Network Reliability Analysis

Uncertainty quantification for post-hazard network performance

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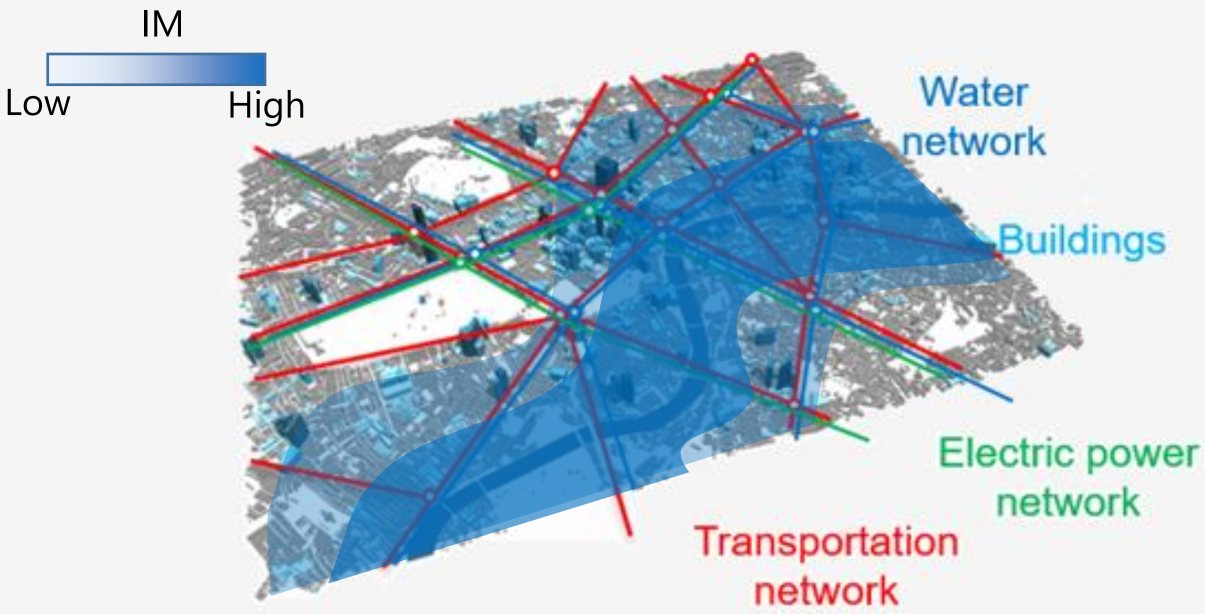
Problem Description

Infrastructure: the basic physical and organizational structures and facilities needed for the operation of a society or enterprise.

How is their performance affected during a hazard ?

In the case of networks usually we are interested in the *flow* capabilities

At a more basic level, we are interested in having all places of interest *at least connected.*



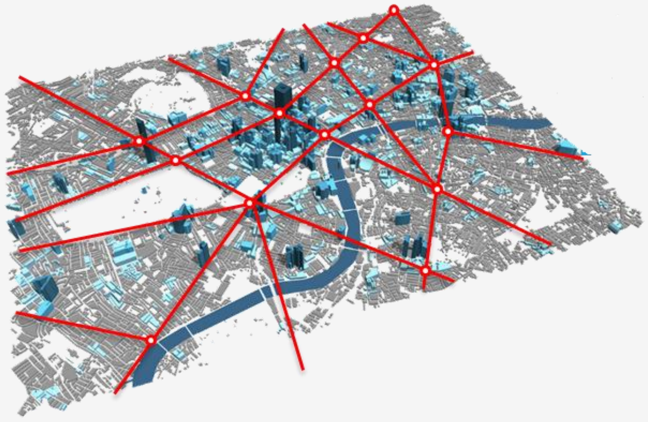
Flood

Infrastructure Representation

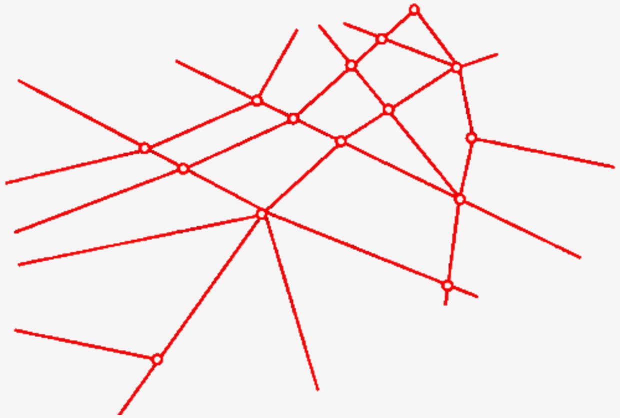
Infrastructure can be modelled as a network of interconnected components

Natural mathematical representation for network models are graphs: $G(N, A)$

Graphs are characterized by their adjacency matrix $\boldsymbol{A} = [a_{ij}]$



Infrastructure Model



Network

$$\mathbf{A} = \begin{array}{c} \left[\begin{matrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{matrix}\right] \\ \begin{matrix} \text{To} & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ \end{array} \quad \text{From}$$

