EERI418 - NOTES: COMPENSATORS

- We will only consider single-input, single-output (SISO) control systems here.

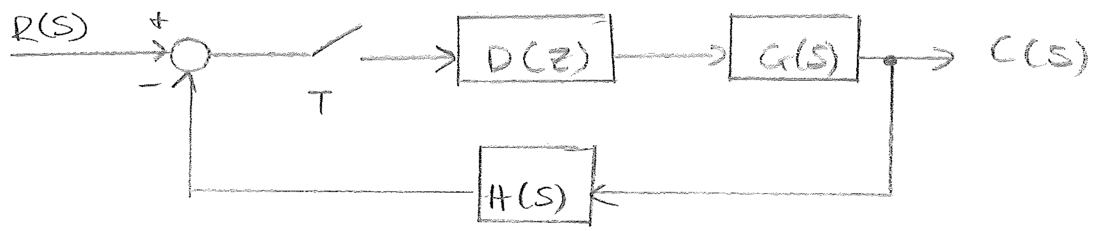
- 

Fig. 1

- For this system $\dfrac{C(z)}{R(z)} = \dfrac{D(z)G(z)}{1 + D(z)\overline{GH}(z)}$

- So the characteristic equation is.

$$Q(z) = 1 + D(z)\overline{GH}(z) = 0$$

- We call the kind of compensation in Fig 1 cascade, or series, compensation.

- We will consider compensation by a first-order device given in the following form.

$$D(z) = \frac{K_d\,(z - z_0)}{(z - z_p)}$$

- The design of D(z) can be performed in the frequency domain using Bode techniques. This implies that we will need to work in the <u>w-plane</u>.

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$$D(w) = D(z)\Big|_{z=[1+(T/2)w]/[1-(T/2)w]}$$

- D(w) will also be first-order

$$D(w) = a_0 \left[\frac{1 + \frac{w}{\omega_{wo}}}{1 + \frac{w}{\omega_{wp}}} \right]$$

where ω_{wo} is the zero location and ω_{wp} is the Pole location in the w-plane.

- The dc-gain of D(z) or D(w) can be found as follows.

$$dc_{gain} = D(z)\Big|_{z=1} = \frac{K_d(1-z_o)}{(1-z_p)}$$

$$= D(w)\Big|_{w=0} = a_0 \left[\frac{1 + \frac{0}{\omega_{wo}}}{1 + \frac{0}{\omega_{wp}}} \right] = a_0$$

$\therefore$ Hence a_0 is the compensator dc-gain.

- To realize the compensator, the designed compensator in D(w) must be transformed back to D(z)

$$- \quad K_d = a_0 \left[\frac{\omega_{wp}(\omega_{wo} + 2/T)}{\omega_{wo}(\omega_{wp} + 2/T)} \right]$$

- $$z_o = \frac{2/T - \omega_{wo}}{2/T + \omega_{wo}}$$

- $$z_p = \frac{2/T - \omega_{wp}}{2/T + \omega_{wp}}$$

The compensator $D(W)$ is classified according to the location of the zero, ω_{wo} relative to the pole, ω_{wp}.

