Gaussian process approximation of a multidisciplinary system and sensitivity analysis to model uncertainty

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19/06/2020

OpenTURNS Users'Day

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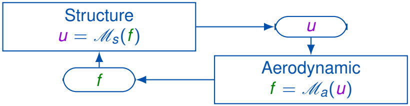
Multidisciplinary analysis (MDA)

- Engineering systems which behavior depends on the interaction between various physical disciplines.
- Example: aircraft wing, interaction between aerodynamic loads and elastic strain of the wing

Multidisciplinary analysis (MDA)

- Modeling: **coupling** between the models that describe each physical phenomenon
- Partitioned approach: non intrusive coupling of each disciplinary solver
- Example



where $\mathscr{M}_s$ is the structural mechanics solver and $\mathscr{M}_a$ the aerodynamic solver.

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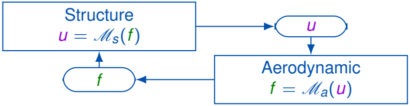


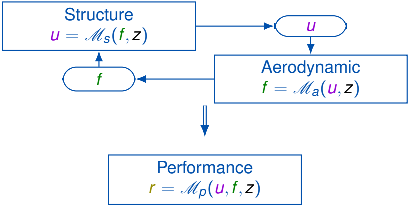
  where $\mathscr{M}_s$ is the structural mechanics solver and $\mathscr{M}_a$ the aerodynamic solver.
- Solution is the displacement field u and the loading vector f that solve the non linear system

$$
\begin{aligned}
u &= \mathscr{M}_s(f) \\
f &= \mathscr{M}_a(u)
\end{aligned}
$$

- Iterative algorithms (fixed point, Newton based etc.) *i.e.* several calls to the disciplinary solvers.

Parametric MDA

- Design variable z (parametric study, optimization, sensitivity analysis, reliabilty etc.)



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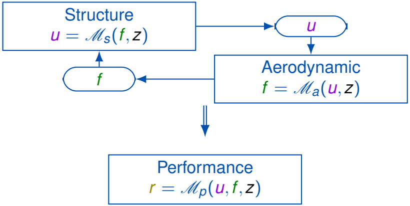

- **Problematic:** solving the MDA for each z implies a huge numerical cost.

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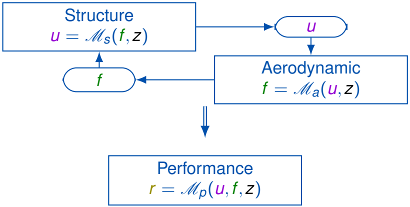

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- **Solution:** construction of an approximation $\hat{r}(u, f, z)$

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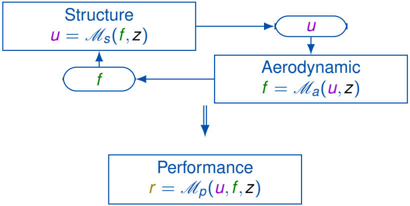

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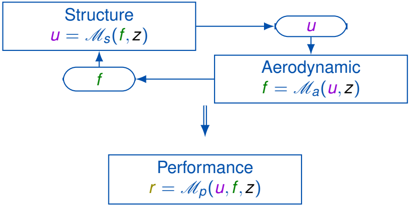
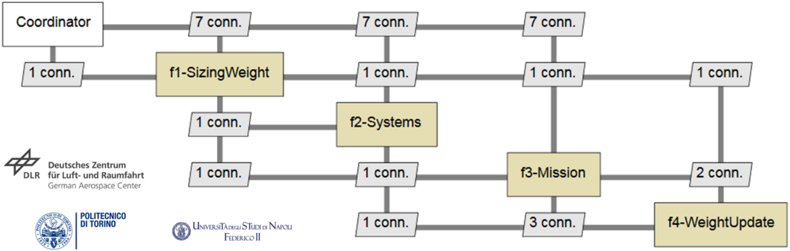


- **Problematic:** solving the MDA for each z implies a huge numerical cost.
- **Solution:** construction of an approximation $\hat{r}(u, f, z)$
    - 1^{st} option: direct approximation of r
    - 2^{nd} option: approximation of each disciplinary solver

