Benodigdhede vir hierdie vraestel:				Oopboek-eksamen:	
Multikeusekaarte:		Nie-programmeerbare sakrekenaar:	X		
Grafiekpapier:		Draagbare rekenaar:			

SEMESTERTOETS / SEMESTER TEST:	1	KWALIFIKASIE/ QUALIFICATION:	**B ING**
MODULEKODE/ MODULE CODE:	**EERI418**	DUUR/ DURATION:	1 ½ UUR / 1 ½ HOUR
MODULE BESKRYWING/ SUBJECT:	BEHEERTEORIE II CONTROL THEORY II	MAKS / MAX:	45
EKSAMINATOR(E)/ EXAMINER(S):	PROF. G VAN SCHOOR	DATUM / DATE:	**04-03-2014**
MODERATOR:	DR. KR UREN	TYD / TIME	**07:30**

VRAAG 1 / QUESTION 1 [15]

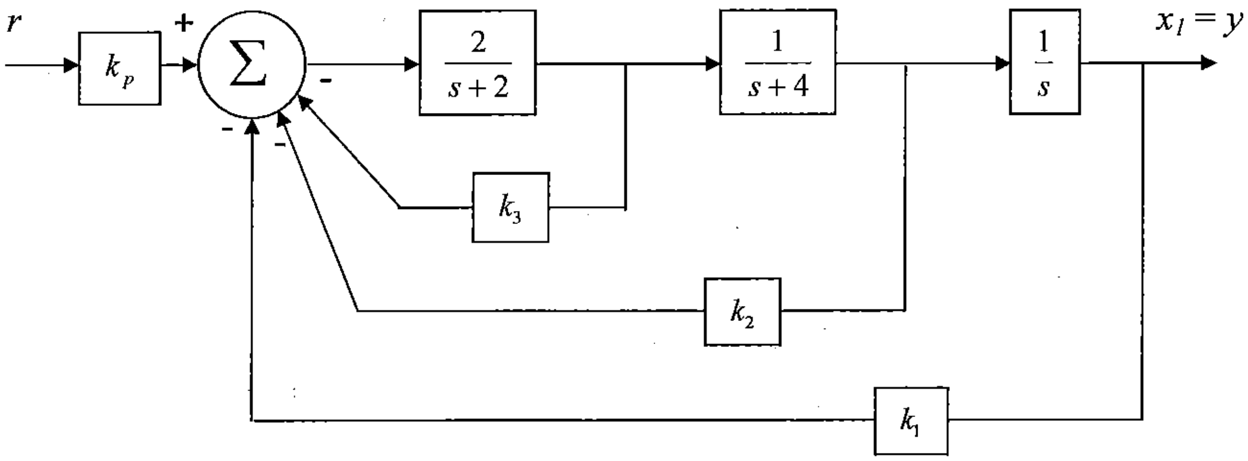
Die blokdiagram van 'n stelsel met toestandsterugvoer word in figuur 1 getoon. Bepaal die winswaardes k_1, k_2, k_3 en k_p sodat: /

The block diagram of a system with state feedback is shown in figure 1. Determine the gains k_1, k_2, k_3 and k_p such that:

(a) die bestendige toestand fout vir 'n trapinset nul is / *the steady state error for a step input is zero;*
(b) die persentasie verbyskiet kleiner as 5 % is en die vestigingstyd kleiner as 0.5 s is. / *the percentage overshoot is less than 5 % and the settling time is less than 0.5 s.*

Benader die stelsel as tweede orde deur die derde wortel 'n orde hoër in frekwensie te kies as die dominante wortels. /

Approximate the system as a second order system by choosing the position of the third root an order in frequency higher than the dominant poles.



