ASSIGNMENT 2

Network Models – Project Management

Justin Doyle

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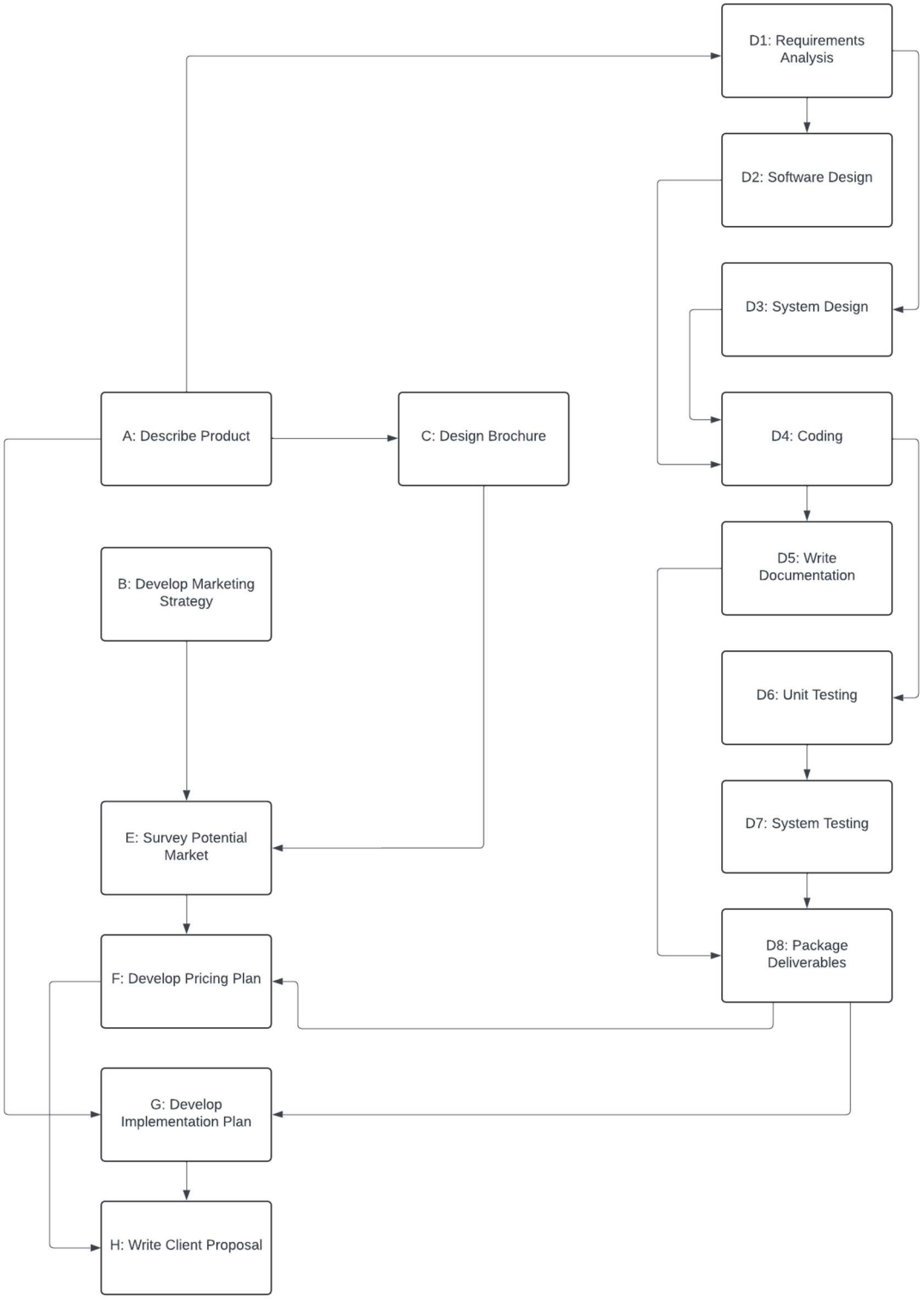
Problem Setup

