NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOMKAMPUS
Fakulteit Ingenieurswese

Benodigdhede vir hierdie vraestel:
Multikeusekaarte: ☐ Nie-programmeerbare sakrekenaar: ☑
Grafiekpapier: ☐ Draagbare rekenaar: ☐

Oopboek-eksamen: ☐

SEMESTERTOETS:	2	GRADE/DIPLOMA:	**B Ing**
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VAK:	BEHEERTEORIE II	MAKS:	35
DOSENT:	DR. K.R. UREN	DATUM:	**24-04-2011**
		TYD:	**08h00**
MODERATOR:	PROF. G. VAN SCHOOR		

TOTAAL: 35

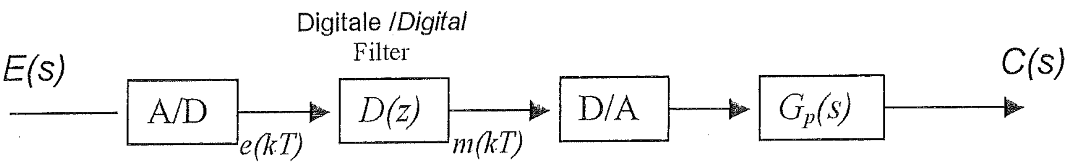
Vraag / *Question* 1

Die digitale filter in Figuur 1 los die volgende verskilvergelyking op: /
The digital filter in Figure 1 solves the following difference equation:

$$m(k) = 0.9m(k-1) + 0.2e(k)$$

Die monstertempo is 1 Hz. / *The sampling rate is 1 Hz.*
Die aanlegoordragsfunksie word gegee deur: / *The plant transfer function is given by:*

$$G_p(s) = \frac{1}{s(s+0.2)}$$



Figuur / *Figure* 1

(a) Bepaal die stelseloordragsfunksie $(\frac{C(z)}{E(z)})$. / *Determine the transfer function of the system* $(\frac{C(z)}{E(z)})$. (8)

(b) Bepaal die gelykstroomwins van die stelsel. / (1)
Determine the DC gain of the system.

(c) Bepaal die z-transform in geslote vorm van die volgende sein: /
Determine the z-transform, in closed form, of the following signal:

$$E(s) = \frac{2(1-e^{-2s})}{s(s+2)}, \quad T = 0.5s$$ (6)

[15]

VRAAG / *QUESTION* 2

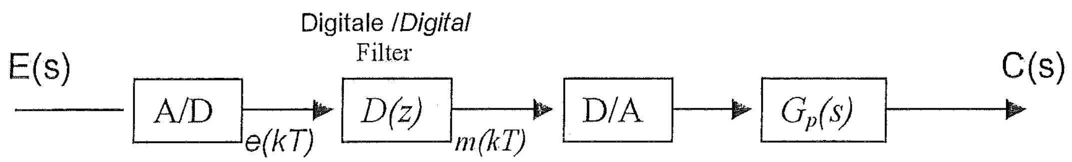
Die digitale filter in Figuur 2 los die volgende verskilvergelyking op: /
The digital filter in Figure 2 solves the following difference equation:

$$m(k) = 0.5m(k-1) + e(k)$$

Die monstertempo is 5 Hz. / *The sampling rate is 5 Hz.*

Die aanlegoordragsfunksie word gegee deur: / *The plant transfer function is given by:*

