Pricing Convertible Bonds

--Value of Stochastic Interest Rate Modelling

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ABSTRACT

This study investigates the value of stochastic interest rate modelling in pricing convertible bonds. We use least square Monte Carlo simulation (LSM) by Longstaff and Schwartz (2001) with a focus on plain vanilla convertible bonds. The results show that there exist pricing differences between constant interest rate model and stochastic interest rate models. The results are statistically and economically significant for convertible bonds that have long maturity or out of the money. Results are also significant when the current interest rate deviate a lot from its long-term mean. For other situations, however, the pricing differences are only statistically significant but economically minor.

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1. Introduction

Pricing derivatives such as vanilla calls and puts using constant interest rate is not a bad choice in modelling since usually the value of those products does not depend on the choice of discounting rate model in a comprehensive manner. A constant interest rate simplifies the calculation and helps to generate succinct closed form solutions. However, convertible bond is a complicated product whose pricing depends explicitly on the dynamic of the interest rate. This raises the need to check if modelling the dynamic of interest rate is of significant value in pricing such products.

This paper follows the approach of Batten, Khaw and Young (2018), uses least square Monte Carlo simulation (LSM) by Longstaff and Schwartz (2001) with a focus on plain vanilla convertible bonds. We account for the impact of three main sources of risk in pricing convertible bonds: stock dynamics, interest rate risk and credit risk. Both Black-Scholes assumption (1973) and Heston's model (1993) are applied for simulation of stock prices. The results from both stock dynamic models consistently show that there exist pricing differences between constant interest rate model and stochastic interest rate models. The results are statistically and economically significant for convertible bonds that have long maturity or out of the money. Results are also significant when the current interest rate deviate a lot from its long-term mean. For other situations, however, the pricing differences are only statistically significant but economically minor.

This study is structured as follows: in section 2 we very briefly review related literature, then in section 3 we discuss the methodology and models under which prices are simulated, section 4 shows some restrictions on parameters when simulating the data, in section 5 we present the results of pricing difference and in section 6 we conclude.

2. Literature Review

Convertible bonds are complex products that cannot theoretically be viewed as combinations of a simple American call option on underlying firms' stock plus a straight bond. Thus, it is hard to generate a generalised closed form solution (Ingersoll 1977, Lewis 1991). As a result, there are rich literature on numeric solutions, for instance, 1). Finite difference method (Ayache et al.2003, Brennan and Schwartz 1977, Takahashi et al. 2001, Tsiveriotis and Fernandes 1998, Yigitbasioglu 2002), 2). finite element method Barone-Adesi et al. 2003), 3). tree model (Chambers and Lu 2007, Hung and Wang 2002, Yagi and Sawaki 2010) and 4). simulation model (Lvov et al. 2004, Wilde and Kind 2005). In this study, we follow Batten, Khaw and Young (2018) and use the LSM model (Longstaff and Schwartz 2001) to simulate convertible bond prices.

Regarding the value of stochastic interest rate modelling in convertible pricing, Batten, Khaw and Young (2018), Brennan and Schwartz (1980), Ammann et al. (2008) and Carayannopoulos (1996) argued that stochastic interest rate modelling is not of significant value. However, these findings are model, data and parameter restricted and Barone-Adesi et al. (2003) and Ho and Pfeffer (1996) show different results. Thus it is worthy to systematically investigate this problem.

3. Methodology

3.1 Main model--least squared Monte Carlo simulation model

The main valuation framework used in this study is Longstaff and Schwartz's (2001) least squared Monte Carlo simulation model (LSM), following Batten, Khaw and Young (2018). We choose LSM because convertible bonds are American-styled, path-dependent and multifactor-affected securities and require backward-induction in simulation-based pricing. LSM uses information (e.g. stock prices) generated by simulation to provide a conditional expected payoff at each decision node via simple regression method.

Following the notation of Longstaff and Schwartz (2001) and Batten, Khaw and Young (2018), we assume a finite time horizon *[0, T]*, where *t=0* indicates today and the maturity of the convertible bond is denoted as *t=T*. The probability space is defined as $(\Omega, \mathcal{F}, \mathbb{P})$. Assume no arbitrage, there is an equivalent martingale measure (risk-neutral measure) $\mathbb{Q}$. Ω is the set of all possible realizations ω of all the state variables (stock price, interest rate, etc.) used to price the convertible bond between time *0* to *T*. $\mathcal{F}_t$ is the sigma algebra of distinguishable events at time *t*. $\mathbb{P}$ is the probability measure corresponding to $\mathcal{F}$. Denote the convertible bond payoff as $C(\omega, s; t, T)$, $\omega \in \Omega$, $s \in (t, T)$, conditional on the holder of the convertible bond optimally exercising the converting option for all decision time *s* after time *t*. Denote the optimal exercising time as τ^*, when the converting option is executed. After τ^*, the convertible bond holder is not entitled to any future coupon and redemption payments and life of the convertible bond ends. Value of the convertible bond is different from a usual option since, in addition to the payoff of exercising (or redemption), the bond holder also has claim on all the coupons accrued over the life of this convertible bond. We assume all accrued interest is paid to the bond holder in this study. Denote the present value at time τ^* of all accrued coupon payments of this convertible bond from *0* to τ^* as $c(\tau^*)$. As a result, the total value of the convertible bond at τ^* is defined as:

$$C_{total}(\omega, \tau^*; t, T) = C(\omega, \tau^*; t, T) + c(\tau^*)$$

where $C_{total}(\omega, \tau^*; t, T)$ is the optimal payoff of the convertible bond at time τ^* subject to all boundary conditions. After deciding the optimal exercising time for all *N* simulated paths, the value of the convertible bond at time *0* can be calculated as:

$$V_0 = \frac{1}{N}\sum_{i=1}^{N} e^{-\int_0^{\tau_i^*} r(\omega_i,s)ds} C_{total}(\omega, \tau_i^*\,; t, T)$$

where V_0 is the value (or price) of the convertible bond at time 0, τ_i^* is the optimal stopping time for each path *i*, and $r(\omega_i, s)$ is the risk-free rate during $s \in (t, T)$ for path *i*.

Outcome	Payoff $C(\omega, \tau^*\,; t, T)$	Boundary condition	Time restriction
Redemption	F	$F > n_T S_T$	For $t = T$
Conversion (at maturity)	$n_T S_T$	$F < n_T S_T$	For $t = T$
Voluntary conversion	$n_{t_k} S_{t_k}$	$n_{t_k} S_{t_k} > F(\omega; t_k)$	
Continuation	0	Otherwise	

The valuation process in this study uses discrete decision time t and t belongs to a set of finite number of stopping times $t_0 \leq t_1 \leq t_2 \ldots \leq t_K$ where $t_0 = 0$ and $t_K = T$. Above table provides the payoff of a convertible bond $C(\omega, s\,; t, T)$ when a certain path ω is realised at time *s*. At maturity $t_K = T$, the payoff $C(\omega, s=T\,; t, T)$ is the maximum of the redemption value F or the conversion value $n_T S_T$, where n_T is the conversion ratio, S_T is the stock price at maturity $t=T$, and F is the face value of the bond.

At each decision time t_k where $k<K$, the convertible bond investor has the right to choose between either to convert the convertible bond into stocks of the underlying company or to hold the bond till next decision time and check again. The conversion value, denoted as $n_{t_k} S_{t_k}$ is observable at time t_k but the continuation value, denoted as $F(\omega; t_k)$ is not observable and has to be estimated.

Given the usual assumption of no arbitrage, by Fundamental Theory of Asset Pricing, the continuation value $F(\omega; t_k)$ can be expressed as expectation of future discounted cash flows under the risk neutral measure $\mathbb{Q}$. To be specific (Longstaff and Schwartz's 2001):

$$F(\omega; t_k) = \mathbb{E}^Q\Big[\sum_{j=k+1}^{K} e^{-\int_{t_k}^{t_j} r(\omega_i,s)ds}\, C(\omega, t_j\,; t_k, T)\,|\mathcal{F}_{t_k}\Big]$$

where $r(\omega_i, s)$ is the risk-free rate and the expectation is conditional on the information set $\mathcal{F}_{t_k}$.

By optimal exercising, the investor must choose the option with higher payoff. If at $t = t_k$, conversion value $n_{t_k} S_{t_k}$ is greater than continuation value $F(\omega; t_k)$, the investor should exercise the option and convert the bond into common shares of the underlying company. Otherwise, the investor should hold the bond for another period and revisit the converting decision at the next decision time $t = t_{k+1}$.

3.2 Stock dynamics

The value of a convertible bond depends on the dynamics of the underlying stock. In this study, we use stock dynamics under both 1). constant volatility model using the assumptions of the Black-Scholes-Merton model (Black and Scholes 1973, Merton 1973) and 2). stochastic volatility model, using Heston's (1993) model assumptions.

The stock dynamic under constant volatility assumptions by Black and Scholes (1973) and Merton (1973) follows geometric Brownian motion:

$$dS_t = \mu S_t dt + \sigma S_t dB_t^S$$

where S_t is the stock price at time t, dS_t is the change of stock price between *[t, t+dt]*, μ is the drift, which is simply risk-free rate under risk-neutral measure, σ is the volatility of instantaneous stock return and B_t^S is the Brownian motion of stock process. Here both μ and σ are assumed to be constant. However, under stochastic interest rate model, the B-S model dynamic under risk-neutral measure $\mathbb{Q}$ can be simply rewritten as

$$dS_t = r_t S_t dt + \sigma S_t dB_t^S$$

where r_t is now the realised interest rate observed at time t, which can be generated using a stochastic interest rate model.

The stochastic volatility model of stock dynamic adopted in this study is Heston's (1993) model, which incorporates a Cox-Ingersoll-Ross process (Cox,

Ingersoll and Ross 1985) as the dynamic of stock volatility. Under Heston's (1993) model, the dynamic of the stock and its volatility follows:

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dB_t^S$$

$$dv_t = \kappa(\bar{v} - v_t)dt + \gamma\sqrt{v_t} dB_t^v \qquad (2\kappa\bar{v} > \gamma^2)$$

$$dB_t^S dB_t^v = \rho_{sv} dt$$

where v_t is the instantaneous variance of stock return at time t, $\bar{v}$ is the long-term variance, κ is the rate v_t reverts to $\bar{v}$, γ is the volatility of volatility, B_t^S is the Brownian motion associated with stock process, B_t^v is the Brownian motion associated with variance process and ρ_{sv} is the correlation between these two Brownian motions. Here κ, $\bar{v}$, γ and ρ_{sv} are all assumed to be constant. The technical condition $2\kappa\bar{v} > \gamma^2$ ensures that variance generated by this model is strictly positive.

Batten, Khaw and Young (2018) used mainly constant elasticity of variance (CEV) model, which is the simplest extension of Black-Scholes model to include the relationship between stock movements and volatility changes associated with leverage effect (Christie 1982, Cox 1975, 1996), and GARCH (1,1) of Bollerslev (1986) to model the behaviour of stock volatility. These models require minimum calculation efforts and fit good to data. However, since this study is based on synthetic convertible bond contracts and relies on simulation method, it is easier to implement Heston's model and simulate stock paths. Further variation of results using real-world data could be conducted using either CEV or GARCH (1,1).

3.3 Interest rate models

Since the purpose of this study is to compare the use of constant and stochastic interest models in pricing convertible bond, we use 1). constant interest rate and 2). stochastic interest rate models including Vasicek's (1977) model and Cox-Ingersoll-Ross (CIR) model (Cox, Ingersoll and Ross 1985).

The Vasicek model assumes interest rate follows an Ornstein–Uhlenbeck process (Ornstein and Uhlenbeck 1930). Specifically:

$$dr_t = \lambda(\bar{r} - r_t)dt + \eta dB_t^r$$

where r_t is the instantaneous interest rate at time t, $\bar{r}$ is the long-term mean of interest rate, λ is the rate r_t reverts to $\bar{r}$, η is the volatility of volatility, B_t^r is the Brownian motion associated with interest rate process.

The CIR model assumes interest rate follows a CIR process (Cox, Ingersoll and Ross 1985). Specifically:

$$dr_t = \lambda(\bar{r} - r_t)dt + \eta\sqrt{r_t}dB_t^r \qquad (2\lambda\bar{r} > \eta^2)$$

where r_t is the instantaneous interest rate at time t, $\bar{r}$ is the long-term mean of interest rate, λ is the rate r_t reverts to $\bar{r}$, η is the volatility of volatility, B_t^r is the Brownian motion associated with interest rate process. The technical condition $2\lambda\bar{r} > \eta^2$ ensures that interest rate generated by this model is strictly positive. Compared with Vasicek model, CIR model is more robust as it constraint the interest rate to be positive. In this study, we present pricing differences between constant interest rate and stochastic interest rate using both Vasicek and CIR models. For complete specification of the model, especially the case that both stochastic volatility model and stochastic interest rate model are used, we define the following parameters:

$$dB_t^s dB_t^r = \rho_{sr}dt$$

$$dB_t^r dB_t^v = \rho_{rv}dt$$

where ρ_{sr} is the correlation between Brownian motions of stock process and interest rate process and ρ_{rv} is the correlation between Brownian motions of interest rate process and stock volatility process.

The constant interest rate used in this study is structured in two different ways. The first way is to use the long-term expectation ($\bar{r}$ in stochastic interest rate model, assumed to be 3%) as the constant interest rate. The second way is to use synthetic one-year average of realised interest rate before initiation of the convertible bond (varying from 1% to 10% depends on specification). These choices reflect two views on the interest rate that will realise during the life of the convertible bond. The first view is that the interest rate will converge back to its long-term mean regardless of current (or recent) realised rate. The second view is that the recent rate will stick for a while until

the maturity of the convertible bond. We will examine the impact of both views on the comparison between stochastic interest rate model and constant interest rate model.

3.4 Credit risk

Following Batten, Khaw and Young (2018) and Ammann, Kind and Wilde (2008), this study applies Tsiveriotis and Fernandes' (1998) model (TF model) to adjust the impact of credit risk on convertible bond pricing. The TF model simply splits payoff of a convertible bond into two components, the bond component and the stock component. The stock component is discounted using the risk-free rate since a company is always able to deliver its common share. The bond component, however, is subject to credit risk and thus should be discounted using credit risk adjusted rate. To be more specific, in this study, any cash flow, either used in the calculation of continuation value $F(\omega; t_k)$ or final value V_0, is discounted differently: cashflows from exercising the conversion option are discounted using risk-free rate since it belongs to stock component and cashflows from coupons or redemption value are discounted using credit risk adjusted rate. We assume a constant credit spread in this study and will examine how different credit spread level affects pricing differences between constant interest rate and stochastic interest rate models. Time varying credit spread is not included since it is not the focus of this study. However, in real world pricing, we can incorporate observable credit spread or CDS spread of certain underlying companies and incorporate this information into pricing.

3.5 Measure of the price difference

To check for both statistical and economic significance of applying stochastic interest rate models in convertible bond pricing, t-statistics for mean difference and percentage price differences are calculated as follows:

$$t = \frac{V_0^{constant} - V_0^{stochastic}}{\sqrt{se^{constant^2} + se^{stocastic^2}}} = \frac{V_0^{constant} - V_0^{stochastic}}{\sqrt{\frac{sd^{constant^2} + sd^{stocastic^2}}{N}}}$$

$$sd^{h^2} = \frac{1}{N}\sum_{i=1}^{N}(e^{-\int_0^{\tau_i^*} r(\omega_i,s)ds} C_{total}(\omega,\tau_i^*;t,T) - V_0)\text{^}2$$

$$percentage\ price\ difference = \frac{V_0^{constant} - V_0^{stochastic}}{V_0^{constant}}$$

where V_0^h is price calculated using either constant interest rate or stochastic interest rate, se^{h^2} is the sample standard error, sd^{h^2} is the sample variance of N simulated price of the convertible bond.

Following the discussion above in section 3.3, we perform the following tests to check for differences between the constant interest rate model and stochastic interest rate models in pricing convertible bonds:

1. Compute comparative statics. We fixed all other parameters and then change one of the parameters to check resulting prices. To be more specific, in constant interest rate model, we check pricing impact of maturity, coupon rate, credit spread, starting interest rate, volatility of the stock and moneyness; in stochastic interest rate models, we also check for correlation between stock process and interest rate process ρ_{sr}, the rate r_t reverts to its long-term mean λ and the volatility of volatility of interest rate process η under both Vasicek model and CIR model.

2. Use long-term mean of interest rate as the interest rate in constant rate model. Based on the view that the interest rate will converge back to its long-term mean regardless of current (or recent) realised rates, we first calculate convertible bonds prices using constant interest rate model in which the constant rate is fixed to be its long-term mean (3% in this study, which is an arbitrarily chosen number), then we calculate prices using stochastic interest rate models but with different time 0 interest rates and compare the differences.

3. Use current (or recent) realised rate as the interest rate in constant rate model. Based on the view that the recent rate will stick for a while until the maturity of the convertible bonds, again, we first calculate convertible bonds prices using constant interest rate model in which the constant rate ranges from 1% to 10% at 1% step, then we calculate prices using stochastic interest rate models with same time 0 interest rates as in the constant rate model ranging from 1% to 10% and compare the differences.

4. Fix both the constant interest rate and the time 0 rate of stochastic model to the long-term mean of interest rate (the arbitrarily chosen 3%) and vary maturity, coupon rate, credit spread and volatility of the stock to check if change in these parameters will affect the pricing differences between constant and stochastic models of interest rate. This test helps to identify if certain characteristics of the contract or the underlying firm's stock would affect the choice between two kinds of models since by nature of the market these characteristics vary among different convertible bonds and test in 2. and 3. focus on similar parameter setting.

We first proceed with B-S assumptions that stocks have constant volatility. Then we repeat all the tests using Heston's model to check if stochastic volatility models have an impact on the comparison between constant interest rate model and stochastic interest rate model in pricing convertible bonds.

4. Data

All data used in this study is generated using original codes in R based on synthetic convertible bond contracts. Since we restrict our attention to plain vanilla convertible bonds, a baseline contract without any other terms is characterised as follows:

Decision time per year	12
Maturity	2 years
Coupon	5% semi-annual compounding
Conversion price	100
Face value	100

We choose a monthly decision time frequency as a result of trading off between computation time and simulation standard error[1]. We restrict the conversion price to be 100, which is the same as the redemption value, implicitly defining a conversion ratio of 1, that is $n_t = 1$. This simplifies analysis on moneyness of the convertible bond. Moneyness is defined as the ratio of the current stock price over conversion price.

Most of the other parameters used in pricing function are arbitrarily chosen, for example, a constant interest rate of 3% and stock volatility of 20%. Some of the parameters have to satisfy certain technical conditions. Except for those technical conditions embedded in stochastic models mentioned above in section 3, the variance-covariance matrix of three Brownian motions corresponding to stock, interest rate and volatility process needs to be positive definite. This constrains the choice of correlation coefficients between these three random series. Based on the leverage effect of Black (1976), correlation between Brownian motions of stock and its volatility should be negative. Also, there is weak evidence that stock volatility should be highly correlated with interest rate. Thus we assume $\rho_{sv} = -0.5$ and $\rho_{rv} = 0$ throughout the whole study while changes the correlation coefficient ρ_{sr} to see how it affect pricing.

[1] The computing time using N=10,000 simulations of a baseline contract of monthly, weekly and daily decision time frequency is 0.31, 1.83 and 45.47 minutes respectively; while standard errors are 0.0119, 0.0125 and 0.0124 respectively.

5. Results and findings

In this section, we proceed as the order suggested in section 3.

5.1. Results under the B-S model of stock dynamics

The prices in this section are calculated using a base setting of parameters as: maturity of 2 years, coupon of 5% with semiannual compounding, conversion price and redemption value at 100, interest rate and credit spread of 3%, volatility of stock at 20%, stock price at time 0 at 100 (at the money), $\rho_{sv} = -0.5$ and $\rho_{rv} = 0$, $\rho_{sr} = 0.1$, the long-term mean of interest rate $\bar{r} = 3\%$, speed of interest rate conversion $\lambda = 0.9$ and instantaneous volatility of interest rate $\eta = 0.1$.

5.1.1 Comparative statics

The results for constant interest model are presented in Table-1 and Figure-1, results for stochastic interest rate models including both Vasicek and CIR models are presented in Table-2 and Figure-2.

Table-1 Comparative statics under B-S model with constant interest rate

Figure-1 Comparative statics under B-S model with constant interest rate

From Table-1 and Figure-1, we can conclude the following results.

1. Moneyness: We can see in Figure-1 that as moneyness goes from deep out of the money to deep in the money (or starting stock price S_0 goes from 20 to 200), prices exhibit a typical payoff structure of a call option. This result is consistent with the nature of a convertible bond contract that it is a call on the underlying firm's stock. When stock is performing bad, that is when the convertible bond is out of the money, investor of the convertible bond will not exercise the option to convert the bond into stock, thus keep the bond and earn on coupons and redemption value which is quite close to the face value of 100. When stock is performing good, that is when the convertible bond is in of the money, investor of the convertible bond will exercise the option to convert the bond into stock, getting the upside of this investment.

2. Maturity: From Table-1 and Figure-1, it is obvious that as maturity increases from 1 year to 10 years, the prices of the convertible bonds increase. The reason for this monotonic increase is that convertible bonds are American-styled securities. Shorter-dated American securities are actually embedded inside longer-dated securities with same contract terms. In our case, since we are doing comparative static analysis and are indeed fixing other parameters, this monotonicity should hold. Notice that as maturity increases, the 95% confidence interval becomes wider. This could be resulting from a more spread out stock price distribution as maturity increases.

3. Coupon: Figure-1 shows that as coupon rate increases, the prices of convertible bonds increase almost linearly. As coupon rate increases, the investor will receive more cash flow at each point in time when there is a coupon payment. Also since coupon amount does not affect continuation value in the backward induction process as it is a separate cash flow from the main payoff of the conversion option, it makes sense that this relationship is approximately linear.

4. Credit Spread: We can see in Figure-1 that prices decrease as credit spread increases. In this study, credit spread enters convertible bond pricing only in terms of discounting factor. Following the TF model (Tsiveriotis and Fernandes 1998), we discount all cash flows from the bond component, coupons and redemption value, of the convertible bond using credit risk adjusted rate. The risk-adjusted rate is simply risk-free interest rate plus the credit spread. Therefore, as credit spread goes up, the discount rate on coupons and redemption value also goes up. However, since continuation value depends on discounted redemption value, which depends on the discounting factor and thus the credit spread, the relationship between should not be linear between credit spread and convertible bond prices.

5. Time 0 interest rate R_0: From Table-1 and Figure-1, we can see that as interest rate increases, the prices of convertible bonds decrease, and the trend is convex. In this setting in which we use B-S assumptions and constant interest rate model, the time 0 interest rate is also the basic risk-free rate used to discount the cash flows. We can see that in this case, prices decrease with increase in interest rate with approximately a convex feature. This could be because of we focus on convertible bonds that are at the money and the bond feature still plays a significant role in pricing.

