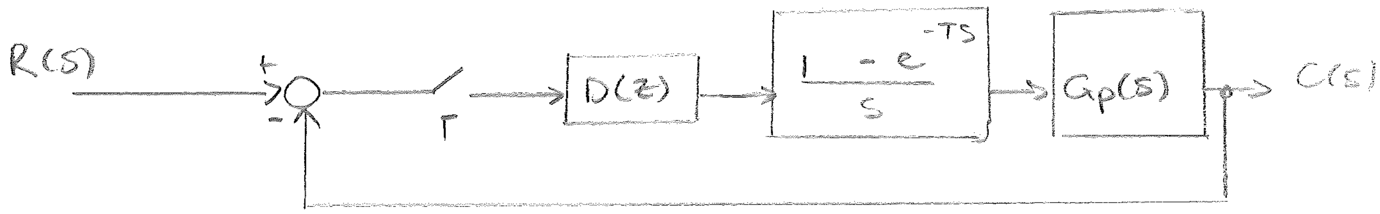
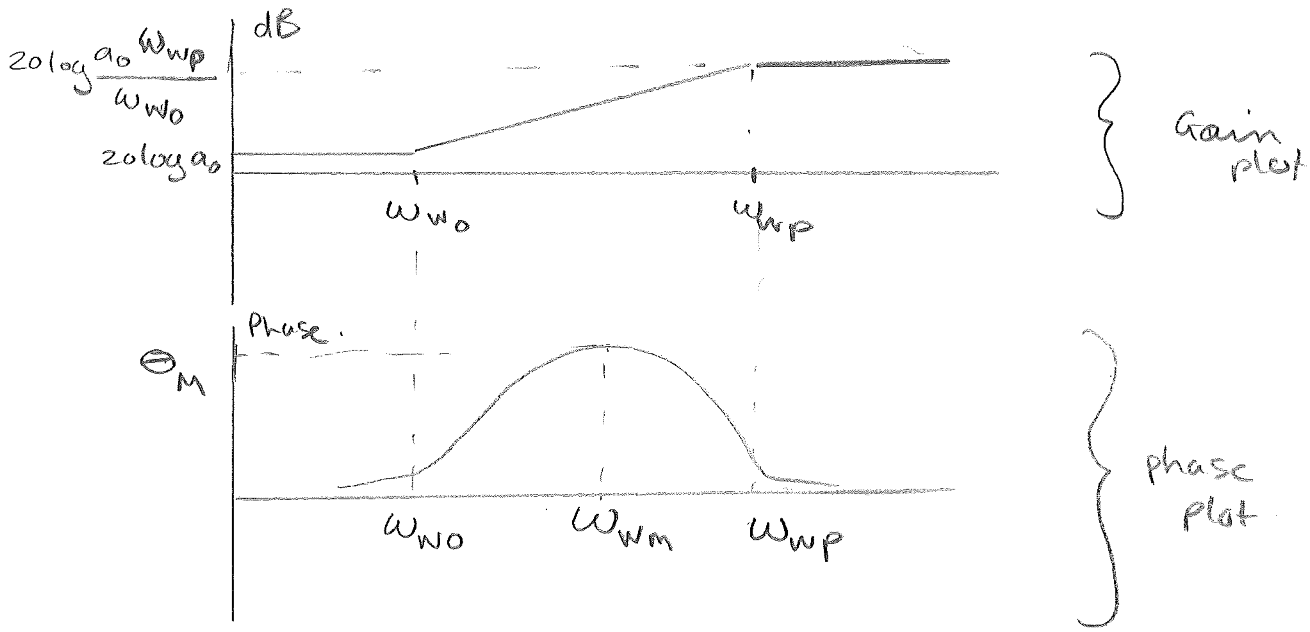
NOTES ON COMPENSATORS: PHASE LEAD

Again consider the digital control system:



A phase lead compensator will now be discussed.

