YUNIBESITI YA BOKONE-BOPHIRIMA
NORTH WEST UNIVERSITY
NOORDWES UNIVERSITEIT

Benodigdhede vir hierdie vraestel:

Multikeusekaarte: ☐ Nie-programmeerbare sakrekenaar: ☑
Grafiekpapier: ☐ Draagbare rekenaar: ☐

Oopboek-eksamen: ☐

SEMESTERTOETS:	1e	GRADE/DIPLOMA:	
VAKKODE:	**EERI 418**	DUUR:	2 URE
VAK:	BEHEERTEORIE II	MAKS:	80
DOSENT:	PROF G VAN SCHOOR	DATUM:	**25-04-2007**
		TYD:	**09h00**
MODERATOR:	PROF CP BODENSTEIN		

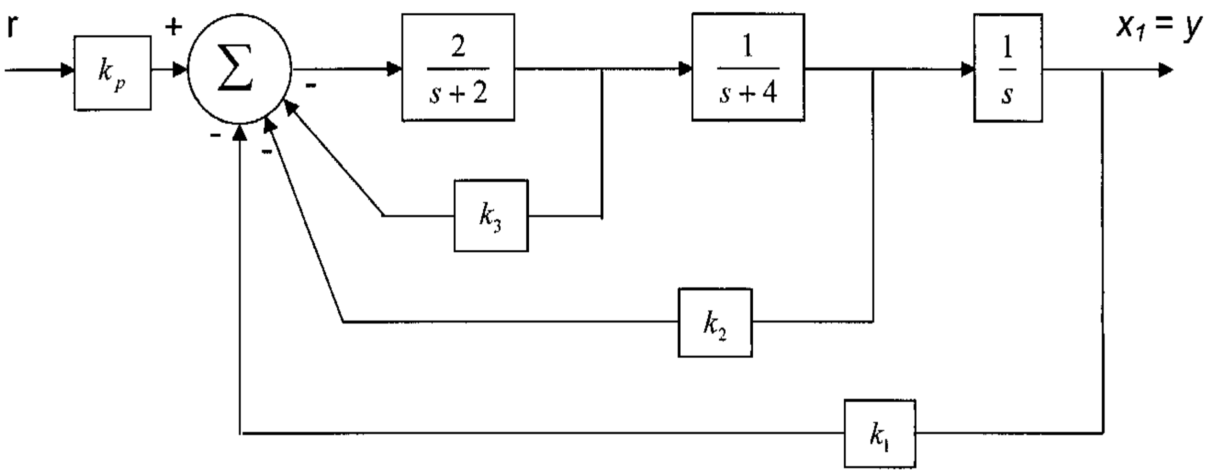
TOTAAL: 80

VRAAG 1/ *QUESTION* 1

Die blokdiagram van 'n stelsel met toestandsterugvoer word in figuur 1 getoon. Bepaal die winswaardes *k1*, *k2*, *k3* en k_p sodat: /
The block diagram of a system with state feedback is shown in figure 1. Determine the gains k1, k2, k3 and k_p *such that:*

(a) die bestendige toestand fout vir 'n trapinset nul is / *the steady state error for a step input is zero;*
(b) die persentasie verbyskiet kleiner as 5 % is en die vestigingstyd kleiner as 0.5 s is / *the percentage overshoot is less than* 5 % *and the settling time less than 0.5 s.*

Gebruik die ITAE optimum polinoom metode. / *Use the ITAE optimum polynomial method.*



Figuur / *Figure* 1

Addisionele inligting / *additional information:*

$$PO = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$T_s = \frac{4}{\zeta\omega_n}$$

[20]

VRAAG 2 / *QUESTION* 2

2.1 'n Stelsel word deur die volgende verskilvergelyking gemodelleer: /
A system is modelled by the following difference equation:

$$y(k+2)+6y(k+1)+5y(k)=3e(k+2)+e(k+1)+2e(k)$$

Bepaal die oordragsfunksie van die stelsel $(\frac{Y(z)}{E(z)})$. /

Determine the transfer function of the system $(\frac{Y(z)}{E(z)})$.

(4)

2.2 Bepaal y(k) vir die stelsel in 2.1 vir 'n eenheidstrapinset deur van magreeksuitbreiding gebruik te maak. Bereken tot die vyfde term (*y(4)*). Aanvaar begintoestande as nul. /

Determine y(k) for the system in 2.1 for a unit step input. Use the power series method and determine up to the fifth term (y(4)). Assume zero initial conditions.

