GEOLOGICAL NOTES

THE EFFECTS OF SHAPE ON CRYSTAL SETTLING AND ON THE RHEOLOGY OF MAGMAS[1]

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ABSTRACT

The effects of the shape of phenocrysts on their settling velocity and on the rheology of magmas are discussed. Spherical crystals fall at a velocity given by Stokes law and need to be present in large concentrations before they have a marked effect on the viscosity of a suspension. Elongate crystals fall at a much smaller velocity, which may be determined from the formulas and graphs given herein. Such elongate crystals can have a dramatic effect on the viscosity of a suspension even at low volume fractions. Suspended crystals and bubbles will not impart a yield stress to the magma unless they form a touching network across the entire suspension. It is suggested, therefore, that most previous inferences of a yield stress in subliquidus magmas are erroneous. Measurements of the subliquidus rheology of magmas need to be accompanied by careful characterization of the microstructure and distribution of the solid phase, on which such rheologies critically depend.

INTRODUCTION

The textures and compositions of igneous rocks are extraordinarily diverse. It has long been recognized that an important mechanism for achieving this diversity is the separation of crystals and melt. Such separation occurs both by the settling of crystals within the convecting magma (Darwin 1844; Bowen 1915; Wager and Deer 1939; Wager and Brown 1968; Brandeis and Jaupart 1986; Cox and Mitchell 1988; Martin and Nokes 1988, 1989) and by *in situ* crystallization coupled with the convective removal of depleted melt (Becker 1897; Campbell 1978; McBirney and Noyes 1979; Sparks and Huppert 1984; Sparks et al. 1984; Langmuir 1989; Kerr et al. 1989; Worster et al. 1990). In the next section we focus on the former mechanism and examine the effects of crystal shape on its efficiency.

It has been suggested that the presence of phenocrysts and exsolved volatiles has a significant effect on the rheology and flow behavior of magmas (Robson 1967; Booth and Self 1973; Gauthier 1973; Walker 1973; Hulme 1974; Sparks et al. 1976; Lipman and Banks 1987). Motivated by this suggestion, we also examine the hydrodynamic effects of suspended crystals and bubbles on the rheology of viscous fluids. In particular, we discuss whether these dispersed phases can cause the suspension to exhibit a yield strength. Our examination of the fluid-mechanical results is then used to comment on previous studies of the rheology of subliquidus magmas and to suggest directions for future research. Our work addresses the continuing concern about whether yield strengths in magmas can prevent the settling of phenocrysts or xenoliths (Cox and Mitchell 1988).

CRYSTAL-SETTLING VELOCITIES

Theory.—The rate of crystal settling from a magmatic body is determined largely by the size of the body and by the settling velocity of isolated crystals. For a spherical crystal of diameter D and density ρ_c, the settling velocity V_s in a magmatic fluid of density ρ_f and dynamic viscosity μ_f is given by the well-known equation (Stokes 1851)

$$V_s = \frac{g(\rho_c - \rho_f)D^2}{18\mu_f} \quad (1)$$

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where g is the acceleration due to gravity. This equation is derived under the assumption that inertial forces can be neglected and is found to be accurate if the Reynolds number

$$Re = \frac{\rho_f V_s D}{\mu_f} \tag{2}$$

is $<\sim 1$ (Batchelor 1967, p. 234). Since even large (1 cm diameter) olivine crystals settling in magmas with viscosities of only 100 poise have Reynolds numbers of less than 10^{-4}, this condition is easily satisfied.

An important limitation on the geological application of (1) is that magmatic crystals are not spherical and may be highly elongate under conditions of rapid growth (e.g., Lofgren 1980). In addition, aggregation (Schwindinger and Anderson 1989) or heterogeneous nucleation may produce composite crystals with irregular shapes and possibly large aspect ratios. Deviations in the settling velocity from (1) are to be due principally to deviations from a spherical shape rather than to crystal roundness or surface roughness (Wadell 1932; Williams 1966; see also later). Hence, we investigate the effects of the shape of a crystal on its settling velocity by considering an ellipsoidal crystal with principal diameters a, b and c. (In Cartesian coordinates the equation for such an ellipsoid can be written $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1/4$.) A simple estimate for the settling velocity V_e of an ellipsoid is provided by the settling velocity V_s of the sphere with equal volume (i.e., where $D^3 = abc$). The error in this estimate is due to the deviation from a spherical shape, as measured by the aspect ratios c/a and b/a. This allows us to define a shape factor

$$S = V_e/V_s \tag{3}$$

which is a function of c/a, b/a and the orientation of the particle. Thus the actual settling velocity of a particle may be found by multiplying the value of S, as calculated below, by the value of V_s, as given by (1).

