Memorandum EERI 418 Semestertoets I
03/03/2011

VRAAG 1

$$G(s) = \frac{10k}{s(s+1)(s+2)}$$

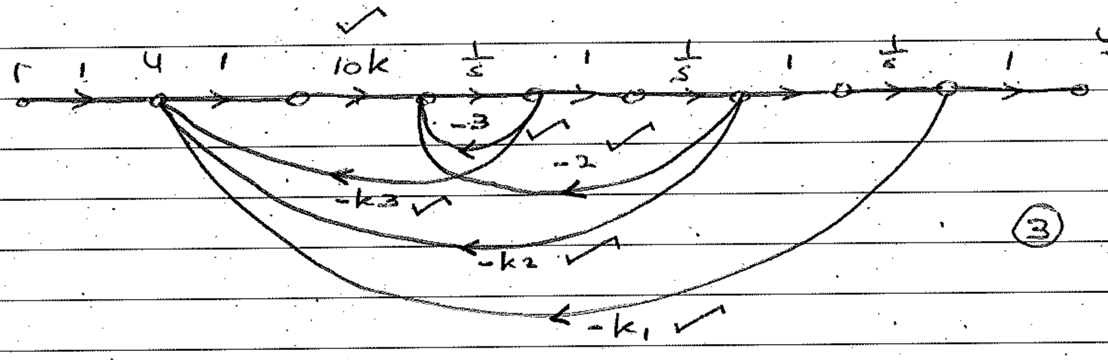
Vir PO $\leq 5\%$ $\zeta \geq 0,7$

$T_s = \frac{4}{\zeta\omega_n} \leq 0,5 \quad \therefore \zeta\omega_n \geq 8 \quad \therefore \omega_n \geq 11,43$ rad/s

kies $\omega_n = 12$ rad/s.

$$G(s) = \frac{10k}{s^3 + 3s^2 + 2s}$$

$$= \frac{10k\,s^{-3}}{1 + 3s^{-1} + 2s^{-2}}$$



(3)

New transfer function:

$$T(s) = \frac{10k/s^3}{1 - \left(-\frac{3}{s} - \frac{k_3 \cdot 10k}{s} - \frac{2}{s^2} - \frac{k_2 \cdot 10k}{s^2} - \frac{k_1 \cdot 10k}{s^3}\right)}$$

$$= \frac{10k}{s^3 + (3 + 10k \cdot k_3)s^2 + (2 + 10kk_2)s + 10kk_1}$$

Vir $e_{ss} = 0 \quad T(0) = 1 \quad \therefore \frac{10k}{10kk_1} = 1 \quad \therefore k_1 = 1$

ITAE polinoom : $s^3 + 1,75\,\omega_n s^2 + 2,15\,\omega_n^2 s + \omega_n^3$

$\therefore 10kk_1 = \omega_n^3 \rightarrow 10k = 12^3 \rightarrow k = 172,8$

$2 + 10kk_2 = 2,15\omega_n^2 \rightarrow 2 + 10kk_2 = 2,15 \cdot 12^2 \rightarrow k_2 = 0,178$

$3 + 10kk_3 = 1,75\omega_n \rightarrow 3 + 10kk_3 = 1,75 \cdot 12 \rightarrow k_3 = 0,0104$

SANDTON
CONVENTION CENTRE

e_{ss} for a ramp $= \frac{1}{k_v}$ since $k_1 = 1$ and $k_v = \lim_{s \to 0} sG'(s)$

$$G'(s) = \frac{10k}{s^3 + (3+10kk_3)s^2 + (2+10kk_2)s}$$ ✓

$$\therefore k_v = \frac{10k}{3+10kk_2} = 5{,}563$$ ✓

$$\therefore e_{ss} = \frac{1}{5{,}563} = 0{,}18$$ ✓

[15]