Adjacency Matrix

Connectivity metrics: Diameter

1) Diameter (Average of shortest paths from i to j)

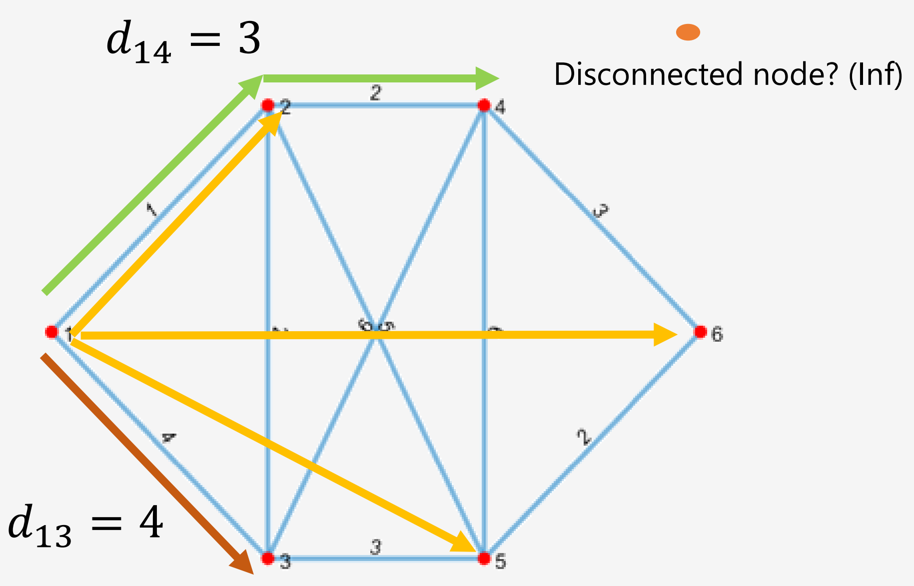
Local $$\delta_i = \frac{1}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n} d_{ij}$$

Global $$\delta = \frac{1}{n}\sum_{i=1}^{n}\delta_i = \frac{1}{n(n-1)}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} d_{ij}$$

2) Eccentricity: ("standard deviation" of δ_i)

Local $$\zeta_i = \sqrt{\frac{1}{(n-1)-1}\left[\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{d_{ij}}{d_{i,opt}} - \bar{\delta}_i\right)^2\right]}$$

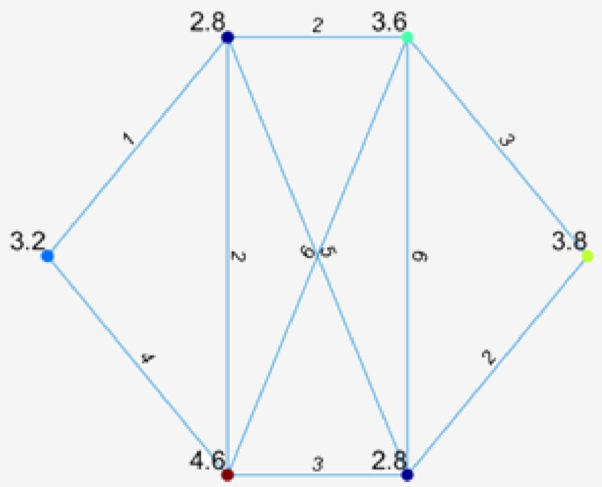
Global $$\zeta = \sqrt{\frac{1}{n(n-1)-1}\left[\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{d_{ij}}{d_{i,opt}} - \bar{\delta}\right)^2\right]}$$

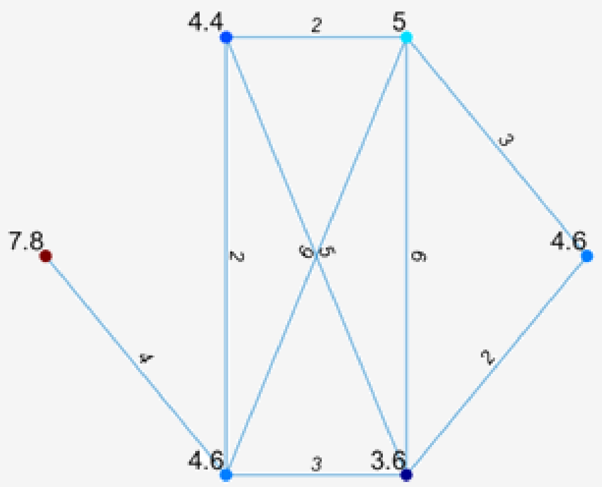


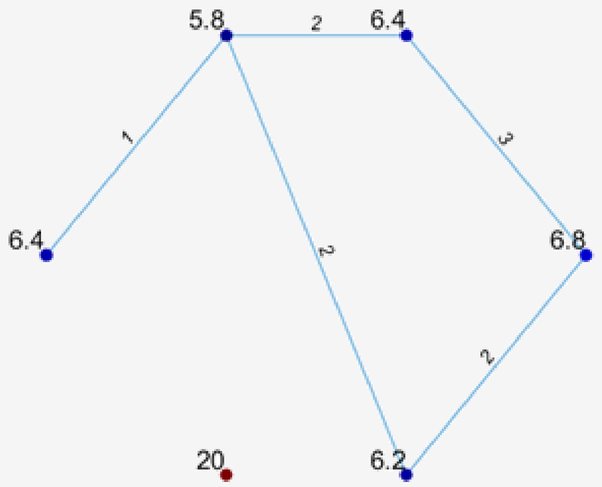
$$\delta_1 = \sum_{j=2}^{6}\delta_{ij} = \frac{(\delta_{12}+\delta_{13}+\delta_{14}+\delta_{15}+\delta_{16})}{5}$$

