EERI 418
CONTROL THEORY II

Multivariable and digital control systems theory

SU 2 Linear mathematical models of systems

Kenny Uren

It all starts here

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOM CAMPUS

General closed-loop system topology

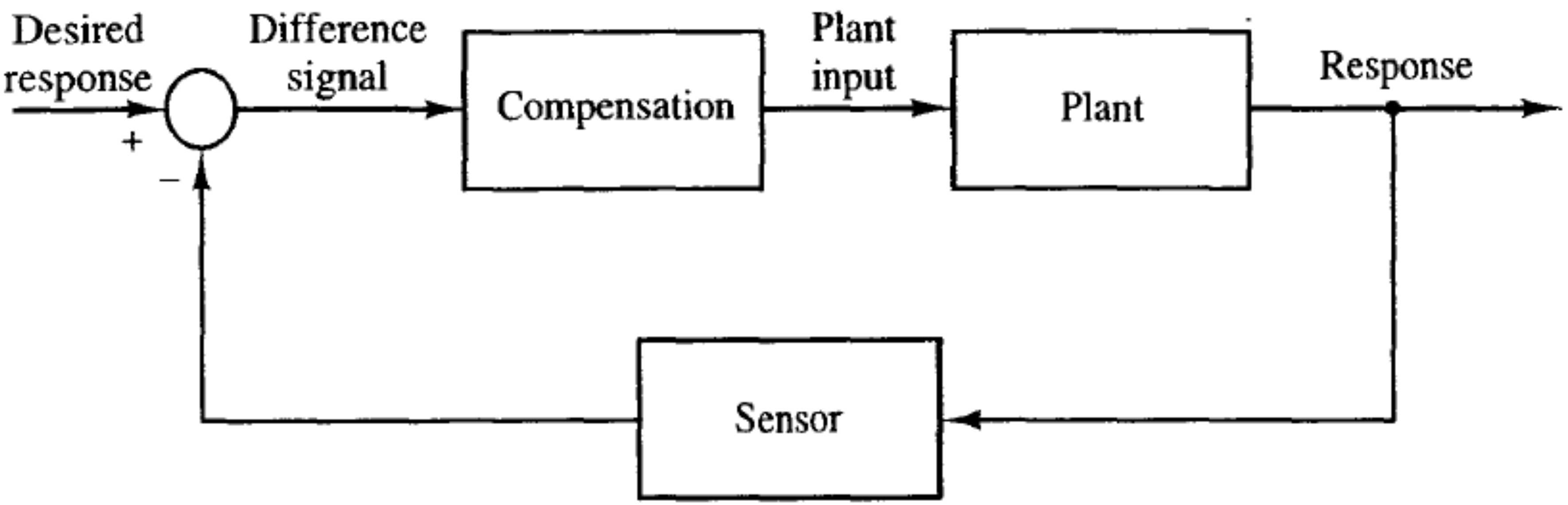


Figure 1-1 Closed-loop system.

Closed-loop terminology

NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOM CAMPUS

- Focus of the work is on
  - Closed-loop systems
  - Containing digital components
- Closed loop
  - System in which the forcing functions (inputs) are determined, at least in part, by the response (outputs) of the system
  - Physical process to be controlled is called the plant.
  - A system called the control actuator is required to drive the plant
  - The sensor (or sensors) measures the response of the plant, which is then compared to the desired response.
- Difference signal/error
  - The difference signal initiates actions that result in the actual response approaching the desired response, which drives the difference signal towards zero.

Compensator/Controller

- Difference signal/Error signal
  - Unacceptable closed-loop response occurs if the plant input is simply the difference in the desired response and the actual response
- Filter
  - The difference signal must be processed (filtered) by another physical system, called a compensator (controller) or simply a filter.
- Sensor
  - Will be an appropriate measuring instrument
- Compensation
  - Will be performed by a digital computer. (Will incorporate dynamics of the system in the digital computer)

