NORTH-WEST UNIVERSITY
YUNIBESITI YA BOKONE-BOPHIRIMA
NOORDWES-UNIVERSITEIT
POTCHEFSTROOM CAMPUS

Benodigdhede vir hierdie vraestel:			
Multikeusekaarte:	☐	Nie-programmeerbare sakrekenaar:	X
Grafiekpapier:	☐	Draagbare rekenaar:	☐

Oopboek-eksamen: ☐

SEMESTERTOETS / SEMESTER TEST:	2	KWALIFIKASIE/ QUALIFICATION:	**B ING**
MODULEKODE/ MODULE CODE:	**EERI418**	DUUR/ DURATION:	1 ½ UUR / 1 ½ HOURS
MODULE BESKRYWING/ SUBJECT:	BEHEERTEORIE II CONTROL THEORY II	MAKS / MAX:	34
EKSAMINATOR(E)/ EXAMINER(S):	PROF. G VAN SCHOOR	DATUM / DATE:	**15-04-2014**
MODERATOR:	DR. KR UREN	TYD / TIME	**07:30**

VRAAG 1 / *QUESTION 1* [6]

Bepaal die z-transform in geslote vorm van die volgende sein: /

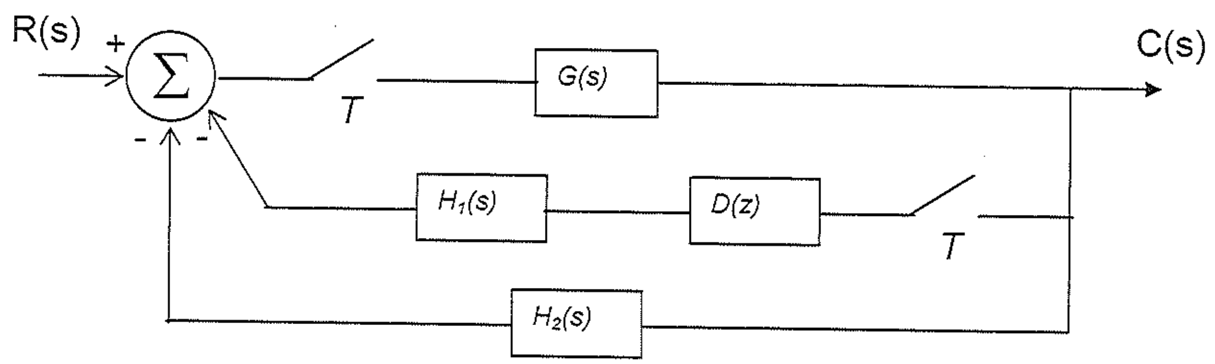
Determine the z-transform, in closed form, of the following signal:

$$E(s) = \frac{2(1-e^{-0.5s})e^{-1.1s}}{s(s+1)}, \qquad T = 0.5\,s$$

VRAAG 2 / *QUESTION 2* [8]

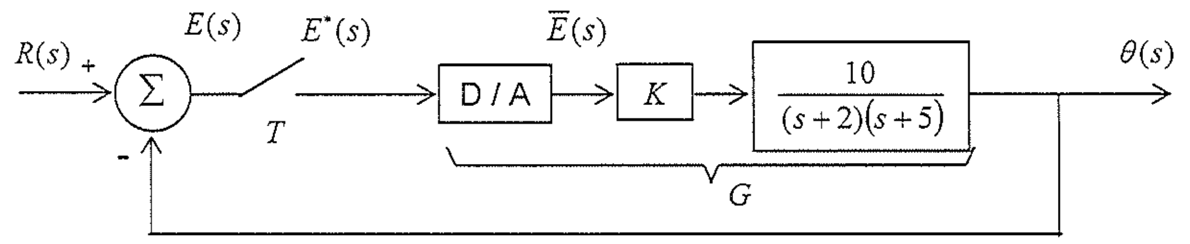
Bepaal die geslotelusoordragsfunksie $(\frac{C(z)}{R(z)})$ vir die stelsel in figuur 1. /

Determine the closed loop transfer function $(\frac{C(z)}{R(z)})$ *for the system in figure 1.*



Figuur / *Figure* 1

VRAAG 3 / *QUESTION 3* [20]



Figuur / *Figure 2*

Beskou die stelsel in figuur 2. / *Consider the system in figure 2.*

3.1 Bepaal die stelseloordragsfunksie ($\frac{\theta(z)}{R(z)}$) in terme van *G(z)*. /

Determine the system transfer function ($\frac{\theta(z)}{R(z)}$) *in terms of G(z).* (1)

3.2 Bepaal die oordragsfunksie vir $K = 20$ en $T = 0.03$ s. Wat is die tipe van die stelsel? /
Determine the transfer function for K = 20 and T = 0.03 s. Find the system type. (5)

3.3 Bepaal die bestendige toestand fout van die diskrete stelsel vir 'n eenheidstrapsinset. /
Determine the steady state error of the discrete system for a unit step input. (4)

