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CVaR-based risk parity model with machine learning☆

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ABSTRACT

This study proposes a risk parity model based on conditional value-at-risk (CVaR), enhanced by integrating machine learning techniques into dynamic portfolio optimization. The CVaR-based risk parity (CVaR-RP) model allocates portfolio tail risk among assets evenly to mitigate downside risk. To enhance the CVaR-RP's predicting accuracy and adaptability to changing market conditions, we use a two-stage training approach within machine learning algorithms to forecast asset price movements. Portfolios are dynamically rebalanced based on these predictions to optimize the trade-off between risk mitigation and return maximization. Numerical analysis shows that the CVaR-RP strategy outperforms volatility-based risk parity and equal-weight strategies. Specifically, with machine learning-driven predictions and dynamic weight adjustments, the CVaR-RP achieves a higher Sharpe ratio, reduced maximum drawdown, and improved Calmar ratio. This research highlights the effectiveness of integrating machine learning methods into CVaR-RP strategies in enhancing returns and mitigating downside risk.

1. Introduction

With the increasing complexity and volatility of global financial markets, achieving effective risk management through optimized portfolio allocation has become a critical challenge in contemporary finance. Traditional asset allocation strategies often lead to imbalanced asset distributions and heightened sensitivity to market fluctuations, ultimately undermining effective risk diversification. To address this challenge, Qian (2005) proposes a risk parity (RP) method to equalize risk contributions across portfolio components, thereby enhancing diversification and strengthening portfolio stability under varying market conditions. Prior research shows that the RP strategy outperforms conventional portfolio strategies, such as equal weight (EW) portfolios, minimum variance portfolios, and fixed 60/40 equity-bond allocations, particularly under shifting market conditions (Maillard et al., 2010; Ruban and Melas, 2010). The RP's superior performance can be attributed to its effectiveness in reducing risk concentration on certain assets within the portfolio, thus providing better risk diversification (Chaves et al., 2011; Bai et al., 2016). However, as the RP model adopts the standard deviation as a risk measure, it may inadequately account for the multidimensional aspects of risk, which could impair its ability to generate adequate risk-adjusted returns under extreme market conditions (Inker, 2011).

To mitigate potential losses under extreme market conditions, Zhao and Fang (2020) introduce a value-at-risk (VaR)-based RP model, in which risk is quantified using VaR metrics. However, VaR is inherently limited as a risk measure, because it fails to account

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for extreme losses beyond the pre-specified VaR threshold, thus underestimating the portfolio's tail risk exposure. Moreover, VaR lacks the sensitivity required to adequately capture market fluctuations, especially in non-linear environments. In contrast, conditional VaR (CVaR) (Rockafellar and Uryasev, 2000) provides a more comprehensive risk measure by capturing expected shortfalls beyond the VaR threshold, offering a more precise evaluation of tail risk exposure. This distinctive characteristic makes CVaR particularly advantageous for portfolios incorporating complex and volatility-sensitive instruments. To enhance the robustness of risk management under extreme market conditions, in this paper, we propose a CVaR-based RP (CVaR-RP) model that evenly allocates portfolio tail risk among assets to achieve balanced diversification of downside risk.

Inspired by the successful application of machine learning algorithms in finance (Sadorsky, 2022), we incorporate these techniques into the CVaR-RP model to forecast asset price movements and dynamically adjust portfolio weights. This approach represents a significant advancement over conventional methods, which typically rely on static assumptions and fixed rebalancing schedules that fail to capture the inherent non-linearity and the time-varying properties of financial markets. In contrast, our machine learning framework constantly adapts to real-time market data, enabling early detection of emerging tail risks and rapid adjustments in portfolio exposure. Machine learning offers a distinct advantage over traditional econometric models due to its ability to model nonlinear processes without requiring prior assumptions about market data. In the context of forecasting future stock price trends, various econometric models have been proposed, including the generalized autoregressive conditional heteroskedasticity (GARCH) model (Awartani and Corradi, 2005) and the autoregressive integrated moving average (ARIMA) model (Mondal et al., 2014). While these models are widely used, they often cannot adequately capture market dynamics and nonlinear relationships (Ariyo et al., 2014; Babu and Reddy, 2014). To address these limitations, machine learning techniques are used to construct nonlinear models for stock price predictions (Kim, 2003; Öğüt et al., 2009; Patel et al., 2015; Cavalcante et al., 2016; Lee et al., 2019). However, traditional machine learning algorithms are constrained by their inherent topology, which may limit their generalization abilities and cause them to become stuck in local optima during the iterative process. In contrast, deep learning models have superior learning and adaptive capabilities, thereby resulting in exceptional performance in predicting and analyzing variables in nonlinear systems. Previous studies demonstrate the effectiveness of deep learning in predicting stock prices, identifying profitable investment opportunities, generating excess returns, and mitigating risk (Persio and Honchar, 2016; Bhandari et al., 2022; Eggebrecht and Lütkebohmert, 2023; Vuletić et al., 2024).

Due to the varying predictive capabilities of machine learning algorithms across economic cycles and asset classes in diverse financial markets (Gu et al., 2020), we consider four traditional machine learning algorithms and four deep learning models in portfolio management strategies. Moreover, we propose a two-stage training approach to enhance the effectiveness of the model training process. In the first stage, the optimal algorithm for each asset class is identified by evaluating the predictive accuracy of each model. In the second stage, these optimal algorithms are retrained and used to forecast asset price movements. Based on these predictions, the weights of a CVaR-RP portfolio are adjusted to enable more effective asset allocation and mitigate the risk of negative returns. This approach enhances portfolio responsiveness to price fluctuations, facilitating more dynamic and effective asset allocations.

Our numerical results indicate that the CVaR-RP model achieves an annualized return of 5.96 %, outperforming both the RP and EW models by 0.84 % and 0.18 %, respectively. Additionally, the CVaR-RP model yields a Sharpe ratio of 48.78 %, significantly surpassing the 12.93 % of the EW model, demonstrating its effectiveness in risk control and return optimization. Machine learning-optimized models significantly outperform their non-optimized counterparts, with the machine learning-enhanced CVaR-RP model delivering a standout annualized return of 17.43 %. Furthermore, the machine learning-enhanced CVaR-RP model demonstrates stronger risk management capabilities, with a lower maximum drawdown of 3.77 % and a higher Calmar ratio of 4.63, indicating superior resilience during market downturns.

Our paper[1] makes the following contributions to prior literature. First, we extend the RP model by adopting CVaR as the risk measure. Previous studies have primarily focused on optimizing RP by adjusting input parameters (William, 2013; Lee and Sohn, 2023; Braga et al., 2023) or by combining it with other models to achieve specific optimization objectives (Kaucic, 2019; Cho and Song, 2023; Di Persio et al., 2023). In contrast, our approach directly addresses tail risk by integrating CVaR into the RP framework, leading to a more effective distribution of tail risk across assets. By focusing on CVaR as a measure of tail risk, our model can effectively mitigate downside risk, representing a significant extension of volatility-based and VaR-based RP models (Zhao and Fang, 2020).

Second, we incorporate machine learning techniques into the CVaR-RP model, enabling dynamic weight adjustments to reflect changing market conditions. The enhanced model not only substantially augments asset allocation robustness and improves risk-adjusted returns, but also offers a complementary perspective to existing methodologies. In contrast to conventional static strategies (Lee and Sohn, 2023) and dynamic strategies that rely on fixed rules for periodic rebalancing (DeMiguel et al., 2009), our approach employs machine learning to forecast stock price fluctuations in real time, effectively capturing nonlinear market dynamics and sudden risks. Consequently, it facilitates a more flexible and precise asset allocation and enhances long-term portfolio performance.

Third, we propose an innovative two-stage training process for machine learning in portfolio optimization. Unlike traditional methods that partition data into training, validation, and test sets (Leippold et al., 2022), our framework uses a two-stage process. The first stage selects the optimal algorithms, while the second stage retrains these models, significantly enhancing adaptability to dynamic market conditions. The proposed two-stage approach avoids look-ahead bias, which is often present in machine learning models

[1] Although the proposed CVaR-based risk parity model with machine learning is presented as a new approach, it remains essentially an engineering of existing ideas rather than a scientific breakthrough, and therefore the results may not be readily generalizable in other contexts or using other datasets.

relying heavily on sample data, ensuring a more realistic representation of market dynamics.

The remainder of the paper is organized as follows. Section 2 describes the CVaR-RP model and its algorithm design. Section 3 elaborates on the machine learning methodology employed for predicting asset price movements, along with the process for optimizing portfolio weights. Section 4 reports the numerical results and robustness checks. Section 5 draws the conclusions.

2. CVaR-based risk parity model

While the traditional RP model (see Appendix A. Volatility-based risk parity model) effectively balances risk contributions across assets, it may fall short in managing tail risks under extreme market conditions. To address this limitation, building upon Zhao and Fang (2020), this study further extends the framework by incorporating CVaR as a robust risk measure, which enhances the model's ability to capture extreme downside risks. The CVaR-RP model is described as follows.

