NORTH-WEST UNIVERSITY
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NOORDWES-UNIVERSITEIT

Benodigdhede vir hierdie vraestel/Requirements for this paper:

Antwoordskrifte/ Answer scripts:	☒	Multikeusekaarte (A5)/ Multi-choice cards (A5):	☐
Presensiestrokies (Invulvraestel)/ Attendance slips (Fill-in paper):	☐	Multikeusekaarte (A4)/ Multi-choice cards (A4):	☐
Rofwerkpapier/ Scrap paper:	☐	Grafiekpapier/ Graph paper:	☐

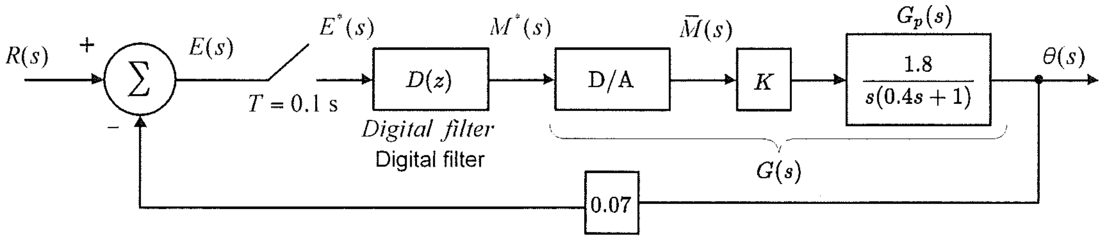
Sakrekenaars / Calculators: **Ja/Yes**
Ander hulpmiddels/ Other resources:

Tipe Assessering/ Type of Assessment:	**Semestertoets 3 Semester test 3**	Kwalifikasie/ Qualification:	**B.ING**
Modulekode/ Module code:	**EERI 418**	Tydsduur/ Duration:	**2 ure/hours**
Module beskrywing/ Module description:	**Beheerteorie II Control theory II**	Maks/ Max:	**35**
Eksaminator(e)/ Examiner(s):	**PROF. K.R. UREN**	Datum/	**16/05/2017**
Moderator(s):	**PROF. G. VAN SCHOOR**	Tyd/Time	**09:00**

Inhandiging van antwoordskrifte/Submission of answer scripts: **Gewoon/Ordinary**

VRAAG 1 / QUESTION 1 [35]

Beskou die digitale beheerstelsel gegee in Figuur 1. In hierdie stelsel, laat $D(z) = 1$. / *Consider the digital control system given in Figure 1. In this system, let $D(z) = 1$.*



Figuur 1/*Figure 1*

1.1 Bepaal die uitdrukking vir $G(z)$. / *Determine the expression for $G(z)$.* (5)

1.2 *Wat is die tipe van die stelsel? / What is the system type?* (1)

1.3 Bepaal die geslote-lus stelsel karakteristieke vergelyking as 'n funksie van die wins K. *Determine the closed-loop system characteristic equation as a function of the gain K.* (2)

1.4 Maak gebruik van die Routh-Hurwitz kriterium om te bepaal vir watter waardes van K die stelsel stabiel sal wees. / *Make use of the Routh-Hurwitz criterion to determine the range of K for stability.* (7)

1.5 Bepaal die frekwensie van ossillasie in die geval van marginale stabiliteit. / *Determine the frequency of oscillation for marginal stability.* (3)

1.6 Bepaal die tydkonstante van die stelsel. Deur die tydkonstante in ag te neem spreek jou uit oor die gekose monsterperiode en maak voorstelle indien nodig. / *Determine the time constant of the system. By considering the time constant, give your opinion of the chosen sampling rate and make recommendations if necessary.* (7)

1.7 Plot die z-vlak wortellokus vir die stelsel en verifieer ook dan jou antwoord in 1.4. / *Plot the z-plane root locus for the system and also verify your answer in 1.4.* (10)

Ekstra inligting: / *Extra info:*

$$
\begin{aligned}
\zeta &= \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}} \\
\omega_n &= \frac{1}{T}\sqrt{\ln^2 r + \theta^2} \\
\tau &= \frac{1}{\zeta\omega_n} = \frac{-T}{\ln r} \\
z &= \frac{1 + (T/2)w}{1 - (T/2)w} \text{ or } w = \frac{2}{T}\frac{z-1}{z+1} \\
\omega_w &= \frac{2}{T}\tan\frac{\omega T}{2}
\end{aligned}
$$