$$G_p(s) = \frac{20}{(s+2)(s+5)}$$



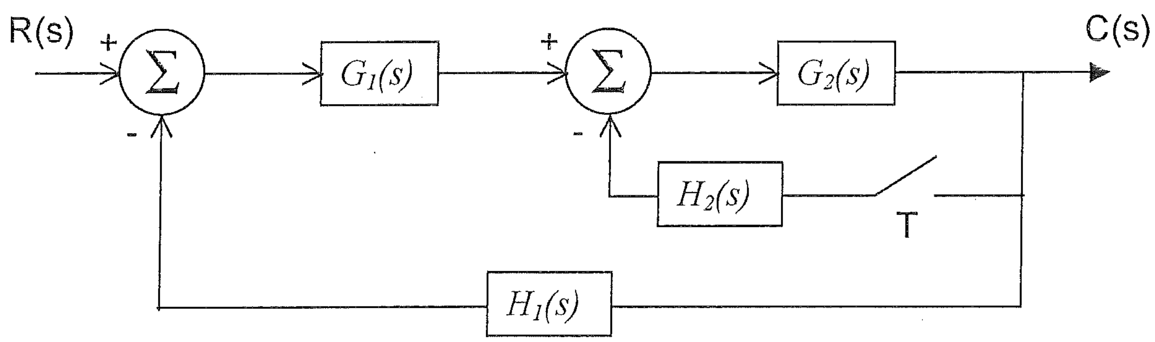
Figuur / Figure 2

Bepaal die stelseloordragsfunksie ($\frac{C(z)}{E(z)}$) indien die verwerkingstyd van die digitale filter van 50 ms ook gemodelleer moet word./

Determine the system transfer function ($\frac{C(z)}{E(z)}$) *when a computational delay of 50 ms also needs to be modelled.* [10]

VRAAG / *QUESTION* 3

Druk *C(z)* in Figuur 3 uit as 'n funksie van *R(z)* en die gegewe oordragsfunksies. /
Express C(z) in Figure 3 in terms of R(z) and the given transfer functions.



Figuur / *Figure* 3

[10]

Table 1. Properties of the z transform

Sequence	Transform
$e(k)$	$E(z) = \sum_{k=0}^{\infty} e(k) z^{-k}$
$a_1 e_1(k) + a_2 e_2(k)$	$a_1 E_1(z) + a_2 E_2(z)$
$e(k-n)u(k-n);\quad n \geq 0$	$z^{-n} E(z)$
$e(k+n)u(k);\quad n \geq 1$	$z^{n}\left[E(z) - \sum_{k=0}^{n-1} e(k) z^{-k}\right]$
$\epsilon^{ak} e(k)$	$E(z\epsilon^{-a})$
$ke(k)$	$-z\dfrac{dE(z)}{dz}$
$e_1(k) * e_2(k)$	$E_1(z)E_2(z)$
$e_1(k) = \sum_{n=0}^{k} e(n)$	$E_1(z) = \dfrac{z}{z-1}E(z)$
Initial value: $e(0) = \lim_{z\to\infty} E(z)$	
Final value: $e(\infty) = \lim_{z\to 1}(z-1)E(z)$, if $e(\infty)$ exists	

Table 2. z-transforms

Sequence	z-Transform
$\delta(k-n)$	z^{-n}
1	$\dfrac{z}{z-1}$
k	$\dfrac{z}{(z-1)^2}$
k^2	$\dfrac{z(z+1)}{(z-1)^3}$
a^k	$\dfrac{z}{z-a}$
ka^k	$\dfrac{az}{(z-a)^2}$
$\sin ak$	$\dfrac{z \sin a}{z^2 - 2z\cos a + 1}$
$\cos ak$	$\dfrac{z(z - \cos a)}{z^2 - 2z\cos a + 1}$
$a^k \sin bk$	$\dfrac{az \sin b}{z^2 - 2az\cos b + a^2}$
$a^k \cos bk$	$\dfrac{z^2 - az\cos b}{z^2 - 2az\cos b + a^2}$