Vraag 2

2.1 $y(k) = y(k-1) + k_p e(k) - k_p e(k-1) + T k_i e(k-1)$

For $k_p = 10$, $k_i = 2$ and $T = 1$

$$y(k) = y(k-1) + 10e(k) - 10e(k-1) + 2e(k-1)$$
$$= y(k-1) + 10e(k) - 8e(k-1)$$ ✓

$$\therefore Y(z) = z^{-1}Y(z) + 10E(z) - 8z^{-1}E(z)$$ ✓
$$Y(z)[1 - z^{-1}] = E(z)[10 - 8z^{-1}]$$ ✓

$$\therefore \frac{Y(z)}{E(z)} = \frac{10 - 8z^{-1}}{1 - z^{-1}} = \frac{10z - 8}{z - 1}$$ ✓ (4)

2.2 $Y(z) = \frac{10z-8}{z-1} \cdot E(z)$ $\quad E(z) = \frac{z}{z-1}$ ✓

$$= \frac{10z^2 - 8z}{z^2 - 2z + 1}$$ ✓

✓ $10 + 12z^{-1} + 14z^{-2} + 16z^{-3} + 18z^{-4}$

$z^2 - 2z + 1 \overline{)\, 10z^2 - 8z}$ ✓

$$\begin{array}{l}
\underline{10z^2 - 20z + 10} \\
12z - 10 \\
\underline{12z - 24 + 12z^{-1}} \\
14 - 12z^{-1} \\
\underline{14 - 28z^{-1} + 14z^{-2}} \\
16z^{-1} - 14z^{-2} \\
\underline{16z^{-1} - 32z^{-2} + 16z^{-3}} \\
18z^{-2} - 16z^{-3}
\end{array}$$ ✓

$\therefore\ y(0) = 10,\ y(1) = 12,\ y(2) = 14,\ y(3) = 16,\ y(4) = 18$

$\therefore\ y(k) = 10 + 2k$ bonus

(5)

(1) Bonus

2.3 $Y(z) = \frac{10z-8}{z-1} \cdot \frac{z}{z-1}$

$$Q(z) = \frac{Y(z)}{z} = \frac{10z-8}{(z-1)^2} = \frac{A}{z-1} + \frac{Bz}{(z-1)^2} = \frac{8}{z-1} + \frac{2z}{(z-1)^2}$$

$$q(k) = y(k-1) = \mathcal{Z}^{-1}(Q(z)) = \mathcal{Z}^{-1}\underbrace{\left(\frac{8}{z-1}\right)}_{P(z)} + 2k$$

Let $P(z) = \frac{8}{z-1} = \frac{R(z)}{z}$ $\therefore$ $p(k) = r(k-1)$

$\therefore\ R(z) = \frac{8(z)}{z-1}$

and $r(k) = 8u(k)$

$\therefore\ p(k) = 8u(k-1)$

Therefore ~~with~~ $q(k) = 8u(k-1) + 2k$

With $q(k) = y(k-1)$, $y(k) = q(k+1)$

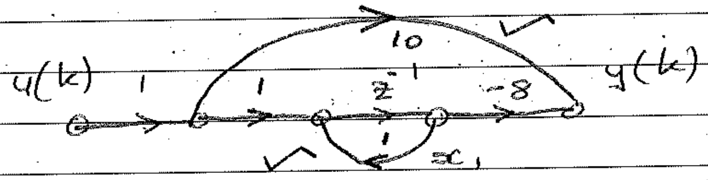
$\therefore\ y(k) = 8u(k) + 2(k+1)$

$= 10u(k) + 2k$ for $k \geq 0$

$\therefore\ y(0) = 10,\ y(1) = 12,\ y(2) = 14,\ y(3) = 16,\ y(4) = 18$

(8)

2.4



$x_1(k+1) = x_1(k) + u(k)$

$y(k) = -8x_1(k) + 10u(k)$

Another option

$x_1(k+1) = x(k) - 8u(k)$

$y(k) = x(k) + 10u(k)$

(3)

[20]

