Data Structures II : Directed graphs

Mauricio Toro
Department of Systems and Informatics
Universidad EAFIT

Cocktail of the day: Colombia

Disclaimer: Keep alcohol out of the hands of minors.

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- 20 ml vodka
- 10 ml blue Curaçao
- 10 ml grenadine
- 10 ml lemon juice
- 60 ml orange juice

Videogames use graphs

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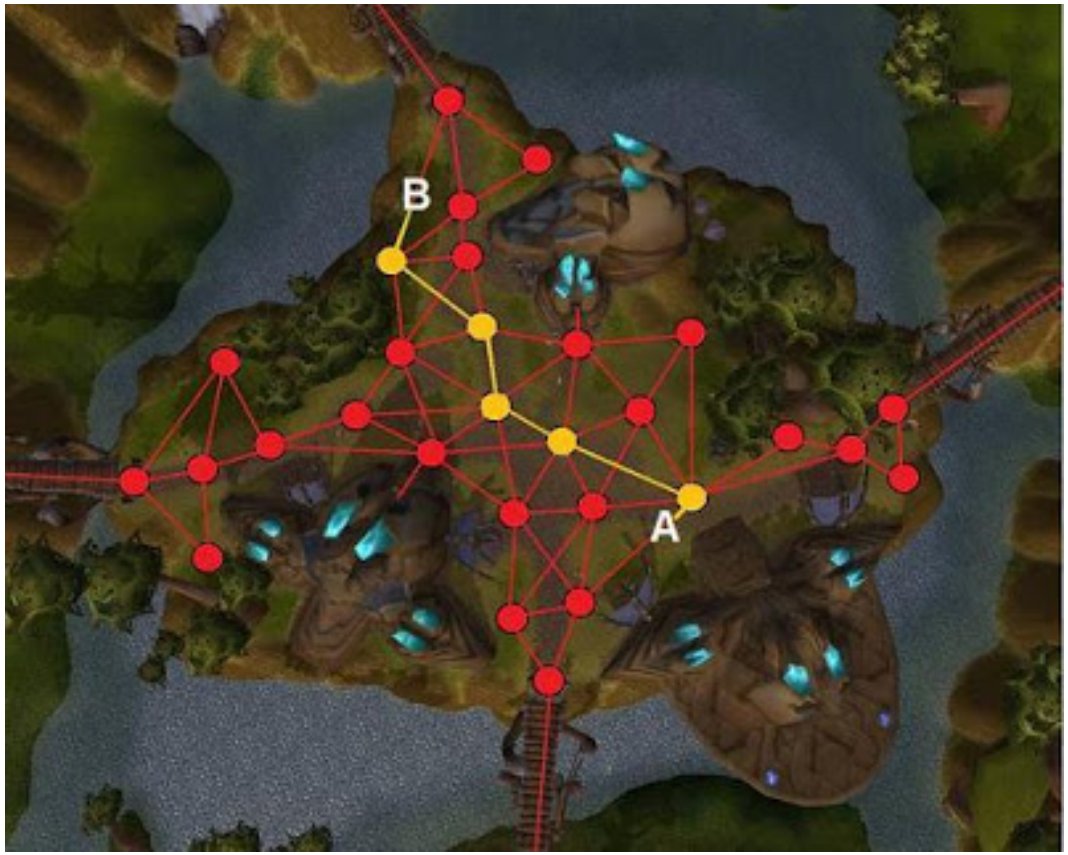
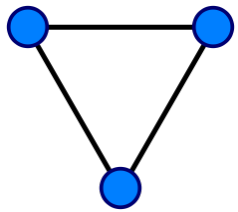


Undirected graph

- An undirected graph (graph for short) G consists of a finite set of vertices V and a set of edges E.
- $G = (V, E)$
- It differs from a directed graph in that each edge in E is an unordered pair of vertices.
- If (v, w) is an undirected edge, then $(v, w) = (w, v)$.

Taken from [Aho77].



