Torpedo Oficial
Fundamentos de Informática II

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Potencias factoriales

$$x^{\underline{k}} = \prod_{0 \le r \le k} (x - r) = x \cdot (x-1) \cdot \dots \cdot (x - k + 1) \qquad x^{\overline{k}} = \prod_{0 \le r \le k} (x + r) = x \cdot (x+1) \cdot \dots \cdot (x + k - 1)$$

$$(-x)^{\underline{k}} = (-1)^k x^{\overline{k}} \qquad (-x)^{\overline{k}} = (-1)^k x^{\underline{k}} \qquad x^{\underline{k}} = (x - k + 1)^{\overline{k}} \qquad x^{\overline{k}} = (x + k - 1)^{\underline{k}}$$

$$n! = n^{\underline{n}} = 1^{\overline{n}} \qquad n^{\underline{k}} = \frac{n!}{(n-k)!} \qquad n^{\overline{k}} = \frac{(n+k-1)!}{(n-1)!} \qquad \frac{\mathrm{d}^k}{\mathrm{d}x^k} x^u = u^{\underline{k}} x^{u-k}$$

$$(x+1)^{\underline{m}} - x^{\underline{m}} = m x^{\underline{m-1}} \qquad (x+1)^{\overline{m}} - x^{\overline{m}} = m (x+1)^{\overline{m-1}} \qquad \sum_{0 \le k < n} k^{\underline{m}} = \frac{n^{\underline{m+1}}}{m+1} \qquad \sum_{0 \le k < n} k^{\overline{m}} = \frac{n^{\overline{m+1}}}{m+1}$$

Funciones generatrices

Las *funciones generatrices* de la secuencia $\langle a_n \rangle_{n \ge 0}$ son:

Ordinaria: $A \stackrel{\text{ogf}}{\longleftrightarrow} \langle a_n \rangle_{n \ge 0}$ si $A(z) = \sum_{n \ge 0} a_n z^n$

Exponencial: $\widehat{A} \stackrel{\text{egf}}{\longleftrightarrow} \langle a_n \rangle_{n \ge 0}$ si $\widehat{A}(z) = \sum_{n \ge 0} a_n \frac{z^n}{n!}$

Sean $A(z) \stackrel{\text{ogf}}{\longleftrightarrow} \langle a_n \rangle_{n \ge 0}$ y $B(z) \stackrel{\text{ogf}}{\longleftrightarrow} \langle b_n \rangle_{n \ge 0}$, $\widehat{A}(z) \stackrel{\text{egf}}{\longleftrightarrow} \langle a_n \rangle_{n \ge 0}$ y $\widehat{B}(z) \stackrel{\text{egf}}{\longleftrightarrow} \langle b_n \rangle_{n \ge 0}$, $p(x)$ un polinomio. Entonces:

$$A(z) \cdot B(z) \stackrel{\text{ogf}}{\longleftrightarrow} \left\langle \sum_{k \ge 0} a_k b_{n-k} \right\rangle_{n \ge 0} \qquad \widehat{A}(z) \cdot \widehat{B}(z) \stackrel{\text{egf}}{\longleftrightarrow} \left\langle \sum_{k \ge 0} \binom{n}{k} a_k b_{n-k} \right\rangle_{n \ge 0}$$

$$\frac{A(z) - a_0 - a_1 z - \dots - a_{m-1} z^{m-1}}{z^m} \stackrel{\text{ogf}}{\longleftrightarrow} \langle a_m, a_{m+1}, \dots \rangle \qquad \frac{\mathrm{d}^m}{\mathrm{d}z^m} \widehat{A}(z) \stackrel{\text{egf}}{\longleftrightarrow} \langle a_m, a_{m+1}, \dots \rangle$$

$$p(z\mathrm{D}) A(z) \stackrel{\text{ogf}}{\longleftrightarrow} \big\langle p(n) a_n \big\rangle_{n \ge 0} \qquad p(z\mathrm{D}) \widehat{A}(z) \stackrel{\text{egf}}{\longleftrightarrow} \big\langle p(n) a_n \big\rangle_{n \ge 0}$$

$$\frac{1}{1-z} \cdot \sum_{n \ge 0} a_n z^n = \sum_{n \ge 0} \Big(\sum_{0 \le k \le n} a_k \Big) z^n$$