Figuur / Figure 1

Addisionele inligting / *additional information:*

$$PO = 100e^{-\zeta\pi / \sqrt{1-\zeta^2}}$$

$$T_s = \frac{4}{\zeta\omega_n}$$

VRAAG 2 / *QUESTION 2* [25]

2.1 'n Stelsel word deur die volgende verskilvergelyking gemodelleer: /

A system is modelled by the following difference equation:

$$x(k) + x(k-1) - x(k-2) = e(k-1) + 2e(k)$$

Bepaal die oordragsfunksie van die stelsel $X(z)/E(z)$. /

Determine the transfer function of the system $X(z)/E(z)$. (5)

2.2 Bepaal $x(k)$ vir die stelsel in 2.1 vir 'n eenheidstrapinset deur van magreeksuitbreiding gebruik te maak. Bereken tot die vierde term ($x(3)$). Aanvaar begintoestande as nul. /

Determine $x(k)$ for the system in 2.1 for a unit step input. Use the power series method and determine up to the fourth term ($x(3)$). Assume zero initial conditions. (5)

2.3 Bepaal $x(k)$ vir die stelsel in 2.1 in geslote vorm vir 'n eenheidstrapinset deur van parsiele breuk uitbreiding gebruik te maak. /

Determine $x(k)$ for the system in 2.1 in closed form for a unit step input using partial fraction expansion.

(7)

2.4 Bepaal die inset vorentoevoer kanonieke toestandsveranderlike model van die stelsel in 2.1. /

Determine the input feedforward canonical state variable model of the system in 2.1. (8)

TOTAAL/*TOTAL* [40]

TABLE 2-2 PROPERTIES OF THE z-TRANSFORM

Sequence	Transform
$e(k)$	$E(z) = \sum_{k=0}^{\infty} e(k)z^{-k}$
$a_1 e_1(k) + a_2 e_2(k)$	$a_1 E_1(z) + a_2 E_2(z)$
$e(k-n)u(k-n); \quad n \geq 0$	$z^{-n} E(z)$
$e(k+n)u(k); \quad n \geq 1$	$z^n \left[E(z) - \sum_{k=0}^{n-1} e(k)z^{-k} \right]$
$\epsilon^{ak} e(k)$	$E(z\epsilon^{-a})$
$ke(k)$	$-z \dfrac{dE(z)}{dz}$
$e_1(k) * e_2(k)$	$E_1(z)E_2(z)$
$e_1(k) = \sum_{n=0}^{k} e(n)$	$E_1(z) = \dfrac{z}{z-1} E(z)$

Initial value: $e(0) = \lim_{z \to \infty} E(z)$

Final value: $e(\infty) = \lim_{z \to 1} (z-1)E(z)$, if $e(\infty)$ exists

TABLE 2-3 z-TRANSFORMS

Sequence	z-Transform
$\delta(k-n)$	z^{-n}
1	$\dfrac{z}{z-1}$
k	$\dfrac{z}{(z-1)^2}$
k^2	$\dfrac{z(z+1)}{(z-1)^3}$
a^k	$\dfrac{z}{z-a}$
ka^k	$\dfrac{az}{(z-a)^2}$
$\sin ak$	$\dfrac{z \sin a}{z^2 - 2z \cos a + 1}$
$\cos ak$	$\dfrac{z(z - \cos a)}{z^2 - 2z \cos a + 1}$
$a^k \sin bk$	$\dfrac{az \sin b}{z^2 - 2az \cos b + a^2}$
$a^k \cos bk$	$\dfrac{z^2 - az \cos b}{z^2 - 2az \cos b + a^2}$

TABLE A8-1 LAPLACE TRANSFORM PROPERTIES

Name	Theorem
Derivative	$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0^+)$
nth-order derivative	$\mathcal{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1}f(0^+) - \cdots f^{(n-1)}(0^+)$
Integral	$\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{F(s)}{s}$
Shifting	$\mathcal{L}[f(t - t_0)u(t - t_0)] = e^{-t_0 s}F(s)$
Initial value	$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$
Final value	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$
Frequency shift	$\mathcal{L}[e^{-at}f(t)] = F(s + a)$
Convolution integral	$\mathcal{L}^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t - \tau)f_2(\tau)\,d\tau = \int_0^t f_1(\tau)f_2(t - \tau)\,d\tau$