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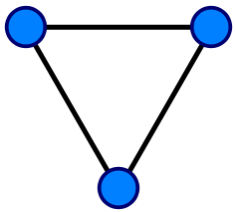
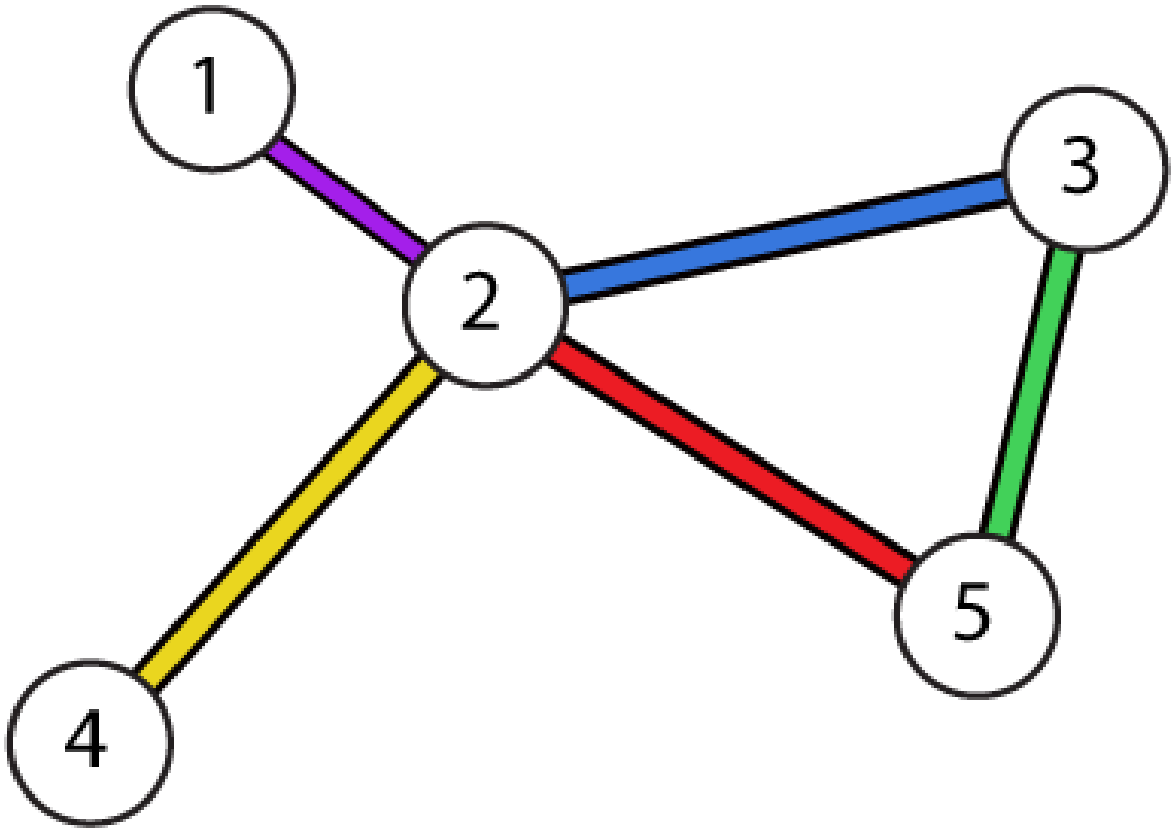


Taken from Wikipedia.

Example of an Undirected graph



Taken from `http://www.alecjacobson.com/`.

Example of a Undirected graph (2)

Simulator:

`https://dl.dropboxusercontent.com/u/4189520/GraphJS/graphjs.html`

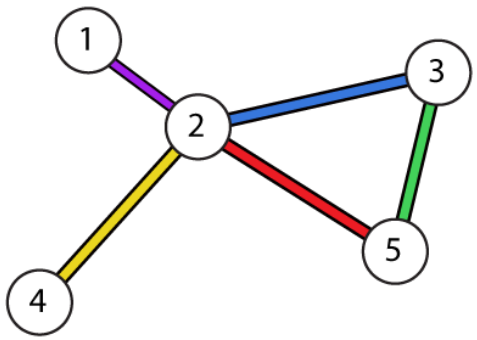
Paths in Undirected graph

- A path in a graph is a sequence of vertices $v_1, v_2, \ldots, v_n$, such that $v_1 \rightarrow v_2, v_2 \rightarrow v_3, \ldots, v_n - 1 \rightarrow v_n$ are arcs.
- The length of a path is the number of arcs on the path, in this case, $n - 1$.
- Remember: If (v, w) is an undirected edge, then $(v, w) = (w, v)$.