Extracción de coeficientes

$$
\begin{aligned}
\left[z^n\right] (\alpha A(z) + \beta B(z)) &= \alpha \left[z^n\right] A(z) + \beta \left[z^n\right] B(z) \\
\left[z^n\right] A(z) \cdot B(z) &= \sum_{0 \le k \le n} a_k b_{n-k} \\
\sum_{0 \le k \le n} \binom{n}{k} a_k &= \left[z^n\right] \frac{1}{1-z} A\left(\frac{z}{1-z}\right)
\end{aligned}
$$

Decimación

$$
\sum_{n \ge 0} a_{2n} z^{2n} = \frac{A(z) + A(-z)}{2} \qquad \sum_{n \ge 0} a_{2n+1} z^{2n+1} = \frac{A(z) - A(-z)}{2}
$$

Si ω es raíz primitiva m-ésima de 1:

$$
\omega = \cos \frac{2\pi}{m} + \mathrm{i} \sin \frac{2\pi}{m}
$$

entonces:

$$
\sum_{0 \le k < m} \omega^{ks} = \begin{cases} 0 & \text{si } m \nmid s \\ m & \text{si } m \mid s \end{cases} \qquad \sum_{t \ge 0} a_{mt+r} z^{mt+r} = \frac{1}{m} \sum_{0 \le k < m} \omega^{-kr} A(\omega^k z)
$$

Algunas series notables

En lo que sigue, $m, n \in \mathbb{N}$, $k \in \mathbb{N}_0$ y $\alpha, \beta \in \mathbb{C}$.

$$
\frac{1}{1-az} = \sum_{n \ge 0} a^n z^n \qquad \frac{1 - z^{m+1}}{1-z} = \sum_{0 \le n \le m} z^n
$$

$$
(1+z)^\alpha = \sum_{n \ge 0} \binom{\alpha}{n} z^n \qquad \binom{\alpha}{n} = \frac{\alpha^{\underline{n}}}{n!} = \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!} \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}
$$

$$
\binom{1/2}{n} = \frac{(-1)^{n-1}}{n 2^{2n-1}} \binom{2n-2}{n-1} \quad (\text{si } n \ge 1) \qquad \binom{-1/2}{n} = \frac{(-1)^n}{2^{2n}} \binom{2n}{n} \qquad \binom{-n}{k} = (-1)^k \binom{n+k-1}{n-1}
$$

$$
\frac{1}{(1-z)^{n+1}} = \sum_{k \ge 0} \binom{n+k}{n} z^k = \frac{1}{n!} \sum_{k \ge 0} (k+1)(k+2)\cdots(k+n) z^k = \frac{1}{n!} \sum_{k \ge 0} (k+1)^{\overline{n}} z^k
$$

$$
\sum_{n \ge 0} \binom{n}{k} z^n = \frac{z^k}{(1-z)^{k+1}}
$$

$$
(\alpha+\beta)(\alpha+\beta+n)^{n-1} = \sum_{0 \le k \le n} \binom{n}{k} (\alpha+k)^{k-1} (\beta+n-k)^{n-k-1}
$$

$$
\mathrm{e}^z = \sum_{n \ge 0} \frac{z^n}{n!} \qquad \mathrm{e}^{u + \mathrm{i} v} = \mathrm{e}^u (\cos v + \mathrm{i} \sin v) \quad (u, v \in \mathbb{C}) \qquad \ln(1-z) = -\sum_{n \ge 1} \frac{z^n}{n}
$$

$$
\sin z = \sum_{n \ge 0} \frac{(-1)^n z^{2n+1}}{(2n+1)!} \qquad \cos z = \sum_{n \ge 0} \frac{(-1)^n z^{2n}}{(2n)!} \qquad \sinh z = \sum_{n \ge 0} \frac{z^{2n+1}}{(2n+1)!} \qquad \cosh z = \sum_{n \ge 0} \frac{z^{2n}}{(2n)!}
$$

