek-a1*fkminus1-a0*fkminus2;
   yk = b2*fk+b1*fkminus1+b0*fkminus2;
   [k, ek, yk]
   fkminus2 = fkminus1;
   fkminus1 = fk;
end
```

2-19.(a) $f_1(k) = g_1 f_1(k-1) - g_2 f_2(k-1) + g_3 e(k)$

$f_2(k) = g_1 f_2(k-1) + g_1 f_1(k-1) + g_4 e(k)$

$y(k) = b_2 e(k) + f_2(k-1)$

(b) (1) $F_1(z) = g_1 z^{-1} F_1(z) - g_2 z^{-1} F_2(z) + g_3 E(z)$

(2) $F_2(z) = g_1 z^{-1} F_2(z) + g_2 z^{-1} F_1(z) + g_4 E(z)$

(3) $Y(z) = b_0 E(z) + z^{-1} F_2(z)$

$\therefore$ (1) $(z - g_1) F_1(z) + g_2 F_2(z) = g_3 z E(z)$

(2) $-g_2 F_1(z) + (z - g_1) F_2(z) = g_4 z E(z)$

$$\therefore F_2(s) = \frac{\begin{vmatrix} z-g_1 & g_3 z E(z) \\ -g_2 & g_4 z E(z) \end{vmatrix}}{\begin{vmatrix} z-g_1 & g_2 \\ -g_2 & z-g_1 \end{vmatrix}} = \frac{(g_4 z^2 - g_1 g_4 z + g_2 g_3 z)}{(z-g_1)^2 + g_2^2} E(z)$$

2.19. (b) $\therefore \dfrac{Y(z)}{E(z)} = b_2 + \dfrac{g_4 z + g_2 g_3 - g_1 g_4}{(z-g_1)^2 + g_2^2}$

Also, $D(z) = b_2 + \dfrac{Re(A) + j\,Im(A)}{z - Re(P) - j\,Im(P)} + \dfrac{Re(A) - j\,Im(A)}{z - Re(P) + j\,Im(P)}$

$= b_2 + \dfrac{\frac{1}{2}(g_4 - j g_3)}{z - g_1 - j g_2} + \dfrac{\frac{1}{2}(g_4 + j g_3)}{z - g_1 + j g_2}$

$= b_2 + \dfrac{g_4 z - g_1 g_4 + g_2 g_3}{(z-g_1)^2 + g_2^2}$ ✓

(c) $D(z) = b_0 + \dfrac{g_2 g_3 z^{-2} + g_4(1 - g_1 z^{-1})}{1 - g_1 z^{-1} - g_1 z^{-1} + g_1^2 z^{-2} + g_2^2 z^{-2}}$

$= b_0 + \dfrac{g_4 z + g_2 g_3 - g_1 g_4}{z^2 - 2 g_1 z + g_1^2 + g_2^2}$

(d)

```
f1kminus1 = 0;
f2kminus1 = 0;
ek = 1;
for k = 0:5
   yk = b0*ek+f2kminus1;
   [k, ek, yk]
   f1k = g1*f1kminus1 - g2*f2kminus1 + g3*ek;
   f2k = g1*f2kminus1 + g2*f1kminus1 + g3*ek;
   f1kminus1 = f1k;
   f2kminus1 = f2k;
end
```

2-20 (a) $\beta_2 = 2$, $\beta_1 = -2.4$, $\beta_0 = 0.72$, $\alpha_1 = -1.4$, $\alpha_0 = 0.98$

(b) $b_2 = 2$, $b_1 = -2.4$, $b_0 = 0.72$, $a_1 = -1.4$, $a_0 = 0.98$

(c) poles : $z = \dfrac{1.4 \pm (1.4^2 - 4(0.98))^{1/2}}{2} = 0.7 \pm j0.7 = 0.99\angle{\pm 45^\circ}$

$D(z) = 2 + \dfrac{A}{z - 0.7 - j0.7} + \dfrac{A^*}{z - 0.7 + j0.7}$

$\therefore A = \left.\dfrac{2z^2 - 2.4z + 0.72}{z - 0.7 + j0.7}\right|_{z = 0.99\angle 45^\circ} = \dfrac{j1.96 - (1.68 + j1.68) + 0.72}{j1.4}$

$= 0.2 + j\,0.6857$

$\therefore$ $g_1 = 0.7$ $\quad g_3 = 1.371$

$g_2 = 0.7$ $\quad g_4 = 0.4$

2-20.(d)

```
num=[2 -2.4 .72];
den=[1 -1.4 0.98];
[r,p,k]=residue(num,den)
```

(e) $\Delta = 1-(0.7z^{-1}+0.7z^{-1}+0.4z^{-2})+0.49z^{-2}$

$= 1-1.4z^{-1}+0.98z^{-2}$

$$D(z) = 2+\frac{1}{\Delta}\left[1.371(0.7)z^{-2}+0.4z^{-1}(1+0.7z^{-1})\right]$$

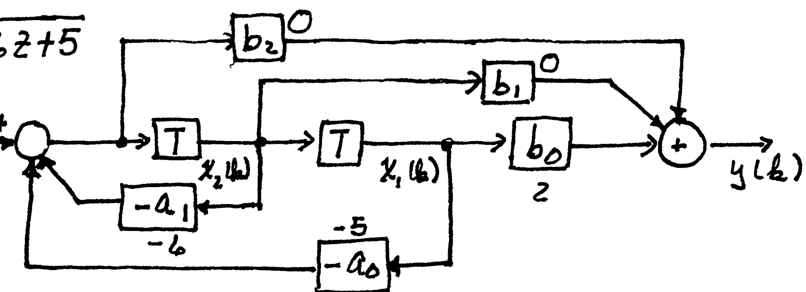
$$= 2+\frac{0.4z-1.24}{z^2-1.4z+0.98} = \underline{\frac{2z^2-2.4z+0.72}{z^2-1.4z+0.98}}$$

2-21.

