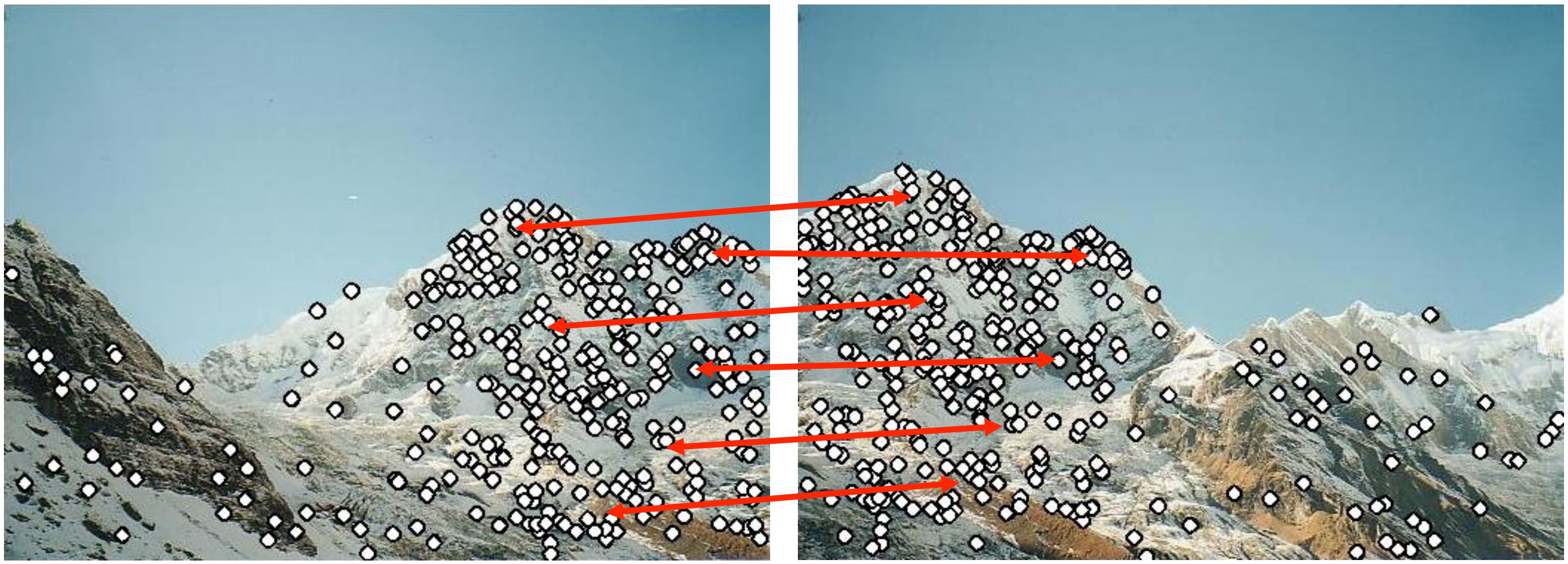
# Keypoint extraction: Corners

# Why extract keypoints?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

# Why extract keypoints?

- Motivation: panorama stitching
  - We have two images – how do we combine them?



Step 1: extract keypoints

Step 2: match keypoint features

# Why extract keypoints?

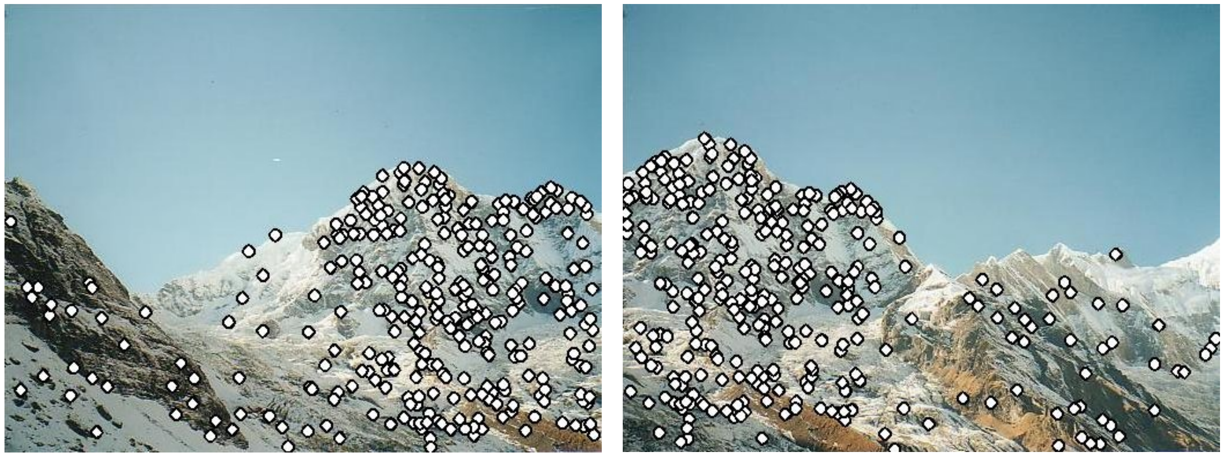
- Motivation: panorama stitching
  - We have two images – how do we combine them?

Step 1: extract keypoints

Step 2: match keypoint features

Step 3: align images

# Characteristics of good keypoints

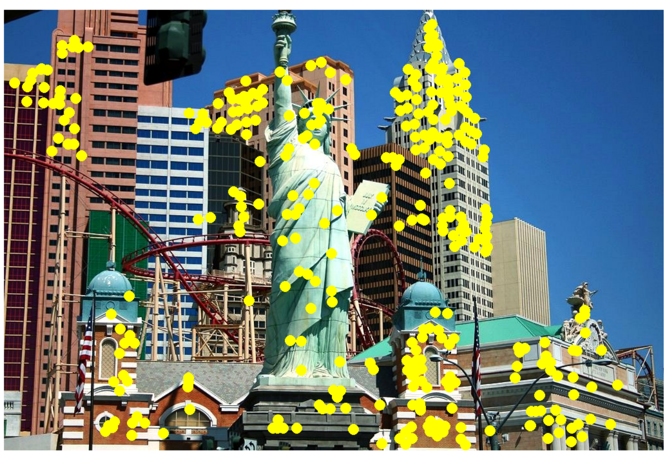


- Repeatability
  - The same keypoint can be found in several images despite geometric and photometric transformations
- Saliency
  - Each keypoint is distinctive
- Compactness and efficiency
  - Many fewer keypoints than image pixels
- Locality
  - A keypoint occupies a relatively small area of the image; robust to clutter and occlusion

