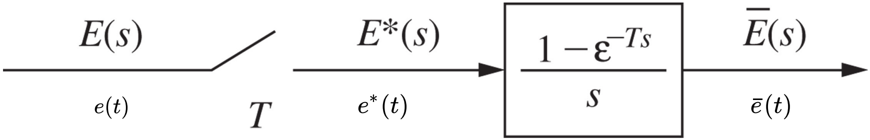
Sampling and Reconstruction

Chapter 3 of Phillips, Nagle and Chakrabortty (Study unit 4)

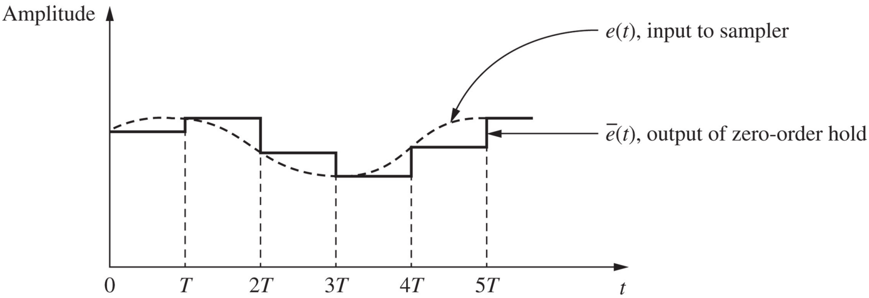
Presented by Prof. KR Uren

Introduction

What is the effect of sampling a continuous-time signal?



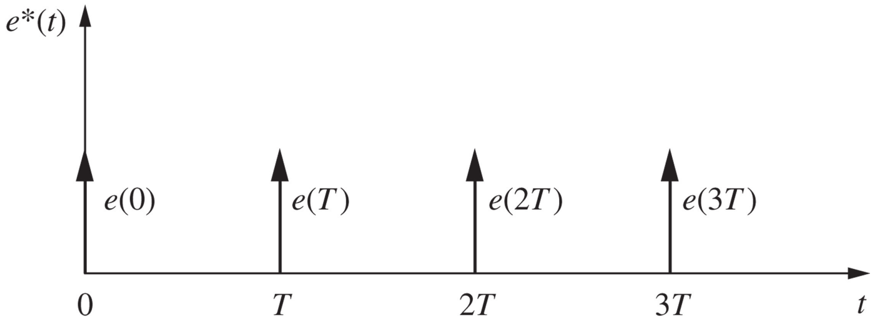
Sampled data systems



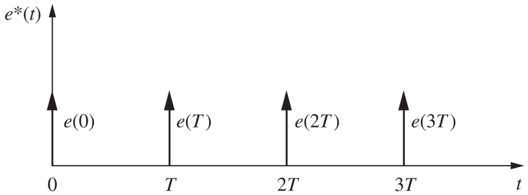
Derive $\bar{E}(s)$:

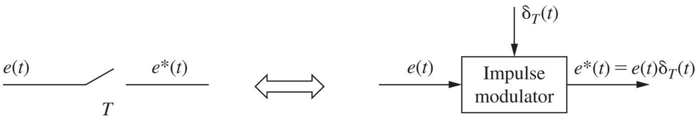
Ideal sampler

$$\begin{aligned} e^*(t) &= \mathscr{L}^{-1}[E^*(s)]e(0)\delta(t) + e(T)\delta(t-T) + \\ &+ e(2T)\delta(t-2T)\ldots \end{aligned} \tag{1}$$



Ideal sampler / Impulse modulator





Def: Starred transform

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The output signal of an ideal sampler is defined as the signal whose Laplace transform is given by

$$E^*(s) = \sum_{n=0}^{\infty} = e(nT)\epsilon^{-nTs} \tag{2}$$

where $e(t)$ is the input signal to the sampler. If $e(t)$ is discontinuous at $t = kT$, where k is an integer, then $e(kT)$ is taken to be $e(kT^+)$. The notation $e(kT^+)$ indicates the value of $e(t)$ as t approaches kT from the right. (at $t = kT + \Delta$, where Δ is made arbitrarily small)

Def: Sample operation

The definition of the sampling operation as specified in (2) together with the zero-order-hold transfer funciton defined by

$$G_{ho}(s) = \frac{1 - \epsilon^{-Ts}}{s} \tag{3}$$

yield the correct mathematical description of the sampler/hold operation defined by (3)

Example 3.1

Determine $E^*(s)$ for $e(t) = u(t)$, the unit step.

Example 3.2

Determine $E^*(s)$ for $e(t) = \epsilon^{-t}$.