```
s1 = 0;
e = 0;
for k=0:5
   s2 = e - s1;
   m = 0.5*s2 - s1;
   [k,m]
   s1 = s2;
   e = e + 1;
end
```

2-22. (a) $\dfrac{Y(z)}{U(z)} = \dfrac{2}{z^2+6z+5}$

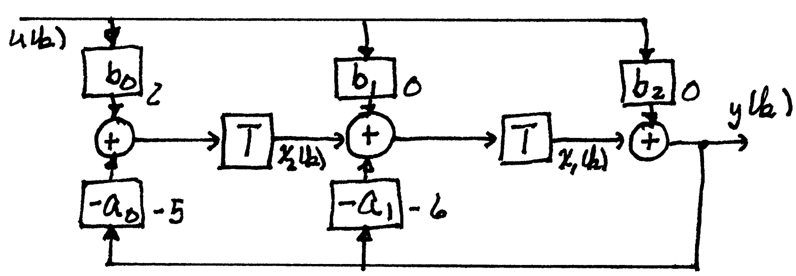
(1) control canonical:



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(k)$$

$$y(k) = [2 \quad 0]\,\underline{x}(k)$$

(2) observer canonical:



$$\underline{x}(k+1) = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix}u(k)$$

$$y(k) = [1 \quad 0]\,\underline{x}(k)$$

2-22. (b) $\dfrac{Y(z)}{U(z)} = \dfrac{z+2}{z^2+6z+5}$ (1) control canonical: $\underline{x}(k+1) =$ same as (a)

$y(k) = [2 \quad 1]\underline{x}(k)$

(2) observer canonical:

$$\underline{x}(k+1) = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\underline{x}(k)$$

(c) $\dfrac{Y(z)}{U(z)} = \dfrac{3z^2+z+2}{z^2+6z+5}$ (1) control canonical: $\underline{x}(k+1) =$ same as (a)

$y(k) = [-13 \quad -17]\underline{x}(k) + 3u(k)$

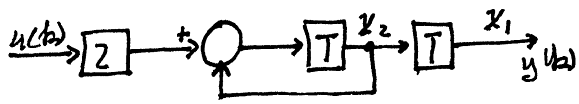
(2) observer canonical:

$$\underline{x}(k+1) = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\underline{x}(k) + 3u(k)$$

2-23. (a) $G(z) = G_1(z)G_2(z) = \dfrac{2}{z^2-z} = \dfrac{2z^{-2}}{1-z^{-1}}$

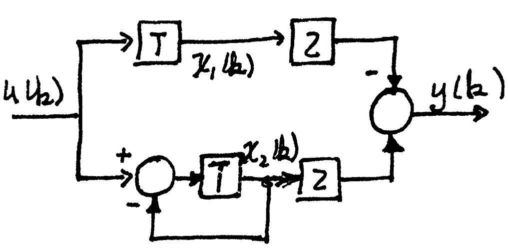
(1)



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\underline{x}(k)$$

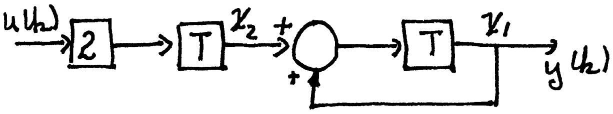
(2) $G(z) = \dfrac{2}{z(z-1)} = \dfrac{-2}{z} + \dfrac{2}{z-1} = G_1(z) + G_2(z)$



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = [-2 \quad 2]\underline{x}(k)$$

(3)



$$\underline{x}(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\underline{x}(k)$$

2-23(b) (1) $zI-A = \begin{bmatrix} z & -1 \\ 0 & z-1 \end{bmatrix}$; $|zI-A| = z^2 - z = \Delta$

$$G(z) = C(zI-A)^{-1}B = \frac{1}{\Delta}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} z-1 & 1 \\ 0 & z \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} z-1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \underline{\frac{2}{z(z-1)}}$$

(2) $zI-A = \begin{bmatrix} z & 0 \\ 0 & z-1 \end{bmatrix}$; $|zI-A| = \Delta = z^2 - z$

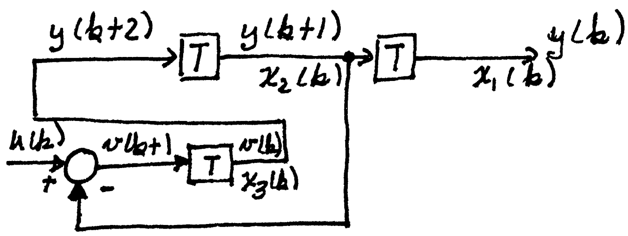
$$G(z) = C(zI-A)^{-1}B = \frac{1}{\Delta}\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} z-1 & 0 \\ 0 & z \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} -1 & 1 \end{bmatrix}\begin{bmatrix} 2z-2 \\ 2z \end{bmatrix} = \underline{\frac{2}{z(z-1)}}$$

(3) $zI-A = \begin{bmatrix} z-1 & -1 \\ 0 & z \end{bmatrix}$; $|zI-A| = z^2 - z = \Delta$

$$G(z) = C(zI-A)^{-1}B = \frac{1}{\Delta}\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} z & 1 \\ 0 & z-1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} z & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \underline{\frac{2}{z(z-1)}}$$

(c)
```
A = [0 1;0 1]; B = [0;2]; C = [1 0]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-24. (a)