Semestertest 3 - Memo 16/05/2017

1.1 $G(s) = \dfrac{1 - e^{-sT}}{s} \cdot K \cdot \dfrac{1,8}{s(0,4s+1)}$

$G(z) = \dfrac{z-1}{z} \cdot K \; \mathcal{Z}\left[\dfrac{4,5}{s^2(s+2,5)}\right]$

$= \dfrac{z-1}{z} \cdot (1,8K) \; \mathcal{Z}\left[\dfrac{2,5}{s^2(s+2,5)}\right]$

$\mathcal{Z}\left[\dfrac{2,5}{s^2(s+2,5)}\right]$

$= \dfrac{z\left[(aT-1+e^{-aT})z + (1-e^{-aT} - aTe^{-aT})\right]}{a(z-1)^2(z-e^{-aT})}$ $\Big|$ $a = 2,5$, $T = 0,1\ s$

$= \dfrac{z\left[(2,5\cdot0,1 - 1 + e^{-2,5\cdot0,1})z + (1 - e^{-2,5\cdot0,1} - 2,5\cdot0,1\, e^{-2,5\cdot0,1})\right]}{2,5(z-1)^2(z-e^{-2,5\cdot0,1})}$

$= \dfrac{z\left[(-0,75 + 0,7788)z + (1 - 0,7788 - 0,1947)\right]}{2,5(z-1)^2(z-0,7788)}$

$= \dfrac{z\left[0,0288z + 0,0265\right]}{2,5(z-1)^2(z-0,7788)}$

$= \dfrac{z(0,0115)(z+0,9201)}{(z-1)^2(z-0,7788)}$

$G(z) = \dfrac{\cancel{z-1}}{\cancel{z}} \cdot 1.8 \cdot K \cdot \dfrac{\cancel{z}(0,0115)(z+0,9201)}{(z-1)^{\cancel{2}}(z-0,7788)}$ (5)

$= \dfrac{0,0207K(z+0,9201)}{(z-1)(z-0,7788)}$ ∎

1.2 How many $\frac{1}{(z-1)}$'s can be taken out from the forward path.

Forward path $= D(z)\,G(z)$

$$= \frac{1 \cdot 0{,}0207K(z+0{,}9201)}{(z-1)(z-0{,}7788)}$$

$\therefore$ Type $= 1$ ✓ (1)

1.3 $Q(z) = 1 + 0{,}07\,G(z) = 0$

$$1 + \frac{0{,}07 \cdot 0{,}0207K(z+0{,}9201)}{(z-1)(z-0{,}7788)} = 0$$

$$(z-1)(z-0{,}7788) + 0{,}0015Kz + 0{,}0013K = 0$$

$$z^2 - 1{,}7788z + 0{,}7788 + 0{,}0015Kz + 0{,}0013K = 0$$

$$z^2 + (0{,}0015K - 1{,}7788)z + (0{,}7788 + 0{,}0013K) = 0$$

✓✓ (2)

1.4 $z = \dfrac{1 + T/2\,w}{1 + T/2\,w} = \dfrac{1 + 0{,}05w}{1 - 0{,}05w}$ ✓

so $Q(w) = \dfrac{(1+0{,}05w)^2}{(1-0{,}05w)^2} + \dfrac{(0{,}0015K - 1{,}7788)(1+0{,}05w)}{(1-0{,}05w)}$

$+ 0{,}7788 + 0{,}0013K = 0$ ✓

$$\Rightarrow (1+0{,}05w)^2 + (1-0{,}05w)(1+0{,}05w)(0{,}0015K - 1{,}7788)$$
$$+ (0{,}7788 + 0{,}0013K)(1-0{,}05w)^2 = 0$$

