UNIVERSIDAD IBEROAMERICANA
PUEBLA

Robótica Aplicada

Profesor: Oliver Ochoa García

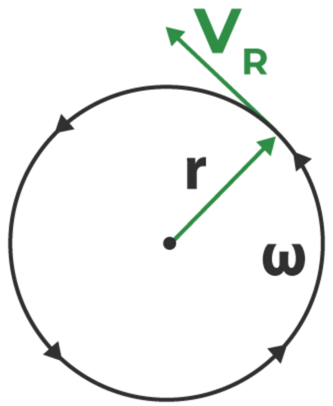
Velocidad angular

$$\omega = \dot{\theta}k$$

Donde
ω velocidad angular
$\dot{\theta}$ Angulo de rotacion
k vector unitario

$$v = \omega \times r$$

Donde
v velocidad linear
r vector del origen



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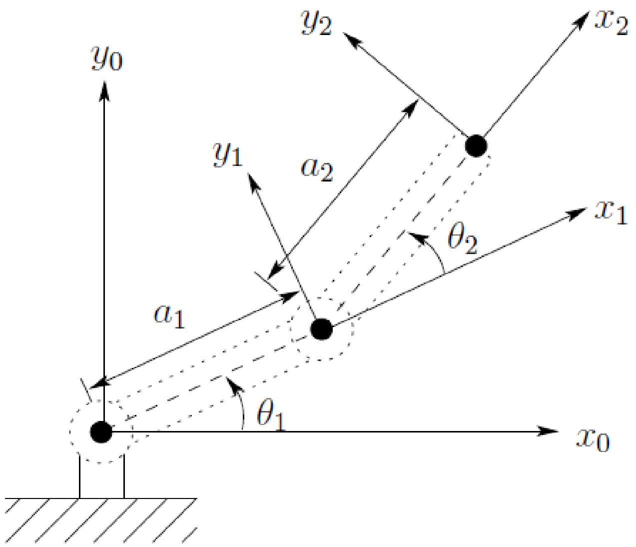
Jacobiana



$$A(t) = f(\theta(t))$$

$$\frac{d}{dt}A(t) = \frac{d}{dt}f(\theta(t))$$

$$\begin{bmatrix} A_1 \\ \vdots \\ A_m \end{bmatrix} = \begin{bmatrix} f_1(\theta) \\ \vdots \\ f_m(\theta) \end{bmatrix} \in \mathbb{R}^m$$

$$\dot{A} = \frac{\partial f}{\partial \theta}(\theta)\dot{\theta}$$

$$\frac{\partial f}{\partial \theta} = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \cdots & \frac{\partial f_1}{\partial \theta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial \theta_1} & \cdots & \frac{\partial f_m}{\partial \theta_n} \end{bmatrix} \in \mathbb{R}^{m \times n}$$

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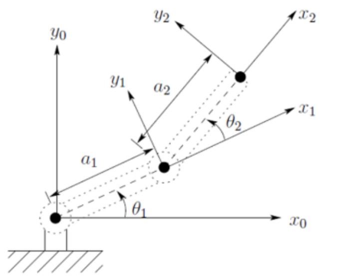
Jacobiana para Velocidada

Eslabón	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\boldsymbol{\theta_1}$
2	a_2	0	0	$\boldsymbol{\theta_2}$

$${}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_i & 0 & a_1 c\theta_1 \\ s\theta_1 & c\theta_i & 0 & a_1 s\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}_{2}T = \begin{bmatrix} c\theta_{12} & -s\theta_{12} & 0 & a_1 c\theta_1 + a_2 c\theta_{12} \\ s\theta_{12} & c\theta_{12} & 0 & a_1 s\theta_1 + a_2 s\theta_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