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u(k)$$

$\underline{y}_o(k) = \begin{bmatrix} x_2(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\underline{x}(k)$; $\underline{y}_o(k)$ = output

(b) $\underline{x}(k+1)$ = same as (a)

$\underline{y}_o(k) = \begin{bmatrix} x_1(k) \\ x_3(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\underline{x}(k)$

(c) $\underline{x}(k+1)$ = same as (a)

$y_o(k) = x_3(k) = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\underline{x}(k)$

(d) $zI-A = \begin{bmatrix} z & -1 & 0 \\ 0 & z & -1 \\ 0 & 1 & z \end{bmatrix}$; $|zI-A| = z^3 - (-z) = z^3 + z = \Delta$

2-24.(d)

$$\operatorname{cof}(zI-A)=\begin{bmatrix} z^2+1 & z^2 & 0 \\ z & z^2 & z \\ 1 & z & z^2 \end{bmatrix};\quad (zI-A)^{-1}=\frac{1}{\Delta}\begin{bmatrix} z^2+1 & z & 1 \\ z^2 & z^2 & z \\ 0 & z & z^2 \end{bmatrix}$$

$$\therefore \frac{Y_0(z)}{U(z)} = C(zI-A)^{-1}B = \frac{1}{\Delta}[0 \quad 0 \quad 1]\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

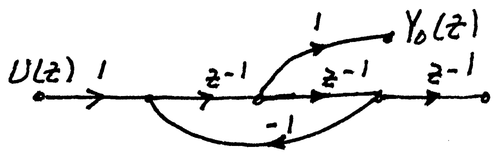
$$= \frac{1}{\Delta}[0 \quad z \quad z^2]\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{z^2}{z^3-z} = \underline{\frac{z}{z^2+1}}$$

(e) $z^2 Y(z) - V(z) = 0 \quad \Rightarrow Y(z) = \frac{1}{z^2} V(z)$

$$z\,V(z) + z\,Y(z) = z\,V(z) + \frac{1}{z} V(z) = U(z)$$

$$\therefore \frac{Y(z)}{U(z)} = \frac{Y_0(z)}{U(z)} = \frac{1}{z+\frac{1}{z}} = \underline{\frac{z}{z^2+1}}$$

(f) From (a):