The exact solution for the flow of a viscous fluid induced by the steady translation of an ellipsoid was derived by Oberbeck (1876). From his solution it may be deduced that, for settling parallel to the a-axis,

$$S = \frac{3}{8}\int_0^\infty \frac{2a^2 + \lambda}{a^2 + \lambda} \frac{(abc)^{1/3} d\lambda}{\sqrt{(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)}} \tag{4}$$

An interchange of a, b and c in (4) gives S for settling parallel to the other principal axes; superposition of the components of motion parallel to each axis gives S for an arbitrary crystal orientation. We now evaluate (4) numerically in order to give both a qualitative and quantitative understanding of the behavior of S and its dependence on aspect ratio and orientation.

Results.—First, we consider the simple case of a spheroid (an ellipsoid in which two of the principal diameters are equal) for which we may take $a = b$. The shape factor is then a function of the single aspect ratio $\gamma = c/a$ which describes the variation from oblate (disc-like) spheroids with $c < a$ to prolate (rod-like) spheroids with $c > a$. The results obtained by evaluation of (4) are shown in figure 1. The uppermost curve segments in the figure indicate the maximum values of the shape factor, which occur when the crystal is oriented so that its horizontal cross-sectional area is a minimum. This is achieved when the c-axis is vertical if $c > a$ and when the c-axis is horizontal if $c < a$. In both these cases slight deviations from a spherical shape can actually lead to an increase in the settling velocity. These maximum velocities will, however, rarely be achieved in a quiescent fluid since, as Komar and Reimers (1978) observed in their experiments, any weak inertial effects due to the Reynolds number not being identically zero will cause an ellipsoid to rotate until it is orientated so that its horizontal cross-sectional area is a maximum (Leal 1980). In this orientation the settling velocity is a minimum and the shape factor is given by

$$S_{\min} = \frac{3\gamma^{1/3}}{8}\left(\frac{\gamma}{\gamma^2 - 1} + \frac{2\gamma^2 - 3}{(\gamma^2 - 1)^{3/2}}\cosh^{-1}\gamma\right) \qquad (\gamma > 1) \ (5a)$$

$$= \frac{3\gamma^{1/3}}{8}\left(\frac{2\gamma}{1 - \gamma^2} + \frac{(2 - 4\gamma^2)}{(1 - \gamma^2)^{3/2}}\cos^{-1}\gamma\right), \qquad (\gamma < 1) \ (5b)$$

where $\gamma = c/a$, as indicated by the lowest two curve segments in figure 1.

The orientation of a crystal in a convecting melt will be influenced by the local gradients in the convective flow. These gradients will tend to align the long c-axis along streamlines in a (simple) shear flow and along the principal axis of strain in a extensional (pure shear) flow; the orientation at each instant defines the appropriate value of S. In the vigorous, unsteady flow characteristic of high-Rayleigh-number convection in magma chambers (Martin et al. 1987; Kerr et al. 1989; Worster et al. 1990) the orientation of the crystal will vary randomly with time. In this case, the appropriate value of the shape factor may be assumed to be given by averaging S over all possible orientations to obtain:

$$S_{\mathrm{mean}} = \frac{\gamma^{1/3}}{(\gamma^2 - 1)^{1/2}} \cosh^{-1}\gamma \qquad (\gamma > 1)\ (6a)$$

$$= \frac{\gamma^{1/3}}{(1 - \gamma^2)^{1/2}} \cos^{-1}\gamma \qquad (\gamma < 1)\ (6b)$$

If the convection is weaker then the settling velocity of the crystal relative to the surrounding melt will vary between $S_{\min}V_s$ and $S_{\max}V_s$ in different regions of the flow, where $S_{\max}$ is given by $3S_{\mathrm{mean}} - 2S_{\min}$ for a spheroidal crystal.

It is an important question how long crystals can remain in suspension in a vigorously convecting body of melt. Martin and Nokes (1988, 1989) show that the mean suspension time of crystals in a melt is governed by the rate at which they settle through the viscous boundary layer at the base of the body of melt. In this region the shear is such that the crystals would be in their slowest-settling orientation. Therefore, in the expressions of Martin and Nokes (1988, 1989) the settling velocity should be calculated using $S_{\min}$.

