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POTCHEFSTROOMKAMPUS

Benodigdhede vir hierdie vraestel/Requirements for this paper:			
Antwoordskrifte/ Answer scripts:	[x]	Multikeusekaarte (A5)/ Multi-choice cards (A5):	[ ]
Presensiestrokies (Invulvraestel)/ Attendance slips (Fill-in paper):	[ ]	Multikeusekaarte (A4)/ Multi-choice cards (A4):	[ ]
Rofwerkpapier/ Scrap paper:	[ ]	Grafiekpapier/ Graph paper:	[ ]

Sakrekenaars / Calculators: Ja/Yes
Ander hulpmiddels/ Other resources:

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Module beskrywing/ Module description:	Beheerteorie II / Control theory II	Maks/ Max:	40
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Inhandiging van antwoordskrifte/Submission of answer scripts: Gewoon/Ordinary

VRAAG 1 / QUESTION 1 [6]

Bepaal die z-transform in geslote vorm van die volgende sein: / *Determine the z-transform in closed form of the following signal:*

$$E(s) = \frac{2\left(1 - e^{-2s}\right)}{s\,(s+2)}, \quad T = 0.5 \text{ s}$$

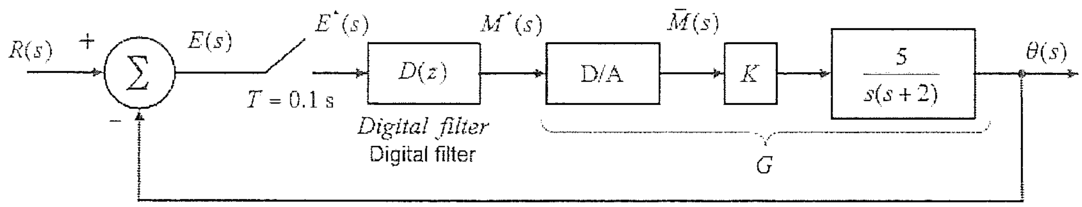
VRAAG 2 / QUESTION 2 [22]

Beskou die stelsel in Fig. 1. / *Consider the system in Fig. 1.*
Die digitale filter los die volgende verskilvergelyking op: / *The digital filter solves the following difference equation:*

$$m(k+1) = e(k+1) - 0.9e(k) + 0.98m(k)$$

2.1 Bepaal die oordragsfunksie $\theta(z)/E(z)$ vir $K = 5$. / *Determine the transfer function $\theta(z)/E(z)$ for $K = 5$.* (7)

2.2 Bepaal ook die stelseloordragsfunksie $\theta(z)/R(z)$. / *Determine the transfer function $\theta(z)/R(z)$.* (3)



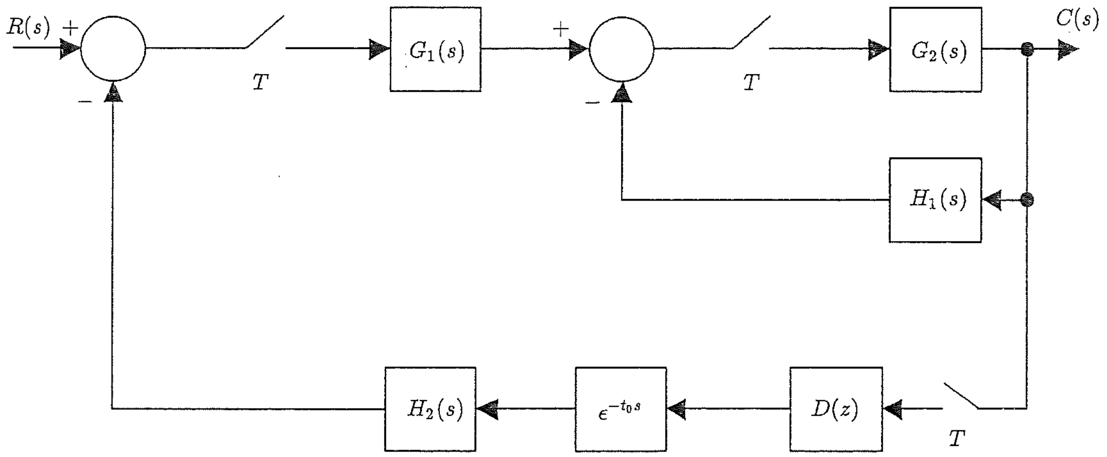
Figuur 1/*Figure 1*

2.3 Bepaal die bestendige toestand fout vir 'n eenheidshellingsinset. / *Determine the steady state error for a unit ramp input.* (6)