Parametric MDA

- MDA can involve many disciplines and many partners (eg. H2020 AGILE project [1])



1 S.Dubreuil et al., Efficient global multidisciplinary optimization based on surrogate models. Multidisciplinary Analysis and Optimization Conference, 2018

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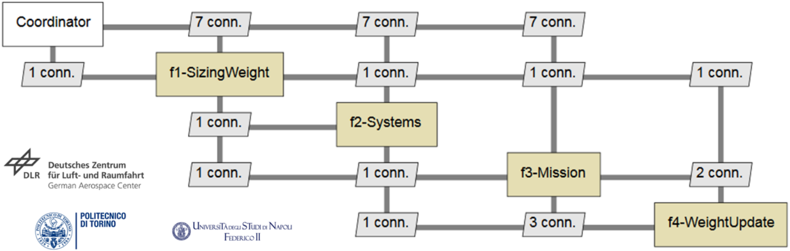

- 2^{nd} option allows to uncouple the MDA and to reduce the computational cost.

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Disciplinary GP

- Construction of a Gaussian process interpolation for each disciplinary solver [2]

$$
\begin{aligned}
u(z) &= \hat{\mathscr{M}}_s(f,z) + \varepsilon_s \\
f(z) &= \hat{\mathscr{M}}_a(u,z) + \varepsilon_a
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2 S. Dubreuil et al. Propagation of modeling uncertainty by polynomial chaos expansion in muldisciplinary analysis. ASME, Journal of Mechanical Design, 2016

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where $\omega \in \Omega$ ($(\Omega, \mathscr{F}, P)$ is a probability space)

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U(\omega, z) &= \hat{\mathscr{M}}_s(F(\omega, z), z) + \varepsilon_s(F(\omega, z), \omega, z) \\
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where $\omega \in \Omega$ (($\Omega, \mathscr{F}, P$) is a probability space) and $(U(\omega, z), F(\omega, z))$ is the new unknown of the problem.

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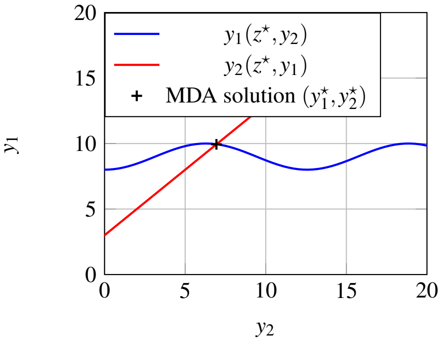
- The performance function $R = \mathscr{M}_p(U(\omega,z),F(\omega,z))$ is also a random variable of unknow probablity distribution [3], [4].

2 S. Dubreuil et al. Propagation of modeling uncertainty by polynomial chaos expansion in muldisciplinary analysis. ASME, Journal of Mechanical Design, 2016
3 S. Dubreuil et al. Extreme value oriented random field discretization based on an hybrid polynomial chaos expansion & kriging approach. Computer Methods in Applied Mechanics and Engineering, 2018
4 S. Dubreuil et al. Toward an efficient global multidisciplinary design optimization algorithm. Structural and Multidisciplinary Optimization, 2020

GP based MDA

- Illustrative example

$$\begin{cases} y_1(z,y_2) = f_1(z,y_2) = z^2 - \cos\left(\frac{y_2}{2}\right) \\ y_2(z,y_1) = f_2(z,y_1) = z + y_1 \end{cases}$$



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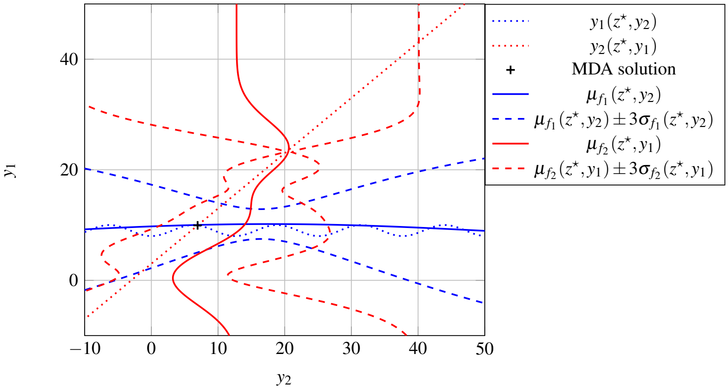

