E11.3 - Observability and controllability

A system is described by the matrix equations

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 2 \end{bmatrix} \mathbf{x} + [0]\, u.$$

Determine whether the system is controllable and observable.

For contrallability: $\mathbb{P}_c = [\, \mathbb{B} \quad \mathbb{A}\mathbb{B} \quad \mathbb{A}^2\mathbb{B} \ \dots \ \mathbb{A}^{n-1}\mathbb{B} \,]$ and $\det(\mathbb{P}_c) \neq 0$

$n = 2$: $\mathbb{P}_c = [\, \mathbb{B} \quad \mathbb{A}\mathbb{B} \,]$

$$\mathbb{A}\mathbb{B} = \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} \Rightarrow 2\times 1$$

$$= \begin{bmatrix} (0\times 0) + (1\times 1) \\ (0\times 0) + (-3\times 1) \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\therefore \mathbb{P}_c = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\det \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix} = (0 \times -3) - (1 \times 1) = 0 - 1 = -1 \neq 0 \quad \therefore \text{contrallable}.$$

For observability: $\mathbb{P}_o = \overset{2}{\begin{bmatrix} \mathbb{C} \\ \mathbb{C}\mathbb{A} \\ \vdots \\ \mathbb{C}\mathbb{A}^{n-1} \end{bmatrix}}$ $\det(\mathbb{P}_o) \neq 0$

$n=2$: $\mathbb{C}\mathbb{A} = \overset{1\times 2}{\begin{bmatrix} 0 & 2 \end{bmatrix}} \overset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}} \Rightarrow 1\times 2$

$= \begin{bmatrix} (0\times 0)+(2\times 0) & (0\times 1)+(2\times -3) \end{bmatrix}$

$= \begin{bmatrix} 0 & -6 \end{bmatrix}$

$\mathbb{P}_o = \begin{bmatrix} 0 & 2 \\ 0 & -6 \end{bmatrix}$ $\det \mathbb{P}_o = (0\times -6) - (2\times 0) = 0$

$\therefore$ not observable.

E11.5 - Observability and controllability

A system is described by the matrix equations

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x} + [0]\, u.$$

Determine whether the system is controllable and observable.

Test for controllability:

$n = 2$

$$\mathbb{P}_c = [\ \mathbb{B} \quad \mathbb{A}\,\mathbb{B}\ ]$$

$$= \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$$

$$\mathbb{A}\,\mathbb{B} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} (0 \times 1) + (1 \times -2) \\ (-1 \times 1) + (-2 \times -2) \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ +3 \end{bmatrix}$$

$$\det \mathbb{P}_c = (1 \times 3) - (-2 \times -2)$$

$$= 3 - 4 = -1 \neq 0$$

$\therefore$ controllable

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Observability.

$$\mathbb{P}_G = \begin{bmatrix} \mathbb{C} \\ \mathbb{C}\mathbb{A} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbb{C} \times \mathbb{A} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times 0) + (0\times -1) & (1\times 1) + (0\times -2) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$\det \mathbb{P}_o = (1\times 1) - (0\times 0)$$

$$= 1 \neq 0$$

$\therefore$ observable.