Taken from [Aho77].

Example of paths in an Undirected graph

- From 1 to 3: $\{1, 2, 3\}$
- From 1 to 3: $\{1, 2, 5, 3\}$
- From 2 to 3: $\{2, 3\}$
- From 2 to 3: $\{2, 5, 3\}$



Taken from Wikipedia.

Representations for Undirected Graphs

- (Symetric) Adjacency Matrix
- (Symetric) Labeled Adjacency Matrix
- Adjacency List
- Labeled Adjacency List

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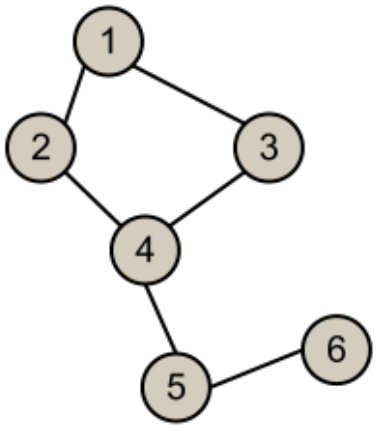
Example of Adjacency Matrix

Undirected Graph & Adjacency Matrix



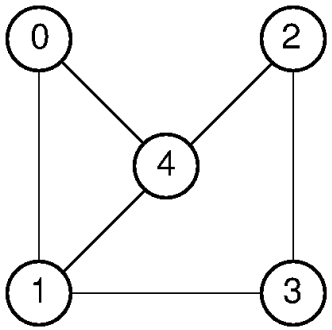
Undirected Graph

	1	2	3	4	5	6
1	0	1	1	0	0	0
2	1	0	0	1	0	0
3	1	0	0	1	0	0
4	0	1	1	0	1	0
5	0	0	0	1	0	1
6	0	0	0	0	1	0

Adjacency Matrix

Taken from `http://www.stoimen.com/blog/wp-content/uploads/2012/08/5.-Undirected-Graph-Adjacency-Matrix.png`

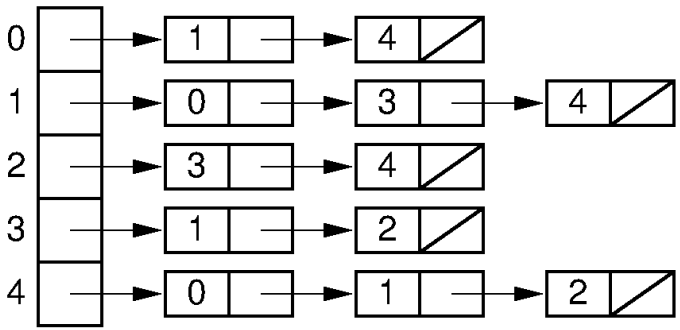
Example of Adjacency List



(a)

	0	1	2	3	4
0		1			1
1	1			1	1
2				1	1
3		1	1		
4	1	1	1		

(b)



(c)

Taken from `http://algoviz.org/OpenDSA/Books/Everything/html/_images/GraphUD.png`

Examples of Undirected graphs

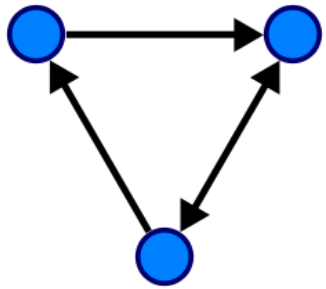
Simulator:

`https://www.cs.usfca.edu/~galles/visualization/DFS.html`

Directed graph

- A directed graph (digraph for short) G consists of a set of vertices V and a set of arcs E.
- $G = (V, E)$
- An arc is an ordered pair of vertices (v, w); $v \in V$ is called the tail and $w \in V$ the head of the arc.

Taken from [Aho77].



Taken from Wikipedia.

