



## OpenTURNS

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‘HPC and Uncertainty Treatment – Examples with Open TURNS and Uranie’

EDF – Phimeca – Airbus Group – IMACS – CEA

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MAISON DE LA SIMULATION



## Overview of OpenTURNS

- What is OpenTURNS?
- Uncertainty methodology
- OpenTURNS features
- Uncertainty quantification with OpenTURNS
- Innovations

## OpenTURNS: Doc and Users

## OpenTURNS in pictures

## OpenTURNS in practice

- OpenTURNS: Basics and example



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# What is Open TURNS?



☐ Partnership since 2005 between:

EDF - R&D

EADS - Innovation Works

Phimeca

IMACS (since 2014)

☐ **Open** source initiative to **T**reat Uncertainties, **R**isks' **N** **S**tatistics

- An *open source* platform dedicated to *uncertainty treatment* in support of probabilistic methods
- *Uncertainty propagation* through a model up to a variable of interest
- *Uncertainty quantification* and *Uncertainty ranking*
- *Meta-model* building
- working on Unix/Linux platform and Windows (since 2010)

☐ Open TURNS includes:

- *C++ scientific library* including the methods for performing uncertainties treatment (statistic, reliability, etc.);
- A *python module* allowing to define in a simple manner the models in an interpreted language;
- A complete documentation;
- A website: [www.openturns.org](http://www.openturns.org).

# Uncertainty methodology (1/2)



## Global Methodology of Treatment of Uncertainties

- developed first at EDF R&D in 1990 and then improved by contributions from other companies

### **Step A** : *Study Specification*

Uncertainty sources, model, variable of interest and criteria

### **Step B** : *Uncertainty Quantification*

Joint probability density function of the input uncertain parameters modeling

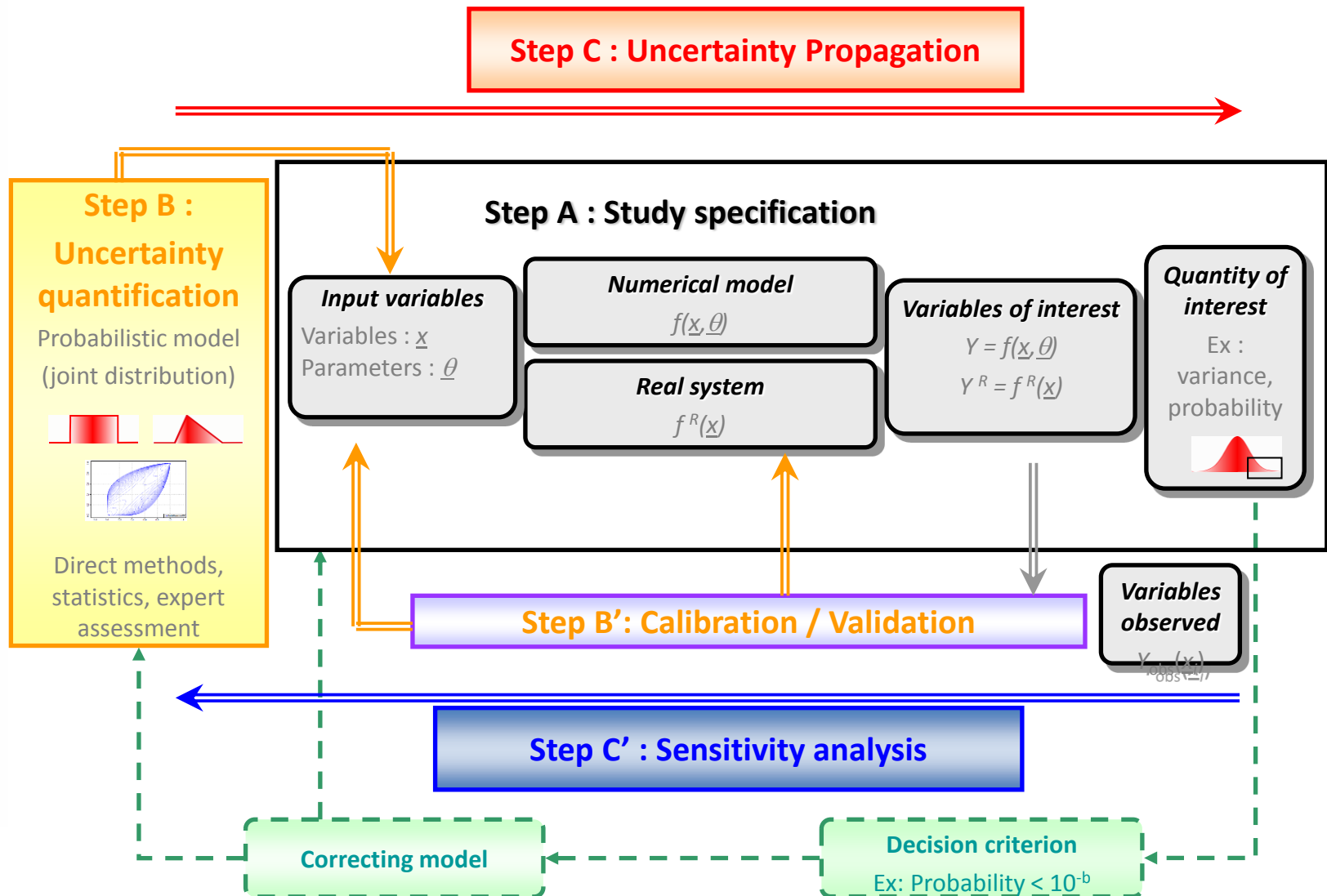
### **Step C** : *Uncertainty Propagation*