Optimum coefficients of T(s) Based on the ITAE Criterion for a Step Input

$$s + \omega_n$$
$$s^2 + 1.4\omega_n s + \omega_n^2$$
$$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$$
$$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$$
$$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$$
$$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.60\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$$

z-transforms

Laplace transform $E(s)$	Time function $e(t)$	z-Transform $E(z)$	Modified z-transform $E(z,m)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1}+\frac{T}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2}\left[\frac{m^2}{z-1}+\frac{2m+1}{(z-1)^2}+\frac{2}{(z-1)^3}\right]$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{1}{s+a}$	ϵ^{-at}	$\frac{z}{z-\epsilon^{-aT}}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$	$\frac{Tz\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}$	$\frac{T\epsilon^{-amT}[\epsilon^{-aT}+m(z-\epsilon^{-aT})]}{(z-\epsilon^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k\epsilon^{-at}$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-\epsilon^{-at}$	$\frac{z(1-\epsilon^{-aT})}{(z-1)(z-\epsilon^{-aT})}$	$\frac{1}{z-1}-\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1-\epsilon^{-at}}{a}$	$\frac{z[(aT-1+\epsilon^{-aT})z+(1-\epsilon^{-aT}-aT\epsilon^{-aT})]}{a(z-1)^2(z-\epsilon^{-aT})}$	$\frac{T}{(z-1)^2}+\frac{amT-1}{a(z-1)}+\frac{\epsilon^{-amT}}{a(z-\epsilon^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\epsilon^{-at}$	$\frac{z}{z-1}-\frac{z}{z-\epsilon^{-aT}}-\frac{aT\epsilon^{-aT}z}{(z-\epsilon^{-aT})^2}$	$\frac{1}{z-1}-\left[\frac{1+amT}{z-\epsilon^{-aT}}+\frac{aT\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}\right]\epsilon^{-amT}$
$\frac{b-a}{(s+a)(s+b)}$	$\epsilon^{-at}-\epsilon^{-bt}$	$\frac{(\epsilon^{-aT}-\epsilon^{-bT})z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}-\frac{\epsilon^{-bmT}}{z-\epsilon^{-bT}}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z\sin(aT)}{z^2-2z\cos(aT)+1}$	$\frac{z\sin(amT)+\sin(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{s}{s^2+a^2}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2-2z\cos aT+1}$	$\frac{z\cos(amT)-\cos(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\epsilon^{-at}\sin bt$	$\frac{1}{b}\left[\frac{z\epsilon^{-aT}\sin bT}{z^2-2z\epsilon^{-aT}\cos(bT)+\epsilon^{-2aT}}\right]$	$\frac{1}{b}\left[\frac{\epsilon^{-amT}[z\sin bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$\epsilon^{-at}\cos bt$	$\frac{z^2-z\epsilon^{-aT}\cos bT}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$	$\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-\epsilon^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$	$\frac{z(Az+B)}{(z-1)(z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT})}$ $A=1-\epsilon^{-aT}\left(\cos bT+\frac{a}{b}\sin bT\right)$ $B=\epsilon^{-2aT}+\epsilon^{-aT}\left(\frac{a}{b}\sin bT-\cos bT\right)$	$\frac{1}{z-1}$ $-\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$ $\frac{+\frac{a}{b}\{\epsilon^{-amT}[z\sin bmT-\epsilon^{-aT}\sin(1-m)bT]\}}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}+\frac{\epsilon^{-at}}{a(a-b)}$ $+\frac{\epsilon^{-bt}}{b(b-a)}$	$\frac{(Az+B)z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})(z-1)}$	$A=\frac{b(1-\epsilon^{-aT})-a(1-\epsilon^{-bT})}{ab(b-a)}$ $B=\frac{a\epsilon^{-aT}(1-\epsilon^{-bT})-b\epsilon^{-bT}(1-\epsilon^{-aT})}{ab(b-a)}$

