Surrogate-based system reliability applied to space-variant problems modeled by random fields

C. AMRANE

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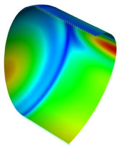
Context

Reliability analysis of space-variant problems

Uncertainty modeling	Problem definition	Uncertainty propagation
Random variables **Stochastic processes**	Computer model	Quantity(ies) of interest : • Response PDF/CDF • Moments • Probability of failure



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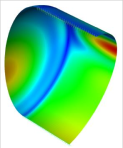

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- Probability of failure :

$$P_f = \mathrm{P}(\min_{\tau \in \mathcal{D}} g(\mathbf{X}, \tau) \leq 0)$$

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- The location $\tau^{(0)}$ may not be well identified
- The failure probability could be under or overestimated
- Spatial random variability may generates multiple critical zones

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$$P_f \simeq \mathrm{P}(\min_{i=1,\dots,p} g(\mathbf{X}, \tau^{(i)}) \leq 0) \simeq \mathrm{P}(\min_{i=1,\dots,p} g_i(\mathbf{X}) \leq 0)$$

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  - Equivalence with series systems :

$$P_f \simeq P_{f,Sys} = \mathrm{P}(\cup_{i=1}^{p} g_i(\mathbf{X}) \leq 0) = \mathrm{P}(g_{comp}(\mathbf{X}) \leq 0)$$

where $g_{comp}(\mathbf{X}) = \min_{j=1,\dots,p} g_j(\mathbf{X})$

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- Consider space-variant problems as series systems
- Apply system reliability approaches

Challenges

- Very high number of locations $p >> 1$
- How many and which locations are to consider ?
- Realistic high-fidelity computational models imply time consuming performance functions

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Approach

- Adaptive search of potential failure zones
- Surrogate model-based system reliability method

Outline

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AK-SYS : Active Learning and Kriging-based SYStem reliability method [1]

Motivations

- Few works consider the use of adaptive learning Kriging when space-variant problems are treated as series systems
- Easy to implement
- Concentrate the numerical effort on regions with a significant probability content

1. Fauriat et Gayton 2014

AK-SYS : Active Learning and Kriging-based SYStem reliability method [1]

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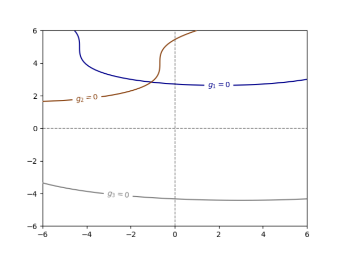
Illustration

$$
\begin{aligned}
P_{f,Sys} &= \mathrm{P}(\cap_{i=1}^{3} g_i(\mathbf{X}) \leq 0) \\
&= \mathrm{P}(\max_i \, g_i(\mathbf{X}) \leq 0) \\
&= \mathrm{P}(g(\mathbf{X})) \leq 0
\end{aligned}
$$



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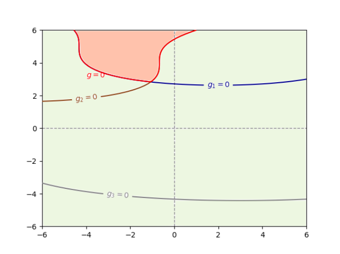

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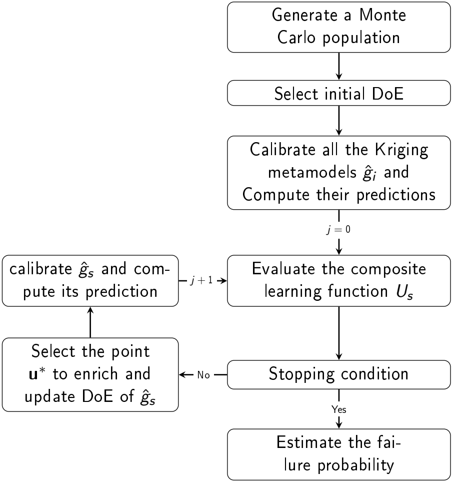

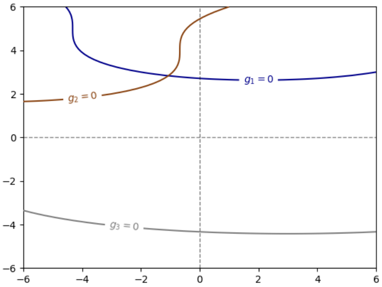

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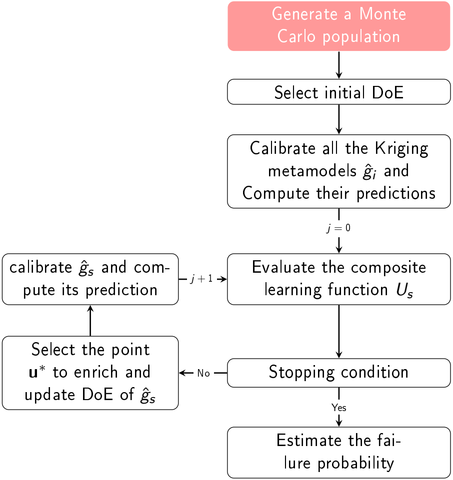