Assume that the return series of n assets follow a multivariate normal distribution. Let $\boldsymbol{\mu}$ represents the mean vector of this multivariate distribution, and $\boldsymbol{\Omega}$denotes the covariance matrix of asset returns. Furthermore, let $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ be the weight vector of a portfolio, where xi represents the proportion of capital allocated to asset i. Given the additive property of multivariate normal distributions, the return of the portfolio, denoted by r_p, also follows a normal distribution; that is, $r_p \sim N(\mathbf{x}^T\boldsymbol{\mu}, \mathbf{x}^T\Omega\mathbf{x})$. The VaR and CVaR can be represented in matrix form as follows:

$$VaR_\alpha = \mathbf{x}^T\boldsymbol{\mu} + \sqrt{\mathbf{x}^T\Omega\mathbf{x}}\varphi^{-1}(\alpha), \tag{1}$$

$$CVaR_\alpha = \mathbf{x}^T\boldsymbol{\mu} + \sqrt{\mathbf{x}^T\Omega\mathbf{x}}\,\frac{\phi(\varphi^{-1}(\alpha))}{1-\alpha}, \tag{2}$$

where $\varphi^{-1}(\alpha)$ denotes the quantile function (inverse cumulative distribution function) of the standard normal distribution, and $\phi(.)$ represents the probability density function of the standard normal distribution.

This study adopts CVaR as the risk measure, recognizing its superior ability to capture extreme tail risks compared to VaR. Unlike VaR, which estimates losses up to a specific threshold, CVaR accounts for the average losses beyond this threshold, offering a more comprehensive assessment of potential extreme losses.

The marginal risk contribution (MRC) of asset i to the portfolio can be expressed as follows:

$$MRC_i(x) = \frac{\partial(CVaR)}{\partial x_i} = r_i + \frac{(\Omega\mathbf{x})_i}{\sqrt{\mathbf{x}^T\Omega\mathbf{x}}}\,\frac{\phi(\varphi^{-1}(\alpha))}{1-\alpha}, \tag{3}$$

where$(\Omega\mathbf{x})_i$represents the ith element of the product of Ω and $\mathbf{x}$. For the CVaR-RP model, the total risk contribution (TRC) of asset i is calculated as follows:

$$TRC_i(x) = x_i\frac{\partial(CVaR)}{\partial x_i} = x_i r_i + x_i\frac{(\Omega\mathbf{x})_i}{\sqrt{\mathbf{x}^T\Omega\mathbf{x}}}\,\frac{\phi(\varphi^{-1}(\alpha))}{1-\alpha}. \tag{4}$$

Furthermore, the sum of the TRCs of all assets is equal to the overall risk of the portfolio:

$$\sum_{i=1}^{n} TRC_i(x) = \sum_{i=1}^{n}\left(x_i r_i + x_i\frac{(\Omega\mathbf{x})_i}{\sqrt{\mathbf{x}^T\Omega\mathbf{x}}}\,\frac{\phi(\varphi^{-1}(\alpha))}{1-\alpha}\right) = CVaR. \tag{5}$$

To fulfill the criteria of the CVaR-RP model, it is essential that the TRC of each asset be equal to each other, which is specified as follows:

$$TRC_i(x) = TRC_j(x) = \frac{CVaR}{n} = \lambda, \forall i, j, \tag{6}$$

where λ represents a constant that ensures uniform risk distribution across all assets.

Given the non-convex nature of the CVaR-RP model, it is not possible to find an explicit analytical solution for Eq. (6). Therefore, we formulate the following optimization problem to determine portfolio weights:

$$\begin{cases} \text{Min } \displaystyle\sum_{i=1}^{n}\left(x_i r_i + x_i\frac{(\Omega\mathbf{x})_i}{\sqrt{\mathbf{x}^T\Omega\mathbf{x}}}\,\frac{\phi(\varphi^{-1}(\alpha))}{1-\alpha} - \lambda\right)^2, \\ s.t.\ \begin{cases} \displaystyle\sum_{i=1}^{n} x_i = 1, \\ 0 \le x_i \le 1. \end{cases} \end{cases} \tag{7}$$

The objective function in Eq. (7) is specified to penalize the squared deviations between each asset's TRC and the target risk contribution λ, ensuring that deviations from equal risk allocation are minimized. Through this approach, the optimization problem iteratively adjusts portfolio weights until the risk contribution of each asset is close to λ. This process ensures that the resulting portfolio fulfills the CVaR-RP model condition, achieving a balanced and stable distribution of risk.

3. Portfolio weight optimization based on machine learning price predictions

The CVaR-RP model simplifies analysis and computation by assuming a static normal distribution of historical returns. While this assumption facilitates practical implementation, it may misrepresent the actual return distribution and fail to capture future shifts in economic conditions or individual asset price movements, particularly those with non-linearities, fat tails, and skewed distributions. To address these limitations, this study integrates machine learning techniques to forecast asset price trends.

3.1. Assets price prediction based on machine learning

3.1.1. Design of the price trend prediction model

In this study, we use various established machine learning and deep learning algorithms to construct models for forecasting asset price movements, leveraging their differing abilities to capture the unique characteristics of various market data. Specifically, the traditional machine learning models used include logistic regression (LR) (Cox, 1958), support vector machines (SVM) (Cortes and Vapnik, 1995), random forest (RF) (Breiman, 2001), and gradient boosting decision tree (GBDT) (Friedman, 2001). These models are widely used in finance due to their robustness in binary classification tasks and ease of application to structured financial data, making them well-suited for predicting price trends.

In addition, we adopt deep learning models to capture inherent non-linearities and complex temporal dynamics in financial data. The deep learning models applied in this study include convolutional neural networks (CNN) (LeCun et al., 1989), recurrent neural networks (RNN) (Elman, 1990), long short-term memory networks (LSTM) (Hochreiter and Schmidhuber, 1997), and the Transformer model (Vaswani et al., 2017). These models are particularly suitable for time-series data, as they can capture sequential dependencies and reveal hidden structures within high-dimensional datasets, providing an advantage in adapting to complex market environments. All these eight machine learning models are applied and trained separately using the price data of each asset, and the models with the highest predictive accuracy are then selected for retraining and forecasting.

The price prediction process, as outlined in Fig. 1, consists of two distinct modules: the data preprocessing module and the machine learning algorithm forecasting module. The data preprocessing module involves preparing the raw data for further analysis. In this stage, the raw data is labeled, normalized, and then partitioned into in-sample and out-of-sample datasets. The in-sample dataset is further split into training and validation subsets. The primary purpose of this module is to transform the raw data into a standardized format and segment it in a way that facilitates the training of machine learning models.

In the machine learning algorithm forecasting module, rises or falls in an asset price are determined using machine learning algorithms. To this end, we propose an innovative two-stage training approach for price prediction. This approach, inspired by the

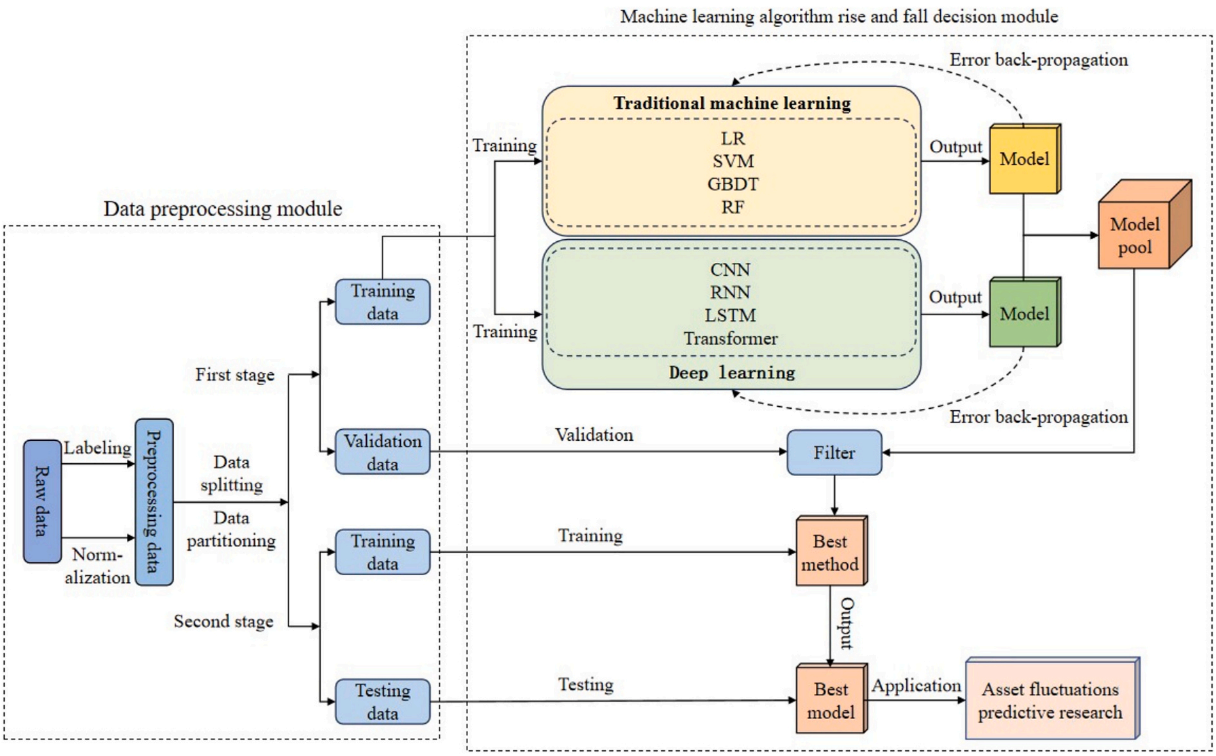

Fig. 1. Overall design framework diagram. The figure shows the overall design framework for predicting asset price movements using traditional machine learning and deep learning algorithms.