Total 35

EERI 418

SEMESTERTOETS: 09/04/2011

V1 (a) $m(k) = 0,9\,m(k-1) + 0,2\,e(k)$

$\therefore M(z) = 0,9z^{-1}M(z) + 0,2E(z)$

$M(z)[1-0,9z^{-1}] = 0,2\,E(z)$

$\therefore \dfrac{M(z)}{E(z)} = \dfrac{0,2}{1-0,9z^{-1}} = \dfrac{0,2z}{z-0,9} = D(z)$

$$\frac{C(z)}{E(z)} = D(z)\cdot G(z)$$

$$G(z) = \mathcal{Z}\left[\frac{1-e^{-sT}}{s}\cdot\frac{1}{s(s+0,2)}\right]$$

$$= \frac{z-1}{z}\cdot\mathcal{Z}\left[\frac{1}{s^2(s+0,2)}\right]$$

$$= \frac{z-1}{z}\,5\cdot\mathcal{Z}\left[\frac{0,2}{s^2(s+0,2)}\right]$$

$$= \frac{z-1}{z}\cdot 5\cdot\frac{z\left[(0,2-1+e^{-0,2})z+(1-e^{-0,2}-0,2e^{-0,2})\right]}{0,2\,(z-1)^2(z-e^{-0,2})}$$

$$= \frac{z-1}{z}\,25\cdot\frac{z[0,01873z+0,01752]}{(z-1)^2(z-0,8187)}$$

$$= \frac{0,4683(z+0,9354)}{(z-1)(z-0,8187)}$$

$$\therefore \frac{C(z)}{E(z)} = D(z)\cdot G(z)$$

$$= \frac{0,2z}{z-0,9}\cdot\frac{0,4683(z+0,9354)}{(z-1)(z-0,8187)}$$

$$= \frac{0,0937z(z+0,9354)}{(z-0,9)(z-1)(z-0,8187)} \quad (8)$$

(b) DC gain $= \infty$ (1)

(c) $E(s) = \dfrac{2(1-e^{-2s})}{s(s+2)}$ $T = 0{,}5\,s.$

$$= \frac{2(1-e^{-4sT})}{s(s+2)}$$

$$= \mathcal{Z}(1-e^{-4sT}) \cdot \mathcal{Z}\left[\frac{2}{s(s+2)}\right]$$

$$E(z) = (1-z^{-4}) \cdot \frac{z(1-e^{-2T})}{(z-1)(z-e^{-2T})}$$

$$= \frac{z^4-1}{z^4} \quad \frac{z(1-e^{-1})}{(z-1)(z-e^{-1})}$$

$$= \frac{(z^2-1)(z^2+1)\, z \cdot 0{,}632}{z^4 \cdot (z-1)(z-0{,}367)}$$

$$= \frac{(z-1)(z+1)(z^2+1)\, z \cdot 0{,}632}{z^4 \cdot (z-1)(z-0{,}367)}$$

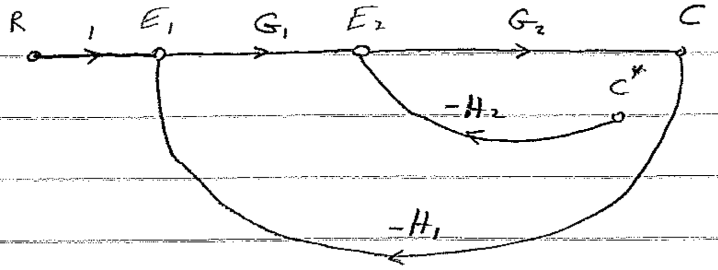
$$= \frac{(z+1)(z^2+1)\, 0{,}632}{z^3 (z-0{,}367)}$$

(6)

[15]