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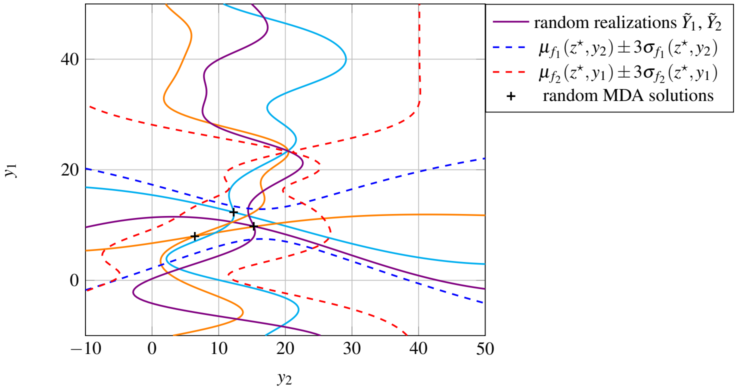

GP based MDA

- Assumption: for each random realization, existence and uniqueness of the solution
  $\Rightarrow$ the solution of the random MDA is the random vector of the coupling variables.

5 Z. Hu and S. Mahadevan, Adaptive Surrogate Modeling for Time-Dependent Multidisciplinary Reliability Analysis, Journal of Mechanical Design, 2017

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- Resolution:
  - 1^{st} option: discretization of the disciplinary GPs (eg. Karhunen Loève) and resolution $\Rightarrow$ high stochastic dimension
  - 2^{nd} option: simplification with perfectly dependent disciplinary GPs.

$$\hat{Y}_i(z) = \mu_{f_i}(z, y_{c(i)}) + \sigma_i(z, y_{c(i)})\xi_i$$

  $\Rightarrow$ the problem is parametrized by one Gaussian variable per coupling variable.

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- Proposed approach:
  1. Sampling the disciplinary solver to compute the disciplinary DoE
     DoE_s structure and DoE_a aerodynamic
  2. Create the disciplinary GPs
     $\hat{U}$ and $\hat{F}$
  3. Solve the random MDA by MCS on perfectly dependent GPs
     $\hat{U} = \mu_{\hat{\mathcal{M}}_s}(z, f) + \sigma_s(z, f)\xi_s$ and $\hat{F} = \mu_{\hat{\mathcal{M}}_a}(z, u) + \sigma_a(z, u)\xi_a$
  4. Construct a polynomial chaos expansion approximation of the random performance function
     $R(U, F) \approx \hat{R}(\hat{U}(\xi_s, \xi_a), \hat{F}(\xi_s, \xi_a)) = \hat{R}(\xi_s, \xi_a)$
  5. Sensitivity analysis with respect to ξ_i to decide which disciplinary GPs should be improved [5]

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Introduction & objectives

Disciplinary surrogate model based MDA

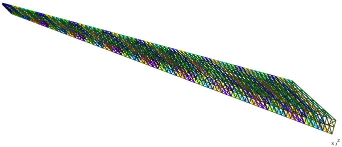
From scalar to vector-valued coupling variable

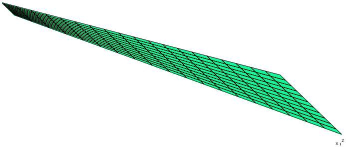
Application

Conclusions

Application to vector-valued coupling variable

- Aero-elasticity problem: $u \in \mathbb{R}^{n_s}$ and $f \in \mathbb{R}^{n_a}$





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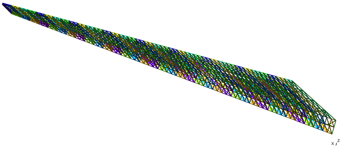