Secuencias notables

Números de Fibonacci:

$$F_{n+2} = F_{n+1} + F_n \quad F_0 = 0, F_1 = 1 \qquad \sum_{n\geq 0} F_n z^n = \frac{z}{1-z-z^2} \qquad \sum_{n\geq 0} F_{n+1} z^n = \frac{1}{1-z-z^2}$$

$$F_n = \frac{\tau^n - \phi^n}{\sqrt{5}} \qquad \tau = \frac{1+\sqrt{5}}{2} \approx 1{,}61803 \quad \phi = \frac{1-\sqrt{5}}{2} = 1 - \tau = -1/\tau \approx -0{,}61803$$

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1 597, 2 584, 4 181, 6 765, 10 946, …

Números de Lucas:

$$L_{n+2} = L_{n+1} + L_n \quad L_0 = 2, L_1 = 1 \qquad \sum_{n\geq 0} L_n z^n = \frac{2-z}{1-z-z^2}$$

$$L_n = F_{n+1} + F_{n-1} \qquad L_n = \tau^n + \phi^n$$

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1 364, 2 207, 3 571, 5 778, 9 349, 15 127, …

Números harmónicos:

$$H_n = \sum_{1\leq k\leq n} \frac{1}{k} \qquad H_n = \ln n + \gamma + \frac{1}{2n} - \frac{1}{12n^2} + \frac{1}{120n^4} - \frac{1}{252n^6} + O(n^{-8}) \qquad \gamma = 0{,}5772156649$$

$$H(z) = \frac{1}{1-z} \ln \frac{1}{1-z}$$

Números de Catalan:

$$C_{n+1} = \sum_{0\leq k\leq n} C_k C_{n-k} \quad C_0 = 1 \qquad C_n = \frac{1}{n+1}\binom{2n}{n} \qquad C(z) = 1 + zC^2(z) \qquad \sum_{n\geq 0} C_n z^n = \frac{1-\sqrt{1-4z}}{2z}$$

1, 1, 2, 5, 14, 42, 132, 429, 1 430, 4 862, 16 796, 58 786, 208 012, 742 900, 2 674 440, 9 694 845, 35 357 670, …

Números de Motzkin:

$$m_{n+2} = m_{n+1} + \sum_{0\leq k\leq n} m_k m_{n-k} \quad m_0 = m_1 = 1 \qquad \sum_{n\geq 0} m_n z^n = \frac{1-z-\sqrt{1-2z-3z^2}}{2z}$$

1, 1, 2, 4, 9, 21, 51, 127, 323, 835, 2 188, 5 798, 15 511, 41 835, 113 634, 310 572, 853 467, 2 356 779, …

Coeficientes binomiales: Cuentan subconjuntos de k elementos tomados de n

$$\binom{n}{0} = 1 \quad \binom{0}{k} = [k=0] \qquad \binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k} \qquad \binom{\alpha}{k} = \frac{\alpha^{\underline{k}}}{k!} \qquad \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$\sum_{k,n} \binom{n}{k} x^k y^n = \frac{1}{1-(1+x)y} \qquad \sum_{k\geq 0} \binom{n}{k} x^k = (1+x)^n \qquad \sum_{n\geq 0} \binom{n}{k} y^n = \frac{y^k}{(1-y)^{k+1}}$$

$$\binom{\alpha}{k} = \frac{\alpha^{\underline{k}}}{k!} \qquad \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Multiconjuntos: Cuentan multiconjuntos de k elementos elegidos entre n

$$\left(\!\binom{n}{0}\!\right) = 1 \quad \left(\!\binom{0}{k}\!\right) = [k=0] \qquad \left(\!\binom{n+1}{k+1}\!\right) = \left(\!\binom{n}{k+1}\!\right) + \left(\!\binom{n+1}{k}\!\right) \qquad \left(\!\binom{n}{k}\!\right) = \binom{n-1+k}{n-1}$$

$$\sum_{k,n} \left(\!\binom{n}{k}\!\right) x^k y^n = \frac{1-x}{1-x-y} \qquad \sum_{k\geq 0} \left(\!\binom{n}{k}\!\right) x^k = \frac{1}{(1-x)^n} \qquad \sum_{n\geq 0} \left(\!\binom{n}{k}\!\right) y^n = \frac{y^{[k>0]}}{(1-y)^k}$$