V2

Teken die oorspronklike seinvloeidiagram:



$$C = G_2 E_2$$
$$E_2 = G_1 E_1 - H_2 C^*$$
$$E_1 = R - H_1 C$$
$$C = G_2 . \left[G_1 E_1 - H_2 C^* \right]$$
$$C = G_1 G_2 E_1 - G_2 H_2 C^*$$
$$= G_1 G_2 \left(R - H_1 C \right) - G_2 H_2 C^*$$
$$\therefore \quad C = G_1 G_2 R - G_1 G_2 H_1 C - G_2 H_2 C^*$$
$$\left[1 + G_1 G_2 H_1 \right] C = G_1 G_2 R - G_2 H_2 C^*$$
$$\therefore \quad C = \frac{G_1 G_2 R}{(1 + G_1 G_2 H_1)} - \frac{G_2 H_2}{(1 + G_1 G_2 H_1)} C^*$$
$$\therefore \quad C^* = \left(\frac{G_1 G_2 R}{1 + G_1 G_2 H_1} \right)^* - \left(\frac{G_2 H_2}{1 + G_1 G_2 H_1} \right)^* C^*$$
$$C^* \left[1 + \left(\frac{G_2 H_2}{1 + G_1 G_2 H_1} \right)^* \right] = \left(\frac{G_1 G_2 R}{1 + G_1 G_2 H_1} \right)^*$$
$$\therefore \quad C(z) = \frac{\frac{G_1 G_2 R}{1 + G_1 G_2 H_1}(z)}{1 + \frac{G_2 H_2}{1 + G_1 G_2 H_1}(z)}$$

(10)

V3.

Oordragsfu sonder monsternemer:

$$T(s) = \frac{G(s)}{1+G(s)\,H(s)}$$

$$= \frac{\frac{20}{s(s+3)}}{1+\frac{20}{s(s+3)}\cdot 0{,}5}$$

$$= \frac{20}{s(s+3)+10}$$

$$= \frac{20}{s^2+3s+10}$$

$$q(s) = s^2+3s+10 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$\therefore\ \omega_n^2 = 10 \quad \therefore\ \omega_n = \sqrt{10}$ rad/s

en $2\zeta\omega_n = 3 \quad \therefore\ \zeta = \frac{3}{2\cdot\omega_n} = \frac{3}{2\cdot\sqrt{10}} = 0{,}474$

$\therefore\ \tau = \frac{1}{\zeta\omega_n} = \frac{2}{3} = 0{,}67$ s.

$\therefore\ \omega_d = \omega_n\sqrt{1-\zeta^2} = \sqrt{10}\cdot\sqrt{1-0{,}474^2}$ rad/s $= 2{,}784$ rad/s

$\omega_d = 2\pi f_d = \frac{2\pi}{T_d} = 2{,}784 \quad \therefore\ T_d = \frac{2\pi}{2{,}784}$ s $= 2{,}256$ s.

$\therefore\ T \leq \frac{\tau}{10}$ en $T < \frac{T_d}{10}$

$\therefore\ T < \frac{0{,}67}{10} = 0{,}067$ s en $T < \frac{2{,}256}{10} = 0{,}2256$ s.

$\therefore$ Kies $T = 0{,}05$ s of $f_s = 20$ Hz.

(10)

Semestertoets 3 EERI 418 : 12/05/2011 (1)

MEMO

VRAAG 1

1.1

$$\frac{\Theta(z)}{R(z)} = \frac{k \cdot D(z)\ G(z)}{1 + 0{,}2k\, D(z) G(z)}$$

met $D(z) = 1$ en $G(z) = \frac{0{,}0117 z + 0{,}011}{(z-0{,}95)(z-0{,}86)} = \frac{0{,}0117\,(z+0{,}94)}{(z-0{,}95)(z-0{,}86)}$ (3)