6. Stock volatility: Figure-1 shows that as stock volatility increases, prices also increase. Since convertible bonds are quite similar to a call option on the underlying firm's stock, as stock volatility goes up, the value of the option should also go up. Thus it makes sense that convertible bonds of more volatile firms worth more than those with lower stock volatility. Notably, as stock volatility goes up, the confidence interval of prices also becomes wider.

Table-2 Comparative statics under B-S model with stochastic interest rate

Figure-2 Comparative statics under B-S model with stochastic interest rate

Comparative statics of moneyness, maturity, coupon, credit spread, R_0 and volatility of stock under stochastic interest rate models, both Vasicek and CIR model, are similar to those under constant interest rate model. Therefore, we limit our attention to new parameters introduced by the use of stochastic interest rate models.

7. ρ_{sr}: From Figure-2 and Table-2, as ρ_{sr} changes from negative 0.5 to positive 0.5, the pricing impact is minor. Although, roughly speaking, prices of convertible bonds calculated with negative ρ_{sr} is on average higher than the prices with positive ρ_{sr}, the change is not significant both from statistical and economical point of view. Thus, at least for at the money convertible bonds, the impact of correlation between stock process and interest rate process on pricing is minor, contradict to Ammann, Kind and Wilde's (2008) finding.

8. $\bar{r}$: We can see from Figure-2 that prices decrease as the long-term mean of interest rate increases. The reason behind it is similar to what R_0 does in constant interest rate model. Under stochastic interest rate models, interest rate converges to its long-term mean $\bar{r}$ and leads to a higher discount rate for all cash flows if the long-term mean is higher. Thus higher $\bar{r}$ results in lower prices.

9. η: Figure-2 shows that as η, the volatility of interest rate, increases, price of convertible bonds will also increase. Similar argument as stock volatility can be used

here. Since convertible bonds are much like a call option, an increase in interest rate volatility increases the insurance value embedded in the convertible bond and thus increases the price.

5.1.2 Results using long-term mean of interest rate as the interest rate in constant rate model

The results are presented below in Table-3.

Table-3 Pricing difference under B-S assumptions using long-term mean of interest rate as the interest rate in constant rate model

The results presented in Table-3 can be concluded as follows:

Under Vasicek model:

1. For at the money convertible bonds (the column with S_0=100), constant interest rate model underprices the bonds. The underpricing is statistically significant but not economically since most of the percentage price difference are below 1%.

2. For slightly in the money convertible bonds (S_0=120, 140), most of the pricing differences are not significant, although for large differences in t=0 interest rate R_0 there are some significant underpricing by constant interest rate model.

3. For out of the money convertible bonds, when R_0<3%, constant interest rate results in significant underpricing while if R_0>3%, the result is reverted, and constant interest rate model overprices the convertible bonds. When out of the money, the convertible bond behaves similarly as a bond and as interest rate goes up, price goes down. Thus when a high R_0 is implemented in Vasicek model for out of the money convertible bonds, the resulting prices are lower and constant interest rate model over prices, vice versa.

4. For deep in the money convertible bonds, the result is mixed but most of the bonds are overpriced under constant interest rate model. Possible explanations are that

1). when interest rate incorporated in Vasicek model is higher compared to our synthetic long-term mean 3%, the discounting on cashflows under Vasicek model is higher, and 2). since under risk-neutral measure, the drift of stock is the risk-free rate generated by either interest rate models, when R_0 is smaller than 3%, cash flow from conversion is lower in Vasicek model compared with constant interest rate model.

Under CIR model:

Most of the results are the same except that

1. Pricing differences between constant and CIR model for at the money and slightly in the money convertible bonds are less significant both statistically and economically.

2. When convertible bonds are deep in the money, the pricing differences are also less significant, and the direction of the differences become more complex. The reason could be that comparing with Vasicek model, under CIR model interest rates are more behaved and revert to long-term mean more quickly. This results in a closer price under CIR model compared with constant interest rate model. We can also see this difference between Vasicek and CIR model later in the discussion of other results in the following sections.

5.1.3 Results using current (or recent) realized rates as the interest rate in constant rate model

The results are presented below in Table-4.

Table-4 Pricing difference under B-S assumptions using current (or recent) realized rates as the interest rate in constant rate model

The results presented in Table-4 can be concluded as follows:

Under Vasicek model:

1. At the money and thereabout convertible bonds are generally underpriced by constant interest rate models but not significantly. When interest rates are around the 3% long-term mean, the resulting underpricing are not significant both statistically and economically.

2. Out of the money convertible bonds again exhibit two extinct regimes. When the interest rate is high, constant interest model underprices the bonds and when the interest rate is low, Vasicek model underprices the bonds. The reasoning is similar compared with the above section: when the current (or recently realized) interest rate is higher compared to the long-term mean of interest rate, using the current level of interest rate in the constant model to price convertible bonds results in significant downward bias. If in the future interest rate does revert back to its lower long-term mean, a stochastic interest rate model would perform better than constant interest rate model by delivering more realistic discount factors.

3. For in the money convertible bonds, the results are again mixed due to the two contradicting argument provided in above section 5.1.2.

Under CIR model:

Again the results are similar compared to Vasicek model but with less significant pricing differences.

5.1.4 Results changing other parameters

Now we focus on convertible bonds priced with 3% interest rate in both constant and stochastic interest rate models since these bonds present the least significant results in the above analysis. We check if changes of other parameters in the contract would affect the pricing difference between two kinds of models. Results are presented in Table-5:

Table-5 Results changing other parameters under B-S model

From Table-5, one can see that under Vasicek model:

1. When changing maturity form 1 year to 10 years, we can observe a clearly magnified underpricing by constant model for all moneyness. The reasoning behind this could be that as maturity becomes longer, there is more uncertainty involved in Vasicek model and by chance, there could be relatively large deviation of interest rate from its long-term mean. This increases the insurance value embedded in the convertible bond's conversion option while constant interest rate model fails to capture the value from volatility in time-varying interest rate into calculation of convertible bonds' prices.

2. Changes in coupon rate, credit spread and stock volatility do not induce more serious pricing difference between constant interest rate model and Vasicek model for at the money and out of the money convertible bonds but do induce pricing differences for deep in the money convertible bonds. However, almost all results are statistically significant.

This induces the problematic part of results analysis for this study: statistically significance is seemingly pseudo because the standard errors for prices are really small. The number of simulation N sufficiently reduces the standard error such that even a really small difference in pricing is still statistically significant. While there is no conventional guideline for economic significance, we have to rely on our subjective judgment on the significance of the results.

Under CIR model, we have similar findings that the pricing difference between constant and CIR model are statistically but not economically significant. Even for maturity which we find impact on pricing difference in Vasicek model, the results under CIR model are not significant, except for deep in the money convertible bonds which we can still find some significant results.

In conclusion, Under B-S assumption of stock dynamics, pricing differences between constant and stochastic interest rate models are large for out of the money or longer-dated convertible bonds; results for at the money bonds are usually not significant and mixed for in the money bonds.

5.2. Results under Heston's model of stock dynamics

The prices in this section are calculated using a base setting of parameters as: maturity of 2 years, coupon of 5% with semiannual compounding, conversion price and redemption value at 100, interest rate and credit spread of 3%, volatility of stock at 20%, stock price at time 0 at 100 (at the money), $\rho_{sv} = -0.5$ and $\rho_{rv} = 0$, $\rho_{sr} = 0.1$, the long-term mean of interest rate $\bar{r} = 3\%$, speed of interest rate conversion $\lambda = 0.9$ and instantaneous volatility of interest rate $\eta = 0.1$. The Hestons' model parameter specifications are: the rate v_t reverts to its mean $\kappa = 0.5$, $\bar{v} = 0.04$ and volatility of volatility $\gamma = 0.1$.

5.2.1 Comparative statics

The results for constant interest rate model are presented in Table-6 and Figure-3, results for stochastic interest rate models including both Vasicek and CIR models are presented in Table-7 and Figure-4.

Table-6 Comparative statics under Heston's model with constant interest rate

Figure-3 Comparative statics under Heston's model with constant interest rate

Table-7 Comparative statics under Heston's model with stochastic interest rate

Figure-4 Comparative statics under Heston's model with stochastic interest rate

The comparative statics analysis shows almost the same results as in section 5.1.1., except that for both Vasicek model and CIR model, changing R_0 does not have much impact on pricing. Possible explanation is that interest rates convert quickly to its long-term mean under both models and thus prices do no change much.

5.2.2 Results using long-term mean of interest rate as the interest rate in constant rate model

The results are presented below in Table-8.

Table-8 Pricing difference under Heston's model using long-term mean of interest rate as the interest rate in constant rate model

The results presented in Table-8 are similar to results presented in Table-3 above, except that now for at the money and slightly in the money convertibles bonds the pricing difference between constant interest rate model and stochastic interest rate model are also statistically significant.

5.2.3 Results using current (or recent) realized rates as the interest rate in constant rate model

The results are presented below in Table-9.

Table-9 Pricing difference under Heston's model using current (or recent) realized rates as the interest rate in constant rate model

From Table-9, we can conclude similar results as we did for analysis about Table-4. However, now most of the pricing differences are now economically insignificant since most of them are less than 1%. A large enough difference between the current interest rate and the long-term mean rate is required for economical significant pricing difference.

5.2.4 Results changing other parameters

Table-10 Results changing other parameters under Heston's model

Table-10 shows similar results comparing with Table-5 that most of the pricing difference between constant interest rate model and stochastic interest rate model are statistically significant but economically significant, except for the case of large maturity in Vasicek model where pricing difference is economically significant.

Thus, through the above analysis, using the stochastic volatility model of stock volatility: 1). does change prices as one can see from Table-1 to Table-10, prices calculated under the same parameter setting but different stock volatility models are different; 2). more importantly, confirms the finding under B-S assumption about the pricing difference of convertible bonds between constant interest rate models and stochastic interest rate models.

6. Conclusions

Following Batten, Khaw and Young (2018), this study examines pricing of plain vanilla convertible bonds using Longstaff and Schwartz's (2001) least squared Monte Carlo simulation model between constant and stochastic (Vasicek and CIR) interest rate models. The main findings of this study are that 1). for current interest rate deviating a lot form its long-term mean, 2). for convertible bonds that are out of the money and 3). for contracts with long maturity, the pricing difference between constant and stochastic interest rate models are statistically and economically significant. For other cases, the pricing differences are statistically but not economically significant.

Potential future research could look at for example how a stochastic credit spread could affect the choice of interest rate modelling in convertible bond pricing.

Appendix—Tables, Figures and Codes

Table-1 Comparative statics under B-S model with constant interest rate

Maturity	95%up	Price	95%low
1	111.99	111.82	111.64
2	116.68	116.41	116.14
3	120.57	120.21	119.85
4	122.86	122.44	122.01
5	126.31	125.79	125.28
6	129.21	128.61	128.01
7	132.41	131.72	131.03
8	133.30	132.55	131.79
9	136.25	135.45	134.66
10	138.60	137.71	136.81

Coupon	95%up	Price	95%low
1	108.82	108.50	108.17
2	110.79	110.48	110.16
3	112.75	112.45	112.16
4	114.72	114.43	114.15
5	116.68	116.41	116.14
6	118.65	118.39	118.13
7	120.62	120.37	120.12
8	122.59	122.35	122.11
9	124.55	124.32	124.10
10	126.52	126.30	126.08

Credit Spread	95%up	Price	95%low
1	118.76	118.51	118.26
2	117.71	117.45	117.19
3	116.68	116.41	116.14

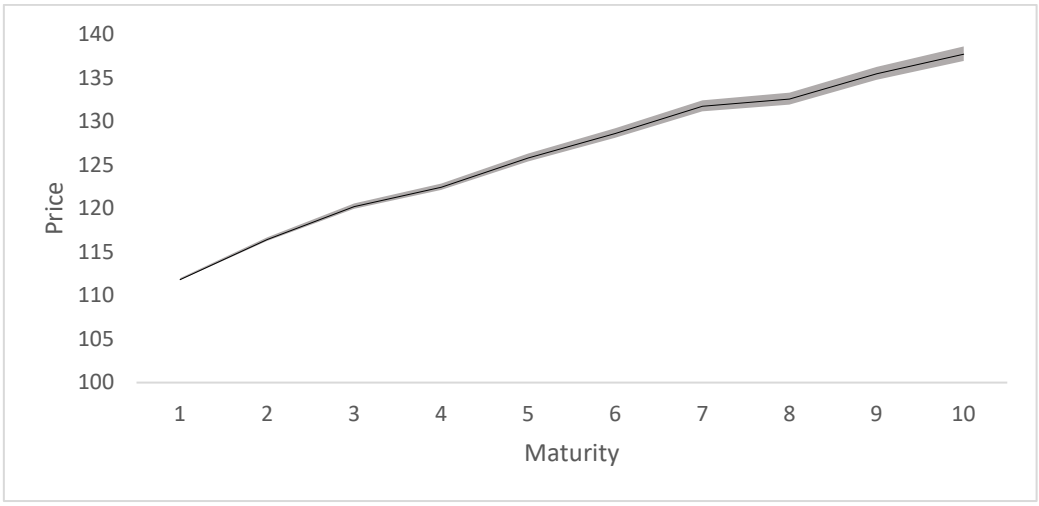
4	115.61	115.33	115.04
5	114.60	114.30	114.01
6	113.25	112.95	112.65
7	112.19	111.88	111.58
8	111.02	110.71	110.40
9	109.71	109.40	109.09
10	107.79	107.49	107.18

R0	95%up	Price	95%low
1	118.00	117.76	117.51
2	117.27	117.02	116.76
3	116.68	116.41	116.14
4	116.10	115.81	115.53
5	115.59	115.29	114.99
6	115.17	114.86	114.54
7	114.84	114.51	114.18
8	114.60	114.26	113.91
9	114.14	113.78	113.42
10	114.12	113.74	113.37

Sigma	95%up	Price	95%low
0.03	103.49	103.36	103.24
0.06	105.12	105.08	105.03
0.09	108.99	108.85	108.71
0.12	111.63	111.47	111.30
0.15	113.60	113.40	113.20
0.18	115.45	115.21	114.97
0.21	117.25	116.96	116.67
0.24	118.97	118.63	118.30
0.27	120.80	120.41	120.03
0.3	122.60	122.16	121.72

Moneyness	95%up	Price	95%low
10	102.97	102.97	102.97
20	102.97	102.97	102.97
30	102.97	102.97	102.97
40	102.98	102.98	102.97
50	103.05	103.03	103.02
60	103.35	103.31	103.28
70	104.44	104.36	104.28
80	106.88	106.74	106.60
90	110.92	110.72	110.51
100	116.68	116.41	116.14
110	123.62	123.28	122.95
120	131.27	130.89	130.51
130	139.67	139.26	138.84
140	148.17	147.73	147.28
150	157.45	156.98	156.51
160	163.78	163.10	162.42
170	173.31	172.62	171.94
180	185.10	184.31	183.53
190	194.84	194.01	193.19
200	211.86	210.85	209.83

Figure-1 Comparative statics under B-S model with constant interest rate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Maturity


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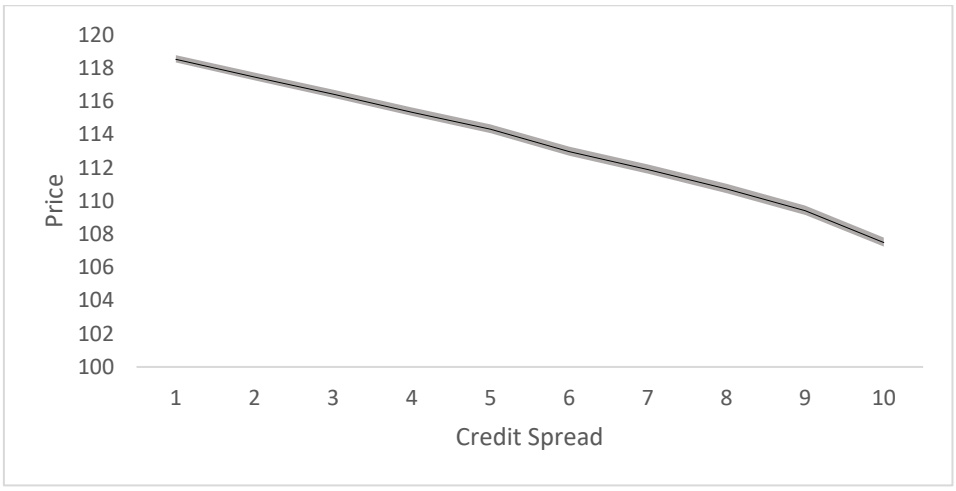
Coupon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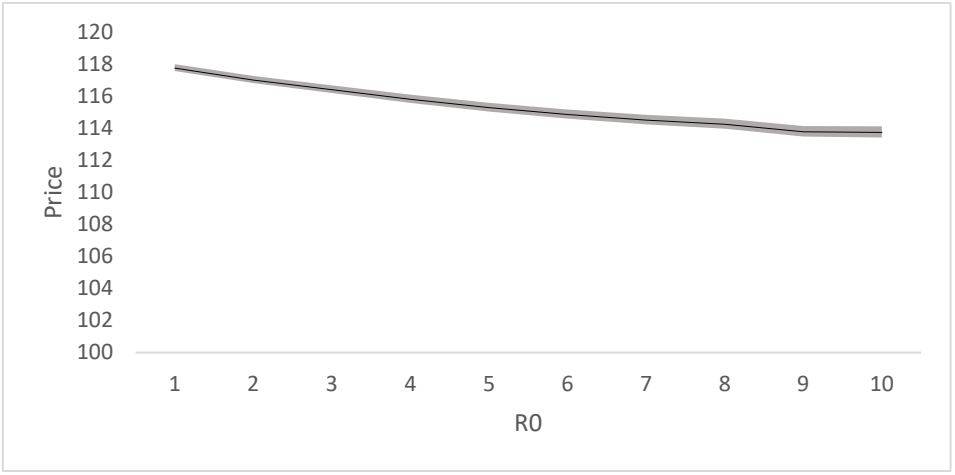
Credit Spread



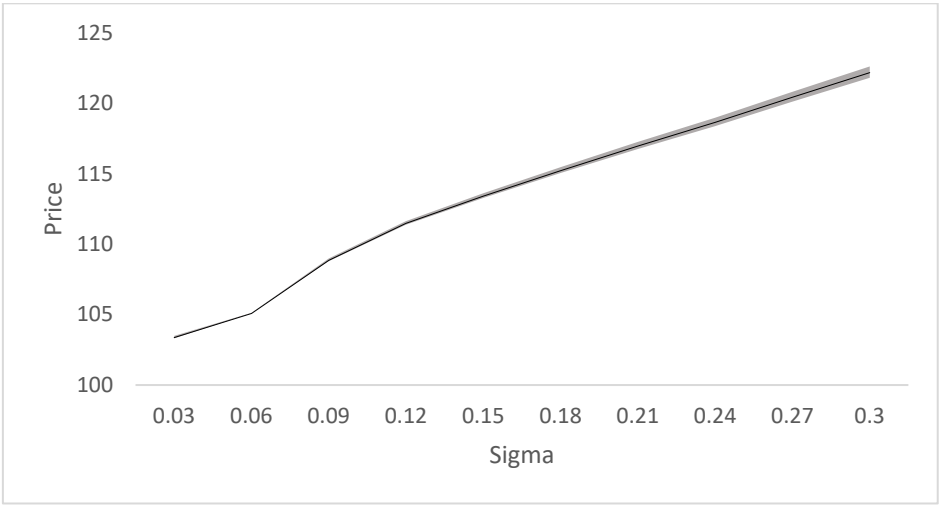


Table-2 Comparative statics under B-S model with stochastic interest rate Vasicek:

Maturity	95%up	Price	95%low
1	112.23	112.06	111.88
2	117.87	117.58	117.29
3	122.05	121.67	121.29
4	125.52	125.05	124.58
5	131.35	130.76	130.16

6	134.78	134.09	133.41
7	139.97	139.17	138.37
8	139.89	139.00	138.10
9	146.06	145.08	144.10
10	152.30	151.19	150.08

Coupon	95%up	Price	95%low
1	109.93	109.59	109.26
2	111.91	111.59	111.27
3	113.89	113.59	113.28
4	115.88	115.58	115.28
5	117.87	117.58	117.29
6	119.85	119.57	119.29
7	121.84	121.57	121.30
8	123.83	123.57	123.30
9	125.82	125.56	125.30
10	127.81	127.56	127.30

Credit Spread	95%up	Price	95%low
1	120.09	119.81	119.54
2	119.01	118.73	118.45
3	117.87	117.58	117.29
4	116.72	116.42	116.12
5	115.67	115.36	115.06
6	114.46	114.15	113.84
7	113.29	112.97	112.65
8	112.08	111.76	111.43
9	110.80	110.47	110.14
10	109.20	108.87	108.54

R0	95%up	Price	95%low
1	118.31	118.03	117.76
2	118.13	117.84	117.56
3	117.87	117.58	117.29
4	117.72	117.42	117.12
5	117.42	117.12	116.81
6	117.40	117.09	116.78
7	117.26	116.94	116.63
8	117.21	116.89	116.56
9	117.08	116.75	116.42
10	116.99	116.65	116.31

Sigma	95%up	Price	95%low
0.03	107.82	107.63	107.44
0.06	109.77	109.60	109.42
0.09	111.55	111.37	111.19
0.12	113.24	113.05	112.85
0.15	114.96	114.73	114.51
0.18	116.69	116.42	116.16
0.21	118.57	118.27	117.97
0.24	120.16	119.81	119.46
0.27	121.85	121.45	121.06
0.3	123.57	123.12	122.68

Moneyness	95%up	Price	95%low
10	103.48	103.31	103.14
20	103.48	103.31	103.14
30	103.48	103.31	103.14
40	103.49	103.32	103.15
50	103.60	103.43	103.26
60	104.08	103.91	103.74

70	105.29	105.12	104.94
80	108.05	107.85	107.66
90	112.21	111.98	111.74
100	117.87	117.58	117.29
110	124.45	124.11	123.76
120	131.47	131.08	130.70
130	139.80	139.36	138.93
140	148.26	147.79	147.32
150	157.08	156.58	156.07
160	162.45	161.98	161.51
170	172.77	172.24	171.71
180	185.51	184.74	183.97
190	191.89	191.17	190.45
200	205.66	204.75	203.85

Rhosr	95%up	Price	95%low
-0.5	118.14	117.80	117.47
-0.4	117.59	117.28	116.96
-0.3	118.13	117.81	117.49
-0.2	118.08	117.77	117.46
-0.1	118.05	117.74	117.44
0	117.49	117.20	116.91
0.1	117.87	117.58	117.29
0.2	117.71	117.43	117.15
0.3	117.67	117.40	117.12
0.4	116.98	116.72	116.45
0.5	117.51	117.24	116.98

lambda	95%up	Price	95%low
0.5	118.43	118.14	117.84
0.6	118.31	118.01	117.72
0.7	118.10	117.81	117.51