(5)

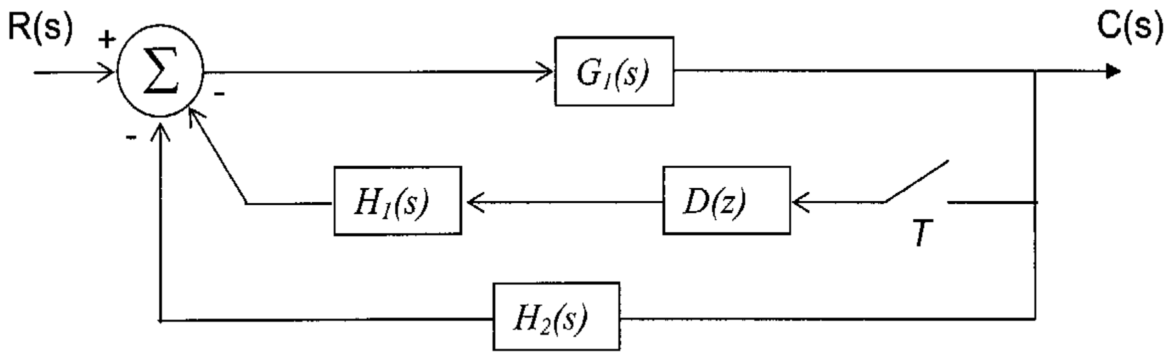
2.3 Bepaal y(k) vir die stelsel in 2.1 in geslote vorm vir 'n eenheidstrapinset deur van parsiele breuk uitbreiding gebruik te maak. /
Determine y(k) for the system in 2.1 in closed form for a unit step input using partial fraction expansion.

(7)

2.4 Bepaal die beheer kanonieke toestandsveranderlike model van die stelsel in 2.1. /
Determine the control canonical state variable model of the system in 2.1.

(8)

2.5 Bepaal vir die stelsel in figuur 2 die uitset C(z) in terme van die inset en die oordragsfunksies. /
Determine for the system in figure 2 the output C(z) in terms of the input and the transfer functions.

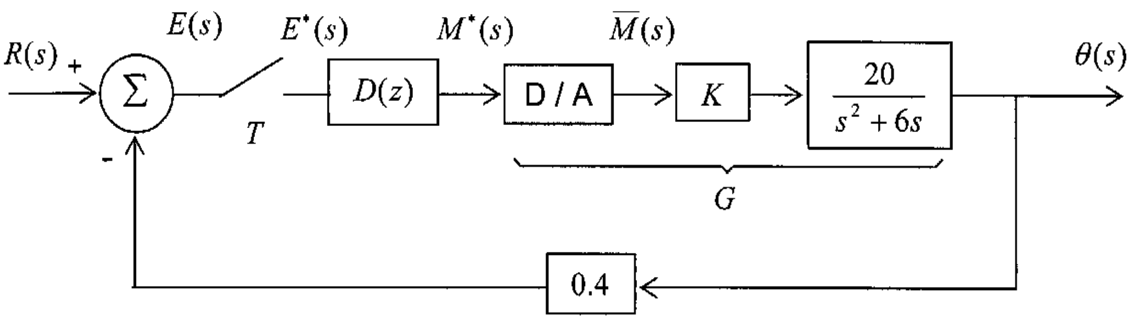


Figuur / *Figure* 2

(6)

[30]

VRAAG 3 / *QUESTION* 3



Figuur / *Figure* 3

Beskou die antenna-beheerstelsel in figuur 3. Die eenheid vir die antennahoek $\theta(t)$ is grade. /
Consider the antenna control system shown in figure 3. The unit for the antenna angle $\theta(t)$ is degrees.

3.1 Bepaal die waardes van $r(t)$ wat hoeke van ±30° vir θ(t) sal gee. /
Determine the values of $r(t)$ that will give the angles of ±30° for $\theta(t)$. (1)

3.2 Bepaal die stelseloordragsfunksie $(\frac{\theta(z)}{R(z)})$ in terme van $G(z)$ en $D(z)$. /
Determine the system transfer function $(\frac{\theta(z)}{R(z)})$ in terms of *G(z)* and *D(z)*. (1)

3.3 Bepaal die oordragsfunksie vir $D(z) = 1$, $K = 20$ en $T = 0.05$ s.
Wat is die tipe van die stelsel? /
Determine the transfer function for $D(z) = 1$, $K = 20$ and $T = 0.05$ s.
Find the system type. (4)

3.4 Bepaal die bestendige toestand fout van die stelsel vir 'n eenheidshellingsinset. /
Determine the steady state error of the system for a unit ramp input. (4)

3.5 Bepaal die demping asook die natuurlike frekwensie van die diskrete stelsel. /
Determine the damping as well as the natural frequency of the discrete system. (5)

3.6 Die filter *D(z)* realiseer nou die volgende verskilvergelyking: /
The filter D(z) now realises the following difference equation:

$$m(k) = e(k) - 0.9e(k-1) + m(k-1)$$

Wat is die tipe van die stelsel nou? /
What is the now the system type?. (3)

3.7 Met *D(z)* soos in 3.6, bepaal weer die bestendige toestand fout van die stelsel vir 'n eenheidshellingsinset. /
For D(z) as in 3.6, again determine steady state error of the system for a unit ramp input. (2)

Addisionele inligting / *additional information:*

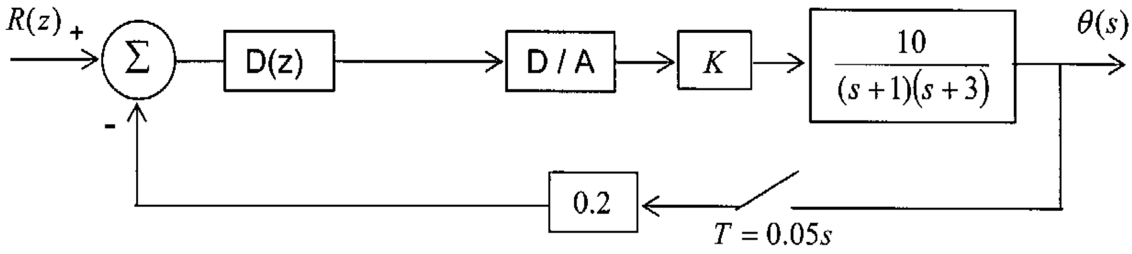
$$\zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}}$$

$$\omega_n = \frac{1}{T}\sqrt{\ln^2 r + \theta^2}$$

[20]

$$\tau = \frac{1}{\zeta\omega_n}$$

VRAAG / *QUESTION* 4



Figuur / *Figure* 4

Beskou die beheerstelsel in figuur 4. /
Consider the control system shown in figure 4.

