ASSIGNMENT 1

The Diet Problem

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Problem Setup

The Diet Problem, as explained in the assignment description, is a classic and easy introduction into the application of linear programming and optimization in the real world. Finding ways to minimize the cost of your own diet based on 7 given nutritional constraints is of great importance to anybody looking to prioritize their food intake. The 7 constraints that we will be using include sodium, energy, protein, vitamin D, calcium, iron, and potassium, each with their respective daily allowances[1]. For this problem, we will multiply each by 7 to create a weekly diet.

I chose to go the route of unprepared foods, as I tend to cook more often than not in my own life. To find costs and nutritional information for foods, I used Amazon Fresh, my main source for groceries. Costs of foods are ever changing in an inflationary environment, so I used the current costs as of 4/4/2024. The five foods that I chose which should allow me to meet all constraints are chicken breast, salmon, white rice, milk, and spinach[2]. The construction of each meal is not as important to the problem, but in theory, I would have two meals a day with each including a separate protein.

Knowing the nutritional information and costs of each of the foods, we can begin to set up our linear programming model. This is a minimization problem where we are looking to minimize the cost per week that we must spend to meet the nutritional constraints that we are provided. In standard form, the problem can be visualized as follows:

Minimize: $Z = 0.74a + 2.83b + 0.09c + 0.79d + 1.00e$

Subject to:

$75a + 65b + 10c + 135d + 65e \leq 35000,$

$-110a - 240b - 160c - 130d - 25e \leq -14000,$

$-24a - 23b - 3c - 8d - 2e \leq -350,$

$-0.1a - 19b - 2.5d \leq -140,$

$-10a - 10b - 320d - 80e \leq -9100,$

$-0.4a - 0.4b - 2c - 2.3e \leq -126,$

$-410a - 410b - 420d - 470e \leq -32900,$

$a, b, c, d, e \geq 0$

where a, b, c, d, and e are Chicken Breast, Salmon, White Rice, Milk, and Spinach respectively.

Results and Problem Alterations

Using Python and PuLP, we are able to easily solve our L.P. model. We find that, with the current restraints, we achieve a minimum cost of $66.38, where our diet includes 23.86 servings of chicken breast, 0 servings of salmon, 58.23 servings of white rice, 55.05 servings of milk, and 0 servings of spinach. Again, these amounts are weekly, so spreading the servings out over the course of 7 days is necessary.

Obviously, this is not an ideal diet, as two of the five food options are excluded from our optimal solution. Knowing this, we can alter the model to ensure that each food is included. Based on the structure of the problem, we can include the constraint that we must have at least 7 servings of each food. In theory, this allows us to consume each food at least once every day. When we make this change, we find that the new optimal solution is a minimum cost of $79.99 weekly, with 39.37 servings of chicken breast, 7 servings of salmon, 45.68 servings of white rice, 25.24 servings of milk, and 7 servings of spinach. This change increases our minimum weekly cost by $13.61 but ensures that we have a wider variety of food options available each day.

Other manners in which we can alter this problem to ensure more variety include variable constraints for different foods (i.e. 21 servings of spinach as opposed to 7 servings of salmon) or including new constraints, like carbohydrates, fats, fibers, etc. A healthy diet is generally more complex that these basic 7 constraints, so there are plenty of changes that can be made to cater the problem to the individual who is trying to maximize their own diet.

Large Language Models and Optimization

Large Language Models are powerful tools that can aid us in this kind of problem. Using ChatGPT, I can give a basic prompt that returns an in-depth analysis of the problem, including nutritional facts, costs, the L.P. model, and code that would allow me to alter the problem. This prompt included the constraints, optimization goals, and framework of the problem. I gave the LLM the flexibility to choose the foods.

The LLM struggled when I gave it so much flexibility. It was unable to find a feasible solution based on the nutritional information and foods it was choosing. When I provided the tool with my own L.P. model and asked it to create Python code that would allow me to find an optimal solution, it was able to do so.