0.8	118.06	117.77	117.47
0.9	117.87	117.58	117.29
1	117.73	117.44	117.15
1.1	117.66	117.38	117.09
1.2	117.65	117.36	117.08
1.3	117.60	117.32	117.03
1.4	117.53	117.25	116.97
1.5	117.48	117.20	116.92

rbar	95%up	Price	95%low
1	118.89	118.61	118.33
2	118.41	118.13	117.84
3	117.87	117.58	117.29
4	117.42	117.12	116.83
5	116.90	116.60	116.30
6	116.54	116.24	115.94
7	116.11	115.80	115.50
8	115.78	115.47	115.16
9	115.28	114.96	114.64
10	114.91	114.59	114.26

eta/volr	95%up	Price	95%low
0.02	116.68	116.41	116.14
0.04	116.87	116.60	116.33
0.06	117.11	116.83	116.55
0.08	117.48	117.20	116.92
0.1	117.87	117.58	117.29
0.12	118.34	118.04	117.74
0.14	118.98	118.67	118.36
0.16	119.65	119.33	119.01
0.18	120.35	120.01	119.67
0.2	121.06	120.71	120.36

CIR:

Maturity	95%up	Price	95%low
1	111.98	111.81	111.64
2	116.65	116.38	116.11
3	120.44	120.08	119.72
4	122.85	122.42	122.00

5	126.42	125.90	125.39
6	129.52	128.92	128.31
7	133.17	132.48	131.78
8	129.92	129.17	128.42
9	135.59	134.80	134.01
10	139.10	138.20	137.30

Coupon	95%up	Price	95%low
1	108.82	108.49	108.17
2	110.78	110.47	110.16
3	112.73	112.44	112.14
4	114.69	114.41	114.13
5	116.65	116.38	116.11
6	118.61	118.35	118.09
7	120.57	120.33	120.08
8	122.54	122.30	122.06
9	124.50	124.27	124.04
10	126.46	126.24	126.02

Credit Spread	95%up	Price	95%low
1	118.74	118.49	118.25
2	117.65	117.40	117.14
3	116.65	116.38	116.11
4	115.61	115.33	115.05
5	114.58	114.29	114.00
6	113.33	113.03	112.73
7	112.29	111.98	111.67
8	111.18	110.87	110.55
9	109.90	109.59	109.27
10	108.12	107.81	107.50

R0	95%up	Price	95%low
1	116.96	116.71	116.45
2	116.80	116.54	116.28
3	116.65	116.38	116.11
4	116.49	116.21	115.93
5	116.37	116.08	115.79
6	116.27	115.97	115.67
7	116.17	115.86	115.55

8	116.14	115.82	115.51
9	116.01	115.69	115.36
10	115.96	115.63	115.29

Sigma	95%up	Price	95%low
0.03	101.55	101.42	101.29
0.06	106.76	106.68	106.60
0.09	109.40	109.26	109.12
0.12	111.67	111.50	111.34
0.15	113.59	113.38	113.18
0.18	115.51	115.26	115.02
0.21	117.26	116.97	116.68
0.24	119.05	118.72	118.38
0.27	120.96	120.57	120.18
0.3	122.78	122.34	121.90

Moneyness	95%up	Price	95%low
10	103.01	102.98	102.95
20	103.01	102.98	102.95
30	103.01	102.98	102.95
40	103.01	102.98	102.95
50	103.07	103.04	103.00
60	103.38	103.33	103.28
70	104.46	104.37	104.29
80	106.89	106.74	106.60
90	110.98	110.77	110.57
100	116.65	116.38	116.11
110	123.57	123.24	122.91
120	131.45	131.06	130.68
130	139.57	139.16	138.74
140	147.36	146.93	146.50
150	156.40	155.95	155.49
160	162.78	162.22	161.67
170	170.74	170.13	169.52
180	185.86	185.06	184.27
190	194.64	193.81	192.98
200	213.34	212.32	211.31

Rhosr	95%up	Price	95%low

-0.5	116.67	116.39	116.11
-0.4	116.17	115.90	115.63
-0.3	116.68	116.40	116.13
-0.2	116.64	116.37	116.09
-0.1	116.70	116.43	116.15
0	116.42	116.15	115.88
0.1	116.65	116.38	116.11
0.2	116.59	116.32	116.06
0.3	116.59	116.32	116.05
0.4	116.08	115.82	115.56
0.5	116.69	116.42	116.15

lambda	95%up	Price	95%low
0.5	116.63	116.36	116.09
0.6	116.65	116.38	116.11
0.7	116.65	116.38	116.11
0.8	116.65	116.38	116.11
0.9	116.65	116.38	116.11
1	116.65	116.38	116.11
1.1	116.66	116.39	116.12
1.2	116.65	116.38	116.11
1.3	116.66	116.39	116.12
1.4	116.69	116.42	116.15
1.5	116.68	116.41	116.14

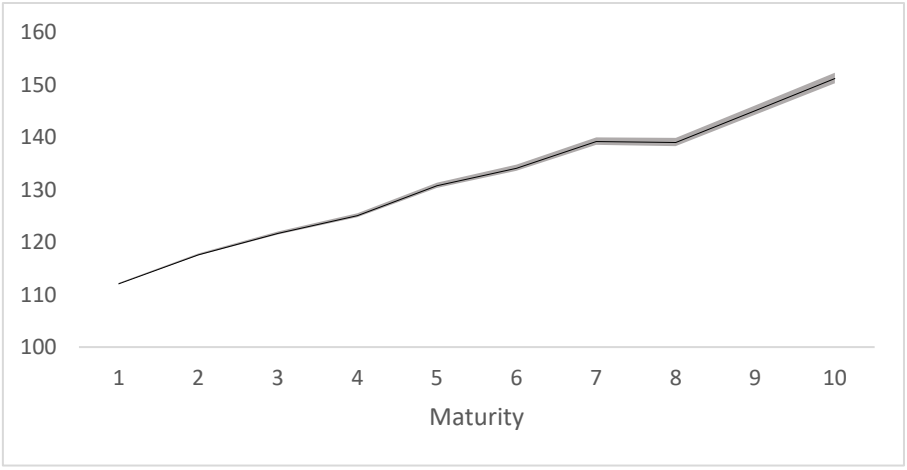
rbar	95%up	Price	95%low
1	117.51	117.25	116.99
2	117.09	116.82	116.56
3	116.65	116.38	116.11
4	116.24	115.97	115.69
5	115.83	115.55	115.26
6	115.40	115.12	114.83
7	115.09	114.80	114.50
8	114.66	114.36	114.06
9	114.31	114.01	113.70
10	113.95	113.63	113.32

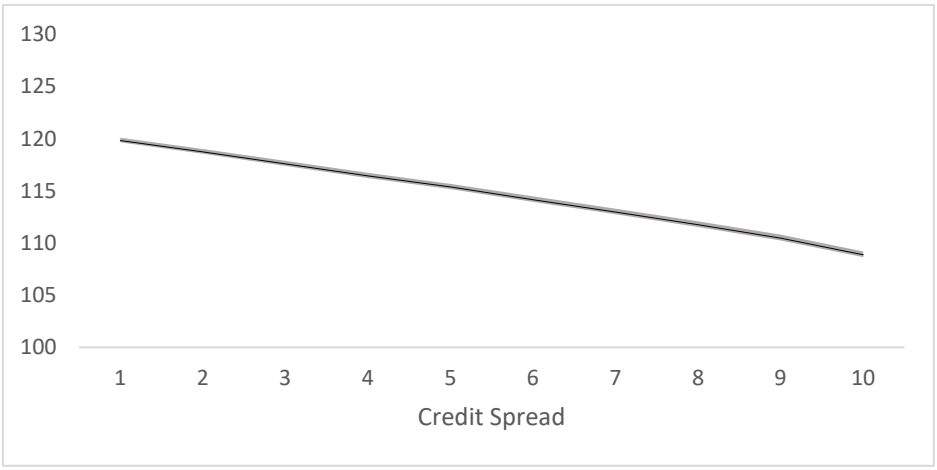
eta/volr	95%up	Price	95%low
0.02	116.69	116.42	116.15

0.04	116.71	116.44	116.16
0.06	116.68	116.41	116.14
0.08	116.66	116.39	116.12
0.1	116.65	116.38	116.11
0.12	116.66	116.39	116.12
0.14	116.69	116.42	116.15
0.16	116.73	116.45	116.18
0.18	116.77	116.50	116.22
0.2	116.76	116.49	116.21

Figure-2 Comparative statics under B-S model with stochastic interest rate

Vasicek:

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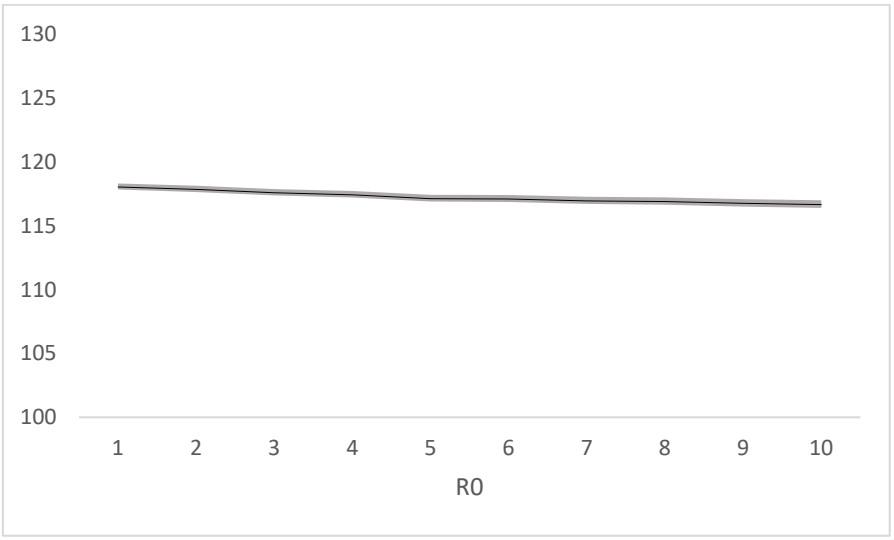
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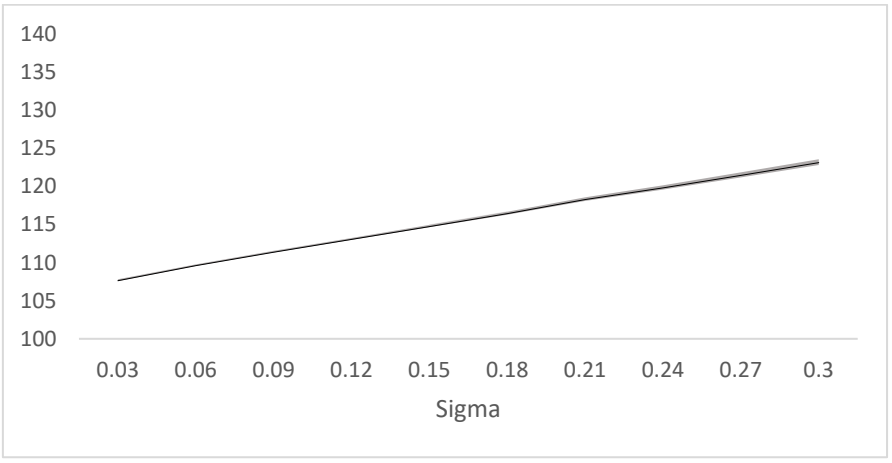
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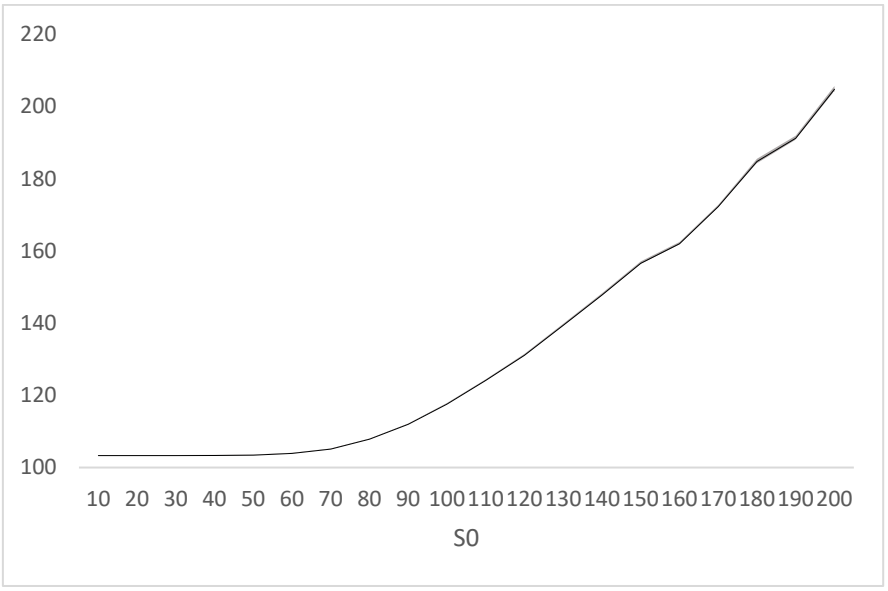
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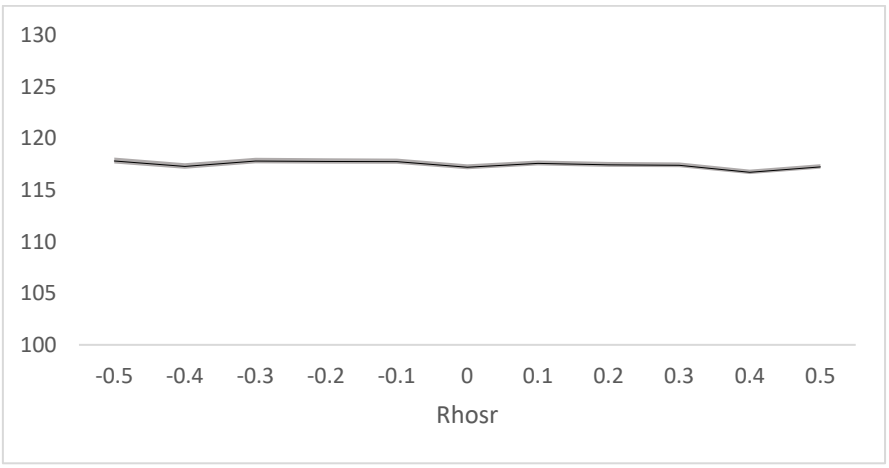
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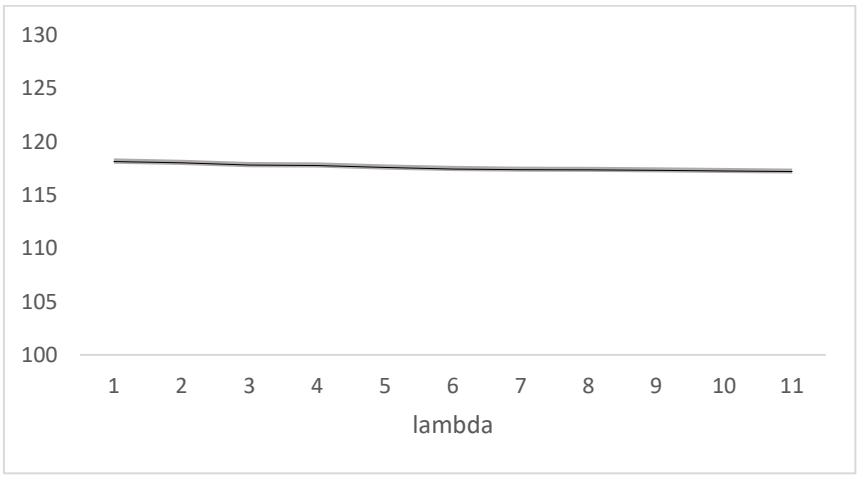
Rhosr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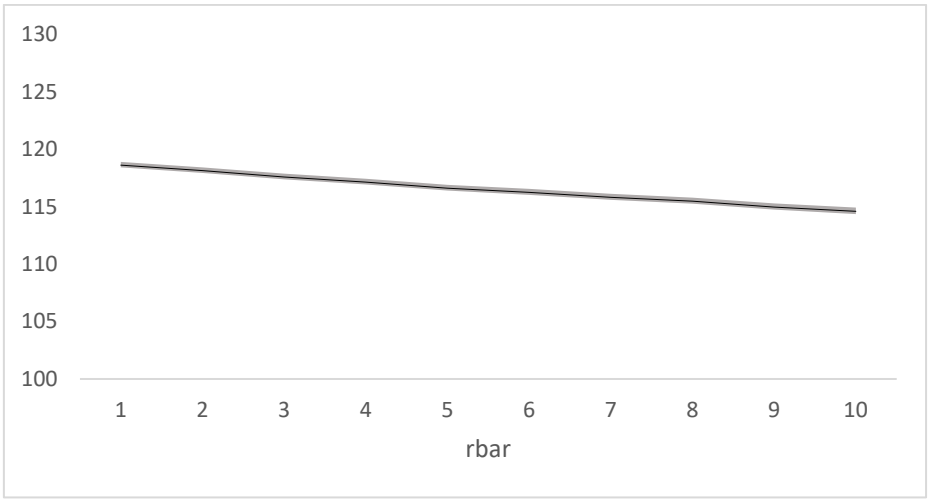
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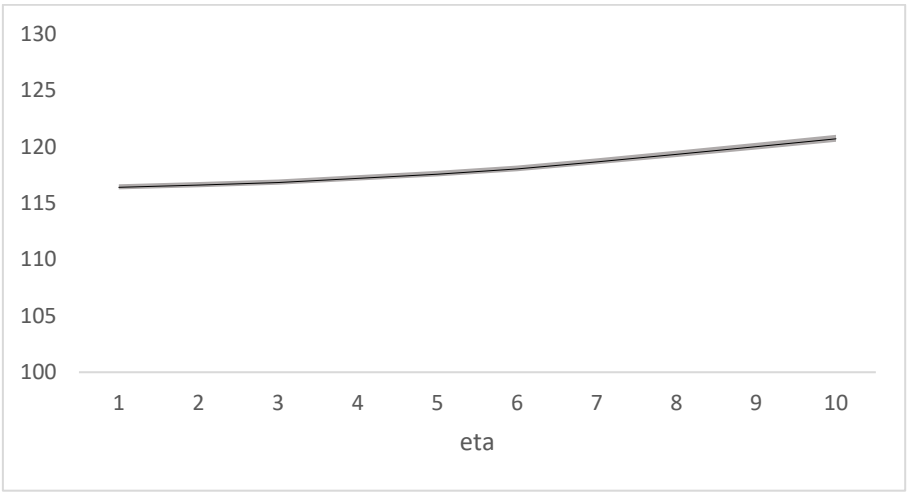
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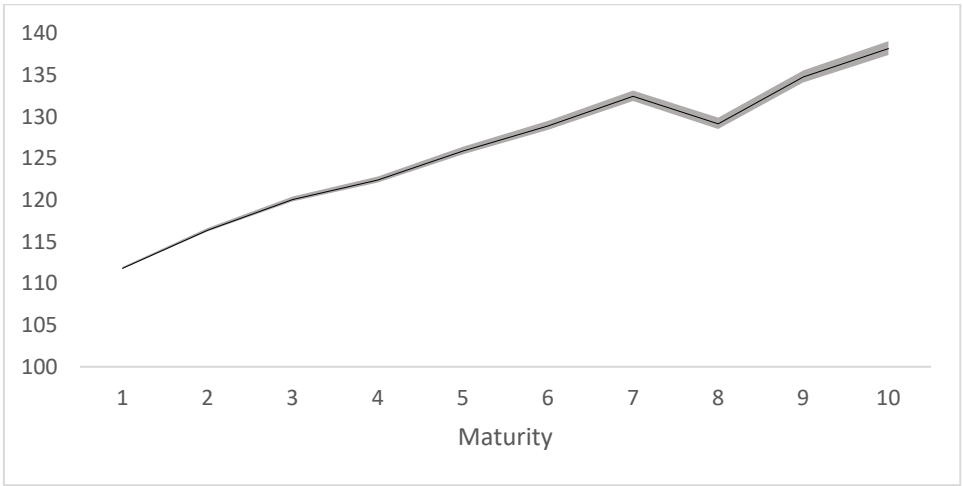
CIR: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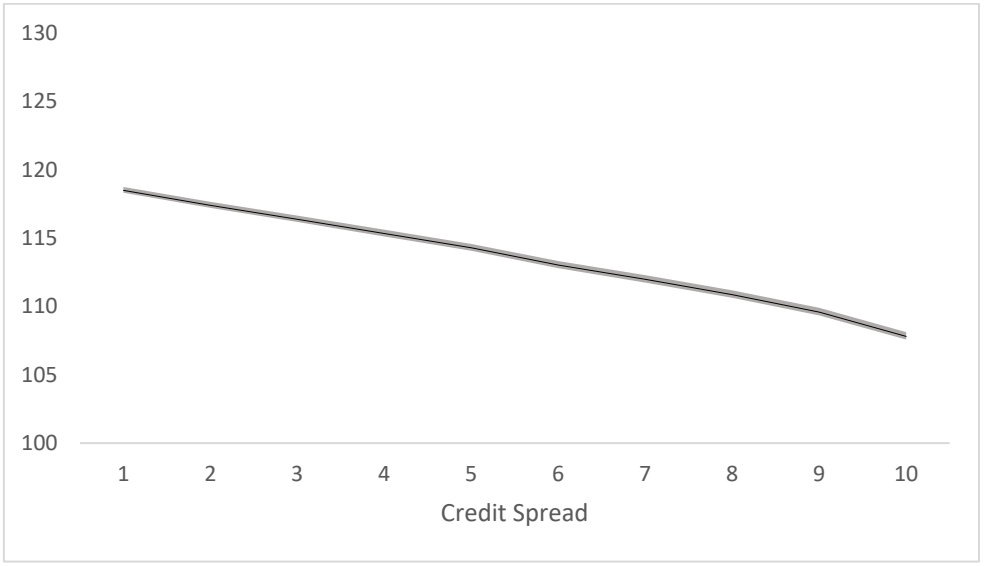
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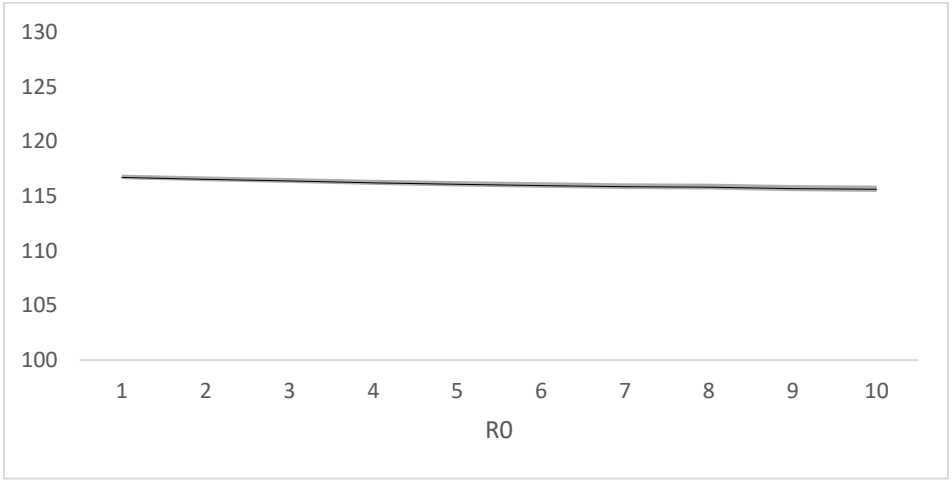
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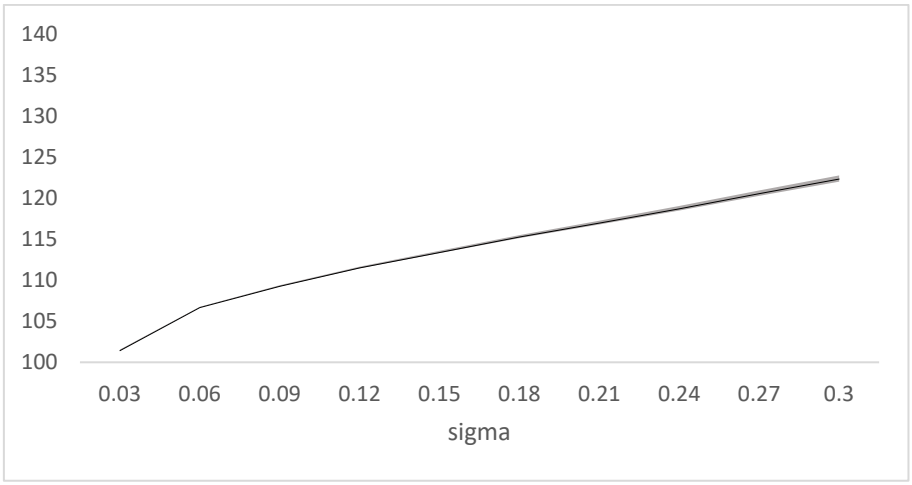
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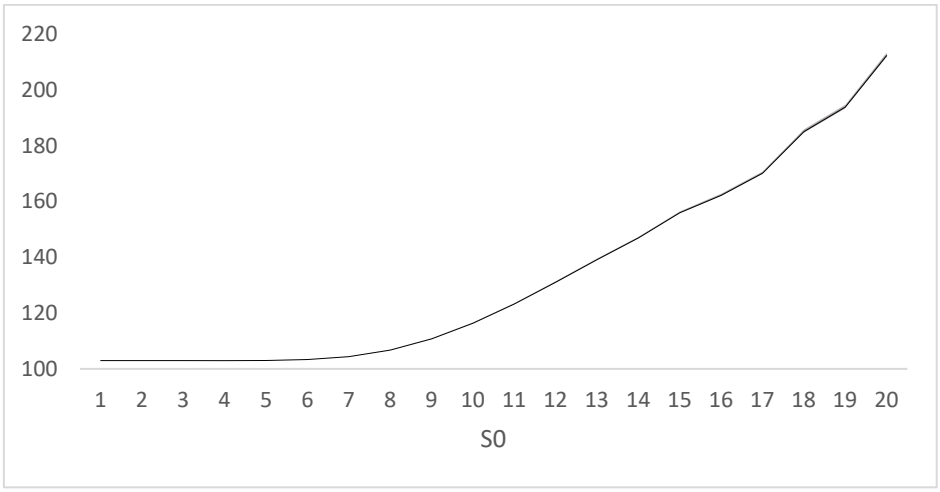
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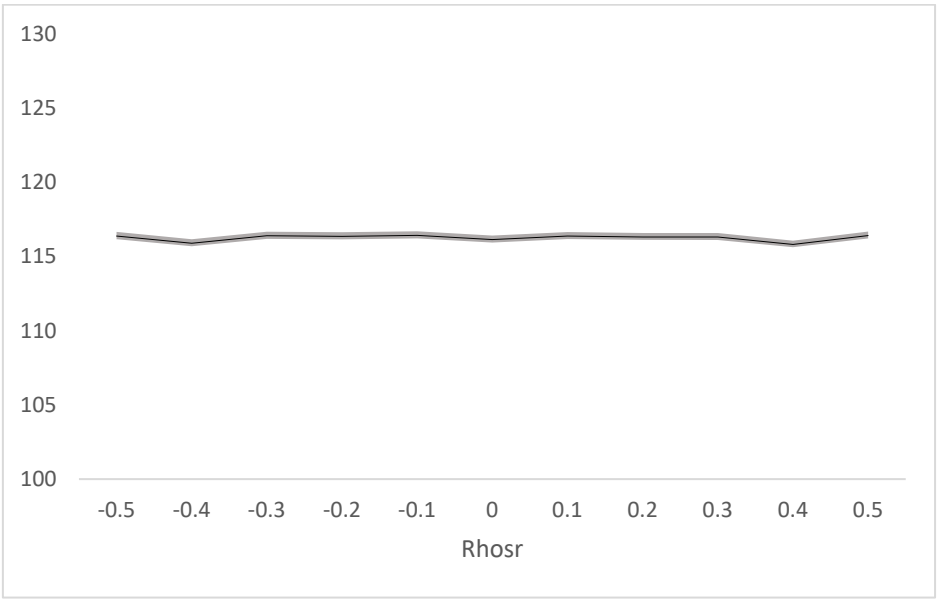
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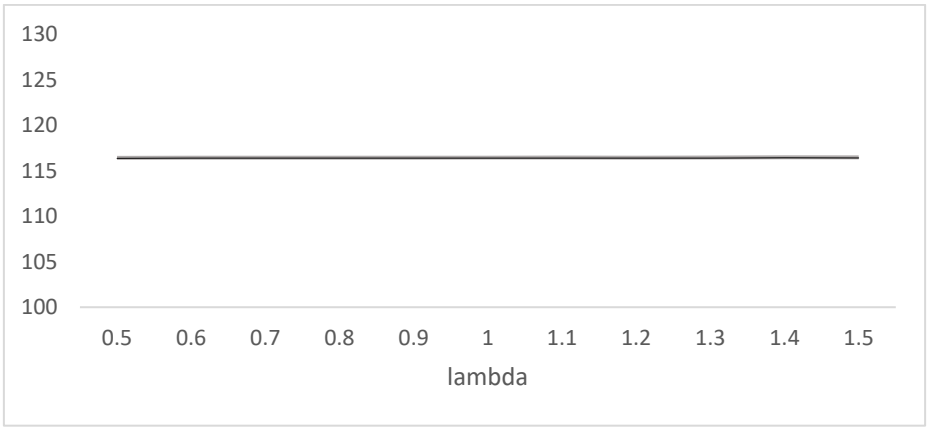
0.4
0.5
Rhosr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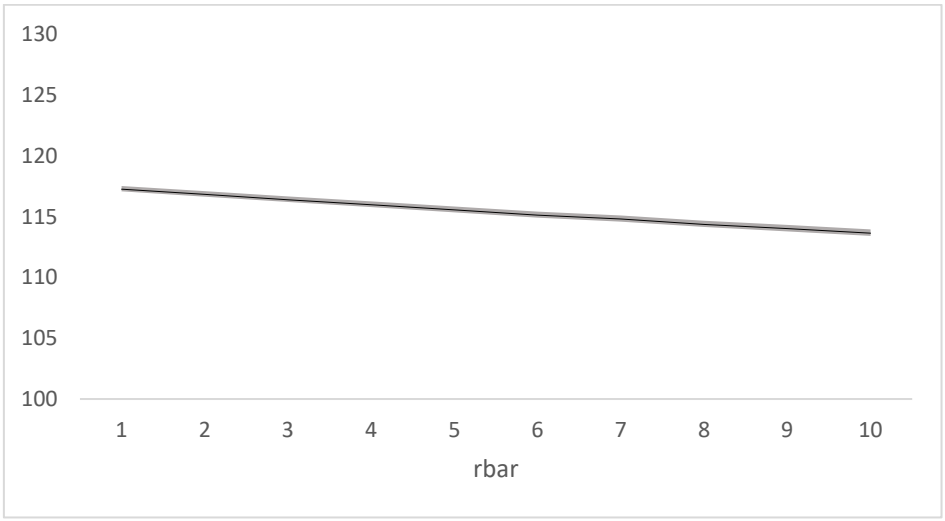
lambda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eta

