NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOM CAMPUS

Benodigdhede vir hierdie vraestel:

Multikeusekaarte:	☐	Nie-programmeerbare sakrekenaar:	X
Grafiekpapier:	☐	Draagbare rekenaar:	☐

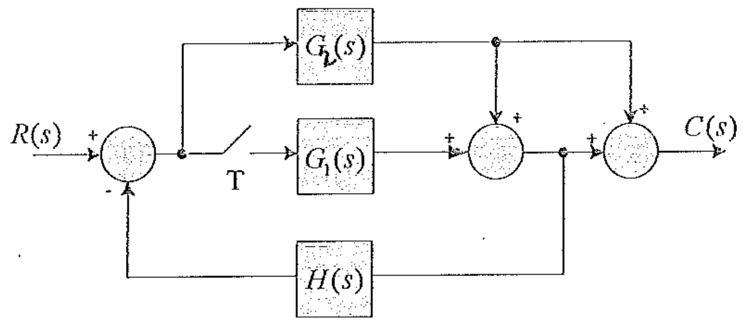
Oopboek-eksamen: ☐

SEMESTERTOETS / SEMESTER TEST:	3	KWALIFIKASIE/ QUALIFICATION:	**B ING**
MODULEKODE/ MODULE CODE:	**EERI418**	DUUR/ DURATION:	1.5 URE / 1.5 HOURS
MODULE BESKRYWING/ SUBJECT:	BEHEERTEORIE II CONTROL THEORY II	MAKS / MAX:	35
EKSAMINATOR(E)/ EXAMINER(S):	DR. K.R. UREN	DATUM / DATE:	**10-05-2013**
MODERATOR:	PROF. G. VAN SCHOOR	TYD / TIME	**9:30**

VRAAG 1 / QUESTION 1 [5]

Lei 'n uitdrukking af vir $C(z)$ vir die stelsel in Fig. 1. /

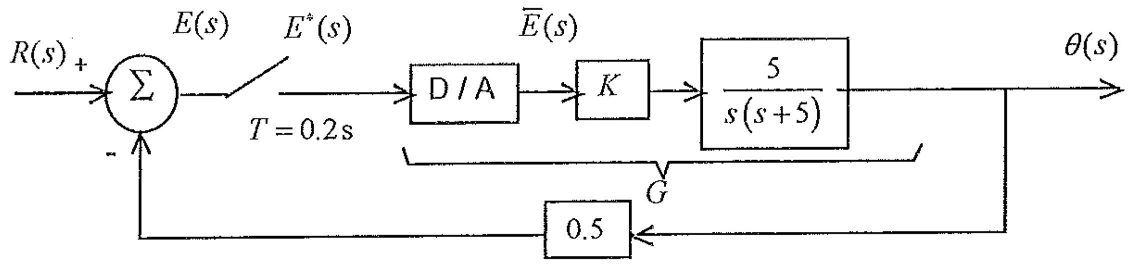
Derive the expression for $C(z)$ for the system in Fig. 1.



Figuur / *Figure* 1

VRAAG 2 / *QUESTION 2* [15]

Beskou die diskrete beheerstelsel in Figuur 2. / *Consider the discrete control system given in Figure 2.*



Figuur / *Figure* 2

2.1 Bepaal die stelseloordragsfunksie $\theta(z)/R(z)$ in terme van *G(z)*. / *Determine the system transfer function $\theta(z)/R(z)$ in terms of G(z).* (1)

2.2 Die oordragsfunksie *G(z)* word gegee deur: / *The transfer function G(z) is given by:*

$$G(z) = \frac{K(0.07358z + 0.05285)}{z^2 - 1.368z + 0.3679}$$

Bepaal die bestendige toestand fout van die diskrete stelsel vir K = 10 vir 'n eenheidshellingsinset. / *Determine the steady state error of the discrete system for K = 10 for a unit ramp input.* (4)

2.3 Bepaal die demping asook die natuurlike frekwensie van die diskrete stelsel. / *Determine the damping as well as the natural frequency of the discrete system.* (4)

2.4 Doen die nodige toetse en spreek jou uit oor die sinvolheid van die keuse van die monsterperiode. / *Do the necessary tests and comment on the choice of the sampling period.* (6)