Beginning this project, we must first specify the total number of hours expected for the best case, the expected case, and the worst case, as well as the number of employees required for each project and the total expected man-hours for each case. Within our team, we have 10 total members: 1 project manager, 3 front-end developers, 3 back-end developers, 2 data scientists, and 1 data engineer. For each task, we will not necessarily use all members of the team/role, so we must convert each task to the number of man hours required to determine total costs. We used our best estimate of what roles will be required for each task as well as the time required to complete each task, which is flexible and can be adjusted as seen fit moving forward. Since we are assuming an arbitrary salary number for each role that is consistent for all roles, man-hours is a suitable metric to use for total costs. For digestibility, however, we will assume a flat rate of $50/hour for each employee when we produce our model results.

taskID	task	predecessorTaskIDs	bestCaseHours	expectedHours	worstCaseHours
A	Describe product		8	12	16
B	Develop marketing strategy		24	40	60
C	Design brochure	A	16	24	32
D	Develop product prototype				
D1	Requirements analysis	A	40	60	80
D2	Software design	D1	60	90	120
D3	System design	D1	60	90	120
D4	Coding	D2, D3	120	200	280
D5	Write documentation	D4	40	60	80
D6	Unit testing	D4	40	80	120
D7	System testing	D6	40	80	120
D8	Package deliverables	D5, D7	24	40	56
E	Survey potential market	B, C	40	60	80
F	Develop pricing plan	D8, E	16	24	32
G	Develop implementation plan	A, D8	20	30	40
H	Write client proposal	F, G	8	12	16

taskID	projectManager	frontendDeveloper	backendDeveloper	dataScientist	dataEngineer
A	1	0	0	1	1
B	1	0	0	1	0
C	0	3	0	0	0
D					
D1	1	0	3	0	0
D2	0	3	3	0	0
D3	0	0	3	0	1
D4	0	3	3	0	0
D5	0	3	3	0	0
D6	0	3	3	0	0
D7	0	0	3	0	1
D8	0	0	3	0	1
E	0	0	0	2	0
F	1	0	0	2	0
G	1	0	0	0	0
H	1	0	0	0	0

taskID	Total bestCase ManHours	Total expected ManHours	Total worstCase ManHours
A	24	36	48
B	48	80	120
C	48	72	96
D			
D1	160	240	320
D2	360	540	720
D3	240	360	480
D4	720	1200	1680
D5	240	360	480
D6	240	480	720
D7	160	320	480
D8	96	160	224
E	80	120	160
F	48	72	96
G	20	30	40
H	8	12	16


D1: Requirements Analysis
D2: Software Design
D3: System Design
D4: Coding
D5: Write Documentation
D6: Unit Testing
D7: System Testing
D8: Package Deliverables
A: Describe Product
C: Design Brochure
B: Develop Marketing Strategy
E: Survey Potential Market
F: Develop Pricing Plan
G: Develop Implementation Plan
H: Write Client Proposal

Model Specification

The primary objective of modeling this project plan for the development of a recommendation system requires determining the critical path and minimizing completion time, and as a result of the assumptions of this problem, the cost as well. The variables that exist within this problem are the start and end times for each task, which are continuous and non-negative. We can represent each of these variables with S_i and E_i respectively.

We have two constraints within the model, which are task durations and precedence relations. For example, task A, as we've determined for our best case, will take 8 total hours, and task C requires completion of A to begin, so task C cannot start until the 8 hours are completed for task A. We can denote the duration for each task within each case as D_i. The representation of the precedence and duration constraints can be seen as follows:

$$S_i \geq E_j \; \forall \; (i,j) \in \text{Precedences},$$

where task j precedes task i

Additionally, we are assuming that our start time is simply 0, so $S_A = 0$.