# Applications

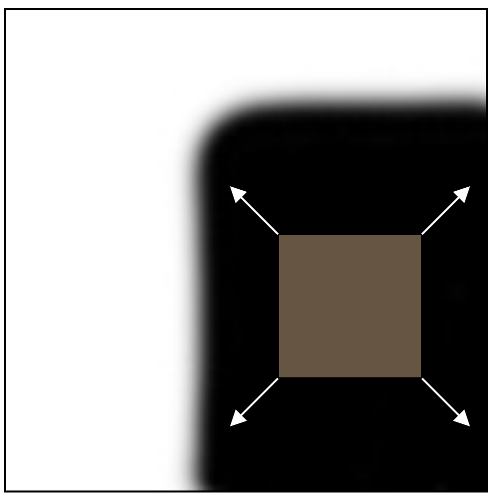
## Keypoints are used for:

- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Indexing and database retrieval
- Object recognition

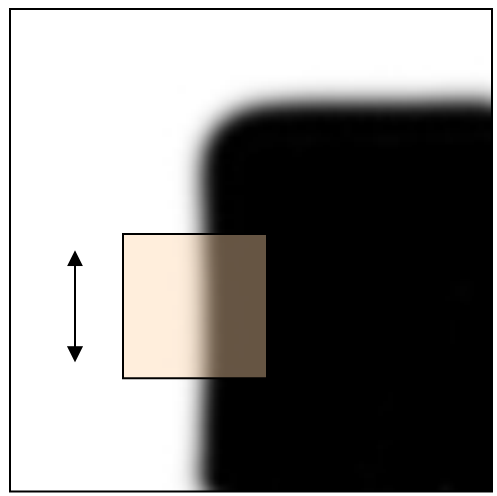


# Corner Detection: Basic Idea

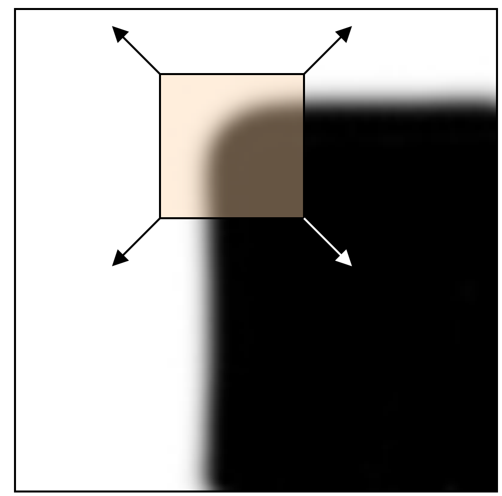
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions



"edge":
no change along the edge direction



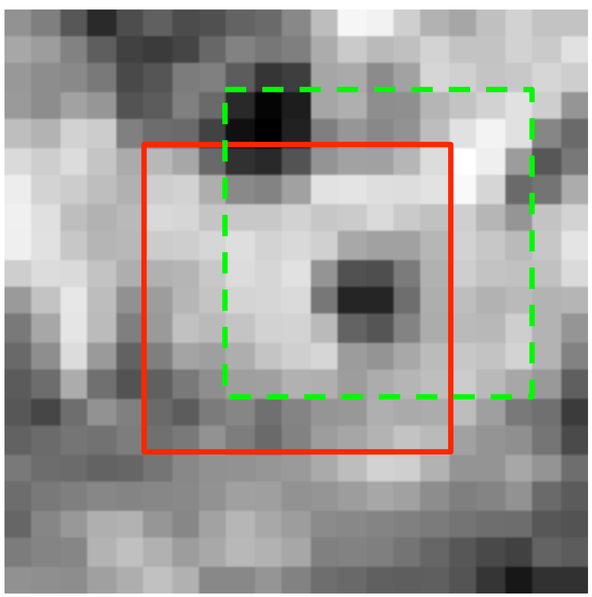
"corner":
significant change in all directions

# Corner Detection: Mathematics

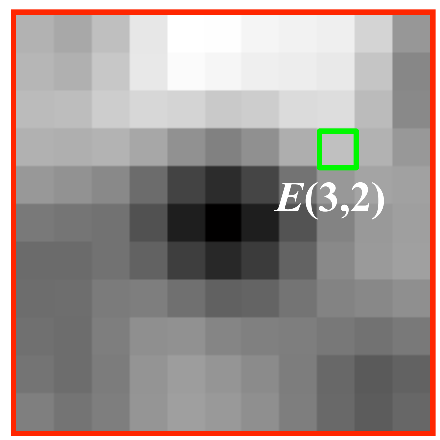
Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$I(x, y)$



$E(u, v)$

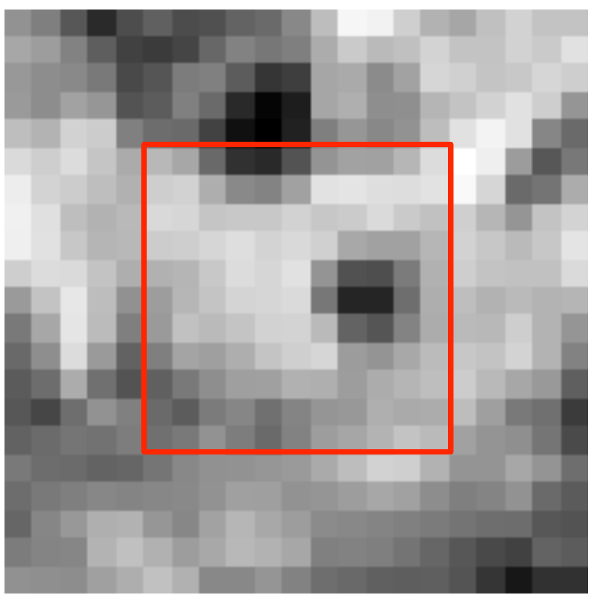


# Corner Detection: Mathematics

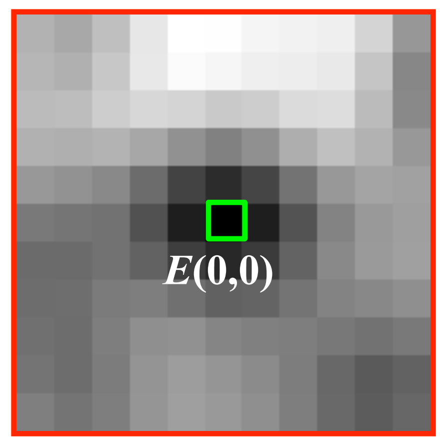
Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$I(x, y)$