Sample simulation

$HI = 0$ | $HI = 0.5$ | $HI = 2$







$\delta = 3.47$ | $\delta = 5.00$ | $\delta = 8.60$

As the hazard intensity increases the diameter increases (locally and globally), i.e: the connectivity drops

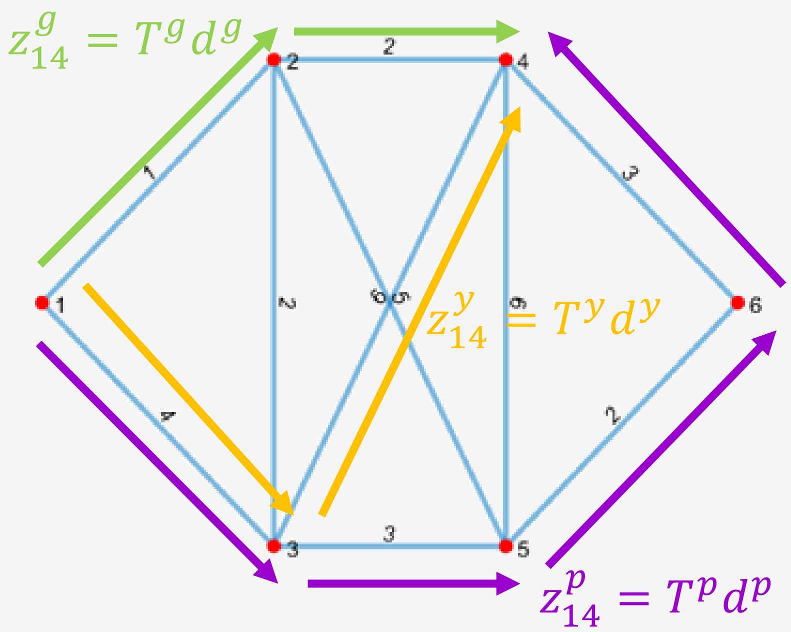
Considering component capacity: traffic routing

$z_{ij}^{*} \coloneqq \min \sum_{(i,j)\in A} c_{ij}\, x_{ij}$	Objective: Minimize traversal cost
$\sum_{j:(s,j)\in A} x_{sj} = T \qquad \sum_{i:(i,t)\in A} x_{it} = T$	Origin and destination constraint
$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = 0 \;\forall\, i \in N\backslash\{s,t\}$	Flow conservation along path
$x_{ij} \le u_{ij} \;\forall\, (i,j) \in A$	Capacity Constraint