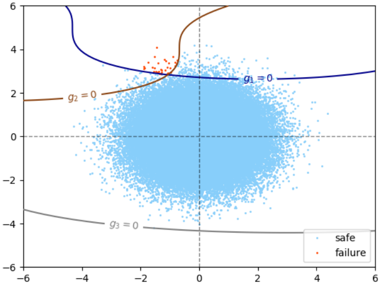

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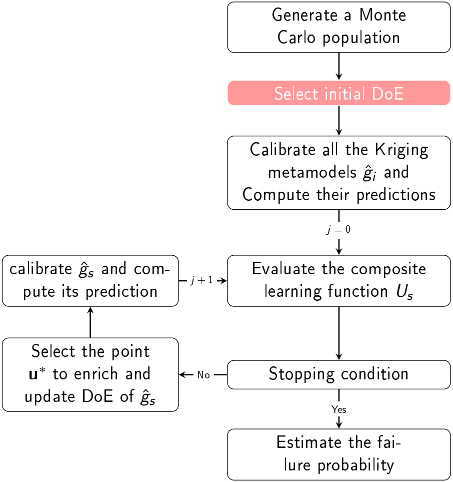

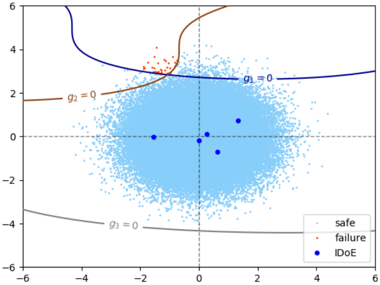

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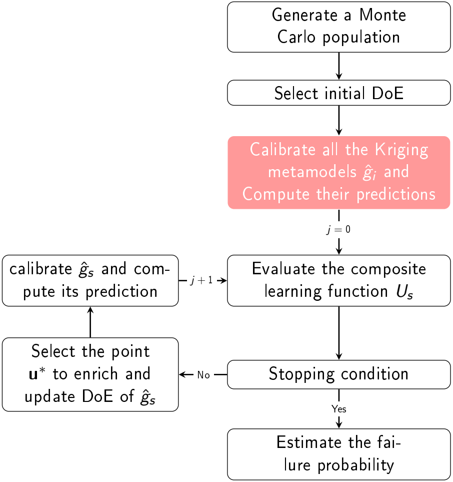

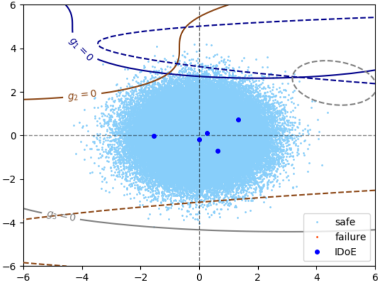

With OpenTURNS [2]

2. Baudin et al. 2016

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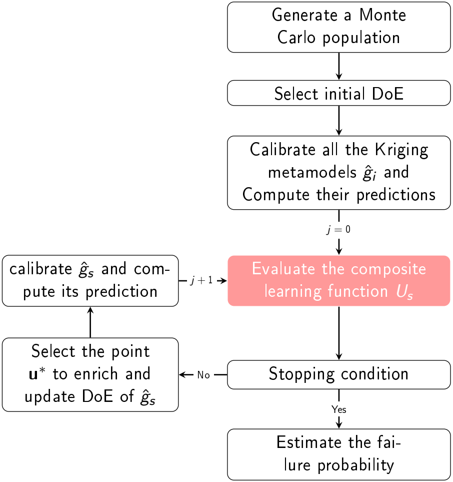

For a given $\mathbf{u}^{(k)}, k = 1, ..., N_{MC}$:

$$U_s(\mathbf{u}^{(k)}) = \frac{|\mu_{\hat{g}_s}(\mathbf{u}^{(k)})|}{\sigma_{\hat{g}_s}(\mathbf{u}^{(k)})}$$

$$s = \underset{i}{\operatorname{argmax}}\ \mu_{\hat{g}_i}(\mathbf{u}^{(k)})$$

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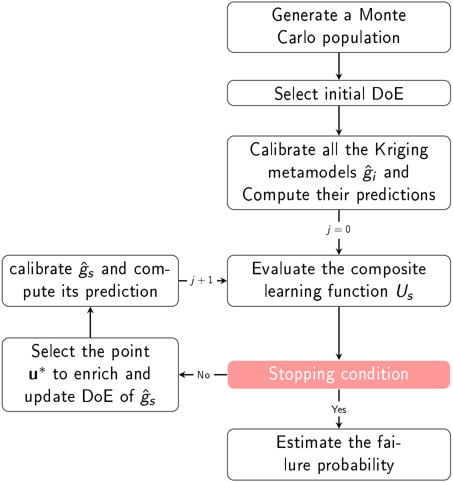

