$U - B$ AND $B - V$ COLORS OF BLACK BODIES

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ABSTRACT

Standard U, B, V response-curves are adopted and are used to operate on 10 Planck curves which range in temperature from $T = \infty$ to $T = 3000°$ K. The same response-curves are used to compute the colors of the B star η UMa and the relatively line-free subdwarf HD 140283. The zero points of the system are thus set to give the same colors as the Johnson-Morgan $U-B$, $B-V$. The line which a black-body radiator would define in the $U-B$, $B-V$ diagram is shown. It is computed that for a star of infinite temperature $B-V = -0.46$, $U = -1.37$ mag. would be observed. The approach of real stars to the black-body relation is discussed. The relation of some high-temperature central stars of planetary nebulae, measured by Abel, to the black-body line is shown.

For many purposes it is important to be able to compare real stars with black bodies. Because of the widespread use of the U, B, V photometric system (Johnson and Morgan 1953), it is important to find out exactly how black bodies of various temperatures would behave in the $U - B$, $B - V$ diagram. The relation given by Bonsack *et al.* (1957) can be shown to be incorrect by the following argument: Two-color measures of some real stars fall above that relation. An example is HD 140283. Stars which have neutral hydrogen as their major opacity source will have radiation in their U wave lengths depressed more than in their B wave lengths relative to a black body (Balmer jump). Metallic-line blanketing will also subtract more from the U continuum than from the B. Cooler stars with H^- opacity and negligible Balmer jumps will have even more metallic-line blanketing, which will move the star below the black-body line in the $U - B$, $B - V$ diagram. Therefore, all real stars—excluding special cases such as emission-line objects—should fall below the upper envelope represented by the $U - B$, $B - V$ black-body relation.

A black-body relation given by Lynds (1957) is much better in this respect. Not enough details were given in that paper, however, to indicate actually how his black-body relation was derived. The line derived here falls about 0.05 mag., in $U - B$, above Lynd's line.

Here a technique which imitates the actual observational procedure is used. A set of response-curves used in the definition of the original Johnson-Morgan system are adopted. Black bodies, as well as real stars of known energy distribution, are operated on by these response functions. The computed results on the real stars are matched to the actual observations of these same stars on the U, B, V system. This insures that the colors we compute for black bodies are actually the colors we would measure if we measured black bodies in the same way that we do stars.

1. *Adopted response functions.*—Table 1 give the response functions adopted here as best representing the behavior of an ordinary U, B, V photometer. They originate from the response of a 1P21 photoelectric cell behind the U, B, V filters (Johnson 1955). Code then included the effect of reflections from two aluminized mirrors and extinction through one air mass (see Melbourne 1959).

2. *Colors on the natural photometer system.*—Black-body curves for the temperatures listed in Table 2 were constructed. The response-curves given in Table 1 were then multiplied into the black-body intensities at wave lengths about 250 A apart. The areas under the resultant curves were planimetered. The ratios of these areas were transformed

to a magnitude scale and are listed in the first two columns of Table 2 as color indices on the "natural" photometer system (C_{U-B} and C_{B-V}).

Energy-curves for HD 140283 were taken from spectrum scans by Melbourne (1959) and for η UMa from monochromatic magnitudes (Washburn Observatory, unpublished). The same process was then followed on these stars as on the black bodies.

3. *Outside-the-atmosphere colors.*—The colors obtained so far are just as they would be obtained from a telescope-photometer combination. Following the normal procedure of

TABLE 1

U, B, V RESPONSE FUNCTIONS*

λ	U	B	V
2941	0 000		
3030	0 066		
3125	0 346		
3226	0 777		
3333	1 300		
3448	1 813		
3571	2 178	0 000	
3704	2.330	0 075	
3846	1 426	0 640	
4000	0 197	2 523	
4167	0 000	2.915	
4348		3 006	
4545		2 683	
4762		2 015	0 000
4878		1.64	0 112
5000		1.286	0 993
5128		0 91	2 104
5263		0.505	2 789
5405		0 20	2 808
5556		0 000	2 486
5714			1 990
5882			1 518
6060			0 930
6250			0 473
6452			0 183
6667			0 099
6897			0 049
7143			0 021
7407			0 000