$x_{ij} = flow\ on\ arc\ (i,j)$

$u_{ij} = capacity\ of\ arc\ (i,j)$

$c_{ij} = unit\ cost\ of\ transiting\ arc\ (i,j)$

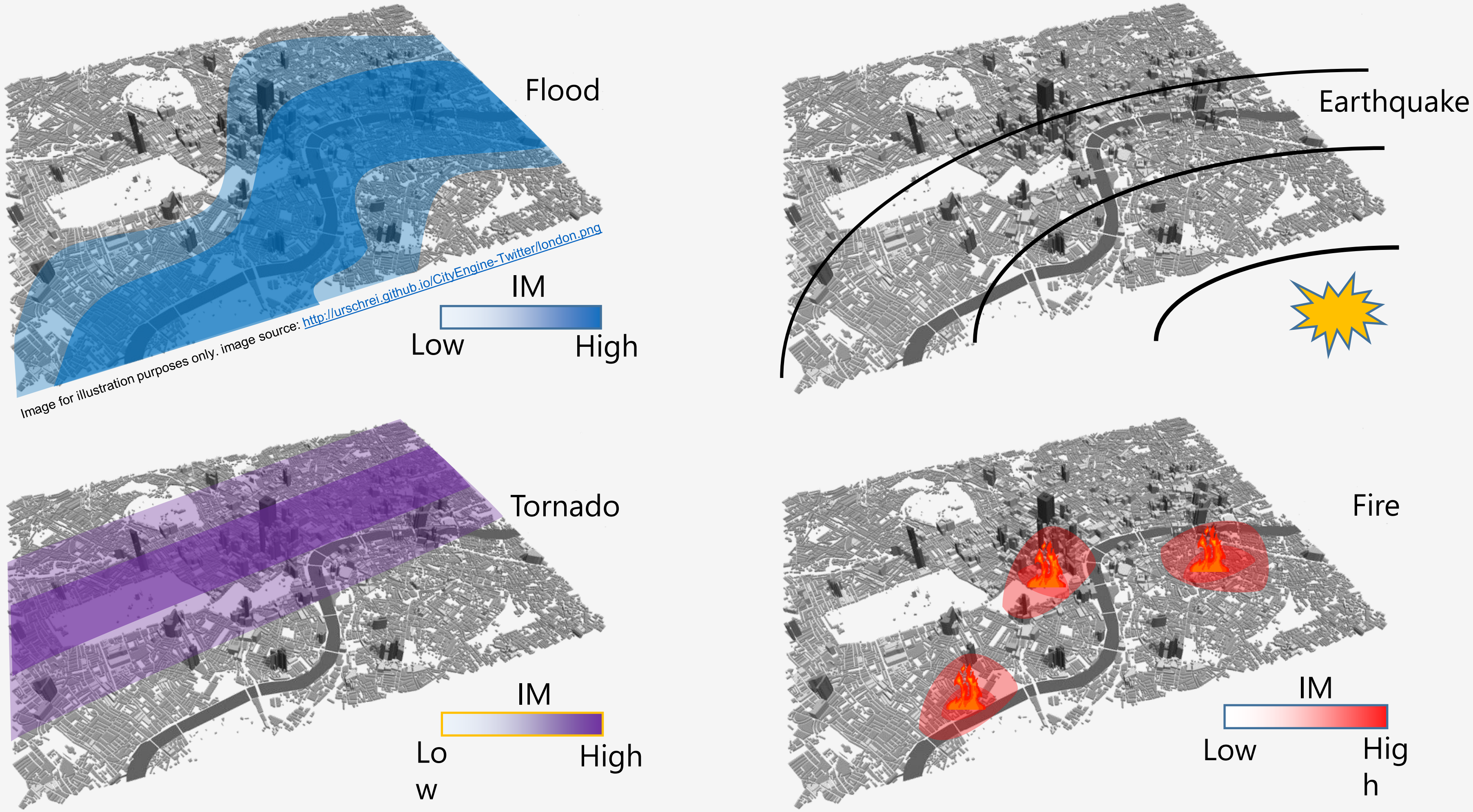


$z_{st,T}^{*} = minimum\ total\ traversal\ cost\ from\ s\ to\ t\ for\ traffic\ T\ (veh.mi.travelled)$

Define new “shortest path” between s and t as:

$$\frac{z_{st,T}^{*}}{\Sigma_{i\neq j} x_{ij}} = \frac{z_{st,T}^{*}}{T} = \delta_{ij}^{mc} = generalized\ shortest\ path\ from\ s\ to\ t\ (shortest\ hyperpath)$$