2.4 Bepaal die oodragsfunksie $\theta(z)/E(z)$ indien die verwerkingstyd van die digitale filter van 0.15 s ook gemodelleer moet word. / *Determine the transfer function $\theta(z)/E(z)$ when a computational delay of 0.15 s also needs to be modelled.* (6)

VRAAG 3/ QUESTION 3 [12]

Bepaal vir die stelsel in Fig. 2 die uitset $C(z)$ in terme van die inset en die oordragsfunksies. / *Determine for the system in Fig. 2 the output $C(z)$ in terms of the input and the transfer functions.*



Figuur 2/*Figure 2*

Tabelle / Tables

TABLE 2-2 Properties of the z-Transform

Sequence	Transform
$e(k)$	$E(z) = \sum_{k=0}^{\infty} e(k)z^{-k}$
$a_1e_1(k) + a_2e_2(k)$	$a_1E_1(z) + a_2E_2(z)$
$e(k-n)u(k-n); \quad n \geq 0$	$z^{-n}E(z)$
$e(k+n)u(k); \quad n \geq 1$	$z^{n}\left[E(z) - \sum_{k=0}^{n-1} e(k)z^{-k}\right]$
$\varepsilon^{akT}e(k)$	$E(z\varepsilon^{-aT})$
$ke(k)$	$-z\dfrac{dE(z)}{dz}$
$e_1(k) * e_2(k)$	$E_1(z)E_2(z)$
$e_1(k) = \sum_{n=0}^{k} e(n)$	$E_1(z) = \dfrac{z}{z-1}E(z)$

Initial value: $e(0) = \lim_{z\to\infty} E(z)$

Final value: $e(\infty) = \lim_{z\to 1}(z-1)E(z)$, if $e(\infty)$ exists

TABLE A5-1 Laplace Transform Properties

Name	Theorem
Derivative	$\mathcal{L}\left[\dfrac{df}{dt}\right] = sF(s) - f(0^+)$
nth-order derivative	$\mathcal{L}\left[\dfrac{d^nf}{dt^n}\right] = s^nF(s) - s^{n-1}f(0^+) - \cdots - f^{(n-1)}(0^+)$
Integral	$\mathcal{L}\left[\int_0^t f(\tau)d\tau\right] = \dfrac{F(s)}{s}$
Shifting	$\mathcal{L}[f(t-t_0)\,u(t-t_0)] = \varepsilon^{-t_0s}F(s)$
Initial value	$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$
Final value	$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$
Frequency shift	$\mathcal{L}[\varepsilon^{-at}f(t)] = F(s+a)$
Convolution integral	$\mathcal{L}^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t-\tau)f_2(\tau)d\tau = \int_0^t f_1(\tau)f_2(t-\tau)d\tau$

TABLE 2-3 *z*-Transforms

Sequence	Transform
$\delta(k-n)$	z^{-n}
1	$\dfrac{z}{z-1}$
k	$\dfrac{z}{(z-1)^2}$
k^2	$\dfrac{z(z+1)}{(z-1)^3}$
a^k	$\dfrac{z}{z-a}$
ka^k	$\dfrac{az}{(z-a)^2}$
$\sin ak$	$\dfrac{z\sin a}{z^2-2z\cos a+1}$
$\cos ak$	$\dfrac{z(z-\cos a)}{z^2-2z\cos a+1}$
$a^k \sin bk$	$\dfrac{az\sin b}{z^2-2az\cos b+a^2}$
$a^k \cos bk$	$\dfrac{z^2-az\cos b}{z^2-2az\cos b+a^2}$