* From Melbourne (1959) for two aluminized reflections and 1 air mass extinction

photoelectric reduction (Arp 1959), these are then compensated for atmospheric extinction by

$$C^0_{U-B} = C_{U-B} - k_{56} \sec z \,, \qquad C^0_{B-V} = C_{B-V} - k_{12} \sec z \,; \tag{1}$$

for $\sec z = 1$ at Mount Wilson and Palomar,

$$\begin{aligned} C^0_{U-B} = C_{U-B} - 0.20 \,, \qquad C^0_{B-V} &= C_{B-V} - (0.110\text{–}0.028) C_{B-V} \,, \\ C^0_{B-V} &= 1.028\, B_{B-V} - 0.110 \,. \end{aligned} \tag{2}$$

This then gives the outside-the-atmosphere colors listed in the fourth and fifth columns of Table 2.

4. *Transforming to the U, B, V system.*—Spectral scans and monochromatic magni-

tudes measure intensity in regions of a star's spectrum which are as free as possible from line absorption. The broad-band U, B, V photometry, however, includes all the absorption lines present in the band pass. Therefore, unless we have unusually line-free stars, the energy-curves we have used here will not be the same as the U, B, V photometer sees.

The subdwarf HD 140283 has, for that reason, been used to set the zero point of the colors computed here. Because the star is so metal-poor, line blanketing is practically negligible. For η UMa, the other check star, the temperature is so hot that few absorption lines appear. The observed colors of the two stars are

$$\text{HD 140283: } B - V = 0.51\,, \qquad U - B = -0.23\,; \tag{3}$$

$$\eta \text{ UMa: } \qquad B - V = -0.21\,,\ U - B = -0.68\,. \tag{4}$$

We do not know what the blanketing-free colors for the latter star are, but Melbourne computes for the first star

$$\Delta(B - V)_M = -0.01\,, \qquad \Delta(U - B)_M = -0.03\,,$$

which gives for HD 140283:

$$(B - V)' = 0.50,\ (U - B)' = -0.26 \text{ (without metallic line blanketing)}\,. \tag{5}$$

It is seen that, if we adopt the transformation,

$$B - V = C^0 + 0.91 \text{ (transformation from adopted response functions to } U, B, V \text{ system)}\,, \tag{6}$$

$$U - B = C^0 - 1.16 \tag{7}$$

TABLE 2*

COMPUTED $U - B$ AND $B - V$ COLORS OF BLACK BODIES AND STARS

T_e (° K)	C_{U-B}	C_{B-V}	C^0_{U-B}	$^0_{B-V}$	$U-B$	$B-V$
Black bodies:						
∞ .	0 03	−1 22	−0 17	−1.37	−1 33	−0 46
25000 .	0 19	−1 00	−0 01	−1 14	−1 17	−0 23
20000. . . .	0 27	−0 94	+0 07	−1 08	−1 09	−0 17
12000	0 52	−0 74	+0 32	−0 87	−0 84	+0 04
8000 . . .	0.84	−0 45	+0.64	−0 57	−0 52	+0.34
6000 . .	1 14	−0 18	+0 94	−0 30	−0 22	+0 61
5000. . .	1 31	−0 02	+1 11	−0 13	−0.05	+0 78
4000 . .	1 81	+0.31	+1 61	+0 31	+0 37	+1 12
3300 .	2.27	+0 62	+2.07	+0 53	+0.83	+1 44
3000	2 58	+0 84	+2 38	+0 75	+1 14	+1 66
Stars:						
HD 140283 .	1 10	−0.30	+0 90	−0 42	−0 26	+0 49
η UMa . . .	0 65	−0 96	+0.45	−1 10	−0 71	−0.19

$\left.\begin{array}{l} B-V=C^0_{B-V}+0.91 \\ U-B=C^0_{U-B}-1.16 \end{array}\right\}$ Empirical transformation, computed colors to observed colors

* Quantities C and C^0 are inside and outside the atmosphere, respectively

we will obtain the observed $B - V$ of η UMa and HD 140283 and the observed (corrected for metallic-line blanketing) $U - B$ of HD 140283 to within 0.01 mag. The predicted and observed $U - B$ colors for η UMa disagree by 0.03 mag. The energy-curve for η UMa, however, included an unknown amount of metallic-line blanketing and an uncertain extrapolation to the ultraviolet.