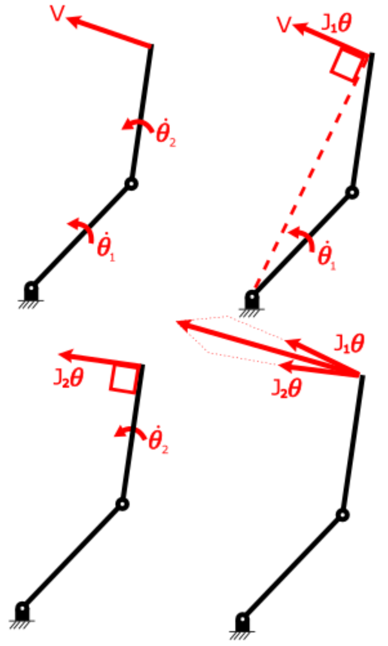
$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

$$v = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \qquad \frac{\partial f}{\partial \theta} = \begin{bmatrix} \dfrac{\partial f_1}{\partial \theta_1} & \dfrac{\partial f_1}{\partial \theta_2} \\ \dfrac{\partial f_2}{\partial \theta_1} & \dfrac{\partial f_2}{\partial \theta_2} \end{bmatrix}$$

$$\frac{\partial x}{\partial \theta_1} = -l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2)) \qquad \frac{\partial x}{\partial \theta_2} = -l_2 \sin(\theta_1 + \theta_2))$$

$$\frac{\partial y}{\partial \theta_1} = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)) \qquad \frac{\partial y}{\partial \theta_2} = l_2 \cos(\theta_1 + \theta_2))$$

$$v = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1 + \theta_2)) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)) \end{bmatrix} \dot{\theta}_1 + \begin{bmatrix} -l_2 \sin(\theta_1 + \theta_2)) \\ l_2 \cos(\theta_1 + \theta_2)) \end{bmatrix} \dot{\theta}_2 = J_1(\theta)\dot{\theta}_1 + J_2(\theta)\dot{\theta}_2$$



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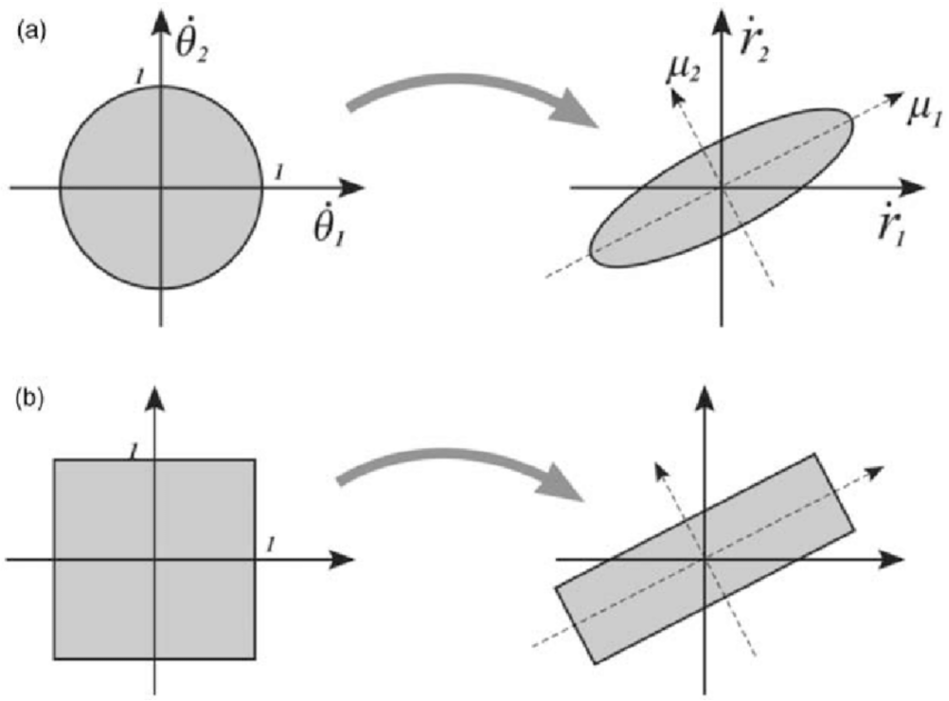
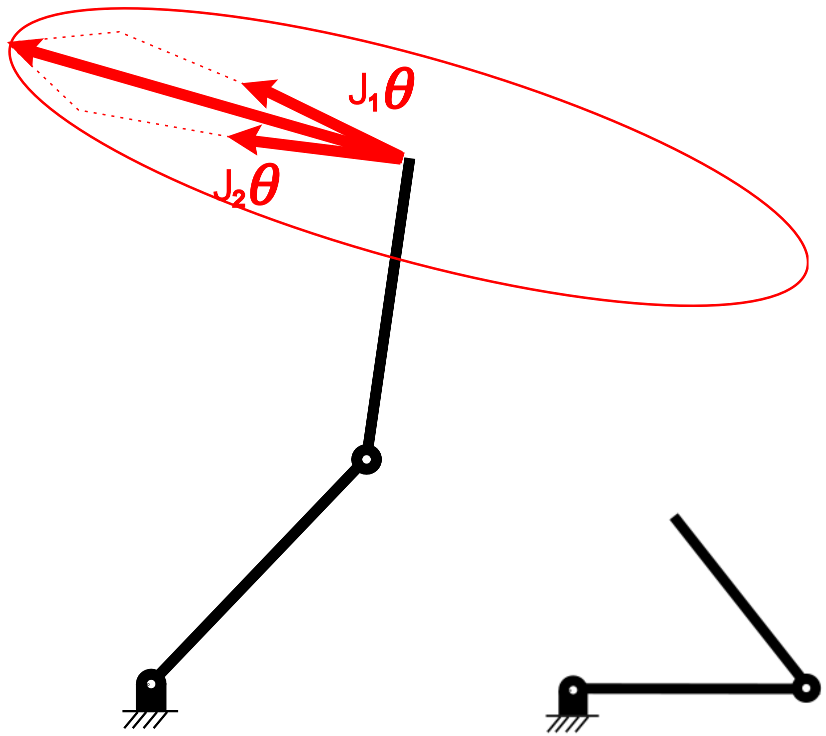
Limites de velocidad por articulacion