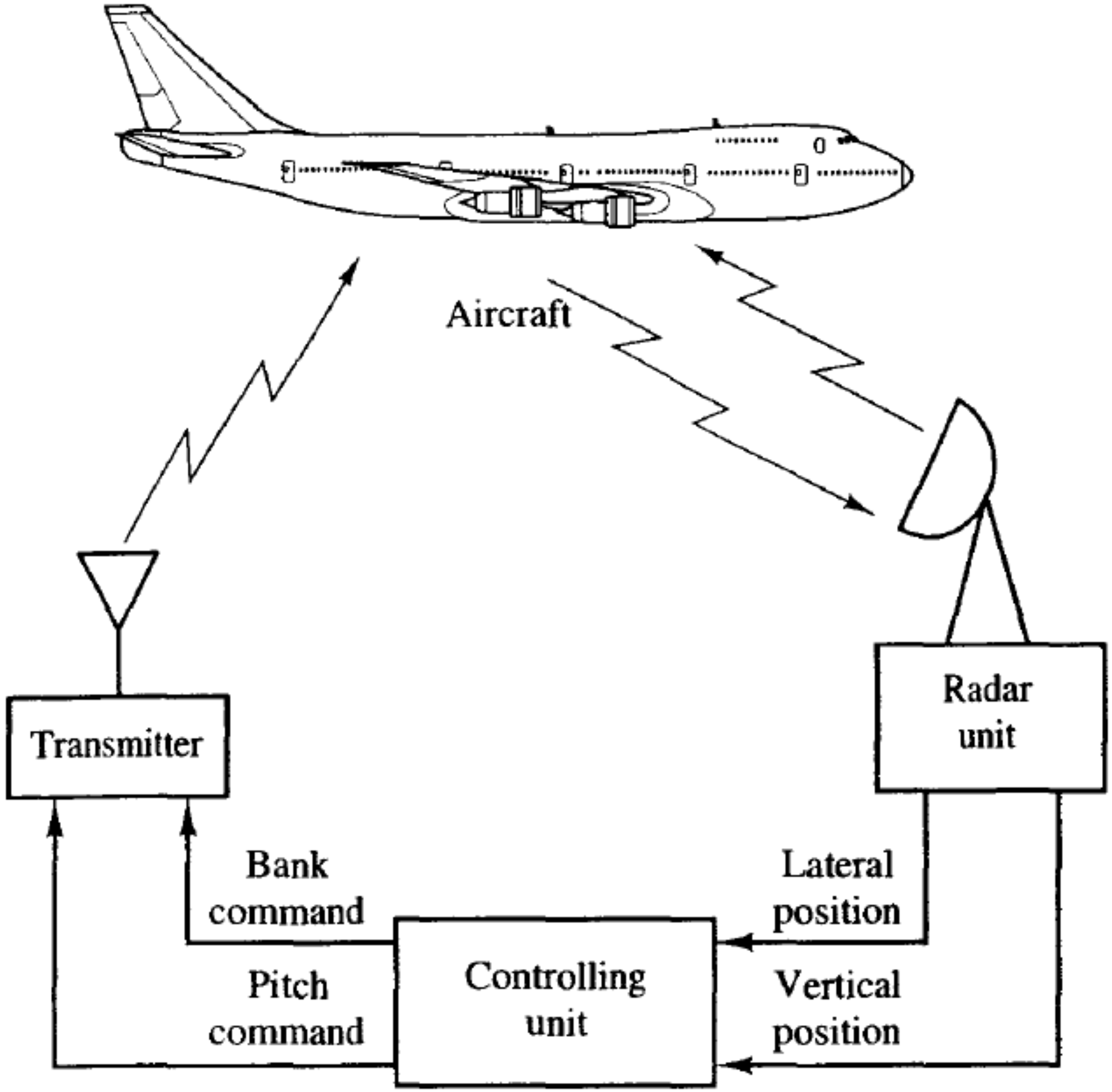
System models

- We will use classical and modern control techniques of analysis and design
- Control system techniques will be designed for <u>linear time-invariant discrete system models</u>.
- Linear systems satisfy the principle of superposition
- Physical systems are inherently nonlinear; however if the system signals do not vary over too wide a range, the system responds linearly.

Discrete system

- It is a system that can change values only at discrete instants in time.