$E(u, v)$



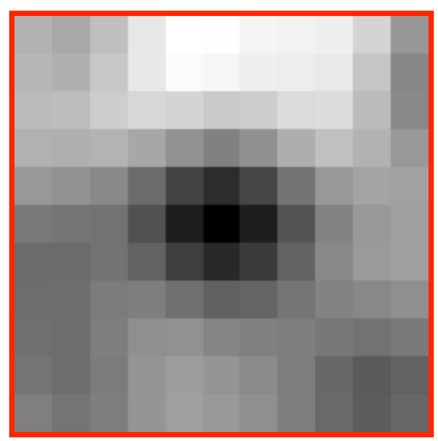
# Corner Detection: Mathematics

Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



# Corner Detection: Mathematics

- First-order Taylor approximation for small motions [*u*, *v*]:

$$I(x+u, y+v) \approx I(x, y) + I_x u + I_y v$$

- Let's plug this into *E*(*u*,*v*):

$$\begin{aligned} E(u,v) &= \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2 \\ &\approx \sum_{(x,y)\in W} [I(x,y) + I_x u + I_y v - I(x,y)]^2 \\ &= \sum_{(x,y)\in W} [I_x u + I_y v]^2 = \sum_{(x,y)\in W} I_x^2 u^2 + 2 I_x I_y u v + I_y^2 v^2 \end{aligned}$$

# Corner Detection: Mathematics

The quadratic approximation can be written as

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a *second moment matrix* computed from image derivatives:

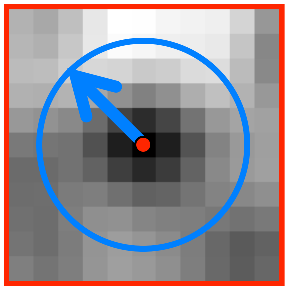
$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$

(the sums are over all the pixels in the window *W*)

# Interpreting the second moment matrix

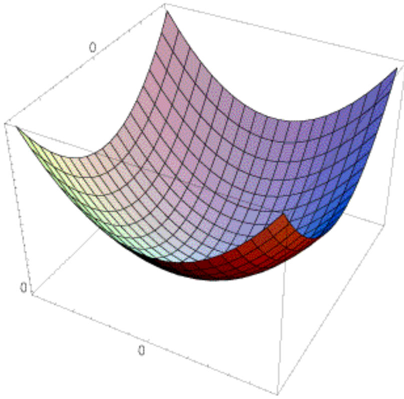
- The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.
  - Specifically, in which directions does it have the smallest/greatest change?

$E(u, v)$



$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

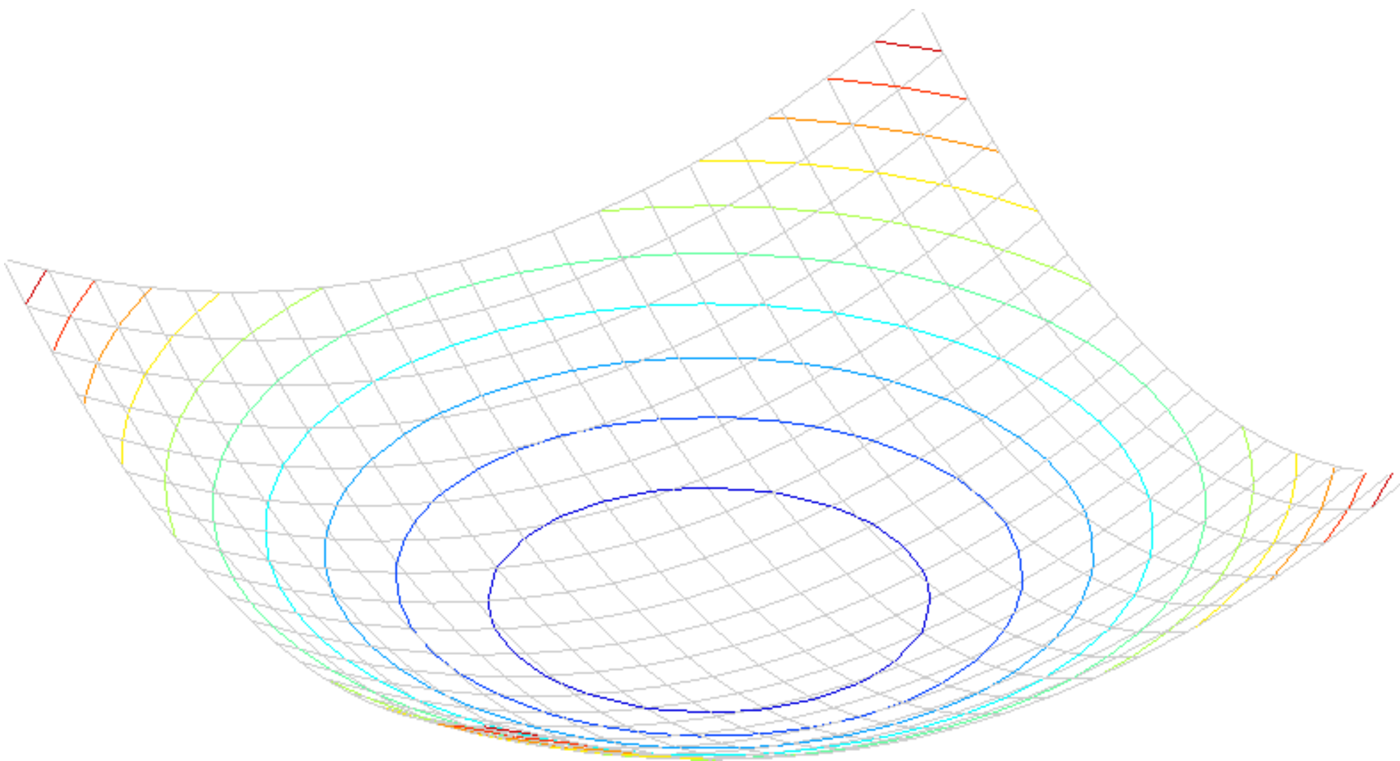
$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$



# Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $\quad [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.



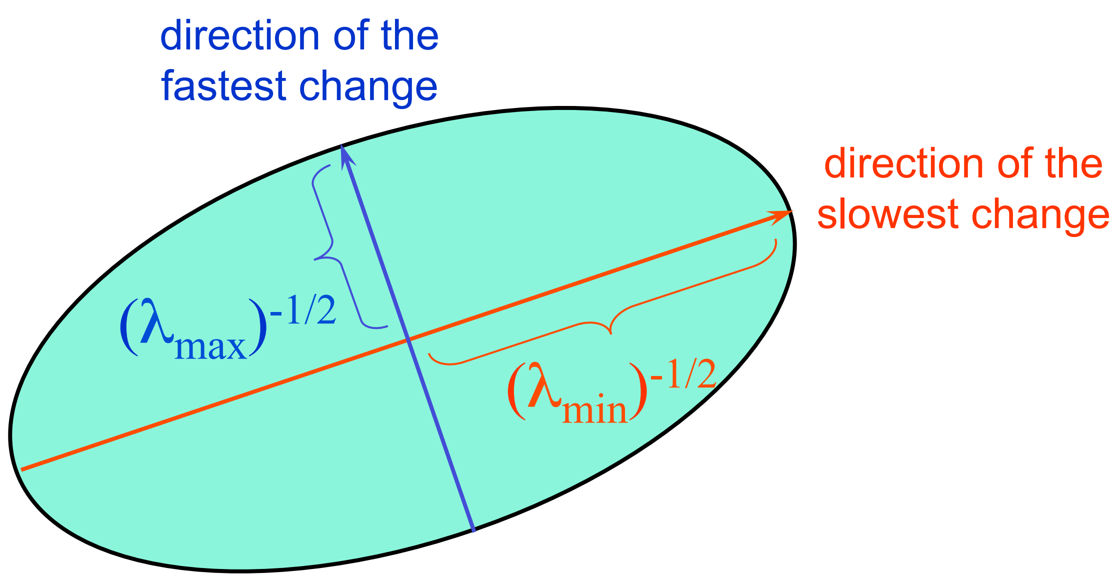
# Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $\quad [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.