1.2 Jury stabiliteitstoets:

$$Q(z) = 1 + 0{,}2 \cdot k \cdot \frac{0{,}0117 z + 0{,}011}{(z-0{,}95)(z-0{,}86)} = 0$$

$$\therefore (z-0{,}95)(z-0{,}86) + 0{,}2k(0{,}0117z + 0{,}011) = 0$$

$$z^2 - 1{,}81z + 0{,}817 + 0{,}00234kz + 0{,}0022k = 0$$

$$z^2 + (0{,}00234k - 1{,}81)z + 0{,}817 + 0{,}0022k = 0$$

① $Q(1) > 0$ $\quad 0{,}007 + 0{,}00454k > 0$

$\therefore k > -1{,}54$ Vermoed $k > 0$

② $(-1)^n Q(-1) > 0$

$(-1)^2 \cdot [1{,}817 + 1{,}81 - 0{,}00014k] > 0$

$\therefore k < 25907$

③ $|a_0| < a_n$

$|0{,}817 + 0{,}0022k| < 1$

$k < 83{,}2$ (5)

1.3. Frekwensie vir marginale stabiliteit $k = 83{,}2$

$Q(z) = z^2 - 1{,}62z + 1$

$$\therefore z = \frac{1{,}62 \pm \sqrt{1{,}62^2 - 4}}{2} = \frac{1{,}62 \pm j1{,}173}{2}$$

$= 0{,}81 \pm j\,0{,}5865$

$= 1\ \angle \pm 0{,}627$

$\therefore \omega T = 0{,}627$, $T = 0{,}05$ $\therefore \omega = 12{,}54$ rad/s

(4)

1.4. Vir $\zeta = 0{,}707$

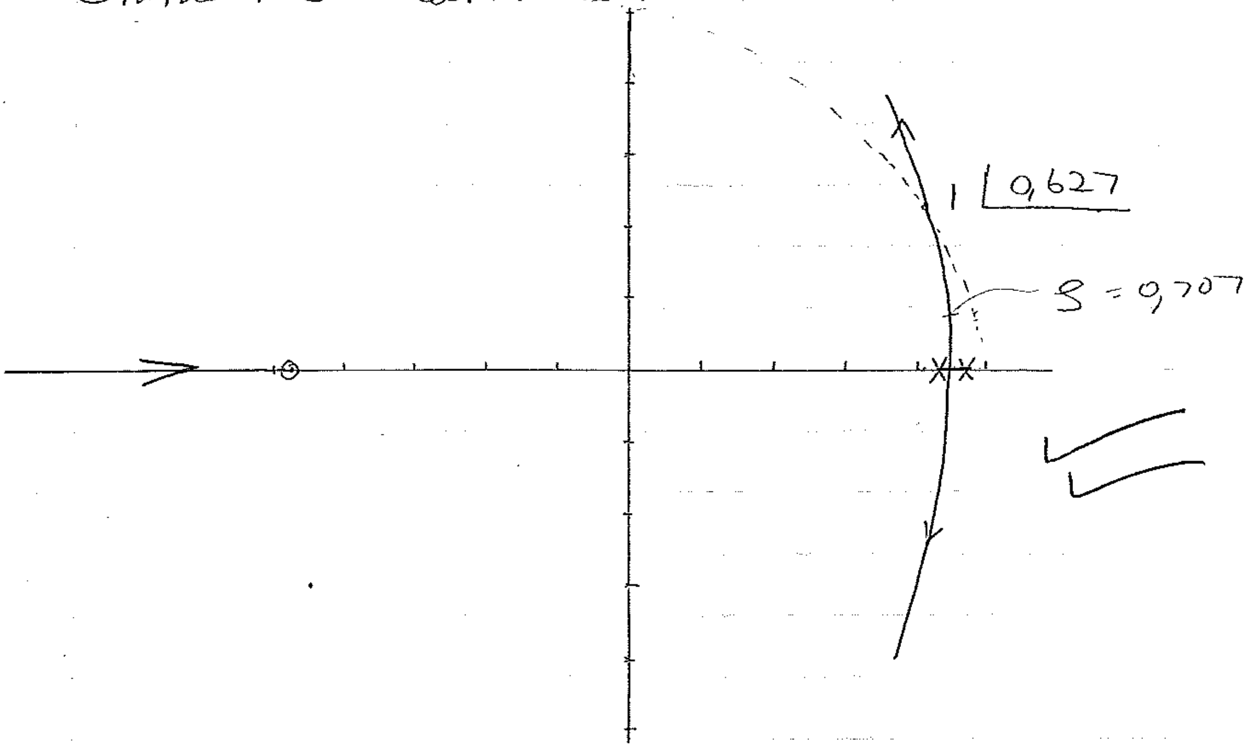
$$z = e^{\sigma T} \angle \sigma T \tan\beta$$