Laplace transform $E(s)$	Time function $e(t)$	z-Transform $E(z)$	Modified z-transform $E(z,m)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1} + \frac{T}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2 z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2}\left[\frac{m^2}{z-1} + \frac{2m+1}{(z-1)^2} + \frac{2}{(z-1)^3}\right]$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a\to 0} (-1)^{k-1} \frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$\lim_{a\to 0} (-1)^{k-1} \frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{1}{s+a}$	ϵ^{-at}	$\frac{z}{z-\epsilon^{-aT}}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$	$\frac{Tz\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}$	$\frac{T\epsilon^{-amT}[\epsilon^{-aT} + m(z-\epsilon^{-aT})]}{(z-\epsilon^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k\epsilon^{-at}$	$(-1)^k \frac{\partial^k}{\partial a^k}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$(-1)^k \frac{\partial^k}{\partial a^k}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-\epsilon^{-at}$	$\frac{z(1-\epsilon^{-aT})}{(z-1)(z-\epsilon^{-aT})}$	$\frac{1}{z-1} - \frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t - \frac{1-\epsilon^{-at}}{a}$	$\frac{z[(aT-1+\epsilon^{-aT})z + (1-\epsilon^{-aT} - aT\epsilon^{-aT})]}{a(z-1)^2(z-\epsilon^{-aT})}$	$\frac{T}{(z-1)^2} + \frac{amT-1}{a(z-1)} + \frac{\epsilon^{-amT}}{a(z-\epsilon^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\epsilon^{-at}$	$\frac{z}{z-1} - \frac{z}{z-\epsilon^{-aT}} - \frac{aT\epsilon^{-aT}z}{(z-\epsilon^{-aT})^2}$	$\frac{1}{z-1} - \left[\frac{1+amT}{z-\epsilon^{-aT}} + \frac{aT\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}\right]\epsilon^{-amT}$

$\frac{b-a}{(s+a)(s+b)}$	$\epsilon^{-at} - \epsilon^{-bt}$	$\frac{(e^{-aT} - \epsilon^{-bT})z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}} - \frac{\epsilon^{-bmT}}{z-\epsilon^{-bT}}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z\sin(aT)}{z^2 - 2z\cos(aT) + 1}$	$\frac{z\sin(amT) + \sin(1-m)aT}{z^2 - 2z\cos(aT) + 1}$
$\frac{s}{s^2+a^2}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2 - 2z\cos aT + 1}$	$\frac{z\cos(amT) - \cos(1-m)aT}{z^2 - 2z\cos(aT) + 1}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\epsilon^{-at}\sin bt$	$\frac{1}{b}\left[\frac{z\epsilon^{-aT}\sin bT}{z^2 - 2z\epsilon^{-aT}\cos(bT) + \epsilon^{-2aT}}\right]$	$\frac{1}{b}\left[\frac{\epsilon^{-amT}[z\sin bmT + \epsilon^{-aT}\sin(1-m)bT]}{z^2 - 2z\epsilon^{-aT}\cos bT + \epsilon^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$\epsilon^{-at}\cos bt$	$\frac{z^2 - z\epsilon^{-aT}\cos bT}{z^2 - 2z\epsilon^{-aT}\cos bT + \epsilon^{-2aT}}$	$\frac{\epsilon^{-amT}[z\cos bmT + \epsilon^{-aT}\sin(1-m)bT]}{z^2 - 2z\epsilon^{-aT}\cos bT + \epsilon^{-2aT}}$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1 - \epsilon^{-at}\left(\cos bt + \frac{a}{b}\sin bt\right)$	$\frac{z(Az+B)}{(z-1)(z^2 - 2z\epsilon^{-aT}\cos bT + \epsilon^{-2aT})}$ $A = 1 - \epsilon^{-aT}\left(\cos bT + \frac{a}{b}\sin bT\right)$ $B = \epsilon^{-2aT} + \epsilon^{-aT}\left(\frac{a}{b}\sin bT - \cos bT\right)$	$\frac{1}{z-1}$ $-\frac{\epsilon^{-amT}[z\cos bmT + \epsilon^{-aT}\sin(1-m)bT]}{z^2 - 2z\epsilon^{-aT}\cos bT + \epsilon^{-2aT}}$ $\frac{+\frac{a}{b}\{\epsilon^{-amT}[z\sin bmT - \epsilon^{-aT}\sin(1-m)bT]\}}{z^2 - 2z\epsilon^{-aT}\cos bT + \epsilon^{-2aT}}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab} + \frac{\epsilon^{-at}}{a(a-b)}$ $+ \frac{\epsilon^{-bt}}{b(b-a)}$	$\frac{1}{ab(b-a)}$ $\frac{(Az+B)z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})(z-1)}$	$A = \frac{b(1-\epsilon^{-aT}) - a(1-\epsilon^{-bT})}{ab(b-a)}$ $B = \frac{a\epsilon^{-aT}(1-\epsilon^{-bT}) - b\epsilon^{-bT}(1-\epsilon^{-aT})}{ab(b-a)}$