We have two different objective functions that we are looking to optimize, but they are both linked via the hours expected to complete the project. The first objective is to minimize total project completion time, which is concluded at the end of the final task's completion. This can be represented as:

$$\text{Min } Z_{\text{Hours}} = \max(E_i)$$

The secondary objective is to minimize cost, which we have defined as man-hours, which is the product of hours for the task and the number of employees required for each task. The specific number of each employee doesn't matter in this case, since we make the assumption that all employees earn the same salary. This can be represented as:

$$\text{Min } Z_{\text{Man-Hours}} = \sum(D_i \text{ x Required Employees}_i)$$

Using everything that we have set forth so far, we can express the full linear program model as follows:

Objective:

$$\text{Min } Z_{Hours} = \max(E_i)$$

Subject To:

$$E_i = S_i + D_{i,}\ \forall_I \in \text{Tasks},$$

$$S_i \geq E_{j,}\ \forall\ (i,j) \in \text{Precedences},$$

$$S_A = 0,$$

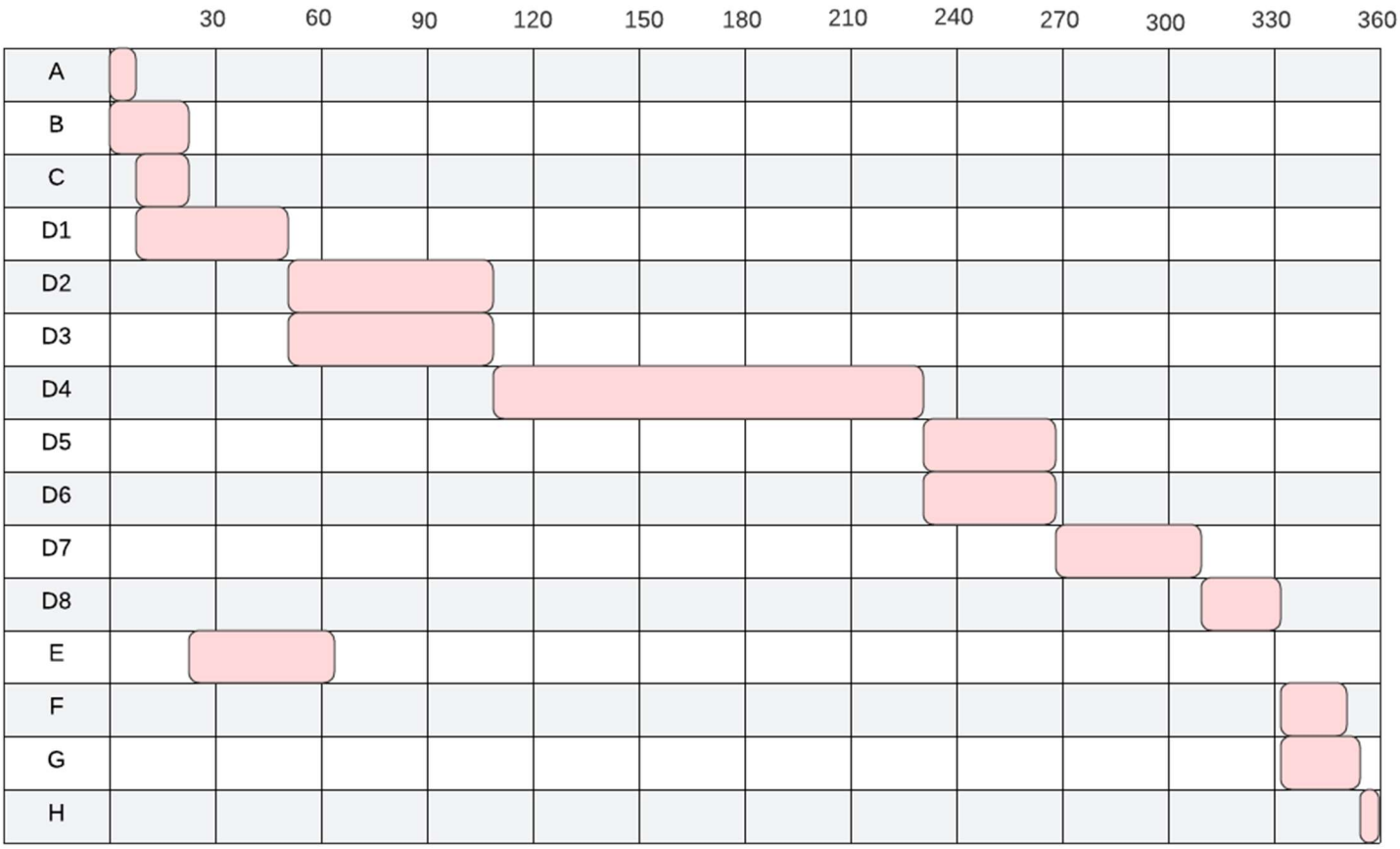
$$S_i, E_i \geq 0,\ \forall_I \in \text{Tasks}$$