$$\beta = 135^\circ \qquad \tan\beta = -1$$

$$\therefore \quad z = e^{\sigma T} \angle -\sigma T = r\angle\theta$$

Vir watter waardes van K lê die pole op 'n radius van $e^{\sigma T}$ en 'n hoek van $-\sigma T$

Grafies m.b.v. wortellokus:



Wegbreekpunte: $K = \dfrac{-(z-0{,}95)(z-0{,}86)}{(0{,}2)(0{,}0117z+0{,}011)}$

	maks				min				
K	0,465	0,462	0,4203		$2{,}6\times10^{4}$	$1{,}07\times10^{4}$	$3{,}15\times10^{3}$	$3{,}16\times10^{3}$	
z	0,9	0,91	0,89		−1	−1,1	−2,8	−2,9	

$z = 0{,}9 \qquad\qquad z = -2{,}8$

By benadering vir $r = e^{\sigma T} = 0{,}825 \quad \sigma T = -0{,}1335 \quad \theta = 11$

pole by $0{,}91 \pm j\,0{,}178$

$$\therefore K = \frac{0{,}19 \cdot 0{,}19}{(0{,}2)(0{,}0117)(1{,}85)} = 8{,}34.$$

(6)

[18]

VRAAG 2

Fase voorloopnetwerk

$\angle G(j\omega_{w1}) < -180° + \phi_m = -180° + 40° = -140°$

$|G(j\omega_{w1})| < 1$ onthou 10 dB lig. a.g.v. $k = 3$

Kies $\omega_{w1} = 8$ rad/s.

$\therefore G(j\omega_{w1}) = 0{,}32 \angle -180°$

Toets $\cos\theta > |G(j\omega_{w1})|$

$\theta = 180° + 40° - (-180°) = 40°$

$\cos 40° > 0{,}32$

$0{,}766 > 0{,}32$ dus Ok

$$a_1 = \frac{1 - a_0 |G(j\omega_{w1})| \cos\theta}{\omega_{w1} |G(j\omega_{w1})| \sin\theta}$$

$$= \frac{1 - 0{,}32 \cdot \cos 40}{8 \cdot 0{,}32 \cdot \sin 40} = 0{,}4587$$

$$b_1 = \frac{\cos\theta - a_0 |G(j\omega_{w1})|}{\omega_{w1} \sin\theta}$$

$$= \frac{0{,}766 - 0{,}32}{8 \cdot \sin 40} = 0{,}0867$$

$$D(w) = \frac{1 + 0{,}4587 w}{1 + 0{,}0867 w} = \frac{1 + \frac{w}{2{,}18}}{1 + \frac{w}{11{,}53}}$$

$$\therefore k_d = a_0 \left[\frac{\omega_{wp} (\omega_{w0} + \frac{2}{T})}{\omega_{w0} (\omega_{wp} + \frac{2}{T})}\right] = \frac{209{,}61}{60} = 3{,}492$$

$$z_0 = \frac{\frac{2}{T} - \omega_{w0}}{\frac{2}{T} + \omega_{w0}} = \frac{16 - 2{,}18}{16 + 2{,}18} = 0{,}76$$

$$z_p = \frac{16 - 11{,}53}{16 + 11{,}53} = 0{,}1624$$

$$D(z) = 3{,}492 \frac{(z - 0{,}76)}{(z - 0{,}1624)}$$

[12]