OTBENCHMARK:

AN OPEN SOURCE PYTHON PACKAGE FOR BENCHMARKING AND VALIDATING UNCERTAINTY QUANTIFICATION ALGORITHMS

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Introduction

Industrial context

- In France, the EDF group operates

 - ▶ 433 hydraulic power plants
 - thousands of wind turbines
- Industrial risk management through the treatment of uncertainties
- Over the past years, a panel of industrials, public research centers and academics:
 - > jointly proposed a generic methodology for uncertainty management
 - ▶ developed OpenTURNS, a tool for the treatment of uncertainties [1]
- Need for a benchmark platform for uncertainty quantification

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Industrial context

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 - ▶ 433 hydraulic power plants
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- Need for a benchmark platform for uncertainty quantification



OpenTURNS

An Open source initiative for the Treatment of Uncertainties, Risks'N Statistics

Plan

Motivations and objectives

Package architecture

Reliability example

Benchmark results

Conclusion

Motivations and objectives

- ullet Uncertain input random vector $oldsymbol{X} \in \mathbb{R}^d$
- Model $g: \mathbb{R}^d \to \mathbb{R}^p$

$$Y = g(X)$$

• Output random variable $Y \in \mathbb{R}^p$

- Uncertain input random vector $\mathbf{X} \in \mathbb{R}^d$ Model $g: \mathbb{R}^d \to \mathbb{R}^p$ $\mathbf{Y} = \mathbf{g}(\mathbf{X})$
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 Model g : ℝ^d → ℝ^p
- Output random variable $Y \in \mathbb{R}^p$

Reliability analysis formulation:

- Failure is characterized by a threshold event E
- The failure probability, denoted p_f , is given by:

$$p_{\mathrm{f}} = \int_{\mathbb{R}^d} \mathbb{1}_{E}(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}) \mathrm{d}\mathbf{x}$$

• Many standard or advanced algorithms for reliability analysis exist [2]

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 - Y = g(X)
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Sensitivity analysis formulation:

- Compute sensitivity indices reflecting the influence of inputs on the output
- e.g., variance decomposition with the Sobol' indices [3]

$$S_i = rac{ ext{Var}\left[\mathbb{E}[Y|X_i]
ight]}{ ext{Var}[Y]} \qquad S_{\mathcal{T}_i} = rac{\mathbb{E}\left[ext{Var}[Y|m{X}_{-i}]
ight]}{ ext{Var}[Y]}$$

- \triangleright S_i is the first-order index, S_{T_i} the total-order index of X_i
- \triangleright X_{-i} stands for X without the *i*-th component

OpenTURNS: www.openturns.org





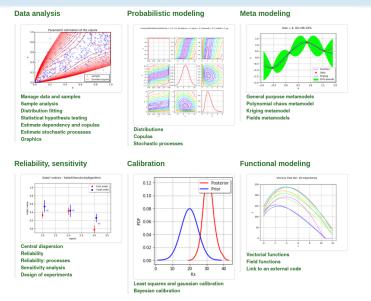




- First release: 2007
- Open source, LGPL licensed, C++/Python library
- Project size (2018): 720 classes, more than 6000 services
- Probabilistic programming paradigm for statistical modeling
- Numerical tools dedicated to the treatment of uncertainties
- Generic coupling to any type of physical model

user@user-desktop:~\$ pip install openturns

OpenTURNS: content



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otbenchmark: from benchmark repositories to a package

What is a benchmark?

"Something that can be measured and used as a standard that other things can be compared with." The Oxford English dictionary

- Many existing benchmark packages for linear algebra, numerical analysis, optimization
- Some uncertainty quantification repositories of benchmark problems

otbenchmark - Journée utilisateurs OT 2021

- Virtual Library of Simulation Experiments¹
- "Black-box Reliability Challenge"³ organized by TNO
 - Online submissions in 2019
 - ▷ Almost 30 reliability benchmark problems

http://www.sfu.ca/ ssurjano/index.html

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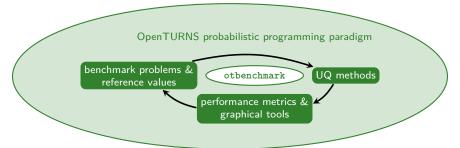
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otbenchmark: benchmark platform for UQ

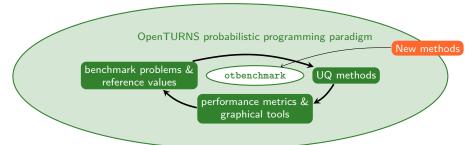
- 26 reliability problems and 4 sensitivity analysis problems so far
- Reference values either computed by exact quadrature methods or large Monte Carlo sampling