Addisionele inligting / *additional information:*

$$\zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}}$$

$$\omega_n = \frac{1}{T}\sqrt{\ln^2 r + \theta^2}$$

$$\tau = \frac{1}{\zeta\omega_n}$$

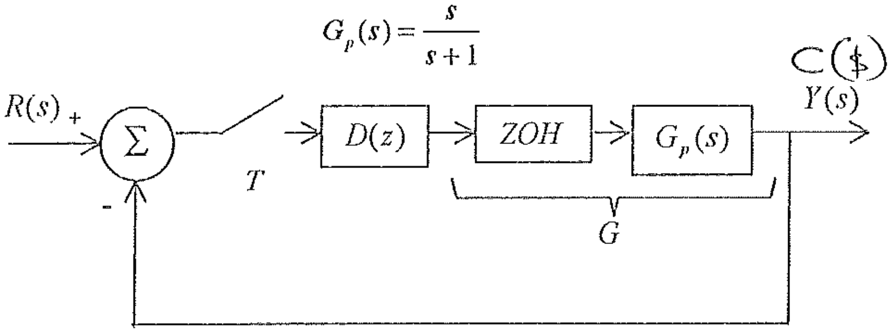
VRAAG 3 / *QUESTION 3* **[15]**

Beskou die stelsel in Figuur 3. Die monsterperiode is 1 sekonde. Die digitale filter word beskryf deur: / *Consider the system of Figure 3. The sampling period is 1 second. The digital filter is described by:*

$$m(kT) = e(kT) - 0.9e[(k-1)T] + m[(k-1)T]$$

Laat die aanleg oordragsfunksie gegee word deur: / *Let the plant transfer function be given by:*

$$G_p(s) = \frac{s}{s+1}$$



Figuur / *Figure* 3

3.1 Bepaal die tipe van die stelsel. / *Determine the system type.* (2)

3.2 Bereken die bestendige toestand respons vir 'n eenheidstrapinset, sonder om $C(z)$ te bepaal. / *Calculate the steady-state response for a unit-step input, without finding $C(z)$.* (3)

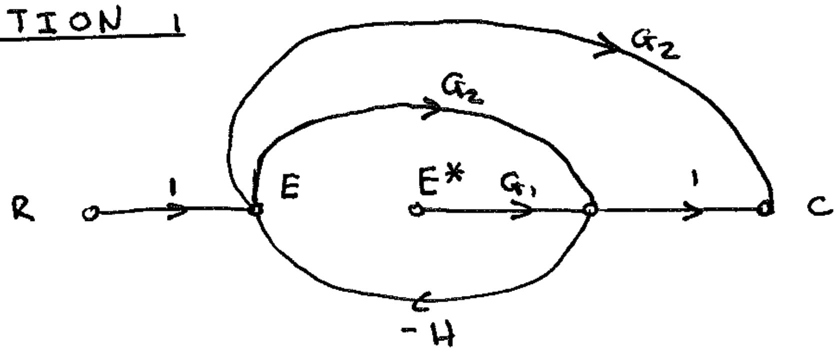
3.3 Bereken die benaderde tyd wat dit neem vir die stelsel om bestendige toestand te bereik. / *Calculate the approximate time for the system to reach steady state.* (5)

3.4 Bereken $c(kT)$ en verifieer so die resultaat van 3.2. / Calculate $c(kT)$ and verify the result of 3.2. (5)

TOTAAL/*TOTAL* [35]

Semestertest 3 - Memo 10-05-2013

QUESTION 1



$$E = R - H\left[\, G_1 E^* + G_2 E \,\right]$$

① $$= R - HG_1E^* - HG_2E$$ ✓

$$\therefore \quad E\left[1 + HG_2\right] = R - HG_1E^*$$

$$E = \frac{R}{1+HG_2} - \frac{HG_1}{1+G_2H}E^*$$

Take the star-transform

$$E^* = \left[\frac{R}{1+HG_2}\right]^* - \left[\frac{HG_1}{1+G_2H}\right]^* E^*$$

② $$E^*\left[1 + \left(\frac{HG_1}{1+G_2H}\right)^*\right] = \left(\frac{R}{1+HG_2}\right)^*$$

$$E^* = \frac{\left(\frac{R}{1+HG_2}\right)^*}{1 + \left(\frac{HG_1}{1+G_2H}\right)^*}$$ ✓

$$C = G_2E + G_1E^* + G_2E$$

③ $$C = 2G_2E + G_1E^*$$ ✓

④ $$C = 2G_2\left[\frac{R}{1+HG_2} - \frac{HG_1}{1+G_2H}E^*\right]$$

✓ $$+ G_1\left[\frac{\left(\frac{R}{1+HG_2}\right)^*}{1+\left(\frac{HG_1}{1+HG_2}\right)^*}\right]$$

$$= \frac{2G_2R}{1+HG_2} - \frac{2G_2HG_1}{1+G_2H}\left[\frac{\left(\frac{R}{1+HG_2}\right)^*}{1+\left(\frac{HG_1}{1+HG_2}\right)^*}\right]$$

$$+ G_1\left[\frac{\left(\frac{R}{1+HG_2}\right)^*}{1+\left(\frac{HG_1}{1+HG_2}\right)^*}\right]$$

$$C(z) = \left[\frac{2G_2R}{1+G_2H}\right](z) -$$

$$\left[\frac{2G_1G_2H}{1+G_2H}\right](z)\left[\frac{\frac{R}{1+HG_2}}{1+\frac{HG_1}{1+HG_2}}\right](z)$$

$$+ \frac{G_1\left(\frac{R}{1+HG_2}\right)(z)}{1+\left[\frac{G_1H}{1+G_2H}\right](z)}.$$ ✓