EER 418 - MEMO   SEMESTER TEST 2   25/04/2017

Question 1 [6]

$$E(s) = \frac{2(1-e^{-2s})}{s(s+2)}, \quad T = 0{,}5\,s$$

$$= \frac{2(1-e^{-4sT})}{s(s+2)} \checkmark$$

$$= \underbrace{\mathcal{Z}(1-e^{-4sT})}_{\checkmark} \cdot \underbrace{\mathcal{Z}\left[\frac{2}{s(s+2)}\right]}_{\checkmark}$$

$$E(z) = \underbrace{(1-z^{-4})}_{\checkmark} \cdot \underbrace{\frac{z(1-e^{-2T})}{(z-1)(z-e^{-1})}}_{\checkmark}$$

$$= \frac{z^4-1}{z^4} \cdot \frac{z(0{,}632)}{(z-1)(z-0{,}368)}$$

$$= \frac{(z^2-1)(z^2+1)z(0{,}632)}{z^4(z-1)(z-0{,}368)}$$

$$\begin{array}{r|l} & z+1 \\ \hline z-1 & z^2-1 \\ & z^2 \\ \hline & -1 \\ & -1 \\ \hline & 0 \end{array}$$

$$\therefore \quad E(z) = \frac{(z+1)(z^2+1)(0{,}632)}{z^3(z-0{,}368)} \checkmark \qquad (6)$$

□

Question 2 [22]

2.1) $m(k+1) = e(k+1) - 0{,}9\,e(k) + 0{,}98\,m(k)$

$$zM(z) = zE(z) - 0{,}9E(z) + 0{,}98M(z)$$

$$(z - 0{,}98)M(z) = (z - 0{,}9)E(z)$$

$$D(z) = \frac{M(z)}{E(z)} = \frac{(z-0{,}9)}{(z-0{,}98)}$$

$$G(z) = \mathcal{Z}\left[\frac{5(1-e^{-sT})}{s} \cdot \frac{5}{s(s+2)}\right]$$

$$= \frac{z-1}{z} \cdot 25\,\mathcal{Z}\left[\frac{1}{s^2(s+2)}\right]$$

$$G(z) = \frac{z-1}{z} \cdot \frac{25}{2}\,\mathcal{Z}\left[\frac{2}{s^2(s+2)}\right]$$

$T = 0{,}1$
$a = 2$

$$= \frac{(z-1)}{\cancel{z}} \cdot 12{,}5 \cdot \frac{\cancel{z}\left[(0{,}2 - 1 + e^{-0{,}2})z + (1 - e^{-0{,}2} - 0{,}2e^{-0{,}2})\right]}{2(z-1)^2(z-e^{-0{,}2})}$$

$$= \frac{12{,}5\left[0{,}0187z + (0{,}0175)\right]}{2(z-1)(z-0{,}8187)}$$

$$= \frac{12{,}5 \cdot 0{,}0187(z + 0{,}936)}{2(z-1)(z-0{,}8187)}$$

$$= \frac{0{,}1169(z+0{,}936)}{(z-1)(z-0{,}8187)}$$

$$\frac{\theta(z)}{E(z)} = D(z) \cdot G(z) = \frac{(z-0{,}9)}{(z-0{,}98)} \cdot \frac{0{,}1169(z+0{,}936)}{(z-1)(z-0{,}8187)}$$ □