- Scenario #1: test a new UQ algorithm on a panel of problems and analyze its performances
- Scenario #2: apply and compare several UQ algorithms available on a given benchmark problem

otbenchmark: benchmark platform for UQ

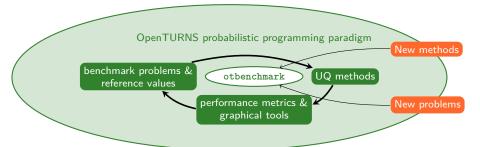
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- Scenario #1: test a new UQ algorithm on a panel of problems and analyze its performances
- Scenario #2: apply and compare several UQ algorithms available on a given benchmark problem

- Generic classes ReliabilityBenchmarkProblem and SensitivityBenchmarkProblem
- Problem-specific classes inheriting methods from the generic classes
- ReliabilityBenchmarkResult to manage the result of a benchmark
- ReliabilityBenchmarkProblemList to perform a benchmark on list of problems
- DrawEvent to provide high-dimension graphical analysis tools (e.g., cross-cuts)

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Reliability example

RP 57: problem definition

Considering the threshold event:

$$E = \{g(\boldsymbol{X}) < 0, \quad \boldsymbol{X} \in \mathbb{R}^2\}$$

with the system limit-state function:

$$g(\mathbf{X}) = \min \left(\max(g_1(\mathbf{X}), g_2(\mathbf{X})), g_3(\mathbf{X}) \right)$$

$$\begin{cases} g_1(\mathbf{X}) = -X_1^2 + X_2^3 + 3 \\ g_2(\mathbf{X}) = 2 - X_1 - 8X_2 \\ g_3(\mathbf{X}) = (X_1 + 3)^2 + (X_2 + 3)^2 - 4 \end{cases}$$

Where $X \in \mathbb{R}^2$ has independent Gaussian marginals:

$$X_1 \sim \mathcal{N}(\mu_1 = 0, \sigma_1 = 1), \quad X_2 \sim \mathcal{N}(\mu_2 = 0, \sigma_2 = 1).$$

For these parameters, the reference probability is

$$p_{\rm f,ref} = 0.0284$$

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Reliability problem definition

```
1  # Import packages
2  import openturns as ot
3  import otbenchmark as otb
4  problem = otb.ReliabilityProblem57()
5  event = problem.getEvent()
6  g = event.getFunction()
7  print("Reference failure probability = %.4f" % (problem.getProbability()))
```

Reference failure probability = 0.0284

FORM approximation

```
meta_algo = otb.ReliabilityBenchmarkMetaAlgorithm(problem)
bench_results = meta_algo.runFORM(ot.AbdoRackwitz())

pf = bench_results.computedProbability

nb_digits = bench_results.numberOfCorrectDigits

nb_simu = bench_results.numberOfFunctionEvaluations

print("FORM failure probability = %.4f" % (pf))

print("FORM nb. good digits = %.4f" % (nb_digits))

print("FORM nb. function calls = %d" % (nb_simu))
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FORM failure probability = 0.4504
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FORM performs poorly on a system limit-state function (as foreseen here)

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Monte Carlo simulation

```
sample_size = 1000
bench_results = meta_algo.runMonteCarlo(sample_size)

pf = bench_results.computedProbability
nb_digits = bench_results.numberOfCorrectDigits
nb_simu = bench_results.numberOfFunctionEvaluations
print("MC failure probability = %.4f" % (pf))
print("MC nb. correct digits = %.4f" % (nb_digits))
print("MC nb. function calls = %d" % (nb_simu))
```

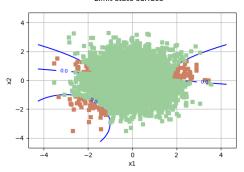
```
MC failure probability = 0.0260
MC nb. good digits = 1.0731
MC nb. function calls = 1000
```

Monte Carlo performs well since p_f is not too rare

Graphical analysis through cross-cuts