3.4 Bepaal die natuurlike frekwensie en die tydkonstante van die diskrete stelsel. /
Determine the natural frequency and the time constant of the discrete system. (5)

3.5 Spreek jou uit oor die sinvollheid van die keuse van die monstertempo. Wat sal die effek van 'n hoër monstertempo op die respons van die stelsel wes. Maak 'n aanbeveling oor die monstertempo wat die diskretiseringsfout sal minimeer, maar nie die modelleringstyd onnodig sal verleng nie. /

Discuss the meaningfulness of the choice of the sampling rate. What will the effect of a higher sampling rate be on the response of the system Make a recommendation on the sampling rate that would minimise the discretisation error without unnecessarily increasing the modelling time. (5)

TOTAAL/*TOTAL* [34]

TABLE 2-2 PROPERTIES OF THE z-TRANSFORM

Sequence	Transform
$e(k)$	$E(z) = \sum_{k=0}^{\infty} e(k)z^{-k}$
$a_1 e_1(k) + a_2 e_2(k)$	$a_1 E_1(z) + a_2 E_2(z)$
$e(k-n)u(k-n);\quad n \geq 0$	$z^{-n}E(z)$
$e(k+n)u(k);\quad n \geq 1$	$z^{n}\left[E(z) - \sum_{k=0}^{n-1} e(k)z^{-k}\right]$
$\epsilon^{ak}e(k)$	$E(z\epsilon^{-a})$
$ke(k)$	$-z\dfrac{dE(z)}{dz}$
$e_1(k) * e_2(k)$	$E_1(z)E_2(z)$
$e_1(k) = \sum_{n=0}^{k} e(n)$	$E_1(z) = \dfrac{z}{z-1}E(z)$

Initial value: $e(0) = \lim_{z \to \infty} E(z)$

Final value: $e(\infty) = \lim_{z \to 1} (z-1)E(z)$, if $e(\infty)$ exists

TABLE 2-3 z-TRANSFORMS

Sequence	z-Transform
$\delta(k-n)$	z^{-n}
1	$\dfrac{z}{z-1}$
k	$\dfrac{z}{(z-1)^2}$
k^2	$\dfrac{z(z+1)}{(z-1)^3}$
a^k	$\dfrac{z}{z-a}$
ka^k	$\dfrac{az}{(z-a)^2}$
$\sin ak$	$\dfrac{z \sin a}{z^2 - 2z\cos a + 1}$
$\cos ak$	$\dfrac{z(z - \cos a)}{z^2 - 2z\cos a + 1}$
$a^k \sin bk$	$\dfrac{az \sin b}{z^2 - 2az\cos b + a^2}$
$a^k \cos bk$	$\dfrac{z^2 - az\cos b}{z^2 - 2az\cos b + a^2}$

TABLE A8-1 LAPLACE TRANSFORM PROPERTIES

Name	Theorem
Derivative	$\mathscr{L}\left[\frac{df}{dt}\right] = sF(s) - f(0^-)$
nth-order derivative	$\mathscr{L}\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^-) - \cdots f^{(n-1)}(0^-)$
Integral	$\mathscr{L}\left[\int_0^t f(\tau)\, d\tau\right] = \frac{F(s)}{s}$
Shifting	$\mathscr{L}[f(t - t_0)u(t - t_0)] = e^{-t_0 s} F(s)$
Initial value	$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$
Final value	$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$
Frequency shift	$\mathscr{L}[e^{-at} f(t)] = F(s + a)$
Convolution integral	$\mathscr{L}^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t - \tau) f_2(\tau)\, d\tau = \int_0^t f_1(\tau) f_2(t - \tau)\, d\tau$

Optimum coefficients of T(s) Based on the ITAE Criterion for a Step Input

$$s + \omega_n$$
$$s^2 + 1.4\omega_n s + \omega_n^2$$
$$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3$$
$$s^4 + 2.1\omega_n s^3 + 3.4\omega_n^2 s^2 + 2.7\omega_n^3 s + \omega_n^4$$
$$s^5 + 2.8\omega_n s^4 + 5.0\omega_n^2 s^3 + 5.5\omega_n^3 s^2 + 3.4\omega_n^4 s + \omega_n^5$$
$$s^6 + 3.25\omega_n s^5 + 6.60\omega_n^2 s^4 + 8.60\omega_n^3 s^3 + 7.45\omega_n^4 s^2 + 3.95\omega_n^5 s + \omega_n^6$$