(5)

Question 2

2.1 $$\frac{\Theta(z)}{R(z)} = \frac{G(z)}{1 + 0,5\,G(z)}$$ (1)

2.2 $$E(z) = \frac{R(z)}{1 + 0,5\,G(z)}$$

$$G(z) = \frac{10\,(0,07358\,z + 0,05285)}{(z-1)(z-0,368)}$$

$$R(z) = \frac{Tz}{(z-1)^2}$$

$$E(z) = \frac{Tz\,(z-1)(z-0,368)}{(z-1)^2\left[(z-1)(z-0,368) + 0,5\cdot 10\,(0,07358\,z + 0,05285)\right.}$$

$$e_{ss} = \lim_{z \to 1} (z-1)\,E(z)$$

$$= \lim_{z \to 1} \frac{Tz\,(z-0,368)}{5(0,07358\,z + 0,05285)}$$

$$= \frac{0,2\,(1-0,368)}{5(0,07358 + 0,05285)}$$

$$= 0,2.$$ (4)

2.3 Damping and Natural freq.

$$Q(z) = z^2 - 1,368 z + 0,368 + 0,3679 z + 0,2642$$
$$= z^2 - z + 0,6322$$

$$z_{1,2} = 0,5 \pm j0,6182$$
$$= 0,7951 \underline{/\pm 0,8907} = r \underline{/\pm\theta}$$

$$\therefore \zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}}$$

$$= \frac{-\ln 0,7951}{\sqrt{(\ln 0,7951)^2 + (0,8907)^2}} = 0,2493$$

$$\omega_n = \frac{1}{T}\sqrt{\ln^2 r + \theta^2} = 4,6 \text{ rad/s} \qquad (4)$$

2.4 $q(s) = 1 + 0,5\,G(s)$
$$= 1 + 0,5 \cdot 10 \frac{5}{s(s+5)} = 0$$
$$\therefore s(s+5) + 25 = 0$$
$$s^2 + 5s + 25 = 0$$
$$s^2 + 5s + 25 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$\therefore \omega_n = 5$ rad/s and $\zeta = \frac{5}{2\omega_n} = \frac{5}{2 \cdot 5} = 0,5$

$$\tau = \frac{1}{\zeta\omega_n} = \frac{1}{0,5 \cdot 5} = 0,4 \text{ s.}$$

$$\frac{\tau}{T} = \frac{0,4}{0,2} = 2$$ (must be at least 5)

$$\omega_d = \omega_n \sqrt{1-\delta^2}$$
$$= 5 \sqrt{1-0{,}5^2}$$
$$= 4{,}33 \text{ rad/s.}$$

$$\therefore \quad T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{4{,}33} \text{ s} = 1{,}45 \text{ s}$$

$$\frac{Td}{T} = \frac{1{,}45}{T} = \frac{1{,}45}{0{,}2} = 7{,}25$$ (Voldoende)

$\therefore$ Sampling period of $\frac{0{,}4}{5} = 0{,}08$ s will be better. (6)

$2\pi = 360°$

$x = 19$

$360x = 19 \times 2\pi$

$x = \frac{19 \times 2\pi}{360} = 0{,}33$

$2\pi = 360$

$0{,}89 = x$

$2\pi x = 360 \cdot 0{,}89$

$x = \frac{360 \cdot 0{,}84}{2\pi} = 50°$

Question 3

3.1 $G(z) = \frac{z-1}{z} \mathcal{Z}\left[\frac{1}{s+1}\right]$

$= \frac{z-1}{z} \frac{z}{z-e^{-1}} = \frac{z-1}{z-0,369}$

$\therefore D(z)G(z) = \frac{z-0,9}{z-0,3679}$ $\therefore$ system typ = 0 (2)

3.2 $D(1)G(1) = \frac{1-0,9}{1-0,3679} = 0,158$

$T(1) = \frac{0,158}{1+0,158} = 0,1364 = c_{ss}(kT)$ (3)

3.3 $1 + G(z) = z - 0,3679 + z - 0,9$

$= z(z-0,634) = 0$

$\therefore \tau = \frac{-T}{\ln r} = \frac{-1}{\ln(0,634)} = 2,19\ s$

$\therefore 4\tau = 8,72\ s$ (5)

3.4 $\frac{D(z)G(z)}{1+D(z)G(z)} = \frac{z-0,9}{z-0,3679+z-0,9}$

$= \frac{z-0,9}{2z-1,2679} = \frac{0,5z-0,45}{z-0,634}$

$\frac{C(z)}{z} = \frac{0,5z-0,45}{(z-1)(z-0,634)} = \frac{0,1366}{z-1} + \frac{0,3634}{z-0,634}$

$c(kT) = 0,1366 + 0,3634(0,634)^k$ (5)

$c_{ss}(kT) = 0,1364$