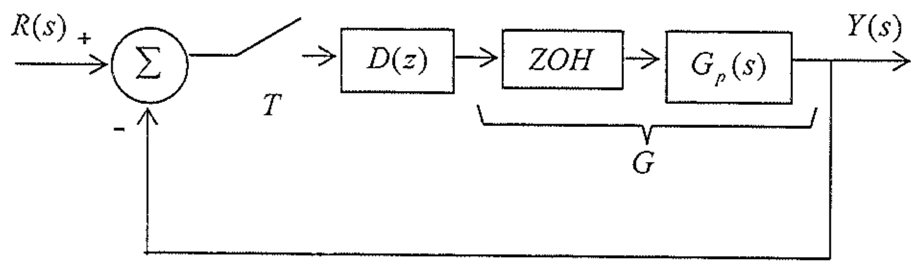
NORTH-WEST UNIVERSITY ®
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOM CAMPUS

Benodigdhede vir hierdie vraestel:			
Multikeusekaarte:	☐	Nie-programmeerbare sakrekenaar:	X
Grafiekpapier:	☐	Draagbare rekenaar:	☐

Oopboek-eksamen:	☐

SEMESTERTOETS / SEMESTER TEST:	3	KWALIFIKASIE/ QUALIFICATION:	**B ING**
MODULEKODE/ MODULE CODE:	**EERI418**	DUUR/ DURATION:	1 ½ UUR / 1 ½ HOURS
MODULE BESKRYWING/ SUBJECT:	BEHEERTEORIE II CONTROL THEORY II	MAKS / MAX:	30
EKSAMINATOR(E)/ EXAMINER(S):	PROF. G VAN SCHOOR	DATUM / DATE:	**16-05-2014**
MODERATOR:	DR. KR UREN	TYD / TIME	**09:10**

VRAAG 1 / *QUESTION 1*



Figuur / *Figure* 1

Die stelsel in figuur 1 het die volgende oordragsfunksie: /
The system in figure 1 has the following transfer function:

$$G_p(s) = \frac{10K}{s(s+4)}$$

Die diskrete oordragsfunksie van die stelsel is soos volg: /
The discrete transfer function of the system is as follows:

$$G(z) = \frac{K(0.06658z + 0.05638)}{z^2 - 1.607z + 0.6065}, \qquad T = 0.125\,s$$

1.1 Konstrueer die benaderde wortellokus in die z-vlak met *D(z)* = 1. /

Construct the approximate root locus in the z-plane with *D(z)* = 1.

Bepaal die waardes van *K* waarvoor die stelsel stabiel sal wees. /

Determine the values of K for which the system will be stable. (10)

Ontwerp nou *D(z)* as 'n fasevoorloop kompensator. Behou krities gedempte pole en verlaag die tydkonstante van die stelsel met 'n faktor 2. /

Design D(z) as a phase lead compensator. Retain critically damped poles and reduce the time constant of the system by a factor 2. (10)

1.2 Figuur 2 toon die bodediagram van *G(jω)* vir *K* = 1. /

Figure 2 shows the bode diagram of G(jω) for K = 1.

Om die bestendige toestand fout te verminder word *K* verhoog na 3. /

To reduce steady state errors the gain K is increased to 3.

Gebruik die gegewe bodediagram om 'n eenheidswins fasenaloopnetwerk *D(z)* te ontwerp wat 'n fasegrens van 40° tot gevolg sal hê. /

Use the given bode diagram to design a unity gain phase lag compensator D(z) that will give a phase margin of 40° *for the system.* (10)