Variable of interest uncertainty assessment

### **Step C'** : *Uncertainty Ranking / sensitivity analysis*

Uncertainty sources ranking with respect to their influence on the variable of interest uncertainty

# Uncertainty methodology (2/2)

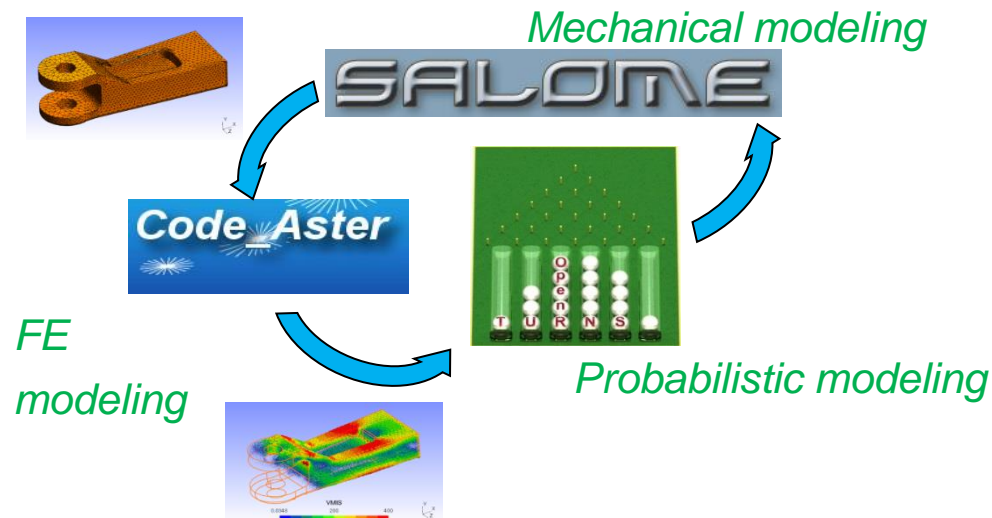


# OpenTURNS features



## Code linked to OpenTURNS

- *Interface with python functions* → to perform complex wrappers without compilation + parallelization functionalities
- *Standard Interface* for the *wrappers of any complexity* (distributed wrapper, binary data) development requiring the development of an external wrapper
- *SalomeMeca compatible* → software including the 3 components to perform a mechanical and probabilistic data models coupling (linked to YACS)
- *GUI of OpenTURNS* within SalomeMeca



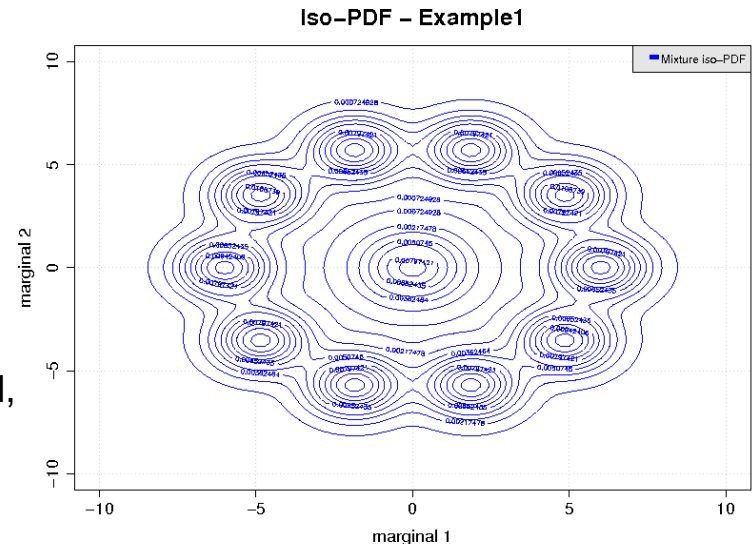
# Uncertainty quantification with OpenTURNS