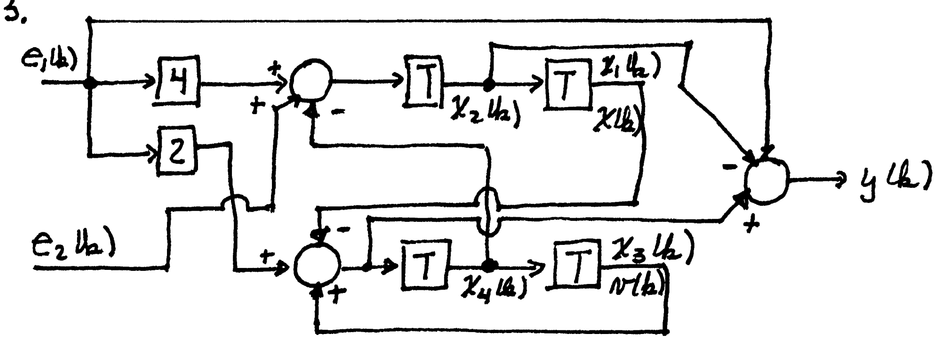


$$\therefore \frac{Y_0(z)}{U(z)} = \frac{z^{-1}}{1+z^{-2}} = \frac{z}{z^2+1}$$

(g)

```
A = [0 1 0;0 0 1;0 -1 0]; B = [0; 0; 1]; C = [0 0 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-25.



$$\underline{x}(k+1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}\underline{x}(k) + \begin{bmatrix} 0 & 0 \\ 4 & 1 \\ 0 & 0 \\ 2 & 0 \end{bmatrix}\underline{e}(k)$$

2-25. $y(k) = x_4(k+1) - x_2(k) + e_1(k) = -x_1(k) + x_3(k) - x_2(k) + e_1(k)$

$\therefore y(k) = [-1 \quad -1 \quad 1 \quad 0]\,\underline{x}(k) + [1 \quad 0]\,\underline{e}(k)$

2-26 (a) $zI - A = \begin{bmatrix} z & -1 \\ 0 & z-3 \end{bmatrix}$; $\Delta = |zI - A| = z(z-3) = \Delta$

$$\frac{Y(z)}{U(z)} = C(zI-A)^{-1}B = \frac{1}{\Delta}[-2 \quad 1]\begin{bmatrix} z-3 & 1 \\ 0 & z \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\Delta}[-2z+6 \quad z-2]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underline{\frac{-z+4}{z(z-3)}}$$

(b) $P = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$; $P^{-1} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$

$$A_w = P^{-1}AP = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

$$= \underline{\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}}$$

$$B_w = P^{-1}B = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underline{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$$C_w = CP = [-2 \quad 1]\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = [-1 \quad 3]$$

$$\therefore \underline{w}(k+1) = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}w(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u(k)$$

$$y(k) = [-1 \quad 3]\,w(k)$$

(c) $zI - A_w = \begin{bmatrix} z-2 & -2 \\ -1 & z-1 \end{bmatrix}$; $\Delta = |zI - A_w| = z^2 - 3z + 2 - 2 = z(z-3)$

$$\frac{Y(z)}{U(z)} = C_w(zI-A_w)^{-1}B_w = \frac{1}{\Delta}[-1 \quad 3]\begin{bmatrix} z-1 & 2 \\ 1 & z-2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\Delta}[-1 \quad 3]\begin{bmatrix} z-1 \\ 1 \end{bmatrix} = \underline{\frac{-z+4}{z(z-3)}}$$

(d) $|zI - A| = \begin{vmatrix} z & 1 \\ 0 & z-3 \end{vmatrix} = \underline{z^2 - 3z}$; $|zI - A_w| = \begin{vmatrix} z-2 & -2 \\ -1 & z-1 \end{vmatrix} = z(z-3)$

$\therefore z_1 = 0,\ z_2 = 3$

$|A| = \begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix} = \underline{0} = z_1 z_2$; $|A_w| = \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = \underline{0}$

$tr\, A = \underline{3} = z_1 + z_2$; $tr\, A_w = \underline{3}$

2-26 (e)

```
A = [0 1;0 3]; B = [1;1]; C = [-2 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)

pause

A = [2 2;1 1]; B = [1;0]; C = [-1 3]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-27. (a) Let z_1, z_2 be the characteristic value of A.

$$zI-A=\begin{bmatrix} z & -1 \\ 0 & z-3 \end{bmatrix}, \therefore |zI-A| = z(z-3); \therefore \underline{z_1=0}, \underline{z_2=3}$$

(b) $$(z_1 I-A)\underline{m}_1 = \begin{bmatrix} 0 & -1 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} m_{11} \\ m_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} -m_{21}=0 \\ -3m_{21}=0 \end{matrix}$$

$$\therefore m_{21}=0, \text{ let } m_{11}=1, \therefore \underline{m}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$(z_2 I-A)\underline{m}_2 = \begin{bmatrix} 3 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} m_{12} \\ m_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow 3m_{12} - m_{22} = 0$$

$$\therefore \text{let } m_{12}=1, m_{22}=3, \therefore \underline{m}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\therefore M = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}, |M| = 3, M^{-1} = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix}$$

$$M^{-1}AM = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$$

(c) $$B_w = M^{-1}B = \begin{bmatrix} 1 & -1/3 \\ 0 & 1/3 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

$$C_w = CM = [-2 \quad 1]\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix} = [-2 \quad 1]$$

$$\therefore \underline{w}(k+1) = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}\underline{w}(k) + \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}u(k)$$

$$y(k) = [-2 \quad 1]\underline{w}(k)$$

(d) See Problem 2-26(c) for the first transfer function.

$$zI-A_w = \begin{bmatrix} z & 0 \\ 0 & z-3 \end{bmatrix}; \quad |zI-A_w| = z(z-3) = \Delta$$

$$\frac{Y(z)}{U(z)} = C_w(zI-A_w)^{-1}B_w = \frac{1}{\Delta}[-2 \quad 1]\begin{bmatrix} z-3 & 0 \\ 0 & z \end{bmatrix}\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

$$= \frac{1}{\Delta}[-2z+6 \quad z]\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix} = \frac{-\frac{4}{3}z+4+\frac{1}{3}z}{\Delta} = \underline{\frac{-z+4}{z(z-3)}}$$