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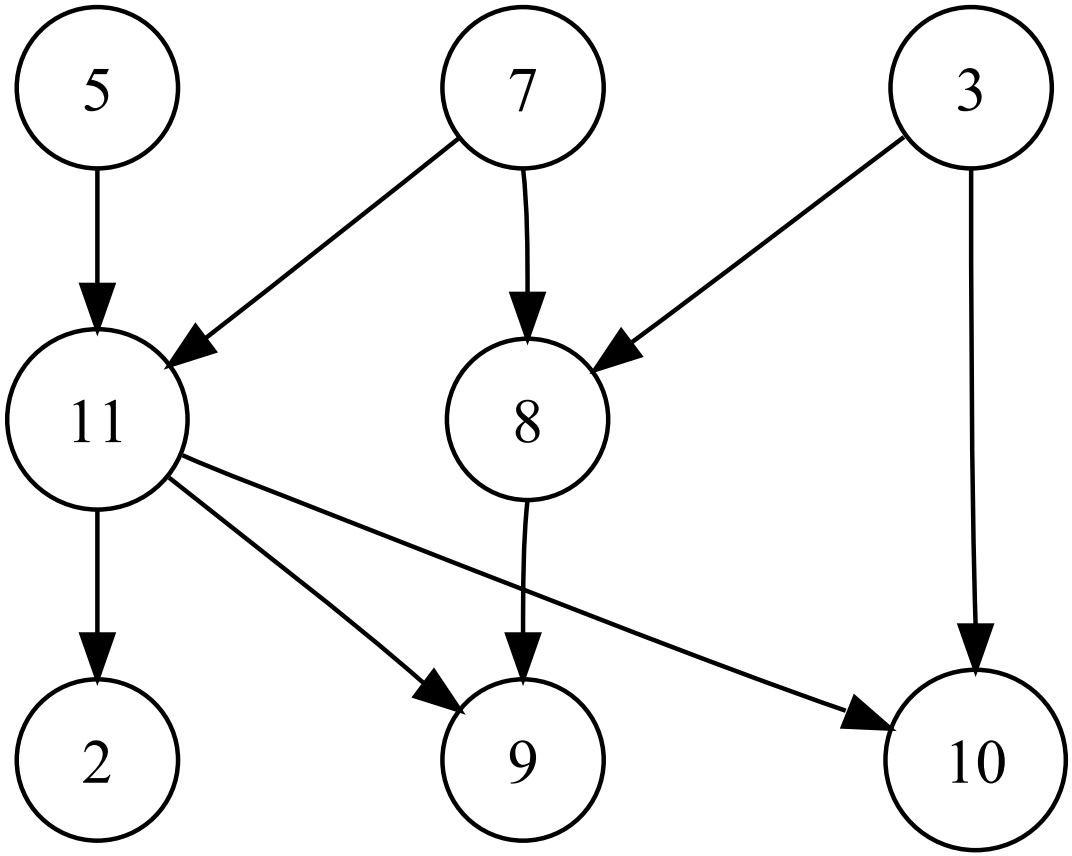

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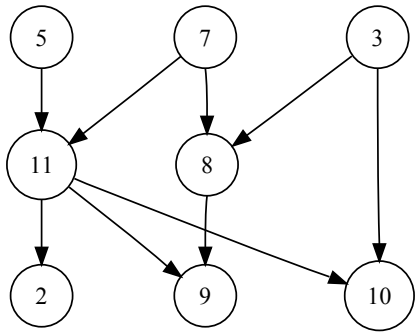
- The length of a path is the number of arcs on the path, in this case, $n - 1$.

Taken from [Aho77].

Example of paths in a directed graph

- From 3 to 9: $\{3, 8, 9\}$
- From 5 to 2: $\{5, 11, 2\}$
- From 3 to 10: $\{3, 10\}$