Even if more energy-curves of stars were available, they would have large and unknown blanketing, and therefore not be generally useful in defining a transformation to the U, B, V system by a number of stars, as is usually done observationally. The transformation here, based primarily on HD 140283, however, seems to be a satisfactory one, and the black-body colors transformed to the U, B, V system should be accurate to within a few hundredths of a magnitude.

Johnson and Morgan (1953) quote a transformation from their original telescope-photometer system:

$$B - V = C_y + 1.04$$

$$U - B = C_u - 1.12, \quad \text{Johnson-Morgan transformation to } U, B, V \text{ from their original "natural" system.}$$

The small differences in color zero points in the transformations are probably due to small differences in minor reflectivity and extinction used between the two systems. For any arbitrary U, B, V system, small differences in filters and 1P21 color sensitivity will also affect the constants of the transformation.

Note on HD 140283.—Roman (1955) listed $U - B = -0.10$ mag. as the observed ultraviolet color index of this star. Melbourne's spectral scans predicted a much bluer $U - B$ color than this, and he asked Abell to remeasure the star. Abell reported $U - B = -0.28$ mag., and this value was published (Melbourne 1959, 1960). Greenstein subsequently asked Abell to recheck this value. The $U - B = -0.28$ mag. value turned out to be a reduction error, and the value should which have been derived was $U - B = -0.14$ mag. The star was low down in the west at this time, and Abell remeasured it, getting a rather uncertain $U - B = -0.19$ mag. Arp got a single measure of $U - B = -0.13$ mag. The value adopted here is $U - B = -0.23$ mag., and the reasons are as follows:

1. Cousins, Eggen, and Stoy (1960) report $U - B = -0.24$ mag. (refractor value in Table X transformed).
2. Cousins, Eggen, and Stoy (1960) report $U - B = -0.23$ mag. from a measure with the 18-inch aluminized Cape reflector.
3. Melbourne's monochromatic magnitudes, taken from his spectral scans, predict $U - B = -0.29$ ($\lambda\lambda$ 3650–4590) and $U - B = -0.18$ ($\lambda\lambda$ 3400–4590). The mean is $U - B = -0.23$ mag. Since this is the energy-curve that is actually operated on in the present paper, it is this $U - B$ color which should be used to fix the zero point in Table 2.
4. When $U - B = -0.23$ mag. is adopted for HD 140283, the other zero-point star, η UMa, has a predicted $U - B$ color within 0.03 mag. of its observed color. This is about the accuracy to be expected and indicates that the position of the black-body line in the $U - B$, $B - V$ diagram is determined here to about that accuracy.

The discordant measures are (1) Roman, $U - B = -0.10$ mag.; (2) Abell, $U - B = -0.14$ mag.; (3) Abell, $U - B = (-0.19)$: mag.; (4) Arp, $U - B = (-0.13)$: mag. This star has been now placed on several observing programs. Careful observations over a few years will probably reveal whether the star is variable (V and $B - V$ measures consistently agree for all observers), or what the cause for the disagreement between measures is.

5. *The black-body relation in the $U - B$, $B - V$ diagram.*—Figure 1 shows the rela-

tion defined by the computed black-body $U - B$, $B - V$ colors given in the last column of Table 2. The curve drawing and integration introduce random errors of the order of ± 0.02 mag. in the colors, and therefore the plotted points have a barely detectable scatter about the mean line drawn through them. For the cooler stars the errors approach 0.1 mag. in U because of the small net U response.

The unreddened $U - B$, $B - V$ relations for luminosity classes I through V (Arp 1958; Johnson 1958) are also drawn in Figure 1. It is apparent that line blanketing and Balmer absorption everywhere depress real stars below the black-body relation. The hottest stars approach the black-body relation most closely in the $U - B$, $B - V$ diagram and have temperatures closely corresponding to black bodies of the same color. (For example, B2 V has $B - V = -0.24$ and $T_e = 22{,}700$; see Arp 1958.) The standard