Second, we consider the case of a general ellipsoid for which we adopt the convention that the principal diameters are defined so that $c > b > a$. The shape factors calculated from (4) for the maximum, minimum and mean settling velocities are shown in figures 2A–C. Previous authors (Albertson 1953; Komar and Reimers 1978) have suggested that S depends primarily primarily on the parameter $a/\sqrt{bc}$. If this suggestion were correct then

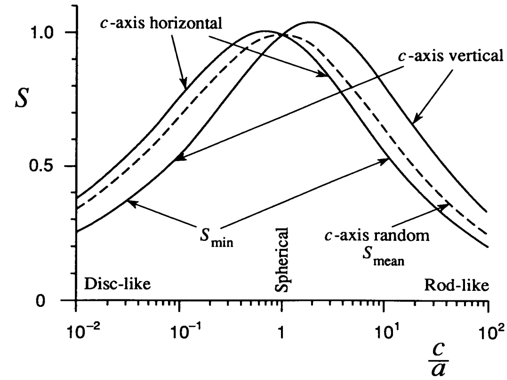

FIG. 1.—The shape factor S for a spheroid with equatorial diameter a (= b) and polar diameter c, covering the variation from oblate (disc-like) to prolate (rod-like) spheroids. Curves are drawn for the cases of the c-axis horizontal and vertical (solid lines) and of an average taken over all orientations of the c-axis (dashed line).

the contours of S would be curves of the form $a/\sqrt{bc}$ = constant (equivalently, $\log(b/a) = -\log(c/a)$ + constant) which are represented as straight lines of slope -1 in our figures. The suggestion is, therefore, clearly incorrect except very close to the disc-like diagonal where b is almost as large as c. (We note also that the empirical correlations of Komar and Reimers are inaccurate by 10–20% due to a failure to correct for viscous drag on the walls of their apparatus.) The figures show, instead, that for small aspect ratios ($<\sim 10$) it would be more accurate to say that the shape factor depends primarily on the maximum aspect ratio c/a and that variations in the ratio b/a from rod-like to disc-like crystals are of secondary importance.

It can be shown theoretically that if c is very much bigger than a then the characteristic lengthscale of Stokes flow around the particle is given by the largest dimension of the particle (i.e., c)—in rough terms, the velocity field near the particle is similar to that produced by the (imaginary) circumscribing spherical particle. In consequence, the viscous drag exerted by the fluid on the particle is proportional to c; the aspect ratios b/c and a/c give only small variations in the constant of proportionality. The balance between the viscous drag and the weight of the particle (which is proportional to abc) implies that the settling velocity of the particle is proportional