Gegee / Given: $z\left[\frac{1-e^{-sT}}{s}\frac{10}{(s+1)(s+3)}\right] = \frac{0.0117z + 0.011}{(z-0.95)(z-0.86)}$

4.1 Skryf die geslotelusoordragsfunksie $(\frac{\theta(z)}{R(z)})$ neer. Neem D(z) = 1. /
Determine the closed loop transfer function $(\frac{\theta(z)}{R(z)})$. Take D(z) = 1. (3)

4.2 Gebruik die Jury stabiliteitstoets om die bereik van K te bepaal vir stabiliteit. /
Use the Jury stability test to determine the range of K for stability. (5)

4.3 Bepaal die frekwensie vir marginale stabiliteit. /
Determine the frequency for marginal stability. (4)

4.4 Teken die benaderde wortellokus vir die stelsel en bepaal die wins *K* vir 'n dempingskonstante van $\varsigma = 0.707$. /
Draw the approximated root locus for the system and determine the gain K for a damping constant of: $\varsigma = 0.707$. (6)

[15]

Table 1. Properties of the z transform

Sequence	Transform
$e(k)$	$E(z) = \sum_{k=0}^{\infty} e(k) z^{-k}$
$a_1 e_1(k) + a_2 e_2(k)$	$a_1 E_1(z) + a_2 E_2(z)$
$e(k-n)u(k-n);\quad n \geq 0$	$z^{-n} E(z)$
$e(k+n)u(k);\quad n \geq 1$	$z^n \left[E(z) - \sum_{k=0}^{n-1} e(k) z^{-k} \right]$
$\epsilon^{ak} e(k)$	$E(z\epsilon^{-a})$
$ke(k)$	$-z \dfrac{dE(z)}{dz}$
$e_1(k) * e_2(k)$	$E_1(z) E_2(z)$
$e_1(k) = \sum_{n=0}^{k} e(n)$	$E_1(z) = \dfrac{z}{z-1} E(z)$

Initial value: $e(0) = \lim_{z \to \infty} E(z)$

Final value: $e(\infty) = \lim_{z \to 1} (z-1)E(z)$, if $e(\infty)$ exists

Table 2. z-transforms

Sequence	z-Transform
$\delta(k-n)$	z^{-n}
1	$\dfrac{z}{z-1}$
k	$\dfrac{z}{(z-1)^2}$
k^2	$\dfrac{z(z+1)}{(z-1)^3}$
a^k	$\dfrac{z}{z-a}$
ka^k	$\dfrac{az}{(z-a)^2}$
$\sin ak$	$\dfrac{z \sin a}{z^2 - 2z \cos a + 1}$
$\cos ak$	$\dfrac{z(z - \cos a)}{z^2 - 2z \cos a + 1}$
$a^k \sin bk$	$\dfrac{az \sin b}{z^2 - 2az \cos b + a^2}$
$a^k \cos bk$	$\dfrac{z^2 - az \cos b}{z^2 - 2az \cos b + a^2}$