1. IF $\omega_{wo} < \omega_{wp}$, the compensator is called phase lead

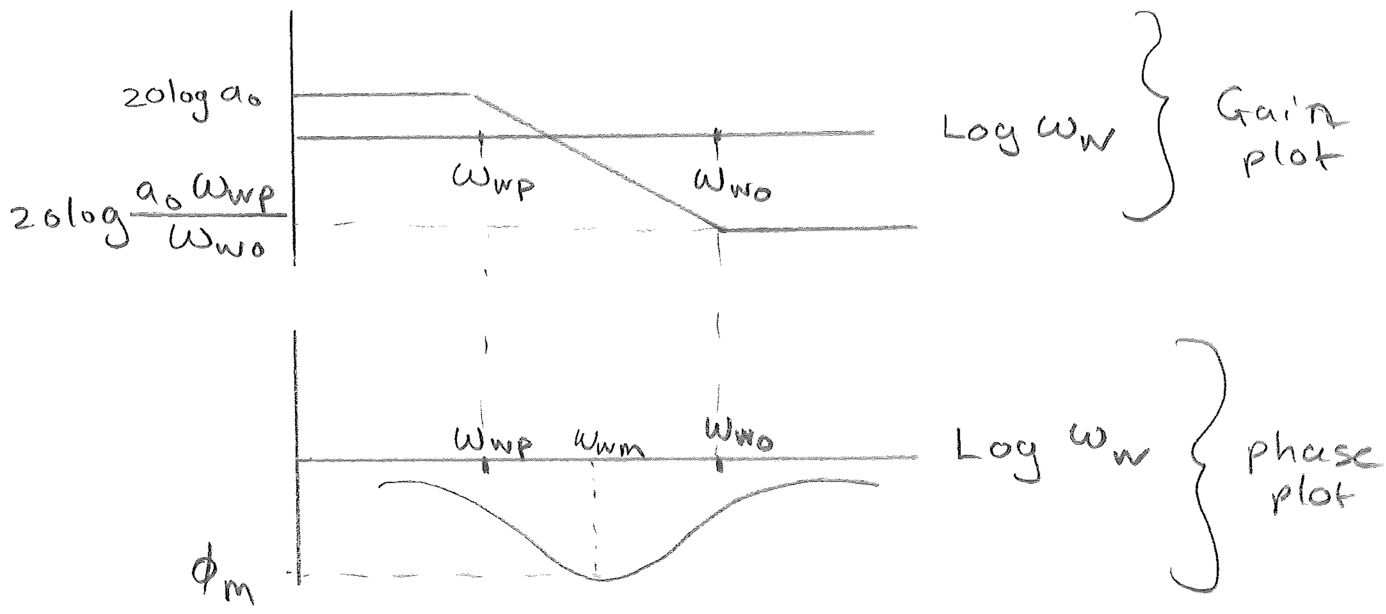
2. IF $\omega_{wo} > \omega_{wp}$, the compensator is called phas lag.

1.1 PHASE-LAG COMPENSATOR DESCRIPTION

For $$D(w) = a_0 \left[\frac{1 + \frac{w}{\omega_{wo}}}{1 + \frac{w}{\omega_{wp}}} \right]$$

$\omega_{wo} > \omega_{wp}$.

- Then $D(w)$ exhibits a negative phase angle, or lag.



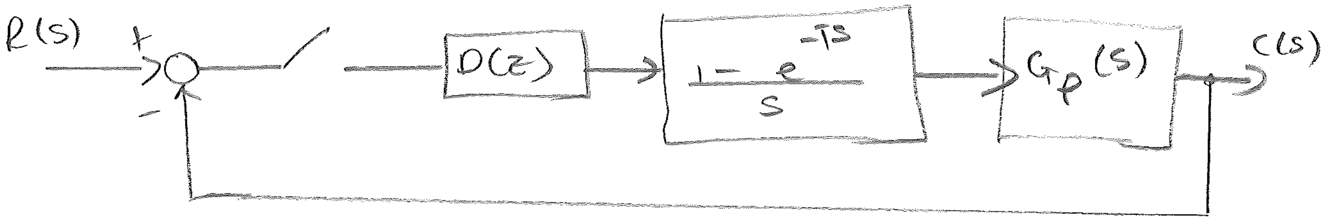
- The dc gain is a_0 and the high frequency gain is $20\log \frac{a_0 \omega_{wp}}{\omega_{wo}}$

- The maximum phase shift is ϕ_M and has a value between 0 and $-90°$ depending on the ratio $\omega_{wo} / \omega_{wp}$

- So in general phase-lag compensaters or filters reduce the high-frequency gain relative to the low-frequency gain and introduces phase lag.

- Since, in general, phase lag tends to destabilize a system, the break frequencies, ω_{wp} and ω_{wo}, must be chosen such that the phase lag does not occur in the vicinity of the $180°$ crossover point of the plant frequency response. $G(j\omega_w)$

$$\text{where} \quad G(w) = \mathcal{Z}\left[\frac{1-e^{-Ts}}{s} G_p(s)\right]\Bigg|_{z=\frac{[1+(T/2)w]}{[1-(T/2)w]}}$$



$$G(z) = \mathcal{Z}\left[\frac{1-e^{-Ts}}{s} G_p(s)\right]$$

$$Q(z) = 1 + D(z)G(z) = 0$$

- For stability it is necessary that the filter introduce a reduced gain in the vicinity of $180°$ crossover.

- Thus, bothe ω_{wp} and ω_{wo} must be much smaller than the $180°$ crossover frequency.

- Normally we design the compensator with a gain of <u>unity</u>!