Diagonalization of M: $\quad M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by *R*



# Interpreting the second moment matrix

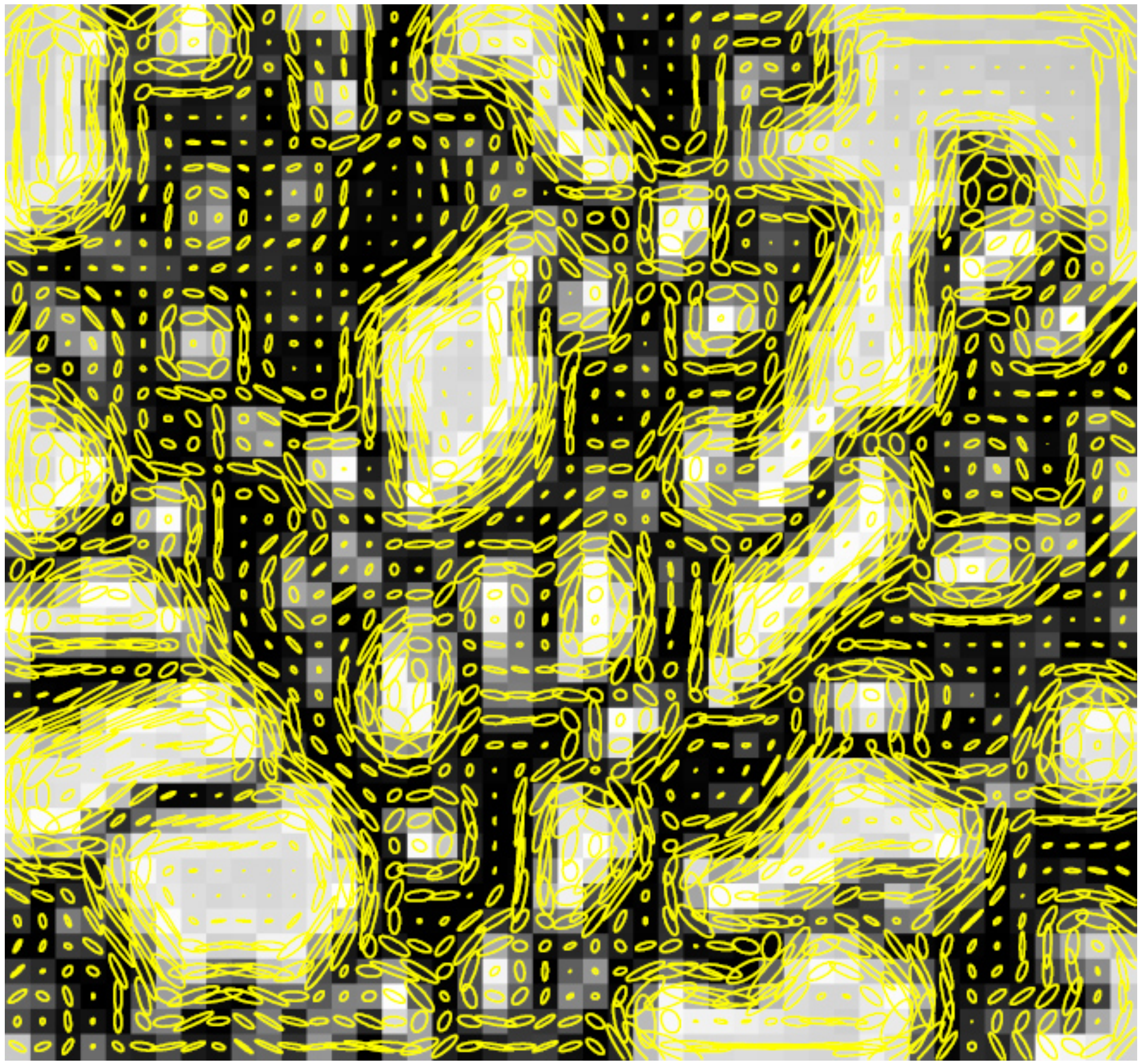
Consider the axis-aligned case (gradients are either horizontal or vertical)

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

If either *a* or *b* is close to 0, then this is **not** a corner, so look for locations where both are large.