Table 3. z-transforms

Laplace transform $E(s)$	Time function $e(t)$	z-Transform $E(z)$	Modified z-transform $E(z,m)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1}+\frac{T}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2}\left[\frac{m^2}{z-1}+\frac{2m+1}{(z-1)^2}+\frac{2}{(z-1)^3}\right]$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{1}{s+a}$	ϵ^{-at}	$\frac{z}{z-\epsilon^{-aT}}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$	$\frac{Tz\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}$	$\frac{T\epsilon^{-amT}[\epsilon^{-aT}+m(z-\epsilon^{-aT})]}{(z-\epsilon^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k\epsilon^{-at}$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-\epsilon^{-at}$	$\frac{z(1-\epsilon^{-aT})}{(z-1)(z-\epsilon^{-aT})}$	$\frac{1}{z-1}-\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1-\epsilon^{-at}}{a}$	$\frac{z[(aT-1+\epsilon^{-aT})z+(1-\epsilon^{-aT}-aT\epsilon^{-aT})]}{a(z-1)^2(z-\epsilon^{-aT})}$	$\frac{T}{(z-1)^2}+\frac{amT-1}{a(z-1)}+\frac{\epsilon^{-amT}}{a(z-\epsilon^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\epsilon^{-at}$	$\frac{z}{z-1}-\frac{z}{z-\epsilon^{-aT}}-\frac{aT\epsilon^{-aT}z}{(z-\epsilon^{-aT})^2}$	$\frac{1}{z-1}-\left[\frac{1+amT}{z-\epsilon^{-aT}}+\frac{aT\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}\right]\epsilon^{-amT}$
$\frac{b-a}{(s+a)(s+b)}$	$\epsilon^{-at}-\epsilon^{-bt}$	$\frac{(e^{-aT}-\epsilon^{-bT})z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}-\frac{\epsilon^{-bmT}}{z-\epsilon^{-bT}}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z\sin(aT)}{z^2-2z\cos(aT)+1}$	$\frac{z\sin(amT)+\sin(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{s}{s^2+a^2}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2-2z\cos aT+1}$	$\frac{z\cos(amT)-\cos(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\epsilon^{-at}\sin bt$	$\frac{1}{b}\left[\frac{z\epsilon^{-aT}\sin bT}{z^2-2z\epsilon^{-aT}\cos(bT)+\epsilon^{-2aT}}\right]$	$\frac{1}{b}\left[\frac{\epsilon^{-amT}[z\sin bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$\epsilon^{-at}\cos bt$	$\frac{z^2-z\epsilon^{-aT}\cos bT}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$	$\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-\epsilon^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$	$\frac{z(Az+B)}{(z-1)(z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT})}$	$\frac{1}{z-1}$
		$A=1-\epsilon^{-aT}\left(\cos bT+\frac{a}{b}\sin bT\right)$	$-\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
		$B=\epsilon^{-2aT}+\epsilon^{-aT}\left(\frac{a}{b}\sin bT-\cos bT\right)$	$\frac{+\frac{a}{b}\{\epsilon^{-amT}[z\sin bmT-\epsilon^{-aT}\sin(1-m)bT]\}}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}+\frac{\epsilon^{-at}}{a(a-b)}$	$\frac{(Az+B)z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})(z-1)}$	$A=\frac{b(1-\epsilon^{-aT})-a(1-\epsilon^{-bT})}{ab(b-a)}$
	$+\frac{\epsilon^{-bt}}{b(b-a)}$		$B=\frac{a\epsilon^{-aT}(1-\epsilon^{-bT})-b\epsilon^{-bT}(1-\epsilon^{-aT})}{ab(b-a)}$

Table 4. Laplace transform properties

Name	Theorem
Derivative	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0^+)$
nth-order derivative	$\mathscr{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^+)$
	$- \cdots f^{(n-1)}(0^+)$
Integral	$\mathscr{L}\left[\int_0^t f(\tau)\, d\tau\right] = \frac{F(s)}{s}$
Shifting	$\mathscr{L}[f(t - t_0)u(t - t_0)] = e^{-t_0 s} F(s)$
Initial value	$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$
Final value	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$
Frequency shift	$\mathscr{L}[e^{-at} f(t)] = F(s + a)$
Convolution integral	$\mathscr{L}^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t - \tau) f_2(\tau)\, d\tau$
	$= \int_0^t f_1(\tau) f_2(t - \tau)\, d\tau$

Table 5 Optimum coefficients of T(s) Based on the ITAE Criterion for a Step Input

$$s + \omega_n$$
$$s^2 + 1.4\omega_n s + \omega_n^2$$
$$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$$
$$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$$
$$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$$
$$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.60\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$$

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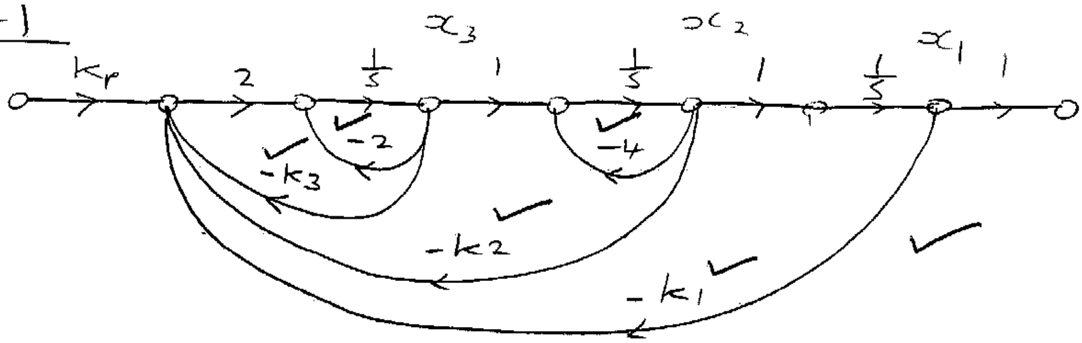
VRAAG 1



$$P_1 = \frac{2k_p}{s^3}$$

$$\Delta = 1 - \left(\frac{-2}{s} - \frac{2k_3}{s} - \frac{4}{s} - \frac{2k_2}{s^2} - \frac{2k_1}{s^3}\right) + \left(\frac{2}{s}\frac{4}{s} + \frac{2k_3}{s}\frac{4}{s}\right)$$

$$= 1 + \frac{6+2k_3}{s} + \frac{2k_2+8+8k_3}{s^2} + \frac{2k_1}{s^3}$$

$$\therefore T(s) = \frac{2k_p}{s^3 + (6+2k_3)s^2 + (2k_2+8k_3+8)s + 2k_1}$$

vir $e_{ss} = 0$ vir 'n eenheidstrap moet $T(0) = 1$

$$\frac{2k_p}{2k_1} = 1 \qquad k_p = k_1$$

ITAE polinoom: $s^3 + 1{,}75\omega_n s^2 + 2{,}15\omega_n^2 s + \omega_n^3$

Vir PO < 5 % $\zeta = 0{,}7$ en vir $T_s = \frac{4}{\zeta\omega_n} < 0{,}5$

$$\omega_n > \frac{4}{0{,}7 \cdot 0{,}5} = 11{,}43 \quad \text{sê } \omega_n = 12 \text{ rad/s}$$