Digital control system example



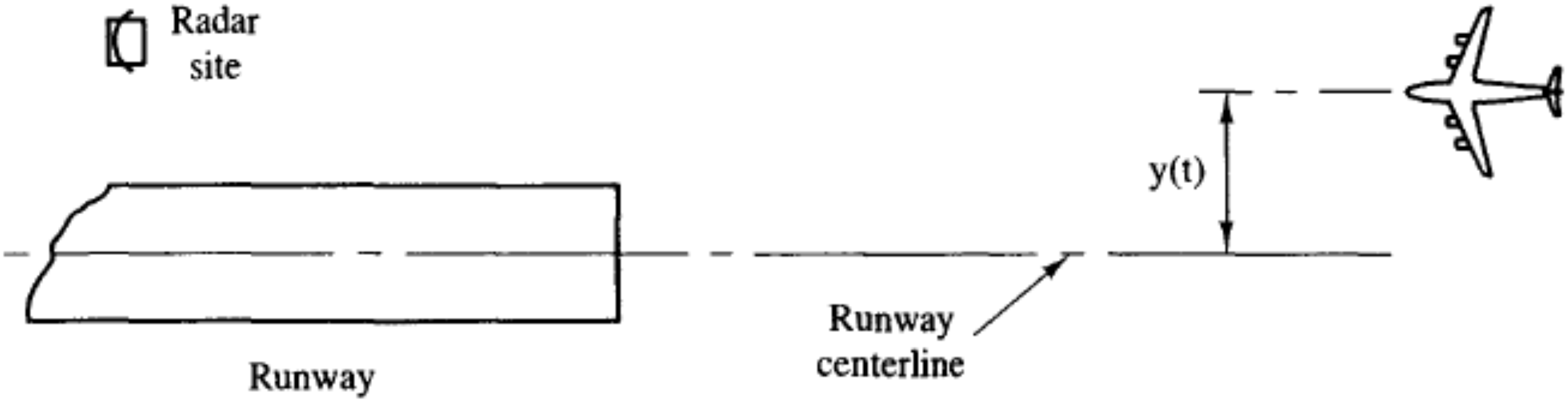
Radar and control unit

- Measures the approximate vertical and lateral positions of the aircraft.
- Then transmits it to the controlling unit
- The controlling unit calculates appropriate pitch and bank commands
- These commands are then transmitted to the aircraft autopilots
- Which in turn cause the aircraft to respond accordingly