$$\min_{i=1,\ldots,N_{MC}} U_s(\mathbf{u}^{(i)}) \geq 2$$

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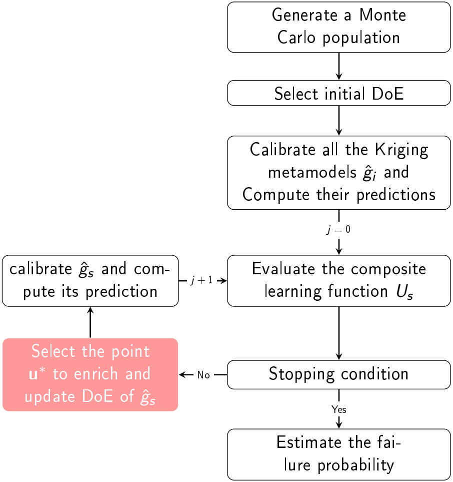

$$\mathbf{u}^* = \min_{k=1,\dots,N_{MC}} U_s(\mathbf{u}^{(k)})$$

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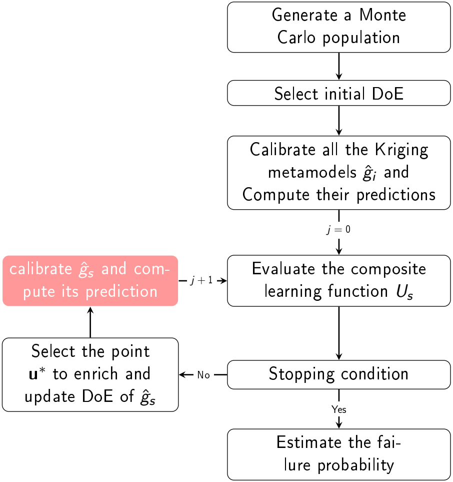
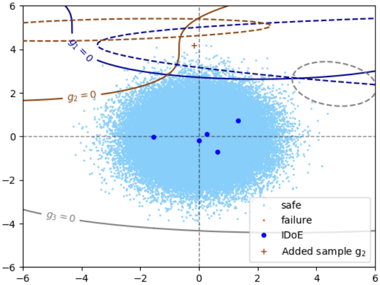




AK-SYS : Active Learning and Kriging-based SYStem reliability method

Generate a Monte Carlo population

↓

Select initial DoE

↓

Calibrate all the Kriging metamodels $\hat{g}_i$ and Compute their predictions

↓ $j = 0$

Evaluate the composite learning function U_s

↓

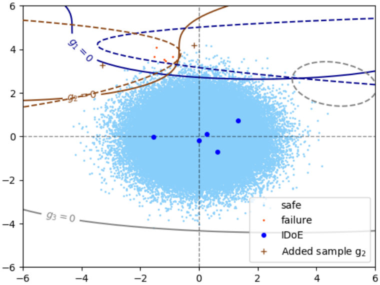
Stopping condition

— No → Select the point $\mathbf{u}^*$ to enrich and update DoE of $\hat{g}_s$

→ calibrate $\hat{g}_s$ and compute its prediction — $j+1$ → Evaluate the composite learning function U_s

↓ Yes

Estimate the failure probability



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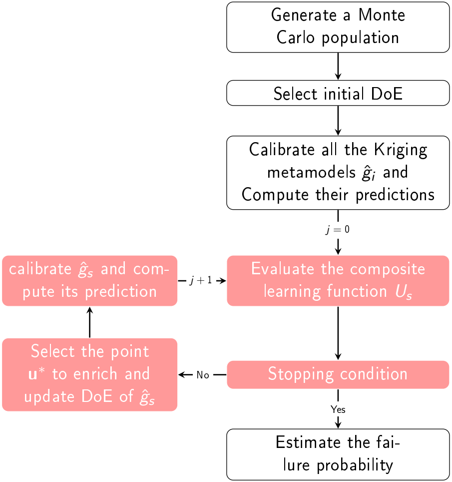

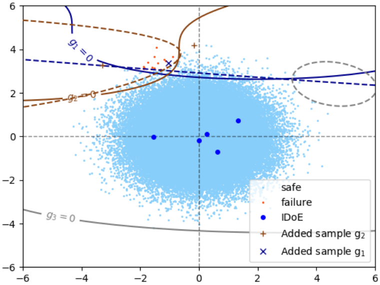

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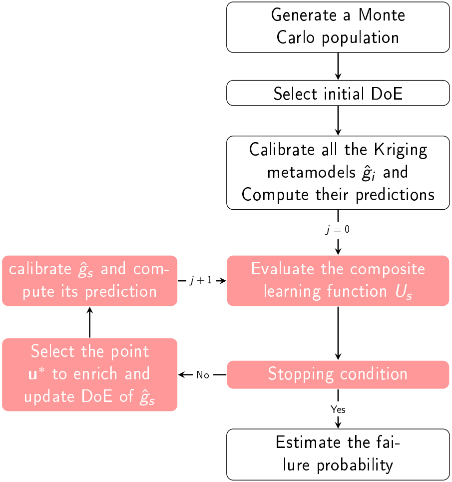