1.2 PHASE-LAG COMPENSATOR DESIGN APPROACH

Step 1: Determine the frequency, ω_{w1} at which the phase angle of $G(j\omega_{w1})$ is approximately $(-180^\circ + \phi_m + 5^\circ)$.

Step 2: choose

$$\omega_{w0} = 0{,}1\,\omega_{w1}$$

to ensure that little phase lag is introduced at ω_{w1}.

Actually, the compensator will introduce approximately 5^0 Phase lag, which has been accounted for in step 1.

Step 3: At ω_{w1}, we want $|D(j\omega_{w1})G(j\omega_{w1})| = 1$ since the gain of the compensator at high frequencies is $a_0\,\omega_{wp}/\omega_{w0}$ when a_0 is the compensator gain), then

$$\frac{a_0\,\omega_{wp}}{\omega_{w0}} = \frac{1}{|G(j\omega_{w1})|}$$

this results to

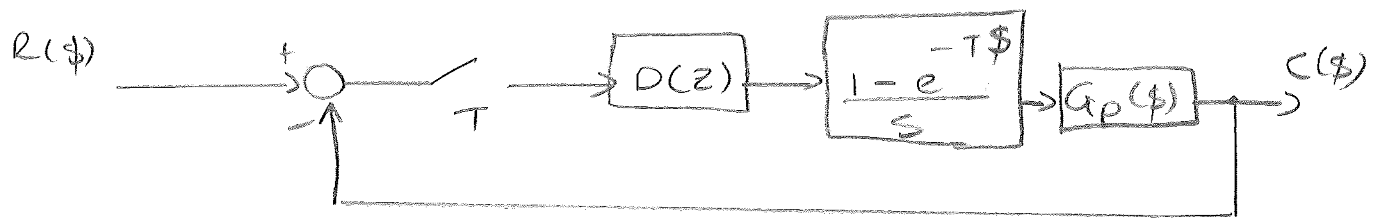
$$\omega_{wp} = \frac{0{,}1\,\omega_{w1}}{a_0\,|G(j\omega_{w1})|}$$

- One ω_{wp} and ω_{w0} are known, $D(z)$ is obtained as describe previously.

- For the case $H(s) \neq 1$ replace $G(j\omega_{w1})$ with $\overline{GH}(j\omega_{w1})$ in step 1.

<u>Ex. 8.1</u>

consider the following control system:



Let $$G_p(s) = \frac{1}{s(s+1)(0{,}5s+1)}$$

$$= \frac{1}{1{,}5s^3 + \frac{3}{2}s^2 + s}$$

and $T = 0{,}05$ s

$$G(z) = \frac{1.372 \times 10^{-5} z^2 + 5.418 \times 10^{-5} z + 1{,}338 \times 10^{-5}}{z^3 - 2{,}95 z^2 + 2{,}901 z - 0{,}9512}.$$

- Suppose that we want to design a unity gain phase-lag compensator to achieve a phase margin of 55°

$s(s+1)(0,5s+1)$
$s(0,5s^2+s+0,5s+1)$
$(0,5s^2+1,5s+s)$

Then:

Step 1 : ϕ of $G(j\omega_w) = -180° + 55° + 5°$
$= -120°$

$\therefore$ From the Bode plot ω_{w1} is the frequency where the phase of $G(j\omega_w)$ is $-120°$

$\omega_{w1} \approx 0,36$ at $-120°$.

At this frequency $|G(j\omega_{w1})|_{dB} = 8.21$

$$20\log|G(j\omega_{w1})| = 8,21$$
$$|G(j\omega_{w1})| = 10^{(8,21/20)}$$
$$\simeq 2,57$$

Then Step 2: $\omega_{w0} = 0,1\,\omega_{w1}$
$= 0,1 \cdot 0,36 = 0,036$

STEP 3: $$\omega_{wp} = \frac{0,1\,\omega_{w1}}{a_0|G(j\omega_{w1})|}$$

$$= \frac{0,1 \cdot 0,36}{1 \cdot (2,57)} = 0,014$$

So $$D(z) = \frac{k_d\,(z - z_0)}{(z - z_p)}$$

$$\therefore\ K_d = a_0 \left[\frac{\omega_{wp}(\omega_{wo} + 2/T)}{\omega_{wo}(\omega_{wp} + 2/T)} \right]$$

$$= 1 \left[\frac{0{,}014\,(0{,}036 + 2/0{,}05)}{0{,}036\,(0{,}014 + 2/0{,}05)} \right]$$