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Matriz Jacobiana

Velocidad por unión
Revolutas rads/s
Prismático m/s

Velocidad Lineal por eje

Velocidad angular por eje

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \mathrm{J} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

Matriz Jacobiana

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Desglose de Matriz

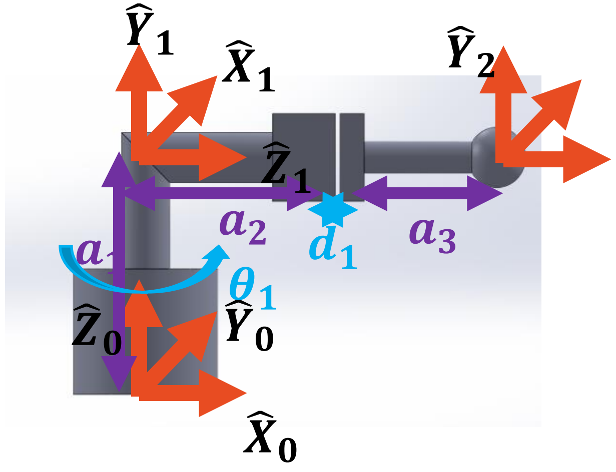
No. De columnas es igual que
numero de uniones

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \overbrace{\begin{bmatrix}\square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\\ \square & \square & \square\end{bmatrix}}^{\text{No. De columnas es igual que numero de uniones}} \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{d}_3\end{bmatrix}$$

	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

n=número total de uniones
i=número de unión revisada

Ejemplo 1



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot (d_2^0 - d_0^0) & R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(d_2^0 - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) & \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_3 \end{bmatrix}$$