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Adjacency Matrix

- The adjacency matrix for G is an $n \times n$ matrix A of booleans, where $A[i, j]$ is `true` if and only if there is an arc from vertex i to j
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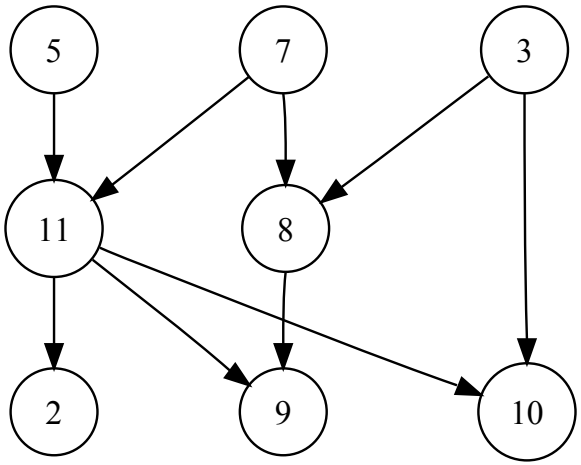
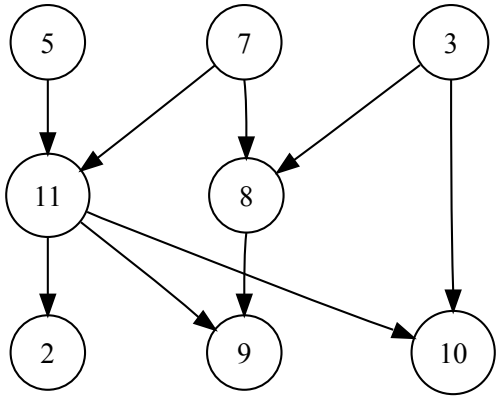
Example of Adjacency Matrix

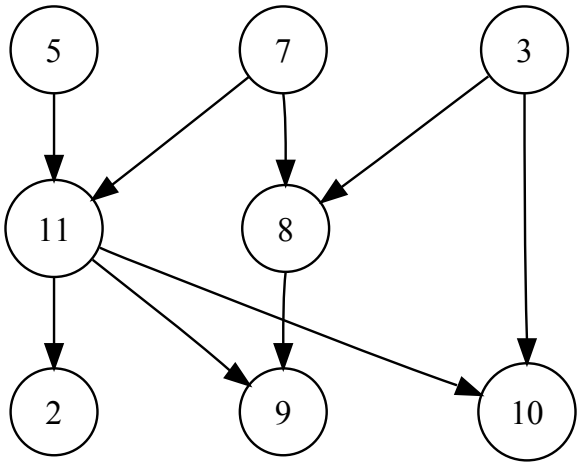


Example of Adjacency Matrix (2)



	2	3	5	7	8	9	10	11
2	0	0	0	0	0	0	0	0
3	0	0	0	0	1	0	1	0
5	0	0	0	0	0	0	0	1
7	0	0	0	0	1	0	0	1
8	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0
11	1	0	0	0	0	0	0	0

Adjacency List



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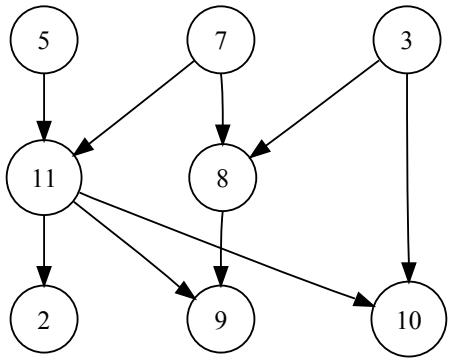

- $1 \rightarrow [\,]$
- $2 \rightarrow [\,]$
- $3 \rightarrow [8, 10]$
- $4 \rightarrow [\,]$
- $6 \rightarrow [\,]$
- $7 \rightarrow [8, 11]$
- $8 \rightarrow [9]$
- $9 \rightarrow [\,]$
- $11 \rightarrow [2, 9, 10]$
- $5 \rightarrow [11]$
- $10 \rightarrow [\,]$

Implementations for Directed Graphs

- (Labeled) Adjacency Matrix
  - Matrix
- (Labeled) Adjacency List
  - Hash table to associate each vertex in a graph with an array of adjacent vertices
  - Singly-linked list of the neighboring vertices of that vertex
  - Object-oriented incidence list structure