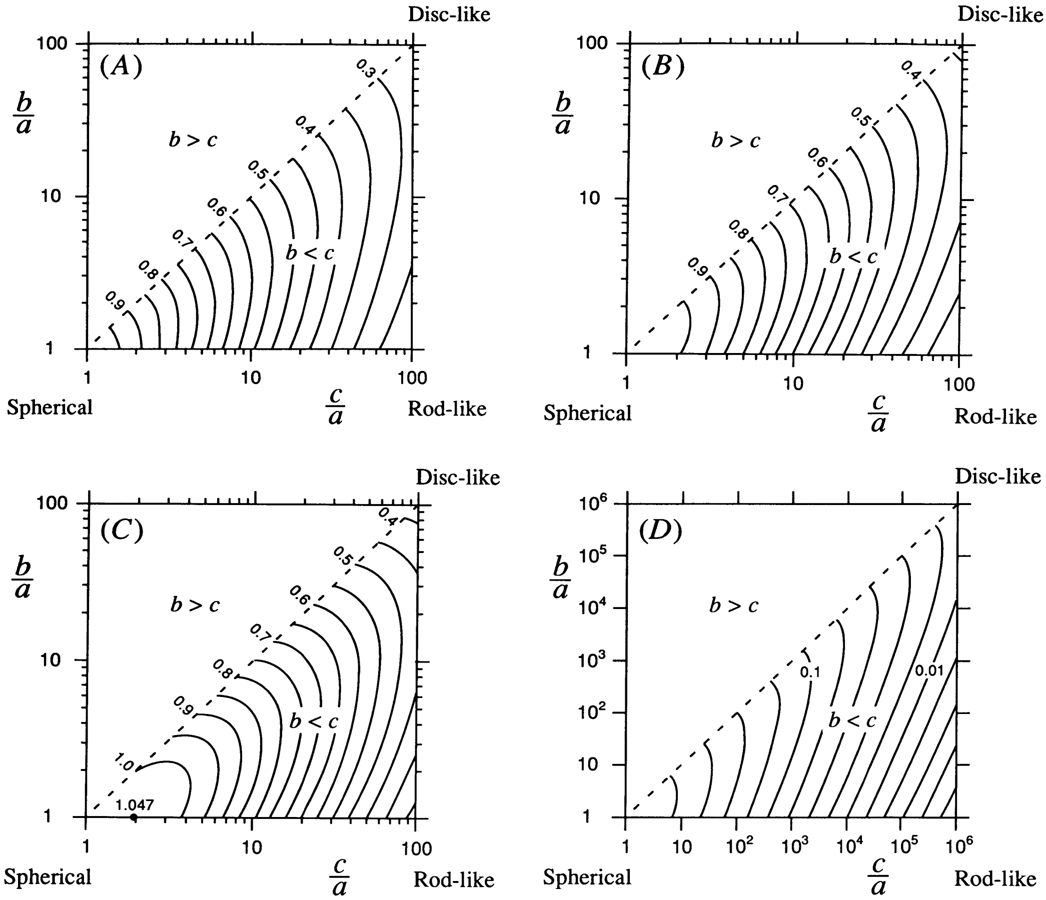

FIG. 2.—Contours of the shape factor S for an ellipsoid with principal diameters $a \leq b \leq c$. (The contours in the region $b > c$ are the reflection of those in $b < c$ but have been omitted to indicate our convention that $b \leq c$.) The cases $a = b = c$, $a = b \ll c$ and $a \ll b = c$ correspond to spherical, rod-like and disc-like ellipsoids respectively. (A) $S_{\min}$ appropriate to an ellipsoid settling with its a-axis vertical. (B) S_{mean} appropriate to an ellipsoid settling with a random orientation. (C) $S_{\max}$ appropriate to an ellipsoid settling with its c-axis vertical. (D) $S_{\min}$, showing the behaviour at large aspect ratios. The contours are logarithmically spaced with each interval corresponding to a factor $10^{1/5}$.

to ab. From (1) the settling velocity of the sphere of equivalent volume is proportional to $(abc)^{2/3}$. We deduce from (3) that

$$S \propto \left(\frac{ab}{c^2}\right)^{1/3}, \tag{7}$$

as may be verified by a full asymptotic analysis of (4). In our figures this result is manifested by a tendency for contours of S to become straight lines of slope 2 at large values of c/a (i.e., curves of constant S are given by $\log(b/a) = 2 \log(c/a) +$ constant). This tendency can be seen more clearly in figure 2D, which shows the variation in S for large aspect ratios.

Comparison with Experiment.—It is of some interest to compare the results of experiments with the predictions of (4). It should be noted, however, that this is primarily a test of the care taken in the experimental measurements since (4) is an exact expression derived from a well-established theoretical solution for the motion of an ellipsoid.

Komar and Reimers (1978) made a series of measurements of the settling velocity of roughly ellipsoidal pebbles in glycerine and attempted to correlate the data against a function of the single parameter $a/\sqrt{bc}$. Komar

(1980) subsequently made further measurements of the settling velocity of cylindrical particles and attempted to correlate both sets of data against a function of either $a/\sqrt{bc}$ or $a/\sqrt{(a^2 + b^2 + c^2)}$. (For the cylindrical particles a and b are taken to be the diameter and c to be the length.) It is clear, first, that such empirical correlations are of very limited use for a particle with two independent aspect ratios, since the exact value, which is given by (4) for an ellipsoidal particle, is a function of both aspect ratios and cannot be expressed in terms of any single parameter (other than the right-hand side of [4]). Second, it would be possible to correlate the settling velocity of a cylinder against any function of the radius to length ratio since this is the only geometric parameter for such a particle. Though the correlations are of little value, it is interesting, nevertheless, to compare these experimental results with the theory.

In figure 3*A* we show the good agreement between the experimental settling velocities of the 51 pebbles used by Komar and Reimers and the theoretical values calculated from equations (1), (3), and (4) and from the tabulated dimensions of the particles. The experimental measurements have been corrected for wall effects (which the original papers omitted to do), since the cylinder in which the measurements were made was only about ten times the diameter of the particles and this leads to a reduction of about 20% in the settling velocity. The exact correction factor is not known for ellipsoidal or cylindrical particles, but is given by Happel and Brenner (1965) for spherical particles. Based on the observation that in Stokes flow the velocity field produced by an translating elongate particle is similar to that produced by the translation of the circumscribing spherical particle, we chose to correct the data by the factor appropriate to a spherical particle, which was calculated using the maximum diameter of the experimental particle. This is the best that can be done and removes most of the wall effects. It is, however, a slight overcorrection of the data, as can be seen in figure 3*B* which shows that the corrected experimental values are a little larger than the theoretical values. The remaining scatter in the data can be attributed to experimental error and to variation in the accuracy of the spherical correction factor. The scatter is too great to discern a systematic variation with Reynolds number. We emphasize again that the theoretical values are exact for ellipsoidal particles and we are essentially testing the experimental technique and data correction for wall effects.