ITAE Polinoom: $s^3 + 1{,}75 \cdot 12\, s^2 + 2{,}15 \cdot 12^2 s + 12^3$

$$= s^3 + 21s^2 + 309{,}6s + 1728$$

$6 + 2k_3 = 21$, $2k_2 + 8k_3 + 8 = 309{,}6$, $2k_1 = 1728$

$k_3 = 7{,}5$ $\quad k_2 = 120{,}8$ $\quad k_1 = 864$

$k_p = 864$

[20]

VRAAG 2

2.1 $y(k+2)+6y(k+1)+5y(k)=3e(k+2)+e(k+1)+2e(k)$

$z^2Y(z)+6zY(z)+5Y(z)=3z^2E(z)+zE(z)+2E(z)$

$\therefore Y(z)[z^2+6z+5]=E(z)[3z^2+z+2]$

$$\therefore \frac{Y(z)}{E(z)}=\frac{3z^2+z+2}{z^2+6z+5}$$

$$=\frac{3+z^{-1}+2z^{-2}}{1+6z^{-1}+5z^{-2}}$$ (4)

2.2 Vir 'n trapinset is $E(z)=\frac{z}{z-1}$

Dus $Y(z)=\frac{(3z^2+z+2)\,z}{(z^2+6z+5)(z-1)}$

$$=\frac{3z^3+z^2+2z}{z^3+5z^2-z-5}$$

$$\begin{array}{r|l} & 3-14z^{-1}+75z^{-2}-374z^{-3}+1875z^{-4} \\ z^3+5z^2-z-5 & 3z^3+z^2+2z \\ & 3z^3+15z^2-3z-15 \\ & -14z^2+5z+15 \\ & -14z^2-70z+14+70z^{-1} \\ & 75z+1-70z^{-1} \\ & 75z+375-75z^{-1}-375z^{-2} \\ & -374+5z^{-1}+375z^{-2} \\ & -374-1870z^{-1}+374z^{-2}+1870z^{-3} \\ & 1875z^{-1}+z^{-2}-1870z^{-3} \end{array}$$ (5)

$\therefore y(0)=3,\ y(1)=-14,\ y(2)=75,\ y(3)=-374,\ y(4)=1875$

2.3 $Y(z)=\frac{(3z^2+z+2)\,z}{(z+5)(z+1)\,(z-1)}$

$$\frac{Y(z)}{z}=\frac{3z^2+z+2}{(z+5)(z+1)(z-1)}=\frac{A}{z+5}+\frac{B}{z+1}+\frac{C}{z-1}$$

$A=\left.\frac{3z^2+z+2}{(z+1)(z-1)}\right|_{z=-5}=\frac{72}{(-4)(-6)}=3$

$B=\left.\frac{3z^2+z+2}{(z+5)(z+1)}\right|_{z=-1}=\frac{-4}{8}=-\frac{1}{2}$

$C=\left.\frac{3z^2+z+2}{(z+5)(z+1)}\right|_{z=1}=\frac{6}{12}=\frac{1}{2}$

$$\therefore \quad \frac{Y(z)}{z} = \frac{3}{z+5} - \frac{\frac{1}{2}}{z+1} + \frac{\frac{1}{2}}{z-1}$$

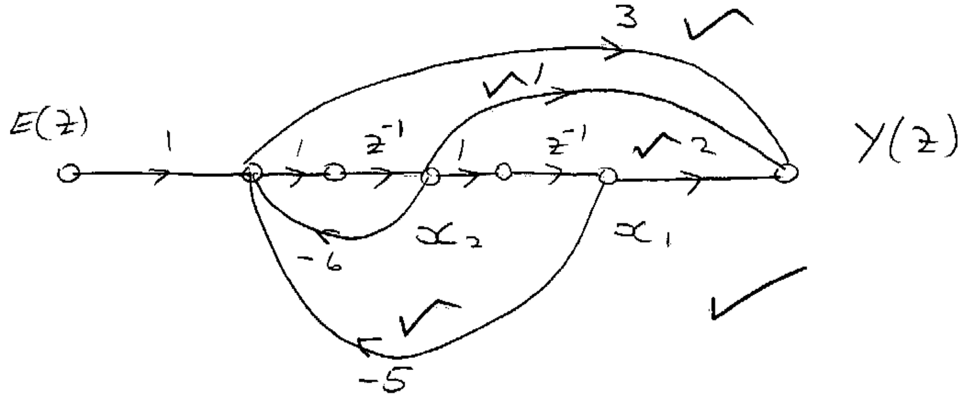
$$\therefore Y(z) = 3\frac{z}{z+5} - \frac{1}{2}\frac{z}{z+1} + \frac{1}{2}\frac{z}{z-1}$$

$$y(k) = 3(-5)^k - \frac{1}{2}(-1)^k + \frac{1}{2}$$

(7)