$n=0$:	1						
$n=1$:	1	1					
$n=2$:	1	2	1				
$n=3$:	1	3	3	1			
$n=4$:	1	4	6	4	1		
$n=5$:	1	5	10	10	5	1	
$n=6$:	1	6	15	20	15	6	1

Cuadro 1: Triángulo de Pascal

Números de Stirling de primera especie: Cuentan el número de permutaciones de n elementos con k ciclos.

$$\begin{bmatrix} n \\ 0 \end{bmatrix} = [n=0] \qquad \begin{bmatrix} n \\ n \end{bmatrix} = 1 \qquad \begin{bmatrix} n+1 \\ k+1 \end{bmatrix} = n \begin{bmatrix} n \\ k+1 \end{bmatrix} + \begin{bmatrix} n \\ k \end{bmatrix}$$

$$z^{\underline{n}} = \sum_{k \geq 0} (-1)^{n-k} \begin{bmatrix} n \\ k \end{bmatrix} z^k \qquad z^{\overline{n}} = \sum_{k} \begin{bmatrix} n \\ k \end{bmatrix} z^k \qquad C(z,u) = \sum_{\substack{n \geq 0 \\ k \geq 0}} \begin{bmatrix} n \\ k \end{bmatrix} u^k \frac{z^n}{n!} = (1-z)^{-u}$$

$n=0$:	1						
$n=1$:	0	1					
$n=2$:	0	1	1				
$n=3$:	0	2	3	1			
$n=4$:	0	6	11	6	1		
$n=5$:	0	24	50	35	10	1	
$n=6$:	0	120	274	225	85	15	1

Cuadro 2: Números de Stirling de primera especie

Números de Stirling de segunda especie: Cuentan el número de particiones de n elementos en k clases.

$$\begin{Bmatrix} n \\ 0 \end{Bmatrix} = [n=0] \qquad \begin{Bmatrix} n \\ n \end{Bmatrix} = 1 \qquad \begin{Bmatrix} n+1 \\ k+1 \end{Bmatrix} = (k+1) \begin{Bmatrix} n \\ k+1 \end{Bmatrix} + \begin{Bmatrix} n \\ k \end{Bmatrix}$$

$$z^n = \sum_{k} \begin{Bmatrix} n \\ k \end{Bmatrix} z^{\underline{k}} \qquad z^n = \sum_{k} (-1)^{n-k} \begin{Bmatrix} n \\ k \end{Bmatrix} z^{\overline{k}} \qquad S_k(y) = \sum_{n \geq 0} \begin{Bmatrix} n \\ k \end{Bmatrix} y^n = \sum_{1 \leq r \leq k} \frac{(-1)^{k-r} r^{k-1}}{(r-1)!(k-r)!} \cdot \frac{y^k}{1-ry}$$

$$S(z,u) = \sum_{\substack{n \geq 0 \\ k \geq 0}} \begin{Bmatrix} n \\ k \end{Bmatrix} u^k \frac{z^n}{n!} = \exp\big(u(\mathrm{e}^z - 1)\big)$$

Números de Lah: Cuentan el número de formas de organizar n elementos en k secuencias.

$$\left\lfloor \begin{matrix} n \\ 0 \end{matrix} \right\rfloor = [n=0] \qquad \left\lfloor \begin{matrix} n \\ n \end{matrix} \right\rfloor = 1 \qquad \left\lfloor \begin{matrix} n+1 \\ k+1 \end{matrix} \right\rfloor = (n+k+1) \left\lfloor \begin{matrix} n \\ k+1 \end{matrix} \right\rfloor + \left\lfloor \begin{matrix} n \\ k \end{matrix} \right\rfloor$$

$n=0$:	1						
$n=1$:	0	1					
$n=2$:	0	1	1				
$n=3$:	0	1	3	1			
$n=4$:	0	1	7	6	1		
$n=5$:	0	1	15	25	10	1	
$n=6$:	0	1	31	90	65	15	1