Hazard Modelling: Θ_{Shape} vs Θ_{IM}



Methodology

Event

Traffic

Storm

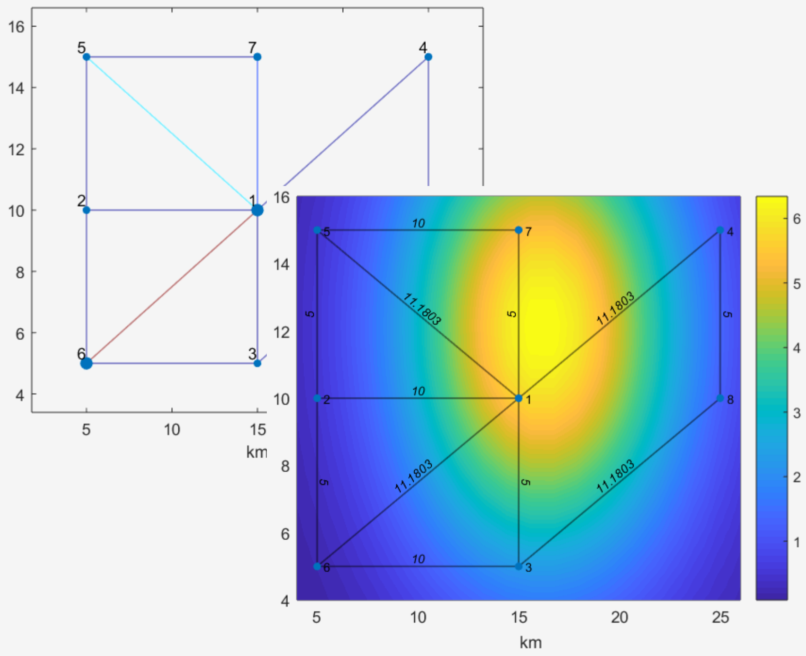
Hazard – Network Model

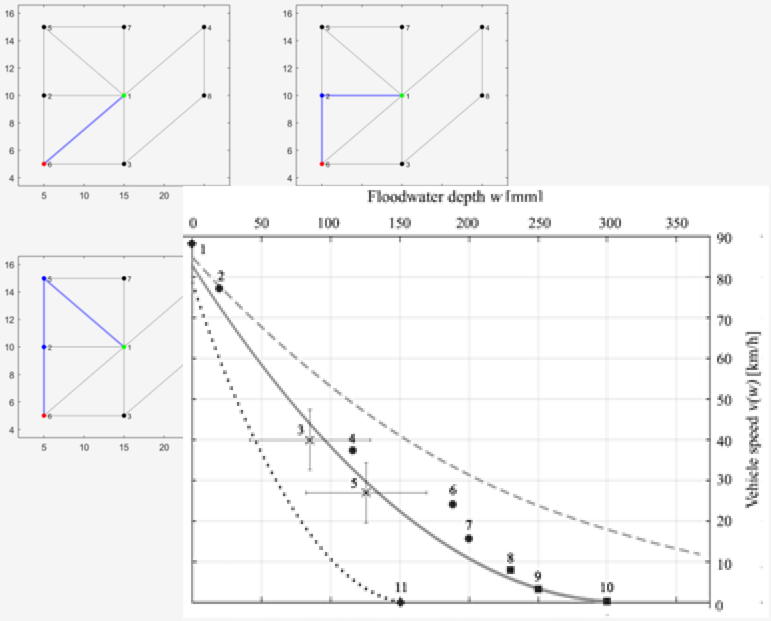
Routing

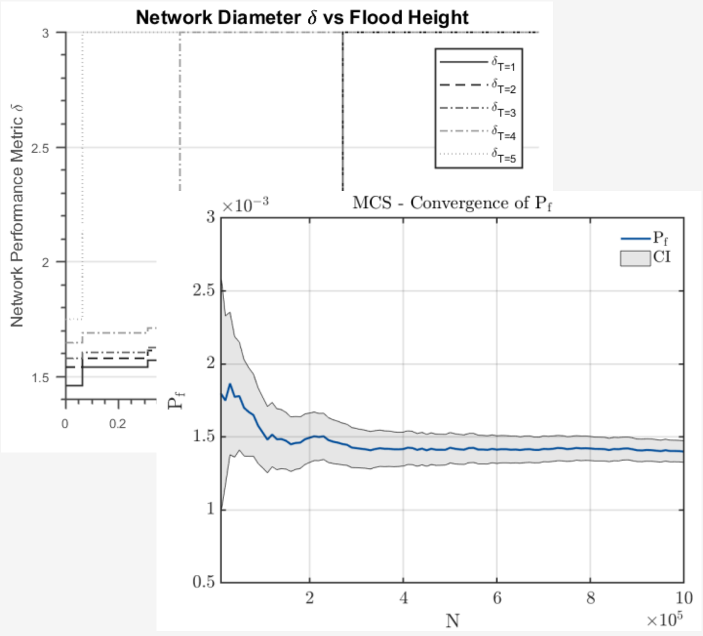
Storm-Capacity

Probabilistic Analysis

- Network Reliability
- Failure Probabilities

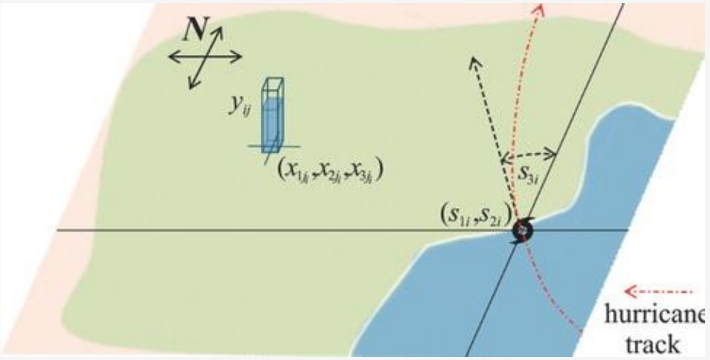




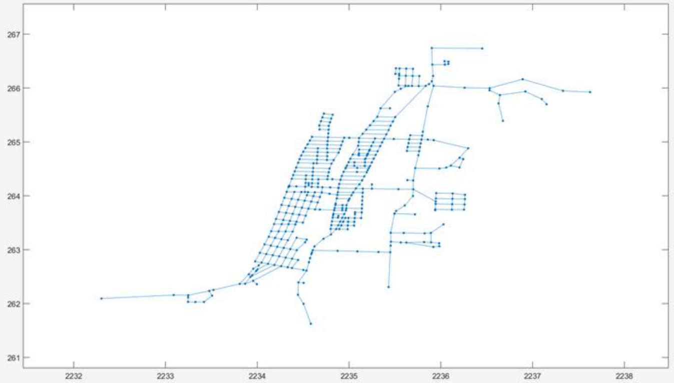


Challenges

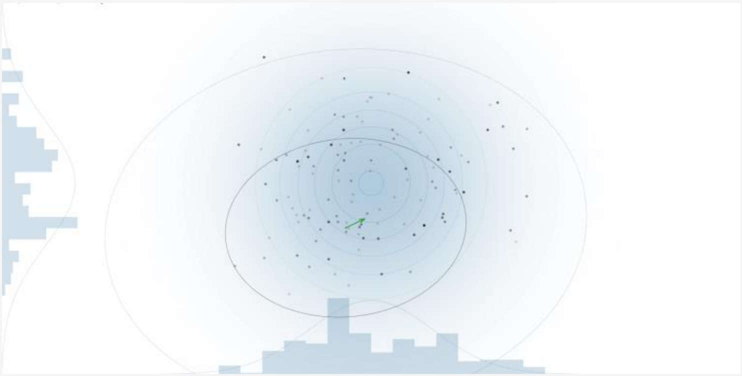
1. Simulation of extreme events



2. Routing in large networks



3. Probabilistic analysis : Computation



The probabilistic analysis implies a large number of simulations, but we want to keep the analysis complex enough to have meaningful results within practical accuracy ranges.