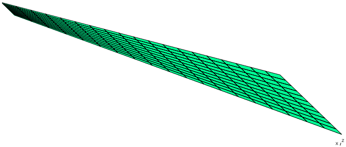

- Reduce order model, disciplinary approximations are assumed to read

$$\hat{Y}_i(z, y_{c(i)}) = \sum_{j=1}^{N_i} \hat{y}_{i_j}(z, y_{c(i)}) Y_{i_j}$$

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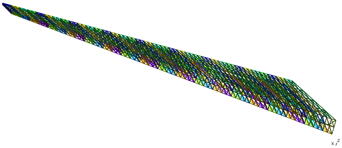

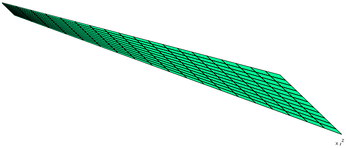

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- POD+Interpolation by Gaussian process

$$\tilde{Y}_i(z, y_{c(i)}) = \sum_{j=1}^{N_i} \left(\mu_{f_{i_j}}(z, y_{c(i)}) + \sigma_{f_{i_j}}(z, y_{c(i)}) \xi_{i_j} \right) Y_{i_j}$$

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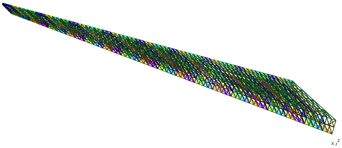
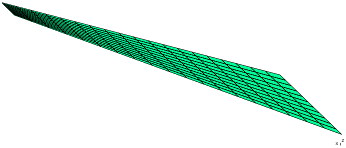

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- The disciplinary GP is parametrized by the random vector $\Xi_i = (\xi_{i_j}, j = 1, \cdots, N_i)$

Application to vector-valued coupling variable

- Resolution: same as for the scalar case

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- Sparse PCE approximation (Least Angle Regression) [6]

$$\hat{R}(\Xi_i, i = 1, \cdots, n_{dis}) = \sum_{\boldsymbol{\alpha} \in \mathscr{A}} a_{\boldsymbol{\alpha}} H_{\boldsymbol{\alpha}}(\Xi)$$

where $\Xi = (\Xi_i, i = 1, \cdots, n_{dis})$, $H_{\boldsymbol{\alpha}}$ the multivariate Hermite polynomial indexed by the multi-index $\boldsymbol{\alpha}$ and $a_{\boldsymbol{\alpha}}$ the unknown coefficients.

6 Blatman, G., Sudret, B., Adaptive sparse polynomial chaos expansion basedon least angle regression,Journal of Computational Physics, 2011

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- Sensitivity analysis of $\hat{R}$ with respect to the random vectors $\Xi_i, i = 1, \cdots, n_{dis}$ [7]

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- Approximation by sparce PCE [8]

$$\hat{S}_{\Xi_i} = \frac{1}{\mathbb{V}(\hat{R})} \sum_{\boldsymbol{\alpha}\in\mathscr{A}_i} a_{\boldsymbol{\alpha}}$$

où $\mathbb{V}(\hat{R}) = \sum_{\alpha\in\mathscr{A}\setminus\{\bar{0}\}} a_{\boldsymbol{\alpha}}^2$ et $\mathscr{A}_i = \{\boldsymbol{\alpha}\in\mathscr{A}\setminus\{\bar{0}\},\ \boldsymbol{\alpha}_{\sim i} = 0\}$

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Application to vector-valued coupling variable

- OpenTURNS

FunctionalChaosSobolIndices ¶

class **FunctionalChaosSobolIndices(**args*)**

Sensitivity analysis based on functional chaos expansion.

Available constructors:

FunctionalChaosSobolIndices(functionalChaosResult)

Parameters:

functionalChaosResult : `FunctionalChaosResult`

A functional chaos result resulting from a polynomial chaos decomposition.