$$= \frac{0{,}561}{1{,}441} = 0{,}389$$

$$z_0 = \frac{2/T - \omega_{wo}}{2/T + \omega_{wo}} = \frac{\frac{2}{0{,}05} - 0{,}036}{2/0{,}05 + 0{,}036}$$

$$= \frac{39{,}964}{40{,}036} = 0{,}9982$$

$$z_p = \frac{2/T - \omega_{wp}}{2/T + \omega_{wp}} = \frac{40 - 0{,}014}{40 + 0{,}014} = 0{,}9993$$

$$D(z) = \frac{0{,}389\,(z - 0{,}9982)}{(z - 0{,}9993)}$$

$$= \frac{0{,}389z - 0{,}388}{z - 0{,}9993}$$

Example 8.1

G_p =

1

0,5

~~1.5~~ s^3 + 1.5 s^2 + s

Continuous-time transfer function.

G_z =

4.014e-05 z^2 + 0.0001547 z + 3.724e-05

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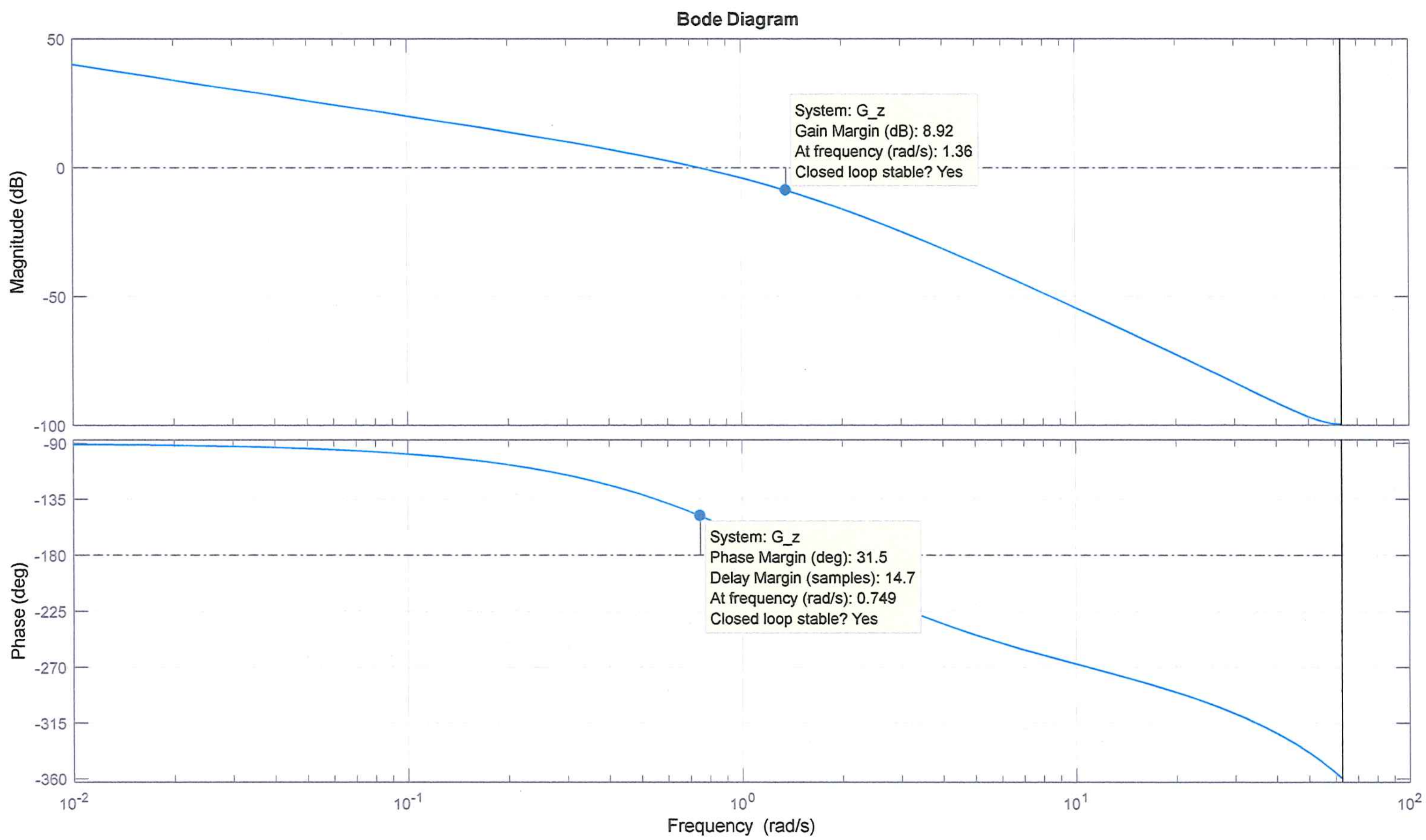
z^3 - 2.856 z^2 + 2.717 z - 0.8607