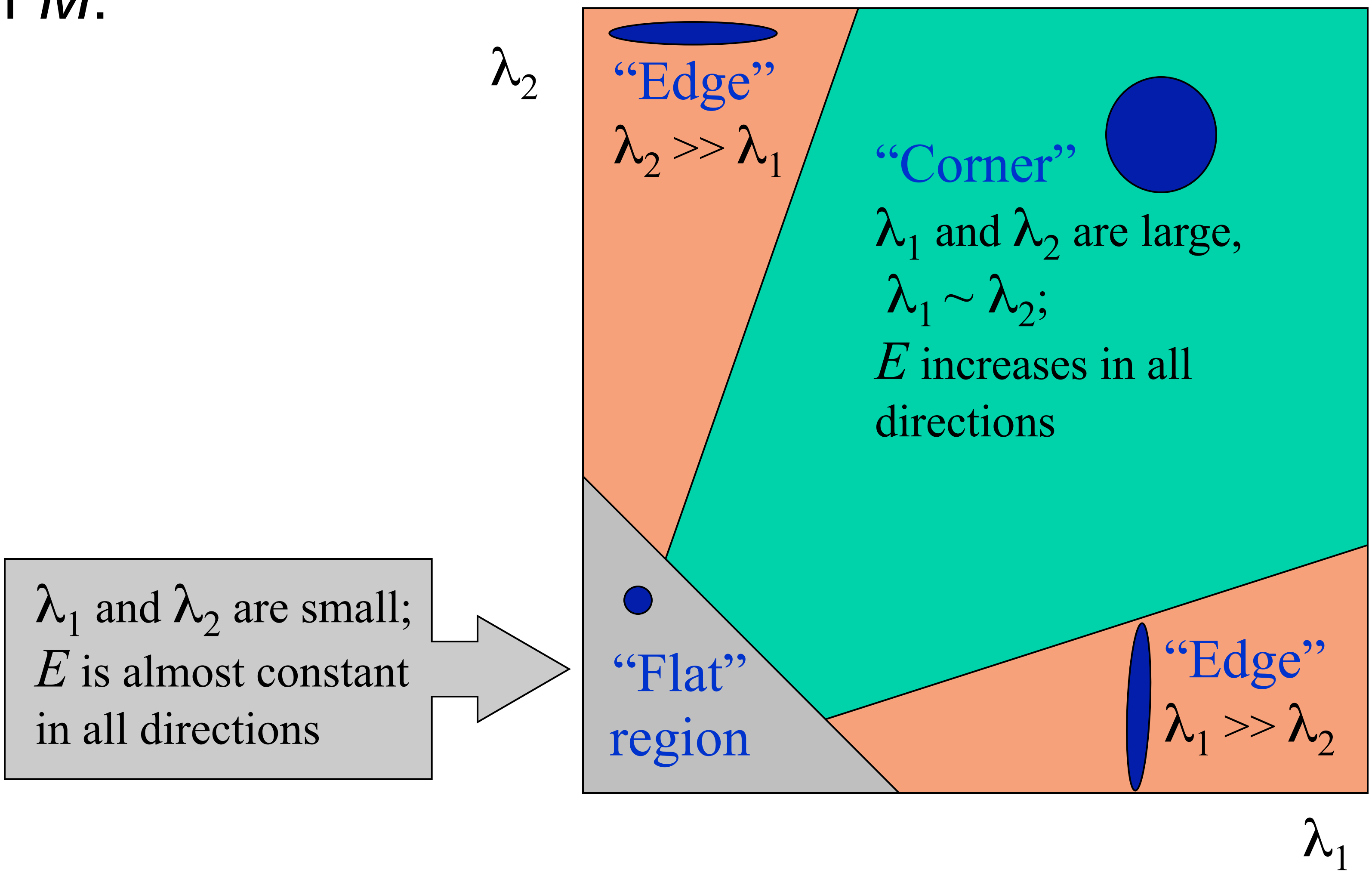
# Visualization of second moment matrices

# Visualization of second moment matrices



# Interpreting the eigenvalues

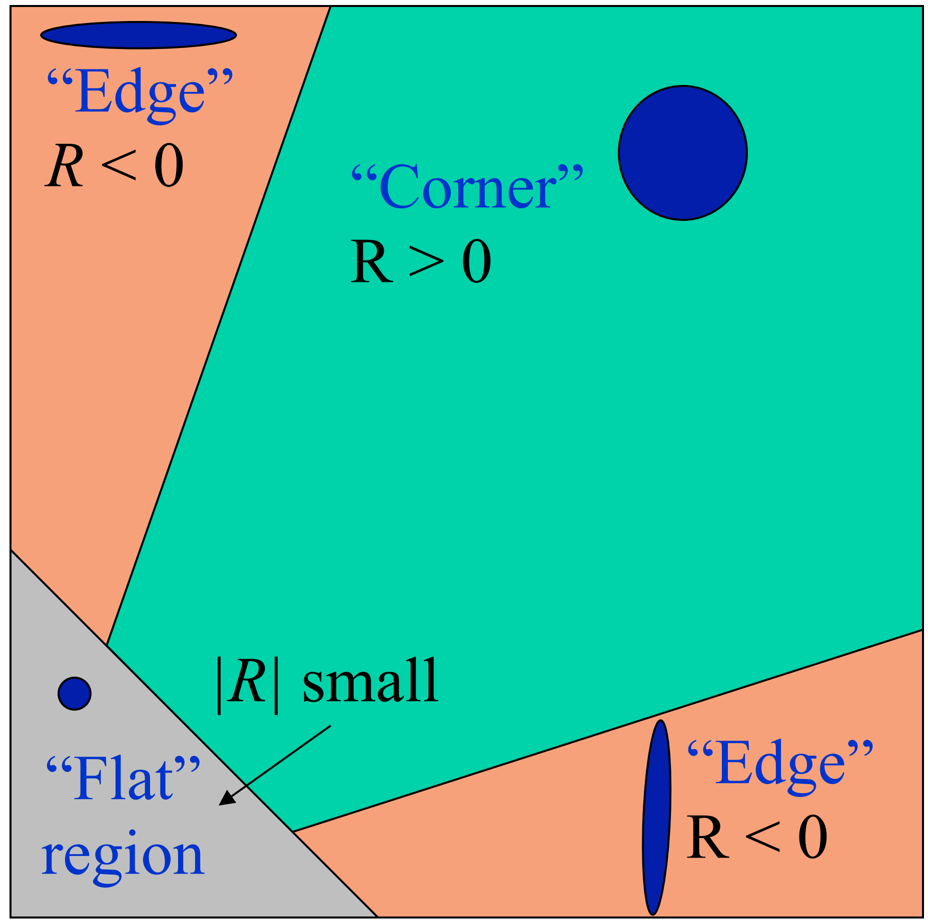
Classification of image points using eigenvalues of *M*:



# Corner response function

$$R = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

$\alpha$: constant (0.04 to 0.06)



# The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel:

$$M = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens.
"A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