- For phase-lead $\omega_{WO} < \omega_{WP}$



- The maximum phase shift, Θ_M, occurs at a frequency ω_{WM}, where ω_{WM} is the geometric mean of ω_{WO} and ω_{WP}

$$\therefore \omega_{WM} = \sqrt{\omega_{WO}\,\omega_{WP}}$$

- Phase-lead compensation introduces phase lead which is a stabilizing effect, but also increases the high-frequency gain relative to the low-frequency gain, which is a destabilizing effect

- The phase lead is introduced in the vicinity of the plant's 180° crossover frequency, in order to increase the system's stability margins.

- The system bandwidth is also increased, resulting in a faster time response.

- Again we design the compensator to have a dc gain of <u>unity</u>

- The design of a phase-lead compensator is a more of a trial-and-error procedure since, in the frequency range that the stabilizing phase lead is added, destabilizing gain is also added.

PHASE-LEAD DESIGN PROCEDURE

- The characteristic eq is given:

$$1 + D(w)\,G(w) = 0$$

- Determine $D(w)$ such that, at some frequency ω_{w1}

$$D(j\omega_{w1})\,G(j\omega_{w1}) = 1 \angle 180^\circ + \phi_M$$

where ϕ_m = desired phase margin.

and, in addition, the system posses ses and adequate gain margin.

- The design equations will now be developed

- Let

$$D(w) = \frac{a_1 w + a_0}{b_1 w + 1}$$

- where a_0 = compensator dc-gain.

- Then $\omega_0 = \dfrac{a_0}{a_1}$, $\omega_p = \dfrac{1}{b_1}$

- Also $|D(j\omega_{w1})| = \dfrac{1}{|G(j\omega_{w1})|}$ must be satisfied

- and

$$\angle D(j\omega_{w_1}) = 180^\circ + \phi_m - \angle G(j\omega_{w_1})$$

- Let the angle associated with $D(j\omega_{w_1})$ be denote by θ

$$\therefore \quad \theta = \angle D(j\omega_{w_1}) = 180^\circ + \phi_m - \angle G(j\omega_{w_1})$$

- so $$a_1 = \frac{1 - a_0 |G(j\omega_{w_1})| \cos\theta}{\omega_{w_1} |G(j\omega_{w_1})| \sin\theta}$$

and

$$b_1 = \frac{\cos\theta - a_0 |G(j\omega_{w_1})|}{\omega_{w_1} \sin\theta}$$

- Remember, if $H(s) \neq 1$ replace $G(j\omega_{w_1})$ with $\overline{GH}(j\omega_{w_1})$

PHASE-LEAD DESIGN STEPS

- The design procedure requires that the compensator dc gain a_0 and the system phase-margin frequency ω_{w1} be chosen

- Then the compensator coefficients a_1 and b_1 are determined.

- The compensator dc gain is usually determined by steady-state specifications for the control system.

- The frequency ω_{w1} can be approximately determined in the following manner:

- Since this is a phase lead

$$\theta > 0$$

when $\theta = 180^\circ + \phi_m - \angle G(j\omega_{w1})$

- Step 1: $\angle G(j\omega_{w1}) < -180^\circ + \phi_m$

Also $|D(j\omega_{w1})| > a_0$

- Step 2: $|G(j\omega_{w1})| < \dfrac{1}{a_0}$

In addition, the coefficient b_1 must be positive, to ensure a stable controller.

- Step 3: $\cos\theta > a_0 |G(j\omega_{w1})|$

So, the phase-margin frequency ω_{w1} must be chosen to satisfy 3 constraints

Ex. 8.2

A phase margin of 55° is to be achieved, and a unity-gain phase-lead compensator will be employed.

So $G_p(s) = \dfrac{1}{s(s+1)(0,5s+1)}$, $T = 0,05s$.