## Estimation from data :

- Distribution fittings (parametric or not)
- Validation Tests (quantitative or graphical)
- Estimation of the dependence : copula, correlation coefficient
- Regression

## Analytical modeling of joint distributions of dimension $n$ :

- Combination Marginals + Copula
- Parametric distributions of dimension  $n$  (normal, student...)
- Truncated distributions
- Stochastic process
- Non parametric distribution of dimension  $n$ : kernel fitting ( $n$ ), Sklar Copula
- Linear combination of PDF
- Linear combination of random variables
- Random sum of independent discrete variables according to a Poisson process
- Etc.





# Uncertainty propagation with OpenTURNS



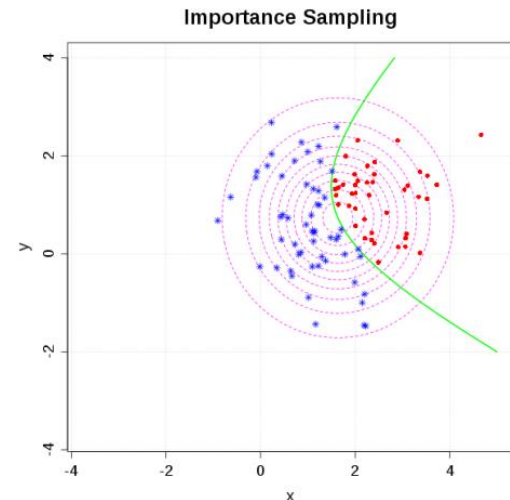
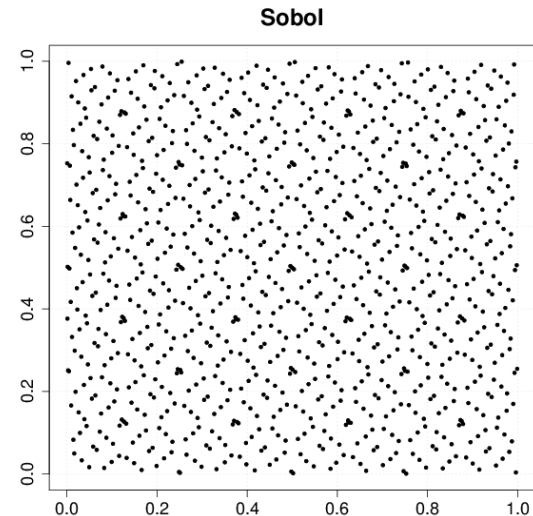
## Sampling data:

- Random generator
- Stratified design of experiment
- Latin Hypercube Sampling
- Low Discrepancy Sequence
- Markov chain



## Probability estimation:

- Isoprobabilistic transformation
- FORM / SORM
- Monte Carlo simulation
- Importance simulation
- Directional simulation
- Latin hypercube simulation
- Simulation algorithms

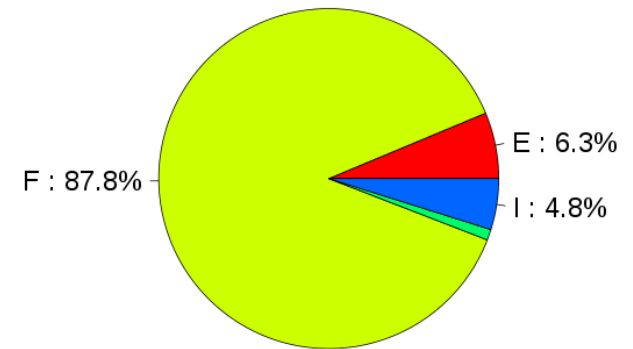


# Uncertainty ranking with OpenTURNS

## Ranking and sensitivity analysis:

- Importance factor from Taylor decomposition
- Ranking from correlation
- Sensitivity analysis
- Importance factor from reliability methods

Importance Factors from Design Point - Unnamed



## Tools

- Optimization algorithm
- Response surface:
  - Parametric approximation
  - Functional chaos expansion
  - Kriging
- Graph

## and Modules...

# Innovative and recently implemented algorithms

## the most recent and efficient algorithms of non uniform distribution generation

- Ziggurat method (2005) for the normal distribution
- sequential reject algorithm (1993) for the binomial distribution,
- Tsang & Marsaglia method (2000) for the gamma distribution,
- Lebrun algorithm (2012) for the MultiNomial distribution,

## the most recent algorithms for evaluating the CDF

- Marsaglia algorithm for the exact statistics of Kolmogorov (2003),
- Benton et Krishnamoorthy algorithm for the distributions non centered Student and non centered Chi2 (2003).