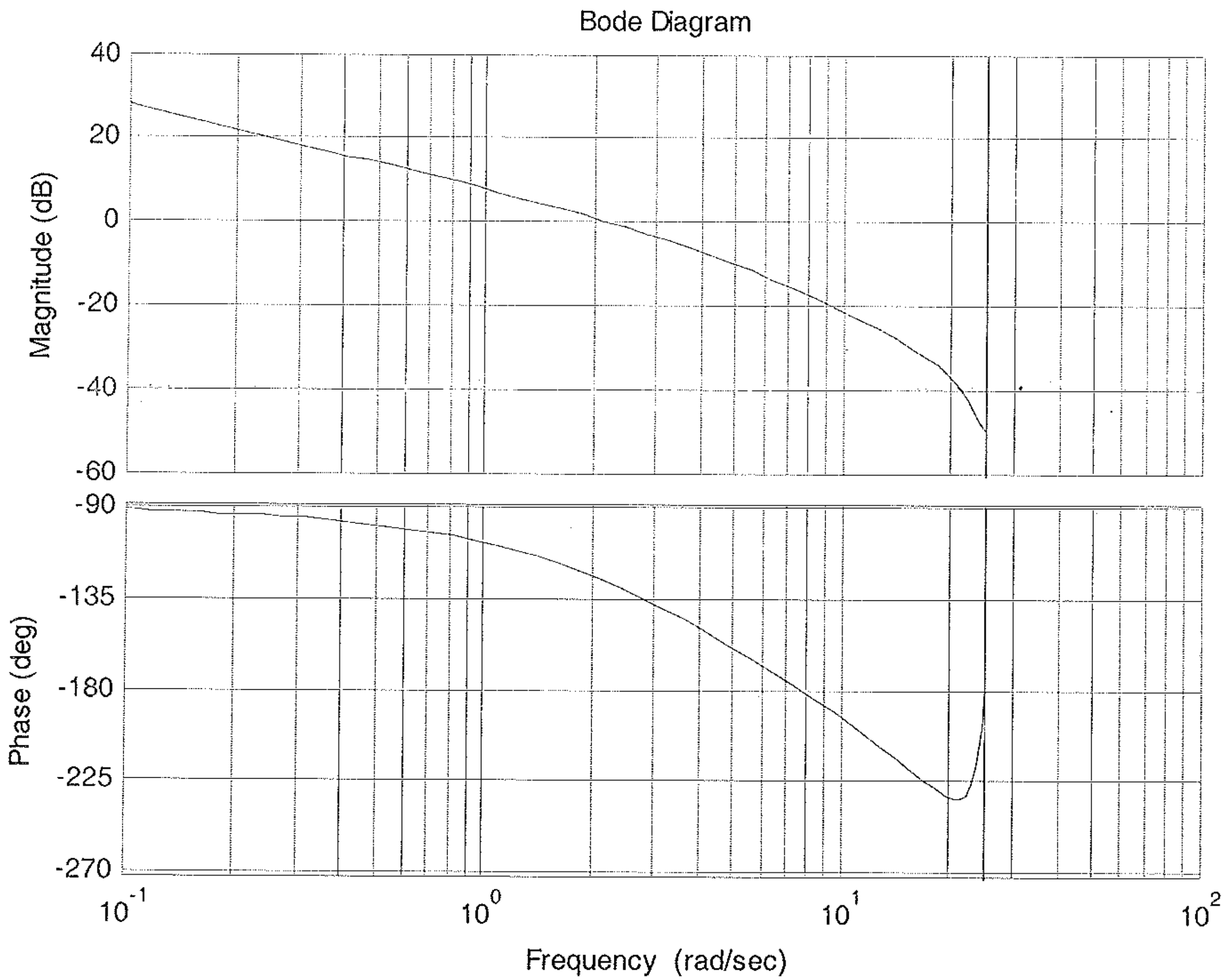
Addisionele inligting / *Additional information:*

$$D(w) = a_0\left[\frac{1+w/(a_0/a_1)}{1+w/(1/b_1)}\right]$$

$$a_1 = \frac{1-a_0|G(j\omega_{w1})|\cos\theta}{\omega_{w1}|G(j\omega_{w1})|\sin\theta},\ b_1 = \frac{\cos\theta - a_0|G(j\omega_{w1})|}{\omega_{w1}\sin\theta}$$

[30]

$$K_d = a_0\left[\frac{\omega_{wp}(\omega_{w0}+2/T)}{\omega_{w0}(\omega_{wp}+2/T)}\right],\ z_0 = \left[\frac{2/T-\omega_{w0}}{2/T+\omega_{w0}}\right],\ z_p = \left[\frac{2/T-\omega_{wp}}{2/T+\omega_{wp}}\right]$$



Figuur / *Figure* 2

TOTAAL/*TOTAL* [30]

TABLE 2-2 PROPERTIES OF THE z-TRANSFORM

Sequence	Transform
$e(k)$	$E(z) = \sum_{k=0}^{\infty} e(k)z^{-k}$
$a_1 e_1(k) + a_2 e_2(k)$	$a_1 E_1(z) + a_2 E_2(z)$
$e(k-n)u(k-n);\quad n \geq 0$	$z^{-n}E(z)$
$e(k+n)u(k);\quad n \geq 1$	$z^n\left[E(z) - \sum_{k=0}^{n-1} e(k)z^{-k}\right]$
$\epsilon^{ak}e(k)$	$E(z\epsilon^{-a})$
$ke(k)$	$-z\dfrac{dE(z)}{dz}$
$e_1(k) * e_2(k)$	$E_1(z)E_2(z)$
$e_1(k) = \sum_{n=0}^{k} e(n)$	$E_1(z) = \dfrac{z}{z-1}E(z)$

Initial value: $e(0) = \lim_{z \to \infty} E(z)$

Final value: $e(\infty) = \lim_{z \to 1} (z-1)E(z)$, if $e(\infty)$ exists

TABLE 2-3 z-TRANSFORMS

Sequence	z-Transform
$\delta(k-n)$	z^{-n}
1	$\dfrac{z}{z-1}$
k	$\dfrac{z}{(z-1)^2}$
k^2	$\dfrac{z(z+1)}{(z-1)^3}$
a^k	$\dfrac{z}{z-a}$
ka^k	$\dfrac{az}{(z-a)^2}$
$\sin ak$	$\dfrac{z \sin a}{z^2 - 2z\cos a + 1}$
$\cos ak$	$\dfrac{z(z - \cos a)}{z^2 - 2z\cos a + 1}$
$a^k \sin bk$	$\dfrac{az \sin b}{z^2 - 2az\cos b + a^2}$
$a^k \cos bk$	$\dfrac{z^2 - az\cos b}{z^2 - 2az\cos b + a^2}$