In figure 4 we analyze the measurements by Komar (1980) of the settling velocity of cylindrical particles in order to test the proposition that our equations can be used to provide a good approximation for the settling velocity of non-ellipsoidal particles. The theoretical settling velocities are those of the spheroids which have the same volume and aspect ratio as the corresponding cylinders (i.e., $\pi ac^2/6 = \pi D^2 L/4$ and $a/c = D/L$). The experimental measurements are corrected, as above, using the correction factor for the circumscribing sphere. The agreement between theory and experiment is good and the scatter and bias is similar to the comparison with ellipsoidal particles in figure 3. Therefore, the difference between theory and experiment is likely to be due largely to experimental technique and data correction. We conclude that the settling velocity of non-ellipsoidal particles is well-approximated by the settling velocity of ellipsoidal particles of equivalent volume and aspect ratios.

Application.—When applied to the largest, and hence most slowly cooled, magmatic intrusions, in which the crystals have aspect ratios of up to 4 (Jackson 1971), figure 2*A–D* indicate corrections of up to about 25% in minimum settling velocity, up to about 15% in mean settling velocity, and up to about 8% in maximum settling velocity. Larger aspect ratios, and hence larger corrections, can be expected in smaller, more rapidly cooled, magmatic bodies. The largest aspect ratios are likely to occur in any lava flows in which degassing causes large undercooling and rapid crystal growth (Sparks and Pinkerton 1978). For example, degassing of the 1984 Mauna Loa lava triggered the growth of microphenocrysts of augite, plagioclase, and olivine with typical aspect ratios of 10 to 15 (Lipman and Banks 1987, fig. 57.24).

THE RHEOLOGY OF MAGMATIC SUSPENSIONS

In a Newtonian fluid the rate of strain is proportional to the applied stress. A suspension of particles, droplets, or gas bubbles in such a fluid, however, may have a much more

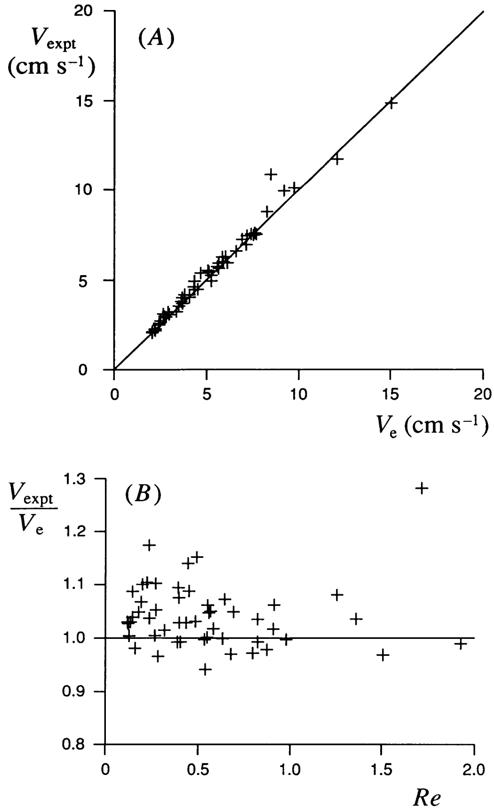

FIG. 3.—(*A*) Comparison of experimental measurements of the settling velocity V_{expt} of ellipsoidal pebbles with the exact theoretical value V_e. The experimental data is taken from Komar and Reimers (1978) and is corrected for wall effects. (*B*) The ratio of the experimental and theoretical values plotted against the Reynolds number based on the largest dimension of each pebble.

