Zadanie 1

a) $2c_n = c_{n-1} + 1 \qquad c_0 = 2$

$$c_n = \frac{1}{2}\left(c_{n-1} + 1\right)$$

$$C(x) = c_0 + x\left(\frac{1}{2}c_0 + \frac{1}{2}\right) + x^2\left(\frac{1}{2}c_1 + \frac{1}{2}\right) + \dots =$$

$$= c_0 + \frac{1}{2}x\left(c_0 + c_1 x + \dots\right) + \frac{1}{2}x\left(1 + x + \dots\right) + \dots =$$

$$= c_0 + \frac{1}{2}x \cdot C(x) + \frac{1}{2}x \cdot \frac{1}{1-x} = 2 + \frac{1}{2}x \cdot C(x) + \frac{x}{2-2x}$$

$$C(x) = 2 + \frac{1}{2}x \cdot C(x) + \frac{x}{2-2x}$$

$$C(x)\left(1 - \frac{1}{2}x\right) = \frac{4-3x}{2-2x}$$

$$C(x) = \frac{4-3x}{(2-2x)\left(1-\frac{1}{2}x\right)}$$

$$C(x) = \frac{A}{2-2x} + \frac{B}{1-\frac{1}{2}x} = \frac{A - \frac{1}{2}Ax + 2B - 2Bx}{(2-2x)\left(1-\frac{1}{2}x\right)}$$

$$-\frac{1}{2}A - 2B = -3$$
$$A + 2B = 4$$
$$\frac{1}{2}A = 1$$

$$\begin{cases} A = 2 \\ B = 1 \end{cases}$$

$$C(x) = \frac{2}{2-2x} + \frac{1}{1-\frac{1}{2}x}$$

$$C(x) = \frac{1}{1-x} + \frac{1}{1-\frac{1}{2}x}$$

$$C(x) = \sum_{k=0}^{\infty}\left(1^k + \left(\frac{1}{2}\right)^k\right)x^k$$

Odp.: $C(x) = \frac{1}{1-x} + \frac{1}{1-\frac{1}{2}x} = \sum_{k=0}^{\infty}\left[1^k + \left(\frac{1}{2}\right)^k\right]x^k$.

Zadanie 5

funkcja tworząca $f(x) = \left(\frac{1-x^{11}}{1-x}\right)^2 \cdot \left(\frac{1}{1-x}\right)^4$

Zadanie 6

funkcja tworząca $f(x) = \left[x^2\left(\frac{1-x^6}{1-x}\right)\right]^5$

Zadanie 7

funkcja tworząca $f(x) = \left(x\left(\frac{1-x^6}{1-x}\right)\right)^8$