$= \dfrac{1}{0,5s^3 + 1,5s^2 + s}$

$G(z) = \dfrac{4.014\times10^{-5} z^2 + 0,0001547 z + 3,724\times10^{-5}}{z^3 - 2,856 z^2 + 2,717 z - 0,8607}$

Step 1: $\angle G(j\omega_{w_1}) \angle -180° + 55°$
$< -125°$

So we must choose $\angle G(j\omega_{w_1}) < -125°$

and $|G(j\omega_{w_1})| < 1$

Curently for a phase of $-127°$ at $0,444$ rad we get a gain of $6,06$ dB

$\therefore \ 20\log|G(j\omega_{w_1})| = 6,06\text{dB}$

$|G(j\omega_{w_1})| = 10^{6,06/20}$
$= 2$

so this does not satify the contraint

So rather choose an arbitrary $\omega_{w_1} = 1{,}2$ rad/s

then $\quad \angle G(j\omega_{w_1}) = -173° < -125°$ ✓

and

$$20 \log |G(j\omega_{w_1})| = -6{,}77 \text{ dB}$$

$$|G(j\omega_{w_1})| = 10^{\frac{-6{,}77}{20}}$$

$$= 0{,}46 \quad < 1 \quad ✓$$

$$\therefore \quad \theta = 180° + 55° - (-173°)$$

$$= 408° \quad \text{or} \quad 48°$$

Consider step 3

The constraint $\quad \cos\theta > a_0 |G(j\omega_{w_1})|$

yields $\quad \cos 48° > 1 \cdot 0{,}46$

$$0{,}669 > 0{,}46 \quad ✓$$

Hence all the constraints are now satisfied

Now $\quad a_1 = \dfrac{1 - a_0 |G(j\omega_{w_1})| \cos\theta}{\omega_{w_1} |G(j\omega_{w_1})| \sin\theta}$

$$= \frac{1 - (1)(0{,}46)\cos 48°}{(1{,}2)(0{,}46)\sin 48°} = \frac{0{,}692}{0{,}4102}$$

$$= 1{,}68 \approx 1{,}7$$

$$b_1 = \frac{\cos\theta - a_0 \, |G(j\omega_{w1})|}{\omega_{w1} \sin\theta}$$

$$= \frac{\cos 48° - (1)(0{,}46)}{(1{,}2)\sin 48°} = \frac{0{,}2091}{0{,}89177} \approx 0{,}24$$

so the transfer function of the lead compensator $D(w)$ is.

$$D(w) = \frac{1 + 1{,}7\,W}{1 + 0{,}24\,W} = \frac{1{,}7}{0{,}24}\left[\frac{1 + \frac{w}{0{,}59}}{1 + \frac{w}{4{,}2}}\right]$$

