Back Substitution:

1) $T(n) = 2T(n - 1) + 1, T(0) = 1$

$T(n) = 2T(n - 1) + 1$
$=2 (2T(n - 2) + 1) + 1$
$= 2 * 2 * T(n - 2) + 2^1 + 1$
$= 2^2T(n - 2) + 2^1 + 1$
$= 2^2(2T(n - 3) + 1) + 2^1 + 1$
$= 2^2 * 2 * T(n - 3) + 2^2 + 2^1 + 1$
$= 2^3T(n - 3) + 2^2 + 3$
$= 2^3(2T(n - 4) + 1) + 2^2 + 3$
$= 2^4T(n - 4) + 2^3 + 2^2 + 1$

We can see a direct correlation between n and the power 2 is raised through back substitution. Therefore, we can determine the upper bound is: **$T(n) = O(2^n)$**

2) *$T(n) = T(n - 2) + n^2$, $T(0) = 1$*

$T(n) = T(n - 2) + n^2$
$= T(n - 4) + (n - 2)^2 + n^2$
$= T(n - 4) + 2n^2 - 4n + 4$
$= T(n - 6) + 2(n - 4)^2 - 4(n - 4) + 4$
$= T(n - 6) + 2(n^2 - 8n + 16) - 4n + 16 + 4$
$= T(n - 6) + 2n^2 - 16n + 32 - 4n + 20$
$= T(n - 6) + 2n^2 - 20n + 52$
…

We can see our largest n value seems to be limiting to n^2, so therefore we can say our loose upper bound is: **$T(n) = O(n^2)$**

3) $T(n) = T(n - 1) + 1/n$, $T(1) = 1$

Consider: Sum of Harmonic Series = $\log(n) + H_n$

$T(n) = T(n - 1) + 1/n$, $T(1) = 1$
$= T(n - 1) + \log(n) + H_n$
$= T(n - 2) + \log(\log(n) + H_n) + H_{n - 1}$
$= T(n - 3) + \log(\log(\log(n) + H_n) + H_{n - 1}) + H_{n - 2}$
…

Since H_n is a constant, we can ignore it in terms of Big O since as n gets sufficiently large, H_n will not make a significant difference. We can see we are logarithmically growing in size, therefore: **$T(n) = O(\log(n))$**

Master Method:

FORMULA: $T(n) = aT(n/b) + n^d$

$a < b^d = O(n^d)$
$a = b^d = O(n^d \times \log(n))$
$a > b^d = O(n^{\log b(a)})$

4) $T(n) = 2T(n/4) + 1$, $T(0) = 1$

$a = 2$, $b = 4$, $d = 0$

$2 < 4^0$? FALSE
$2 = 4^0$? FALSE
$2 > 4^0$? TRUE

We get: $O(n^{\log 4(2)})$. $\log_4(2)$ simplifies to ½

ANSWER: $O(\sqrt{n})$

5) $T(n) = 2T(n/4) + \sqrt{n}$, $T(0) = 1$

$a = 2$, $b = 4$, $d = ½$

$2 < \sqrt{4}$? FALSE
$2 = \sqrt{4}$? TRUE

ANSWER: $O(\sqrt{n} \times log(n))$

6) $T(n) = 2T(n/4)\ n^2$, $T(0) = 1$

$a = 2$, $b = 4$, $d = 2$

$2 < 4^2$? TRUE

ANSWER: $O(n^2)$

7) $T(n) = 10T(n/3) + n^2$, $T(0) = 1$

$a = 10$, $b = 3$, $d = 2$

$10 < 3^2$? FALSE
$10 = 3^2$? FALSE
$10 > 3^2$? TRUE

$O(n^{\log 3(10)})$

$\log_3(10) = \log_2(10) / \log_2(3) = 3.3219280949 / 1.5849625007 = 2.10$ when rounded to 2 decimals

ANSWER: $O(n^{2.1})$

8) $T(n) = 2T(2n/3) + 1$, $T(0) = 1$

$a = 2$, $b = 3/2$, $d = 0$

$2 < \frac{3}{2}^0$? FALSE
$2 = \frac{3}{2}^0$? FALSE
$2 > \frac{3}{2}^0$? TRUE

$O(n^{\log_{\frac{3}{2}}(2)})$
$\log_{\frac{3}{2}}(2) = \log_2(2) / \log_2(1.5) = 1 / 0.5849625007 = 1.71$ when rounded to two decimals

ANSWER: $O(n^{1.71})$