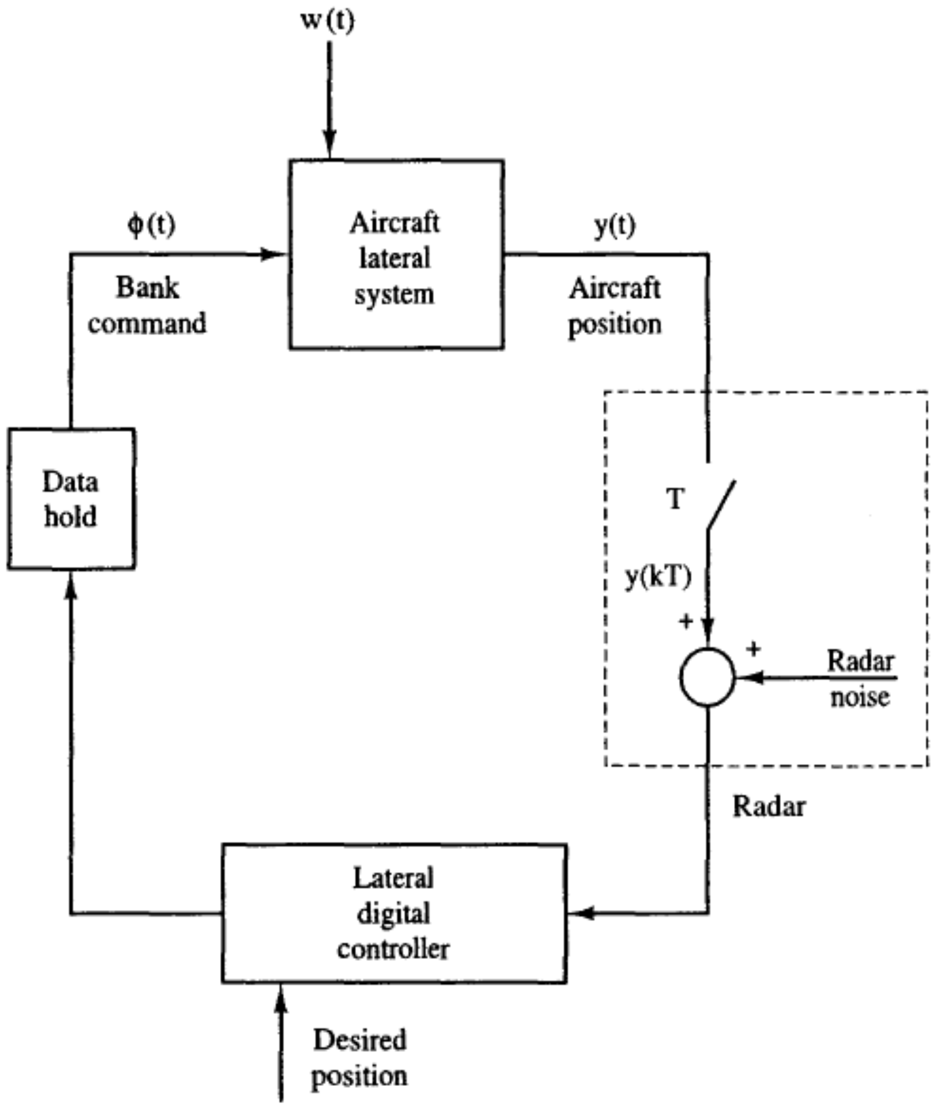
Control unit

- The control unit is a digital computer
- Lateral control system: Controls lateral position of aircraft
- Vertical control system: Controls the altitude of aircraft
- These two control systems are independent (decoupled)

Lateral control problem



Lateral control



(b)

Lateral control problem

lateral position, $y(t)$

The control system attempts to force $y(t)$ to zero

radar unit measures $y(t)$ every 0.05 s

Thus $y(kT)$ is the sampled value of $y(t)$

$T = 0.05$ s and $k = 0, 1, 2, 3, \ldots.$

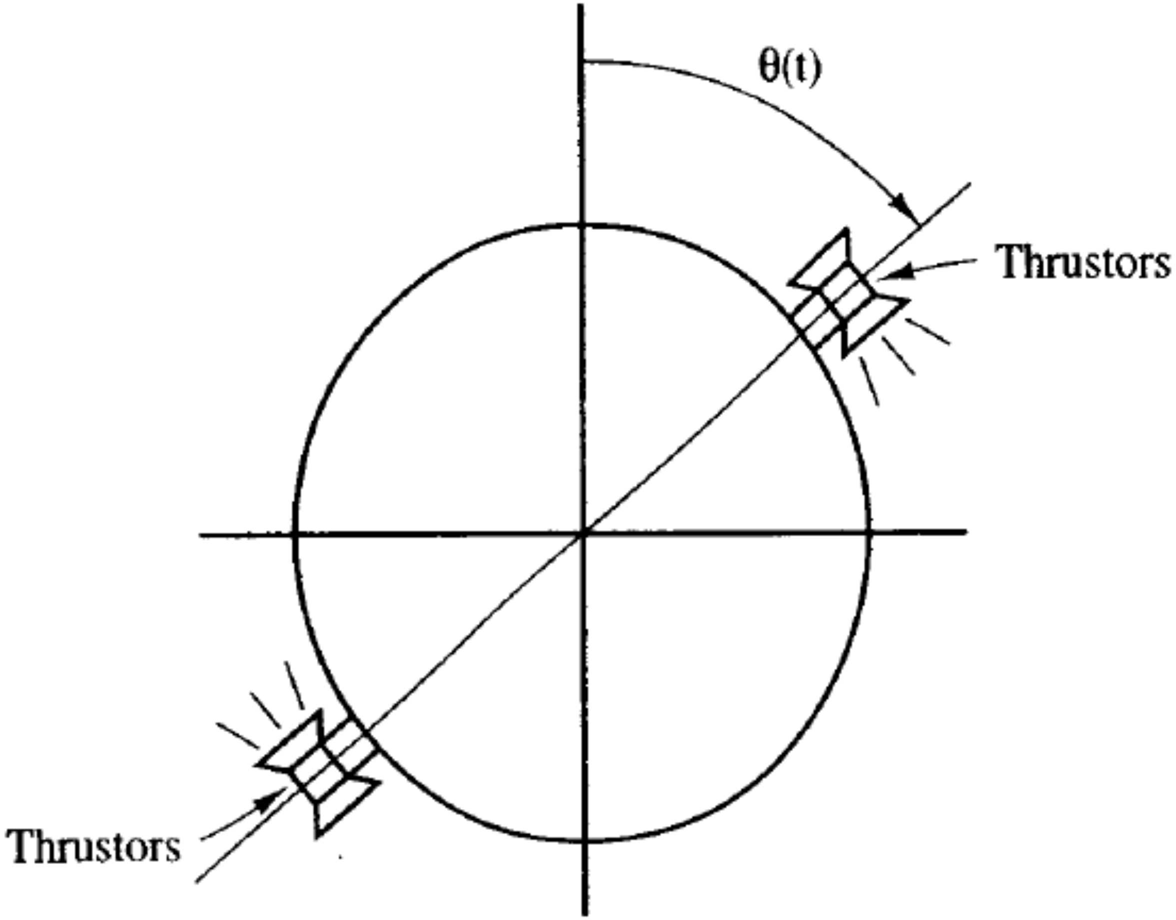
digital controller
generates the discrete bank commands $\phi(kT)$.