(3)

$$Q(w) = (8{,}894\times10^{-3} - 2{,}9\times10^{-7}K)w^2 + (0{,}02212 - 1{,}334\times10^{-4}K)w + 2{,}78\times10^{-3}K = 0$$

$$\begin{array}{c|cc}
w^2 & (8{,}894\times10^{-3} - 2{,}9\times10^{-7}K) & 2{,}78\times10^{-3}K \\
w^1 & (0{,}02212 - 1{,}334\times10^{-4}K) & 0 \\
w^0 & a & b
\end{array}$$

$$a = -\frac{\begin{vmatrix} (8{,}894\times10^{-3} - 2{,}9\times10^{-7}K) & 2{,}78\times10^{-3}K \\ (0{,}02212 - 1{,}334\times10^{-4}K) & 0 \end{vmatrix}}{(0{,}02212 - 1{,}334\times10^{-4}K)}$$

$$= 2{,}78\times10^{-3}K.$$

$\therefore$ (1) $2{,}78\times10^{-3}K > 0$

$K > 0$

(2) $0{,}02212 - 1{,}334\times10^{-4}K > 0$

$K < 165{,}817$

(3) $8{,}894\times10^{-3} - 2{,}9\times10^{-7}K <$

$-2{,}9\times10^{-7}K < -8{,}894\times10^{-3}$

$K < 30668{,}965.$

$0 < K < 165{,}817$

(7)

1.5. $\left(8{,}894 \times 10^{-3} - 2{,}9 \times 10^{-7}(165{,}817)\right)w^2 + 2{,}78 \times 10^{-3}(165{,}817) = 0$

$8{,}8459 \times 10^{-3} w^2 + 0{,}46097 = 0$

$w^2 + 52{,}113 = 0$

$w_w = \pm j\, 7{,}2188$ ✓

$\omega_w = \frac{2}{T} \tan \frac{wT}{2}$ ✓ $\Rightarrow \omega = \frac{2}{T} \tan^{-1}\left(\frac{w_N T}{2}\right)$

$= \cancel{\frac{2}{0{,}1} \tan \frac{T}{}}$

$= \frac{2}{0{,}1} \tan^{-1}\left(\frac{7{,}2 \cdot 0{,}1}{2}\right)$

$= \frac{2}{0{,}1} \tan^{-1}(0{,}36)$

$= 6{,}9$ rad/s. ✓

(3)

1.6. K is not given, so student can choose a value for k.

Choose.

$K = 20$ ✓

$Q(z) = z^2 - 1{,}7498 + 0{,}8055$

$z = 0{,}8749 \pm j\,0{,}2$

$= 0{,}89747 \angle 0{,}2247$ rad ✓

$\tau = -\frac{T}{\ln r} = 0{,}92$ s discrete time constant.

✓

$G(s) = \frac{20\,(1{,}8)}{s(0{,}4s+1)}$ ✓

$2\zeta w_n = 2{,}5$

$Q(s) = 1 + 0{,}07\, G(s) = 0$ $\qquad \tau = 1/\zeta w_n = 0{,}8$ s.

$s^2 + 2{,}5s + 63 = 0$

$T = \frac{0,8}{5} = 0,16\,s$ is a ~~bett~~ well chosen sampling period.

Currently the system is operating at $T = 0,15$, so the sample rate is OK.

(7)

1.7. Open-loop transferfunction

$$G(z)H(z) = 0$$

$$\frac{(0,07) \cdot 0,0207K\,(z+0,920)}{(z-1)(z-0,7788)} = 0$$

$$\Rightarrow \quad \frac{1,45 \times 10^{-3}\,K\,(z+0,920)}{(z-1)(z-0,7788)} = 0$$

$n_p = 2 \qquad n_z = 1$

① Asymptotes: $n_p - n_z = 2 - 1 = 1$

② Asymptote angles: $\phi_A = \frac{(2k+1)\pi}{n_p - n_z}$ $\quad k = 0, \ldots, n_p - n_z - 1 = 0$

$$= \frac{\pi}{1} = 180°.$$

Breakaway points:

$$D(z)\frac{dN(z)}{dz} - N(z)\frac{d(D(z))}{dz} = 0$$

$$z^2 + 1,839z - 2,415 = 0$$

$\therefore$ $z = 0,8862$ and $z = -2,725$.