Table-3 Pricing difference under B-S assumptions using long-term mean of interest rate as the interest rate in constant rate model

Panel A: Convertible bond price under constant interest rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
Long-term r										
3%	102.97	102.98	103.31	106.74	116.41	130.89	147.73	163.10	184.31	210.85
Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	-0.021 ***	-0.021 ***	-0.022 ***	-0.021 ***	-0.014 ***	-0.003	0.001	0.059 ***	-0.019 ***	0.033 ***
2%	-0.012 ***	-0.012 ***	-0.014 ***	-0.015 ***	-0.012 ***	-0.003	0.000	0.056 ***	0.034 ***	0.040 ***
3%	-0.003 ***	-0.003 ***	-0.006 ***	-0.010 ***	-0.010 ***	-0.001	0.000	0.064 ***	-0.002	0.029 ***
4%	0.005 ***	0.005 ***	0.002 ***	-0.006 ***	-0.009 ***	-0.002	-0.002	0.066 ***	-0.004	0.026 ***
5%	0.014 ***	0.014 ***	0.010 ***	-0.001	-0.006 ***	-0.003	0.000	0.077 ***	0.001	0.020 ***
6%	0.023 ***	0.022 ***	0.017 ***	0.004 ***	-0.006 ***	-0.003	0.002	0.116 ***	0.031 ***	0.015 ***
7%	0.031 ***	0.031 ***	0.025 ***	0.008 ***	-0.005 ***	-0.005 ***	0.001	0.063 ***	0.021 ***	0.010 ***
8%	0.040	0.039	0.032	0.013	-0.004	-0.007	0.000	0.136	0.043	0.012

	***	***	***	***	**	***		***	***	***
9%	0.048	0.048	0.039	0.016	-0.003	-0.006	-0.007	0.039	0.036	0.019
	***	***	***	***		***	***	***	***	***
10%	0.056	0.056	0.047	0.020	-0.002	-0.006	0.003	0.080	0.001	0.021
	***	***	***	***		***		***		***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	-0.018	-0.018	-0.017	-0.011	-0.003	0.000	0.005	0.042	0.018	0.028
	***	***	***	***			**	***	***	***
2%	-0.009	-0.009	-0.008	-0.005	-0.001	-0.002	0.004	0.011	-0.011	0.000
	***	***	***	***			*	***	***	
3%	0.000	0.000	0.000	0.000	0.000	-0.001	0.005	0.063	-0.004	-0.007
							***	***		**
4%	0.009	0.009	0.008	0.005	0.002	-0.003	0.006	0.000	0.014	-0.013
	***	***	***	***			***		***	***
5%	0.017	0.017	0.016	0.009	0.003	-0.003	0.006	0.071	0.009	0.020
	***	***	***	***			***	***	***	***
6%	0.026	0.026	0.023	0.014	0.004	-0.003	0.003	0.079	0.012	0.020
	***	***	***	***	**			***	***	***
7%	0.034	0.034	0.031	0.018	0.005	-0.003	-0.004	0.087	0.014	-0.005
	***	***	***	***	***		*	***	***	
8%	0.043	0.042	0.038	0.023	0.005	-0.007	-0.007	0.069	-0.035	-0.037
	***	***	***	***	***	***	***	***	***	***
9%	0.051	0.051	0.045	0.027	0.006	-0.007	0.001	0.076	0.003	-0.045
	***	***	***	***	***	***		***		***

10%	0.059	0.059	0.052	0.031	0.007	-0.005	-0.006	0.090	-0.030	0.013
	***	***	***	***	***	**	***	***	***	***

Table-4 Pricing difference under B-S assumptions using current (or recent) realized rates as the interest rate in constant rate model

Panel A: Convertible bond price under constant interest rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	106.83	106.83	107.01	109.55	117.76	131.47	148.41	169.85	184.20	206.38
2%	104.88	104.88	105.12	108.08	117.02	131.19	148.08	170.49	185.20	206.81
3%	102.97	102.98	103.31	106.74	116.41	130.89	147.73	173.10	184.31	210.85
4%	101.11	101.11	101.55	105.49	115.81	130.77	147.67	160.05	181.22	209.69
5%	99.27	99.28	99.87	104.28	115.29	129.91	146.82	165.61	185.76	216.45
6%	97.47	97.49	98.24	103.16	114.86	129.88	146.27	152.88	182.41	211.53
7%	95.71	95.74	96.68	102.18	114.51	130.10	147.62	166.89	184.22	205.43
8%	93.98	94.02	95.17	101.26	114.26	129.99	144.14	164.30	191.02	220.68
9%	92.29	92.33	93.70	100.50	113.78	129.75	145.69	159.06	199.99	217.80
10%	90.63	90.68	92.28	99.74	113.74	129.00	151.43	151.01	200.16	227.12
Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	0.016	0.016	0.013	0.005	-0.002	0.001	0.005	0.041	-0.020	0.012
	***	***	***	***			***	***	***	***

2%	0.006 ***	0.006 ***	0.004 ***	-0.003 ***	-0.007	0.000	0.003 **	0.041 ***	0.038 ***	0.022 ***
3%	-0.003 ***	-0.003 ***	-0.006 ***	-0.010 ***	-0.010	-0.001	0.000 *	0.064 ***	-0.002	0.029 ***
4%	-0.013 ***	-0.013 ***	-0.015 ***	-0.018 ***	-0.014	-0.003	-0.002	-0.010 ***	-0.022 *	0.021
5%	-0.023 ***	-0.023 ***	-0.024 ***	-0.024 ***	-0.016 ***	-0.010	-0.006 **	0.035 ***	0.009	0.046
6%	-0.032 ***	-0.033 ***	-0.033 ***	-0.031 ***	-0.019 ***	-0.011	-0.008 ***	0.000 ***	0.021 ***	0.018 *
7%	-0.042 ***	-0.042 ***	-0.042 ***	-0.036 ***	-0.021 ***	-0.011	0.000 ***	0.028 ***	0.021 ***	-0.017 ***
8%	-0.052 ***	-0.052 ***	-0.051 ***	-0.041 ***	-0.023 ***	-0.014	-0.025 *	0.090 ***	0.077 ***	0.056 ***
9%	-0.062 ***	-0.062 ***	-0.059 ***	-0.045 ***	-0.026 ***	-0.015	-0.021	-0.045 ***	0.112 ***	0.051
10%	-0.072 ***	-0.072 ***	-0.067 ***	-0.049 ***	-0.026 ***	-0.021	0.028 ***	-0.054 ***	0.080	0.091

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	0.019 ***	0.019 ***	0.018 ***	0.015 ***	0.009 ***	0.005 **	0.009 ***	0.024 ***	0.017 ***	0.007 **
2%	0.010 ***	0.010 ***	0.009 ***	0.007 ***	0.004 ***	0.001	0.006 ***	-0.004 ***	-0.006 ***	-0.020 ***
3%	0.000	0.000	0.000	0.000	0.000	-0.001	0.005	0.063	-0.004	-0.007

	***	***	***	***	***		***	***		***
4%	-0.010	-0.010	-0.009	-0.007	-0.003	-0.004	0.005	-0.082	-0.002	-0.019
	***	***	***	***	***		***	***	***	***
5%	-0.019	-0.019	-0.018	-0.014	-0.007	-0.011	0.000	0.029	0.017	0.045
	***	***	***	***	***		***	***	***	
6%	-0.029	-0.029	-0.027	-0.020	-0.010	-0.011	-0.007	-0.043	0.001	0.023
	***	***	***	***	***		***	***	***	
7%	-0.039	-0.039	-0.036	-0.025	-0.012	-0.009	-0.004	0.054	0.014	-0.032
	***	***	***	***	***			***	***	***
8%	-0.049	-0.049	-0.044	-0.030	-0.014	-0.014	-0.032	0.019	0.002	0.009
	***	***	***	***	***			***	***	***
9%	-0.059	-0.059	-0.053	-0.033	-0.017	-0.016	-0.013	-0.005	0.081	-0.011
	***	***	***	***	***		**	***		***
10%	-0.069	-0.069	-0.061	-0.037	-0.017	-0.020	0.019	-0.043	0.051	0.083
	***	***	***	***	***			***	***	**

Table-5 Results changing other parameters under B-S model

Maturity										
Panel A: Convertible bond price under constant interest rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
Maturity										
1	103.96	103.96	103.98	104.90	111.82	126.31	144.77	168.41	188.99	209.49
2	102.97	102.98	103.31	106.74	116.41	130.89	147.73	173.10	184.31	210.85

3	102.05	102.11	103.19	108.71	120.21	136.07	157.03	183.92	201.27	222.51
4	101.18	101.42	103.40	110.36	122.44	136.55	155.48	175.64	191.83	205.60
5	100.37	100.80	103.89	112.21	125.79	142.48	154.75	169.68	191.93	235.70
6	99.60	100.34	104.28	114.18	128.61	142.95	158.47	187.56	215.89	243.20
7	98.91	99.87	105.29	116.10	131.72	151.13	174.62	197.90	227.71	240.36
8	98.29	99.73	105.96	115.93	128.55	138.24	152.70	173.13	214.08	225.47
9	97.72	99.43	106.11	117.67	135.45	153.37	173.28	191.21	181.43	197.79
10	97.16	99.90	107.80	121.02	137.71	157.09	177.99	198.89	196.09	209.18

Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Maturity										
1	-0.001 ***	-0.001 ***	-0.001 ***	-0.002 ***	-0.002 *	0.001	0.003 *	-0.002	0.006 **	-0.002
2	-0.003 ***	-0.003 ***	-0.006	-0.010 ***	-0.010 ***	-0.001 ***	0.000 ***	0.064 ***	-0.002 ***	0.029 ***
3	-0.007 ***	-0.008 ***	-0.014 ***	-0.017 ***	-0.012 ***	-0.004 ***	0.025 ***	0.038 ***	0.005 ***	0.018 ***
4	-0.011 ***	-0.014 ***	-0.026 ***	-0.030 ***	-0.021 ***	-0.017 ***	0.005 ***	0.029 ***	-0.005 ***	-0.034 ***
5	-0.018 ***	-0.024 ***	-0.037 ***	-0.051 ***	-0.039 ***	-0.031 ***	-0.068 ***	-0.079 ***	-0.055 ***	-0.023 ***
6	-0.021 ***	-0.030 ***	-0.048 ***	-0.051 ***	-0.043 ***	-0.041 ***	-0.048 ***	0.030 ***	0.068 ***	0.096 ***
7	-0.028 ***	-0.044	-0.062 ***	-0.073 ***	-0.057 ***	-0.035 ***	-0.012 ***	-0.001 ***	0.002 ***	-0.075 ***
8	-0.034	-0.055	-0.072	-0.090	-0.081	-0.115	-0.096	-0.049	0.093	-0.030

	***	***	***	***	***	***	***	***	***	***
9	-0.039	-0.065	-0.088	-0.097	-0.071	-0.057	-0.043	-0.039	-0.167	-0.160
	***	***	***	***	***	***	***	***	***	***
10	-0.045	-0.074	-0.100	-0.103	-0.098	-0.084	-0.071	-0.072	-0.173	-0.279
	***	***	***	***	***	***	***	***	***	***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Maturity										
1	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.004	0.005 *	0.006 **
2	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	0.005 ***	0.063 ***	-0.004 ***	-0.007 ***
3	0.000 ***	0.000 ***	-0.001 ***	0.000 ***	0.001 ***	0.005 ***	0.002 ***	0.009 ***	0.001 ***	-0.002 ***
4	0.000 ***	0.000 ***	-0.001 ***	-0.001 ***	0.000 ***	0.004 ***	0.002 ***	-0.007 ***	0.010	-0.004 ***
5	0.000 ***	-0.001 ***	-0.001	-0.004 ***	-0.001 ***	-0.001 ***	-0.024 ***	-0.004 ***	-0.072 ***	0.051 ***
6	0.000 ***	-0.001 ***	-0.002 ***	-0.002 ***	-0.002 ***	-0.003 ***	0.002 ***	0.045 ***	-0.004 ***	0.020 ***
7	-0.001 ***	-0.002 ***	-0.003 ***	-0.002 ***	-0.006 ***	-0.001 ***	-0.004 ***	-0.016 ***	-0.010 ***	-0.021 ***
8	-0.001 ***	-0.002 ***	-0.006 ***	-0.005 ***	-0.005 ***	-0.011 ***	-0.002 ***	-0.033 ***	0.063 ***	-0.011 ***
9	-0.001 ***	-0.002 ***	-0.005 ***	-0.007 ***	0.005 ***	-0.006 ***	0.006 ***	0.016 ***	0.009 ***	-0.082 ***

10	-0.001 ***	-0.003 ***	-0.004 ***	-0.011 ***	-0.004 ***	-0.001 ***	-0.013 ***	-0.006 ***	-0.143 ***	-0.114 ***

Coupon										
Panel A: Convertible bond price under constant interest rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
Coupon										
1	91.55	91.76	94.35	102.29	114.94	130.78	148.05	165.84	185.16	208.81
2	94.41	94.59	97.02	104.69	117.07	132.72	149.79	167.42	186.65	210.50
3	97.26	97.43	99.70	107.09	119.20	134.65	151.54	169.00	188.14	212.19
4	100.12	100.27	102.37	109.50	121.34	136.59	153.28	170.58	189.63	213.88
5	102.97	103.11	105.05	111.90	123.47	138.52	155.03	172.16	191.12	215.56
6	105.83	105.95	107.72	114.30	125.60	140.46	156.77	173.73	192.60	217.25
7	108.69	108.79	110.40	116.71	127.73	142.39	158.52	175.31	194.09	218.94
8	111.54	111.63	113.08	119.11	129.86	144.33	160.26	176.89	195.58	220.63
9	114.40	114.46	115.75	121.51	131.99	146.26	162.01	178.47	197.07	222.32
10	117.26	117.30	118.43	123.92	134.12	148.20	163.75	180.05	198.56	224.01
Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
Coupon										
1	-0.004 ***	-0.004 ***	-0.007 ***	-0.013 ***	-0.010 ***	-0.001	0.002	0.051 ***	-0.002	0.023 ***
2	-0.003 ***	-0.004 ***	-0.007 ***	-0.012 ***	-0.010 ***	-0.001 ***	0.001 ***	0.054 ***	-0.002 ***	0.024 ***