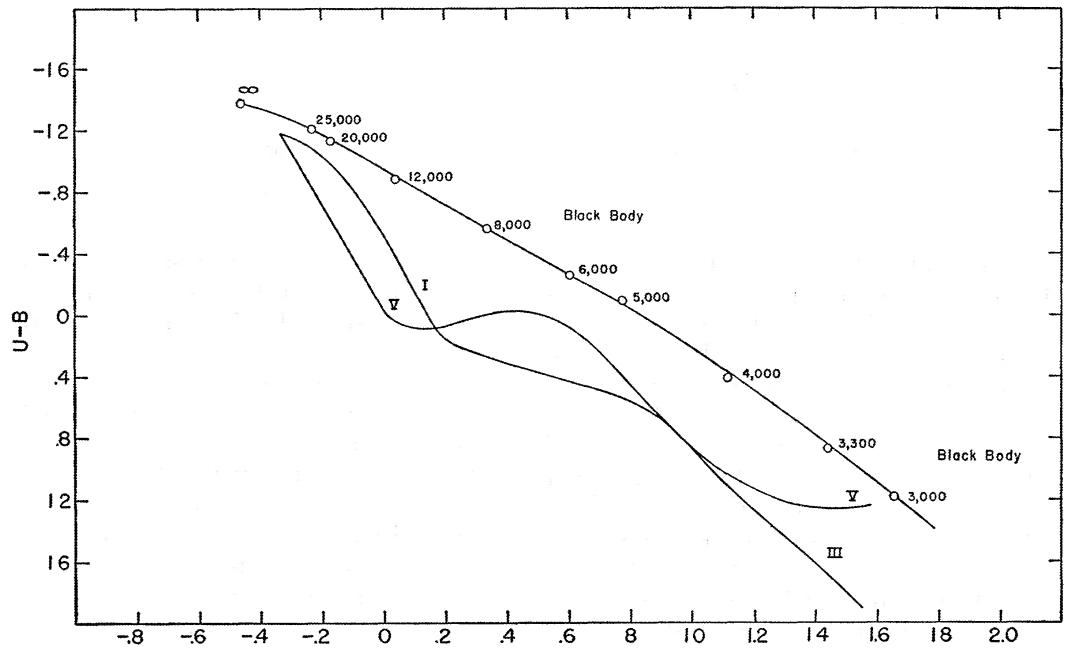

FIG. 1.—The $U - B$, $B - V$ relation for black bodies. The plotted points are taken from the last two columns of Table 2. The standard Johnson relations for stars of luminosity classes I–V are also shown.

relation even at its bluest, however, is about 0.05 mag. deficient from the black-body relation in $U - B$. This apparently represents still some residual Balmer jump and/or line blanketing.

In order to check the black-body $U - B$, $B - V$ relation which has been semi-empirically derived here, we can consider an approximate theoretical treatment such as that given in Russell, Dugan, and Stewart (1938, p. 733) or Aller (1953, p. 168). From Stefan's law we have, for the monochromatic magnitude of a black body of radius R and effective temperature T,

$$M_\lambda = C_\lambda - 5 \log R + \frac{1.561}{\lambda T} + X_\lambda , \tag{8}$$

where X is a correction term of increasing importance as $\lambda T \rightarrow \infty$. If we make the approximation that the broad-band U, B, V photometric system can be approximated by an effective wave length ($\lambda = \lambda_e = \lambda_U, \lambda_B, \lambda_V$, etc.), then, for U: $\lambda_e = 3590$, B: $\lambda_e = 4425$, V: $\lambda_e = 5500$, centroid of area under response-curves, and, using $T_e = 5784°$ K and $M_V = +4.84$ for the sun (Kron and Stebbins 1957), we can evaluate the constant C_V in equation (8), obtaining

$$C_V = -0.06 . \tag{9}$$

Then, using the $B - V = 0.61$, $U - B = -0.26$ for a black body of $T = 6000°$ K as computed in Table 2, we can compute C_B and C_U and obtain

$$B - V = -0.56 + \frac{6900}{T} + X_B - X_V , \tag{10}$$

$$U - B = -1.62 + \frac{8200}{T} + X_U - X_B . \tag{11}$$

These two formulae give a very good representation of the black-body line in Figure 1. Only at very high temperatures and very low temperatures does the real line bend away from the approximation in equations (10) and (11). This occurs because when there is a strong gradient of light-intensity through the broad photometric band the effective wave length is changed. To illustrate this "broad-band" effect at $T = \infty$:

