RLC Circuits and Resonant Behavior

Jairo Portillo

PHY 338K Electronic Techniques
Department of Physics
The University of Texas at Austin
Austin, TX 78712, USA

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Abstract

In this lab, we were to become acquainted with a RLC circuit and a trap filter RLC circuit. We observed and measured the behavior of the input and output voltages, then plotting the transfer function for both configurations. We are able to model the behavior of the circuits and find for out configurations that the resonant frequency is at 129.949 kHz.

1 Preperation

In preparation for this lab, we has to review phasor diagrams and AC circuit analysis. We also derived the transfer functions for both configurations of the RLC circuit. Using Kirchhoff's laws and the impedance of capacitors and inductors, we can find the transfer function. We simply use the relation

$$T(\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_{out}}{Z_{in} + Z_{out}}$$

where Z_{out} is the impedance for the output voltage and V_{in} is the impedance for the input voltage. For the series RLC, the transfer function can be found with

$$T(\omega) = \frac{Z_C}{Z_C + R} = \frac{R}{R + \frac{1}{j\omega C} + j\omega L}$$

with much algebra it can be simplified to

$$T(\omega) = \frac{R}{\sqrt{R^2 + \omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C}}}$$

where R is the resistance, C is the capacitance, L is the inductance, and ω is the frequency. For the notch/trap circuit, we find the tranfer function to be

$$T(\omega) = \frac{j\omega L + \frac{1}{j\omega C}}{R + \frac{1}{j\omega C} + j\omega L} = \sqrt{\frac{\omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C}}{R^2 + \omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C}}}$$

In order for the circuit to give us the best results we needed to achieve a circuit with a quality factor greater than 2. The quality factor tells us how damped the circuit is. The quality factor is the ratio between the resonance frequency to the bandwidth. The frequency for the RLC is

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The quality factor is given by

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_o L}{R} = \sqrt{\frac{L}{CR^2}}$$

Using $50\mu H$ inductor, 10Ω resistor, and a $0.03\mu F$ capacitor, we get a quality factor of 4.08 and a resonance frequency of 129.949 kHz. We found these values using a multimeter and finding the spec sheet for the inductor.

2 Lab work

2.1 Apparatus

Our apparatus was a signal generator connected with a Tee and BNC to channel one of the oscilloscope. The Tee allowed a connection for the input of the circuit. The output of the circuits was connected to channel two of the oscilloscope. The TTL output of the signal generator would be connected to the external trigger slot of the scope. This was how we arranged the resistor, inductor and capacitor.
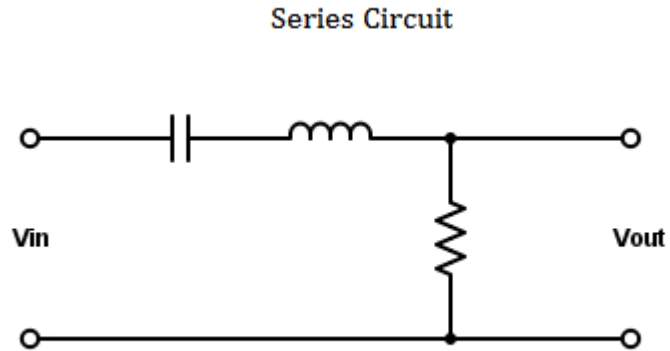
Series Circuit

Figure 1: Our series circuit without oscilloscope or signal generator pictured.

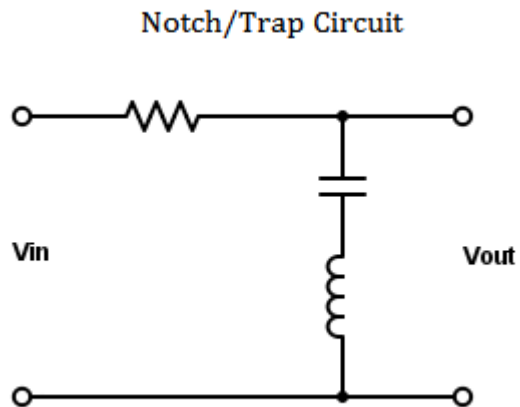This is our arrangement of circuit elements for our notch circuit.

