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Two Elliptical Copulas: Gaussian and Student Copulas

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Gaussian Copula

The Gaussian copula, derived from the multivariate normal distribution, is perhaps the most commonly used copula family mainly due to its simplicity. The n-dimensional multivariate Gaussian copula with correlation matrix $\rho n \times n$ can be expressed as (Nelsen (2006)):

$$C_\rho(u_1, ..., u_n) = F_\rho^n(F^{-1}(u_1), ..., F^{-1}(u_n))$$

whose density function is:

$$c(u_1, \ldots, u_n) = \frac{1}{\sqrt{det\rho}} exp\left(-\frac{1}{2}\right.$$

$$\begin{aligned} \text{where :} \quad F^n &= \text{Multivariate Gaussian CD} \\ y(u_i) &= F^{-1}(u_i) \end{aligned}$$

t-Copula

The t-copula, also known as Student copula, is an elliptical copula based on the Student distribution that can be represented as:

$$C_{\nu,\rho}(u_1, \ldots, u_n) = t^n_{\nu,\rho}(t_\nu^{-1}$$

$$\begin{aligned} \text{where :} \quad t^n &= \text{Multivariate Student CDF} \\ \rho &= \text{shape matrix} \\ \nu &= \text{degrees of freedom} \end{aligned}$$

and

$$t^n_{\nu,\rho}(x) = \frac{1}{\sqrt{det\rho}} \frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)(\pi\nu)^{n/2}} \times \int_{-\infty}^{x_1}$$

For $\nu > 2$, the shape matrix in the above equation is proportional to the correlation matrix (Malevergne and Sornette (2003)). The density function of the t-copula can be expressed as (Malevergne and Sornette (2003)):

$$c(u_1, \ldots, u_n) = \frac{1}{\sqrt{det\rho}} \frac{\Gamma\left(\frac{\nu+n}{2}\right)\left(\Gamma\left(\frac{\nu}{2}\right)\right)}{\left(\Gamma\left(\frac{\nu+1}{2}\right)\right)^n}$$

$$\begin{aligned} \text{where:} \quad y_k &= t_\nu^{-1}(u_k) \\ t_\nu &= \text{univariate Student distributio} \end{aligned}$$

Figure 1 shows the bivariate copula density functions of the Gaussian copula and t-copula for different parameters. As shown, the density function is wider for lower correlations (compare Figures 1(a) and 1(b)). Additionally, note the difference between the density functions for different values of degrees of freedom (Figures 1(c) and 1(d)). Both Gaussian and t-copulas are elliptical; however, they represent different tail dependencies, which describes the significance of the dependence in lower left quantile or upper right quantile of a multivariate distribution function.

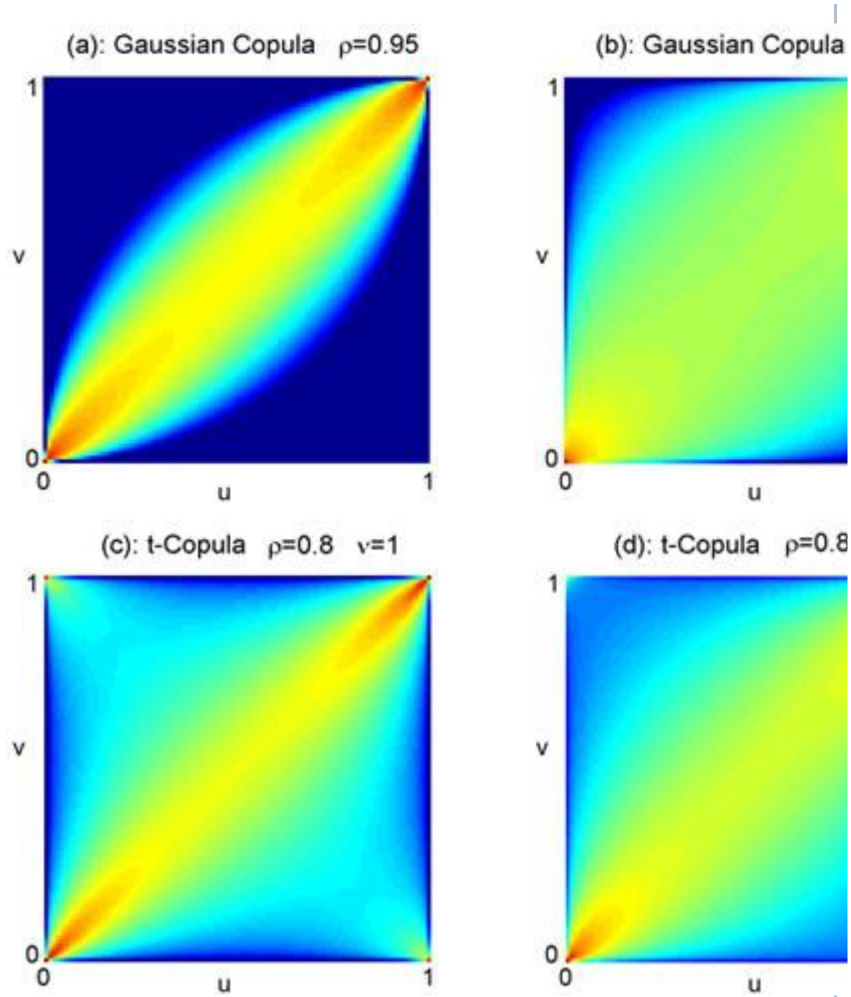

Figure 1: Bivariate copula density functions of the Gaussian copula and t-copula for different parameters.

Bibliography

Embrechts, P., Lindskog, F., McNeil, A., 2001. Modelling Dependence with Copulas and Applications to Risk Management. Department of Mathematics, ETHZ, Zurich, Switzerland.

Malevergne, Y., Sornette, D., 2003. Testing the gaussian copula hypothesis for financial assets dependences. In: Quantitative Finance Volume 3, 231-250.

Nelsen, R., 2006. An Introduction to Copulas (Springer Series in Statistics). Springer Verlag, New York.