Cuadro 3: Números de Stirling de segunda especie

$$z^{\overline{n}} = \sum_k \left\lfloor \begin{matrix} n \\ k \end{matrix} \right\rfloor z^{\underline{k}} \qquad z^{\underline{n}} = \sum_k (-1)^{n-k} \left\lfloor \begin{matrix} n \\ k \end{matrix} \right\rfloor z^{\overline{k}} \qquad \left\lfloor \begin{matrix} n \\ k \end{matrix} \right\rfloor = \frac{n!}{k!} \binom{n-1}{k-1}$$

$$L(z, u) = \sum_{\substack{n \geq 0 \\ k \geq 0}} \left\lfloor \begin{matrix} n \\ k \end{matrix} \right\rfloor u^k \frac{z^n}{n!} = \exp\left(uz(1-z)^{-1}\right)$$

$n=0$:	1						
$n=1$:	0	1					
$n=2$:	0	1	1				
$n=3$:	0	6	6	1			
$n=4$:	0	24	36	12	1		
$n=5$:	0	120	240	120	20	1	
$n=6$:	0	720	1800	1200	300	30	1

Cuadro 4: Números de Lah

Números de Bell: Número total de formas de particionar n elementos.

$$B_n = \sum_{0 \leq k \leq n} \left\{ \begin{matrix} n \\ k \end{matrix} \right\} \qquad B_0 = 1 \quad B_{n+1} = \sum_{0 \leq k \leq n} \binom{n}{k} B_k$$

$$\widehat{B}(z) = \sum_{n \geq 0} B_n \frac{z^n}{n!} = \mathrm{e}^{\mathrm{e}^z - 1}$$

1, 1, 2, 5, 15, 52, 203, 877, 4 140, 21 147, 115 975, 678 570, 4 213 597, 27 644 437, 190 899 322, 1 382 958 545, …

Números de Bell ordenados: Número de formas de particionar n elementos, el orden de las particiones importa ("competencias con empate").

$$R_n = \sum_{k \geq 0} \frac{k^n}{2^{k+1}} \qquad 2R_n = [n = 0] + \sum_{1 \leq k \leq n} \binom{n}{k} R_{n-k}$$

$$\widehat{R}(z) = \sum_{n \geq 0} R_n \frac{z^n}{n!} = \frac{1}{2 - \mathrm{e}^z}$$

1, 1, 3, 13, 75, 541, 4 683, 47 293, 545 835, 7 087 261, 102 247 563, 1 622 632 573, 28 091 567 595, 526 858 348 381, …

Método simbólico

Teorema (Método simbólico, OGF; objetos no rotulados)**.** *Sean $\mathscr{A}$ y $\mathscr{B}$ clases de objetos, con funciones generatrices ordinarias respectivamente $A(z)$ y $B(z)$. Entonces funciones generatrices ordinarias enumeran:*

1. $\mathscr{A}+\mathscr{B}$: $A(z)+B(z)$ *2.* $\mathscr{A}\times\mathscr{B}$: $A(z)\cdot B(z)$ *3.* $\mathscr{A}^\bullet$: $zA'(z)$ *4.* $\mathscr{A}\circ\mathscr{B}$: $A(B(z))$ *5.* $\textsc{Seq}(\mathscr{A})$: $1/(1-A(z))$
6. $\textsc{Set}(\mathscr{A})$: $\prod_{n\geq 0}(1+z^n)^{a_n}=\exp\left(\sum_{k\geq 1}(-1)^{k+1}A(z^k)/k\right)$ *7.* $\textsc{MSet}(\mathscr{A})$: $\prod_{n\geq 1}(1-z^n)^{-a_n}=\exp\left(\sum_{k\geq 1}A(z^k)/k\right)$
8. $\textsc{Cyc}(\mathscr{A})$: $\sum_{n\geq 1}\frac{\phi(n)}{n}\ln\frac{1}{1-A(z^n)}$