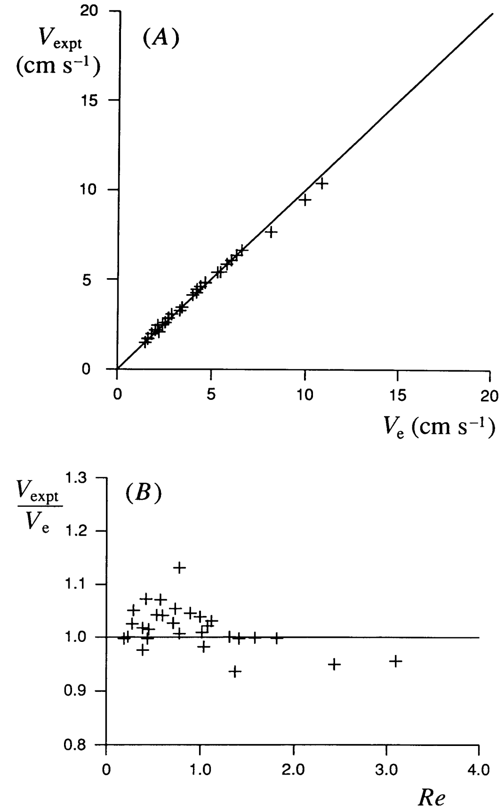

FIG. 4.—(*A*) Comparison of experimental measurements of the settling velocity V_{expt} of cylindrical rods with the theoretical value V_e for ellipsoidal particles of the same volume and aspect ratio. The experimental data is taken from Komar (1980) and is corrected for wall effects. (*B*) The ratio of the experimental and theoretical values plotted against the Reynolds numbered based on the largest dimension of each rod.

complex rheology. Under appropriate conditions, the suspension may show shear-thinning (stress increasing more slowly than strain), shear-thickening, time-dependent, and flow-dependent behavior. A thorough review of the rheology of particulate suspensions given by Jeffrey and Acrivos (1976). Some of the more complicated behavior occurs in slurries of micron-sized clay particles in water but disappears when we consider millimetre-sized dispersions in viscous magmas. We confine our discussion to the magmatic application and summarize below the relevant results.

The Effect of Spherical Particles.—Einstein (1906, 1911) calculated the viscosity $\mu(\phi)$ of a dilute suspension of rigid spherical particles, where ϕ is the volume fraction of the particles. He showed that if the suspension is sufficiently dilute that interactions between the particles can be neglected then

$$\mu(\phi) = \left(1 + \frac{5}{2}\phi\right)\mu_f, \tag{8}$$

where μ_f is the viscosity of the suspending Newtonian fluid. When the suspension is not dilute ($\phi > 2\%$ [Happel and Brenner 1965]), the interactions between particles become important and the viscosity is a function,

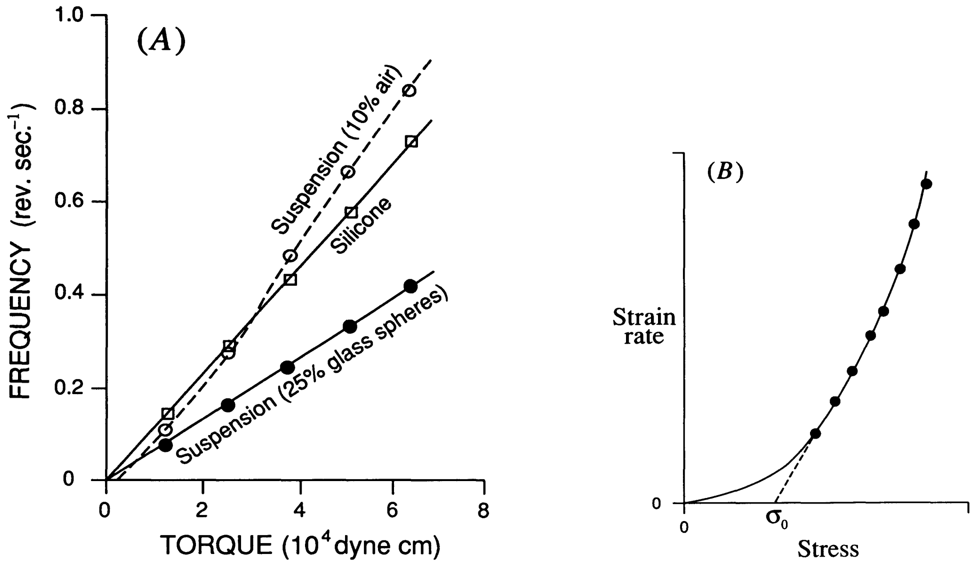

FIG. 5.—(*A*) Experimental measurements taken from figure 13 of Shaw et al. (1968) of the angular frequency of a rotational viscometer produced by an applied torque. The three curves correspond to a pure silicone fluid and to suspensions of 10% air by volume and of 25% glass spheres by volume in the same silicone fluid. (*B*) Illustration of how extrapolation (dashed line) of experimental data (solid circles) to zero strain rates can give an apparent yield stress σ_0. The true rheology (solid line) is shear thinning with a Newtonian limit at small shear rates.

