Dynamical Systems MA3081

Exercise sheet 1

Exercise 1 (ODE solving)**.** Solve the following ordinary differential equations:

1. $\ddot{x} - 2\dot{x} + x = 0$; hint: linear ODE;
2. $\ddot{x} + x = 0$; hint: linear ODE;
3. $t\dot{x} - \sqrt{1 - x^2} = 0$; hint: separation of variables;
4. $\dot{x} - \frac{\sqrt{1+x}}{t^2+4} = 0$; hint: separation of variables;
5. $\dot{x} + \sin t \cdot x = \cos t$; hint: separation of variables, variation of constants.

Exercise 2 (Matrix exponential)**.** Compute the matrix exponential $\exp(tA)$ of the following matrices:

1. $A = \begin{pmatrix} 0.879708 & 0.257095 & 0.598457 \\ 0.257095 & 0.592012 & 0.370489 \\ 0.598457 & 0.370489 & 0.036444 \end{pmatrix}$;
2. $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$;
3. $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

Exercise 3 (Implicit function theorem)**.** Consider the following polynomial $p \in C^1(I, \mathbb{R})$ as an element of the continuous real-valued functions on $I \subset \mathbb{R}$, where I is some reasonable compact interval:

$$p_\alpha \colon x \mapsto x^2 + \alpha.$$

Show that for $\alpha < 0$ any root of p_α persists under small perturbations. As a more specific, simpler case, consider the same p_α and an $f \in C^1(I, \mathbb{R})$. Show that for ε with $|\varepsilon|$ sufficiently small any $p_\alpha + \varepsilon f$ has (i) as many roots as p_α, which are (ii) close to those of p_α. What can we say for $\alpha = 0$? And what about $\alpha > 0$?

Numerical excursions: get yourself a free copy of julia (www.julialang.org), it runs on all platforms. Alternatively, you can use it in your browser without a local copy: www.juliabox.com. Its usage is somewhat similar to MATLAB and Python, its performance competes with C++. For instance, a matrix is defined by

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>> A = [1.2 2.3 3.4; 4.5 5.6 6.7; 7.8 8.9 9.0].
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You could then use it for support in Exercise 2.