# The Harris corner detector

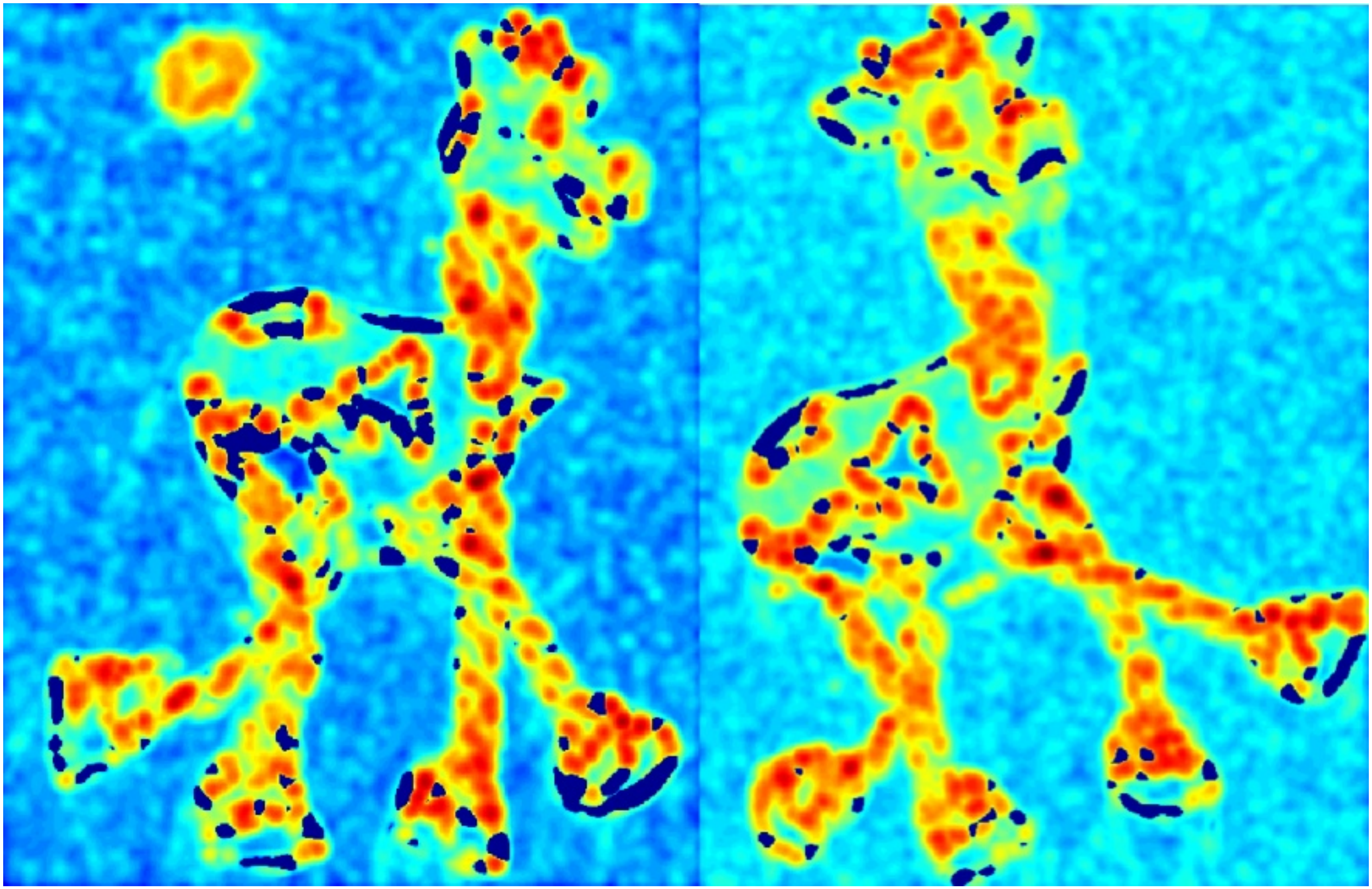
1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel
3. Compute corner response function *R*

C.Harris and M.Stephens.
"A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

# Harris Detector: Steps

# Harris Detector: Steps

Compute corner response $R$



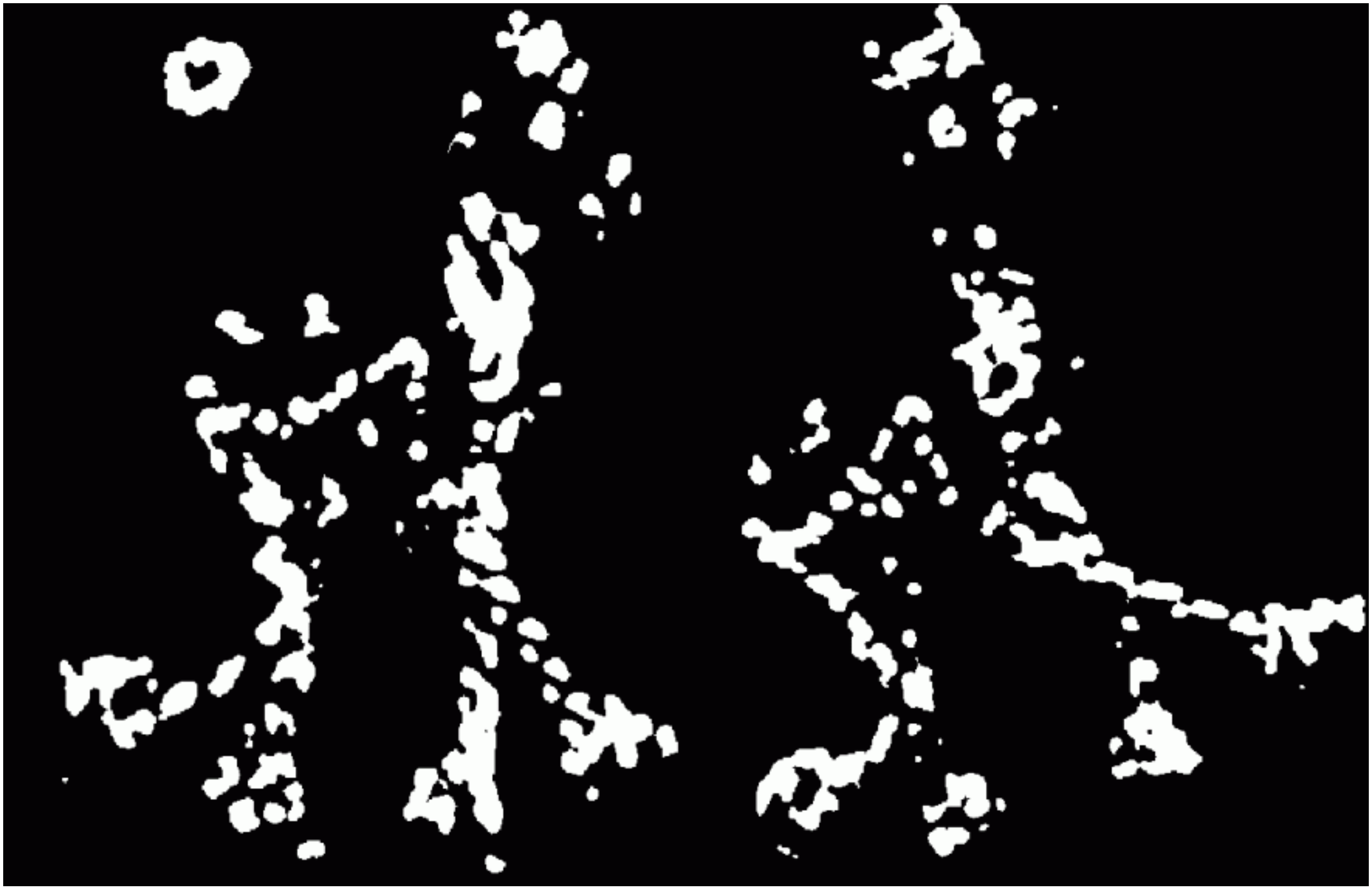
# The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel
3. Compute corner response function *R*
4. Threshold *R*
5. Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens.
"A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