$$\lim\,(X_B - X_V)\,|_{T\to\infty} = 2.5 \log \frac{\lambda_V}{\lambda_B} \text{ from a series expansion of } X \text{ as } T \to \infty ;$$

$$\lim\,(B - V)\,|_{T\to\infty} = -0.56 + 0 + 0.23 = -0.33 \text{ mag. ;}$$

$$\lim\,(U - B)\,|_{T\to\infty} = -1.62 + 0 + 0.23 = -1.39 \text{ mag .}$$

And, although the color index of a black body of infinite temperature is thus predicted more or less correctly, it is not so accurate as the $(B - V)_{T=\infty} = -0.46$ and $(U - B)_{T=\infty} = -1.37$ computed in Table 2.

6. *The relation of real stars to the black-body line.*—The unblanketed position of HD 140283 is shown by the circled cross in Figure 2. It falls only 0.03 mag. below the black-body line in $U - B$. This is consistent with its small Balmer jump and the removal of its small line blanketing. However, Melbourne (1959) fits a model atmosphere of $T_e = 5450°$ K to this star. In the figures it can be seen that HD 140283 falls close to the black-body line at a much higher temperature—around 6700° K. The reason for this star having a color temperature which is much hotter than its effective temperature is found from the vectors in Figure 2.

(*a*) *Gray body.*—In a black body the radiation is emitted from a discrete surface at some specific temperature. In a real star the emergent radiation is composed of light from a number of layers, all at different temperatures. Starting from a black body of $T_e = 5450°$ K, the first step toward a real star, therefore, is to allow for the gray-body approximation (Chandrasekhar 1945). Plotting the gray-body radiation for $T_e = 5450°$ K (Wooley and Stibbs 1953, p. 55) against the black-body radiation-curve for the same temperature enables one to compute that the gray-body curve is broadened to the extent that the observed $U - B$ and $B - V$ colors would appear about 0.04 mag. bluer. This move is shown by the short initial vector in Figure 2.

(The correction to gray-body radiators was also computed for $T_e = 4000°$ K, where $\Delta(B - V) = -0.10$ mag. and $\Delta(U - B) = -0.22$ mag. At $T_e = 20000°$ K, the gray-body and black-body curves are very close and there is negligible correction to the $U - B$, $B - V$ colors.)

(*b*) *Negative hydrogen-ion absorption.*—Making a further step toward a real star, we replace the neutral opacity coefficient of the gray body with the opacity dependence on wave length that is appropriate for the negative hydrogen-ion absorption at a temperature of $\theta = 0.92$ (Aller 1953, p. 190; Wooley and Stibbs 1953, p. 65). We make the assumption that the final effect on the emergent flux can be approximated by reducing the gray-body intensity at each wave length by the proportional absorption of the negative hydrogen. This gives $\Delta(B - V) \approx -0.19$ mag. and $\Delta(U - B) \approx -0.17$ mag. The effect is represented by the large vector in Figure 2. The size of this vector demonstrates that the major cause of the high color temperature of HD 140283 is that the negative hydrogen ion, which is the principal opacity source at this temperature, absorbs more in the red than in the blue and therefore causes the energy of the star to emerge more in the blue-wave-length regions. The end of this second vector in Figure 2 extends the farthest to the blue and should represent approximately a real star with no line absorption whatsoever.