2-27. (e)

```
A = [0 1;0 3]; B = [1;1]; C = [-2 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)

A = [0 0;0 -3]; B = [.6667; .3333]; C = [-2 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-28. (a) $\dfrac{Y(z)}{U(z)} = \dfrac{-z+4}{z(z-3)}$; from Problem 2-26(c)

(b) $Y(z) = \dfrac{(-z+4)z}{z(z-3)(z-1)}$

$\dfrac{Y(z)}{z} = \dfrac{-z+4}{z(z-1)(z-3)} = \dfrac{4/3}{z} + \dfrac{-3/2}{z-1} + \dfrac{1/6}{z-3}$

$\therefore y(k) = \begin{cases} 4/3 - 3/2 + 1/6 = \underline{0}, & k=0 \\ \underline{-3/2 + 1/6\,(3)^k}, & k \geq 1 \end{cases}$ $\quad \therefore y(0) = \underline{0} \quad y(2) = -\frac{3}{2} + \frac{3}{2} = \underline{0}$

$y(1) = -\frac{3}{2} + \frac{1}{2} = \underline{-1}$

(c) From Problem 2.26(c),

$\Phi(z) = z(zI-A)^{-1} = z\begin{bmatrix} \frac{z-3}{z(z-3)} & \frac{1}{z(z-3)} \\ 0 & \frac{z}{z(z-3)} \end{bmatrix} = z\begin{bmatrix} \frac{1}{z} & \frac{-1/3}{z} + \frac{1/3}{z-3} \\ 0 & \frac{1}{z-3} \end{bmatrix}$

$\therefore \Phi(k) = \begin{bmatrix} \delta(k) & -\frac{1}{3}\delta(k) + \frac{1}{3}(3)^k \\ 0 & (3)^k \end{bmatrix}$

(d) $y(k) = \sum_{j=0}^{k-1} C\,\Phi(k-1-j)\,Bu(j) = \sum_{j=0}^{k-1} [-2 \quad 1]\,\Phi(k-1-j)\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \sum_{j=0}^{k-1} [-2 \quad 1]\begin{bmatrix} \frac{2}{3}\delta(k-1-j) + \frac{1}{3}(3)^{k-j-1} \\ (3)^{k-j-1} \end{bmatrix} = \sum_{j=0}^{k-1} \left[-\frac{4}{3}\delta(k-j-1) + \frac{1}{3}(3)^{k-j-1}\right]$

$= \underline{\sum_{j=0}^{k-1} \left[-\frac{4}{3}\delta(k-1-j) + \frac{1}{3}(3)^{k-1-j}\right]}$

$y(0) = \underline{0}$; $\quad y(1) = -\frac{4}{3}\delta(0) + \frac{1}{3}(3)^0 = -\frac{4}{3} + \frac{1}{3} = \underline{-1}$

$y(2) = -\frac{4}{3}\delta(1) + \frac{1}{3}(3)^1 - \frac{4}{3}\delta(0) + \frac{1}{3}(3)^0 = 1 - \frac{4}{3} + \frac{1}{3} = \underline{0}$

(e) $\underline{x}(1) = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $y(1) = [-2 \quad 1]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underline{-1}$

$\underline{x}(2) = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$; $y(2) = [-2 \quad 1]\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \underline{0}$

2-29. (a) From Problem 2-30(a),

2-30.(a) $\frac{Y(z)}{U(z)} = C(zI-A)^{-1}B = [1 \quad 2]\begin{bmatrix} \frac{1}{z-1} & 0 \\ 0 & \frac{1}{z-0.5}\end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$= [1 \quad 2]\begin{bmatrix} \frac{2}{z-1} \\ \frac{1}{z-0.5}\end{bmatrix} = \frac{2}{z-1} + \frac{2}{z-0.5} = \underline{\frac{4z-3}{(z-1)(z-0.5)}}$

(b) $P = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $P^{-1} = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$

$\therefore A_w = P^{-1}AP = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/4 \\ -1/2 & 1/4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3/4 & -1/4 \\ -1/4 & 3/4 \end{bmatrix}$

$B_w = P^{-1}B = \begin{bmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$

$C_w = CP = [1 \quad 2]\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = [3 \quad 1]$

$\therefore \underline{w}(k+1) = \begin{bmatrix} 3/4 & -1/4 \\ -1/4 & 3/4 \end{bmatrix}\underline{w}(k) + \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}u(k)$

$y(k) = [3 \quad 1]\underline{x}(k)$

(c) $zI - A_w = \begin{bmatrix} z-3/4 & 1/4 \\ 1/4 & z-3/4 \end{bmatrix}$, $|zI-A_w| = z^2 - 1.5z + \frac{9}{16} - \frac{1}{16} = z^2 - 1.5z + 0.5 = \Delta$

$\frac{Y(z)}{U(z)} = C_w(zI-A_w)^{-1}B_w = [3 \quad 1]\frac{1}{\Delta}\begin{bmatrix} z-3/4 & -1/4 \\ -1/4 & z-3/4 \end{bmatrix}\begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$

$= \frac{1}{\Delta}[3z-2.5 \quad z-1.5]\begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix} = \frac{4z-3}{(z-1)(z-0.5)}$

(d) $|zI-A| = \begin{vmatrix} z-1 & 0 \\ 0 & z-0.5 \end{vmatrix} = \underline{z^2 - 1.5z + 0.5}$; $|zI-A_w| = \underline{z^2 - 1.5z + 0.5}$

$\therefore z_1 = 1$, $z_2 = 0.5$

$|A| = \begin{vmatrix} 1 & 0 \\ 0 & 0.5 \end{vmatrix} = \underline{0.5} = z_1 z_2$; $|A_w| = \begin{vmatrix} 3/4 & -1/4 \\ -1/4 & 3/4 \end{vmatrix} = \frac{9}{16} - \frac{1}{16} = \underline{0.5}$

$\text{tr}\, A = \underline{1.5} = z_1 + z_2$; $\text{tr}\, A_w = \underline{1.5}$

2-30.(a) $zI - A = \begin{bmatrix} z-1 & 0 \\ 0 & z-0.5 \end{bmatrix}$; $|zI-A| = \Delta = (z-1)(z-0.5)$

$(zI-A^{-1}) = \frac{1}{\Delta}\begin{bmatrix} z-0.5 & 0 \\ 0 & z-1 \end{bmatrix} = \begin{bmatrix} \frac{1}{z-1} & 0 \\ 0 & \frac{1}{z-0.5} \end{bmatrix}$

2-30 (a) $\therefore \Phi(k) = \mathcal{Z}^{-1}\begin{bmatrix} \frac{z}{z-1} & 0 \\ 0 & \frac{z}{z-0.5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0.5^k \end{bmatrix}$

$\therefore \underline{x}(k) = \Phi(k)\,\underline{x}(0) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5^k \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2(0.5)^k \end{bmatrix}$

(b) $y(k) = C\underline{x}(k) = [1 \quad 2]\begin{bmatrix} 1 \\ 2(0.5)^k \end{bmatrix} = \underline{1+4(0.5)^k}$

(c) $\Phi(0) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5^0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

(d) $x(k)\Big|_{k=0} = \begin{bmatrix} 1 \\ 2(0.5)^k \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(e) From (b), $y(0) = \underline{5}$ $\quad y(2) = \underline{2}$

$y(1) = \underline{3}$ $\quad y(3) = \underline{1.5}$

$y(0) = C\underline{x}(0) = [1 \quad 2]\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \underline{5}$

$x(1) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad y(1) = [1 \quad 2]\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \underline{3}$

$x(2) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \quad y(2) = [1 \quad 2]\begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \underline{2}$

$\underline{x}(3) = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{bmatrix} 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.25 \end{bmatrix}, \quad y(3) = [1 \quad 2]\begin{bmatrix} 1 \\ 0.25 \end{bmatrix} = \underline{1.5}$

(f)