```
sample_size = 2000
drawEvent = otb.DrawEvent(event)
cloud = drawEvent.drawSampleCrossCut(sample_size)
bounds = ot.Interval([-4, -4], [4, 4])
graph = drawEvent.drawLimitStateCrossCut(bounds)
graph.add(cloud)
graph
```

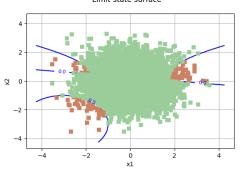
Limit state surface



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Limit state surface



otbenchmark can also cross algorithms and problems and compare performances

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Benchmark results

Benchmark results: crossing problems and algorithms

Table: Estimation of p_f for all the problems using 5 algorithms ($n_{\rm max}=10^4$ calls).

	P _{f.ref}	Monte Carlo	FORM	SORM	FORM-IS	Subset
RP8	7.840e-04	9.000e-04	6.599e-04	7.837e-04	7.737e-04	8.863e-04
RP14	7.520e-03	9.000e-04	7.003e-04	6.988e-04	7.598e-04	8.720e-04
RP22	4.160e-03	3.500e-03	6.210e-03	4.391e-03	4.259e-03	4.117e-03
RP24	2.860e-03	3.600e-03	6.209e-03	6.209e-03	2.749e-03	2.486e-03
RP25	6.140e-06	1.000e-04	2.105e-03	1.064e-05	4.644e-05	3.415e-05
RP28	1.460e-07	_	2.850e-08	0.000e + 00	1.332e-07	1.756e-07
RP31	1.800e-04	2.300e-03	2.275e-02	2.275e-02	3.319e-03	3.919e-03
RP33	2.570e-03	1.600e-03	1.350e-03	1.350e-03	2.322e-03	2.718e-03
RP35	3.540e-03	3.000e-03	1.350e-03	2.134e-03	2.377e-03	3.430e-03
RP38	8.100e-03	8.500e-03	7.902e-03	8.029e-03	8.146e-03	7.848e-03
RP53	3.130e-02	3.260e-02	1.180e-01	2.986e-02	3.143e-02	2.971e-02
RP55	5.600e-01	5.660e-01	5.000e-01	1.093e-05	5.645e-01	5.655e-01
RP54	9.980e-04	1.100e-03	5.553e-02	3.552e-03	9.767e-04	9.611e-04
RP57	2.840e-02	2.950e-02	4.504e-01	0.000e + 00	2.746e-02	2.772e-02
RP75	1.070e-02	1.030e-02	0.000e + 00	0.000e + 00	0.000e + 00	9.409e-03
RP89	5.430e-03	5.000e-03	2.009e-09	2.009e-09	9.002e-05	5.460e-03
RP107	2.920e-07	_	2.867e-07	2.867e-07	2.896e-07	2.337e-07
RP110	3.190e-05	_	3.167e-05	3.167e-05	3.078e-05	7.116e-06
RP111	7.650e-07	_	0.000e + 00	0.000e + 00	0.000e + 00	7.308e-07
RP63	3.790e-04	1.000e-04	1.000e+00	0.000e + 00	0.000e + 00	4.063e-04
RP91	6.970e-04	1.000e-03	6.984e-04	7.001e-04	6.964e-04	6.838e-04
RP60	4.560e-02	4.860e-02	4.484e-02	4.484e-02	4.503e-02	4.230e-02
RP77	2.870e-07	_	6.687e-02	6.687e-02	4.002e-07	3.683e-07
Four-branch serial system	2.186e-03	2.900e-03	0.000e + 00	0.000e + 00	0.000e + 00	2.428e-03
R-S	7.865e-02	7.870e-02	7.865e-02	7.865e-02	7.792e-02	7.633e-02
Axial stressed beam	2.920e-02	2.690e-02	2.998e-02	2.933e-02	2.867e-02	2.936e-02

Benchmark results: performance metrics

Figure: Log relative error using 5 algorithms ($n_{\rm max}=10^4$ calls).



Performance metrics

For a reference value $p_{f,ref}$ and its approximation $p_{f,apx}$

- Absolute error: $\epsilon = |p_{\mathsf{f},\mathsf{ref}} p_{\mathsf{f},\mathsf{apx}}|$
- Relative error: $\eta = \frac{|p_{\rm f,ref} p_{\rm f,apx}|}{|p_{\rm f,ref}|}$
- ullet Log relative error: $N = log_{10}\left(rac{1}{\eta}
 ight)$

Conclusion

Conclusion: otbenchmark, a benchmark platform for UQ

- otbenchmark's ambition: gather and standardize UQ problems in a single Python package
- Any user can bring, either his new UQ algorithm or new use-case (with reference values!)
- Wide variety of reliability and sensitivity analysis problems
- Performance metrics and graphical tools for a smart analysis
- Agile development workflow to ensure a robust benchmark
- More problems to come, and more fields of uncertainty quantification to analyze (e.g., central tendency, calibration)

Welcome to participate!

- GitHub public repository¹
- Unit-tests and continuous integration using Circle-CI
- otbenchmark 0.1.1 package available on PyPI



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- [1] M. Baudin, A. Dutfoy, B. Iooss, and A.-L. Popelin. OpenTURNS: An Industrial Software for Uncertainty Quantification in Simulation. In R. Ghanem, D. Higdon, and H. Owhadi, editors, *Handbook of Uncertainty Quantification*, pages 2001–2038. Springer International Publishing, Cham, 2017.
- [2] Vincent Chabridon. Reliability-oriented sensitivity analysis under probabilistic model uncertainty Application to aerospace systems. PhD thesis, 04 2019.
- [3] B. looss and P. Lemaître. A Review on Global Sensitivity Analysis Methods. In G. Dellino and C. Meloni, editors, *Uncertainty Management in Simulation-Optimization of Complex Systems: Algorithms and Applications*, chapter 5, pages 101–122. Springer US, Boston, MA, 2015.

Thank you