traditional dataset splitting method employed by Leippold et al. (2022), enhances the training process by incorporating a dynamic training and validation mechanism. The introduction of a two-stage training approach is motivated by its ability to overcome the limitations inherent in conventional single-stage approaches when applied to complex and dynamic market data. In the first stage, general feature representations are derived through data augmentation techniques (ValizadehAslani et al., 2024). In the second stage, the model is meticulously refined for particular tasks (Ye et al., 2023). This process is pivotal in enhancing both the stability and precision of forecasts. In the first stage, models are trained on the in-sample training dataset and evaluated based on the validation dataset. Both traditional machine learning algorithms and deep learning algorithms are used to capture patterns in historical stock data. Based on their performance on the validation dataset, the most accurate model for each asset is selected. In the second stage, the selected model is retrained on the full in-sample dataset and tested on the out-of-sample dataset to ensure its ability to generalize to unobservable data. The final, optimized model is then used to forecast future price movements, which in turn inform asset weight adjustments.

3.1.2. Prediction accuracy measures

To evaluate the performance of machine learning algorithms in predicting asset price movements, a comprehensive accuracy analysis is performed. Accuracy, defined as the ratio of correctly predicted samples to the total samples, provides an indication of a model's capacity to correctly identify price movement patterns across datasets. However, accuracy alone fails to reflect the potential losses incurred from the two types of prediction errors. Therefore, receiver operating characteristic (ROC) curve analysis (Hosmer Jr. et al., 2013) is also utilized to assess the accuracy of price predictions more robustly. The ROC curve plots the true positive rate (TPR) against the false positive rate (FPR) at various threshold levels, serving as a diagnostic tool to measure the effectiveness of binary classification models. A model demonstrating strong classification performance will generate an ROC curve closer to the upper left corner, indicating greater predictive power.

To select the optimal machine learning model for each asset, we evaluate multiple candidate models based on both accuracy and ROC. This ensures the chosen model is not only capable of making correct predictions, but also able to handle the uncertainties inherent in financial data.

3.2. Portfolio weight optimization based on machine learning

3.2.1. Determination of initial weights

To fulfill the conditions of the CVaR-RP, it is necessary to satisfy Eq. (7).

Setting $\beta_i = MRC_i(\mathbf{x}) = r_i + \frac{(\mathbf{\Omega x})_i}{\sqrt{\mathbf{x}^T\mathbf{\Omega x}}}\frac{\phi\left(\varphi^{-1}(\alpha)\right)}{1-\alpha}$, we obtain:

$$x_i\beta_i = \lambda. \tag{8}$$

Combining Eq. (8) with $\sum_{i=1}^{n} x_i = 1$, we obtain $\sum_{i=1}^{n} \frac{\lambda}{\beta_i} = 1$. Thus, we can derive the following:

$$\lambda = \frac{1}{\sum_{i=1}^{n} 1\Big/\beta_i}. \tag{9}$$

Furthermore, substituting Eqs. (8) into (9), and solving for the equation yields:

$$x_i = \frac{1/\beta_i}{\sum_{i=1}^{n} 1\Big/\beta_i}. \tag{10}$$

It is challenging to solve Eq. (10) due to the reciprocal dependency, where β_i on the right-hand side of the equation is a function of x_i on the left-hand side, and the weight x_i is in turn dependent on β_i. To address such problems, a common approach is to establish a loop between Eqs. (8) and (10). In this loop, matrix Ω iteratively evolves into a higher power matrix $\Omega^n\mathbf{x}$, aiming to identify the eigenvector associated with the largest eigenvalue in absolute terms. This paper incorporates an iterative approach to optimize this algorithm, thereby solving Eq. (10) to determine the weights of each asset. Eq. (8) represents the condition that must be met for the weights to be optimal. Thus, updating the weights via Eq. (10) can fulfill this condition. This process can be described as follows:

Step 1: We assign an arbitrary set of initial values, $\mathbf{x}^{(0)}$, to the portfolio weights $\mathbf{x}$. An example of this could be equal weights, $\left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$, where n represents the number of assets.

Step 2: Combining $\begin{cases} \sum_{i=1}^{n} \dfrac{1/\beta_i}{\sum_{i=1}^{n} 1/\beta_i} = 1 \\ x_i\beta_i = x_j\beta_j, \forall i, j \end{cases}$ with Eq. (10), the iteration formula can be expressed as: $x_i\beta_i = \frac{1}{n}$, $i = 1, 2, \ldots, n$. This implies that the TRC of each asset to the portfolio is the same, each being $\frac{1}{n}$. By substituting the current portfolio weights $x_i^{(k)}$ into the iteration formula, the beta value of each asset class can be calculated as $\beta_i^{(k)} = \frac{1}{n x_i^{(k)}}$, where k represents the number of iterations.

Step 3: We set K as the maximum number of iterations allowed to ensure computational efficiency, and define the convergence threshold as ε, where ε is a small positive number. If the convergence condition is not met within K iterations, the algorithm will stop and return the latest weight estimates. If the condition $\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i^{(k)}\cdot\beta_i^{(k)}-\frac{1}{n}\right)^2}<\varepsilon$ is met, exit the iteration and use x_i^* as the final weight for the ith asset. If the number of iterations exceeds K, the iteration stops. If the aforementioned conditions are not met, calculate the new weights as $x_i^{(k+1)}=\frac{1/\beta_i^{(k)}}{\sum_{i=1}^{n}1/\beta_i^{(k)}}$ and revert to the previous step.

3.2.2. The process of weight optimization

Given the dynamic nature of financial markets, with constant influxes of new information and events, integrating machine learning algorithms into price prediction allows for more flexible portfolio weight adjustments. When machine learning identifies potential risks or opportunities, investors can promptly rebalance portfolios to reflect updated market conditions. This enhances their ability to manage volatility and uncertainty, achieving a more effective balance between risk and return. In this approach, the weights in the CVaR-RP portfolio are optimized based on the machine learning model selected over the validation dataset. The steps are given as follows:

Step 1: We solve for the asset weights of the CVaR-RP portfolio for the test dataset. These weights can be expressed in the matrix form as follows:

$$\mathbf{x}_{m\times n}=\begin{pmatrix} x_{t+1,1} & x_{t+1,2} & \cdots & x_{t+1,n} \\ x_{t+2,1} & x_{t+2,2} & \cdots & x_{t+2,n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{t+m,1} & x_{t+m,2} & \cdots & x_{t+m,n} \end{pmatrix}_{m\times n}, \tag{11}$$

where t represents the length of the training dataset, m denotes the length of the test dataset, and n represents the number of assets.

Step 2: Based on the results obtained from the evaluation methods for price prediction accuracy described in Section 3.1.2, the machine learning algorithm with the highest level of accuracy is selected. Using the sequence data from the training dataset, the price predictions for stock price movements in the test dataset are obtained. These predictions, expressed in the matrix form, are as follows:

$$\widehat{\mathbf{Y}}_{m\times n}=\left[\mathbf{Y}_{t+1,n},\mathbf{Y}_{t+2,n},\cdots,\mathbf{Y}_{t+m,n}\right]. \tag{12}$$

The trends of the n assets in the ith month of the test dataset are as follows:

$$\mathbf{Y}_{t+i,n}=\left[y_{t+i,1},y_{t+i,2},\cdots,y_{t+i,n}\right]. \tag{13}$$

where $y_{t+i,n}=0$ or $y_{t+i,n}=1$, and $i\in[1,m]$.