Table 3. z-transforms

Laplace transform $E(s)$	Time function $e(t)$	z-Transform $E(z)$	Modified z-transform $E(z,m)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1}+\frac{T}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2}\left[\frac{m^2}{z-1}+\frac{2m+1}{(z-1)^2}+\frac{2}{(z-1)^3}\right]$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{1}{s+a}$	ϵ^{-at}	$\frac{z}{z-\epsilon^{-aT}}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$	$\frac{Tz\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}$	$\frac{T\epsilon^{-amT}[\epsilon^{-aT}+m(z-\epsilon^{-aT})]}{(z-\epsilon^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k\epsilon^{-at}$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-\epsilon^{-at}$	$\frac{z(1-\epsilon^{-aT})}{(z-1)(z-\epsilon^{-aT})}$	$\frac{1}{z-1}-\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1-\epsilon^{-at}}{a}$	$\frac{z[(aT-1+\epsilon^{-aT})z+(1-\epsilon^{-aT}-aT\epsilon^{-aT})]}{a(z-1)^2(z-\epsilon^{-aT})}$	$\frac{T}{(z-1)^2}+\frac{amT-1}{a(z-1)}+\frac{\epsilon^{-amT}}{a(z-\epsilon^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\epsilon^{-at}$	$\frac{z}{z-1}-\frac{z}{z-\epsilon^{-aT}}-\frac{aT\epsilon^{-aT}z}{(z-\epsilon^{-aT})^2}$	$\frac{1}{z-1}-\left[\frac{1+amT}{z-\epsilon^{-aT}}+\frac{aT\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}\right]\epsilon^{-amT}$
$\frac{b-a}{(s+a)(s+b)}$	$\epsilon^{-at}-\epsilon^{-bt}$	$\frac{(\epsilon^{-aT}-\epsilon^{-bT})z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}-\frac{\epsilon^{-bmT}}{z-\epsilon^{-bT}}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z\sin(aT)}{z^2-2z\cos(aT)+1}$	$\frac{z\sin(amT)+\sin(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{s}{s^2+a^2}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2-2z\cos aT+1}$	$\frac{z\cos(amT)-\cos(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\epsilon^{-at}\sin bt$	$\frac{1}{b}\left[\frac{z\epsilon^{-aT}\sin bT}{z^2-2z\epsilon^{-aT}\cos(bT)+\epsilon^{-2aT}}\right]$	$\frac{1}{b}\left[\frac{\epsilon^{-amT}[z\sin bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$\epsilon^{-at}\cos bt$	$\frac{z^2-z\epsilon^{-aT}\cos bT}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$	$\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-\epsilon^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$	$\frac{z(Az+B)}{(z-1)(z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT})}$ $A=1-\epsilon^{-aT}\left(\cos bT+\frac{a}{b}\sin bT\right)$ $B=\epsilon^{-2aT}+\epsilon^{-aT}\left(\frac{a}{b}\sin bT-\cos bT\right)$	$\frac{1}{z-1}$ $-\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$ $+\frac{\frac{a}{b}\{\epsilon^{-amT}[z\sin bmT-\epsilon^{-aT}\sin(1-m)bT]\}}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}+\frac{\epsilon^{-at}}{a(a-b)}+\frac{\epsilon^{-bt}}{b(b-a)}$	$\frac{(Az+B)z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})(z-1)}$	$A=\frac{b(1-\epsilon^{-aT})-a(1-\epsilon^{-bT})}{ab(b-a)}$ $B=\frac{a\epsilon^{-aT}(1-\epsilon^{-bT})-b\epsilon^{-bT}(1-\epsilon^{-aT})}{ab(b-a)}$

Table 4. Laplace transform properties

Name	Theorem
Derivative	$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0^+)$
nth-order derivative	$\mathcal{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^+) - \cdots f^{(n-1)}(0^+)$
Integral	$\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{F(s)}{s}$
Shifting	$\mathcal{L}[f(t - t_0)u(t - t_0)] = e^{-t_0 s} F(s)$
Initial value	$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$
Final value	$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$
Frequency shift	$\mathcal{L}[e^{-at} f(t)] = F(s + a)$
Convolution integral	$\mathcal{L}^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t - \tau) f_2(\tau)\,d\tau = \int_0^t f_1(\tau) f_2(t - \tau)\,d\tau$

Table 5 Optimum coefficients of T(s) Based on the ITAE Criterion for a Step Input

$$s + \omega_n$$
$$s^2 + 1.4\omega_n s + \omega_n^2$$
$$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$$
$$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$$
$$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$$
$$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.60\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$$

Semestertoets 2 - Memo

Vraag 1

a) $$m(k) = 0{,}9\, m(k-1) + 0{,}2\, e(k)$$

$$\therefore \quad M(z) = 0{,}9\, z^{-1} M(z) + 0{,}2\, E(z) \quad ✓$$

$$M(z)\left[1 - 0{,}9 z^{-1}\right] = 0{,}2\, E(z) \quad ✓$$

$$D(z) = \frac{M(z)}{E(z)} = \frac{0{,}2}{1 - 0{,}9 z^{-1}} = \frac{0{,}2\, z}{z - 0{,}9}$$

$$\frac{C(z)}{E(z)} = D(z)\, G(z) \quad ✓$$

$$G(z) = \mathcal{Z}\left[\frac{1 - e^{-sT}}{s} \cdot \frac{1}{s(s+0{,}2)}\right] ✓$$