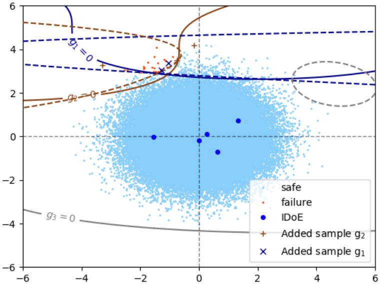

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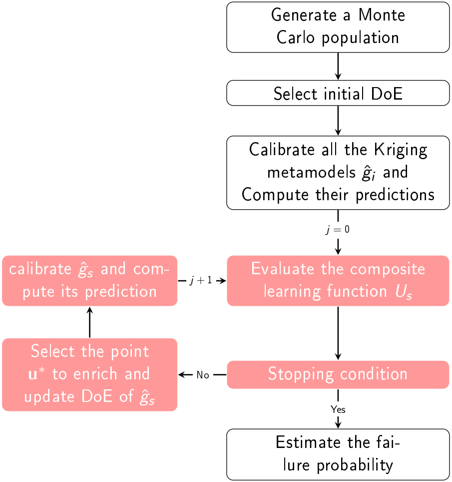

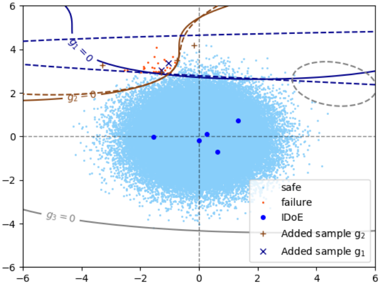

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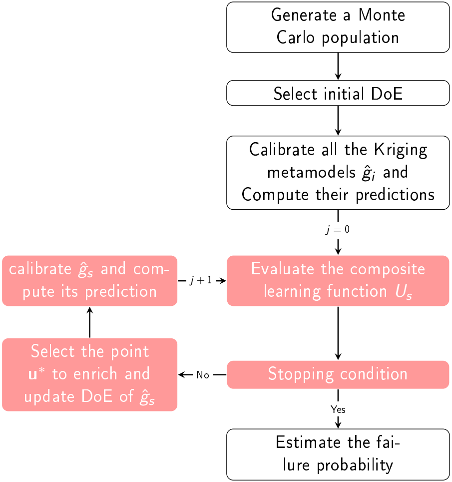

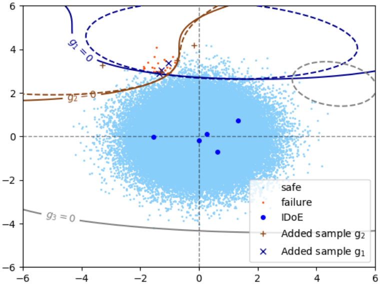

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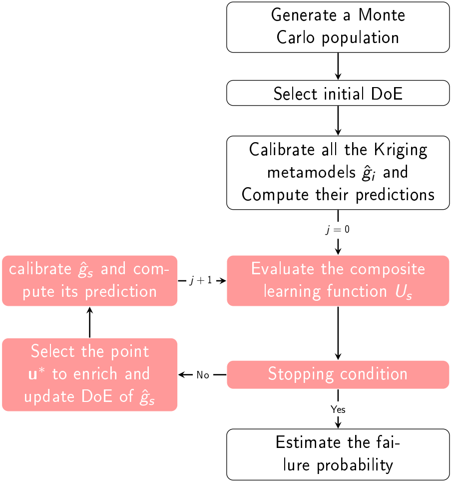