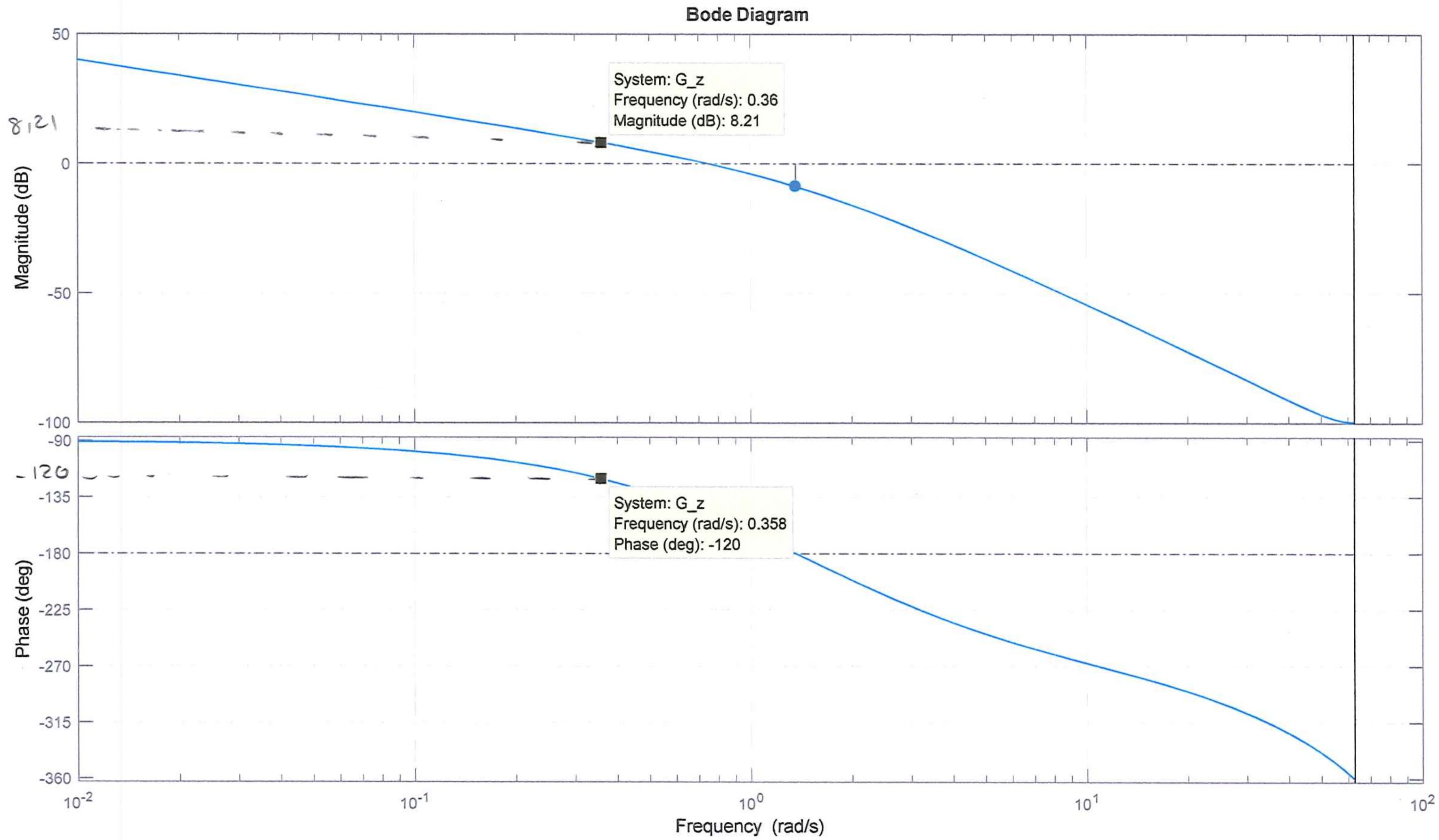
Sample time: 0.05 seconds

Discrete-time transfer function.