3	-0.003 ***	-0.004 ***	-0.007 ***	-0.011 ***	-0.010 ***	-0.001 ***	0.001 ***	0.058 ***	-0.002 ***	0.026 ***
4	-0.003 ***	-0.003 ***	-0.006 ***	-0.011 ***	-0.010 ***	-0.001 ***	0.000 ***	0.061 ***	-0.002 ***	0.027 ***
5	-0.003 ***	-0.003 ***	-0.006 ***	-0.010 ***	-0.010 ***	-0.001 ***	0.000 ***	0.064 ***	-0.002 ***	0.029
6	-0.003 ***	-0.003 ***	-0.005 ***	-0.010 ***	-0.010 ***	-0.002 ***	-0.001 ***	0.068 ***	-0.002 ***	0.030
7	-0.003 ***	-0.003 ***	-0.005 ***	-0.009 ***	-0.010 ***	-0.002 ***	-0.002 ***	0.071 ***	-0.002 ***	0.032
8	-0.003 ***	-0.003 ***	-0.005 ***	-0.009 ***	-0.010 ***	-0.002 ***	-0.002 ***	0.074 ***	-0.002 ***	0.033 **
9	-0.003 ***	-0.003 ***	-0.004 ***	-0.009 ***	-0.010 ***	-0.002 ***	-0.003 ***	0.077 **	-0.002 ***	0.035 ***
10	-0.003 ***	-0.003 ***	-0.004 ***	-0.008 ***	-0.010 ***	-0.002 ***	-0.003 ***	0.080	-0.002 ***	0.036 ***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Coupon										
1	0.000	0.000	0.000	0.000	0.000	-0.001	0.004 *	0.046 ***	-0.003	-0.006
2	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	0.005 *	0.051 ***	-0.003 ***	-0.006 ***
3	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	0.005 *	0.055 ***	-0.004 ***	-0.006 ***
4	0.000	0.000	0.000	0.000	0.000	-0.001	0.005	0.059	-0.004	-0.007

	***	***	***	***	***	***	***	***	***	***
5	0.000	0.000	0.000	0.000	0.000	-0.001	0.005	0.063	-0.004	-0.007
	***	***		***	***	***	***	***	***	***
6	0.000	0.000	0.000	0.000	0.000	-0.001	0.006	0.067	-0.004	-0.007
	***	***	***	***	***	***	***	***	***	***
7	0.000	0.000	0.000	0.000	0.000	-0.002	0.006	0.071	-0.005	-0.008
	***		***	***	***	***	***	***	***	***
8	0.000	0.000	0.000	0.000	0.000	-0.002	0.006	0.074	-0.005	-0.008
	***	***	***	***	***	***	***	***	***	***
9	0.000	0.000	0.000	0.000	0.000	-0.002	0.006	0.078	-0.005	-0.008
	***	***	***	***	***	***	***	**	***	***
10	0.000	0.000	0.000	0.000	0.000	-0.002	0.006	0.082	-0.005	-0.009
	***	***	***	***	***	***	***		***	***

Credit Spread

Panel A: Convertible bond price under constant interest rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Credit Spread										
1%	106.83	106.92	108.50	114.65	125.60	140.17	156.07	173.12	191.46	214.28
2%	104.88	105.00	106.77	113.25	124.58	139.46	155.52	172.51	191.10	215.98
3%	102.97	103.11	105.05	111.90	123.47	138.52	155.03	172.16	191.12	215.56
4%	101.11	101.26	103.40	110.58	122.51	137.75	154.21	171.69	190.48	215.04
5%	99.27	99.45	101.75	109.25	121.56	136.93	153.61	171.43	190.31	215.03
6%	97.47	97.67	100.16	107.91	120.61	136.14	152.86	170.89	189.73	215.80
7%	95.71	95.93	98.60	106.60	119.59	135.33	152.28	170.68	188.60	216.10
8%	93.98	94.21	97.07	105.32	118.63	134.56	151.75	169.85	180.34	216.00

9%	92.29	92.54	95.55	104.08	117.74	133.78	151.05	169.22	184.22	213.91
10%	90.63	90.91	94.08	102.87	116.76	132.93	150.42	167.85	180.92	211.82

Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Credit Spread										
1%	-0.003 ***	-0.003 ***	-0.005 ***	-0.010 ***	-0.011 ***	-0.004 **	0.003	0.051 ***	0.051 ***	0.003
2%	-0.003 ***	-0.003 ***	-0.006 ***	-0.010 ***	-0.011	-0.003	0.000 ***	0.014 ***	-0.035 ***	0.042 ***
3%	-0.003 ***	-0.003 ***	-0.006 ***	-0.010 ***	-0.010 ***	-0.001 ***	0.000 ***	0.064 ***	-0.002 ***	0.029 ***
4%	-0.003 ***	-0.003 ***	-0.006 ***	-0.010 ***	-0.009 ***	-0.002 ***	0.003 ***	0.020 ***	0.027	0.028 ***
5%	-0.003 ***	-0.003 ***	-0.006 ***	-0.011 ***	-0.009 ***	-0.006 ***	0.000 ***	0.032 ***	0.010 *	0.022 ***
6%	-0.003 ***	-0.003 ***	-0.006 ***	-0.010 ***	-0.011 ***	-0.004 ***	-0.004 ***	0.027 ***	0.014	-0.002 ***
7%	-0.003 ***	-0.003 ***	-0.006 ***	-0.011 ***	-0.010 ***	-0.008 ***	-0.007 ***	-0.009 ***	0.001 ***	0.034 ***
8%	-0.003 ***	-0.003 ***	-0.006 ***	-0.012 ***	-0.009 ***	-0.004 ***	-0.001 ***	0.020 ***	-0.001 ***	0.017 ***
9%	-0.003 ***	-0.003 ***	-0.006 ***	-0.012 ***	-0.010 ***	-0.064 ***	0.012 ***	-0.016 ***	-0.014 ***	0.027 ***
10%	-0.003 ***	-0.003 ***	-0.006 ***	-0.013 ***	-0.013 ***	-0.024 ***	0.016 ***	0.054 ***	0.041 ***	0.008 ***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Credit Spread										
1%	0.000	0.000	0.000	0.000	0.000	-0.001	0.003	0.047 ***	0.000	0.026 ***
2%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 *	0.004 ***	0.015 ***	-0.020 ***	-0.003 **
3%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	0.005 ***	0.063 ***	-0.004 ***	-0.007 ***
4%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	0.010 ***	-0.006 ***	0.029	0.022 ***
5%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.003 ***	0.001 ***	0.000 ***	-0.006 ***	0.017 ***
6%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	-0.003 ***	0.041 ***	0.012 ***	-0.010 ***	-0.039 ***
7%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	-0.004 ***	0.010 ***	-0.023 ***	0.006 ***	0.043 ***
8%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	0.001 ***	-0.001 ***	0.015 ***	-0.041 ***	0.003 ***
9%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.002 ***	0.006 ***	0.013 ***	-0.031 ***	-0.005 ***	0.012
10%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.003 ***	0.000 ***	-0.013 ***	0.023 ***	0.015 ***	-0.028

Sigma

Panel A: Convertible bond price under constant interest rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Sigma										
5%	102.97	103.03	104.38	110.31	121.73	137.00	153.52	170.48	187.98	216.36
10%	102.97	103.05	104.55	110.64	122.03	137.55	153.14	169.84	188.87	216.38
15%	102.97	103.08	104.76	111.15	122.58	137.87	153.66	171.42	189.98	217.42
20%	102.97	103.11	105.05	111.90	123.47	138.52	155.03	172.16	191.12	215.56
25%	102.98	103.20	105.55	112.72	124.57	139.15	156.16	173.78	192.46	214.71
30%	102.98	103.31	106.12	113.73	125.80	140.49	157.04	174.90	193.44	213.51
35%	102.98	103.47	106.82	114.91	127.13	141.89	158.59	175.80	194.63	214.61
40%	102.99	103.72	107.72	116.23	128.65	143.35	159.88	177.49	195.59	214.87
45%	102.99	104.03	108.64	117.65	130.34	144.99	161.56	179.08	196.93	216.12
50%	103.01	104.38	109.73	119.14	132.04	146.83	163.15	180.80	198.64	217.64

Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Sigma										
5%	-0.003 ***	-0.003 ***	-0.003 ***	-0.003 ***	-0.093 ***	0.007 ***	0.013 ***	0.012 ***	0.012 ***	0.012 ***
10%	-0.003 ***	-0.003 ***	-0.003 ***	-0.002 ***	0.002 ***	-0.052 ***	-0.012 ***	-0.018 ***	0.019 ***	0.019 ***
15%	-0.003 ***	-0.003 ***	-0.003 ***	-0.001 ***	0.002 ***	0.003 ***	-0.036 ***	-0.012 ***	-0.013	-0.004 ***
20%	-0.003 ***	-0.003 ***	-0.003 ***	-0.001 ***	0.001 ***	0.002 ***	0.000 ***	0.034 ***	-0.010 ***	-0.005
25%	-0.003 ***	-0.003 ***	-0.003 ***	-0.001 ***	0.002 ***	0.003 ***	0.004	-0.003 ***	0.033 ***	-0.020 ***

30%	-0.003 ***	-0.003 ***	-0.003 ***	-0.001 ***	0.001 ***	0.003 ***	0.004 ***	0.006 **	-0.002 ***	0.042 ***
35%	-0.003 ***	-0.003 ***	-0.002 ***	-0.002 ***	0.001 ***	0.001 ***	0.003 ***	0.004 ***	0.006 *	0.024 ***
40%	-0.003 ***	-0.003 ***	-0.002 ***	-0.002 ***	0.000 ***	0.003 ***	0.005 ***	0.005 ***	0.007 ***	0.005
45%	-0.003 ***	-0.003 ***	-0.002 ***	0.000 ***	0.001 ***	0.004 ***	0.003 ***	0.004 ***	0.000 ***	0.008 ***
50%	-0.003 ***	-0.003 ***	-0.003 ***	-0.001 ***	0.001 ***	0.001 ***	0.006 ***	0.007 ***	0.009 ***	0.011 ***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Sigma										
5%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	0.009 ***	-0.016 ***	-0.016 ***	-0.015 ***	-0.015 ***
10%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.002 ***	-0.035 ***	-0.037 ***	-0.041 ***	0.009 ***	0.009 ***
15%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.002 ***	0.001 ***	-0.016 ***	-0.012 ***	-0.010	-0.003 ***
20%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.001 ***	0.001 ***	-0.003 ***	0.003 ***	0.016 ***	-0.012
25%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.001 ***	0.001 ***	0.002	0.005 ***	0.000 ***	0.019 ***
30%	0.000 ***	0.000 ***	0.000 ***	0.000 ***	0.001 ***	0.001 ***	0.001 ***	0.001 **	-0.001 ***	0.056 ***
35%	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.003	0.001	0.002

	***	***	***	***	***	***	***	***	*	***
40%	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.001	0.003	0.001
	***	***	***	***	***	***	***	***	***	
45%	0.000	0.000	0.000	0.001	0.000	0.001	0.003	0.003	0.000	0.004
	***	***	***	***	***	***	***	***	***	***
50%	0.000	0.000	0.000	0.000	0.001	0.000	0.002	0.001	0.003	-0.001
	***	***	***	***	***	***	***	***	***	***

Table-6 Comparative statics under Heston's model with constant interest rate

Moneyness	95%up	Price	95%low
10	102.97	102.97	102.97
20	102.97	102.97	102.97
30	103.00	102.99	102.98
40	103.14	103.11	103.08
50	103.75	103.69	103.62
60	105.16	105.05	104.93
70	107.92	107.73	107.54
80	112.18	111.90	111.62
90	117.40	117.04	116.67
100	123.92	123.47	123.01
110	131.30	130.75	130.20
120	139.15	138.52	137.89
130	147.25	146.55	145.84
140	155.79	155.03	154.26
150	164.23	163.42	162.60
160	173.02	172.16	171.29
170	182.71	181.78	180.85
180	192.10	191.12	190.13
190	201.15	200.13	199.11
200	216.87	215.56	214.25

Maturity	95%up	Price	95%low
1	114.96	114.72	114.48
2	123.92	123.47	123.01
3	131.58	130.91	130.24
4	138.09	137.22	136.35
5	145.55	144.42	143.28
6	149.92	148.63	147.35
7	154.63	153.10	151.57
8	160.63	158.83	157.04
9	161.35	159.42	157.50
10	170.96	168.58	166.19

Coupon	95%up	Price	95%low
1	115.45	114.94	114.43
2	117.57	117.07	116.58

3	119.69	119.20	118.72
4	121.81	121.34	120.87
5	123.92	123.47	123.01
6	126.04	125.60	125.15
7	128.16	127.73	127.30
8	130.28	129.86	129.44
9	132.40	131.99	131.58
10	134.52	134.12	133.72

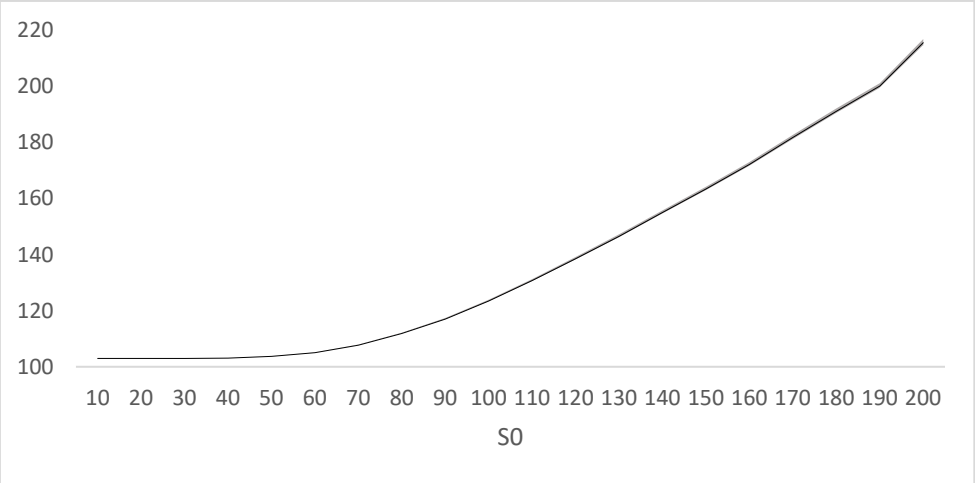
Credit Spread	95%up	Price	95%low
1	126.03	125.60	125.17
2	125.02	124.58	124.13
3	123.92	123.47	123.01
4	122.98	122.51	122.04
5	122.04	121.56	121.07
6	121.10	120.61	120.11
7	120.10	119.59	119.09
8	119.15	118.63	118.11
9	118.27	117.74	117.22
10	117.30	116.76	116.22

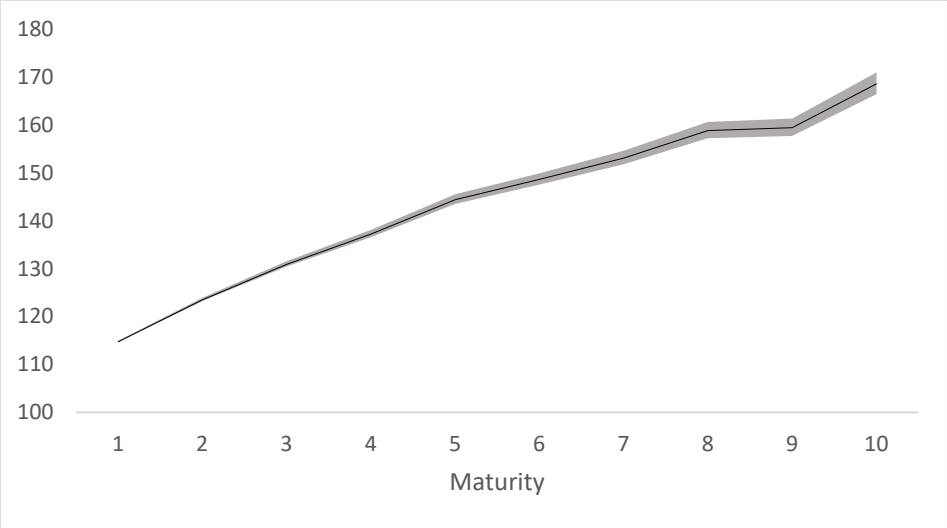
R0	95%up	Price	95%low
1	124.88	124.46	124.03
2	124.43	123.99	123.54
3	123.92	123.47	123.01
4	123.71	123.23	122.76
5	123.51	123.01	122.52
6	123.15	122.64	122.13
7	123.03	122.50	121.98
8	122.88	122.34	121.80
9	122.96	122.40	121.84
10	122.91	122.34	121.77

Sigma	95%up	Price	95%low
0.03	122.02	121.62	121.22
0.06	122.22	121.82	121.41
0.09	122.43	122.02	121.61
0.12	122.67	122.25	121.83
0.15	123.01	122.58	122.15

0.18	123.55	123.11	122.66
0.21	124.23	123.77	123.30
0.24	124.86	124.37	123.88
0.27	125.55	125.04	124.53
0.3	126.33	125.80	125.26

Figure-3 Comparative statics under Heston's model with constant interest rate



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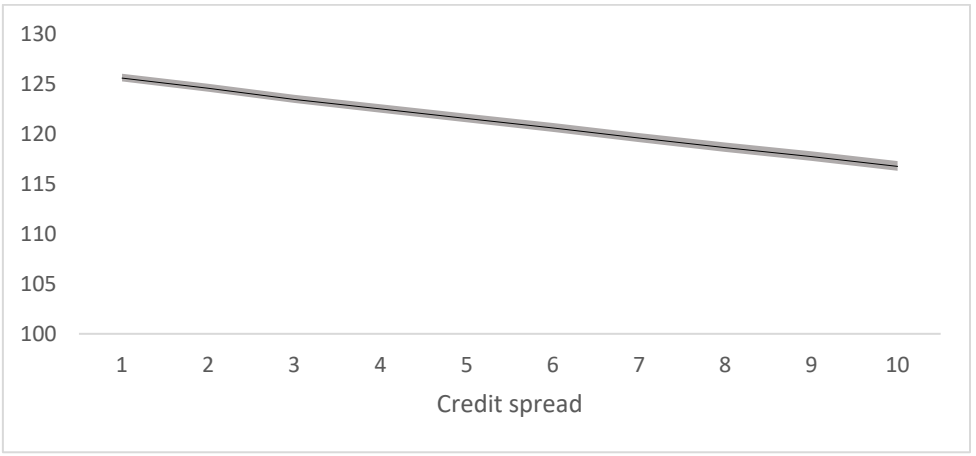
Coupon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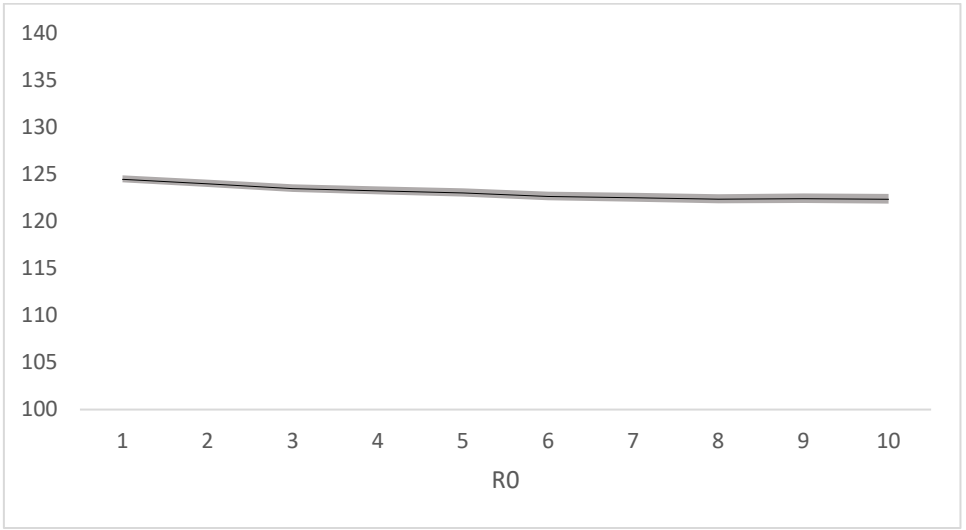
Credit spread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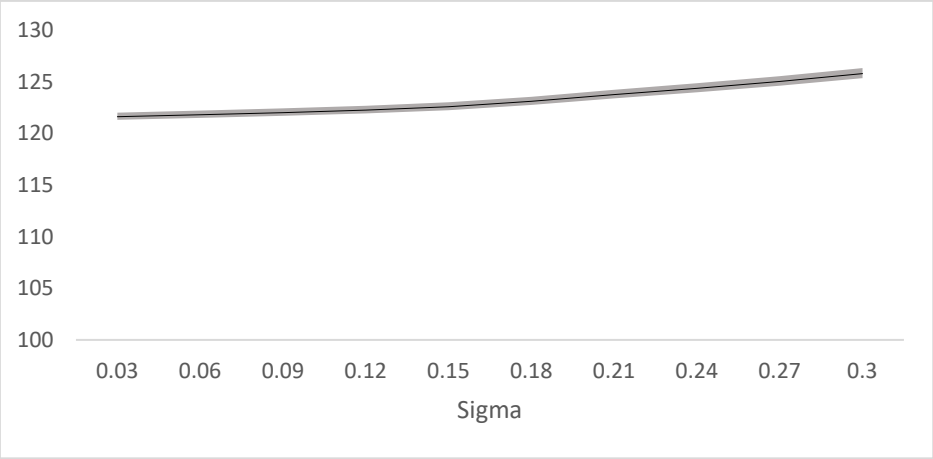


Table-7 Comparative statics under Heston's model with stochastic interest rate Vasicek:

Moneyness	95%up	Price	95%low
10	103.48	103.31	103.14
20	103.48	103.31	103.14
30	103.50	103.33	103.16
40	103.73	103.56	103.39
50	104.49	104.32	104.14
60	106.05	105.86	105.67
70	108.81	108.58	108.34
80	113.20	112.90	112.59
90	118.53	118.15	117.77
100	124.81	124.35	123.89
110	132.08	131.53	130.98
120	139.69	139.06	138.43
130	147.84	147.13	146.43
140	156.42	155.64	154.86
150	164.73	163.89	163.04
160	173.23	172.33	171.44
170	182.53	181.57	180.61
180	191.93	190.90	189.88
190	200.77	199.70	198.63
200	213.29	212.07	210.84

Maturity	95%up	Price	95%low
1	115.16	114.92	114.68
2	124.81	124.35	123.89
3	132.83	132.15	131.47
4	140.23	139.34	138.45
5	149.59	148.41	147.24
6	154.21	152.86	151.52
7	163.25	161.56	159.87
8	168.39	166.46	164.53
9	169.96	167.86	165.76
10	180.10	177.59	175.08