which is in general unknown, both of ϕ and of the size distribution of the spheres. Nevertheless, the exemplary experiments of Krieger (1972) show that a monodisperse suspension of spheres is Newtonian for volume fractions at least as large as 50% when Brownian motion and electroviscous effects are negligible (as is indeed the case in magmas due to the large size of the crystals and the high viscosity of the magma). This result is also observed to apply to polydisperse suspensions of spheres, as can be seen, for example, in the measurements by Shaw et al. (1968, fig. 13, reproduced here as fig. 5*A*) of the rheology of a 25% by volume suspension of glass spheres in silicone. These measurements show an effective Newtonian viscosity of approximately 1.8 μ_f; that this value is larger than the 1.625 μ_f predicted by (8) is due to interactions between the particles at these high concentrations. In conclusion, the Newtonian behavior of suspensions for $\phi <$ 50% shows that yield stresses are not observed when ϕ is less than the approximate value required for the existence of a packed framework of spheres across the suspension.

The Effect of Crystal Shape.—When the suspended particles are not spherical, the following results apply: First, the viscosity of a suspension will become very much larger for a given volume fraction as the particles become increasingly elongate (Batchelor 1967, p. 253). This effect is related to the observation, made in the preceding analysis of crystal settling, that the drag exerted by a particle is proportional to its longest dimension, whereas the volume of the particle is also dependent on the shorter dimensions. As a result, the total viscous dissipation per volume fraction of particles is an increasing function of the particle aspect ratio. Second, the viscosity of the suspension will depend on the orientation of the crystals and hence will be different in an extensional (pure shear) flow and in a (simple) shear flow (Weinberger and Goddard 1974). In an extensional flow the crystals are aligned along the direction of extension, and the increased viscosity is dominantly due to the difficulty of maintaining velocity gradients in the direction of the long axis; in a shear flow the crystals may tumble due to hydrodynamic interactions, but will spend most of the time with their long axis perpendicular to the velocity gradient and will have a smaller effect on the viscosity of the suspension. Third, as in the case of spherical particles, the suspension has no yield strength except when ϕ is large.

The Effect of Gas Bubbles.—The rheological properties of emulsions and foams have been discussed recently by Jaupart and Vergniolle (1989, appendix A). We note here three important points. First, an emulsion has no yield stress for volume fractions less than the value of approximately 60% required for cubic close packing of the spherical bubbles. At larger volume fractions the bubbles form a touching framework that possesses a yield strength proportional to surface tension (Princen 1985). Second, for small stresses and small volume fractions, surface tension prevents deformation of the bubbles from a spherical shape. Under such conditions, a dilute suspension of bubbles has a Newtonian viscosity equal to

$$\mu(\phi) = (1 + \phi)\mu_f \qquad (9)$$

(Taylor 1932). Third, for sufficiently large stresses, the bubbles are deformed and stretched along streamlines. Since the viscosity of the gas is much less than that of the fluid, the stretched bubbles lubricate the flow and cause the viscosity to be a decreasing function of the applied stress (i.e., the rheology is non-Newtonian and shear-thinning).

These observations explain the measurements by Shaw et al. (1968; see fig. 5*A*) of the viscosity of a silicone fluid containing a volume fraction of 10% air, which was incorrectly extrapolated (as illustrated in fig. 5*B*) to infer a yield stress. At the lowest shear rate the suspension of bubbles has a viscosity of 1.1 μ_f, while at higher shear rates the bubbles deform to yield a suspension viscosity less than that of the silicone.

Yield Strengths in Magmas?—The conclusion to be drawn from the preceding fluid-mechanical studies is that a solid or gaseous suspension in a Newtonian fluid will only have a yield stress if the volume fraction is sufficient to form a touching framework across the suspension. This makes sound physical sense. For a suspension to have a yield stress, it must be capable of supporting a small, non-zero stress without deforming continuously. The stress can be supported if there is a framework of particles or bubbles in contact and unable to move relative to each other due to their rigidity and surface tension, respectively. Conversely, if the suspension is sufficiently dilute that the particles are not in contact and have the geometric freedom to move relative to each other, then there is no hydrodynamic reason why they should not do so when subjected to an applied stress. Such a suspension may have a large apparent viscosity but will not have a yield strength.