$$= 7\left[\frac{1 + \frac{w}{0{,}59}}{1 + \frac{w}{4{,}2}}\right]$$

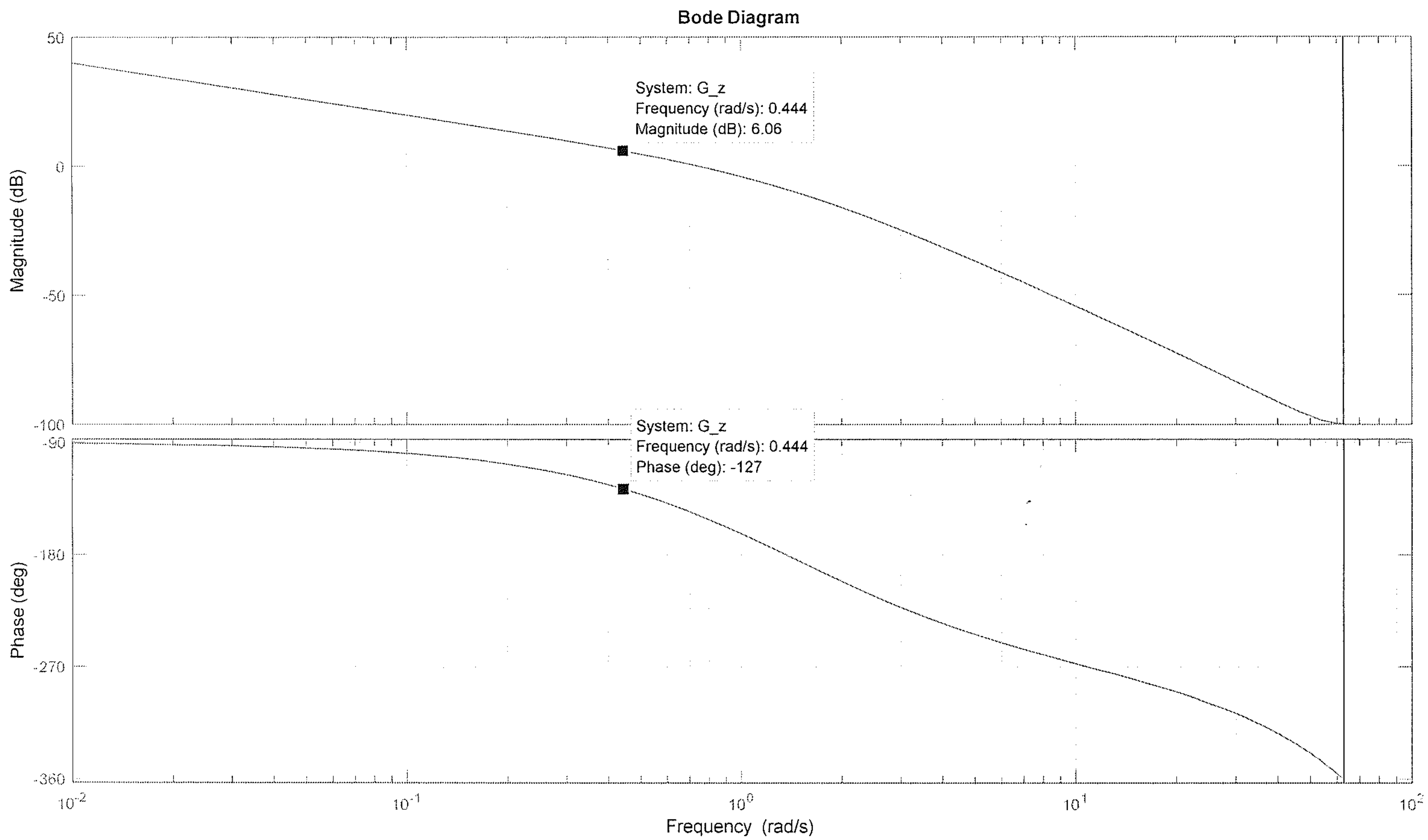
$\therefore \ \omega_{wo} = 0{,}59$ and $\omega_{wp} = 4.2$ and $a_0 = 7.$

$$\therefore \ z_0 = \frac{2/T - \omega_{wo}}{2/T + \omega_{wo}} = \frac{\frac{0{,}05}{2} - 0{,}59}{\frac{0{,}05}{2} + 0{,}59} = \frac{-0{,}565}{0{,}615} = -0{,}919$$