$\therefore$ $y(0) = 3$, $y(1) = -14$, $y(2) = 75$, $y(3) = -374$, $y(4) = 1875$

2.4 Beheerkanonieke vorm:



$$x_1(k+1) = x_2$$

$$x_2(k+1) = e(k) - 6x_2(k) - 5x_1(k)$$

$$y(k) = 2x_1(k) + x_2(k) + 3(e(k) - 6x_2(k) - 5x_1(k))$$

$$= 2x_1(k) + x_2(k) + 3e(k) - 18x_2(k) - 15x_1(k)$$

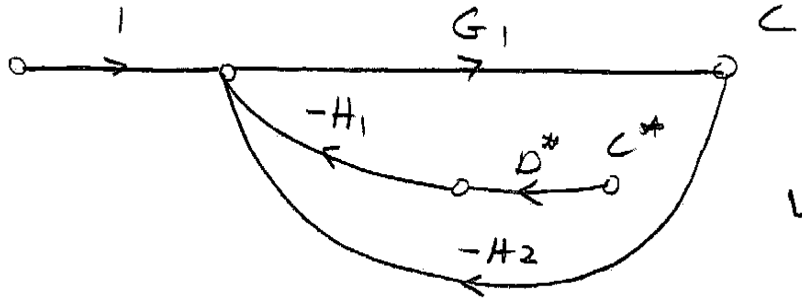
$$= -13x_1(k) - 17x_2(k) + 3e(k)$$

$$\mathbb{X}(k+1) = \begin{bmatrix} 0 & 1 \\ -5 & -6 \end{bmatrix} \mathbb{X}(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e(k)$$

(8)

$$y(k) = \begin{bmatrix} -13 & -17 \end{bmatrix} \mathbb{X}(k) + 3e(k)$$

.2.5



Insette R, C^*

Uitset C

$$C = G_1(R - H_1 D^* C^* - H_2 C)$$

$$C + G_1 H_2 C = G_1 R - G_1 H_1 D^* C^*$$

$$C = \frac{G_1 R - G_1 H_1 D^* C^*}{(1 + G_1 H_2)}$$

$$C^* = \left(\frac{G_1 R}{1 + G_1 H_2}\right)^* - \left(\frac{G_1 H_1}{1 + G_1 H_2}\right)^* D^* C^*$$

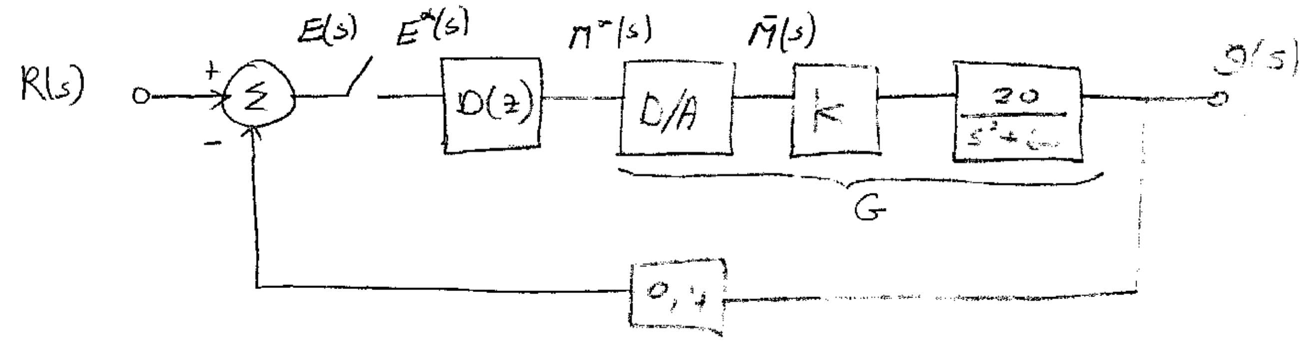
$$C^*\left[1 + \left(\frac{G_1 H_1}{1 + G_1 H_2}\right)^* D^*\right] = \left(\frac{\overline{G_1 R}}{1 + G_1 H_2}\right)^*$$

$$C^* = \frac{\left(\frac{\overline{G_1 R}}{1 + G_1 H_2}\right)^*}{1 + \left(\frac{G_1 H_1}{1 + G_1 H_2}\right)^* D^*}$$

$$C(z) = \frac{\frac{G_1 R}{1 + G_1 H_2}(z)}{1 + \frac{G_1 H_1}{1 + G_1 H_2}(z) D(z)} \qquad (6)$$

[30]

VRAAG 3



3.1 As die stelselfout nul is, is: $r(t) - 0{,}4\,\theta(t) = 0$

Vir $\theta(t) = 30°$

$$r(t) = 0{,}4 \cdot \theta(t) = 0{,}4 \cdot 30 = 12$$

Vir $\theta(t) = -30°$ $\quad r(t) = -12$ (1)