In the last 20 years there has been a number of measurements of the rheology of subliquidus magmas, both in the field (Shaw et al. 1968; Pinkerton and Sparks 1978) and in the laboratory (Shaw 1969; Murase et al. 1984; McBirney and Murase 1984; Ryerson et al. 1988). These papers have inferred yield strengths and attributed them to the presence of suspended crystals or gas bubbles. In the light of our theoretical understanding we are forced to question why yield strengths are reported at moderate or low fractions of particles and bubbles.

It might be argued that the rheology of the suspending magma itself is non-Newtonian and displays a yield strength. If this were the case then the observations of a yield stress are due to the fluid and not to the presence of crystals or volatiles. However, as McBirney and Murase (1984, p. 351) state, "no unambiguous evidence has yet been reported that crystal-free liquids above their liquidus temperatures have measurable yield strengths." Further, Barnes and Walters (1985) argue that polymeric melts do not exhibit yield stresses, and we see no physical reason why melts should behave differently. We conclude that this explanation of the geological measurements is unlikely.

The second, and much more likely, possibility is that the yield strengths are an artifact of the extrapolation of experimental data to zero shear rates. It is known theoretically that both the deformation of bubbles and the orientation of crystals in a flow field cause shear-thinning behavior. Such behavior is indeed observed experimentally in magmas and encourages misinterpretation of the data in terms of a fictitious yield strength (fig. 5*B*). This point is illustrated by the 10% bubbles curve of Shaw et al. (1968), which indicates a fictitious yield strength by extrapolation (fig. 5*A*). We suggest similarly, that the yield strengths inferred by Ryerson et al. (1988, fig. 12 and eqn. 17), particularly those at moderate values of ϕ, are likely to be artifacts of the quadratic approximation of their data. This conclusion is supported by the observation by Ryerson et al. that their data was as adequately represented by a power-law rheology with no yield strength. Extrapolation artifacts have also been found for polymer solutions (Barnes and Walters 1985) and cannot be eliminated entirely given that any experiment must have a limited sensitivity and minimum observable strain rate.

The third possibility is that a touching framework of crystals and/or bubbles exists across the entire suspension. As has been noted by many authors, this is a viable mechanism for imparting a yield strength to a suspension, since such a framework would be capable of supporting small but finite stresses without continuous deformation. (The contiguity factor discussed by Ryerson et al. [1988] is irrelevant since the existence or otherwise of such a framework cannot be predicted from the continguity factor; indeed, a framework can exist at arbitrarily small contiguity.) For example, a touching framework is quite likely in the lava described by Gauthier (1973, p. 94), which consisted of 44.6% crystals, 32.6% gas bubbles, and only 22.8% melt. In general, a connected lattice of crystals can form at much smaller values of ϕ if the crystals are highly elongate due to the melt being rapidly cooled or degassed (Sparks and Pinkerton 1978). A connected lattice is also more likely to form if the crystals nucleate heterogeneously (Lofgren 1983). The availability of nucleation sites on pre-existing crystals or, in an experimental context, on the solid boundaries of the apparatus will strongly encourage heterogeneous nucleation and lattice formation. Variations in such crystal microstructures can explain why "no simple relation has yet been found between yield strength and the volumetric proportion of crystals" (McBirney and Murase 1984 p. 352). We note that the destruction of these microstructures at increasingly high shear rates can account for the observation by Ryerson et al. (1988, p. 3429), among others, that "the magmas always display greater deviations from Newtonian behaviour during acceleration from rest rather than deceleration from a higher strain rate regime."

CONCLUSIONS

We have evaluated the settling velocities of non-spherical crystals relative to the settling velocities of spherical crystals of equivalent volume. Actual settling velocities may readily be calculated from equations (1) and (3) and figures 1 and 2. The settling velocities of elongate crystals and of spherical crystals of equal volume are found to be significantly different, showing that calculated settling velocities need to be based on the aspect ratios as well as the volume of the crystals.

Suspended phases cannot impart a yield strength to a Newtonian fluid unless they form a connected network across the suspension. A likely explanation for measured yield stresses in subliquidus magmas is false extrapolation to zero shear rates. Tests of this hypothesis will require improved techniques for measurement at low strain rates. An alternative explanation is that a connected lattice of crystals was present in the experimental sample. If this is the case then the measured rheology will depend on the detailed microstructure of this lattice and the distribution and arrangement of the solid phases should be characterized carefully in future measurements.

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