## PhD results

- Sparse chaos expansion polynomials : G. Blatman (EDF/R&D/MMC) (2010)
- Accelerated simulation algorithm for the evaluation of low probabilities : M. Munoz (EDF/R&D/MRI) : (current dev)
- Copulas for order statistics distributions: R. Lebrun (EADS) , Richard Fischer (EDF) (2013)



- The internal data models is based on the *multidimensional cumulative density function*



- *the sampling approach* : estimation of the output variable statistical characteristics from a large numerical sample
- and *the analytical approach* : exact partial or total solving of some problems using the probabilistic modeling and the implementation in Open TURNS of the function  $\mathbb{R}^n \rightarrow \mathbb{R}^p$  algebra up to the order 2:
  - exact determination of *the distribution of the sum of random variables* thanks to the characteristic functions implemented for each distribution
  - exact representation of the distribution of a random variable such that *its pdf is a linear combination of pdf*
  - exact determination of the distribution of a random variable defined as *the random sum according to a Poisson process of iid discrete random variables* (for ex, cumulated failure times when failures follow a Poisson process)
  - *particular manipulations on distributions* : extraction of marginals, extraction of dependence structure, etc.



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# OpenTURNS: Doc and Users



## *Several guides* intended for users

- **Installation** : on windows, linux, from anaconda, from sources
- **API Reference (Sphinx documentation)** : Python docstring of most of the *objects, arguments and methods* in OpenTURNS and available in HTML.
- **Examples Guide** : application of the whole Global Methodology on *classical mechanical examples*
- **Reference Guide** : *Theory* of the methods implemented within OpenTURNS
- **Contribute** : how to contribute to OpenTURNS, core code, modules ...

## *... and a sympathetic community :*

- Openturns.org : official web site
- a particular page share to communicate about the software
- the annual Users Day

# OpenTURNS: Doc and Users



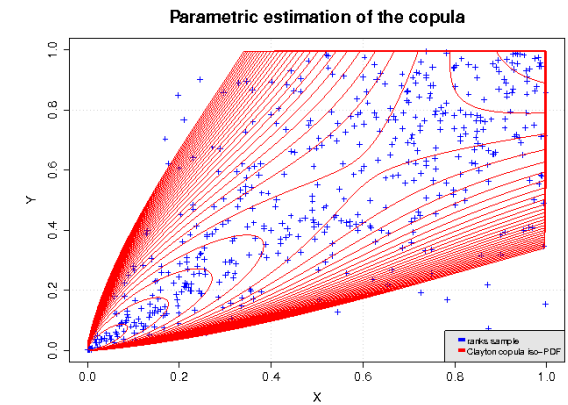
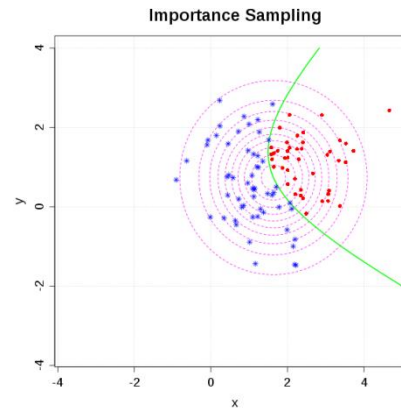
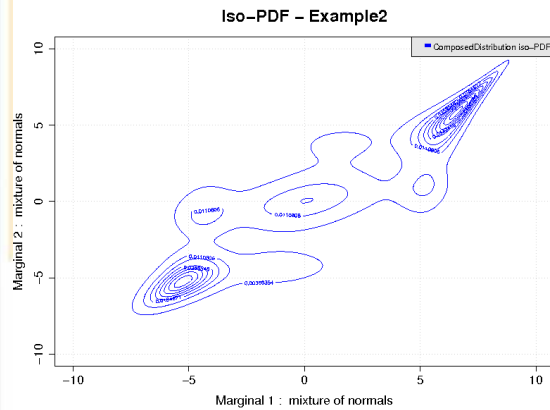
- ✉ Mailing list for users: [users@openturns.org](mailto:users@openturns.org):
  - Ask any question relative to the installation and use of Open TURNS
- ✉ Bug tracking :
  - <http://trac.openturns.org/wiki>
  - Communication about the correction of already identified bugs in the Trac and the new ones.



