BEHEERTEORIE II TUTORIAAL
7 Februarie 2017

VRAAG 1/ *QUESTION* 1

'n Stelsel word deur die volgende toestandsruimtemodel beskryf: /

A system can be described by the following state space model:

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -1 \end{bmatrix}\mathbf{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u \qquad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{X}$$

Gebruik toestandsveranderlike terugvoer en inkorporeer 'n verwysingsinset $u = -\mathbf{KX} + \alpha r$. Selekteer die winste $\mathbf{K}$ en α sodat die stelsel 'n vinnige respons het met 'n verbyskiet van kleiner as 1 % en 'n vestigingstyd van kleiner as 1 s en 'n bestendige toestand fout van 0 vir 'n trapinset. /

Use state variable feedback and incorporate a reference input $u = -\mathbf{KX} + \alpha r$. *Select the gains* $\mathbf{K}$ *and* α *so that the system has a rapid response with an overshoot of less than 1 % and a settling time of less 1 s and a steady state error of zero for a step input.*

Neem aan die stelsel is beheerbaar asook waarneembaar. Benader die stelsel as tweede orde deur die derde wortel 'n orde hoër in frekwensie te kies as die dominante wortels. /
Assume the system is controllable and observable. Approximate the system as a second order system by choosing the position of the third root an order in frequency higher than the dominant poles.

$$PO = 100e^{-\zeta\pi / \sqrt{1-\zeta^2}}$$

Addisionele inligting / *Additional information:* $T_s = \dfrac{4}{\zeta\omega_n}$ [20]

BEHEERTEORIE II TUTORIAAL
7 Februarie 2017

VRAAG 1/ *QUESTION* 1

'n Stelsel word deur die volgende toestandsruimtemodel beskryf: /

A system can be described by the following state space model:

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -1 \end{bmatrix}\mathbf{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u \qquad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{X}$$

Gebruik toestandsveranderlike terugvoer en inkorporeer 'n verwysingsinset $u = -\mathbf{KX} + \alpha r$. Selekteer die winste $\mathbf{K}$ en α sodat die stelsel 'n vinnige respons het met 'n verbyskiet van kleiner as 1 % en 'n vestigingstyd van kleiner as 1 s en 'n bestendige toestand fout van 0 vir 'n trapinset. /

Use state variable feedback and incorporate a reference input $u = -\mathbf{KX} + \alpha r$. *Select the gains* $\mathbf{K}$ *and* α *so that the system has a rapid response with an overshoot of less than 1 % and a settling time of less 1 s and a steady state error of zero for a step input.*

Neem aan die stelsel is beheerbaar asook waarneembaar. Benader die stelsel as tweede orde deur die derde wortel 'n orde hoër in frekwensie te kies as die dominante wortels. /
Assume the system is controllable and observable. Approximate the system as a second order system by choosing the position of the third root an order in frequency higher than the dominant poles.

$$PO = 100e^{-\zeta\pi / \sqrt{1-\zeta^2}}$$

Addisionele inligting / *Additional information:* $T_s = \dfrac{4}{\zeta\omega_n}$ [20]

Tutorial test 1 - Memo

EERI 418

A system is described by:

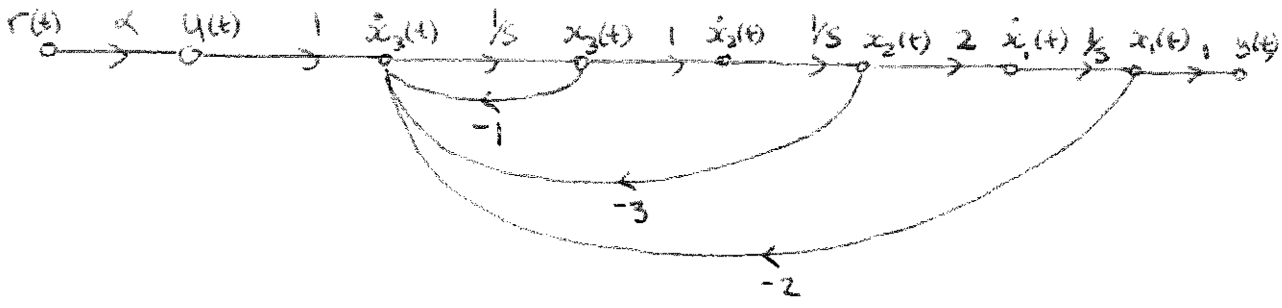
$$\dot{\mathbb{X}} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -1 \end{bmatrix} \mathbb{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbb{X}$$