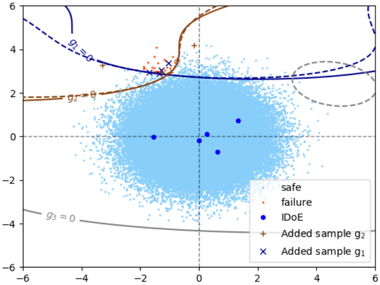

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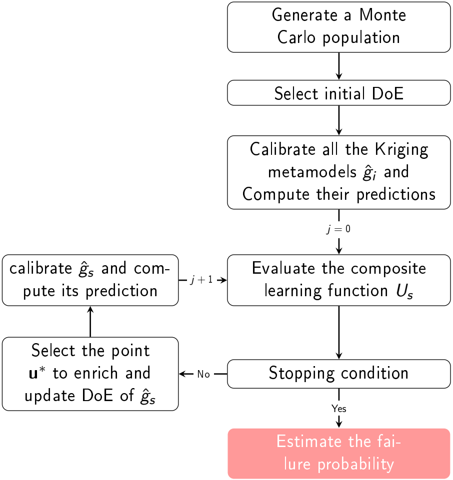
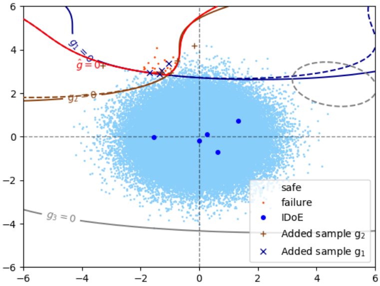
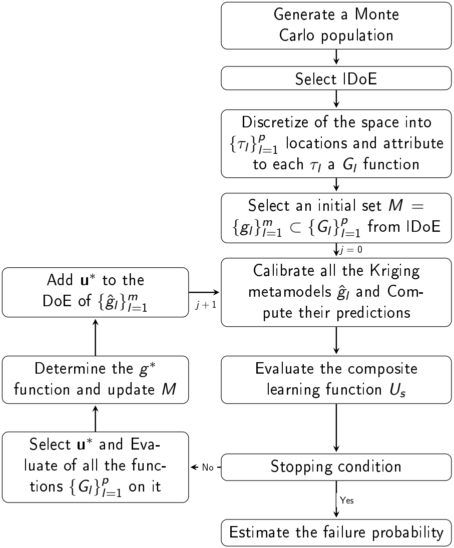




$$\hat{P}_f = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} I\{\min_{j=1,\ldots,m} \hat{g}_j(\mathbf{x}^{(i)}) \leq 0\}$$

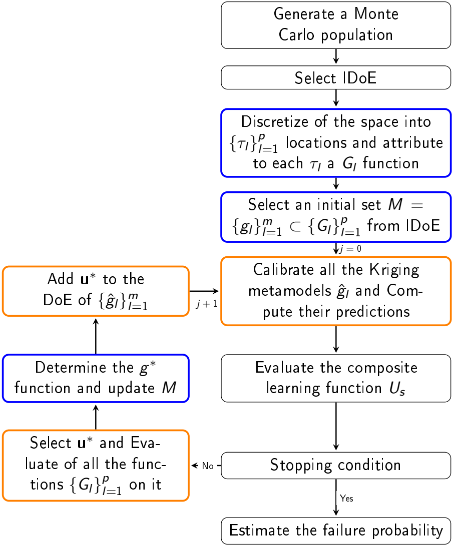
Outline

AK-SYSs for reliability analysis of space-variant problems



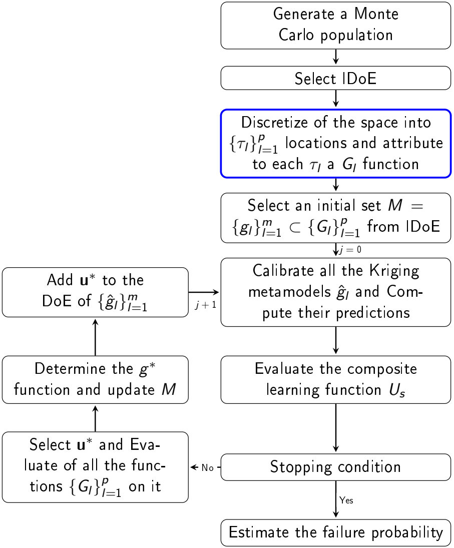
- Based on the AK-SYS framework

AK-SYSs for reliability analysis of space-variant problems



- Based on the AK-SYS framework
- Combines an adaptive search of critical zones with the adaptive enrichment process of AK-SYS
- The performance functions are evaluated on $\mathbf{u}^*$ and all the Kriging models are updated at each iteration

AK-SYSs for reliability analysis of space-variant problems



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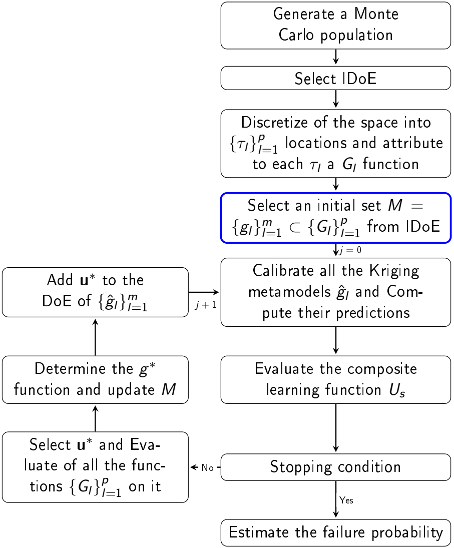