Teorema (Método simbólico, EGF; objetos rotulados)**.** *Sean $\mathscr{A}$ y $\mathscr{B}$ clases de objetos, con funciones generatrices exponenciales $\widehat{A}(z)$ y $\widehat{B}(z)$, respectivamente. Entonces funciones generatrices exponenciales enumeran:*

1. $\mathscr{A}+\mathscr{B}$: $\widehat{A}(z)+\widehat{B}(z)$ *2.* $\mathscr{A}\star\mathscr{B}$: $\widehat{A}(z)\cdot\widehat{B}(z)$ *3.* $\mathscr{A}^\bullet$: $z\widehat{A}'(z)$ *4.* $\mathscr{A}\circ\mathscr{B}$: $\widehat{A}(\widehat{B}(z))$ *5.* $\textsc{Seq}(\mathscr{A})$: $1/(1-\widehat{A}(z))$
6. $\textsc{MSet}(\mathscr{A})$: $\exp(\widehat{A}(z))$ *7.* $\textsc{Cyc}(\mathscr{A})$: $-\ln(1-\widehat{A}(z))$ *8.* $\mathscr{A}^\square\star\mathscr{B}$: $\int_0^z\widehat{A}'(u)\cdot\widehat{B}(u)\,\mathrm{d}u$

Fórmula de inversión de Lagrange

Teorema. *Sean $f(u)$ y $\phi(u)$ series formales de potencias en u, con $\phi(0)=1$. Hay una única serie formal $u=u(t)$ tal que $u=t\phi(u)$. El valor $f(u(t))$ expandido en serie alrededor de $t=0$ es:*

$$\left[t^n\right]\left\{f(u(t))\right\}=\frac{1}{n}\left[u^{n-1}\right]\left\{f'(u)\phi(u)^n\right\}$$

Principio de Inclusión y Exclusión

Sea Ω un conjunto de objetos, $\mathscr{P}$ un conjunto de propiedades de los objetos. Para $\mathscr{S}\subseteq\mathscr{P}$ sea $N(\supseteq\mathscr{S})$ el número de objetos con las propiedades en $\mathscr{S}$, y e_t el número de objetos con exactamente t propiedades.

$$N_r=\sum_{|\mathscr{S}|=r}N(\supseteq\mathscr{S})\qquad N(z)=\sum_r N_r z^r\qquad E(z)=\sum_t e_t z^t\qquad E(z)=N(z-1)$$

$$e_0=E(0)=N(-1)\qquad e_t=\frac{E^{(t)}(0)}{t!}=\frac{N^{(t)}(-1)}{t!}\qquad \mathbb{E}[t]=\frac{N_1}{N_0}\qquad \operatorname{var}[t]=\frac{2N_2+N_1}{N_0}-\frac{N_1^2}{N_0^2}$$

Fórmula de Euler-Maclaurin

$$\sum_{1\leq k<a}f(k)=\int_1^a f(z)\,\mathrm{d}z+\gamma_f+B_1 f(a)+\sum_{1\leq k\leq n}\frac{B_{2k}}{(2k)!}f^{(2k-1)}(a)+R_n(a)$$

$$\gamma_f=\lim_{a\to\infty}\sum_{1\leq k<a}f(k)-\int_1^a f(z)\,\mathrm{d}z\qquad |R_n(a)|\leq\frac{|B_{2n+2}|}{(2n+2)!}\,|f^{(2n+2)}(a)|$$

$$B_0(x)=1\qquad B_n'(x)=nB_{n-1}(x)\qquad \int_0^1 B_n(x)\,\mathrm{d}x=[n=0]\qquad B_n=B_n(0)=B_n(1)\text{ si }n\neq 1$$

$$B(x,y)=\sum_{n\geq 0}B_n(x)y^n=\frac{y\mathrm{e}^{xy}}{\mathrm{e}^y-1}$$

$$B_n(x+y)=\sum_{0\leq k\leq n}\binom{n}{k}B_{n-k}(x)y^k$$

Análisis complejo

Derivadas

$$f(z) = f(x + \mathrm{i}y) = u(x, y) + \mathrm{i}v(x, y) \qquad \frac{\mathrm{d}f}{\mathrm{d}z} = \frac{\partial u}{\partial x} + \mathrm{i}\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - \mathrm{i}\frac{\partial u}{\partial y} \qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Teorema de Cauchy

