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# Dynamical Systems MA3081

Exercise sheet 12

**Exercise 1** (1D maps)**.** (a) Show that the tent map

$$g\colon x \mapsto \begin{cases} 2x, & x \in [0, 1/2), \\ 2(1-x), & x \in [1/2, 1], \end{cases}$$

and the logistic map $f\colon x \mapsto 4x(1-x)$ are topologically conjugate via the conjugacy $h(x) = \sin^2(\pi x/2)$.

(b) Prove that topological transitivity is preserved by topological conjugacy. More precisely, if two dynamical systems are topologically conjugate then either both are topologically transitive or both are not topologically transitive.

**Exercise 2** (1D maps)**.** Let $\mu > 0$. Show that the map $f\colon x \mapsto \mu x(1-x)$ is conjugate to a map $g\colon x \mapsto x^2 + c$ for a suitable value of $c$.

**Exercise 3** (Attractors)**.** (a) Prove in detail $(T1)$ and $(T2)$ of Theorem 18.4 from the lecture notes.

(b) Consider the planar vector field

$$\begin{aligned} x' &= x - x^3, \\ y' &= -y. \end{aligned}$$

Find a trapping region $\mathcal{U}$ and an attracting set $\mathcal{A}$. Is $\mathcal{A}$ an attractor?