```
A = [1 0;0 .5]; B = [2; 1]; C = [1 2];
x=[1; 2];
u = 0;
for k = 0:3
   x1 = A*x + B*u;
   y = C*x;
   [k,y]
   x = x1;
end
```

2-31. (a) $zI - A = \begin{bmatrix} z-1.1 & -1 \\ 0.3 & z \end{bmatrix}; \quad |zI-A| = \Delta = z^2 - 1.1z + 0.3 = (z-0.5)(z-0.6)$

$(zI-A)^{-1} = \frac{1}{\Delta}\begin{bmatrix} z & 1 \\ -0.3 & z-1.1 \end{bmatrix}$

2-31 (a) $\Phi(k) = \mathcal{Z}^{-1}[z(zI-A)^{-1}] = \mathcal{Z}^{-1}\left(z\begin{bmatrix} \frac{z}{(z-0.5)(z-0.6)} & \frac{1}{(z-0.5)(z-0.6)} \\ \frac{-0.3}{(z-0.5)(z-0.6)} & \frac{z-1.1}{(z-0.5)(z-0.6)} \end{bmatrix}\right)$

$$= \mathcal{Z}^{-1}\left(z\begin{bmatrix} \frac{-5}{z-.5} + \frac{6}{z-.6} & \frac{-10}{z-.5} + \frac{10}{z-.6} \\ \frac{3}{z-.5} + \frac{-3}{z-.6} & \frac{6}{z-.5} + \frac{-5}{z-.6} \end{bmatrix}\right)$$

$$= \begin{bmatrix} -5(0.5)^k + 6(0.6)^k & -10(0.5)^k + 10(0.6)^k \\ 3(0.5)^k - 3(0.6)^k & 6(0.5)^k - 5(0.6)^k \end{bmatrix}$$

$$\therefore \underline{x}(k) = \Phi(k)\underline{x}(0) = [\quad]\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -15(0.5)^k + 14(0.6)^k \\ 9(0.5)^k - 7(0.6)^k \end{bmatrix}$$

(b) $y(k) = C\underline{x}(k) = [1 \quad -1]\,\underline{x}(k) = -24(0.5)^k + 21(0.6)^k$

(c) $\Phi(0) = \begin{bmatrix} -5+6 & -10+10 \\ 3-3 & 6-5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

(d) $\underline{x}(k)\Big|_{k=0} = \begin{bmatrix} -15+14 \\ 9-7 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

(e) From (b), $\underline{y(0) = -3}$ $\quad \underline{y(2) = 1.56}$

$\underline{y(1) = 0.6}$ $\quad \underline{y(3) = 1.536}$

$y(0) = C\underline{x}(0) = [1 \quad -1]\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \underline{-3}$

$\underline{x}(1) = \begin{bmatrix} 1.1 & 1 \\ -0.3 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix}$; $y(1) = [1 \quad -1]\begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix} = \underline{0.6}$

$\underline{x}(2) = \begin{bmatrix} 1.1 & 1 \\ -0.3 & 0 \end{bmatrix}\begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 1.29 \\ -0.27 \end{bmatrix}$; $y(2) = [1 \quad -1]\begin{bmatrix} 1.29 \\ -0.27 \end{bmatrix} = \underline{1.56}$

$\underline{x}(3) = \begin{bmatrix} 1.1 & 1 \\ -0.3 & 0 \end{bmatrix}\begin{bmatrix} 1.29 \\ -0.27 \end{bmatrix} = \begin{bmatrix} 1.149 \\ -0.387 \end{bmatrix}$; $y(3) = [1 \quad -1]\begin{bmatrix} 1.149 \\ -0.387 \end{bmatrix} = \underline{1.536}$

(f)