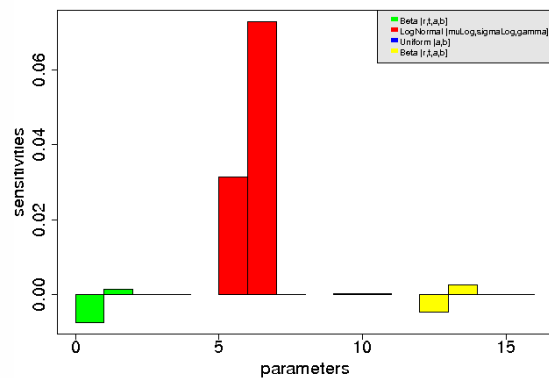
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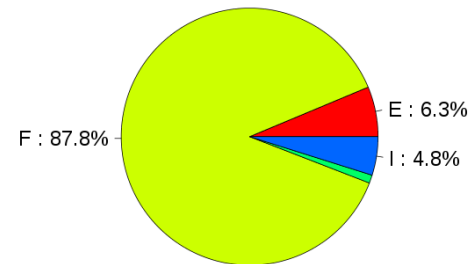
# OpenTURNs in pictures



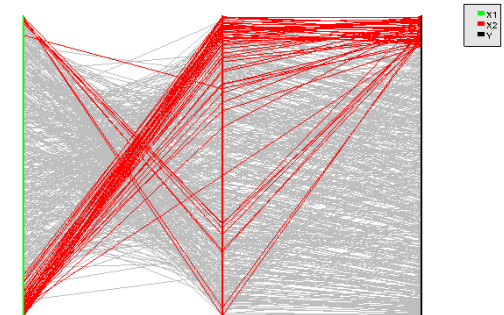
ORM – Event Probability Sensitivities – Marginal parameters – Even



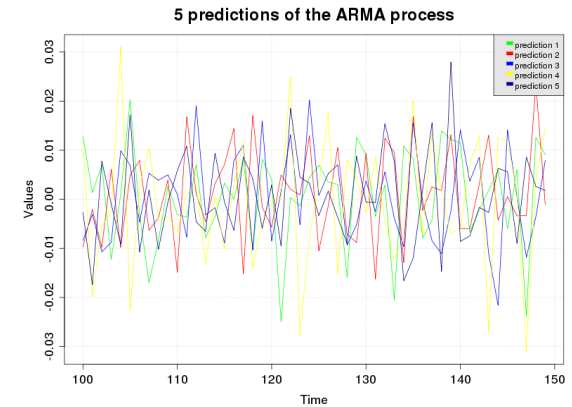
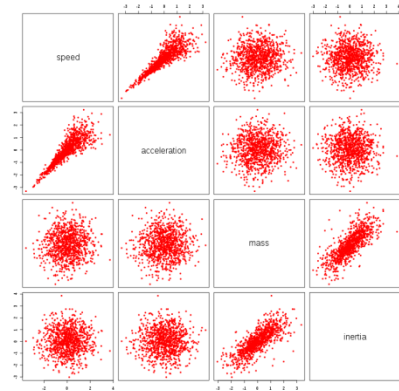
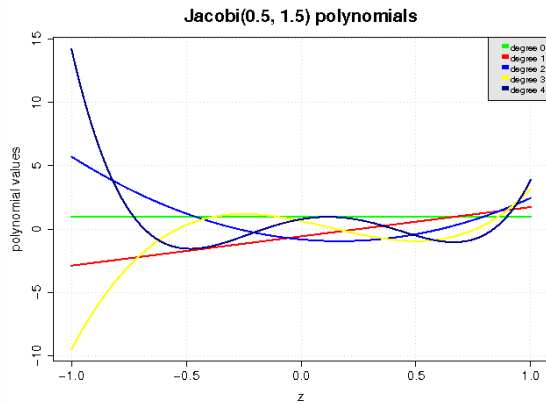
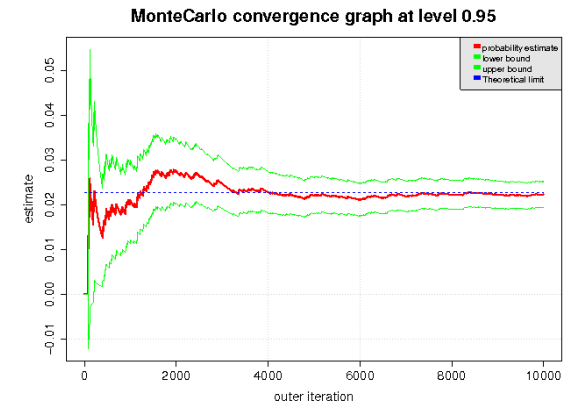
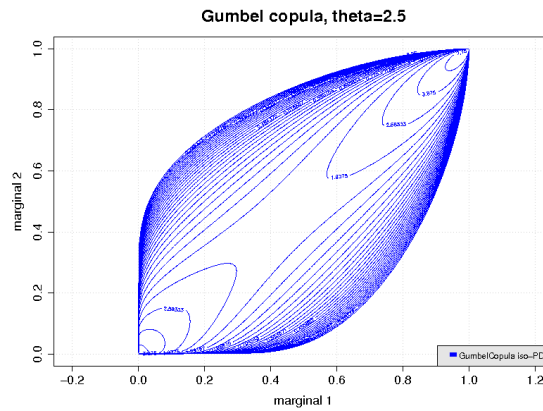
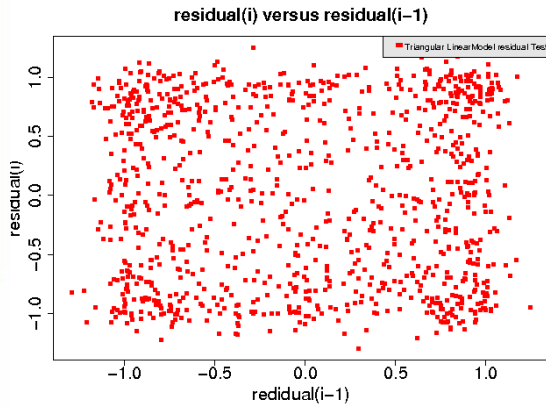
Importance Factors from Design Point - Unnamed