Bode diagram of uncompensated system, and will be given in exam.



Determining ω_{w1} by looking at the phase of -120^0



Designed phase lag controller

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D_z = tf([0.389 -0.388],[1 -0.9993],0.05)


D_z =


  0.389 z - 0.388
  ---------------
    z - 0.9993

Sample time: 0.05 seconds

Discrete-time transfer function.

DG_z = D_z*G_z

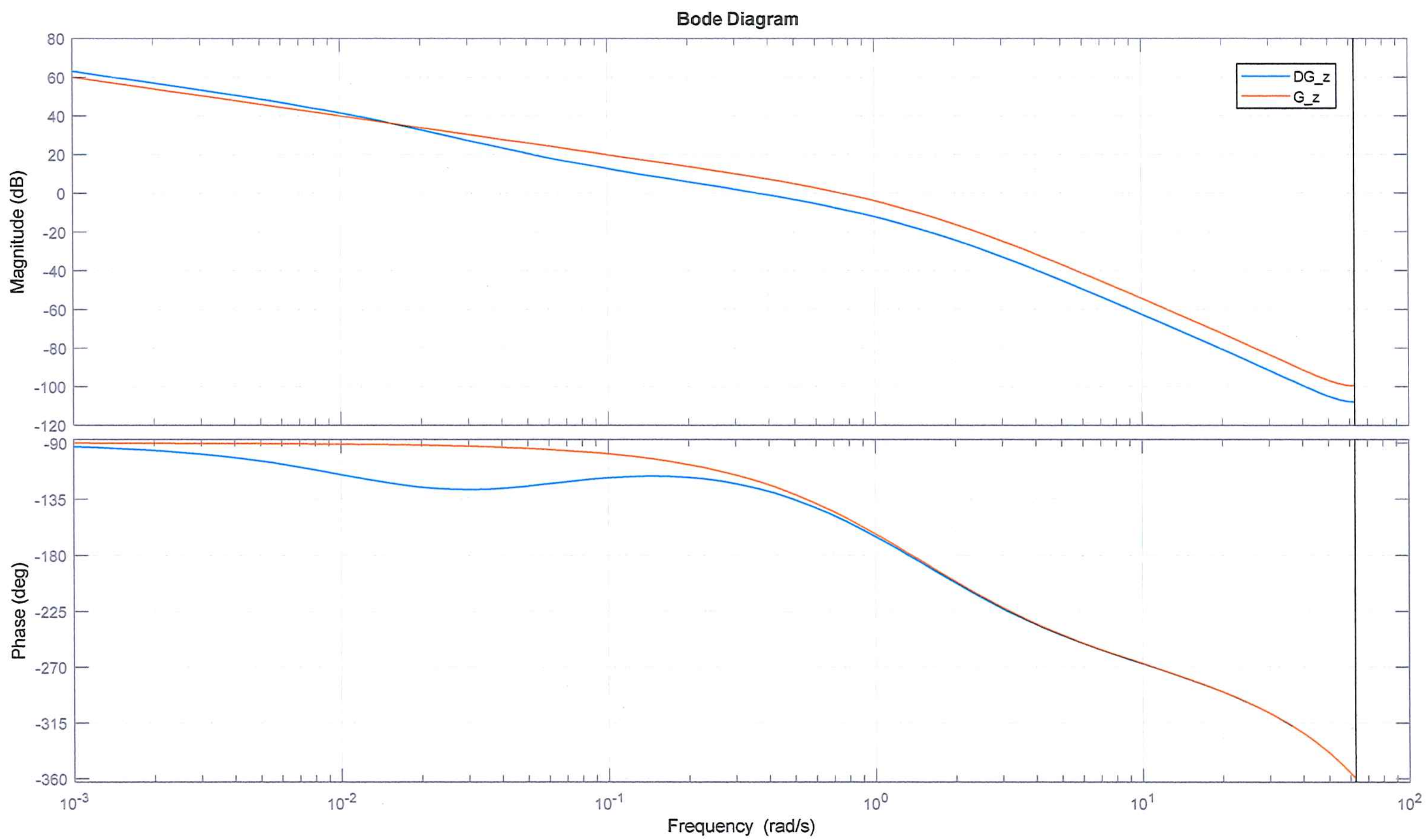
DG_z =


  1.561e-05 z^3 + 4.46e-05 z^2 - 4.553e-05 z - 1.445e-05
  ------------------------------------------------------
     z^4 - 3.855 z^3 + 5.571 z^2 - 3.576 z + 0.8601