Solution

For each of the cases, we have provided a Gantt Chart to visualize the critical path, which allows us to see the order of completion for each task and the general requirements for completion time for each. Additionally, we have provided the results for minimizing total man-hours for each case, which is then converted into a U.S. dollar amount assuming that each employee is earning $50 per hour.

Best Case

Total Hours



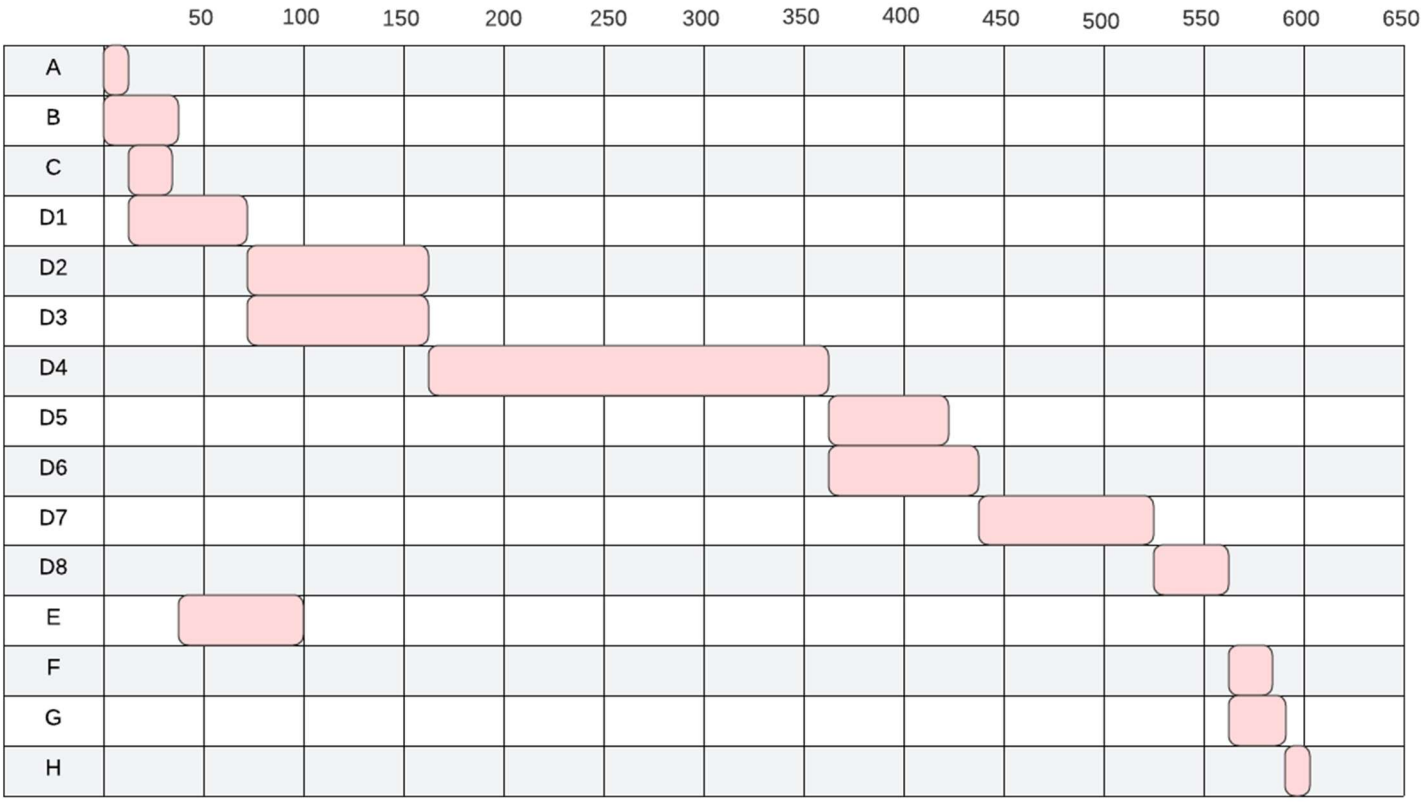
Total Man-Hours

Project Requirement: 1,816 hours

Total Cost (Assuming $50/hour salary): $90,800

Expected Case

Total Hours



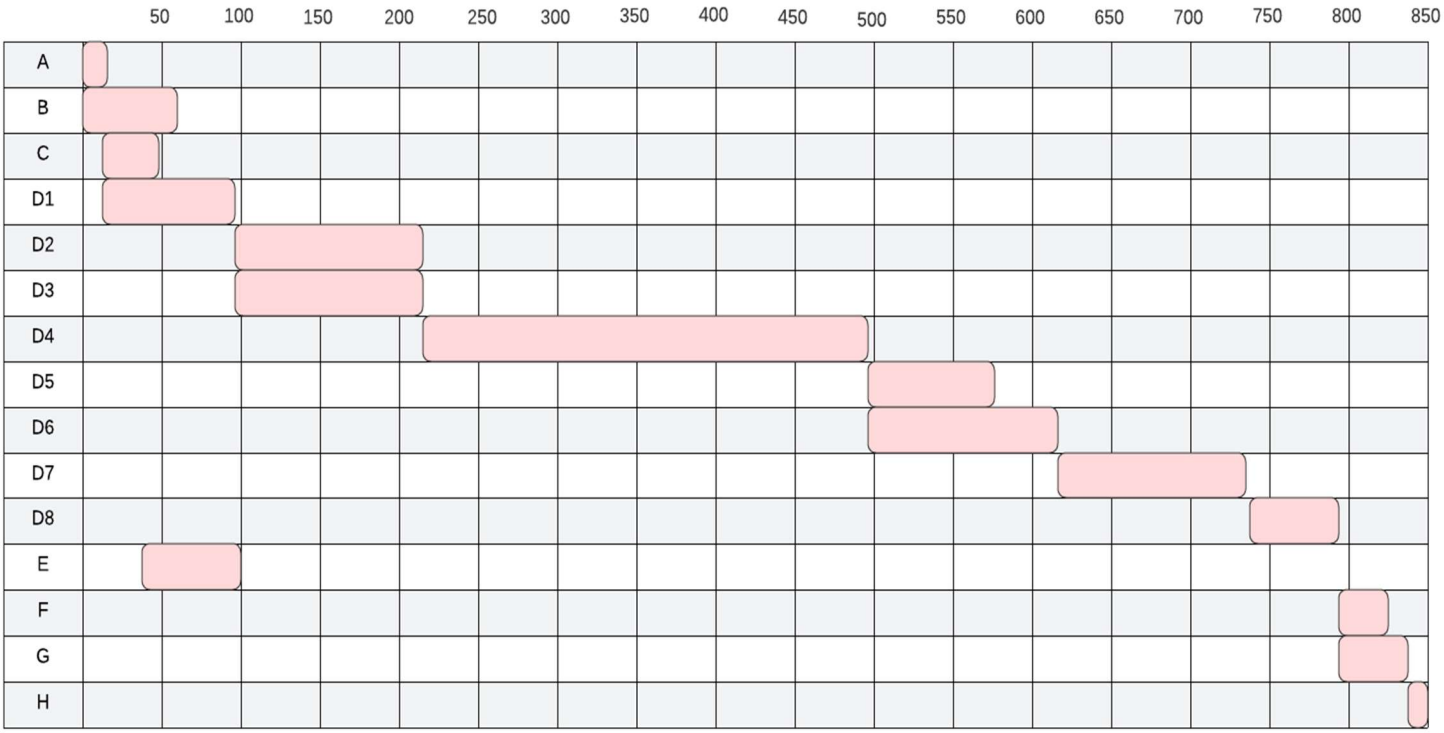
Total Man-Hours

Project Requirement: 3,060 hours

Total Cost (Assuming $50/hour salary): $153,000

Worst Case

Total Hours



Total Man-Hours

Project Requirement: 4,304 hours

Total Cost (Assuming $50/hour salary): $215,200

Overview

The results of each of the models provide some valuable insight into the general employment costs for the project. The largest proportion of the team's time will be spent completing Task D, which encompasses the development of the product prototype. Additionally, using the Gantt Charts, we can conceptualize the flow of the project. If we were to assume that, given no external factors, the project would be completed based on the expected amount of time provided in the overview, we would look to charge anywhere between $150,000 and $160,000 for the project. This would cover all employment fees for the 10 workers, but does not include the software licensing and cloud hosing fees. Using this case, we can also assume that the project would be completed in 604 hours at a total of 3,060 man-hours.