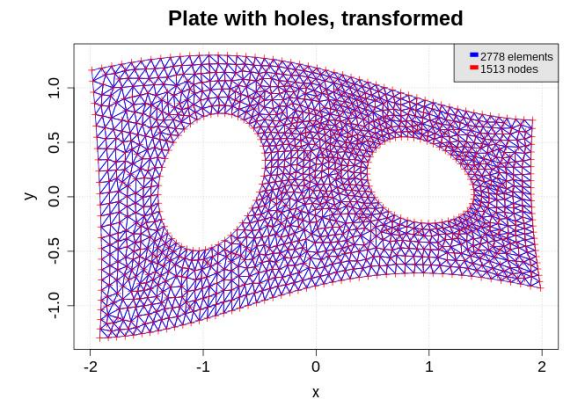
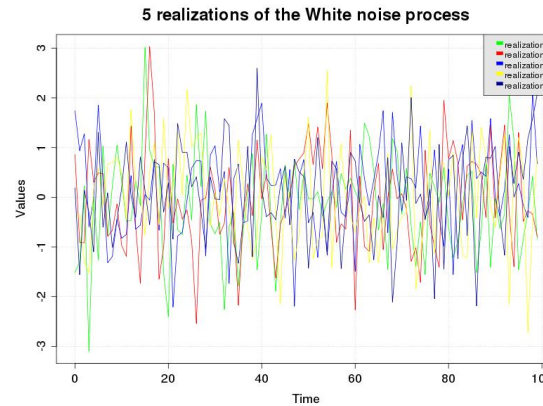
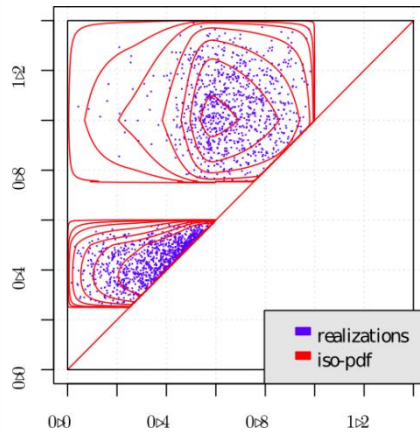
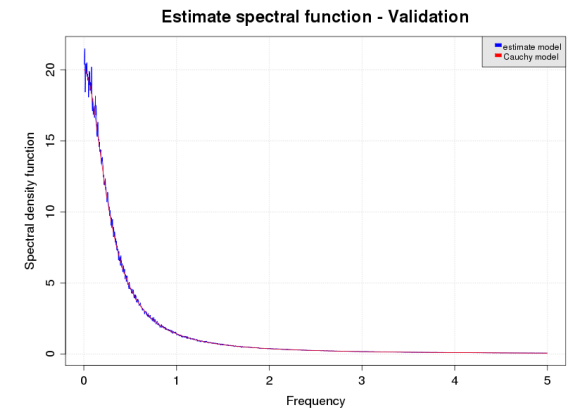
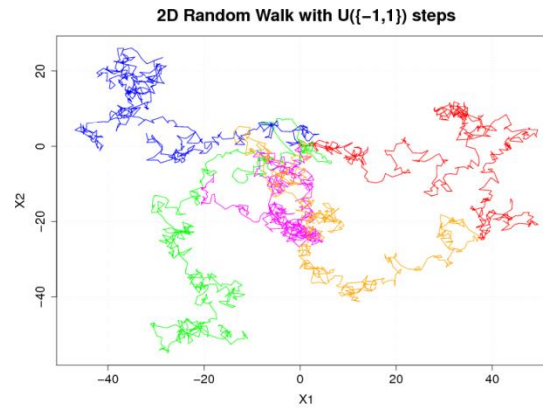
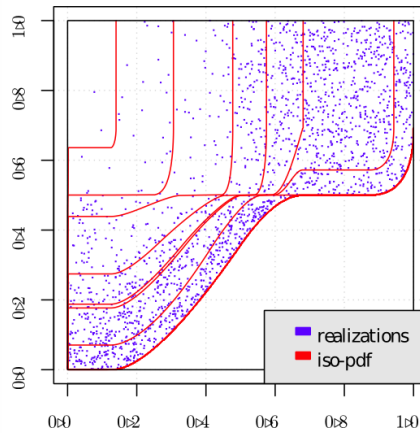
Cobweb graph – [Y] vs [X1,X2]