Coupon	95%up	Price	95%low
1	116.29	115.79	115.28
2	118.42	117.93	117.43
3	120.55	120.07	119.59
4	122.68	122.21	121.74
5	124.81	124.35	123.89
6	126.94	126.49	126.04
7	129.07	128.63	128.20
8	131.20	130.77	130.35
9	133.33	132.92	132.50
10	135.47	135.06	134.65

Credit Spread	95%up	Price	95%low
1	126.93	126.49	126.05
2	125.88	125.43	124.98
3	124.81	124.35	123.89
4	123.74	123.27	122.80
5	122.80	122.31	121.83
6	121.74	121.24	120.75
7	120.72	120.22	119.72
8	119.77	119.26	118.75
9	118.87	118.35	117.83
10	117.91	117.37	116.84

R0	95%up	Price	95%low
1	125.03	124.59	124.14
2	124.90	124.45	124.00

3	124.81	124.35	123.89
4	124.71	124.24	123.77
5	124.72	124.24	123.76
6	124.73	124.24	123.74
7	124.76	124.25	123.75
8	124.75	124.24	123.72
9	124.84	124.31	123.78
10	124.91	124.37	123.84

Sigma	95%up	Price	95%low
0.03	122.66	122.27	121.87
0.06	123.00	122.60	122.19
0.09	123.26	122.85	122.43
0.12	123.57	123.15	122.73
0.15	124.05	123.61	123.18
0.18	124.50	124.05	123.60
0.21	124.98	124.51	124.04
0.24	125.64	125.16	124.67
0.27	126.40	125.89	125.38
0.3	127.25	126.71	126.17

rhosr	95%up	Price	95%low
-0.2	124.68	124.19	123.71
-0.15	124.66	124.18	123.70
-0.1	124.69	124.22	123.74
-0.05	124.82	124.35	123.88
0	124.15	123.69	123.23
0.05	124.83	124.36	123.90
0.1	124.81	124.35	123.89
0.15	124.84	124.38	123.93
0.2	124.84	124.39	123.94

lambda	95%up	Price	95%low
0.5	125.28	124.81	124.35
0.6	125.11	124.64	124.18
0.7	125.00	124.54	124.07
0.8	124.87	124.41	123.95
0.9	124.81	124.35	123.89
1	124.75	124.29	123.83

1.1	124.66	124.20	123.75
1.2	124.66	124.20	123.74
1.3	124.58	124.12	123.66
1.4	124.59	124.13	123.67
1.5	124.58	124.12	123.66

rbar	95%up	Price	95%low
1	125.65	125.20	124.75
2	125.19	124.74	124.28
3	124.81	124.35	123.89
4	124.42	123.96	123.49
5	124.14	123.67	123.19
6	123.80	123.33	122.85
7	123.54	123.05	122.56
8	123.29	122.79	122.30
9	123.08	122.58	122.08
10	122.77	122.27	121.76

eta	95%up	Price	95%low
0.02	124.07	123.61	123.15
0.04	124.08	123.62	123.17
0.06	124.37	123.91	123.45
0.08	124.51	124.06	123.60
0.1	124.81	124.35	123.89
0.12	125.27	124.81	124.34
0.14	125.58	125.11	124.64
0.16	126.14	125.67	125.19
0.18	126.71	126.23	125.74
0.2	127.22	126.73	126.23

CIR:

Moneyness	95%up	Price	95%low
10	103.01	102.98	102.95
20	103.01	102.98	102.95
30	103.02	102.99	102.96
40	103.16	103.12	103.08
50	103.78	103.72	103.65

60	105.22	105.10	104.98
70	107.94	107.75	107.56
80	112.25	111.97	111.68
90	117.55	117.18	116.81
100	124.07	123.61	123.15
110	131.35	130.80	130.25
120	139.09	138.46	137.83
130	147.39	146.69	145.98
140	155.68	154.92	154.15
150	164.34	163.52	162.70
160	173.14	172.27	171.40
170	183.19	182.24	181.29
180	192.41	191.41	190.40
190	203.09	201.99	200.90
200	217.77	216.42	215.06

Maturity	95%up	Price	95%low
1	114.92	114.68	114.44
2	124.07	123.61	123.15
3	131.52	130.86	130.19
4	138.12	137.25	136.38
5	145.64	144.51	143.38
6	149.74	148.46	147.19
7	155.61	154.06	152.51
8	161.04	159.24	157.44
9	161.05	159.13	157.21
10	170.90	168.54	166.18

Coupon	95%up	Price	95%low
1	115.58	115.07	114.56
2	117.70	117.20	116.71
3	119.82	119.34	118.85
4	121.94	121.47	121.00
5	124.07	123.61	123.15
6	126.19	125.74	125.29
7	128.31	127.87	127.44
8	130.43	130.01	129.58
9	132.55	132.14	131.73
10	134.68	134.28	133.87

Credit Spread	95%up	Price	95%low
1	126.02	125.59	125.16
2	125.03	124.59	124.14
3	124.07	123.61	123.15
4	123.04	122.57	122.10
5	122.07	121.58	121.10
6	121.10	120.60	120.11
7	120.09	119.58	119.08
8	119.16	118.65	118.13
9	118.23	117.71	117.18
10	117.30	116.76	116.23

R0	95%up	Price	95%low
1	124.13	123.69	123.26
2	124.07	123.62	123.17
3	124.07	123.61	123.15
4	124.01	123.54	123.08
5	124.03	123.55	123.07
6	124.05	123.56	123.07
7	124.11	123.61	123.11
8	124.17	123.66	123.14
9	124.18	123.65	123.13
10	124.26	123.73	123.19

Sigma	95%up	Price	95%low
0.03	122.02	121.62	121.23
0.06	122.20	121.80	121.40
0.09	122.43	122.03	121.62
0.12	122.65	122.23	121.82
0.15	122.98	122.55	122.12
0.18	123.57	123.13	122.68
0.21	124.25	123.79	123.32
0.24	124.91	124.42	123.93
0.27	125.70	125.18	124.67
0.3	126.45	125.92	125.38

rhosr	95%up	Price	95%low
-0.4	122.90	122.46	122.02
-0.3	123.95	123.48	123.02

-0.2	123.99	123.53	123.06
-0.1	124.09	123.62	123.16
0	123.77	123.31	122.85
0.1	124.07	123.61	123.15
0.2	123.99	123.53	123.07
0.3	123.85	123.40	122.95
0.4	123.10	122.67	122.23

lambda	95%up	Price	95%low
0.5	124.03	123.57	123.12
0.6	124.06	123.60	123.14
0.7	124.06	123.60	123.15
0.8	124.06	123.60	123.15
0.9	124.07	123.61	123.15
1	124.06	123.60	123.15
1.1	124.07	123.61	123.15
1.2	124.08	123.62	123.16
1.3	124.08	123.62	123.16
1.4	124.09	123.63	123.17
1.5	124.09	123.63	123.17

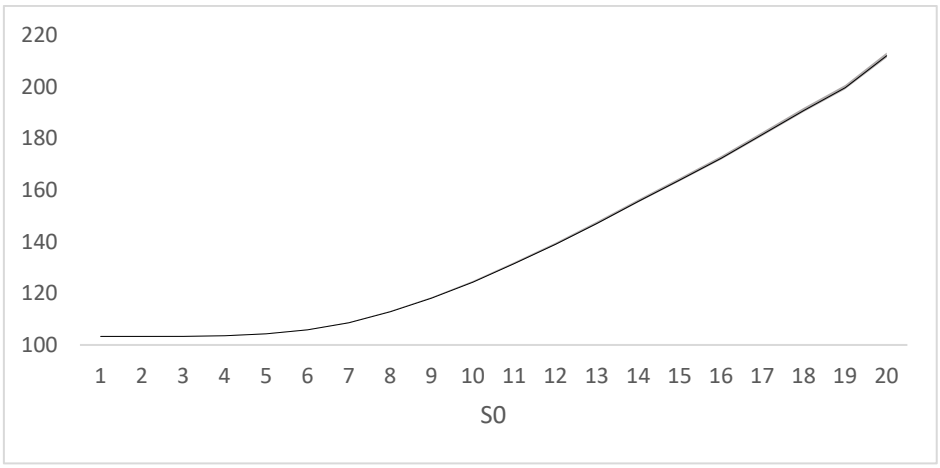
rbar	95%up	Price	95%low
1	124.71	124.26	123.82
2	124.35	123.90	123.45
3	124.07	123.61	123.15
4	123.76	123.29	122.83
5	123.50	123.02	122.55
6	123.16	122.68	122.21
7	122.93	122.44	121.96
8	122.63	122.14	121.65
9	122.38	121.88	121.39
10	122.17	121.67	121.17

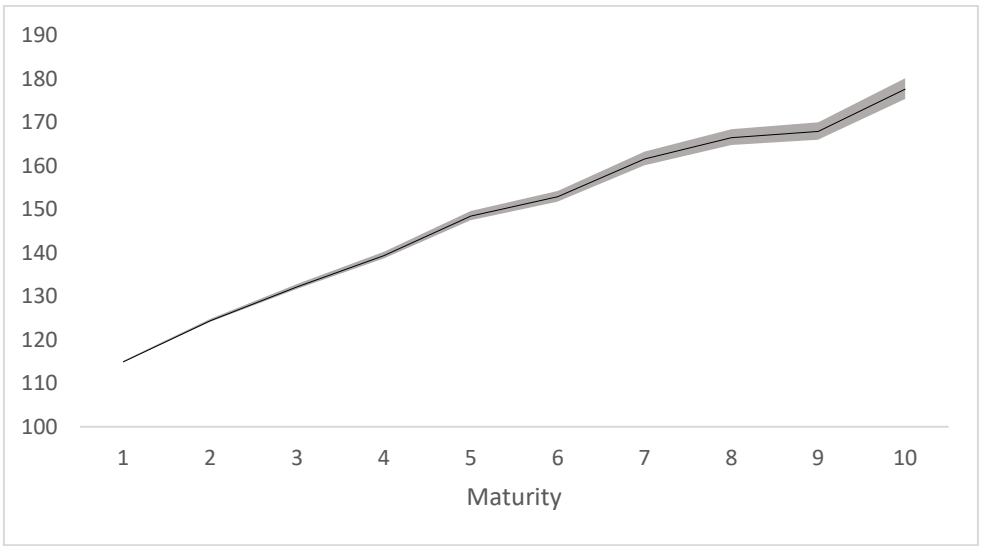
eta	95%up	Price	95%low
0.02	124.00	123.54	123.08
0.04	124.07	123.61	123.15
0.06	124.07	123.61	123.15
0.08	124.07	123.61	123.15
0.1	124.07	123.61	123.15

0.12	124.06	123.60	123.14
0.14	124.07	123.62	123.16
0.16	124.07	123.61	123.16
0.18	124.08	123.63	123.17
0.2	124.10	123.65	123.19

Figure-4 Comparative statics under Heston's model with stochastic interest rate

Vasicek:



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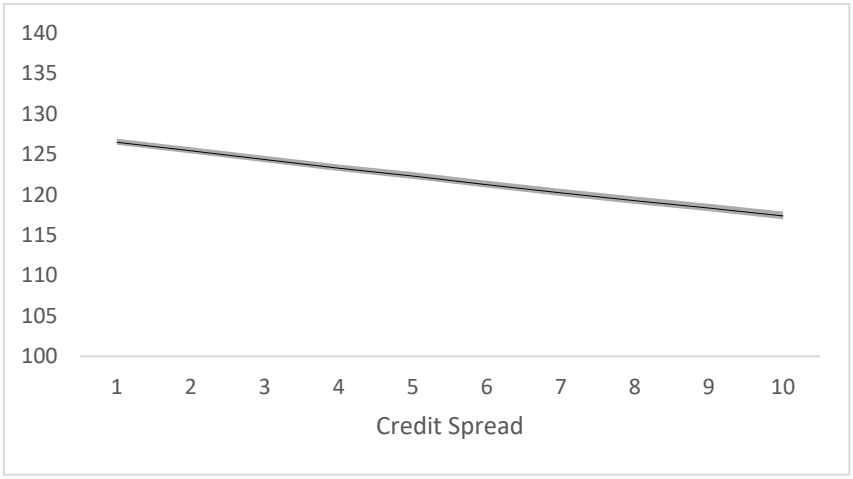
Coupon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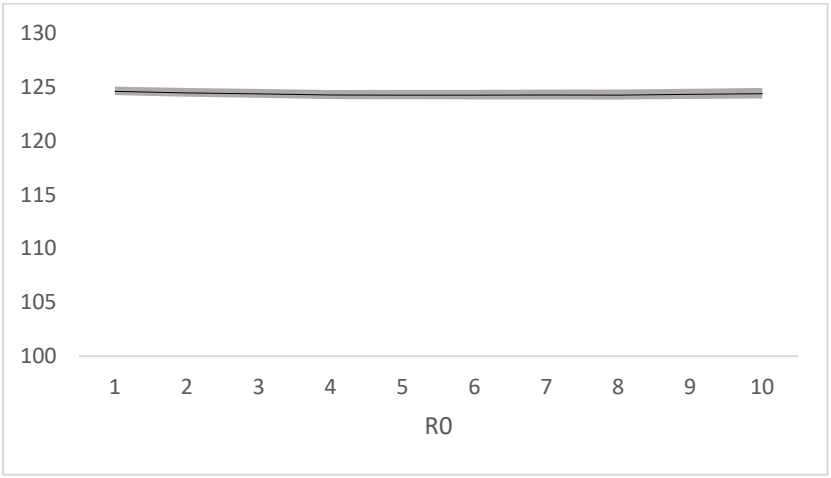
Credit Spread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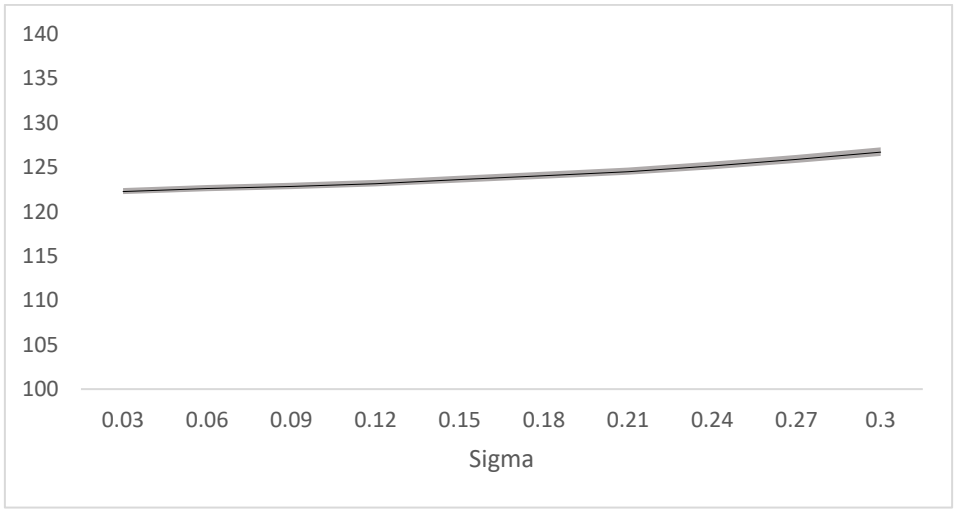
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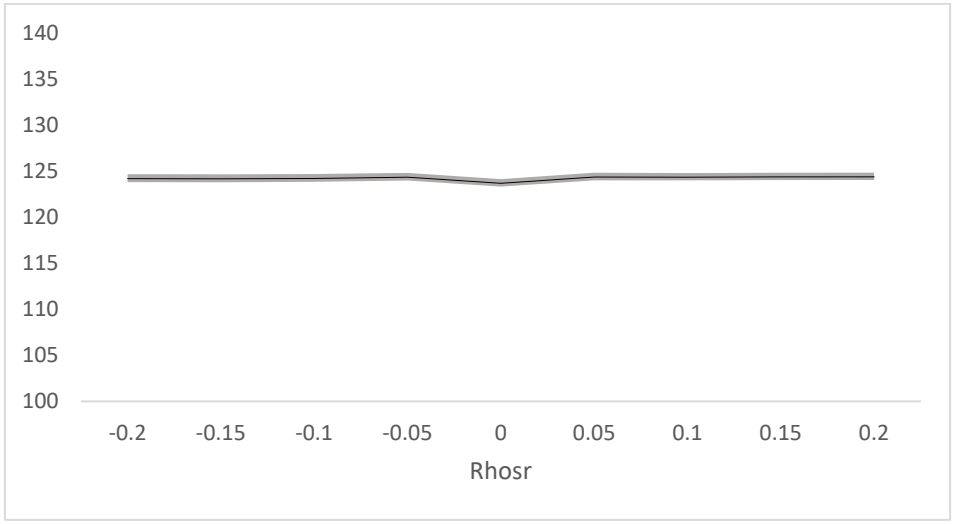
0.3
Sigma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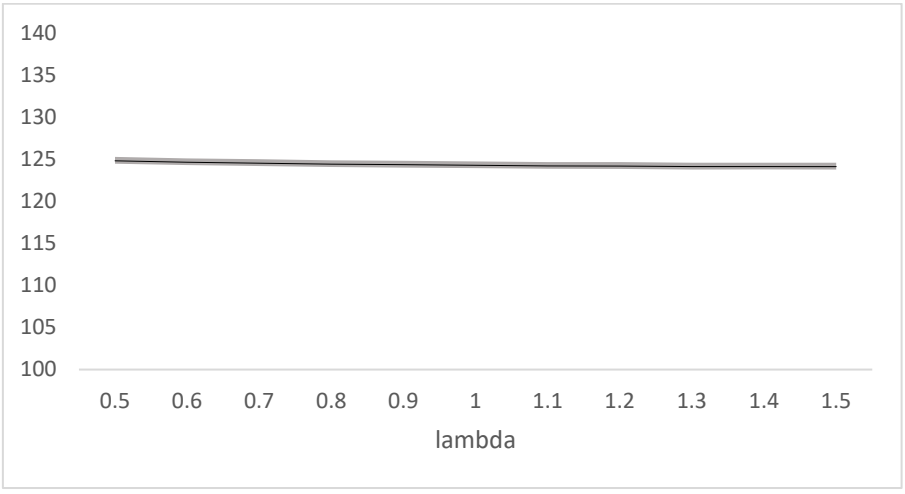
0.2
Rhosr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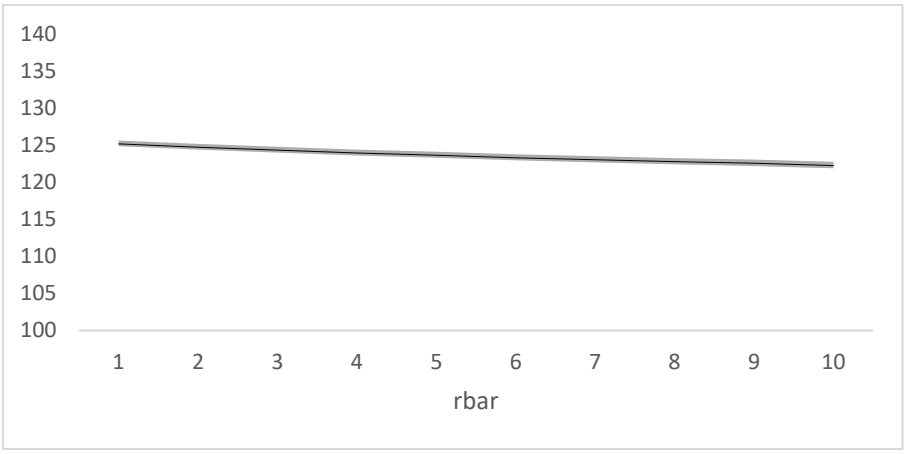
lambda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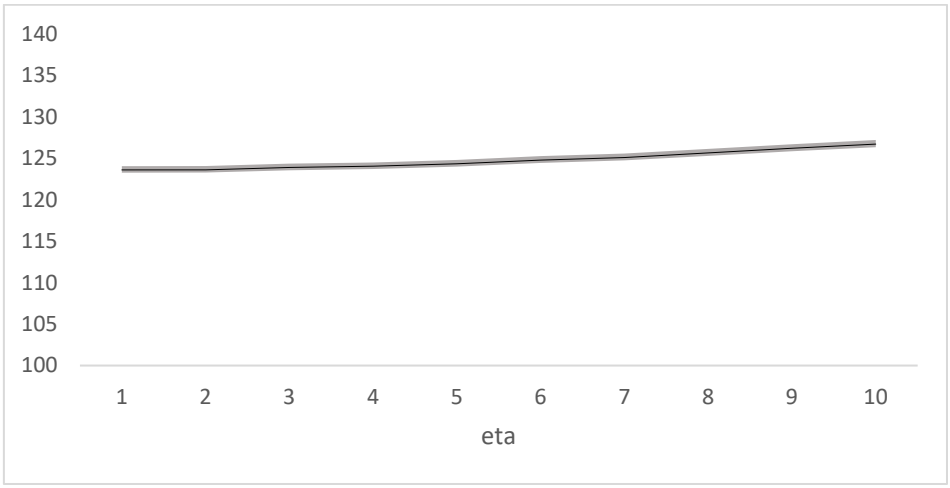
rbar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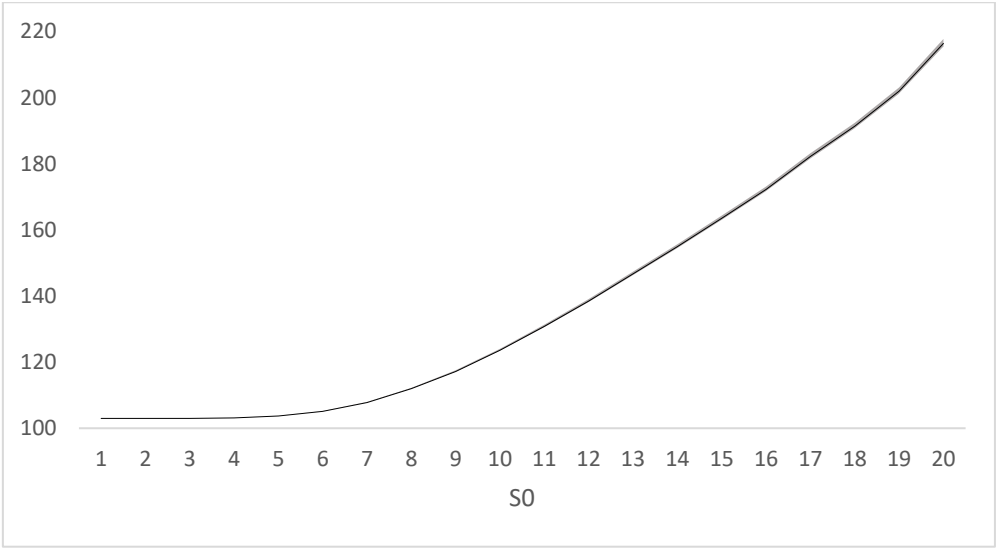
eta