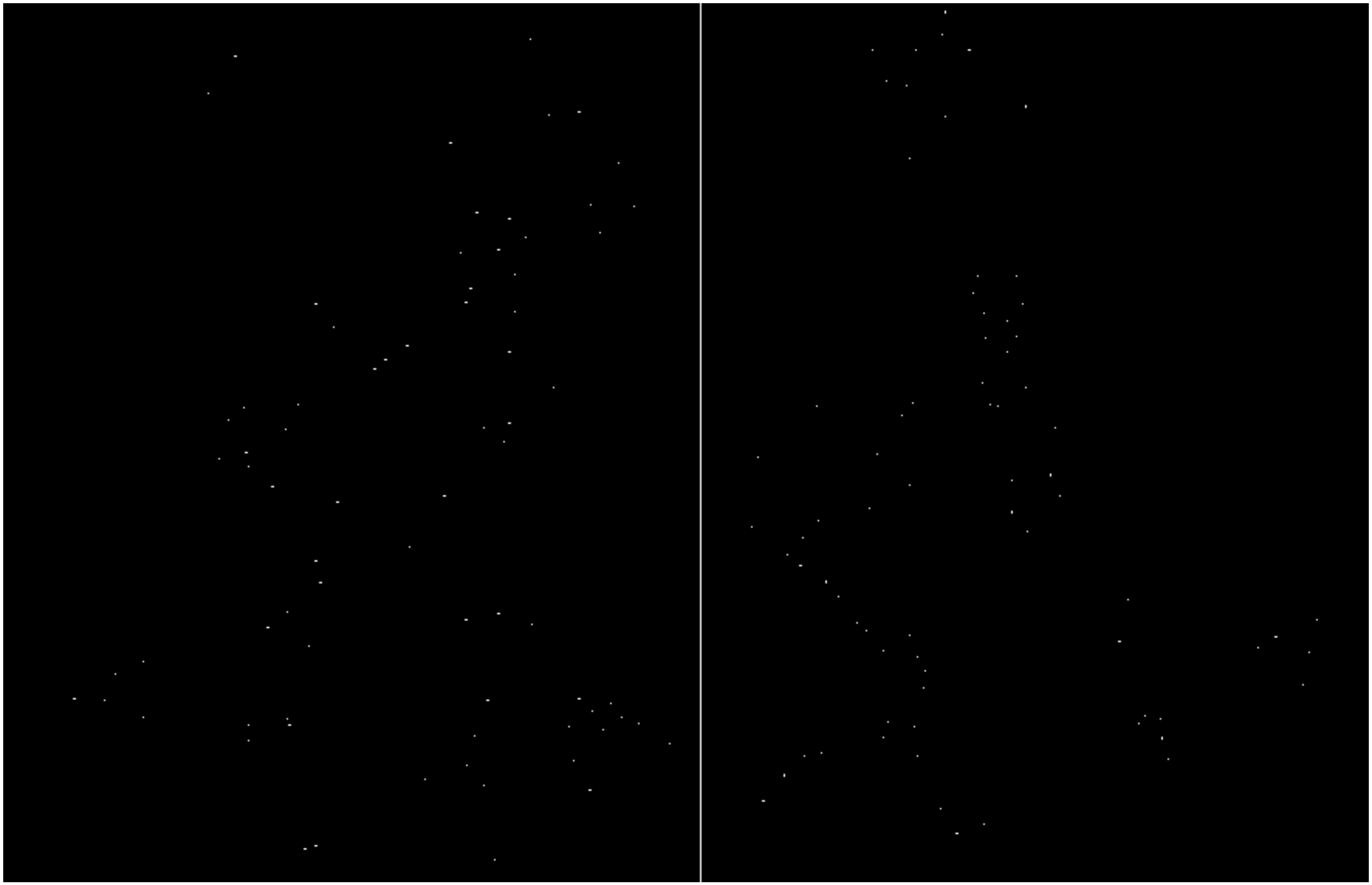
# Harris Detector: Steps

Find points with large corner response: $R >$ threshold



# Harris Detector: Steps

Take only the points of local maxima of $R$

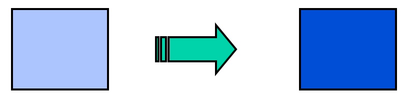


# Harris Detector: Steps

# Robustness of corner features

- What happens to corner features when the image undergoes geometric or photometric transformations?

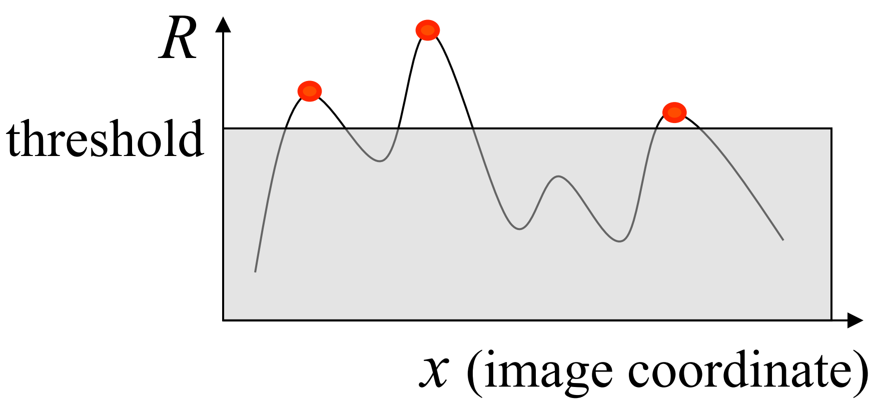
# Affine intensity change

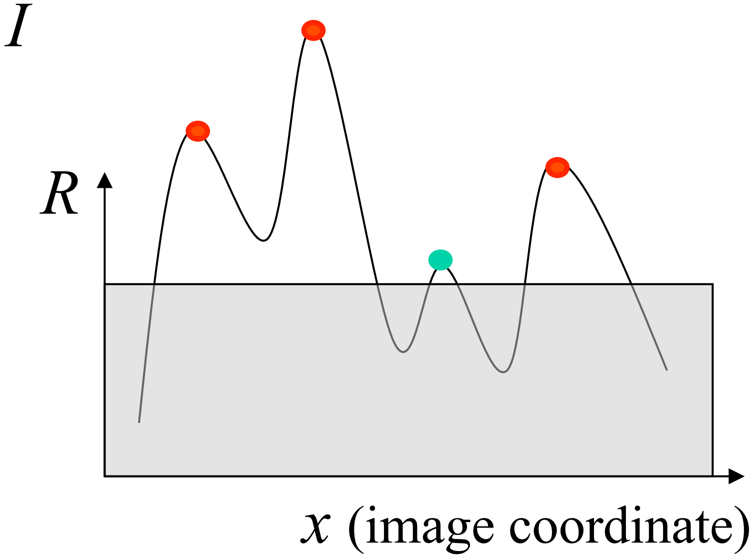


$$I \rightarrow a\,I + b$$

- Only derivatives are used =>
invariance to intensity shift $I \rightarrow I + b$