Another important form of $E^*(s)$

$$E^*(s) = \frac{1}{T} \sum_{\infty}^{-\infty} E(s + jn\omega_s) + \frac{e(0)}{2} \tag{4}$$

where ω_s is the radian sampling frequency, that is, $\omega_s = 2\pi/T$

Example 3.5

Given $e(t) = 1 - \epsilon^{-t}$, determine $E^*(s)$ using the definition of the starred transform.

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Fourier transform

The Fourier transform is defined by

$$\begin{aligned} \mathscr{F}[e(t)] &= E(j\omega) \int_{-\infty}^{\infty} e(t)\epsilon^{-j\omega t} dt \qquad (5) \\ &= \int_{-\infty}^{0} e(t)\epsilon^{-j\omega t} dt + \int_{0}^{\infty} e(t)\epsilon^{-j\omega t} dt \end{aligned}$$

and if $e(t) = 0$ for $t < 0$, then

$$\mathscr{F}[e(t)] = \int_{0}^{\infty} e(t)\epsilon^{-j\omega t} dt = \mathscr{L}[e(t)]|_{s=j\omega} \qquad (6)$$

Fourier spectrum

A plot of the Fourier transform $E(j\omega)$ is called *frequency spectrum* of $e(t)$. A common procedure for showing the frequency spectrum is to express $E(j\omega)$ as

$$E(j\omega) = |E(j\omega)|\epsilon^{j\theta(j\omega)} = |E(j\omega)|\angle\theta(j\omega) \tag{7}$$

and plot $|E(j\omega)|$ versus ω (amplitude spectrum) and $\theta(j\omega)$ versus $\theta(j\omega)$ (phase spectrum).

Frequency response

If we have an input $e(t)$ which is an impulse function $\delta(t)$ and we have a system $G(s)$ and an output $y(t)$, then the Fourier transform

$$Y(j\omega) = G(j\omega)E(j\omega) \tag{8}$$

and $G(j\omega)$ is called the *frequency response*.

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Property 1:

$E^*(s)$ is periodic [1] in s with period $j\omega_s$.

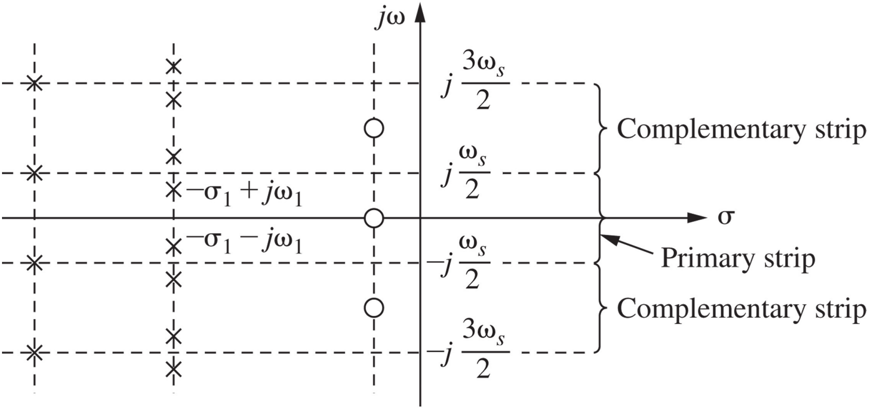
$$E^*(s + jm\omega_s) = E^*(s)$$

Property 2:

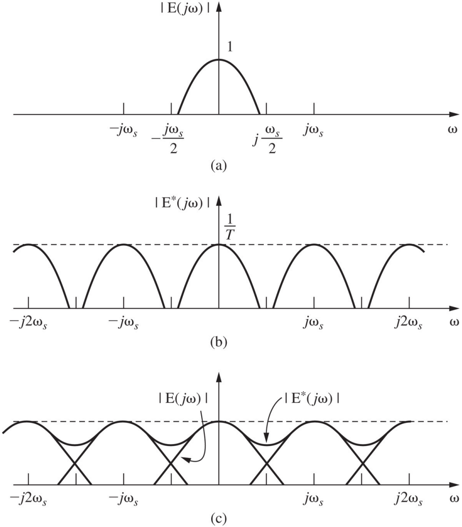
If $E(s)$ has a pole at $s = s_1$, the $E^*(s)$ must have poles at $s = s_1 + jm\omega_s$, $m = 0, \pm 1, \pm 2, \ldots$

[1] By definition, a continuous-time signal $x(t)$ is periodic if $x(t) = x(t + T)$, $T > 0$ for all t, where the constant T is the period.

Properties of $E^*(s)$



Effect of sampling period



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