4. a) $\Diamond b \Rightarrow (a \mathbin{U} b)$

$\pi \models \Diamond b$

$\exists j \geq 0.\ \pi[j\ldots] \models b$

if a = true:

$\exists j \geq 0.\ \pi[j\ldots] \models b$ and $\forall 0 \leq i < j.\ \pi[i\ldots] \models \text{true}$

$\exists j \geq 0.\ \pi[j\ldots] \models b$ and $\forall 0 \leq i < j.\ \pi[i\ldots] \models a$

$\pi \models a \mathbin{U} b$

$\pi \models \Diamond b \Rightarrow (a \mathbin{U} b)$

$\exists \pi.\ \pi \models \Diamond b \Rightarrow (a \mathbin{U} b)$

if a = ¬true:

$\exists j \geq 0.\ \pi[j\ldots] \models b$ and $\forall 0 \leq i < j.\ \pi[i\ldots]\ |\!\neq \neg\text{true}$

$\neg(\exists j \geq 0.\ \pi[j\ldots] \models b$ and $\forall 0 \leq i < j.\ \pi[i\ldots] \models a)$

$\pi\ |\!\neq a \mathbin{U} b$

$\pi\ |\!\neq \Diamond b \Rightarrow (a \mathbin{U} b)$

$\exists \pi.\ \pi\ |\!\neq \Diamond b \Rightarrow (a \mathbin{U} b)$

$\neg(\forall \pi.\ \pi \models \Diamond b \Rightarrow (a \mathbin{U} b))$

So $\Diamond b \Rightarrow (a \mathbin{U} b)$ is satisfiable, but not valid.

b) $O(a \vee \Diamond a) \Rightarrow \Diamond a$

$\pi \models O(a \vee \Diamond a)$

$\pi[1\ldots] \models a \vee \Diamond a$

if $\pi[1\ldots] \models a$:

$\exists j \geq 0.\ \pi[j\ldots] \models a$

$\pi \mid= \Diamond a$

$\pi \mid= O(a \vee \Diamond a) \Rightarrow \Diamond a$

if $\pi[1\ldots] \mid= \Diamond a$:

$\exists j \geq 0.\ \pi[j + 1\ldots] \mid= a$

$\exists j' \geq 0.\ \pi[j'\ldots] \mid= a$

$\pi \mid= \Diamond a$

$\pi \mid= O(a \vee \Diamond a) \Rightarrow \Diamond a$

So $O(a \vee \Diamond a) \Rightarrow \Diamond a$ is valid.