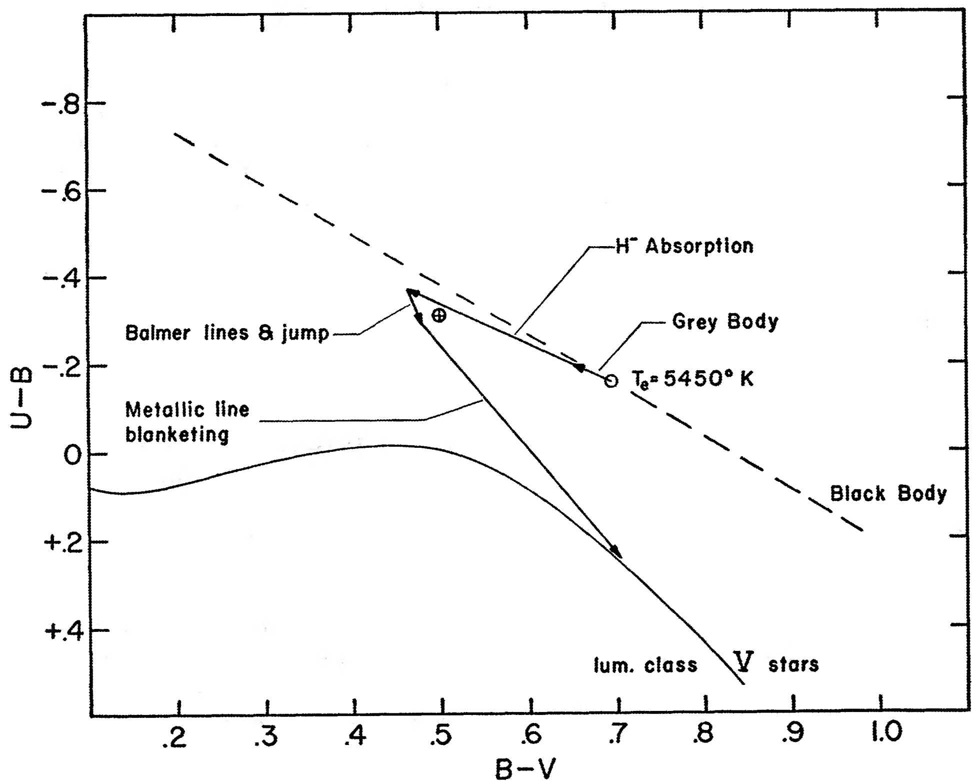

FIG. 2.—The observed position, minus its small amount of line blanketing, of HD 140283 is shown by the circled cross. The open circle shows the position of a black body of the same T_e. The first vector shows the effect of the gray-body correction to the black-body point. The second vector shows the effect of negative hydrogen-ion absorption in place of the neutral gray-body opacity. The third vector is an estimate of the small Balmer jump and Balmer line correction for HD 140283. Finally, the blanketing path, which projects stars of normal metal abundance onto the luminosity class V relation, is shown.

(*c*) *Balmer lines and jump.*—HD 140283 has about 0.1 mag. Balmer jump, as can be seen from inspection of Melbourne's (1959) spectrum scans. It is hard to estimate exactly, but probably a few hundredths of a magnitude are also subtracted from the B-wave-length region by the confluence of the Balmer lines. A correction of roughly $\Delta(U - B) = +0.08$, $\Delta(B - V) = +0.02$ has been guessed for this. This is additional to the hydrogen and metallic-line blanketing corrected for by Melbourne. The third small vector represents the estimated effect of this Balmer absorption. The end of this vector should then be close to the observed position of HD 140283. Figure 2 shows that, indeed, HD 140283 is observed to be within a few hundredths of a magnitude of this point, which is satisfying agreement in view of the approximate nature of the calculation.

(*d*) *Metallic-line blanketing.*—The further effect in stars of normal chemical composition is for the metallic-line blanketing to redden the star along the final large vector shown in Figure 2. The slope of this vector is from Melbourne (1959) and Eggen and Sandage (1959). If the star is of normal metal abundance, the final vector will terminate on the observed $U - B$, $B - V$ relation for luminosity class V stars.

Therefore, middle-type main-sequence stars of normal metal abundance have roughly the same $B - V$ color indices as do black bodies of the same effective temperature. But this is a fortuitous addition of four different effects. In different temperature ranges and different chemical compositions, the color temperatures and effective temperatures can differ by more than 1000° K.