(7)

2.2) $$\frac{\Theta(z)}{R(z)} = \frac{D(z)\,G(z)}{1 + D(z)G(z)}$$

$$= \frac{0{,}1169\,(z-0{,}9)(z+0{,}936)}{(z-1)(z-0{,}98)(z-0{,}8187) + 0{,}1169\,(z-0{,}9)(z+0{,}936)} \quad \square$$

(3)

2.3) $$E(z) = R(z) - \Theta(z)$$
$$= R(z) - D(z)\,G(z)\,E(z)$$

$$E(z) + D(z)\,G(z)\,E(z) = R(z)$$
$$E(z)\,[1 + D(z)G(z)] = R(z)$$

$$E(z) = \frac{R(z)}{1 + D(z)G(z)}$$

$$E(z) = \frac{(z-0{,}98)(z-1)(z-0{,}8187)\,R(z)}{[(z-0{,}98)(z-1)(z-0{,}8187) + 0{,}1169(z-0{,}9)(z+0{,}936)]}$$

For a unit-ramp input

$$\therefore\ R(s) = 1/s^2 \Rightarrow R(z) = \frac{Tz}{(z-1)^2} \qquad T = 0{,}1s$$

$$= \frac{0{,}1\,z}{(z-1)^2}$$

$$\therefore\ E(z) = \frac{(z-0{,}98)(z-0{,}8187)\cdot 0{,}1\cdot z}{(z-1)\,[(z-0{,}98)(z-1)(z-0{,}8187) + 0{,}1169(z-0{,}9)(z+0{,}936)]}$$

$$E(\infty) = \lim_{z \to 1} (z-1)\, E(z) \checkmark$$

$$= \lim_{z \to 1} \frac{(z-0{,}98)(z-0{,}8187)\cdot 0{,}1 \cdot z}{(z-0{,}98)(z-1)(z-0{,}8187) + 0{,}1169(z-0{,}9)(z+0{,}936)}$$

$$= \frac{0{,}0003626}{0 + 0{,}0226} = 0{,}016 \quad \square$$

✓✓ (6)

2.4) Computational delay of the filter : 0,15 s

$\therefore$ $k = 1$ and $\Delta = 0{,}05$ $\Rightarrow$ $m = 0{,}5$ ✓

$$\frac{C(z)}{E(z)} = z^{-1} \cdot D(z) \cdot \underbrace{5 \cdot \mathcal{Z}_m \left[\frac{5(1-e^{-sT})}{s(s)(s+2)} \right]_{m=0,5}}_{G(z)} \checkmark$$

$$G(z) = 5 \, \frac{z-1}{z} \, \mathcal{Z}_m \left[\frac{1}{s^2(s+2)} \right]_{m=0,5} \checkmark$$

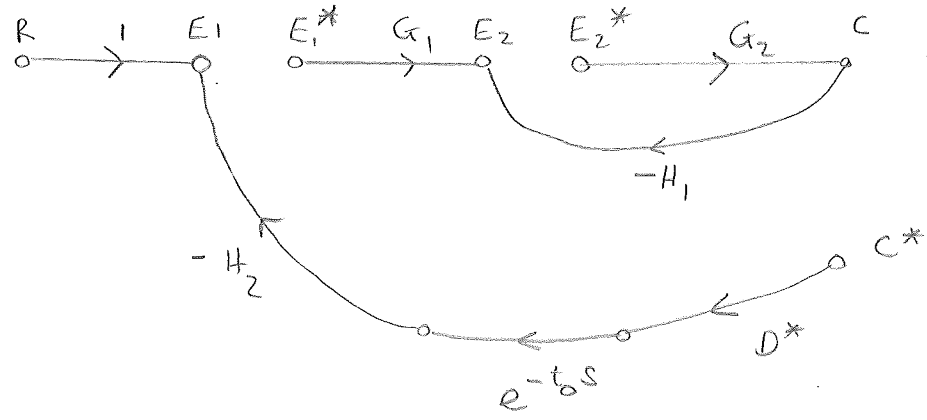
$$= \frac{5}{2} \, \frac{z-1}{z} \, \mathcal{Z}_m \left[\frac{2}{s^2(s+2)} \right]_{m=0,5} \checkmark$$