3.2
$$E = R - 0{,}4\,\theta$$
$$= R - 0{,}4\, G D^* E^*$$
en $$\theta = G \cdot D^* E^*$$

$$\therefore \quad E^* = R^* - 0{,}4\ G^* D^* E^* \quad \therefore E^* = \frac{R^*}{1 + 0{,}4\,G^* D^*}$$
$$\theta^* = G^* D^* E^*$$
$$= \frac{G^* D^*}{1 + 0{,}4\, G^* D^*}\ R^*$$

$$\therefore \quad \theta(z) = \frac{G(z) \cdot D(z)}{1 + 0{,}4\, G(z)\, D(z)} \cdot R(z)$$ (1)

3.3
$$\frac{\theta(z)}{R(z)} = \frac{G(z) \cdot D(z)}{1 + 0{,}4\, G(z) \cdot D(z)}$$

$$G(z) = \mathcal{Z}\left[\frac{1 - e^{-sT}}{s} \cdot k \cdot \frac{20}{s(s+6)}\right]$$

$$= \frac{z-1}{z} \cdot k \cdot \mathcal{Z}\left[\frac{20}{s^2(s+6)}\right]$$

$$= \frac{z-1}{z}\,\frac{20k}{6} \cdot \mathcal{Z}\ \frac{6}{s^2(s+6)}$$

Met $k = 20$, $T = 0{,}05$

$$G(z) = 0{,}4536 \frac{z + 0{,}905}{(z-1)(z-0{,}7408)}$$

(4)

Stelsel is van tipe 1

3.4 $$e_{ss}(kT) = \lim_{z\to 1} (z-1)\, E(z)$$

$$E(z) = \frac{R(z)}{1 + 0{,}4 \cdot G(z) \cdot D(z)}$$

$$R(z) = \frac{0{,}05\, z}{(z-1)^2}$$

$$E(z) = \frac{0{,}05 z}{(z-1)^2} \left[\frac{1}{1 + 0{,}4 . 0{,}544 \frac{z + 0{,}905}{(z-1)(z-0{,}7408)}} \right]$$

$$= \frac{0{,}05 z}{(z-1)^2} \left[\frac{(z-1)(z-0{,}7408)}{(z-1)(z-0{,}7408) + 0{,}1814(z+0{,}905)} \right]$$

$$= \frac{0{,}05 z}{(z-1)} \frac{z - 0{,}7408}{z^2 - z - 0{,}7408 z + 0{,}7408 + 0{,}1814 z + 0{,}164}$$

$$= \frac{0{,}05 z}{(z-1)} \frac{z - 0{,}7408}{z^2 - 1{,}559 z + 0{,}905}$$

$$\therefore\ e_{ss}(kT) = \lim_{z \to 1} \frac{0{,}05\, z(z - 0{,}7408)}{z^2 - 1{,}559 z + 0{,}905}$$

$$= \frac{0{,}01296}{0{,}346}$$

$$= 0{,}037^{\circ}$$

(4)

3.5 Komplekse pole is by: $z = 0{,}78 \pm j0{,}545$

$$= 0{,}95 \angle \pm 0{,}61 \text{ rad}$$

$$\therefore\ \zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}} = \frac{-\ln 0{,}95}{\sqrt{\ln^2 0{,}95 + 0{,}61^2}} = 0{,}084$$

$$\omega_n = \frac{1}{T}\sqrt{\ln^2 r + \theta^2} = \frac{1}{0{,}05} \cdot \sqrt{\ln^2 0{,}95 + 0{,}61^2}$$

$$= 12{,}23 \text{ rad/s}$$

(5)

3.6 $m(kT) = e(kT) - 0{,}9\, e(kT-T) + m(kT-T)$

$\therefore\ M(z) = E(z) - 0{,}9 z^{-1} E(z) + z^{-1} M(z)$

$\therefore\ M(z)\left[1 - z^{-1}\right] = E(z)\left[1 - 0{,}9 z^{-1}\right]$

$\therefore\ \dfrac{M(z)}{E(z)} = D(z) = \dfrac{z - 0{,}9}{z - 1}$ (3)

Tipe van stelsel is nou 2.

3.7 $e_{ss\ ramp} = 0$ aangesien dit 'n tipe 2 stelsel is. (2)

[20]

VRAAG 4

4.1 $\dfrac{\Theta(z)}{R(z)} = \dfrac{k\, D(z)\, G(z)}{1 + 0{,}2k\, D(z)\, G(z)}$

met $D(z) = 1$ en $G(z) = \dfrac{0{,}0117z + 0{,}011}{(z-0{,}95)(z-0{,}86)}$

$= \dfrac{0{,}117(z+0{,}94)}{(z-0{,}95)(z-0{,}86)}$ (3)

4.2 Jury stabiliteitstoets:

$Q(z) = 1 + 0{,}2 \cdot k \cdot \dfrac{0{,}0117z + 0{,}011}{(z-0{,}95)(z-0{,}86)} = 0$

$\therefore (z-0{,}95)(z-0{,}86) + 0{,}2k(0{,}0117z + 0{,}011) = 0$

$z^2 - 1{,}81z + 0{,}817 + 0{,}00234kz + 0{,}0022k = 0$

$z^2 + (0{,}00234k - 1{,}81)z + 0{,}817 + 0{,}0022k = 0$

① $Q(1) > 0$ $\quad 0{,}007 + 0{,}00454k > 0$

$\therefore k > -1{,}54$ Vermoed $k > 0$

② $(-1)^n Q(-1) > 0$

$(-1)^2 \cdot [1{,}817 + 1{,}81 - 0{,}00014k] > 0$

$\therefore k < 25907$

③ $|a_0| < a_n$

$|0{,}817 + 0{,}0022k| < 1$

$k < 83{,}2$ (5)

