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Límites

Calcule, en caso de existir, los siguientes límites:

1. $\lim\limits_{(x,y)\to(0,0)} \frac{sen(xy)}{sen(x)sen(y)}$

2. $\lim\limits_{(x,y)\to(0,0)} \frac{8x^3+y^3}{2x+y}$

3. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y^3}{x^4+y^2}$

4. $\lim\limits_{(x,y)\to(0,0)} \frac{x^4-2y^3}{x^2+|y|}$

5. $\lim\limits_{(x,y)\to(0,0)} \frac{sen(x^y)sen(y^2)}{x^6+|y|}$

6. $\lim\limits_{(x,y)\to(0,0)} \frac{y^4}{x^2+2y^2-2xy}$

7. $\lim\limits_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^2}$

Solución

1. $\lim\limits_{(x,y)\to(0,0)} \frac{sen(xy)}{sen(x)sen(y)}$

$= \lim\limits_{(x,y)\to(0,0)} \frac{sen(xy)}{xy} \cdot \frac{x}{sen(x)} \cdot \frac{y}{sen(y)} = 1$ (por álgebra de límites)

2. $\lim\limits_{(x,y)\to(0,0)} \frac{8x^3+y^3}{2x+y}$

$= \lim\limits_{(x,y)\to(0,0)} \frac{(2x+y)(4x^2-2xy+y^2)}{2x+y}$

$= \lim\limits_{(x,y)\to(0,0)} 4x^2 - 2xy + y^2 = 0$

3. $0 \leq \left|\frac{x^2y^3}{x^4+y^2}\right| = \frac{x^2|y|^3}{x^4+y^2} \leq x^2|y|$

$$\lim_{(x,y)\to(0,0)} 0 = \lim_{(x,y)\to(0,0)} x^2|y| = 0$$

Por Teorema de acotamiento, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y^3}{x^4+y^2} = 0$

4. $0 \le \left|\frac{x^4-2y^3}{x^2+|y|}\right| \le \frac{x^4}{x^2+|y|} + \frac{2|y|^3}{x^2+|y|} \le x^2 + 2y^2$

$$\lim_{(x,y)\to(0,0)} 0 = \lim_{(x,y)\to(0,0)} x^2 + 2y^2 = 0$$

Por Teorema de acotamiento, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4-2y^3}{x^2+|y|} = 0$

5. $0 \le \left|\frac{sen(x^y)sen(y^2)}{x^6+|y|}\right| = \frac{|sen(x^y)||sen(y^2)|}{x^6+|y|} \le \frac{|sen(y^2)|}{x^6+|y|} \le \frac{y^2}{x^6+|y|} \le |y|$

$$\lim_{(x,y)\to(0,0)} 0 = \lim_{(x,y)\to(0,0)} |y| = 0$$

Por Teorema de acotamiento, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{sen(x^y)sen(y^2)}{x^6+|y|} = 0$

6. $0 \le \left|\frac{y^4}{x^2+2y^2-2xy}\right| = \frac{y^4}{(x-y)^2+y^2} \le y^2$

$$\lim_{(x,y)\to(0,0)} 0 = \lim_{(x,y)\to(0,0)} y^2 = 0$$

Por Teorema de acotamiento, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{y^4}{x^2+2y^2-2xy} = 0$

7. Se sabe que $xy \le x^2 + y^2 \;\Rightarrow\; x^2y \le x^4 + y^2$

$$0 \le \left|\frac{x^3y}{x^4+y^2}\right| = \frac{|x|x^2|y|}{x^4+y^2} \le \frac{|x|(x^4+y^2)}{x^4+y^2} = |x|$$

$$\lim_{(x,y)\to(0,0)} 0 = \lim_{(x,y)\to(0,0)} |x| = 0$$

Por Teorema de acotamiento, $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^2} = 0$