Step 3: By multiplying the corresponding elements of the asset weight matrix $\mathbf{x}_{m\times n}$ by the machine learning asset price trend prediction result matrix $\widehat{\mathbf{Y}}_{m\times n}$, we obtain the following matrix:

$$\mathbf{x}'_{m\times n}=\begin{pmatrix} x'_{t+1,1} & x'_{t+1,2} & \cdots & x'_{t+1,n} \\ x'_{t+2,1} & x'_{t+2,2} & \cdots & x'_{t+2,n} \\ \cdots & \cdots & \cdots & \cdots \\ x'_{t+m,1} & x'_{t+m,2} & \cdots & x'_{t+m,n} \end{pmatrix}_{m\times n}. \tag{14}$$

In other words, $\mathbf{x}'_{m\times n}=\mathbf{x}_{m\times n}\circ\widehat{\mathbf{Y}}_{m\times n}$. The matrix $\mathbf{x}'_{m\times n}$ has already adjusted the weights of assets predicted to decrease in price, specifically those in the ith month of the test dataset (denoted as $y_{t+i,n}=0$), to zero. Now, the remaining weights are redistributed among the assets predicted to rise in price (indicated as $y_{t+i,n}=1$), thereby leading to the optimized weight matrix $\mathbf{x}^*_{m\times n}$. In this matrix$\mathbf{x}^*_{m\times n}$, the non-zero elements $\mathbf{x}^*_{i,j}$ can be represented as $\mathbf{x}^*_{i,j}=\frac{\mathbf{x}'_{t+m,j}}{\sum_{s=1}^{n}\mathbf{x}'_{t+i,s}}$, where j denotes the jth asset, $j\in[1,n]$.

4. Numerical analysis

4.1. Data

This study selects three stock indices, one bond index, and two commodity indices as assets for portfolio analysis for the following reasons. On the one hand, the HS300, Hang Seng (HSZS), and NASDAQ indices represent a range of equity markets, including mainland Chinese stocks, Hong Kong equities, and U.S. growth stocks. This combination provides a balanced representation of regional economic forces and distinct market dynamics. On the other hand, the ZZJR bond index, focused on Chinese financial bonds, contributes stability to the portfolio by reducing volatility and delivering steady income, offsetting the higher risk associated with equities. Additionally, the chemical CFCI (CI-CFCI) and agricultural CFCI (AP-CFCI) indices offer exposure to key commodity markets, providing both inflation protection and diversification. Together, these indices encompass a broad range of financial assets, balancing growth potential and risk management.

A 95 % confidence level is used for the CVaR analysis. The in-sample period covers January 2005 to December 2019, while the out-of-sample period spans January 2020 to January 2023, including both bull and bear market phases, particularly the significant declines after June 2015. This dataset provides a robust level of representativeness and timeliness, enabling an assessment of strategy performance during sharp market downturns. The lookback period, a timeframe within which the historical data of investment assets are observed and analyzed to assess their performance and volatility, is set to three months in numerical experiments. Data is sourced from the RESSET financial research database.

4.2. Data description and exploration

To comprehensively assess the statistical properties of various financial assets, we present both descriptive statistics and exploratory data analysis in Table 1.[2] Descriptive statistics such as mean return, standard deviation, skewness, and kurtosis for each asset, help establish an initial understanding of the characteristics and risk profiles associated with these assets. To examine whether the return series satisfies the Independence and Identically Distributed (I.I·D) assumption, we conduct two key tests, following Ziggel et al. (2014). The Ljung-Box test is used to detect autocorrelation in the time series, which would violate the assumption of independence. On the other hand, the Kolmogorov-Smirnov (KS) test is applied to compare the return distributions between the first and second halves of the sample period, assessing the stability of distributional properties over time and hence the identically distributed aspect. Furthermore, we conduct the augmented Dickey-Fuller (ADF) (Dickey and Fuller, 1981) unit root test to confirm the stationarity of each return series. We then conduct the autoregressive conditional heteroskedasticity (ARCH) (Engle, 1982) test for conditional heteroskedasticity to verify the presence of volatility clustering. Finally, the correlation analysis between assets provides initial insights into the interrelationships and potential diversification benefits among these assets.

Table 1 indicates substantial variation in average daily returns across six assets: HS300 records the highest, whereas AP-CFCI records the lowest. Except for ZZJR and AP-CFCI, most assets exhibit daily return standard deviations between 1.00 % and 1.70 %, reflecting elevated volatility. Inter-asset correlations remain low, facilitating partial natural hedging and diversification. ZZJR presents skewness of 0.796 and kurtosis of 24.875, consistent with fat-tailed distributions. Ljung-Box tests confirm significant autocorrelation for most assets, particularly persistent in ZZJR. KS tests demonstrate pronounced distributional shifts over the sample period, underscoring time-varying return dynamics. Despite non-normality and heteroscedasticity, GARCH models assume normality to simplify estimation while capturing key volatility dynamics. ADF tests verify stationarity at the 1 % level, and ARCH tests justify the use of GARCH for volatility estimation.

4.3. Numerical analysis of the CVaR-based risk parity model

4.3.1. Calculation of CVaR

We estimate the conditional volatility for the following month by using a three-month lookback period. Prior research shows that low-order GARCH models effectively capture the dynamics of financial time series (Bollerslev, 1986). To simplify analysis and reduce computational complexity, we employ the GARCH (1) model under the assumption of normality for subsequent evaluations.

For the monthly return series of each asset, we estimate the GARCH (1) model to capture volatility dynamics. Residuals are then scrutinized using a comprehensive set of diagnostic tests, namely the ARCH-LM test, the Jarque-Bera test, and the Ljung-Box test, to evaluate model adequacy,[3] Although some months fail certain tests, likely due to the fat tails and leptokurtic characteristics identified in our exploratory analysis, the normality assumption simplifies the modeling process. Despite these limitations, the results remain robust, supporting the model's efficacy in capturing volatility dynamics.

Following the estimation of asset price volatility using the GARCH (1) model, the quantile regression method is applied to calculate the VaR values (Chen and Chen, 2002). With these VaR values, CVaR is subsequently computed according to Eq. (2). All six assets pass the Kupiec test (Kupiec, 1995), indicating that the observed frequency of CVaR exceedances is consistent with the expected level. Furthermore, the Christoffersen test (Ziggel et al., 2014) suggests that, with the exception of AP-CFCI, these exceedances are largely serially independent.[4] These findings suggest that the model accurately captures both the magnitude and timing of tail risks, supporting its effectiveness in risk forecasting.

4.3.2. Cumulative returns of different models

We consider multiple asset allocation strategies, including EW, RP, and CVaR-RP. EW provides a simple asset allocation method, making it an ideal strategy under the condition of limited data availability. RP seeks to equalize risk contributions from each asset, while CVaR-RP incorporates CVaR to explicitly address tail risks. We first construct portfolios based on these strategies. We then conduct an analysis of their cumulative returns,[5] and examine their performance in terms of return and risk.

Table 2 provides a comparative analysis of the performance metrics of the EW, RP, and CVaR-RP strategies. The CVaR-RP strategy demonstrates superior risk-adjusted performance, reflected in its Sharpe ratio of 48.78 %, significantly surpassing other strategies both in terms of return and drawdown management. This highlights its efficacy in mitigating risks amid volatile market conditions. In contrast, while the RP strategy achieves the lowest annualized volatility, it underperforms the CVaR-RP in terms of returns, emphasizing the added value of addressing tail-risk in the conventional RP framework.

[2] A detailed description of the price trends for various assets is provided in Appendix Fig. A1.
[3] The proportion of months during which all three tests yield non-significant results is shown in Appendix Fig. A2.
[4] The computed CVaR estimates demonstrate strong reliability across most assets as shown in Appendix Table A1.
[5] The cumulative returns of all three portfolios are shown in Appendix Fig. A3.

Table 1
Descriptive statistics and exploratory analysis of various assets.

Asset	HS300	HSZS	NASDAQ	ZZJR	CI-CFCI	AP-CFCI
Mean (‰)	0.326	0.06	0.367	0.172	0.185	0.168
Std (%)	1.662	1.484	1.373	0.109	1.429	0.858
Max	0.089	0.118	0.112	0.017	0.102	0.045
Min	−0.097	−0.131	−0.132	−0.01	−0.082	−0.048
Skewness	−0.509	−0.211	−0.41	0.796	−0.031	−0.101
Kurtosis	4.041	8.190	7.904	24.875	2.388	2.677
Ljung-Box test	39.502***	14.514	53.129***	495.561***	62.322***	28.575***
Kolmogorov-Smirnov test	0.082***	0.038**	0.039**	0.081***	0.084***	0.043***
ADF	−15.246***	−13.325***	−17.080***	−10.818***	−17.190***	−46.548***
ARCH	442.770***	938.140***	1144.400***	79.789***	428.680***	418.400***
Correlation with HS300	1	–	–	–	–	–
Correlation with HSZS	0.536	1	–	–	–	–
Correlation with NASDAQ	0.113	0.223	1	–	–	–
Correlation with ZZJR	−0.024	−0.053	0.02	1	–	–
Correlation with CI-CFCI	0.279	0.339	0.062	−0.058	1	–
Correlation with AP-CFCI	0.228	0.250	0.041	−0.053	0.531	1
Observations	4385	4385	4385	4385	4385	4385

This table reports summary statistics of the daily returns of the six assets (HS300, HSZS, NASDAQ, ZZJR, CI-CFCI, and AP-CFCI), the results of the augmented Dickey-Fuller (ADF) test and the autoregressive conditional heteroskedasticity (ARCH) effect test. Mean refers to the mean return. Std stands for the standard deviation. Min and Max represent the lowest and highest values observed, respectively. Skewness indicates the degree of asymmetry of return distributions, while Kurtosis measures the heaviness of the tails compared to a normal distribution. Ljung-Box Test checks for autocorrelation in time series data; Kolmogorov-Smirnov (KS) Test examines whether the return distributions from the first and second halves of the sample period are statistically different. ADF and ARCH refer to the ADF unit root test and the ARCH Lagrange multiplier test, respectively. Correlation rows show the correlation of each asset with other assets. ***, **, and * denote significance at 1 %, 5 %, and 10 %, respectively. N refers to the number of observations for each asset.