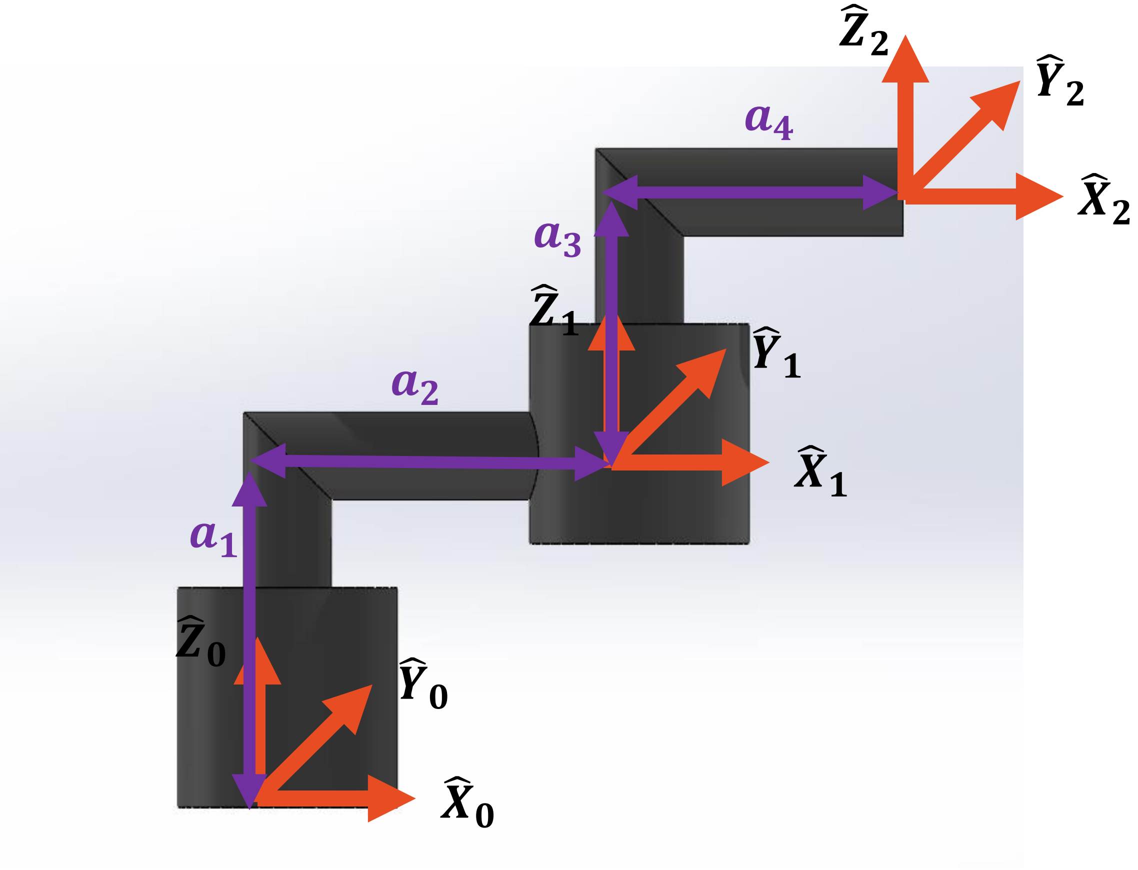
Recordatorio producto cruz

$$\vec{a} = (a_1, a_2, a_3)$$

$$\vec{b} = (b_1, b_2, b_3)$$

$$\vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

Ejercicio 1

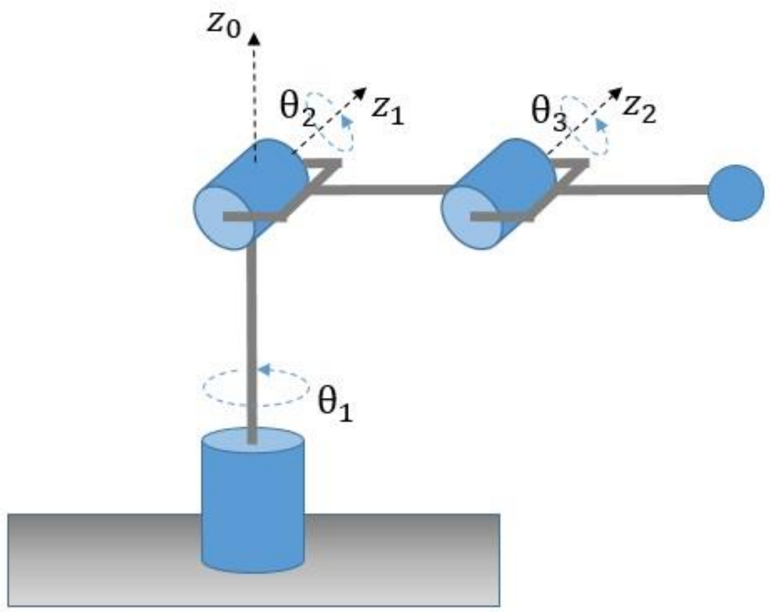


	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\omega_x\\\omega_y\\\omega_z\end{bmatrix}=\begin{bmatrix}\quad & \quad\\ & \\ & \\ & \\ & \\ & \end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\end{bmatrix}$$

Ejercicio 2

	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times\left(d_n^0 - d_{i-1}^0\right)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$



$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}$$