Methods

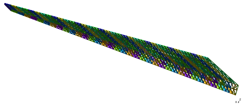
`getClassName`(self)	Accessor to the object's name.
`getFunctionalChaosResult`(self)	Accessor to the functional chaos result.
`getId`(self)	Accessor to the object's id.
`getName`(self)	Accessor to the object's name.
`getShadowedId`(self)	Accessor to the object's shadowed id.
`getSobolGroupedIndex`(self, *args)	Get the grouped Sobol first order indices.
`getSobolGroupedTotalIndex`(self, *args)	Get the grouped Sobol total order indices.
`getSobolIndex`(self, *args)	Get the Sobol indices.
`getSobolTotalIndex`(self, *args)	Get the total Sobol indices.
`getVisibility`(self)	Accessor to the object's visibility state.
`hasName`(self)	Test if the object is named.
`hasVisibleName`(self)	Test if the object has a distinguishable name.
`setName`(self, name)	Accessor to the object's name.
`setShadowedId`(self, id)	Accessor to the object's shadowed id.
`setVisibility`(self, visible)	Accessor to the object's visibility state.
`summary`(self)	Summary accessor.

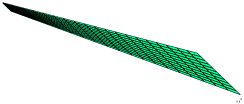
Preliminary results on an aero-elasticity test case

- Linear structural solver $KU = F$
- Potential flow solver (Vortex Lattice Method) $A\Gamma = B$.
- System to solve

$$\begin{aligned} KU &= F(\Gamma) \\ A(U)\Gamma &= B \end{aligned}$$

(+interpolation operation between the two meshes)





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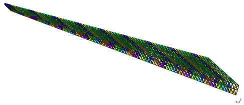
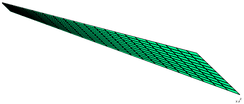
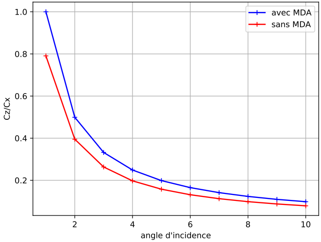
$$KU = F(\Gamma)$$
$$A(U)\Gamma = B$$

  (+interpolation operation between the two meshes)
- Design variable, angle of attack $z \in [1^\circ, 10^\circ]$
- Quantity of interest, lift to drag ratio $R = \frac{C_z}{C_x}$







Preliminary results on an aero-elasticity test case

- Disciplinary GPs

$$\hat{U}(z,\hat{\gamma}_j, j=1,\cdots,N_\Gamma) = \sum_{i=1}^{N_U} \hat{u}_i(z,\hat{\gamma}_j, j=1,\cdots,N_\Gamma) U_i$$
$$\hat{\Gamma}(z,\hat{u}_i, i=1,\cdots,N_U) = \sum_{j=1}^{N_\Gamma} \hat{\gamma}_j(z,\hat{u}_i, i=1,\cdots,N_U) \Gamma_j$$

where $N_\Gamma = 4$ and $N_U = 3$.

Preliminary results on an aero-elasticity test case

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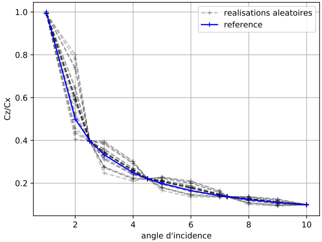
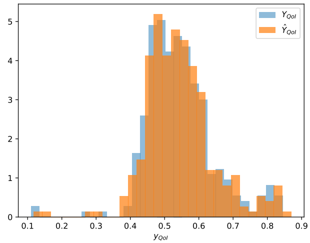
- In practice these disciplinary GPs are constructed using a common disciplinary DoE (solving 5 MDA). Uncoupling this construction is a current evolution of the method.
- Solution of the random MDA by MCS



Preliminary results on an aero-elasticity test case

- Sensitivity analysis at angle of attack $z = 2^{\circ}$
- Sparse PCE approximation (300 resolutions of the random MDA are used to identify the PCE coefficients)
- Histogram of the quantity of interest, comparison MC PCE



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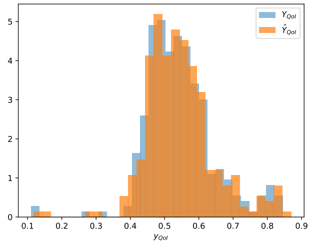