Table 2
Performance of EW, RP, and CVaR-RP.

	EW	RP	CVaR-RP
Annualized return	5.78 %	5.12 %	5.96 %
Annualized volatility	44.64 %	11.50 %	12.22 %
Sharpe ratio	12.93 %	44.48 %	48.78 %
Maximum drawdown	32.79 %	2.82 %	3.22 %
Calmar ratio	0.18	1.81	1.85

This table reports the performance of equal weight (EW), risk parity (RP), and CVaR-based risk parity (CVaR-RP) strategies. Sharpe ratio refers to the risk-adjusted return. Maximum drawdown is defined as the largest potential loss an investment has experienced over a specific timeframe, and Calmar ratio measures the risk-adjusted performance of an investment by comparing its annualized return to its maximum drawdown.

4.4. Numerical analysis of the CVaR-based risk parity model with machine learning

Given that fat tails and skewness are typically observed in asset returns, the normal distribution assumption in the CVaR-RP model may lead to an underestimation of extreme risks. Additionally, the GARCH model, which depends on historical data for volatility forecasting, often lags in responsiveness to sudden market shifts and unforeseen events, limiting its ability to accurately reflect current market dynamics. To address these limitations, machine learning methods are introduced to predict asset price movements, allowing the model to capture complex data patterns beyond traditional distributional assumptions.

To gauge the effectiveness of machine learning optimization, we compare six investment strategies, including EW, RP, and CVaR-RP strategies as benchmarks, and machine learning-optimized versions: refined EW (Re_EW), refined RP (Re_RP), and refined CVaR-RP (Re_CVaR-RP) strategies. To this end, a two-stage data partitioning approach is employed to separate in-sample and out-of-sample datasets. The specific process of data partitioning is illustrated in Fig. 2.

4.4.1. Solving for initial asset allocation weights

In this experiment, the initial allocation weights for the six assets are set to [1/6, 1/6, 1/6, 1/6, 1/6, 1/6]. The maximum number of iterations is set to 3000, and the convergence threshold is set to 10^{-6}. By substituting these initial allocation weights into the iterative formula $x_i\beta_i = \frac{1}{n}, i = 1, 2, \ldots, n$, the time-varying investment weights for each asset in the testing dataset are obtained, which are shown in Fig. 3.

It is noteworthy that the time-varying weights of ZZJR are at a higher level due to the bond's low-risk characteristics, aligning with the principle of RP weight allocation. This suggests that to equalize the risk contribution of low-risk assets with that of high-risk assets, it might be necessary to allocate more funds to the low-risk assets.

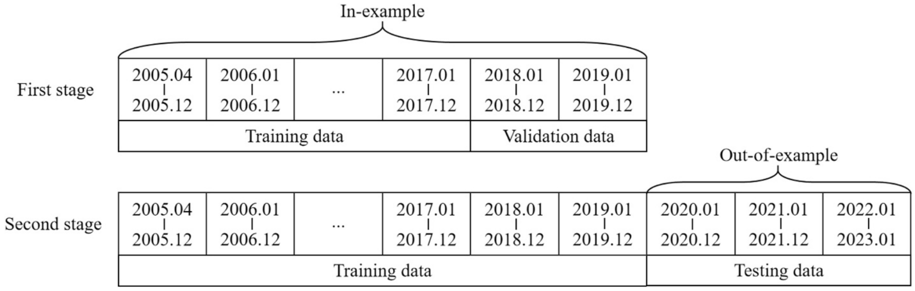

Fig. 2. Dataset division: two-stage training approach. The figure shows the specific division of the dataset when employing a two-stage training approach for price prediction in the numerical analysis.

4.4.2. Machine learning-based assets price forecasting

Machine learning is applied to predict future prices based on historical data for each asset.[6] The dataset includes monthly observations of closing prices, opening prices, highest prices, lowest prices, and monthly returns for the six assets from January 2005 to January 2023. To improve the accuracy and robustness of the model, cross-validation and grid search methods are used to find the optimal hyperparameters for all machine learning models. In addition, for each deep learning approach, this paper sets the network depth to two layers, employs rectified linear unit (ReLU) as the activation function to accelerate the training process of models, and cross-entropy loss as the loss function to effectively optimize and train models. In order to enhance the flexibility of the model in adapting to market dynamics, a two-stage training approach is employed to predict asset price movements.

In the first stage, eight machine learning algorithms are employed to predict the monthly price changes of each asset in the validation dataset. To facilitate prediction, the monthly price changes in the sample are first transformed into labeled data. Specifically, price changes less than 0 are labeled as 0, while those greater than or equal to 0 are labeled as 1. To comprehensively evaluate the models, we plot ROC curves based on prediction accuracy results of LR, SVM, RF, GBDT, CNN, RNN, LSTM, and Transformer for each asset.[7] From Fig. 4, we note that LSTM is optimal for HS300 and ZZJR, CNN is optimal for HSZS, Transformer is optimal for AP-CFCI, and RNN is optimal for NASDAQ and CI-CFCI. This result validates the feasibility and effectiveness of selecting the optimal machine learning model for each asset separately in the first stage. Thus, we optimize portfolio weights using the machine learning algorithm with the highest accuracy for each asset.

In the second stage, each asset's optimal machine learning algorithm, is re-trained on the training dataset. The trained models are then used to generate predictions on the validation dataset. Based on these predictions and the calculated weights for the EW, RP, and CVaR-RP strategies, assets predicted to decline have their weights set to zero, while the remaining weights are allocated to assets predicted to rise, in proportion to their original weights. Fig. 5 presents charts comparing allocation results before and after weight optimization on the testing dataset.

4.4.3. Analysis of the CVaR-based risk parity model with machine learning

To evaluate the performance of portfolios under different asset allocation strategies, we calculate their returns and cumulative returns.[8] Table 3 presents performance metrics for each portfolio. The results indicate that incorporating machine learning optimization significantly enhances all three strategies, with optimized versions achieving notably higher annualized returns and improved risk-adjusted performance.

In summary, machine learning algorithms offer an effective means to optimize asset allocation within the Re_CVaR-RP strategy, mitigating downside risks by accounting for the extreme volatility and tail risks of individual assets. This approach enhances the portfolio's resilience against extreme events, leading to improved risk diversification and more stable returns. To evaluate the effectiveness of the proposed CVaR-RP model with machine learning-enhanced weight optimization, we compare it with the traditional Fama-French three-factor model (Fama and French, 1993). As shown in Appendix Table A3, after accounting for market, size, and value factors, the proposed model still yields a significant Alpha of 0.013, indicating significant excess returns beyond conventional factors.

4.5. Robustness checks

4.5.1. Impacts of CVaR on portfolio optimization

In this section, we examine the advantages achieved by incorporating CVaR into portfolio optimization. Specifically, we apply the CVaR methodology to the EW portfolio, referred to as CVaR-based EW (CVaR-EW) portfolio,[9] for both risk assessment and

[6] The price movement series for these assets is shown in Appendix Fig. A4.

[7] The performance of different machine learning models across asset classes is shown in Appendix Table A2.

[8] A comparison of cumulative returns over the 37-month period is shown in Appendix Fig. A5.

[9] The CVaR-EW strategy starts with an equal allocation of assets. It then calculates the portfolio's CVaR and optimizes the weights to minimize extreme losses, enhancing the portfolio's risk-adjusted performance.