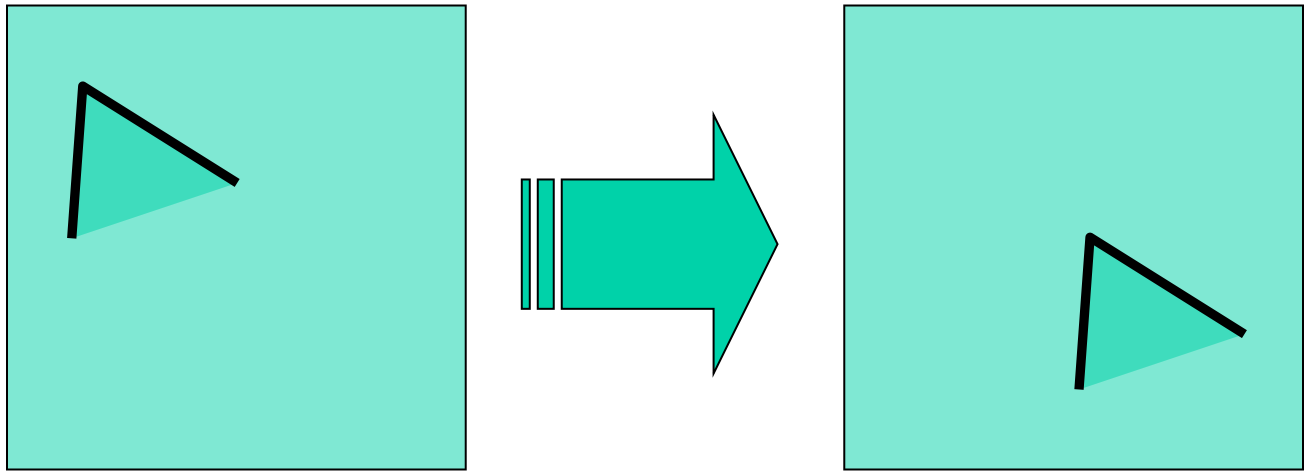
- Intensity scaling: $I \rightarrow a\,I$




*Partially invariant* to affine intensity change

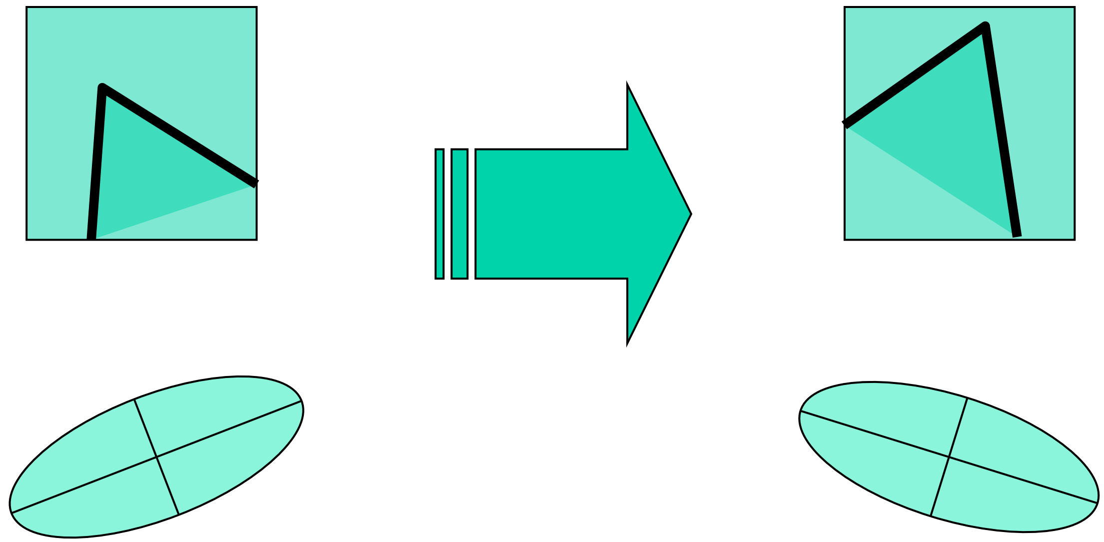
# Image translation



- Derivatives and window function are shift-invariant

Corner location is *covariant* w.r.t. translation

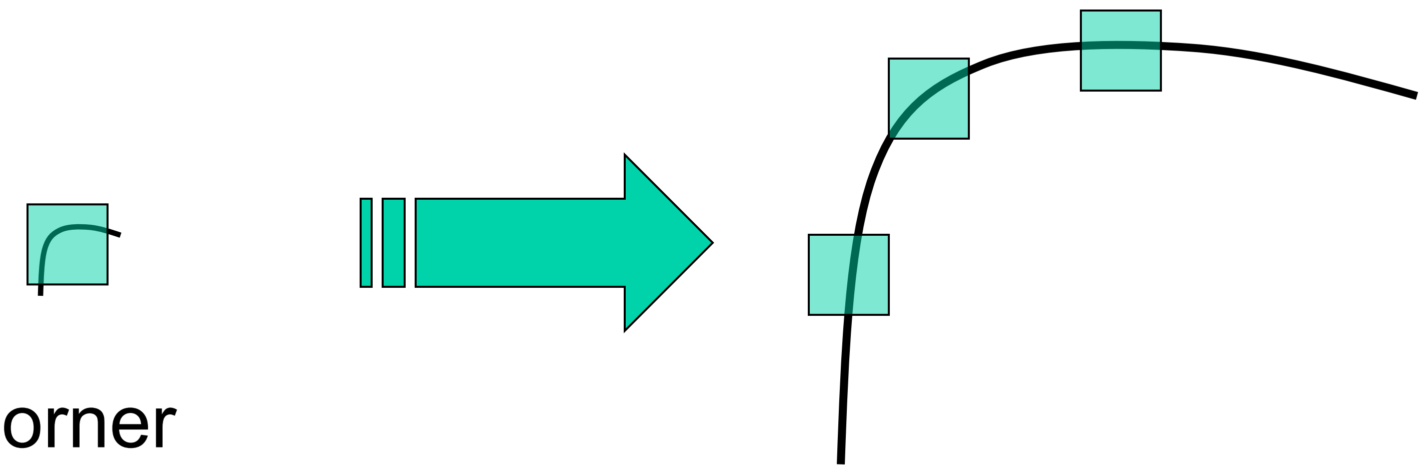
# Image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

# Scaling



Corner

All points will be classified as edges

Corner location is not covariant to scaling!