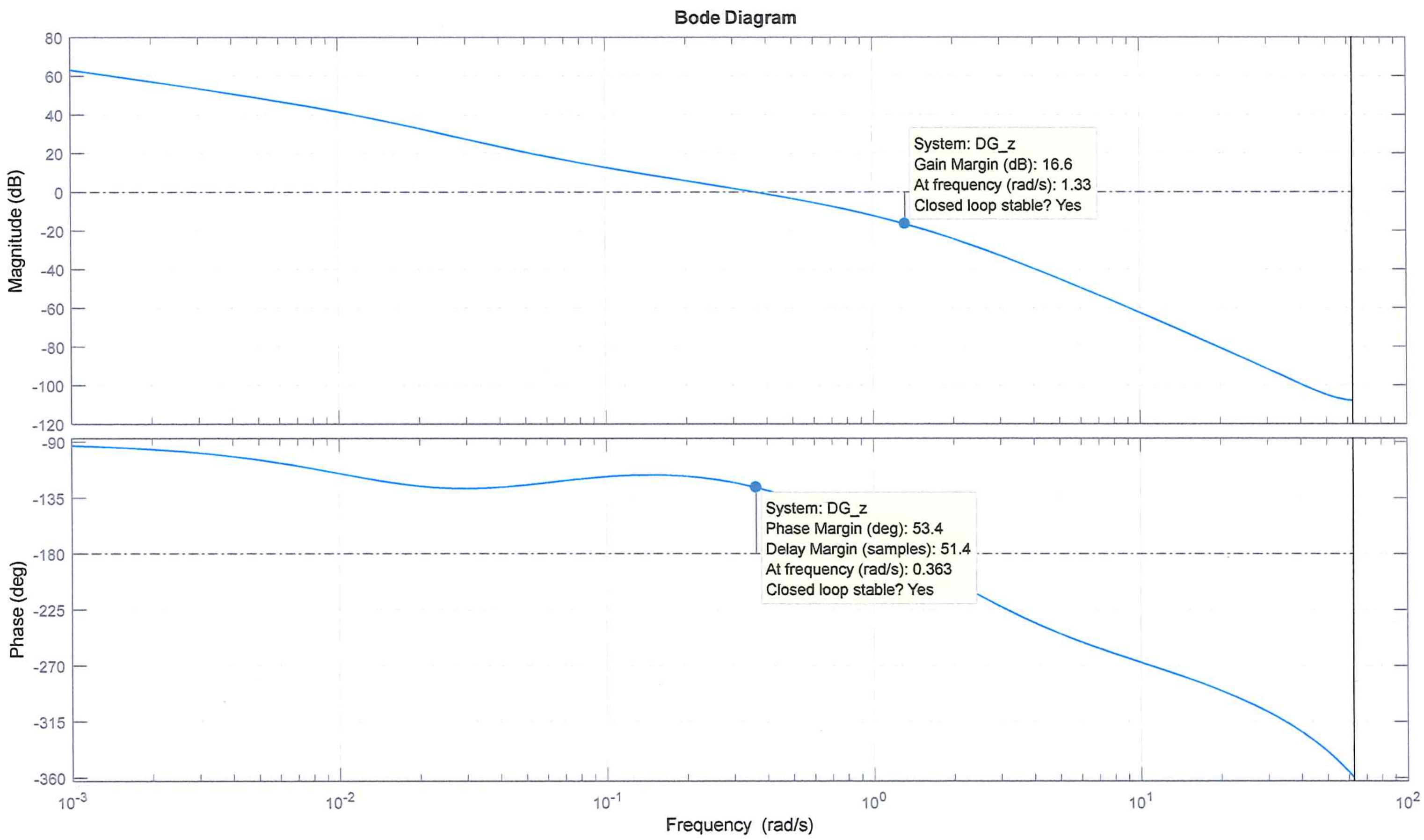

Sample time: 0.05 seconds

Discrete-time transfer function.
```

Compensated system DG_z



Check stability margins of compensated system



The phase margin is around 55 degrees and a gain margin of about 16 dB.