LLMs have the capability and more to solve these types of problems, but there are some limitations. For example, since these LLMs don’t have access to live web searches, they are limited in its knowledge of more complex packaged foods and costs. Additionally, since it is only programmed based on its historical data set, it cannot adapt to changes in costs. Finally, the user must be ultra specific in the prompts in order for the LLM to be able to solve the problem. It struggles with having freedom, so restricting what it does from step to step is key. However, in conjunction with human capabilities and guidance, it can be an invaluable tool in decision analytics.

Appendix

[1] Nutritional Constraints

Component	Max/Min	Daily Amount and measure
Sodium	Maximum	5,000 milligrams (mg)
Energy	Minimum	2,000 Calories (kilocalories, kcal)
Protein	Minimum	50 grams (g)
Vitamin D	Minimum	20 micrograms (mcg)
Calcium	Minimum	1,300 milligrams (mg)
Iron	Minimum	18 milligrams (mg)
Potassium	Minimum	4,700 milligrams (mg)

[2] Nutritional Facts

Chicken Breast: $14.79 / 20 Servings = $0.74 per serving

Nutrition Facts

About 20 servings per container

Serving size **4 oz (112g)**

Amount per serving

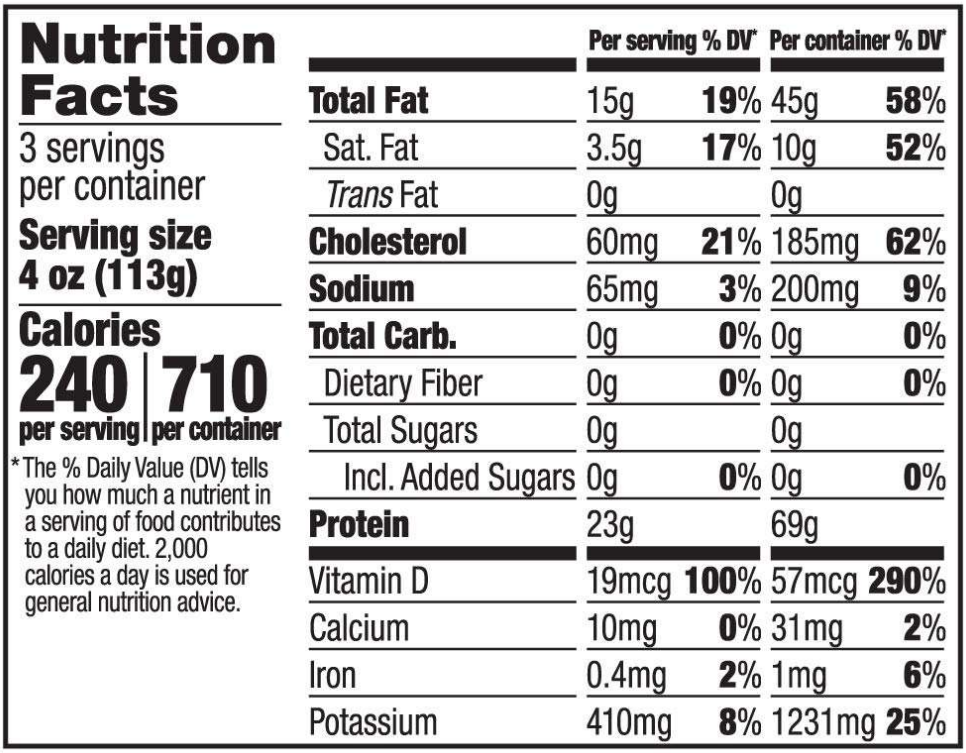
Calories **110**

	% Daily Value*
Total Fat 1g	**1%**
Saturated Fat 0g	**1%**
Trans Fat 0g	
Cholesterol 70mg	**24%**
Sodium 75mg	**3%**
Total Carbohydrate 0g	**0%**
Dietary Fiber 0g	**0%**
Total Sugars 0g	
Includes 0g Added Sugars	**0%**
Protein 24g	

Vit. D 0.1mcg 0%	Calcium 10mg 0%
Iron 0.4mg 2%	Potas. 410mg 8%

* The % Daily Value tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

Salmon: $8.49 / 3 Servings = $2.83 per serving



Nutrition Facts

3 servings per container

Serving size 4 oz (113g)

