1) $b_n = 3b_{n-1} - 2b_{n-2}$ $\quad b_0 = 2 \quad b_1 = 3$

$r^2 = 3 \cdot r - 2$

$r^2 - 3r + 2 = 0$

$\Delta = 9 - 8 = 1$

$$\begin{cases} r_1 = \frac{3-1}{2} = 1 \\ r_2 = \frac{3+1}{2} = 2 \end{cases}$$

$b_n = \alpha_1 \cdot 1^n + \alpha_2 \cdot 2^n$

$2 = \alpha_1 \cdot 1^0 + \alpha_2 \cdot 2^0$

$2 = \alpha_1 + \alpha_2$

$3 = \alpha_1 \cdot 1^1 + \alpha_2 \cdot 2^1$

$3 = \alpha_1 + 2\alpha_2$

$$\begin{cases} 2 = \alpha_1 + \alpha_2 \quad | \cdot (-1) \\ 3 = \alpha_1 + 2\alpha_2 \end{cases}$$

$$\begin{cases} -2 = -\alpha_1 - \alpha_2 \\ 3 = \alpha_1 + 2\alpha_2 \end{cases}$$

$1 = 0 + \alpha_2$

$\alpha_2 = 1$

$2 = \alpha_1 + 1$

$$\begin{cases} \alpha_1 = 1 \\ \alpha_2 = 1 \end{cases}$$

$b_n = 1 \cdot 1^n + 1 \cdot 2^n$

Odp: $b_n = 1^n + 2^n$.

(c) $a_{n+1} = 3a_n + 2n + 1 \quad a_0 = 0$

$k = n+1, \quad n = k-1$

$a_k = 3a_{k-1} + 2k - 1$

I część jednorodna

$a_k = 3a_{k-1}$

$\lambda = 3$

$a_k^{(0)} = \alpha_1 \cdot 3^k$

II część niejednorodna

$f(k) = 2k-1$

$a_k = b_1 k + b_0$

$b_1 k + b_0 = 3[b_1(k-1) + b_0] + 2k - 1$

dla $k=0$ dla $k=1$ $\begin{cases} b_0 = -3b_1 + 3b_0 - 1 \\ b_1 + b_0 = 3b_0 + 1 \end{cases}$

$\begin{cases} 1 = -3b_1 + 2b_0 \\ -1 = -b_1 + 2b_0 \quad | \cdot (-1) \end{cases}$

$\begin{cases} 1 = -3b_1 + 2b_0 \\ 1 = +b_1 - 2b_0 \end{cases}$

$2 = -2b_1$

$b_1 = -1$

$b_0 = 3 + 3b_0 - 1$

$-2b_0 = 2$

$b_0 = -1$

$\begin{cases} b_0 = -1 \\ b_1 = -1 \end{cases}$

$a_k^{(s)} = -k - 1$

rozwiązanie ogólne rekurencji niejednorodnej

$a_k = \alpha_1 \cdot 3^k - k - 1$

$0 = \alpha_1 - 1$

$\alpha_1 = 1$

$a_k = 1 \cdot 3^k - k - 1$

$a_{n+1} = 3^{n+1} - n - 2$

$a_n = 3^n - n - 1$

Odp: $a_n = 3^n - n - 1$.

b) $b_n = 3b_{n-1} + 5n^2$

I/ część jednorodna

$b_n = 3b_{n-1}$

$r = 3$

$a_n^{(0)} = \alpha_1 \cdot 3^n$

II/ część niejednorodna

$f(n) = 5n^2$

$a_n = b_2 n^2 + b_1 n + b_0$

$b_2 n^2 + b_1 n + b_0 = 3\left[b_2(n-1)^2 + b_1(n-1) + b_0\right] + 5n^2$

$$\begin{array}{l} \text{dla } n=0 \\ \text{dla } n=1 \\ \text{dla } n=2 \end{array} \begin{cases} b_0 = 3b_2 - 3b_1 + 3b_0 \\ b_2 + b_1 + b_0 = 3b_0 + 5 \\ 4b_2 + 2b_1 + b_0 = 3b_2 + 3b_1 + 3b_0 + 20 \end{cases}$$

$$\begin{cases} 0 = 3b_2 - 3b_1 + 2b_0 \\ 0 = -b_2 - b_1 + 2b_0 + 5 \\ 0 = -b_2 + b_1 + 2b_0 + 20 \end{cases}$$

$$\begin{cases} 0 = 3b_2 - 3b_1 + 2b_0 \\ 0 = -b_2 - b_1 + 2b_0 + 5 \\ 0 = 2b_1 + 15 \end{cases}$$

$$\begin{cases} b_1 = -7\frac{1}{2} \\ 0 = -b_2 + 2b_0 + 5 + 7\frac{1}{2} \\ 0 = 3b_2 + 2b_0 + 22\frac{1}{2} \end{cases}$$

$$\begin{cases} b_1 = -7\frac{1}{2} \\ 0 = -b_2 + 2b_0 + 12\frac{1}{2} \\ 0 = 4b_2 + 10 \end{cases}$$

$$\begin{cases} b_1 = -7\frac{1}{2} \\ b_2 = -2\frac{1}{2} \\ -2b_0 = 2\frac{1}{2} + 12\frac{1}{2} \end{cases}$$

$b_0 = -7\frac{1}{2}$

$b_1 = -7\frac{1}{2}$

$b_2 = -2\frac{1}{2}$

$a_n^{(0)} = -2\frac{1}{2}n^2 - 7\frac{1}{2}n - 7\frac{1}{2}$

Odp: $a_n = \alpha_1 \cdot 3^n - 2\frac{1}{2}n^2 - 7\frac{1}{2}n - 7\frac{1}{2}$