```
A = [1.1 1;-0.3 0]; B = [1; 1]; C = [1 -1];
x=[-1; 2];
u = 0;
for k = 0:3
   x1 = A*x + B*u;
   y = C*x;
   [k,y]
   x = x1;
end
```

2-32.

$$\underline{x}(k+1) = \begin{bmatrix} -a_{n-1} & 1 & 0 & \cdots & 0 \\ -a_{n-2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ -a_0 & 0 & 0 & \cdots & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} a_{n-1} \\ b_{n-2} \\ \vdots \\ b_0 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0 \quad 0 \quad \cdots \quad 0]\,\underline{x}(k)$$

2-33. $\underline{x}(k+1) = A\,\underline{x}(k)$; $\underline{x}(k) = \Phi(k)\,\underline{x}(0)$

$\therefore \Phi(k+1)\,\underline{x}(0) = A\,\Phi(k)\,\underline{x}(0)$

Since this is true for any $\underline{x}(0)$, $\therefore$ $\underline{\Phi(k+1) = A\,\Phi(k)}$

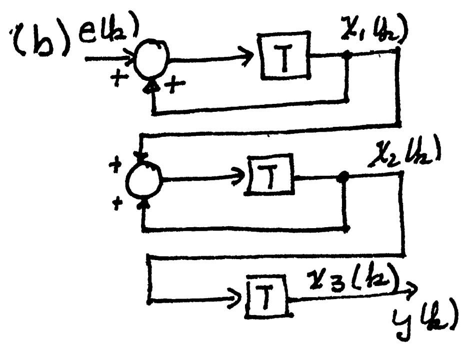
2-34 (a) $zI - A = \begin{bmatrix} z-1 & 0 & 0 \\ -1 & z-1 & 0 \\ 0 & -1 & z \end{bmatrix}$; $\Delta = z^3 - 2z^2 + z = z(z-1)^2$

$$\operatorname{cof}(zI-A) = \begin{bmatrix} z(z-1) & z & 1 \\ 0 & z(z-1) & z-1 \\ 0 & 0 & (z-1)^2 \end{bmatrix}, \quad (zI-A)^{-1} = \begin{bmatrix} \frac{1}{z-1} & 0 & 0 \\ \frac{1}{(z-1)^2} & \frac{1}{z-1} & 0 \\ \frac{1}{z(z-1)^2} & \frac{1}{z(z-1)} & \frac{1}{z} \end{bmatrix}$$

$$G(z) = C(zI-A)^{-1}B = [0 \quad 0 \quad 1](zI-A)^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{z(z-1)^2} & \frac{1}{z(z-1)} & \frac{1}{z} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{z(z-1)^2} = \underline{\frac{1}{z^3 - 2z^2 + z}}$$

(b)



(c) $\Delta = 1 - z^{-1} - z^{-1} + z^{-2} = 1 - 2z^{-1} + z^{-2}$

$$\therefore G(z) = \frac{z^{-3}}{\Delta} = \frac{1}{z^3 - 2z^2 + z}$$

2-34. (d)

```
A = [1 0 0;1 1 0;0 1 0]; B = [1;0;0]; C = [0 0 1]; D = 0;
[num, den] = ss2tf(A,B,C,D)
```

2-35. $C_W(zI-A_W)^{-1}B_W + D_W = CP[zI - P^{-1}AP]^{-1}P^{-1}B + D$

$$= CP[zP^{-1}IP - P^{-1}AP]^{-1}P^{-1}B + D$$

$$= CP[P^{-1}(zI-A)P]^{-1}P^{-1}B + D$$

$$= CPP^{-1}(zI-A)^{-1}PP^{-1}B + D \quad ; \text{ since } (ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$

$$= C(zI-A)^{-1}B + D$$

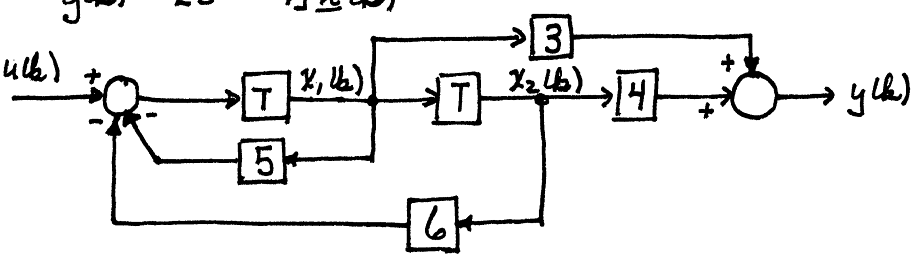
2-36. (a)