For each $\mathbf{u}^{(i)}, i = 1, ..., N_{IDoE}$:

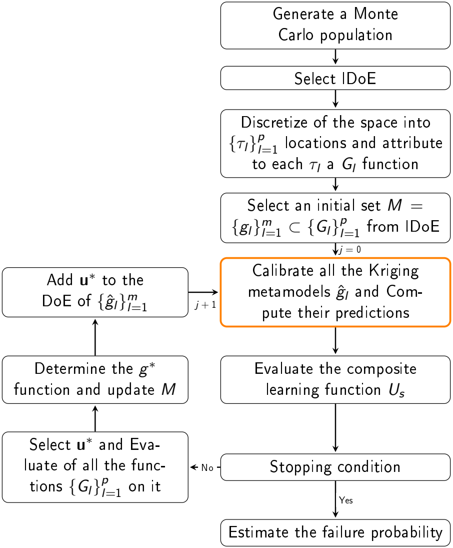
$$l^* = \underset{l}{\text{argmin}} \; G_l(\mathbf{u}^{(i)})$$

$$g^* = G_{l^*}$$

If $g^* \notin M$ then :

$$M = M \cup \{g^*\}$$

AK-SYSs for reliability analysis of space-variant problems



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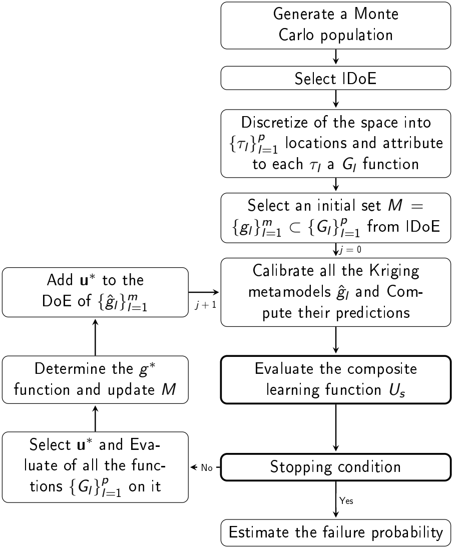

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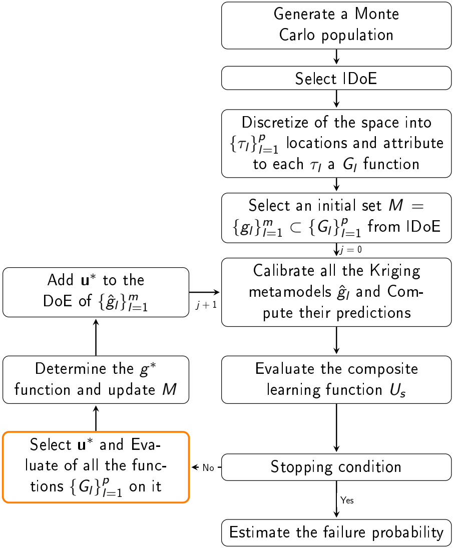

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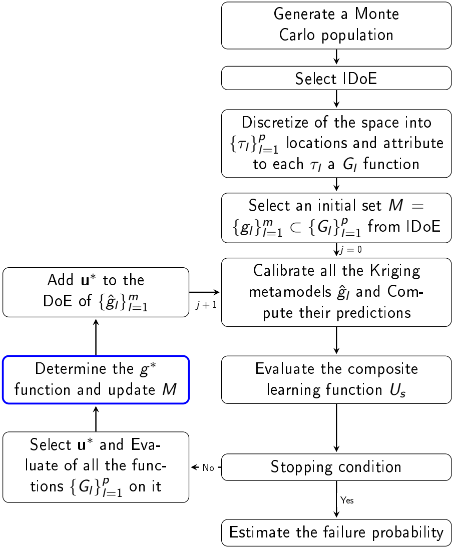


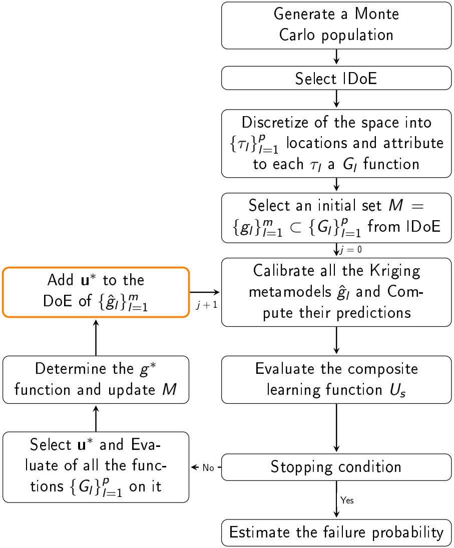
$$l^* = \underset{l}{\operatorname{argmin}}\; G_l(\mathbf{u}^*)$$

$$g^* = G_{l^*}$$

If $g^* \notin M^{(j)}$ then :