$$= \frac{z-1}{z} \cdot \mathcal{Z}\left[\frac{1}{s^2(s+0{,}2)}\right]$$

$$= \frac{z-1}{z} \cdot 5 \cdot \mathcal{Z}\left[\frac{0{,}2}{s^2(s+0{,}2)}\right]$$

$$= \frac{z-1}{z} \cdot 5 \cdot \frac{z\left[(0{,}2 - 1 + e^{-0{,}2})z + (1 - e^{-0{,}2} - 0{,}2\, e^{-0{,}2})\right]}{0{,}2(z-1)^2(z - e^{-0{,}2})}$$

$$= \frac{z-1}{z} \cdot 25 \cdot \frac{z\left[0{,}01873\, z + 0{,}01752\right]}{(z-1)^2(z - 0{,}8187)}$$

$$= \frac{0{,}4683\left[z + 0{,}9354\right]}{(z-1)(z-0{,}8187)} \quad ✓✓$$

$$\therefore \quad \frac{C(z)}{E(z)} = D(z) \cdot G(z) = \frac{0{,}2\, z}{z - 0{,}9} \cdot \frac{0{,}4683\left[z + 0{,}9354\right]}{(z-1)(z-0{,}81857)}$$

$$= \frac{0{,}0937\, z\,(z + 0{,}9354)}{(z-0{,}9)(z-1)(z-0{,}8187)} \quad ✓✓$$

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b) DC gain $= \infty$ ✓ 1

c) $E(s) = \dfrac{2(1-e^{-2s})}{s(s+2)}$, $T = 0{,}5s$

$$= \frac{2(1-e^{-4sT})}{s(s+2)} ✓$$

$$= \mathcal{Z}(1-e^{-4sT}) ✓ \cdot \mathcal{Z}\left[\frac{2}{s(s+2)}\right] ✓$$

$$E(z) = (1-z^{-4}) ✓ \cdot \frac{z(1-e^{-2T})}{(z-1)(z-e^{-2T})} ✓$$

$$= \frac{z^4-1}{z^4} \cdot \frac{z(1-e^{-1})}{(z-1)(z-e^{-1})}$$

$$= \frac{(z^2-1)(z^2+1)\, z \cdot 0{,}632}{z^4 \cdot (z-1)(z-0{,}367)}$$

$$= \frac{(z-1)(z+1)(z^2+1)\, z \cdot 0{,}632}{z^4 \cdot (z-1)(z-0{,}367)}$$

$$= \frac{(z+1)(z^2+1)\, 0{,}632}{z^3(z-0{,}367)} ✓$$

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Vraag 2

$$\frac{C(z)}{E(z)} = D(z) \cdot G(z)$$

Determine $D(z)$:

$$m(k) = 0,5\, m(k-1) + e(k)$$
$$M(z) = 0,5\, z^{-1} M(z) + E(z)$$
$$M(z)\left[1 - 0,5 z^{-1}\right] = E(z)$$
$$\frac{M(z)}{E(z)} = \frac{1}{1-0,5 z^{-1}} = \frac{z}{z-0,5}$$

$$G(z) = \mathcal{Z}\left[\frac{1-e^{+sT}}{s} \cdot \frac{20}{(s+2)(s+5)}\right] \qquad T = 0,2s$$

$$= \frac{z-1}{z} \cdot 20 \cdot \mathcal{Z}\left[\frac{1}{s(s+2)(s+5)}\right]$$

Parsial fraction expansion.

$$G(z) = \frac{z-1}{z} \cdot 20 \cdot \mathcal{Z}\left[\frac{0,1}{s} - \frac{0,17}{s+2} + \frac{0,067}{s+5}\right]$$

$$= \frac{z-1}{z} \cdot 20 \left[\frac{0,1 z}{z-1} - \frac{0,17 z}{z-e^{-2 \cdot 0,2}} + \frac{0,067 z}{z-e^{-5 \cdot 0,2}}\right]$$

$$= 2 - \frac{3,4(z-1)}{z-0,6703} + \frac{3,4(z-1)}{z-0,3679}$$

$$= \frac{0,2740\,(z-0,5670)}{(z-0,6703)(z-0,3679)}$$

$$\frac{C(z)}{E(z)} = \frac{0,2740\,(z-0,5670)}{(z-0,6703)(z-0,3679)} \cdot \frac{z}{z-0,5}$$