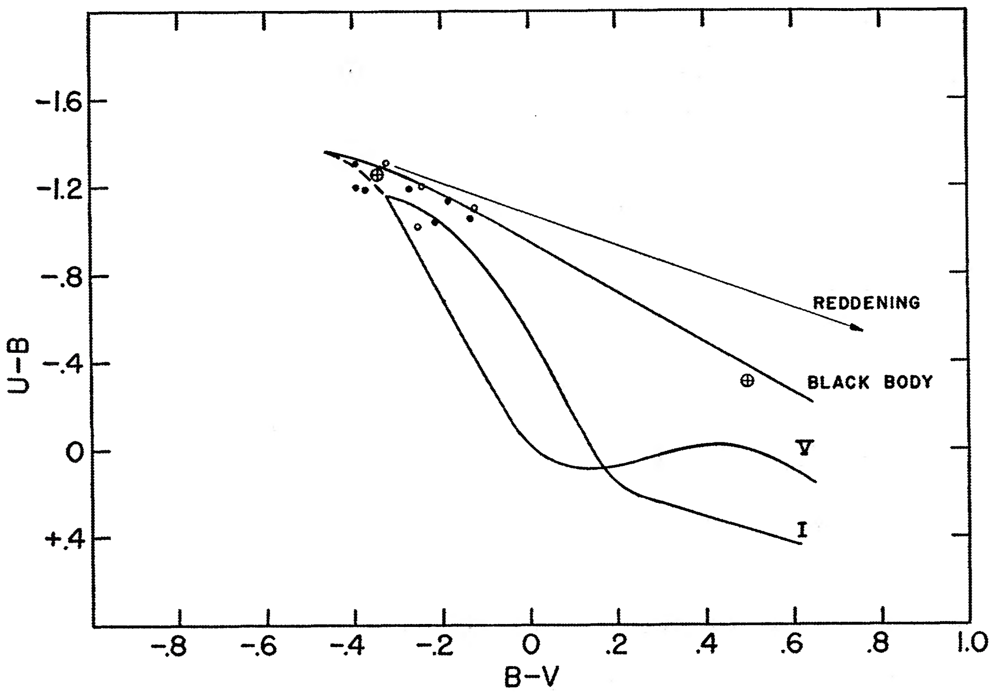

FIG. 3.—Stars that have $U - B$, $B - V$ colors near those of black bodies. The small open circles are Abel's measures of central stars of planetary nebulae, which he computes to have minimum temperatures of over 100000° K. The small filled circles are the same kind of objects with lower computed minimum temperatures. The circled cross on the right shows the unblanketed position of HD 140283. The circled cross on the left shows the position of the bluest Johnson-Morgan standard, BD+28°4211.

Only when we go to very hot stars, do we encounter real stars which approach black-body radiators. There, as we have seen, as we go to higher temperatures, the gray-body and black-body colors approach each other. The opacity source changes to neutral hydrogen and finally to electron scattering, which is independent of wave length. The Balmer jump eventually disappears for very hot stars and metallic-line blanketing also finally becomes inappreciable.

An example of a real star which fulfils these last conditions is BD+28°4211: $B - V = -0.34$, $U - B = -1.26$. This star falls 0.00 mag. deficient in $U - B$ from the black-body line and is, therefore, extremely close to a black body in two-color behavior. Oke (unpublished) states that spectrum scans of this star indicate negligible Balmer jump and only traces of line absorption. This description would check closely with the zero deviation from the black-body relation observed here.

NOTE.—The redistribution of radiation absorbed by the negative hydrogen ion and metallic-line opacity has not been taken into account in the treatment made here. The effect is not large, it probably serves only to raise the temperature a few hundred degrees (see Sandage and Eggen 1959, for the case of metallic-line blanketing). Systematic effects arising from the consideration of opacity sources separately and then cumulatively are not serious, so long as only one opacity source is dominant. Of course, the overriding point is that the present discussion is only a qualitative treatment, indicating the approximate direction which these various sources of radiation absorption move a star in the $U - B$, $B - V$ diagram.

7. *Nuclei of planetary nebulae.*—Two-color measures by Abell (unpublished) of the central stars of some planetary nebulae (only planetaries with thin nebulosity) are plotted in Figure 3. Abell computes very high minimum temperatures for these objects. It is seen that they do indeed fall into the narrow region between real stars and black bodies which converges toward high temperature. Given some probable reddening, Abell's $U - B$, $B - V$ measures confirm his computed high temperatures. The group of planetary nuclei plotted here are all those occurring in galactic longitudes 200°–300°. In the longitude range from 20° to 80° the points fall to the right of the reddening line in Figure 3 (not plotted). If the latter energy-curves are well-behaved, a very unusual reddening line would be needed to explain their position in the diagram. This is a puzzling result which is unexplained at present.

Thanks are due George Preston, who pointed out the effect of H^- absorption, and to Kraft and Preston for discussion of gray bodies and opacity sources. Thanks are also due Frank Breuckel for the computations of the black-body colors.

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