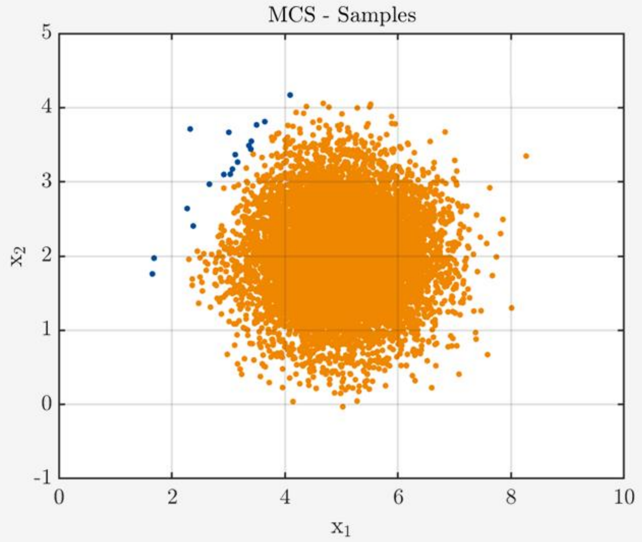
Luckily we can apply some techniques from Uncertainty Quantification.

1) Importance Sampling for events of interest

Approach 1: Importance Sampling

Importance sampling over the whole range of intensities.

Only few values are informative otherwise

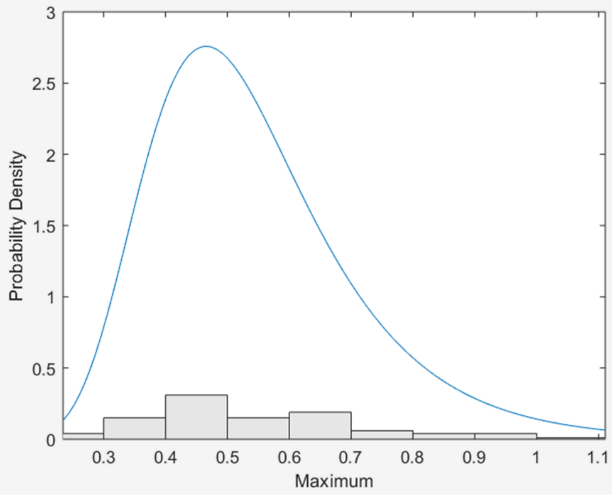


Approach 2: GEV Function

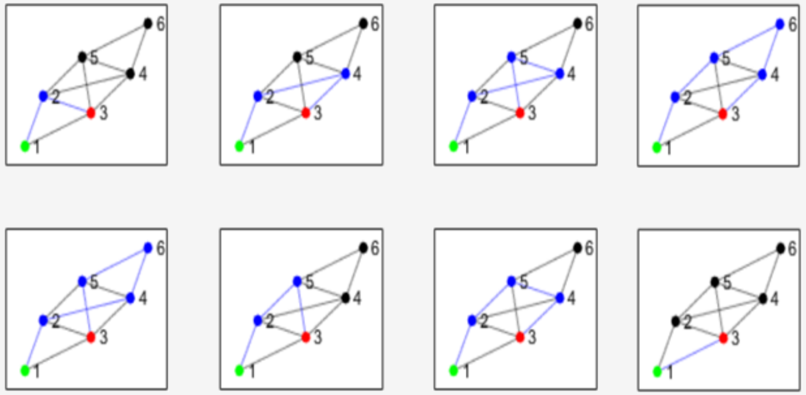
MATLAB implements a MLE fit for the parameters (μ, σ, k) of Generalized extreme value distributions Type I II and III.

The parameter space can be explored, confidence intervals obtained.