$$\therefore \quad K = \frac{-(z-1)(z-0,7788)}{1,45\times10^{-3}(z+0,9201)}$$

For $z = 0,8862 \Rightarrow K = 4,67$

$z = -2,725 \Rightarrow K = 4987,058$

Jury:

$$Q(z) = z^2 + (1,45\times10^{-3}K - 1,7788)z + (0,7788 + 1,335\times10^{-3})K$$

z^0 z^1 z^2

$Q(1) > 0$

$K > 0$

$(-1)^n Q(-1) = Q(-1)$

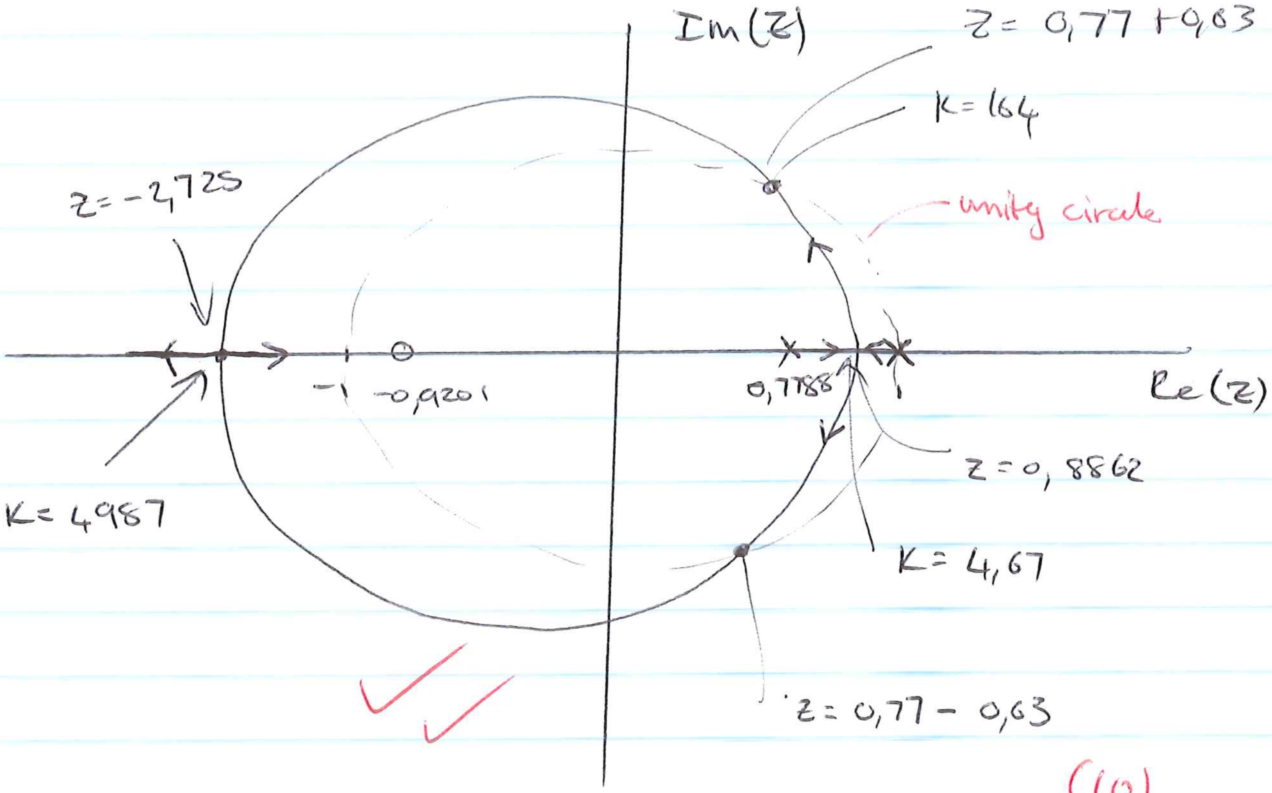
$K < 30935,65$

$|a_0| < a_n$