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Vraag 2

Remember $5\,Hz \rightarrow \frac{1}{5} = T$

$T = 0{,}2$ = sampling period

For $t_0 = 50\,ms = \frac{50 \times 10^{-3}}{0{,}2}\,T$

$= 0{,}25\,T$

$mT = 0{,}75 \times 0{,}2$
$= 0{,}15$

$D(z) = \frac{z}{z - 0{,}5}$

$\therefore \Delta = 0{,}25$ $\quad m = 0{,}75$

$$\frac{C(z)}{E(z)} = D(z) \cdot G(z,m)$$

$$G(z,m) = \mathcal{Z}_m \left[\frac{1 - e^{sT}}{s} \cdot \frac{20}{(s+2)(s+5)} \right] \qquad T = 0{,}2s$$

$$= \frac{z-1}{z} \cdot 20\, \mathcal{Z}_m \left[\frac{0{,}1}{s} - \frac{0{,}17}{s+2} + \frac{0{,}067}{s+5} \right]$$

$$= \frac{z-1}{z} \cdot 20 \left[\frac{0{,}1}{z-1} - \frac{0{,}17\, e^{-2 \cdot 0{,}75 \cdot 0{,}2}}{z - e^{-2 \cdot 0{,}2}} + \frac{0{,}067\, e^{-5 \cdot 0{,}75 \cdot 0{,}2}}{z - e^{-5 \cdot 0{,}2}} \right]$$

$$= \frac{2}{z} - \frac{0{,}1259\,(z-1)}{z\,(z - 0{,}6703)} + \frac{3{,}149\,(z-1)}{z\,(z - 0{,}3679)}$$

$$= \frac{5{,}039\, z^2 - 7{,}1905\, z + 2{,}5683}{z\,(z - 0{,}6705)(z - 0{,}3679)}$$

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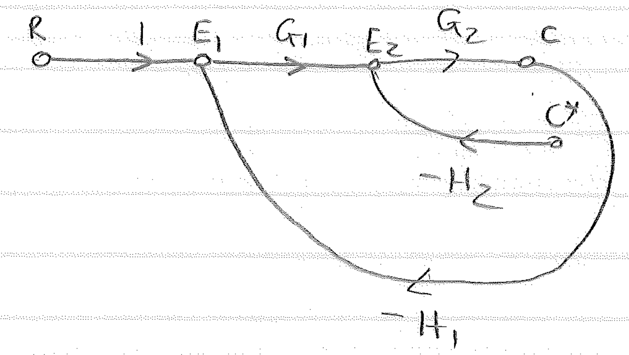
Vraag 3

Draw the original signal-flow diagram:



✓✓

$$C = G_2 E_2$$
$$E_2 = G_1 E_1 - H_2 C^*$$
$$E_1 = R - H_1 C$$

$$C = G_2 \cdot [G_1 E_1 - H_2 C^*]$$ ✓
$$= G_1 G_2 (R - H_1 C) - G_2 H_2 C^*$$
$$C = G_1 G_2 R - G_1 G_2 H_1 C - G_2 H_2 C^*$$ ✓

$$\therefore C = \frac{G_1 G_2 R}{(1 + G_1 G_2 H_1)} - \frac{G_2 H_2}{(1 + G_1 G_2 H_1)} C^*$$ ✓

$$C^* = \left(\frac{G_1 G_2 R}{1 + G_1 G_2 H_1}\right)^* - \left(\frac{G_2 H_2}{1 + G_1 G_2 H_1}\right)^* C^*$$ ✓

$$C(z) = \frac{\dfrac{G_1 G_2 R}{1 + G_1 G_2 H_1}(z)}{1 + \dfrac{G_2 H_2}{1 + G_1 G_2 H_1}(z)}$$ ✓✓

$$C^* \left[1 + \left(\frac{G_2 H_2}{1 + G_1 G_2 H_1}\right)^*\right] = \left(\frac{G_1 G_2 R}{1 + G_1 G_2 H_1}\right)^*$$ ✓✓ 10