$$= 2{,}5 \, \frac{z-1}{z} \left[\frac{0{,}1}{(z-1)^2} + \frac{-0{,}9}{2(z-1)} + \frac{e^{-0{,}1}}{2(z-e^{-0{,}2})} \right] \quad \square$$

✓✓ (6)

Question 3 [12]

Step 1 : DRAW ORIGINAL SIGNAL-FLOW:



INPUTS : R, E_1^*, E_2^*, C^*

OUTPUTS : E_1, E_2, C

Step 2 : WRITE OUTPUTS IN TERMS OF INPUTS.

input input

① $E_1 = R - \left(H_2 e^{-t_0 s} D^*\right) C^*$

input outpt → not in ideal form

② $E_2 = G_1 E_1^* - H_1 C$ $\qquad C = G_2 E_2^*$ ← input

input input.

$\therefore \quad E_2 = G_1 E_1^* - H_1 G_2 E_2^*$

input.

③ $C = G_2 E_2^*$

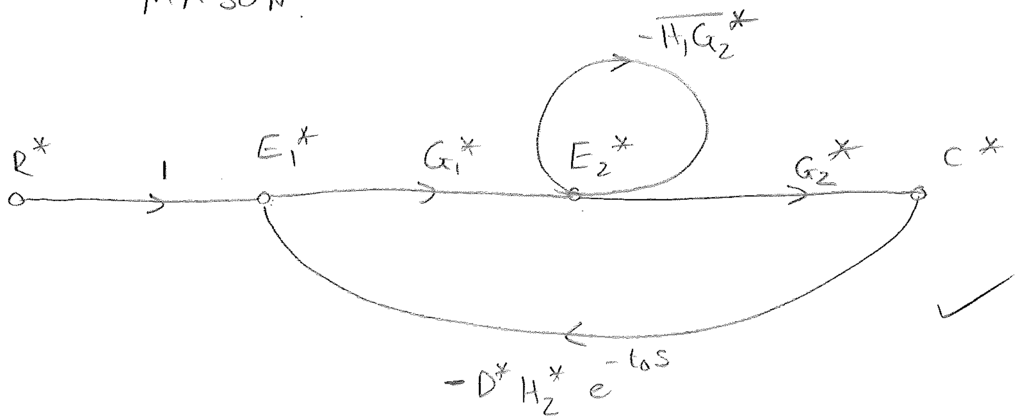
Step 3 : TAKE THE STAR TRANSFORM

① $E_1^* = R^* - H_2^* e^{-t_0 s} D^* C^*$ ✓

② $E_2^* = G_1^* E_1^* - \overline{H_1 G_2}^* E_2^*$ ✓

③ $C^* = G_2^* E_2^*$ ✓

Step 4 : DRAW SAMPLED SIGNAL-FLOW AND USE MASON.



✓

$$\frac{C^*}{R^*} = \frac{G_1^* G_2^*}{1 - \left[- G_1^* G_2^* H_2^* D^* e^{-t_0 s} - \overline{H_1 G_2}^* \right]}$$

$$= \frac{G_1^* G_2^*}{1 + G_1^* G_2^* H_2^* D^* e^{-t_0 s} + \overline{H_1 G_2}^*}$$ ✓

$$\frac{C(z)}{R(z)} = \frac{G_1(z) G_2(z)}{1 + G_1 G_2 H_2(z,m) D(z) + \overline{H_1 G_2}(z)}$$ ✓

✓

□ (12)