- Results of the sensitivity analysis (coefficients of variation are estimated by bootstrap)

variable	Mean $\hat{S}_{\Xi_i}$	Coeff. var. $\hat{S}_{\Xi_i}$	Mean $\hat{S}^T_{\varepsilon_i}$	Coeff. var. $\hat{S}^T_{\varepsilon_i}$
Ξ_U	5.2×10^{-3}	11.0%	1.5×10^{-2}	8.3%
Ξ_Γ	$\boxed{9.8 \times 10^{-1}}$	0.12%	$\boxed{9.9 \times 10^{-1}}$	0.06%

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- Reduction of the variance of the quantity of interest from 18% to 3%
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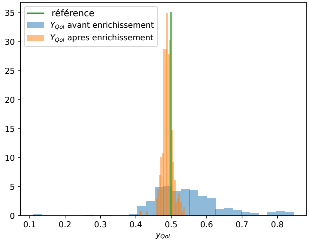

Preliminary results on an aero-elasticity test case

- What if we do not follow the sensitivity analysis conclusion and improve the structural disciplinary GP

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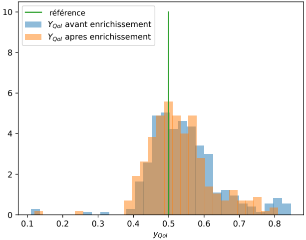

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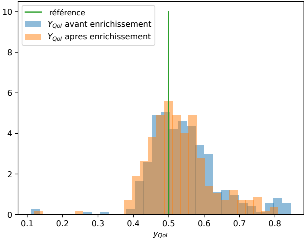

- Sensitivity analysis to model uncertainty is useful :-)

Preliminary results on an aero-elasticity test case

- Lift to drag ratio is an "aerodynamic" quantity of interest and the conclusion is to improve the aerodynamic model... I don't need PCE and sensitivity analysis to draw this conclusion!

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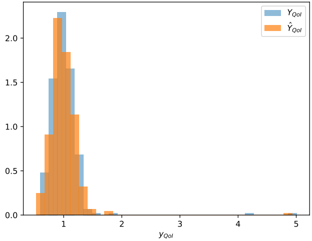
- Let's try with a "structural" quantity of interest. We consider the wing twist at tip.
- Histogram of the quantity of interest, comparison MC PCE



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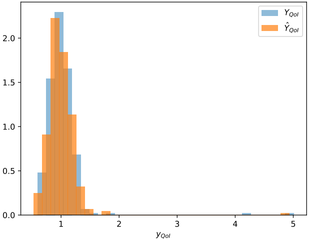
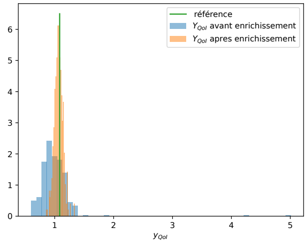


variable	Mean $\hat{S}_{\Xi_i}$	Coeff. var. $\hat{S}_{\Xi_i}$	Mean $\hat{S}^T_{\varepsilon_i}$	Coeff. var. $\hat{S}^T_{\varepsilon_i}$
Ξ_U	1.6×10^{-1}	13.3%	3.5×10^{-1}	18.5%
Ξ_Γ	$\boxed{6.5 \times 10^{-1}}$	9.7%	$\boxed{8.4 \times 10^{-1}}$	2.5%

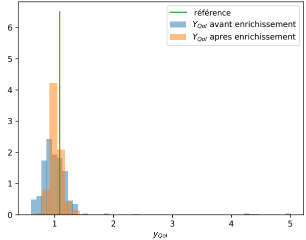
Preliminary results on an aero-elasticity test case

- Improvement of the aerodynamic disciplinary GP allows to reduce the variance of the QoI from 34% to 7%
- Histogram of the quantity of interest after disciplinary enrichment



Preliminary results on an aero-elasticity test case

- If we improve the structural disciplinary GP the reduction is only from 34% to 28%
- Histogram of the quantity of interest after disciplinary enrichment



Conclusions

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- Uncertainty propagation thanks to perfectly dependent GPs
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- Disciplinary GPs improvement
- Efficient Monte Carlo resolution of the random MDA
- How to uncouple the construction of the disciplinary GPs in the vector-valued case?

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