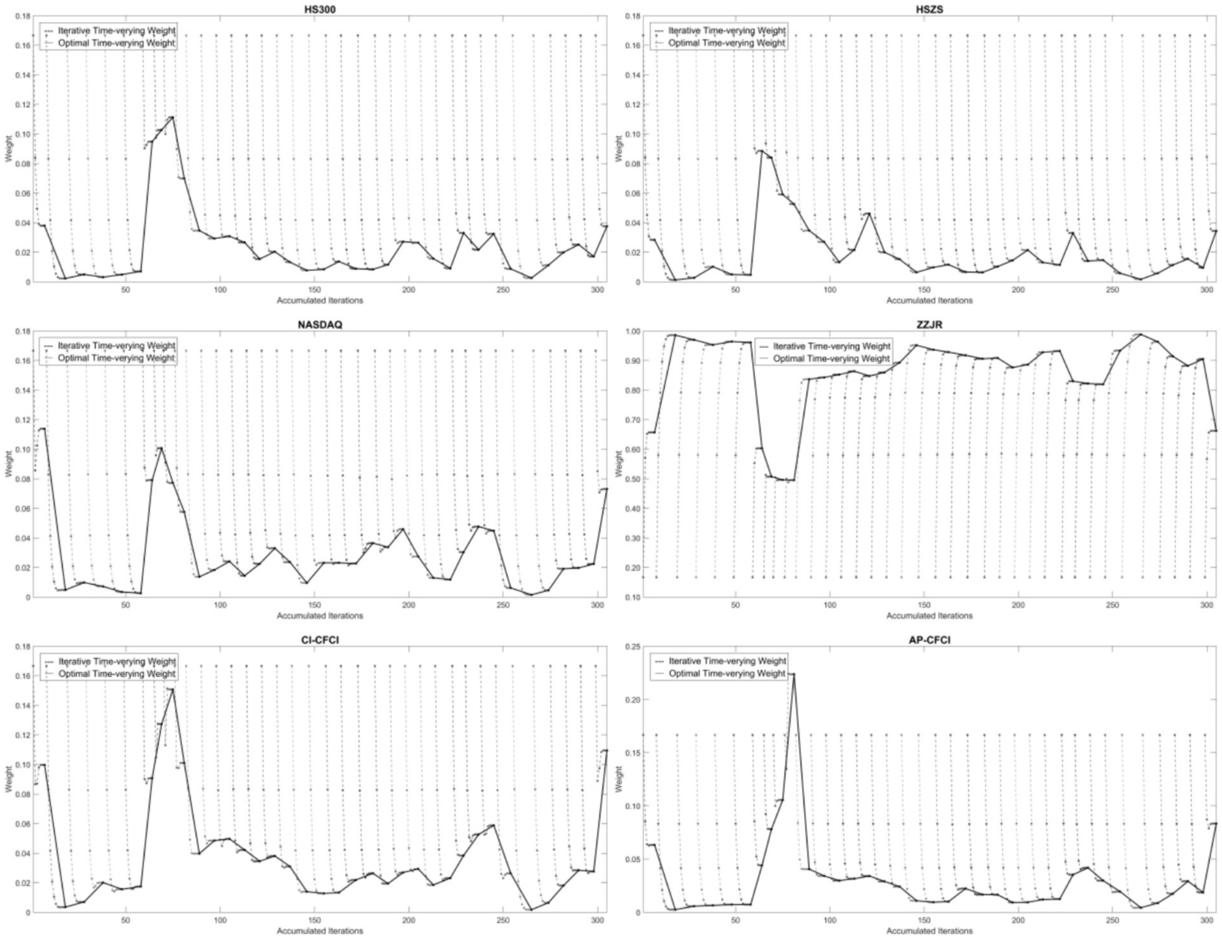

Fig. 3. Time-varying portfolio weights. The figure plots the time-varying portfolio weights for each asset within the test dataset. Each subplot contains 37 dashed lines (one for each month, totaling 37 months) and one solid line. Each dashed line consists of several data points, with their starting point at 1/6 and their endpoint at the optimal weight value. The data points along the dashed lines represent the changing trajectory of weights during the iterative process from the initial value to the optimal value. In each subplot, the solid line connects the optimal weight values for 37 months. The horizontal axis represents the cumulative number of iterations, which is the sum of iteration counts for calculating the optimal weights for each asset each month (the iteration count k for solving the optimal weights in each month may vary). The vertical axis represents the weight values.

optimization. We then compare its performance with that of other strategies, including the traditional EW and the CVaR-RP strategies. Table 4 reports the results of the performance of the CVaR-EW strategy and other strategies (EW and CVaR-RP) over the entire dataset. The results show that applying CVaR enhances risk-adjusted returns, confirming that incorporating CVaR into portfolio optimization leads to a better trade-off between risk and return than traditional methods.

4.5.2. *Alternative lookback periods*

In this section, we conduct robustness tests for the Re_CVaR-RP model by using alternative lookback periods. This approach allows for a more precise evaluation of the model's robustness under varying conditions, ensuring its ability to maintain robust performance across diverse market scenarios. Appendix Fig. A6 illustrates the cumulative returns of the six strategies with a 6-month lookback period (T = 6) and a 12-month lookback period (T = 12).

Table 5 shows that the Re_CVaR-RP strategy generates annualized returns of 17.43 % and 18.01 % for T = 6 and T = 12, respectively. Additionally, it exhibits annualized volatilities of 29.49 % and 29.92 %, signifying its capacity to balance risk while delivering robust returns over varying lookback durations. In terms of Sharpe ratio, the Re_CVaR-RP strategy showcases exceptional risk-adjusted performance with Sharpe ratios of 59.10 % and 60.20 % for T = 6 and T = 12, respectively. These findings highlight the strategy's ability to generate high returns relative to its comparatively low risk exposure. Furthermore, the Re_CVaR-RP strategy maintains lower maximum drawdown levels when compared with other machine learning-enhanced strategies, demonstrating its ability to mitigate losses due to market fluctuations. The notably high Calmar ratio of the Re_CVaR-RP strategy is indicative of its outstanding performance in effectively managing downside risk while capturing upside returns. This shows the effectiveness of

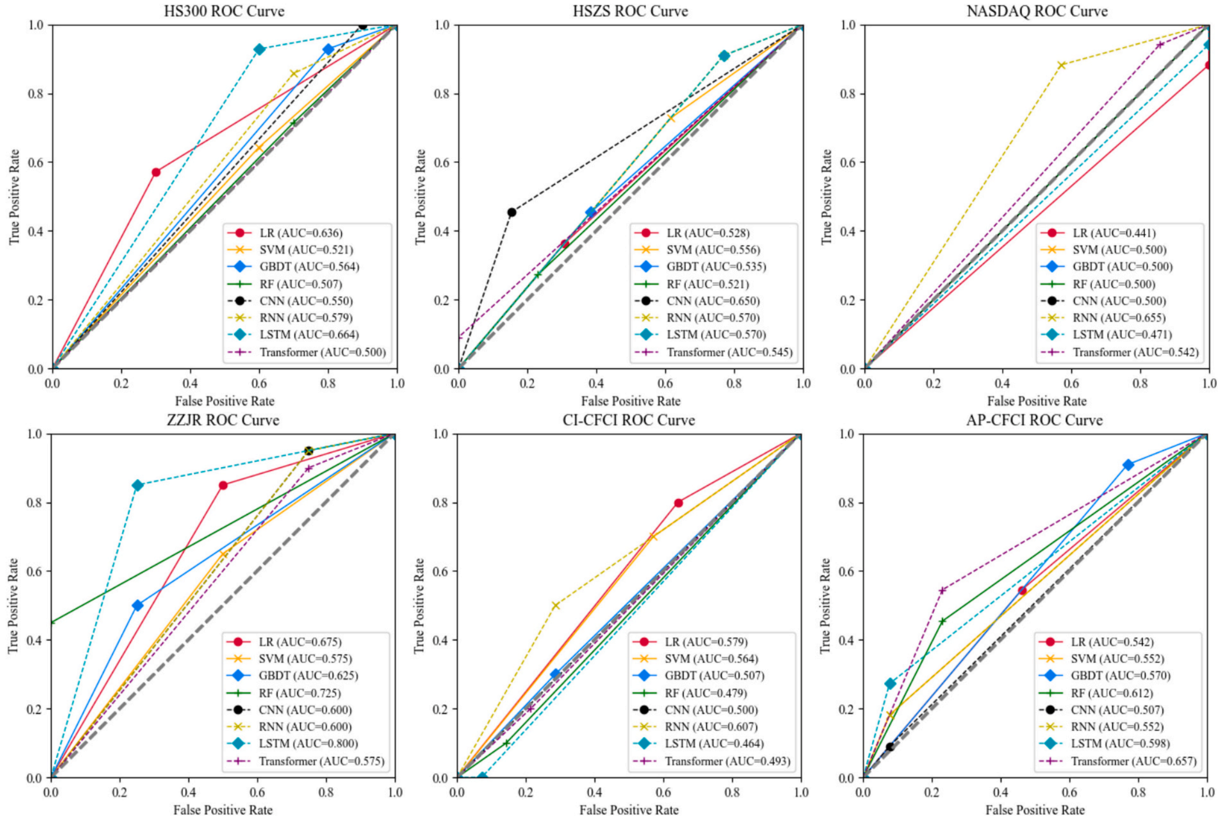

Fig. 4. ROC curves of various machine learning algorithms. This figure shows receiver operating characteristic (ROC) curves for the prediction accuracy results of LR, SVM, RF, GBDT, CNN, RNN, LSTM, and Transformer machine learning algorithms for each asset. The ROC curve plots the true positive rate (TPR) against the false positive rate (FPR) at various threshold levels.

incorporating machine learning techniques into the CVaR-RP model in achieving better returns and reducing risk.