# OpenTURNS in pictures



# OpenTURNS in pictures





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# Basic commands in OT (1 / 3)



## Importation of OpenTURNS functionalities

```
>import openturns as ot      # import openturns module with an alias, here ot
```

## Mathematical objects

```
>a = ot.Point(3)      # Vector of 3 components of dimension 1
> S = ot.Sample(2, 3) # Vector of 2 components of dimension 3
>b = ot.Matrix(5 ,7)   # Matrix with 5 rows and 7 columns
>d = ot.Tensor(3, 4, 5)      # Tensor à 3 rows, 4 columns et 5 pages
>d[2, 1, 3] = -2.0         # assign the value -2 to the 3rd row, 2nd column and
                           #4th page, of the tensor d
```

# Basic commands in OT (2/3)



## Methods

```
>a = ot.Point(3)
>a[0] = 2.
>a[1] = -3.
>a[2] = 5.
>norm_a = a.norm()                                     # Euclidean norm of the vector a

>mat = ot.SquareMatrix(2)                               # Squared matrix of order 2
>mat[0, 0] = -2.
>mat[0, 1] = 3.
>mat[1, 0] = 0.
>mat[1, 1] = 1.
>det_mat = mat.computeDeterminant()                     # Determinant of the matrix mat

>y = ot.Point(2)
>y[0] = 1.
>y[1] = 5.
>x = mat.solveLinearSystem(y)                           # Solve the system mat*x=y
```

# Basic commands in OT (3/3)



## Methods

```
mat = ot.SquareMatrix(2)
mat[0, 0] = -2.
mat[0, 1] = 3.
mat[1, 0] = 0.
mat[1, 1] = 1.
```

```
det_mat = mat.computeDeterminant()
```

Objet relative to the method

Name of the method:

- begins with a lower case (*compute*)
- if it is composed of several words the following begin with a capital (*Determinant*)

