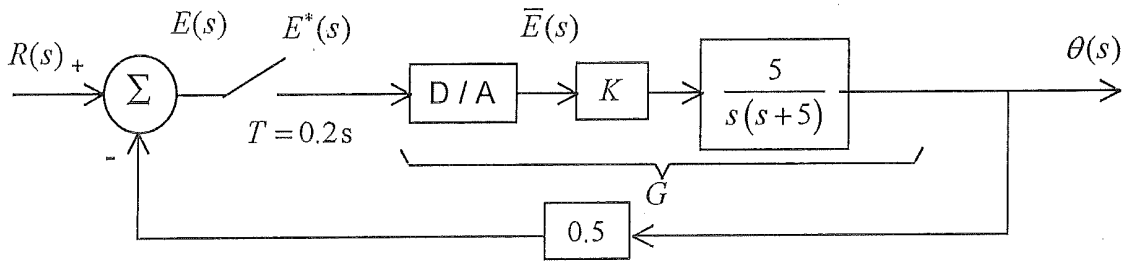
NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOMKAMPUS
Fakulteit Ingenieurswese

Benodigdhede vir hierdie vraestel:				Oopboek-eksamen:	
Multikeusekaarte:	☐	Nie-programmeerbare sakrekenaar:	☑		
Grafiekpapier:	☐	Draagbare rekenaar:	☐		

SEMESTERTOETS:	3	GRADE/DIPLOMA:	
VAKKODE:	**EERI 418**	DUUR:	1 UUR
VAK:	BEHEERTEORIE II	MAKS:	30
DOSENT:	DR KR UREN	DATUM:	**22-05-2012**
		TYD:	**08h00**
MODERATOR:	PROF G VAN SCHOOR		

TOTAAL: 30

VRAAG 1/ *QUESTION* 1



Figuur / *Figure* 1

Beskou die beheerstelsel in figuur 1. / *Consider the control system in figure 1.*

(a) Bepaal die stelseloordragsfunksie $(\frac{\theta(z)}{R(z)})$ in terme van *G(z)*. /

Determine the system transfer function $(\frac{\theta(z)}{R(z)})$ *in terms of G(z).* (1)

(b) Die oordragsfunksie *G(z)* word gegee deur: / *The transfer function G(z) is given by:*

$$G(z)=\frac{K(0.07358z+0.05285)}{z^2-1.368z+0.3679}$$

Bepaal die bestendige toestand fout van die diskrete stelsel vir *K* = 10 vir 'n eenheidshellingsinset. /
Determine the steady state error of the discrete system for K = 10 for a unit ramp input. (4)

(c) Bepaal die demping asook die natuurlike frekwensie van die diskrete stelsel. /
Determine the damping as well as the natural frequency of the discrete system. (4)

(d) Doen die nodige toetse en spreek jou uit oor die sinvolheid van die keuse van die monsterperiode./
Do the necessary tests and comment on the choice of the sampling period. (6)

(e) Gebruik die Jury stabiliteitstoets om die bereik van *K* te bepaal vir stabiliteit. /
Use the Jury stability test to determine the range of K for stability. (5)

(f) Bepaal die frekwensie vir marginale stabiliteit. /
Determine the frequency for marginal stability. (4)

(g) Teken die benaderde wortellokus vir die stelsel en bepaal die wins *K* vir 'n dempingskonstante van $\varsigma = 0.707$. /
Draw the approximated root locus for the system and determine the gain K for a damping constant of: $\varsigma = 0.707$. (6)