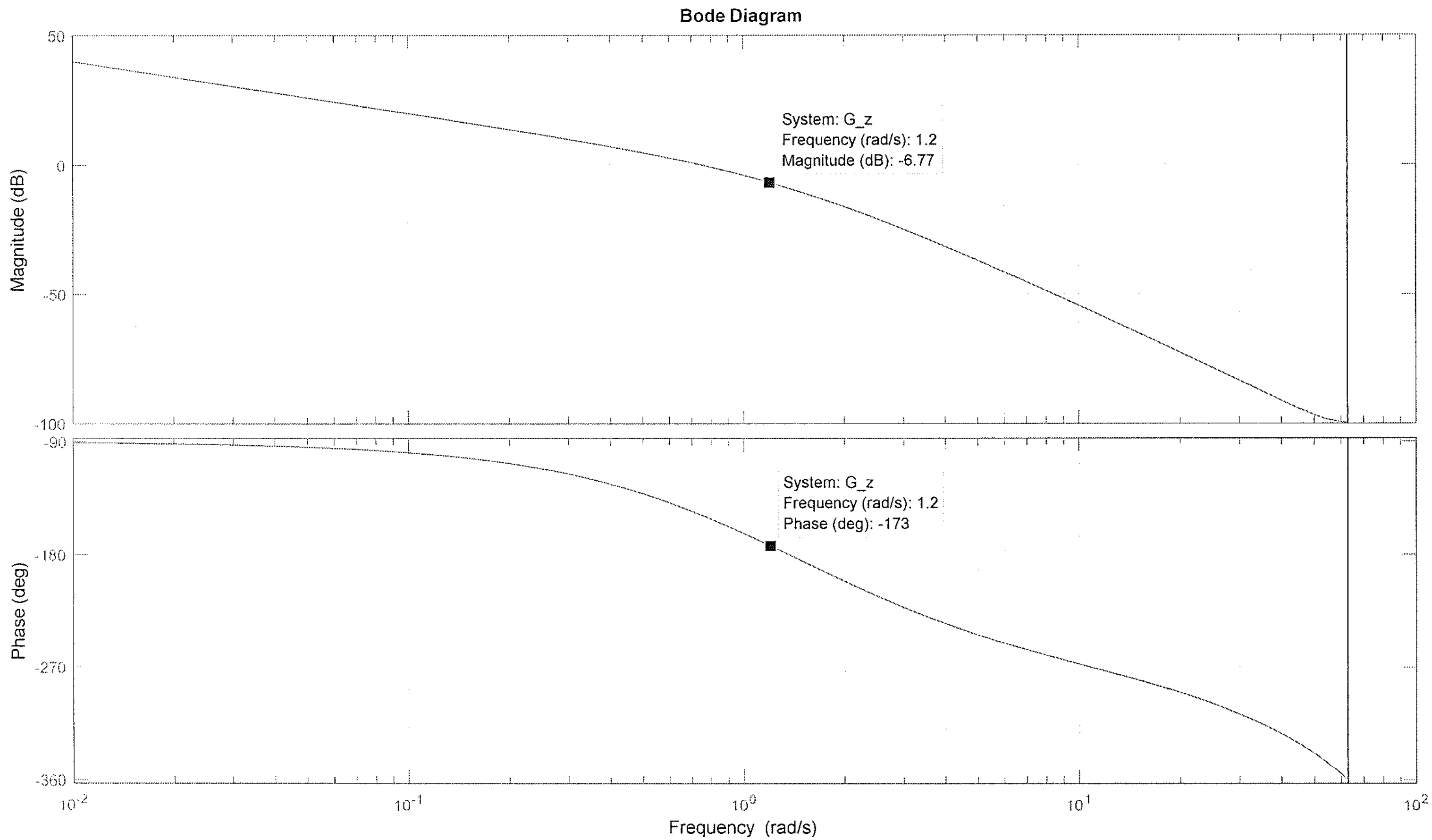
$$z_p = \frac{2/T - \omega_{wp}}{2/T + \omega_{wp}} = \frac{\frac{0{,}05}{2} - 4{,}2}{\frac{0{,}05}{2} - 4{,}2} = \frac{-4{,}175}{-4{,}175} = 1$$