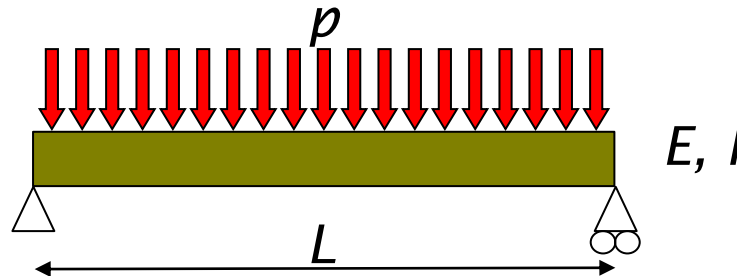
« () » at the end of the method

« . » between the objet and the method

# Example of uncertainty propagation



## ⌘ Bending beam under uniform loading



## ⌘ Maximal displacement

$$v_{\max} = \frac{5}{384} \frac{pL^4}{EI}$$

## ⌘ Probabilistic model

parameter	Symbol	Distribution	Mean	Standard Deviation
Length [mm]	$L$	Lognormal	5000	50
Young modulus [MPa]	$E$	Lognormal	30000	4500
Inertia [mm <sup>4</sup> ]	$I$	Lognormal	10 <sup>9</sup>	10 <sup>8</sup>
Load [N/mm]	$p$	Lognormal	10	3



# Model function



## Defining a Function

```
class myfunction(ot.OpenTURNPythonFunction):

    def __init__(self):
        ot.OpenTURNPythonFunction.__init__(self, 4, 1)

    def _exec(self, X):
        dep_max = 5.0 / 384.0 * X[3] * X[0] ** 4 / (X[1] * X[2])
        return [dep_max]

depmax = ot.Function(myfunction())

# ou

Depmax = ot.SymbolicFunction(['x1', 'x2', 'x3', 'x4'],
                              ['5. / 384 * x4 * x1 / (x2 * x3)'])
```

# Defining the derivatives



## Centered Finite difference

```
pas = ot.Point(4)
pas[0] = 5.0
pas[1] = 30.0
pas[2] = 1.0e6
pas[3] = 0.01

myGradient = ot.CenteredFiniteDifferenceGradient(pas,
                                                  depmax.getEvaluation ())
myHessian = ot.CenteredFiniteDifferenceHessian(pas,
                                                depmax.getEvaluation())

depmax.setGradient(myGradient)
depmax.setHessian(myHessian)
```

# Defining the probabilistic model



## 1 – Define the marginals

```
moy = ot.Point([5000.0, 300000, 1.0e9, 10.0])  
  
et = ot.Point([50.0, 4500.0, 1.0e8, 3.0])  
  
binf = 0.0  
  
L1 = ot.LogNormalMuSigma(moy[0], et[0], binf).getDistribution()  
L1.setName("L")  
...
```

# Defining the probabilistic model



## 2 – Define the correlation

The variables are independent

➔ Use the independent copula.

```
Copula = ot.IndependentCopula(4)
```

# Defining the probabilistic model



## 3 – Define the composed distribution

- Collection of distributions:

```
Collection = ot.DistributionCollection(4)
Collection[0] = ot.Distribution(L1)
Collection[1] = ot.Distribution(L2)
Collection[2] = ot.Distribution(L3)
Collection[3] = ot.Distribution(L4)
```

- Association of the collection and copula to create the composed distribution.

```
Modelproba = ot.ComposedDistribution(Collection, Copula)
```

# Propagation using MC simulations



## ☐ Defining the DOE

- Sampling of input values (here 1000 values)

```
Input = Modelproba.getSample(1000)
```

- Evaluation of the output

```
DepMC = depmax(Input)
```

# Result of the MC simulation



## ☐ Evaluation of the 4 first statistical moments

```
Mean = DepMC.computeMean()  
Covariance = DepMC.computeCovariance()  
stdev= DepMC.computeStandardDeviationPerComponent()  
Skewness = DepMC.computeSkewnessPerComponent()  
Kurtosis = DepMC.computeKurtosisPerComponent()
```

```
Terminal  
Fichier Édition Affichage Terminal Onglets Aide  
willaume@cadillac-l64:~/TP exemple$ python TPexemple.py  
MOMENTS  
Moyenne MC = 2.83656020137  
Covariance = 1.12843577652  
Stdev = 1.06227857765  
Asymetrie = 1.24539518003  
Aplatissement = 5.81957545311  
willaume@cadillac-l64:~/TP exemple$
```

# Histogram and empirical CDF



## 📊 Graphs on the sample DepMC

```
from openturns.viewer import View
```

```
hist = ot.VisualTest.DrawHistogram(DepMC)
View(hist, bar_kwarg={'label':'DepMC'})
```

```
CDF = ot.VisualTest.DrawEmpiricalCDF(DepMC,
                                     DepMC.getMin()[0] - 1.0, DepMC.getMax()[0] + 1.0)
View(CDF, step_kwarg={'label':'DepMC'})
```