4.5.3. Different training methods

In this section, we compare the proposed two-stage training method with the traditional training partitioning approach to evaluate the effectiveness and robustness of the two-stage approach under diverse data conditions, highlighting its potential advantages over traditional methodologies.[10] Table 6 reports the performance of the two training methods. It shows that the two-stage training approach demonstrates clear advantages over the traditional training method. The two-stage training method consistently achieves higher annualized returns and Sharpe ratios, indicating better risk-adjusted performance. For example, the Re_EW strategy yields an annualized return of 23.59 % with the two-stage method compared to 17.16 % under traditional training. Furthermore, the two-stage approach yields lower volatilities and drawdowns, highlighting its greater risk management capability. The Re_CVaR-RP strategy shows improved risk metrics, including a maximum drawdown of 3.77 %, compared to 5.71 % for the traditional method, validating the effectiveness of this approach in enhancing portfolio stability.

Overall, the two-stage training approach demonstrates significant robustness and effectiveness in portfolio optimization. Compared to the traditional training method, this approach exhibits superior performance in terms of various metrics, indicating enhanced effectiveness in risk management and return maximization.

4.5.4. Different distribution assumptions

Given that asset returns in real financial markets often exhibit skewness and fat-tail characteristics, the normality assumption may fail to adequately capture tail risk, thereby impacting the accuracy of CVaR calculations and asset allocation outcomes. To address this concern, we conduct robustness tests over two distinct periods. First, we compare the performance of the CVaR-RP model under normal and t-distribution assumptions across the entire dataset. Subsequently, on the testing dataset, we make the same comparison while also introducing the Re_CVaR-RP model to assess its adaptability under future market conditions.

Table 7 indicates that although the model based on the normal distribution assumption demonstrates superior performance in

[10] The traditional training partitioning approach is illustrated in Appendix Fig. A7.

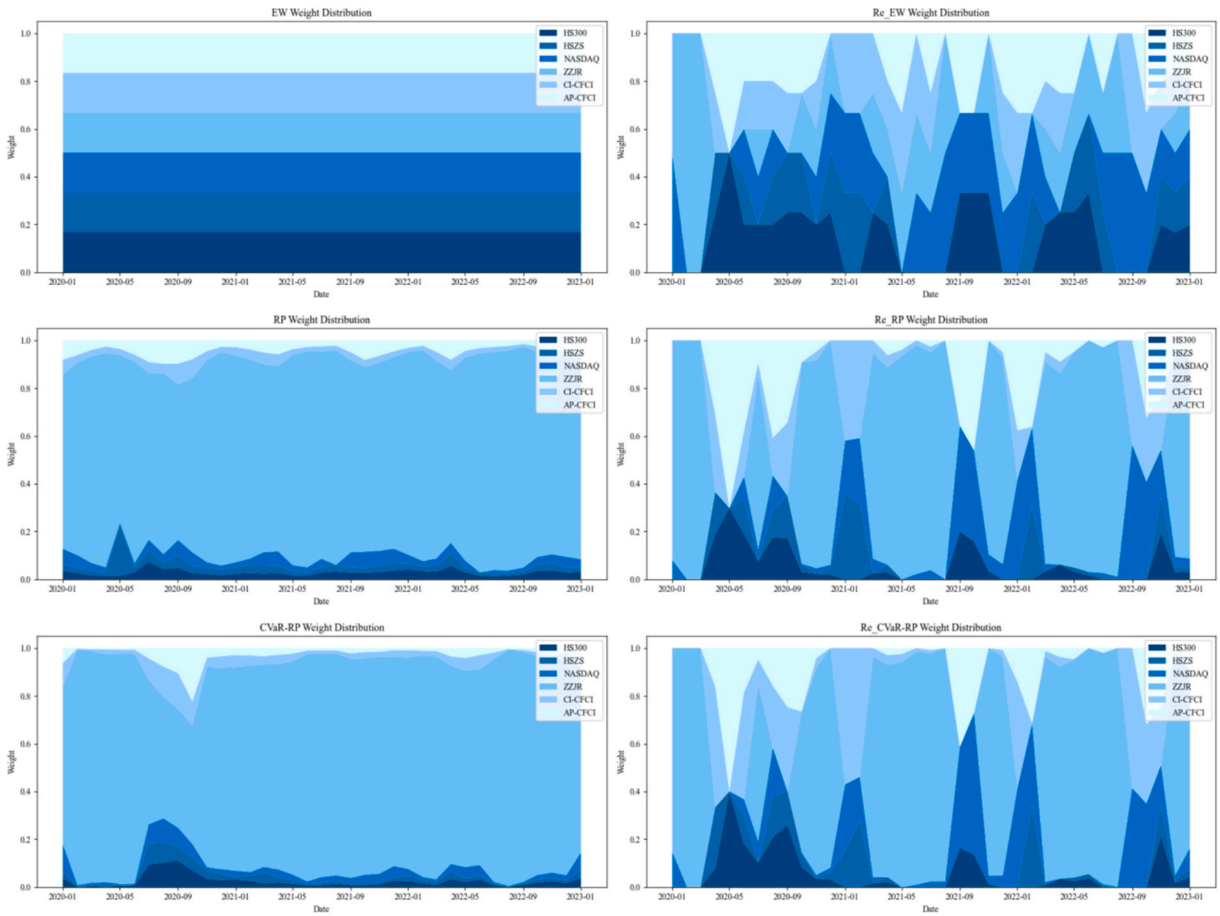

Fig. 5. Original weights and optimized configuration distribution diagram. This figure shows asset allocation weights before and after optimization on the test dataset. Each color band represents a different asset in the portfolio. The y-axis shows the weight proportion for each asset, while the x-axis represents the timeline within the test period. The left column shows the original allocations based on the equal weight (EW), CVaR-based EW (CVaR-EW), and CVaR-based risk parity (CVaR-RP) strategies, while the right column displays optimized allocations.

Table 3
Performance of various strategies with and without machine learning.

	Re_EW	Re_RP	Re_CVaR-RP	EW	RP	CVaR-RP
Annualized return	23.59 %	14.44 %	17.43 %	4.29 %	4.34 %	6.30 %
Annualized volatility	34.41 %	27.71 %	29.49 %	40.95 %	9.04 %	11.23 %
Sharpe ratio	68.57 %	52.12 %	59.10 %	10.48 %	48.02 %	56.07 %
Maximum drawdown	5.63 %	4.75 %	3.77 %	18.67 %	1.77 %	1.92 %
Calmar ratio	4.19	3.04	4.63	0.23	2.46	3.27

This table reports the performance of equal weight (EW), risk parity (RP), CVaR-based risk parity (CVaR-RP), refined EW (Re_EW), refined RP (Re_RP), and refined CVaR-based RP (Re_CVaR-RP) strategies. Sharpe ratio refers to the risk-adjusted return of an investment. Maximum drawdown is defined as the largest potential loss an investment has experienced over a specific timeframe, and Calmar ratio measures the risk-adjusted performance of an investment by comparing its annualized return to its maximum drawdown.

backtesting, it may underestimate tail risks under actual market conditions, thereby introducing a potential systemic bias. Consequently, future research could benefit from investigating more flexible modeling approaches, such as normal mixture distributions or non-parametric methods, to enhance the model's adaptability and robustness in real-world market settings.

4.5.5. Analysis based on individual stock portfolios

To rigorously evaluate the robustness and applicability of the proposed model under more complex market structures, this section extends the empirical analysis from index-based portfolios to individual stock portfolios. We consider a total of 10 representative A-share listed companies in China for the period from January 4, 2010, to December 31, 2024. These firms represent a wide range of sectors, including banking, consumer goods, energy, transportation, and industrial manufacturing. The stocks include China

Table 4
Performance of EW, CVaR-EW, and CVaR-RP.

	EW	CVaR-EW	CVaR-RP
Annualized return	5.78 %	9.52 %	5.96 %
Annualized volatility	44.64 %	51.61 %	12.22 %
Sharpe ratio	12.93 %	18.45 %	48.78 %
Maximum drawdown	32.79 %	30.49 %	3.22 %
Calmar ratio	0.18	0.31	1.85

This table reports the performance of equal weight (EW), CVaR-based EW (CVaR-EW), and CVaR-based risk parity (CVaR-RP) strategies. Sharpe ratio refers to the risk-adjusted return. Maximum drawdown is defined as the largest potential loss an investment has experienced over a specific timeframe, and Calmar ratio measures the risk-adjusted performance of an investment by comparing its annualized return to its maximum drawdown.

Table 5
Performance of various strategies for alternative looking back periods.