4.3 Frekwensie vir marginale stabiliteit $k = 83{,}2$

$Q(z) = z^2 - 1{,}62z + 1$

$\therefore z = \dfrac{1{,}62 \pm \sqrt{1{,}62^2 - 4}}{2} = \dfrac{1{,}62 \pm j1{,}173}{2}$

$= 0{,}81 \pm j\,0{,}5865$

$= 1 \angle \pm 0{,}627$

$\therefore \omega T = 0{,}627$, $T = 0{,}05$ $\therefore \omega = 12{,}54$ rad/s

(4)

4.4 Vir $\zeta = 0,707$

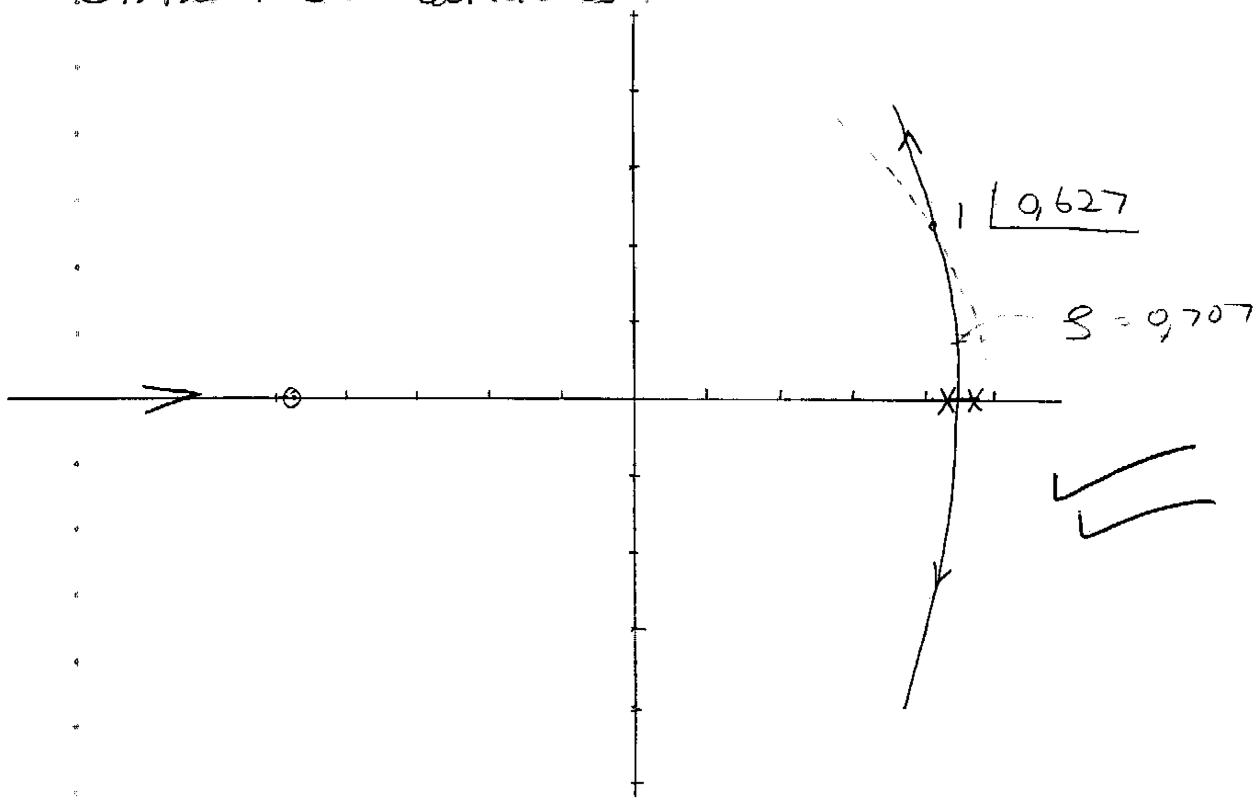
$$z = e^{\sigma T} \angle \sigma T \tan\beta$$

$$\beta = 135° \qquad \tan\beta = -1$$

$$z = e^{\sigma T} \angle -\sigma T = r\angle\theta$$

Vir wotter waardes van K lê die pole op ṅ radius van $e^{\sigma T}$ en ṅ hoek van $-\sigma T$

Grafies m.b.v wortellokus.



Wegbreekpunte: $K = \dfrac{-(z-0,95)(z-0,86)}{(0,2)(0,0117z+0,011)}$

maks

K	9465	0,462	0,42
z	0,9	0,91	0,89

min

$2,6\times10^4$	$1,07\times10^4$	$3,15\times10^3$	$3,16\times10^3$
-1	$-1,1$	$-2,8$	$-2,9$

$z = 0,91$ $\qquad z = -2,8$

By benadering vir $r = e^{\sigma T} = 0,825$ $\quad \sigma T = -0,1335$ $\quad \theta = 11°$

pole by $0,91 \pm j\,0,178$

$\therefore K = \dfrac{0,19 \cdot 0,19}{(0,2)(0,0117)(1,85)} = 8,34.$

(6)

[18]