TABLE A8-1 LAPLACE TRANSFORM PROPERTIES

Name	Theorem
Derivative	$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0^+)$
nth-order derivative	$\mathcal{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^+) - \cdots f^{(n-1)}(0^+)$
Integral	$\mathcal{L}\left[\int_0^t f(\tau)\,d\tau\right] = \frac{F(s)}{s}$
Shifting	$\mathcal{L}[f(t - t_0)u(t - t_0)] = e^{-t_0 s} F(s)$
Initial value	$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$
Final value	$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$
Frequency shift	$\mathcal{L}[e^{-at} f(t)] = F(s + a)$
Convolution integral	$\mathcal{L}^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t - \tau) f_2(\tau)\,d\tau = \int_0^t f_1(\tau) f_2(t - \tau)\,d\tau$

Optimum coefficients of T(s) Based on the ITAE Criterion for a Step Input

$$s + \omega_n$$
$$s^2 + 1.4\omega_n s + \omega_n^2$$
$$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$$
$$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$$
$$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$$
$$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.60\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$$

z-transforms

Laplace transform $E(s)$	Time function $e(t)$	z-Transform $E(z)$	Modified z-transform $E(z,m)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1}+\frac{T}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2}\left[\frac{m^2}{z-1}+\frac{2m+1}{(z-1)^2}+\frac{2}{(z-1)^3}\right]$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{1}{s+a}$	ϵ^{-at}	$\frac{z}{z-\epsilon^{-aT}}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$	$\frac{Tz\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}$	$\frac{T\epsilon^{-amT}[\epsilon^{-aT}+m(z-\epsilon^{-aT})]}{(z-\epsilon^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k\epsilon^{-at}$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-\epsilon^{-at}$	$\frac{z(1-\epsilon^{-aT})}{(z-1)(z-\epsilon^{-aT})}$	$\frac{1}{z-1}-\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1-\epsilon^{-at}}{a}$	$\frac{z[(aT-1+\epsilon^{-aT})z+(1-\epsilon^{-aT}-aT\epsilon^{-aT})]}{a(z-1)^2(z-\epsilon^{-aT})}$	$\frac{T}{(z-1)^2}+\frac{amT-1}{a(z-1)}+\frac{\epsilon^{-amT}}{a(z-\epsilon^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\epsilon^{-at}$	$\frac{z}{z-1}-\frac{z}{z-\epsilon^{-aT}}-\frac{aT\epsilon^{-aT}z}{(z-\epsilon^{-aT})^2}$	$\frac{1}{z-1}-\left[\frac{1+amT}{z-\epsilon^{-aT}}+\frac{aT\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}\right]\epsilon^{-amT}$
$\frac{b-a}{(s+a)(s+b)}$	$\epsilon^{-at}-\epsilon^{-bt}$	$\frac{(\epsilon^{-aT}-\epsilon^{-bT})z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}-\frac{\epsilon^{-bmT}}{z-\epsilon^{-bT}}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z\sin(aT)}{z^2-2z\cos(aT)+1}$	$\frac{z\sin(amT)+\sin(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{s}{s^2+a^2}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2-2z\cos aT+1}$	$\frac{z\cos(amT)-\cos(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\epsilon^{-at}\sin bt$	$\frac{1}{b}\left[\frac{z\epsilon^{-aT}\sin bT}{z^2-2z\epsilon^{-aT}\cos(bT)+\epsilon^{-2aT}}\right]$	$\frac{1}{b}\left[\frac{\epsilon^{-amT}[z\sin bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$\epsilon^{-at}\cos bt$	$\frac{z^2-z\epsilon^{-aT}\cos bT}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$	$\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-\epsilon^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$	$\frac{z(Az+B)}{(z-1)(z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT})}$	$\frac{1}{z-1}$
		$A=1-\epsilon^{-aT}\left(\cos bT+\frac{a}{b}\sin bT\right)$	$-\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
		$B=\epsilon^{-2aT}+\epsilon^{-aT}\left(\frac{a}{b}\sin bT-\cos bT\right)$	$\frac{+\frac{a}{b}\{\epsilon^{-amT}[z\sin bmT-\epsilon^{-aT}\sin(1-m)bT]\}}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}+\frac{\epsilon^{-at}}{a(a-b)}$	$\frac{(Az+B)z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})(z-1)}$	$A=\frac{b(1-\epsilon^{-aT})-a(1-\epsilon^{-bT})}{ab(b-a)}$
	$+\frac{\epsilon^{-bt}}{b(b-a)}$		$B=\frac{a\epsilon^{-aT}(1-\epsilon^{-bT})-b\epsilon^{-bT}(1-\epsilon^{-aT})}{ab(b-a)}$