z-transforms

Laplace transform $E(s)$	Time function $e(t)$	z-Transform $E(z)$	Modified z-transform $E(z,m)$
$\frac{1}{s}$	$u(t)$	$\frac{z}{z-1}$	$\frac{1}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$	$\frac{mT}{z-1}+\frac{T}{(z-1)^2}$
$\frac{1}{s^3}$	$\frac{t^2}{2}$	$\frac{T^2z(z+1)}{2(z-1)^3}$	$\frac{T^2}{2}\left[\frac{m^2}{z-1}+\frac{2m+1}{(z-1)^2}+\frac{2}{(z-1)^3}\right]$
$\frac{(k-1)!}{s^k}$	t^{k-1}	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$\lim_{a\to 0}(-1)^{k-1}\frac{\partial^{k-1}}{\partial a^{k-1}}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{1}{s+a}$	ϵ^{-at}	$\frac{z}{z-\epsilon^{-aT}}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$	$\frac{Tz\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}$	$\frac{T\epsilon^{-amT}[\epsilon^{-aT}+m(z-\epsilon^{-aT})]}{(z-\epsilon^{-aT})^2}$
$\frac{(k-1)!}{(s+a)^k}$	$t^k\epsilon^{-at}$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{z}{z-\epsilon^{-aT}}\right]$	$(-1)^k\frac{\partial^k}{\partial a^k}\left[\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}\right]$
$\frac{a}{s(s+a)}$	$1-\epsilon^{-at}$	$\frac{z(1-\epsilon^{-aT})}{(z-1)(z-\epsilon^{-aT})}$	$\frac{1}{z-1}-\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}$
$\frac{a}{s^2(s+a)}$	$t-\frac{1-\epsilon^{-at}}{a}$	$\frac{z[(aT-1+\epsilon^{-aT})z+(1-\epsilon^{-aT}-aT\epsilon^{-aT})]}{a(z-1)^2(z-\epsilon^{-aT})}$	$\frac{T}{(z-1)^2}+\frac{amT-1}{a(z-1)}+\frac{\epsilon^{-amT}}{a(z-\epsilon^{-aT})}$
$\frac{a^2}{s(s+a)^2}$	$1-(1+at)\epsilon^{-at}$	$\frac{z}{z-1}-\frac{z}{z-\epsilon^{-aT}}-\frac{aT\epsilon^{-aT}z}{(z-\epsilon^{-aT})^2}$	$\frac{1}{z-1}-\left[\frac{1+amT}{z-\epsilon^{-aT}}+\frac{aT\epsilon^{-aT}}{(z-\epsilon^{-aT})^2}\right]\epsilon^{-amT}$
$\frac{b-a}{(s+a)(s+b)}$	$\epsilon^{-at}-\epsilon^{-bt}$	$\frac{(\epsilon^{-aT}-\epsilon^{-bT})z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})}$	$\frac{\epsilon^{-amT}}{z-\epsilon^{-aT}}-\frac{\epsilon^{-bmT}}{z-\epsilon^{-bT}}$
$\frac{a}{s^2+a^2}$	$\sin(at)$	$\frac{z\sin(aT)}{z^2-2z\cos(aT)+1}$	$\frac{z\sin(amT)+\sin(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{s}{s^2+a^2}$	$\cos(at)$	$\frac{z(z-\cos(aT))}{z^2-2z\cos aT+1}$	$\frac{z\cos(amT)-\cos(1-m)aT}{z^2-2z\cos(aT)+1}$
$\frac{1}{(s+a)^2+b^2}$	$\frac{1}{b}\epsilon^{-at}\sin bt$	$\frac{1}{b}\left[\frac{z\epsilon^{-aT}\sin bT}{z^2-2z\epsilon^{-aT}\cos(bT)+\epsilon^{-2aT}}\right]$	$\frac{1}{b}\left[\frac{\epsilon^{-amT}[z\sin bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}\right]$
$\frac{s+a}{(s+a)^2+b^2}$	$\epsilon^{-at}\cos bt$	$\frac{z^2-z\epsilon^{-aT}\cos bT}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$	$\frac{\epsilon^{-amT}[z\cos bmT-\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$1-\epsilon^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$	$\frac{z(Az+B)}{(z-1)(z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT})}$	$\frac{1}{z-1}$
		$A=1-\epsilon^{-aT}\left(\cos bT+\frac{a}{b}\sin bT\right)$	$-\frac{\epsilon^{-amT}[z\cos bmT+\epsilon^{-aT}\sin(1-m)bT]}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
		$B=\epsilon^{-2aT}+\epsilon^{-aT}\left(\frac{a}{b}\sin bT-\cos bT\right)$	$\frac{+\frac{a}{b}\{\epsilon^{-amT}[z\sin bmT-\epsilon^{-aT}\sin(1-m)bT]\}}{z^2-2z\epsilon^{-aT}\cos bT+\epsilon^{-2aT}}$
$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab}+\frac{\epsilon^{-at}}{a(a-b)}$	$\frac{(Az+B)z}{(z-\epsilon^{-aT})(z-\epsilon^{-bT})(z-1)}$	$A=\frac{b(1-\epsilon^{-aT})-a(1-\epsilon^{-bT})}{ab(b-a)}$
	$+\frac{\epsilon^{-bt}}{b(b-a)}$		$B=\frac{a\epsilon^{-aT}(1-\epsilon^{-bT})-b\epsilon^{-bT}(1-\epsilon^{-aT})}{ab(b-a)}$