In this approach we are already looking at periodical maximums



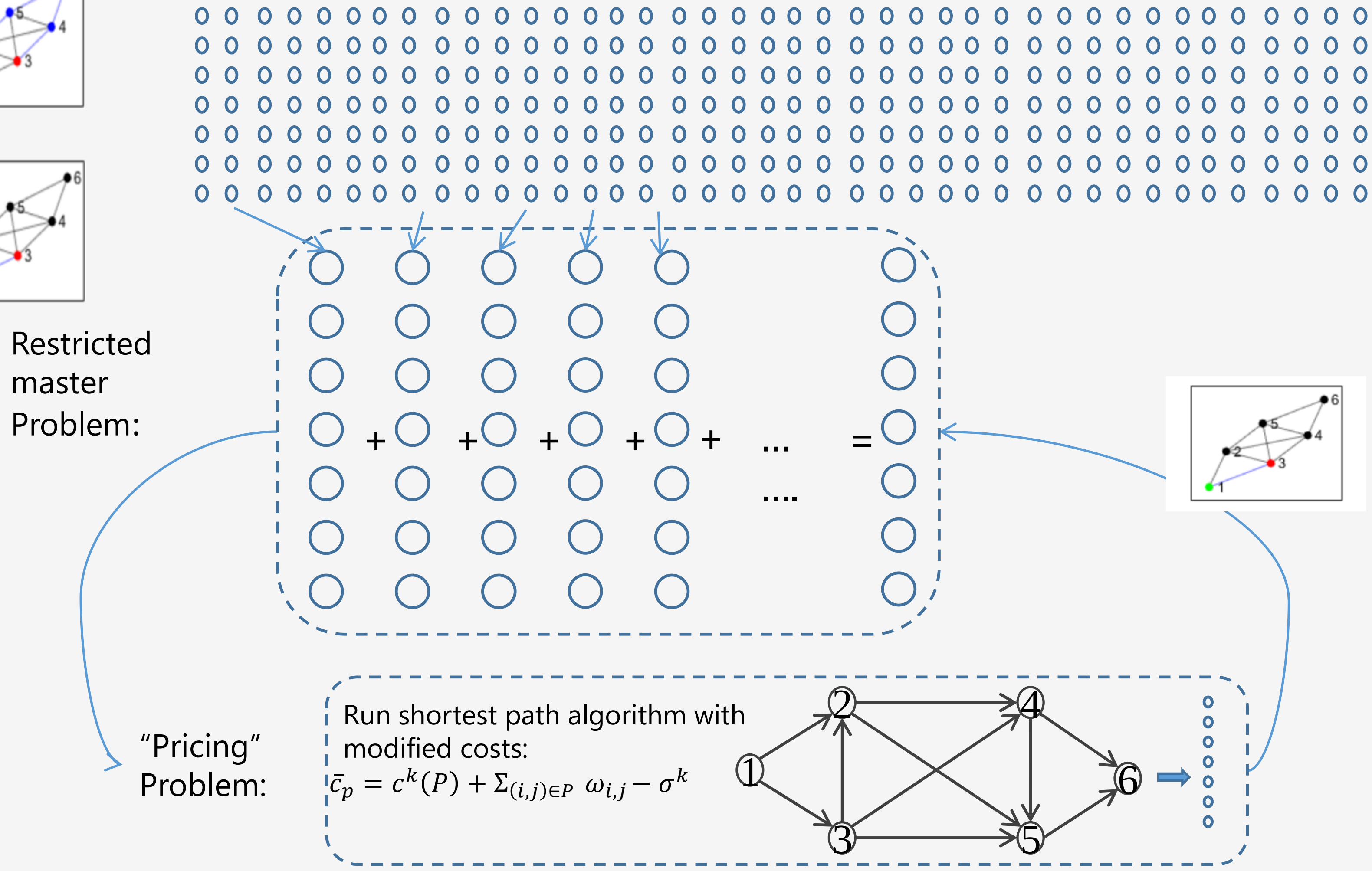
2) Surrogate model for network evaluation



The initial search space is exponentially large (NP-Hard)

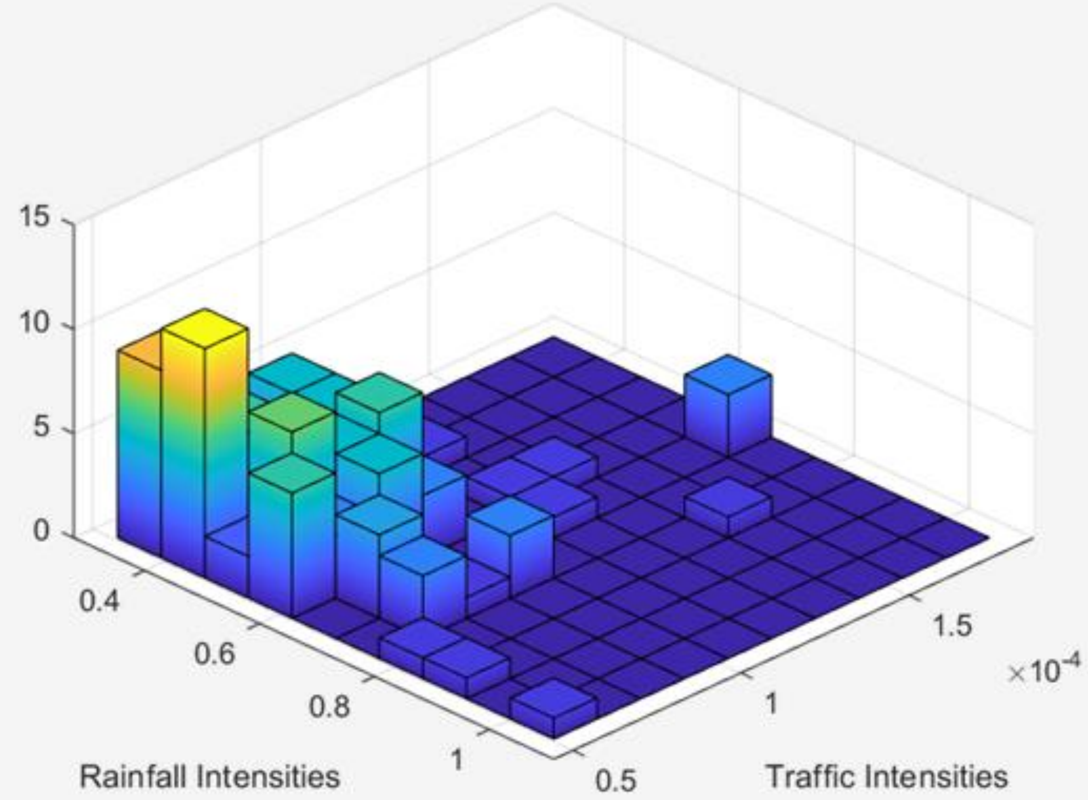
We use column generation to add relevant paths