Notch/Trap Circuit

Figure 2: Our notch/trap circuit.

2.2 Data Collection

In the first portion of the lab, we studied the behavior of the series RLC circuit. We measured the input and output voltages at various frequencies

to find the transfer function. We then compared our data to the derived transfer function above.

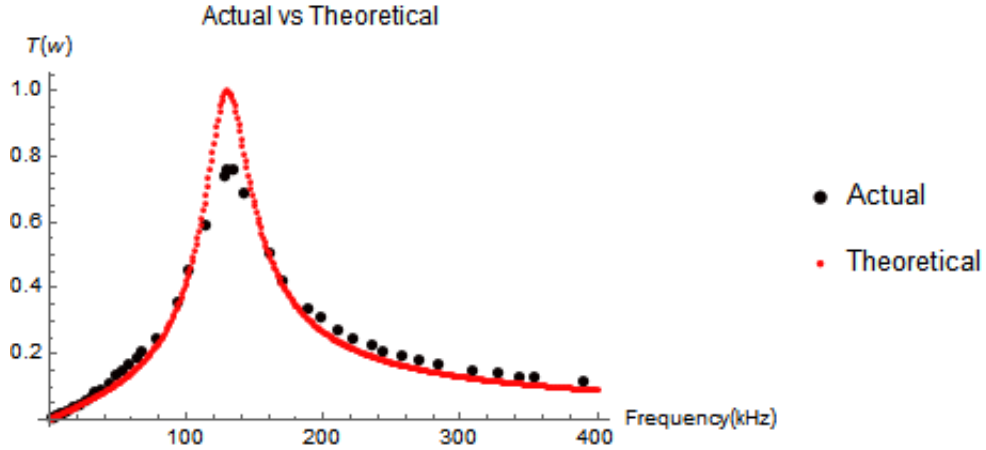$$T(\omega) = \frac{R}{\sqrt{R^2 + \omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C}}}$$

Figure 3: Our series circuit data compared to the theoretical results.

We can see from Figure 3 that our data followed the theoretical model though we did not reach the 1 ratio for the transfer function. However the peak of the transfer function occurs at approximately 130 kHz based on the graph and 128 kHz based on the raw data. This is very close to our calculated resonance frequency of 129.95 kHz. We likely didn't reach the peak due to the internal impedance of the generator and oscilloscope. We then found the phase shifts at resonance frequency and twice the resonance frequency.
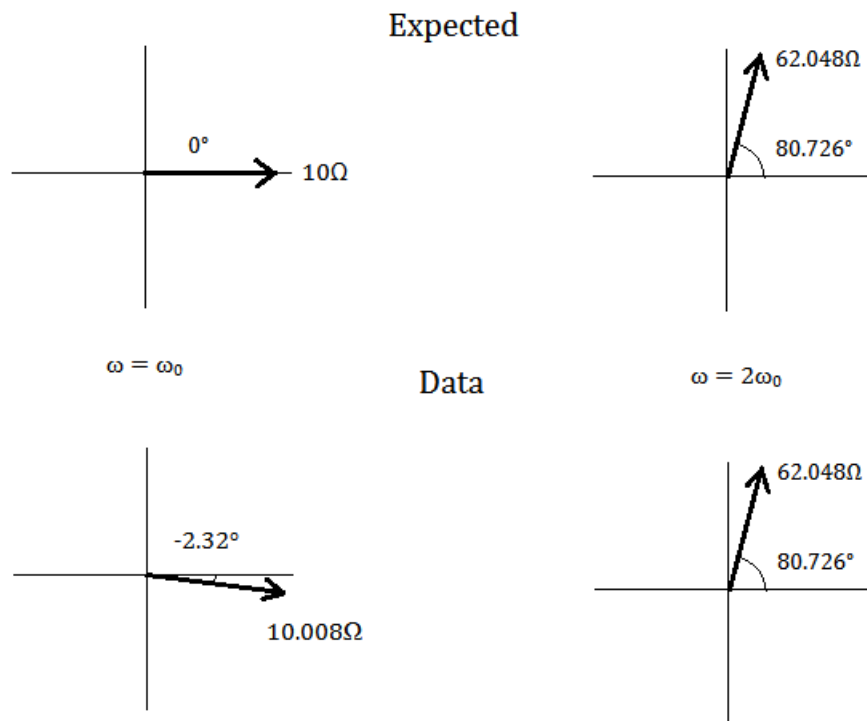
Expected