There are obviously factors that can affect this estimation. If we were to include more contractors within the team, we could certainly predict that the expected time of completion would decrease. For example, if we had an additional front and back-end developer, the development of the product prototype would likely be shortened. There is the tradeoff, however, of additional man-hours that must be factored in. Ideally, the insertion of an additional employee would reduce duration enough to outpace the increase in man-hours, but this is the risk that must be calculated beforehand. Using this case, let's examine how this would affect total completion time and man-hours assuming that each of the tasks within Task D are reduced by 10 hours for the expected case.

Updated Expected Case

Total Hours: 544 hours

Total Man-Hours: 3,590 man-hours

As a result of increasing the number of developers by 1 for both front and back-end for task D projects, we reduced the overall time of completion by 60 hours, but increase the overall number of man-hours by 530. As we can see, while more contractors will certainly increase the efficiency of the team, it will also come at a monetary cost, which may be necessary for the prospective client.

Appendix

See Github repository for the code and output for all cases.

References

Dinwiddie, George. *Software Estimation Without Guessing: Effective Planning in an Imperfect World.* Dallas, TX: Pragmatic Bookshelf. [ISBN-13:978-1680506983]

Falk, Kim. 2019. *Practical Recommender Systems.* Shelter Island, NY: Manning. [ISBN-13: 9781617292705]

Hillier, Frederick S., and Gerald J. Lieberman. 2021. *Introduction to Operations Research* (eleventh edition). New York: McGraw-Hill. [ISBN-13: 978-125987299-0] Online resources, including Chatper 22 Project Management withPERT/CPM at https://highered.mheducation.com/sites/dl/free/1259872998/1126268/Hillier_IOR_11e_Ch022_WebChapter.pdfLinks to an external site.

Majumdar, Partha. 2021. *Linear Programming for Project Management Professionals: Explore Concepts, Techniques, and Tools to Achieve Project Management Objectives.* India: BPB Publications. [ISBN-13: 978-9355511164]

Williams, H. Paul. 2013. *Model Building in Mathematical Programming* (fifth edition). New York: Wiley. Section 5.3.7 Critical Path Analysis, pages 94–98. [ISBN-13: 978-1-118-44333-0] We provide a Python PuLP solution to Williams' home building example using an activities dictionary and derived start_time and end_time decision variables: Williams-Critical-Path-Analysis.pyDownload Williams-Critical-Path-Analysis.py

Wisniewski, Mik, and Jonathan H. Klein. 2001. *Linear Programming/Critical Path Analysis.* New York: Palgrave. [ISBN-13:978-0-333-76355-1] Chapter 8, Critical Path Network Analysis Techniques, pages 119–147. Available from Course Reserves.