Dynamical Systems MA3081

Exercise sheet 4

Exercise 1 (Stability, attraction, and asymptotic stability)**.** Recall the definition of stability of an equilibrium first, and rephrase it using the term "continuity". Recall that asymptotic stability of an equilibrium x^* required that x^* be (i) stable and (ii) locally attractive, i.e., there exists a neighborhood $\mathcal{U}$ of x^* such that for any $x \in \mathcal{U}$ one has $\lim_{t\to\infty} \phi_t(x) = x^*$. Is it possible to have an attractive, yet unstable equilibrium? Give an example (by any means: equation or phase portrait), or prove that attractivity implies stability.

Exercise 2 (Topological equivalence)**.** Show that two hyperbolic linear systems

$$\dot{x} = Ax, \qquad\qquad \dot{y} = By,$$

are topologically equivalent if and only if they have equal number of eigenvalues with positive and negative real parts, respectively.

Exercise 3 (Dynamics of the pendulum)**.** Consider the dynamical system induced by $\ddot{\varphi} + \sin\varphi = 0$.

- What is the phase space of the pendulum? Draw a phase portrait.
- Find all equilibria and determine their stability type.
- For the hyperbolic ones, compute the stable and unstable manifolds (as good as you can).

Numerical excursions: Work on the pendulum equation: draw the phase portrait, solve the equation numerically, plot solutions, animate solutions as a real pendulum. What do you observe for different numerical integrators? From an accuracy point of view, what is crucial for numerical integrators to obtain faithful solutions?