62.048Ω

0°

80.726°

10Ω

$\omega = \omega_0$

Data

$\omega = 2\omega_0$

62.048Ω

-2.32°

80.726°

10.008Ω

Figure 4: Phasor diagrams for the resonance frequency ω_0 and twice the resonance frequency $2\omega_0$.

The magnitude of our phase diagrams from our data was nearly exact for both frequencies however the phase shift at resonance frequency is not the same as what is expected. This may be due to some error in the oscilloscope finding the phase shift.

The power dissipated can be found using

$$V = IR \text{ and } P = IV$$

Given a 10 rms-volts at resonance frequency, we find the current with $I = \frac{V}{Z}$ where Z is the total impedence. This gives a current of 1 Amps. Using $P = I_{rms}^2 R$, we find that the power is 10 J. In order to find the power

using the phase shift we must use

$$P = V_{rms} I_{rms} \cos(\phi)$$

In order to find the phase shift we use

$$\phi = \tan^{-1}\left(\frac{Z_L - Z_C}{R}\right)$$

At resonance frequency, we see that $\phi = 0$ and $\cos(0) = 1$. Thus our power using phase shift is again 10 J.

Finally, we also studied and compared the behavior of the transfer function of the trap circuit.
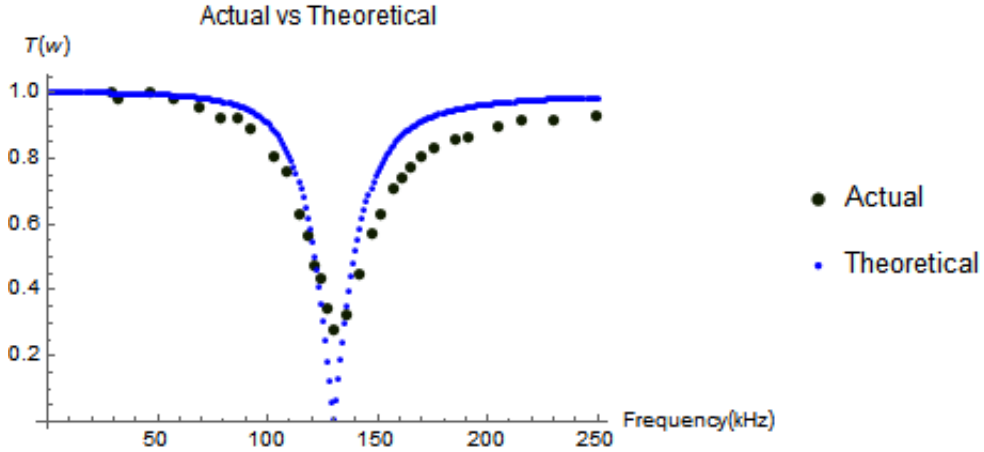

Figure 5: Our trap circuit data compared to the theoretical results.

We can see from Figure 5 that our data followed the theoretical model though as the series circuit it did not approach the lowest point. However the peak of the transfer function occurs at approximately 130 kHz based on the graph and raw data. This is also very close to our calculated resonance frequency of 129.95 kHz.

3 Summary and conclusions

We studied the behavior of RLC circuits. We found that the resonance frequency for out circuit was 129.96 kHz. We were able to accurately model the behavior of the transfer function of both configuration to their theoretical

models. The calculated phase diagrams at resonance and two times the resonance were relatively close to the expected diagrams. The hypothetical power of a 10 rms-volt signal at resonance was found to be 10 J.