4. a) ♦b ⇒ (a U b)

π|= ♦b

∃j ≥ 0. π[j...] |= b

if a = true:

∃j ≥ 0. π[j...] |= b and ∀0 ≤ i < j. π[i...] |= true

∃j ≥ 0. π[j...] |= b and ∀0 ≤ i < j. π[i...] |= a

π|= a U b

π|= ♦b ⇒ (a U b)

∃π. π|= ♦b ⇒ (a U b)

if a = ¬true:

∃j ≥ 0. π[j...] |= b and ∀0 ≤ i < j. π[i...] |≠ ¬true

¬(∃j ≥ 0. π[j...] |= b and ∀0 ≤ i < j. π[i...] |= a)

π|≠ a U b

π|≠ ♦b ⇒ (a U b)

∃π. π|≠ ♦b ⇒ (a U b)

¬(∀π. π|= ♦b ⇒ (a U b))

So ♦b ⇒ (a U b) is satisfiable, but not valid.

b) O(a ∨ ♦a) ⇒ ♦a

π|= O(a ∨ ♦a)

π[1…] |= a ∨ ♦a

if π[1…] |= a:

∃j ≥ 0. π[j...] |= a

π|= ♦a

π|= O(a ∨ ♦a) ⇒ ♦a

if π[1…] |= ♦a:

∃j ≥ 0. π[j + 1...] |= a

∃j’ ≥ 0. π[j’...] |= a

π|= ♦a

π|= O(a ∨ ♦a) ⇒ ♦a

So O(a ∨ ♦a) ⇒ ♦a is valid.