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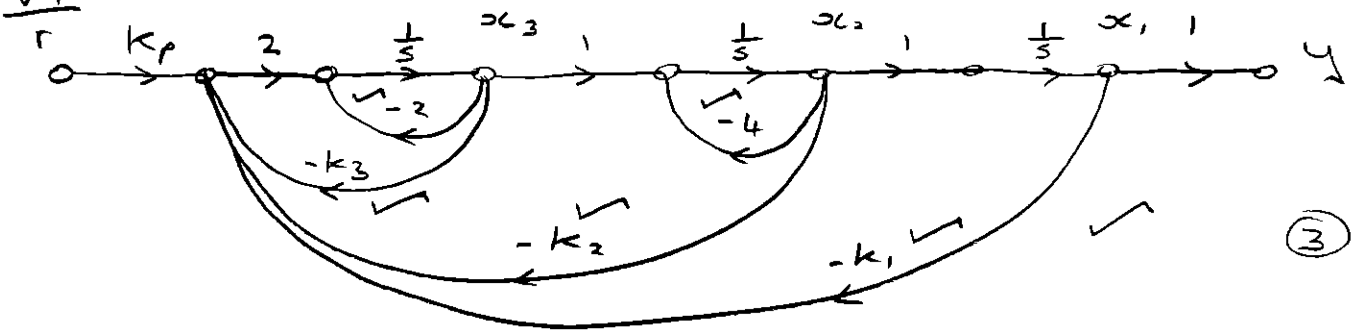
MEMORANDUM

V1



(3)

$$T(s) = \frac{2k_p}{s^3 + (6+2k_3)s^2 + (2k_2 + 8k_3 + 8)s + 2k_1}$$

Vir $e_{ss} = 0$ vir 'n eenheidstrapinset moet $T(0) = 1$

$$\therefore \frac{2k_p}{2k_1} = 1 \qquad \therefore k_p = k_1$$

Vir PO < 5% $\zeta \simeq 0,7$ en $T_s = \frac{4}{\zeta\omega_n} < 0,5$

$\omega_n > 11,43$ sê $\omega_n = 12$ rad/s

Volgens dominante pool benadering:

$$q(s) = (s^2 + 2\zeta\omega_n s + \omega_n^2)(s + 10\zeta\omega_n)$$
$$= s^3 + 12\zeta\omega_n s^2 + (\omega_n^2 + 20\zeta^2\omega_n^2)s + 10\zeta\omega_n^3$$

$\therefore$ $6 + 2k_3 = 12\zeta\omega_n = 12 \cdot 0,7 \cdot 12 = 100,8$

$\therefore$ $k_3 = 47,4$

$2k_2 + 8k_3 + 8 = \omega_n^2 + 20\zeta^2 \cdot \omega_n^2 = 12^2 + 20 \cdot 0,7^2\, 12^2$

$= 1555,2$

$\therefore$ $2k_2 = 1168$ $\therefore$ $k_2 = 584$

en $2k_1 = 10 \cdot \zeta \cdot \omega_n^3 = 10 \cdot 0,7 \cdot 12^3 = 12096$

$\therefore$ $k_1 = 6048$

$\therefore$ $k_p = 6048$

[15]