Addisionele inligting / *additional information:*

$$\zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}}$$

$$\omega_n = \frac{1}{T}\sqrt{\ln^2 r + \theta^2} \qquad [30]$$

$$\tau = \frac{1}{\zeta \omega_n}$$

Semester test 3 - Memo 22 May 2012

a) $\frac{\Theta(z)}{R(z)} = \frac{G(z)}{1 + 0{,}5\,G(z)}$ ✓ (1)

b) $E(z) = \frac{R(z)}{1 + 0{,}5\,G(z)}$ ✓

$$G(z) = \frac{10\,(0{,}07358\,z + 0{,}05285)}{(z-1)(z-0{,}368)}$$

$$R(z) = \frac{T\,z}{(z-1)^2}$$

$$E(z) = \frac{T\,z\,(z-1)(z-0{,}368)}{(z-1)^2\left[(z-1)(z-0{,}368) + 0{,}5\cdot 10\cdot(0{,}07358\,z + 0{,}05285)\right]}$$ ✓

$$e_{ss} = \lim_{z\to 1}\,(z-1)\,E(z)$$ ✓

$$= \lim_{z\to 1}\,\frac{T\,z\,(z-0{,}368)}{5\,(0{,}07358\,z + 0{,}05285)}$$

$$= \frac{0{,}2\,(1-0{,}368)}{5\,(0{,}07358 + 0{,}05285)}$$ ✓ (4)

$$= 0{,}2$$

c) Damping and natural frequency

$$Q(z) = z^2 - 1{,}368\,z + 0{,}368 + 0{,}3679\,z + 0{,}2642$$

$$= z^2 - z + 0{,}6322$$ ✓

$$z_{1,2} = 0{,}5 \pm j\,0{,}6182 = 0{,}7951\,\angle \pm 0{,}8907 = r\,\angle \pm\theta$$

$$\therefore \zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}} = \frac{-\ln 0{,}7951}{\sqrt{(\ln 0{,}7951)^2 + 0{,}8907^2}} = 0{,}2493$$ ✓

$$\omega_n = \frac{1}{T}\sqrt{\ln^2 r + \theta^2} = 4{,}6 \text{ rad/s}$$ ✓ (4)

d) Choice of the sampling period

Continous characteristic equation

$$q(s) = 1 + 0,5\,G(s)$$
$$= 1 + 0,5 \cdot 10 \cdot \frac{5}{s(s+5)} = 0$$

$$\therefore\ s(s+5) + 25 = 0 \quad \checkmark$$
$$\therefore\ s^2 + 5s + 25 = 0$$
$$= s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$
$$\therefore\ \omega_n = 5\ rad/s \ \checkmark \quad \text{and} \quad \zeta = \frac{5}{2\zeta\omega_n} = \frac{5}{2.5} = 0,5 \ \checkmark$$

$$\tau = \frac{1}{\zeta\omega_n} = \frac{1}{0,5 \cdot 5} = 0,4\,s \quad \checkmark$$

$$\frac{\tau}{T} = \frac{0,4}{0,2} = 2 \quad (\text{must be at least } 5) \quad \checkmark$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 5\sqrt{1-0,5^2} = 4,33\ rad/s$$

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{4,33}\,s = 1,45\,s \ \checkmark$$

$$\frac{T_d}{T} = \frac{1,45}{0,2} = 7,25 \ (\text{Voldoende}) \ \checkmark$$

Master periode van $\frac{0,4}{5} = 0,08$ s is beter ✓ (6)

e) Jury

$$Q(z) = 1 + 0{,}5\,G(z) = 1 + 0{,}5\,k\,\frac{0{,}07358\,z + 0{,}05285}{z^2 - 1{,}368z + 0{,}368}$$

$$Q(z) = z^2 - 1{,}368z + 0{,}368 + 0{,}5k \cdot 0{,}07358z + 0{,}5k \cdot 0{,}05285 \quad \checkmark$$

$$= z^2 + (0{,}0368k - 1{,}368)z + (0{,}0264k + 0{,}368)$$

a_2 a_1 a_0 ✓

① $Q(1) > 0$ $\quad k > 0$ ✓

② $(-1)^2 Q(-1) > 0$ $\quad \therefore 1 - 0{,}0104k + 1{,}736 > 0$

$\therefore k < 268{,}08$ ✓

③ $|a_0| < a_2$ $\quad \therefore |(0{,}0264k + 0{,}368)| < 1$

$\therefore k < 23{,}94$ ✓

(5)