$w(t)$ wind input, which certainly affects the position of the aircraft.

Control system design

- To effect the design problem, it is necessary to know the mathematical relationships between the wind input, the bank command input, and the lateral position.
- These mathematical relationships are referred to as the mathematical model
- Th McDonell-Douglas Corporation F4 aircraft,
  - The model of the lateral system is a ninth-order ordinary nonlinear differential equation.
  - For the control of the bank command a ninth-order ordinary linear differential equation was used.

Satellite model



Servomotor model

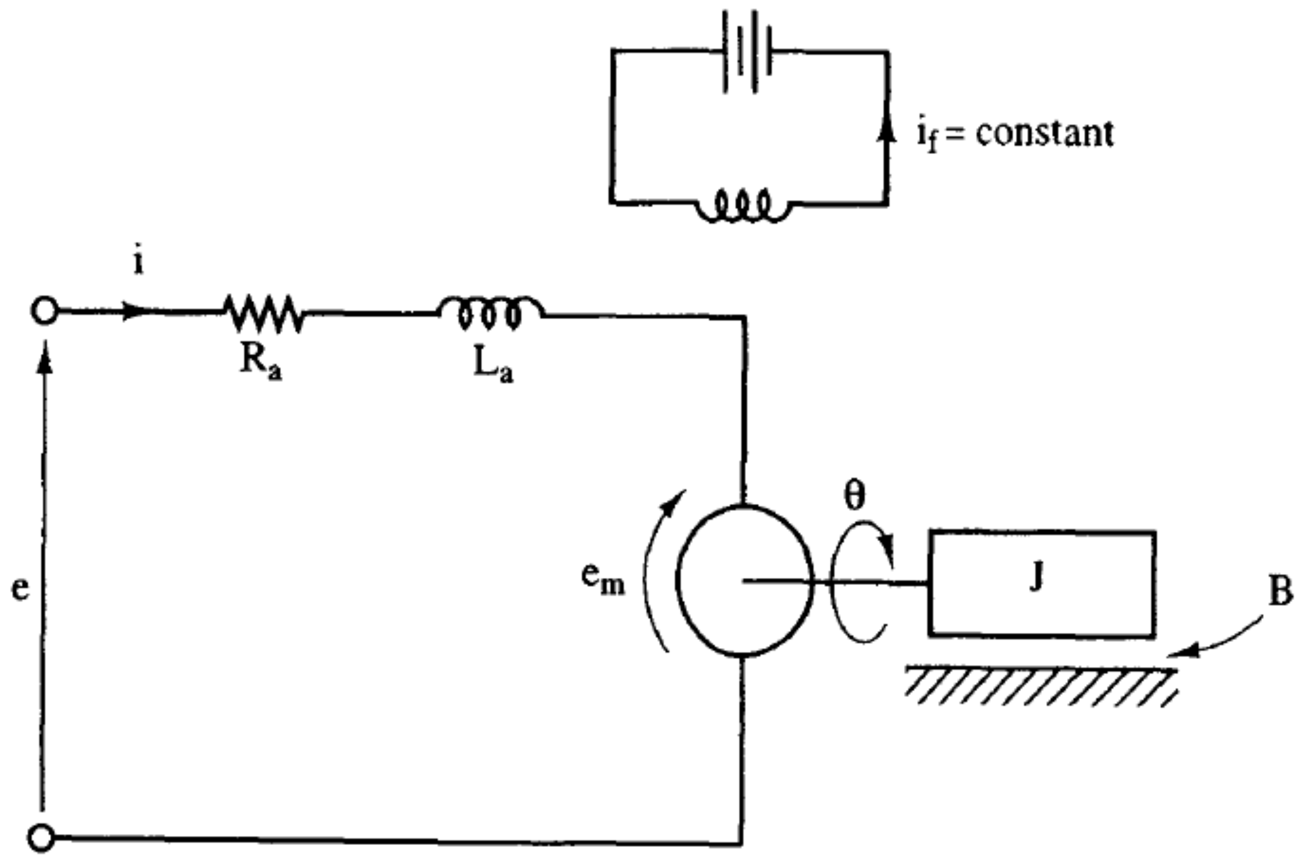
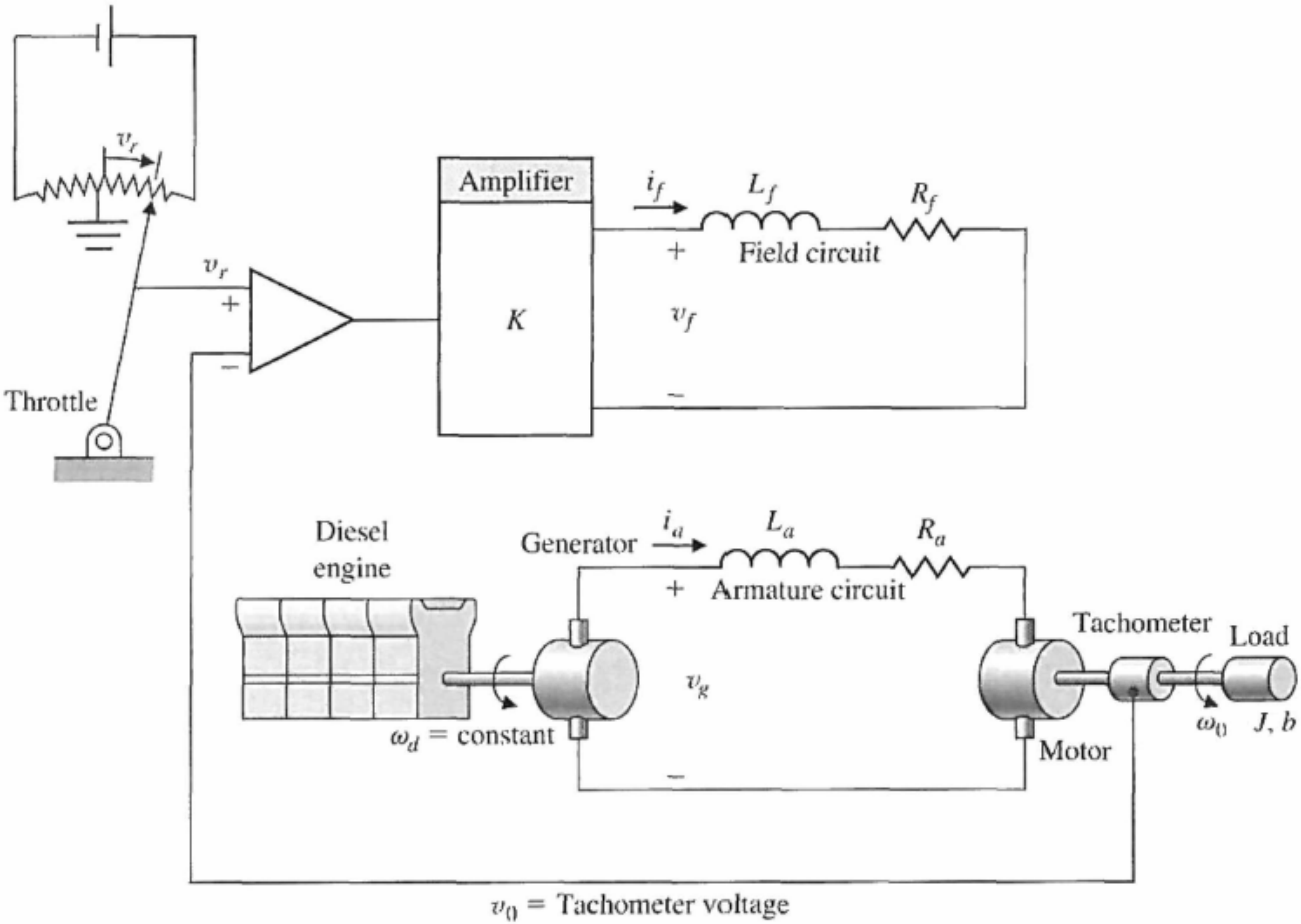


Figure 1-6 Servomotor system.

Develop model



END