V2

2.1 $$x(k) + x(k-1) - x(k-2) = e(k-1) + 2e(k)$$

$$\therefore X(z) + z^{-1}X(z) - z^{-2}X(z) = z^{-1}E(z) + 2E(z)$$

$$z^2 X(z) + z X(z) - X(z) = z E(z) + 2z^2 E(z)$$

$$\therefore X(z)\left[z^2 + z - 1\right] = E(z)\left[z + 2z^2\right]$$

$$\frac{X(z)}{E(z)} = \frac{2z^2 + z}{z^2 + z - 1} \qquad (4)$$

2.2. $$X(z) = \frac{2z^2 + z}{z^2 + z - 1} \cdot E(z)$$

$$= \frac{2z^2 + z}{z^2 + z - 1} \cdot \frac{z}{z-1}$$

$$= \frac{2z^3 + z^2}{z^3 - 2z + 1}$$

$$\begin{array}{r|l}
 & 2 + z^{-1} + 4z^{-2} + 0z^{-3} + 7z^{-4} \\
\hline
z^3 - 2z + 1 & 2z^3 + z^2 \\
 & 2z^3 \quad - 4z + 2 \\
\hline
 & z^2 + 4z - 2 \\
 & z^2 \quad - 2 + z^{-1} \\
\hline
 & 4z \quad - z^{-1} \\
 & 4z \quad - 8z^{-1} + 4z^{-2} \\
\hline
 & 7z^{-1}
\end{array}$$

$$x(0) = 2, \quad x(1) = 1, \quad x(2) = 4, \quad x(3) = 0 \qquad (5)$$

2.3 $$\frac{X(z)}{z} = \frac{2z^2 + z}{(z-1)(z-0{,}618)(z+1{,}618)}$$

$$= \frac{A}{z-1} + \frac{B}{z-0{,}618} + \frac{C}{z+1{,}618}$$

$$= \frac{3}{z-1} - \frac{1{,}618}{z-0{,}618} + \frac{C}{z+1{,}618}$$

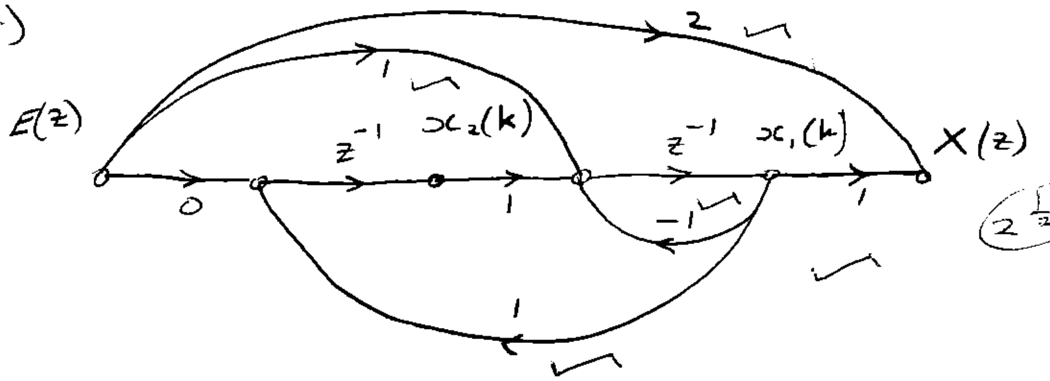
$$\therefore X(z) = 3 \cdot \frac{z}{z-1} - 1{,}618\frac{z}{z-0{,}618} + 0{,}618\frac{z}{z+1{,}618}$$

$$\therefore x(k) = 3 - 1{,}618(0{,}618)^k + 0{,}618(-1{,}618)^k$$

(7)

2.4

$$\frac{X(z)}{E(z)} = \frac{2z^2+z}{z^2+z-1} = \frac{2+z^{-1}}{1+z^{-1}-z^{-2}}$$



$2\frac{1}{2}$

$$\therefore \quad x_1(k+1) = -x_1(k) + x_2(k) + e(k)$$
$$x_2(k+1) = x_1(k)$$

$$\therefore \quad \mathbb{X}(k+1) = \underbrace{\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}}_{\mathbb{A}} \mathbb{X}(k) + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\mathbb{B}} e(k)$$

$$x(k) = x_1(k) + 2e(k)$$

$$\therefore \quad x(k) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{\mathbb{C}} \mathbb{X}(k) + \underbrace{2}_{\mathbb{D}} e(k) \qquad (8)$$

[25]