$0 < k < 23{,}94$ ✓

f) Vir $k = 23{,}94$ ✓

$Q(z) = z^2 - 0{,}487z + 1$ ✓

$\therefore z_{1,2} = 0{,}2435 \pm j0{,}9699$

$= 1 \angle \pm 1{,}3248$ ✓

$\therefore \Theta = 1{,}3248$ rad $= \omega T$ ✓

$\therefore \omega = \frac{1{,}3248}{0{,}2}$ rad/s $= 6{,}624$ rad/s ✓

(4)

g) Rootlocus

Open-loop transfer function

$$= 0,5\ G(z) = \frac{0,5k(0,07358z + 0,05285)}{z^2 - 1,368z + 0,368}$$

$$= \frac{0,0368k(z + 0,7183)}{(z-1)(z-0,368)}$$

Wegbrandpunte by $\frac{\partial G(z)}{\partial z}$ ∴ $z = 0,648$, $-2,08$

Marginale stabiliteit by $k = 23,94$

$z = 0,2435 \pm j\,0,9699$

Vir $\zeta = 0,707$ $\theta = \cos^{-1}(0,707) = 45°$ $z = e^{\sigma T}/\sigma T \angle\beta$

$\beta = \theta + 90° = 135°$ $\tan\beta = -1$ $z = e^{\sigma T}/-\sigma T$

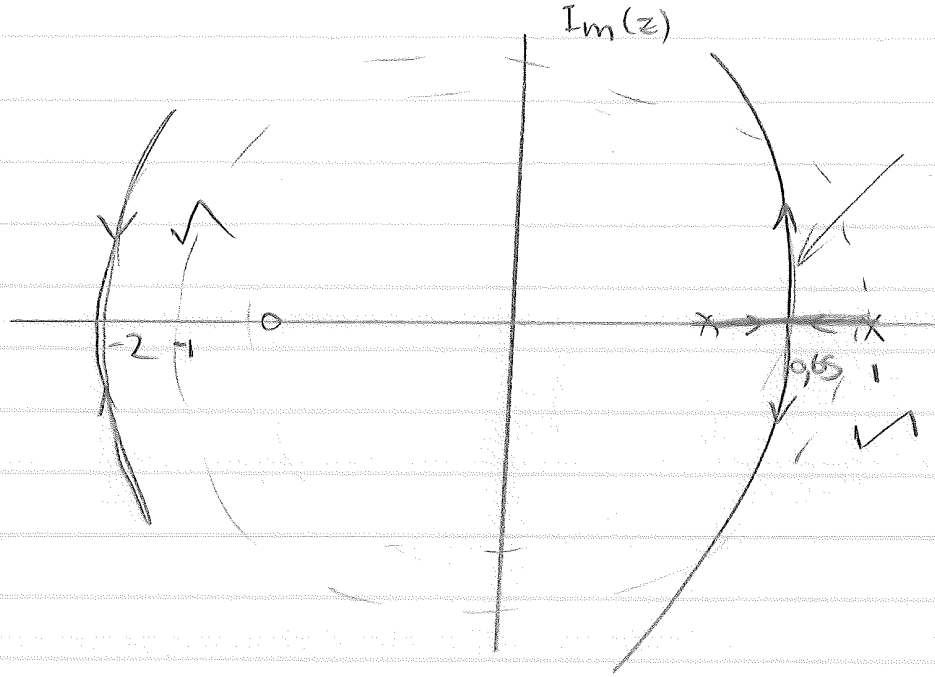
For what value of k will the pole lye o the radius $e^{\sigma T}$ with an angle $-\sigma T$

For $z = 0,6724/0,3969$ this is true

$$\therefore\ 1 + 0,5kG(z) = 0$$

$$\therefore\ 1 + \frac{0,0368k(z + 0,7183)}{(z-1)(z-0,368)}$$

$$k = \left| \frac{(z-1)(z-0,368)}{0,0368(z+0,7183)} \right| = \frac{0,46 \cdot 0,34}{0,0368 \cdot 1,28} = 3,32$$



$0,607 \pm j0,262$

$e^{-0,607} \angle 0,662$