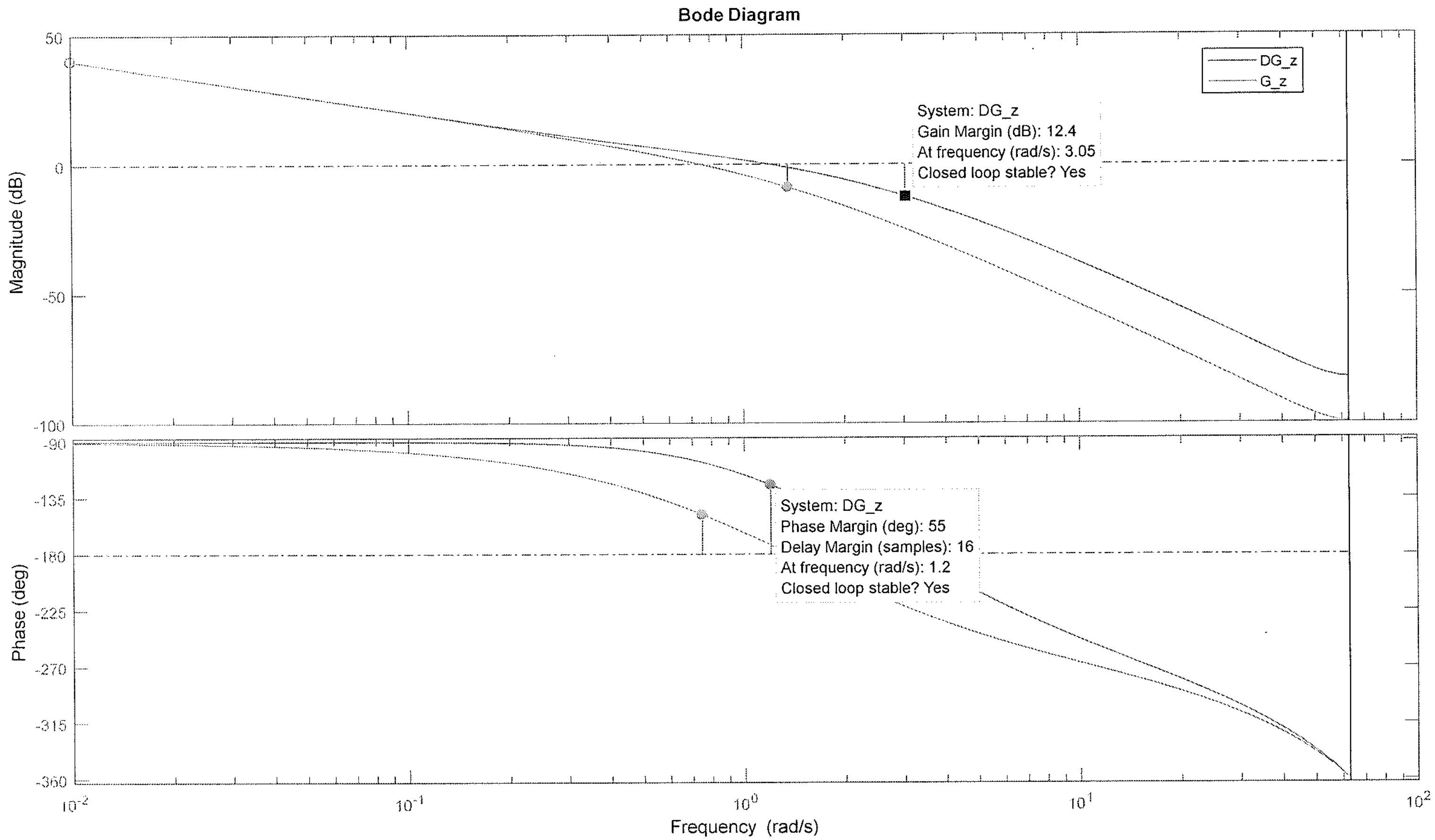
$$D(z) = \frac{7\,(z - 0{,}92)}{(z-1)} = \frac{7z - 6{,}44}{z - 1}$$

For $\omega_{w1} = 0.444$ rad/s the phase is -127 degrees which is smaller than -125 degrees but the gain is of $G(\omega_{w1})$ is not smaller than one, in fact it is 2.



Choose $\omega_{w1} = 1.2$ rad/s, then the gain is -6.77 and phase is -173 degrees


Bode Diagram
50
0
-50
-100
Magnitude (dB)
System: G_z
Frequency (rad/s): 1.2
Magnitude (dB): -6.77
-90
-180
-270
-360
Phase (deg)
System: G_z
Frequency (rad/s): 1.2
Phase (deg): -173
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/s)



Bode Diagram
50
0
-50
-100
Magnitude (dB)
DG_z
G_z
System: DG_z
Gain Margin (dB): 12.4
At frequency (rad/s): 3.05
Closed loop stable? Yes
-90
-135
-180
-225
-270
-315
-360
Phase (deg)
System: DG_z
Phase Margin (deg): 55
Delay Margin (samples): 16
At frequency (rad/s): 1.2
Closed loop stable? Yes
10^-2
10^-1
10^0
10^1
10^2
Frequency (rad/s)