Si f es holomorfa sobre la curva simple cerrada γ y en su interior:

$$\int_\gamma f(z)\,\mathrm{d}z = 0 \qquad \frac{1}{2\pi\mathrm{i}}\int_\gamma \frac{f(z)}{(z-z_0)^{n+1}}\,\mathrm{d}z = f^{(n)}(z_0)$$

Si f es holomorfa sobre γ, y salvo singularidades aisladas z_k es holomorfa a su interior:

$$\frac{1}{2\pi\mathrm{i}}\int_\gamma f(z)\,\mathrm{d}z = \sum_{z_k} \mathrm{res}(f, z_k)$$

Para el residuo en un polo simple z_0; si $f(z) = g(z)/h(z)$, $g(z_0) \neq 0$ y $h(z)$ tiene un cero simple en z_0:

$$\mathrm{res}(f, z_0) = \lim_{z\to z_0} (z - z_0) f(z) = \frac{g(z_0)}{h'(z_0)}$$

En un polo de orden m en z_0:

$$\mathrm{res}(f, z_0) = \frac{1}{(m-1)!} \lim_{z\to z_0} \frac{\mathrm{d}^{m-1}}{\mathrm{d}z^{m-1}} \left((z - z_0)^m f(z)\right)$$

Funciones Gamma y Beta

$$\Gamma(z) = \int_0^\infty t^{z-1}\mathrm{e}^{-t}\,\mathrm{d}t \qquad \Gamma(z+1) = z\Gamma(z) = z! \qquad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z} \qquad \Gamma(1/2) = \sqrt{\pi}$$

$$B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,\mathrm{d}t = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

Lista de series frecuentes

$\sum_{n\geq 0} z^n$	$\frac{1}{1-z}$	$\sum_{n\geq 0} nz^n$	$\frac{z}{(1-z)^2}$
$\sum_{n\geq 0} n^2 z^n$	$\frac{z+z^2}{(1-z)^3}$	$\sum_{n\geq 1} \frac{z^n}{n}$	$-\ln(1-z)$
$\sum_{n\geq 0} \frac{z^n}{n!}$	e^z	$\sum_{n\geq 0} \frac{(-1)^n z^n}{n!}$	e^{-z}
$\sum_{n\geq 0} \frac{z^{2n}}{(2n)!}$	$\cosh z$	$\sum_{n\geq 0} \frac{z^{2n+1}}{(2n+1)!}$	$\sinh z$
$\sum_{n\geq 0} \frac{(-1)^n z^{2n}}{(2n)!}$	$\cos z$	$\sum_{n\geq 0} \frac{(-1)^n z^{2n+1}}{(2n+1)!}$	$\sin z$
$\sum_{k\geq 0} \binom{\alpha}{k} z^k$	$(1+z)^\alpha$	$\sum_{n\geq 0} \binom{n}{k} z^n$	$\frac{z^k}{(1-z)^{k+1}}$
$\sum_{k\geq 0} \left(\!\!\binom{n}{k}\!\!\right) z^k$	$\frac{1}{(1-z)^n}$	$\sum_{n\geq 0} \left(\!\!\binom{n}{k}\!\!\right) z^n$	$\frac{z^{[k>0]}}{(1-z)^{k+1}}$
$\sum_{n\geq 0} \binom{n+k}{k} z^n$	$\frac{1}{(1-z)^{k+1}}$	$\sum_{n\geq 0} \binom{2n}{n} z^n$	$\frac{1}{\sqrt{1-4z}}$
$\sum_{n\geq 0} C_n z^n$	$\frac{1-\sqrt{1-4z}}{2z}$	$\sum_{n\geq 1} H_n z^n$	$\frac{1}{1-z} \ln \frac{1}{1-z}$
$\sum_{n\geq 0} F_n z^n$	$\frac{z}{1-z-z^2}$	$\sum_{n\geq 0} F_{n+1} z^n$	$\frac{1}{1-z-z^2}$