CIR: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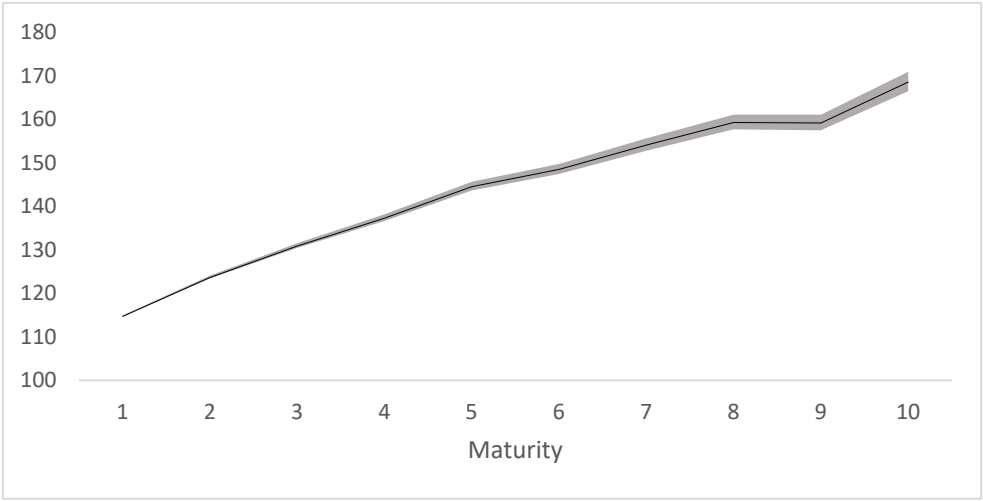
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Maturity


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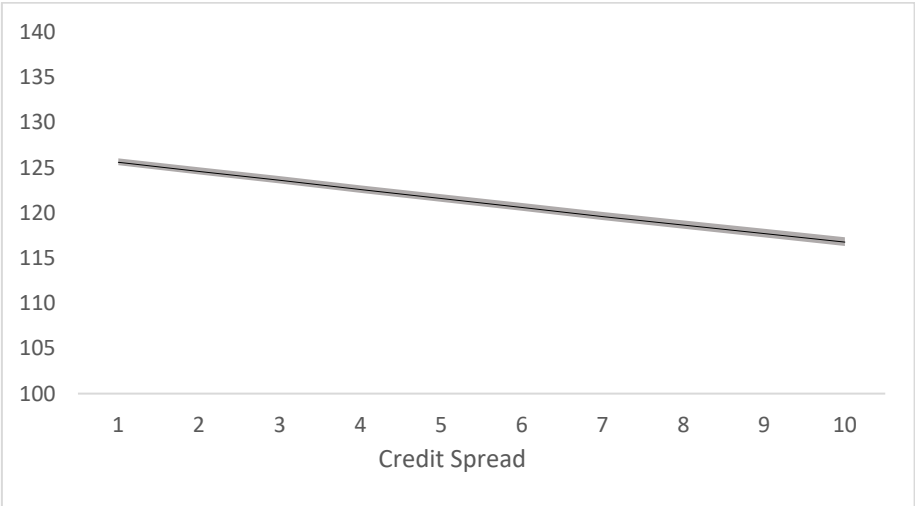
Coupon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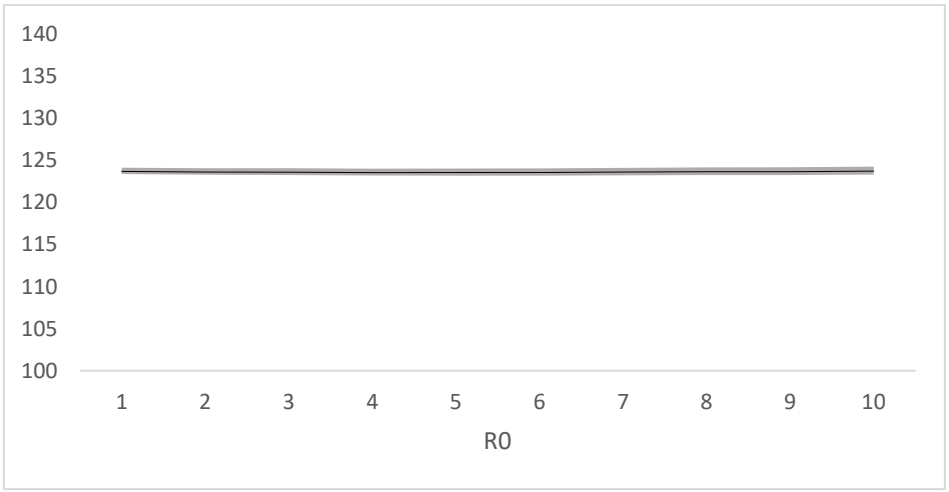
Credit Spread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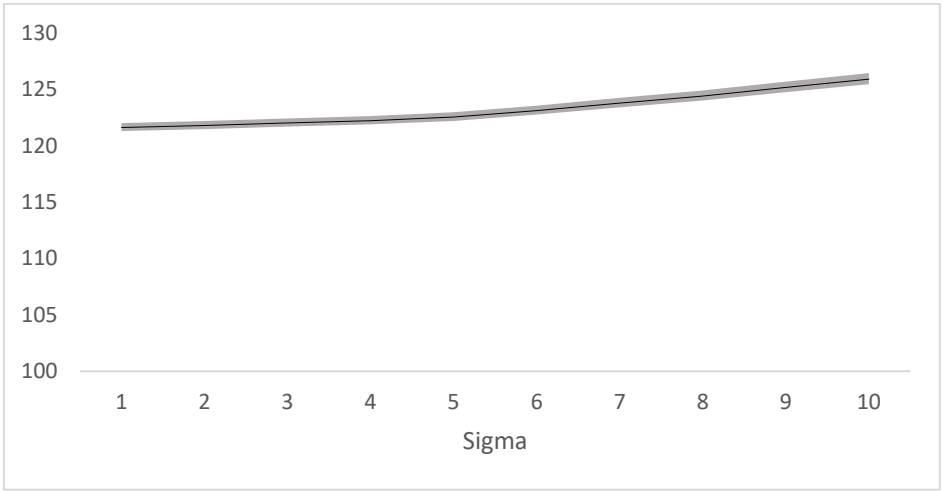
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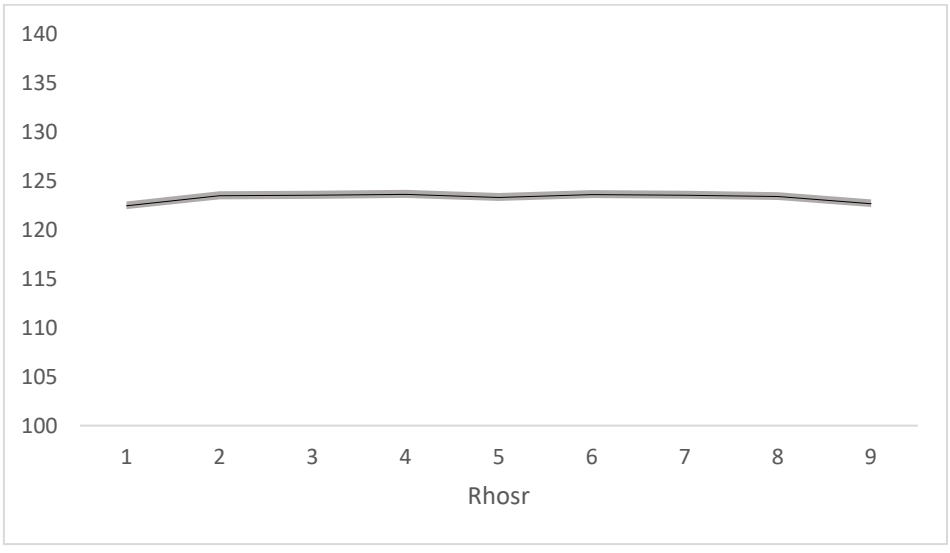
Sigma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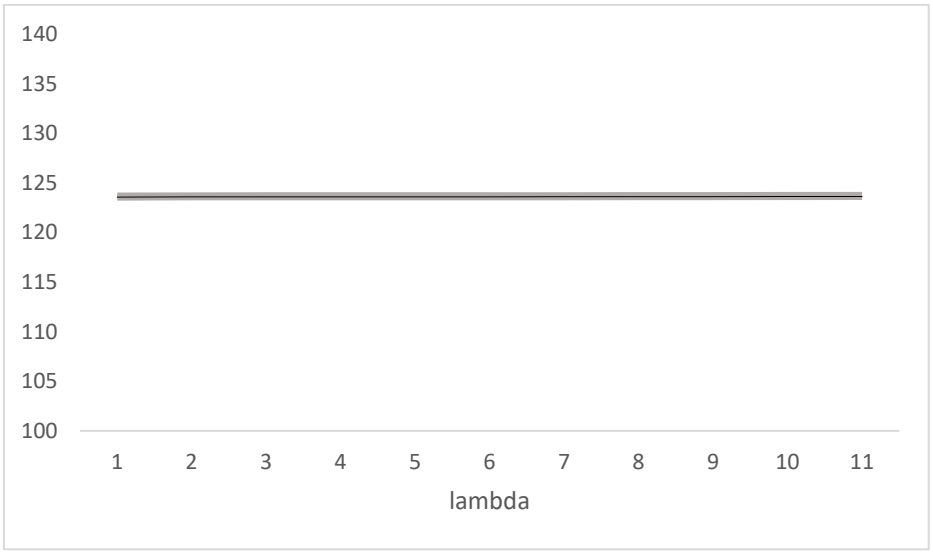
Rhosr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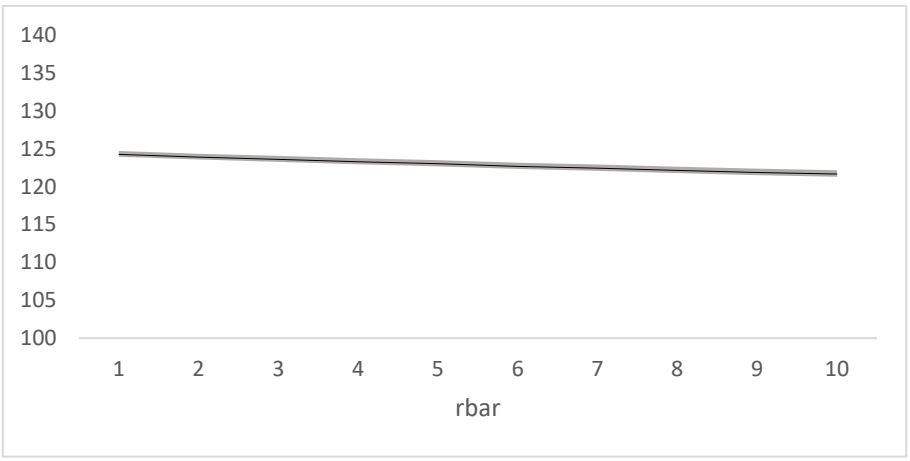
lambda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100
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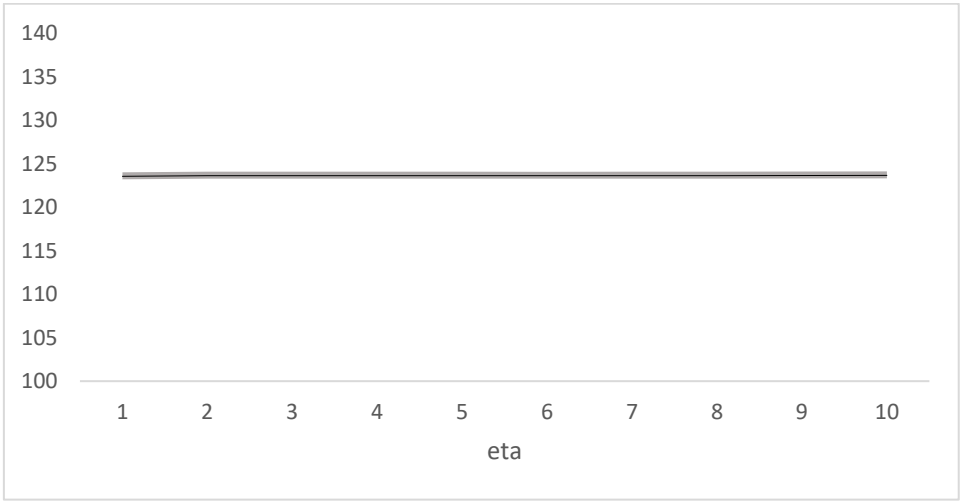
rbar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1 2 3 4 5 6 7 8 9 10
eta

Table-8 Pricing difference under Heston's model using long-term mean of interest rate as the interest rate in constant rate model

Panel A: Convertible bond price under constant interest rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
Long-term r										
3%	102.97	103.11	105.05	111.90	123.47	138.52	155.03	172.16	191.12	215.56
Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	-0.021 ***	-0.021 ***	-0.021 ***	-0.016 ***	-0.009 ***	-0.002 ***	0.001 ***	0.004 ***	0.008 *	0.024 ***
2%	-0.012 ***	-0.013 ***	-0.014 ***	-0.012 ***	-0.008 ***	-0.003 ***	-0.001 ***	0.002 ***	0.007 ***	0.019 ***
3%	-0.003 ***	-0.004 ***	-0.008 ***	-0.009 ***	-0.007 ***	-0.004 ***	-0.004 ***	-0.001 ***	0.001 ***	0.016 ***
4%	0.005 ***	0.004 ***	-0.001 ***	-0.006 ***	-0.006 ***	-0.005 ***	-0.007 ***	-0.003 ***	0.001 ***	0.015 ***
5%	0.014 ***	0.012 ***	0.005 ***	-0.002 ***	-0.006 ***	-0.006 ***	-0.009 ***	-0.007 ***	-0.004 ***	0.013 ***
6%	0.023 ***	0.021 ***	0.012 ***	0.001 ***	-0.006 ***	-0.008 ***	-0.007 ***	-0.010 ***	-0.005 ***	-0.022 ***
7%	0.031 ***	0.029 ***	0.018 ***	0.004 ***	-0.006 ***	-0.008 ***	-0.010	-0.014 ***	-0.010 ***	-0.030 ***
8%	0.040 ***	0.037 ***	0.024 ***	0.007 ***	-0.006 ***	-0.010 ***	-0.013	-0.014 ***	-0.012 ***	0.013

9%	0.048 ***	0.045 ***	0.030 ***	0.010 ***	-0.007 ***	-0.012	-0.016 ***	-0.015 ***	-0.019 ***	-0.041 ***
10%	0.056 ***	0.052 ***	0.035 ***	0.012 ***	-0.007 ***	-0.015 ***	-0.019 ***	-0.019 ***	-0.033 ***	-0.029 ***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	-0.018 ***	-0.017 ***	-0.015 ***	-0.008 ***	-0.002 ***	0.002 ***	0.006 ***	0.005 ***	0.005 *	0.010 ***
2%	-0.009 ***	-0.009 ***	-0.007 ***	-0.004 ***	-0.001 ***	0.002 ***	0.003 ***	0.003 ***	0.004 ***	0.007 ***
3%	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	-0.001 ***	0.000 ***	0.001 ***	-0.001 ***	-0.002 ***	-0.004 ***
4%	0.009 ***	0.008 ***	0.006 ***	0.003 ***	-0.001 ***	-0.001 ***	-0.001 ***	-0.004 ***	-0.005 ***	-0.012 ***
5%	0.017 ***	0.017 ***	0.012 ***	0.005 ***	-0.001 ***	-0.003 ***	-0.001 ***	-0.007 ***	-0.013 ***	-0.008 ***
6%	0.026 ***	0.025 ***	0.018 ***	0.008 ***	-0.001 ***	-0.005 ***	-0.002 ***	-0.008 ***	-0.014 ***	-0.021 ***
7%	0.034 ***	0.033 ***	0.024 ***	0.011 ***	-0.001 ***	-0.007 ***	-0.005	-0.013 ***	-0.029 ***	-0.016 ***
8%	0.043 ***	0.041 ***	0.031 ***	0.014 ***	-0.002 ***	-0.008 ***	-0.009	-0.012 ***	-0.032 ***	-0.034
9%	0.051 ***	0.049 ***	0.036 ***	0.016 ***	-0.002 ***	-0.010	-0.012 ***	-0.014 ***	-0.031 ***	-0.032 ***
10%	0.059	0.057	0.042	0.019	-0.002	-0.013	-0.015	-0.019	-0.038	-0.030

	***	***	***	***	***	***	***	***	***	***

Table-9 Pricing difference under Heston's model using current (or recent) realized rates as the interest rate in constant rate model

Panel A: Convertible bond price under constant interest rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	106.83	106.91	108.36	114.03	124.46	138.58	154.56	171.84	190.24	208.85
2%	104.88	104.98	106.68	112.92	123.99	138.56	154.64	171.87	190.42	210.22
3%	102.97	103.11	105.05	111.90	123.47	138.52	155.03	172.16	191.12	215.56
4%	101.11	101.28	103.55	110.96	123.23	138.63	155.05	172.89	190.75	206.98
5%	99.27	99.51	102.15	110.13	123.01	138.54	155.48	173.98	191.38	219.02
6%	97.48	97.78	100.78	109.32	122.64	138.81	155.80	173.12	191.40	223.32
7%	95.71	96.09	99.44	108.63	122.50	138.77	155.62	173.42	191.79	227.88
8%	93.99	94.45	98.17	107.92	122.34	139.05	155.38	173.76	204.15	230.53
9%	92.29	92.86	97.04	107.28	122.40	138.82	155.62	174.17	204.67	213.28
10%	90.63	91.30	95.94	106.70	122.34	139.28	155.95	173.17	186.97	235.18
Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	0.016	0.015	0.010	0.003	-0.001	-0.002	-0.002	0.002	0.004	-0.007
	***	***	***	*						*
2%	0.006	0.005	0.001	-0.003	-0.004	-0.003	-0.003	0.000	0.003	-0.006
	***	***	***	***						***

3%	-0.003 ***	-0.004 ***	-0.008 ***	-0.009 ***	-0.007	-0.004	-0.004 *	-0.001	0.001	0.016 ***
4%	-0.013 ***	-0.014 ***	-0.016 ***	-0.014 ***	-0.008	-0.004	-0.006 ***	0.001	-0.001	-0.026 ***
5%	-0.023 ***	-0.023 ***	-0.023 ***	-0.019 ***	-0.010	-0.006 *	-0.006 ***	0.004 **	-0.003 ***	0.028 ***
6%	-0.032 ***	-0.033 ***	-0.030 ***	-0.023 ***	-0.013	-0.006 **	-0.002 ***	-0.005 ***	-0.004 ***	0.013 ***
7%	-0.042 ***	-0.042 ***	-0.038 ***	-0.026 ***	-0.014	-0.006 **	-0.006 ***	-0.006 ***	-0.006 ***	0.026 ***
8%	-0.052 ***	-0.052 ***	-0.045 ***	-0.030 ***	-0.015	-0.007 ***	-0.011 ***	-0.004 ***	0.053 ***	0.077 ***
9%	-0.062 ***	-0.061 ***	-0.050 ***	-0.033 ***	-0.016	-0.010 ***	-0.012 ***	-0.003 ***	0.048 ***	-0.052 ***
10%	-0.072 ***	-0.070 ***	-0.056 ***	-0.036 ***	-0.017	-0.010 ***	-0.013 ***	-0.013 ***	-0.056 ***	0.057 ***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
T0 Interest Rate										
1%	0.019 ***	0.019 ***	0.016 ***	0.011 *	0.006	0.003	0.003	0.004	0.000	-0.021 *
2%	0.010 ***	0.009 ***	0.008 ***	0.005 ***	0.003	0.002	0.001	0.001	0.000	-0.018 ***
3%	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	-0.001	0.000	0.001 *	-0.001	-0.002	-0.004 ***
4%	-0.010	-0.010	-0.008	-0.006	-0.003	0.000	-0.001	0.000	-0.007	-0.054

	***	***	***	***			***			***
5%	-0.019	-0.019	-0.016	-0.011	-0.004	-0.003	0.002	0.003	-0.012	0.008
	***	***	***	***		*	***	**	***	***
6%	-0.029	-0.028	-0.023	-0.015	-0.007	-0.003	0.003	-0.003	-0.012	0.014
	***	***	***	***		**	***	***	***	***
7%	-0.039	-0.038	-0.031	-0.019	-0.009	-0.005	-0.001	-0.005	-0.026	0.039
	***	***	***	***		**	***	***	***	***
8%	-0.049	-0.047	-0.037	-0.023	-0.011	-0.004	-0.006	-0.003	0.034	0.033
	***	***	***	***		***	***	***	***	***
9%	-0.059	-0.056	-0.044	-0.026	-0.010	-0.008	-0.008	-0.002	0.038	-0.043
	***	***	***	***		***	***	***	***	***
10%	-0.069	-0.065	-0.049	-0.029	-0.011	-0.008	-0.009	-0.013	-0.061	0.056
	***	***	***	***		***	***	***	***	***

Table-10 Results changing other parameters under Heston's model

3. fixed r=0.03,ATM

Maturity										
Panel A: Convertible bond price under constant interest rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
Maturity										
1	103.96	103.96	104.09	106.21	114.72	129.06	146.64	165.88	186.64	207.18
2	102.97	103.11	105.05	111.90	123.47	138.52	155.03	172.16	191.12	215.56
3	102.07	103.07	107.84	117.34	130.91	146.54	164.25	183.23	203.56	225.00
4	101.33	103.85	111.12	122.56	137.22	153.56	170.90	189.30	206.64	227.45

5	100.83	104.99	114.60	128.26	144.42	162.53	181.93	202.64	225.14	247.77
6	100.48	106.44	118.04	132.36	148.63	166.80	184.36	204.74	224.95	241.35
7	100.39	108.37	121.48	136.89	153.10	171.34	190.19	210.07	230.96	251.04
8	100.63	109.51	123.52	140.33	158.83	178.03	199.77	220.94	244.89	267.64
9	100.57	111.32	125.33	141.79	159.42	178.07	197.23	215.76	231.78	252.00
10	101.09	113.50	130.00	149.27	168.58	212.04	239.75	264.55	295.23	243.97

Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Maturity										
1	-0.001 ***	-0.001 ***	-0.001 ***	-0.002 ***	-0.002	-0.001	0.000	0.004 *	0.005 *	0.007 ***
2	-0.003 ***	-0.004 ***	-0.008 ***	-0.009 ***	-0.007 ***	-0.004 ***	-0.004 ***	-0.001 ***	0.001 ***	0.016 ***
3	-0.008 ***	-0.011 *	-0.014 ***	-0.016 ***	-0.009 ***	-0.007 ***	0.005 ***	0.010 ***	0.014 ***	0.016 ***
4	-0.013 ***	-0.019 ***	-0.023 ***	-0.024 ***	-0.015 ***	-0.008 ***	-0.007 ***	-0.001 ***	-0.003 ***	0.006 ***
5	-0.020 ***	-0.028 ***	-0.028 ***	-0.029 ***	-0.028 ***	-0.023 ***	-0.021 ***	-0.021 ***	-0.019 ***	-0.020 ***
6	-0.026 ***	-0.034 ***	-0.037 ***	-0.032 ***	-0.028 ***	-0.021 ***	-0.004 ***	0.007 ***	0.012 ***	0.007 ***
7	-0.032	-0.041 ***	-0.041 ***	-0.042 ***	-0.055 ***	-0.057 ***	-0.059 ***	-0.054 ***	-0.051 ***	-0.059 ***
8	-0.039 **	-0.055 ***	-0.057 ***	-0.054 ***	-0.048 ***	-0.049 ***	-0.039 ***	-0.045 ***	-0.037 ***	-0.051 ***
9	-0.048	-0.051	-0.056	-0.054	-0.053	-0.053	-0.048	-0.027	-0.050	-0.053

	***	***	***	***	***	***	***	***	***	***
10	-0.055	-0.065	-0.064	-0.049	-0.053	0.055	0.077	0.076	0.048	-0.255
	***	***	***	***	***	***	***	***	***	***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Maturity										
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.006
	***	***	***	***				*	*	***
2	0.000	0.000	0.000	-0.001	-0.001	0.000	0.001	-0.001	-0.002	-0.004
	***	***	***	***	***	***	***	***	***	***
3	0.000	0.000	-0.001	-0.001	0.000	0.001	0.003	0.004	0.009	0.015
	***	*	***	***	***	***	***	***	***	***
4	0.000	-0.001	-0.001	-0.002	0.000	0.002	0.003	0.003	0.000	0.005
	***	***	***	***	***	***	***	***	***	***
5	0.000	-0.001	-0.001	0.000	-0.001	-0.001	0.002	0.003	0.008	0.005
	***	***	***	***	***	***	***	***	***	***
6	-0.001	-0.002	0.000	-0.004	0.001	0.002	0.001	0.005	0.008	-0.002
	***	***	***	***	***	***	***	***	***	***
7	-0.002	-0.003	-0.003	0.000	-0.006	-0.001	-0.003	0.001	-0.005	-0.007
		***	***	***	***	***	***	***	***	***
8	-0.001	-0.001	-0.002	-0.004	-0.003	-0.002	0.003	0.000	-0.006	0.001
	**	***	***	***	***	***	***	***	***	***
9	-0.002	-0.001	-0.003	-0.002	0.002	0.003	-0.006	-0.010	-0.005	-0.007
	***	***	***	***	***	***	***	***	***	***
10	-0.003	-0.004	-0.003	0.001	0.000	0.101	0.013	-0.009	0.011	-0.005
	***	***	***	***	***	***	***	***	***	***

Coupon										
Panel A: Convertible bond price under constant interest rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
Coupon										
1	91.55	91.76	94.35	102.29	114.94	130.78	148.05	165.84	185.16	208.81
2	94.41	94.59	97.02	104.69	117.07	132.72	149.79	167.42	186.65	210.50
3	97.26	97.43	99.70	107.09	119.20	134.65	151.54	169.00	188.14	212.19
4	100.12	100.27	102.37	109.50	121.34	136.59	153.28	170.58	189.63	213.88
5	102.97	103.11	105.05	111.90	123.47	138.52	155.03	172.16	191.12	215.56
6	105.83	105.95	107.72	114.30	125.60	140.46	156.77	173.73	192.60	217.25
7	108.69	108.79	110.40	116.71	127.73	142.39	158.52	175.31	194.09	218.94
8	111.54	111.63	113.08	119.11	129.86	144.33	160.26	176.89	195.58	220.63
9	114.40	114.46	115.75	121.51	131.99	146.26	162.01	178.47	197.07	222.32
10	117.26	117.30	118.43	123.92	134.12	148.20	163.75	180.05	198.56	224.01
Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model										
Stock Price	20	40	60	80	100	120	140	160	180	200
Coupon										
1	-0.004 ***	-0.005 ***	-0.009 ***	-0.010 ***	-0.007 **	-0.004	-0.003	0.000	0.003	0.015 ***
2	-0.003 ***	-0.005 ***	-0.009 ***	-0.010 ***	-0.007 ***	-0.004 ***	-0.003 ***	0.000 ***	0.002	0.015
3	-0.003 ***	-0.005 ***	-0.008 ***	-0.010 ***	-0.007 ***	-0.004 ***	-0.003 ***	0.000 ***	0.002 ***	0.016
4	-0.003	-0.005	-0.008	-0.009	-0.007	-0.004	-0.004	-0.001	0.002	0.016

	***	***	***	***	***	***	***	***	***	*
5	-0.003	-0.004	-0.008	-0.009	-0.007	-0.004	-0.004	-0.001	0.001	0.016
	***	***	***	***	***	***	***	***	***	***
6	-0.003	-0.004	-0.007	-0.009	-0.007	-0.004	-0.004	-0.001	0.001	0.017
	***	***	***	***	***	***	***	***	***	***
7	-0.003	-0.004	-0.007	-0.008	-0.007	-0.004	-0.004	-0.002	0.000	0.017
	***	***	***	***	***	***	***	***	***	***
8	-0.003	-0.004	-0.007	-0.008	-0.007	-0.004	-0.005	-0.002	0.000	0.017
	***	***	***	***	***	***	***	***	***	***
9	-0.003	-0.004	-0.007	-0.008	-0.007	-0.004	-0.005	-0.002	0.000	0.018
	***	***	***	***	***	***	***	***	***	***
10	-0.003	-0.004	-0.006	-0.008	-0.007	-0.004	-0.005	-0.003	-0.001	0.018
	***	***	***	***	***	***	***	***	***	***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Coupon										
1	0.000	0.000	-0.001	-0.001	-0.001	0.000	0.001	0.000	-0.001	-0.003
	***	***	***	***	**					***
2	0.000	0.000	-0.001	-0.001	-0.001	0.000	0.001	0.000	-0.001	-0.003
	***	***	***	***	***	***	***	***		
3	0.000	0.000	-0.001	-0.001	-0.001	0.000	0.001	-0.001	-0.001	-0.004
	***	***	***	***	***	***	***	***	***	
4	0.000	0.000	0.000	-0.001	-0.001	0.000	0.001	-0.001	-0.001	-0.004
	***	***	***	***	***	***	***	***	***	*
5	0.000	0.000	0.000	-0.001	-0.001	0.000	0.001	-0.001	-0.002	-0.004
	***	***	***	***	***	***	***	***	***	***

6	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	-0.001 ***	0.000 ***	0.001 ***	-0.001 ***	-0.002 ***	-0.004 ***
7	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	-0.001 ***	0.000 ***	0.001 ***	-0.001 ***	-0.002 ***	-0.004 ***
8	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	-0.001 ***	0.000 ***	0.001 ***	-0.001 ***	-0.002 ***	-0.004 ***
9	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	0.000 ***	0.001 ***	-0.001 ***	-0.002 ***	-0.005 ***
10	0.000 ***	0.000 ***	0.000 ***	0.000 ***	-0.001 ***	0.001 ***	0.001 ***	-0.001 ***	-0.002 ***	-0.005 ***

Credit Spread

Panel A: Convertible bond price under constant interest rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Credit Spread										
1%	106.83	106.92	108.50	114.65	125.60	140.17	156.07	173.12	191.46	214.28
2%	104.88	105.00	106.77	113.25	124.58	139.46	155.52	172.51	191.10	215.98
3%	102.97	103.11	105.05	111.90	123.47	138.52	155.03	172.16	191.12	215.56
4%	101.11	101.26	103.40	110.58	122.51	137.75	154.21	171.69	190.48	215.04
5%	99.27	99.45	101.75	109.25	121.56	136.93	153.61	171.43	190.31	215.03
6%	97.47	97.67	100.16	107.91	120.61	136.14	152.86	170.89	189.73	215.80
7%	95.71	95.93	98.60	106.60	119.59	135.33	152.28	170.68	188.60	216.10
8%	93.98	94.21	97.07	105.32	118.63	134.56	151.75	169.85	180.34	216.00
9%	92.29	92.54	95.55	104.08	117.74	133.78	151.05	169.22	184.22	213.91
10%	90.63	90.91	94.08	102.87	116.76	132.93	150.42	167.85	180.92	211.82

Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Credit Spread										
1%	-0.003 ***	-0.004 ***	-0.007 ***	-0.009 ***	-0.007 ***	-0.004	-0.005	-0.001	0.001	0.006
2%	-0.003 ***	-0.004 ***	-0.007 ***	-0.009 *	-0.007	-0.003	-0.005	-0.002	0.001	0.018 ***
3%	-0.003 ***	-0.004 ***	-0.008 ***	-0.009 ***	-0.007 ***	-0.004 ***	-0.004	-0.001	0.001	0.016 ***
4%	-0.003 ***	-0.005 ***	-0.007 ***	-0.009 ***	-0.006 ***	-0.003 ***	-0.005 *	-0.001 *	0.002 *	0.020 ***
5%	-0.003 ***	-0.005 ***	-0.008 ***	-0.009 ***	-0.006 ***	-0.003 ***	-0.002 ***	0.001 ***	0.003 ***	0.006
6%	-0.003 ***	-0.005 ***	-0.008 ***	-0.009 ***	-0.005 ***	-0.003 ***	-0.003 ***	-0.002 ***	0.003 ***	0.022 ***
7%	-0.003 ***	-0.005 ***	-0.008 ***	-0.009 ***	-0.005 ***	-0.004 ***	-0.002 ***	0.003 ***	-0.001 ***	0.106 ***
8%	-0.003 ***	-0.005 ***	-0.008 ***	-0.009 ***	-0.005 ***	-0.003 ***	-0.001 ***	0.000 ***	-0.048 ***	0.085 ***
9%	-0.003 ***	-0.005 ***	-0.008 ***	-0.008 ***	-0.005 ***	-0.003 ***	-0.003 ***	0.000 ***	0.017 ***	0.060 ***
10%	-0.003 ***	-0.005 ***	-0.008 ***	-0.009 ***	-0.005 ***	-0.004 ***	-0.002 ***	-0.004 ***	0.008 ***	0.087 ***

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Credit Spread										
1%	0.000	0.000	0.000	-0.001	0.000	0.001	0.000	0.000	-0.006	-0.006

	***	***	***	***	***					
2%	0.000	0.000	0.000	-0.001	0.000	0.001	0.000	-0.003	-0.006	0.009
	***	***	***	*						***
3%	0.000	0.000	0.000	-0.001	-0.001	0.000	0.001	-0.001	-0.002	-0.004
	***	***	***	***	***	***				***
4%	0.000	0.000	0.000	0.000	-0.001	0.000	0.001	-0.001	-0.005	0.000
	***	***	***	***	***	***	*	*	*	***
5%	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.001	-0.008	0.007
	***	***	***	***	***	***	***	***	***	
6%	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	-0.001	0.001
	***	***	***	***	***	***	***	***	***	***
7%	0.000	0.000	-0.001	-0.001	0.000	0.001	0.000	0.002	-0.001	0.013
	***	***	***	***	***	***	***	***	***	***
8%	0.000	0.000	-0.001	-0.001	0.000	0.001	0.001	0.000	-0.060	0.010
	***	***	***	***	***	***	***	***	***	***
9%	0.000	0.000	-0.001	-0.001	0.000	0.002	0.001	-0.001	-0.049	0.013
	***	***	***	***	***	***	***	***	***	***
10%	0.000	0.000	-0.001	-0.001	0.000	0.002	0.001	-0.005	-0.068	-0.012
	***	***	***	***	***	***	***	***	***	***

Sigma

Panel A: Convertible bond price under constant interest rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Sigma										
5%	102.97	103.03	104.38	110.31	121.73	137.00	153.52	170.48	187.98	216.36
10%	102.97	103.05	104.55	110.64	122.03	137.55	153.14	169.84	188.87	216.38

15%	102.97	103.08	104.76	111.15	122.58	137.87	153.66	171.42	189.98	217.42
20%	102.97	103.11	105.05	111.90	123.47	138.52	155.03	172.16	191.12	215.56
25%	102.98	103.20	105.55	112.72	124.57	139.15	156.16	173.78	192.46	214.71
30%	102.98	103.31	106.12	113.73	125.80	140.49	157.04	174.90	193.44	213.51
35%	102.98	103.47	106.82	114.91	127.13	141.89	158.59	175.80	194.63	214.61
40%	102.99	103.72	107.72	116.23	128.65	143.35	159.88	177.49	195.59	214.87
45%	102.99	104.03	108.64	117.65	130.34	144.99	161.56	179.08	196.93	216.12
50%	103.01	104.38	109.73	119.14	132.04	146.83	163.15	180.80	198.64	217.64

Panel B: Convertible bond price change under stochastic interest rate model (Vasicek) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Sigma										
5%	-0.003 ***	-0.004 ***	-0.008 ***	-0.010 ***	-0.007 ***	-0.002	-0.005	-0.011 ***	-0.009 ***	0.011 ***
10%	-0.003 ***	-0.004 ***	-0.007 ***	-0.009 ***	-0.008 ***	-0.001	-0.012 ***	-0.010 **	-0.013 ***	0.009 **
15%	-0.003 ***	-0.004 ***	-0.007 ***	-0.009 ***	-0.008 ***	-0.002 ***	-0.009 ***	-0.003 ***	-0.005 ***	0.010
20%	-0.003 ***	-0.004 ***	-0.008 ***	-0.009 ***	-0.007 ***	-0.004 ***	-0.004 ***	-0.001 ***	0.001 ***	0.016 ***
25%	-0.003 ***	-0.004 ***	-0.007 ***	-0.009 ***	-0.007 ***	-0.006 ***	-0.002 ***	0.000 ***	0.002 ***	0.021 ***
30%	-0.003 ***	-0.005 ***	-0.007 ***	-0.009 ***	-0.007 ***	-0.005 ***	-0.003 ***	-0.002 ***	-0.003 ***	0.000 ***
35%	-0.003 ***	-0.005 ***	-0.007 ***	-0.009 ***	-0.009 ***	-0.005 ***	0.001 ***	-0.003 ***	-0.001 ***	0.002 *
40%	-0.003	-0.005	-0.006	-0.008	-0.008	-0.006	-0.002	0.000	-0.001	0.003

	***	***	***	***	***	***	***	***	***	*
45%	-0.003	-0.005	-0.007	-0.007	-0.008	-0.008	-0.002	0.000	-0.002	0.002
	***	***	***	***	***	***	***	***	***	
50%	-0.003	-0.005	-0.006	-0.008	-0.008	-0.008	-0.005	-0.001	-0.002	0.001
	***	***	***	***	***	***	***	***	***	

Panel C: Convertible bond price change under stochastic interest rate model (CIR) compare to constant rate model

Stock Price	20	40	60	80	100	120	140	160	180	200
Sigma										
5%	0.000	0.000	-0.001	0.000	0.000	0.001	0.000	0.007	0.007	-0.001
	***	***	***	***	***			***	***	***
10%	0.000	0.000	0.000	-0.001	-0.001	0.001	0.001	0.000	0.004	-0.002
	***	***	***	***	***		***	**	***	**
15%	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.000	0.005	0.006
	***	***	***	***	***	***	***	***	***	
20%	0.000	0.000	0.000	-0.001	-0.001	0.000	0.001	-0.001	-0.002	-0.004
	***	***	***	***	***	***	***	***	***	***
25%	0.000	0.000	0.000	-0.001	-0.001	-0.002	0.000	0.001	-0.004	0.019
	***	***	***	***	***	***	***	***	***	***
30%	0.000	0.000	0.000	-0.001	-0.001	-0.002	0.000	0.001	-0.001	0.003
	***	***	***	***	***	***	***	***	***	***
35%	0.000	0.000	0.000	0.000	-0.001	0.001	-0.001	0.000	0.000	0.001
	***	***	***	***	***	***	***	***	***	*
40%	0.000	0.000	0.000	0.000	-0.001	-0.001	0.000	-0.001	0.000	0.000
	***	***	***	***	***	***	***	***	***	*
45%	0.000	0.000	0.000	-0.001	0.000	-0.001	0.000	0.001	-0.002	0.001
	***	***	***	***	***	***	***	***	***	

50%	0.000 ***	0.000 ***	-0.001 ***	-0.001 ***	0.000 ***	0.000 ***	-0.001 ***	0.000 ***	-0.001 ***	0.002

Sample codes (stochastic interest rate under Heston's model)

```
cb_sto_he<-
function(deltat,maturity,coupon,converprice,f,r,creditspread,s,sigma,rhosv,rhosr,rhorv,k
appa,vbar,gamma,n,simulation,modelchoice,lambda,theta,eta){
 set.seed(simulation)

 #number of periods
 t<-maturity/deltat+1

 #generate brownian motions
 M<-matrix(data= c(1,rhosv,rhosr,
                   rhosv,1,rhorv,
                   rhosr,rhorv,1),nrow=3,ncol=3)
 rns<-matrix(0,n,t)
 rnv<-matrix(0,n,t)
 rnr<-matrix(0,n,t)

 for (i in 1: n){
  for (j in 1: t){
   Q<-mvrnorm(n=1,mu=c(0,0,0),Sigma=M)
   rns[i,j]=Q[1]
   rnv[i,j]=Q[2]
   rnr[i,j]=Q[3]
  }
```

```
}

#generate interest rate paths
rf<-matrix(0,n,t)
rf[,1]<-r
for(i in 1:n){
  for(j in 1:(t-1)){
    if (modelchoice==1)
    {rf[i,j+1] <- rf[i,j]+lambda*(theta-rf[i,j])*deltat+eta*sqrt(deltat)*rnr[i,j]}
    else
    {rf[i,j+1] <- rf[i,j]+lambda*(theta-
rf[i,j])*deltat+eta*sqrt(rf[i,j])*sqrt(deltat)*rnr[i,j]+eta^2/4*deltat*((rnr[i,j])^2-1)}
  }
}

rc<-matrix(0,n,t)
for(i in 1:n){
  for(j in 1:t){
    rc[i,j] <- rf[i,j]+creditspread
  }
}

#generate simulated stock paths
stock<-matrix(0,n,t)
```

```
stock[,1]<-s
v<-matrix(0,n,t)
v[,1]<-sigma^2
#Milstein Scheme
for(i in 1:n){
  for(j in 2:t){
    v[i,j] <- v[i,j-1]-kappa*(v[i,j-1]-vbar)*deltat+gamma*sqrt(v[i,j-1]*deltat)*rnv[i,j-1]+gamma^2/4*deltat*((rnv[i,j-1])^2-1)
  }
}

for(i in 1:n){
  for(j in 2:t){
    stock[i,j] <- stock[i,j-1]+(rf[i,j-1]-v[i,j-1]/2)*deltat+sqrt(v[i,j-1]*deltat)*rns[i,j-1]
  }
}

#generate convert decision matrix and cf marix for y in lsm
ex<-matrix(0,n,t)
cf<-matrix(0,n,t)
#final period
for(i in 1:n){if (stock[i,t]>f){ex[i,t]<-1}
  if (stock[i,t]>f){cf[i,t]<-stock[i,t]}
  if (stock[i,t]<=f){ex[i,t]<-0}
```

```
  if (stock[i,t]<=f){cf[i,t]<-f}
}
#find location of excersise
find1<-function(k){
  q=1
  while (q<=t){
    if (ex[k,q]==1){return(q)}
    if (ex[k,q]==1){break}
    else {q=q+1}
    if (q==t){if(ex[k,t]==0){return(-1)}
    }
  }
}

#calculate sum of interest rate
sumdf<-function(k,l){
  a=find1(k)
  sumr=0
  if(a==-1){
    for(p in l:(t-1))
    {sumr=sumr+rc[k,p]}
  }
  else{
```

```
    for(p in l:(a-1))
    {sumr<-sumr+rf[k,p]}
  }
  return(sumr)
}
sumdf(1,t-1)
#construct cash flow matrix used in lsm before final period
for(j in 1:(t-2)){
  for(i in 1:n){

    if (find1(i)==-1){cf[i,t-j]<-f*exp(-deltat*sumdf(i,t-j))}
    else {
      a=find1(i)
      cf[i,t-j]<-stock[i,a]*exp(-deltat*sumdf(i,t-j))}
  }

  lm.cf=lm(cf[,t-j]~stock[,t-j]+stock[,t-j]^2+stock[,t-j]^3)
  pre_cf=predict(lm.cf,data.frame(x_1=stock[,t-j],x_2=stock[,t-j]^2,x_3=stock[,t-j]^3))
  for(i in 1:n){
    if (stock[i,t-j]>pre_cf[i]){ex[i,t-j]=1}
  }
}
#cashflow form stock and f
```

```
for (i in 1:n){
  if (find1(i)==-1){cf[i,1]<-f*exp(-deltat*sumdf(i,1))+coupon}
  else {a=find1(i)
  cf[i,1]<-stock[i,a]*exp(-deltat*sumdf(i,j))}
}



#cash flow from coupon, assuming perperiod coupon


#discount rate of coupon
dis<-function(i,j){
  sumdis=0
  for(p in 1:(j-1)){
    sumdis<-sumdis+rc[i,p]
  }
  return(sumdis*deltat)
}


pvcoupon<-function(i){
  s<-0
  a<-find1(i)
  if (a>=0){
    b=floor(((a-1)*deltat*2))
```

```
   if (b==0){s<-coupon/2*(a*2*deltat)*exp(-dis(i,a))}
   else {
     u=0
     for (k in 1:b){
       p<-k/2/deltat+1
       u=u+exp(-dis(i,p))
     }
     s=coupon/2*u+coupon/2*((a-b/2/deltat)*2*deltat)*exp(-dis(i,a))
   }
  }
  else{
   u=0
   for (k in 1:(maturity*2)){
     p<-k/2/deltat+1
     u=u+exp(-dis(i,p))
   }
   s=coupon/2*u
  }
  return(s)
}
#total cashflow
totalcf<-matrix(0,n,1)
for (i in 1:n){totalcf[i,1]<-cf[i,1]+pvcoupon(i)}
```

```
  #result price and standard error
  price<-mean(totalcf[,1])
  se<-sd(totalcf[,1])/sqrt(n)
  up_95<-mean(totalcf[,1])+1.96*sd(totalcf[,1])/sqrt(n)
  down_95<-mean(totalcf[,1])-1.96*sd(totalcf[,1])/sqrt(n)

  up_99<-mean(totalcf[,1])+2.576*sd(totalcf[,1])/sqrt(n)
  down_99<-mean(totalcf[,1])-2.576*sd(totalcf[,1])/sqrt(n)

  up_90<-mean(totalcf[,1])+1.645*sd(totalcf[,1])/sqrt(n)
  down_90<-mean(totalcf[,1])-1.645*sd(totalcf[,1])/sqrt(n)

  result=c(price,se,up_99,down_99,up_95,down_95,up_90,down_90)
  return(result)
}
```

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