$$M^{(j+1)} = M^{(j)} \cup \{g^*\}$$

AK-SYSs for reliability analysis of space-variant problems



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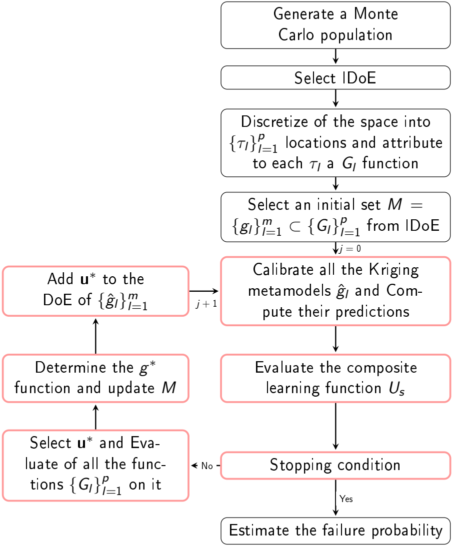

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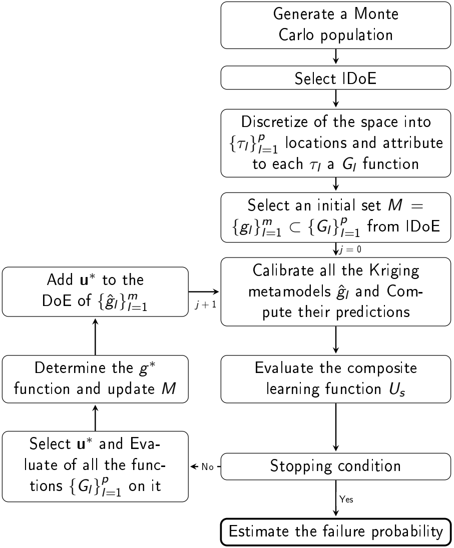

$$\hat{P}_f = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} I\{\min_{l=1,\ldots,m} \hat{g}_l(\mathbf{u}^{(i)}) \leq 0\}$$

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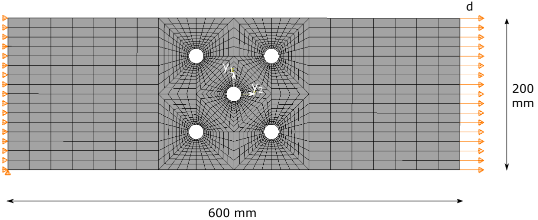
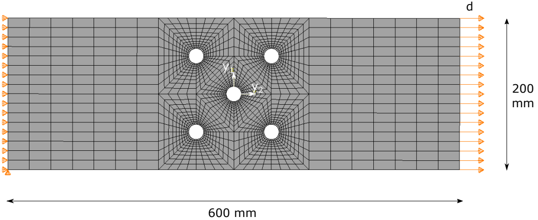
Application example



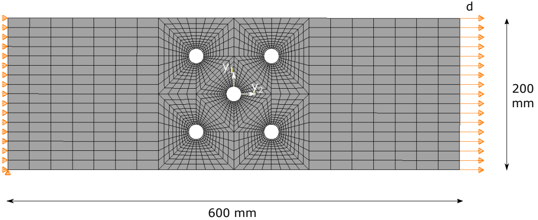
Application example



Case 1 : Spatial random material property

- $\nu = 0.3$
- $E \sim LN(200, 50) GPa$
- $d = 0.5 mm$

Application example



Case 1 : Spatial random material property

- $\nu = 0.3$
- $E \sim LN(200, 50)GPa$
- $d = 0.5mm$

Case 2 : Spatial random material property and boundary conditions

- $\nu = 0.3$
- $E \sim LN(200, 50)GPa$
- $d \sim N(0.5, 0.05)mm$

Application example

- The Karhunen-Loève (KL) expansion [2] is used to model a stationary Gaussian stochastic field of E and d ;
- The KL expansion is truncated to $r_E = 18$ for E and to $r_d = 6$;
  - $\rightarrow$ 18 RVs for case 1
  - $\rightarrow$ 24 RVs for case 2
- For each realization of the random field :

$$G_j(\mathbf{u}) = \sigma_y - \sigma_j \tag{1}$$

where $\sigma_y = 800\ MPa$ is the yield stress of the material, σ_j is the Von Mises stress calculated with the FEM on the j^{th} node and $j = 1, ..., 1946$.