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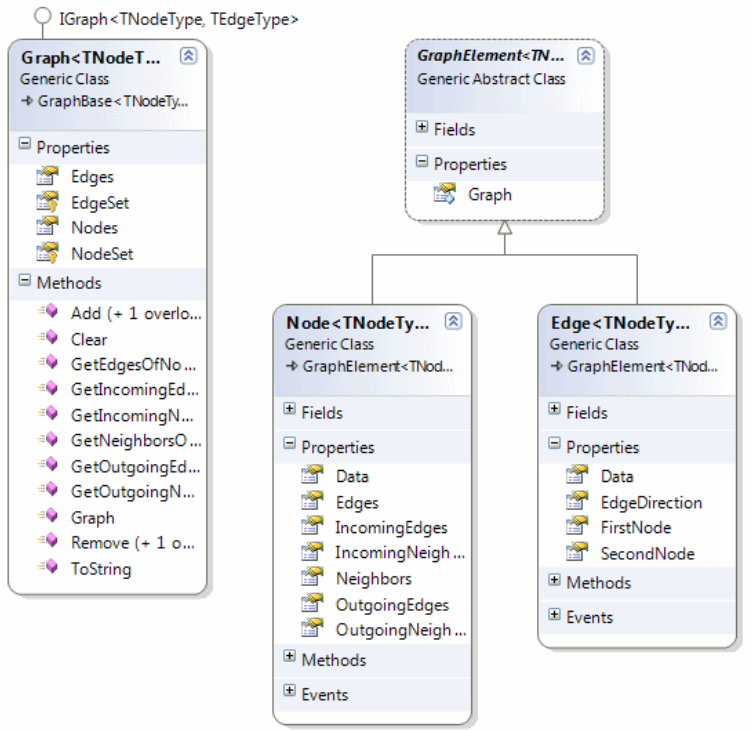
Object-oriented graph implementation



Taken from `http://www.codeproject.com/Articles/53577/Animation-of-graph-algorithms-with-WPF-D`

Quiz questions

- An undirected graph (*graph* for short) G consists of a set of vertices V and a set of arcs E where pairs in E are unordered.
- A directed graph (*digraph* for short) G consists of a set of vertices V and a set of arcs E.
- A path in a digraph is a sequence of vertices $v_1, v_2, \ldots, v_n$

References

- Please learn how to reference images, trademarks, videos and fragments of code.
- Avoid plagiarism

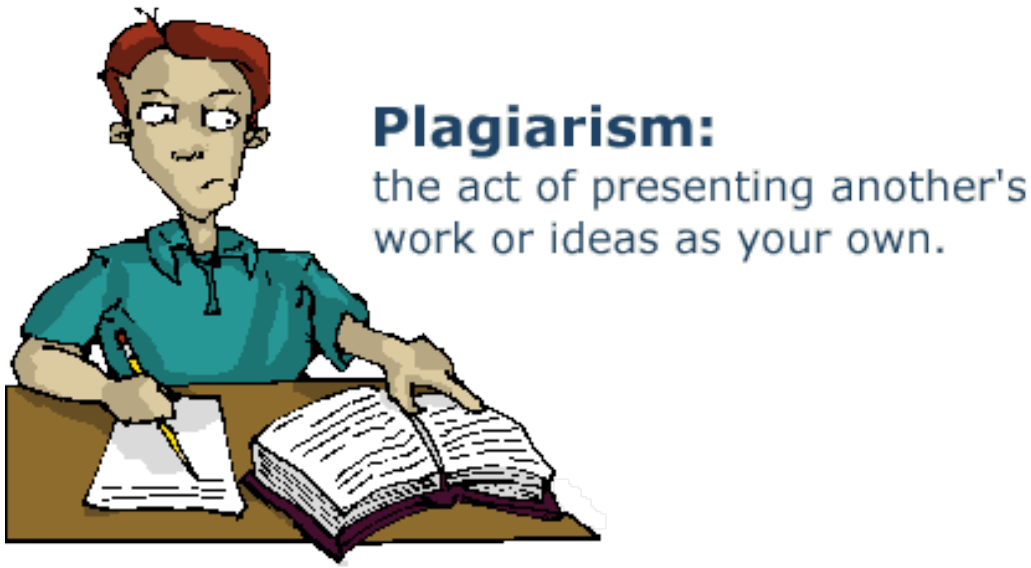


Figure: Figure about plagiarism, University of Malta [Uni09]

References

University of Malta.

Plagarism — The act of presenting another's work or ideas as your own, 2009.

[Online; accessed 29-November-2013].

Further reading

- Alfred Aho, Estructuras de Datos y Algoritmos. Capítulo 6: Grafos dirigidos.
- Alfred Aho, Estructuras de Datos y Algoritmos. Capítulo 7: Grafos no dirigidos. Páginas 314 - 321.