	Lookback period	Annualized return	Annualized volatility	Sharpe ratio	Maximum drawdown	Calmar ratio
Re_EW	T = 6	23.59 %	34.10 %	68.57 %	5.63 %	4.19
	T = 12	19.96 %	36.50 %	54.69 %	8.44 %	2.36
Re_RP	T = 6	14.44 %	27.71 %	52.12 %	4.75 %	3.04
	T = 12	13.09 %	25.75 %	50.84 %	4.99 %	2.63
Re_CVaR-RP	T = 6	17.43 %	29.49 %	59.10 %	3.77 %	4.63
	T = 12	18.01 %	29.92 %	60.20 %	3.51 %	5.13
EW	T = 6	9.43 %	38.33 %	24.61 %	16.92 %	0.56
	T = 12	3.91 %	36.48 %	10.71 %	19.30 %	0.20
RP	T = 6	4.30 %	8.91 %	48.20 %	1.54 %	2.79
	T = 12	4.14 %	7.61 %	54.45 %	1.72 %	2.41
CVaR-RP	T = 6	5.95 %	11.72 %	50.73 %	1.96 %	3.03
	T = 12	6.94 %	12.56 %	55.23 %	2.34 %	2.96

This table reports the performance of the equal weight (EW), risk parity (RP), CVaR-based risk parity (CVaR-RP), refined EW (Re_EW), refined RP (Re_RP), and refined CVaR-based RP (Re_CVaR-RP) strategies, with a 6-month lookback period (T = 6) and with a 12-month lookback period (T = 12). Sharpe ratio refers to the risk-adjusted return of an investment. Maximum drawdown is defined as the largest potential loss an investment has experienced over a specific timeframe, and Calmar ratio measures the risk-adjusted performance of an investment by comparing its annualized return to its maximum drawdown.

Table 6
Performance of different training approaches.

	Two-stage training approach			Traditional training approach		
	Re_EW	Re_RP	Re_CVaR-RP	Re_EW	Re_RP	Re_CVaR-RP
Annualized return	23.59 %	14.44 %	17.43 %	17.16 %	13.64 %	16.95 %
Annualized volatility	34.41 %	27.71 %	29.49 %	37.20 %	28.41 %	30.76 %
Sharpe ratio	68.57 %	52.12 %	59.10 %	46.12 %	47.99 %	55.09 %
Maximum drawdown	5.63 %	4.75 %	3.77 %	8.43 %	7.00 %	5.71 %
Calmar ratio	4.19	3.04	4.63	2.04	1.95	2.97

This table reports the performance of the two-stage and traditional training methods, including annualized return, volatility, Sharpe ratio, maximum drawdown, and Calmar ratio, for the refined EW (Re_EW), refined RP (Re_RP), and refined CVaR-based RP (Re_CVaR-RP) strategies. Sharpe ratio refers to the risk-adjusted return of an investment. Maximum drawdown is defined as the largest potential loss an investment has experienced over a specific timeframe, and Calmar ratio measures the risk-adjusted performance of an investment by comparing its annualized return to its maximum drawdown.

Construction Bank (CCB), Industrial and Commercial Bank of China (ICBC), Kweichow Moutai (Moutai), Wuliangye Yibin (Wuliangye), PetroChina, China Petroleum & Chemical Corporation (Sinopec), CRRC Corporation Limited (CRRC), China Life Insurance Company Limited (China Life), China Railway Group (CRG), and (First Automobile Works) FAW Jiefang.

Following the data partitioning in Fig. 2, a two-stage training approach is used, and the algorithm with the highest in-sample accuracy (bolded in Appendix Table A4) is selected for second stage forecasting to improve generalization and robustness. In the empirical analysis of individual stock portfolios, we adopt the same analytical framework as that used in the index-based setting. Specifically, we construct three strategies based on machine learning predictions: Re_CVaR_RP, Re_RP, and Re_EW. These strategies are compared with benchmark approaches, including EW, RP, and CVaR_RP. As illustrated in Table 8, the Re_CVaR_RP strategy demonstrates the highest annualized return of 44.90 % and achieves superior risk-adjusted performance, with a Sharpe ratio of 54.73 % and a Calmar ratio of 3.31 %. These results significantly outperform those of the conventional CVaR_RP and RP strategies. The findings suggest that the proposed model remains effective even when applied to individual stocks. It maintains strong predictive accuracy and stability, enhances portfolio performance, and provides effective downside risk control.

Table 7
Performance of various models under different distribution assumptions.

	Entire dataset		Testing dataset			
	Normal distribution	t distribution	Normal distribution		t distribution	
	CVaR-RP	CVaR-RP	CVaR-RP	Re_CVaR-RP	CVaR-RP	Re_CVaR-RP
Annualized return	5.96 %	5.25 %	6.30 %	17.43 %	5.01 %	14.34 %
Annualized volatility	12.22 %	13.30 %	11.23 %	29.49 %	11.97 %	28.03 %
Sharpe ratio	48.78 %	39.50 %	56.07 %	59.10 %	41.83 %	51.17 %
Maximum drawdown	3.22 %	3.50 %	1.92 %	3.77 %	2.13 %	6.59 %
Calmar ratio	1.850	1.501	3.270	4.630	2.353	2.176

This table reports the performance of the model under the normal and t-distribution assumptions, in terms of annualized return, volatility, Sharpe ratio, maximum drawdown, and Calmar ratio, for the CVaR-based RP (CVaR-RP) and refined CVaR-based RP (Re_CVaR-RP) strategies. Sharpe ratio refers to the risk-adjusted return of an investment. Maximum drawdown is defined as the largest potential loss an investment has experienced over a specific timeframe, and Calmar ratio measures the risk-adjusted performance of an investment by comparing its annualized return to its maximum drawdown.

Table 8
Cumulative returns of portfolio strategies based on individual stocks.

	Re_EW	Re_RP	Re_CVaR-RP	EW	RP	CVaR-RP
Annualized return	39.42 %	36.25 %	44.90 %	8.30 %	9.61 %	12.01 %
Annualized volatility	79.40 %	68.45 %	82.04 %	60.72 %	57.87 %	64.00 %
Sharpe ratio	49.65 %	52.95 %	54.73 %	13.67 %	16.61 %	18.76 %
Maximum drawdown	15.36 %	12.75 %	13.55 %	17.79 %	17.42 %	17.06 %
Calmar ratio	2.57	2.84	3.31	0.47	0.55	0.70

This table reports the performance of equal weight (EW), risk parity (RP), CVaR-based risk parity (CVaR-RP), refined EW (Re_EW), refined RP (Re_RP), and refined CVaR-based RP (Re_CVaR-RP) strategies, based on individual stocks. Sharpe ratio refers to the risk-adjusted return of an investment. Maximum drawdown is defined as the largest potential loss an investment has experienced over a specific timeframe, and Calmar ratio measures the risk-adjusted performance of an investment by comparing its annualized return to its maximum drawdown.

While the proposed model exhibits strong predictive stability and effective risk control at the index and stock levels, extending it to high-dimensional portfolios presents additional challenges. As the number of assets increases, the covariance matrix may become ill-conditioned, making weight estimation unstable and overly sensitive to input data.[11] Future research may address these issues through feature selection, dimensionality reduction techniques such as PCA or autoencoders, and robust or sparse optimization methods to enhance model scalability, generalization, and applicability in large-scale asset allocation contexts.

5. Conclusion

This study proposes a machine learning-enhanced CVaR-RP strategy to improve portfolio performance and risk management. By incorporating CVaR into the RP framework, the model achieves a balanced distribution of tail risk, increasing resilience to extreme market events. The integration of machine learning for asset weight optimization enhances the model's adaptability, enabling CVaR-RP portfolios to dynamically respond to changing market conditions and manage downside risks more effectively.

Based on six representative assets, we show that the proposed model outperforms traditional strategies, achieving higher returns, improved Sharpe ratios, and reduced drawdowns. Robustness checks, including the analysis of CVaR effectiveness, model performance with varying lookback periods, and the performance of the two-stage training method relative to traditional training methods, further validate the model's stability and adaptability.

These findings highlight the potential of combining machine learning with advanced risk measures like CVaR to create more resilient and adaptive portfolio strategies. The machine learning-enhanced CVaR-RP model not only captures higher returns in bullish markets but also demonstrates stronger defensive capabilities during downturns, enabling investors to achieve stable performance across diverse market conditions. This approach provides valuable insights for financial institutions seeking to enhance portfolio resilience, optimize risk-adjusted returns, and refine risk management practices.

CRediT authorship contribution statement

Jiliang Sheng: Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Lanxi Chen:** Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis. **Huan Chen:** Methodology, Investigation, Data curation, Conceptualization. **Yunbi An:** Writing – review & editing, Writing – original draft, Validation, Supervision,

[11] In Appendix Fig. A8, we show how the optimization success rate and extreme weight frequency change with the number of assets in the portfolio.

Methodology, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.pacfin.2025.102857.

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