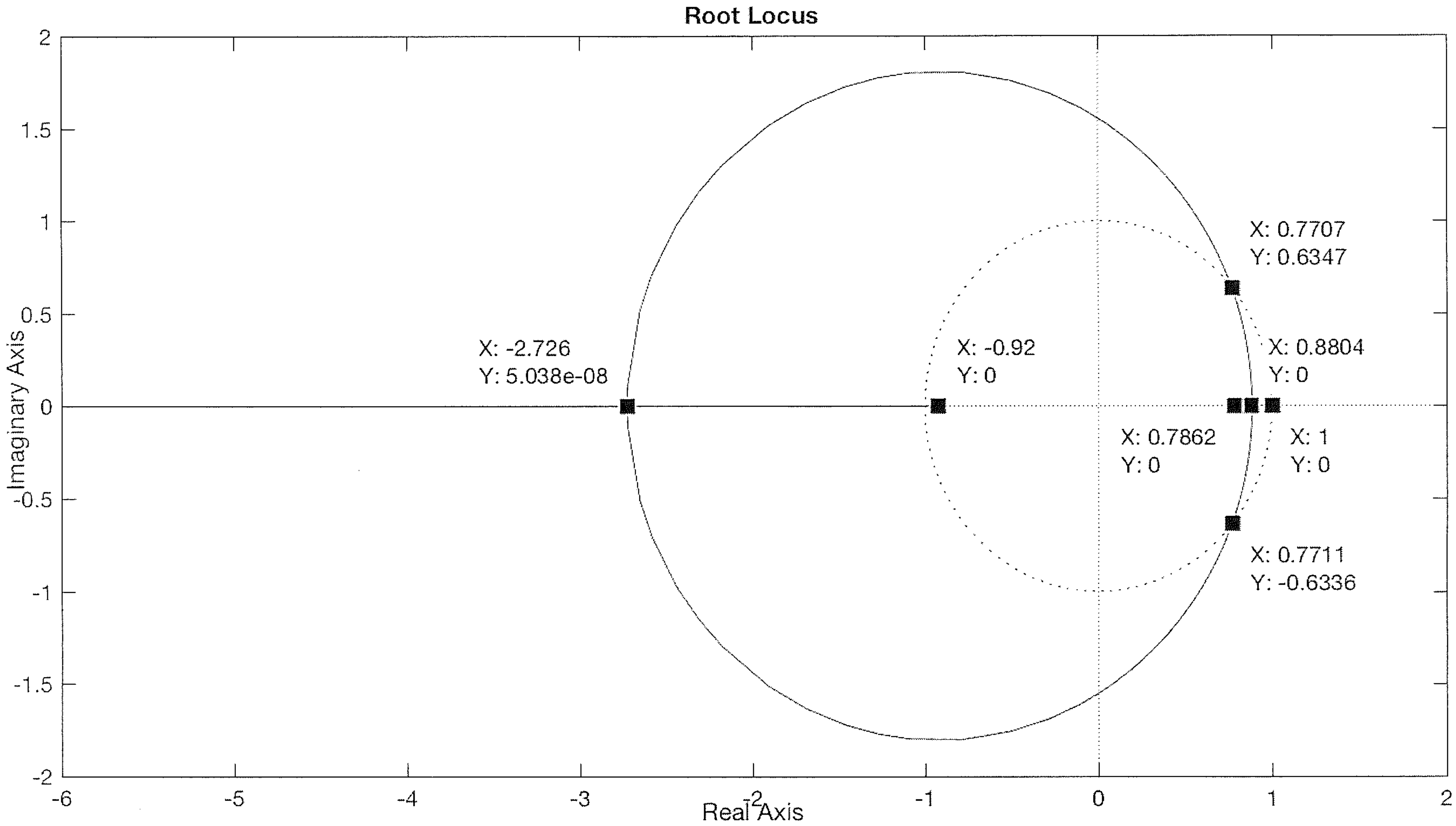
$K < 165,69$

$\therefore$ $0 < K < 165,9$ $\therefore$ checks with Routh-Hurwitz

7


Im(z)
z = 0,77 + 0,63
K = 164
unity circle
z = -2,725
-1
-0,9201
0,7788
1
Re(z)
K = 4,987
z = 0,8862
K = 4,67
z = 0,77 - 0,63

(10)


Root Locus
X: 0.7707
Y: 0.6347
X: -2.726
Y: 5.038e-08
X: -0.92
Y: 0
X: 0.8804
Y: 0
X: 0.7862
Y: 0
X: 1
Y: 0
X: 0.7711
Y: -0.6336
Imaginary Axis
Real Axis