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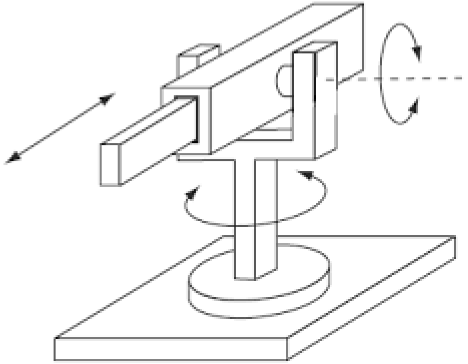
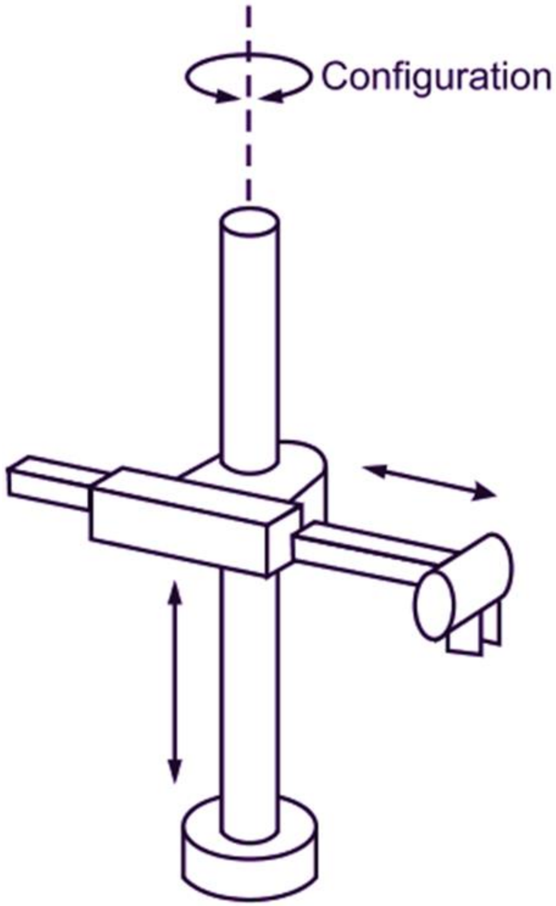
Ejercicio 3

	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_n^0-d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$



$$\begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\\\omega_x\\\omega_y\\\omega_z\end{bmatrix}=\begin{bmatrix} & & \\ & & \\ & & \\ & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\\\dot{d}_3\end{bmatrix}$$

Ejercicio 4



	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix} \times (d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix} \qquad\qquad \end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{d}_2\\ \dot{d}_3\end{bmatrix}$$