JP – 764

I Semester M.C.A. (Two Years Course) Examination, July 2023
(CBCS) (2020 – 2021 and Onwards)
COMPUTER SCIENCE
1MCA 2 : Discrete Mathematics

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer* ***any five*** *from Part –* ***A*** *and* ***any four*** *from Part –* ***B****.*

PART – A

Answer **any five** questions. **Each** question carries **six** marks. **(5×6=30)**

1. a) Prove that, for any three sets A, B and C
$A \times (B \cup C) = (A \times B) \cup (A \times C)$. **3**

   b) Prove that "Null set is a subset of every set". **3**

2. a) Find inverse, converse and contrapositive of the following implication. "If the weather is sunny, then I will go to school." **3**

   b) Let x be the set of factors of 12 and let "≤" be the relation divisor i.e., $x \leq y$ if and only if x divides y. Draw the Hasse diagram of $(X, \leq)$. **3**

3. a) Let $f : R \rightarrow R$, determine whether f is invertible and if so, determine f^{-1} where $f = \{(x, y) \mid 2x = 3y = 7\}$. **3**

   b) Show that $p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$. **3**

4. Obtain an explicit form for the following sequences $\{a_n\}$ defined recursively by $a_n = 2a_{n-1} + 1$ for $n \geq 2$ with $a_1 = 3$.

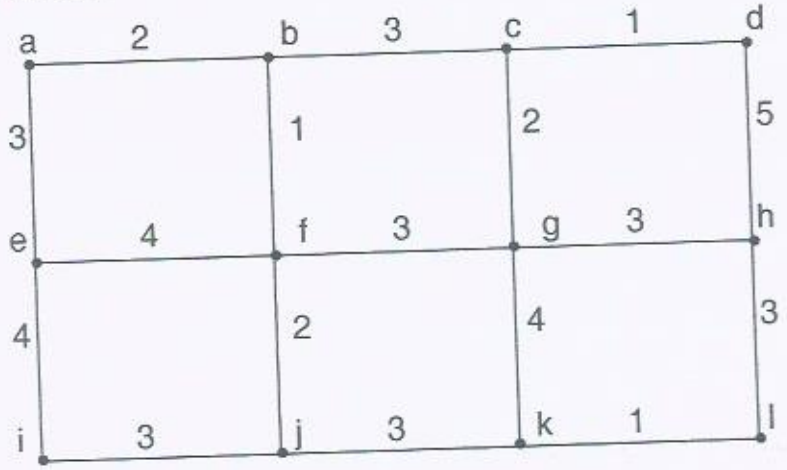
5. What is the expected value of the sum of the numbers that appear when a pair of fair dice is rolled ?

P.T.O.

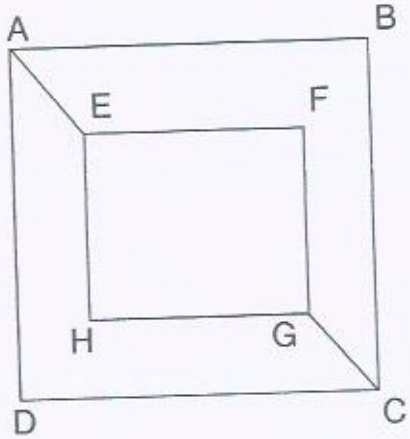
6. a) How many 5-digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 and no digit appears more than once ? **3**

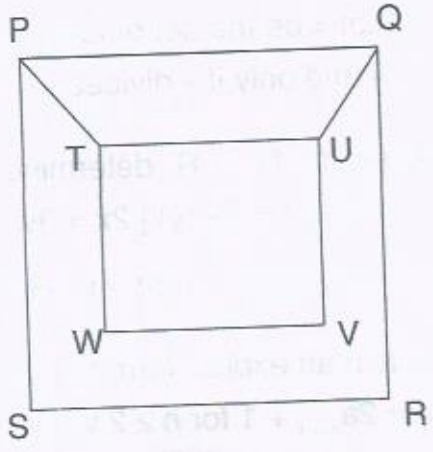
   b) In how many ways a five member committee can be formed out of ten people if two particular people must be included ? **3**

7. Use Kruskal's algorithm to find a minimum spanning tree in the weighted graph given below.



8. Examine whether the following pair of graphs are isomorphic or not. Justify your answer.





PART – B

Answer **any four** questions. **Each** question carries **ten** marks. **(4×10=40)**

9. a) There is a total of 200 students in Class XI. 120 of them study Mathematics, 50 students study Commerce and 30 students study both Mathematics and Commerce. Find the number of students who

   i) Study Mathematics but not Commerce.

   ii) Study Commerce but not Mathematics.

   iii) Study Mathematics or Commerce. **5**

   b) Prove that $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$. **5**

10. a) By mathematical induction, prove that the sum of cubes of 'n' natural numbers is equal to $\frac{n^2(n+1)^2}{4}$ for all 'n' natural numbers. **5**

    b) Define a relation R on set A = {1, 2, 3, 4, 5} by (x, y) $\in$ R if R = {(a, b) : |a – b| is even}, then show that R is an equivalence relation. **5**

11. a) Prove that for any proposition p, q, r, the following compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. **5**

    b) A committee of 8 people is formed from two mathematicians and ten economists. In how many ways can it be done, if the committee must include at least one mathematician ? **5**

12. a) State and prove Baye's theorem. **5**

    b) The probability distribution of a discrete random variable is given below : **5**

X	– 2	– 1	0	1	2	3
P(X)	0.1	K	0.2	2K	0.3	K

Find :

i) K

ii) Mean

iii) Variance.

13. a) Three coins are tossed in succession. Find out the probabilities of occurrences.
    i) two consecutive heads
    ii) two heads and
    iii) two heads in the following order :
    head, tail, head. 5

    b) State and prove Hand Shaking theorem. 5

14. a) Prove that in any undirected graph, the number of odd degree vertices is even. 5

    b) Using Dijkstra's algorithm, find the shortest path between a to z in the weighted graph. 5