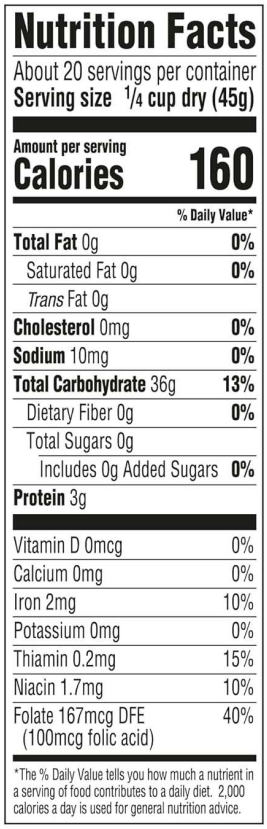
Calories

240 per serving | **710** per container

*The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

	Per serving	% DV*	Per container	% DV*
Total Fat	15g	**19%**	45g	**58%**
Sat. Fat	3.5g	**17%**	10g	**52%**
Trans Fat	0g		0g	
Cholesterol	60mg	**21%**	185mg	**62%**
Sodium	65mg	**3%**	200mg	**9%**
Total Carb.	0g	**0%**	0g	**0%**
Dietary Fiber	0g	**0%**	0g	**0%**
Total Sugars	0g		0g	
Incl. Added Sugars	0g	**0%**	0g	**0%**
Protein	23g		69g	
Vitamin D	19mcg	**100%**	57mcg	**290%**
Calcium	10mg	**0%**	31mg	**2%**
Iron	0.4mg	**2%**	1mg	**6%**
Potassium	410mg	**8%**	1231mg	**25%**

White Rice: $1.79 / 20 Servings = $0.09 per serving



Nutrition Facts

About 20 servings per container
Serving size 1/4 cup dry (45g)

Amount per serving
Calories **160**

	% Daily Value*
Total Fat 0g	**0%**
Saturated Fat 0g	**0%**
Trans Fat 0g	
Cholesterol 0mg	**0%**
Sodium 10mg	**0%**
Total Carbohydrate 36g	**13%**
Dietary Fiber 0g	**0%**
Total Sugars 0g	
Includes 0g Added Sugars	**0%**
Protein 3g	
Vitamin D 0mcg	0%
Calcium 0mg	0%
Iron 2mg	10%
Potassium 0mg	0%
Thiamin 0.2mg	15%
Niacin 1.7mg	10%
Folate 167mcg DFE (100mcg folic acid)	40%

*The % Daily Value tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

Milk: $6.29 / 8 Servings = $0.79 per serving

Nutrition Facts

About 8 servings per container

Serving size 1 cup (240mL)

Amount per serving

Calories 130

	% Daily Value*
Total Fat 5g	**6%**
Saturated Fat 3g	**15%**
Trans Fat 0g	
Polyunsaturated Fat 0g	
Monounsaturated Fat 1.5g	
Cholesterol 20mg	**7%**
Sodium 135mg	**6%**
Total Carbohydrate 13g	**5%**
Dietary Fiber 0g	**0%**
Total Sugars 12g	
Includes 0g Added Sugars	**0%**
Protein 8g	**16%**

Vitamin D 2.5mcg 15% • Calcium 320mg 25%

Iron 0mg 0% • Potassium 420mg 8%

Vitamin A 150mcg 15% • Riboflavin 0.4mg 30%

Vitamin B12 1.3mcg 50% • Phosphorus 240mg 20%

*The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

Spinach: $2.99 / 3 Servings = $1.00 per serving

Spinach

Nutrition Facts

3 servings per container

Serving size	**2 Cups (85g)**
Amount per serving	
Calories	**25**
	% Daily Value*
Total Fat 0g	**0%**
Saturated Fat 0g	**0%**
Trans Fat 0g	
Cholesterol 0mg	**0%**
Sodium 65mg	**3%**
Total Carbohydrate 3g	**1%**
Dietary Fiber 2g	**7%**
Total Sugars 0g	
Includes 0g Added Sugars	**0%**
Protein 2g	
Vitamin D 0mcg	0%
Calcium 80mg	6%
Iron 2.3mg	15%
Potassium 470mg	10%
Vitamin A 400mcg	45%
Vitamin C 24mcg	25%

*The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.