Use the structure obtained as a surrogate to evaluate faster

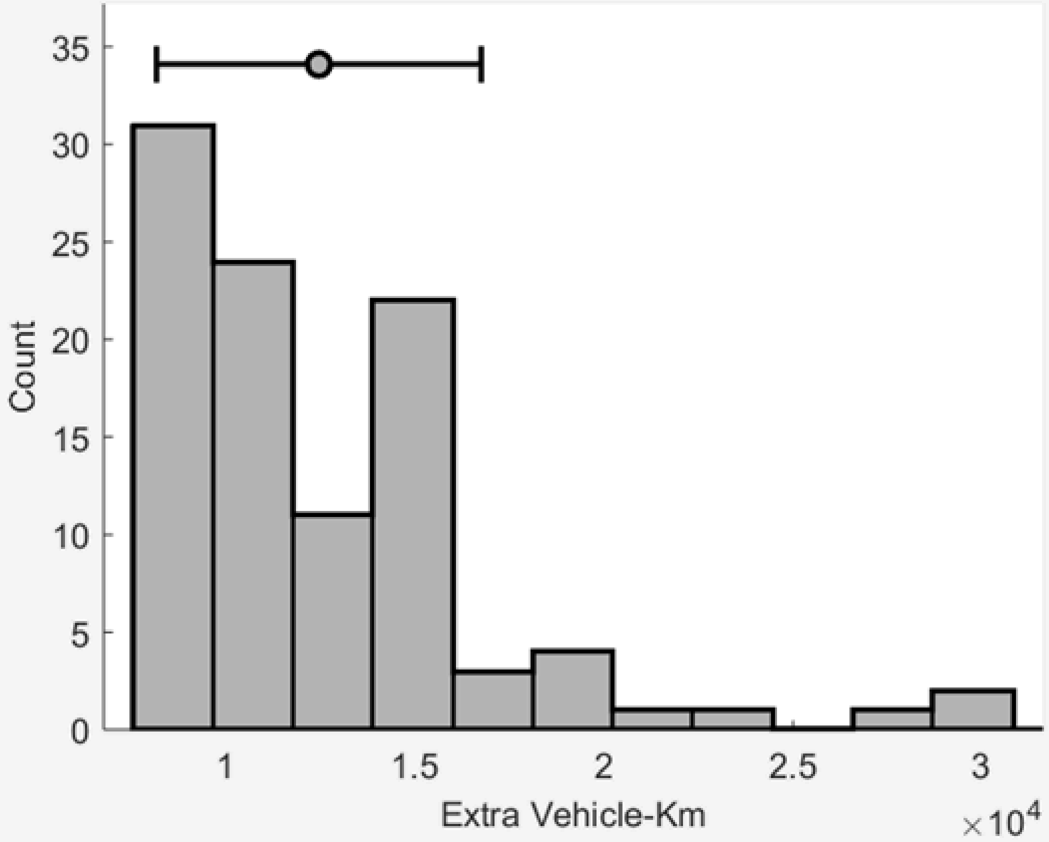


3) Parameter space exploration with markov chain monte carlo

Distribution of Hazard and Traffic Intensities Sampled

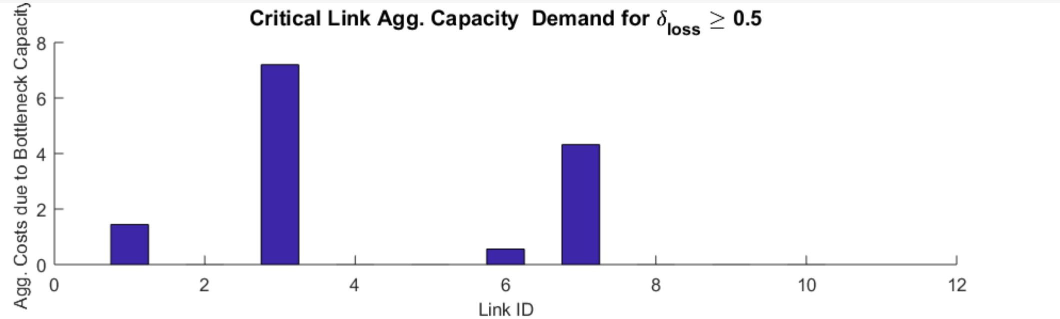


Distribution of extra operational costs



Future Work

- Obtain capacity budget per link



- Surrogate model for the fragility function
- Apply to real scenario (make less synthetic)
- Define costs limit as failure to get failure probabilities
- Graduate

Thanks!

Questions? Comments?