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Dynamical Systems MA3081

Exercise sheet 3

Exercise 1 (Stability analysis for 2D linear time-continuous systems)**.** For $A \in \mathbb{R}^{2\times 2}$, discuss the (stability) type of the origin in terms of trace and determinant of A. Then indicate in the $(\mathrm{trace}A, \det A)$–plane where the origin is a (stable/unstable) node/saddle/focus/center and where degenerate cases occur.

Exercise 2 (Logistic map)**.** Consider the logistic map $[0,1] \ni x \mapsto rx(1-x)$, $0 \leq r \leq 4$, and its induced time-discrete (non-invertible) dynamical system.

1. Find all the fixed points and determine their stability.
2. Show that the logistic map has a 2-periodic cycle for $r > 3$.

Exercise 3 (Persistence of hyperbolic equilibria)**.** Consider the dynamical system induced by $\dot{x} = f(x)$ for $f \in C^1(\mathbb{R}^n, \mathbb{R}^n)$, and assume x^* is a hyperbolic equilibrium. Show that it persists together with its stability type under continuously differentiable perturbations of f.

Exercise 4 (Cubic vector fields on the line)**.** Classify all possible flows on the line induced by the equation

$$\dot{x} = a_0 + a_1 x + a_2 x^2 + x^3$$

by their phase portraits. Do not solve any equations!

Numerical excursions: Set up the time-discrete dynamical system given by the logistic map. A classic way to get a feeling for asymptotic dynamics is the following. Take some (random) initial condition, iterate the dynamical system, say, 10,000 times, and then plot the subsequent 500 iterates. Try to build some intuition first: why would one omit the first iterates from the plot, what do you expect to see for different invariant sets (fixed points, periodic orbits, etc.)? Try different parameters and different initial conditions. Do you see different results for same parameter but different initial conditions? On a different note, try to generate the plot for Exercise 1 in Julia, send it to Daniel and win a free coffee!