```
n = [0 3 4];
d = [1 5 6];
[A,B,C,D] = tf2ss(n,d)
```

$$\underline{x}(k+1) = \begin{bmatrix} -5 & -6 \\ 1 & 0 \end{bmatrix} \underline{x}(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = [3 \quad 4]\,\underline{x}(k)$$

(b)



(c) The control canonical form with the states renumbered.

CHAPTER 3

3-1. (a) $E^*(s) = \sum_{n=0}^{\infty} e(nT)\epsilon^{-nTs}$ (b) $E(z) = \sum_{n=0}^{\infty} e(nT)z^{-n}$

(c) $E^*(s) = E(z)\big|_{z=\epsilon^{sT}}$

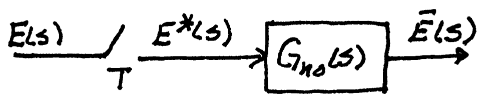
3-2. (a) 1. No frequencies in $e(t)$ greater than $\omega_s/2$.

2. An ideal low-pass filter follows the sampler.

(b) None

(c) The signal can be recovered to a sufficient degree of accuracy.

3-3.



$E_1(s) \longrightarrow \bar{E}_1(s)$ $\qquad$ $E_2(s) \longrightarrow \bar{E}_2(s)$

Let $E = E_1 + E_2$

$\therefore\ E^* = (E_1 + E_2)^* = E_1^* + E_2^*$

$\therefore\ \bar{E} = G_{ho}(E_1^* + E_2^*) = G_{ho}E_1^* + G_{ho}E_2^*$

3-4. $E^*(s) = \sum$ residues of $E(\lambda)\dfrac{1}{1-\epsilon^{-T(s-\lambda)}}$

(a) $$E^*(s) = \frac{20}{(s+5)(1-\epsilon^{-T(s-\lambda)})}\bigg|_{\lambda=-2} + \frac{20}{(s+2)(1-\epsilon^{-T(s-\lambda)})}\bigg|_{\lambda=-5}$$

$$= \frac{20/3}{1-\epsilon^{-T(s+2)}} + \frac{-20/3}{1-\epsilon^{-T(s+5)}}$$

(b) $$E^*(s) = \frac{5}{(s+1)(1-\epsilon^{-T(s-\lambda)})}\bigg|_{\lambda=0} + \frac{5}{s(1-\epsilon^{-T(s-\lambda)})}\bigg|_{\lambda=-1}$$

$$= \frac{5}{1-\epsilon^{-Ts}} - \frac{5}{1-\epsilon^{-T(s+1)}}$$

3-4.(c) $E^*(s) = \left.\frac{\lambda+2}{(\lambda+1)(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=0} + \left.\frac{\lambda+2}{\lambda(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=-1} = \underline{\frac{2}{1-\epsilon^{-Ts}} - \frac{1}{1-\epsilon^{-T(s+1)}}}$

(d) $(\text{residue})_{\lambda=0} = \frac{d}{d\lambda}\left[\frac{\lambda+2}{(\lambda+1)(1-\epsilon^{-T(s-\lambda)})}\right]_{\lambda=0}$

$$= \left.\frac{(\lambda+1)(1-\epsilon^{-T(s-\lambda)}) - (\lambda+2)(\lambda+1)(-T\epsilon^{-T(s-\lambda)}) - (\lambda+2)(1-\epsilon^{-T(s-\lambda)})}{(\lambda+1)^2(1-\epsilon^{-T(s-\lambda)})^2}\right|_{\lambda=0}$$

$$= \frac{-(1-\epsilon^{-Ts}) + 2T\epsilon^{-Ts}}{(1-\epsilon^{-Ts})^2}$$

$(\text{residue})_{\lambda=-1} = \left.\frac{\lambda+2}{\lambda^2(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=-1} = \frac{1}{1-\epsilon^{-T(s+1)}}$

$\therefore E^*(s) = \underline{\frac{-(1-\epsilon^{-Ts}) + 2T\epsilon^{-Ts}}{(1-\epsilon^{-Ts})^2} + \frac{1}{1-\epsilon^{-T(s+1)}}}$

(e) $E^*(s) = \sum_{\text{residues}} \frac{\lambda^2+5\lambda+6}{\lambda(\lambda+4)(\lambda+5)(1-\epsilon^{-T(s-\lambda)})} = \underline{\frac{3/10}{1-\epsilon^{-Ts}} + \frac{-1/2}{1-\epsilon^{-T(s+4)}} + \frac{6/5}{1-\epsilon^{-T(s+5)}}}$

(f) $s = -1 \pm j2$

$E^*(s) = \left.\frac{2}{(\lambda+1+j)(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=-1+j2} + \left.\frac{2}{(\lambda+1-j2)(1-\epsilon^{-T(s-\lambda)})}\right|_{\lambda=-1-j2}$

$= \underline{\frac{-j/2}{1-\epsilon^{-T(s+1-j2)}} + \frac{j/2}{1-\epsilon^{-T(s+1+j2)}}}$

3-5.(a) $E^*(s) = 1 + \epsilon^{aT}\epsilon^{-Ts} + \epsilon^{2aT}\epsilon^{-2Ts} + \cdots = 1 + \epsilon^{(a-s)T} + [\epsilon^{(a-s)T}]^2 + \cdots$

$= \underline{\frac{1}{1-\epsilon^{(a-s)T}}}$

(b) $e(t) = \epsilon^{a(t-2T)}u(t-2T)$

$E^*(s) = \epsilon^{-2Ts} + \epsilon^{aT}\epsilon^{-3Ts} + \epsilon^{2aT}\epsilon^{-4Ts} + \cdots$

$= \epsilon^{-2Ts}(1 + \epsilon^{aT}\epsilon^{-Ts} + \epsilon^{2aT}\epsilon^{-2Ts} + \cdots) = \underline{\frac{\epsilon^{-2Ts}}{1-\epsilon^{(a-s)T}}}$

(c) From (b), $E^*(s) = \underline{\frac{\epsilon^{-2Ts}}{1-\epsilon^{(a-s)T}}}$

(d) $E^*(s) = \epsilon^{aT/2}\epsilon^{-Ts} + \epsilon^{3aT/2}\epsilon^{-2Ts} + \epsilon^{5aT/2}\epsilon^{-3Ts} + \cdots$