We need to use state variable feedback control having the following control law:

$$u(t) = -\mathbb{K}\mathbb{X} + \alpha r$$
$$= -\begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \alpha r(t)$$

Step 1 : Start by drawing a signal flow diagram of the open-loop system



Step 2: Design parameters. 2

a) For $P.O. = 1\%$ then $\zeta = 0,826$

but $P.O. < 1\%$ $\therefore \zeta > 0,826$

$\therefore$ choose $\boxed{\zeta = 0,9}$

NB. If the damping ζ increase the P.O. will decrease.

Remember
$$\zeta = \sqrt{\frac{\left[\ln\left(\frac{P.O.}{100}\right)\right]^2}{\pi^2 + \left[\ln\left(\frac{P.O.}{100}\right)\right]^2}}$$

b)
$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{(0,9)\,\omega_n} = \frac{4,445}{\omega_n}$$

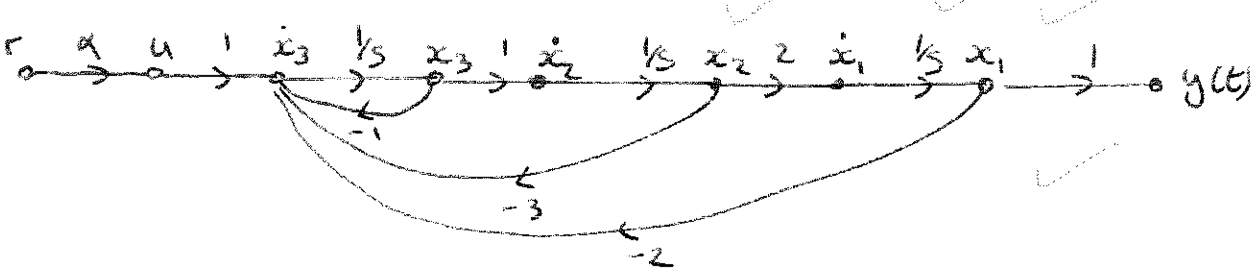
Design for $T_s < 1$

$$\frac{4,445}{\omega_n} < 1$$

$$4,445 < \omega_n$$

$\therefore$ Choose $\boxed{\omega_n = 4,5}$

Step 3: Redraw signal flow for closed loop system.



3.

Derive the closed loop transfer function using Mason, then

$$T(s) = \frac{2\alpha}{s^3 + s^2(1+k_3) + s(3+k_2) + (4+2k_1)}$$

The second order approximation is done by choosing the 3rd root an order greater

$$\therefore \quad (s^2 + 2\zeta\omega_n s + \omega_n^2)(s + 10\zeta\omega_n)$$

$$\Rightarrow \quad s^3 + (12\zeta\omega_n)s^2 + (20\zeta^2\omega_n^2 + \omega_n^2)s + (10\zeta\omega_n^3)$$

$$\therefore \quad 1 + k_3 = 12\zeta\omega_n \Rightarrow k_3 = 47{,}6$$

$$3 + k_2 = 20\zeta^2\omega_n^2 + \omega_n^2 \Rightarrow k_2 = 345{,}3$$

$$4 + 2k_1 = 10\zeta\omega_n^3 \Rightarrow k_1 = 408{,}0625$$

For $e_{ss} = 0$ for a step input

$$y_{ss} = \lim_{s \to 0} s\,T(s)\,R(s)$$

$$\therefore \quad \frac{2\alpha}{4 + 2k_1} = 1$$

$$\therefore \alpha = 410{,}0625$$