2. Ghanem, Member et Spanos 1991

Application example

Abaqus coupling with python

- Create the input files
  - → values of Young's modulus in each element of the discretized space
  - → values of the applied load
- Execute Abaqus

```
command2 = ['C:\Appli\SIMULIA\Commands/abaqus.bat', 'cae','noGUI=runmodel.py']
process2 = subprocess.call(command2,shell=True, stdout=subprocess.PIPE,stderr=subprocess.PIPE)
```

- Read the output file
  - → contain the values of the QoI
  - → a python script is used to read the file

Application example

Case 1 : Spatial random material property

Probabilistic results

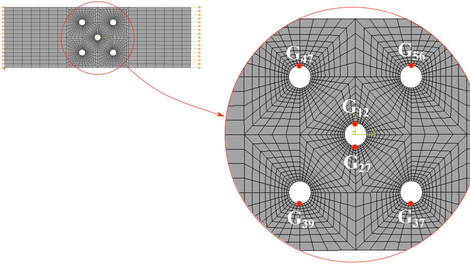
Method	$\mathrm{N}_{\text{calls}}$	$P_{f,Sys}$	RAE
MC	50000	0.00156	–
FORM	2855	0.00202	29.7%
AK-MCS	190	0.00154	1.28%
AK-SYSs	166	0.00156	0.00%

Application example

Case 1 : Spatial random material property

Probabilistic results

Method	N_{calls}	$P_{f,Sys}$	RAE
MC	50000	0.00156	–
FORM	2855	0.00202	29.7%
AK-MCS	190	0.00154	1.28%
AK-SYSs	166	0.00156	0.00%



Application example

Case 2 : Spatial random material property and boundary conditions

Probabilistic results

Method	N_{calls}	Coefficient of variation of N_{calls}	P_{Sys}	RMAE
MC	50000	–	0.00452	–
FORM	10802	–	0.00540	19.50%
AK-MCS	654.75	10.59%	0.00347	23.14%
AK-SYSs	566.20	4.99%	0.00417	7.72%

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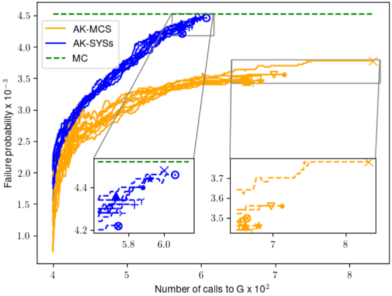

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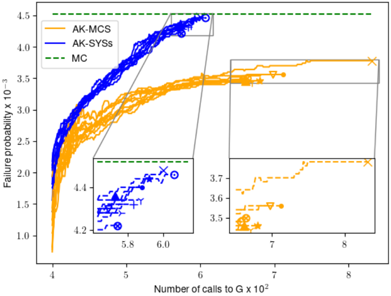
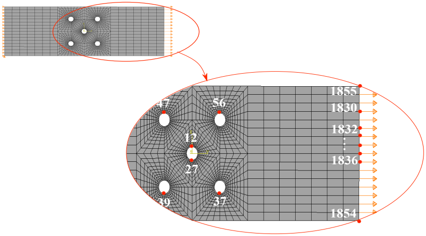


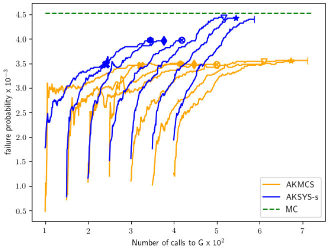


Application example

Case 2 : Spatial random material property and boundary conditions

Results of the sensitivity to the IDoE size

Size of the IDoE		100	150	200	250	300	350	400
AK-MCS :	P_{Sys}	0.00346	0.0035	0.00348	0.00348	0.00344	0.00356	0.00356
	Added Samples	222	263	264	248	309	322	312
AK-SYSs :	P_{Sys}	0.00348	0.00398	0.00396	0.00398	0.00444	0.00442	0.00444
	Added Samples	142	194	176	167	215	194	187
Initial set of G		39, 1834, 27, 47, 12, 37, 1832, 56	39, 1834, 27, 47, 12, 37, 1832, 56, 1854	39, 1834, 27, 47, 12, 37, 1832, 56, 1854, 1855, 1836	39, 1834, 27, 47, 12, 37, 1832, 56, 1854, 1855, 1836	39, 1834, 27, 47, 12, 37, 1832, 56, 1854, 1855, 1836	39, 1834, 27, 47, 12, 37, 1832, 56, 1854, 1855, 1836	39, 1834, 27, 47, 12, 37, 1832, 56, 1854, 1855, 1836
Added G		1833	1835, 1836, 1833	1835	1835, 1833	1835, 1833, 1830	1835, 1833, 1830	1835, 1833, 1830



Outline

Conclusions and perspectives

- This presentation exposes an extension of AK-SYS to space-variant problems named AK-SYSs
- Locate the critical zones whiteout prior assumptions on their locations
- AK-SYSs outperforms AK-MCS, performed on a composite performance function, in terms of accuracy, robustness and computational cost.
- Sensitivity to the IDoE's size
  - More recent system reliability methods, such as AK-SYSi [3] and EEK-SYS [4] could be investigated
- Spatial correlation between the LSFs should be considered

3. Yun et al. 2019
4. Jiang et al. 2020

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