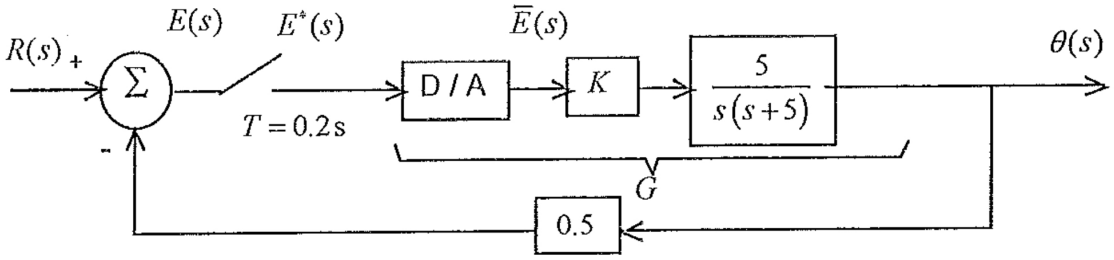

1 Bepaal die stelseloordragsfunksie $\theta(z)/R(z)$ in terme van *G(z). / Determine the system transfer function* $\theta(z)/R(z)$ *in terms of G(z).* (1)

2 Die oordragsfunksie *G(z)* word gegee deur: */ The transfer function G(z) is given by:*

$$G(z)=\frac{K(0.07358z+0.05285)}{z^2-1.368z+0.3679}$$

Bepaal die bestendige toestand fout van die diskrete stelsel vir *K* = 10 vir 'n eenheidshellingsinset. */ Determine the steady state error of the discrete system for K = 10 for a unit ramp input.* (4)

3 Bepaal die demping asook die natuurlike frekwensie van die diskrete stelsel. */ Determine the damping as well as the natural frequency of the discrete system.* (4)

4 Doen die nodige toetse en spreek jou uit oor die sinvolheid van die keuse van die monsterperiode. */ Do the necessary tests and comment on the choice of the sampling period.* (6)

Addisionele inligting / *additional information:*

$$\zeta=\frac{-\ln r}{\sqrt{\ln^2 r+\theta^2}}$$

$$\omega_n=\frac{1}{T}\sqrt{\ln^2 r+\theta^2}$$

$$\tau=\frac{1}{\zeta\omega_n}$$

1 $$\frac{\Theta(z)}{R(z)} = \frac{G(z)}{1 + 0{,}5\,G(z)} \qquad (1)$$

.2 $$E(z) = \frac{R(z)}{1 + 0{,}5\,G(z)}$$ ✓

$$G(z) = \frac{10\,(0{,}07358\,z + 0{,}05285)}{(z-1)(z-0{,}368)}$$

$$R(z) = \frac{Tz}{(z-1)^2}$$

$$E(z) = \frac{T\,z\,(z-1)(z-0{,}368)}{(z-1)^2\left[(z-1)(z-0{,}368) + 0{,}5\cdot 10\,(0{,}07358\,z + 0{,}05285)\right.}$$ ✓

$$e_{ss} = \lim_{z\to 1} (z-1)\,E(z)$$ ✓

$$= \lim_{z\to 1} \frac{Tz\,(z-0{,}368)}{5(0{,}07358\,z + 0{,}05285)}$$

$$= \frac{0{,}2\,(1-0{,}368)}{5(0{,}07358 + 0{,}05285)}$$

$$= 0{,}2.$$ ✓ (4)

3 Damping and Natural freq.

$$Q(z) = z^2 - 1,368z + 0,368 + 0,3679z + 0,2642$$
$$= z^2 - z + 0,6322$$

$$z_{1,2} = 0,5 \pm j0,6182$$

$$= 0,7951 \angle \pm 0,8907 = r \angle \pm\theta \quad \checkmark$$

$$\therefore \zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}}$$

$$\checkmark = \frac{-\ln 0,7951}{\sqrt{(\ln 0,7951)^2 + (0,8907)^2}} = 0,2493 \checkmark$$

$$\omega_n = \frac{1}{T}\sqrt{\ln^2 r + \theta^2} = 4,6 \text{ rad/s} \checkmark \qquad (4)$$

4 $$q(s) = 1 + 0,5\,G(s)$$
$$= 1 + 0,5 \cdot 10 \frac{5}{s(s+5)} = 0$$
$$\therefore s(s+5) + 25 = 0$$
$$s^2 + 5s + 25 = 0 \quad \checkmark$$
$$s^2 + 5s + 25 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\therefore \omega_n = 5 \text{ rad/s} \quad \text{and} \quad \zeta = \frac{5}{2\omega_n} = \frac{5}{2 \cdot 5} = 0,5 \quad \checkmark$$

$$\tau = \frac{1}{\zeta\omega_n} = \frac{1}{0,5 \cdot 5} = 0,4 \text{ s.} \quad \checkmark$$

$$\frac{\tau}{T} > \frac{0,4}{0,2} = 2 \quad \checkmark \text{ (must be at least 5)}$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$
$$= 5\sqrt{1-0,5^2}$$
$$= 4,33 \text{ rad/s.} \quad \checkmark$$

$$\therefore T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{4,33} \text{ s} = 1,45 \text{ s}$$

$$\frac{T_d}{T} = \frac{1,45}{T} = \frac{1,45}{0,2} = 7,25 \quad \text{(Voldoende)}$$

$\therefore$ Sampling period of $\frac{0,4}{5} = 0,08$ s will be better. $\checkmark$ (6)